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# Optimally Managing a Stochastic Renewable Resource under General Economic Conditions

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# Optimally Managing a Stochastic Renewable Resource under General Economic Conditions*

Bruce McGough, Andrew J. Plantinga, and Christopher Costello

## Abstract

Empirical evidence indicates that environmental fluctuations have important effects on fisheries production. However, existing analytical solutions of stochastic fisheries models have been produced only under highly simplified economic and biological conditions. The main contribution of this paper is to derive under general conditions a policy function for the management of a stochastic fishery. Our model includes general specifications of demand and cost relationships and a stochastic biological growth function with serially-correlated shocks. Applying methods from the theory of dynamic stochastic general equilibrium modeling and multivariate linear expectational difference equations, we derive a linear approximation of the solution to the model. Our main result is a reduced-form expression for an approximation to optimal escapement, which is shown to be a function of the current stock, past environmental shocks, and model parameters. This theoretically-grounded policy function has intuitive appeal, yields insights into comparative statics, and provides a theoretically-grounded, practical starting point for fisheries management.

**KEYWORDS:** dynamic stochastic general equilibrium, fisheries management, renewable resources, stochasticity

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# 1 Introduction

In many countries, fisheries are highly regulated in order to curtail the open access use of the resource. Despite this, overexploitation of fish stocks is a chronic problem worldwide (Food and Agriculture Organization 2004; Worm et al. 2006). The management of fisheries is challenging, in part, because of the complexity of underlying biological relationships that are influenced by temporal variability in climate and ocean temperatures (Barber and Chavez 1983; Mann and Lazier 1996; McGowan et al. 1998; Finney et al. 2000). Productivity shocks also have been shown to be serially correlated (Chavez et al. 2003; Pyper and Peterman 1998; Korman et al. 1995). These features render the derivation of the economically-optimal management rule considerably more difficult. Yet, only after identifying efficient catch levels can property-rights approaches, such as catch shares, be effectively used to enhance biological and economic outcomes (Costello et al. 2008). In previous papers, analytical solutions have been produced only for highly simplified economic environments and i.i.d. productivity shocks. To gain tractibility, researchers have been forced to sacrifice both economic and biological realism.

This paper proposes a general solution to the management of a stochastic fishery. In the following section, we apply techniques from the macroeconomics literature on real business cycles (e.g., Kydland and Prescott (1982) and Farmer (1999)) to solve a model with general specifications of demand and cost relationships and a stochastic stock-recruitment function with serially-correlated shocks. In contrast to previous studies, we derive an analytical expression for the escapement[1] rule in a general framework. Two key steps are required to solve the model. The first is to log-linearize the Euler equation and the stochastic biological growth function about a deterministic steady state. If the steady state is robust to perturbations, then adding small stochastic shocks will not cause the system to deviate from the neighborhood in which the approximation is valid. The second step is to apply results from the literature on multivariate linear expectational difference equations (see Evans and Honkapohja (2001)) to solve for the rational expectations solution of the model. Our key result is a reduced-form, linear approximation of the optimal escapement rule, shown to be a function of the current stock, past environmental shocks, and model parameters.

Early papers on fisheries economics (Gordon 1954; Scott 1955; Smith 1969; Beddington et al. 1975; Levhari et al. 1981) derived management rules within

[1]Escapement refers to the stock of fish that remains after harvest and, thus, "escapes" to the next period.

a deterministic framework. In later papers, authors account for the effects of environmental variability by introducing stochastic growth (stock-recruitment) functions.[2] The seminal paper is Reed (1979) who shows, under the conditions of his model, that a stochastic fish population should be harvested to achieve the same escapement level in every period. This constant escapement level is independent of the current stock size.

Accommodating environmental variability in fisheries models has come at a cost. While concise management rules can be derived from deterministic models under standard economic assumptions, general analytical results have not been obtained for stochastic models. Reed (1979), for example, places strong restrictions on the demand and cost functions by assuming that prices and unit fishing costs do not depend on the level of harvest. Similarly, Clark and Kirkwood (1986) and Weitzman (2002) assume marginal profits are independent of harvest. Mirman and Spulber (1985) analyze a more general model with downward-sloping inverse demand and harvest-dependent marginal costs, but they are unable to obtain an explicit solution for the optimal harvesting policy.[3] Other authors adopt specific functional forms and provide results based on numerical simulations (Sethi et al. 2005; Singh et al. 2006).[4] Among these studies, only Singh et al. (2006) allow for serially-correlated productivity shocks.

The main contribution of this paper is to provide new analytical results for the general problem of fisheries management under uncertainty. Section 3 presents a series of propositions that demonstrate how the optimal escapement rule derived from the linearization changes with different specifications of demand, marginal costs, risk preferences, and the distribution of environmental shocks. For example, Reed's constant escapement policy is shown to be a special case obtained with perfectly elastic demand, harvest-independent marginal costs, and i.i.d. shocks. Under more general conditions, escapement is a function of the current stock, and harvests are smoothed over time to balance its effects on prices, unit fishing costs, and the future stock. Numerical simulations are used, in section 4, to explore further how optimal management is affected by changes in parameters characterizing the economic environment and the uncertainty. We show, for example, that greater positive serial cor-

[2]See Reed (1979), Clark and Kirkwood (1986), Mirman and Spulber (1985), Roughgarden and Smith (1996), Weitzman (2002), Sethi et al. (2005), and Singh et al. (2006).

[3]In their paper, the optimal harvest rule in their Equation (17) is a function of $\bar{h}(x)$, which is implicitly defined by their Equation (16).

[4]Pindyck (1984) considers a general model of renewable resource markets. He adopts specific functional forms for demand, cost, and production functions and derives explicit harvest policies.

relation in the productivity shock increases the marginal impact of the shock on escapement levels. Intuitively, higher serial correlation and a positive productivity shock realization implies a higher expected return on non-harvested fish, so optimal escapement levels increase.

Our approach of log-linearizing the Euler equation has three important advantages. First, it facilitates an analytical solution (even for very complex problems), so studying the properties of the solution is straightforward. Second, the solution has practical appeal because it fits within the class of harvest policies often applied in the real world (e.g., constant escapement). Third, this method applies equally in situations when numerical approaches (e.g., dynamic programming) are computationally intractable.[5] The rule is derived in closed form and is straightforward to calculate from the primitives of the problem: production function of fish, demand, cost, and stochasticity. Because the rule is derived by log-linearizing the Euler equation, it represents a first-order approximation of the fully optimal rule. However, our section 5 demonstrates with numerical simulations that it provides solutions that are nearly identical to those derived from a non-linear model. We conclude that, for practical purposes, our escapement rule can be treated as the optimal policy function and used to gain insights into optimal fisheries policy. In the final section, we summarize our findings and discuss application of our methods to the broader class of renewable resource problems.

# 2 Model

We model a price-setting regulator facing a downward sloping (inverse) demand curve given by $p(q)$. Resource stock, $s_t$, is known at the beginning of period $t$, at which time harvest, $h_t$, is chosen. The remaining stock available for reproduction is the "escapement," $x_t = s_t - h_t$. Following Reed (1979), Weitzman (2002), Costello et al. (2001), Costello and Polasky (2008), and others, we treat escapement (rather than harvest) as the control variable. This choice is for mathematical convenience and, by the identity $x \equiv s - h$, does not affect results or interpretation.

Resource production is a random variable given by

$$s_{t+1} = z_{t+1} f(x_t), \tag{1}$$

[5]More specifically, many non-linear solutions methods require that the modeler identify an equilibrium with the solution to an associated planner's problem; however, in case of distortions, the equilibrium may not be efficient, and a constrained planner may be difficult to identify. A pertinent case (not examined here) is market equilibrium in a many-agent fishery with externalities.

where $f' > 0$, $f'' < 0$, and $z_t$ is a stationary, possibility serially correlated, multiplicative production shock, with small positive support and unconditional mean equal to one.[6] Production shocks have state transitions

$$z_t = v_t z_{t-1}^{\rho}, \tag{2}$$

where $v_t$ is independently and identically distributed with mean 1, and $\rho$ is the serial correlation. We assume $x^K$ exists with $f(x^K) = x^K$, which represents the unharvested deterministic steady state; i.e., the carrying capacity of the resource in the absence of environmental variability.

Given a stock level $s_t$ and escapement decision $x_t$, net economic surplus from harvesting in period $t$ is given by

$$S(s_t, x_t) = \int_0^{h_t} p(\omega)d\omega - \int_{x_t}^{s_t} c(\omega, s_t - \omega)d\omega. \tag{3}$$

Here, marginal harvest cost of a unit of stock depends on stock (i.e., a stock effect on cost) and on harvest; thus, $c = c(s, h)$, where we make the standard assumptions that $c_1 < 0$ and $c_2 \geq 0$. The regulator may have non-neutral risk preferences over surplus, which we model with a constant relative risk aversion as follows:

$$U(S) = \frac{1}{1-\sigma} S^{1-\sigma},$$

where $\sigma = 0$ implies risk neutrality.

The objective is to choose a contingency plan for escapement levels that maximizes the discounted expected future stream of surplus, accounting for risk by guarding against surplus variation. Specifically,

$$\begin{aligned} \max_{\{x_t\}} \quad & E \sum_{t=0}^{\infty} \delta^t U\left(S(s_t, x_t)\right) \\ \text{s.t} \quad & s_t = z_t f(x_{t-1}) \\ & z_t = v_t z_{t-1}^{\rho} \\ & x_0 \text{ given}, \end{aligned} \tag{4}$$

for discount factor $\delta$.

[6] The restriction on the size of the shock's support depends on the curvature of the nonlinear system.

The Euler equation[7] associated with an interior solution[8] to this programming problem is given by

$$S_t^{-\sigma}\left(p(s_t - x_t) - c(x_t, s_t - x_t)\right) = \delta f'(x_t) * \tag{5}$$
$$E_t\left(z_{t+1}S_{t+1}^{-\sigma}\left(p(s_{t+1} - x_{t+1}) - c(s_{t+1}, 0) - \int_{x_{t+1}}^{s_{t+1}} c_2(\omega, s_{t+1} - \omega)d\omega\right)\right),$$

where $S_t = S(s_t, x_t)$. This condition is familiar; at the optimum, the marginal utility of escapement in period $t$ must equal the discounted expected marginal return in the subsequent period. To intuit this equation, set $\sigma = 0$ and $z_{t+1} = 1$. Then allowing an additional fish to escape in period $t$ reduces surplus by the left hand side of (5). The expected increase in (discounted) surplus in period $t+1$ then must account for the fish stock's growth, $f'(x_t)$, the additional revenue obtained by harvesting the new "larger" fish tomorrow, $p(s_{t+1} - x_{t+1})$, minus the additional cost incurred by harvesting this fish. The additional cost has two terms: $c(s_{t+1}, 0)$ captures the fact that this fish can be harvested first (i.e., when stock is high and harvest is zero), and the second term accounts for the added cost of harvesting future fish due to the harvesting of the first fish. Provided the right concavity conditions are met, the Euler equations, together with a transversality condition, are necessary and sufficient to characterize the optimal contingency plan.

The Euler equation (5) and the production function (1) define the model's implied time-path of stock and escapement.[9] Note immediately the familiar special case without costs, downward-sloping demand, risk, or shocks. In that case $\sigma = 0$, $p$ is constant, $c = 0$, and $z = 1$, and we obtain $f'(x) = 1/\delta$. More generally, solutions to nonlinear systems of expectational difference equations are intractable analytically, but analytical approaches have been developed for rigorous approximation of their solution (Woodford 1986). Following Woodford (1986), we log-linearize the model around the deterministic steady-state, which is implicitly defined by the following equation:

$$p(f(x) - x) - c(x, f(x) - x) = \tag{6}$$
$$\delta f'(x)(p(f(x) - x) - c(f(x), 0)) - \int_x^{f(x)} c_2(\omega, f(x) - \omega)d\omega;$$

[7]See Chapter 4 of Stokey and Lucas (1989) for the derivation of the Euler equation in discrete-time.

[8]Some specifications of the model lead to corner solutions that indicate a zero optimal harvest level. We restrict attention to specifications with solutions near an interior steady state, so that the optimal harvest level is positive. In this case, the interior Euler equation is the relevant first-order condition for our purposes.

[9]Again, these dynamics are subject to the transversality condition, which will require that they remain bounded in a natural sense. See Stokey and Lucas (1989) for details.

where $x$ is the optimal deterministic steady-state escapement, and the associated stock is $s = f(x)$.

Log-linearization allows us to express the variables of this model as percent deviations from the steady state, which we will denote with a *tilde* (e.g., $\tilde{x}_t$ is the percentage deviation of $x_t$ from the deterministic steady state $x$: $\tilde{x}_t \equiv log(x_t) - log(x)$). Log-linearizing the system (1) and (5), we obtain

$$\begin{aligned} a\tilde{x}_t + b\tilde{s}_t &= dE_t\tilde{x}_{t+1} + eE_t\tilde{s}_{t+1} + g\tilde{z}_t \\ \tilde{s}_t &= m\tilde{x}_{t-1} + \tilde{z}_t, \end{aligned} \tag{7}$$

where the parameters $a$, $b$, $d$, $e$, $g$, and $m$ are defined in the Appendix. Note that these parameters are scalar functions of the original model parameters and are straightforward to calculate given specific functional forms.

The system (7) has a closed-form, intuitive solution that is summarized in the following proposition.

**Proposition 1** *If the Euler equations (plus transversality) are sufficient for optimality and the support of the productivity shocks is small, then there exist scalars $\bar{a}$ and $\bar{b}$, defined in the Appendix, so that the optimal escapement plan is well-approximated by:*[10]

$$\tilde{x}_t = \bar{a}\tilde{x}_{t-1} + \bar{b}\tilde{z}_t. \tag{8}$$

*Further, $\tilde{x}_t$ is a covariance-stationary process.*

The scalars $\bar{a}$ and $\bar{b}$ are defined by the model's parameters and are computed in the Appendix. A brief description of the solution method used to prove this proposition and to compute these scalars is warranted. Equations (7) form a system of expectational difference equations capturing the linear approximation to the optimal solution to the recursive programming problem (4). Under the imposed convexity assumptions, this problem will have a unique solution, which is reflected in the "saddle-path stability" of the system (7).[11] The solution method used to compute (8) involves identifying the restrictions needed to guarantee that a process satisfying (7) is on the stable manifold. In simple linear models, such as the one above, these restrictions are obtained using matrix decomposition methods. For details on solving this model, see the Appendix, and for solving general expectational difference equations, see Blanchard and Kahn (1980).

[10] By "well-approximated" we mean that the linearized plan will deviate from the nonlinear plan by a residual that is proportional to the square of the productivity innovation's support, which is assumed small.

[11] This saddle-path stability property is entirely analogous to the continuous time stability properties of the dynamic solution to the standard Ramsey problem.

Equation 8 completely characterizes the linear approximation to the optimal contingency plan in the fishery. Notice that because $\tilde{x}_t$ and $\tilde{z}_t$ represent percent deviations from steady state, the magnitude of parameters $\bar{a}$ and $\bar{b}$ will govern return-to-steady state dynamics. To satisfy the transversality conditions, and thereby rule out explosive responses to shocks, the absolute value of $\bar{a}$ must be less than one. This implies that $\tilde{x}_t$ returns asymptotically to zero following a one-time shock. Thus, harvests are smoothed, in contrast to a constant escapement policy. Equivalently, the relationship in 8 can be re-cast as a linear function of *stock* and escapement (by combining 7 and 8), as follows:

$$\tilde{x}_t = A\tilde{s}_t + B\tilde{z}_t, \tag{9}$$

where $A = \frac{\bar{a}}{m}$ and $B = \bar{b} - \frac{\bar{a}}{m}$. The next section of this paper is devoted to interpreting this fundamental equation and extracting its general characteristics.

# 3 Characterizing optimal escapement

In the sequel, we refer to Equation 9 as the "optimal" state-contingent escapement plan. It is optimal conditional on the log-linearization, but to the extent that the problem is nonlinear, may deviate from the fully optimal policy (which can typically only be solved numerically).[12] While the mathematical form of that plan is simple (percent deviation from steady state is a linear function of the most recent deviation of stock and the most recent environmental shock), the coefficients ($A$ and $B$) are complicated functions of the rest of the model parameters and of deterministic steady-state values. While these complicated functions are difficult to analyze in general, several special cases of this model reveal useful insights into the optimal policy function.

It is helpful to define the following conditions:

A.1 Shocks are i.i.d. ($\rho = 0$)

A.2 Demand is perfectly elastic ($p' = 0$)

A.3 Risk neutrality ($\sigma = 0$)

A.4 Marginal harvest costs are not a direct function of the quantity harvested ($c_2 = 0$)

[12] The extent of this deviation is the focus of Section 5.

Condition A.1 regards the environmental shocks, and excludes the possibility that shocks are correlated across periods. Conditions A.2-A.4 concern the economic environment and exclude what would be considered typical economic conditions (downward-sloping demand, risk aversion, nonlinear harvest cost). In what follows it is helpful to classify any given manifestation of this problem along two dimensions: (1) whether the shocks are i.i.d. (Condition A.1); and (2) whether Conditions A.2-A.4 hold. If A.2-A.4 hold then profits are a linear function of harvest, which we refer to as the case of linear profits.

## 3.1 Is 'constant escapement' optimal?

Much of the previous literature focuses on the class of harvest policies characterized by "constant escapement." A constant escapement rule is one in which the regulator harvests down to a (time-invariant) pre-determined level every period. Constant escapement policies are not only practically appealing, they have been shown to be optimal under certain economic conditions (see Reed (1979), Roughgarden and Smith (1996), Weitzman (2002)). A constant escapement policy has commensurately been adopted in many managed fisheries worldwide. Yet the literature lacks a decisive analysis or conclusion about the conditions under which the constant escapement policy is, or is not, optimal. The generality of the setup and result derived above allows us to provide that analysis. The result is summarized with the following propositions:

**Proposition 2 (Reed)** *Under the model given in Equation 4, constant escapement is optimal if Conditions A.1-A.4 hold.*

This proposition confirms the result of Reed (1979) that A.1-A.4 are sufficient for constant escapement to be optimal. Establishing necessity is more subtle: because the effect of violating one of the assumptions may interact with the effect of violating another assumption, it is not possible to show in general that A.1 - A.4 are jointly necessary for constant escapement to be the optimal policy. Taken individually, however, the assumptions are necessary, as the following proposition shows.

**Proposition 3** *For $i = 1, \ldots, 4$, if condition A.i is violated, and the rest of the conditions hold, then constant escapement is not optimal.*

Taken together, these propositions reveal that despite the ubiquity and appeal of the constant escapement policy, individual deviation from conditions A.1 – A.4 will overturn the constant escapement result. This finding is in line with emerging papers that find special cases (which violate A.1–A.4 in one way or

another) in which escapement should not be held constant (see e.g. Mirman and Spulber (1985), Singh et al. (2006), Carson et al. (2009)).[13]

## 3.2 Independently distributed stocks

While Proposition 3 provides that a violation of one of the conditions A.1 – A.4 will over-turn Reed's constant escapement result, it gives little insight into the nature of the new optimal rule. In this and the next section, we relax the restrictions imposed by conditions A.1 – A.4 and consider the resulting impact on the optimal policy parameters $A$ and $B$. We begin by considering nonlinear profits with i.i.d. productivity shocks. We have the following result:

**Proposition 4** *Under A.1 but without requiring A.2-A.4, optimal escapement is dependent at most on the current stock:* $\tilde{x}_t = A\tilde{s}_t$.

Proposition 4 implies that when environmental shocks are i.i.d., the optimal escapement policy depends only on the current size of the stock and, in particular, does not depend separately on the productivity shock.[14] The dependence on the stock is a result of the nonlinear dependence of profits on harvest, and, for example, may be intuited as follows: consider the case of a downward sloping inverse demand (violation of A.2), and suppose stock is high. A constant escapement rule would dictate selling all excess stock today, and thus facing a drastically reduced price. The optimal rule suggests smoothing behavior by saving some of the excess stock today (i.e. increasing escapement), thereby decreasing the magnitude of the price reduction. That current productivity shocks do not separately impact the escapement decision follows from the fact that they are i.i.d. and so provide no information about future productivity shocks. Rather, the effects of the current shock are transmitted through the current stock $s_t$.

## 3.3 Linear profits

In the previous section we assumed that environmental shocks were i.i.d., but we allowed for profits to exhibit a nonlinear dependence on harvest. Here, we switch those conditions, allowing environmental shocks to be correlated across periods but constraining ourselves to linear profits. The following proposition characterizes optimal harvest under those conditions:

[13]Non-constant escapement can also be achieved by changing the informational timing in the model (Clark and Kirkwood 1986).

[14]Of course the optimal policy is statistically dependent on the productivity shock through the dependence of the stock on $z_t$.

Table 1: The Optimal Escapement Policy Under Alternative Representations of Profits and Uncertainty

| Shocks | Linear profits | Nonlinear profits |
|---|---|---|
| None | constant esc. | stock dep. |
| i.i.d. | constant esc. (Prop. 2) | stock dep. (Prop. 4) |
| Correlated | shock dep. (Prop. 5) | stock and shock dep. (Prop. 1) |

**Proposition 5** *Under A.2-A.4 but without requiring A.1, optimal escapement depends at most on the productivity shock:* $\tilde{x}_t = B\tilde{z}_t$.

Proposition 5 implies that the current size of the environmental shock is the only information that is required to set the current escapement. In other words, it rules out the possibility that the size of the resource stock is of any use in setting the escapement policy. Intuitively, the current environmental shock is useful because it facilitates the prediction of the next period's shock, and thus provides information about the next period's production. A special case is when the shocks are uncorrelated (i.i.d.) in which case Proposition 2 obtains.

## 3.4 Summary of escapement plans

It is instructive to summarize our results thus far. Table 1 characterizes the nature of optimal escapement policies under different assumptions about profits and the distribution of environmental shocks.

The first row of Table 1 simply reproduces well-known results of optimal exploitation of renewable resources in a deterministic environment (see, e.g., Clark (1990)). If the profits are linear in harvest, a bang-bang result obtains and the optimal escapement is independent of time. On the other hand, if are profits nonlinear, the escapement level will depend on the stock size, implying that harvests are smoothed in order to maximize surplus.

The second row of the table presents the optimal escapement plans under i.i.d. production shocks. With linear profits, the optimal escapement is again constant, a result that was obtained by Reed (1979) and is also proven in Proposition 2 of this paper. However, as in the deterministic case, when shocks are i.i.d and profits are nonlinear, we find via Proposition 4 that the optimal escapement depends on the current stock size.

The final row of the table allows the environmental shocks to be correlated across time so that this period's shock contains some information valuable for

the prediction of next period's shock. We find that when profits are linear, the escapement function depends only on the most recent shock (Proposition 5). In the most general case, when profits are nonlinear and the productivity shock is forecastable, escapement will depend both on the stock and shock, as shown by Proposition 1.

Two insights emerge from this summary. First, regardless of one's assumption about the nature of the productivity shocks, if profits are linear, the optimal escapement function depends, at most, on the most recent productivity shock (and never on the current stock size). Second, regardless of one's assumption about profits, if the shocks are uncorrelated across periods, the optimal escapement function depends, at most, on the current stock size (and never on the productivity shock). We thus find that only in case of serially correlated shocks *and* nonlinear profits will the optimal escapement rule exhibit a dual dependence on stocks and shocks: in this case, the predictive role of the productivity shock must be separated from its impact on the stock level.

# 4 Comparative statics

In the previous section we established that the optimal escapement rule is given, in general, by $\tilde{x}_t = A\tilde{s}_t + B\tilde{z}_t$ (equation 9). In this section, we develop intuition for this rule by evaluating how this functional relationship changes as the demand, cost, and production specifications are varied. Specifically, we present comparative statics for the coefficients $A$ and $B$, which, as shown in the appendix, are functions of model parameters and deterministic steady-state values. Because $A$ and $B$ are too complicated to analyze in their general form, we present a numerical analysis. Our results are easier to interpret if the escapement rule is rewritten in terms of levels of the variables ($x_t$, $s_t$, $z_t$):

$$x_t = \bar{C} + \bar{A}s_t + \bar{B}z_t,$$

where the constants $\bar{A}$, $\bar{B}$, and $\bar{C}$ depend on the specific functions and parameter values. In terms of the previously defined parameters, they are $\bar{C} = x(1 - A - B)$, $\bar{A} = A(x/s)$, and $\bar{B} = B(x/z)$. $\bar{A}$ and $\bar{B}$ provide the marginal effect of the stock and productivity shock on the optimal level of escapement. We examine how these marginal effects are altered as parameters of the model change.

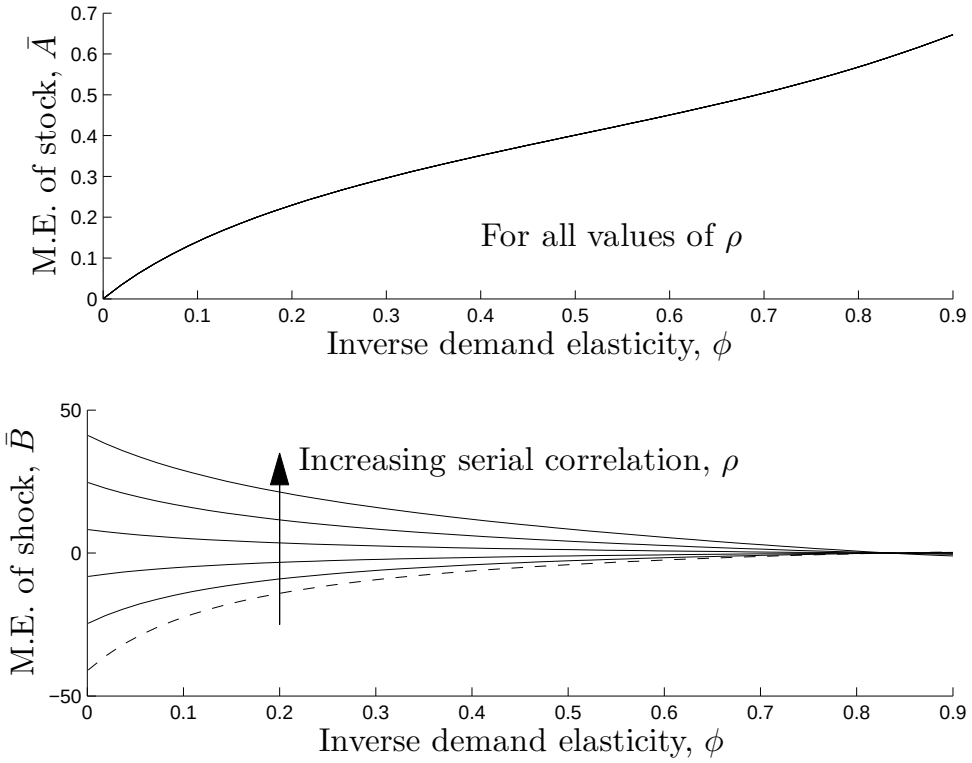


Figure 1: Top Panel: Dependence of marginal effect of stock ($\bar{A}$) on inverse demand elasticity ($\phi$). Bottom Panel: Dependence of marginal effect of productivity shock ($\bar{B}$) on inverse demand elasticity ($\phi$). Plots are drawn for various levels of serial correlation ($\rho$).

For this exploratory analysis we adopt the following parameterization:

$$\begin{array}{rrcl} \text{Demand:} & p(s-x) & = & 10(s-x)^{-\phi} \\ \text{Marginal Cost:} & c(x,s-x) & = & x^{-\theta} + W(s-x)^{\psi} \\ \text{Production:} & f(x) & = & 10x^{\alpha}. \end{array} \tag{10}$$

We assume constant price elasticity of demand; the inverse of the demand elasticity is constrained to the unit interval ($0 < \phi < 1$), implying elastic demand for output $q$. Inelastic demand needs to be ruled out, because with non-decreasing costs, profits can always be raised by reducing output, indicating an optimal harvest close to zero. Marginal cost has two additive effects: (1) a *stock effect*, where marginal harvest cost depends on fish density (Reed (1979) adopts this functional form); and (2) a *harvest effect* where marginal

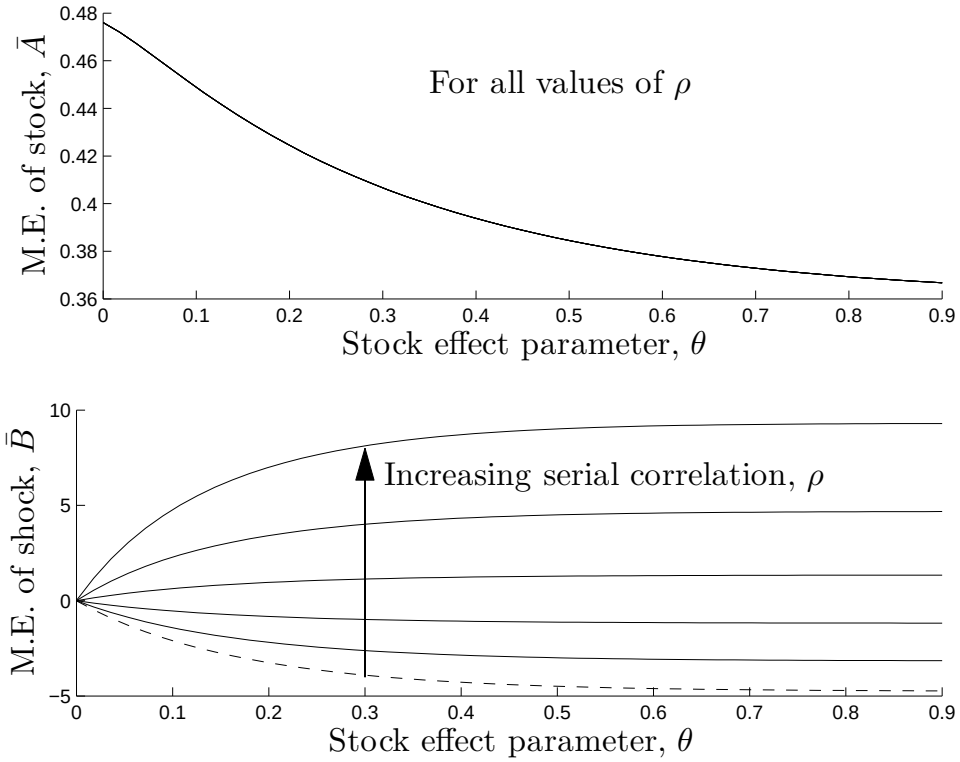


Figure 2: Top Panel: Dependence of marginal effect of stock ($\bar{A}$) on stock effect parameter ($\theta$). Bottom Panel: Dependence of marginal effect of productivity shock ($\bar{B}$) on stock effect parameter ($\theta$). Plots are drawn for various levels of serial correlation ($\rho$).

costs may increase in harvest volume. Changes in $\psi$ have the same qualitative effects on $\bar{A}$ and $\bar{B}$ as do changes in $\phi$, so we set $W = 0$. Finally, the production function is extremely simple, but retains the desired properties outlined above. We require that it exhibit diminishing marginal returns, so $0 < \alpha < 1$. The discount factor is set at $\delta = 1/1.05$, and we consider a risk-neutral decision-maker ($\sigma = 0$).

We first consider, in Figure 1, how $\bar{A}$ and $\bar{B}$ change as we vary $\phi$ from 0 to 0.9 and the serial correlation $\rho$ (see equation (2)) from -0.9 to 0.9. Specifically, for six values of $\rho$, the values of $\bar{A}$ and $\bar{B}$ are plotted against the varying demand elasticity; thus, six plots appear in each panel. First consider the top panel. Notice that the plots are identical across serial correlation values ($\rho$). Given the timing of the model, the productivity shock $z_t$ is observed before

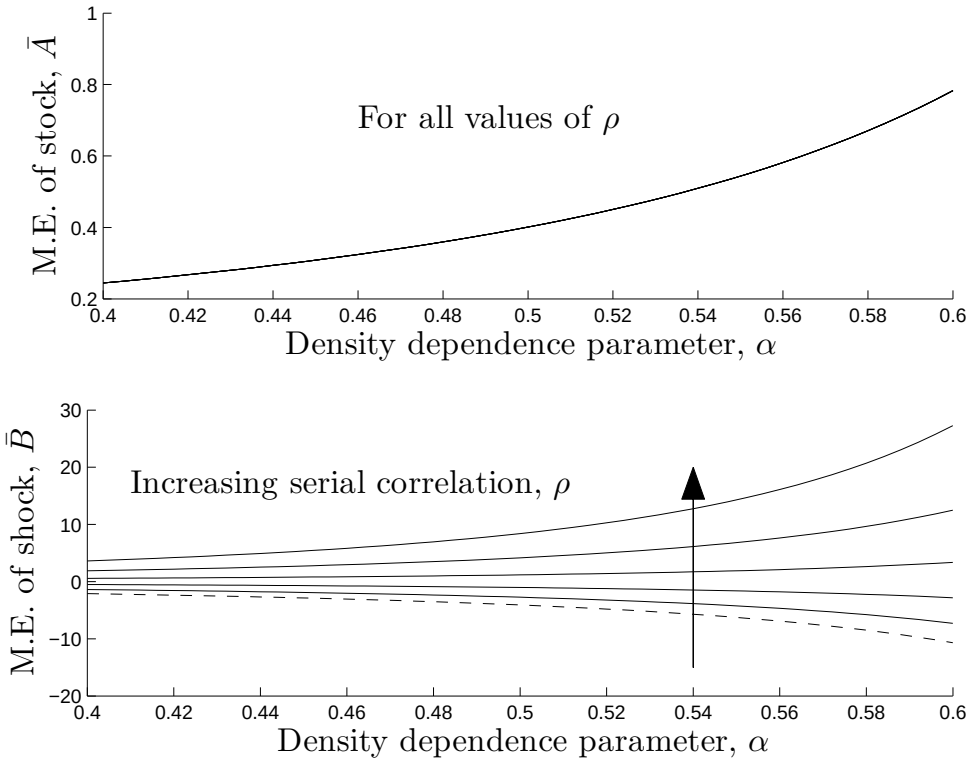


Figure 3: Top Panel: Dependence of marginal effect of stock ($\bar{A}$) on biological density dependence ($\alpha$). Bottom Panel: Dependence of marginal effect of productivity shock ($\bar{B}$) on biological density dependence ($\alpha$). Plots are drawn for various levels of serial correlation ($\rho$).

escapement $x_t$ is chosen; thus the value of $\rho$ only affects the forecastability of the productivity shock, and does not influence the marginal effect of the stock. Next, notice that $\bar{A}$ equals zero when demand is perfectly elastic ($\phi = 0$), corresponding to the result in Reed (1979) that escapement is independent of the stock level when price is constant. The marginal effect of stock on escapement increases as $\phi$ increases. As demand becomes more inelastic, harvesting an extra unit of the stock has ever greater effects on price. Therefore, to avoid depressing current prices, more of the stock is allowed to escape to the next period.

Now turn to the bottom panel of Figure 1, which shows how $\bar{B}$, the marginal effect of the current shock $z_t$ on escapement, varies with $\phi$, and again plotted for different values of $\rho$. The dashed plot corresponds to $\rho = -.9$ and for

fixed demand elasticity values, as $\rho$ increases, so does $\bar{B}$. First notice that the sign of the marginal effect coincides with the sign of the serial correlation: if $\rho > 0$, then a positive shock in the current period indicates positive expected future shocks; escapement is increased because the expected "rate of return" on future stocks is high. However, as demand becomes more inelastic ($\phi$ increases), escapement is reduced to avoid depressing future prices. The relationship is reversed when $\rho < 0$. Now, a positive shock in the current period implies a relatively low return on future stocks, and escapement is reduced. As demand becomes more inelastic, escapement increases to avoid depressing current prices. These effects are magnified as $\rho$ increases in absolute value and future shocks are more highly correlated with the current shock. In contrast, when $\rho = 0$, the current shock provides no information about future shocks and, as in Reed (1979), it has no effect on optimal escapement.

In Figure 2 we explore the effects of the cost parameter $\theta$ on $\bar{A}$ and $\bar{B}$, again for varying $\rho$, just as in Figure 1. For a given level of the stock, marginal costs decline as $\theta$ increases. Likewise, for given $\theta$, marginal costs fall as the stock increases (the stock effect). For the parameter values we consider and in the neighborhood of our steady state, the stock effect is smaller at larger values of $\theta$. The top panel shows that the marginal effect of the stock on escapement declines as the stock effect diminishes. As $\theta$ increases, more of an additional unit of stock is harvested (less escapes) in the current period for two reasons: first, the marginal cost of harvesting today is reduced; and second, the cost reduction tomorrow obtained by allowing a fish to escape today is diminished. The autoregressive parameter $\rho$ has no effect on the marginal effect of the stock for the same reasons discussed above.

In the lower panel of Figure 2, we show the marginal effect of the productivity shock on escapement for different values of $\theta$ and $\rho$. As in the bottom panel of Figure 1, the sign of the marginal effect coincides with the sign of the serial correlation: if $\rho > 0$, then a positive shock in the current period indicates a high return on future stocks and the marginal effect of the stock on escapement is positive. A higher $\theta$ indicates lower marginal costs in the future and reinforces this effect, raising escapement even more. In contrast, when $\rho < 0$, diminishing future marginal costs compete against the effect of low expected future returns on stocks. This explains the asymmetric response to $\theta$ and $\rho$ evident in the figure.

No particular significance should be assigned to the result that $\bar{B} = 0$ when $\theta = 0$. If, for instance, we choose a lower value of $\phi$, $\bar{B}$ remains positive or negative at $\theta = 0$. As demand becomes more elastic, escapement increases ($\rho > 0$) or decreases ($\rho < 0$), just as in the bottom panel of Figure 1. If, instead, we choose large values of $\phi$, $\bar{B}$ becomes zero at a positive value of $\theta$.

Finally, we consider the effect of the biological production parameter $\alpha$ on $\bar{A}$ and $\bar{B}$ (Figure 3). Larger values of $\alpha$ correspond to higher productivity of the stock. As well, the marginal productivity of the stock is increasing in $\alpha$. Due to this latter effect, $\bar{A}$ is increasing in $\alpha$ (top panel). When an additional unit of the stock provides a higher future return, more of the stock is allowed to escape. As above, this relationship does not change with different values of $\rho$. The top panel was produced for a fixed value of $\phi$. As indicated by Figure 1, escapement also increases as demand becomes more inelastic. Thus, we see that higher productivity and more inelastic demand have reinforcing effects on harvesting smoothing. Finally, as seen in Figures 1 and 2, $\bar{B}$ is positive for $\rho > 0$ and negative for $\rho < 0$ (bottom panel). Higher values of $\alpha$ increase the expected future productivity of the stock and escapement is increased ($\rho > 0$) or decreased ($\rho < 0$).

# 5 Efficiency of the linearized model

By log-linearizing the Euler equation about the deterministic steady state, we have derived a state-contingent policy whose properties are easily studied and that is both intuitively appealing and easy to implement. A natural question is to what extent this simple policy and its associated stocks and profits deviate from the fully optimal (i.e. non-linearized) policy function. While this question is most certainly empirical, and will depend on the specifics of any given problem, here we examine this question over a large range of reasonable parameter values for the model in Equations 10.

The first step in comparing our linearized policy with the fully optimal policy involves solving for the optimal policy function. As noted above, while special cases of our problem have known analytical solutions, a fully optimal analytical solution to the general problem has never been found. We thus rely on numerical dynamic optimization techniques to derive the optimal solution. Specifically, we use value function iteration (Judd 1998), which involves iterating on the two-dimensional value function, until the (also two-dimensional) policy function converges. By choosing a finer and finer grid, this method allows us to get arbitrarily close to the fully optimal policy (albeit at considerable computational expense). The Bellman equation for our problem is:

$$V_t(s_t, z_t) = \max_{x_t} S(s_t, x_t) + \delta E_t V_{t+1}(s_{t+1}, z_{t+1}). \tag{11}$$

Upon convergence, the infinite-horizon policy function, $x_t^*(s_t, z_t)$ gives the optimal escapement as a function of stock and the most recent environmental

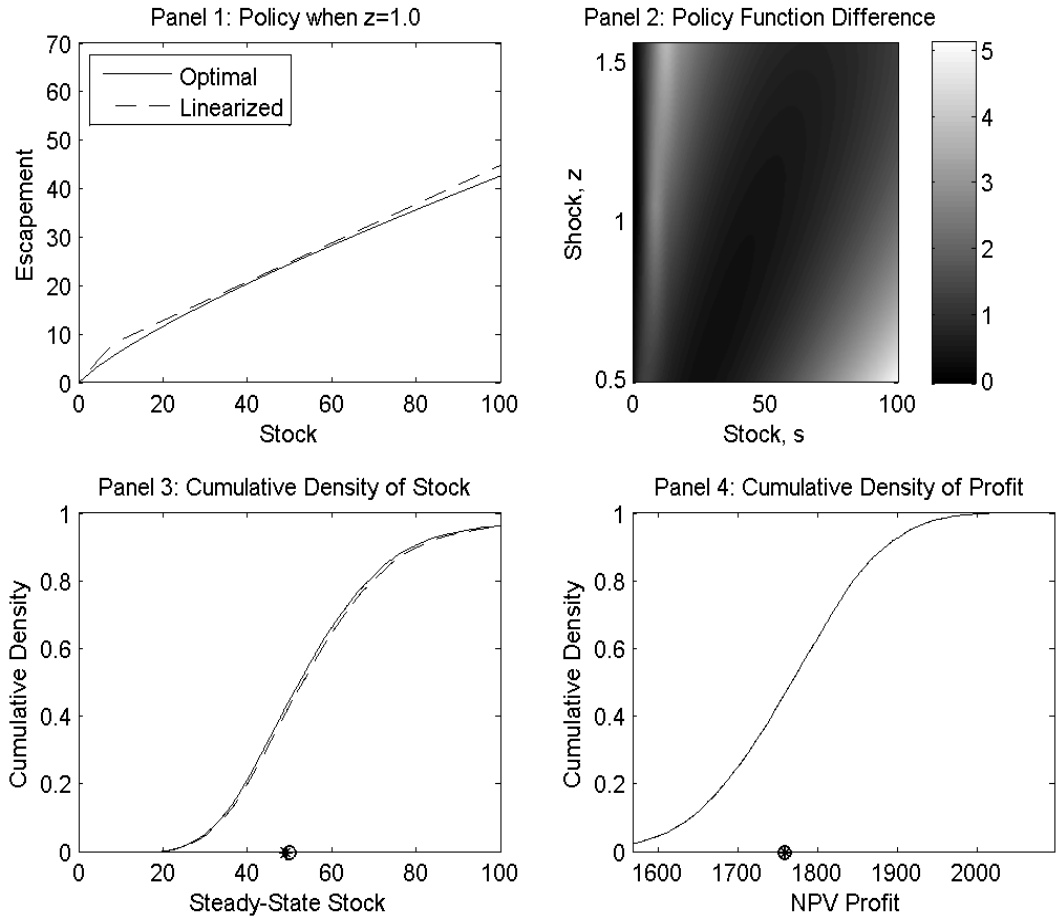


Figure 4: Diagnostics of the optimal nonlinear policy vs. the linearized model. Panel 1 shows the sample policy function. Panel 2 shows the difference between policy functions. Panel 3 shows the density of steady state stock. Panel 4 shows the density of profit.

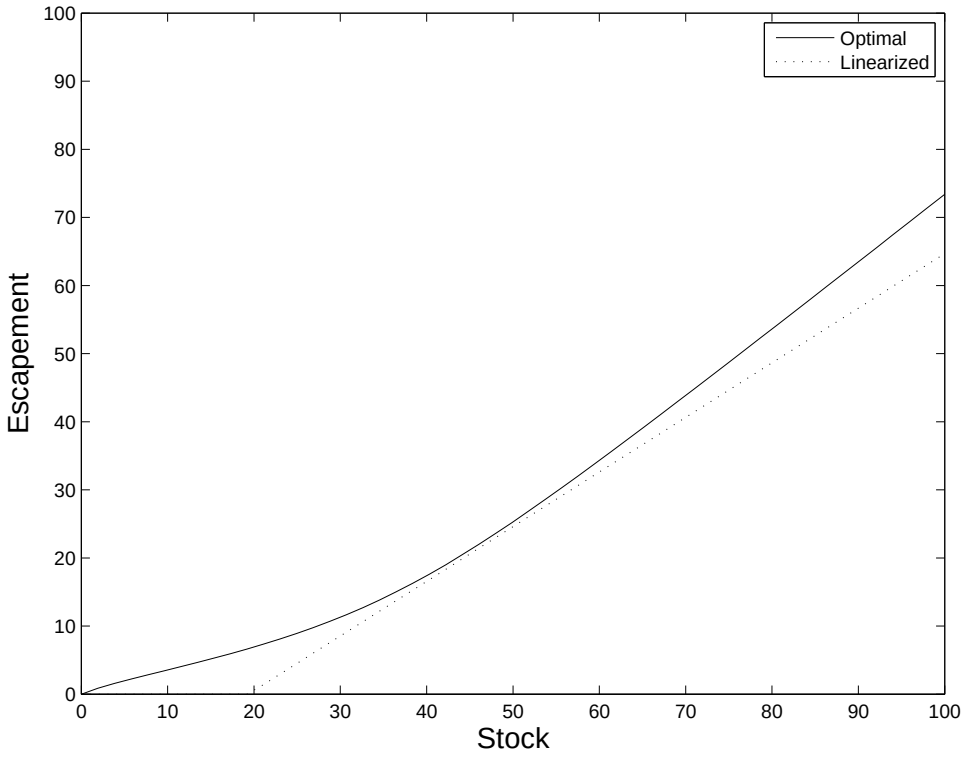


Figure 5: Fully optimal nonlinear rule (solid) and the linearized rule (dotted) for a case in which extinction under the linearized rule may be possible.

shock with no presumption about the functional form of this relationship. In contrast, by log-linearizing the Euler equation, we obtain an explicit policy that is linear in the two state variables: $x_t = \bar{C} + \bar{A}s_t + \bar{B}z_t$. The final detail required to implement our comparison is a specification of the statistical distribution of the environmental shocks. Recall equation (2),

$$z_t = v_t z_{t-1}^{\rho}.$$

For the purposes of this experiment, we assume $v_t \sim N(1, \sigma_v^2)$, and we use standard deviation $\sigma_v = 0.2$. To guard against the possibility that $z_t = 0$, which implies extinction of the resource, we truncate this distribution slightly above zero.

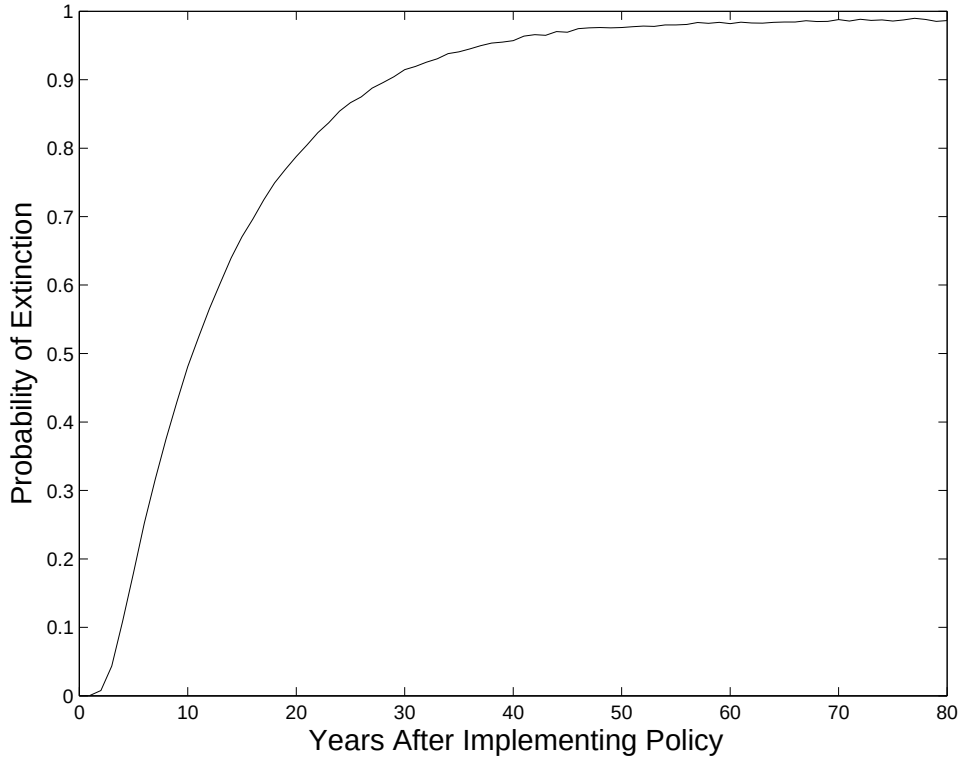


Figure 6: Probability of extinction over time from strict application of the linear rule when extinction is possible.

## 5.1 Base case comparison

Our base case parameters include a downward-sloping demand curve ($\phi = 0.5$), a stock effect on harvest cost ($\theta = 0.35$), and positive serial correlation ($\rho = 0.5$). The optimal policy, derived numerically, is tangent to the linearized policy around the deterministic steady state, but is strictly concave rather than linear. Panel 1 of Figure 4 plots the two policy functions for the case where $z_t = 1.0$. Overall, the policies are nearly coincident. When the stock deviates significantly from the deterministic steady state, the optimal policy is to escape slightly fewer fish than would be called for under the linearized policy. The deviation in policy functions, represented by the light shade in panel two, depends on the particular combination of $z_t$ and $s_t$. Near the deterministic steady state (approximately $z_t = 1$, $s_t = 50$ for these parameters), the difference is negligible. Deviations of up to 5 are possible, if $z_t$ is extremely low and $s_t$ is extremely high. Deviations of the converse ($z_t$ high and $s_t$ low)

are misleading because the linearized policy suggests a policy where $x_t > s_t$, which violates the non-negativity constraint on harvest (thus the dashed line in Panel 1 has a kink where it intersects the $45^o$ line).

Under the base case parameters, our conclusion is that the two policy functions are very close, at least around the deterministic steady state. A related question arises: How different are the dynamics of the two systems? We already noted that the optimal policy leaves slightly lower escapement than the linearized policy. Do these differences magnify as a result of system dynamics, or are they dampened? To examine this question, we calculate the long-run steady-state distribution of stocks, $s_t$, under each policy function using a Markov transition model for this (optimally controlled) stochastic process. The third panel of Figure 4 plots the c.d.f. of the long-run steady state distribution of stock under the linearized and optimal policies. The circle and star on the horizontal axis shows the mean of each. Two results emerge. The first is that the steady-state distributions of stock are nearly identical. The second is that the long-run mean stock under the linearized policy is slightly above that of the optimal policy. This makes intuitive sense upon reflection that the escapement of the latter is always slightly lower than the former. Not surprisingly, profits follow a similar pattern. The fourth panel of Figure 4 plots the c.d.f. of the steady-state distribution of annual profits under each policy. Note here that expected profits under the optimal policy ($*$ on horizontal axis) is larger than under the linearized policy ($O$ on axis), though by less than 1%.

## 5.2 Comparison over general parameter values

Provided that the assumptions of this model are met, the linearized policy continues to perform extremely well over a wide range of reasonable parameter values.[15] Expected net present value of the fishery under the linearized model was typically less than 0.1% smaller than the value under the optimal policy; in no case did the deviation exceed 1%. Perhaps equally relevant is the effect on the long-run steady-state distribution of stock. We found that the linearized policy can either yield larger, or smaller expected stocks than the optimal policy, but the deviations were small - typically less than 0.5%, and never greater than 3%. Distributions of stocks were nearly coincident.

An important caveat remains. For the linearization to be valid, we require that perturbations do not catapult the system far away from the deterministic steady state. An example of a clear violation of this criterion is when following

[15]We conduct the experiment described above for a full factorial of parameter values in the ranges $\phi \in (0, 0.7)$, $\theta \in (0, 0.9)$, and $\rho \in (-0.7, 0.7)$.

the linearized policy leads to extinction of the stock. Importantly, because the only source of stochasticity is a multiplicative shock to production, only two avenues can lead to extinction. The first is that $z_t = 0$ for some $t$, a case we rule out by assumption. The second possibility is $x_t = 0$ for some $t$. Provided the discount rate is sufficiently small, forced extinction could never be optimal. However, because the linearized policy does not explicitly account for possibilities such as extinction, the linearized policy could lead to forced extinction.

Consider, for example, the model above with $\phi = .7$, $\theta = 0$, $\rho = 0$, and standard deviation of shocks $\sigma_v = 0.3$. In this case, the linearized policy intersects the stock axis, even for $z = 1$ (Figure 5). While the policy performs very well near the deterministic steady state, the large standard deviation on stochastic shocks can occasionally reduce stocks to unsafe levels (e.g. if $z = 1$, then $s \leq 20$ would lead to immediate extinction under a strict application of the linearized policy). How likely is this outcome? Figure 6 shows the probability of extinction under strict application of the linearized policy; the stock is likely to go extinct within 20-30 years. Applying the fully optimal policy (solid line in Figure 5) results in a probability of extinction of zero.

While this example illustrates our caveat, we found that it is the exception, not the rule. For over 90% of the cases examined in this experiment, extinction did not occur under either policy. While our framework does not technically apply in the extinction case (we restrict attention to interior steady states), we offer the following suggestion: If the linearized policy function does not intersect the $s$ axis (i.e. if $\bar{C} + \bar{B}z > 0$, $\forall z$), then forced extinction is not possible. If $\bar{C} + \bar{B}z < 0$, then the possibility exists, and some precaution is warranted when stocks approach dangerously low levels.

# 6 Conclusion

Most fisheries are characterized by nonlinear economic relationships and inherent environmental stochasticity. In combination, these two features significantly complicate analytical models of optimal fishery management. As such, earlier authors have either ignored one of these complications or resorted to numerical analysis. In this paper, we apply to the stochastic fisheries problem tools of dynamic stochastic general equilibrium and multivariate linear expectational difference equations developed in the macroeconomic literature. By adopting these techniques, we are able to solve the fisheries harvest problem in a general model that includes serially-correlated shocks, downward sloping demand, quantity-dependent marginal costs, and risk aversion. The solution,

based on a linear approximation, yields a simple policy function. Optimal escapement is a linear function of the current stock and productivity shock, with coefficients that depend on the root parameters of the model and steady-state values.

The properties of this linear policy function are easily studied, and we establish a number of new theoretical results. We present the sufficient conditions for the optimality of a constant escapement policy that is currently applied in many actual fisheries. These include i.i.d. productivity shocks, perfectly elastic demand, risk neutrality, and harvest-independent marginal costs. Further, we show that if any one of these conditions fails to hold, then constant escapement is no longer optimal. The assumption of a constant price may be reasonable for fisheries that represent a small share of a global market, such as national fisheries for tuna, crab, and squid. Even when a species is caught elsewhere, however, demand in local markets for live fish can be inelastic. As well, single fisheries may represent large shares of the global market, as with Bristol Bay sockeye salmon, Alaskan halibut, and Peruvian anchoveta. Further, as noted above, evidence indicates that productivity fluctuations in ocean fisheries exhibit serial dependence. We show that downward-sloping demand, and other nonlinearities in profits, causes optimal escapement to depend on the current stock. With serially-correlated shocks, optimal escapement also becomes a function of the current shock.

Numerical analysis helps us understand how the model parameters affect the optimal policy function. Two general insights emerge. The first is that nonlinear profits induce the optimal smoothing of harvests. Consider, for example, a fishery with a stock above its steady-state value. With downward-sloping demand and harvest-dependent marginal costs, the manager should not harvest all of the surplus at once, as under a constant escapement policy, but rather smooth the harvest over time to avoid depressing prices and raising marginal costs. The second insight is that serial correlation in the productivity shocks provides the manager with information about the future "rate of return" on escaped fish. When the correlation is positive, a positive current shock signals positive shocks in the future, and escapement should be increased to take advantage of higher future productivity. With negative correlation, a positive current shock indicates lower future productivity and escapement should be curtailed, a result whose intuition is echoed by Costello et al. (2001) and Carson et al. (2009).

To solve the model, we linearize the Euler equation and production function around a deterministic steady-state. Thus, a natural question is whether the resulting approximate solution departs considerably from the fully optimal solution. Markov Chain simulations reveal that the linearized policy performs

almost identically to the fully optimal program (profits less than .1% smaller). This finding has practical implications for fisheries management. When the assumptions needed for constant escapement are violated, our approach can be used to derive simple policy rules. We demonstrate that the model can be solved for general specifications of demand, costs, and risk preferences, indicating that our approach can accommodate whatever economic information is available for the fishery (provided the required convexity assumptions are met). Moreover, while we used a standard representation of the stochastic stock-recruitment relationship, the approach is flexible in this regard. In particular, the shock need not be multiplicative nor be limited to a first-order autoregressive process.

While we analyze a standard model of a stochastic fishery, the methods are applicable to a broader class of renewable resource problems. The model considered here can be extended to represent capital, entry-exit, and other common features of fisheries. As well, the approach can be adapted to other natural resources such as forests, wetlands, and wildlife species. McGough et al. (2004) use these methods to study price dynamics in a rational timber market. In the macroeconomics literature, these methods were developed to study general equilibrium with imperfect markets and endogenous policy responses. Clear parallels to renewable resource markets exist, including fisheries with incomplete property rights and endogenous regulation, e.g. Homans and Wilen (1997). The techniques presented here can be used to approximate the market equilibrium when numerical methods are intractable.

# 7 Appendix

## The linearization

The log-linearization technique employed here is standard, and can be found, for example, in Evans and Honkapohja (2001). The parameters of equation (7) are given by $a = a_1 - a_2$ where

$$\begin{aligned} a_1 &= \left( -\sigma S^{-\sigma-1} \frac{\partial S}{\partial x}(p-c) + S_t^{-\sigma}(c_2 - c_1 - p') \right) x \\ a_2 &= \delta f'' S^{-\sigma} \left( p - c - \int_x^s c_2(\omega, s-\omega) d\omega \right) x, \end{aligned}$$

and

$$\begin{aligned}
b &= \left(-\sigma S^{-\sigma-1}\frac{\partial S}{\partial s}(p-c) + S_t^{-\sigma}\left(p' - c_2\right)\right)x, \\
d &= \delta f'\left(-\sigma S_t^{-\sigma-1}\frac{\partial S}{\partial x}\left(p - c - \int_x^s c_2(\omega, s-\omega)d\omega\right) - S^{-\sigma}\left(p' - c_2\right)\right)x \\
e &= \delta f'\Big\{-\sigma S_t^{-\sigma-1}\frac{\partial S}{\partial \tilde{s}}\left(p - c - \int_x^s c_2(\omega, s-\omega)d\omega\right), \\
&\quad + S^{-\sigma}\left(p' - c_1 - \int_x^s c_{22}(\omega, s-\omega)d\omega\right)\Big\}s, \\
g &= \delta f'\left(S_t^{-\sigma}(p - c - \int_x^s c_2(\omega, s-\omega)d\omega)\right)\rho.
\end{aligned}$$

Here $m = xf'(x)/f(x)$,

$$\begin{aligned}
\frac{\partial S}{\partial x} &= c(x, s-x) - p(s-x) \\
\frac{\partial S}{\partial s} &= p(\tilde{s} - x) - c(s, 0) - \int_x^{\tilde{s}} c_2(\omega, s-\omega)d\omega,
\end{aligned}$$

and everywhere, $p$ ane $p'$ are evaluated at $s - x$, in $a_1$ and $b$ the functions $c$, $c_1$ and $c_2$ are evaluated at $(x, s-x)$, in $a_2, d, e$, and $g$ the functions $c$ and $c_1$ are evaluated at $(s, 0)$ and $c_2$ at $(x, s-x)$.

## Proof of Proposition 1

To prove this proposition, we solve equation (7) for its unique covariance stationary solution, which is precisely the linear approximation to the unique process satisfying the Euler equations and the transversality condition.[16]

For notational simplicity, we drop the tildes on the variables. Write (7) as

$$(a - em)x_t + bmx_{t-1} + bz_t = dE_t x_{t+1} + (e\rho + g)z_t. \tag{12}$$

Now let $\xi_t$ be the manager's forecast error for escapement. Then $\xi_t = x_t - E_{t-1}x_t$. Because the manager is rational, his expected forecast error is zero, that is, $\xi_t$ forms a martingale difference sequence. Our task is to find the

[16]In the parlance of expectational difference equations, the system (7) is said to be "determinate," which, in this linear case, means there is a unique non-explosive solution. This "determinacy" follows from the fact that we have imposed sufficient convexity assumptions on the model to guarantee that the programming problem has a unique solution.

correct restriction on this sequence so that (7) is not explosive. To this end, replace $E_t x_{t+1}$ in (12) with $x_{t+1} - \xi_{t+1}$. We may then form the stacked system

$$\begin{pmatrix} (a-em) & bm & b-(e\rho+g) \\ 1 & 0 & 0 \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} x_t \\ x_{t-1} \\ z_t \end{pmatrix} = \begin{pmatrix} d & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{t+1} \\ x_t \\ z_{t+1} \end{pmatrix} - \begin{pmatrix} d & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_{t+1} \\ v_{t+1} \end{pmatrix}.$$

Letting $w_t = (x_t, x_{t-1}, z_t)'$, $\varepsilon_t = (\xi_t, v_t)'$, we may write this in VAR(1) form as $w_t = Dw_{t-1} + F\varepsilon_t$. Now write $D = S(\lambda_1 \oplus \lambda_2 \oplus \rho)S^{-1}$ where the $\lambda_i$ are endogenous eigenvalues of $D$ written in decreasing order of magnitude, and the columns of $S$ are the associated eigenvectors. The saddle-path stability of the system is reflected in the fact that the modulus of $\lambda_1$ is larger than one while the modulus of $\lambda_2$ is less than one. For this VAR(1) system to be non-explosive, it must be the case that the forecast error is chosen is such as way as to impose that the vector $w_t$ remain in the direct sum of the eigenspaces spanned by the second and third columns of $S$, i.e. the spaces for which the associated eigenvalues are contracting. This is accomplished by letting $\hat{w}_t = S^{-1}w_t$ and setting $\hat{w}_{1t} = 0$. This equality places a linear restriction on the entries in $w_t$, namely $S^{11}x_t + S^{12}x_{t-1} + S^{13}z_t = 0$, where $S^{ij}$ is the $ij^{\text{th}}$ entry of $S^{-1}$. We may solve this linear restriction for $x_t$. For notational simplicity, we make the following definitions:

$$R_1 = \frac{d}{a-em}; \;\; R_2 = -\frac{bm}{a-em}; \;\; R_3 = \frac{e\rho - b + g}{a-em}. \tag{13}$$

Algebra shows

$$\bar{a} = \frac{1-\sqrt{1-4R_1R_2}}{2R_1} \;\text{ and }\; \bar{b} = \frac{R_3}{1-R_1(\bar{a}+\rho)}. \;\blacksquare \tag{14}$$

## Proof of Proposition 2

This proof and the remaining proofs rely on the algebraic dependence of $A$ and $B$ , which are easily computed to be

$$A = \frac{\bar{a}}{m} \;\text{ and }\; B = \bar{b} - A.$$

Constant escapement obtains if and only if $A = B = 0$. Now notice that A.1 implies $\rho \cdot e = 0$ and $g = 0$ and A.2, A.3 and A.4 imply $b = 0$. Thus $R_3 = 0$ so that $\bar{a}$ and $\bar{b}$ are zero. ■

## Proof of Proposition 3

Using the notation from the previous proof, it suffices to show that if A.$i$ is violated, with A.$j$ holding for $j \neq i$, then $R_3 \neq 0$. In case $i = 2, 3$, or 4, this follows from the facts that 1. if $\rho = 0$ then $e \cdot \rho + g = 0$; and 2. the condition $p > c$ (which is necessary for optimality), together with one of the assumptions A.$i$, implies $b \neq 0$. In case $\rho \neq 0$, and A.2, A.3, and A.4 holding, we have that

$$e\rho + g = \delta f' \rho (p - (c + c_1)).$$

This expression can not be zero because $p > c$ and we have assumed decreasing marginal costs of harvest, i.e. $c_1 < 0$. ■

## Proof of Proposition 4

Optimal escapement is given by equation (8) above. To prove this result, it suffices to show that $\bar{b}m = \bar{a}$, because, by (8), this implies that

$$x_t = \bar{b}\,(mx_{t-1} + z_t) = \bar{b}s_t,$$

where the last equality follows from the linearized production function. To demonstrate $\bar{b}m = \bar{a}$, use $\rho = 0$ to obtain

$$\begin{aligned} \bar{b}m &= \frac{R_3 m}{1 - R_1 \bar{a}} && (15) \\ &= \frac{2R_3 m}{1 + \sqrt{1 - 4R_1 R_2}} && (16) \\ &= \frac{2R_3 m \left(1 - \sqrt{1 - 4R_1 R_2}\right)}{4R_1 R_2} = \frac{R_3}{R_2} m\bar{a}. && (17) \end{aligned}$$

By the definitions above,

$$\frac{R_3}{R_2} = \frac{b - g}{bm},$$

and since $g = 0$ whenever $\rho = 0$, it follows that $(R_3/R_2)m = 1$. ■

## Proof of Proposition 5

Notice that $p' = 0, \sigma = 0$, and $c_2 = 0$ implies that $b = 0$ and $d = 0$. By the above relations, we get that $R_1 = 0$ and $R_2 = 0$. Now notice that

$$\tilde{x}_t = R_1 E_t \tilde{x}_{t+1} + R_2 \tilde{x}_{t-1} + R_3 z_t.$$

Because $R_1 = R_2 = 0$, the result obtains. ■

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