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# FORM FACTORS AND METHODS OF CALCULATING THE STRENGTH OF WOODEN BEAMS



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# FORM FACTORS AND METHODS OF CALCULATING

## THE STRENGTH OF WOODEN BEAMS<sup>1</sup>

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Wood is a complex structural material, consisting essentially of fibers of cellulose cemented together by lignin. It is the shape, size, and arrangement of these fibers, together with their chemical and physical composition, that governs the strength of wood, and accounts for the large difference in properties along and across the grain.

The fibers are essentially long hollow tubes tapering towards the ends, which are closed. They are about one-eighth of an inch long in softwoods and one twenty-fourth of an inch long in hardwoods, with a central diameter about one-hundredth of the length. Besides these vertical fibers, which are oriented with their longer dimension lengthwise of the tree and comprise the principal part of what we call wood, all species, except palms and yuccas, contain horizontal strips of cells known as rays, which are oriented radially and are an important part of the tree's food transfer and storage system. Among different species the rays differ widely in their size and prevalence.

From the strength standpoint, this arrangement of fibers results in an anisotropic structure with three principal axes of symmetry (longitudinal, radial, and tangential), which account for three Young's moduli varying as much as 150 to 1, three shear moduli varying 20 to 1, six Poissons' ratios varying 40 to 1, and other properties varying with grain direction. Not all of these wood properties have yet been thoroughly evaluated.

The engineer must depend, in his strength calculations, on various formulas, many of which are of long standing. The derivation of these formulas, is, of course, based on certain assumptions, and the reasonableness of these assumptions in a wide range of application is attested to by their satisfactory use. On the other hand, it is essential that the assumptions upon which the formulas are based, and their limitations, be kept constantly in mind, to avoid serious error.

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Let us in this connection examine the usual flexure theory

$$M = S \frac{I}{c} ; \quad (1)$$

where  $M$  is the bending moment;  
 $S$  is the unit stress in extreme fiber;  
 $I$  is the moment of inertia; and  
 $c$  is the distance from neutral axis to outermost fiber.

In this theory it is assumed, among other things, that a plane section remains a plane section, that the stress does not exceed the proportional limit, and that the maximum stress for a material is a constant. In estimating the bending strength of wood, any errors in performance are frequently attributed to errors in the first two of these assumptions. On the contrary, it has been found that the ordinary assumption as to distribution of stress holds quite well up to the proportional limit, and that it is the maximum and proportional limit stress that varies, being dependent on the size and form of the beam. As long as the results of tests in static bending are applied to members of the same kind, form, size, and condition of loading the same proportional limit and maximum stress obtains and but little error results even when the stresses exceed the proportional limit. When applied to radically different sections, however, the same proportional limit and maximum stress does not obtain and large errors may result. These errors, in most instances, are away from the side of safety.

Standard strength tests of different species of wood are commonly made on specimens 2 by 2 inches in cross section. Since, as has been mentioned, test results are, for many properties, intimately related to method, some of these data are consequently strictly applicable only to specimens of the size employed. In the usual flexure formula the factor by which the proportional limit and maximum stress,  $S$ , based on the results of standard tests, must be multiplied in calculating the strength of any section, is known as the form factor of that section. The bending formula is thus simply expressed:

$$M = FS \frac{I}{c} \quad (2)$$

where  $F$  represents the form factor.

Form factors for wood have been investigated and established from theoretical and empirical considerations at the Forest Products Laboratory, and are applicable to both proportional limit and ultimate stress calculations.

To obtain a better picture of the principles of the form factor theory, a few specific examples will be of interest. Consider, for instance, a prism, 2 by 2 by 8 inches loaded centrally parallel to the grain until the maximum load,  $P$ , is obtained. Assuming the stress is uniformly distributed over the area  $A$ , the maximum unit crushing strength,  $C$ , is  $P/A$ . Now let this same specimen be loaded eccentrically through knife edges, so that the compressive stress is zero on one side, and a

maximum on the other. It may be shown that this occurs when the eccentricity is  $\frac{t}{6}$ , where  $t$  is the width of the specimen. If the maximum unit crushing strength of the piece is  $C$ , it follows that the maximum eccentric load  $P_1$  should be  $\frac{P}{5}$ . In actual tests, however, it was found that the eccentrically loaded specimen carried, not one-half, but over two-thirds the load sustained by the centrically loaded specimen. For some reason the fiber stress had gone far beyond what might be expected on the basis of the usual assumptions.

Again, although there is every reason to believe that the ordinary assumption as to distribution of stress in bending holds well to the proportional limit, a wood I-beam of a certain size and form, for example, may have an elastic limit stress 30 percent less than a solid rectangular beam made of the same material. Such an I-beam would have a proportional limit form factor,  $F_E$ , of 0.70.

Consider as a further example, a beam of square section with the diagonal vertical. In the ordinary beam formula, (1) it is seen that the bending moment sustained by such a beam, other things being equal, is directly proportional to  $I$ , the moment of inertia, and inversely proportional to  $c$ , the distance from the neutral axis to the extreme fiber in compression. It happens that the moment of inertia of a square about a neutral axis perpendicular to its sides is the same as the moment of inertia about a diagonal. The distance  $c$ , from the neutral axis to the extreme fiber in compression for a beam with the diagonal vertical, however, is  $\sqrt{2}$  times as great as that for the beam with the sides vertical. If we assume the fibers fail at the same stress  $S$  in the two beams of the same material, we would expect the maximum load for the beam with sides vertical to be 1.414 times that of a beam with the diagonal vertical. Tests have shown, however, that the sustained loads are practically equal. A factor of 1.414 then must be applied to the usual formula in calculating the strength of a beam with the diagonal vertical when using stress values  $S$  determined by standard tests of square specimens. Hence such a beam of square section with the diagonal vertical has a form factor  $F$  equal to 1.414 and the formula becomes

$$M = 1.414 \frac{SI}{c} \quad (3)$$

It is thus seen that form factors may be greater or less than unity, depending on the type of section. Ignoring the form factor may result in errors of calculation of as much as 50 percent. In some cases the calculations may be over-conservative, in others over-optimistic.

How can such factors influencing strength be accounted for? Obviously, the fiber stress at proportional limit and the modulus of rupture are not a constant for a given piece of wood, but vary with the form and shape of the piece.

In analyzing the form factor effect it is necessary to consider the structure and characteristics of timber. The strength of wood in tension and compression parallel to the grain is very different, the tensile strength ranging from two to four times the compressive strength. When a wood beam of normal wood fails it gives way first at the surface

on the compression side and these fibers lose some of their ability to sustain load. The adjacent fibers receive a greater stress and with this redistribution of stress the neutral axis moves toward the tension side and shortens the arm of the internal resisting couple, giving an increasing stress in tension. This process continues until tension failure occurs.

It is well established by test that the compressive stress in the outer fibers of a wooden beam both at proportional limit and ultimate are definitely higher than the compressive stresses in a piece subjected to longitudinal compression uniformly distributed over the cross section. Actually, survey of the data from tests in both the green and dry conditions on some 160 woods, including hardwood and coniferous species, shows that in every instance the computed stress at proportional limit in bending exceeds the ultimate in longitudinal compression.

Many theories have been advanced to account for this phenomenon. The most prominent explanation is that the fiber stresses and strains are not proportional to the distances from the neutral axis, but this is not verified by the special studies at the Forest Products Laboratory. These studies, rather, have not only indicated that the stresses within the proportional limit are very nearly proportional to the distances from the neutral axis, but also have shown that a greater fiber stress parallel to the grain is actually developed in a beam than in a compression specimen.

Newlin and Trayer account for this ability of wood to take greater compressive stress in a beam by the assumption that the minute wood fibers, say one-eighth of an inch long and one eight-hundredth of an inch wide, act as miniature Euler columns more or less bound together. These fibers, when all uniformly stressed in compression offer little support to one another, but when the stress is nonuniformly distributed as in a bent beam the fibers near the neutral axis, being less stressed, will not buckle, and will therefore lend lateral support to the extreme fibers causing them to take a higher load. The study of form factors thus becomes one of evaluating the supporting action under different conditions.

In considering this explanation of the supporting action of wood fibers it would appear that it is not the individual fibers, as such, that are free to act as miniature Euler columns, since these are cemented together by lignin. The action, rather, relates to the buckling of the cell walls, and the twisting of the column representing the common junction of several fibers.

Whatever the exact nature of the supporting action, it appears reasonable in all instances that the least stressed fibers are free to lend the greatest supporting action, and that from the standpoint of position, the ability of fibers to lend supporting action decreases as their distance from the most highly stressed compression fibers is increased.

### Form Factor (Height Factor) of Rectangular Beam

When the height of a beam of rectangular cross section is increased, the modulus of rupture is decreased. Such a beam has a form factor,  $F$ , less than unity. For rectangular beams, the form factor may be expressed by the following formula:

$$F = 1 - 0.07(\sqrt{\frac{d}{2}} - 1), \quad (4)$$

where  $d$  is the depth of the beam. For specimens 2 inches deep, as used in standard tests, it may be noted that this formula gives a form factor of unity. For a beam 8 inches thick, the form factor  $F$  becomes 0.93; and for a beam 1 inch thick, the form factor is 1.02.

This variation of unit strength with depth of beam is in harmony with the supporting action theory, since the deeper the beam the less rapidly does the stress decrease from the outer side toward the neutral surface and consequently the less able is the material adjacent to the outer surface to support that at this surface.

### Form Factor of a Circular Section

For a circular section the form factor,  $F$ , has been found by a series of tests to be 1.18. In comparing a beam of circular section with one of square section, it is found that for the same area, the section modulus,  $I/c$ , of the square is approximately 118 percent of that for the circle. The modulus of rupture of the beams of circular section, as calculated by the usual formula, was found to be about 115 percent of that of the matched beams of 2-inch square section. This shows that a beam of circular section and one with a square section of equal area will sustain practically equal loads.

These facts suggest a simplified procedure for calculating the strength of timbers or posts of circular section. The strength value in bending of round timbers of any species may be considered as identical with that of square timbers of the same grade and cross-sectional area. Tapered timbers should be assumed as of uniform diameter, the point of measurement being one-third the span from the small end, but in no case should the diameter at this point be assumed to be more than one and one-half times the small end diameter.

Likewise, the strength of round columns may be assumed to be equal to that of square columns of equal cross-sectional area. For long, tapered columns, the cross section of the equivalent square column should be taken as equal to the cross-sectional area of the round timber measured at a point one-third of its length from the small end. The stress at the small end must not exceed the allowable stress for short columns.

## I-Beams and Box Beams

Beams and girders of I and box section are of little importance in general wood construction, and are mainly of value for specialized uses such as airplane design. Such shapes place a large proportion of the material at a distance from the neutral axis, thereby increasing the moment of inertia, which theoretically should materially increase the resisting moment. While this makes for efficient practice with metal, the net gain in bending strength with wood is small because of the reduction in extreme fiber stress, or in other words, because of a fractional form factor. Although with wood the increase in bending strength for a given cross-sectional area is thus not very important, other advantages, such as increase in overall width by using I and box sections, are realized, materially increasing the resistance of the beam to lateral buckling.

In an I-beam, only the fibers in a width equal to the width of the web get the complete supporting action that obtains in a solid beam. The supporting action for the fibers outside the web is limited to the depth of the compression flange. Even a casual inspection of such sections on the basis of the supporting action theory would lead one to expect fractional form factors.

It is difficult to evaluate the exact amount of the supporting action given the extreme fibers in compression in developing a means of determining the form factor. An empirical curve was eventually worked out to appraise the amount of supporting action for I and box sections, taking into account the ability of fibers to lend support by virtue of their (a) condition of stress and (b) distance from the extreme compressive fibers. The method established checked actual test results with exceptional accuracy.

The form factor for fiber stress at proportional limit,  $F_E$ , may be determined by substituting in the following formula:<sup>1</sup>

$$F_E = 0.58 + 0.42 \left( \frac{K(t_2 - t_1)}{t_2} + \frac{t_1}{t_2} \right) \quad (5)$$

in which  $F_E$  represents the form factor within the proportional limit;  $K$ , a coefficient depending on the ratio of flange depth to total depth of beam;  $t_1$ , the thickness of the web of an I-beam, or the combined thickness of the two webs of a box beam; and  $t_2$ , the overall width of beam.

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<sup>1</sup>An algebraic expression for determining the form factor for stress at proportional limit,  $F_E$ , has been derived by T. R. C. Wilson. This expression, which gives values nearly identical to those found with the method described above (formula 5) is as follows:

$$F_E = 0.60 + 0.40 \left[ r^2(6 - 8r + 3r^2) \frac{t_2 - t_1}{t_2} + \frac{t_1}{t_2} \right]$$

where  $r$  is the ratio of the thickness of the compression flange to the total depth of beam.

Values of K in the form factor formula are as follows:

Ratio of depth of compression flange to depth of beam	Values of K	Ratio of depth of compression flange to depth of beam	Values of K
0.10	0.085	0.60	0.875
.15	.155	.65	.920
.20	.230	.70	.950
.25	.315	.75	.970
.30	.400	.80	.985
.35	.490	.85	.995
.40	.575	.90	.998
.45	.660	.95	1.000
.50	.740	1.00	1.000
.55	.810		

The form factor for modulus of rupture,  $F_u$ , may be obtained by substituting in the following formula:

$$F_u = 0.50 + 0.50 \left( \frac{K(t_2 - t_1)}{t_2} + \frac{t_1}{t_2} \right) \quad (6)$$

in which the values of K are as given in the preceding table, and the legend is the same as for the formula for determining the proportional limit form factor (5).

#### Horizontal Shear in Beams

The calculations of horizontal shear in beams is another important problem where the use of the usual formula, without correction, may give misleading results with timber. While the error is on the side of safety, the result is often such as to make economical design difficult, or to provide larger sizes than necessary. In certain uses, for example, such as floor beams of highway bridges and railway ties, usual design methods predict stresses that are two or three times the ultimate shearing strength of the material, yet these members are able to carry their loads without failure.

The reason for these discrepancies is that, because of the shear distortion in the vicinity of the base of checks or fissures that are present in practically all large beams, the upper and lower portions of a beam act partly as two independent beams. The result is that part

of the end reaction or vertical shear is resisted internally by each half of the beam independently, and consequently is not associated with the shearing stress at the neutral plane, and part of the end reaction is taken by the beam as a whole in accordance with the usual assumption.

An extensive series of tests, supplemented by theoretical consideration and analysis, was made to develop methods of evaluating the extent of the two-beam action, and to work out design methods. This study led to the following recommendations for design:

1. Use the ordinary shear formula and the working stresses recommended for timber.

2. In calculating the reactions for use in the ordinary shear formula, (a) neglect all loads within the height of the beam from both supports; (b) place the heavy concentrated moving load at three times the height of the beam from the support; and (c) then treat all loads in the usual manner.

3. If a timber does not qualify under the foregoing recommendations, which under certain conditions may be over-conservative, the reactions for the concentrated loads should be checked with the following more precise equation:

$$R' = \frac{10P (L - x) \left(\frac{x}{h}\right)^2}{9L \left[2 + \left(\frac{x}{h}\right)^2\right]} \quad (7)$$

in which  $R'$  is the reaction to be used as due to the load  $P$ ;  $L$  the span in inches;  $x$  the distance in inches from the reaction to the load  $P$ ; and  $h$ , the height of the beam in inches.

If the load is a rolling or moving one the value of  $x$  for use in formula (7) may be found from the following equation:

$$Z^3 + 6Z = 4\mathcal{L} \quad (8)$$

where  $Z = x/h$  and  $\mathcal{L} = L/h$ .  $Z$  can be most conveniently evaluated from (8) by successive trials, knowing that if  $\mathcal{L}$  is between 12 and 21,  $Z$  will be between 3 and 4.

The above formula (7) for determining the reactions to be used in the ordinary shear formula for rectangular beams in conjunction with previously published shear stresses includes a 10/9 factor to compensate for the fact that there was approximately 10 percent of two-beam action in the test beams from which the safe shear stresses were derived.

As an example of the significance of the modification of the method of calculating shear, the following example will be of interest.

Consider a 5 by 16-inch timber with a 16-foot span, and of a grade having a unit shear stress of 100 pounds per square inch. Let us investigate the maximum single moving load this timber would accommodate by means of the various formulas.

1. By the regular shear formula, ignoring double-beam action, and placing the load adjacent to the support for maximum shear, the maximum load  $P$  is found to be 5,330 pounds.

2. By the recommended procedure of locating the concentrated load three times the depth of the beam from the support, the maximum load  $P_1$  is found to be 7,110 pounds.

3. By using the more exact method involved in formula (7) the maximum load  $P_2$  is found to be 7,820 pounds.

It may thus be seen that the newer methods available for calculating strength are of great importance in the proper design and use of timber.

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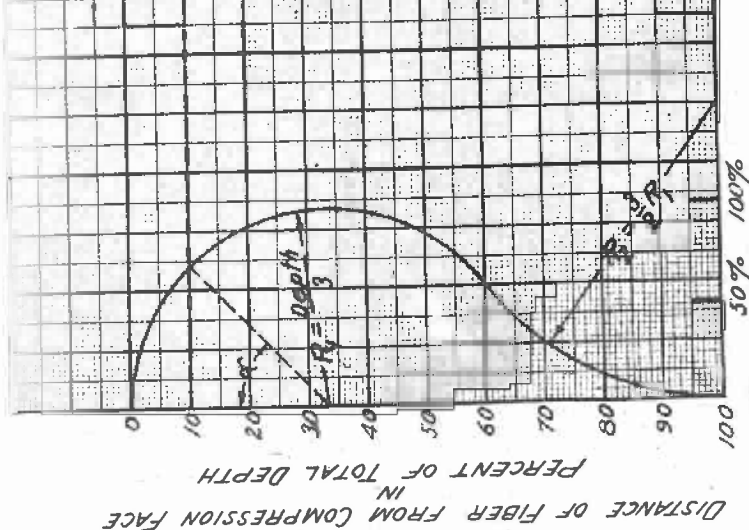
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Figure 1

The curves evaluate the amount of supporting action contributed by the less stressed compressive fibers of a beam to the higher stressed fibers. The support that can be rendered is dependent on the (1) amount of stress already carried by the supporting fibers and (2) the distance of the supporting fibers from the higher stressed fibers.

CURVE A

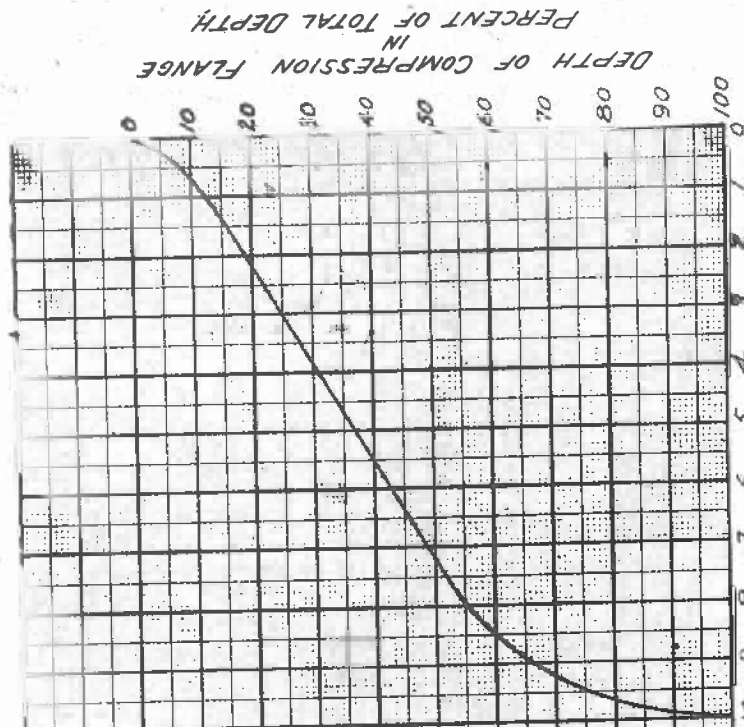
EMPIRICAL RELATION OF POSITION OF FIBER  
TO ITS SUPPORTING INFLUENCE  
AREA REPRESENTS SUPPORTING ABILITY OF  
RECTANGULAR BEAM



RELATIVE SUPPORTING INFLUENCE  
ON  
EXTREME FIBER IN COMPRESSION

CURVE B

RELATION OF DEPTH OF COMPRESSION FLANGE IN PER CENT OF  
TOTAL DEPTH TO SUPPORTING RATIO K IN FORMULA BELOW  
K = RATIO OF CURVE A AREA ABOVE HORIZONTAL REPRESENTING  
GIVEN FLANGE-DEPTH RATIO TO TOTAL CURVE A AREA



SUPPORTING RATIO  $K = 0.293 \left( \frac{L}{S} - \cos \alpha \sin \alpha \right)$   
IN FORMULA

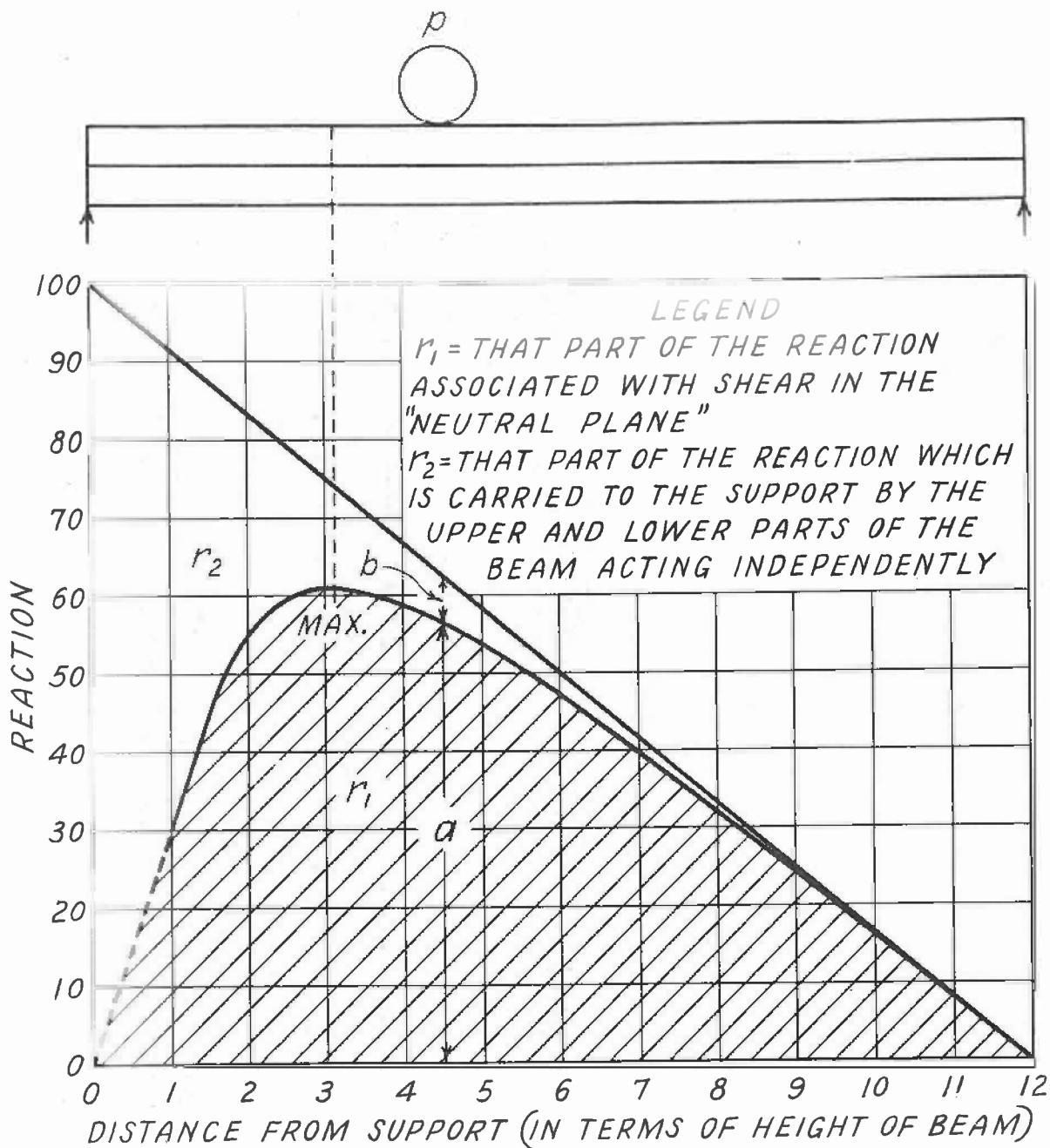
$$F_e = 0.58 + 0.42 \left[ 0.293 \left( \frac{L}{S} - \cos \alpha \sin \alpha \right) \left( \frac{t_1}{t_2} + \frac{t_2}{t_1} \right) \right]$$

WHERE  $F_e$  = STRESS FACTOR AT ELASTIC LIMIT

CURVES SHOWING EMPIRICAL RELATIONS IN STRESS FACTOR THEORY  
BASED ON ASSUMPTION THAT  
LESS STRESSED FIBERS LEND SUPPORT TO HIGHER STRESSED FIBERS

Figure 2

Diagram showing the relative proportion of the end reaction of a simple wooden beam (1) associated with shear in the neutral plane and (2) associated with double-beam action, in which the upper and lower parts act independently. The double-beam action in checked beams in effect relieves the stress along the neutral plane, and safely permits the use of higher loading without danger of shear failure than is indicated by the usual shear formula.



REACTION DIAGRAM  
FOR A  
SPAN HEIGHT RATIO OF 12

ZM22620F