An Analysis Technique for Testing Log Grades

Carl A. Newport and William G. O'Regan

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13. ABSTRACT *(Maximum 200 words)*

Workable log grading systems are needed as aids to marketing timber and other phases of forest management. How can one system be evaluated and compared with other systems? The question of comparisons arose soon after the Western Pine Section of the Forest Service's National Log Grade Project was established in 1958. The analytical technique described in this report was developed as an objective, statistically sound method of evaluating log-grading systems. It has been used as the major analytical procedure in developing an advanced system for grading ponderosa pine and sugar pine saw logs in trees. Computations were done first on an IBM 701 computer. These operations demonstrated the usefulness of the technique and confirmed some anticipated limitations.

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Contents

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William G. O'Regan has been mathematical statistician of the Pacific Southwest Station since 1957. He earned both his B.S. (in 1949) and Ph.D. (in 1962) degrees at the University of California, Berkeley. He studied at the London School of Economics under a Fulbright grant, 1951-52, and was a postdoctoral fellow at Harvard University, 1962-63.

Workable log grading systems are needed as aids to marketing timber and other phases of forest management. How can one system be evaluated and compared with other systems? The question of comparisons arose soon after the Western Pine Section of the Forest Service's National Log Grade Project was established in 1958.

The analytical technique described in this report[1] was developed as an objective, statistically sound method of evaluating log-grading systems. It has been used as the major analytical procedure in developing an advanced system for grading ponderosa pine (*Pinus ponderosa* Dougl.) and sugar pine (*Pinus lambertiana* Dougl.) saw logs in trees (Gaines 1962). Computations were done first on an IBM 701 computer and later reprogrammed and completed on an IBM 704 computer. These operations demonstrated the usefulness of the technique and confirmed some anticipated limitations.

The Western Pine Log Grade Project later began developing other procedures for evaluating log grading systems. It appears that the technique described in this report will in time be superseded by other methods that have the advantages of greater flexibility, broader application, and easier interpretation.

This report is published to make available the full description of a technique that has proved useful in a major log grade development project.[2] It briefly explains the technique, includes an example of its application, and provides details of the statistical analysis. Log grade researchers possibly may wish to use the technique, at least to compare with alternative methods.

Objectives of Log Grade Testing

A log grading system is a set of specifications used to segregate a given lot of logs into two or more grades. The specifications are drawn up in many ways, but generally include limitations of knot size, knot character, clear surface area, and other outward log characteristics. These characteristics are difficult to define and combine in numerical expression. Consequently, most log grading systems are developed from a more or less arbitrary selection of specifications for the grades within the system. In a few instances, a correlation analysis is used to isolate the significant characteristics controlling quality.

Regardless of the manner of selecting the grade specifications—arbitrary or by correlation analysis—they need testing to determine the effectiveness of stratification. Therefore, in any analysis of log grade testing, we must answer four questions:

1. Does the grading system separate the logs into grades that are distinctly different in average value of end products?
2. Is the variability around the average value for each log grade reasonably small?
3. If the differences in value are not significant, can it be shown that the end-product grade yields are significantly different?
4. If segregation of value is satisfactory, does the system predict gross value within reasonable limits of error?

A log grading system is satisfactory if the answers to the first three questions are "yes." The last question requires the application of the proposed

[1] The authors gratefully acknowledge the assistance of Edward M. Gaines in updating and revising this report.

[2] Brief summaries of the technique are given and Gaines (1962).

system to a sample of logs drawn from the same population as the logs used to develop and test the specifications. The performance of each log grade in a proposed system may vary with factors other than the grade, such as sawing method and equipment and local market conditions. This question of performance calculations for log grade systems and their accuracy in application is not investigated here. We shall only say that a grading system that stratifies logs into value groups, as determined by the tests proposed in this paper, will give better prediction of value than no log grading system at all. And the system doing the best job of stratification by the following tests will do the best job of prediction.

The first three questions above are the concern of this analysis method. They have purposely been stated in a way which permits the use of statistical methods in deciding upon an answer. Statistics cannot provide the exact answer, but they can provide strong evidence in support of the answers. And statistics must be used with knowledge of their limitations when applied to the kind of data available for log grade testing.

Necessary Data

The data needed in log grade testing come from end-product recovery studies. The end products may be lumber, veneer, ties, or other products or combinations of products. Generally each end-product use class is divided into quality groups which are called log grades. For example, logs suitable for lumber may be classed as lumber logs ("saw logs"), and a log grading system can be used to divide the lumber use class into quality groups. A lumber grade recovery study is needed to test a lumber-log grading system.

Each log is graded by the system or systems to be tested. The lumber grade recovery is recorded for each log as it is sawed. Prices, either current or average, for each lumber grade are applied to the recovery data to obtain the value of each log. This, of course, is an oversimplification of the data collection job, but illustrates that the data must give the grade, volume, and value of each study log to be used in testing.

Testing Procedure

A test of any log grading system can be made by determining the answers to the first three key questions mentioned above. Briefly, the procedure we followed was to:

1. Determine the total lumber recovery and total value recovery for each log from a random sample of the complete range of logs for each log grade. Note that unit value is not used.
2. Convert these volumes and values to logarithms.
3. Compute regressions for each log grade of the form $y = a + bx + cx^2$ in which y = logarithm of value and x = logarithm of volume.
4. Test the quadratic term of this regression to determine its significance.
5. Test the regressions in their appropriate quadratic or linear form for significant differences between log grades.
6. Compute the variability around each regression that is different from every other and check this against a standard. In the example in this paper we used 7 percent as a standard. We later found that maximum acceptable variability depends upon the number of log grades and the range from lowest to highest lumber-grade price. We therefore modified the concept of a standard to that given in step 9.
7. Test for differences in the yields of key lumber grades when any two log grade value differences are not significant.
8. Choose the log grading system (if several are being tested) which has significant difference in either value or grade yields for all grades and which has lowest or nearly the lowest variability.
9. Modify the chosen system in successive steps to reduce variability. Such modification requires that complete diagrams be made of the study logs before sawing. The reduction in variability, if any, can be weighed against the increased

complexity of the grade specifications after each modification. This step would be a matter of judgment. After each modification, repeat steps 3 through 6 to determine whether variability has been reduced significantly. An acceptable standard of variability becomes one that cannot be reduced by reasonable modifications of specifications.

Analysis of the White Fir Grading System

Study Data

A lumber recovery study of old-growth white fir (*Abies concolor* Gord. & Glend. [Lindl.]) (Wise and May 1958) provided typical data for testing the application of this analysis technique. The data were conveniently available on IBM punch cards (Miller 1956). The tentative log grading system in the study had been applied to the logs at the study mill before sawing began. Specifications for the log grades are found in the appendix.

The logs milled in this study (table 1) were sawed by a single band headsaw, an edger, and a trimmer in a mill which has a capacity of about 25,000 board feet per 8-hour day. An attempt was made to produce the maximum amount of the traditionally "better" lumber grades, regardless of current market or company considerations.

The following standard Western Pine Association and average prices for white fir lumber were used:

Lumber grade	1954 net average price per M
C & Better	$149.69
D Select	135.72
Moulding & Better	115.98
Number 3 Clear	113.99
Number 1 Shop	105.08
Number 2 Shop	80.11
Number 3 Shop	61.61
Numbers 1 & 2 Common	77.72
Number 3 Common	67.24
Number 4 Common	51.97
Number 5 Common	40.16
Numbers 1 & 2 Dimension	71.31
Number 3 Dimension	56.40
Number 4 Dimension	33.31

The lumber from each log was followed through the rough-dry stage of manufacture. A sample of each rough-dry grade was surfaced to get information on change in grade. However, the values in this report have been computed using the surfaced-dry lumber prices and rough-dry volumes because of the difficulty of computation by individual logs. An electronic computer program has been developed to make this calculation easier (Newport and Leach 1959).

The log length distribution was:

	Percent of total number of logs
16-foot	60
14-foot	17
12-foot	21
10-foot	2
	100

More than 90 percent of the grade 1 logs and 80 percent of the grade 2 logs were 16 feet long. It was assumed that the shorter logs were cut for reasons other than to improve log quality.

Logs more than 50 percent defective in scaling were considered culls and excluded from this study.

Preliminary Calculations and Observations

We suspected a greater variability in defective log values than in sound log values. For this reason we divided each of the four log grades into sound and defective groups. From this point on we will be considering seven grades instead of four, because we found only one sound grade 1 log.

The variability in log value tends to increase as volume increases. Statistical analysis is based upon the requirement that variance be homogeneous. In this case, we do not want the variance to be a function of size or class, but we find that the variance around a regression of log value on lumber recovery volume for all sound logs tends to become greater as volume per log increases (fig. 1).

A logarithmic transformation of the dependent variable would tend to destroy the relationship between variance and average value of Y. We hoped a similar transformation on X would render the resulting relationship (between log Y and log X) linear. But a polynominal approximation approach was taken because of uncertainty about the shape of the logarithmic relationship. This approach called for the testing of at least the quadratic term in X.

Table 1.—Number of study logs, by top-diameter group, soundness, and log grade

Diameter group (inches)	Grade 1		Grade 2		Grade 3		Grade 4		Total
	Sound	Defective	Sound	Defective	Sound	Defective	Sound	Defective	
6-9	--	--	--	--	13	--	1	--	14
10-13	--	--	--	--	29	4	5	--	38
14-17	--	--	--	--	27	3	21	4	55
18-21	--	--	2	--	19	6	13	9	49
22-25	--	--	4	3	12	5	19	22	65
26-29	1	1	5	3	2	10	14	24	60
30-33	--	1	--	5	--	--	4	17	27
34-37	--	3	--	5	--	--	1	20	29
38-41	--	9	--	14	--	--	--	13	36
42-45	--	5	--	2	--	--	--	3	10
46-49	--	2	--	2	--	--	--	2	6
50-53	--	1	--	--	--	--	--	4	5
54-57	--	--	--	1	--	--	--	--	1
Totals	1	23	11	35	102	28	78	118	395

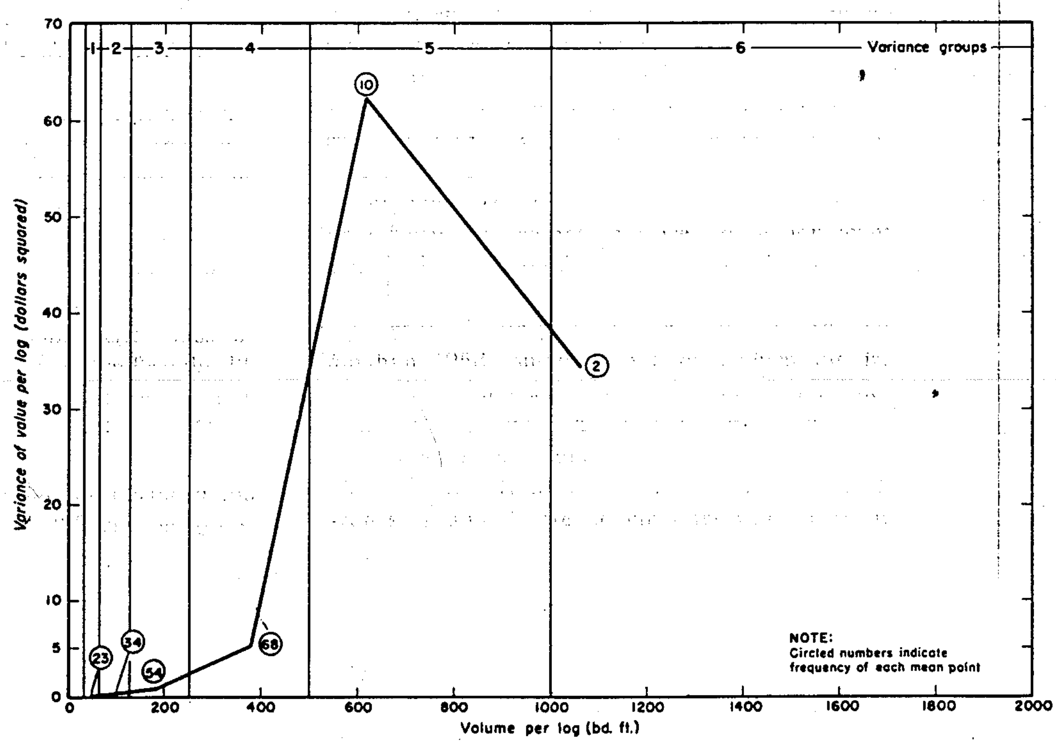

Figure 1. — Variance of value per log around a single regression for sound logs of all grades for white fir.

Results

1. Does the grading system separate the logs into grades that are distinctly different in average value of end products?

The log grade specifications were used to group the logs into four grades. In this analysis each grade was subdivided into two classes: (a) sound, if the gross scale equaled the net scale and (b) defective, if it did not. Because we found only one sound grade 1 log, our analysis was limited to seven grade-defect classes:

1_d —Grade 1 - defective
2_s —Grade 2 - sound
2_d —Grade 2 - defective
3_s —Grade 3 - sound
3_d —Grade 3 - defective
4_s —Grade 4 - sound
4_d —Grade 4 - defective

After the dollar value and lumber recovery volume for each log were converted to logarithms an equation of the general quadratic form:
$y = a + bx + cx^2$ in which y = logarithm of log value and x = logarithm of lumber recovery was computed for each grade-defect class.

The analysis to answer the above question was carried out within this model.

Tests, by defect class, led to the conclusion that the quadratic coefficient was not different from zero. The logarithmic equations therefore reduced to the linear form: $y = a + bx$. [3]

Tests were carried out be defect class because of the evident homogeneity of variance within defect class and the equally evident heterogeneity across defect class. We found that within defect class the linear regression coefficients were significantly different from each other. We also found that for both defect classes of grade 3, unit slope of Y on X was acceptable.

In answer to our first question we concluded that white fir has seven distinctively different log grade-defect classes as previously listed.

The accepted linear logarithmic regression for each of these grades has been converted to its arithmetic form:

$$Y_{1d} = \frac{X^{1.130246}}{25.714}$$

$$Y_{2s} = \frac{X^{1.146710}}{31.982}$$

$$Y_{2d} = \frac{X^{1.206757}}{49.068}$$

$$Y_{3s} = \frac{X}{14.558}$$

$$Y_{3d} = \frac{X}{15.192}$$

$$Y_{4s} = \frac{X^{1.054615}}{20.135}$$

$$Y_{4d} = \frac{X^{1.161435}}{39.773}$$

in which Y = value per log and X = volume per log.

We are also more accustomed to thinking in terms of value per unit of volume. The above equations have been used to compute the value per thousand board feet for various volumes for each log grade (table 2).[4] This information also is shown graphically in figures 2 through 6. The individual log data are plotted on these graphs.

The differences in value per thousand board feet for a given log size not only need to be significant statistically but should also be of enough real magnitude to justify grading. The report of the National Log Grade Working Group (Newport, Lockard, and Vaughan 1958) suggests that for a given log size the value per thousand for a log grade should differ from the next lower grade value per thousand by 10 percent of the higher. The log grade values in this study are given for selected log sizes in table 2. Grades 1_d and 2_s are the only ones which meet the suggested standard. It is necessary to give further consideration to the differences between grades 2_s, 2_d, 3_s, 3_d, 4_s and 4_d. This procedure will be followed through later.

[3] The quadratic terms were retained in the regressions used for testing and developing the Improved System for Grading Ponderosa and Sugar Pine Saw Logs (Gaines 1962). Some of the quadratic coefficients were significant, and we decided to retain the quadratic term in all regressions to ensure consistency in the subsequent covariance analyses and other comparisons that we made.

[4] It may be noted that the regressions of value on volume can be converted to regressions of value per thousand board feet by dividing both sides by $\frac{X}{1000}$. The arithmetic form reduces to $V = \frac{1000Y}{X} = \frac{X^{(b-1)}}{A}$ in which V is value per thousand board feet of lumber.

Table 2.—Values per thousand board feet for each log grade at selected volume of lumber recovered

Grade	Value[1] when volume of lumber recovered per log is—			
	100 bd. ft.	500 bd. ft.	1,000 bd. ft.	1,500 bd. ft.
1_d	--	$(87.37)	$ 95.60	$100.50
2_s	--	77.80	(86.20)	--
2_d	--	73.70	85.00	92.40
3_s	$68.70	68.70	(68.70)	--
3_d	65.82	65.82	(65.82)	--
4_s	63.50	69.80	72.40	--
4_d	52.50	68.60	76.70	82.00

[1] *Values in parentheses are from beyond the range of the actual study data.*

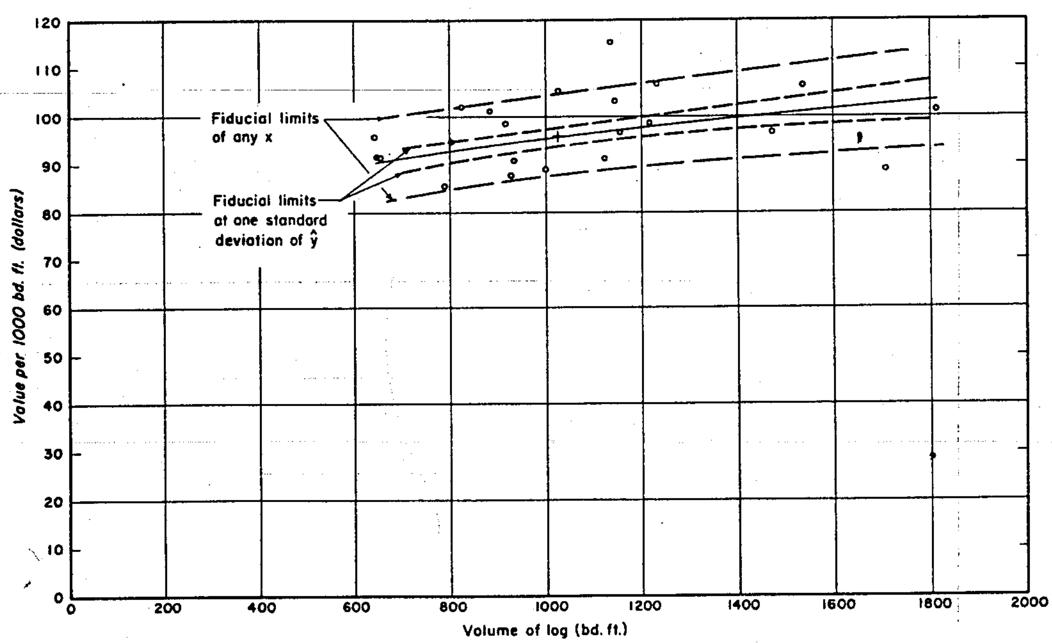

Figure 2. — Value per M board ft. for grade 1 white fir logs.

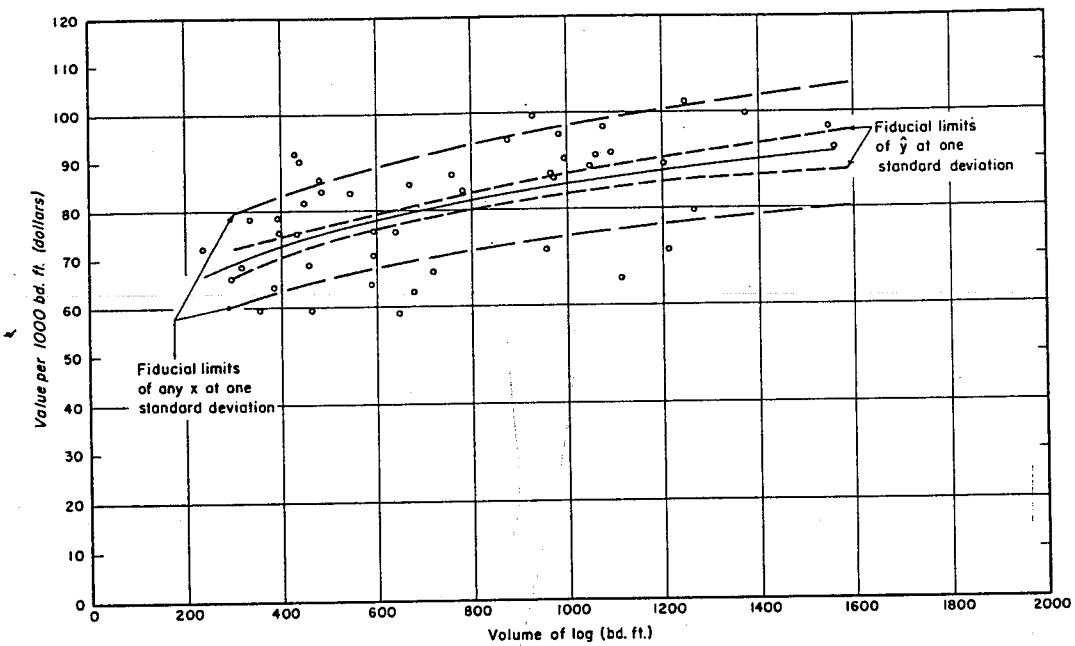

Figure 3. — Value per M board ft. for grade 2 (sound and defective combined) white fir logs.

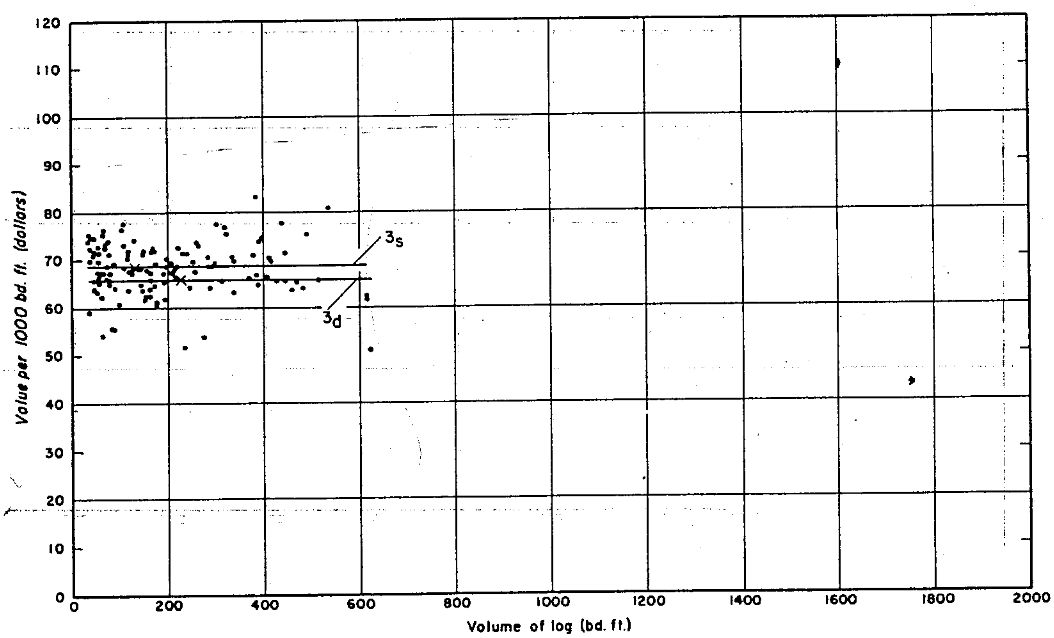

Figure 4. — Value per M for grades 3_s and 3_d white fir logs.

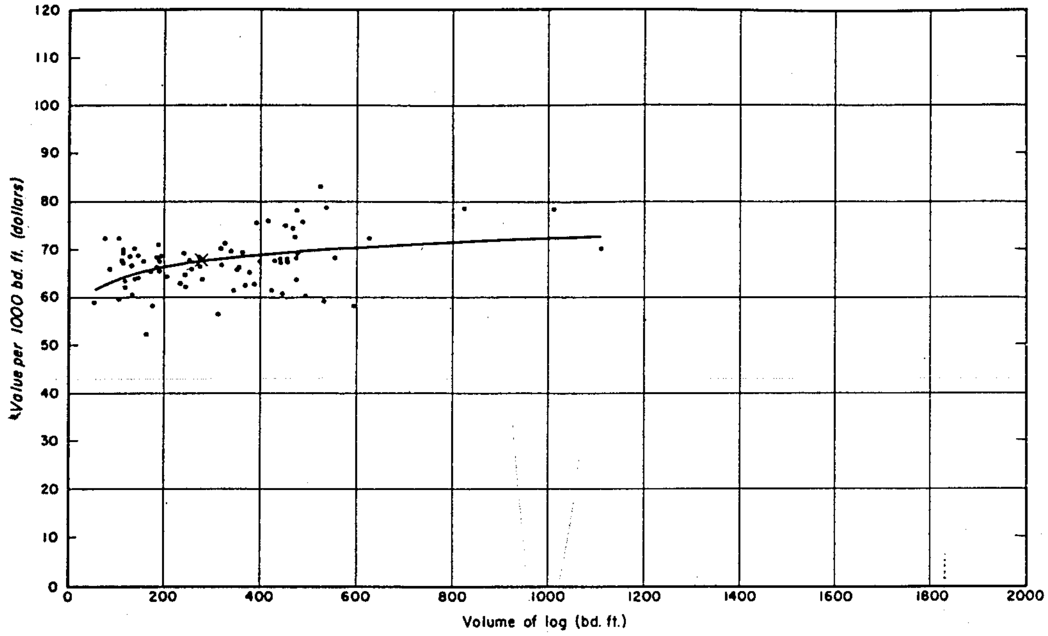

Figure 5. — Value per M for grade 4_s white fir logs.

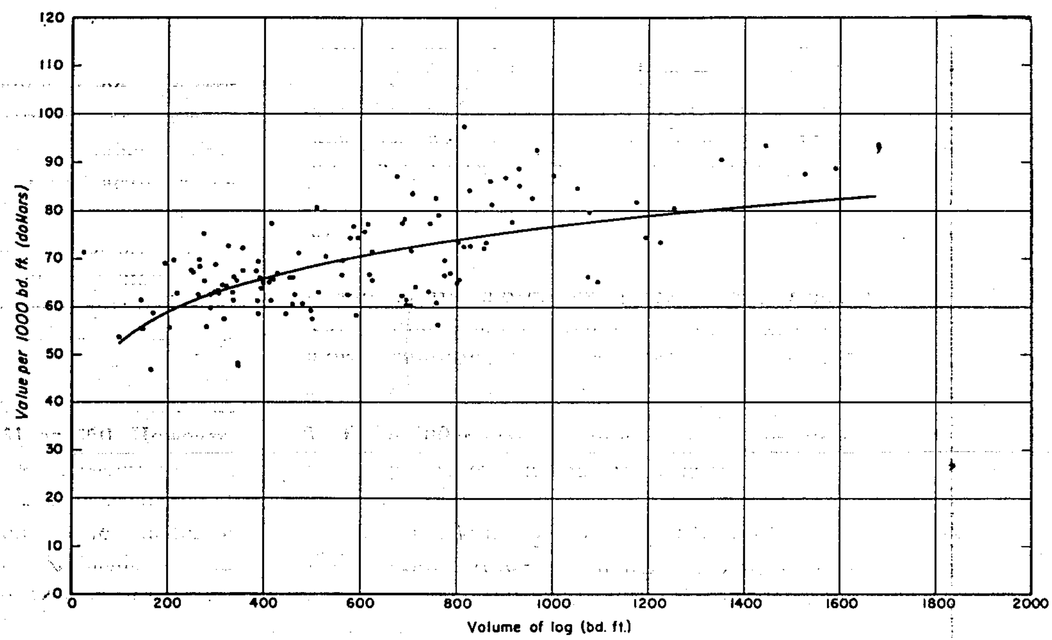

Figure 6. — Value per M for grade 4_d white fir logs.

2. Is the variability around the mean value for each log grade reasonably small?

We have already pointed out that the variability of value within a log grade tends to become greater as log size increases. But we have found that it tends to be a constant percent of the average value for all sizes within each log grade or at all levels of volume in the regression. (This tendency, in fact, is what makes the logarithmic transformation appropriate.) The square root of variance is a measure of dispersion. These are stated as a percent of regression value for each log grade:[5]

	Percent
Grade 1 — defective	8.7
Grade 2 — sound	10.0
Grade 2 — defective	15.6
Grade 3 — sound	7.6
Grade 3 — defective	11.1
Grade 4 — sound	8.6
Grade 4 — defective	12.0

A limit of 7 percent was recommended by the National Log Grade Working Group (Newport, Lockard, and Vaughn 1958). None of these variation percentages meets this standard although grade 3_s is very close.

Further investigation was made of the 7 percent variability standard. We discovered that such a standard must be accompanied by other restrictions in the analysis. An important factor in the magnitude of the percent variability is the relationship between the range and the level of the lumber grade prices used in the analysis. For example, a one hundred dollar difference from the lowest to highest lumber grade price used in an analysis will result in the same absolute variability around log grade regressions regardless of whether the lowest lumber grade price is $1 or $50. However, the percent variability would be considerably different for these two price levels. Furthermore, the percent variability is a function of the number of lumber grades and the physical relationship between them. We finally concluded that a satisfactory maximum acceptable variability cannot be set unless a rigid set of lumber prices and lumber grades is also specified. Therefore, a log grading system cannot be developed to meet a universal standard of accuracy in value estimation unless lumber grades and lumber grade prices are universally standard for all log grade analysis.

In this analysis of white fir grades we suggest that the 7 percent standard is compatible with the lumber grades and with the level and range of lumber grade prices. Obviously then, a change in lumber grades or prices might make a log grading system appear to be unsatisfactory. For this reason we recommend that our model for analysis be used with a method of measuring progressive reasonable improvement in grade specifications in terms of relative reduction in pooled variance.

Assume that reductions in pooled variance can be obtained by successive revisions of the existing log grading system which has the lowest variance. If the first major and reasonable revision reduces the variance considerably, further revisions should be tried, using generally less reasonable and/or more complex grade specifications.[6] A record of the variance after each revision should show a tendency for the pooled variance to be reduced less and less by successive revisions. A balance must then be made between specification complexity and variability. This procedure for log grade testing is valid for any lumber grades and any price range and level.

In figures 2 and 3 the fiducial limits for any x and $\hat{y}$ are shown for grade 1_d and grade 2 (sound and defective combined). The limits for x show graphically how the absolute variability increases as log size increases.

3. If the differences in value are not significant, can it be shown that the end-product grade yields are significantly different?

The value of a log is the weighted average value of the various grades of lumber it produces. Before using any set of prices in an analysis they should be examined to make certain that no two lumber prices are the same or nearly the same. Actual market prices frequently are about the same for two physically different lumber grades. For purposes of log grade analysis there is no reason why a set of market prices cannot be adjusted so that differences (say at least $5) exist between any two lumber grades. Even with this restriction two log grades

[5] The computation of these figures is as follows: The variance about the linear logarithmic regression $Y_{1d} = 1.130246X - 1.410162$ (where y = logarithm of log value and X = logarithm of log volume) is .001314. The square root of this is the logarithm .03625. The antilog of this is 1.087. A diviation of .03625 in the logarithm of the value means that the value has been multiplied (or divided) by 1.087. Therefore, we may say that one standard deviation in the logarithm corresponds to a percentage standard deviation of 8.7 percent in the value. This is the coefficient of variation.

[6] In practice, the revisions will include those which are merely different but not necessarily less reasonable or more complex.

that do not show differences in the value analysis can actually differ in lumber grade yields. But the reverse is not true. When value differences between log grades are significant there must be significant differences in lumber grade yields, even though we do not know which lumber grades differ.

The values of the lumber grades used in the analysis are shown elsewhere. Four differences between lumber grades are less than $5, but in view of the large number of grades we decided to use this set of actual average prices without adjustment.

Log grades 2_s and 2_d did not show significant difference in value per log, but the two variances are different. An inspection of the lumber grade yields indicated no differences. However, since the different variances indicate population differences, the two grades were kept.

The value analysis of grades 3_s, 3_d, 4_s, and 4_d has already shown that the values per log are significantly different. This is strong evidence that each also has different individual lumber grade yields in most size classes.

Sometimes lumber grade yields for two log grades are not as obviously different as they were in this study. If so, the following steps can be taken. Select a lumber grade whose yield in both log grades under comparison exceeds 10 percent of the total yield for most log sizes. Compute regressions of lumber tally in this lumber grade on total lumber tally for each of the two log grades. Test for significance of difference in the slopes and levels of the two regressions. The test may be repeated for other lumber grades which make up 10 percent or more or the total lumber yield. If no one lumber grade makes up 10 percent of yield, then combine two or more related grades, such as No. 1 and No. 2 shop or C and D selects.

If one or more of the lumber grades or grade combinations tested have significantly different yields, then the log grades should be considered sufficiently different. In essence, this means that some set of lumber price data other than the set used in the value analysis would give significant differences between these log grades. Since we cannot be absolutely certain as to the suitability of the price data for the value analysis, this lumber grade analysis is used to make certain that lumber grade differences do not exist when value differences do not.

Conclusions

The answers to the three questions put to the white fir data were not all "yes." We concluded that the grading system was not satisfactory, but we did obtain some favorable answers. Therefore, we suggest the following steps for improving the specifications. After revised grades are applied, the testing technique can be repeated:

1. Defect should be considered in grading logs. Even though defect cannot be "scaled out" in the application of the grades to 16-foot logs in the standing tree, every known indicator of defect should be used to grade the logs as apparently sound or defective within each log grade.
2. All statements regarding the amount of certain lumber grades that a log in a grade must produce should be omitted. This is necessary because foresters cannot be expected to apply consistently specifications that require judgment of lumber grade recovery.
3. Log diagrams should be inspected in order to find additional factors that could be used to make a better segregation of logs into value groups. Diagrams were not made of all the logs in this study. Consequently, no attempt has been made to improve the specifications to any greater extent, although that has been shown to be the logical next step.

Statistical Analysis

We have mentioned the need for a transformation of the data to improve the form of the statistical model and to make the variance homogeneous. An analysis of the interim southern pine log grades (Southern Forest Experiment Station 1953) was based upon the assumption that the variance of log value within a log grade is proportional to the log volume (lumber tally). In that analysis homogeneous variance was obtained by weighting the observations by the inverse of log volume. In an earlier suggestion for comparing log grading systems (Forest Products Laboratory 1958), it was assumed that the variance of log value varies directly as the square of the log volume. Homogeneous variance is obtained by weighting the observations by the inverse of the square of log volume.

Study of the plotted Dinuba data suggested that the standard deviation of log value at a given log volume was proportional to the expected log value at that log volume. A linear relationship between the standard deviation of a value and the expected value of a variable indicates the need for a logarithmic transformation of the variable (Kempthorne 1952; Snedecor 1956).

Transforming log value to logarithms, however, could lead to difficulties in the form of the relationship between log value and log volume. The theoretical curvilinear relationship between log value and log volume led us to hope that a logarithmic transformation of both variables might lead to homogeneous variances about the regression line (or curve) and a simpler regression line (or curve). Therefore, logarithmic transformation of both variables was chosen. The plotted transformed data seemed to indicate a linear relationship, but some grades showed a slight indication of curvilinearity.

General Model

The basic data were transformed from arithmetic terms so that:

Y_{ijk} = logarithm of total value of the lumber recovered from the i^{th} log in the j^{th} log grade, k^{th} defect class.

X_{ijk} = logarithm of volume of lumber recovered from the i^{th} log in the j^{th} log grade, k^{th} defect class.

with

$i = 1, 2, 3 \ldots\ldots\ldots\ldots n_{jk}$

j = 1, 2, 3 and 4 corresponding to log grades 1, 2, 3 and 4.

k = 1 for logs with no scaling deductions (sound) and 2 for logs with scaling deductions (defective).

The general quadratic regression model is as follows:

$$Y_{ijk} = \mu + \lambda_1 G_{1ijk} + \lambda_2 G_{2ijk} + \lambda_3 G_{3ijk} + \lambda_4 G_{4ijk} + \delta_1 D_{1ijk} + \delta_2 D_{2ijk} + \beta_{jk} X_{ijk} + \theta_{jk} X^2_{ijk} + \epsilon_{ijk}$$

in which

Y_{ijk}, X_{ijk} are defined as before

μ = general effect

G_{1ijk} = 1 for all Y_{ijk} with j = 1, and = 0 for all other Y_{ijk}.[7]

G_{2ijk} = 1 for all Y_{ijk} with j = 2, and = 0 for all other Y_{ijk}.

G_{3ijk} = 1 for all Y_{ijk}, with j = 3, and = 0 for all other Y_{ijk}.

G_{4ijk} = 1 for all Y_{ijk} with j = 4, and = 0 for all other Y_{ijk}.

λ_j = constant effect of j^{th} grade.

D_{1ijk} = 1 for all Y_{ijk} with k = 1, and = 0 for all other Y_{ijk}.

D_{2ijk} = 1 for all Y_{ijk} with k = 2, and = 0 for all other Y_{ijk}.

[7] The sample contained no sound grade 1 logs. The necessary modification of the model is obvious.

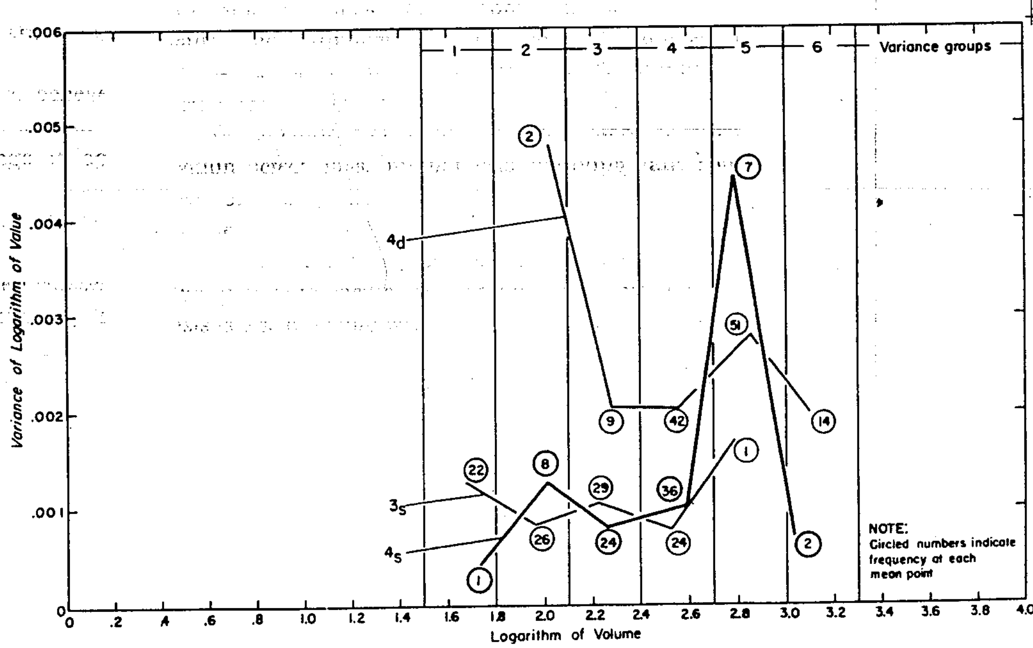

Figure 7. — Variance of logarithm of value around linear regressions for grades 3_s, 4_s, and 4_d white fir logs.

δ_k = constant effect of k^{th} defect class.[8]

β_{jk} = linear regression coefficient for j^{th} grade, k^{th} defect class.

θ_{jk} = quadratic regression coefficient for j^{th} grade, k^{th} defect class.

ϵ_{ijk} = random component.

The G_{pijk} and the D_{1ijk} are dummy variables which, along with the λ_j and δ_k, modify the μ so that each grade defect class has a potentially different intercept. The β_{jk} and θ_{jk} allow for different quadratic curves for each grade-defect class.

We make the following additional assumption: $\epsilon_{ijk} = NID\ (o, \sigma^2)$. That is, that following the transformation we have random elements that are normally and independently distributed, with mean zero and constant variance.

In general, we would like to decide whether the regression of Y on X is quadratic, linear, or nonexistent. Given the degree of the regression relationship, we would like to decide whether there is a single regression of Y on X (that is, grades and defect classes are not different at all); whether there are parallel regressions; *or whether there are separate* regressions for at least some classes.

The usual tests of these hypotheses depend, for their strict validity, upon the homogeneous variance assumption, which could break down in one or both of two cases:

1. The variance is variable within grade-defect class. That is, the dependent variable does not have constant variance about the regression curve, within class.
2. Or the variance between classes is not constant, even though it is constant within class.

Visual inspection of figure 7 leads us to believe that the logarithmic transformation apparently does "homogenize" the variance over the range of log sizes within a given grade-defect class.

Given that each grade-defect class has the variance in logarithm of log value independent of the logarithm of log volume, do the grade-defect classes have homogeneous variance? Or, for example, is the variance about the regression curve in sound logs of grade 2 the same as that for defective logs in grade 1? Bartlett's test (Snedecor 1956) indicated that over all the variance was heterogeneous (chi square = 36.25, with 6 degrees of freedom). However, when Bartlett's test was applied separately to the defective group and then to the nondefective group, nonsignificant chi squares resulted. We then conclude that within the two defect classes we have homogeneous variance about the regression curves and between grade classes. This type of variance will allow us to make the usual tests of regression coefficients within defect classes. But we would also like to test hypotheses about regression coefficients within grade classes, but over defect classes. For instance, given that grade 2 logs in both defect classes have linear-in-logarithm regressions of log value on log volume, are the regression lines identical, parallel, or intersecting? If only two variances are involved, a test of their homogeneity can be achieved by use of the F ratio. Furthermore, experience tells us that defective logs are more variable than sound logs, so that we can reduce the F test to a 1-tailed test by taking the ratio of estimated variance, defective logs, overestimated variance, sound logs, and using the upper end of the F distribution for our region of rejection.

	Defect Class 1		Defect Class 2		
Grade	Est. var.	df	Est. var.	df	Var. ratio
2	.001729	8	.003974	32	2.298
3	.001026	99	.002177	25	2.122
4	.001257	75	.002326	115	1.850

These ratios are large enough to indicate that tests across defect classes within grade, if made at all, must be considered as approximations. "Typically, the ordinary t - test, incorrectly applied if $\sigma_1 \neq \sigma_2$, causes more than the expected number of rejections if Ho is true" (Snedecor 1956, p.98).

We conclude that hypotheses can safely be tested within defect class, but that tests involving data from two or more defect classes—if made at all—should be viewed with some reservations.

We first tested, by defect class, the hypothesis that the quadratic coefficients are all zero. This hypothesis is tested as follows:

Source of variation	df	Sum of squares	Mean square
Three individual quadratic regressions (k-1)	182	.209666	.001152
Reduction due to quadratic regressions	6	16.930394	

[8]As defined the λ's and δ's are not linearly independent. Linear restrictions

$$\sum_{j=1}^{4} n_j \lambda_j = 0$$

$$\sum_{k=1}^{2} n_k \delta_k = 0$$

suffice to remove this difficulty.

Reduction due to linear regressions	3	16.924516	
Additional reduction due to quadratic regressions	3	.005878	.001959

$$F = \frac{.001959}{.001152} = 1.70 \; N.S.$$

Four individual quadratic regressions (k-2)	191	.474405	.002484
Reduction due to quadratic regressions	8	15.598224	
Reduction due to linear regressions	4	15.576544	
Additional reduction due to quadratic	4	.021680	.005420

$$F = \frac{.005420}{.002484} = 2.18 \; N.S.$$

The acceptance of these hypotheses is equivalent to choosing the model

$$Y_{ijk} = \mu + \sum_{p=1}^{m_k} \lambda_p G_{pjk} + \sum_{l=1}^{2} \delta_l D_{ijk} + \beta_{jk} X_{ijk} + \epsilon_{ijk}$$

with variables and parameters defined as before.

The β_{jk} are obviously different from zero on the evidence of the preceding table. We can ask questions about the β_{jk} (within defect class) and about the λ_p. In fact, if we are to be guided by our conclusions that the variance in defect-class 2 is larger than that in defect-class 1, we should restate the model as follows:

$$Y_{ijk} = \mu_k + \sum_{p=1}^{m_k} \lambda_{pk} G_{pjk} + \beta_{jk} X_{ijk} + \epsilon_{ijk}$$

This, in effect, gives us seven separate regressions (one for each grade-defect class).

We have concluded that $\epsilon_{ijk} \rightarrow NID\,(0_1 \sigma_k^2)$, that is, that variance is a function of defect class.

We are at liberty then to test hypothesis within defect class, but will have some reservations about any test which includes both defect classes.

Our first effort will be to reduce the number of parameters within defect class.

We first test the hypotheses that there is only one equation per defect class:

$$\lambda_{nk} = 0$$

$$\beta_{jk} = \beta_k$$

Source	df	Sum of squares	Mean square
Defect Class 1			
About single regression line	189	0.8854	
About three regression lines	185	0.2156	0.001159
Additional reduction due to individual regression lines	4	0.6698	0.167450

$$F_{4,185} = 144.48$$

Source	df	Sum of squares	Mean square
Defect Class 2			
About single regression line	201	0.7370	
About four regression lines	195	0.4961	0.002544
Additional reduction due to fitting individual regression lines	6	0.2409	0.040150

$$F_{6,195} = 15.79$$

The results of the tests are such that we conclude that a simple regression line will not do in either defect class.

There might be some reason to suppose that the regression of the logarithm of log value on the logarithm of log volume has a slope of unity. We test, then, by defect class this hypotheses:

$$\beta_{jk} = 1 \text{ for both } k$$

Source	df	Sum of squares	Mean square
Defect Class 1			
About parallel regression lines $\beta_{j1} = 1$	188	0.2361	
About individual regression lines	185	0.2156	0.00116
Additional reduction due to fitting individual regression lines	3	0.0206	0.00687

$$F_{3,185} = 5.92$$

		Defect Class 2	
About parallel regression lines $\beta_{j2} = 1$	199	0.7545	
About individual regression lines	195	0.4961	0.00254
Additional reduction due to fitting individual regression lines	4	0.2581	0.06453

$$F_{4,195} = 25.41$$

The magnitude of the F ratios is such that we conclude that separate, parallel regression lines of unit slope fit neither defect class very well. It might be, however, that separate parallel regressions not limited to unit slope might fit one or both sets of data. We test then

$$\beta_{jk} = \beta_k \quad \text{for both k.}$$

Source	df	Sum of squares	Mean square
		Defect Class 1	
About parallel regression lines $\beta_{j1} = \beta$	187	0.2327	
About individual regression lines	185	0.2156	0.00116
Additional reduction due to fitting individual regression lines	2	0.0171	0.00855

$$F_{2,185} = 7.37$$

		Defect Class 2	
About parallel regression lines $\beta_{j2} = \beta_j$	198	0.5373	
About individual regression lines	195	0.4961	0.00254
Additional reduction due to fitting individual regression lines	3	0.0412	0.01380

$$F_{3,195} = 5.433$$

The F ratios are such that we conclude that in each defect group there are at least two grades whose regression slopes are not equal.

We have, now, selected the following form of the model:

$$Y_{ijk} = \mu_{jk} + \beta_{jk} X_{ijk} + \epsilon_{ijk}$$

$$\epsilon_{ijk} \rightarrow NID\,(0, \sigma_k^2)$$

That is, we have seven separate regression lines of logarithm of log value on logarithm of log volume, with variance about the regression line constant for grades within defect classes, but different over defect classes.

Further simple F tests, indicated that for grade 3, both defect classes, the slope of value on volume was not different from unity.

The final regressions for each of the seven grade-defect classes are presented in arithmetic form elsewhere.

Summary

An analytical technique that may be used in evaluating log-grading systems is described. It provides answers to these questions:

1. Does the system separate logs into grades that are distinctly different in average value of end product?
2. Is the variation around average value for each grade reasonably small?
3. If value differences are not significant, are the end-product grade yields distinctly different?

The technique also provides means of comparing two or more grading systems, or a proposed change with the system from which it was developed, in terms of the same three questions.

The total volume and computed value of lumber from each sample log are the basic data used. From them we computed quadratic logarithmic regressions of value on volume within each log grade. We used covariance techniques to evaluate the significance of differences between log grades within a system, and between grading systems. We computed and compared estimated values from the regressions to evaluate further the effectiveness of a system in segregating logs into distinct value groups.

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Appendix

Specifications for Log Grades

Grade 1 - Select. — Logs are at least 90 percent surface clear, and straight and generally smooth in appearance. Spiral grain does not exceed 1 in 5. Any one of the following is admitted:

1. One knot in the central zone larger than 3 inches in diameter.
2. Two scattered knots in the central zone less than 3 inches in diameter or four scattered pin knots.
3. Any number of knots of any size within a foot of one end.
4. Concentrated grouping of knots of any size or other defect or blemish affecting not more than one-fourth of the circumference for a length of 6 feet from one end (logs having fire scars covering a larger area should be graded as having been long butted).
5. A line of knots less than 3 inches in diameter for the full length of the log (one larger knot permitted) that affects a strip of the circumference not wider than three-tenths of the log diameter inside bark at the small end. A straight-grained frost crack would be permitted in an otherwise high grade log.

Grade 2 - Shop. — Logs are 50 percent surface clear in length or circumference. Also included are shop-type logs upon which the blemishes and knots are so distributed as to produce factory cuttings. On such shop-type logs, 50 percent or more of the surface should be in clear areas, at least 8 feet long, and 6 inches or more wide between knots and blemishes.

Grade 3 - High Common. — Logs are less than 50 percent surface clear, having any combination of knots or blemishes which are not permitted on the higher grades. Any number of knots not over 3 inches diameter inside bark are allowed.

Grade 4 - Low Common. — Logs do not qualify for grades 1, 2, or 3, but are considered merchantable by Forest Service regional standards.

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Newport, Carl A., and O'Regan, William G.
1963. An analysis technique for testing log grades.
Berkeley, Calif., Pacific SW. Forest & Range Expt. Sta. pp., illus. (U.S. Forest Serv. Res. Paper PSW-P3)

An analytical technique that may be used in evaluating log-grading systems is described. It also provides means of comparing two or more grading systems, or a proposed change with the system from which it was developed. The total volume and computed value of lumber from each sample log are the basic data used.

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