AN ABSTRACT OF THE THESIS OF

ROBERT W. FOOTE for the degree of MASTER OF SCIENCE

in GEOPHYSICS presented on AUGUST 9, 1985

Title: CURIE-POINT ISOTHERM MAPPING AND INTERPRETATION FROM AEROMAGNETIC MEASUREMENTS IN THE NORTHERN OREGON CASCADES

Redacted for Privacy

Abstract approved: Richard W. Couch

During the summer and fall of 1982, personnel from the Geophysics Group in the School of Oceanography at Oregon State University conducted an aeromagnetic survey in the northern Oregon Cascades to assess geothermal potential and study the thermal evolution of the Cascade volcanic arc.

Total field and low-pass filtered magnetic anomaly maps obtained from the survey data show high amplitude positive and negative anomalies associated with volcanic cones and shallow source bodies along the axis of the High Cascades. Spectral analysis of the aeromagnetic data yielded source depths and depths-to-the-bottom of the magnetic sources. The magnetic source bottom, in the northern Oregon Cascades, is interpreted as the depth to the Curie-point isotherm.

The northern Oregon study area shows shallow Curie-point isotherm depths of 5 to 9 km below sea level (BSL) beneath the axis of the High Cascades from the southern boundary (44° N latitude) to near Mt. Wilson (45° N latitude). A smaller region of shallow Curie-point depths of 6 to 9 km BSL lies west of Mt. Wilson (45° N latitude,
122° W longitude). The shallow Curie-point isotherm suggests the emplacement of relatively recent intrusive bodies in the upper crust beneath the axis of the High Cascades and west of Mt. Wilson.

A major northeast trending structure observed in magnetic and residual gravity anomalies near Mt. Wilson, is the northernmost extent of shallow Curie-point depths and high geothermal gradients mapped in the northern Oregon Cascades. This northeast trending structure appears to mark a division between high intrusive activity in localized areas south of Mt. Wilson and intrusive activity confined beneath the major cones north of Mt. Wilson.
Curie-Point Isotherm Mapping and Interpretation from Aeromagnetic Measurements in the Northern Oregon Cascades

by

Robert W. Foote

A THESIS submitted to Oregon State University in partial fulfillment of the requirements for the degree of Master of Science

Completed on August 9, 1985
Commencement June 1986
ACKNOWLEDGEMENTS

This dissertation marks the end of an effort that began one spring day, 1980, in northcentral Finland. I was entered in a local 10 km cross-country ski race and halfway out during the race found the wax to have changed from Rex green to Swix yellow. I was never good at waxing for marginal conditions and shortly found myself being passed on the downhills by little kids. After eight years of competitive ski racing, I decided a career change was in order.

My sincere thanks to Dr. Bruce Nolf at Central Oregon College for teaching me to think quantitatively, solve problems in the field, and for sharing his enthusiasm for geology, geophysics and learning. Milt, Carole, Corrinne, Colleen, and Caryn Quam were the best of friends and family during my competitive years in Bend and later in school. Their support and encouragement made hectic times more bearable and my progress is a credit to them.

A deepest thanks to my major professor and friend, Dr. Richard Couch for taking a chance on a geologist and providing me the opportunity to study geophysics. His advice, encouragement and prodding, always much appreciated, developed my interest in the potential fields and digital signal processing.

Thanks to Bruce Dubendorff for freely giving his time, explaining Green's functions to me and for all those nights working on somebody's homework. Thanks to Haraldur Audunsson, Fa Dwan, and Osvaldo Sanchez for being great office partners.
My thanks to Dr. Ed Taylor, Dr. Cyrus Field, Dr. Bill Taubeneck and Dr. Bill Menke for being outstanding educators and their enthusiasm for teaching and sharing their valuable time was always much appreciated.

Sara Culley provided much needed help with the final stages of the manuscript and was a good friend during the tumultuous times of thesis writing. My thanks to Donna Moore for typing the manuscript and to Steve Troseth for drafting the figures and also for being a good friend aboard the "Yucamoros". Jeb Bowers, Michael Kelsay and Sean Shanahan all were a valuable asset in answering my endless hardware and software questions.

My parents, Wilson and Eleanor Foote, provided me the opportunity to go to school and their assistance was deeply appreciated. My thanks to the Texaco Fellowship for providing graduate financial assistance and the opportunity to work on non-funded research.

To my best friend, Jay Bowerman, my deepest thanks for the best of times we had racing, training, and making one or the other of us suffer. I would like to dedicate my work to Jay -- he never taught me how to wax for marginal conditions.
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INTRODUCTION</strong></td>
</tr>
<tr>
<td><strong>GEOLOGIC OVERVIEW OF THE STUDY AREA</strong></td>
</tr>
<tr>
<td><strong>PREVIOUS GEOPHYSICAL STUDIES</strong></td>
</tr>
<tr>
<td><strong>TOTAL FIELD MAGNETIC ANOMALY MAP</strong></td>
</tr>
<tr>
<td><strong>LOW-PASS FILTERED MAPS</strong></td>
</tr>
<tr>
<td><strong>DEPTH TO THE MAGNETIC SOURCE TOP USING SPECTRAL ANALYSIS</strong></td>
</tr>
<tr>
<td><strong>CURIE-POINT ISOTHERM MAPPING BY SPECTRAL ANALYSIS</strong></td>
</tr>
<tr>
<td><strong>SUMMARY AND DISCUSSION OF CURIE-POINT ISOTHERM DEPTHS IN THE NORTHERN OREGON CASCADES</strong></td>
</tr>
<tr>
<td><strong>REFERENCES CITED</strong></td>
</tr>
<tr>
<td><strong>APPENDICES</strong></td>
</tr>
<tr>
<td><strong>A. THE NORTHERN OREGON AEROMAGNETIC SURVEY</strong></td>
</tr>
<tr>
<td>Data Collection</td>
</tr>
<tr>
<td><strong>B. DIGITAL FILTERING TECHNIQUES</strong></td>
</tr>
<tr>
<td>PART I</td>
</tr>
<tr>
<td>PART II</td>
</tr>
<tr>
<td>PART III</td>
</tr>
<tr>
<td><strong>C. TECHNIQUES OF ANALYSIS FOR THE NORTHERN OREGON AEROMAGNETIC DATA</strong></td>
</tr>
<tr>
<td>Mean Depth to Source Calculations from the Energy Spectrum</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Generalized geology of the Pacific Northwest and the northern Oregon Cascade study area.</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Topography Cascade Range, Northern Oregon</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Total field aeromagnetic anomaly map Cascade Range, Northern Oregon</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>Total field aeromagnetic anomaly map of the Mt. Hood Area, Oregon Cascades.</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>Total field aeromagnetic anomaly map Mt. Jefferson Area, Oregon Cascades.</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>Three-dimensional representation of the total field aeromagnetic anomaly map.</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Spatial domain representation of the two-dimensional unit-sample responses for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.</td>
<td>23</td>
</tr>
<tr>
<td>8</td>
<td>Magnitude of the frequency response for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>Magnitude of the frequency response in decibels for (a) truncated ideal low-pass filter and (b) optimal linear phase filter.</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>Phase angle plotted as a function of frequency for the optimal two-dimensional FIR filters.</td>
<td>27</td>
</tr>
<tr>
<td>11</td>
<td>Low-pass filtered aeromagnetic anomaly map of the northern Oregon Cascades - 15 km wavelength cutoff.</td>
<td>28</td>
</tr>
<tr>
<td>12</td>
<td>Low-pass filtered aeromagnetic anomaly map with wavelengths shorter than 15 km removed.</td>
<td>28a</td>
</tr>
<tr>
<td>13</td>
<td>Spatial domain representation of the two-dimensional unit-sample responses for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>Magnitude of the frequency response for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.</td>
<td>31</td>
</tr>
</tbody>
</table>
Figure 15: Magnitude of the frequency response in decibels for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.

Figure 16: Low-pass filtered aeromagnetic anomaly map of the northern Oregon Cascades - 15 km wavelength cutoff.

Figure 17: Low-pass filtered aeromagnetic anomaly map with wavelengths shorter than 35 km removed.

Figure 18: Radially averaged spectrum of the south 512 x 512 grid in the northern Oregon Cascades.

Figure 19: Radially averaged spectrum of the north 512 x 512 grid in the northern Oregon Cascades.

Figure 20: Radially averaged spectrum of the south-southwest 256 x 256 grid in the northern Oregon Cascades.

Figure 21: Radially averaged spectrum of the south-south 256 x 256 grid in the northern Oregon Cascades.

Figure 22: Radially averaged spectrum of the south-southeast 256 x 256 grid in the northern Oregon Cascades.

Figure 23: Radially averaged spectrum of the southwest 256 x 256 grid in the northern Oregon Cascades.

Figure 24: Radially averaged spectrum of the south 256 x 256 grid in the northern Oregon Cascades.

Figure 25: Radially averaged spectrum of the southeast 256 x 256 grid in the northern Oregon Cascades.

Figure 26: Radially averaged spectrum of the west-central 256 x 256 grid in the northern Oregon Cascades.

Figure 27: Radially averaged spectrum of the central 256 x 256 grid in the northern Oregon Cascades.

Figure 28: Radially averaged spectrum of the east-central 256 x 256 grid in the northern Oregon Cascades.

Figure 29: Radially averaged spectrum of the northwest 256 x 256 grid in the northern Oregon Cascades.

Figure 30: Radially averaged spectrum of the north 256 x 256 grid in the northern Oregon Cascades.
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>Radially averaged spectrum of the northeast 256 x 256 grid in the northern Oregon Cascades.</td>
<td>53</td>
</tr>
<tr>
<td>32</td>
<td>Radially averaged spectrum of the north-northwest 256 x 256 grid in the northern Oregon Cascades.</td>
<td>54</td>
</tr>
<tr>
<td>33</td>
<td>Radially averaged spectrum of the north-north 256 x 256 grid in the northern Oregon Cascades.</td>
<td>55</td>
</tr>
<tr>
<td>34</td>
<td>Radially averaged spectrum of the north-northeast 256 x 256 grid in the northern Oregon Cascades.</td>
<td>56</td>
</tr>
<tr>
<td>35</td>
<td>Calculations of minimum and maximum Curie-point depths for (A) 256 x 256 subgrids that did not resolve the source bottom, (B) the south-south subgrid, (C) the south subgrid, and the west-central (WC) subgrid.</td>
<td>62</td>
</tr>
<tr>
<td>36</td>
<td>Radially averaged spectrum for the west 128 x 128 subgrid with (a) 128 estimates, and (b) 512 estimates.</td>
<td>63</td>
</tr>
<tr>
<td>37</td>
<td>Radially averaged spectrum for the east 128 subgrid appended with zeros to grid size 512 x 512.</td>
<td>64</td>
</tr>
<tr>
<td>38</td>
<td>Computed depths to the magnetic source bottom, constrained by magnetic source top depths for the west 128 x 128 and east 128 x 128 subgrids.</td>
<td>66</td>
</tr>
<tr>
<td>39</td>
<td>Depth to Curie Point Cascade Range, Northern Oregon</td>
<td>72</td>
</tr>
<tr>
<td>Appendix Figure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A 1</td>
<td>Flightline map of the northern Oregon aeromagnetic survey area.</td>
<td>83</td>
</tr>
<tr>
<td>A 2</td>
<td>Crossing errors for the northern Oregon aeromagnetic survey.</td>
<td>85</td>
</tr>
<tr>
<td>B 1.1</td>
<td>Input sequence mapped by transformation into output sequence y(m,n) (after Oppenheim and Schafer, 1975).</td>
<td>88</td>
</tr>
<tr>
<td>B 3.1</td>
<td>Tolerance specifications for one-dimensional low-pass filter (after Oppenheim and Schafer, 1975, pg. 196).</td>
<td>100</td>
</tr>
<tr>
<td>B 3.2</td>
<td>Mapping of the one-dimensional frequency to two-dimensions (after McClellen, 1973).</td>
<td>106</td>
</tr>
</tbody>
</table>
Appendix

Figure

B 3.3  Spacial domain representation of a finite-area data sequence \(x(m,n)\), padded with zeros up to size \((M-1,N-1)\) to obtain a linear convolution.  

B 3.4  Filtered output from a linear convolution with constant group delay in both the \(m\) and \(n\) directions.  

Page

109

110
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Source depths from spectral analysis of aeromagnetic anomalies.</td>
<td>57</td>
</tr>
<tr>
<td>2</td>
<td>Curie-point isotherm depths from spectral analysis of aeromagnetic data and corresponding surface heat flow and geothermal gradients.</td>
<td>68</td>
</tr>
</tbody>
</table>
BEGINNING IN 1975, THE GEOPHYSICS GROUP IN THE SCHOOL OF
OCEANOGRAPHY AT OREGON STATE UNIVERSITY, UNDER THE DIRECTION OF
DR. RICHARD COUCH, UNDERTOOK A DETAILED POTENTIAL FIELD ANALYSIS
OF THE CASCADE RANGE IN OREGON AND NORTHERN CALIFORNIA. SUPPORTED
BY A GRANT FROM THE U. S. GEOLOGICAL SURVEY EXTRAMURAL GEOTHERMAL
PROGRAM, PERSONNEL FROM THE GEOPHYSICS GROUP CONDUCTED GRAVITY AND
AEROMAGNETIC SURVEYS WITH THE INTENT TO ASSESS THE GEOTHERMAL
POTENTIAL AND MORE QUANTITATIVELY UNDERSTAND THE THERMAL EVOLUTION
OF THE CASCADE VOLCANIC ARC. THE RESULTS OF THIS STUDY, INCLUDING
THE DEVELOPMENT OF NEW ACQUISITION AND PROCESSING TECHNIQUES, ARE
SUMMARIZED IN UNITED STATES GEOLOGICAL SURVEY OPEN-FILE REPORTS
82-933 AND 82-932.

DURING THE SUMMER AND FALL OF 1982, AN AEROMAGNETIC SURVEY
WAS FLOWN OVER THE AREA OUTLINED IN FIGURE 1. THE GENERALIZED
TOPOGRAPHY OF THE STUDY AREA IS SHOWN IN FIGURE 2. THE NORTHERN
OREGON STUDY AREA EXTENDS FROM JUST NORTH OF THE THREE SISTERS
(44°15' N LATITUDE) TO NORTH OF THE COLUMBIA RIVER (45°45' N LATI-
TUE). INCLUDED IN THE STUDY AREA ARE THE TWO HIGHEST OREGON COM-
POSITE CONES, MT. HOOD (11,235 FT.) AND MT. JEFFERSON (10,497 FT.),
BOTH SITES OF RECENT VOLCANISM, THE HOLOCENE BASALTIC LAVA FLOWS NEAR
MCKENZIE AND SANTIAM PASS AND THE GEOTHERMAL HOT SPRINGS AT BREITENBUSH.
THE RECENT VOLCANISM AND SURFACE MANIFESTATION OF THIS GEOTHERMAL HEAT
Figure 1. Generalized geology of the Pacific Northwest and the northern Oregon Cascade study area.
make the northern Oregon Cascades an attractive potential site for geothermal energy development.

From previous gravity studies (Pitts, 1979; Braman, 1981; Veen, 1981; and Couch et al. 1982) ample evidence indicates the strata-volcanoes of the High Cascade part of the Oregon Cascade Range occupy a 60 km wide north-south trending graben-like structure. This "Cascade Graben" extends from the Klamath Graben in the south to the vicinity of Mt. Hood where its continuation is poorly known. Taylor (1981) suggested a chain of Pliocene age volcanoes (the Plio-Cascades) has subsided in the High Cascade part of the graben and were buried beneath the Pleistocene platform rocks of the present day High Cascades. This graben then has had an important effect on the development of the Oregon Cascade volcanic arc.

The objectives of this study are to interpret the northern Oregon aeromagnetic data to help clarify structural relationships there and to map the Curie-point isotherm. Some questions to be addressed in this study are:

1. Does the Cascade Graben continue north past Mt. Hood and across the Columbia River? Does the magnetic data support changes in crustal structure between Mt. Hood and Mt. Jefferson as does the gravity data?

2. Does the northward trend of the relatively shallow Curie-point isotherm mapped by Connard (1979) and Connard et al. (1983) south of the study area continue into the northern Oregon Cascades?

The total field aeromagnetic anomaly map shows anomalies caused by variations in magnetic susceptibility in the upper crust. High amplitude anomalies often have a high correlation with topography, but other anomalies are caused by lava flows or buried intrusives such
as dikes, plugs, sills, and other hypabyssal bodies. The polarity of
the anomalies, used with the magnetic time scale (Harland et al.
1984) and other geologic information (eg. radiometric age dating),
provides constraints on rock ages. The total field aeromagnetic
anomaly map also delineates structures that are buried by non-magnetic
sediments or overburden.

In order to resolve the deeper structures, the high frequency
or short wavelength anomalies due to topography and shallow
sources are removed using digital signal processing techniques.
These filtered magnetic anomaly maps contain only long wavelength
sources and sources due to the deeper crustal structures.

Spectral analysis techniques have been developed by Spector
(1968), Spector and Grant (1970), Shuey et al. (1977), Boler (1978),
Connard (1979), Connard et al. (1982) and McLain (1981) that estimate
the depth to the top and bottom of magnetic sources. Because the
source bottom in the Oregon High Cascades is believed to indicate
the Curie-Point isotherm depth (the temperature at which rocks lose
their ferromagnetic properties, approx. 580°C, Nagata, 1961), esti-
mates of the Curie-Point depth provide temperature gradients and data
for thermal models. These thermal models are used for assessment
of geothermal potential and can lead to a better understanding of
the evolution of the Oregon Cascade volcanic arc.
Figure 1 shows the location of the study area in relation to the major geologic provinces of the Pacific Northwest. The Cascade Range in northern Oregon is bounded on the west by the Willamette Valley and Coast Range and on the east by the Deschutes-Umatilla Plateau (Dicken, 1965). South of the study area, the Cascade Range continues from the Three Sisters volcanic center to Mt. Lassen. North of the study area, the Washington Cascades continue north to British Columbia.

The Cascade Range in Oregon can be subdivided into two physiographic provinces: (1) the Western Cascades Range and (2) the High Cascade Range (Baldwin, 1981). The distinction is not entirely geographical; a temporal and chemical inhomogeneity exists between the two mountain ranges. The Western Cascades are composed of a thick pile of silicic tuffs, lesser amounts of iron-rich basaltic to silicic lavas, calc-alkaline lavas and associated tuffs ranging in age from late Eocene to mid-late Miocene (Peck et al., 1964). The younger High Cascade rocks, from late Miocene to the present, form a platform of basaltic and basaltic andesite lavas capped by basaltic andesite to dacitic volcanoes (Taylor, 1981). This dissertation will follow the informal time-stratigraphic division put forth by Priest et al. (1983) which separates the Western and High Cascades into early and late episodes. This informal usage is an attempt to avoid the confusion caused by stratigraphic units being correlated with type localities over large distances in the Cascades.
The oldest exposed rocks in the study area are the volcanic rocks of the early Western Cascades (40-18 m.y.B.P.). They are chiefly siliceous ash-flow tuffs, debris flows and lava flows. Peck et al. (1964) referred to these rocks, in regional mapping, as the Little Butte Volcanic Series. In the Breitenbush area, these moderately altered ash-flow tuffs and epiclastics are called the Breitenbush Formation (Hammond et al. 1980).

The late Western Cascades episode (18-9 m.y.B.P.) consisted of calc-alkaline lavas and debris flows of intermediate composition. There are also locally abundant basalt and basaltic andesite lava flows (Priest et al. 1983). These rocks are mapped as the Sardine Formation (Peck et al. 1964; White, 1980). The uppermost basaltic and basaltic andesite lava and ash-flow tuffs have been mapped as the Rhododendron Formation by Hammond et al. (1980) and the Outerson Formation and Outerson volcanics by White (1980) and Hammond et al. (1980), respectively.

Stratigraphically below and interfingering up into the Rhododendron Formation are two formations of the Columbia River Basalt Group. Identified by Beeson et al. (1976), these tholeiitic flood basalts reach 550 meters thick in the upper Clackamas River drainage. The two formations are the Grande Rhonde Basalt and the overlying Frenchman Springs member of the Wanapum Basalt. The lower one-third of the Columbia River basalts in the study area have reversed polarity while the upper two-thirds possess normal polarity (Hammond et al. 1980).

The overlying rocks of the early High Cascades (9-4 m.y.B.P.) are basalt and basaltic andesites that probably erupted from
sources within and adjacent to the borders of the High Cascade province (White and McBirney, 1978; Hammond et al. 1980; Taylor, 1981). On the west side of the High Cascades, these lavas are the upper part of the Outerson Formation of White (1980) and lower part of the Boring Lavas of Peck et al. (1964). On the east side, the Deschutes Formation appears compositionally and temporally correlative to the early High Cascades episode (Taylor, 1981).

Between 4 and 5 m.y.B.P. faulting uplifted the Western Cascades relative to the High Cascades. Rocks west of a major fault near Belknap Hot Springs have been displaced two thousand feet upward relative to rocks of equivalent age on the east side (Armstrong et al. 1975). On the east side of the Cascades, faulting along Green Ridge, of equivalent age to the fault on the west side of the Cascades, displaced rocks downward west of Green Ridge. Basaltic lavas from the late High Cascade episode (4-0 m.y.B.P.) flowed down incised river valleys in the Western Cascades. Later eruptions of overlapping basaltic and basaltic andesite lavas built a Pleistocene platform. These platform lavas formed the "shields" beneath the prominent Quaternary composite cones making up the major peaks (Taylor, 1981).
PREVIOUS GEOPHYSICAL STUDIES

Thiruvathukal (1968) located 265 gravity stations in the northern Oregon Cascade study area. Based on a 3rd-order least squares polynomial surface fit to the data, Thiruvathukal obtained a crustal thickening that increases from about 35 km at the Columbia River to 40 km at the southern part of the survey area. Braman (1981) computed free-air, Bouguer, and residual gravity anomaly maps for the study area. He also computed a west-to-east crustal model constrained by gravity, seismic, and well data. Couch and Gemperle (1979) completed a gravity survey of the Mt. Hood area. Hassemer and Peterson (1977) reported principal facts for a gravity survey of the Breitenbush area; Couch and Gemperle (1982) used that data and their own additional measurements to map fault structures for a geothermal resource assessment.

A regional study in the Pacific Northwest by Dehlinger et al. (1965) found Pn velocities lower west of the Cascades (7.67 km/s) than east of the Cascades (8.0 km/s). Leaver et al. (1984) developed a P-wave crustal velocity model for the Oregon Cascades using a reversed refraction profile and three earthquakes. They measured Pn velocities of 7.70 km/s at a depth of 44 km increasing to 8.0 km/s at 100 km depth. Weaver et al. (1982) using 10 locatable earthquakes and 55 teleseisms, studied the P wave velocity structure of the crust and upper mantle beneath Mt. Hood and found no discernable velocity anomaly associated with the volcano.

Flanagan and Williams (1982) interpreted aeromagnetic measurements over Mt. Hood and found the bulk of the mountain to be composed
of magnetically similar andesites. Boler (1978), Connard (1979), Connard et al. (1983), and McLain (1981) mapped the Curie-point isotherm using the technique of Spector (1968), Spector and Grant (1970) in the Vale area of southeastern Oregon, and in the southern and central Oregon Cascades respectively. Bodvarsson et al. (1974) reported magnetotelluric measurements near Sisters, Oregon that had resistivities of 330 to 360 ohm-meters, typical of mafic tertiary volcanics. Goldstein et al. (1982) conducted an electromagnetic and magnetotelluric survey at Mt. Hood and found evidence for a moderately conductive north-south structure and a zone of intrusives concealed on the southeast flank.

Blackwell et al. (1978), (1982) reported a rapid increase in heat flow from 40 mW/m² in the Willamette Valley and Western Cascades to over 100 mW/m² in the High Cascades. Their data shows a narrowing of the greater than 100 mW/m² zone north of Mt. Jefferson and a small area of over 100 mW/m² enclosing Mt. Hood.

Investigators show the High Cascades in the northern Oregon study area to be an area of high heat flow and recent volcanism. However, there is no geophysical evidence for large magmatic bodies associated with the major volcanoes. In fact, geologic and geophysical evidence suggests magmatic emplacement in the form of dikes, sills, plugs, and other small intrusions (Taylor, 1981; Couch et al. 1982). Emplacement of these molten and partially molten intrusive bodies would raise the temperature of the surrounding rocks above the Curie-Point. By mapping these elevated Curie-Point isotherms in the northern Oregon Cascades, this study can provide a better
understanding of the thermal and magmatic evolution, as well as the geothermal potential, of the northern Oregon Cascades.
Figure 3 shows the total field magnetic anomaly map of the northern Oregon Cascade study area. Flight elevations were 7,000 feet west of the High Cascade axis and 5,000 feet east of the axis with an overlap of five miles. The 5,000 foot flight data were upward continued and merged with the 7,000 foot flight data. Topographically high areas such as Mt. Hood and Mt. Jefferson were flown at 11,000 feet and are shown in figures 4 and 5. A three-dimensional representation of the total field magnetic anomaly map is shown in figure 6. The regional geomagnetic field (IGRF 80), updated to survey time, was removed along with diurnal variations measured from a base station at Corvallis, Oregon thereby yielding the total field magnetic anomaly map (Couch et al. 1985).

The magnetic anomalies shown in figures 3 through 6 represent variations in the magnetization of the upper crustal rocks. The rocks of the survey area consist of mainly Cenozoic volcanic rocks with high thermoremanent magnetism. Thermoremanent magnetism results when magnetic minerals, such as magnetite and titanomagnetites, are cooled below the Curie-point (approximately 580° C) in the presence of an external magnetic field. The magnetic remanence acquired in this manner is fairly stable (Nagata, 1961). Lavas or intrusives cooled in a normal polarity field enhance the present earth's geomagnetic field expressing positive anomalies. Sources showing reversed magnetic anomalies are at least 730,000 years old (Harland et al. 1984). Prudent interpretation of magnetic anomalies can provide age constraints.
TOTAL FIELD AEROMAGNETIC ANOMALY MAP
Mount Hood Area, Oregon Cascades

Contour interval 50 nanoteslas
Estimated RMS uncertainty in measurements 4 nanoteslas
Flight elevation 1,000 feet
10R6 (1980)

Figure 4
TOTAL FIELD AEROMAGNETIC ANOMALY MAP
Mount Jefferson Area, Oregon Cascades

Figure 5
Figure 6. Three-dimensional representation of the total field aeromagnetic anomaly map.
High amplitude anomalies may result from high topography relative to the flight elevation. This is particularly evident along the axis of the High Cascades where Mt. Washington (7,794 ft.), Three-Fingered Jack (7,841 ft.), Mt. Jefferson (10,497 ft.), and Mt. Hood (11,235 ft.) all show high amplitude positive anomalies. Alternatively, high amplitude anomalies may result from sources that have high magnetic mineral content, such as mafic intrusives of flows. Long wavelength or regional magnetic anomalies may be due to long wavelength topography and deeper crustal sources.

South of 45° N latitude, Black Crater (7,251 ft.), Mt. Washington, Three-Fingered Jack, and Mt. Jefferson exhibit high amplitude positive anomalies. Taylor (1981) described Black Crater as a late Pleistocene basaltic andesite volcano. The positive polarity supports this volcano as being younger than 730,000 years B.P. Mt. Washington and Three-Fingered Jack, both heavily dissected basaltic andesite volcanos, show positive polarity rocks suggesting there may have been some intrusive activity less that 730,000 years B.P. Mt. Jefferson is also composed of normal polarity rocks.

The axis of the High Cascades shows up very prominently as a north-south trending line of high amplitude positive and negative anomalies. This trend continues from the southern border of the study area north up to the vicinity of Mt. Wilson where it becomes less defined. The positive polarity fields surrounding Mt. Washington, Three-Fingered Jack, and south of Mt. Jefferson are interpreted to be upper parts of the shield or platform rocks upon which the respective peaks sit. The high amplitude negative anomalies at Santiam Pass, near Sand Mountain, and north of Three-Fingered Jack
probably represent near-surface buried intrusives. The Sand Mountain chain of cinder cones erupted basalt flows about 3,800 years B.P. (Taylor, 1981). Basalts of this age would have normal polarity, but it appears their polarity has been masked by a large negative anomaly source beneath them. The larger negative anomaly near Santiam Pass is interpreted to be a near surface intrusion. A similar masking effect is observed in the vicinity of Belknap Crater, where young basaltic lavas erupted from Belknap Crater (1,500 years B.P.), Little Belknap Crater (2,900 years B.P.), and Yapoah Cone (Taylor, 1981) have their normal polarity "swamped" by negative anomaly sources buried beneath them. The north-south trending positive anomalies along the west edge of the study area are due to sources in the Western Cascades. The diminished amplitudes of the Western Cascades compared to the High Cascades may be due not only to differences in relief, but to the abundance of volcanic debris and ash flows, alteration, and the more silicic composition of the Western Cascades.

The regional negative anomaly south of 45° N latitude is probably made up of two sources. To the east of Green Ridge, magnetic sources in the Deschutes Formation which makes up the bulk of the near surface rocks, appears to be predominantly reversed. In the vicinity of the High Cascades, the large negative anomaly field is interpreted to represent the Plio-Pleistocene basalt and basaltic andesite lava flows that make up the platform rocks (Couch et al. 1985).

North of 45° N latitude, a large positive anomaly is associated with Mt. Wilson. North of Mt. Wilson, the number of large amplitude
anomalies diminish sharply. Mt. Hood, shown in figure 4, is made up of normal polarity rocks and has been interpreted by Williams et al. (1982) to be less than 700,000 years old. The positive anomalies shown around the base of Mt. Hood in figures 3 and 4, may be units of the Columbia River Basalt Group which were encountered in drill holes near the base of the mountain (Beeson et al. 1981, Couch et al. 1985). Mt. Hood fills part of the Mt. Hood subsidence block which is part of the The Dalles syncline (Williams et al. 1982). This syncline may have provided a passage way for Columbia River Basalts to flow through the Miocene Cascades from their sources in the eastern part of the Columbia Plateau (Beeson et al. 1981). The positive polarity anomaly associated with Mt. Hood and the Mt. Hood subsidence block is superimposed on a southwest-northeast trending negative anomaly. This negative anomaly has been interpreted by Flanagan and Williams (1982) to be structural relief in the Columbia River Basalts due to the Dalles syncline passing under Mt. Hood.

The large positive anomalies in the north-west part of the study area are due to a combination of Western Cascade and Columbia River Basalt sources. The positive anomalies in the north-east corner are interpreted to be from the Columbia River Basalts.

The prominent north-south trending lineament 122° 15' W longitude, between the positive and negative anomalies, shows good agreement with the western boundary fault of the Cascade Graben mapped by Couch et al. (1982). The total field magnetic anomaly map lends support to geophysical evidence (Connard, 1979; Couch et al. 1982; Connard et al. 1983; and Couch and Foote, 1983) and geologic mapping by Priest et al. (1983) that a major fault exists
west of the postulated High Cascades-Western Cascades fault boundary. Smaller faults such as the one at Green Ridge and the Sisters-Tumalo Fault system are not detectable in the magnetic anomalies. The positive anomaly at Green Ridge is interpreted as a buried intrusion.
LOW-PASS FILTERED MAPS

Magnetic anomalies associated with topography and shallow sources often mask long wavelength anomalies that may be due to deeper sources. Removal of the shorter wavelengths by filtering operations can aid in the geologic interpretation. Following Connard (1979), McLain (1981), Connard et al. (1983), and Huppunen (1983) low-pass filtered maps were generated with wavelengths removed shorter than 15 km and 35 km, respectively. The low-pass filters used were optimal two-dimensional, linear phase finite impulse response filters. The filtering operation is described in Appendix B.

The filter specifications \( N, \omega_p, \omega_s, \delta_1, \delta_2 \) (defined in Appendix B) were the inputs to PROGRAM FIR. Using approximate design relationships (Rabiner, 1973), \( \omega_p, \omega_s, \delta_1, \delta_2 \) returned an initial estimate \( \hat{N} \) for \( N \), the length of the one-dimensional filter impulse response. PROGRAM FIR computed the filter impulse response coefficients \( h(n) \) and the frequency response \( H(e^{j\omega}) \) for \( \omega_p, \omega_s, \delta_1, \delta_2 \), and the estimate \( \hat{N} \). Several iterations were done until the filter's frequency response met or exceeded the specifications. The specifications used in this study were equal ripple in the passband and stopband, where

\[
0.994 < |H(e^{j\omega})| < 1.006
\]

in the passband and

\[
|H(e^{j\omega})| < -44 \text{ dB}
\]
in the stopband. The transition width \((\omega_s - \omega_p)\) was made as narrow as possible until filter size \(N\) reached 255.

The frequency response of the one-dimensional optimal FIR filter was then transformed into an optimal two-dimensional FIR filter by the McClellan transformation (described in Appendix B, part III). The impulse response of the two-dimensional filter, magnitude of the frequency response, gain, and the phase angle are shown in figures 7 through 10 and figures 13 through 15.

Figures 7a and 7b compare the unit-sample response of the "ideal low-pass filter" truncated by a rectangular window with an optimal two-dimensional linear phase FIR filter. These optimal linearly convolved filters designed in this dissertation are an improvement over the frequency sampling filters used by Connard (1979), Connard et al. (1983), and McLain (1981). A frequency sampling filter, linearly convolved, is at best comparable to the truncated ideal low-pass filter. To make the filters causal or realizable, the impulse response was delayed 127 x 127 grid points. The ripples in figure 7a propagate further and with greater amplitude than in the optimal filter in figure 7b. The magnitude of the frequency response for the truncated and optimal low-pass filter are shown in figure 8. The Gibbs oscillations are quite pronounced in the truncated filter in figure 8a for both bands. The wide transition band along the base is quite apparent. The optimal filter, on the other hand, shows virtually no ripple in either the pass or stopbands, and it shows a narrow transition band. Figure 9 shows the gain \((20\times\log_{10}(|H(e^{j\omega})|))\) plotted as a function of frequency. The effect which the Gibbs oscillations and a wider transition band has on the attenuation becomes more pronounced.
Figure 7. Spacial domain representation of the two-dimensional unit-sample responses for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.
Figure 8. Magnitude of the frequency response for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.
Figure 9. Magnitude of the frequency response in decibels for (a) truncated ideal low-pass filter and (b) optimal linear phase filter.
minimum stopband attenuation for the truncated filter is less than 20 dB and which is very poor. The attenuation for the optimal filter in figure 9b, on the other hand, is less than -44 dB and more than a factor of 16 better.

The phase angle as a function of frequency is plotted in figure 10 for the optimal filter. For linear phase filters the group delay (-dθ/dω) gives the delay of the filtered output in number of grid points. The group delay, for the filters used in this study, is 127 x 127 grid points.

The optimal two-dimensional filter in figure 7b was used to generate the map shown in figure 11. All wavelengths shorter than 15 km were removed. The correlation between topography and the anomalies, while diminished, is still present along the High Cascades. Wavelengths longer than 15 km are observed associated with Mt. Washington, the Mt. Jefferson-Three-Fingered Jack area, Olallie Butte, and Mt. Wilson. While deeper sources may also be present, the anomalies associated with Mt. Washington, Mt. Jefferson, and Three-Fingered Jack may be due to positive polarity sources on the upper part of the shield or platform. The northward trend of the Western Cascades and the northeast trend of the lineament in the Mt. Wilson area, identified on the total field magnetic anomaly map, are also evident in figure 11. Topographic features whose magnetic anomalies have been filtered out include Black Butte, Belknap Crater, and smaller cinder cones. Magnetic sources due to these cones are either very shallow or entirely attributed to topography. Figure 12 is a three-dimensional representation of figure 11.
Figure 10. Phase angle plotted as a function of frequency for the optimal two-dimensional FIR filters.
Figure 12. Low-pass filtered aeromagnetic anomaly map with wavelengths shorter than 15 km removed.
The magnetic anomalies associated with the High Cascades are very prominent as are the anomalies in the Western Cascades. The north and northeast trending positive anomalies, while less prominent than the anomalies in the High Cascades, stand out in comparison to the regional negative anomaly field to the north and south of Mt. Wilson.

In order to remove the remaining correlation between magnetic anomalies and topography, and to attempt to discern deeper sources, an optimal two-dimensional filter with a 35 km cutoff wavelength was designed. The optimal filter, shown in figures 13 through 15 with an equivalent truncated ideal low-pass filter for comparison, was used to remove all wavelengths shorter than 35 km. The resulting map in figure 16 shows that most of the correlation between sources and topography has disappeared. However, there is a narrow magnetic high along the High Cascades which again may be caused by positive polarity sources in the upper part of the platform. The north and northeast trending positive anomalies are now very pronounced as are the negative anomaly fields in the Mt. Hood area and along the High Cascades. The three-dimensional representation of this map is shown in figure 17.

North-south trends in regional magnetic anomalies mapped by Connard (1979) and Connard et al. (1983) in the Central Oregon Cascades appear to continue into the northern Oregon study area as far north as 45° N latitude. This change in trend to a northeasterly direction is similar to changes in trend in residual gravity anomalies mapped by Couch et al. (1982) near Mt. Hood. While regional magnetic anomalies may be due in part to other sources, such as long wavelength features in the topography, the magnetic anomaly maps suggest there may be
Figure 13. Spatial domain representation of the two-dimensional unit-sample responses for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.
Figure 14. Magnitude of the frequency response for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.
Figure 15. Magnitude of the frequency response in decibels for the (a) truncated ideal low-pass filter and (b) optimal linear phase FIR filter.
Figure 17. Low-pass filtered aeromagnetic anomaly map with wavelengths shorter than 35 km removed.
changes in upper crustal structure between Mt. Hood and Mt. Jefferson. If a change in upper crustal structure is present, it may be reflected in the Curie-point isotherm depths mapped in the northern Oregon study area.
Spector and Grant (1970) developed a method for estimating depths to the magnetic source top from spectral analysis of magnetic anomalies. The method of Spector and Grant outlined below and in Appendix C, is based on the assumption that magnetic anomalies consist of a number of independent ensembles of rectangular, vertical-sided prisms. If this assumption is correct, the energy spectrum of the magnetic anomaly map, in polar coordinates, for an ensemble of vertical-sided prisms with vertical magnetization (rotation-to-the-pole), in a vertical geomagnetic field is given as,

$$<E(r,\theta)> = 4\pi^2 M^2 e^{-2hr} <(1-e^{-tr})^2> <s^2(r)> \tag{1}$$

where $< >$ indicates the expected value,

$M =$ average magnetic moment/unit depth

$h =$ depth to the top of the prism

$t =$ thickness of the prism

$s =$ factor for the horizontal size of the prism averaged over theta

The assumptions implied in the above equation include

(1) the anomaly field $f(x,y)$ is the realization of a stationary, random process,

(2) the magnetization is zero, except in discrete uniformly magnetized bodies randomly located,

(3) the parameters of the prism (eg. depth, width, length, thickness, magnetization, and body center location) are characterized by a joint frequency distribution, and
(4) the magnetization direction is approximately parallel to the earth's geomagnetic field direction (within ± 20°) (Connard, 1979; McLain, 1981; Connard et al. 1983).

Assumption (1) may or may not be violated since the survey area encompasses more than one geologic-physiographic province such as the Western and High Cascades (Naidu, 1970). Assumption (2) may also be violated where anomalies are aligned into lineaments (Shuey et al. 1977), such as along the axis of the High Cascades. Assumption (3) is simplistic and may not represent the true geologic picture. Assumption (4) appears reasonable for this study area because the anomaly patterns appear "normal".

Assuming the distribution of prism depths falls into the range $0.75 \bar{h} < h < 1.25 \bar{h}$ for $r < 1/\bar{h}$, where $\bar{h}$ is the mean ensemble depth, Spector and Grant (1970) show that $<e^{-2hr}> = e^{-2hr}$ is the dominating factor in the power spectrum. Hence a logarithm plot of the energy spectrum, representing ensembles of prisms as a function of $r$, should approximate a straight line whose slope $-2h$ represents the average depth to an ensemble of magnetic prisms. However, given the above assumptions, effects due to the shape factor $s^2(r)$ and large depth variations of individual prisms, the depth estimates to the top of the magnetic source represent approximate estimates.

To apply depth-to-the-source-top calculations for the northern Oregon study area, the anomaly map in figure 3 was digitized to a uniformly spaced grid of size 720 rows x 512 columns (see Appendix A). After gridding, the gridded data were detrended, tapered to zero at the grid boundaries and transformed to the frequency domain. A radially averaged energy spectrum was computed for the frequency
elements \((k,l)\) within bands according to their wavenumber, i.e., \(0.5 < (k^2 + l^2)^{1/2} \leq 1.5\) were averaged, \(1.5 < (k^2 + l^2)^{1/2} \leq 2.5\) were averaged and so forth, for all elements up to the Nyquist wavenumber. The frequency element \((0,0)\) was not averaged with any other elements (Boler, 1978). Finally, plots of the energy spectrum, normalized with respect to the \((0,0)\) frequency element were computed for grid sizes \(512 \times 512\) and \(256 \times 256\). Figure 18 and 19 show spectral plots for two overlapping \(512 \times 512\) \((128 \text{ km} \times 128 \text{ km})\) grids covering the northern Oregon study area. The slopes were computed by fitting lines to linear portions of the spectra. The uncertainty associated with the computed lines represents a measure of how well the lines fit the data and not a measure of the accuracy of the depth estimate (Boler, 1978). The spectral plots for both grids show pronounced linear slopes indicating distinct magnetized source tops are being resolved. This contrasts with the \(512 \times 512\) spectral plots by Connard (1979), McLain (1981), and Connard et al. (1983) that are exponential in shape. Connard and Connard et al. attribute this to sources from distinct geologic provinces being included in one spectral plot, thus violating assumption (1). Connard and Connard et al. also point out that the extensive volcanism in the Central Oregon study area, including the emplacement of intrusive bodies at all levels in the crust, would make distinct layers unresolvable. The southern \(512 \times 512\) spectral plot shows average depth to source tops of \(1.7\) and \(1.0-0.8\) km above sea level (ASL). The \(1.7\) km (ASL) depth is most likely due to the elevated topography in the southern one-half of the survey area. The northern \(512 \times 512\) spectral plot resolves an average depth-to-source...
Figure 18. Radially averaged spectrum of the south 512 x 512 grid in the northern Oregon Cascades.
Figure 19. Radially averaged spectrum of the north 512 x 512 grid in the northern Oregon Cascades.
top of 1.0 km (ASL), but also resolves deeper source tops of 0.2 km (ASL) and 1.7 km below sea level (BSL).

Fifteen 256 x 256 (64 km x 64 km) overlapping spectral plots were computed to obtain more resolution in a specific area. The 256 x 256 spectral plots are shown in figures 20 through 34 and in the upper right hand corner of each plot are the locations of each subgrid in relation to the total grid. The 256 x 256 subgrids consistently resolved source depths of 1.0 km (ASL). This may represent the depth to the top of the platform rocks or younger lava flows that cover the High Cascades. This source depth information is summarized in Table 1 for the 512 x 512 and 256 x 256 size grids and subgrids respectively.
Figure 20. Radially averaged spectrum of the south-southwest 256 x 256 grid in the northern Oregon Cascades.
Figure 21. Radially averaged spectrum of the south-south 256 x 256 grid in the northern Oregon Cascades.

\[ f_{\text{max}} = 0.033 \pm 0.004 \]

\[ -2.7 \pm 0.2 \]

\[ -1.6 \pm 0.02 \]
Figure 22. Radially averaged spectrum of the south-southeast 256 x 256 grid in the northern Oregon Cascades.
Figure 23. Radially averaged spectrum of the southwest 256 x 256 grid in the northern Oregon Cascades.
Figure 24. Radially averaged spectrum of the south 256 x 256 grid in the northern Oregon Cascades.
Figure 25. Radially averaged spectrum of the southeast 256 x 256 grid in the northern Oregon Cascades.
Figure 26. Radially averaged spectrum of the west-central 256 x 256 grid in the northern Oregon Cascades.
Figure 27. Radially averaged spectrum of the central 256 x 256 grid in the northern Oregon Cascades.
Figure 28. Radially averaged spectrum of the east-central 256 x 256 grid in the northern Oregon Cascades.
Figure 29. Radially averaged spectrum of the northwest 256 x 256 grid in the northern Oregon Cascades.
Figure 30. Radially averaged spectrum of the north 256 x 256 grid in the northern Oregon Cascades.
Figure 31. Radially averaged spectrum of the northeast 256 x 256 grid in the northern Oregon Cascades.
Figure 32. Radially averaged spectrum of the north-northwest 256 x 256 grid in the northern Oregon Cascades.
Figure 33. Radially averaged spectrum of the north-north 256 x 256 grid in the northern Oregon Cascades.
Figure 34. Radially averaged spectrum of the north-northeast 256 x 256 grid in the northern Oregon Cascades.
<table>
<thead>
<tr>
<th>Grid Location</th>
<th>Mean Terrain Clearance of Survey in km (estimated)</th>
<th>Mean Elevation of Terrain in km ASL (estimated)</th>
<th>Source Depths km from Sea-Level (-below, +above)</th>
<th>Deep</th>
<th>Intermediate</th>
<th>Shallow</th>
</tr>
</thead>
<tbody>
<tr>
<td>N (512 grid)</td>
<td>1.6</td>
<td>1.1</td>
<td>-1.2 ± .7</td>
<td>.02 ± .1</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>S (512 grid)</td>
<td>1.5</td>
<td>1.2</td>
<td>--</td>
<td>.09 ± .2</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>SSW (256 grid)</td>
<td>1.5</td>
<td>1.2</td>
<td>--</td>
<td>.03 ± .2</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>SS (256 grid)</td>
<td>1.3</td>
<td>1.4</td>
<td>--</td>
<td>0.0 ± .2</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>SSE (256 grid)</td>
<td>1.6</td>
<td>1.1</td>
<td>-1.6 ± .4</td>
<td>--</td>
<td>.1</td>
<td></td>
</tr>
<tr>
<td>SW (256 grid)</td>
<td>1.5</td>
<td>1.2</td>
<td>-5.5 ± .6</td>
<td>-0.2 ± .1</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>S (256 grid)</td>
<td>1.2</td>
<td>1.5</td>
<td>--</td>
<td>.03 ± .1</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>SE (256 grid)</td>
<td>1.6</td>
<td>1.1</td>
<td>-2.1 ± .3</td>
<td>.07 ± .03</td>
<td>.03</td>
<td></td>
</tr>
<tr>
<td>WC (256 grid)</td>
<td>1.7</td>
<td>1.0</td>
<td>-1.2 ± .3</td>
<td>--</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>C (256 grid)</td>
<td>1.3</td>
<td>1.4</td>
<td>--</td>
<td>0.0 ± .1</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>EC (256 grid)</td>
<td>1.7</td>
<td>1.0</td>
<td>--</td>
<td>0.0 ± .1</td>
<td>.03</td>
<td></td>
</tr>
<tr>
<td>NW (256 grid)</td>
<td>1.8</td>
<td>0.9</td>
<td>-1.5 ± .4</td>
<td>--</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>N (256 grid)</td>
<td>1.2</td>
<td>1.5</td>
<td>--</td>
<td>0.1 ± .2</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>NE (256 grid)</td>
<td>1.5</td>
<td>1.2</td>
<td>-0.7 ± .1</td>
<td>--</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>NNNW (256 grid)</td>
<td>2.1</td>
<td>0.6</td>
<td>--</td>
<td>.05 ± .04</td>
<td>.03</td>
<td></td>
</tr>
<tr>
<td>NN (256 grid)</td>
<td>1.5</td>
<td>1.2</td>
<td>-1.0 ± .4</td>
<td>--</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>NNE (256 grid)</td>
<td>1.7</td>
<td>1.0</td>
<td>-0.4 ± .2</td>
<td>--</td>
<td>.02</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Source depths from spectral analysis of aeromagnetic anomalies.
CURIE-POINT ISOTHERM MAPPING BY SPECTRAL ANALYSIS

The magnetized parts of the earth's crust usually have a lower boundary that is either a lithologic discontinuity, or a temperature horizon below which the rocks are too hot to sustain magnetization. This temperature is the Curie temperature of ferromagnetic minerals and the temperature horizon is termed the Curie-point isotherm. In geologic provinces, where the heat flow is low, such as the Canadian Shield, and in the U.S. east of the Rockies, the magnetic source bottom may represent a lithologic boundary. In areas of high heat flow, such as along belts of young volcanic rocks, the magnetic source bottom is interpreted to represent the temperature at which magnetic materials lose their acquired magnetic remanence (Curie point). The high heat flow and recent volcanism in the High Cascades portion of this study area suggest the magnetic source bottom depth is the Curie-point isotherm.

The need for an assessment of geothermal resources and a better understanding of the thermal evolution of the High Cascade Range led to studies in Central Oregon (Connard, 1979; Connard et al. 1983), Southern Oregon (McLain, 1981), and Northern California (Huppunen, 1983). Different techniques for estimating the depth to the magnetic source bottom, used by different investigators, have met with varying success. Connard (1979) and McLain (1981) review these techniques and their results. This study employs the method developed by Spector (1968), Spector and Grant (1970), Smith et al. (1974), Boler (1978), Connard (1979), and McLain (1981) to resolve the Curie-point isotherm in the Cascade Range. The method is outlined in Appendix C.
If the dimensions of the study area are large enough so that the long wavelength anomalies due to the magnetic source bottom are resolved, the \((1-e^{-tr})\) factor in combination with the \(e^{-2hr}\) factor in equation (1) introduces a peak in the energy spectrum (Spector and Grant, 1970). Shuey et al. (1977) determined that the spectrum of the map area contains depth information to a maximum depth of \((LX/2\pi)\) where LX is one side of the map in kilometers. The size of the study area is 128 km by 180 km and for the 512 x 512 grids (128 km x 128 km), the maximum resolvable depth for this study area is approximately 20 km. Any attempt to "see" below this depth would require a larger area. Magnetic source bottoms deeper than \(LX/2\pi\) have a peak in the energy spectrum at a frequency less than the fundamental frequency and are not resolvable.

When a spectral peak does occur in the spectrum, the frequency of the spectral peak \(f_{max}\), in cycles/km, is related by,

\[
 f_{max} = \frac{1}{2\pi(d-h)} \ln \left( \frac{d}{h} \right) \tag{2}
\]

where \(\overline{d}\) is the average depth to the source bottom and \(h\) is the average depth to the top of the source (Connard, 1979). Solving for \(\overline{d}\),

\[
 \overline{d} = \frac{\overline{c}}{1-\exp(-2\pi tf_{max})} \tag{3}
\]

The \(\overline{c}\) in the above expression means the Curie-point isotherm depth cannot be determined by \(f_{max}\) alone. Some estimate of mean ensemble thickness is required, thereby setting a minimum Curie-point isotherm
Following Shuey et al. (1977), Connard (1979), McLain (1981), and Connard et al. (1983) a minimum source thickness of \( t = 5 \) km was used. Since the magnetic source bottoms are a long wavelength signal, they are observed at the low frequency end of the energy spectrum. To obtain a good estimate of the shape of the spectral peak, a large study area provides more spectral information in the lower frequencies. In fact, Shuey et al. (1977) have suggested a study area of shortest dimension to be \( 2\pi D \) where \( D \) is the maximum resolvable magnetic source bottom depth. Unfortunately, the larger the study area, the more regional the Curie-point isotherm determination becomes. To better resolve the undulations of the source bottom, a moving window technique of variable size developed by McLain (1981), is used to enhance these smaller scale source depths.

Another factor that may affect the location of the spectral peak is the shape of the magnetic source body. Because a regional anomaly gradient could be mistaken for a source body, and needs to be removed, the data set is detrended before computing the energy spectrum (Connard, 1979). In these detrended spectral plots, the energy of the DC term \( E(0) \) will always be less than the energy of the fundamental frequency \( E(1) \). Spectral plots with a peak at the fundamental frequency indicate that the magnetic source bottom is not being resolved.

Two overlapping \( 512 \times 512 \) (128 km x 128 km) grids covered the entire survey area. The radially averaged spectra of the two \( 512 \times 512 \) grids (figures 18 and 19), covering the northern Oregon study area, fail to exhibit spectral peaks. However, the recent volcanism in the McKenzie-Santiam Pass area (Taylor, 1981) suggests
elevated Curie-point isotherm depths may exist but were not resolved by the 512 x 512 size grids. Connard (1979) and Connard et al. (1983) mapped elevated Curie-point isotherms in the Central Oregon Cascades narrowing to the north but appearing to continue on into the northern Oregon study area. Using a smaller window size of 256 x 256 and moving the window over the study area, three of the fifteen 256 x 256 subgrids were able to resolve the source bottom. They are the south-south, south, and west-central subgrids shown in figures 21, 24, and 26, respectively. Additional 256 x 256 subgrids, not shown in spectra, were computed along the axis of the High Cascades to constrain the location of the magnetic source bottom. Figure 35 shows the range of allowable Curie-point isotherm depths for the 256 x 256 grids. Assuming a minimum source thickness of 5 km, the Curie-point isotherm depths were between 5 and 9 km below sea level (BSL) for the south-south and south 256 x 256 subgrids and between 6 and 9 km for the west-central subgrid. The possibility that the magnetic source bottoms are locally even shallower warranted the use of a smaller window. A window size of 128 x 128 grid points was moved over the areas where the 256 x 256 subgrids showed a spectral peak and over some of the volcanic cones. Only two 128 x 128 subgrids resolved the source bottom. Their spectral plots are shown in figures 36 and 37. The spectral plot in figure 36a illustrates the difficulty in determining the frequency of the spectral peak. The frequency of the spectral peak lies approximately in the interval

\[ f_{\text{avg}}(2) - \left( \frac{f_{\text{avg}}(2) - f_{\text{avg}}(1)}{2} \right) \leq f_{\text{max}} \leq \left( \frac{f_{\text{avg}}(3) - f_{\text{avg}}(2)}{2} \right) + f_{\text{avg}}(2) \]
Figure 35. Calculations of minimum and maximum Curie-point depths for (A) 256 x 256 subgrids that did not resolve the source bottom, (B) the south-south subgrid, (C) the south subgrid, and the west-central (WC) subgrid.
Figure 36. Radially averaged spectrum for the west 128 x 128 subgrid with (a) 128 estimates, and (b) 512 estimates.
Figure 37. Radially averaged spectrum for the east 128 subgrid appended with zeros to grid size 512 x 512.
for the spectral plot shown in figure 36a, $f_{\text{max}}$ is in the interval

$$0.053 < f_{\text{max}} < 0.081$$

For spectral peaks in this range, and using depth to magnetic source tops as a constraint, the spectral peak may be due either to a shallow Curie-point or a lithologic boundary, i.e., a magnetic layer overlying non-magnetic sediments. Thus a more constrained spectral peak is desired.

A technique used in this study for increasing the number of estimates in the lower frequencies and to constrain the interval in which the frequency of the spectral peak lies, is to append zeros, up to size 512 x 512, on the ends of the 128 x 128 subgrid. Recomputing the forward two-dimensional Fourier transform of the appended 512 x 512 subgrid, has the effect of increasing the frequency estimates by a factor of four for 128 x 128 subgrids and by a factor of two for 256 x 256 subgrids appended to 512 x 512. The 128 x 128 subgrid in figure 36a, appended with zeros to size 512 x 512 is shown in figure 36b. The frequency of the spectral peak now lies between the interval

$$0.059 < f_{\text{max}} < 0.066$$

The spectral plots of the west 128 subgrid, shown in figure 36b, and the east 128 subgrid in figure 37 were computed using the above technique. Using the depths to the magnetic source top as a constraint, figure 38 shows the computed magnetic source bottom depths are 1.1 and 0.7 km below sea level (BSL), respectively. These depths are interpreted to be lithologic boundaries. Residual gravity anomalies
Figure 38. Computed depths to the magnetic source bottom, constrained by magnetic source top depths for the west 128 x 128 and east 128 x 128 subgrids.
in the east 128 area show mass deficiencies of less than -12 mgals. The west 128 subgrid shows both negative and positive residual anomalies (Braman, 1981; Couch et al. 1982). The magnetic source bottom being resolved may represent platform-like lavas overlying low density sediments or volcanic debris and ash-flows.

Table 2 summarizes the depth to the magnetic source bottom information derived from the estimated frequencies and depths to source tops for the 256 x 256 and 128 x 128 subgrids described above. The corresponding geothermal gradients shown are assuming a Curie-point temperature of 580°C (Nagata, 1961). Average heat flow values were calculated assuming a conductivity of 1.7 Wm⁻¹°C⁻¹ for basement rocks in the Cascades Mountains (Blackwell et al. 1978).

The Curie-point temperature of 580°C for magnetite (Fe₃O₄) is often assumed in Curie-point depth analyses (Bhattacharyya and Leu, 1975; Shuey et al. 1977). However, the Curie-point is dependent upon bulk rock composition, temperature and oxygen fugacity at the time of last equilibration and can thus have a wide range (Mayhew et al. 1985). Haggarty (1978) found that Curie-point temperatures for titanomagnetites that experienced low temperatures of oxidation, are about 300°C. Lower crustal volcanic rocks from the Honshu Arc, Japan, a subduction-derived volcanic arc similar to the Cascade volcanic arc, were found to have high magnetization values and a Curie-point temperature near 550°C (Wasilewski and Mayhew, 1982). Rocks containing a dominance of magnetite, i.e., formed in a more oxidizing environment, are associated with higher Curie-points (580°C) than rocks with ilmenite and titanomagnetite
<table>
<thead>
<tr>
<th>Grid Location</th>
<th>SSW (256)</th>
<th>SS (256)</th>
<th>SSE (256)</th>
<th>SW (256)</th>
<th>S (256)</th>
<th>SE (256)</th>
<th>WC (256)</th>
<th>C (256)</th>
<th>EC (256)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Size</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
</tr>
<tr>
<td>Mean Terrain</td>
<td>1.2</td>
<td>1.4</td>
<td>1.1</td>
<td>1.2</td>
<td>1.5</td>
<td>1.1</td>
<td>1.0</td>
<td>1.4</td>
<td>1.0</td>
</tr>
<tr>
<td>Elevation</td>
<td>.026-.033</td>
<td>.019</td>
<td>.019</td>
<td>.026-.036</td>
<td>.019</td>
<td>.026</td>
<td>.019</td>
<td>.019</td>
<td>.019</td>
</tr>
</tbody>
</table>

**MINIMUM CURIE-DEPTHS (5 km thick sources)**

| Curie Depth  | >12 | 7.7 | >12 | >12 | 7.4 | >12 | >9 | >12 | >9 | >12 | >12 |
| km BFL       | km BSL | T<sub>c</sub>=580 | T<sub>c</sub>=580 | Vertical Temp. Gradient °C/km | <57 | 90 | <57 | <57 | 95 | <57 | 83 | <56 | <58 |
| Surface Heat Flow mW/m² | <97 | 153 | <97 | <97 | 161 | <97 | 141 | <95 | <99 |

**MAXIMUM CURIE-DEPTH (depth to source top controlled)**

| Depth BFL to Source Top (km) | 2.74<sup>±</sup>2 | 2.4<sup>±</sup>1 | -- | -- | -- | -- | -- | -- | -- | -- |
| Curie Depth | 11.7 | -- | -- | -- | -- | -- | -- | -- | -- | -- |
| km BFL | 8.9 | -- | -- | -- | -- | -- | -- | -- | -- | -- |
| Vertical Temp. Gradient °C/km | 56 | -- | -- | -- | -- | -- | -- | -- | -- | -- |
| Surface Heat Flow | 95 | -- | -- | -- | -- | -- | -- | -- | -- |

**Table 2.** Curie-point isotherm depths from spectral analysis of aeromagnetic data and corresponding surface heat flow and geothermal gradients.
<table>
<thead>
<tr>
<th>Grid Location</th>
<th>NW (256)</th>
<th>N (256)</th>
<th>NE (256)</th>
<th>NW (256)</th>
<th>NN (256)</th>
<th>NNE (256)</th>
<th>W 128</th>
<th>E 128</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Size</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>64x64 km</td>
<td>32x32 km</td>
<td>32x32 km</td>
</tr>
<tr>
<td>Mean Terrain Elevation</td>
<td>.9</td>
<td>1.5</td>
<td>1.2</td>
<td>.6</td>
<td>1.2</td>
<td>.8</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Frequency of Spectral Peak ( f_{\text{max}} )</td>
<td>&lt;.019</td>
<td>&lt;.019</td>
<td>&lt;.019</td>
<td>&lt;.019</td>
<td>&lt;.019</td>
<td>&lt;.019</td>
<td>.0625</td>
<td>.071</td>
</tr>
</tbody>
</table>

**MINIMUM CURIE-DEPTH (5 km thick source)**

<table>
<thead>
<tr>
<th>Curie Depth ( \text{km BFL} )</th>
<th>&gt;12</th>
<th>&gt;12</th>
<th>&gt;12</th>
<th>&gt;12</th>
<th>&gt;12</th>
<th>&gt;12</th>
<th>3.8</th>
<th>3.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>KM BSL</td>
<td>&gt;9</td>
<td>&gt;9</td>
<td>&gt;9</td>
<td>&gt;9</td>
<td>&gt;9</td>
<td>&gt;9</td>
<td>1.1</td>
<td>0.7</td>
</tr>
<tr>
<td>( T_c = 580 )</td>
<td>( T_c = 580 )</td>
<td>( T_c = 580 )</td>
<td>( T_c = 580 )</td>
<td>( T_c = 580 )</td>
<td>( T_c = 580 )</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Vertical Temp. Gradient ( °C/km )</td>
<td>&lt;59</td>
<td>&lt;55</td>
<td>&lt;57</td>
<td>&lt;60</td>
<td>&lt;57</td>
<td>59</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Surface Heat Flow ( \text{m W/m}^2 )</td>
<td>&lt;100</td>
<td>&lt;94</td>
<td>&lt;97</td>
<td>&lt;103</td>
<td>&lt;97</td>
<td>&lt;101</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

**MAXIMUM CURIE-DEPTH**

| Depth BFL to Source Top | -- | -- | -- | -- | -- | -- | -- | -- |
| Curie-depth \( \text{km BFL} \) | -- | -- | -- | -- | -- | -- | -- | -- |
| km BSL | -- | -- | -- | -- | -- | -- | -- |
| Vertical Temp. Gradient \( °C/km \) | -- | -- | -- | -- | -- | -- | -- |
| Surface Heat Flow \( \text{mW/m}^2 \) | -- | -- | -- | -- | -- | -- | -- |

Table 2. continued ....
(300° C), which formed in a more reducing environment (Mayhew et al. 1985).

Heat flow studies in Oregon (Blackwell et al. 1978), the Cascade Range (Blackwell et al. 1982) and the central and northern Oregon Cascade Range (Black et al. 1983), which included measurements in 170 drill holes, demonstrated a rapid increase in heat flow from 40 mW/m² in the Willamette Valley and Western Cascades to over 100 mW/m² in the High Cascades. The average heat flow in the High Cascades portion of the study area is 104 mW/m² (Black et al. 1983). Thermal conductivities were measured from well cuttings and core samples where possible (Black et al. 1983). The results of Blackwell et al. (1978), Blackwell et al. (1982), and Black et al. (1983) are compared in the next section with the computed Curie-point depths and temperature gradients for the northern Oregon study area.
SUMMARY AND DISCUSSION OF CURIE-POINT ISOThERM DEPTHS IN THE NORTHERN OREGON CASCADES

The computed Curie-point isotherm depths summarized in table 2, are shown plotted in figure 39 on a topographic map of the northern Oregon study area. Corresponding heat flow values based on a Curie-point temperature of 580°C, and conductivities from Blackwell et al. (1978) are also shown. Thermal springs, listed in Bowen and Peterson (1970) and located in the study area, are indicated in the figure by the solid triangles.

The hexagons and circles in figure 39 represent the locations of computed 64 km x 64 km and 32 km x 32 km subgrids, respectively used to map and constrain the extent of the Curie-point isotherm. Only the 64 km x 64 km subgrids resolved what is interpreted to be the Curie-point isotherm. All the 64 km x 64 km subgrids located inside the Curie-point isotherm contours resolved the magnetic source bottom. Contours and stippling outline areas of relatively shallow Curie-point isotherm depths of 5 - 9 km below sea level (BSL) along the axis of the High Cascades. The large areas of shallow Curie-point isotherm depths extend from the southern boundary of the study area to the vicinity of Mt. Wilson. A smaller region of elevated Curie-point depths is located west of Mt. Wilson. Outside of these contours, the magnetic source bottom was not resolved and the Curie-point depths are deeper than the maximum resolvable depth. The mapped Curie-point isotherm contours agree with Connard's (1979) and Connard et al.'s (1983) possible extension of shallow Curie-point depths into the northern Oregon study area. This extension was
suggested by them on the basis of Blackwell et al.'s. (1978) heat flow data for the area and their structural interpretation.

The heat flow values shown in figure 39 range from 90 mW/m² to 160 mW/m² along the axis of the High Cascades and 100 mW/m² to 140 mW/m² in the smaller region to the west of Mt. Wilson. These values compare favorably to heat flow values computed by Blackwell et al. (1978), Blackwell et al. (1982) and Black et al. (1983) from drill hole measurements. Thus the 580°C Temperature used for the Curie-point temperature and the computed Curie-point isotherm depths yield heat flow values that agree very well with those observed. This study area, as well as the southern Oregon (McLain, 1981), and central Oregon (Connard, 1979; Connard et al. 1983) study areas support the argument that the Curie-point temperature is closer to 580°C than to 300°C. A Curie-point temperature of 550°C suggested by Wasilewski and Mayhew (1985) does not change significantly the agreement with the observed heat flow.

The elevated Curie-point isotherm depths shown in figure 39 may represent localized areas of intrusive magmatic activity. This seems especially likely along the axis of the High Cascades where recent volcanism and high heat flow suggest relatively shallow sources at depth. These computed Curie-point depths have a temperature gradient of between 55°C and 90°C/km. Gradients such as these, if continued downward, would result in partial melting at about 700°C around depths of 10 km (Blackwell et al. 1982). As was discussed earlier, there is no geophysical or geological evidence for large areas of partial melt in the upper crust. An alternative explanation for these high gradients and lack of melting in the upper crust was proposed by
R. Couch (personal commun., 1984). Volcanic pipes or conduits of magma from the lower crust and upper mantle could, upon reaching the upper crust, branch out laterally in the form of sills, dikes and other intrusive bodies. This intrusive activity in the upper crust would elevate the temperatures of the surrounding rocks resulting in a "perched" thermal anomaly. The Curie-point isotherm depth contours may reflect these thermal anomalies and they may then be interpreted as localized zones of late magmatic intrusions where heat conduction has elevated the temperature and consequently the Curie-point isotherm depth.

The interpretation of the total field and low-pass filtered aeromagnetic anomaly maps indicates that structures trend north-south in the southern part of the survey area and then turn to a northeast trend in the vicinity of Mt. Wilson. Faults and lineaments mapped by Couch et al. (1982) also show a change in structure from a roughly north-south trend south of 45° N latitude to a northeast trend in the vicinity of Mt. Wilson. The magnetic anomalies and the residual gravity anomalies are consistent in that they do not show either the Cascade or High Cascade Grabens to be a continuous north-south trending structure between Mt. Jefferson and Mt. Hood.

The onset of northeast trending structure in the vicinity of Mt. Wilson coincides with the northern extent of the shallow Curie-point isotherm depths. Except for the small region of shallow Curie-point depths west of Mt. Wilson, the Curie-point depth deepens to greater than 9 km BSL toward the vicinity of Mt. Hood. Blackwell et al. (1982) and Black et al. (1983) show a decline in the heat flow values from over 100 mW/m² near Mt. Wilson to lower values in the
vicinity of Mt. Hood. Except for two measurements of over 100 mW/m² at Mt. Hood, the heat flow drops to the 60 mW/m² to 70 mW/m² range near the Columbia River.

An interpretation and analysis of the aeromagnetic data, combined with residual gravity anomalies, Curie-point isotherm depths, and heat flow measurements, in the northern Oregon Cascades lend support to a change in crustal structure in the vicinity of Mt. Wilson. South of Mt. Wilson, the areal extent of the shallow Curie-point depths indicate relatively large quantities of magma have intruded into the upper crust in the form of dikes, sills, and other intrusive bodies, elevating the geothermal gradient. North of Mt. Wilson, higher temperatures appear confined to Mt. Hood. Heat flow measurements to the west of Mt. Wilson by Blackwell et al. (1982) and Black et al. (1983) indicate a small change to a north-west trend in the 100 mW/m² heat flow contour. This change in direction of the heat flow contour may be the surface expression of the small area of elevated Curie-point depths west of Mt. Wilson. Austin Hot Springs is located above the eastern half of this small thermal anomaly.

Further geophysical studies in the northern Oregon Cascades are warranted to investigate this apparent change in residual gravity and aeromagnetic anomalies, and in Curie-point isotherm depths and heat flow measurements between Mt. Jefferson and Mt. Hood. If future work supports a change in crustal structure, then the character of volcanism changes from localized areas of high intrusive activity south of Mt. Wilson, to intrusive activity concentrated solely beneath the major peaks north of Mt. Wilson.
REFERENCES CITED


Bhattacharyya, B.K., 1966, Continuous Spectrum of the Total Magnetic Field Anomaly Due to a Rectangular Prismatic Body: Geophysics, v. 31, p. 97-112.


Swick, C.H., 1932, First and Second Order Triangulation in Oregon, U.S. Dept. of Commerce, Coast and Geodetic Survey Special Publication no. 75.


APPENDICES
APPENDIX A
THE NORTHERN OREGON AEROMAGNETIC SURVEY

Data Collection

During the summer and fall of 1982, personnel from the Geophysics group at Oregon State University conducted an aeromagnetic survey of the Cascade Range in northern Oregon. The survey extended from near McKenzie Pass (44°15' N latitude) to just north of the Columbia River (45°45' N latitude) covering approximately 18,400 square kilometers. The flightlines of the survey, oriented east-west, were spaced at one mile intervals, and north-south tie lines were spaced at five mile intervals. A flightline map for the northern Oregon survey is shown in figure A 1. Measurements, made with a total field proton precession magnetometer, were spaced at approximately 140 meter intervals along the survey lines. The survey was flown at constant barometric elevations of 5,000, 7,000, and 11,000 feet above sea level. These elevations were selected to yield planar surfaces at a minimum elevation consistent with topography.

A microwave range-range navigation system, consisting of two ground-based radar transponders and a receiver-transmitter in the aircraft provided horizontal position control for the survey. The navigation system determined the range to each transponder every two seconds. The transponders were placed on or within 30 meters of a geodetic station. The State Plane Coordinates and the latitude and longitude of the triangulation station, obtained from the U.S. Coast and Geodetic Survey and based on the North American Datum of
AEROMAGNETIC SURVEY FLIGHTLINES

Mt. Hood area:
Flight elevation 11,000 feet

Mt. Jefferson area:
Flight elevation 11,000 feet

Combined flightlines from 5,000 foot and 7,000 foot flight elevations

Figure A. 1. Flightline map of the northern Oregon aeromagnetic survey area.
of 1927 (Swick, 1932) give the horizontal coordinates of the transponders with an uncertainty of less than a meter. The elevation uncertainty is less than 3 meters. The determination of the aircraft's horizontal position was not sensitive to errors in elevation. A 300 milli-second delay between the aircraft location determination and the corresponding magnetic reading corrected for the distance between the aircraft and the magnetic sensor towed 20 meters behind the aircraft. The estimated root-mean square uncertainty in the horizontal position of the measurements is 15 meters.

During the entire survey, a base station recorded the outputs of a proton-precession magnetometer and a pressure altimeter identical to those in the aircraft. The base station data provided corrections for the magnetic and pressure altitude measurements. The estimated root-mean square uncertainty in the data is 6 nanoteslas. Larger uncertainties are noted near Three-Fingered Jack. Figure (A 2) shows a histogram of the crossing errors for this survey. Gemperle and Bowers (1977), Gemperle et al. (1978), and Couch (1978) describe the measurements and data acquisition procedures.

The regional geomagnetic field, subtracted from the measurements to yield the magnetic anomalies, was determined from the International Geomagnetic Reference Field of 1980, updated to survey time, and then modified to allow anomaly values in this survey to match anomaly values in the survey of Connard et al. (1983) adjacent to the southern boundary. The minimum-curvature gridding algorithm of Briggs (1975) was used to reduce the aeromagnetic measurements to an equally-spaced grid. The gridded data subsequently were upward continued, where appropriate, by operations in the frequency
Figure A 2. Crossing errors for the northern Oregon aero-magnetic survey.
domain and machine contoured to yield the total field aeromagnetic anomaly map. Boler (1978), Gemperle et al. (1978), Couch (1979), and McLain (1981), describe the processing and reduction of aeromagnetic measurements.
APPENDIX B
DIGITAL FILTERING TECHNIQUES

Part I

Digital filtering of potential field data is an essential technique for the enhancement of wavelengths that are of a particular interest. Depending on the purpose of the investigation, the objective may be to reveal regional trends or remove the regional to accentuate the shorter wavelengths that are due to surface or near surface structures. In the past this was accomplished in the space domain by convolving the data with a set of coefficients that characterize a filter. With the advent of fast machine algorithms for computing the Fourier Series, it became computationally efficient to perform fast convolution in the frequency domain.

Oppenheim and Schafer (1975) provide an excellent review of one-dimensional signal analysis. The following discussion is based on their definitions, derivations and notation, but is applied to two-dimensional signals for use in geophysics.

In discrete-space system theory, signals are represented by sequences of real numbers. Let $x(m,n)$ be a two-dimensional sequence where $m$ and $n$ are integer variables. In a formal sense, $x(m,n)$ refers to the $m,n$th number in the sequence. However, it is more convenient to refer to $x(m,n)$ as the sequence $x(m,n)$. The sequence $x(m,n)$ represents a sampled version of the continuous two-dimensional signal or waveform $x(s,t)$; i.e.,

$$x(m,n) = x(mT,nT) = x(s,t)\big|_{s=mT, t=nT}$$
where \( T \) is the sampling interval or period.

A useful two-dimensional sequence used in digital signal processing is the unit-sample sequence defined as the sequence that is zero except at the origin; ie.,

\[
\delta(m,n) = \begin{cases} 
0 & m,n \neq 0 \\
1 & m=n = 0
\end{cases}
\]

\( \delta(m,n) \) is often referred to as the discrete-space impulse or simply as an impulse.

A sequence \( y(m,n) \) is referred to as a delayed or shifted version of the sequence \( x(m,n) \) if it has values

\[
y(m,n) = x(m-n_0, n-n_0)
\]

where \( n_0 \) is an integer.

An arbitrary sequence can be expressed as a sum of scaled and delayed unit-samples; ie.,

\[
x(m,n) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} x(k,l)\delta(m-k, n-l)
\]

A discrete-space system is a transformation that maps an input sequence \( x(m,n) \) into an output sequence \( y(m,n) \). This can be depicted as in figure (B 1.1).

\[
x(m,n) \xrightarrow{T[ \ ]} y(m,n)
\]

Figure B 1.1. Input sequence mapped by transformation into output sequence \( y(m,n) \) (after Oppenheim and Schafer, 1975).
Two constraints being imposed on $T[\ ]$ are that it is a linear system and shift-invariant. A linear system is defined in the following manner. If $x_1(m,n)$ and $x_2(m,n)$ are the inputs to a linear system and $y_1(m,n)$, $y_2(m,n)$ are their respective outputs, then a system is linear if and only if an input sequence $ax_1(m,n) + bx_2(m,n)$ yields the output sequence $ay_1(m,n) + by_2(m,n)$ where $a$ and $b$ are arbitrary coefficients. A shift-invariant system is defined as a system that if an input sequence $x(m,n)$ produces an output sequence $y(m,n)$, then an input sequence $x(m-n_0, n-n_0)$ would produce the output sequence $y(m-n_0, n-n_0)$.

In the case of a linear, shift-invariant system, a convolution relationship exists between the input sequence and the output sequence $y(m,n)$. If $x(m,n)$, represented by Eq. (B 1.2) is operated on by the system, as in figure (B 1.1), we can write

$$y(m,n) = T[x(k,l)s(m-k, n-i)]. \quad (B \ 1.3)$$

By linearity,

$$y(m,n) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} x(k,l)h(m-k, n-l)]. \quad (B \ 1.4)$$

Let $h(m,n)$ be the response of the system to an impulse $\delta(m,n)$. The sequence $h(m,n)$ is called the unit-sample response or simply the impulse response. If $h(m,n)$ is the response of the system to $\delta(m,n)$, then by shift-invariance, $h(m-k, n-l)$ is the response to $\delta(m-k, n-l)$. Thus (B 1.4) becomes

$$y(m,n) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} x(k,l)h(m-k, n-l) \quad (B \ 1.5)$$

Equation (B 1.5) is called the convolution sum in two-dimensions and
is commonly denoted by

\[ y(m,n) = x(m,n) \ast h(m,n) \quad (B \, 1.6) \]

By a substitution of variables into equation (B 1.5), the alternative expression

\[ y(m,n) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} h(k,1)x(m-k,n-l) \quad (B \, 1.7) \]

\[ = h(m,n) \ast x(m,n) \quad (B \, 1.8) \]

is obtained. In other words, the order in which the sequence is convolved is unimportant. A fundamental property of linear shift-invariant systems is that the response of the system to a sinusoidal input is sinusoidal of the same frequency as the input, with amplitude and phase determined by the system (Oppenheim and Schafer, 1975, pg. 19). If \( x(m,n) = e^{j\omega_1 m} e^{j\omega_2 n} \) is inserted into equation (B 1.7), the output is

\[ y(m,n) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} h(k,1) e^{j\omega_1 (m-k)} e^{j\omega_2 (n-l)} \]

\[ = e^{j\omega_1 m} e^{j\omega_2 n} \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} h(k,1) e^{-j\omega_1 k} e^{-j\omega_2 l} \]

Defining

\[ H(e^{j\omega_1}, e^{j\omega_2}) = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} h(k,1) e^{-j\omega_1 k} e^{-j\omega_2 l} \quad (B \, 1.9) \]

we can write

\[ y(m,n) = H(e^{j\omega_1}, e^{j\omega_2}) e^{j\omega_1 m} e^{j\omega_2 n} \quad . \quad (B \, 1.10) \]
From equation (B 1.10), the output of the system from a sinusoidal input is the input multiplied by a complex weighting factor which represents the change in complex amplitude as a function of frequency \( \omega_1, \omega_2 \). \( H(e^{j\omega_1}, e^{j\omega_2}) \) is called the frequency response of the system which has impulse response \( h(m,n) \).

\( H(e^{j\omega_1}, e^{j\omega_2}) \) is, in general, complex and can be broken down into its real and imaginary parts,

\[
H(e^{j\omega_1}, e^{j\omega_2}) = \text{Re}[H(e^{j\omega_1}, e^{j\omega_2})] + j\text{Im}[H(e^{j\omega_1}, e^{j\omega_2})] \quad (B 1.11)
\]

or in polar form as

\[
H(e^{j\omega_1}, e^{j\omega_2}) = |H(e^{j\omega_1}, e^{j\omega_2})| e^{j\theta(\omega_1, \omega_2)} \quad (B 1.12)
\]

where \( |H(e^{j\omega_1}, e^{j\omega_2})| \) is the magnitude of the frequency response and \( \theta \) is the phase angle of each \( \omega \). The frequency response is a continuous function of \( \omega_1, \omega_2 \) and is doubly periodic with period \( 2\pi \); i.e.,

\[
H(e^{j\omega_1}, e^{j\omega_2}) = H(e^{j(\omega_1+2\pi), e^{j(\omega_2+2\pi)}').
\]

Since the frequency response is periodic, any frequency interval of length \( 2\pi \) is sufficient to completely describe this response.

Periodic functions of \( \omega_1, \omega_2 \) can be represented by a two-dimensional Fourier Series. \( H(e^{j\omega_1}, e^{j\omega_2}) \) is expressed in equation (B 1.9) as a Fourier Series where the impulse response \( h(m,n) \) are the Fourier coefficients. The impulse response can then be obtained from the periodic function \( H(e^{j\omega_1}, e^{j\omega_2}) \) by the relation used to obtain the Fourier coefficients; i.e.,

\[
h(m,n) = 1/4\pi^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} H(e^{j\omega_1}, e^{j\omega_2}) e^{j\omega_1 m} e^{j\omega_2 n} d\omega_1 d\omega_2 \quad (B 1.13)
\]
and,

$$H(e^{j\omega_1}, e^{j\omega_2}) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} h(m,n) e^{-j\omega_1 m} e^{-j\omega_2 n}$$  \hspace{1cm} (A 1.14)$$

Equations (B 1.13) and (B. 1.14) are a Fourier transform pair. Equation (B 1.14) is the forward transform and equation (B 1.13) the inverse transform.

The Fourier transform pair is not restricted to the unit-sample response or system response, it is applicable to any sequence provided that the series of equation (B 1.15) converges (Oppenheim and Schafer, 1975, pg. 21). The Fourier transform pair for a general sequence is

$$X(e^{j\omega_1}, e^{j\omega_2}) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x(m,n) e^{-j\omega_1 m} e^{-j\omega_2 n}$$  \hspace{1cm} (B 1.15)$$

$$x(m,n) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(e^{j\omega_1}, e^{j\omega_2}) e^{j\omega_1 m} e^{j\omega_2 n} d\omega_1 d\omega_2$$  \hspace{1cm} (B 1.16)$$

In order to represent a sequence by a Fourier Series, the sequence \(x(m,n)\) must satisfy the following Dirichlet conditions. If \(X(e^{j\omega_1}, e^{j\omega_2})\) is periodic with period \(2\pi\), and if between \(-\pi\) and \(+\pi\) it is single valued, has a finite number of maxima and minima, a finite number of discontinuities and if \(\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(e^{j\omega_1}, e^{j\omega_2}) d\omega_1 d\omega_2\) is finite, the Fourier Series in equation (B 1.15) converges to \(X(e^{j\omega_1}, e^{j\omega_2})\), where \(X(e^{j\omega_1}, e^{j\omega_2})\) is continuous and to the midpoint at the discontinuities (after Boas, 1983, pg. 313).

For geophysical data, the input sequence \(x(m,n)\) is generally real. For a real sequence the symmetry properties become particularly useful. The Fourier transform of a real sequence is conjugate
symmetric; i.e.,

\[ X(e^{j\omega_1}, e^{j\omega_2}) = X^* (e^{-j\omega_1}, e^{-j\omega_2}). \]

Expressing \( X(e^{j\omega_1}, e^{j\omega_2}) \) in terms of its real and imaginary parts,

\[ X(e^{j\omega_1}, e^{j\omega_2}) = \text{Re}[X(e^{j\omega_1}, e^{j\omega_2})] + j\text{Im}[X(e^{j\omega_1}, e^{j\omega_2})] \]

it follows that

\[ \text{Re}[X(e^{j\omega_1}, e^{j\omega_2})] = \text{Re}[X(e^{-j\omega_1}, e^{-j\omega_2})] \]

and

\[ \text{Im}[X(e^{j\omega_1}, e^{j\omega_2})] = -\text{Im}[X(e^{-j\omega_1}, e^{-j\omega_2})]. \]

The real part of the Fourier transform is an even function of \( \omega_1, \omega_2 \) and the imaginary part is odd. Similarly in polar coordinates, the magnitude is an even function of \( \omega_1, \omega_2 \) and the phase is odd.

Potential fields, such as the earth's magnetic and gravitational field, represent smooth continuous functions that must be sampled in order to be studied. An understanding of the relationship between a discrete-space signal and a continuous function derived by periodic sampling is essential because of the effects sampling may have on the analysis. Oppenheim and Schafer (1975, pg. 29) give the relation between the frequency response of the discrete-time sequence and the Fourier transform of the continuous-time waveform in a one-dimensional sense. The argument presented below applies equally well to two-dimensional applications in geophysics since the sampling is often done in a one-dimensional sense (e.g. aero-magnetic flightlines).
\[ X(e^{j\omega}) = \frac{1}{T} \sum_{r=-\infty}^{+\infty} X_a(j\omega/T + j2\pi r/T) \quad (B\ 1.17) \]

Equation (B 1.17) shows the periodic frequency response of the discrete-time sequence to consist of the sum of an infinite number of frequencies from the frequency response of the continuous-time waveform. In the case where the frequency response of the continuous-time waveform is bandlimited between \( |\omega| \leq \pi/T \), equation (B 1.17) shows that in the frequency range \( |\omega| \leq \pi/T \),

\[ X(e^{j\omega}) = \frac{1}{T} X_a(j\omega). \]

The frequency response of the discrete-time sequence is then identical to the sampled continuous waveform to within a scale factor. If the frequency response of the continuous-time waveform is greater than \( \pi/T \); i.e., \( |\omega| > \pi/T \), then higher frequencies of the continuous-time waveform are folded into the lower frequencies of the discrete-time signal. This folding of higher frequencies is called aliasing and once the energy enters a sampled sequence, it cannot be distinguished from the true signal. The way to avoid aliasing is to adjust the sampling frequency so it forces the frequency response of the waveform to be bandlimited between \( |\omega| \leq 1/T_{\text{new}} \), where \( 1/T_{\text{new}} \) is the adjusted sampling frequency. A sampled discrete-time signal will then be an exact reconstruction of its continuous-time waveform if the sampling frequency is at least twice the highest frequency present in the continuous waveform (Shannon, 1948). This sampling frequency is referred to as the Nyquist frequency. In geophysical applications, to design a sampling strategy, the highest frequency
of the continuous-space waveform must be known. Knowledge of the highest frequency present is rarely met in practice, so a sampling strategy might be to sample the continuous waveform at four times the estimated highest frequency in the survey area. A low-pass filter (anti-aliasing filter) applied to the sampled sequence, with a filter bandwidth 2.5-2.7 times the highest frequency to be resolved, should prevent distortion of the resolvable frequencies provided a suitable filter is used.
The Fourier analysis techniques summarized in the preceding chapter apply to continuous two-dimensional functions and sequences of infinite area. In practice, geophysical data are two-dimensional sequences of finite area, so an alternative Fourier representation will be developed that applies to finite-area sequences. This alternative Fourier representation is called the Discrete Fourier Transform (DFT). The DFT is a Fourier representation of a finite-area sequence which is itself a finite-area sequence corresponding to samples equally spaced in the frequency domain. In the previous chapter, the Fourier transform of an infinite sequence was a continuous function of frequency. The DFT is important in geophysical applications thanks to fast machine algorithms for computing the DFT (Cooley and Tukey, 1965).

For a finite, discrete-area sequence that is non-zero for a finite area in the (m,n) plane, the Discrete Fourier transform pairs are

\[
X(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m,n) e^{-j2\pi mk/M} e^{-j2\pi nl/N} \quad (B 2.1)
\]

\[
x(m,n) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} X(k,l) e^{j2\pi mk/M} e^{j2\pi nl/N} \quad (B 2.2)
\]

The sampled data \(x(m,n)\) and the Fourier Series coefficients \(X(k,l)\) represent finite-area sequences (Oppenheim and Schafer, 1975, pg. 117).
For geophysical interpretations of potential field data such as gravity and magnetics, filtering out undesirable frequencies can enhance and isolate wavelengths due to structures of interest. To filter a data set \( x(m,n) \) of length \( M_1, N_1 \) with a filter function \( h(m,n) \) of length \( M_2, N_2 \), a linear convolution is desired, i.e.,

\[
y(m,n) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} x(p,q) h(m-p, n-q)
\]  

(B 2.3)

Linear convolution in the space domain results in an output \( y(m,n) \) with size \( M_3 = M_1 + M_2 - 1 \) and \( N_3 = N_1 + N_2 - 1 \). However, two-dimensional linear convolution becomes less efficient after grid size 10 x 10 compared to multiplication in the frequency domain (Mersereau et al., 1976) using fast machine algorithms for the computation of the two-dimensional DFT (Clayton, personal commun., listed in McLain, 1981). Let \( X(k,l) \) and \( H(k,l) \) be the two-dimensional DFTs of \( x(m,n) \) and \( h(m,n) \) with sizes \( M_1, N_1 \) and \( M_2, N_2 \) respectively, augmented with zeros if necessary, so both sequences are of sizes \( M \times N \). Then doing the fast convolution,

\[
Y(k,l) = X(k,l) \ast H(k,l)
\]  

(B 2.4)

and taking the inverse DFT gives the filtered output

\[
y(m,n) = \frac{1}{M N} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} Y(k,l) e^{-j2\pi mk/M} e^{-j2\pi nl/N}
\]  

(B 2.5)

The filtered output \( y(m,n) \) is of area \( M \times N \) and represents a circular convolution in two-dimensions. For a discussion on the effects of circular convolution, see Oppenheim and Schafer, 1975, pg. 105). This circular convolution aliases information back into the signal thereby distorting the desired filtered output. To obtain
a linear convolution and avoid the aliasing inherent in a circular convolution, the circular (fast) convolution must have the same end result as a linear convolution. As was shown above for the linear convolution, the area of the filtered output \( y(m,n) \) was \( M_1 + M_2 - 1 \) and \( N_1 + N_2 - 1 \). For multiplication in the frequency domain \( M \) and \( N \) must be chosen so that \( M > M_1 + M_2 - 1 \) and \( N > N_1 + N_2 - 1 \). For finite area sequences \( x(m,n) \) and \( h(m,n) \), this involves augmenting the sequences in the spacial domain with zeros up to or exceeding lengths \( M \) and \( N \). The filtered output will then be equivalent to the filtered output from a linear convolution in the spacial domain (Oppenheim and Schafer, 1975, pg. 120).
Because the two-dimensional filter impulse response sequence \( h(m,n) \) is a finite-area sequence, it possesses certain desirable properties from the point of view of two-dimensional filter design. Two-dimensional finite impulse response (FIR) sequences are always stable, and with appropriate delay can be made realizable. Furthermore, their frequency response can be designed to have exactly linear phase.

Linear phase filters are important in geophysical applications where certain anomalies in gravitational and magnetic fields are associated with sources in the crust. Processing with linear phase filters maintains the position of the anomaly relative to its source, whereas non-linear phase filters induce a phase distortion. The filters designed in this paper will approximate a circular function, ie.,

\[
H(e^{j\omega_1}, e^{j\omega_2}) = H(e^{j\sqrt{\omega_1^2 + \omega_2^2}/2})
\]

The process to design FIR filters involves the following basic steps:
1. Decide on the appropriate set of specifications for the digital filter.
2. Solve the approximation problem to determine the coefficients of the impulse response \( h(m,n) \).
3. Evaluate the filtered output to verify that the performance meets the specifications.
For the design of two-dimensional FIR filters, the specifications take the form of a tolerance scheme. This tolerance scheme is depicted in figure (B 3.1) for a one-dimensional low-pass filter.

![Tolerance specifications for one-dimensional low-pass filter](image)

Figure (B 3.1). Tolerance specifications for one-dimensional low-pass filter (after Oppenheim and Schafer, 1975, pg. 196).

In the passband, the magnitude of the frequency response approximates one within the tolerance $\pm \delta_1$; i.e.,

$$1 - \delta_1 \leq |H(e^{j\omega_1}, e^{j\omega_2})| \leq 1 + \delta_1 \quad 0 \leq \omega_1, \omega_2 \leq \omega_p$$

In the transition band $(\omega_s - \omega_p)$, the magnitude response drops smoothly from the passband $\omega_p$ to the stopband $\omega_s$ where the magnitude response tolerance is within $\pm \delta_2$; i.e.,

$$|H(e^{j\omega_1}, e^{j\omega_2})| \leq \delta_2 \quad \omega_s \leq \omega_1, \omega_2 \leq \pi$$
The approximation problem also implies a decision about what type of two-dimensional FIR filter will be used. This decision includes the possibility of window filters, frequency sampling, or optimal FIR filter designs. Rabiner and Gold (1975) provide an excellent discussion and comparison of the above three filter designs. This paper will concentrate on the design and implementation of an optimal two-dimensional FIR filter.

McClellen (1973) developed a technique that consists of transforming a one-dimensional filter into a two-dimensional filter by a change of variables. This "McClellen transformation" (Mersereau et al. 1976) includes such filters as low-pass and band pass circularly symmetric filters. The advantage of this technique is that the majority of the effort is in designing the one-dimensional filter. To aid in the design, a machine algorithm developed by McClellen et al. (1973) designs an optimal one-dimensional linear phase FIR filter from user-supplied specifications $\delta_1, \delta_2, \omega_p, \omega_s$, and $N$.

The design problem is formulated as a Chebyshev approximation problem where the approximating function $P(e^{j\omega})$, a sum of $r$ cosine functions, minimizes the maximum weighted error in approximating a desired frequency response; i.e.,

$$||E(e^{j\omega})|| = \max_{0 \leq \omega \leq \pi} W(e^{j\omega})|D(e^{j\omega}) - P(e^{j\omega})|$$  \hspace{1cm} (B 3.1)

Where $W(e^{j\omega})$ is a positive weight function and $||E(e^{j\omega})||$ denotes minimum value. For realizable or causal FIR filters ($h(m,n) = 0$ for $m, n < 0$), whose symmetrical impulse response is odd, the frequency response
\[ H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n} \]  
\hspace{2cm} (B 3.2)

can be written in the form,
\[ H(e^{j\omega}) = e^{-j\omega(N-1)/2} \left[ \sum_{n=0}^{(N-1)/2} a(n) \cos(\omega n) \right] \]  
\hspace{2cm} (B 3.3)
(Rabiner and Gold, 1975, pg. 81). The term \( e^{-j\omega(N-1)/2} \) contains the phase and group delay information. The group delay is defined by Oppenheim and Schafer (1975, pg. 19) as the negative of the first derivative of the phase with respect to frequency \( \omega \). The term
\[ \sum_{n=0}^{(N-1)/2} a(n) \cos(\omega n) \]
is the frequency response, in this case a real function. The frequency response in equation (B 3.3) is a sum of cosine functions and is defined to be \( P(e^{j\omega}) \), i.e.,
\[ P(e^{j\omega}) = \sum_{n=0}^{(N-1)/2} a(n) \cos(\omega n) \]  
\hspace{2cm} (B 3.4)
where
\[ a(0) = h((N-1)/2) \text{ and } a(n) = 2h((N-1)/2 - n) \]
\[ n = 1, 2, \ldots (N-1)/2 \]

The user defines \( D(e^{j\omega}) \), a desired (real) frequency response of the filter and \( W(e^{j\omega}) \), the weighting function on the approximation error. \( W(e^{j\omega}) \) enables the relative size of the error in different frequency bands to be chosen. The Chebyshev approximation problem now becomes finding the set of coefficients \( a(n) \) that minimize the maximum error \( E(e^{j\omega}) \) over the frequency bands where the approximation is being performed. Parks and McClellen (1972) reformulated a
theorem of approximation theory, the alternation theorem, to obtain a solution to equation (B.3.1).

If \( P(e^{j\omega}) \) is a linear combination of \( r \) cosine functions,

\[
P(e^{j\omega}) = \sum_{n=0}^{r-1} a(n) \cos(\omega n)
\]

then in order for \( P(e^{j\omega}) \), on the closed frequency interval \([0, \pi]\), to be the unique best Chebyshev approximation to \( D(e^{j\omega}) \), it is necessary and sufficient that the error function \( E(e^{j\omega}) \) exhibit on \([0, \pi]\) at least \((r+2)\) extremal frequencies (alternations). Thus

\[
E(e^{j\omega_i}) = -E(e^{j\omega_{i+1}}), \quad i = 1, 2 \ldots r,
\]

and

\[
E(e^{j\omega_i}) = \max_{0 \leq \omega \leq \pi} [E(e^{j\omega})]
\]

with \( \omega_0 < \omega_1 < \ldots < \omega_{r+1} \) and \( \omega_i \) contained in \([0, \pi]\). This alternation theorem gives a set of necessary and sufficient conditions on the weighted error function \( E(e^{j\omega}) \) such that if these conditions are met, the coefficients \( a(n) \) give the unique best approximation to the desired frequency response \( D(e^{j\omega}) \).

A computer program (PROGRAM FIR) based on the algorithm by McClellen et al. (1973) and modified for use on a Data General Eclipse mini-computer was used to compute the one-dimensional optimal FIR filter coefficients. The general optimal filter design used in this study consisted of the following steps:

1. An initial estimate of the filter length \( N \) was made for a specified \( \omega_p, \omega_s, \delta_1, \) and \( \delta_2 \) using approximate design relationships for low-pass filters (Rabiner, 1973).

2. The estimated filter length \( N, \omega_p, \omega_s, \delta_1 \) and \( \delta_2 \) were inputs
into PROGRAM FIR. The output is the optimal filter unit-sample response.

Steps 1 and 2 can be repeated in an iteration sense until the output of step 2 meets the users specifications.

The one-dimensional filter computed by PROGRAM FIR must be transformed into a two-dimensional filter for application in the spacial domain. This is accomplished by a transformation attributed to McClellen (1973) that transforms an optimal one-dimensional filter into a two-dimensional filter preserving the optimal characteristics. A two-dimensional, linear phase FIR filter with impulse response

\[ h(m,n) \quad m = 0, 1, \ldots, M-1 \quad n = 0, 1, \ldots, N-1 \]

has a frequency response defined by the two-dimensional DFT as

\[
H(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} h(m,n) e^{-j2\pi mk} e^{-j2\pi n l/M} \tag{B 3.5}
\]

If the impulse response \((M, N)\) is odd and symmetric; i.e.,

\[ h(m,n) = h(M-1-1, n) \quad m = 0, 1, \ldots, (M-1) \]
\[ h(m,n) = h(m, N-1-1) \quad n = 0, 1, \ldots, (N-1) \]

then the frequency response can be rewritten as

\[
H(f_1, f_2) = e^{-j2\pi[M-1/2]f_1} e^{-j2\pi[N-1/2]f_2} x \sum_{p=0}^{(M-1)/2} \sum_{q=0}^{(N-1)/2} a(p,q) \cos(2\pi f_1 p) \cos(2\pi f_2 q)
\]

where
\[ a(0,0) = h((M-1)/2, (N-1)/2) \]
\[ a(0,q) = 2h((M-1)/2, (N-1)/2-q) \]  
\[ a(p,0) = 2h((M-1)/2-p, (N-1)/2) \]
\[ a(p,q) = 2h((M-1)/2-p, (N-1)/2-q) \]

Set
\[ \hat{H}(f_1, f_2) = \sum_{p=0}^{(M-1)/2} \sum_{q=0}^{(N-1)/2} a(p,q) \cos(2\pi f_1 p) \cos(2\pi f_2 q) \]  \( \text{(B 3.6)} \)

For the one-dimensional DFT, the frequency response can be rewritten as in equation (B 3.3) as
\[ H(f) = e^{-jf2\pi (N-1)/2} \sum_{n=0}^{(N-1)/2} a(n) \cos(2\pi fn) \]  \( \text{(B 3.7)} \)

and
\[ \hat{H}(f) = \sum_{n=0}^{(N-1)/2} a(n) \cos(2\pi fn) \]  \( \text{(B 3.8)} \)

The frequency response \( H(f) \) can be rewritten as a trigonometric polynomial in \( f \)
\[ \hat{H}(f) = \sum_{n=0}^{(N-1)/2} \hat{a}(n) [\cos(2\pi f)]^n \]  \( \text{(B 3.9)} \)

by the choice of suitable coefficients \( \hat{a}(n) \). Taking the McClelllen transformation of variables,
\[ \cos(2\pi f) = A \cos(2\pi f_1) + B \cos(2\pi f_2) \]
\[ + C \cos(2\pi f_1)\cos(2\pi f_2) + D \]  \( \text{(B 3.10)} \)

and substituting into equation (B 3.9), equation (B 3.9) transforms
\[
\hat{H}(f) = \sum_{p=0}^{(M-1)/2} \sum_{q=0}^{(N-1)/2} a(p,q) \cos(2\pi f_1)^m \cos(2\pi f_2)^n
\]

which can be rewritten uniquely in the same form as \( \hat{H}(f_1, f_2) \) where

\[
\hat{H}(f) = \sum_{p=0}^{(M-1)/2} \sum_{q=0}^{(N-1)/2} a(p,q) \cos(2\pi f_1 p) \cos(2\pi f_2 q) = \hat{H}(f_1, f_2) \quad (B\,3.11)
\]

Thus the McClellen transformation preserves the proper form for a two-dimensional, linear phase FIR filter (McClellen, 1973). By setting \( A = B = C = -D = 0.5 \), and solving for \( f_2 \) in terms of \( f_1 \), the transformation gives

\[
f = 1/2\pi \cos^{-1} \left[ \frac{\cos(2\pi f) + 0.5 - 0.5 \cos(2\pi f_1)}{0.5 + 0.5 \cos(2\pi f_1)} \right] \quad (B\,3.12)
\]

Figure (B 3.2) show how the one-dimensional frequency \( f \) maps to the two-dimensional frequency domain \( f_1, f_2 \). Contour lines representing the one-dimensional frequency response are mapped so that the transformed two-dimensional frequency response is a constant.
equal to the one-dimensional frequency at \( f \). The resulting contour lines have approximately circular symmetry and are suitable for the design of circularly symmetric two-dimensional filters. Along the line \( f_2 = 0 \), the two-dimensional frequency response is a one-dimensional frequency response of \( f_1 \). For \( f_2 = 0 \), equation (B 3.10) becomes

\[
\cos(2\pi f) = \cos(2\pi f_1),
\]

where the two-dimensional frequency response along \( f_2 = 0 \), is exactly the same as the optimal one-dimensional response in \( f \) used to generate the two-dimensional filter. Thus along \( f_2 = 0 \), the minimized maximum error of the one-dimensional filter is a lower bound for the error of the two-dimensional filter. Since the transformation preserves the minimized maximum error, the resulting two-dimensional filter is optimal in the Chebyshev sense (McClellen, 1973).

In the previous section, the importance of performing a linear convolution during a filtering operation was emphasized. If the data grid \( x(m,n) \) is of the area \((M_1,N_1)\), and the transformed optimal two-dimensional filter is size \((M_2,N_2)\) where \( M_2 = N_2 \), the linearly convolved output will be a grid of area \([(M_1+M_2-1,N_1+N_2-1)]\). In order to ensure that multiplication in the frequency domain (circular convolution) has the effect of a linear convolution, \( M \) and \( N \) must be chosen so that \( M > M_1 + M_2 - 1 \) and \( N > N_1 + N_2 - 1 \) (Oppenheim and Schafer, 1975, pf. 119). For example if the data grid of area \((256,256)\) is linearly convolved, in the spacial domain, with the filter of area \((128,128)\), the filtered output has an area of
For multiplication in the frequency domain, the spatial domain data and filter grids must be padded with zeros up to or exceeding an area of (383,383) as in figure (B 3.3). Fast machine algorithms for computing the two-dimensional DFT require the grid dimensions of the input grids to be a power of two. Additional padding from (383,383) to (512,512) is required.

The filtered output from a linear convolution contains a linear phase shift corresponding to a finite delay. For a filter impulse response that is of area \((M_2,N_2)\) where \(M_2, N_2\) is odd and symmetrical, the filtered output is delayed by an integral number of samples. In this case, the phase shift corresponds to a delay of \([(M_2-1)/2, (N_2-1)/2]\) grid points. Figure (B 3.4) shows the delayed sequence \(y(m,n)\) after filtering. For finite-area grids or sequences, it is desirable to remove the delay from the filtered output so that the final filtered grid \(y(m,n)\) is the same size as the original input grid \(x(m,n)\).
Figure B 3.3. Spatial domain representation of a finite-area data sequence $x(m,n)$, padded with zeros up to size $(M-1,N-1)$ to obtain a linear convolution.
Figure B 3.4. Filtered output from a linear convolution with constant group delay in both the m and n directions.
APPENDIX C

TECHNIQUES OF ANALYSIS FOR THE NORTHERN OREGON AEROMAGNETIC DATA

This study applies the techniques developed by Spector (1968), Spector and Grant (1970), Shuey et al. (1977), Boler (1978), Connard (1979), Connard et al. (1983), and McLain (1981) for determining magnetic source depths and depth-to-the-source bottom and depth of the Curie-point isotherm. Information about the Curie-point isotherm depth is helpful in assessing the geothermal potential of the northern Oregon Cascades study area.

In order to apply this Curie-point technique to aeromagnetic data, the earth's magnetic field must be sampled in order to be analyzed. Sampling theory predicts the highest frequency that can be resolved sampling a continuous signal for a given sampling rate or frequency. This relationship between the continuous-space waveform of the earth's magnetic field and the discrete-space sequence \( x(m,n) \) representing the sampled magnetic field is discussed in Appendix B, Part I. The sampled data can be represented in the frequency domain by the Discrete Fourier Transform (DFT) as,

\[
F(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X(m,n) e^{-j2\pi mu/M} e^{-j2\pi mv/N}
\]

where \( M \) and \( N \) are the total number of sampled points in the \( m \) and \( n \) direction.
Mean Depth to Source Calculations from the Energy Spectrum

Bhattachryya (1966) developed the Fourier transform for the total field magnetic anomaly of a vertical sized prism with horizontal top and bottom as,

\[ F(u,v) = 2\pi Me^{-h(u^2+v^2)^{1/2}} \cdot (1-e^{-t(u^2+v^2)^{1/2}}) \]
\[ \cdot S(u,v) \cdot R_p(u,v) \cdot R_g(u,v) \]

where

- \( M \) = magnetic moment/unit depth
- \( h \) = depth to the top of the prism
- \( t \) = thickness of the prism
- \( S \) = factor for horizontal size of the prism
- \( R_p \) = factor for magnetization direction of the prism
- \( R_g \) = factor for geomagnetic field direction

The energy spectrum of the magnetic anomaly was described by Spector (1968) as,

\[ E(u,v) = F(u,v) \cdot F^*(u,v) \]

where \( F^*(u,v) \) denotes the complex conjugate of \( F(u,v) \).

Transcribing the energy spectrum in polar wavenumber coordinates in the \( u, v \) frequency plane, Spector (1968) showed that,

\[ E(r,\theta) = 4\pi M^2e^{-2hr} \cdot (1-e^{-tr})^2s^2(r,\theta)R_p(\theta)R_g(\theta) \]

where \( r = (u^2+v^2)^{1/2} \) and \( \theta = \tan^{-1}(u/v) \).

Spector and Grant (1970) developed a model that assumed magnetic anomalies are due to a number of independent ensembles of rectangular
vertical-sided parallelepipeds or prisms. Each ensemble has a joint
frequency distribution for depth $h$, width $a$, length $b$, depth extent
t. Further, the direction of magnetization $R_g(e)$ is assumed constant
because the geomagnetic field direction does not vary appreciably
over the study area. For $R_g(e)$ constant, Spector and Grant (1970)
show the energy spectrum becomes,

$$<E(r,\theta)> = 4\pi^2M^2 <R^2(\theta)> \cdot e^{-2hr} \cdot \langle 1-e^{-tr} \rangle^2 \cdot \langle R_p^2(\theta) \rangle \cdot \langle S^2(r,\theta) \rangle$$

where $< >$ indicates the expected value from an ensemble of vertical-
sided prisms. Assuming the difference between inclination and declin-
ation of the magnetization of the rocks is not too large ($<20^\circ$),
rotation-to-the-pole gives,

$$<E(r,\theta)> = 4\pi M^2 e^{-2hr} \cdot \langle 1-e^{-tr} \rangle^2 \cdot \langle S^2(r,\theta) \rangle$$

Because of the magnetic latitude rotation-to-the-pole has virtually
no effect on the anomalies in the northern Oregon study area and can
be considered an optional step in the analysis.

Finally, taking the average with respect to $\theta$ gives,

$$<E(r)> = 4\pi^2M^2 e^{-2hr} \cdot \langle 1-e^{-tr} \rangle^2 \cdot \langle S^2(r) \rangle$$

Spector (1968) show for $\Delta h/h < .25$, $e^{-2hr} = e^{-2hr}$. Spector and
Grant (1970) state that $e^{-2hr}$ is the dominating term in the power
spectrum and the logarithm of this line approximates a straight line
whose slope is $-2h$. Thus depths to the magnetic source top can be
computed from the slope of the logarithm of the energy spectrum.

If the magnetic source bottom is being resolved, the $(1-e^{-tr})$
term in combination with $e^{-2hr}$ introduces a spectral peak in the
energy spectrum. The frequency of the spectral peak shifts toward lower frequencies with increasing values of t. Maximizing \((1 - e^{-tr})^2 \cdot e^{-2hr}\) with respect to r, McLain (1981) shows the derivation for

\[
 f_{\text{max}} = \frac{1}{2\pi (\bar{d} - h)} \ln \left( \frac{\bar{d}}{h} \right)
\]

where \(\bar{h}\) is the mean depth to the source top, \(\bar{d}\) is the depth to the bottom of the source, and \(f_{\text{max}}\) is the spatial frequency of the spectral peak. Solving for \(\bar{d}\), the above equation becomes

\[
 \bar{d} = \frac{\bar{\epsilon}}{1 - \exp(-2\pi \bar{\epsilon} f_{\text{max}})}
\]

where \(\bar{\epsilon}\) is the mean ensemble thickness \((\bar{d} - h)\). Several of these curves are shown in figure 35 and figure 38. Unfortunately, depth-to-the-source-bottom calculations depend on an estimate of \(\bar{\epsilon}\). Following the suggestion of Shuey et al. (1977), and implementation by Connard (1979), Connard et al. (1983), and McLain (1981) a minimum source thickness of five kilometers was used for estimating the Curie-point isotherm.

The shape factor \(S\) also influences the spectrum. (Shuey et al., 1977). Spector and Grant (1970) give the shape factor as

\[
 S(r, \theta) = \frac{\sin(a r \cos \theta)}{a r \cos \theta} \frac{\sin(b r \cos \theta)}{b r \cos \theta}
\]

for a rectangular body whose dimensions are \(2a \times 2b\). Shuey et al. (1977) show for a magnetic source body whose dimensions are comparable
to the dimensions of the survey area, the spectral peak is displaced toward the zero frequency. Regional anomaly gradients would be indistinguishable from those generated by such a body so the anomalies are detrended before computing the energy spectrum (Connard, 1979). Spectral estimates with a one-point peak at the fundamental frequency indicate the source bottom is not being resolved. Therefore, spectral peaks consisting of one point peaks are inadequate for estimating the depth to the source bottom. For grids that were unable to resolve the source bottom, the source bottom must be deeper than \( \frac{L_X}{2\pi} \), where \( L_X \) is the length of one side of the survey area in kilometers.