AN ABSTRACT OF THE THESIS OF

Myung Hun Lee for the degree of Doctor of Philosophy

in Electrical and Computer Engineering presented on May 7, 1976

Title: Statistics of speckle propagation through the turbulent atmosphere

Abstract Approved: Redacted for privacy

J. Fred Holmes

This work is concerned with the analysis of the first and second-order statistics of received irradiance of a single mode laser beam after scattering from the diffuse target. The technique is based upon the extended Huygens-Fresnel formulation and includes the effects of the turbulent atmosphere on the laser beam as it propagates back to the receiver. Formulation have been developed for both the focused and collimated cases. It is first assumed in the analysis that phase perturbation of spherical wave is the dominant effect due to the atmosphere. Utilizing this assumption, it is shown that the fields at the receiver are Gaussian and that the space-averaged, spatial power spectral density at the receiver is "white." Based on these results, it is assumed that the field statistics at the receiver are jointly Gaussian. This appears to be a reasonable assumption and allows a closed-form solution for the variance and covariance to be derived. For a point detector, it is found that the normalized variance is unity and independent of the turbulence strength, Cn^2, and that the transverse correlation length becomes proportional to ρ_o as Cn^2 increases. The time delayed

covariance function for the received intensity is formulated using the joint Gaussian assumption and invoking Taylor's Hypothesis. From this, the slope of the time delayed covariance function and the temporal frequency spectra are evaluated. An additional formulation is developed in which the amplitude perturbation term is included and its effect on the probability distribution and the covariance of irradiance is examined. The conditions for validity of the jointly Gaussian assumption are discussed and an analysis of the near received irradiance in the presence of "glints" (specular reflector) is presented using the phase dominance assumption.

Statistics of Speckle Propagation Through the Turbulent Atmosphere

by

Myung Hun Lee

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

June 1976

APPROVED:

Redacted for privacy

Professor of Electrical and Computer Engineering in charge of major

Redacted for privacy

Head of Department of Electrical and Computer Engineering

Redacted for privacy

Dean of Graduate School

Date thesis is presented May 7, 1976

Typed by Beverly Kyler for Myung Hun Lee

ACKNOWLEDGMENT

There never seem to exist the proper words with which one says thank you to all those who helped in the preparation of the thesis. I would like to thank everyone who allowed me to obtain its full value by sharing it with me in the Department of Applied Physics at Oregon Graduate Center where the research for this dissertation was conducted.

However, a very special thank you is due my adviser, Dr. J. Fred Holmes for his constant willingness to help and provide the interaction so necessary for research. Without his enthusiasm and support, the work surely would not have culminated in the way that it did. I also wish to thank Dr. J. R. Kerr for his many discussions and advice.

I would like to express gratitude to the Oregon Graduate Center, U. S. Army Electronics Command, U. S. Air Force Weapon Laboratory, and Defense Advanced Research Projects Agency of the D.O.D. for financial support during the period of research. Finally, I would like to thank Beverly Kyler for excellent typing and cooperation throughout the preparation of this thesis.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

I. BACKGROUND AND INTRODUCTION

It is well known that the scattering from the rough surface or a random diffuser for all types of wave-motion produces a pattern of light and dark patches, called speckle, which are the result of spatial interference effects.

Work in the area of speckle statistics by other researchers has primarily been concentrated on the nature and statistics of the target surface,[1-6] propagation of the speckle field without turbulence[7-18] and the effects of speckle on image quality.[19-28] The only work that appears to have been done on speckle propagation through the turbulent atmosphere involves a vertical path.[29] In that analysis, the assumption is made that the entire target lies within a single isoplanatic patch and consequently the work applies only to space objects as a target. Some analytic work on propagation of fields from a diffuse source through the turbulent atmosphere has been done by NOAA.[30] In that analysis, the source is assumed to also be temporally incoherent. This effectively neglects the speckle and makes the work not strictly applicable to the case of a laser illuminated, diffuse target as a source. When EM wave propagates through the turbulent atmosphere, it is distorted by the result of variations in the refractive index, which is mainly caused by temperature fluctuations in the atmosphere. In order to understand the effect of turbulence on the propagation of the speckle field, an analysis has been made of the first and the second order statistics of the received intensity after scattering from a diffuse target. The treatment is based on the extended Huygens-Fresnel formulation and includes the effects of the turbulent atmosphere on the single mode laser beam as it propagates to the target and on the speckle as it propagates back to the receiver. Formulations have been developed for the illuminating laser beam focused on the target and also for the collimated case. A general formulation for the covariance is developed which when reduced to its simplest form is represented by a threefold integral. Under certain conditions, phase perturbation of the waves is the dominant effect due to the atmosphere. Utilizing this as an assumption, it is shown that the fields at the receiver are Gaussian distributed, and that the space averaged, spatial power spectral density is "white." Based on this result, it is assumed that the field statistics at the receiver are jointly Gaussian. This appears to be a reasonable assumption and has the advantage of allowing a closed form solution for the second order statistics of the irradiance to be derived. By considering the general formulations for the covariance, it can be shown that this assumption is valid at least for weak turbulence and saturated turbulence conditions. The relationship between the statistics of received intensity and the crosswind components has been formulated from the covariance by utilizing Taylor's Hypothesis. From this, the slope of the time lagged, covariance function and the temporal frequency spectra have been evaluated using the joint Gaussian assumption at the receiver.

The application of interest for the above work is to aid in the understanding of the performance of coherent adaptive optical systems in the presence of target and turbulence-induced speckle and to provide a theoretical basis for remote crosswind sensing using a pulsed laser in conjunction with a diffuse target.

II. THEORETICAL DESCRIPTION

Significant progress has been made in the physical and analytical understanding of turbulence effects on the dynamics of target-reflected radiation. The important quantities are the variance of irradiance (σ_I^2), covariance of irradiance [$C_I(p)$], mutual coherence function ($\Gamma(p)$], probability distribution function of irradiance, and spatial and temporal power spectrum. These quantities are of interest in the plane of the active laser transceiver, and include turbulence effects on both the illuminating radiation from the transmitter to the target and on the scattered radiation over the return path (Fig. 1).

The treatment to be given below utilizes primarily the extended Huygens-Fresnel principle as applied to a turbulent path.[31,32] Until otherwise stated, the laser is assumed to be a coherent (TEM_{00}) source, collimated or focused, with a perfectly diffuse target. In all cases, we attempt to show clearly the assumptions and approximations that are made and to discuss their implications. An important feature is the definition of distinct (asymptotic) parameter conditions applying to any given configuration; there are generally six such conditions, representing the possible permutations of inequalities between the three pertinent parameters: Fresnel zone size $(L/k)^{\frac{1}{2}}$, coherence radius (ρ_o) and speckle size in the absence of turbulence. Each such condition will in general carry a distinct physical and analytical interpretation. It will be seen that cases of strong scintillations ("saturation" or multiple scattering) are included in these conditions, so that the treatment is general.

We first consider (Sec. II.A) the most common situation, i.e.,

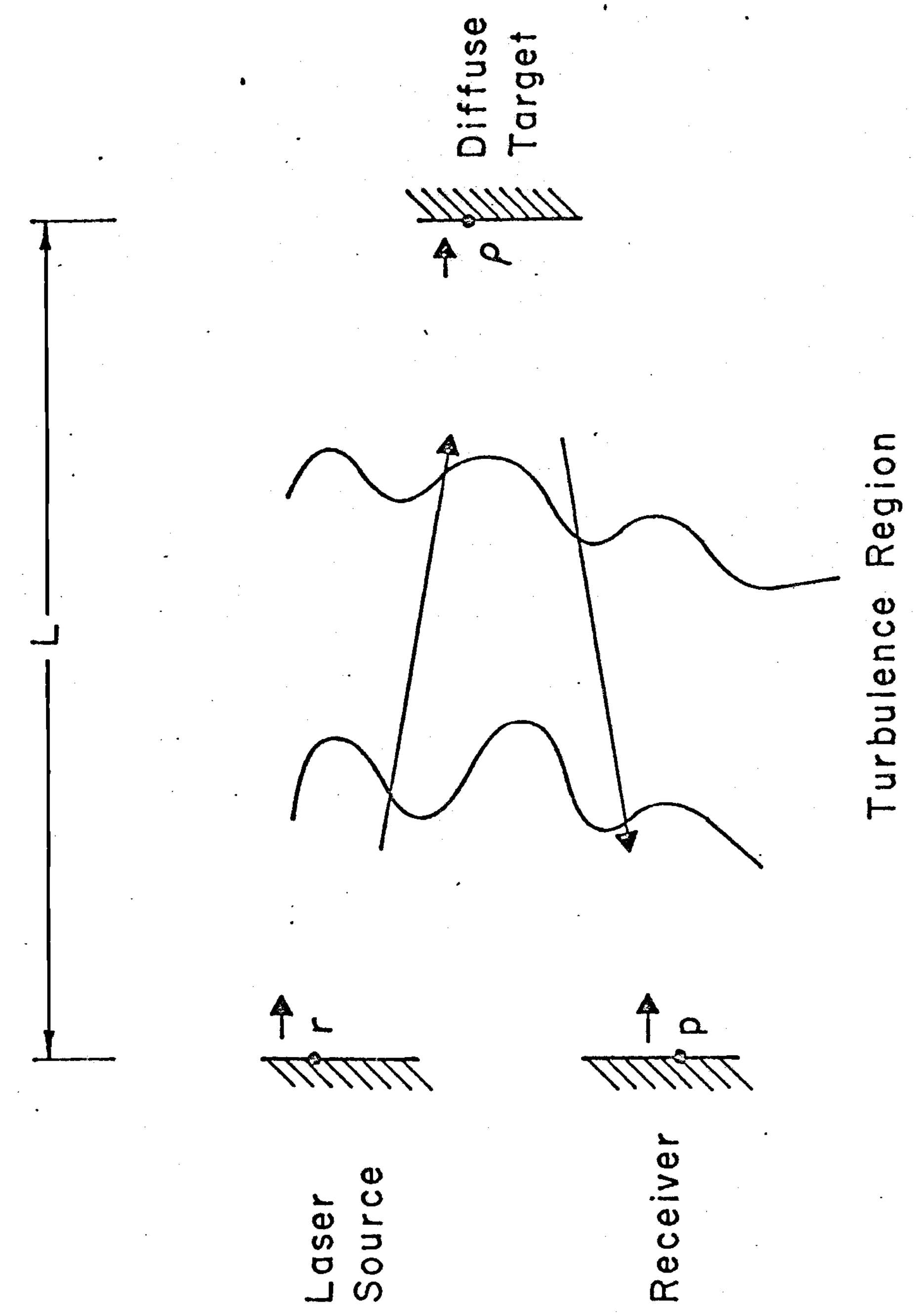

Figure 1. Illuminator, Target, and Receiver Configuration

that in which the primary effect of the turbulence on the reflected radiation arises through the perturbation of the phase term in the associated Green's function. The implications of this assumption in terms of the field statistics are also explained.

We then generalize the development (Sec. II.B) to include the effects of the amplitude perturbation term in the Green's function. The implications are again explored in detail. In Sec. II.C, we treat the first-order statistics of irradiance for a target containing one or more glints.

II.A. BASIC IRRADIANCE STATISTICS AND MUTUAL COHERENCE FUNCTION

Previous work on speckle statistics has primarily been concentrated on the nature and statistics of the target surface, propagation of the speckle field without turbulence, and effects of speckle on image quality. Speckle propagation through turbulence has been considered over a vertical path for the purposes of speckle interferometry.[34]

In the present section an analysis is given of the first and second order statistics of the received intensity (irradiance) after scattering from a diffuse target. The treatment is based on the extended Huygens-Fresnel formulation and includes the effects of the turbulent atmosphere on the laser beam as it propagates to the target and on the speckle as it propagates back to the receiver. Formulations are given for both the focused and collimated cases. The analysis also includes the mutual coherence function (MCF).

The source, target, and receiver configuration is shown in Figure 1. The present analysis is confined to the case of a TEM_{00} laser illuminator. The source and target are assumed to be much smaller than the path length (L), and the distance between the receiver and source is greater than the source size and much smaller than the path length. These geometric conditions confine the problem to small angles and ensure that the outgoing and returning radiation experience independent turbulence regions; the latter limitation is thought to be inessential owing to the diffuse target characteristics.

1. Mean Irradiance at Receiver

To find the mean irradiance, we need no assumptions other than that of a diffuse target.

We write the source amplitude distribution as

$$U_o(\bar{r}) = U_o \exp\left(-\frac{r^2}{2\alpha_o^2} - \frac{ikr^2}{2F}\right) \tag{1}$$

where α_o and F are the characteristic beam radius and focal length respectively. The field at the target is written from the extended Huygens-Fresnel principle[1,2] as

$$U(\bar{\rho}) = \frac{ke^{ikL}}{2\pi iL} \int U_o(\bar{r}) \exp\left[\frac{ik|\bar{\rho} - \bar{r}|^2}{2L} + \psi_1 (\bar{\rho},\bar{r})\right] d\bar{r} \tag{2}$$

where ψ_1 describes the effects of the random medium on the propagation of a spherical wave. Combining Eqs. (1) and (2), we have

$$U(\bar{\rho}) = \frac{ke^{ik\left[L + \frac{\rho^2}{2L}\right]} U_o}{2\pi iL} \int \exp\left[-\frac{r^2}{2\alpha_o^2} + \frac{ik}{2L}\left(1 - \frac{L}{F}\right) r^2 - \frac{ik}{L}\bar{\rho}\cdot\bar{r} + \psi_1 (\bar{\rho},\bar{r})\right] d\bar{r} \tag{3}$$

In particular, this applies to the special cases of a focused (L = F) or collimated ($F \to \infty$) beam respectively.

The field at the receiver is written by reapplying the Huygens-Fresnel principle to the field at the target:

$$U(\bar{P}) = \frac{ke^{ik\left[L + \frac{P^2}{2L}\right]}}{2\pi iL} \int U'(\bar{\rho}) \exp\left[\frac{ik}{2L}(\rho^2 - 2\bar{P}\cdot\bar{\rho}) + \psi_2(\bar{P},\bar{\rho})\right] d\bar{\rho} \tag{4}$$

where $U'(\bar{\rho})$ is the field solution after reflection from the target, and ψ_2 represents the turbulence effect from the target to the receiver.

The mean intensity at the receiver is then

$$<I(\bar{p})> = <|U(\bar{p})|^2> = \left(\frac{k}{2\pi L}\right)^2 \iint d\bar{\rho}_1 d\bar{\rho}_2 <U'(\bar{\rho}_1)U'^*(\bar{\rho}_2)>$$

$$\cdot \exp\left[\frac{ik}{2L}\left((\rho_1^2 - \rho_2^2) - 2\bar{p}\cdot(\bar{\rho}_1-\bar{\rho}_2)\right)\right]$$

$$\cdot < \exp\left[\psi_2(\bar{p},\bar{\rho}_1) + \psi_2^*(\bar{p},\bar{\rho}_2)\right] > \tag{5}$$

Through the assumption of a diffuse target, the reflected beam suffers a random phase delay from point-to-point over the target, so that

$$<U'(\bar{\rho}_1)U'^*(\bar{\rho}_2)> = <I(\bar{\rho}_1)> \delta(\bar{\rho}_1-\bar{\rho}_2) \tag{6}$$

Using this in Eq. (5), the mean intensity becomes

$$<I(\bar{p})> = \left(\frac{k}{2\pi L}\right)^2 \int d\bar{\rho}_1 <|U(\bar{\rho}_1)|^2> < \exp\left[\psi_2(\bar{p},\bar{\rho}_1) + \psi_2^*(\bar{p},\bar{\rho}_1)\right]> \tag{7}$$

where the mean exponential term is unity from considerations of energy conservation.[32] The resultant mean intensity at the receiver is then simply

$$<I(\bar{p})> = \left(\frac{k}{2\pi L}\right)^2 \int d\bar{\rho} <|U(\bar{\rho})|^2> \tag{8}$$

To complete the solution, we use Eq. (3) with Eq. (8). We note that the structure function gives us ($r = |\bar{r}_1-\bar{r}_2|$)

$$< \exp\left[\psi_1(\bar{\rho},\bar{r}_1) + \psi_1^*(\bar{\rho},\bar{r}_2)\right]> = e^{-\left(\frac{r}{\rho_o}\right)^{5/3}} \tag{9}$$

For the focused beam, we then have

$$<|U(\bar{\rho})|^2> = \left(\frac{k}{2\pi L}\right)^2 |U_o|^2 \iint d\bar{r}_1 d\bar{r}_2 \exp\left[-\frac{r_1^2 + r_2^2}{2\alpha_o^2} - \frac{ik}{L}\bar{\rho}\cdot(\bar{r}_1 - \bar{r}_2) - \left(\frac{r}{\rho_o}\right)^{5/3}\right] \tag{10}$$

Carrying out the integration indicated in Eq. (8), involving the Fourier-Bessel integral, we have finally

$$<I(\bar{p})> = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2} \tag{11}$$

The result for the collimated beam is identical, and in fact could be deduced for an arbitrary beam focus (Eq. (1)) through conservation of energy:

$$<I(\bar{p})> = \left(\frac{k}{2\pi L}\right)^2 \int d\bar{r}\, <|U(\bar{r})|^2>$$

$$= \left(\frac{k}{2\pi L}\right)^2 2\pi |U_o|^2 \int^{\infty} r e^{-r^2/\alpha_o^2} dr = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2} \tag{12}$$

Thus the mean intensity at the receiver (illuminator) plane is uniform and independent of turbulence level.

2. Correlation Function of Irradiance

In order to calculate the correlation function or covariance of irradiance, we assume for the present section that the perturbation Green's function (wave structure function) is dominated by the phase perturbation (phase structure function).[35] This will be true for many cases of interest and will be relaxed in subsequent sections, where the actual implications of the assumption are pointed out.

The correlation function of the intensity at receiver points $\bar{p}_1$ and $\bar{p}_2$ is given by

$$B_I(\bar{p}_1,\bar{p}_2) = \langle I_1(\bar{p}_1)I_2(\bar{p}_2)\rangle = \langle U(\bar{p}_1)U^*(\bar{p}_1)U(\bar{p}_2)U^*(\bar{p}_2)\rangle \tag{13}$$

Utilizing the extended Huygens-Fresnel principle, the correlation function can be expressed as

$$B_I(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^4 \iiiint d\bar{\rho}_1 d\bar{\rho}_2 d\bar{\rho}_3 d\bar{\rho}_4 \ \langle U(\bar{\rho}_1)U^*(\bar{\rho}_2)U(\bar{\rho}_3)U^*(\bar{\rho}_4)\rangle$$

$$\cdot \ \exp[ik(R_1 - R_2 + R_3 - R_4)] \ H(\bar{\rho}_1,\bar{\rho}_2,\bar{\rho}_3,\bar{\rho}_4;\bar{p}_1,\bar{p}_2) \tag{14}$$

where H is the fourth order mutual coherence function given by

$$H = \langle \exp[\psi(\bar{\rho}_1,\bar{p}_1) + \psi^*(\bar{\rho}_2,\bar{p}_1) + \psi(\bar{\rho}_3,\bar{p}_2) + \psi^*(\bar{\rho}_4,\bar{p}_2)] \rangle \tag{15}$$

and

$$R_1 = |\bar{p}_1 - \bar{\rho}_1|$$
$$R_2 = |\bar{p}_1 - \bar{\rho}_2|$$
$$R_3 = |\bar{p}_2 - \bar{\rho}_3|$$
$$R_4 = |\bar{p}_2 - \bar{\rho}_4|$$

Under the assumption of dominant phase perturbations,

$$H = \langle \exp[i\phi(\bar{\rho}_1,\bar{p}_1) - i\phi(\bar{\rho}_2,\bar{p}_1) + i\phi(\bar{\rho}_3,\bar{p}_2) - i\phi(\bar{\rho}_4,\bar{p}_2)] \rangle \tag{16}$$

After reflection from the diffuse target, the fields are Gaussian and spatially incoherent. Therefore, the fields at the target can be expressed as

$$<U(\bar{\rho}_1)U^*(\bar{\rho}_2)U(\bar{\rho}_3)U^*(\bar{\rho}_4)> = <U(\bar{\rho}_1)U^*(\bar{\rho}_2)> <U(\bar{\rho}_3)U^*(\bar{\rho}_4)>$$

$$+ <U(\bar{\rho}_1)U^*(\bar{\rho}_4)> <U^*(\bar{\rho}_2)U(\bar{\rho}_3)>$$

$$= <I(\bar{\rho}_1)> <I(\bar{\rho}_3)> \delta(\bar{\rho}_1-\bar{\rho}_2)\ \delta(\bar{\rho}_3-\bar{\rho}_4)$$

$$+ <I(\bar{\rho}_1)> <I(\bar{\rho}_3)> \delta(\bar{\rho}_1-\bar{\rho}_4)\ \delta(\bar{\rho}_3-\bar{\rho}_2) \tag{17}$$

Utilizing (17) and (14) the correlation function can be expressed as

$$B_I(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2\ d\bar{\rho}_4\ <I(\bar{\rho}_2)> <I(\bar{\rho}_4)>$$

$$+ \left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2\ d\bar{\rho}_4\ <I(\bar{\rho}_2)> <I(\bar{\rho}_4)>\ \exp\left[\frac{jk}{L}(\bar{\rho}_2-\bar{\rho}_4)\cdot(\bar{p}_1-\bar{p}_2)\right]$$

$$\cdot\ H(\bar{\rho}_2,\bar{\rho}_2,\bar{\rho}_4,\bar{\rho}_4;\bar{p}_1,\bar{p}_2) \tag{18}$$

where use has been made of

$$R_1-R_2 \simeq \frac{1}{2L}\left[\rho_1^{\ 2} - \rho_2^{\ 2} - 2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}_1\right]$$

$$R_3-R_4 \simeq \frac{1}{2L}\left[\rho_3^{\ 2} - \rho_4^{\ 3} - 2(\bar{\rho}_3 - \bar{\rho}_4)\cdot\bar{p}_1\right]$$

The fourth order mutual coherence function (see Appendix A) in Eq. (18) is given by[32,36,37]

$$H(\bar{\rho}_2,\bar{\rho}_2,\bar{\rho}_4,\bar{\rho}_4;\bar{p}_1,\bar{p}_2) = H(\bar{\rho}_1,\bar{\rho}_2,\bar{\rho}_3,\bar{\rho}_4;\bar{p}_1,\bar{p}_2,\bar{p}_3,\bar{p}_4)\Bigg|_{\substack{\bar{\rho}_1 = \bar{\rho}_2 \\ \bar{\rho}_3 = \bar{\rho}_4 \\ \bar{p}_3 = \bar{p}_1 \\ \bar{p}_4 = \bar{p}_2}}$$

$$= \left. e^{\frac{1}{2}(D_{12} - D_{13} + D_{14} + D_{23} - D_{24} + D_{34})} \right|_{\substack{\bar{\rho}_1 = \bar{\rho}_2 \\ \bar{\rho}_3 = \bar{\rho}_4 \\ \bar{p}_3 = \bar{p}_1 \\ \bar{p}_4 = \bar{p}_2}} \quad (19)$$

where the wave phase structure function D_{ij} are given by

$$D_{ij} = \frac{16}{3} \frac{1}{\rho_o^{5/3}} \int_o^1 dt \left| t(\bar{p}_j - \bar{p}_i) + (1-t)(\bar{\rho}_j - \bar{\rho}_i) \right|^{5/3} \quad (20)$$

where $\rho_o = (0.545\ C_n^{\,2} L k^2)^{-3/5}$ is the turbulence-induced coherence scale and $C_n^{\,2}$ = Structure constant of index of refraction $(m^{-2/3})$. Using this in (18) and making the change of variables

$\bar{\rho} = \bar{\rho}_2 - \bar{\rho}_4$, $\bar{p} = \bar{p}_1 - \bar{p}_2$

and

$2\bar{R} = \bar{\rho}_2 + \bar{\rho}_4$

and recognizing that the first term in (18) equals $\langle I(\bar{p}_1)\rangle\langle I(\bar{p}_2)\rangle$, the covariance for the focused case is given by

$$C_I(\bar{p}_1, \bar{p}_2) = \left(\frac{k}{2\pi L}\right)^4 \left(\frac{k}{L}\right)^4 |U_o|^4 \left(\frac{\alpha_o^{\,2}}{2}\right)^2 e^{-2\left(\frac{p}{\rho_o}\right)^{5/3}}$$

$$\cdot \iint d\bar{\rho}\, d\bar{R}\, J_o\left(\frac{k}{L} r_1 \left|\bar{R} + \frac{\bar{\rho}}{2}\right|\right) J_o\left(\frac{k}{L} r_2 \left|\bar{R} - \frac{\bar{\rho}}{2}\right|\right) e^{\frac{ik}{L} \bar{\rho} \cdot \bar{p}}$$

$$\cdot \iint r_1 r_2 dr_1 dr_2 \exp\left[-\frac{(r_1^2 + r_2^2)}{4\alpha_o^2} - \frac{(r_1^{5/3} + r_2^{5/3})}{\rho_o^{5/3}} - \frac{1}{\rho_o^{5/3}}\right.$$

$$\left.\left[2\rho^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{\rho}t + (1-t)\bar{\rho}|^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{\rho}t - (1-t)\bar{\rho}|^{5/3}\right]\right]$$

(21)

The covariance for the collimated case is obtained from (21) by replacing the dr_1dr_2 integration by

$$\iint r_1 r_2 dr_1 dr_2 \exp\left[-(r_1^2 + r_2^2)\left(\left(\frac{1}{2\alpha_o}\right)^2 + \left(\frac{k\alpha_o}{2L}\right)^2\right) - \frac{r_1^{5/3} + r_2^{5/3}}{\rho_o^{5/3}}\right.$$

$$\left.- \frac{1}{\rho_o^{5/3}}\left[2\rho^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{\rho}t + (1-t)\bar{\rho}|^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{\rho}t - (1-t)\bar{\rho}|^{5/3}\right]\right]$$

(22)

In order to further reduce the number of integrations, the zero order Bessel functions must be expanded to functionally separate the $\bar{R}$ and $\bar{\rho}$ dependence. This can be accomplished by utilizing the following identities:

$$J_o\left(\frac{k}{L} r_1 |\bar{R} \pm \frac{\bar{\rho}}{2}|\right) = \sum_{m=0}^{\infty} \epsilon_m (\mp 1)^m \; J_m\left(\frac{k}{L} r_1 R\right) J_m\left(\frac{k}{L} r_1 \frac{\rho}{2}\right) \cos m\phi$$

(23)

where

$\phi = \theta_R - \theta_\rho$

$\epsilon_o = 1$

and

$\epsilon_m = 2$ for $m \neq 0$

The θ_R integration is then given by

$$\int_0^{2\pi} J_o\left(\frac{k}{L} r_1 |\bar{R} + \frac{\bar{\rho}}{2}|\right) J_o\left(\frac{k}{L} r_2 |\bar{R} - \frac{\bar{\rho}}{2}|\right) d\theta_R$$

$$= 2\pi \sum_{m=0}^{\infty} (-1)^m \epsilon_m J_m\left(\frac{k}{L} r_1 R\right) J_m\left(\frac{k}{L} r_2 R\right)$$

$$\cdot J_m\left(\frac{k}{L} r_1 \frac{\rho}{2}\right) J_m\left(\frac{k}{L} r_2 \frac{\rho}{2}\right) \qquad (24)$$

Using the Fourier-Bessel Integral,

$$\int_0^{\infty} r_1 dr_1 f(r_1) \int_0^{\infty} R dR\, J_m\left(\frac{k}{L} r_1 R\right) J_m\left(\frac{k}{L} r_2 R\right) = \left(\frac{L}{k}\right)^2 f(r_2) \qquad (25)$$

and the covariance for the focused case becomes

$$C_I(\bar{p}) = \left(\frac{1}{2\pi}\right)^3 \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o}{2}\right)^2 \exp\left[-2\left(\frac{p}{\rho_o}\right)^{5/3}\right] \int_o^{\infty} r_2 dr_2$$

$$\cdot \exp\left[-\frac{r_2^2}{2\alpha_o^2} - \frac{2r_2^{5/3}}{\rho_o^{5/3}}\right] \int d\bar{\rho}\, J_o\left(\frac{k}{L} r_2 \rho\right) \cdot \exp\left[\frac{ik}{L} \bar{p} \cdot \bar{\rho} - \frac{1}{\rho_o^{5/3}}\right.$$

$$\left.\left[2\rho^{5/3} - \frac{8}{3}\int_o^1 dt\, |\bar{p}t + (1-t)\bar{\rho}|^{5/3} - \frac{8}{3}\int_o^1 dt\, |\bar{p}t - (1-t)\bar{\rho}|^{5/3}\right]\right]$$

(26)
(Focused)

where use has been made of

$$\sum_{m=0}^{\infty} (-1)^m \epsilon_m J_m^2 (x) = J_o(2x) \tag{27}$$

The variance then is given by

$$\sigma_I^2 = \left(\frac{1}{2\pi}\right)^2 \left(\frac{k}{L}\right)^4 |U_o|^4 \left(\frac{\alpha_o^2}{2}\right)^2 \tag{28}$$

and the normalized variance

$$\sigma_{I_N}^2 = \frac{\sigma_I^2}{\langle I\rangle^2}$$

is unity.

The same technique yields the covarance for the collimated case. It is given by

$$C_I(\bar{p}) = \left(\frac{1}{2\pi}\right)^3 \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o^2}{2}\right)^2 \exp\left[-2\left(\frac{p}{\rho_o}\right)^{5/3}\right] \int_o^{\infty} r_2 dr_2$$

$$\cdot \exp\left[-2r_2^2\left[\left(\frac{1}{2\alpha_o}\right)^2 + \left(\frac{k\alpha_o}{2L}\right)^2\right] - \frac{2r_2^{5/3}}{\rho_o^{5/3}}\right] \int d\bar{\rho}\, J_o\left(\frac{k}{L} r_2 \rho\right) \cdot \exp\left[\frac{ik}{L}\bar{p}\cdot\bar{\rho}\right.$$

$$\left.- \frac{1}{\rho_o^{5/3}}\left[2\rho^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{p}t + (1-t)\bar{\rho}|^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{p}t - (1-t)\bar{\rho}|^{5/3}\right]\right]$$

(collimated) (29)

and the normalized variance is again unity.

The above results are based on the assumptions of a diffuse target and phase perturbations dominating the turbulence effects. Unfortunately the expressions that result, reduced to their simplest form, involve threefold integrations and except for showing that the normalized variance is unity, allow little physical insight into the nature of the problem. Physical interpretation is not difficult but will be clarified in the discussion of further simplifications below. It suffices to point out here that the covariance fundamentally involves two scales: the turbulence-induced coherence scale (ρ_o) and the speckle scale in the absence of turbulence (α_o and $L/k\alpha_o$ for the focused and collimated case respectively).

Also, we note that within the present assumptions, the normalized variance of intensity is always unity independent of turbulence strength. This agrees with a physical model of identical-frequency, randomly phased oscillators summed to represent any given point in the receiver field: the model applies regardless of whether target speckle (α_o or $L/k\alpha_o$) or "atmospheric speckle" (ρ_o) dominates.

3. Mutual Coherence Function

The mutual coherence function (MCF) may be very important in analyzing the operation of a coherent optical adaptive system, and can be readily derived given the assumption of a diffuse target but without assuming dominance of the phase perturbation term. We write

$$\Gamma(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^2 \iint d\bar{\rho}_1 d\bar{\rho}_2 < U(\bar{\rho}_1)U^*(\bar{\rho}_2) > \exp\left\{ik\left[R_5(\bar{\rho}_1,\bar{p}_1) - R_6(\bar{\rho}_2,\bar{p}_2)\right]\right\} < \exp\left[\psi_2(\bar{p}_1,\bar{\rho}_1) + \psi_2^*(\bar{p}_2,\bar{\rho}_2)\right] > \tag{30}$$

where $R_5(\bar{\rho}_1,\bar{p}_1)$, $R_6(\bar{\rho}_2,\bar{p}_2)$ are the distances from $\bar{\rho}_1$ to $\bar{p}_1$ and $\bar{\rho}_2$ to $\bar{p}_2$ respectively.

By the Fresnel approximation

$$R_5(\bar{\rho}_1,\bar{p}_1) - R_6(\bar{\rho}_2,\bar{p}_2) \cong \frac{p_1^2 - p_2^2 + \rho_1^2 - \rho_2^2}{2L} - \frac{\bar{p}_1 \cdot \bar{\rho}_1 - \bar{p}_2 \cdot \bar{\rho}_2}{L} \tag{31}$$

Finally, from (30) and (31),

$$\Gamma(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^2 \exp\left[\frac{ik(p_1^2-p_2^2)}{2L}\right]\iint d\bar{\rho}_1 d\bar{\rho}_2 \quad \langle U(\bar{\rho}_1)U^*(\bar{\rho}_2)\rangle$$

$$\cdot \exp\left\{ik\left(\frac{\rho_1^2-\rho_2^2}{2L} - \frac{\bar{p}_1\cdot\bar{\rho}_1-\bar{p}_2\cdot\bar{\rho}_2}{L}\right)\right\} \langle \exp\left[\psi_2(\bar{p}_1,\bar{\rho}_1) + \psi_2^*(\bar{p}_2,\bar{\rho}_2)\right]\rangle \tag{32}$$

Since the wave is incoherent after reflection from the diffuse target, the coherence function at that plane can again be represented by the Dirac delta function as given in (6). Using this in (32), $\Gamma(\bar{p}_1,\bar{p}_2)$ can be simplified to

$$\Gamma(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^2 \exp\left[\frac{ik(p_1^2-p_2^2)}{2L}\right]\int d\bar{\rho} \ \langle I(\bar{\rho})\rangle \exp\left\{-\frac{ik}{L}(\bar{p}_1-\bar{p}_2)\cdot\bar{\rho}\right\}$$

$$\cdot\ e^{-\left(\frac{p}{\rho_o}\right)^{5/3}} \tag{33}$$

In the absence of turbulence, this equation is entirely identical to the Van Cittert-Zernike theorem of coherence theory,[39] which is identical to a result obtained by Goodman for the mutual coherence function of a pulsed optical radar.[40]

To complete the solution, we utilize the mean intensity at the target. For the focused case:

$$\langle I(\bar{\rho})\rangle = \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\,2}}{2} \int_0^\infty r\,dr\, J_o\left(\frac{k}{L}\,\rho r\right) e^{-\frac{r^2}{4\alpha_o^{\,2}} - \left(\frac{r}{\rho_o}\right)^{5/3}} \qquad \text{(focused)}$$

we thus have

$$\Gamma(p_1,p_2) = \left(\frac{k}{2\pi L}\right)^2 \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\,2}}{2} \int d\bar{\rho} \int_0^\infty r\,dr\, J_o\left(\frac{k}{L}\,\rho r\right)$$

$$\cdot\, e^{-\frac{r^2}{4\alpha_o^{\,2}} - \left(\frac{r}{\rho_o}\right)^{5/3}} \exp\left[-\frac{ik}{L}\,\bar{\rho}\cdot\bar{p} - \left(\frac{p}{\rho_o}\right)^{5/3}\right]$$

$$\cdot\, \exp\left[\frac{ik(p_1^2-p_2^2)}{2L}\right]$$

$$= \frac{1}{2\pi}\left(\frac{k}{L}\right)^4 |U_o|^2 \frac{\alpha_o^{\,2}}{2} \int_0^\infty \rho\,d\rho \int_0^\infty r\,dr\, J_o\left(\frac{k}{L}\,\rho r\right) J_o\left(\frac{k}{L}\,\rho p\right)$$

$$\cdot\, e^{-\frac{r^2}{4\alpha_o^{\,2}} - \left(\frac{r}{\rho_o}\right)^{5/3} - \left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)} \tag{34}$$

where $p = |\bar{p}_1-\bar{p}_2|$. From the Fourier-Bessel integral formula,

$$\int_0^\infty \rho J_o\left(\frac{k}{L}\,r\rho\right) J_o\left(\frac{k}{L}\,p\rho\right) d\rho = \left(\frac{L}{k}\right)^2 \frac{1}{\sqrt{rp}}\,\delta(r-p) \tag{35}$$

Equation (34) can then be simplified and it becomes

$$\Gamma(\bar{p}_1,\bar{p}_2) = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\,2}}{2}\, e^{-\frac{p^2}{4\alpha_o^{\,2}} - 2\left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)}$$

$$= \langle I(p)\rangle\, e^{-\frac{p^2}{4\alpha_o^{\,2}} - 2\left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)} \qquad \text{(focused)} \qquad (36)$$

Using $\langle I(\bar{\rho})\rangle$ for the collimated case in (33) and simplifying yields

$$\Gamma(\bar{p}_1,\bar{p}_2) = \langle I(\bar{p})\rangle\, e^{-p^2\left[\left(\frac{1}{2\alpha_o}\right)^2 + \left(\frac{k\alpha_o}{2L}\right)\right]^2 - 2\left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-\rho_2^2\right)}$$

$$\text{(collimated)} \qquad (37)$$

These results for the MCF will be used further below.

Now let

$$\bar{p}_2 = \bar{p}_1 + \bar{p}$$

in (36), divide by $\langle I(\bar{p})\rangle$ and integrate over $d\bar{p}_1$ to obtain the space averaged, normalized mutual intensity

$$\Gamma_N(\bar{p}) = \int d\bar{p}_1\, \Gamma_N(\bar{p}_1,\bar{p}_1 + \bar{p})$$

$$= e^{-\frac{p^2}{4\alpha_o^{\,2}} - 2\left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}p^2}$$

$$\cdot \int d\bar{p}_1\, e^{\frac{ik}{L}\,\bar{p}\cdot\bar{p}_1}$$

$$= 2 \ (2\pi)^2 \left(\frac{L}{K}\right)^2 e^{-\frac{p^2}{4\alpha_o^2} - 2\left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L} p^2} \delta(\overline{p}) \qquad \text{(focused)} \tag{38}$$

The space averaged, power spectral density then becomes

$$S_{\Gamma_N \Gamma_N} = 2 \left(\frac{L}{K}\right)^2 \qquad \text{(focused)} \tag{39}$$

A similar result is obtained for the collimated case and it is concluded that at least on the average (spatial), the fields at the receiver are incoherent.

It may be noted that the MCF's of Eq.(36) and (37) imply a "white" or constant spatial power spectrum for the (complex) amplitude. This is of course an idealization resulting from assuming delta-function rather than wavelength-sized phase correlation for the field upon reflection off the target. The more interesting spectrum, however, is that of the irradiance, as discussed below.

4. Probability Distribution

In order to formulate the probability distribution for the scintillating energy at the receiver, we evaluate the n^{th} moment of the intensity in terms of $\langle I(\overline{p})\rangle$. We again assume that the phase perturbations are the dominant turbulence effect, and write the second moment as

$$<I^2(\overline{p})> = \left(\frac{k}{2\pi L}\right)^4 \iiiint d\overline{\rho}_1 d\overline{\rho}_2 d\overline{\rho}_3 d\overline{\rho}_4$$

$$\cdot <U(\overline{\rho}_1)U^*(\overline{\rho}_2)U(\overline{\rho}_3)U^*(\overline{\rho}_4)> \exp\left[\frac{ik}{2L}\left(\rho_1^2 - \rho_2^2\right.\right.$$

$$\left.\left. + \rho_3^2 - \rho_4^2 - 2(\overline{\rho}_1 - \overline{\rho}_2) \cdot \overline{p} - 2(\overline{\rho}_3 - \overline{\rho}_4) \cdot \overline{p}\right)\right]$$

$$\cdot <\exp\left[i\left(\phi(\overline{\rho}_1, \overline{p}) - \phi(\overline{\rho}_2, \overline{p}) + \phi(\overline{\rho}_3, \overline{p}) - \phi(\overline{\rho}_4, \overline{p})\right)\right]> \qquad (40)$$

Since the fields after reflection from the diffuse target are jointly Gaussian and uncorrelated,

$$<U(\overline{\rho}_1)U^*(\overline{\rho}_2)U(\overline{\rho}_3)U^*(\overline{\rho}_4)> = <U(\overline{\rho}_1)U^*(\overline{\rho}_2)> <U(\overline{\rho}_3)U^*(\overline{\rho}_4)>$$

$$+ <U(\overline{\rho}_1)U^*(\overline{\rho}_4)> <U^*(\overline{\rho}_2)U(\overline{\rho}_3)> .$$

$$= I(\overline{\rho}_1)\delta(\overline{\rho}_1 - \overline{\rho}_2)\, I(\overline{\rho}_3)\delta(\overline{\rho}_3 - \overline{\rho}_4) + I(\overline{\rho}_1)\delta(\overline{\rho}_1 - \overline{\rho}_4) I(\overline{\rho}_3)\delta(\overline{\rho}_3 - \overline{\rho}_2) \qquad (41)$$

Using (40) in (41) yields

$$<I^2(\overline{p})> = 2\left(\frac{k}{2\pi L}\right)^4 \iint d\overline{\rho}_2 d\overline{\rho}_4 <I(\overline{\rho}_2)> <I(\overline{\rho}_4)> \qquad (42)$$

Since

$$<I(\overline{p})> = \left(\frac{k}{2\pi L}\right)^2 \int d\overline{\rho}_2 <I(\overline{\rho}_2)>$$

the second moment becomes

$$<I^2(\overline{p})> = 2<I(\overline{p})>^2 \qquad (43)$$

The nth moment of the intensity can be expressed as

$$<I^N(\bar{p})> = \left(\frac{k}{2\pi L}\right)^{2N} \int \cdots\cdot \int d\bar{\rho}_1 \;\cdots\cdot\; d\bar{\rho}_{2N}$$

$$\cdot <U\left(\bar{\rho}_1\right)U^*\left(\bar{\rho}_2\right) \;\cdots\cdot\; U\left(\bar{\rho}_{2N-1}\right)U^*\left(\bar{\rho}_{2N}\right)>$$

$$\cdot \exp\left[\frac{ik}{2L}\left(\rho_1^2-\rho_2^2 \;\cdots\cdot + \rho_{2N-1}^2 - \rho_{2N}^2 - 2\left(\bar{\rho}_1-\bar{\rho}_2\right) \cdot \bar{p} - \right.\right.$$

$$\left.\left. \cdots\cdot -2\left(\bar{\rho}_{2N-1}-\bar{\rho}_{2N}\right) \cdot \bar{p}\right)\right]$$

$$\cdot <\exp\left[i\left(\phi\left(\bar{\rho}_1,\bar{p}\right) - \phi\left(\bar{\rho}_2,\bar{p}\right) \cdots\cdot + \phi\left(\bar{\rho}_{2N-1},\bar{p}\right) - \phi\left(\bar{\rho}_{2N},\bar{p}\right)\right)\right]> \tag{44}$$

Because of the jointly Gaussian and incoherent nature of the source, and the occurrence of only paired difference terms in the integrand, when

$$<U\left(\bar{\rho}_1\right)U^*\left(\bar{\rho}_2\right) \;\cdots\cdot\; U^*\left(\bar{\rho}_{2N}\right)>$$

is expanded and the integrations involving the delta functions are performed, the n^{th} moment reduces to

$$<I^N(\bar{p})> = N!\left(\frac{k}{2\pi L}\right)^{2N} \int \cdots\cdot \int d\bar{\rho}_1 \;\cdots\; d\bar{\rho}_N$$

$$\cdot <I\left(\bar{\rho}_1\right)> \;\cdots\cdot\; <I\left(\bar{\rho}_N\right)> = N! \;<I\left(\bar{p}\right)>^N \tag{45}$$

The probability density function for the intensity therefore is exponential and

$$p_I(\alpha) = \frac{e^{-\frac{\alpha}{\langle I\rangle}}}{\langle I\rangle} \tag{46}$$

where α is greater than or equal to zero. It is thus concluded that the field and amplitude at the receiver are normally and Rayleigh distributed respectively, given the assumption that phase perturbations dominate the turbulence effects.

5. Simplification for Weak and Strong Turbulence

Since, as shown above, the fields at the receiver are Gaussian and spatially "white," it is tempting to assume that the receiver fields are also jointly Gaussian. This turns out to be a good approximation in many situations, and in this section the implications and conditions for validity of this added assumption are explored. This leads to a simple, straightforward interpretation of the terms in the covariance of intensity.

The jointly Gaussian assumption yields

$$\begin{aligned} B_I &= \langle U(\bar{p}_1)U^*(\bar{p}_1)\rangle\langle U(\bar{p}_2)U^*(\bar{p}_2)\rangle + \langle U(\bar{p}_1)U^*(\bar{p}_2)\rangle\langle U^*(\bar{p}_1)U(\bar{p}_2)\rangle \\ &\equiv \langle I(\bar{p}_1)\rangle\langle I(\bar{p}_2)\rangle + |\Gamma(\bar{p}_1,\bar{p}_2)|^2 \end{aligned} \tag{47}$$

It follows that the covariance of intensity is given by

$$C_I(\bar{p}_1,\bar{p}_2) = B_I(\bar{p}_1,\bar{p}_2) - \langle I(\bar{p}_1)\rangle\langle I(\bar{p}_2)\rangle = |\Gamma(\bar{p}_1,\bar{p}_2)|^2 \tag{48}$$

Finally, utilizing the mutual coherence result (Eq.(36)) the normalized covariance function of irradiance for the focused case can thus be written:

$$C_{I_N}(\bar{p}) \equiv \frac{C_I(\bar{p})}{\sigma_I^{\,2}} = e^{-\frac{p^2}{2\alpha_o^{\,2}} - 4\left(\frac{p}{\rho_o}\right)^{5/3}} \qquad \text{(focused)} \qquad (49)$$

where the normalized variance is unity as before.

For the collimated case, the same variance is obtained, and the normalized covariance is

$$C_{I_N}(\bar{p}) = e^{-4\left(\frac{p}{\rho_o}\right)^{5/3} - \frac{1}{2}\left[\left(\frac{1}{\alpha_o}\right)^2 + \left(\frac{k\alpha_o}{L}\right)^2\right] p^2} \qquad \text{(collimated)} \qquad (50)$$

The covariance scale lengths are obvious from these results. Either the "atmospheric speckle" (ρ_o) or the "target speckle" (speckle in the absence of turbulence) will predominate, depending upon which is smaller (strong and weak turbulence respectively). We point out in passing that a third covariance scale $(L/k)^{\frac{1}{2}}$ may also enter, but this scale is lost within the present assumption of dominant phase perturbations. Figures 2-6 show the correlation scale for various cases.

We note that the spatial power spectrum of irradiance may be readily obtained by transforming Eqs.(49,50). However, a more important quantity in the operation of e.g. an adaptive optical system may be the temporal spectrum. This spectrum, which will be derived in a later section, depends only on the atmospheric speckle term; the target speckle field will not translate with the transverse wind.

We now explore the conditions for validity of the jointly Gaussian assumption. The simple multiplicative terms for the covariance scales in Eqs.(49,50) are replaced by a more complicated interrelationship in Eqs.(26,29). It may therefore be surmised that the jointly Gaussian assumption is valid under conditions of weak and strong turbulence, when target and atmospheric speckle terms respectively predominate, but that the jointly Gaussian description is not correct in the range of intermediate turbulence effects when both scales are important and interact. We now show that this is indeed the case.

Weak Turbulence

For the weak turbulence case, $\rho_o >> \sqrt{L/k}$ and the term

$$J_o\left(\frac{k}{L}\, r\rho\right) e^{-\frac{1}{\rho_o^{5/3}}\left[2\rho^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{\rho}t + (1-t)\bar{\rho}|^{5/3} - \frac{8}{3}\int_o^1 dt\,|\bar{\rho}t - (1-t)\bar{\rho}|^{5/3}\right]} \simeq J_o\left(\frac{k}{L}\, r\rho\right) \tag{51}$$

in (26) and (29). The covariances then become identical to those derived using the jointly Gaussian assumption.

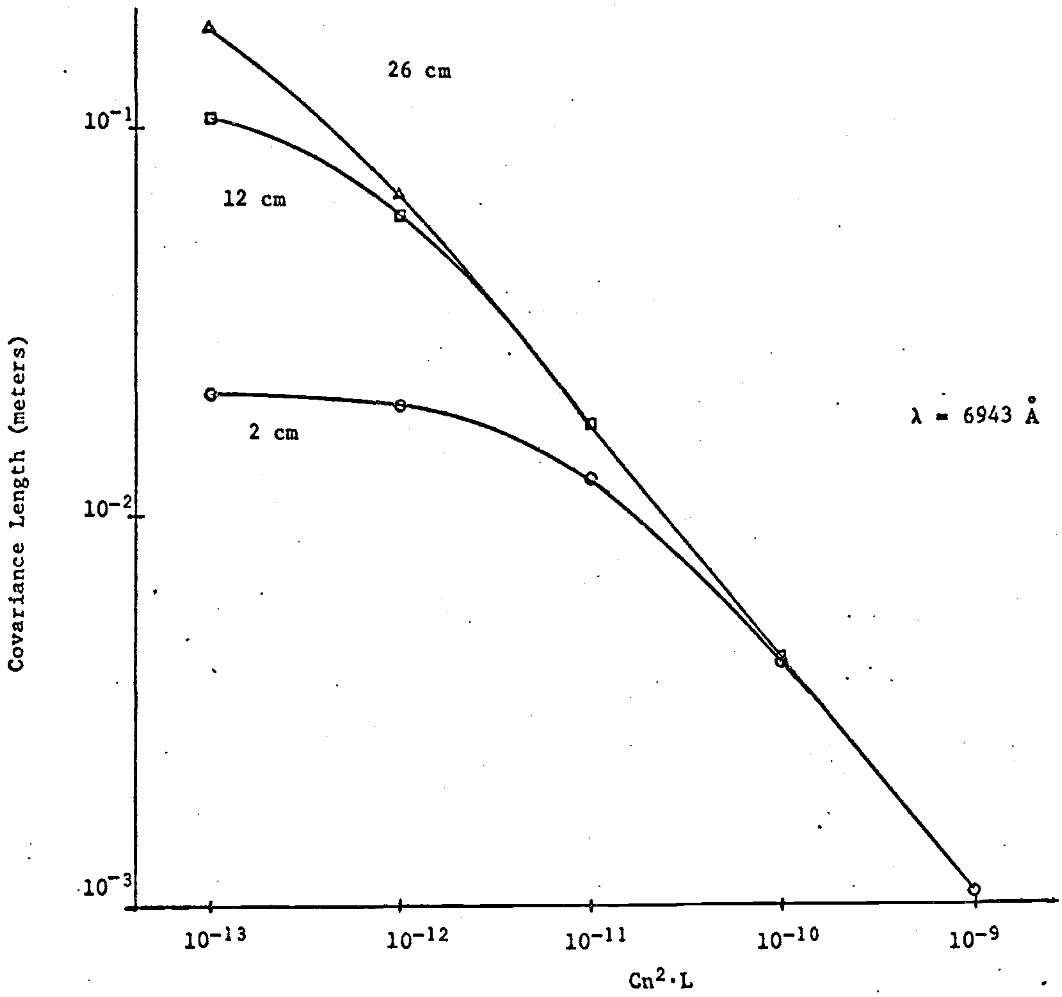

Figure 2. Covariance Length for Various Transmitter Diameters, Focused Case

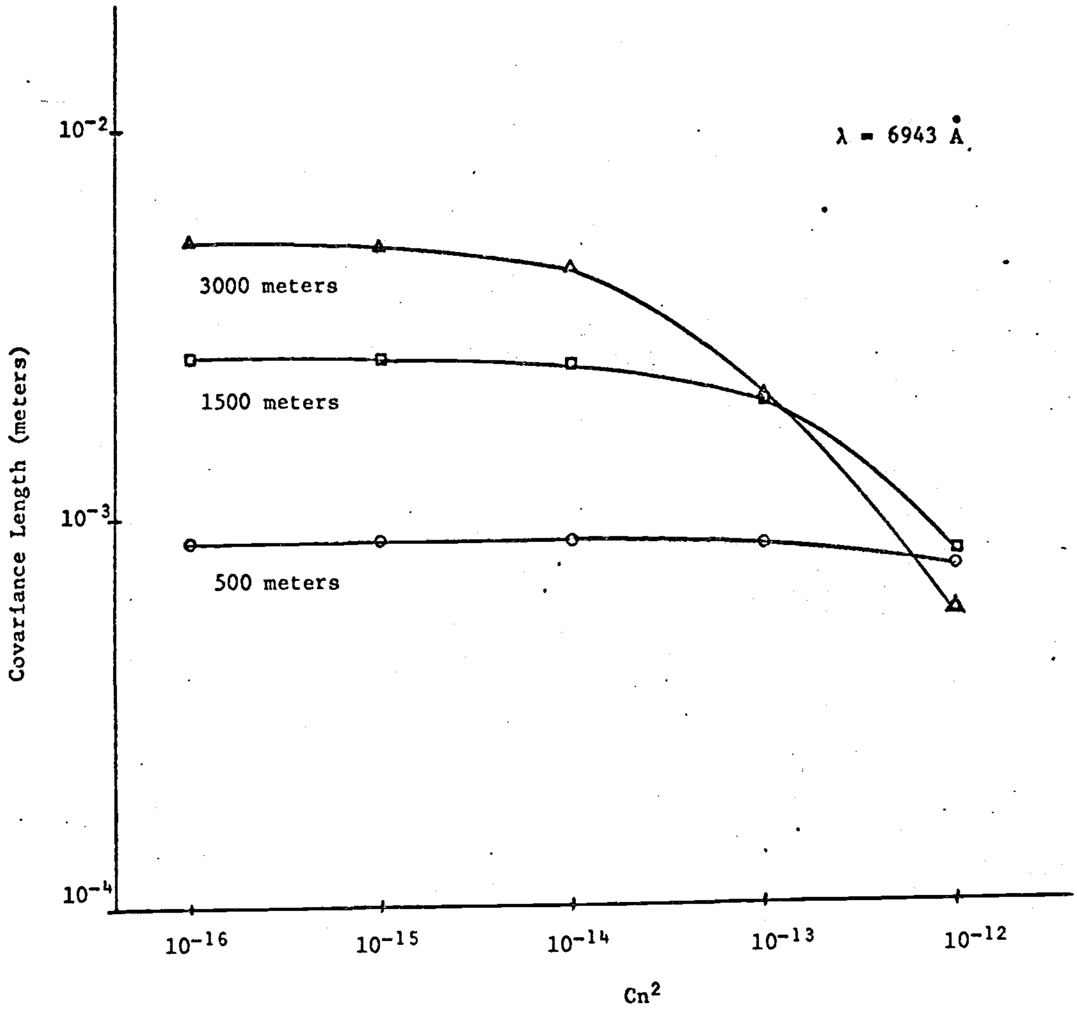

Figure 3. Covariance Length, Collimated Case, 26 cm Aperture

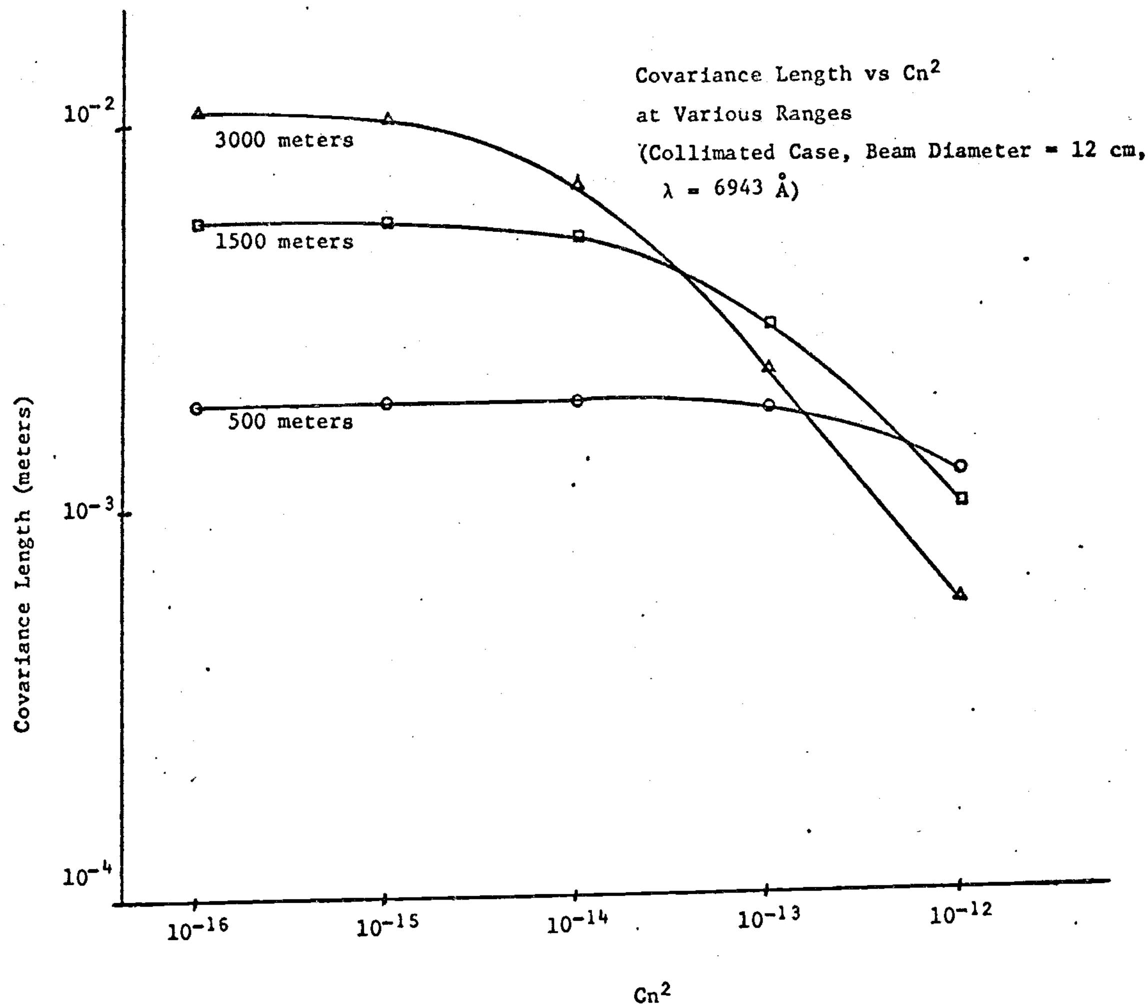

Figure 4. Covariance Length, Collimated Case, 12 cm Aperture

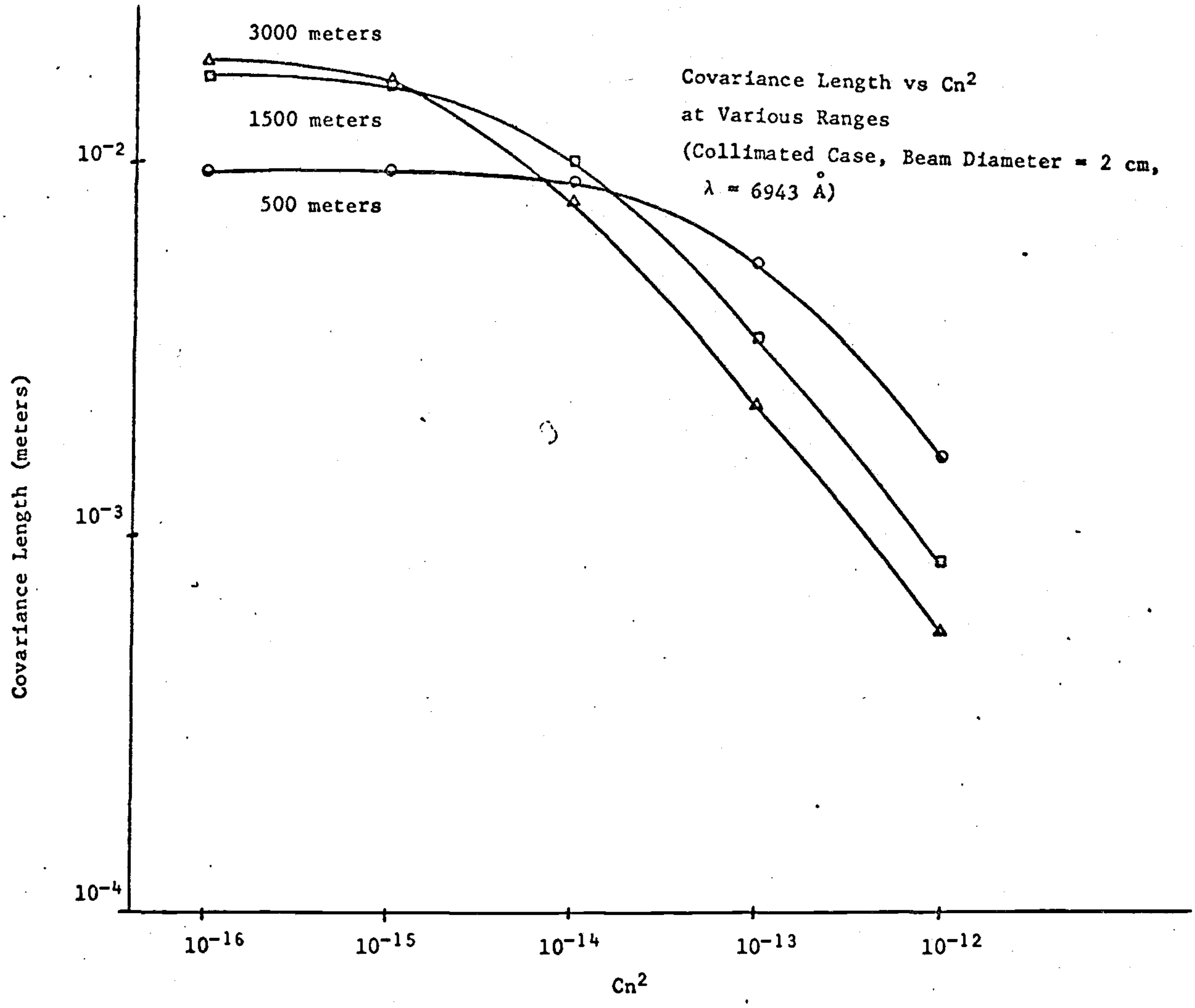

Figure 5. Covariance Length, Collimated Case, 2 cm Aperture

Strong Turbulence

In the strong turbulence case, $\rho_o << \sqrt{L/k}$, which corresponds to multiple scattering, or "saturation of scintillations" for a point source. Let us consider Eq.(26) with $\rho_o \to 0$. The only interesting range of the argument (p) is $0 \leq p \lesssim \rho_o$. The Bessel term ($\sim \rho J_o$ in polar coordinates) is appreciable only for $\rho \not\approx 0$, i.e. $\rho >> \rho_o$ or $\rho >> p$; and because of the latter condition the final bracket in the equation is zero. Hence the condition (51) is again obtained, and the covariances again become identical to those derived using the jointly Gaussian assumption. The atmosphere has "decoherentized" the field in a manner similar to that of the diffuse reflector itself.

6. Time-Lagged Covariance and Temporal Spectrum of Scintillations

In order to obtain the relationship between the statistics of the received intensity and the cross wind, the time delayed covariance function must first be formulated. Then the slope of the time delayed, covariance function and the temporal frequency spectra can be evaluated.

For this development we assume that the fields are jointly Gaussian at the receiver and consequently

$$C_I(\bar{p}_1,\bar{p}_2,\tau) = |\Gamma(\bar{p}_1,\bar{p}_2,\tau)|^2 \tag{52}$$

Using the extended Huygens-Fresnel principle

$$\Gamma(\overline{p}_1,\overline{p}_2,\tau) = \left(\frac{k}{2\pi L}\right)^2 e^{\frac{ik(p_1^2-p_2^2)}{2L}} \iint d\overline{\rho}_1 d\overline{\rho}_2$$

$$\cdot <U\left(\overline{\rho}_1,0\right)U^*\left(\overline{\rho}_2,\tau\right)> \exp\left[\frac{ik}{2L}\left(\rho_1^2 - \rho_2^2 - 2\overline{p}_1 \cdot \overline{\rho}_1\right.\right.$$

$$\left.\left. - 2\overline{p}_2 \cdot \overline{\rho}_2\right)\right] <\exp\left[\psi_2\left(\overline{p}_1,\overline{\rho}_1,0\right) + \psi_2^*\left(\overline{p}_2,\overline{\rho}_2,\tau\right)\right]> \tag{53}$$

Due to the diffuse target

$$<U(\overline{\rho}_1,0)U^*(\overline{\rho}_2,\tau)> = <U(\overline{\rho}_1,0)U^*(\overline{\rho}_2,\tau)>\delta(\overline{\rho}_1-\overline{\rho}_2) \tag{54}$$

Using (54) in (53) and utilizing the extended Huygens-Fresnel principle to express the fields incident on the target, the time delayed mutual coherence function at the receiver becomes for the focused case

$$\Gamma\left(\overline{p}_1,\overline{p}_2,\tau\right) = \left(\frac{k}{2\pi L}\right)^4 \left|U_o\right|^2 e^{\frac{ik}{2L}\left(p_1^2-p_2^2\right)} \iiint d\overline{r}_1 d\overline{r}_2 d\overline{\rho}_1$$

$$\cdot \exp\left[-\frac{ik}{L}\ \overline{\rho}_1\left(\overline{p}_1-\overline{p}_2\right) - \frac{\left(r_1^2 + r_2^2\right)}{2\alpha_o^2} - \frac{ik}{2L}\left(r_1^2 - r_2^2\right)\right.$$

$$\left. + \frac{ik}{2L}\left(\overline{\rho}_1 - \overline{r}_1\right)^2 - \frac{ik}{2L}\left(\overline{\rho}_2 - \overline{r}_2\right)^2\right] <\exp\left[\psi_1\left(\overline{\rho}_1,\overline{r}_1,0\right)\right.$$

$$\left. + \psi_1^*\left(\overline{\rho}_1,\overline{r}_2,\tau\right)\right] <\exp\left[\psi_2\left(\overline{p}_1,\rho_1,0\right) + \psi_2^*\left(\overline{p}_2,\overline{\rho}_1,\tau\right)\right]> \tag{55}$$

The first ensemble average in (55) corresponds to the time delayed mutual coherence function for spherical waves originating at two points $\overline{r}_1$ and $\overline{r}_2$ in the transmitter plane and propagating to a single point $\overline{\rho}_1$ in the target plane. The second ensemble average corresponds to the time delayed mutual coherence function for a spherical wave originating at the point $\overline{\rho}_1$ in the target plane and propagating to two points $\overline{p}_1$ and $\overline{p}_2$ in the receiver plane. Performing the integrations in (55) it becomes

$$\Gamma(\overline{p},\tau) = \left(\frac{k}{2\pi L}\right)^2 \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\,2}}{2} \int d\overline{\rho} \int_o^\infty rdr\, J_o\!\left(\frac{k}{L}\rho r\right)$$

$$\cdot \exp\left[-\frac{r^2}{4\alpha_o^{\,2}} - i\frac{k}{L}\,\overline{p}\cdot\overline{\rho}\right] F(\overline{r},0,\tau)F(0,\overline{p},\tau)$$

$$= \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\,2}}{2}\, e^{-\frac{p^2}{4\alpha_o^{\,2}} + \frac{ik}{2L}\left(p_1{}^2 - p_2{}^2\right)} F(\overline{p},0,\tau)F(0,\overline{p},\tau) \tag{56}$$

where the time delayed mutual coherence functions $F(\overline{p},0,\tau)$ and $F(0,\overline{p},\tau)$ can be obtained from the mutual coherence function for a spherical wave by invoking Taylor's Hypothesis.[35]

This mutual coherence function is given by[32]

$$F(\overline{r},\overline{\rho}) = \exp\left[-\frac{2.91}{2} Lk^2 \int_o^1 C_n^{\,2}(w)\left|w\overline{\rho} + (1-w)\overline{r}\right|^{5/3} dw\right] \tag{57}$$

where $\overline{r}$ is the transmitter aperture vector $\overline{r}_1 - \overline{r}_2$, $\overline{\rho}$ is the target aperture vector $\overline{\rho}_1 - \overline{\rho}_2$ and w is the distance from the source to the field point normalized by the total path length L. For the uniform turbulence case, we note that $k^2 L C_n^{\,2} \sim \rho_o^{-5/3}$. The time delayed mutual coherence function[41] can be obtained from (57) by replacing

$\overline{w\rho}$ by $\overline{w\rho} - \overline{V}(w)\tau$, where $\overline{V}(w)$ is the transverse wind:

$$F(\overline{r},\overline{\rho},\tau) = \exp\left[-\frac{2.91}{2} Lk^2 \int_0^1 C_n{}^2(w) \left|w\overline{\rho} - \overline{V}(w)\tau + (1-w)\overline{r}\right|^{5/3} dw\right] \tag{58}$$

and consequently

$$F(\overline{p},0,\tau) = \exp\left[-\frac{2.91}{2} Lk^2 \int_0^1 C_n{}^2(w) \left|(1-w)\overline{p} - \overline{V}(w)\tau\right|^{5/3} dw\right] \tag{59}$$

and

$$F(0,\overline{p},\tau) = \exp\left[-\frac{2.91}{2} Lk^2 \int_0^1 C_n{}^2 (1-z) \left|z\overline{p} - \overline{V}(1-z)\tau\right|^{5/3} dz \right. \tag{60}$$

where z is the normalized distance from the target to the field point.

The normalized, time delayed covariance function for the focused case and an arbitrary distribution of $C_n{}^2$ along the path is thus given by

$$C_{I_N}(\overline{p},\tau) = e^{-\frac{p^2}{2\alpha_o{}^2}}$$

$$\exp\left[- 5.82\, Lk^2 \int_0^1 C_n{}^2(w) \left|(1-w)\overline{p} - \overline{V}(w)\tau\right|^{5/3} dw\right]$$

(focused) (61)

The slope at zero time delay can now be found from (61).

$$\left.\frac{\partial C_{I_N}}{\partial \tau}\right|_{\tau = 0} = M_{I_N}(\bar{p}) = 9.7\ Lk^2 p^{2/3} \left[\int_0^1 C_n^{\,2}(w)(1-w)^{2/3}\ \bar{V}(w)\cdot\hat{p}\ dw \right] \cdot C_{I_N}(\bar{p}) \tag{62}$$

It is found that the slope is linearly related to the component of wind along $\bar{p}$ and is proportional to a weighted average of the crosswind along the propagation path. The crosswind weighting function can be found from (62) by letting

$\bar{V}(w) = \hat{p}\ \delta(w - w_o)$

This yields

$$W(w_o) = 9.7\ Lk^2 p^{2/3}\ C_n^{\,2}(w_o)\ (1-w_o)^{2/3}\ C_{I_N}(\bar{p}) \tag{63}$$

The wind weighting function is zero at the target and at least for uniform turbulence varies as $(1 - w_o)^{2/3}$ along the path with the maximum weighting occurring at the receiver. This result is similar to the result for the cw system[33] in that the weighting is zero at the target (source) and peaked up toward the receiver due to the source size being large with respect to a Fresnel zone and to the receiver size. But in the cw system the weighting is zero at the receiver which is not the same as the result obtained for the pulsed system.

The normalized covariance of the intensity is equivalent to the normalized covariance of the log-amplitude for a plane or spherical wave in the case of weak turbulence. In that case, the wind weighting function should be zero at the receiver. However, in the beam wave case or diffuse source case, the normalized covariance of the intensity does not bear a simple relationship to the normalized log-amplitude covariance and the wind weighting function may indeed, as predicted by the theory, not be zero at the receiver. This result is consistent with the theory developed for the passive system.[30]

If the turbulence and crosswind are uniform, then the slope at zero time delay and the component of the crosswind along $\overline{p}$ are related by

$$\overline{V}\cdot\hat{p} = \frac{1}{5.82\ Lk^2p^{2/3}\ C_n{}^2\ C_{I_N}(\overline{p},0)}\ M_{I_N}(\overline{p},0) \tag{64}$$

The factor multiplying the slope in (64) is the calibration factor that relates the measured slope of the time delayed covariance function to the path averaged crosswind. It should be noted that it is a function of turbulence level. The proper relationships for the collimated case can be obtained from (61) and (62) by including the factor

$$\exp\left[-\frac{1}{2}\,p^2\left(\frac{k\alpha_o}{2L}\right)^2\right]$$

The power spectral density can be obtained from (61). Letting

$\overline{p} = 0$ and assuming uniform turbulence and crosswind, (61) becomes

$$C_{I_N}(0,\tau) = e^{-5.82\, Lk^2 C_n^{\,2}\, |\overline{V}|^{5/3}\, \tau^{5/3}} = e^{-10.67\left|\frac{\overline{V}}{\rho_o}\right|^{5/3} \tau^{5/3}} \tag{65}$$

Taking the Fourier transformation of (65) yields the power spectral density.

$$S_I(w) = 2\int_0^{\infty} e^{-10.67\left|\frac{\overline{V}}{\rho_o}\right|^{5/3} \tau^{5/3}} \operatorname{Cos}(w\tau)\, d\,\tau$$

$$= \frac{6}{5}\,\frac{\rho_o}{|\overline{V}|\ (10.67)^{3/5}}\left[\Gamma(3/5) - \frac{X^2}{2!}\,\frac{\Gamma(9/5)}{(10.67)^{6/5}} + \ldots\right.$$

$$\left. + \ldots\ (-1)^{n-1}\,\frac{X^{2(n-1)}}{2(n-1)!}\,\frac{\Gamma\left(\frac{3}{5} + \frac{6}{5}\,(n-1)\right)}{(10.67)^{6(n-1)/5}} + \ldots\right] \tag{66}$$

where

$$x = \frac{w\rho_o}{|\overline{V}|}$$

The normalized power spectral density is plotted versus the parameter X in Figure 6.

Summary of the above work: under the assumption that the wave structure function is approximately equal to the phase structure function, a general formulation for the normalized covariance of the intensity was developed for the focused and collimated cases that involved a threefold integration. Also, the normalized variance was

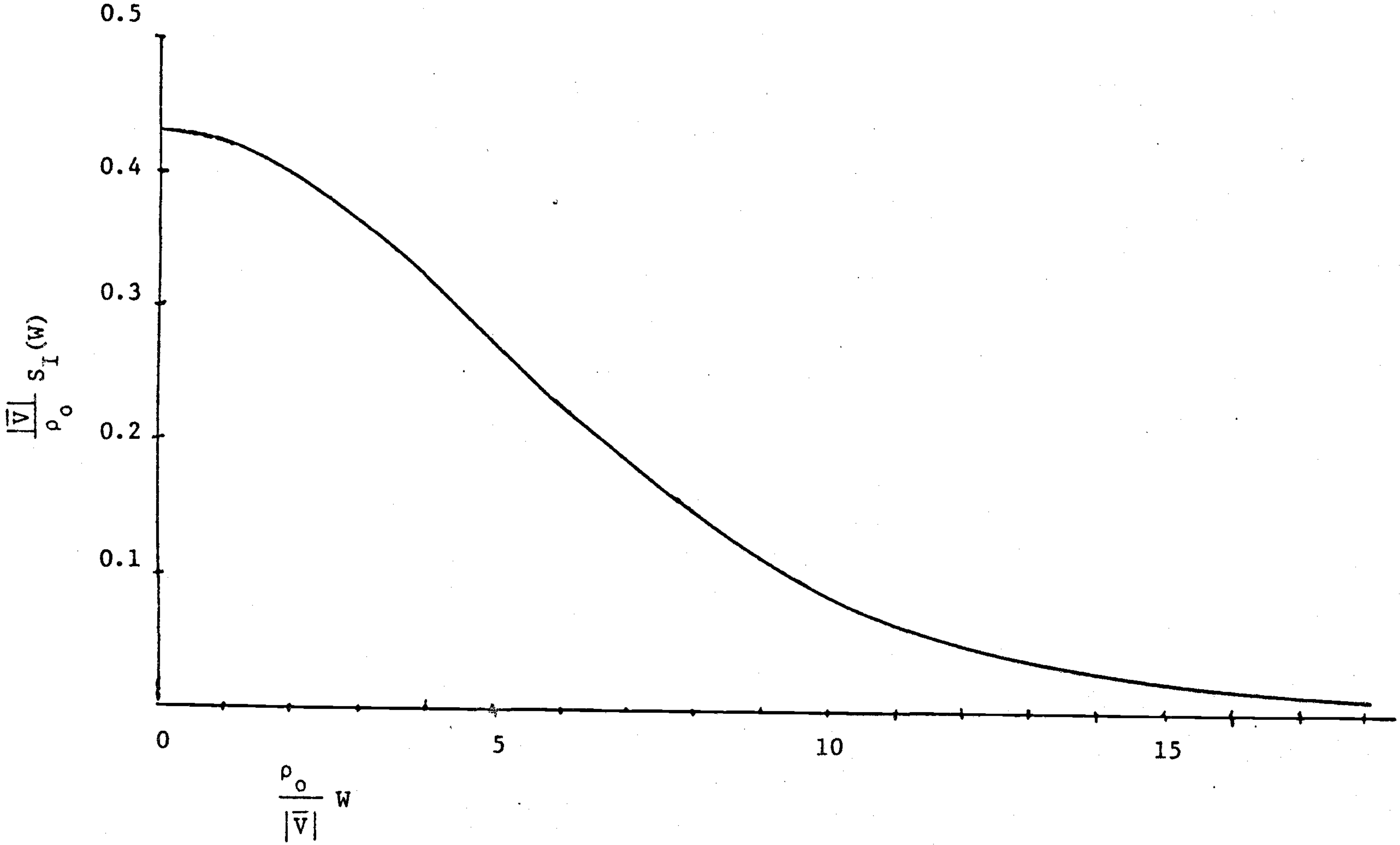

Figure 6. Temporal Power Spectral Density, Received Intensity

found to be unity and the probability density function for the field at the receiver was found to be Gaussian and, on a space averaged basis, incoherent. In the case of weak or saturated turbulence, a closed form solution was found for the normalized covariance of the intensity; under the assumption that the fields at the receiver are jointly normal, a closed form solution was developed for the normalized covariance of the intensity that is identical to that derived from the general formulation under weak or saturated turbulence conditions. In addition, formulations were developed for the time delayed covariance function and its slope, the wind weighting function and the temporal power spectral density. The following work remains to be completed: extension of the work to the case of a finite receiving aperture, development of a general formulation that is free of the phase domination assumption and experimental conformation of the theoretical results.

7. Higher Order Modes

The analysis of the previous section assumed the source to be a single mode (TEM_{00}) laser beam. The same techniques may be applied to higher order modes. In the following, a TEM_{10} beam is selected as the source. Then

$$U_{10}(r) = U_o\left(1 - \frac{r^2}{\alpha_o^2}\right) e^{-\left(\frac{r^2}{2\alpha_o^2} + \frac{ik}{2F} r^2\right)}$$

The mean intensity at receiver can be described as,

$$\langle I(\overline{p})\rangle = \left(\frac{k}{2\pi L}\right)^2 \quad d\overline{r} \ \langle I(\overline{r})\rangle$$

$$= \frac{|U_o|^2}{2\pi} \left(\frac{k}{L}\right)^2 \frac{\alpha_o^2}{2} \tag{67}$$

this is identical to the TEM_{00} mode case. In order to calculate the co-variance function, the assumption of joint Gaussian fields at the receiver is applied. The mutual coherence function is calculated using Eq.(33)

$$\Gamma(\overline{p}_1,\overline{p}_2) = \left(\frac{k}{2\pi L}\right)^2 e^{\frac{ik(p_1^2-p_1^2)}{2L}} \int d\overline{\rho} \ \langle I(\overline{\rho})\rangle_{10} \ e^{-\frac{ik}{L}\overline{\rho}\cdot\overline{\rho} - \left(\frac{p}{\rho_o}\right)^{5/3}} \tag{68}$$

The mean intensity at target can be defined,

$$\langle I(\overline{\rho})\rangle_{10} = \left(\frac{k}{2\pi L}\right)^2 \iint d\overline{r_1} \ d\overline{r_2} \ U_{10}(\overline{r_1}) \ U_{10}^*(\overline{r_2})$$

$$\times \exp\left[-\frac{ik}{L}\overline{p}\cdot(\overline{r_1}-\overline{r_2}) + \frac{ik}{2L}(r_1^2-r_2^2) - \left(\frac{|\overline{r_1}-\overline{r_2}|}{\rho_o}\right)^{5/3}\right] \tag{69}$$

The next work is concerned with the mathetical manipulation of expressions necessary to get the integration into a form in which satisfactory closed formulation can be made. The integration is divided each term for convenience and evaluated separately. The variables $\overline{r_1}, \overline{r_2}$ are transformed $\bar{r}$ and $\bar{R}$ defined

$$\bar{r} = \overline{r_1} - \overline{r_2} \qquad 2\bar{R} = \overline{r_1} + \overline{r_2}$$

$$<I(\bar{\rho})>_{10} = \left(\frac{k}{2\pi L}\right)^2 |U_o|^2 \iint \left\{1 - \frac{r^2 + 4R^2}{2\alpha_o^{\,2}} + \frac{1}{\alpha_o^{\,4}} \left[\left(R^2 + \frac{r^2}{4}\right)^2 - |\bar{R}-\bar{r}|^2\right]\right\}$$

$$x \exp\left[-\frac{r^2 + 4R^2}{4\alpha_o^{\,2}} + \frac{ik}{L}(1 - L/F)\bar{R}\cdot\bar{r} - \frac{ik}{L}\bar{r}\cdot\bar{\rho} - \left(\frac{r}{\rho_o}\right)^{5/3}\right] d\bar{r}\, d\bar{R}$$

$$= <I(\bar{\rho})>_A + <I(\bar{\rho})>_B + <I(\bar{\rho})>_C + <I(\bar{\rho})>_D \tag{70}$$

Each term can be computed as

$$<I(\bar{\rho})>_A = \left(\frac{k}{L}\right)^2 |U_o|^2 \int_0^\infty\!\!\int_0^\infty RrJ_o\left[\frac{k}{L}(1 - L/F)Rr\right] J_o\left(\frac{k}{L}\rho r\right) e^{-\frac{r^2+4R^2}{4\alpha_o^{\,2}} - \left(\frac{r}{\rho_o}\right)^{5/3}} dRdr$$

$$= \frac{\alpha_o^{\,2}}{2}\left(\frac{k}{L}\right)^2 |U_o|^2 \int_0^\infty J_o\left(\frac{k}{L}\rho r\right) e^{-\frac{\alpha_o^{\,2}}{4}\frac{k^2}{L^2}(1 - L/F)^2 r^2 - \frac{r^2}{4\alpha_o^{\,2}} - \left(\frac{r}{\rho_o}\right)^{5/3}} rdr \tag{71}$$

$$<I(\bar{\rho})>_B = -\left(\frac{k}{L}\right)^2 \frac{|U_o|^2}{2\alpha_o^{\,2}} \int_0^\infty J_o\left(\frac{k}{L}\rho r\right) e^{-\frac{r^2}{4\alpha_o^{\,2}} - \left(\frac{r}{\rho_o}\right)^{5/3}} rdr$$

$$x \int_0^\infty R(r^2 + 4R^2) J_o\left[\frac{k}{L}(1 - L/F)Rr\right] e^{-\frac{R^2}{\alpha_o^{\,2}}} dR \tag{72}$$

$$= -\left(\frac{k}{L}\right)^2 \frac{|U_o|^2}{4} \int_0^\infty J_o\left(\frac{k}{L}\rho r\right) \left\{ r^2 + 4\alpha_o^2 \left[1 - \frac{\alpha_o^2}{4}\frac{k^2}{L^2}(1 - L/F)^2 r^2\right]\right\}$$

$$\times\ e^{-\frac{r^2}{4\alpha_o^2} - \frac{\alpha_o^2}{4}\frac{k^2}{L^2}(1 - L/F)^2 r^2 - \left(\frac{r}{\rho_o}\right)^{5/3}} rdr \tag{73}$$

$$\langle I(\overline{\rho})\rangle_c = \left(\frac{k}{L}\right)^2 |U_o|^2 \left(\frac{1}{\alpha_o^2}\right)^2 \int_0^\infty J_o\left(\frac{k}{L}\rho r\right) e^{-\frac{r^2}{4\alpha_o^2} - \left(\frac{r}{\rho_o}\right)^{5/3}} rdr$$

$$\times \int_0^\infty \left(R^5 + \frac{R^3 r^2}{2} + \frac{Rr^4}{16}\right) J_o\left[\frac{k}{L}(1 - L/F)Rr\right] e^{-\frac{R^2}{\alpha_o^2}} dR$$

$$= \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{1}{\alpha_o^4} \int_0^\infty J_o\left(\frac{k}{L}\rho r\right) F_c(r)\ e^{-\frac{r^2}{4\alpha_o^2} - \left(\frac{r}{\rho_o}\right)^{5/3} - \frac{\alpha_o^2}{4}\frac{k^2}{L^2}(1 - L/F)^2 r^2} rdr \tag{74}$$

where

$$F_c(r) = \alpha_o^6 \left\{1 - \frac{\alpha_o^2}{2}\frac{k^2}{L^2}(1 - L/F)^2 r^2 + \frac{1}{8}\left[\frac{\alpha_o^2}{2}\frac{k^2}{L^2}(1 - L/F)^2 r^2\right]^2\right\}$$

$$+ \frac{\alpha_o^4}{4}\left[1 - \frac{\alpha_o^2}{4}\frac{k^2}{L^2}(1 - L/F)^2 r^2\right] r^2 + \frac{\alpha_o^2}{32} r^4$$

and

$$\langle I(\overline{\rho})\rangle_d = -\left(\frac{k}{2\pi L}\right)^2 |U_o|^2 \left(\frac{2}{W^2}\right)^2 \iint |\overline{R}\cdot\overline{r}|^2 \exp\left[-\frac{r^2 + 4R^2}{4\alpha_o^2} + \frac{ik}{L}(1 - L/F)\right.$$

$$\left.\overline{R}\cdot\overline{r} - \frac{ik}{L}\overline{r}\cdot\overline{\rho} - \left(\frac{r}{\rho_o}\right)^{5/3}\right] d\overline{r}\ d\overline{R}$$

$$= -\left(\frac{k^2}{L}\right)|U_o|^2 \int_o^\infty \left[\frac{1}{4} - \frac{\alpha_o^2}{8}\frac{k^2}{L^2}(1 - L/F)^2 r^2\right] r^2 J_o\left(\frac{k}{L}\rho r\right)$$

$$\exp\left[-\frac{r^2}{4\alpha_o^2} - \left(\frac{r}{\rho_o}\right)^{5/3} - \frac{\alpha_o^2}{4}\frac{k^2}{L^2}(1 - L/F)^2 r^2\right] rdr \tag{75}$$

The mean irradiance at target can be written without repeating the arithmetic calculation.

$$<I(\bar{\rho})> = \frac{\alpha_o^2}{2}\left(\frac{k}{L}\right)^2 |U_o|^2 \int_o^1 \left\{1 - \frac{1}{2}\left[\frac{\alpha_o^2}{2}\frac{k^2}{L^2}(1 - L/F)^2 + \frac{1}{2\alpha_o^2}\right] r^2\right\}^2$$

$$\times J_o\left(\frac{k}{L}\rho r\right) \exp\left[-\frac{r^2}{4\alpha_o^2} - \left(\frac{r}{\rho_o}\right)^{5/3} - \frac{\alpha_o^2}{4}\frac{k^2}{L^2}(1 - L/F)^2 r^2\right] rdr \tag{76}$$

By using this formula, the mutual coherence function is calculated.

$$\Gamma(\bar{p}_1, \bar{p}_2) = \left(\frac{1}{2\pi}\right)\frac{\alpha_o^2}{2}\left(\frac{k}{L}\right)^2 |U_o|^2 e^{+\frac{ik(p_1^2 - p_2^2)}{2}}$$

$$\left\{1 - \frac{1}{2}\left[\frac{\alpha_o^2}{2}\frac{k^2}{L^2}(1 - L/F)^2 + \frac{1}{2\alpha_o^2}\right] p^2\right\}^2$$

$$\times \exp\left\{-\frac{p^2}{4\alpha_o^2} - 2\left(\frac{p}{\rho_o}\right)^{5/3} - \frac{\alpha_o^2}{4}\frac{k^2}{L^2}(1 - L/F)^2 p^2\right\} \tag{77}$$

The covariance of irradiance is calculated by the Gaussian Assumption

$$C_{IN}(\overline{p}) = |\Gamma_N(\overline{p}_1,\overline{p}_2)|^2 = \left\{1 - \frac{1}{2}\left[\frac{\alpha_o^{\ 2}}{2}\ \frac{k^2}{L^2}\ (1 - L/F) + \frac{1}{2\alpha_o^{\ 2}}\right] p^2\right\}^2$$

$$\times \exp\left\{- \frac{p^2}{2\alpha_o^{\ 2}} - 4\left(\frac{p}{\rho_o}\right)^{5/3} - \frac{\alpha_o^{\ 2}}{2}\frac{k^2}{L^2}\ (1 - L/F)^2 p^2\right\} \tag{78}$$

The focused case

$$C_{IN}(\overline{p}) = \left(1 - \frac{p^2}{4\alpha_o^{\ 2}}\right)^2 e^{- \frac{p^2}{2\alpha_o^{\ 2}} - 4\left(\frac{p}{\rho_o}\right)^{5/3}} \tag{79}$$

The collimated case

$$C_{IN}(\overline{p}) = \left\{1 - \frac{1}{2}\left[\frac{\alpha_o^{\ 2}}{2}\frac{k^2}{L^2} + \frac{1}{2\alpha_o^{\ 2}}\right] p^2\right\}^2 e^{- \frac{p^2}{2\alpha_o^{\ 2}} - 4\left(\frac{p}{\rho_o}\right)^{5/3} - \frac{\alpha_o^{\ 2}}{2}\frac{k^2}{L^2}p^2} \tag{80}$$

II.B. THE GENERAL COVARIANCE FUNCTION

In a previous section, the mean irradiance and mutual coherence function were derived for the case of a diffuse target. However, the analysis of the intensity covariance was based on the assumption of neglecting the amplitude term in the perturbation Green's function. This latter assumption is not always valid, as can be seen by considering the case where the illuminated target spot is small: the scintillations at the receiver should then approximate those of a point source, with log normal statistics and a covariance scale on the order $(L/k)^{\frac{1}{2}}$.

In this section we include the amplitude perturbation term and examine the effect on the probability distribution of the irradiance. We then derive the general covariance function.

1. Moments and Probability Distribution of Irradiance

The second moment of irradiance can be written from Eqs.(14) and (17) and the mutual coherence function (15). We now generalize the latter to include amplitude perturbation terms (Appendix A), resulting in

$$<I^2(\bar{\rho})> = 2\left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2 d\bar{\rho}_4 \; <I(\bar{\rho}_2)><I(\bar{\rho}_4)> \; e^{4C_\chi(|\bar{\rho}_2-\bar{\rho}_4|)} \tag{81}$$

where C_χ is the point-source log amplitude covariance function given in the first-order theory[35,42,43] by

$$C_\chi(|\bar{\rho}_2-\bar{\rho}_4|) = 4\pi^2k^2 \int_0^\infty \int_0^L u\Phi(u) J_o\left(\frac{|\bar{\rho}_2-\bar{\rho}_4|}{L} us\right) \sin^2\left[\frac{u^2 s(L-s)}{2kL}\right] ds du \tag{82}$$

and Φ is the Kolmogorov spectrum[35]

$$\Phi(u) = 0.033\ C_n^{\ 2} u^{-11/3} \tag{83}$$

We assume for the present that $C_\chi \ll 1$, which will be true[35] for both weak and saturated scintillations (weak and strong turbulence scattering respectively). It will not be true for the intermediate case, however, and we will lift the restriction in calculating the covariance below. Also, we point out that the function C_χ has been derived phenomenologically for the saturated or strong scattering case by Yura and Clifford[44-46] so that in principle we are not limited to the first-order expression (82).

We generalize (81) to the nth moment:

$$\langle I^n(\bar{p})\rangle = n!\left(\frac{k}{2\pi L}\right)^{2n} \int\cdots\int d\bar{\rho}_1\ldots d\bar{\rho}_n \left[\prod_{i=1}^{n} \langle I(\bar{\rho}_i)\rangle\right] \exp\left[2\sum_{i\neq j}^{n} C_{\chi_{ij}}\right] \tag{84}$$

We write this as

$$\langle I^n(\bar{p})\rangle = n!\langle I(\bar{p})\rangle^n F_n \tag{85}$$

where

$$F_1 = 1$$

$$F_2 = \frac{\iint d\bar{\rho}_2 d\bar{\rho}_4 \langle I(\bar{\rho}_2)\rangle\langle I(\bar{\rho}_4)\rangle\, e^{4C_\chi(|\bar{\rho}_2-\bar{\rho}_4|)}}{\left[\int d\bar{\rho}_2\langle I(\bar{\rho}_2)\rangle\right]^2}$$

$$F_n = \frac{\int\cdots\int d\bar{\rho}_1 d\bar{\rho}_2,\ldots d\bar{\rho}_n\left[\prod_{i=1}^{n}\langle I(\bar{\rho}_i)\rangle\right]\exp\left[2\sum_{i\neq j}^{n} C_\chi(|\bar{\rho}_i-\bar{\rho}_j|)\right]}{\left[\int d\bar{\rho}\langle I(\bar{\rho})\rangle\right]^n} \tag{86}$$

Our assumption on C_χ yields

$$e^{4C_\chi(|\bar{\rho}_2-\bar{\rho}_4|)} = 1+4C_\chi(|\bar{\rho}_2-\bar{\rho}_4|) \tag{87}$$

We thus simplify F_2 to

$$F_2 = 1+\frac{4\iint d\bar{\rho}_2 d\bar{\rho}_4 <I(\bar{\rho}_2)><I(\rho_4)>C_\chi(|\bar{\rho}_2-\bar{\rho}_4|)}{\left[\int d\bar{\rho}_2<I(\bar{\rho}_2)>\right]^2} \tag{88}$$

For the denominator for the focused case, we write from Eq.(10):

$$<I(\bar{\rho})> = \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2}\int_0^\infty rdrJ_o\left(\frac{k}{L}\rho r\right) e^{-\frac{r^2}{4\alpha_o^2}-\left(\frac{r}{\rho_o}\right)^{5/3}} \tag{89}$$

which integrates to

$$\int<I(\bar{\rho})>d\bar{\rho} = 2\pi|U_o|^2 \frac{\alpha_o^2}{2} \tag{90}$$

The integral in the numerator of (88) is straightforward but laborious, we state here the result for F_2:

$$F_2 = 1+(4\pi)^2k^2\int_0^\infty udu\int_0^L ds\, e^{-\frac{u^2s^2}{2\alpha_o^2k^2}-2\left(\frac{us}{k\rho_o}\right)^{5/3}} \Phi(u)\sin^2\left[\frac{u^2s(L-s)}{2kL}\right] \tag{91}$$

where the approximation relates to the assumption (87) on C_χ. We then have the second moment from Eq.(85):

$$<I^2(\bar{\rho})> = 2<I(\bar{\rho})>^2F_2 \tag{92}$$

and the n^{th} moment is

$$\langle I^n \rangle = n!\langle I \rangle^n \left\{1+ \frac{n(n-1)}{2} (F_2-1)\right\} \tag{93}$$

Hence nonunity F_2 represents the departure from an exponential intensity distribution.

We finally derive the probability distribution using

$$\langle e^{-sI} \rangle = 1-s\langle I \rangle + \frac{s^2\langle I^2 \rangle}{2!} - \frac{s^3\langle I^3 \rangle}{3!} + \ldots \tag{94}$$

We let $\langle I \rangle \equiv I_o$ and find the probability as

$$P_I(I) = \frac{1}{I_o} e^{-\frac{I}{I_o}} \quad 1 + (F_2-1)\left[1- \frac{2I}{I_o} + \frac{I_2}{2I_o{}^2}\right] \tag{95}$$

so that we have a first-order correction to the exponential distribution.

We note that F_2 (and F_n) will be unity for weak turbulence and for very strong turbulence ($\rho_o \to 0$). Also, F_2 will be unity for a small focused source (α_o), which physically relates to a large target spot. Conversely, F_2 will depart from unity and therefore show the effect of the amplitude perturbation term for the case of intermediate turbulence and a large source (small target spot). Physically this is the case of a quasi-point-source attempting to scintillate in the usual manner for a point source in the first-order theory,[35] but nevertheless interacting with the speckles created by the diffuse target.

The exponential distribution applies for the weak turbulence case simply because the behavior is that of a diffuse source in a vacuum (speckles). It applies to a large target-spot, implying that the phase-perturbation-dominance assumption of Section II.A is then applicable, i.e.

that the target (vacuum) speckle mechanism dominates. It applies for strong turbulence because the field again has the nature of that from a diffuse source (atmospheric speckle ρ_o). These considerations will be clarified below.

2. Covariance and Variance of Intensity

We now derive the general covariance function, dropping the requirement that $C_\chi \ll 1$.

The covariance is the second term of Eq.(18):

$$C_I(\overline{p}_1,\overline{p}_2) = \left(\frac{k}{2\pi L}\right)^4 \iint d\overline{\rho}_2 d\overline{\rho}_4 \; \langle I(\overline{\rho}_2)\rangle\langle I(\overline{\rho}_4)\rangle \left\{ e^{\frac{ik}{L}(\overline{p}_1-\overline{p}_2).(\overline{\rho}_1-\overline{\rho}_2)} H(\overline{\rho},\overline{p}) + e^{4C_\chi(\overline{\rho},\overline{p})} - 1 \right\} \quad \text{where } \overline{\rho} = \overline{\rho}_2-\overline{\rho}_4,\; \overline{p} = \overline{p}_1-\overline{p}_2. \tag{96}$$

The full coherence function replacing Eq.(19) is (Appendix A)

$$H(\overline{\rho},\overline{p}) = e^{\left\{-2\left(\frac{p}{\rho_o}\right)^{5/3} - \frac{1}{\rho_o^{5/3}}\left[2\rho^{5/3} - \frac{8}{3}\int_0^1 dt\,|\overline{p}t + (1-t)\overline{\rho}|^{5/3} - \frac{8}{3}\int_0^1 dt\,|\overline{p}t - (1-t)\overline{\rho}|^{5/3}\right] + 2C_\chi(\overline{\rho},\overline{p}) + 2C_\chi(\overline{\rho},-\overline{p})\right\}} \tag{97}$$

where[47]

$$C_\chi(\overline{\rho},\overline{p}) = 0.132\pi^2 k 2 C_n^{\,2} L \int_0^1 dt \int_0^\infty du\; u^{-8/3} \sin^2\left[\frac{u^2 L t(1-t)}{2k}\right] . J_o\left[u|t\overline{p} + (1-t)\overline{\rho}|\right] \tag{98}$$

The latter expression (98) assumes that multiple scattering (saturation) does not apply but can be modified for such a case.[45] The irradiance at the target $\langle I(\bar{\rho})\rangle$ is given by (89) for the focused case and with an additional $(k\alpha_o/2L)^2$ term in the exponent for the collimated case as before.

We again omit tedious algebra and integrations and state the result for the focused case:

$$C_I(\bar{p}) = \left(\frac{1}{2\pi}\right)^3 \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o^2}{2}\right)^2 \int_0^\infty r_2 dr_2\, e^{-\frac{r^2}{2\alpha_o^2} - 2\frac{r_2^{5/3}}{\rho_o^{5/3}}}$$

$$\cdot \int d\bar{\rho}\, J_o\left(\frac{k}{L} r_2 \rho\right) e^{\frac{ik}{L}\bar{\rho}\cdot\bar{p}}\, H(\bar{\rho},\bar{p})$$

$$+ \left(\frac{1}{2\pi}\right)^2 \left(\frac{K}{L}\right)^4 |U_o|^4 \left(\frac{\alpha_o^2}{2}\right)^2 \left\{\frac{1}{2\pi}\left(\frac{K}{L}\right)^2 \int_0^\infty r_2 dr_2\, e^{\frac{r_2^2}{-2\alpha_o^2} - 2\left(\frac{r_2}{\rho_o}\right)^{5/3}}\right.$$

$$\left. \times \int d\bar{\rho}\, J_o\left(\frac{K}{L} r_2 \rho\right) e^{4C_\chi(\bar{\rho},\bar{p})} - 1\right\} \tag{99}$$

The variance is given by

$$C_I(o) = \sigma_I^2 = \left(\frac{1}{2\pi}\right)^2 \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o^2}{2}\right)^2 \int_0^\infty r_2 dr_2\, e^{-\frac{r_2^2}{2\alpha_o^2} - \frac{2r_2^{5/3}}{\rho_o^{5/3}}}$$

$$\cdot \int_0^\infty \rho d\rho J_o\left(\frac{k}{L} r_2 \rho\right) \left\{ 2e^{4C_\chi(\rho)} - 1\right\} \tag{100}$$

These general results involve five-fold integrals (more in the saturated case[45]) and have not been numerically evaluated. A comparison with Eqs.(26) and (28) show that the amplitude perturbation term simply introduces the C_χ terms in Eq.(99), i.e. the log amplitude covariance for a point source. A qualitative interpretation will be given below.

In order to compare these results with those of Section II.B.1, we again let $C_\chi \ll 1$, and we find that (80) and (63) yield

$$\sigma^2_{I_N} = \frac{\sigma^2_I}{\bar{I}^2}$$

$$= 1 + 1.056\pi^2 k^2 C_n^2 L \int_0^1 dt \int_0^\infty du\, u^{-8/3} \exp\left\{-\frac{1}{2\alpha_o{}^2}\left[\frac{u}{k} L(1-t)\right]^2 - \frac{2}{\rho_o{}^{5/3}}\left[\frac{u}{k} L(1-t)\right]^{5/3}\right\} \sin^2\left[\frac{u^2 L t(1-t)}{2k}\right] \tag{101}$$

This is consistent with Eq.(92), and again shows the departure from the unity normalized variance obtained when C_χ cannot be taken as zero. The general formulations of variance, covariance, higher order moments and probability density function of irradiance have been analyzed including log-amplitude perturbation effects. In addition, in case of small log-amplitude perturbations, these statistical properties are reduced to close expressions that can be compared with those of previous sections.

II.C. TARGET GLINTS

As a preliminary examination of the effects of target structure, we present in this section an analysis of the mean receiver irradiance in the presence of "glints" (specular reflectors). We assume that the target is otherwise diffuse, and that phase perturbations predominate. The transmitter is again a TEM_{00} focused or collimated source.

We again start with the field at the receiver as described by Eqs.(1,2). The field reflected at the target is represented as

$$U(\bar{\rho}) = U_{diffuse}(\bar{\rho}) + U_{glint}(\bar{\rho}) = U_d(\bar{\rho}) + U_g(\bar{\rho}) \tag{102}$$

where

$$U_{glint} = \sum_{m=1}^{M} U_{incident}(\bar{\rho}_m)\, a_m \exp\left\{ \frac{-(\bar{\rho}-\bar{\rho}_m)^2}{\Delta\rho_m^{\;2}} \right\}$$

ρ_m = position of m^{th} glint

$\Delta\rho_m$ = width of m^{th} glint

a_m = complex strength of m^{th} glint

Then the irradiance is

$$I(\bar{p}) = \left(\frac{k}{2\pi L}\right)^2 \iint \left[U_d(\bar{\rho}_1) + U_g(\bar{\rho}_1)\right]\left[U_d^*(\bar{\rho}_2) + U_g^*(\bar{\rho}_2)\right]$$

$$\cdot \exp\left\{ \frac{ik}{2L}\left[(\bar{\rho}_1-\bar{p})^2 - (\bar{\rho}_2-\bar{p})^2\right] + \psi_2(\bar{\rho}_1,\bar{p}) + \psi_2^*(\bar{\rho}_2,\bar{p}) \right\} d\bar{\rho}_1, d\bar{\rho}_2$$

$$= \left(\frac{k}{2\pi L}\right)^2 \iint \left[U_d(\bar{\rho}_1) + U_g(\bar{\rho}_1)\right]\left[U_d^*(\bar{\rho}_2) + U_g^*(\bar{\rho}_2)\right]$$

$$\cdot \exp\left\{ \frac{ik}{2L}\left[\rho_1^2 - \rho_2^2 - 2(\bar{\rho}_1-\bar{\rho}_2)\bar{p}\right] + \psi_2(\bar{\rho}_2,\bar{p}) + \psi_2^*(\bar{\rho}_2,\bar{p}) \right\} d\bar{\rho}_1\, d\bar{\rho}_2 \tag{103}$$

We then simplify the field at the target:

$$< \left[U_d(\bar{\rho}_1)+U_g(\bar{\rho}_1)\right] \left[U_d^*(\bar{\rho}_2)+U_g^*(\rho_2)\right] >$$

$$= <U_d(\rho_1)U_d^*(\bar{\rho}_2)> + <U_d(\bar{\rho}_1)U_g^*(\bar{\rho}_2)> + <U_g(\bar{\rho}_1)U_d^*(\bar{\rho}_2)> + <U_g(\bar{\rho}_1)U_g^*(\rho_2)> \tag{104}$$

Since

$$<U_d(\bar{\rho}_1)U_g^*(\bar{\rho}_2)> = <U_d(\bar{\rho}_1)><U_g^*(\rho_2)> = 0$$

$$<U_g(\bar{\rho}_1)U_d^*(\bar{\rho}_2)> = <U_g(\bar{\rho}_1)><U_d^*(\bar{\rho}_2)> = 0$$

$$<U_d(\bar{\rho}_1)U_d^*(\bar{\rho}_2)> = <I(\bar{\rho}_1)> \delta\ (\bar{\rho}_1-\bar{\rho}_2)$$

$$< e^{\psi_2(\bar{\rho}_1,\bar{p})+\psi_2^*(\bar{\rho}_2,\bar{p})} >= e^{-\left(\frac{|\bar{\rho}_1-\bar{\rho}_2|}{\rho_o}\right)^{5/3}} \tag{105}$$

we have

$$<I(\bar{p})> = <I(\bar{p})>_d + <I(\bar{p})>_g \tag{106}$$

where $<I(\bar{p})>_d$ is the diffuse term analyzed in Section II.A and $<I(\bar{p})>_g$ is the glint term:

$$<I(\bar{p})>_g = \left(\frac{k}{2\pi L}\right)^2 \iint d\bar{\rho}_1 d\bar{\rho}_2\ <U_g(\bar{\rho}_1)U_g^*(\bar{\rho}_2)> \cdot \exp\left\{\frac{ik}{2L}\left[\rho_1^2-\rho_2^2-2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}\right] - \frac{|\bar{\rho}_1-\bar{\rho}_2|^{5/3}}{\rho_o^{5/3}}\right\} \tag{107}$$

We now describe the complex correlation of the glint field at the target:

$$<U_g(\rho_1)U_g^*(\bar{\rho}_2)> = \sum_{m=1}^{M} |a_m|^2 <I(\bar{\rho}_m)> \exp\left\{-\frac{\rho_1^2+\rho_2^2-2\bar{\rho}_m\cdot(\bar{\rho}_1+\bar{\rho}_2)+2\rho_m^2}{\Delta\rho_m^2}\right\}$$

$$+ \sum_{m_1\neq m_2}^{M} a_{m_1} a_{m_2} <U(\bar{\rho}_{m_1})U^*(\bar{\rho}_{m_2})>$$

$$\cdot\exp\left\{-\frac{\rho_1^2-2\bar{\rho}_{m_1}\cdot\bar{\rho}_1+\rho_{m_1}^2}{\Delta\rho_{m_1}^2} - \frac{\rho_2^2-2\bar{\rho}_{m_2}\cdot\bar{\rho}_2+\rho_{m_2}^2}{\Delta\rho_{m_2}^2}\right\} \tag{108}$$

where the focused case

$$<I(\bar{\rho}_m)> = \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2}\int_0^\infty r\,dr\, J_o\left(\frac{k}{L}\rho_m r\right) e^{-\frac{r^2}{4\alpha_o^2}-\left(\frac{r}{\rho_o}\right)^{5/3}} \tag{109}$$

The corresponding expression for the collimated case has an additional $(k\alpha_o/2L)^2$ term in the exponent. The coherence function can be readily shown to be

$$<U(\rho_{m_1})U^*(\rho_{m_2})> = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2} e^{-\left[\frac{\alpha_o}{2}\frac{k}{L}|\bar{\rho}_{m_1}-\bar{\rho}_{m_2}|\right]^2-\frac{ik}{2L}\left(\rho_{m_1}^2-\rho_{m_2}^2\right)}$$

$$\cdot\int d\bar{r}\, e^{-\frac{r^2}{4\alpha_o^2}-\frac{ik}{2L}\bar{r}\,(\bar{\rho}_{m_1}+\bar{\rho}_{m_2})} F(\bar{r},\bar{\rho}_{m_1}-\bar{\rho}_{m_2}) \tag{110a}$$

$$<U(\rho_{m_1})U^*(\rho_{m_2})> = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2}\int d\bar{r}\, e^{-\frac{r^2}{4\alpha_o^2}-\left[\frac{\alpha_o}{2}\frac{k}{L}|\bar{r}-(\bar{\rho}_{m_1}-\bar{\rho}_{m_2})|\right]^2}$$

$$e^{-\frac{ik}{2L}\bar{r}\cdot\left(\bar{\rho}_{m_1}+\bar{\rho}_{m_2}\right)-\frac{ik}{2L}\left(\rho_{m_1}^2-\rho_{m_2}^2\right)} F(\bar{r},\bar{\rho}_{m_1}-\bar{\rho}_{m_2}) \tag{110b}$$

where the mutual coherence function F is

$$F(\bar{r},\bar{\rho}_{m_1}-\bar{\rho}_{m_2}) = F(\bar{r},\bar{\rho}) = e^{-\frac{1}{\rho_o^{5/3}}\frac{8}{3}\int_0^1 dt\,|\bar{p}t + (1-t)\bar{\rho}|^{5/3}} \tag{111}$$

Finally, the mean receiver irradiance (diffraction pattern) due to glints is expressed from (107,108) as

$$<I(\bar{p})>_g = \left(\frac{k}{2\pi L}\right)^2 \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} a_{m_1} a^*_{m_2} <U(\bar{\rho}_{m_1})U^*(\bar{\rho}_{m_2})> \cdot$$

$$\cdot\iint d\bar{\rho}_1 d\bar{\rho}_2 \exp\left\{\frac{ik}{2L}\cdot\left[\rho_1^2-\rho_2^2-2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}\right] - \frac{(\bar{\rho}_1-\bar{\rho}_{m_1})^2}{\Delta\rho_{m_1}^2} - \frac{(\bar{\rho}_2-\bar{\rho}_{m_2})^2}{\Delta\rho_{m_2}^2} - \left(\frac{\rho}{\rho_o}\right)^{5/3}\right\} \tag{112}$$

where $<U(\bar{p}_{m_1})U^*(\bar{\rho}_{m_2})>$ is given by Eqs.(93). We now show specific cases.

1. Single glint (M = 1)

From Eq.(112) we have

$$<I(\bar{p})>_g = \left(\frac{k}{2\pi L}\right)^2 <I(\bar{\rho}_m)> |a_m|^2 \cdot\iint d\bar{\rho}_1 d\bar{\rho}_2$$

$$\exp\left\{\frac{ik}{2L}\left[\rho_1^2-\rho_2^2-2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}\right] - \frac{\rho_1^2+\rho_2^2-2\bar{\rho}_m\cdot(\bar{\rho}_1+\bar{\rho}_2)+2\rho_m^2}{\Delta\rho_m^2} - \left(\frac{\rho}{\rho_o}\right)^{5/3}\right\} \tag{113}$$

With the following change of variables:

$$\left.\begin{aligned}\bar{\rho}_1-\bar{\rho}_2 &= \bar{\rho}\\ \bar{\rho}_1+\bar{\rho}_2 &= 2\bar{R}\end{aligned}\right\} \quad \rho_1^2+\rho_2^2 = \frac{1}{2}(\rho^2+4R^2) \tag{114}$$

we have

$$\langle I(\overline{p})\rangle_g = \left(\frac{k}{2\pi L}\right)^2 \langle I(\overline{\rho}_m)\rangle |a_m|^2 \; e^{-2\left(\frac{\rho_m}{\Delta\rho_m}\right)^2} \cdot$$

$$\cdot \int d\overline{R} \; e^{\frac{ik}{L}\overline{\rho}\cdot\overline{R} + \frac{4\overline{\rho}_m{}^1\overline{R}}{\Delta\rho_m{}^2} - \frac{2R^2}{\Delta\rho_m{}^2}}$$

$$\cdot \int d\overline{\rho} \; e^{-\frac{ik}{L}\overline{\rho}\cdot\overline{p} - \frac{p^2}{2\Delta\rho_m{}^2} - \left(\frac{\rho}{\rho_o}\right)^{5/3}} \qquad (115)$$

The first integration yields

$$2\pi \int_o^\infty R \; J_o \left[\left|\frac{k}{L}\overline{p} - \frac{i4\rho_m}{\Delta\rho_m{}^2}\right| R\right] e^{-2\left(\frac{R}{\Delta\rho_m}\right)^2} dR \; .$$

$$= 2\pi \frac{\Delta\rho_m{}^2}{4} e^{-\frac{\Delta\rho_m{}^2}{8}\left[\left(\frac{k}{L}p\right)^2 - \left(\frac{4\rho_m}{\Delta\rho_m}\right)^2\right] + \frac{ik}{L}\overline{\rho}_m\cdot\overline{p}} \qquad (116)$$

which with (115) gives

$$\langle I(\overline{p})\rangle_g = \left(\frac{k}{L}\right)^2 \langle I(\overline{\rho}_m)\rangle |a_m|^2 \; \frac{\Delta\rho_m{}^2}{4}$$

$$\cdot \int_o^\infty \rho \; J_o\left[\frac{k}{L}\;|\overline{p}-\overline{\rho}_m|\rho\right] \; e^{-\frac{1}{4}\left[\frac{2}{\Delta\rho_m{}^2} + \frac{\Delta\rho_m{}^2}{2}\left(\frac{k}{L}\right)^2\right]\rho^2 - \left(\frac{\rho}{\rho_o}\right)^{5/3}} d\rho \qquad (117)$$

This may be usefully normalized by setting $\overline{p} = \overline{\rho}_m$.

The result (117) may be interpreted as follows. The maximum occurs at $\overline{p} = \overline{\rho}_m$, as expected. The scale of the diffraction pattern ($|\overline{p} - \overline{\rho}_m|$) is seen from the Bessel term to be $L/k\rho_{max}$, where ρ_{max} is determined from the exponential as the smallest of $\Delta\rho_m$, $L/k\Delta\rho_m$, and ρ_o respectively. The corresponding $I(\bar{p})$ scales are $L/k\Delta\rho_m$ (diffraction from the glint), $\Delta\rho_m$ (geometric reflection term), and ρ_o (effective coherent glint size in strong turbulence); the largest will predominate in each case.

2. Arbitrary Number of Glints (M)

This involves lengthy but straightforward calculation. The result is

$$<I(\bar{p})>_g = \sum_{i=1}^{M} <I(\bar{p})>_{g_i} + \frac{1}{2} \sum_{i\neq j}^{M} F_{i,j}(I) \tag{118}$$

where

$$F_{i,j}(I) = \left(\frac{k}{L}\right)^2 \text{Re}\left[<U(\bar{\rho}_{m_i})U^*(\bar{\rho}_{m_j})>a_{m_i} a^*_{m_j} (\Delta\rho^2_{ij+}) \right.$$
$$\cdot \exp\left\{-\frac{|\bar{\rho}_{m_j} - \bar{\rho}_{m_i}|^2}{\Delta\rho_{m_i}^2 + \Delta\rho_{m_j}^2}\right\}$$
$$\cdot \int \rho d\rho\, J_o(|\bar{\gamma}|\rho) \exp\left\{-\frac{1}{4}\left[\frac{1}{\Delta\rho^2_{ij+}} + \Delta\rho^2_{ij+}\left(\frac{k}{L} + \frac{i}{\Delta\rho^2_{ij-}}\right)^2\right]\rho^2\right.$$
$$\left.-\left(\frac{\rho}{\rho_o}\right)^{5/3}\right\} \tag{119}$$

and

$$\frac{1}{\Delta\rho^2_{ij\pm}} = \frac{1}{\Delta\rho_i^2} \pm \frac{1}{\Delta\rho_j^2}$$

$$\bar{\gamma} = \frac{k}{L}\left[\bar{p} - \left(\bar{\rho}_{m_i} + \frac{\left(\bar{\rho}_{m_j} - \bar{\rho}_{m_i}\right)\Delta\rho_{m_i}^{\ 2}}{\Delta\rho_{m_i}^{\ 2} + \Delta\rho_{m_j}^{\ 2}}\right)\right] + i\ 2\ \frac{\Delta\rho_{m_i}^{\ 2}\left(\bar{\rho}_{m_j} - \bar{\rho}_{m_i}\right)}{\Delta\rho_{m_i}^{\ 2} + \Delta\rho_{m_j}^{\ 2}} \tag{120}$$

Each pair of glints results in a cross-term bright spot in the receiver irradiance field, located at

$$\bar{p}_{ij} = \left(\bar{\rho}_{m_i} + \frac{\left(\bar{\rho}_{m_j} - \bar{\rho}_{m_i}\right)\Delta\rho_{m_i}^{\ 2}}{\Delta\rho_{m_i}^{\ 2} + \Delta\rho_{m_j}^{\ 2}}\right) \tag{121}$$

This is illustrated in Figure 7.

The analysis of the diffraction pattern in the presence of one or more glints has been made using the assumption of jointly Gaussian fields for single mode laser beam source. The locations of single and cross-term bright spots have been calculated from the properties of Bessel functions and the corresponding geometrical figures were presented. These results can be interpreted as reasonable optical phenomena.

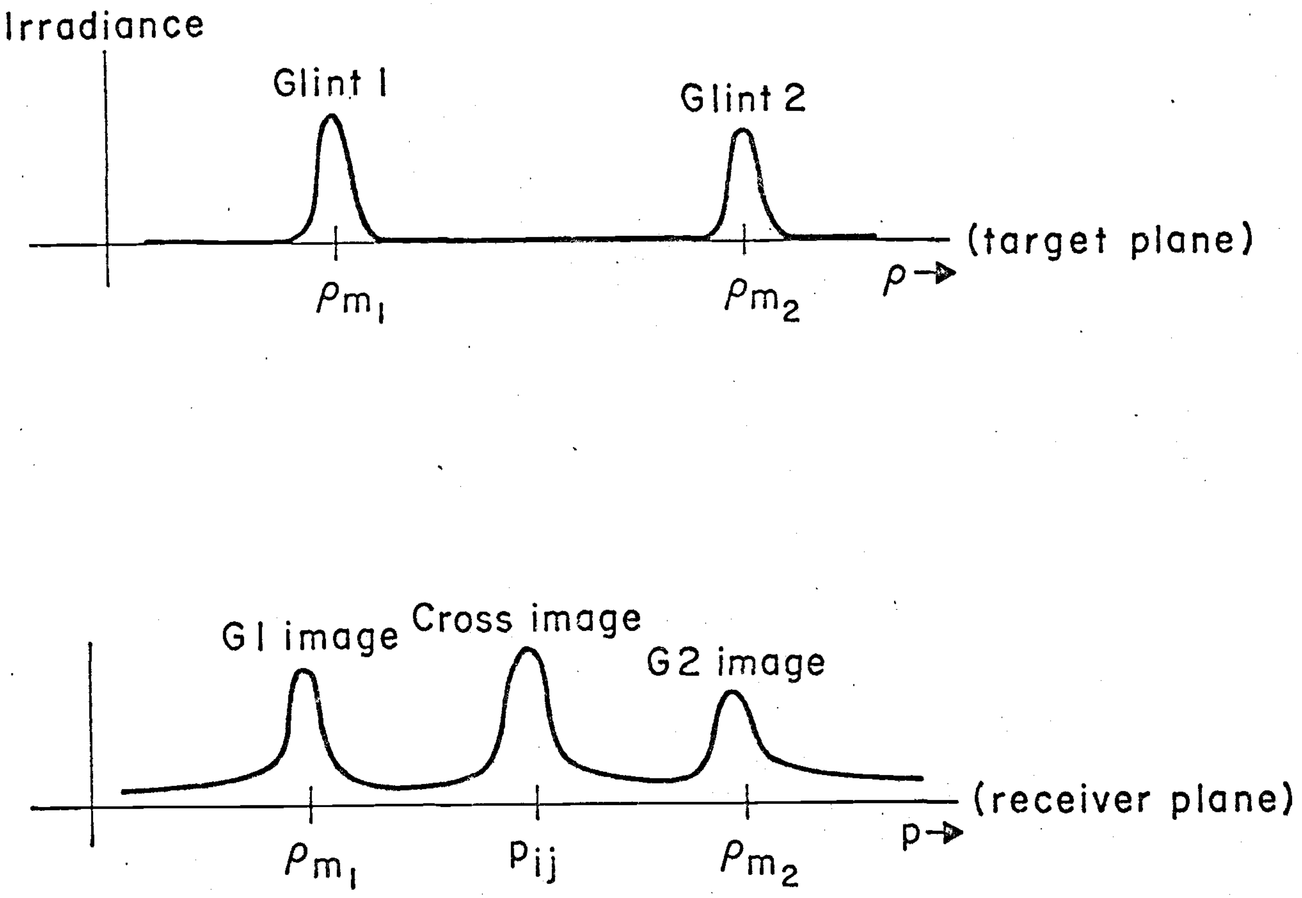

Figure 7. Irradiance distribution for two glints.

BIBLIOGRAPHY

1. Bennett, H. E., 1963, "Specular Reflectance of Aluminized Ground Glass and the Height Distribution of Surface Irregularities," J. Opt. Soc. Am., 53, 1389.

2. Porfens, J. O., 1963, "Relation Between the Height Distribution of a Rough Surface and the Reflection at Normal Incidence," J. Opt. Soc. Am., 53, 1394.

3. Greenwood, D. P., 1972, "The Scattering from Curved Rough Surfaces of an EM Wave Transmitted Through a Turbulent Medium," IEEE Trans. On Antennas and Prop., AP-20, 19.

4. Kurtz, C. N., 1972, "Transmitted Characteristics of Surface and the Design of Nearly Band-Limited Binary Diffusers," J. Opt. Soc. Am., 62, 982.

5. Sprague, R. A., 1972, "Surface Roughness Measurement Using White Light Speckle," Appl. Opt., 11, 2811.

6. Nagata, K., Tamotsu Unehara, and Jien Nishiwaki, 1973, "The Determination of rms Roughness and Correlation Length of Rough Surface by Measuring Spatial Coherence Function," Japan J. Appl. Phys., 12, 694.

7. Reynolds, G. O. and Thomas J. Skinner, 1964, "Mutual Coherence Function Applied to Imaging Through a Random Medium," J. Opt. Soc. Am., 54, 1302.

8. Gould, G., et al, 1964, "Coherent Detection of Light Scattered From a Diffusing Reflecting Surface," Appl. Opt., 3, 648.

9. Goldfischer, L. I., 1965, "Autocorrelation Function and Power Density of Laser-Produced Speckle Patterns," J. Opt. Soc. Am., 55, 247.

10. Goodman, J. W., 1965, "Some Effects of Target-Induced Scintillation on Optical Radar Performance," Proc. IEEE, 53, 1688.

11. Lowenthal, S., and Henri Arsenault, 1970, "Image Formation for Coherent Diffuse Objects: Statistical Properties," J. Opt. Soc. Am., 60, 1478.

12. Crane, R. B., 1970, "Use of a Laser-Produced Speckle Pattern to Determine Surface Roughness," J. Opt. Soc. Am., 60, 1658.

13. Miller, M. G., A. M. Schneiderman, and P. F. Kellen, 1975, "Second-Order Statistics of Laser-Speckle Patterns," J. Opt. Soc. Am., 65, 779.

14. Fujii, H. and T. Asakura, 1974, "Effect of Surface Roughness on the Statistical Distribution of Image Speckle Intensity," Opt. Comm., 11, 35.

15. Pederson, H. M., 1974, "The Roughness Dependence of Partially Developed Monochromatic Speckle Patterns," Opt. Comm., 12, 156.

16. Agarwal, G. S., 1975, "Scattering from Rough Surfaces," Opt. Comm., 14, 161.

17. Nagata, K. and Tamotu Unebara, 1973, "Spatial Correlation of Gaussian Beam in Moving Ground Glass," Japan J. Appl. Phys., 12, 694.

18. Takai, N., 1974, "Statistics of Dynamic Speckles Produced by a Moving Diffuser Under the Gaussian Beam Laser Illumination," Japan J. Appl. Phys., 13, 2025.

19. Kirkpatrick, P. and Hussein M. A. El-Sum, 1956, "Image Formation by Reconstruction Wavefront I. Physical Principles and Methods of Refinement," J. Opt. Soc. Am., 46, 825.

20. Leith, E. N. and J. Upatnieks, 1962, "Reconstructed Wavefronts and Communication Theory," J. Opt. Soc. Am., 52, 1123.

21. Leith, E. N., and J. Upatnieks, 1964, "Wavefront Reconstruction with Diffused Illumination and Three-Dimensional Objects," J. Opt. Soc. Am., 54, 1295.

22. Upatnieks, J., 1967, "Improvement of Two-Dimensional Image Quality in Coherent Optical Systems," Appl. Opt., 11, 1905.

23. Inloe, L. H., 1967, "Noise-Like Structure in the Image of Diffusely Reflecting Objects in Coherent Illumination," Bell System Tech. J., 46, 1479.

24. Deitz, P. H., 1975, "Image Information by Means of Speckle-Pattern Processing," J. Opt. Soc. Am., 65, 279.

25. Estes, L. E. and R. Boucher, 1975, "Temporal and Spatial-Intensity-Inferterometer Imaging Through a Random Medium," J. Opt. Soc. Am., 65, 760.

26. Lahart, M. J. and A. S. Marathay, 1975, "Image Speckle Patterns of Weak Diffusers."

27. Fujii, H. and T. Asakura, 1974, "A Contrast Variation of Image Speckle Intensity Under Illumination of Partially Coherent Light," Opt. Comm., 12, 32.

28. Goodman, J. W., 1975, "Dependence of Image Speckle Contrast on Surface Roughness."

29. Elbaum, M., M. King, and M. Greenebaum, 31 October 1974, "Laser Correlography: Transmission of High-Resolution Object Signatures Through the Turbulent Atmosphere," Riverside Research Institute, Technical Report T-1/306-3-11, 80 West End Avenue, New York, New York 10023.

30. Clifford, S. F., G. R. Ochs, and Ting-i Wang, September 1974, "Theoretical Analysis and Experimental Evaluation of a Prototype Passive Sensor to Measure Crosswinds," U.S. Department of Commerce, National Oceanic and Atmospheric Administration, Technical Report ERL 312-WPL 35, Boulder, Colorado 80302.

31. Lutomirski, R. F. and Yura, H. T., "Propagation of a Finite Optical Beam in a Inhomogeneous Medium", Applied Optics, 10, 1652, July 1971.

32. Yura, H., "Mutual Coherence Function of a Finite Cross Section Optical Beam Propagating in a Turbulent Medium", Applied Optics, 11, 1399, June 1972.

33. Kerr, J. R., et al, "Propagation of Multiwavelength Laser Radiation through Atmospheric Turbulence", RADC Technical Report, August 1975.

34. Elbaum, M., et al, "Laser Correlography: Transmission of High-Resolution Object Signatures through the Turbulent Atmosphere", Riverside Research Institute, Technical Report T-1/306-3-11, October 31, 1974.

35. Lawrence, R. S. and J. W. Strohbehn, "A Survey of Clean-Air Propagation Effects Relevant to Optical Communications", Proc. IEEE, 58, 1523, October 1970.

36. Fried, D. L., "Effects of Atmospheric Turbulence on Static and Tracking Optical Heterodyne Receivers", Optical Science Consultants, Technical Report TR-027, August 1971.

37. Fried, D. L., "Atmospheric Modulation Noise in an Optical Heterodyne Receiver", IEEE Trans. on Quantum Electronics, OE-3, 213, June 1967.