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Title: DYNAMIC RESPONSE OF STEEL STRUCTURES WITH SEMIRIGID CONNECTIONS

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The nonlinear response of a ten-story steel structure with semirigid girder connections is studied under conditions of dynamic loading. The dynamic loading used in this study is the North-South component of the May 18, 1940 El Centro, California earthquake. To deal with the complexity of the problem the structure is idealized by a series of equivalent masses, lumped at the floor levels and restrained by weightless members. The physical model used to represent individual members consists of a flexible central beam with springs attached at both ends. All connections have the capability of exhibiting bilinear hysteresis curves.

A numerical procedure is used to integrate the governing differential equations. The analysis is accomplished by a step-by-step procedure in which the structure is assumed to respond linearly during each time increment. However, the member properties may be

changed from one interval to the next. Thus the nonlinear response is obtained as a sequence of linear responses of successively differing systems.

Semirigid girder connections affect the properties of a structure in three ways: (a) by altering the relative girder to column stiffness (b) by changing the strength or yield deformation characteristics, and (c) by decreasing the stiffness of the structure. The effects that these variables have on structural response are determined. The ground motion characteristics, intensity and duration, are also investigated.

Dynamic Response of Steel Structures with Semirigid Connections

by

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DYNAMIC ANALYSIS OF STEEL STRUCTURES WITH SEMIRIGID CONNECTIONS

I. INTRODUCTION

Hechtman and Johnston (12) in a progress report published in 1947 by AISC recommend that a dependable percentage of end restraint can be used in design for several types of semirigid connections. However, before high-speed computers became available the analysis of structures with semirigid connections was difficult and time consuming. In the early 1960's analyses of such structures for static loading became feasible through the use of matrix methods and high-speed computers (10;15, p. 13; 22, p. 143). Little is known, however, as to how semirigid connections affect the dynamic response of structures.

The object of this research is a computer investigation of the nonlinear response of steel structures with semirigid connections subjected to seismic loading. In particular, the investigation is to consider the effects of semirigid connections on structure deflections and natural frequencies by comparing the response of structures with connections of varying rigidity.

To deal with the complexity of the problem, certain assumptions have been introduced. The structure is idealized by a series of equivalent masses, lumped at the floor levels and restrained by weightless members. Connections are simulated by inserting springs at both ends of each member. The spring stiffness is varied from zero, for a pin connection, to infinity, for a rigid connection, through the use of fixity factors. All connections have the capability of exhibiting bilinear hysteresis curves.

The governing differential equations are integrated in matrix form by a numerical integration technique, using a step-by-step procedure in which the structure is assumed to respond linearly during each time increment. However, the member properties may be changed from one interval to the next. Thus the nonlinear response is obtained as a sequence of linear responses of successively differing systems.

The seismic loading used in this investigation is the North-South component of the May 18, 1940 El Centro, California earthquake accelerogram. This earthquake is believed representative of strong earthquakes in the western part of the United States and its accelerogram is the strongest yet recorded.

A rational method of design for seismic loading on structures was first introduced by Naito in Japan in the early 1920's. His method consisted of assuming a horizontal force proportional to the weight of each floor or element in the structure. This method with refinements in the 1930's by Biot (3; 4, p. 262) and Freeman (7) and others became the accepted basis of design concepts and building codes.

Introduction of the elastic response spectrum concept provided a simple means of calculating elastic internal member forces and deflections. From such analyses it became obvious that inelastic action was taking place in structures for even moderate earthquake forces, and to build structures to exhibit only elastic deformation is not economically practical. Further studies in this area led to the conclusion that the maximum deflections based on elastic considerations can be quite different from the results based on nonlinear considerations. Therefore it became necessary to consider nonlinear effects for structures in order to prevent limits on the height of structures that can be built in zones of high earthquake forces.

Among the problems encountered in an investigation such as this is the shape that the hysteresis curve will assume under dynamic loading. Most investigators attribute an increase in stiffness from a few percent to as much as 40 percent for dynamic deformation as compared to static deformation. Due to this uncertainty investigators usually assume a hysteresis relationship for dynamic analysis computed from static tests. A comparison of static and dynamic hysteresis curves is given by Hanson (11, p. 87).

The computer program written for this investigation is applicable to any rectangular building frame and any seismic excitation. However, with slight modifications it can be used for any two-dimensional structure and any type of dynamic loading. Rayleigh damping is supplied as a percent of critical damping of either the mass or the stiffness matrix. Static loads are also considered. The physical model used to represent individual members consists of a flexible beam with springs attached at both ends. This model is particularly effective in calculating plastic deformation for either static or dynamic loading.

The stiffness matrix for the structure is generated using a coding procedure devised by Tezcan (22, p. 143; 23, p. 445). This method when used with a carefully selected systematic procedure of numbering the degrees of freedom produces a stiffness matrix in band form that reduces the computation time and the required computer storage considerably.

Information on static tests of semirigid connections were obtained from reports by J. F. Baker (1), Pippard and Baker (19), Munse, Bell and Chesson (16, p. 729), Batho (2), Hechtman and Johnston (12) and many others. These investigations report the stiffness properties of several types of riveted, bolted, and welded semirigid connections. A complete listing of publications concerning semirigid connections is given by Gere (8).

An extensive review of the literature on research work in earthquake engineering is given by E. P. Hollis (13) and E. Rosenblueth (21, p. 923).

II. METHOD OF ANALYSIS

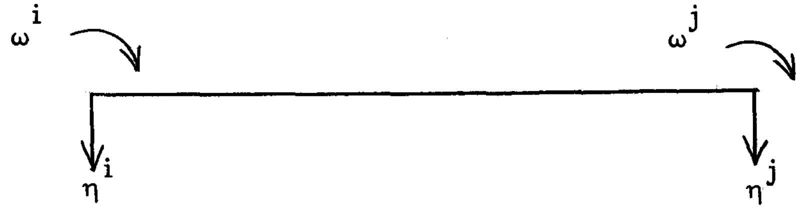
In this chapter the method of analysis used in the investigation is developed. The nonlinear analysis is accomplished by a step-by-step procedure in which the structure is assumed to respond linearly during each time increment. However, the member properties may be changed from one interval to the next. Thus the nonlinear response is obtained as a sequence of linear responses of successively differing systems.

A flow chart of the digital computer program, along with the limitations and assumptions used in the study, are given at the end of this chapter.

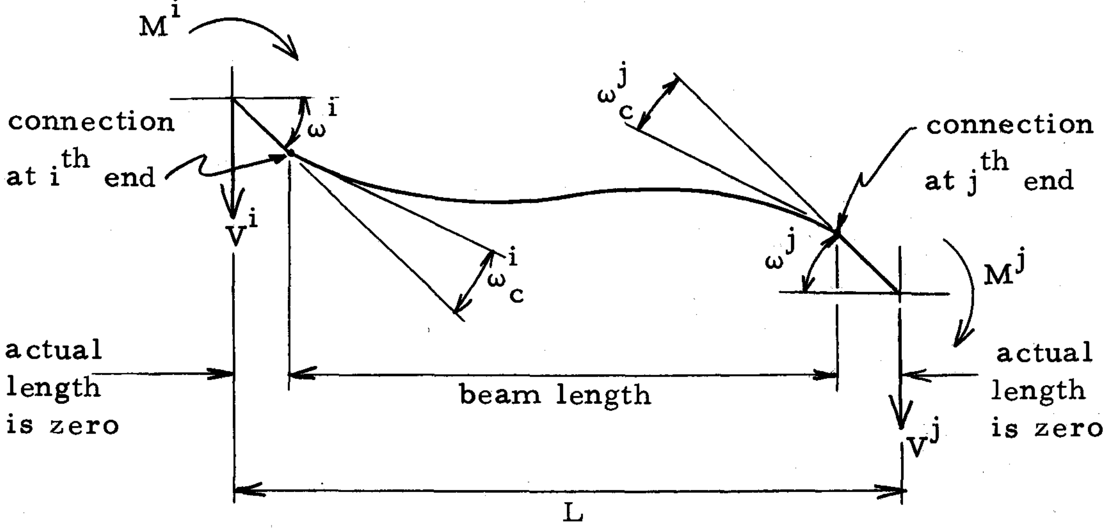
Individual Member Stiffness Matrix

The stiffness of structural elements is assumed linear during all time intervals. Hence, the general relationship between member-end moments and member-end rotation apply whether the structure is elastic or partially elastic-partially plastic. The difference between the two states is the connection rigidities that are used in computing deformations.

Neglecting axial deformations, the force-displacement relationship is shown in Figure 2. 1. A definition of terms used in this investigation is given in Table 2. 1. The force-displacement



(a) Sign convention for member displacements. Directions as shown are considered positive.



(b) Force and displacement quantities for semirigidly connected members.

Figure 2. 1. Member force and displacement quantities and sign convention.

relationship for a member is given in matrix form as follows

$$\{q\} = [k]\{d\} \tag{2.1}$$

where

$\{q\}$ - member force vector

$\{d\}$ - member displacement vector

$[k]$ - member stiffness matrix

Table 2.1. List of symbols used in Figures 2.1 and 2.2 and associated developments.

M^i, M^j	- bending moment at i and j ends of a member.
V^i, V^j	- shear at i and j ends of a member.
η^i, η^j	- displacement at i and j ends of a member.
ω^i, ω^j	- rotation at i and j ends of a member.
ω_c^i, ω_c^j	- total rotation of the connection at i and j ends of a member.
$\omega_{ce}^i, \omega_{ce}^j$	- elastic rotation of the connection at i and j ends of a member.
$\omega_{cn}^i, \omega_{cn}^j$	- plastic rotation of the connection at i and j ends of a member.
k_c^i, k_c^j	- stiffness of the connection at the i and j ends of a member.
M_{Yo}^i, M_{Yo}^j	- bending moment at i and j ends of a member at which plastic deformation begins.
$\theta_{Yo}^i, \theta_{Yo}^j$	- rotation at i and j ends of a member at which plastic deformation begins.

These matrices are of the form

$$\{q\} = \begin{bmatrix} M^i \\ M^j \\ V^i \\ V^j \end{bmatrix} \qquad \{d\} = \begin{bmatrix} \omega^i \\ \omega^j \\ \eta^i \\ \eta^j \end{bmatrix} \qquad (2.2)$$

$$[k] = \frac{2EI}{L}\begin{bmatrix} 2\gamma_{11} & \gamma_{12} & \frac{2\gamma_{11}+\gamma_{12}}{L} & -\frac{2\gamma_{11}+\gamma_{12}}{L} \\ \gamma_{12} & 2\gamma_{22} & \frac{2\gamma_{22}+\gamma_{12}}{L} & -\frac{2\gamma_{22}+\gamma_{12}}{L} \\ \frac{2\gamma_{11}+\gamma_{12}}{L} & \frac{2\gamma_{22}+\gamma_{12}}{L} & \frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} & -\frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} \\ -\frac{2\gamma_{11}+\gamma_{12}}{L} & -\frac{2\gamma_{22}+\gamma_{12}}{L} & -\frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} & \frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} \end{bmatrix}$$

(2.3)

The elements of Equation 2.3 are developed in Appendix A. In Equation 2.3

E - modulus of elasticity of the member

I - moment of inertia of the member

L - length of the member

$$\gamma_{11} = \left(\frac{3\nu^i}{4-\nu^i\nu^j}\right)$$

$$\gamma_{12} = \left(\frac{3\nu^i\nu^j}{4-\nu^i\nu^j}\right) \qquad (2.4)$$

$$\gamma_{22} = \left(\frac{3\nu^j}{4-\nu^i\nu^j}\right)$$

ν^i, ν^j - the connection fixity factors for the i and j ends of the member respectively.

The connection fixity factors are used to express the stiffness of the connection k_c as a function of the stiffness of the beam k_m in the form

$$k_c = Pk_m \qquad (2.5)$$

where

$$k_m = \frac{4EI}{L} \qquad (2.6)$$

and

$$P = \frac{3}{4}\left(\frac{\nu}{1-\nu}\right) \qquad (2.7)$$

The value of ν varies from zero for a pinned connection to 1.0 for a rigid connection. Using Equation 2.5, the connection stiffnesses given in tests by Baker (1) and others can be related to the fixity factor used in the stiffness matrix.

Using the above representation for semirigid connections yields results very similar to those obtained by Giberson (9). Giberson introduces springs into the member as it commences plastic deformation. The springs, initially rigid, are capable of exhibiting a curvilinear hysteresis loop. The stiffness of the spring k_s used by Giberson is related to the beam stiffness k_m as in the form

$$k_s = fk_m$$

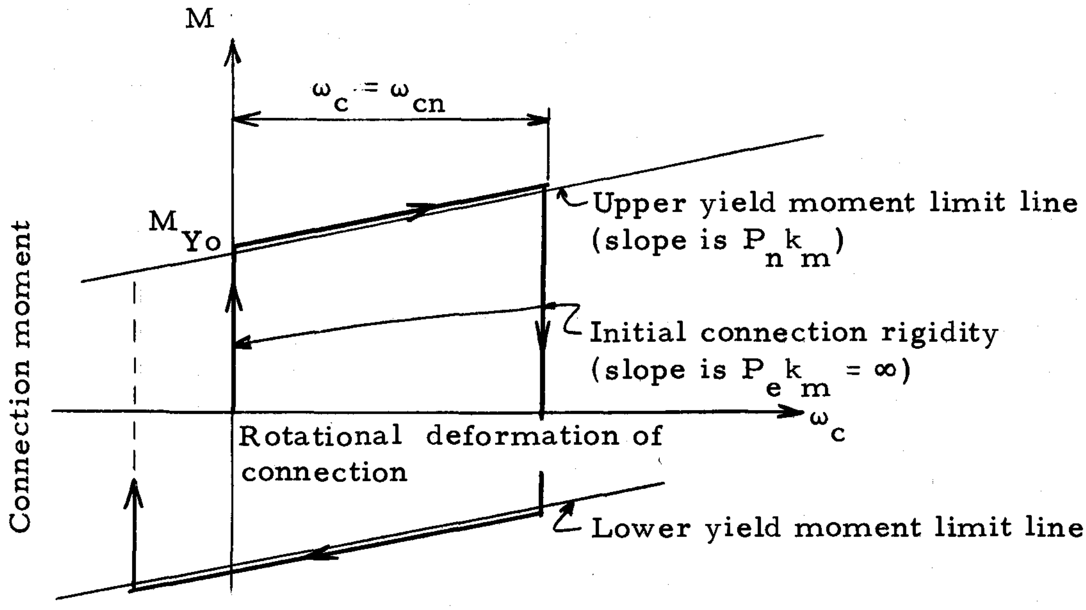
or

$$f = P$$

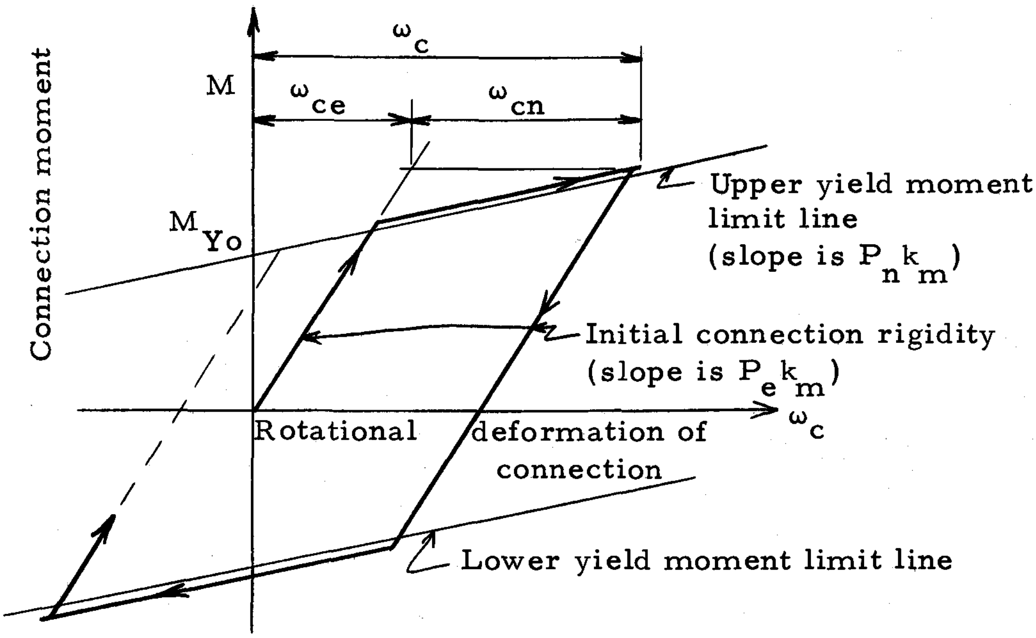
The two methods will yield identical results for structural members with connections that are initially rigid and that exhibit the same hysteresis curves. The approach investigated herein lends itself to modification for use with curvilinear hysteresis loops.

Hysteresis Loops

Bilinear moment-rotation loops that display the connection characteristics assumed for this study are shown in Figure 2. 2 along with pertinent nomenclature. To provide a form of bilinear moment resistance at the member end, the rigidity of the connection is changed from $P_e k_m$ for the fully elastic region of connection deformation to $P_n k_m$ in the range of nonlinear connection deformation. Typical values of $P_e k_m$ and $P_n k_m$ were determined from previous static tests published by J. F. Baker (1).



(a) Rigid connection bilinear moment-rotation hysteresis loop.



(b) Semirigid connection bilinear moment-rotation hysteresis loop.

Figure 2. 2. Bilinear moment-rotation hysteresis loops.

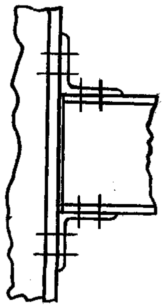
Connection rigidities reported by J. F. Baker (1) for 12 inch beams vary from 150×10^6 inch-lbs for a somewhat flexible connection of the type shown in Figure 2.3a to 500×10^6 inch-lbs for a fairly rigid connection of the type shown in Figure 2.3c. Fixity factors calculated from Baker's work indicated values in the elastic range of deformation of ν_e from 0.7 to 0.9.

In this investigation initial connection fixity factors of 0.60, 0.80 and 1.00 are considered. These values are considered to be representative of the fixity factors encountered in moment resisting frames.

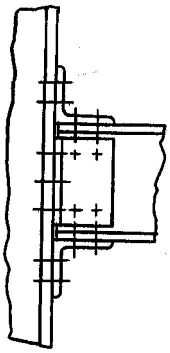
The slope $P_n k_m$ of the hysteresis loop in Figure 2.2 is a function of strain hardening during plastic deformation. The connection rigidities reported by Baker in this range of deformation corresponded to fixity factors of $\nu_n = 0.10$ to $\nu_n = 0.15$.

The mathematical approach used in this investigation uses a one-component beam model whereas a two-component beam model was used by Clough and Benuska (5). In Clough and Benuska's model one component was an elastic-perfectly plastic beam while the second component was a fully elastic beam. As the rotation of a member end surpassed the yield rotation, the contribution to the stiffness from the elastic-perfectly plastic component for this end was set equal to zero.

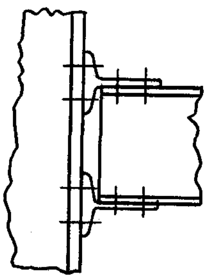
The author's one-component beam model consisted of a flexible



(a) Flange connection.



(b) Flange-web connection.



(c) Split I connection

Figure 2. 3. So-called rigid connections tested by J. F. Baker (1).

member with springs attached at both ends. The rigidity of the springs was altered as the member entered the plastic range of deformation to simulate a bilinear hysteresis loop.

It can be shown that the two models will yield different results (deformations and internal member forces), with the one-component beam model being more realistic from a physical point of view and also more versatile.

As shown in Figure 2.2 the connection moment crosses over the yield moment limit line before changing its state of stiffness. Likewise the connection moment decreases parallel to the yield moment limit line, or backtracks, for one time increment before changing back to the initial stiffness. The crossing over and backtracking results from assuming that the connection stiffness remains constant throughout each time increment.

Ductility Factor

Failure of a member is closely associated with the nonlinear displacement that takes place during plastic deformation. A ductility factor, defined such that it becomes a measure of this nonlinear yielding, is defined as

$$\mu_n = \begin{cases} M/M_{Yo} & M \leq M_Y \\ 1 + \dfrac{\omega_{cn}}{\theta_{Yo}} & M > M_Y \end{cases} \tag{2.8}$$

where

μ_n - the ductility factor which defines nonlinear or permanent rotational deformation of a connection.

Because no elastic deformation of the connections takes place for structures with rigid connections, the definition given in Equation 2.8 also defines the total connection deformation. This is not the case when semirigid connections are considered, as can be seen in Figure 2.3b. To determine the total rotational deformation that a connection undergoes, a second definition of ductility factor is given and calculated in this investigation as

$$\mu_t = 1 + \frac{\omega_c}{\theta_{Yo}} \tag{2.9}$$

where

μ_t is a ductility factor which defines the total rotational deformation of a connection.

For members with rigid connections μ_t equals μ_n for $M \geq M_Y$.

The expression for initial yield rotation θ_{Yo} is derived in Appendix B and given in Equation B.2 as

$$\theta_{Yo} = \frac{M_{Yo}L}{6EI}\left[\frac{4-\nu_e^i\nu_e^j}{\nu_e^i(2+\nu_e^j)}\right] \tag{2.10}$$

In this investigation connection moments are calculated at the end of each time increment; hence, the ductility factors are calculated in terms of connection moments rather than connection rotations. This is first done for μ_n as follows: From Figure 2.3b the non-linear connection deformation ω_{cn} during any time interval, with $M > M_y$, is seen equal to the total deformation of the connection ω_c minus the elastic deformation of the connection ω_{ce} or

$$\omega_{cn} = \frac{M-M_y}{P_n k_m} - \frac{M-M_y}{P_e k_m} \tag{2.11}$$

which is rewritten in terms of fixity factors as

$$\omega_{cn} = \frac{4}{3}\left(\frac{\nu_e^i - \nu_n^i}{\nu_e^i \nu_n^i}\right)\frac{M-M_y}{k_m} \tag{2.12}$$

The ductility factor μ_n can then be calculated from Equation 2.8 as

$$\mu_n = \begin{cases} M/M_{yo} & M \leq M_y \\ 1 + 2\left(\dfrac{2+\nu_e^j}{4-\nu_e^i \nu_e^j}\right)\left(\dfrac{\nu_e^i - \nu_n^i}{\nu_n^i}\right)\left(\dfrac{M-M_y}{M_{yo}}\right) & M > M_y \end{cases} \tag{2.13}$$

The ductility factor μ_t given in Equation 2.9 is next determined as follows: From Figure 2.3b it is seen that the total connection deformation ω_c is equal to

$$\omega_c = \begin{cases} \dfrac{M}{P_e k_m} & M \leq M_y \\ \dfrac{M_y}{P_e k_m} + \dfrac{M - M_y}{P_n k_m} & M > M_y \end{cases} \tag{2.14}$$

The ductility factor μ_t given in Equation 2.9 is then calculated in terms of fixity factors as

$$\mu_t = \begin{cases} 1 + 2(1-\nu_e^i)\left(\dfrac{2+\nu_e^j}{4-\nu_e^i\nu_e^j}\right)\dfrac{M}{M_{yo}} & M \leq M_y \\ 1 + 2\left(\dfrac{2+\nu_e^j}{4-\nu_e^i\nu_e^j}\right)\left[(1-\nu_e^i)+\dfrac{\nu_e^i}{\nu_n^i}(1-\nu_n^i)\left(\dfrac{M-M_y}{M_{yo}}\right)\right] & M > M_y \end{cases} \tag{2.15}$$

Two ductility factors are calculated in the computer program used in this investigation, μ_n as given in Equation 2.13 and μ_t as given in Equation 2.15. With the ductility thus defined both the nonlinear deformation and the total connection deformation can be determined.

Damping

For this investigation damping was assumed to be Rayleigh damping which is composed of both stiffness proportional viscous damping and mass proportional viscous damping as given by the following equation:

$$[C] = \alpha[M] + \beta[K] \tag{2.16}$$

where

[C] - the damping matrix

[M] - the mass matrix

[K] - the system stiffness matrix

α - a scalar quantity that indicates the fraction of mass used for damping

β - a scalar quantity that indicates the fraction of stiffness used for damping

The fraction of mass and stiffness conventionally used in damping is determined as some percent of critical damping in the fundamental mode. This percent of critical damping has been related to α and β by O'Kelley (18) as follows

$$\xi_n^m = \frac{\alpha}{2\omega_n} \qquad n = 1, 2, \ldots N \tag{2.17}$$

and

$$\xi_n^s = \frac{\beta\omega_n}{2} \tag{2.18}$$

where

ξ_n^m - the percent of mass proportional critical damping in the n^{th} mode,

ξ_n^s - the percent of stiffness proportional damping in the n^{th} mode,

ω_n - the n^{th} circular frequency.

For the fundamental mode with $n = 1$, α and β are

$$\alpha = 2\omega_1 \xi_1^m \tag{2.19}$$

$$\beta = \frac{2\xi_1^s}{\omega_1} \tag{2.20}$$

Thus the damping matrix takes the following form

$$[C] = 2\omega_1 \xi_1^m [M] + \frac{2\xi_1^s}{\omega_1}[K] \tag{2.21}$$

Equations of Motion

The equations of motion governing structural response for earthquake excitation can be expressed in matrix form as

$$[M]\{\Delta\ddot{u}\} + [C]\{\Delta\dot{u}\} + [K]\{\Delta u\} = - [M]\{1\}\Delta\ddot{u}_g \tag{2.22}$$

where

$\{\Delta u\}$ - the change in the displacement vector

$\{\Delta\dot{u}\}$ - the change in the velocity vector

$\{\Delta\ddot{u}\}$ - the change in the acceleration vector

$\Delta\ddot{u}_g$ - the change in the ground acceleration

The integration of nonlinear equations can be accomplished by means of a numerical procedure using finite increments of time.

The numerical procedure used here is that presented by R. W. Clough and E. L. Wilson (6). In the procedure the following assumptions are used

1. The acceleration, velocity and displacement at the beginning of each time interval are known.
2. The acceleration is linear within each time interval.
3. The member stiffness properties remain constant throughout each time interval but properties may change from one interval to the next.

With these assumptions, a set of equations relating the incremental acceleration, velocity and displacement during the t^{th} time interval is found to be

$$\{\Delta \dot{u}_t\} = \frac{3}{\Delta t}\{\Delta u_t\} + \{B_{t_o}\} \tag{2. 23}$$

$$\{\Delta \ddot{u}_t\} = \frac{6}{(\Delta t)^2}\{\Delta u_t\} + \{A_{t_o}\} \tag{2. 24}$$

where

$$\{A_{t_o}\} = -\frac{6}{\Delta t}\{\dot{u}_{t_o}\} - 3\{\ddot{u}_{t_o}\} \tag{2. 25}$$

$$\{B_{t_o}\} = -3\{\dot{u}_{t_o}\} - \frac{\Delta t}{2}\{\ddot{u}_{t_o}\} \tag{2. 26}$$

t_o - the time at the beginning of the t^{th} time interval.

Substitution of Equations 2. 23 through 2. 26 into Equation 2. 22 results in

$$\{\Delta u_t\} = \left[\frac{6}{(\Delta t)^2}[M] + \frac{3}{\Delta t}[C] + \{K\}\right]^{-1}\left\{-[M](\Delta\ddot{u}_g\{1\} + \{A_{t_o}\}) - [C]\{B_{t_o}\}\right\} \tag{2. 27}$$

Since all values on the right hand side of Equation 2. 27 are known at the beginning of each time interval, the displacement at the end of that interval is determined directly. The change in velocity and acceleration are found from Equations 2. 23 and 2. 24 and added to the total at the beginning of the increment to determine their values at the end of the increment. From these new totals, vectors {A} and {B} are computed for the increment. Thus the right hand side of Equation 2. 27 is known for the next interval.

Determination of Time Increment

Little has been done to date to determine what effect the magnitude of the time increment, used in approximate solutions, has on calculations of multi-story structural response. In general, the smaller the time increment the more accurate the solution, as long as round-off and truncation errors are controlled. But as the accuracy increases, cost also increases. It is desirable therefore to determine a time increment that yields only the accuracy required, thereby minimizing cost. One criteria frequently used to determine a suitable time increment Δt is

$$\Delta t \approx \frac{T_{min}}{10} \tag{2.28}$$

where T_{min} is the shortest natural period of vibration. Newmark (17, p. 67), in considering stability for approximate solutions, concluded that a time interval of 1/5 and 1/6 of the shortest natural period of vibration would limit the error to tolerable amounts. For a twenty-story structure Giberson (9) used $T_{20}/6$.

The effect that varying the magnitude of the time increment has on a ten-story building, using the program developed for this investigation, is given in Table 2.2. The time increment used throughout this investigation was 0.01 second or approximately $T_{10}/10$.

Table 2.2. Time increment vs error.

Criteria	Δt seconds	Max[1] error
$\frac{T_{min}}{5}$	.02	1.3%
$\frac{T_{min}}{10}$	.01	.5%
$\frac{T_{min}}{20}$	.005	----

[1]Maximum percent difference in deflection assuming $T_{min}/20$ results in "correct" deflections. Values of deflection were calculated for one sec. of the El Centro earthquake.

Calculation of Horizontal Shear and Deflection of Each Floor

After the equations of motion are solved, the equivalent horizontal static force vector and member deformation vector are calculated as follows

$$\{\Delta s\} = [K]\{\Delta u\} \tag{2.29}$$

and

$$\{\Delta d\} = [F]\{\Delta s\} \tag{2.30}$$

where

$\{\Delta s\}$ - a vector of the changes in the horizontal static force.

$\{\Delta d\}$ - a vector of the changes in the internal member-end deformations.

$[F]$ - the force matrix.

The moments and shears of the ends of the members are determined by using their individual member stiffness matrices as follows

$$\{q^i\} = [k^i]\{d^i\} \tag{2.31}$$

where

$\{q^i\}$ - the internal member force vector for the i^{th} member,

$[k^i]$ - the member stiffness matrix for the i^{th} member,

$\{d^i\}$ - the member-end deformation vector of the i^{th} member.

The Computer Program

A flow chart of the digital computer program used in this investigation is shown in Figure 2.4. In Step 6 of Figure 2.4 it is necessary to determine if a connection has changed state during the time interval. To do this the procedure given in Figure 2.5 is used. If at least one connection has changed state during this increment, a new stiffness matrix is generated in Step 2 of Figure 2.4.

In developing the computer program, the class of structures to be analyzed was restricted to those with the following properties

1. The foundation is infinitely rigid.
2. Only plane frames are considered.
3. The structures are moment-resisting frames; thus girders and columns provide the only stiffness of the frame.
4. Axial, torsional and shear deformations are neglected in all members.
5. All members can yield only at the ends and must yield according to the bilinear bending moment-rotation hysteresis loop shown in Figure 2.3.
6. All mass is concentrated at the floor levels.
7. The mass at each floor level moves only horizontally; hence, rotational inertia associated with joint rotation is neglected.

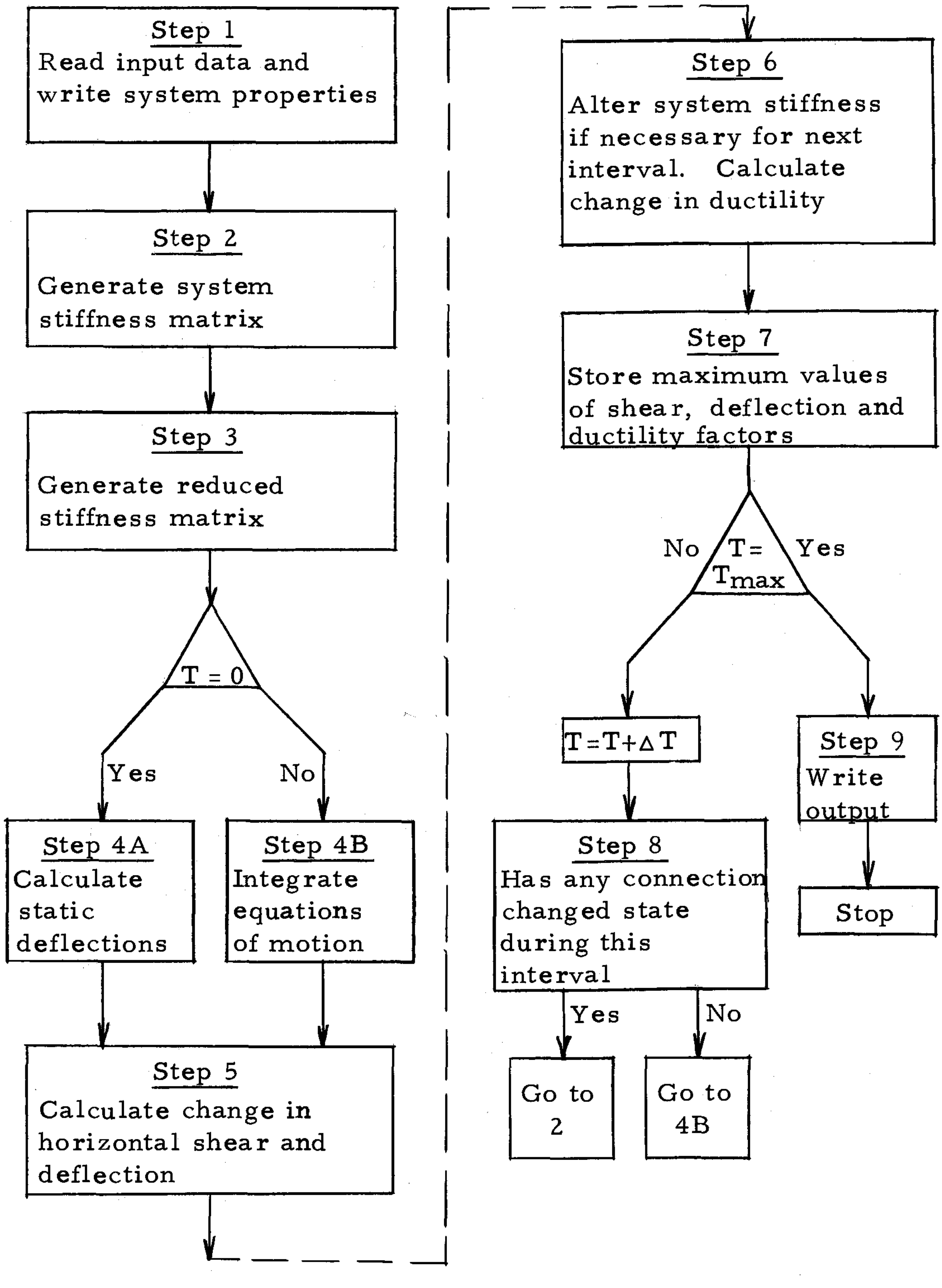


Figure 2. 4. Digital computer program flow chart for calculation of nonlinear response for structures subjected to earthquake excitation.

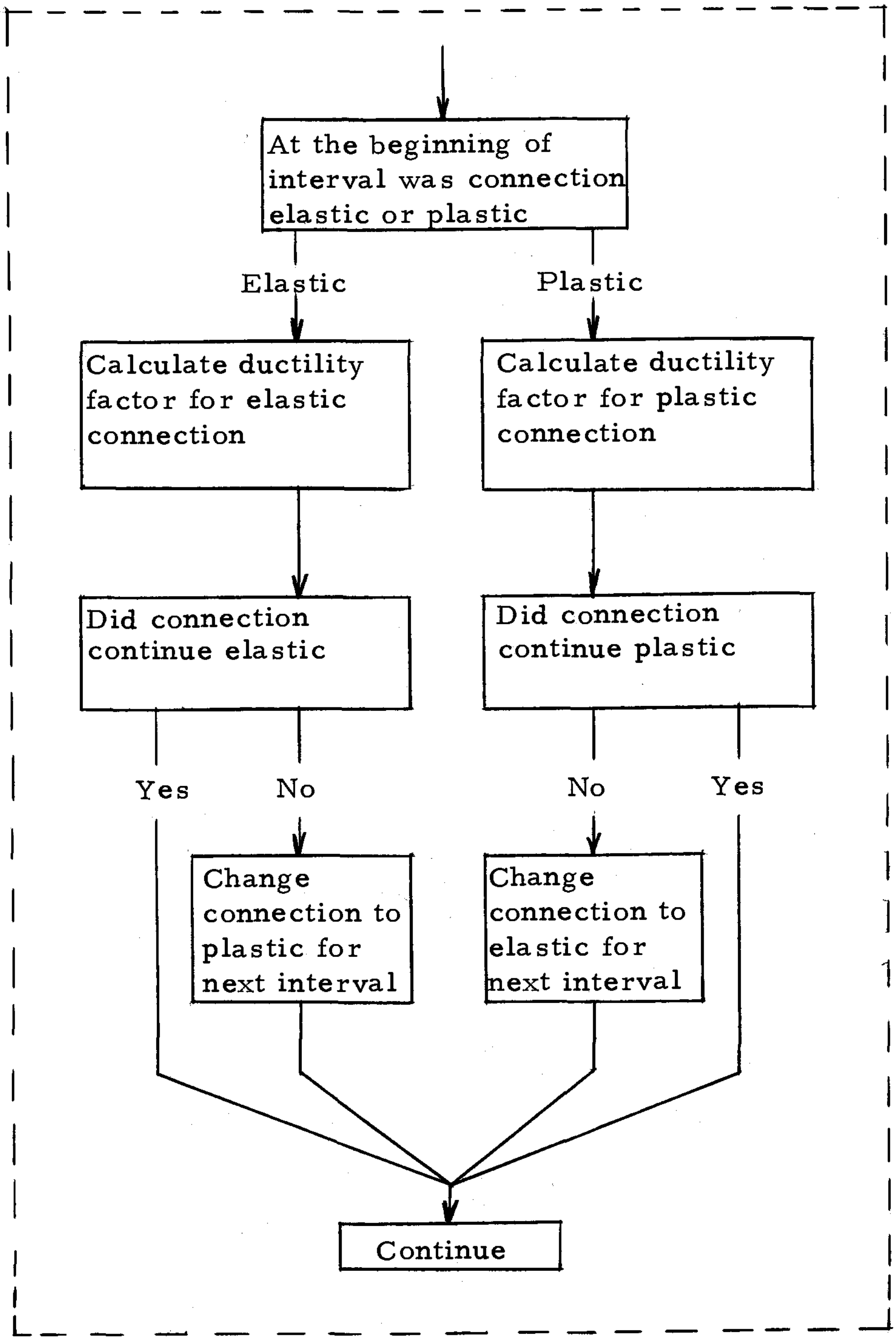


Figure 2. 5. Detailed flow chart of Step 6.

8. The building is laid out in a rectangular grid pattern, but the length per bay, the number of bays, the height per story and the number of stories are variable.

9. The connection moment must not exceed twice the yield moment at any connection. In nonlinear analysis this will seldom happen and does not happen in any of the examples presented in this report.

Since the computer program is written to analyze structures subjected to earthquake excitation, the properties and limitations of this section of the program will also be detailed.

10. The time history of the accelerogram must be digitized into pairs of coordinates, time and ground acceleration, of an assumed piecewise linear accelerogram.

11. An amplificator or scale factor was used which allows the intensity of the earthquake to be varied.

Earthquake Accelerogram

The accelerogram[1] used in the investigation is the North-South component of the May 18, 1940 El Centro earthquake. This earthquake is believed representative of strong earthquakes in the western

[1]The accelerogram used herein was supplied by Paul C. Jennings, Assistant Professor of Applied Mechanics, California Institute of Technology.

part of the United States and its accelerogram, given in Appendix D, is the strongest yet recorded.

Studies by Clough and Benuska (5) indicate that the structural response depends primarily on the peak acceleration impulse in the ground motion and that continuing motions of smaller amplitude have only a small effect on the maximum response. Therefore, in order to avoid excessive computational time, the duration of the earthquake used in this analysis was primarily limited to the first four to eight seconds of the El Centro earthquake. This same time duration was used extensively by Clough and Benuska (5).

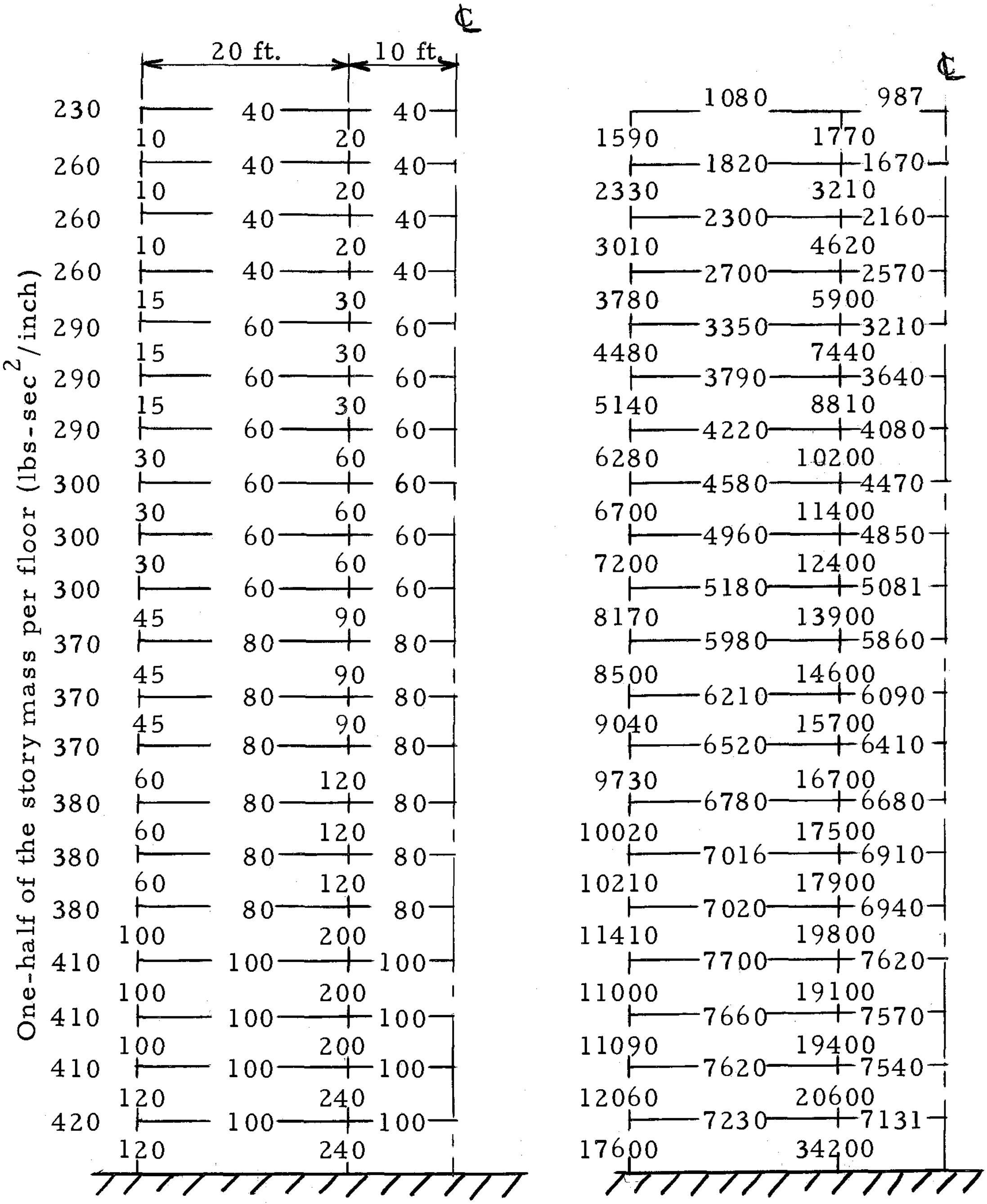
III. TESTING OF THE COMPUTER PROGRAM

To verify that the digital computer program yielded accurate solutions to the problems being studied, three types of errors, (a) gross, (b) round-off, and (c) truncation errors were considered.

Gross errors or mistakes are most effectively checked by comparing results obtained herein to results obtained for the same problem by other investigators. If the results of the independent solutions are the same or approximately the same, it is assumed that no gross errors exist. To check for gross errors in the digital computer program presented, the twenty-story structure shown in Figure 3. 1 was analyzed for the limiting case of rigid connections and the results compared as shown in Figure 3. 2 to those obtained by Clough and Benuska (5) and Giberson (9) for the same structure.

Clough and Benuska's method of analysis consisted of using a member with two components acting in parallel; one component was elastic-perfectly plastic while the second component was fully elastic. As the rotation of a member end surpassed the yield rotation, the contribution from the elastic-perfectly plastic component for that end was considered to be zero.

Giberson's method and the method used in this study uses a one-component member with springs attached at the ends. The rigidity of the springs are altered as the member enters the plastic range of



Moment of inertia of members times 10^{-3} (in^4)

Yield moments times 10^{-3} (in-lbs)

Figure 3. 1. Structural properties of test frame. The height between floors is 12 feet except for the first floor which is 15 feet. This data is taken from Giberson (9, p. 54).

A comparison of results obtained with the computer program developed for this study and the results obtained by Clough and Benuska (5) and by Giberson (9) for a 20 story-building. (Values of ductility factor for both Clough and Giberson were modified to correspond to the definition given in Equation 2. 13.

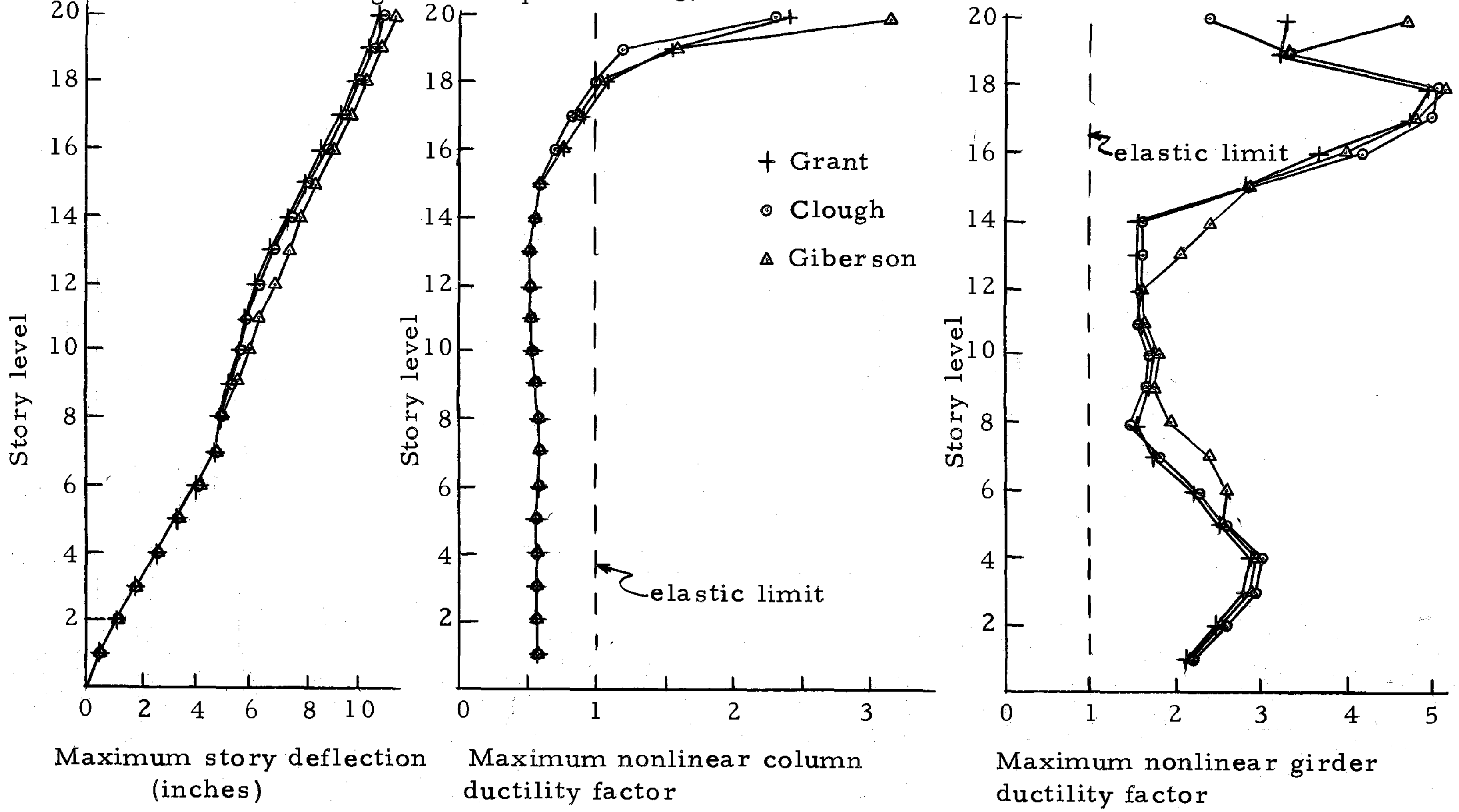


Figure 3. 2. Comparison of results.

deformation to simulate a bilinear hysteresis loop.

Giberson in comparing the two models, concluded that the one-component model was more realistic from a physical point of view and that it was also more versatile.

Following the work of Clough and Benuska it was assumed that the properties of structural symmetry could be used by pinning the structure at the center line and thereby analyzing only one-half of the structure. This assumption, though correct in the elastic range of deformation, introduces error during plastic deformation. Giberson, after studying the effects of this assumption concluded that the resulting error was small.

Due to the differences cited, the comparisons as shown in Figure 3. 2, cannot be expected to be identical. It is believed that the results given by the computer program described here compared well with results obtained by Clough and Benuska and Giberson and the differences shown can be explained by the differences in the mathematical models and by the difference in the assumed structural behavior.

A further check on the computer program developed for this investigation was made in the following manner. Since the results obtained from an analysis in the elastic range of deformations using Giberson's and the writer's program should yield similar results, a comparison of elastic response results were made for the first second of the El Centro earthquake for the twenty-story structure shown in

Figure 3. 1. This comparison showed a difference in results of less than one-half of one-percent. It was therefore concluded that gross errors were eliminated from the computer program.

When large numbers of computations are made, it is a difficult task to insure that round-off and truncation errors have not contaminated the results. Three procedures by which these errors were evaluated were: (a) checking results with results obtained by independent methods, (b) the use of double precision arithmetic, and (c) the use of error parameters.

An error parameter was introduced into the computer program to help monitor the magnitude of the round-off and truncation error. It was based on the fact that the sum of the moments at each joint should at the end of each time increment be zero. Hence, the moment residue accumulated at the joints is due solely to round-off and truncation error. The error parameter thus defined, however, is not affected by round-off and truncation errors that occur in the solution of the equations of motion (Equation 2. 27) and the subsequent change of the horizontal static force vector (Equation 2. 29). To evaluate these errors, higher precision arithmetic[2] was used for all variables

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[2]The computer used for this investigation, The Control Data 3300, has a fixed word length of 22 bits. For single precision floating point arithmetic, two words are used which results in precision to 11 decimal places. The higher precision arithmetic used here utilized three words. This increased the time of computation about 450 percent and for this reason was used sparingly in this investigation.

in both these equations and the results were compared to the results obtained using single precision arithmetic. The results of the structural response for the first one-half second of the El Centro earthquake were essentially identical, and the error parameter was zero to eight decimal places in both cases. When the structure was subjected to four seconds of the El Centro earthquake, the error parameter was zero to five decimal places.

Consequently, it is believed that gross errors have been eliminated and round-off and truncation errors minimized such that the computer program provides the accuracy required.

An iteration procedure given by Kreyszig (14, p. 458) was used in this investigation to calculate the circular frequencies and the corresponding characteristic vectors for the structure. After each iteration a bound on the circular frequency, or a maximum possible error, was determined. The iteration process was continued until the maximum error, or bound, was less than some predetermined percent of the circular frequency.

Using a maximum relative error of . 0001 percent of the calculated circular frequency, the first five natural periods, calculated from the circular frequencies, were obtained for the structure shown in Figure 3. 1. These values differed by less than one-half of one percent for the same values calculated by Giberson (9). It was therefore concluded that the computer program written to obtain the natural periods provided the required precision and accuracy.

IV. RESULTS

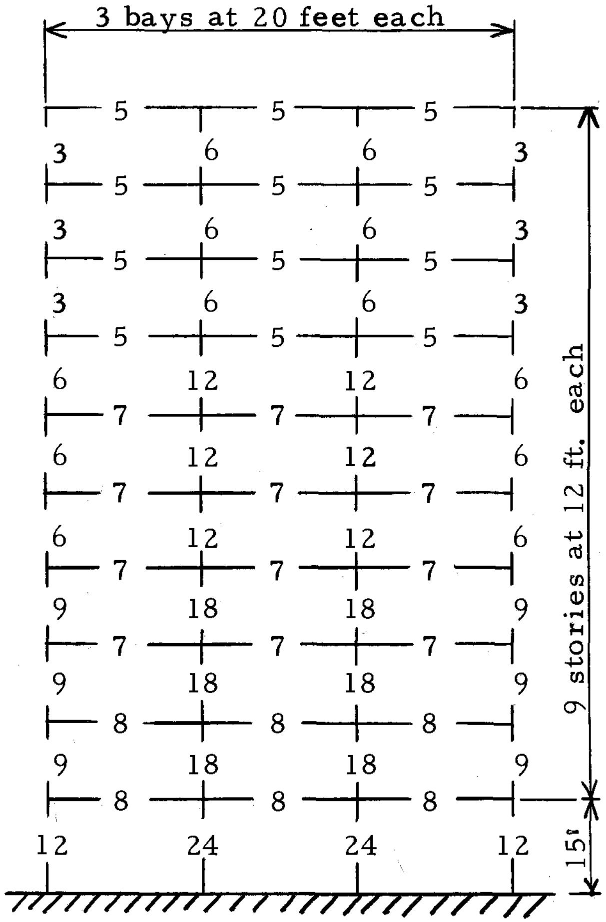
Design of the Building Used in the Investigation

In order to evaluate the influence of semirigid connections on the dynamic response of structures, a ten-story steel structure was designed in accordance with the Uniform Building Code (24) of 1967. The building was designed for vertical gravity loads plus static lateral forces using approximate procedures. From the resulting internal member forces, relative member properties were obtained. These were then used as the initial member properties in a more exact computer analysis of determining member sizes and internal member forces. This second set of internal member forces, designated the design forces, were used to determine the final relative member stiffness as shown in Figure 4. 1.

The overall stiffness of the structure was altered slightly by multiplying the stiffness of every member by a constant factor such that the resulting fundamental period was equal to 2. 2 seconds. This followed the work of Clough and Bensuka (5) and permitted a comparison with their results.

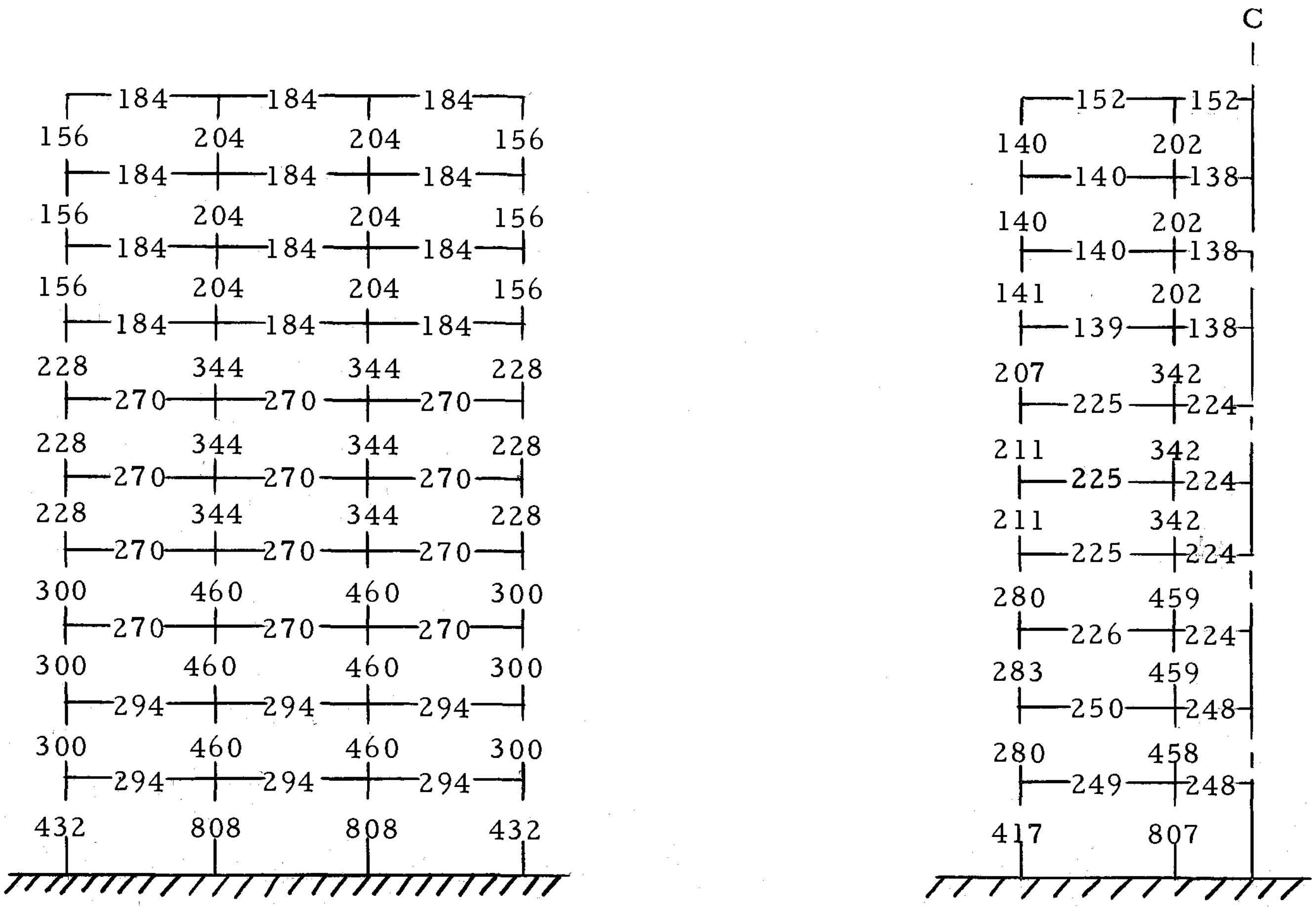
The yield moments of the individual members were determined by multiplying the maximum design moment for a particular member by four for the columns and by two for the girders. For this reason the resulting yield moments shown in Figure 4. 2a are designated the

Weight per floor (kips)	Story level
60	10
80	9
80	8
80	7
80	6
80	5
80	4
80	3
80	2
80	1



Number shown by each member is the relative member stiffness.

Figure 4. 1. Building properties and relative member stiffness.



(a) 2/4 yield moments (ft-kips)

(b) Modified yield moments (ft-kips)

Figure 4. 2. Yield moments of ten-story steel structure.

2/4 yield moments.

Effect of Semirigid Girder Connections on the Fundamental Period

The effect of girder connection rigidity on the fundamental period was determined by varying the girder connection fixity factor from 1. 0 to 0. 4 and calculating the resulting period. The results are shown in Figure 4. 3. The range of typical girder connection fixity factors is also noted. It is seen that within this range of fixity factors the fundamental period can increase by as much as 25 percent of that for rigid connections.

Effect of Semirigid Girder Connections on Nonlinear Response

Semirigid girder connections influence the dynamic response of structures in the following three ways. By decreasing the girder connection fixity factors (a) the relative stiffnesses of the girders are reduced (b) the strength or yield moments of the girder connections are reduced, and (c) the overall stiffness of the structure is reduced or the period of vibration is increased. These effects are isolated and investigated independently.

Ground motion characteristics also have a fundamental influence on the dynamic response of structures, thus the influence of the intensity and duration of the ground motion on the dynamic response

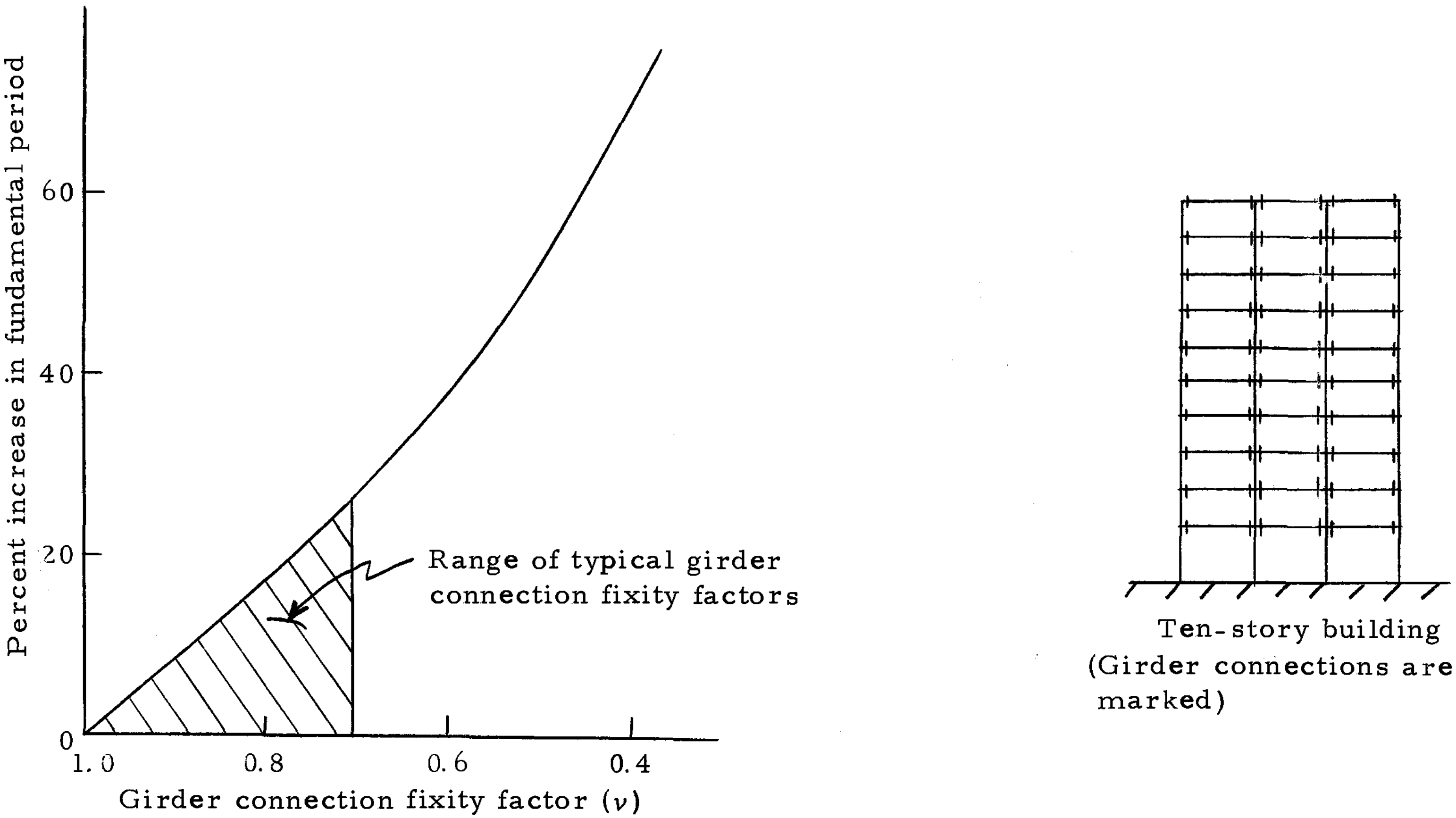


Figure 4. 3. Percent increase in fundamental period for girder connections of varying fixity for ten-story steel structure shown in Figure 4. 1. Fundamental period is 2. 2 seconds when building has girder connections that are rigid. Column splices are assumed rigid in all cases.

were investigated.

When the rigidity of the girder connections is changed, there is a resulting change in the fundamental period. To isolate the effect of changing the connection rigidity, or fixity factor, the relative member stiffness of every member in the structure was multiplied by a factor (I*) such that the fundamental period remains constant. Table 4. 1 shows the multiplication factor (I*) along with all periods that result when girder connection fixity factors of 1. 0, 0. 8 and 0. 6 were used.

Table 4. 1. Periods of vibration (T) with structural stiffness constant (I*).

	Girder connection fixity factor	Rigid	Semirigid		
		ν = 1. 0	ν = 0. 8	ν = 0. 6	ν = 0. 8
	I*	56. 1	75. 5	106. 2	46. 5
Periods of vibration	T_1	2. 20	2. 20	2. 20	2. 80
	T_2	0. 79	0. 78	0. 76	0. 99
	T_3	0. 45	0. 44	0. 42	0. 56
	T_4	0. 31	0. 29	0. 28	0. 37
	T_5	0. 23	0. 21	0. 20	0. 27
	T_6	0. 20	0. 19	0. 17	0. 24
	T_7	0. 18	0. 16	0. 14	0. 21
	T_8	0. 15	0. 13	0. 12	0. 17
	T_9	0. 13	0. 11	0. 10	0. 14
	T_{10}	0. 10	0. 09	0. 08	0. 12

The basic quantities used to measure the nonlinear response of multi-story buildings were the maximum deformation of the connection, the maximum lateral floor displacements and the maximum horizontal story shear that existed at each story level. Connection deformations are measured by the nonlinear and the total ductility factors.

The story shear was calculated by summing the horizontal story forces from the top of the structure to the story under consideration. The horizontal story forces are equivalent static forces placed at each floor level which produce a given deflected shape determined from dynamic analysis. Thus the maximum story shear at the bottom floor is the base shear that exists for this structure and given earthquake.

The damping matrix used in this investigation consisted of a fraction of the mass matrix. The damping coefficient was selected such that the damping ratio was equal to five percent of critical damping in the fundamental mode of vibration.

In all cases the fixity factor for both girder and column connections was equal to 0.10 during plastic deformation.

Column connections or splices were considered initially rigid in all tests.

Approximate Solution Using Symmetry

In order to use symmetry (see Appendix C) to reduce the amount of calculations required in analyzing the structure, the static gravity loads must be removed from the girders. To account for this reduction in internal member forces, the individual member moments determined for static gravity loading only, were subtracted from the yield moments shown in Figure 4. 2a. This resulted in the modified yield moments shown in Figure 4. 2b. A comparison of the maximum response results obtained from four seconds of the El Centro earthquake using these two sets of conditions are shown in Figures 4. 4 and 4. 5. These results indicate that the approximate solution yields accurate values for the maximum lateral floor deflection but overestimates the maximum nonlinear ductility factors and the maximum story shear. These results, although not exact, were considered to be sufficiently close. Thus it was decided to use symmetry and the modified yield moments with the static gravity loads removed for the remaining analyses in order to reduce the required computation time.

Influence of Intensity of Ground Motion

The ground motion record used in this investigation was the 1940 El Centro, California earthquake accelerogram. To obtain the influence of earthquake intensity, the earthquake acceleration was multiplied by a scale factor S. Thus a modified earthquake accelerogram of intensity S was defined. The results of using the El Centro earthquake of intensity 1. 0 and 1. 5 are shown in Figures 4. 6

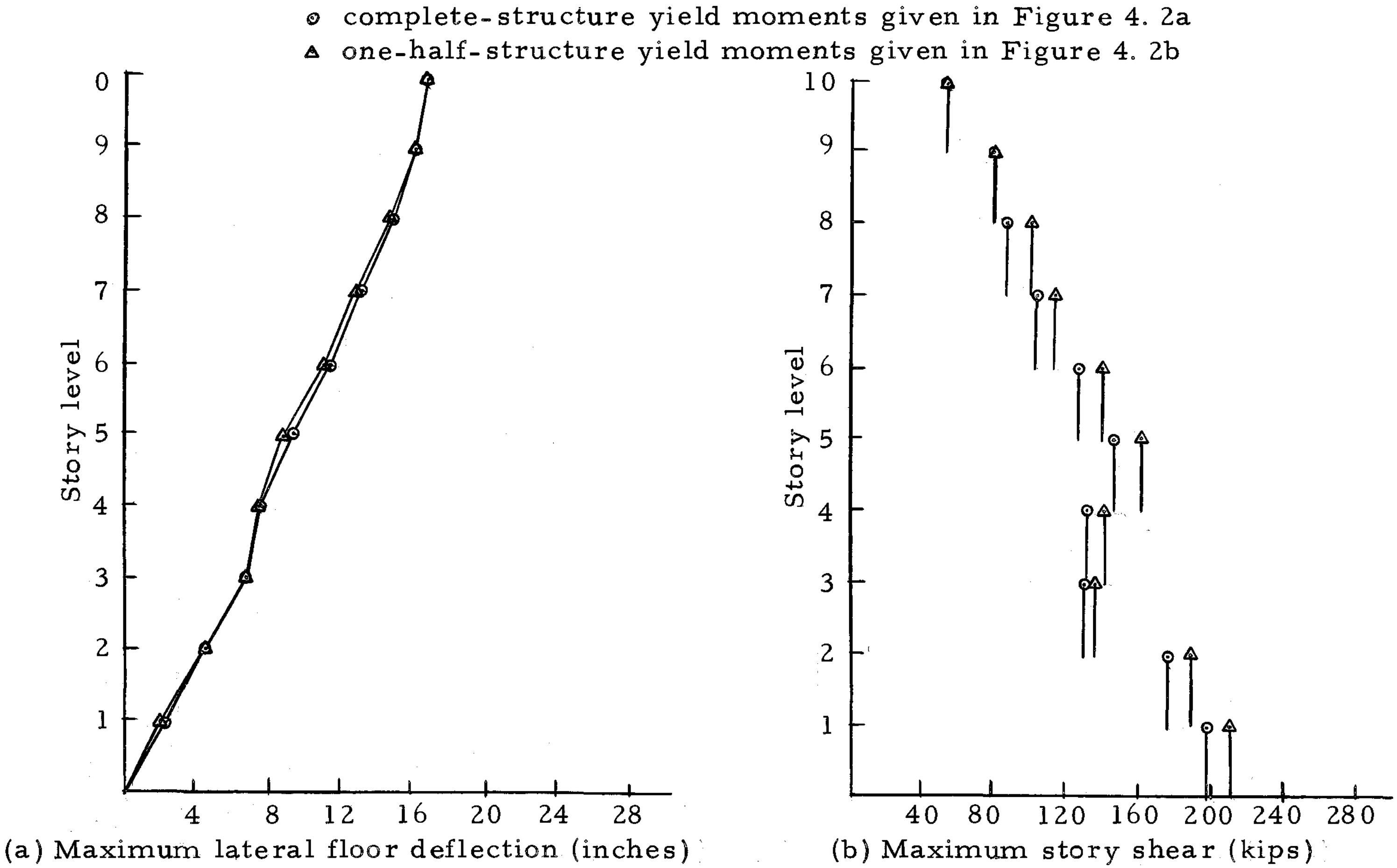
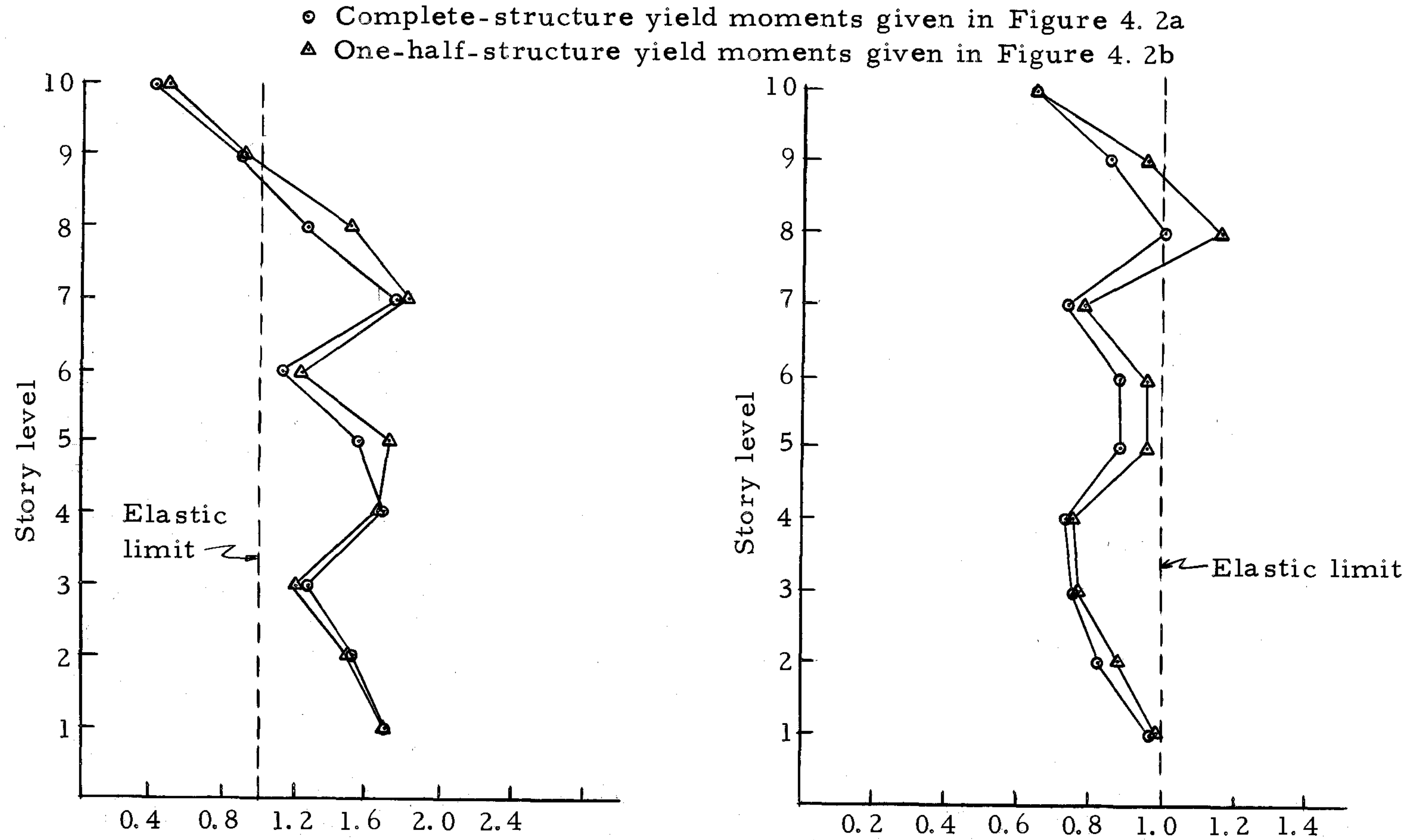


Figure 4. 4. Comparison of dynamic response of exact and approximate solutions. Duration of El Centro earthquake is 4 seconds with a scale factor of 1. 5. Girder connection fixity factor is 0. 8. Fundamental period is 2. 2 seconds.



(a) Maximum girder nonlinear ductility factor

(b) Maximum column nonlinear ductility factor

Figure 4. 5. Comparison of dynamic response of exact and approximate solutions. Duration of El Centro earthquake is 4 seconds with a scale factor of 1. 5. Girder connection fixity factor is 0. 8. Fundamental period is 2. 2 seconds.

through 4. 9.

To illustrate the effect that earthquake intensity has on the dynamic response, the ratio of the maximum structural response for an earthquake of scale 1. 5 to scale 1. 0 is also shown. From Figure 4.6b it is observed that the resulting ratio of maximum lateral floor deflection is between 1. 5 and 1. 8. Likewise Figure 4. 7b shows that the ratio of the maximum story shear for an earthquake of scale 1. 5 to 1. 0 varies from 1. 0 to 1. 35. Figure 4. 8b shows that the ratio of the maximum girder nonlinear ductility factor varies from approximately 1. 0 to 2. 0. Figure 4. 9b shows that the ratio of maximum column nonlinear ductility factor varies from about 1. 1 to 1. 8.

From studying the results it is evident that this building, designed using the Uniform Building Code (24) of 1967, is designed to adequately withstand the El Centro earthquake with a scale factor of 1. 0. For this excitation only slight plastic rotation occurs in the girder connections and none in the column connections. This is true for the structure with rigid or semirigid connections.

Since this study was intended primarily as an investigation of the nonlinear response of structures under dynamic loading, it was concluded that a scale factor of 1. 5 would be used in all of the tests to insure that the response would include nonlinear deformations.

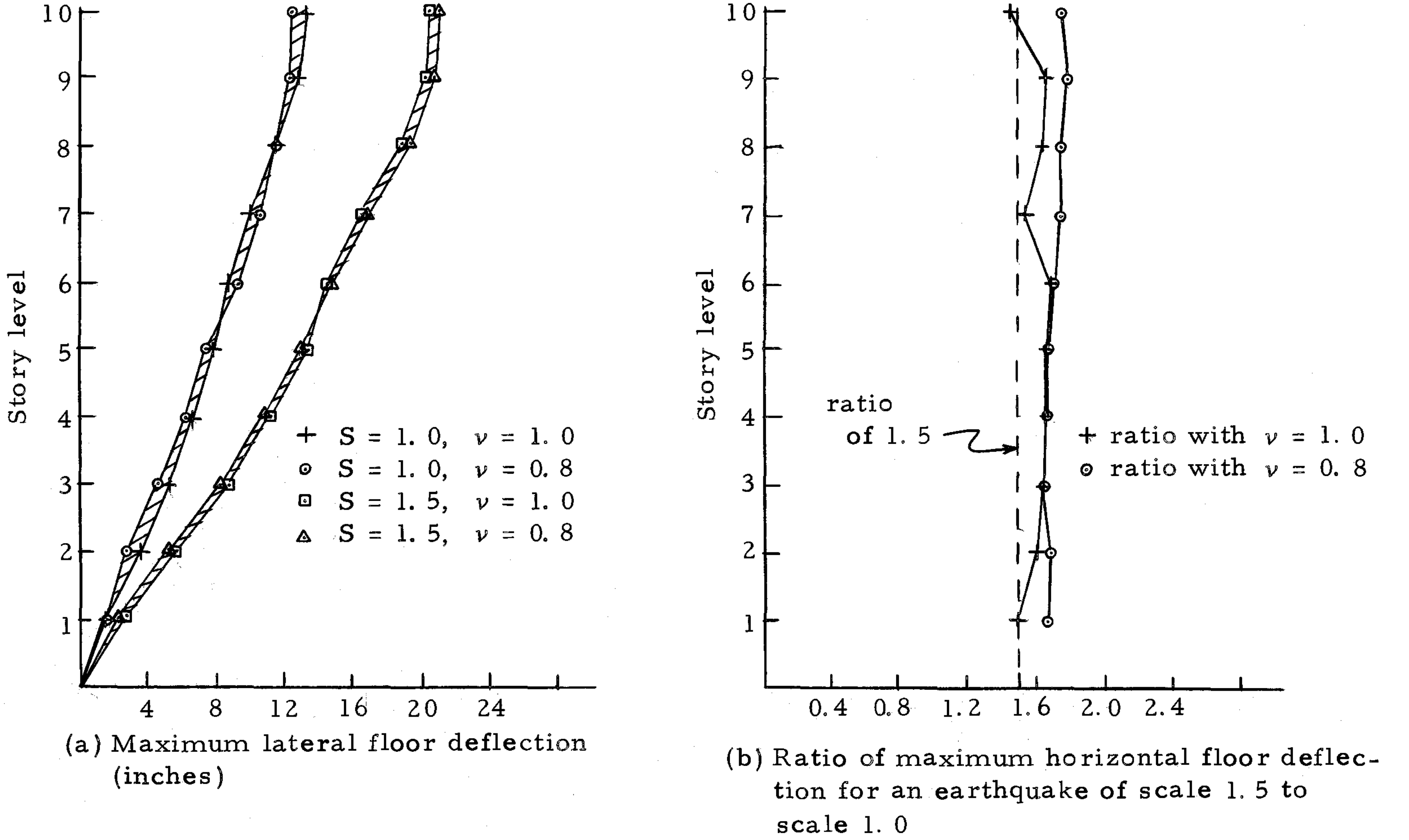
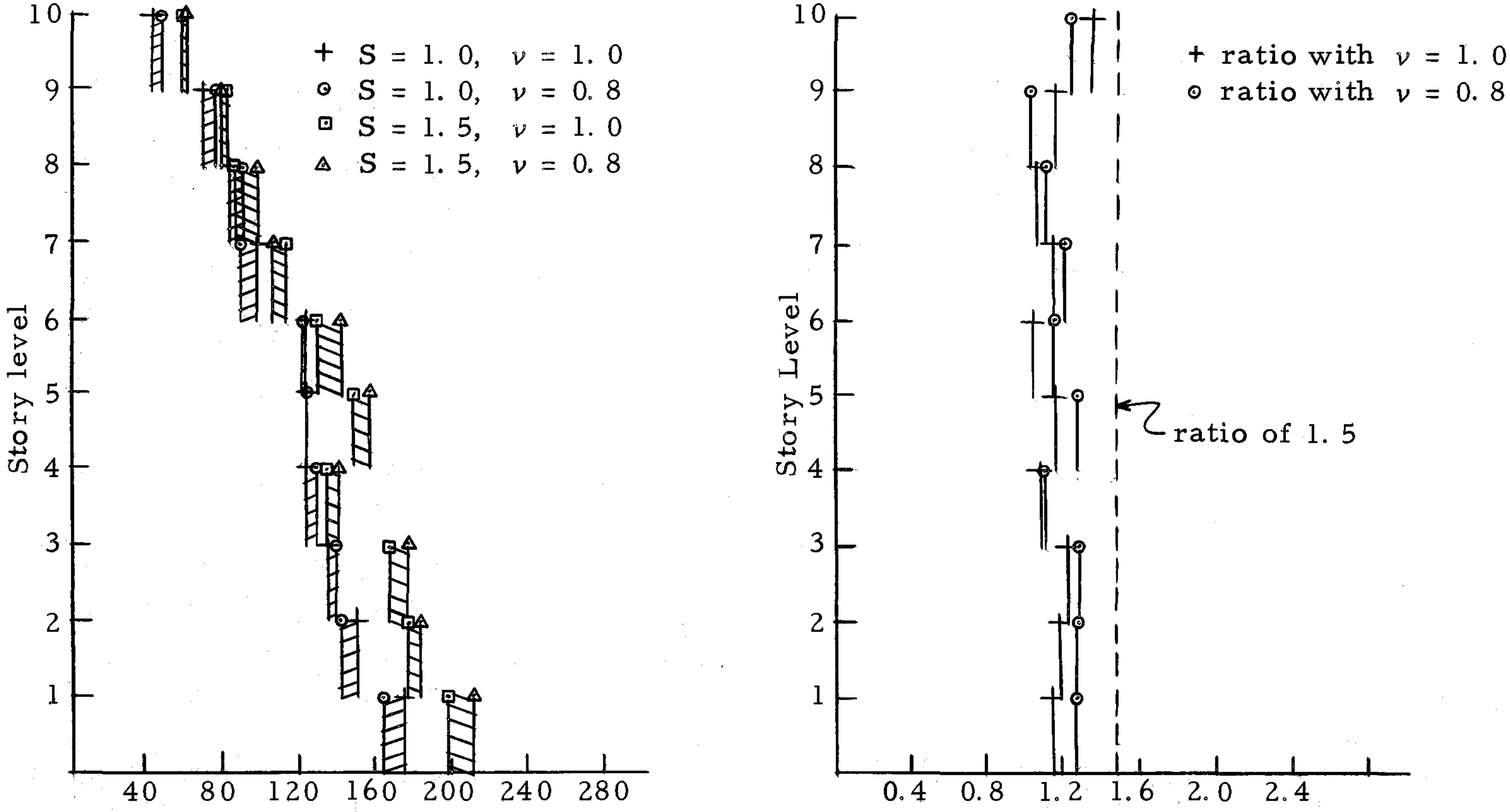


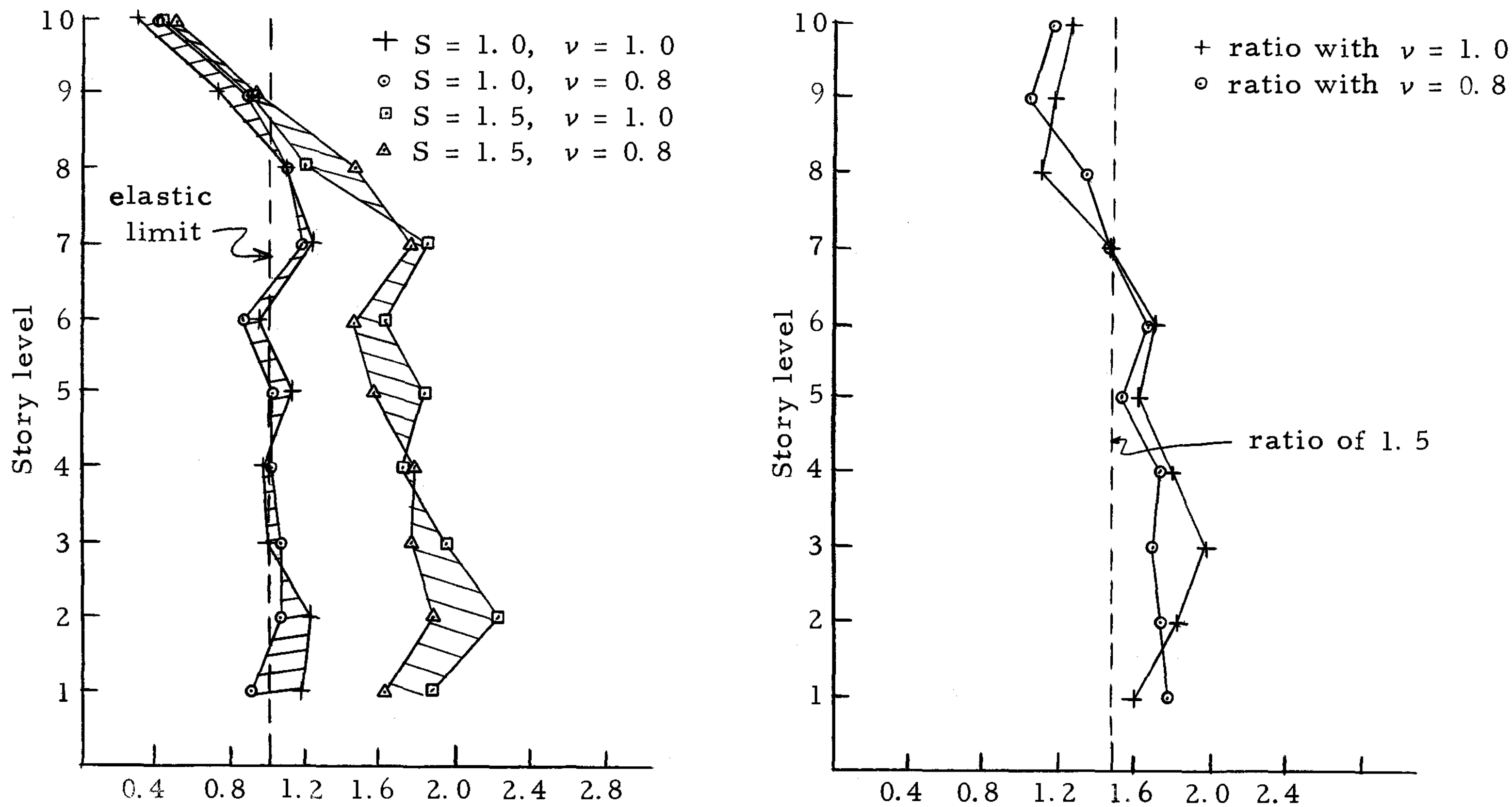
Figure 4. 6. Influence of intensity of El Centro earthquake on dynamic response. Yield moments used are given in Figure 4. 2b. Fundamental period is 2. 2 seconds.



(a) Maximum horizontal story shear force (kips)

(b) Ratio of maximum horizontal story shear for an earthquake of scale 1.5 to scale 1.0

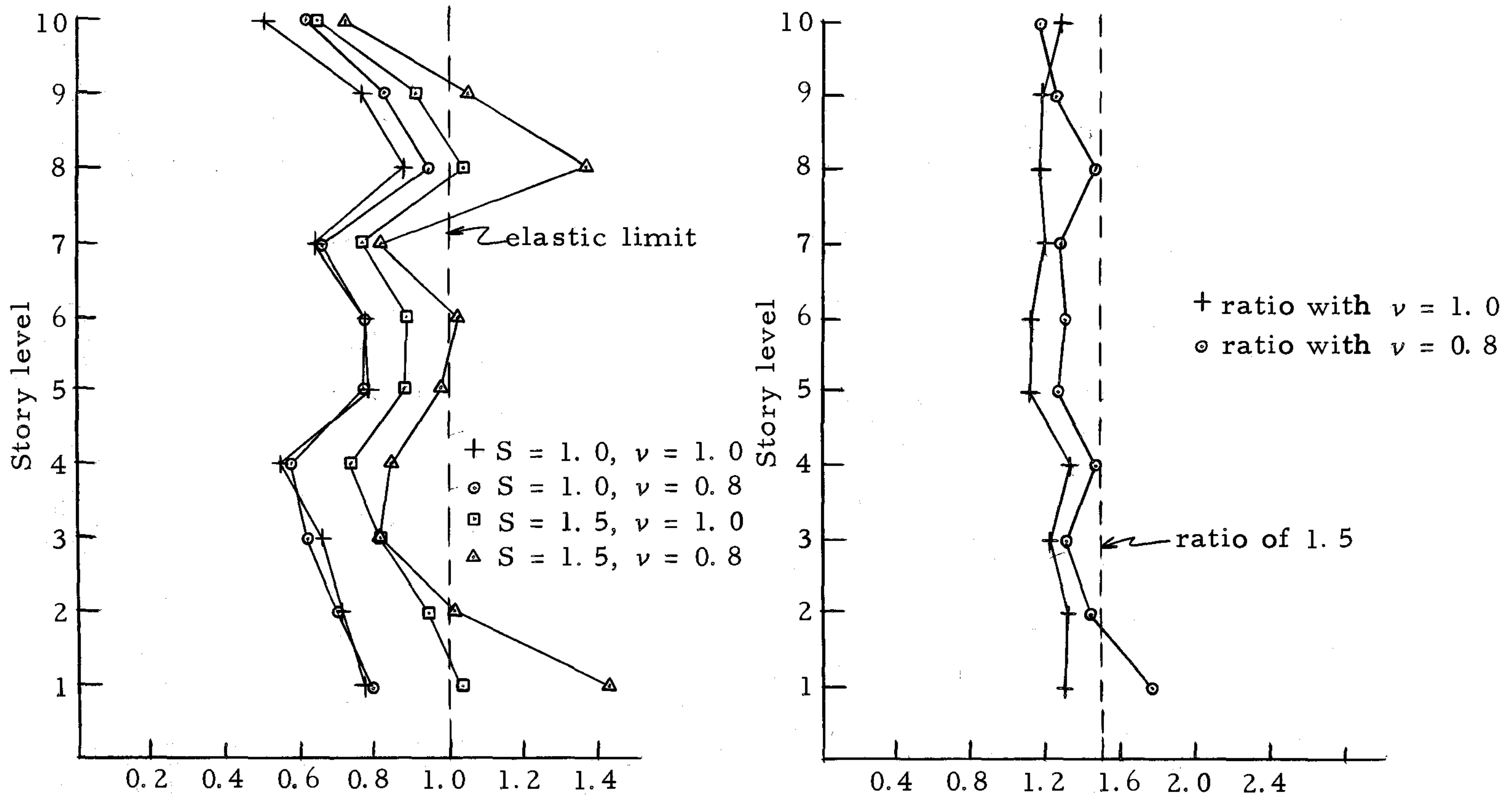
Figure 4.7. Influence of intensity of El Centro earthquake on dynamic response. Yield moments used are given in Figure 4.2b. Fundamental period is 2.2 seconds.



(a) Maximum girder nonlinear ductility factor

(b) Ratio of maximum girder nonlinear ductility for an earthquake of scale 1. 5 to scale 1. 0

Figure 4. 8. Influence of intensity of El Centro earthquake on dynamic response. Yield moments used are given in Figure 4. 2b. Fundamental period is 2. 2 seconds.



(a) Maximum column nonlinear ductility factor

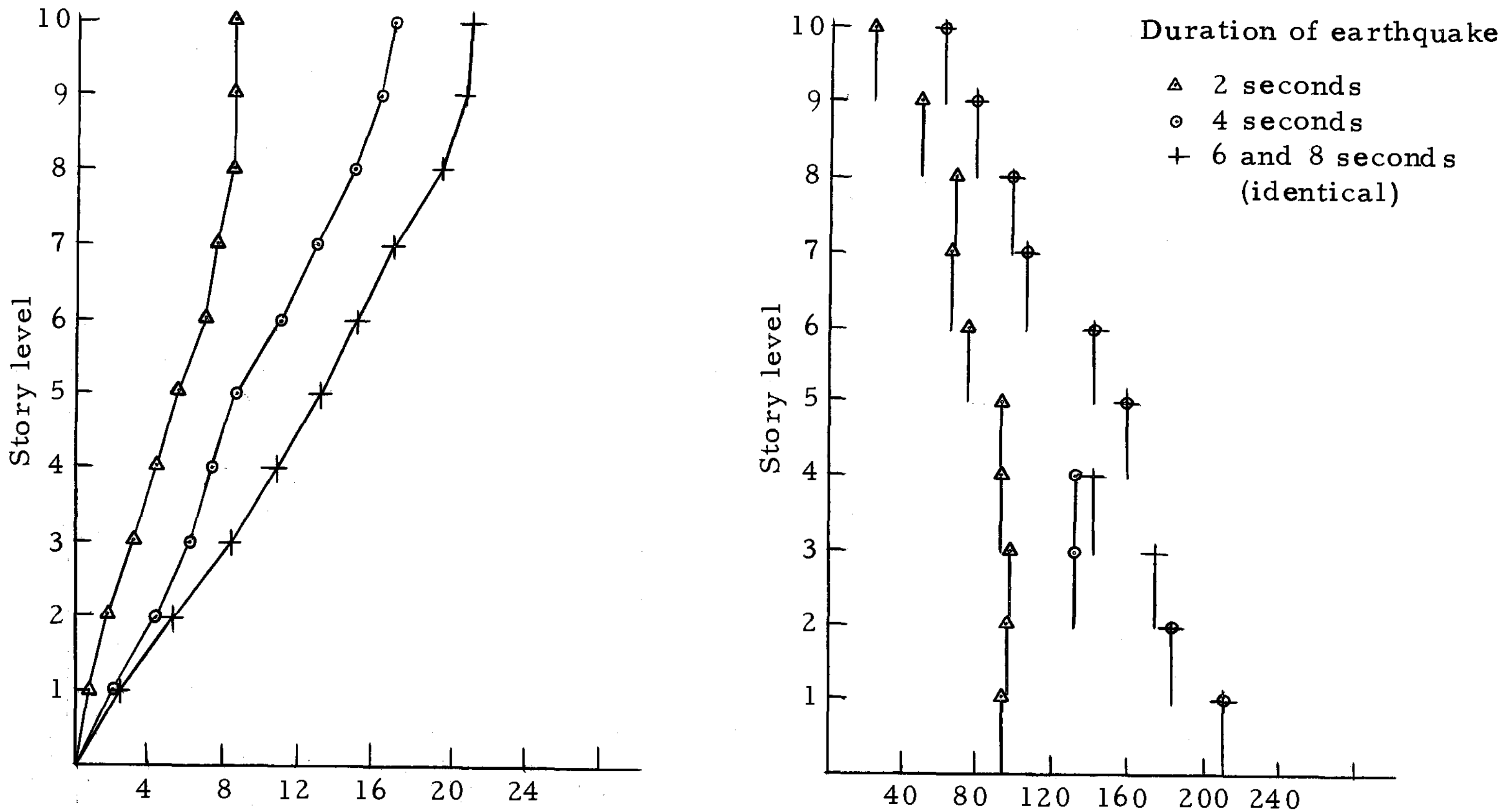
(b) Ratio of maximum column nonlinear ductility for an earthquake of scale 1. 5 to scale 1. 0.

Figure 4. 9. Influence of intensity of El Centro earthquake on dynamic response. Yield moments used are given in Figure 4. 2b. Fundamental period is 2. 2 seconds.

Influence of Duration of Ground Motion

To determine the influence of the duration of the ground motion, the maximum response values at the end of each second of earthquake time were printed by the computer. The results at two second intervals up to a maximum of eight seconds are shown in Figures 4. 10 and 4. 11. Three separate tests, each carried out to a total of eight seconds and considering both rigid and semirigid girder connections, produced results similar to those shown in Figure 4. 10 and 4. 11. In no case did any maximum response occur between six and eight seconds of earthquake time. It was found that the time at which the maximum responses occurred decreased as the period of vibration increased and as the initial yield moments decreased. By increasing the period of vibration from 2. 2 to 2. 8 seconds or by lowering the yield moments by 33 percent the maximum response generally occurred in four seconds or less rather than six seconds as indicated by the case shown. Reducing the girder connection fixity factors while maintaining the period of vibration and yield moments was found to have no effect on the time at which the maximum responses occurred.

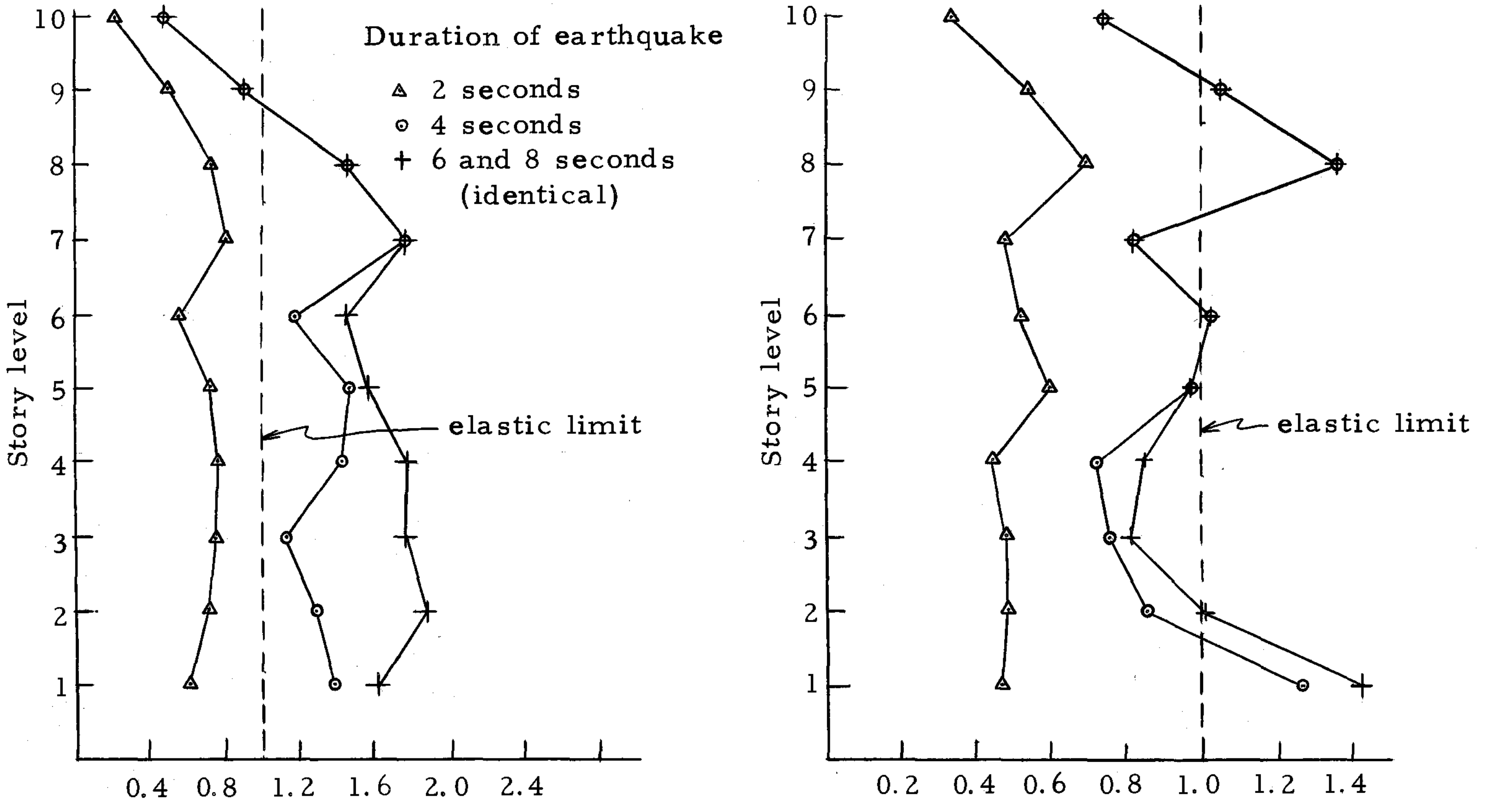
Clough and Benuska (5) found that the maximum structural response depends primarily on the peak acceleration impulse in the ground motion and it is not affected strongly by continuing motions of



(a) Maximum lateral story deflection (inches)

(b) Maximum story shear (kips)

Figure 4. 10. Influence of duration of earthquake on dynamic response. Scale factor of El Centro earthquake is 1. 5. Yield moments used are given in Figure 4. 2b. Fundamental period is 2. 2 seconds. Girder fixity factor in elastic deformation is 0. 80.



(a) Maximum girder nonlinear ductility factor

(b) Maximum column nonlinear ductility factor

Figure 4.11. Influence of duration of earthquake on dynamic response. Scale factor of El Centro earthquake is 1.5. Yield moments used are given in Figure 4.2b. Fundamental period is 2.2 seconds. Girder fixity factor in elastic deformation is 0.80.

smaller amplitudes. Generally, this conclusion was borne out in this investigation. Thus it was concluded the analyses would be limited to an earthquake duration of six seconds.

Influence of Girder Connection Fixity Factor on Structural Response

The influence of girder connection fixity factors on structural response was isolated as previously outlined by multiplying the relative stiffness of each member by a factor such that the fundamental period remained at 2. 2 seconds as the connection fixity factor was varied. The results of the responses thus determined are given in Figures 4. 12 through 4. 14. These results show that the maximum lateral floor deflection and the maximum story shear are changed only slightly by using semirigid girder connections. The maximum nonlinear ductility factor is generally decreased for girders and increased for columns and the maximum total ductility factor is increased for girders.

Increasing the girder connection flexibility alters the stiffness relationship between girders and columns, with the girders becoming relatively more flexible. Thus, yielding is decreased in the girders and increased in the columns as expected. However, the overall stiffness and strength of the structure remains unchanged. Therefore, little variation would be expected in the maximum lateral deflection.

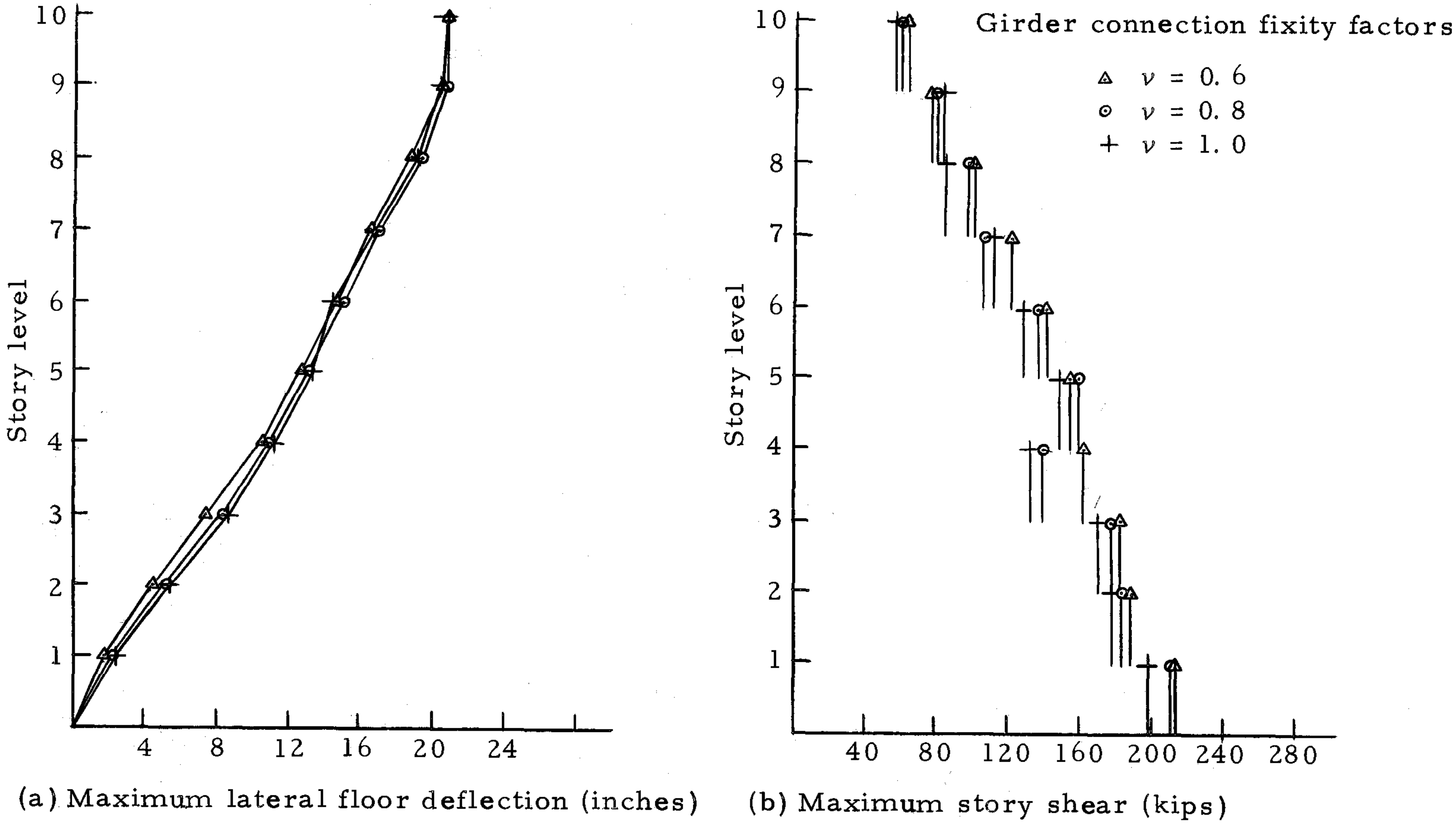
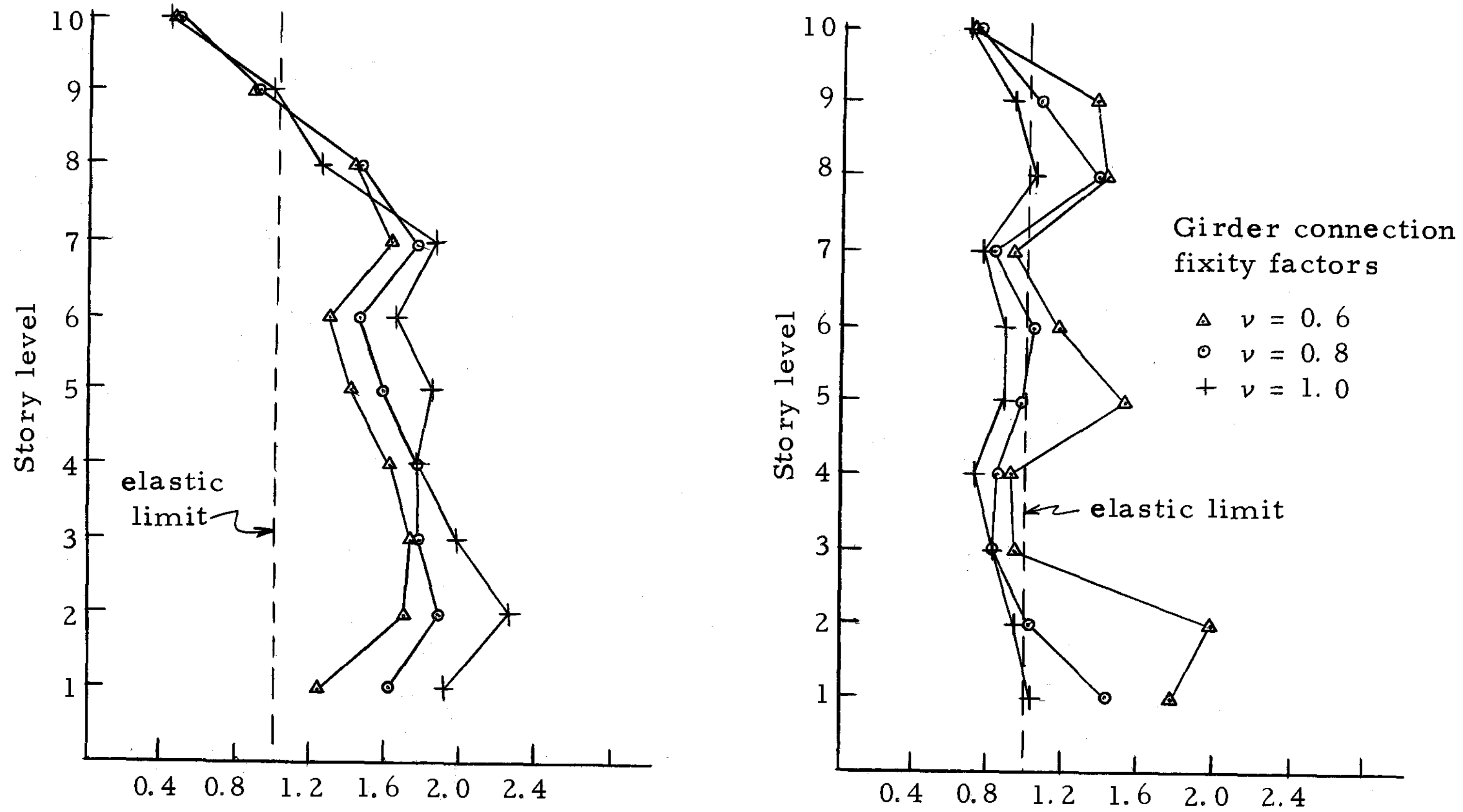


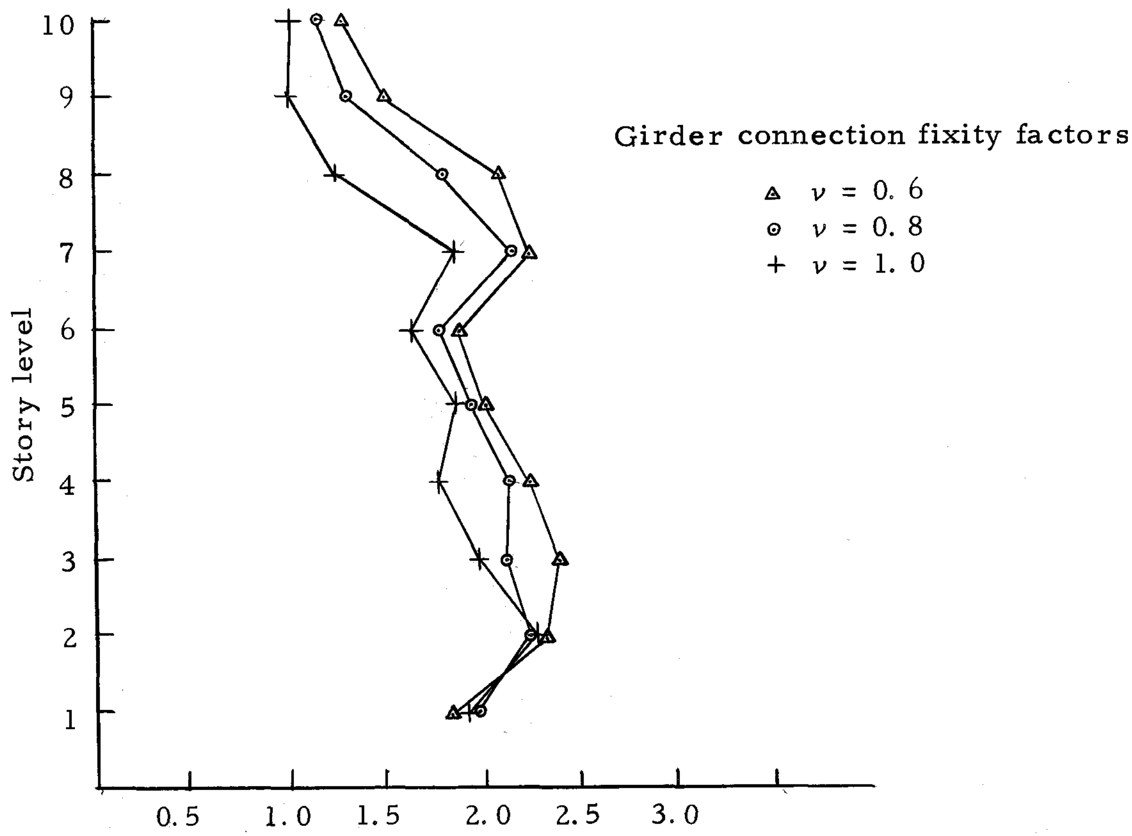
Figure 4. 12. Influence of girder connection fixity factor on dynamic response. Duration of El Centro earthquake is six seconds and scale factor is 1. 5. Yield moments used are those given in Figure 4. 2b. Fundamental period is 2. 2 seconds.



(a) Maximum girder nonlinear ductility factor

(b) Maximum column nonlinear ductility factor

Figure 4. 13. Influence of girder connection fixity factor on dynamic response. Duration of El Centro earthquake is six seconds and scale factor is 1. 5. Yield moments used are those given in Figure 4. 2b. Fundamental period is 2. 2 seconds.



(a) Maximum girder total ductility factor

Figure 4.14. Influence of girder connection fixity factor on dynamic response. Duration of El Centro earthquake is six seconds and scale factor is 1.5. Yield moments used are those given in Figure 4.2b. Fundamental period is 2.2 seconds.

Influence of Girder Connection Initial Yield Moments on Structural Response

The effect that the initial yield moments have on structural response was obtained by comparing the responses of two tests in which the values of the initial yield moments were the only variables. The results, given in Figures 4. 15 and 4. 16, indicate that a reduction in the initial girder connection yield moments decrease the maximum lateral floor deflection, the maximum story shear and the maximum column nonlinear ductility requirements and increases the maximum girder nonlinear ductility requirements.

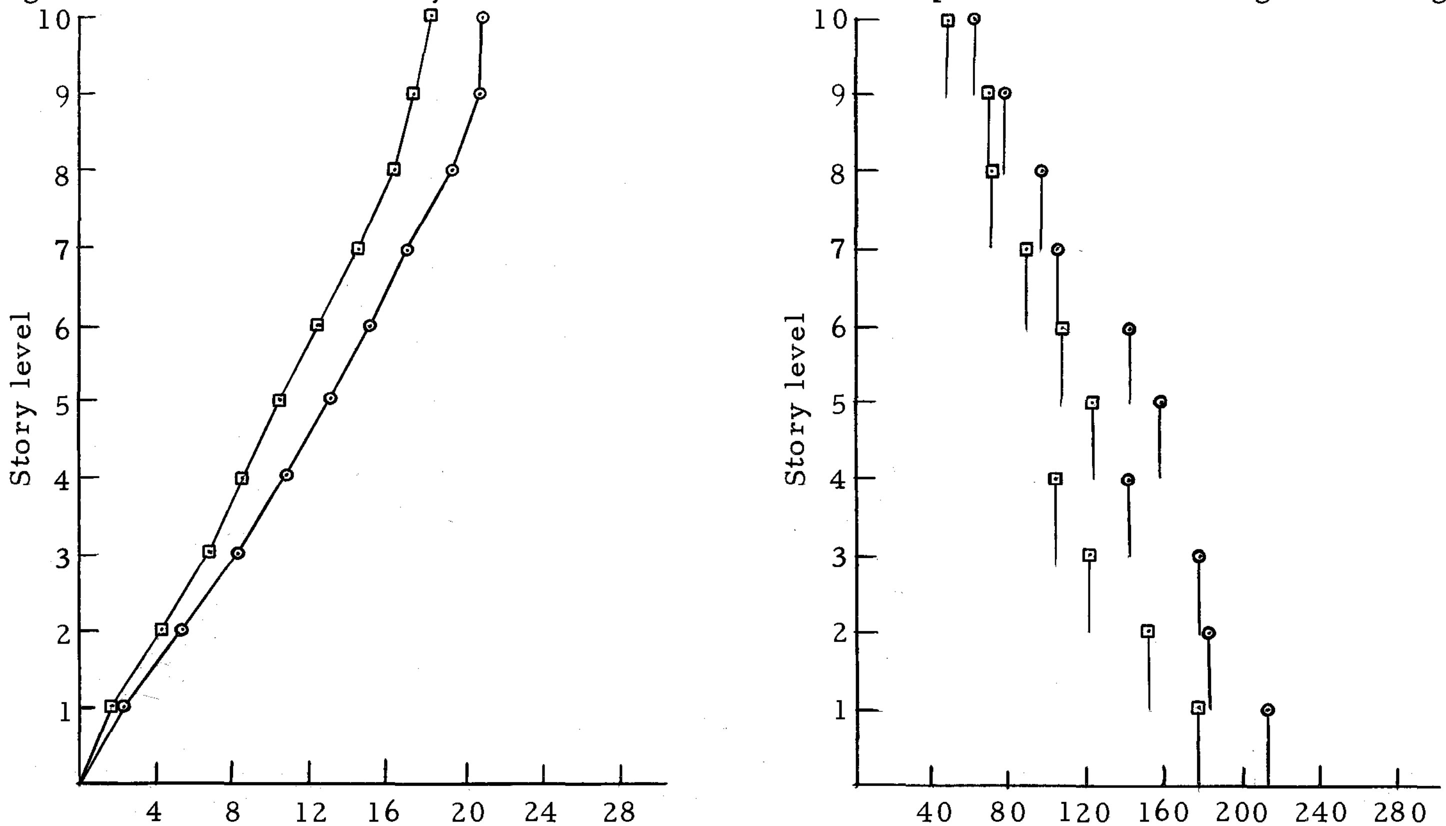
Influence of the Period of Vibration on Structural Response

The influence of the period of vibration on structural response is shown in Figures 4. 17 and 4. 18. The single variable in these results was the period of vibration of the structure which changes from 2. 2 to 2. 8 seconds. These results show that with this change in period the maximum lateral floor deflection increases while the maximum girder nonlinear ductility requirements decrease and the maximum story shear and column nonlinear ductility factor remain essentially unchanged and mixed.

Summary of Results and Conclusions

Semirigid girder connections can affect the nonlinear response

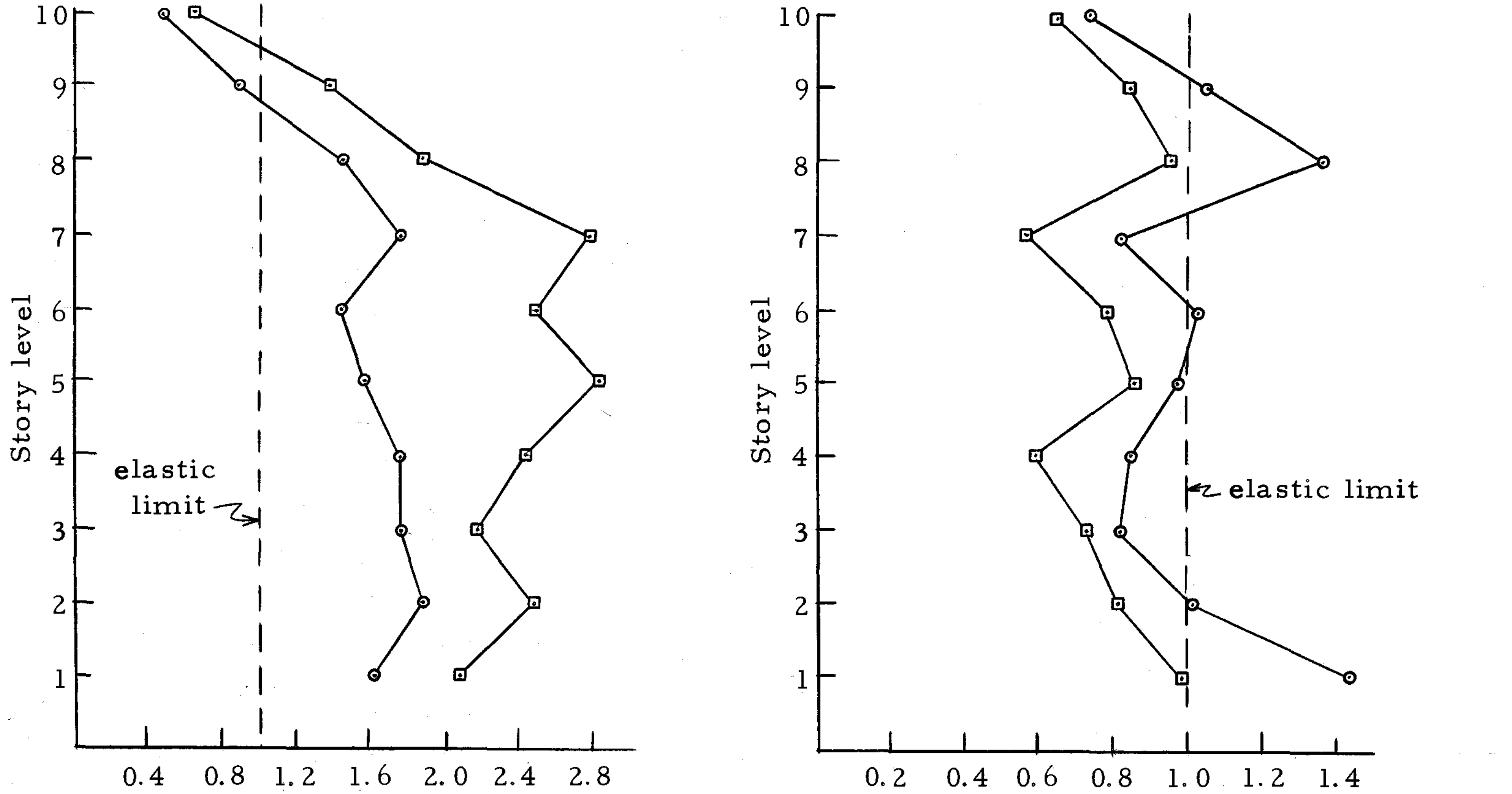
⊙ girder connection initial yield moments given in Figure 4. 2b.
⊡ girder connection initial yield moments are reduced 33 percent from those given in Figure 4. 2b.



(a) Maximum lateral floor deflection (inches) (b) Maximum horizontal story shear (kips)

Figure 4. 15. Influence of initial girder connection yield moment on dynamic response. Duration of El Centro earthquake is six seconds with a scale factor of 1. 5. Column yield moments used are those given in Figure 4. 2b. The fixity factor in elastic deformation is 0. 8 and the fundamental period is 2. 2 seconds.

⊙ girder connection initial yield moments are given in Figure 4. 2b.
⊡ girder connection initial yield moments are reduced 33 percent of those given in Figure 4. 2b.



(a) Maximum girder nonlinear ductility factor (b) Maximum column nonlinear ductility factor

Figure 4. 16. Influence of initial girder connection yield moment on dynamic response. Duration of El Centro earthquake is six seconds with a scale factor of 1. 5. Yield moments used are given in Figure 4. 2b. The fixity factor in elastic deformation is 0. 8 and the fundamental period is 2. 2 seconds.

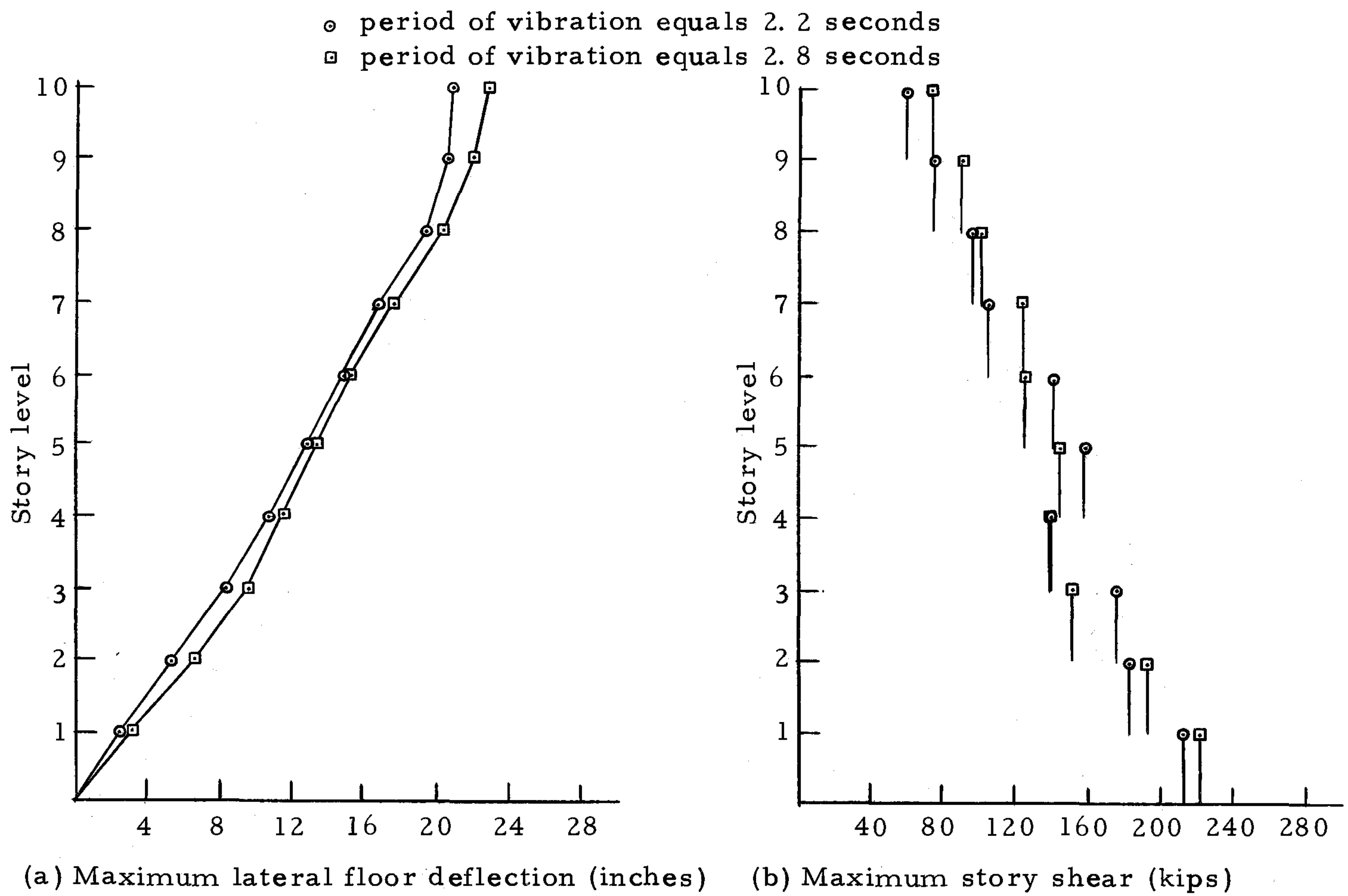
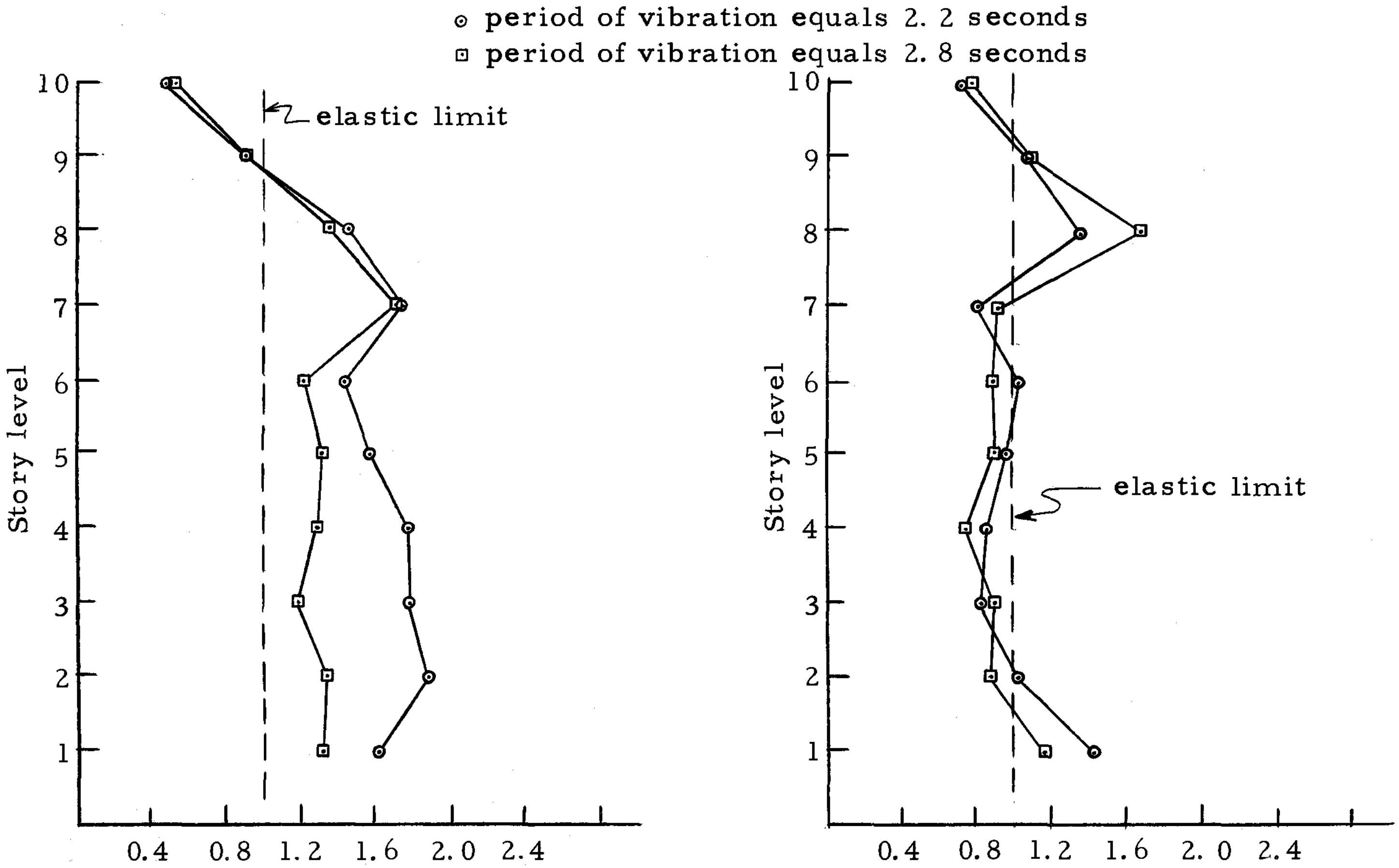


Figure 4. 17. Influence of period of vibration on structural response. Duration of El Centro earthquake is six seconds with a scale factor of 1. 5. The fixity factor in elastic deformation is 0. 80. Yield moments used are those given in Figure 4. 2b.



(a) Maximum girder nonlinear ductility factor (b) Maximum column nonlinear ductility factor

Figure 4. 18. Influence of period of vibration on structural response. Duration of El Centro earthquake is six seconds with a scale factor of 1. 5. The fixity factor in elastic deformation is 0. 80. Yield moments used are those given in Figure 4. 2b.

of a structure in the following three ways: (a) by reducing the relative stiffness of girders to columns with decreasing fixity factors, (b) by reducing the girder connection strength, or yield moments and (c) by reducing the overall stiffness of the structure or increasing the fundamental period of vibration. These three effects were isolated and investigated independently. The results are summarized in Table 4.2. In column (a) the effect of reducing the relative stiffness of the girder to columns is shown. This is done by reducing the girder connection fixity factors while holding all other variables constant. In column (b) the effect of reducing the strength of the girder connections is shown. This is accomplished by reducing the girder connection yield moments while holding all other variables constant. Column (c) shows the effect of reducing the overall stiffness of the structure or increasing the period of vibration while holding all other variables constant.

Because of large axial forces in the lower columns of tall multistory buildings and the unknown requirements needed to prevent column instability, it is generally concluded that column ductility requirements must be minimized.

It is of interest to note from Table 4.2 that reducing the girder connection yield moments will result in smaller maximum responses except for the girder nonlinear ductility factors. Thus, reducing the yield moments in semirigid girder connections may be an effective method of minimizing the ductility requirements of the columns of controlling the points of plastic deformation, and of partially controlling the maximum lateral deflections.

Table 4. 2. Summary of the effects of semirigid connections on structural responses.

	Decreasing girder connection fixity factors (a)	Decreasing girder connection yield moments (b)	Increasing period of vibration (c)
Maximum lateral deflection	no change	decrease	increase
maximum story shear	slight increase	decrease	mixed
maximum girder nonlinear ductility	decrease	increase	decrease
maximum column nonlinear ductility	increase	decrease	mixed

No general conclusions can be reached from testing only one structure and one earthquake. However, these results indicate that semirigid girder connections will have a significant effect on the structural responses obtained, and that through a proper choice of the strength and stiffness properties of the connections, these responses may be altered to produce a beneficial effect on the structure.

BIBLIOGRAPHY

1. Baker, J. F. The steel skeleton. Vol. 1. Cambridge University Press, 1954. 208 p.

2. Batho, C. and H. C. Rowan. Investigations on beam and stanchion connections. In: Second report of the Department of Scientific and Industrial Research of Great Britain. Steel Structures Research Committee. London, 1934. p. 61-137.

3. Biot, M. A. Analytical and experimental methods in engineering seismology. Transactions of the American Society of Civil Engineers 108:262-268. 1943.

4. Biot, M. A. Theory of elastic systems vibrating under transient impulse with an application to earthquake proof buildings. Proceedings of the National Academy of Sciences 19:262-268. 1933.

5. Clough, R. W., K. L. Benuska and T. Y. Lin and Associates. FHA study of seismic design criteria for high-rise buildings. Washington, D. C., U. S. Federal Housing Administration. August 1966. (HUD TS-3)

6. Clough, R. W. and E. L. Wilson. Dynamic response by step-by-step matrix analysis. In: Symposium on the Use of Computers in Civil Engineering, Lisbon, Portugal, 1962. Portugal, Laboratorio Nacional de Engenharia Civil, 1962. p. 29. 1-29. 22.

7. Freeman, J. R. Earthquake damage and earthquake insurance. New York, McGraw-Hill, 1932.

8. Gere, James M. Moment distribution. Princeton, N. J. Van Nostrand, 1963. 378 p.

9. Giberson, Melbourne F. The response of nonlinear multi-story structures subjected to earthquake excitation. Ph. D. thesis. Pasadena, California Institute of Technology, 1967. 232 numb. leaves.

10. Grant, Joseph. Matrix analysis of structures with semirigid connections. Master's thesis. Corvallis, Oregon State University, 1965. 34 numb. leaves.

11. Hanson, Robert D. Comparison of static and dynamic hysteresis curves. Journal of the American Society of Civil Engineers, Engineering Mechanics Division 92:87-112. Oct. 1966.

12. Hechtman, Robert A. and Bruce G. Johnston. Riveted semi-rigid beam-to-column building connections. New York, American Institute of Steel Construction, 1947. (Committee on Steel Structures Research. Progress Report no. 1)

13. Hollis, E. P. Bibliography of engineering seismology. 2d ed. San Francisco, Earthquake Engineering Research Institute, 1966.

14. Kreyszig, Erwin. Advanced engineering mathematics. New York, Wiley, 1964. 841 p.

15. Monforton, G. R. and T. S. Wu. Matrix analysis of semirigidly connected frames. Journal of the American Society of Civil Engineers, 89:13-42. Dec. 1963.

16. Munse, William H., Wyatt G. Bell and Eugene Chesson, Jr. Behavior of riveted and bolted beam-to-column connections. Transactions of the American Society of Civil Engineers 126: 729-763. 1961.

17. Newmark, N. M. A method of computation for structural dynamics. Journal of the American Society of Civil Engineers, Engineering Mechanics Division 85:67-94, July, 1959. no. EM3, 1959. p. 67-94.

18. O'Kelley, M. E. J. Vibration of viscously damped linear dynamic systems. Ph. D. thesis. Pasadena, California Institute of Technology, 1964. 219 numb. leaves.

19. Pippard, A. J. S. and J. F. Baker. The analysis of engineering structures. 2d ed. London, E. Arnold and Company, 1943. 627p.

20. Rathbun, J. Charles. Elastic properties of riveted connections. Transactions of the American Society of Civil Engineers 101: 524-563. 1936.

21. Rosenblueth, E. Earthquake resistant design. Applied Mechanics Review 14:923-926. 1961.

22. Tezcan, Semih S. Computer analysis of plane and space structures. Journal of the American Society of Civil Engineers, Structural Division 92:143-173, April 1966.

23. Tezcan, Semih S. Discussion of: Simplified formulation of stiffness matrices, by Peter M. Wright. Journal of the American Society of Civil Engineers, Structural Division 89:445-449, Dec. 1963.

24. Uniform Building Code. Vol. 1, 1967. International conference of Building Officials, Pasadena, California. 595 p.

APPENDICES

APPENDIX A

Derivation of the Individual Stiffness Matrix for Members with Semirigid Connections

From basic engineering texts, the general fourth-order differential equation for a structural member with loads applied only at the nodes is given as

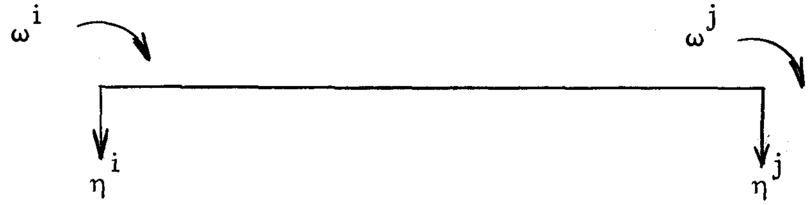
$$y^{IV} = 0 \tag{A. 1}$$

which has as its solution and first three derivatives

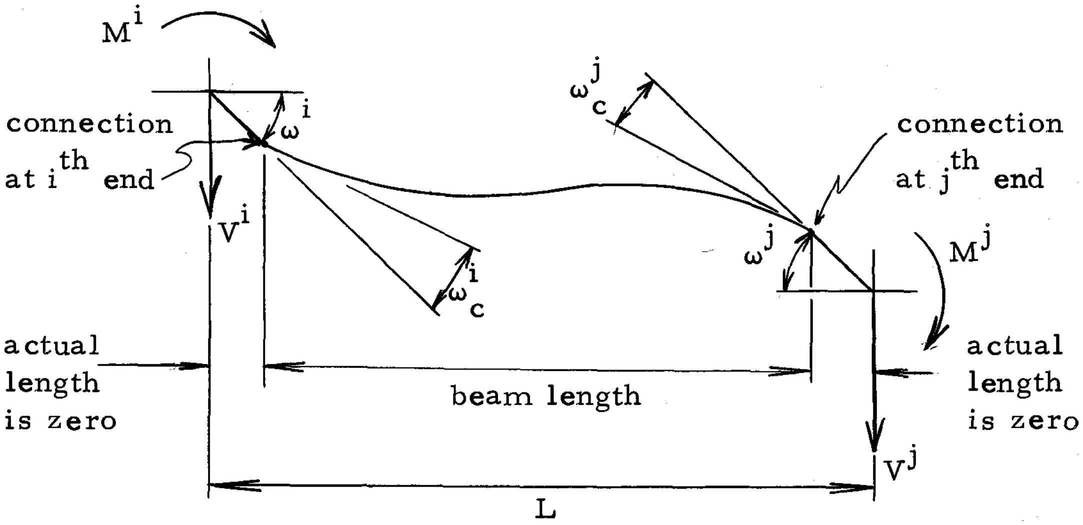
$$\begin{aligned} y &= Ax^3 + Bx^2 + Cx + D \\ y^I &= 3Ax^2 + 2Bx + C + 0 \\ y^{II} &= 6Ax + 2B + 0 + 0 \\ y^{III} &= 6A + 0 + 0 + 0 \end{aligned} \tag{A. 2}$$

From the geometric boundary conditions shown in Figure A. 1

$$\begin{aligned} \eta^i &= y(0) \\ \eta^j &= y(L) \\ \omega^i &= y^I(0) + \omega_c^i \\ \omega^j &= y^I(L) + \omega_c^j \end{aligned} \tag{A. 3}$$



(a) Sign convention for member displacements. Directions as shown are considered positive.



(b) Force and displacement quantities for semirigidly connected members.

Figure A. 1. Member force and displacement quantities and sign convention.

where ω_c^i, ω_c^j are the rotations of the connections at the i and j ends of a member and

$$\omega_c^i = \frac{M^i}{k_c^i}$$

and

$$\omega_c^j = \frac{M^j}{k_c^j} \qquad (A.4)$$

The connection stiffness is defined as

$$k_c^i = \frac{3EI}{L}\left(\frac{\nu^i}{1-\nu^i}\right)$$

and

$$k_c^j = \frac{3EI}{L}\left(\frac{\nu^j}{1-\nu^j}\right) \qquad (A.5)$$

where

k_c^i, k_c^j - the stiffness of the connection at the i and j ends of a member.

ν^i, ν^j - the fixity factor at the i and j ends of a member.

then

$$\omega_c^i = \frac{M^i}{3\frac{EI}{L}\left(\frac{\nu^i}{1-\nu^i}\right)}$$

and

$$\omega_c^j = \frac{M^j}{3\frac{EI}{L}\left(\frac{\nu^j}{1-\nu^j}\right)} \qquad (A.6)$$

Substituting $M^i = -EIy^{II}(0)$ into Equation A. 6 results in

$$\omega_c^i = -y^{II}(0)\frac{L}{3}\left(\frac{1-\nu^i}{\nu^i}\right) \qquad (A.7)$$

Likewise substituting $M^j = +EIy^{II}(L)$ into Equation A. 6 results in

$$\omega_c^j = +y^{II}(L)\frac{L}{3}\left(\frac{1-\nu^j}{\nu^j}\right) \qquad (A.8)$$

Solving for Equation A. 3 in terms of Equation A. 2 and writing in matrix form gives

$$\begin{bmatrix}\omega^i\\ \omega^j\\ \eta^i\\ \eta^j\end{bmatrix} = \begin{bmatrix}0 & -2\beta^i & +1 & 0\\ (+3L^2+6L\beta^j) & (+2L+2\beta^j) & +1 & 0\\ 0 & 0 & 0 & +1\\ +L^3 & +L^2 & +L & +1\end{bmatrix}\begin{bmatrix}A\\ B\\ C\\ D\end{bmatrix} \qquad (A.9)$$

or

$$\{d\} = [M]\{A\}$$

where

$\{d\}$ - the individual member displacement vector

$\{A\}$ - a vector of undetermined constants

[M] - a matrix of coefficients of the undetermined constants {A}.

$$\beta^i = \frac{L}{3}\left(\frac{1-\nu^i}{\nu^i}\right)$$

$$\beta^j = \frac{L}{3}\left(\frac{1-\nu^j}{\nu^j}\right)$$

From the expression for moment and shear at the boundary, the following equations can be obtained.

$$\begin{aligned} M^i &= -\ EIy^{II}(0) \\ M^j &= +\ EIy^{II}(L) \\ V^i &= +\ EIy^{III}(0) \\ V^j &= -\ EIy^{III}(L) \end{aligned} \qquad (A.\ 10)$$

where

E - modulus of elasticity

I - moment of inertia

which when solved using Equation A. 2 in matrix form results in

$$\begin{bmatrix} M^i \\ M^j \\ V^i \\ V^j \end{bmatrix} = EI \begin{bmatrix} 0 & -2 & 0 & 0 \\ +6L & +2 & 0 & 0 \\ +6 & 0 & 0 & 0 \\ -6 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \qquad (A.\ 11)$$

or

$$\{q\} = [N]\{A\}$$

where

$\{q\}$ - a vector of internal member forces,

$[N]$ - a matrix of coefficients of $\{A\}$.

Solving for vector $\{A\}$ in Equation A. 9

$$\{A\} = [M]^{-1}\{d\} \qquad (A.12)$$

and substituting into Equation A. 11

$$\{q\} = [k]\{d\} \qquad (A.13)$$

where

$$[k] = [N][M]^{-1} \qquad (A.14)$$

then clearly $[k]$ is the stiffness matrix of the individual member, since it relates the internal member forces $\{q\}$ to the internal member displacements $\{d\}$.

Inverting the $[M]$ matrix gives

$$[M]^{-1} = \frac{\begin{bmatrix} L^2\left(\frac{2+\nu^j}{3\nu^j}\right) & L^2\left(\frac{2+\nu^i}{3\nu^i}\right) & 2L\left(\frac{\nu^i+\nu^j+\nu^i\nu^j}{3\nu^i\nu^j}\right) & -2L\left(\frac{\nu^i+\nu^j+\nu^i\nu^j}{3\nu^i\nu^j}\right) \\ -2L^3\frac{1}{\nu^j} & -L^3 & -L^2\left(\frac{2+\nu^j}{\nu^j}\right) & L^2\left(\frac{2+\nu^j}{\nu^j}\right) \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \end{bmatrix}}{+L^4\left(\frac{4-\nu^i\nu^j}{3\nu^i\nu^j}\right)} \qquad (A.15)$$

The last two rows of Equation A. 15 are left blank since the last two columns of the $[N]$ matrix are zero. Multiplying $[N][M]^{-1}$ results in

$$[k] = \frac{2EI}{L} \begin{bmatrix} 2\gamma_{11} & \gamma_{12} & \frac{2\gamma_{11}+\gamma_{12}}{L} & -\frac{2\gamma_{11}+\gamma_{12}}{L} \\ \gamma_{12} & 2\gamma_{22} & \frac{2\gamma_{22}+\gamma_{12}}{L} & -\frac{2\gamma_{22}+\gamma_{12}}{L} \\ \frac{2\gamma_{11}+\gamma_{12}}{L} & \frac{2\gamma_{22}+\gamma_{12}}{L} & \frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} & -\frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} \\ -\frac{2\gamma_{11}+\gamma_{12}}{L} & -\frac{2\gamma_{22}+\gamma_{12}}{L} & -\frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} & \frac{2(\gamma_{11}+\gamma_{12}+\gamma_{22})}{L^2} \end{bmatrix} \qquad (A.16)$$

where

$$\gamma_{11} = \frac{3\nu^i}{4-\nu^i\nu^j}$$

$$\gamma_{12} = \frac{3\nu^i\nu^j}{4-\nu^i\nu^j} \qquad (A.17)$$

$$\gamma_{22} = \frac{3\nu^j}{4-\nu^i\nu^j}$$

APPENDIX B

Yield Rotation of Members with Semirigid Connections

To determine the yield rotation of members with semirigid connections Equation B. 1 was determined from Equation A. 16 of Appendix A.

$$\begin{bmatrix} M^i \\ M^j \end{bmatrix} = \frac{2EI}{L} \begin{bmatrix} 2\gamma_{11} & \gamma_{12} \\ \gamma_{12} & 2\gamma_{22} \end{bmatrix} \begin{bmatrix} \theta^i \\ \theta^j \end{bmatrix} \qquad \text{(B. 1)}$$

where

$$\theta^i = \omega^i - \frac{\eta^j - \eta^i}{L}$$

$$\theta^j = \omega^j - \frac{\eta^j - \eta^i}{L}$$

Other terms are defined in Appendix A.

To determine a unique value of yield rotation it is assumed that both ends of a member undergo the same rotation or that $\theta^i = \theta^j$. This assumption is made only in calculating the yield rotation and does not affect the force-displacement relationship of a member that occurs during earthquake excitation. The initial yield rotation θ_{Yo} thus defined is given in terms of initial yield moment M_{Yo} as

$$\theta^i_{Yo} = \frac{M^i_{Yo} L}{6EI} \left[\frac{4 - \nu^i \nu^j}{\nu^i (2+\nu^j)}\right]$$

and (B. 2)

$$\theta^j_{Yo} = \frac{M^j_{Yo} L}{6EI} \left[\frac{4 - \nu^j \nu^i}{\nu^j (2+\nu^i)}\right]$$

For members with rigid connections $\nu^i = \nu^j = 1.0$. Equation B. 2 reduces to

$$\theta_{Yo} = \frac{M_{Yo} L}{6EI} \qquad \text{(B. 3)}$$

which was the yield rotation used by Clough and Benuska (5).

APPENDIX C

Treatment of Symmetry

The number of calculations involved in analyzing a structure is reduced considerably when symmetry can be used. For lateral loading (earthquake loading) removing the static loads, cutting the structure at the center line, and pinning the cut ends is correct for a structure with rigidly connected members undergoing elastic deformation. Additional adjustments are required when nonlinear deformations or members with semirigid connections are considered. The additional adjustments must insure that the structure maintains an equivalent force-deformation relationship before and after being cut. Thus the required adjustments can be determined by comparing the relationship between the member-end rotation θ and the member-end moment M of the complete member to the cut member. From Equation B. 1 (Appendix B), the member-end rotation can be written

$$\begin{bmatrix} \theta^i \\ \theta^j \end{bmatrix} = \frac{L}{6EI} \begin{bmatrix} \frac{2}{\nu^i} & -1 \\ -1 & \frac{2}{\nu^j} \end{bmatrix} \begin{bmatrix} M^i \\ M^j \end{bmatrix} \qquad (C.\ 1)$$

where:

θ^i, θ^j - member-end rotation at i and j ends of a member

M^i, M^j - bending moment at i and j ends of a member

ν^i, ν^j - fixity factor at i and j ends of a member

L - length of the member

E - modulus of elasticity of the member

I - moment of inertia of the member

For a complete member $\nu^i = \nu^j = \nu$; and since the structure is subjected to lateral loads only $M^i = M^j = M$ and $\theta^i = \theta^j = \theta$. Thus the member-end rotation θ is given by

$$\theta = \frac{ML}{6EI}\left(\frac{2-\nu}{\nu}\right) \tag{C. 2}$$

For the cut member $\nu^i = \nu'$, $M^i = M'$, $M^j = 0$, $\theta^i = \theta'$ and length of $L/2$. Thus the member-end rotation θ' is found to be

$$\theta' = \frac{M'L/2}{6EI}\left(\frac{2}{\nu'}\right) \tag{C. 3}$$

Equating the two cases, with primes indicating properties of the cut member, results in

$$\theta' = \theta$$

$$\frac{M'L/2}{6EI}\left(\frac{2}{\nu'}\right) = \frac{ML}{6EI}\left(\frac{2-\nu}{\nu}\right) \tag{C. 4}$$

but for the cut member to yield exact results M' must equal M.

Hence, solving Equation C. 4 for ν' results in

$$\nu' = \frac{\nu}{2-\nu} \tag{C. 5}$$

Therefore by using the modified fixity factor ν' as given in Equation C. 5 for the cut member, symmetry can be utilized and the structure analyzed exactly. It is evident from Equation C. 5 that for members with rigid connections $(\nu = 1.0)$ these adjustments are not required as the adjusted connection is also rigid $(\nu' = 1.0)$.

APPENDIX D

Earthquake Accelerogram

```
C     EARTHQUAKE RECORD OF THE EL CENTRO MAY 18, 1940 NORTH SOUTH
C     COMPONENT. 37 CARDS, 8.0705 SECONDS, INDEX NUMBER 102.
C
C     THE NUMBERS GIVEN IN PAIRS ARE THE TIME AND ACCELERATION
C     COORDINATES OF THE ASSUMED PIECEWISE LINEAR RECORD
C
  1  0.0000 0.010849  0.0415 0.001076  0.0969 0.015909  0.1606-0.000114 102
  2  0.2215 0.018989  0.2630 0.000144  0.2907 0.005942  0.3322-0.001239 102
  3  0.3738 0.020093  0.4291-0.023781  0.4707 0.007659  0.5814 0.042514 102
  4  0.6229 0.009414  0.6645 0.013899  0.7198-0.008849  0.7198-0.025697 102
  5  0.7891-0.038741  0.7891-0.056885  0.8721-0.023290  0.8721-0.034306 102
  6  0.9413-0.040222  0.9413-0.060310  0.9967-0.078909  1.0659-0.066680 102
  7  1.0659-0.038168  1.0936-0.042997  1.1683 0.089755  1.3151-0.169618 102
  8  1.3843-0.082867  1.4120-0.082899  1.4397-0.094595  1.4812-0.088552 102
  9  1.5089-0.108024  1.5366-0.128015  1.6279 0.114491  1.7027 0.235583 102
 10  1.7996 0.142810  1.8550 0.177739  1.9242-0.261035  2.0072-0.319447 102
 11  2.2149 0.295277  2.2703 0.263464  2.3201-0.298405  2.3948 0.005427 102
 12  2.4502 0.286599  2.5194-0.046936  2.5748 0.151683  2.6523 0.207718 102
 13  2.7077 0.108776  2.7686-0.032552  2.8932 0.103399  2.9762-0.080328 102
 14  3.0676 0.052030  3.1285-0.154743  3.2116 0.006525  3.2531-0.206060 102
 15  3.3860 0.192717  3.4192-0.093731  3.5300 0.170805  3.5992-0.035974 102
 16  3.6684 0.036536  3.7376-0.073690  3.8345 0.031194  3.9037-0.183359 102
 17  4.0145 0.022732  4.0560-0.043532  4.1058 0.021611  4.2221-0.197259 102
 18  4.3135-0.176217  4.4159 0.146007  4.4713-0.004767  4.6180 0.257285 102
 19  4.6651-0.204520  4.7565 0.060822  4.8312-0.273350  4.9696 0.177930 102
 20  5.0389 0.030128  5.1081 0.218380  5.1994 0.026756  5.2327 0.125226 102
 21  5.3019 0.129057  5.3296 0.108947  5.3434-0.023904  5.4541 0.172352 102
 22  5.5095-0.102185  5.6064 0.014120  5.6895-0.194989  5.7725-0.024240 102
 23  5.8002-0.005080  5.8085-0.027507  5.8694-0.057361  5.8833-0.032748 102
 24  5.9248 0.021653  5.9802 0.010855  6.0134 0.023531  6.0854-0.066594 102
 25  6.1324 0.001412  6.1740 0.049333  6.1878 0.014980  6.1878-0.020012 102
 26  6.2294-0.038186  6.2792 0.020746  6.3263-0.005856  6.3678-0.060317 102
 27  6.3816-0.016263  6.4093 0.020006  6.4591-0.017613  6.4785-0.003371 102
 28  6.5201 0.004376  6.5339-0.004058  6.5616-0.009909  6.5754-0.001753 102
 29  6.6031-0.017065  6.6446 0.037338  6.6862 0.045734  6.7139 0.038587 102
 30  6.7277 0.000994  6.7692-0.028841  6.7692 0.001615  6.8108 0.011307 102
 31  6.8523 0.002207  6.9077 0.009298  6.9907-0.099620  7.0738 0.036017 102
 32  7.1208 0.007864  7.1430-0.027790  7.1485 0.002662  7.1707 0.027273 102
 33  7.2260 0.057693  7.2953-0.049270  7.3700 0.029739  7.4060 0.010926 102
 34  7.4254 0.018690  7.4614-0.025396  7.5251-0.034766  7.5721 0.003698 102
 35  7.5998-0.062804  7.6413-0.028096  7.6690-0.019689  7.6912 0.006866 102
 36  7.7521-0.005480  7.7936-0.060326  7.8351-0.035726  7.8767-0.071648 102
 37  7.9597-0.014024  7.9874-0.005615  8.0013 0.022241  8.0705 0.046827 102
```

APPENDIX E

Computer Program

```
      PROGRAM MAINDEFL
C     THIS PROGRAM CALCULATES THE RESPONSE OF NONLINEAR MULTI-STORY
C     STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION
      DIMENSION Q(50,11),S(550),NCODE(80,4),EPW(80,2),CCM(2),ERR(80),
     1YLIM(80,2),CM(80,2),RG(80,2),EG(80,2),DFACT(80,2),RK(10,10),FIX(4)
     2,C(10,10),XX(10,10),IFLEX(20),A(20),B(20),ACC(20),VEL(20),PRE(20),
     3DEFL(20),CDEF(20),XM(20),CSF(20),SF(20),XMLF(20),XMLD(20),ET(5),
     4CCMDF(60),XL(80),XI(80),W(80),R(80,3),EIOL(80),SM(4,4),EM(5),P(80)
     5,TR(80,2)
      COMMON Q,S,NCODE,XI,XL,W,RG,SM,FIX,R,E,CM,EIOL,ERR,DFACT,XMLO,XMLF
     2,TR
   20 FORMAT (5I3,E12.4)
C     NSF - NUMBER, DEGREES OF FREEDOM FOR COMPLETE STRUCTURE
C     NMF - NUMBER, DEGREES OF FREEDOM FOR INDIVIDUAL MEMBER
C     ME  - NUMBER OF MEMBERS
C     NB  - ONE-HALF THE BAND WUDTH OF STIFFNESS MATRIX
C     NM  - NUMBER OF LUMPED MASSES
C     E   - MODULUS OF ELASTICITY OF LINEAR STRUCTURAL MEMBERS
      READ 20, NSF,NMF,ME,NB,NM,E
      WRITE (61,25) (NSF,NMF,ME,NB,NM,E)
   25 FORMAT (1H1,30HDEGREE OF FREEDOM FOR SYSTEM =,I4/28H DEG OF FREEDO
     1M PER MEMBER =,I3/26H TOTAL NUMBER OF MEMBERS =I4/22H ONE-HALF BAN
     2D WIDTH =,I3/19H NUMBER OF STORYS =              ,I3/24H MODULUS OF E
     3ELASTICITY =,E12.4,13H KIPS/INCH**2)
      READ 21, TT, TMAX, DPM,DPS,P(1),PRD
   21 FORMAT (6F10.0)
C     TT   - TIME INCREMENT (SEC)
C     TMAX- DURATION OF EARTHQUAKE USED (SEC)
C     DPM - PERCENT OF CRITICAL DAMPING IN THE FUNOAMENTAL MODE
C     PROPORATIONAL TO THE MASS MATRIX
C     DPS - PERCENT OF CRITICAL DAMPING IN THE FUNDAMENTAL MODE
C     PROPORATIONAL TO THE STIFFNESS MATRIX
C     P(1)  FIXITY FACTOR FOR MEMBER ONE DURING NONLINEAR DEFORMATION
C     PRD -FUNDAMENTAL PERIOD OF STRUCTURE (SEC)
      WRITE (61,22) TT,TMAX,DPM,DPS,P(1),PRD
   22 FORMAT (1H , 21HTIME INCREMENT (SECS),F8.4/20H MAXIMUM TIME (SECS)
     1,F8.4/ 20H OAMPING COEFFICIENT /10X,18H MASS PROPORTIONAL,F8.4/
     2 10X, 23H STIFFNESS PROPORTIONAL,F8.4/ 32H RIGIDITY IN PLASTIC DEF
     3ORMATION ,F8.4/ 25H FUNDAMENTAL PERIOD (SEC) , F8.4)
      NA = NSF*NB
      NM1 = NM + 1
      NB1 = NB - 1
      GG = 1.0
      TIMER = 1.0
      SCALE = 1.5
C     IFLEX(I) DEGREE OF FREEDOM ASSOCIATED WITH THE ITH FLOOR
C     XM(I)    MASS OF THE ITH FLOOR
      READ 32, (IFLEX(I), XM(I),I = 1,NM)
   32 FORMAT (5(I4,F7.0))
      WRITE (61,33) (I,IFLEX(I),XM(I), I = 1,NM)
   33 FORMAT (1H0, 41HDEG OF FREEDOM ASSOCIATED WITH STORY MASS /
     16H STORY,4X,8HDEG FREE,4X,13HWEIGHT (KIPS)/(I4,6XI4,8XE12.4))
      DO 40 I = 1,NMF
      DO 38 J = 1,NMF
   38 SM(I,J) = 0.0
   40 FIX(I) = 0.0
      WRITE (61,15)
   15 FORMAT (1H0,6HMEMBER,5X,4HCODE,9X,6HLENGTH,5X,7HINERTIA,6X,17HRIGI
     1IDITY FACTORS,4X,12HUNIFORM LOAD,2X,11HYIELD LIMIT /26X4HFEET,7X,
     25HIN**4,7X5HL-END,7X5HR-END,7X6HKIP/FT,7X7HFT*KIPS)
   42 FORMAT (3X,4I3,6F9.0)
   43 FORMAT (1H ,I4,I5,3I4,7E12.4)
      DO 41 I = 1,ME
      P(I) = P(1)
C     NCODE(I,J) TEZCAN CODING NUMBERS
C     XL(I)      LENGTH OF THE I TH MEMBER (FT)
C     XI(I)      MOMENT OF INERTIA OF THE I TH MEMBER (IN**4)
C     EG(I,1)  FIXITY FACTOR IN ELASTIC DEFORMATION OF 1 CONNECTION FOR
C     THE I TH MEMBER
C     EG(I,2)  FIXITY FACTOR IN ELASTIC DEFORMATION OF 2 CONNECTION FOR
C     THE I TH MEMBER
C     W(I)       UNIFORM LOAD IN KIPS ON THE I TH MEMBER
C     YLIM(I,1) YIELD MOMENT OF THE CONNECTION FOR THE I TH MEMBER AT
C     THE 1 END
   41 READ 42, (NCODE(I,J),J=1,NMF),XL(I),XI(I),EG(I,1),EG(I,2),W(I),
     1 YLIM(I,1)
      RRR = 0.8
      DO 4444 I  = 1,10
      EG(I+20,1) = RRR
      EG(I+20,2) = RRR
      EG(I+30,1) = RRR/(2. - RRR)
      P(I+30) = P(I+30)/(2. - P(I+30))
 4444 CONTINUE
      DO 46 I = 1,ME
      XI(I) = XI(I)*46.5
   46 WRITE (61,43) I,(NCODE(I,J),J=1,NMF),XL(I),XI(I),EG(I,1),EG(I,2),
     1 W(I), YLIM(I,1),P(I)
      JET = 2
C     READ FIRST CARD OF THE ACCELERATION RECORD
C     ET  - TIME OF EARTHQUAKE
C     EM  - MAGNITUDE OF EARTHQUAKE ACCELERATION
      READ 12, (ET(I), EM(I), I = 2,5)
   12 FORMAT (3X, 4(F8.4, F9.5))
      TIME = 0.
      GL = EM(2)
      KCJ = -1
      DO 170 I = 1,ME
      EIOL(I) = 2.0*XI(I)*E/(XL(I)*144.)
      ERR(I) = 0.0
      YLIM(I,2) = YLIM(I,1)
      DO 160 J = 1,2
      DFACT(I,J) = 0.0
      TR(I,J) = 0.0
      EPW(I,J) = 0.0
      RG(I,J) = EG(I,J)
  160 CM(I,J) = 0.0
  170 CONTINUE
      DO 180 I = 1,NM
```

```
      XM(I) = XM(I)/32.2
      ACC(I) = -GL*32.2*SCALE
      VEL(I) = 0.0
      DEFL(I) = 0.0
      SF(I) = 0.0
      XMLF(I) = 0.0
  180 XMLD(I) = 0.0
      DO 184 I = 1,NSF
  184 Q(I,NM1) = 0.0
  185 DO 30 J = I,NA
   30 S(J) = 0.0
      DO 36 J = 1,NM
      DC 34 I = 1,NSF
   34 Q(I,J) = 0.0
      K = IFLEX(J)
   36 Q(K,J) = 1.0
      CALL STIFF (ME,GG,NB,NM1)
      CALL BAND (NSF,NB,NM1)
      DO 210 I = 1,NM
      K = IFLEX(I)
      DO 210 J = 1,NM
  210 RK(I,J) = Q(K,J)
      CALL MATINV (RK,NM)
      IF (TIME) 220,220,380
STEP 5A CALCULATE STATIC OEFLECTIONS
  220 DO 250 I = 1,NSF
  250 CCMDF(I) = Q(I,NM1)
      GG = 0.0
  380 DO 390 N = 1,NM
      DO 385 J = 1,NM
  385 XX(N,J)=RK(N,J)*(1.+3.*PRD*DPS/(3.142*TT))
  390 XX(N,N)=XX(N,N)+XM(N)*(6./TT**2+12.568*DPM*3.0/(PRD*TT))
      CALL MATINV (XX,NM)
      KCJ =  KCJ + 1
      IF(TIME) 552,552,400
  400 IF(JET-5) 402,401,401
  401 JET = 1
      ET(1) = ET(5)
      EM(1) = EM(5)
C     CALCULATE CHANGE IN DEFLECTION OF FLOORS
      READ 12, (ET(I), EM(I), I = 2,5)
  402 IF (TIME - ET(JET+1)) 404,404,403
  403 JET = JET + 1
      GO TO 400
  404 GACC = ((EM(JET+1)-EM(JET))/(ET(JET+1)-ET(JET)))*(TIME-ET(JET))+
     1EM(JET)- GL
      GL = GACC + GL
       GACC = GACC*32.2*SCALE
      DO 408 N = 1,NM
      A(N) = -6.0*VEL(N)/TT - 3.0*ACC(N)
  408 B(N) = -3.0*VEL(N) - TT*ACC(N)/2.0
      DO 420 N = 1,NM
      PRE(N) = -XM(N)*(GAEC + A(N)+12.568*DPM*B(N)/PRD)
      DO 420 J = 1,NM
  420 PRE(N) = PRE(N) - PRD*DPS*RK(N,J)*B(J)/3.142
      DO 460 I = 1,NM
      CDEF(I) = 0.0
      DO 450 J = 1,NM
  450 CDEF(I) = CDEF(I) + XX(I,J)*PRE(J)
      VEL(I) = VEL(I) + 3.0*CDEF(I)/TT + B(I)
      ACC(I) = ACC(I) + 6.0*CDEF(I)/TT**2 + A(I)
  460 DEFL(I) = DEFL(I) + CDEF(I)
C     CALCULATE CHANGE IN EQUIVLENT STATIC FORCES AND ADD TO TOTAL
      DO 520 I = I,NM
      CSF(I) = 0.0
      DO 510 J = 1,NM
  510 CSF(I)=RK(I,J)*CDEF(J) + CSF(I)
  520 SF(I) = SF(I) + CSF(I)
C     CALCULATE CHANGE IN DEFLECTION AT EACH DEGREE OF FREEDOM ADD TO TOTAL
      DO 540 I = 1,NSF
      CCMDF(I) = 0.0
      DO 540 J = 1,NM
  540 CCMDF(I) = CCMDF(I) + Q(I,J)*CSF(J)
  552 CJ = 0.
C     COMPUTE CHANGE IN INTERNAL MEMBER FORCES
      DO 730 I = 1,ME
      CCM(1) = 0.0
      CCM(2) = 0.0
      DO 590 J = 1,2
      L = 3 - J
      ZZ = L - J
      IF (NCODE(I,J)) 570,570,560
  560 K = NCODE(I,J)
      CCM(J) = CCM(J) + 2.0*R(I,J)*CCMDF(K)*EIOL(I)
      CCM(L) = CCM(L) + R(I,3)*CCMDF(K)*EIOL(I)
  570 IF (NCODE(I,J+2)) 590,590,580
  580 K = NCODE(I,J+2)
      CCM(J) = CCM(J) +(CCMDF(K)*(2.0*R(I,J) + R(I,3))*EIOL(I)/XL(I))*ZZ
      CCM(L) = CCM(L) +(CCMDF(K)*(R(I,3) + 2.0*R(I,L))*EIOL(I)/XL(I))*ZZ
  590 CONTINUE
      CM(I,1) = CM(I,1) + CCM(1)
      CM(I,2) = CM(I,2) + CCM(2)
C     HAS CONNECTION CHANGED STATE IN THIS INTERVAL
      DO 720 J = 1,2
      K = 3-J
C     WAS CONNECTION ELASTIC OR PLASTIC IN LAST INTERVAL
      IF (EG(I,J).EQ.RG(I,J)) 610,650
C     DID ELASTIC CONNECTION CHANGE TO PLASTIC IN THIS INTERVAL.
  610 DDD = ABS(CM(I,J)/(YLIM(I,J) ))
      TTT=1.+2.*(2.+EG(I,K))/(4.-EG(I,J)*EG(I,K))*(1.-EG(I,J))*ABS(CM(I,
     2J)/YLIM(I,J))
      IF(CM(I,J).GE.(YLIM(I,J)+EPW(I,J)).AND.CCM(J).GT.0.0) 645,630
  630 IF(CM(I,J).LE.(-YLIM(I,J)+EPW(I,J)).AND.CCM(J).LT.0.) 645,710
C     CHANGED TO PLASTIC
  645 RG(I,J) = P(I)
      CJ = CJ + 1.
      GO TO 710
C     HAS CONNECTION CHANGED FROM PLASTIC TO ELASTIC IN THIS INTERVAL.
```

```
  650 EPW(I,J) = EPW(I,J) + CCM(J)
      DDD=1.+2.*(2.+EG(I,K))/(4.-EG(I,J)*EG(I,K))*((EG(I,J)-RG(I,J))/ RG
     2(I,J))*ABS(EPW(I,J)/YLIM(I,J))
      TTT=1.+2.*(2.+EG(I,K))/(4.-EG(I,J)*EG(I,K))*(1.-EG(I,J)+EG(I,J)*
     2(1.-RG(I,J))*ABS(EPW(I,J)/YLIM(I,J))/RG(I,J))
      IF(ABS(CM(I,J)+CCM(J)).GT.ABS(CM(I,J))) 710,700
C     CHANGED TO ELASTIC
  700 RG(I,J) = EG(I,J)
      CJ = CJ + 1.
C     COMPUTE DUCTILITY FACTOR
  710 IF(TTT.GT.TR(I,J)) 712,714
  712 TR(I,J) = TTT
  714 IF (DDD.GT.OFACT(I,J)) 715,720
  715 DFACT(I,J) = DDD
  720 CONTINUE
  730 CONTINUE
      BIG = 0.0
      DO 830 I = 1,NM
      IF(XMLD(I) - ABS(DEFL(I)*12.)) 815,820,820
  815 XMLD(I)= ABS(DEFL(I))*12.0
  820 NVM = NM + 1 - I
      BIG = BIG + SF(NVM)
      IF(XMLF(NVM) - ABS(BIG)) 825,830,830
  825 XMLF(NVM)= ABS(BIG)
  830 CONTINUE
      IF (TIME.GE.(TIMER-.001)) 834,836
  834 CALL ERROR (TIME,KCJ,ME,NSF,NM)
      TIMER = TIMER + 1.0
  836 CONTINUE
      IF(TIME.GE.(TMAX - .001)) 900,850
  850 TIME = TIME + TT
      IF (CJ) 400,400,185
C     WRITE OUTPUT
  900 CALL ERROR (TIME,KCJ,ME,NSF,NM)
      STOP
      END
```

```
      SUBROUTINE STIFF(M,B,NB,NM1)
      DIMENSION Q(50,11),S(550),NCODE(80,4),XI(80),XL(80),W(80),RG(80,2)
     1 ,SM(4,4), FIX(4), R(80,3),CM(80,2),EIOL(80)
      COMMON Q,S,NCODE,XI,XL,W,RG, SM,FIX,R,E,CM,EIOL
      DO 11 I = 1,M
      R(I,1) = 3.0*RG(I,1)/(4.0 - RG(I,1)*RG(I,2))
      R(I,2) = 3.0*RG(I,2)/(4.0 - RG(I,1)*RG(I,2))
      R(I,3) = R(I,1)*RG(I,2)
      SM(1,1) = (2.0*R(I,1)*EIOL(I))
      SM(1,2) = (   R(I,3)*EIOL(I))
      SM(1,3) = (SM(1,1) + SM(1,2))/XL(I)
      SM(1,4) = -SM(1,3)
      SM(2,2) = (2.*R(I,2)*EIOL(I))
      SM(2,3) = (SM(1,2) + SM(2,2))/XL(I)
      SM(2,4) = -SM(2,3)
      SM(3,3) = (SM(1,3) + SM(2,3))/XL(I)
      SM(3,4) = -SM(3,3)
      SM(4,4) = SM(3,3)
      FIX(1) =-((2.*R(I,1)-R(I,3))*W(I)*XL(I)**2/12.)*B
      FIX(2) =  ((2.*R(I,2)-R(I,3))*W(I)*XL(I)**2/12.)*B
C     GENERATE SYSTEM STIFFNESS MATRIX IN SINGLE ARRAY
      DO 10 LG = 1,4
      IF (NCODE(I,LG)) 10,10,4
    4 K = NCODE(I,LG)
      Q(K,NM1)=Q(K,NM1) - FIX(LG)
      IF (LG-2)14,14,16
   14 CM(I,LG) = CM(I,LG) + FIX(LG)
   16 DO 8 J = 1,4
      IF (NCODE(I,J)) 8,8,5
    5 L = NCODE(I,J)
      M1 = K + (NB-1)*L
      IF (K-L) 7,7,6
    6 M1 = L + (NB-1)*K
    7 S(M1) = S(M1) + SM(LG,J)
    8 CONTINUE
   10 CONTINUE
   11 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATINV(A,N)
C     MATRIX INVERSION
      DIMENSION A(10,10)
      DO 5 K = 1,N
      COM = A(K,K)
      A(K,K) = 1.0
      DO 2 J = 1,N
    2 A(K,J) = A(K,J) / COM
      DO 5 I = 1,N
      IF (I-K) 3,5,3
    3 COM = A(I,K)
      A(I,K) = 0.0
      DO 4 J = 1,N
    4 A(I,J)= A(I,J) - COM * A(K,J)
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BAND (NSF, NB, NLC)
C     SELECTED INVERSION OF BAND MATRIX
      DIMENSION S( 550),Q( 50,11)
      COMMON Q,S
C     DETERMINE TRIANGULAR BAND MATRIX IN SINGLE ARRAY
      NB1 = NB - 1
      DO 80 J = 1,NSF
      MC = J*NB
      DD 80 I = 1,NB
      MR = J*NB + (I - 1)*NB1
      IF (NSF*NB - MR)80,70,70
   70 LL = NB - I
      AA = 0.0
      IF(LL) 78,78,72
   72 DO 74 K = 1,LL
      KC = MC - K
      KR = MR - K
   74 AA = AA + S(KC)*S(KR)
      IF (MC - MR) 78,76,78
   76 S(MC) = (S(MC) - AA)**0.5
      GD TD 79
   78 S(MR) = (S(MR) - AA)/S(MC)
   79 CONTINUE
   80 CONTINUE
C     DETERMINE INTERMEDIATE LOAD MATRIX
      DD 96 J = 1,NSF
       MC = J*NB
      DO 96 JJ = 1,NLC
      AA = 0.0
      DO 94 K = 1,NB1
      IF (J-K)94,94,92
   92 MK = MC - K
      JK = J - K
      AA = AA + S(MK)*Q(JK,JJ)
   94 CONTINUE
   96 Q(J,JJ) = (Q(J,JJ) - AA)/S(MC)
C     DETERMINE DISPLACEMENT MATRIX AND SELECTED FLEXABILITY MATRIX
      DO 116 J = 1,NSF
      NI = NSF - J + 1
      MC = NI*NB
      DO 116 JJ = 1,NLC
      AA = 0.0
      DO 114 K = 1,NB1
      MR = MC + K*NB1
      IF(NSF*NB-MR) 114,114,112
  112 NJ = NI + K
      AA = AA + S(MR)*Q(NJ,JJ)
  114 CONTINUE
  116 Q(NI,JJ) =(Q(NI,JJ) - AA)/S(MC)
      RETURN
      END
```

```
      SUBROUTINE ERROR(TIME,KCJ,ME,NSF,NM)
      DIMENSION S(550),Q(50,11),NCODE(80,4),XI(80),XL(80),W(80),RG(80,2)
     I,SM(4,4),FIX(4),R(80,3),CM(80,2),EIOL(80),ERR(80),DFACT(80,2),
     2 XMLD(20),XMLF(20),TR(80,2)
      COMMON Q,S,NCODE,XI,XL,W,RG,SM,FIX,R,E,CM,EIOL,ERR,DFACT,XMLD,XMLF
     2,TR
      WRITE (61,905)
  905 FORMAT (1H1,5X,6HMEMBER,5X,28HMAXIMUM NONLINEAR DUCTILITY ,6X23HMA
     2XIMUM TOTAL DUCTILITY /20X 5H1-END,7X5H2-END,17X5H1-END,7X5H2-END)
      DO 920 I = 1,ME
  920 WRITE (61,930) I, (DFACT(I,J), J = 1,2),(TR(I,L),L = 1,2)
  930 FORMAT (5X,I4,7X,2E12.4 ,10X,2E12.4)
      ERMAX = 0.0
      DO 925 I = 1,ME
      DO 925 J = 1,2
      IF (NCODE(I,J)) 925,925,922
  922 L = NCODE(I,J)
      ERR(L) = ERR(L) + CM(I,J)
  925 CONTINUE
      DO 945 J = 1,NSF
      IF (ABS(ERR(J))- ERMAX) 945,945,942
  942 ERMAX = ABS(ERR(J))
      JOINT = J
  945 ERR(J) = 0.0
      WRITE (61,999) KCJ, ERMAX, JOINT, TIME
  999 FORMAT (1H1, 34HNO OF TIMES BUILDING CHANGED STATE  ,I4//14H
     1ERROR IS ,E12.4, 9H AT JOINT, I4, 14H AT TIME EQUAL ,F8.4,5H SEC.)
  940 FORMAT (1H0,1X,5HSTORY,4X,26HMAXIMUM LATERAL DEFLECTION,4X,19HMAXI
     1MUM STORY SHEAR /1H ,6HNUMBER,15X8H(INCHES),16X,6H(KIPS))
      WRITE (61,940)
      WRITE (61,950) (I,XMLD(I),XMLF(I),I = 1,NM)
  950 FORMAT (I5,14X, E12.4, 11X, E12.4)
      RETURN
      END
```

```
      SUBROUTINE NATFRQ(FLEX,N)
C     INTERATION PROCEDURE USED IN CALCULATING FREQUENCES AND
C     CHARACTERISTIC SHAPES
C     N = DYNAMIC DEGREES OF FREEDOM (NO. OF MASSES)
C     NFQ = NUMBER OF FREGUENCYS REQUIRED
C     ILOOP = THE MAXIMUM NUMBER OF ITERATIONS TO CALCULATE ANY ONE FREQUENCY.
C    IF FREQUENCY IS NOT CALCULATED IN 'ILOOP' ITERATIONS, SOLUTION IS STOPPED.
C     DELTA = RELATIVE PERCENT ERROR
   25 FORMAT (2I5,E12.4)
   35 FORMAT(1H1,32HDIAGONAL ELEMENTS OF MASS MATRIX,15H-WEIGHT IN KIPS)
   37 FORMAT ( 1H0, 23HTHE FLEXIBILITY MATRIX ,10H (FT/KIPS))
   40 FORMAT ( 1H ,10E12.4)
   41 FORMAT ( 6E12.4)
  180 FORMAT (1H0, 14HEIGEN VALUE ( ,I2, 4H) = ,E12.4,5X 8HPERIOD = ,
     2E12.4,4H SEC/24H MAXIMUM PERCENT ERROR = ,E12.4/ 23H NUMBER OF ITE
     3RATIONS =  ,I4/ 12H MODE SHAPE ,10E12.4/(12X 10E12.4))
  400 FORMAT (1H1, 16HDIAGONAL ELEMENT, I3, 8H IS ZERO )
      DIMENSION XM(20,20),FLEX(20,20), DYN(20,20), EVECT(20), AVECT(20)
      DIMENSION X(20,20), DYN1(20,20), SWEEP(20,20), XMASS(20)
      READ 25, NFQ,ILOOP,DELTA
      DO 10 I = 1,N
      DO 10 J = 1,N
      XM(I,J) = 0.0
   10 X(I,J) = 0.0
      READ 41, (XM(I,I), I = 1,N)
      WRITE (61,35)
      WRITE (61,40)  (XM(I,I), I = 1,N)
      WRITE (61,37)
      DO 20 I = 1,N
      XM(I,I) = XM(I,I)/32.2
   20 WRITE (61,40)  (FLEX(I,J), J = 1,N)
      DO 22 I = 1,N
   22 XM(I,I) = SQRTF(XM(I,I))
      DO 27 I = 1,N
      DO 26 J = 1,N
   26 DYN(I,J) = XM(I,I) * FLEX(I,J)*XM(J,J)
   27 XMASS(I) = XM(1,1)/XM(I,I)
      DO 28 I = 1,N
      DO 28 J = 1,N
   28 DYN1(I,J) = DYN(I,J)
      DO 350 LX = 1,NFQ
      KHECK = 0
      DO  50 K = 1,N
   50 EVECT(K) = 1.0
   80 DO 100 J = 1,N
      AVECT(J) = 0.0
      DO  100 I = 1,N
  100 AVECT(J) = AVECT(J) + DYN1(J,I)*EVECT(I)
      KHECK = KHECK + 1
      IF (KHECK.GT.ILOOP) 1005,1015
 1005 PD = (2.0*3.142)/SQRTF(1.0/EXACT)
      WRITE (61,1010) KHECK,DIFF ,PD
 1010 FORMAT (1H0, 27HSOLUTION HAS NOT CONVERGED. /53H TWO EIGENVALUES M
     2AY HAVE VERY NEARLY THE SAME VALUE. /23H NUMBER OF ITERATIONS = ,
     3I4/ 34H MAXIMUM PERCENT ERROR IN PERIOD = ,E12.4/ 30H APPROXIMATE
     4VALUE OF PERIOD = ,E12.4,4H SEC)
      GO TO 999
 1015 EVMAX = AVECT(1)
      EVMIN = AVECT(1)
      DO 110 I = 1,N
      IF (EVMAX - AVECT(I)/EVECT(I)) 102,104,104
  102 EVMAX = AVECT(I)/EVECT(I)
  104 IF (AVECT(I)/EVECT(I) - EVMIN) 106,110,110
  106 EVMIN = AVECT(I)/EVECT(I)
  110 EVECT(I) = AVECT(I)/AVECT(1)
      EXACT = AVECT(1)
      BOUND = ABSF(EVMAX - EVMIN)
      DIFF = ABSF(BOUND/EXACT)*100.
      IF (DIFF - 0ELTA) 150,80,80
  150  PD = (2.0*3.14)/SQRTF(1.0/EXACT)
      DO 160 I = 1,N
  160 AVECT(I) = EVECT(I) * XMASS(I)
      WRITE (61,180) LX,EXACT,PD,DIFF ,KHECK,(AVECT(I), I = 1,N)
      IF (LX - NFQ) 185,999,999
  185 DO 190 J = 1,N
  190 X(LX,J) = EVECT(J)
      LZ = LX - 1
      IF(LZ) 290,290,200
  200 DO 240 J = 1,LZ
      PIVOT = X(LX,J)
      DO 240 I = 1,N
  240 X(LX,I) = X(LX,I) - X(J,I)*PIVOT
      PIVOT = X(LX,LX)
      IF (PIVOT) 250,380,250
  380 WRITE (61,400)  LX
      GO TO 999
  250 DO 260 J = 1,N
  260 X(LX,J) = X(LX,J) /PIVOT
      DO 280 J = 1,LZ
      PIVOT = X(J,LX)
      DO 280 I = 1,N
  280 X(J,I) = X(J,I) - X(LX,I)*PIVOT
  290 DO 300 I = 1,N
      DO 300 J = 1,N
  300 SWEEP(I,J) = - X(I,J)
      DO 320 I = 1,LX
  320 SWEEP(I,I) = 0.0
      LV =  LX + 1
      DO 330 I = LV,N
  330 SWEEP(I,I) = 1.0
      DO 350 K = 1,N
      DO 350 J = 1,N
      DYN1(K,J) = 0.0
      DO 350 I = 1,N
  350 DYN1(K,J) = DYN1(K,J) + DYN(K,I)*SWEEP(I,J)
  999 RETURN
      END
```