LOW-PASS TO BAND-PASS TRANSFORMATION
IN POLAR FORM

by
GERALD DEAN EWING

A THESIS
submitted to
OREGON STATE UNIVERSITY

in partial fulfillment of
the requirements for the
degree of
ELECTRICAL ENGINEER

June 1962
APPROVED:

Redacted for privacy

Professor of Electrical Engineering
In Charge of Major

Redacted for privacy

Head of Department of Electrical Engineering

Redacted for privacy

Chairman of School Graduate Committee

Redacted for privacy

Dean of Graduate School

Date thesis is presented __May__, 1962__

Typed by __Jolene Wuest__
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INTRODUCTION

In the design of active or passive band-pass filters it is often convenient to specify the pole locations of a low-pass filter, which has the desired response characteristics, and then transform these known pole locations to the band-pass case. This technique when employed in the design of passive band-pass filters has great utility since no analytical pole transformations need to be computed once the passive low-pass prototype filter has been designed. The desired passive band-pass filter is obtained simply by resonating each inductor in the low-pass filter with a series capacitor and each capacitor with a parallel inductor at the specified midband frequency. This procedure is presented by most authors of works on modern network synthesis. (1, p. 602 - 607)

The active band-pass filter or band-pass amplifier with single tuned-circuit interstages is not as simple to design as the passive case, because the actual pole locations in the band-pass complex frequency plane must be computed in order to solve for the $L$ and $R$ values of the parallel tuned-circuits forming the interstages. Many designers who have computed these pole locations from the known low-pass poles, which give rise to a desired frequency characteristic, have met with
considerable computational difficulty in obtaining the exact
solution of the complex frequency low-pass to band-pass
transform equation. (3, p. 225 - 228) If certain approxima-
tions can be made, which are usually justified if the ratio of
bandwidth to center-band frequency is small, the difficulty
can be avoided. The narrow-band case has been adequately
treated in the literature. (4, p. 382 - 389)

In the wideband case the narrow-band approximations
no longer give useful results and one must solve the transform
equation or resort to an experimental or physical analog
approach. (3, p. 181 - 198)

In this thesis exact expressions are derived in polar
form for the real and imaginary parts of a complex frequency
in the band-pass plane in terms of a given complex frequency
in the low-pass plane. These expressions, while quite
formidable-looking, can be evaluated with a minimum of compu-
tational effort and skill as will be illustrated in the wide band-
width R.F. amplifier design example given.

DERIVATION OF THE TRANSFORM EQUATION

To derive the transform equation it will be expedient to
start with the impedance of an inductor as a function of the
low-pass complex frequency variable p,
\[ Z_1 = L_p \]  \hspace{1cm} (1)

Now, if the response characteristics of a prototype low-pass filter are to be transformed to a band-pass filter, it is evident that the impedance of every branch must remain invariant with the transformation. For example, an inductor in a low-pass filter must be resonant with a capacitor at the mid-band frequency of the band-pass filter to form the corresponding branch impedance element. That is, the inductor-capacitor combination forms a series resonant circuit at the mid-band frequency of the band-pass filter replacing the inductor in the low-pass filter.

The impedance of a series inductor-capacitor combination as a function of the band-pass complex frequency variable \( s \) is given by

\[ Z_b = Ls + \frac{1}{Cs} , \]  \hspace{1cm} (2)

If \( \omega_0 \) is the mid-band angular frequency then

\[ C = \frac{1}{\omega_0^2}L . \]  \hspace{1cm} (3)

Substituting equation (3) into (2) we get

\[ Z_b = L( s + \frac{\omega_0^2}{s} ) . \]  \hspace{1cm} (4)
Since impedance is to remain invariant in the transformation we have

\[ Z_1 = Z_b. \]

From this relation and from equations (1) and (4) the desired transform equation is given by

\[ p = s + \omega_0^2/s. \]  \hspace{1cm} (5)

Equation (5) can also be derived by considering the admittance function of a capacitor in terms of the low-pass complex frequency variable \( p \). In this case a capacitor in the low-pass prototype filter is resonant at \( \omega_0 \) with an inductor connected in parallel with the capacitor to form the corresponding branch admittance element in the band-pass filter.

Equation (5) expresses \( p \) as a function of \( s \), which is a band-pass to low-pass transformation. However, a low-pass to band-pass transformation is required; therefore, equation (5) must be solved for \( s \) as a function of \( p \).

From equation (5) \[ s^2 - ps + \omega_0^2 = 0 \] or

\[ s = p/2 \pm \frac{1}{2}(p^2 - 4\omega_0^2)^{1/2}. \]  \hspace{1cm} (6)

For every point in the \( p \)-plane there are two points in the \( s \)-plane as indicated by equation (6). If \( p \) is equal to zero then \( s \) is equal to \( \pm j\omega_0 \). That is, the origin of the \( p \)-plane transforms over into two points in the \( s \)-plane located on the imaginary axis at \( \pm \omega_0 \).
Figure 1 shows a semicircle in the p-plane mapped over onto the s-plane by equation (6).

Figure 1
(Complex Frequency Planes)

The points A, B, and C on the semicircular contour in the p-plane correspond to the primed points in the s-plane. A semicircle in the p-plane does not, in general, map into two semicircles in the s-plane.

The distortion involved in the transformation is dependent upon the relative magnitude of $\omega_0$ as compared to the radius of the given semicircle in the p-plane. The larger $\omega_0$ is compared to the radius of the semicircle the smaller the distortion involved and the closer one comes to a narrow
band approximation. The narrow band approximation usually
gives sufficient accuracy if the bandwidth $b$ is less than approx-
imately 30% of the mid-band frequency $\omega_0$. (3, p. 226) The
distortion of the semicircle and pole locations in the transfor-
mation, in this case, is negligible.

Another important result of the transformation is that the
magnitude of the radius of the semicircle, which is equal to
the 3 db bandwidth of a maximally flat low-pass filter having
all its poles located on the semicircle, is equal to the distance
between the points $A'$, $C'$ or $A''$, $C''$. This means that the
bandwidth of a filter is invariant in a low-pass to band-pass
transformation.

CONSERVATION OF BANDWIDTH

The following is a proof that the real bandwidth in a
low-pass to band-pass transformation is conserved. (2,
p. 343 - 344) In figure 2 the magnitude of the transfer
functions of a low-pass and band-pass filter are shown with
the cutoff frequencies designated on the frequency axis.
Substituting these cutoff frequencies into equation (5) one obtains
\[ \omega_b = \omega_2 - \frac{\omega_0^2}{\omega_2} \]  \hspace{2cm} (7)
and
\[ -\omega_b = \omega_1 - \frac{\omega_0^2}{\omega_1} \]  \hspace{2cm} (8)

Now, multiplying equations (7) and (8) through by \( \omega_2 \) and \( \omega_1 \) respectively and then subtracting the second from the first gives
\[ \omega_b (\omega_2 + \omega_1) = \omega_2^2 - \omega_1^2 \]  \hspace{2cm} (9)

By factoring the right side of equation (9) the desired proof is completed by showing that
\[ \omega_b = \omega_2 - \omega_1 \]  \hspace{2cm} (10)

That is, the real bandwidth of the prototype low-pass filter is equal to the bandwidth of the correspondingly derived band-pass filter.
MID-BAND AND CENTER-BAND FREQUENCIES

To prove that the mid-band frequency \( \omega_c \), as given in the low-pass to band-pass transformation, is equal to the geometric mean of the upper and lower cutoff frequencies \( \omega_2 \) and \( \omega_1 \) respectively, we again start with equation (5)

\[ p = s + \frac{\omega_0^2}{s} \]

Substituting the real bandwidth \( \omega_b \) and the corresponding upper cutoff frequency \( \omega_2 \) into equation (5) yields

\[ \omega_b = \omega_2 - \frac{\omega_0^2}{\omega_2} \]

Now, solving for \( \omega_0^2 \) gives

\[ \omega_0^2 = \omega_2^2 - \omega_2 \omega_b \]

but, from equation (10), \( \omega_b = \omega_2 - \omega_1 \) we have

\[ \omega_0^2 = \omega_2^2 - \omega_2 (\omega_2 - \omega_1) \quad \text{(11)} \]

By simplifying equation (11)

\[ \omega_0^2 = \omega_1 \omega_2 \]

and taking the square root the desired proof is obtained in the form of equation (12).

\[ \omega_0 = (\omega_1 \omega_2)^{1/2} \quad \text{(12)} \]
If a band-pass filter or amplifier is to just accommodate a given symmetrical double sideband signal the carrier frequency should be made equal to \( \omega_c \), the arithmetic center-band frequency, and not \( \omega_o \) in order to maintain the spectral symmetry of the signal. With this fact in mind we now wish to derive an expression for \( \omega_o \) in terms of the desired center-band frequency \( \omega_c \) and the bandwidth \( \omega_b \), since \( \omega_o \) is a quantity needed to solve the transform equation.

By definition
\[
\omega_c = \frac{\omega_b}{2} + \omega_1 \tag{13}
\]

or
\[
\omega_1 = \omega_c - \frac{\omega_b}{2} \tag{14}
\]

Solving equation (10) for \( \omega_2 \) gives
\[
\omega_2 = \omega_b + \omega_1 \tag{15}
\]

and then substituting equation (14) into (15) and simplifying results in
\[
\omega_2 = \omega_c + \frac{\omega_b}{2} \tag{16}
\]

Now, substituting equation (14) and (16) into equation (12) one obtains
\[
\omega_o = \left[ \left( \omega_c - \frac{\omega_b}{2} \right) \left( \omega_c + \frac{\omega_b}{2} \right) \right]^{1/2}
\]
or
\[
\omega_o = \left[ \frac{\omega_c}{2} - \left( \frac{\omega_b}{2} \right)^2 \right]^{1/2} \tag{17}
\]
DERIVATION OF THE POLAR FORM

With reference to figure 1, consider a general point B in the \( p \)-plane expressed in polar coordinates by \( b \) and \( \Theta \), which is to be transformed into points \( B' \) and \( B'' \) in the \( s \)-plane given by the rectangular coordinates \((\alpha', \omega')\) and \((\alpha'', \omega'')\). To accomplish this transformation we start by substituting \( b \, e^{j\Theta} \) for \( p \) in equation (6),

\[
s = \frac{b}{2} \, e^{j\Theta} \pm \left[ \left( \frac{b}{2} \right)^2 \, e^{j2\Theta} - \omega_0^2 \right]^{1/2}
\]

Since we are only interested in poles in the left half plane for stability reasons \( \Theta \) will be restricted to the range \( \frac{\pi}{2} < |\Theta| < \pi \).

By the use of Euler's equation,

\[
s = \frac{b}{2} (\cos\Theta + j\sin\Theta) \pm (X + jY)^{1/2}
\]

where

\[X = \left( \frac{b}{2} \right)^2 \cos 2\Theta - \omega_0^2\] \hspace{1cm} (19)

and

\[Y = \left( \frac{b}{2} \right)^2 \sin 2\Theta\] \hspace{1cm} (20)

From equation (20) we see that \( Y < 0 \) for all positive values of \( \Theta \) in the given range.
For values of \( \Theta \) in the range \( \frac{\pi}{2} < \Theta \leq \frac{3\pi}{4} \) equation (19) yields \( x < 0 \).

For the range \( \frac{3\pi}{4} \leq \Theta \leq \pi \)

\[
x < 0 \text{ if } \omega_s^2 > \left(\frac{b}{2}\right)^2 \cos 2\Theta
\]
or

\[
x > 0 \text{ if } \omega_s^2 < \left(\frac{b}{2}\right)^2 \cos 2\Theta
\]

For the condition \( x < 0 \):

equation (18) can be expressed in the form

\[
s = \frac{b}{2} (\cos\Theta + jsin\Theta) \pm \left[\left(x^2 + y^2\right)^2 e^{j\phi} / 2\right]^{1/2}
\]

where

\[
\phi = \tan^{-1} \frac{y}{x}, \quad x > 0.
\]

Extracting the indicated square root in equation (21) results in

\[
s = \frac{b}{2} (\cos\Theta + jsin\Theta) \pm \left(x^2 + y^2\right)^{1/4} e^{j\phi / 2}
\]

By a second application of Euler's equation gives

\[
s = \frac{b}{2} (\cos\Theta + jsin\Theta) \pm \left(x^2 + y^2\right)^{1/4} e^{j\phi / 2 + j\phi / 2}, \quad (23)
\]

Grouping the real and imaginary parts after substituting

\[
sin \frac{\phi}{2} \text{ for } \cos \frac{\phi + \pi}{2} \text{ and } \cos \frac{\phi}{2} \text{ for } \sin \frac{\phi + \pi}{2}
\]
in equation (23) yields the desired results:

\[
s' = \sigma' + j\omega' = \frac{b}{2} \cos\Theta - (x^2 + y^2)^{1/4} \sin \frac{\phi}{2} + j\left[\frac{b}{2} \sin\Theta + (x^2 + y^2) \cos \frac{\phi}{2}\right] \quad (24)
\]

\[
s'' = \sigma'' + j\omega'' = \frac{b}{2} \cos\Theta + (x^2 + y^2)^{1/4} \sin \frac{\phi}{2} + j\left[\frac{b}{2} \sin\Theta - (x^2 + y^2) \cos \frac{\phi}{2}\right] \quad (25)
\]

Recalling that equations (24) and (25) are only valid for \( x < 0 \),

we now impose the condition \( x > 0 \) on equation (18) and proceed
as before.

For the condition \( X > 0 \):

\[
s = \frac{b}{2} \left( \cos \Theta + j \sin \Theta \right) \pm \left[ \left( X^2 + Y^2 \right)^{1/2} \right]^{1/2} e^{j \Phi} \quad (26)
\]

where

\[
\Phi = \tan^{-1} \left( \frac{Y}{X} \right) < 0.
\quad (27)
\]

Extracting the indicated square root in equation (26) results in

\[
s = \frac{b}{2} \left( \cos \Theta + j \sin \Theta \right) \pm \left( X^2 + Y^2 \right)^{1/4} e^{j \Phi/2}.
\]

By again applying Euler's equation and grouping the real and imaginary parts yields the desired results:

\[
s' = \sigma + j \omega = \frac{b}{2} \cos \Theta - (X^2 + Y^2)^{1/4} \cos \frac{\Phi}{2} + j \left( \frac{b}{2} \sin \Theta - (X^2 + Y^2)^{1/4} \sin \frac{\Phi}{2} \right) \quad (28)
\]

\[
s'' = \sigma + j \omega = \frac{b}{2} \cos \Theta + (X^2 + Y^2)^{1/4} \cos \frac{\Phi}{2} + j \left( \frac{b}{2} \sin \Theta + (X^2 + Y^2)^{1/4} \sin \frac{\Phi}{2} \right) \quad (29)
\]

Since the poles \( b \neq \Theta \) and \( b \neq \Theta \) in the p-plane always occur as complex conjugates, except for the one on the real axis if it exists, only one application of the pair of transform equations previously derived need be used per given pole pair due to symmetrical properties of the mapping shown in figure 3.
The symmetrical properties just illustrated can greatly reduce the amount of computation necessary to design a band-pass amplifier as will be shown in the example to follow.

PARALLEL TUNED-CIRCUIT ANALYSIS

Now we wish to relate the transformed pole locations in the s-plane to the element values of a parallel tuned-circuit as shown in figure 4.

![Parallel Tuned-Circuit Diagram](image)

**Figure 4**

This circuit is of interest since it represents the equivalent circuit of all interstages in the class of band-pass amplifiers under consideration.

Solving for the impedance as a function of $s$,

$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{C s + 1/Ls + 1/R}$$

or

$$Z(s) = \frac{(1/C)s}{s^2 + (1/RC)s + 1/LC}. \quad (30)$$
Now, solving for the current transfer ratio

\[
\frac{I_R(s)}{I(s)} = \frac{Z(s)}{R} = \frac{1}{RC} \frac{s}{s^2 + (1/RC)s + 1/LC}.
\]

(31)

By equating the denominator of equation (30) or (31) to zero the characteristic equation of the network is obtained.

\[
s^2 + (1/RC)s + 1/LC = 0
\]

or

\[
s = -1/2RC \pm \left[ \frac{(1/2RC)^2 - 1/LC}{1/LC} \right]^{1/2}.
\]

Given the general condition that \(s\) have both a real and imaginary part gives rise to the inequality

\[
1/LC > (1/2RC)^2; \text{ therefore,}
\]

\[
s = -1/2RC \pm j\left[ 1/LC - (1/2RC)^2 \right]^{1/2}.
\]

(32)

Now, by equating the real and imaginary parts of equation (32) to the real and imaginary parts respectively of a transformed pole in the \(s\)-plane given by \(v + j\omega\), we obtain the desired expressions for \(R\) and \(L\) when given \(C\).

\[
R = \frac{1}{2C |v|}.
\]

(33)

\[
L = \frac{1}{C(v^2 + \omega^2)}.
\]

(34)
DESIGN PROCEDURE

A design procedure will now be given for a general band-pass amplifier based upon the conditions and equations previously defined. Figure 5 shows the cascade arrangement of the unilateral active devices, which will most likely be pentode vacuum tubes or transistors, and the single tuned-circuit interstages.

![Diagram of a band-pass amplifier](image)

**Figure 5**

The resistor $R$, which appears as a lumped value of resistance connected in parallel with $L$ and $C$ in the equivalent circuit, is made up of several parts which are as follows: the real parts of the input and output impedances of the active devices and a fixed auxiliary resistor. The purpose of the auxiliary resistor is to set the total resistance $R$ to the designed value given by equation (33). Likewise, the input, output and stray capacities of the active and
passive elements constitute C. Since the largest gain-bandwidth product of the amplifier is desired, an auxiliary capacitor is not usually provided. Once C has been determined the value of L follows from equation (34).

In order to proceed with a design the following specifications must be given:

1. the center-band frequency $f_c = \omega_c/2\pi$
2. the band width $f_b = \omega_b/2\pi$
3. the type of response; e.g. maximally flat, equal ripple, etc., (3, p. 202-219)
4. the skirt selectivity which specifies the minimum number of poles in the complex frequency plane, (skirt selectivity in db per octave is equal to six times the number of pole pairs located in the s-plane.)
5. gain, noise figure, impedance levels, type of active elements etc. An excellent treatment of these subjects are given in reference (3).

An orderly design procedure is given as follows.

1. Find the complex frequency poles in polar coordinates ($B/\Theta$) that characterize the specified
real bandwidth $f_o$, the type of response and the skirt selectivity of the prototype low-pass filter.

2. From equation (17) calculate the mid-band frequency

$$f_o = \frac{\omega_o}{2\pi}.$$ 

$$f_o = \left[ f_c^2 - \left( \frac{f_b}{2} \right)^2 \right]^{1/2}$$

3. For a given complex frequency pole pair

$$(B \angle \Theta \text{ and } B \angle -\Theta)$$

where $90^\circ < \Theta < 180^\circ$ on the $P/2\pi$ - plane calculate $X$, $Y$, and $\Phi$ from equations (19), (20) and (22).

$$X = (B/2)^2 \cos 2\Theta - f_o^2$$

$$Y = (B/2)^2 \sin 2\Theta$$

$$\Phi = \tan^{-1} \frac{Y}{X}$$

4. If $\Phi$ is positive use equation (24) to calculate the real and imaginary parts of the complex frequency

$$s'/2\pi = \delta + j\phi$$

where

$$\delta (B, \Theta) = \frac{B}{2} \cos \Theta - \left( X^2 + Y^2 \right)^{1/4} \sin \frac{\Phi}{2}$$

$$\phi (B, \Theta) = \frac{B}{2} \sin \Theta + \left( X^2 + Y^2 \right)^{1/4} \cos \frac{\Phi}{2}$$

$$\delta (B, -\Theta) = \frac{B}{2} \cos \Theta + \left( X^2 + Y^2 \right)^{1/4} \sin \frac{\Phi}{2}$$

$$\phi (B, -\Theta) = -\frac{B}{2} \sin \Theta + \left( X^2 + Y^2 \right)^{1/4} \cos \frac{\Phi}{2}$$

5. If $\Phi$ is negative use equation (28) to calculate:

$$\delta (B, \Theta) = \frac{B}{2} \cos \Theta - \left( X^2 + Y^2 \right)^{1/4} \cos \frac{\Phi}{2}$$

$$\phi (B, \Theta) = \frac{B}{2} \sin \Theta - \left( X^2 + Y^2 \right)^{1/4} \sin \frac{\Phi}{2}$$
\[ \delta(B, -\Theta) = B/2 \cos\Theta + (x^2 + y^2)^{1/4} \cos \phi/2 \]
\[ f(B, -\Theta) = -B/2 \sin\Theta - (x^2 + y^2)^{1/4} \sin \phi/2 \]

6. From equations (33) and (34) respectively and the previously computed pole locations in the \( s/2\pi \) - plane calculate

\[ R = \frac{1}{4\pi |\delta|} \]

and

\[ L = \frac{1}{4\pi^2 f_r^2} \]

where the resonant frequency of the parallel tuned-circuit interstage

\[ f_r = (\delta^2 + \phi^2)^{1/2} \]

A WIDE BANDWIDTH R.F.
AMPLIFIER DESIGN EXAMPLE.

Suppose a nontunable wide bandwidth R.F. amplifier is needed to drive a long coaxial cable which connects a log-periodic or spear point 5 to 55 mc. antenna to a remotely located communication receiving center. The amplifier is to be mounted on the antenna itself in order to give the best possible system signal to noise ratio and provide sufficient gain to make up for coaxial cable transmission line losses. The amplifier specifications are given as follows:
1. The center-band frequency $f_c = 30$ mc.
2. The bandwidth $f_b = 50$ mc.
3. A maximally flat band-pass frequency response will be required.
4. A 24 db. per octave skirt selectivity is required.
5. The amplifier must have an input and output impedance of 93 ohms and a power gain greater than or equal to 40 db.

The previously given design procedure will now be applied to the above specifications.

1. The given skirt selectivity requires 4 poles ($4 \times 6 = 24$ db.) in the $p$-plane which will result in 4 pole pairs in the $s$-plane. From the given bandwidth, gain and number of poles need we find from reference (3, p. 235 - 237) that four type 5847 pentode vacuum tubes connected as shown in figure 6 should suffice. The 5847 has a $g_m$ equal to 12.5 millimhos, an input capacitance of 7 pf. and an output capacitance of 3 pf.

For the 4-pole maximally flat response the poles are located on the $p/2\pi$-plane at $(B, \theta_1)$, $(B, -\theta_1)$, $(B, \theta_2)$ and $(B, -\theta_2)$ where $B = f_b$, $\theta_1 = 157.5^\circ$ and $\theta_2 = 112.5^\circ$. 

WIDE BANDWIDTH R.F. AMPLIFIER

NOTE: ALL UNLABELED CAPS. ARE BYPASS CAPS.

Figure 6
2. The mid-band frequency \( f_0 = \left[ (30)^2 - (50/2)^2 \right]^{1/2} \)
\( (275)^{1/2} = 16.6 \) mc.

3. The pole locations in the s/\( 2\pi \)-plane are now calculated. For \( \Theta_1 \):
\( X_1 = \frac{50}{2} \cos 315^\circ - 275 = 625 \cos 45^\circ - 275 = 442 - 275 = 167 \)
\( Y_1 = 625 \sin 315^\circ = -625 \sin 45^\circ = -442 \)
\( \Phi_1 = \tan^{-1} \frac{-442}{167} = -69.3^\circ \)
\( \delta(B, \Theta_1) = 25 \cos 157.5^\circ - \left[ (167)^2 + (442)^2 \right]^{1/4} \cos 34.65^\circ \)
\( = -25 \cos 22.5^\circ - 21.7 \cos 34.65^\circ = -23.1 \)
\( = -17.85 = -40.95 \) mc.
\( f(B, \Theta_1) = 25 \sin 157.5^\circ - 21.7 \sin 34.65^\circ \)
\( = 25 \sin 22.5^\circ + 21.7 \sin 34.65^\circ = 9.58 + 12.33 = 21.91 \) mc.
\( \delta(B, -\Theta_1) = -23.1 + 17.85 = 5.25 \) mc.
\( f(B, -\Theta_1) = -9.58 + 12.33 = 2.78 \) mc.

For \( \Theta_2 \):
\( X_2 = 625 \cos 225^\circ - 275 = -625 \cos 45^\circ - 275 = -442 - 275 = -717 \)
\( Y_2 = 625 \sin 225^\circ - 625 \sin 45^\circ = -442 \)
\( \Phi_2 = \tan^{-1} \frac{-442}{-717} = 31.65^\circ \)
\( \delta(B, \Theta_2) = 25 \cos 112.5^\circ - \left[ (717)^2 + (442)^2 \right]^{1/4} \sin 15.82^\circ \)
\( = -25 \cos 67.5^\circ - 29.0 \sin 15.82^\circ = -9.57 - 7.91 = \)
\( -17.48 \) mc.
\[ f(B, \Theta_2) = 25 \sin 67.5^\circ + 29.0 \cos 15.82^\circ = 23.1 + 27.9 = 51.0 \text{ mc}. \]

\[ \delta(B, -\Theta_2) = -9.57 + 7.9i = -1.66 \text{ mc}. \]

\[ f(B, -\Theta_2) = -23.1 + 27.9 = 4.8 \text{ mc}. \]

4. In order to calculate the resistance and inductance values for each stage the total interstage capacitance must be known. The capacitance for the first three stages is equal to the sum of the input, output and stray capacitance of the 5847 tube and circuit.

\[ C = 7 + 3 + 5 = 15 \text{ pf}. \]

For the output stage the capacity is adjusted by the addition of \( C_4 \) to give the proper impedance match to the 93 ohm coaxial cable.

The output stage capacity

\[ C_0 = C_4 + 8 \text{ pf}. \]

to make \( R_4 = 93 + 93 = 186 \text{ ohms}. \) The plate resistance of the 5847 is so much larger than 186 ohms it may be neglected.

Now that the interstage capacitance has been determined we will calculate the resonant frequency, inductance and resistance for each stage.
By assigning the pole with the largest value of \( f \) to the output stage we can achieve an impedance match to the coaxial cable with the smallest possible value of padding capacitor \( C_4 \).

This will result in the greatest overall gain-bandwidth product.

For the first stage:

\[
\begin{align*}
\frac{f_{r1}}{2} &= \left[ (-17.48)^2 + (51.0)^2 \right]^{1/2} = (2906)^{1/2} = 53.9 \text{ mc.} \\
L_1 &= \frac{1}{4\pi^2 (2906) 15} = 0.58 \mu \text{h,} \\
R_1 &= \frac{1}{4\pi \left| -1748 \right| (15) 10^{-6}} = 304 \text{ ohms}
\end{align*}
\]

For the second stage:

\[
\begin{align*}
\frac{f_{r2}}{2} &= \left[ (-1.66)^2 + (4.8)^2 \right]^{1/2} = (25.76)^{1/2} = 5.08 \text{ mc.} \\
L_2 &= \frac{1}{4\pi^2 (25.76) 15} = 65.6 \mu \text{h.} \\
R_2 &= \frac{1}{4\pi \left| -1.66 \right| (15) 10^{-6}} = 3200 \text{ ohms}
\end{align*}
\]

For the third stage:

\[
\begin{align*}
\frac{f_{r3}}{2} &= \left[ (-5.25)^2 + (2.48)^2 \right]^{1/2} = (33.70)^{1/2} = 5.80 \text{ mc.} \\
L_3 &= \frac{1}{4\pi^2 (33.7) 15} = 50 \mu \text{h.} \\
R_3 &= \frac{1}{4\pi \left| -5.25 \right| (15) 10^{-6}} = 101 \text{ ohms}
\end{align*}
\]

For the fourth and last stage:

\[
\begin{align*}
\frac{f_{r4}}{2} &= \left[ (-40.95)^2 + (21.91)^2 \right]^{1/2} = (2158)^{1/2} = 46.8 \text{ mc.}
\end{align*}
\]
\[ R_4 = \frac{1}{4\pi \left| -40.95 \right| \times 10^6 \cdot C_0} = 186 \text{ ohms} \]

\[ C_0 = \frac{1}{4\pi (40.95)(186) \times 10^6} = 10.5 \text{ pf.} \]

\[ C_4 = C_0 - 8 = 10.5 - 8 = 2.5 \text{ pf.} \]

\[ L_4 = \frac{1}{4\pi^2 (2158) \times 10.5} = 1.12 \, \mu \text{h.} \]

The poles and zeros in the \( s/2\pi \)-plane and the corresponding frequency characteristics of the wide band-width R.F. amplifier are shown in figure 7 on the following page.
Figure 7
BIBLIOGRAPHY


