A STUDY TO MEASURE ANNUAL MORTALITY CAUSED BY THE MOUNTAIN PINE BEETLE IN LODGEPOLE PINE ON THE WYOMING, RANGERHEAD AND GALLATIN NATIONAL FORESTS.

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A Pilot Survey to Measure Annual Mortality Caused by the Mountain Pine Beetle in Lodgepole Pine on the Beaverhead and Gallatin National Forests, Montana 1978

by

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## Contents

Abstract ................................................................. ii
Introduction ............................................................ 1
Site Description .......................................................... 1
Methods ................................................................. 2
   Survey Design ......................................................... 2
   Aerial Sketchmapping ............................................... 2
   Stratification ........................................................ 2
   Aerial Photography .................................................. 2
   Ground Truth Acquisition .......................................... 2
Results ................................................................. 3
Discussion and Recommendations .................................. 5
Acknowledgement ...................................................... 6
References ............................................................. 6
Appendixes

A—Timetable and estimated costs of the survey,
   Beaverhead and Gallatin National Forests, 1977 ................. 7

B—Formulae and procedures used for determining
   multistage sampling estimates ................................... 9

C—Formulae used for calculating linear regressions
   for each stratum .................................................. 13
Abstract

A multistage sampling survey to measure annual and cumulative mortality of lodgepole pine by mountain pine beetle was conducted on the Beaverhead and Gallatin National Forests, 1978-79. The survey area, 270,000 acres, was stratified into three intensity classes from aerial sketchmapping. This was followed by large scale (1:6,000) aerial photography, photographic interpretation, and ground truth measurements.

Estimates of the number of lodgepole killed by mountain pine beetle in 1977 were 1,270,617 trees and 24.2 million cubic feet. The number of total standing dead within the survey boundaries was estimated at 3,117,888 trees and 62.2 million cubic feet. This total standing dead represents 31.4 percent of the total basal area.
Introduction

Mountain pine beetle\(^1\) is the most destructive insect to pine type in the western United States and Canada. In 1977, epidemic mountain pine beetle infestations occurred over more than 600,000 acres throughout the western half of Montana.

Recently, multistage sampling surveys have been developed to provide accurate and timely mountain pine beetle loss information to land managers and program administrators (Hostetler and Young, 1979; Klein, et al., 1979). The objective of this pilot survey was to implement these multistage sampling methods for measuring annual and cumulative mortality of lodgepole and ponderosa pine by mountain pine beetle over three widely separated and distinct infestations. Contracting problems and early snow prevented surveying two of the infestations (Bennett, 1979). This report is limited to the survey of the Beaverhead/Gallatin infestation.

Site Description

The survey area covered approximately 270,000 acres of mountain pine beetle infested lodgepole pine on portions of the Beaverhead and Gallatin National Forests, including State and private ownerships, in southwestern Montana (figure 1). Lodgepole pine is the predominant tree species with lesser amounts of Douglas-fir, Engelmann spruce, subalpine fir, whitebark and limber pines, and aspen. The survey covered a wide diversity of terrain from a flat plateau (elevation 6,600 ft.) south and west of West Yellowstone, Montana, to the steep slopes of Gallatin Canyon (elevation 5,800-9,000 ft.).

\(^1\)Dendroctonus ponderosae Hopk.
Methods

Survey Design

The survey design was essentially the same, with some modifications, as those used in Regions 2 and 4 (Hostetler and Young, 1979; Klein, et al., 1979). The design entailed a two-stage sampling system stratified into three intensity classes, or strata. Initial stratification was from 1978 aerial sketchmap surveys. This was followed by large scale (1:6,000) aerial photography, photographic interpretation, and ground truth measurements. Photo plots were selected by a systematic random process for each strata. Ground truth plots were selected by probability proportional to size (pps). This procedure has been shown to be effective in reducing sampling error when using multistage sampling (Langley, 1978).

These ground samples permitted adjustment of the large photo sample and provided necessary data to determine ratio estimators for number of trees and volume killed. Additional data was collected from the ground plots to estimate the cumulative number of trees and volume killed, including 1978 attacks.

Aerial Sketchmapping

Those portions of the Beaverhead and Gallatin National Forests likely to have mountain pine beetle faders (1977 attacks) were sketchmapped July 31 to August 3, 1978. Approximately 24 hours of flying time were required to survey the 1.7 million acres at a cost of about $2,000. The number of faders were estimated and delineated into recognizable areas (polygons) and recorded on 1:126,720 scale planimetric maps. The sketchmapping was aimed at mountain pine beetle detection, although other insect and disease damage was concurrently mapped on the Beaverhead portion.

Stratification

The stratification procedures were the same as those used by both Klein, et al. (1979), and Hostetler and Young (1979). Each of the 322 polygons were placed into one of three strata according to their density of faders per acre (figure 1 and table 1). Strata used were: (1) \( \leq 4.9 \) trees/acre; (2) 5.0-9.9 trees/acre; and (3) \( \geq 10.0 \) trees/acre. Outlying areas of widely scattered faders were not included in the survey.

Aerial Photography

In an attempt to secure a timely aerial photography contract, flight lines, and photo plot locations were fixed prior to boundary determination and stratification. Flight lines corresponded to north-south section lines throughout the infestation and the surrounding area, with photo plots at 1-mile intervals (east-west section lines) along flight lines. This resulted in a disproportionate distribution of photo plots per stratum: (1) 94, (2) 24, (3) 26. One hundred thirty-four plots were not used because they fell outside the final infestation boundary.

Stereo pairs were taken near each predetermined point at an approximate scale of 1:6,000. Altitude of the aircraft was recorded at each photo point and later used in scale adjustment. Each stereo pair was then indexed and located on topographic (1:24,000) and standard Forest (1:126,720) maps. Topographic maps were used to determine the proper scale of each pair; the Forest maps for orientation and allocation into strata.

Ground Truth Acquisition

From the photo plots, 56 2.5-acre ground plots were selected by pps sampling. To reduce travel between plots, two ground plots were selected (pps) from each of 28 selected photo plots (table 1). Each ground plot was then located on standard Forest maps, topographic maps, and small scale aerial photography (1:80,000). Finally each plot was scribed onto the emulsion side of one transparency per stereo pair.

Excessive snow prevented ground truth acquisition until spring, 1979. Data from all 56
Figure 1. Survey area of lodgepole pine mortality caused by mountain pine beetle, Beaverhead and Gallatin National Forests, 1977.
Within each variable plot all "IN" trees 5" diameter at breast-height and greater were recorded by species, diameter, and one of the following damage classes: healthy tree, unknown mortality, 1978 MPB attack, 1977 MPB attack, MPB attacked prior to 1977, unsuccessful MPB attack, or other bark beetles. Heights were taken of the first two trees per species. A 1/300-acre fixed plot was established at each variable plot center to measure regeneration (trees less than 5" DBH). Finally, all three persons cruised the entire 2.5-acre plot and recorded the diameter of all faders (1977 attacks). Height of the first two trees in each inch diameter class was also measured for volume estimations.

Analysis consisted of: (1) generating the number and volume of lodgepole pine killed by mountain pine beetle in 1977, 1978, and total standing dead; (2) determining the relationship between photointerpreter counts of faders and counts of faders made on the ground; and (3) determining a basal area ratio of total dead to total living lodgepole pine.

Formulae used to determine the number and volume of trees killed in 1977 and the relationship between photo and ground counts were modified and taken from Hostetler and Young (1979, pg. 20-24) and are shown in appendices B and C. The variable plot data was analyzed by the computer program INDIDS (Bousfield, 1979) to calculate and display trees and volume per acre and damage classes for all species. These figures were then multiplied by 2.5 acres per plot and substituted into the same formulae used to derive the 1977 estimates. This generated estimates for the number and volume of trees killed in 1977, 1978, and total standing dead.

Results

The total number of trees killed in the survey area in 1977 was 1,270,617 containing a volume of 24.2 million cubic feet (table 2).
### Table 2—Estimates of number of trees and volume of lodgepole pine killed by the mountain pine beetle in 1977 on the Beaverhead and Gallatin National Forests, 1979 (determined from 100 percent cruise data)

<table>
<thead>
<tr>
<th>Strata</th>
<th>Acres</th>
<th>No. of faders</th>
<th>Standard error</th>
<th>Volume per tree (cu. ft.)</th>
<th>Total volume</th>
<th>Standard error (M cu. ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>179,375</td>
<td>487,938 (7.23)*</td>
<td>33,842</td>
<td>22.16</td>
<td>10,370</td>
<td>1,602 (15.5)*</td>
</tr>
<tr>
<td>2</td>
<td>52,231</td>
<td>462,406 (7.78)*</td>
<td>36,023</td>
<td>18.49</td>
<td>8,449</td>
<td>1,508 (17.6)*</td>
</tr>
<tr>
<td>3</td>
<td>38,649</td>
<td>340,273 (7.69)*</td>
<td>26,179</td>
<td>15.63</td>
<td>5,317</td>
<td>965 (18.2)*</td>
</tr>
<tr>
<td>Total</td>
<td>270,255</td>
<td>1,270,617 (4.4)*</td>
<td>55,931</td>
<td>19.07</td>
<td>24,237</td>
<td>2,403 (9.92)*</td>
</tr>
</tbody>
</table>

*Percent standard error

Relative sampling errors were 4.4 and 9.9 percent for number of faders and cubic foot volume, respectively. Correlations between photo counts and ground counts were very good for all three strata as well as the combined data. The coefficient of determination ($R^2$) values for stratum 1, 2, 3, and combined were .82, .94, .94, and .90, respectively. Linear regression slope ($b$) estimates indicated the photo counts underestimated the ground counts in each of the strata and the combined data: (1) $b = 1.50$, (2) $b = 1.15$, (3) $b = 1.53$, and (combined) $b = 1.37$.

By substituting variable plot data (faders) into the estimator formulae, we found nearly the same number of trees (1,118,821) and associated volume (21.2 million cubic feet) as from the 100 percent cruise data. However, the percent standard error values were higher, 11.5 and 12.9, respectively (table 3).

### Table 3—Estimates of number of trees and volume of lodgepole pine killed by the mountain pine beetle in 1977 on the Beaverhead and Gallatin National Forests, 1979 (determined from variable plot data)

<table>
<thead>
<tr>
<th>Strata</th>
<th>Acres</th>
<th>No. of faders</th>
<th>Standard error</th>
<th>Volume per tree (cu. ft.)</th>
<th>Total volume</th>
<th>Standard error (M at. ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>179,375</td>
<td>408,013 (22.96)*</td>
<td>93,686</td>
<td>21.04</td>
<td>5,362</td>
<td>1,736 (58.0)*</td>
</tr>
<tr>
<td>2</td>
<td>52,231</td>
<td>400,084 (20.72)*</td>
<td>82,881</td>
<td>18.30</td>
<td>7,320</td>
<td>1,599 (141.0)*</td>
</tr>
<tr>
<td>3</td>
<td>38,649</td>
<td>310,724 (8.96)*</td>
<td>27,835</td>
<td>16.96</td>
<td>5,268</td>
<td>887 (67.69)*</td>
</tr>
<tr>
<td>Total</td>
<td>270,256</td>
<td>1,118,821 (11.45)*</td>
<td>128,145</td>
<td>18.92</td>
<td>21,173</td>
<td>2,726 (12.88)*</td>
</tr>
</tbody>
</table>

*Percent standard error

### Table 4—Estimates of number of trees and volume of lodgepole pine killed by the mountain pine beetle in 1978 on the Beaverhead and Gallatin National Forests, 1979 (determined from variable plot data)

<table>
<thead>
<tr>
<th>Strata</th>
<th>Acres</th>
<th>No. of faders</th>
<th>Standard error</th>
<th>Volume per tree (cu. ft.)</th>
<th>Total volume</th>
<th>Standard error (M at. ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>179,375</td>
<td>270,266 (29.8)*</td>
<td>80,539</td>
<td>19.8</td>
<td>5,362</td>
<td>1,736 (20.4)*</td>
</tr>
<tr>
<td>2</td>
<td>52,231</td>
<td>491,070 (31.5)*</td>
<td>154,896</td>
<td>16.9</td>
<td>8,306</td>
<td>2,432 (15.5)*</td>
</tr>
<tr>
<td>3</td>
<td>38,649</td>
<td>439,890 (62.5)*</td>
<td>274,601</td>
<td>20.8</td>
<td>9,159</td>
<td>7,739 (339.6)*</td>
</tr>
<tr>
<td>Total</td>
<td>270,255</td>
<td>1,200,726 (27.1)*</td>
<td>325,400</td>
<td>19.0</td>
<td>22,828</td>
<td>8,296 (36.3)*</td>
</tr>
</tbody>
</table>

*Percent standard error

### Table 5—Estimates of number of trees and volume of the total standing dead lodgepole pine killed by the mountain pine beetle through 1978 on the Beaverhead and Gallatin National Forests, 1979 (determined from variable plot data)

<table>
<thead>
<tr>
<th>Strata</th>
<th>Acres</th>
<th>No. of faders</th>
<th>Standard error</th>
<th>Volume per tree (cu. ft.)</th>
<th>Total volume</th>
<th>Standard error (M cu. ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>179,375</td>
<td>959,835 (31.5)*</td>
<td>194,714</td>
<td>20.6</td>
<td>5,362</td>
<td>1,736 (54.9)*</td>
</tr>
<tr>
<td>2</td>
<td>52,231</td>
<td>1,209,730 (51.5)*</td>
<td>194,200</td>
<td>19.9</td>
<td>8,306</td>
<td>2,432 (15.5)*</td>
</tr>
<tr>
<td>3</td>
<td>38,649</td>
<td>914,123 (37.3)*</td>
<td>274,601</td>
<td>20.8</td>
<td>9,159</td>
<td>7,739 (140.0)*</td>
</tr>
<tr>
<td>Total</td>
<td>270,255</td>
<td>3,117,888 (14.1)*</td>
<td>341,329</td>
<td>19.9</td>
<td>22,828</td>
<td>8,296 (19.9)*</td>
</tr>
</tbody>
</table>

*Percent standard error

The basal area ratio of total dead to total green lodgepole for all strata was 42.7 percent. The basal area ratio of total dead lodgepole to the green residual stand was 31.4 percent (table 6).
Table 6.—Estimates of basal area in percent by stratum and species composition in survey area, Beaverhead and Gallatin National Forests, 1978

<table>
<thead>
<tr>
<th>Strata</th>
<th>Lodgepole pine</th>
<th>Whitebark/Limber pine</th>
<th>Douglas-fir</th>
<th>Spruce</th>
<th>Subalpine fir</th>
<th>Aspen</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Live</td>
<td>Dead</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>43.74</td>
<td>27.86</td>
<td>0.46</td>
<td>0.62</td>
<td>18.78</td>
<td>3.77</td>
<td>4.77</td>
</tr>
<tr>
<td>2</td>
<td>34.56</td>
<td>30.73</td>
<td>0.19</td>
<td>0</td>
<td>12.45</td>
<td>12.96</td>
<td>8.49</td>
</tr>
<tr>
<td>3</td>
<td>47.03</td>
<td>37.25</td>
<td>0.70</td>
<td>0.30</td>
<td>6.69</td>
<td>3.64</td>
<td>4.39</td>
</tr>
<tr>
<td>Weighted Mean</td>
<td>42.05</td>
<td>31.35</td>
<td>0.65</td>
<td>0.5</td>
<td>13.51</td>
<td>6.36</td>
<td>5.72</td>
</tr>
</tbody>
</table>

Discussion and Recommendations

This survey was extremely effective in estimating the annual and cumulative mortality of lodgepole pine, within the designated boundaries of the infestation, with very good levels of confidence. Other methods of mountain pine beetle detection such as aerial sketchmapping and trend plot surveys are less expensive but yield mortality estimates lacking statistical reliability (table 7).

There are extensive areas of old dead lodgepole pine which were not included within the boundaries of this survey. Due to depletion of desirable host material, there are few, if any, faders in these areas. Therefore, the estimated number of standing dead lodgepole within our defined survey is substantially less than the total standing dead throughout the “entire” outbreak on the Beaverhead and Gallatin Forests.

It is suggested that similar multistage sampling surveys be conducted State-wide every 5 years where major bark beetle infestations call for exact loss information. The following modifications are recommended:

1. Include all outlying areas of “scattered” faders within the boundaries of the infestation.

2. Delineate and stratify the infestation before fixing flight lines and photo plot locations. Require aerial photographers to photograph the predetermined location of each plot.

3. Aerial photography should be taken as soon as possible. Our late (October) photography resulted in long shadows and brown ground cover making photointerpretation difficult stereoscopically, and nearly impossible, monoscopically. It also delayed ground truth acquisition until spring.

Table 7.—Estimated number of faders and compared costs of three different survey methods to determine the number of mb-killed lodgepole in 1977 on the Beaverhead and Gallatin National Forests

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Sketchmapping</th>
<th>Multistage</th>
<th>Trend plot survey, 1977</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$37,270</td>
<td>467,938</td>
<td>(7.23)*</td>
</tr>
<tr>
<td>2</td>
<td>$45,400</td>
<td>462,606</td>
<td>(7.79)*</td>
</tr>
<tr>
<td>3</td>
<td>651,350</td>
<td>340,273</td>
<td>(7.68)*</td>
</tr>
<tr>
<td>Total</td>
<td>1,384,020</td>
<td>1,270,817</td>
<td>(4.4)*</td>
</tr>
<tr>
<td>Cost</td>
<td>$1,400</td>
<td>$37,000</td>
<td>$2,000</td>
</tr>
</tbody>
</table>

*Relative sampling error

Tannock, Scott, and Oscar J. Dooling, 1978. Forest Insect and Disease Conditions, 1977, in the Northern Region. USDA Forest Service, Northern Region, Missoula, Montana.
4. Acquire ground truth data immediately following aerial photography and interpretation. Ground truth acquisition can be performed the following spring, however, degrees of fading make year of attack differentiation more difficult. Also access can be severely hampered by deep snowpack.

5. It is not necessary to measure heights during the 100 percent cruise. Regression estimators developed from local forest inventory plots can be used to predict heights for given measured diameters. Volume determinations can now be made for each tree. This greatly reduces the amount of time spent at each ground plot.

Acknowledgement

The authors thank Ken Gibson, Mark McGregor, Bill Klein, and Bob Young for their assistance throughout the survey. Special thanks go to the seasonal employees whose dedication and conscientiousness helped complete the survey.

References


### Timetable and estimated costs of the survey, Beaverhead and Gallatin National Forests, 1977

<table>
<thead>
<tr>
<th>Activity</th>
<th>Dates</th>
<th>Mandays</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aerial Sketchmapping</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Sketchmapping</td>
<td>7/31 - 8/3, 1978</td>
<td>3</td>
<td>$2,000</td>
</tr>
<tr>
<td>b. Digitizing</td>
<td>8/7-11, 1978</td>
<td>4</td>
<td>225</td>
</tr>
<tr>
<td>c. Stratification</td>
<td>10/4-5, 1978</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td><strong>Aerial Photography</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Aerial photography</td>
<td>10/3-5, 1978</td>
<td></td>
<td>3,200</td>
</tr>
<tr>
<td>c. Processed film received</td>
<td>10/30, 1978</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. Photo and map indexing</td>
<td>10/31 - 11/8, 1978</td>
<td>14</td>
<td>700</td>
</tr>
<tr>
<td><strong>Photo Interpretation</strong></td>
<td>11/9 - 12/19, 1978</td>
<td>19</td>
<td>950</td>
</tr>
<tr>
<td><strong>Ground Crew Training</strong></td>
<td>5/14-15, 1979</td>
<td>16</td>
<td>1,600</td>
</tr>
<tr>
<td><strong>Ground Truth Acquisition</strong></td>
<td>5/16 - 6/28, 1979</td>
<td>235</td>
<td>18,200 salaries</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8,225 travel</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3,000 helicopter</td>
</tr>
<tr>
<td><strong>Data Computation and Analysis</strong></td>
<td>8/29 - 11/2, 1979</td>
<td>17</td>
<td>900</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>7/31 - 11/2, 1979</td>
<td>312</td>
<td>$39,300</td>
</tr>
</tbody>
</table>
I. Notation

Subscripts:
- $h = 1, \ldots, m_h$ denotes the number of strata that are sampled
- $i = 1, \ldots, m_h$ denotes the sampled 40-acre photo plots
- $j = 1, \ldots, n_h$ denotes the 40-acre photo plots selected for ground sampling
- $k = 1, 2$ denotes the 2.5-acre subplots selected from each 40-acre photo plot

$A_h$: Number of acres in $h^{th}$ stratum

$M_h$: Number of primary sample units in the $h^{th}$ stratum

\[ M_h = \frac{A_h}{40 \text{ acres}} \]

$m_h$: Number of photo plots sampled in the $h^{th}$ stratum

$n_h$: Number of photo plots selected for ground sampling in the $h^{th}$ stratum

$x_{hi}$: Number of faders counted in the $i^{th}$ photo plot in the $h^{th}$ stratum

\[ x_h = \sum_{i=1}^{m_h} x_{hi} \]

$x_{hjk}$: Number of faders counted from the photo in the $k^{th}$ subplot of the $j^{th}$ photo plot in the $h^{th}$ stratum

$Y_{hjk}$: Number of faders counted on the ground, in the $k^{th}$ subplot of the $j^{th}$ photo plot in the $h^{th}$ stratum

$\hat{y}_{jh}$: Estimate of the number of faders in the $j^{th}$ photo plot of the $h^{th}$ stratum

$\hat{Y}_h$: Estimate of the number of faders in the $h^{th}$ stratum

S.E.$\hat{Y}$: Standard error of the estimate

R.S.E.$\hat{Y}$: Relative standard error of the estimate,

\[ R.S.E.\hat{Y} = \frac{\text{S.E.}\hat{Y}}{\hat{Y}} \cdot 100, \text{ expressed as a percent} \]
II. Estimators

\[ \hat{Y}_h = \frac{A_h}{(40)(m_h)(n_h)} \sum_{j=1}^{n_h} \hat{y}_{hj} \]

where

\[ \hat{y}_{hj} = \frac{2 \sum_{k=1}^{2} \frac{(x_{hj})(\hat{y}_{hjk})}{x_{hjk}}}{(x_{hj})(2)} \]

\[ \text{S.E.} \hat{Y}_h = \sqrt{ \left( \frac{A_h}{(40)(m_h)} \right)^2 \sum_{j=1}^{m_h} \left( \hat{y}_{hj} - \hat{Y}_h \right)^2 \left( \frac{n_h}{n_h - 1} \right) } \]

Since \( \hat{Y}_h \) is the mean of the \( \hat{y}_{hj} \) values, the above formula can be reduced to the following more easily calculated form:

\[ \text{S.E.} \hat{Y}_h = \sqrt{ \left( \frac{A_h}{(40)(m_h)} \right)^2 \sum_{j=1}^{m_h} \left( \frac{n_h}{n_h - 1} \right) } \]

\[ \text{R.S.E.} \hat{Y}_h = \frac{\text{S.E.} \hat{Y}_h}{\hat{Y}_h} \times 100, \text{ expressed as a percent} \]

\[ \hat{Y} = \sum_{h=1}^{2} \hat{Y}_h \]

\[ \text{S.E.} \hat{Y} = \sqrt{ \sum_{h=1}^{3} \text{S.E.} \hat{Y}_h^2 } \]

\[ \text{R.S.E.} \hat{Y} = \frac{\text{S.E.} \hat{Y}}{\hat{Y}} \times 100, \text{ expressed as a percent} \]
III. Comments

The sample design consists of a systematic random selection of the \( m_h \) photo plots from the \( M_h \) possible photo plots in the \( h^{th} \) stratum. Photo plots from each stratum were selected independently. Plots for ground sampling were selected in two stages both using probabilities proportional to an estimate of size. Since there is high correlation between the photo counts and what is expected to be on the ground, the photo counts provide a good estimate of size. The \( n_h \) samples were selected from the \( m_h \) photos using the total photo count (\( x_h \)) and the individual photo count (\( x_{hi} \)) following the procedure outlined in Cochran (1977, p. 251) and using sampling with replacement. Once the \( n_h \) samples were selected, the same procedure was used to sample 2 of the 36 2.5-acre subplots.

The estimators used for this survey are modifications of the basic probability proportional to size:

\[
\hat{Y}_{pps} = \frac{1}{n} \sum_{i=j}^{n} \frac{y_i}{z_i}
\]

where \( y_i \) represents the ground data and \( z_i \) represents the probability of a particular photo plot being selected. The variance for that function is as follows:

\[
V(\hat{Y}) = \left[ \frac{\sum_{i=1}^{n} \left( \frac{y_i}{z_i} - \hat{Y} \right)^2}{n(n-1)} \right]
\]
Formulae used for calculating linear regressions for each stratum (taken from Hostetler and Young, 1979, pg. 24).

Note: Linear regressions were based on matched data pairs of faders per acre on the ground and on photos of the 2.5-acre subplots.

Regression estimates of slope (b) and y-intercept (a)

\[
b = \frac{\sum x_i y_i - (\sum x_i \sum y_i)}{\sum x_i^2 - (\sum x_i)^2/n}
\]

\[a = \bar{y} - bx\]

where, \(n\) = number of data pairs

- \(x_i\) = number of faders counted on photo in \(i^{th}\) subplot
- \(y_i\) = number of faders counted on ground in \(i^{th}\) subplot

\[
\bar{x} = \frac{\sum x_i}{n}
\]

\[
\bar{y} = \frac{\sum y_i}{n}
\]