

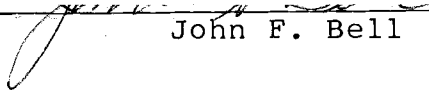
AN ABSTRACT OF THE THESIS OF

Jaime Fernando Sales Luis for the degree of Master of
Science in Forest Management presented on June 22,
1984.

Title: Growth Growing-Stock Relationships in Young
Douglas-fir (*Pseudotsuga menziesii* (Mirb.)
Franco).

Abstract approved: _____

Signature redacted for privacy.


John F. Bell

In the thinning theory, the hypothesis that the stand growth is unaffected by the density over a broad range of densities is generally accepted today.

In this study three different measures of stand density were used in basal area growth and volume growth multiple regressions in even-aged managed stands, searching for the optimum density level.

The three measures of stand density selected were: Basal Area, Relative Density Index (Drew and Flewelling, 1979) and Stand Density Index (Reineke, 1933).

The population for the study consisted of a natural young Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) stand, at Hoskins in the Oregon Coast Range, which has been repeatedly thinned at different intensities.

The graphical solutions of the developed regression equations showed that for the basal area growth all the three density measures studied led to similar optimum growth estimates and supported the widely accepted Langsaeter-Moller hypothesis.

From a management standpoint one can say that, within the studied age interval (23 to 36 years), the maximum basal area growth lies between 175 to 225 ft²/ac of basal area.

The shape of the volume growth curves shows in most cases an increasing tendency with no signs of a maximum within the range of densities represented in this study.

The different shapes of the basal area growth and of the volume growth curves reflect the fact that young stands of Douglas-fir maintain a fairly rapid height growth rate.

The results of this study should essentially serve as a guide for potential stand growth, as one knows the existing differences between growth and yield on small research plots and on large managed forests.

GROWTH GROWING-STOCK RELATIONSHIPS IN YOUNG
DOUGLAS-FIR (Pseudotsuga menziesii
(Mirb.) Franco)

by

Jaime Fernando Sales Luis

A THESIS

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GROWTH GROWING-STOCK RELATIONSHIPS IN YOUNG
DOUGLAS-FIR (*Pseudotsuga menziesii* (Mirb.) Franco)

INTRODUCTION

Stand density control through thinnings is probably the forester's most important tool from a management standpoint.

In the thinning theory, the hypothesis formulated by Langsaeter (1941) cited by Smith (1962) that

"the total production of cubic-foot volume by a stand of given age and composition on a given site is, for all practical purposes, constant and optimum for a wide range of density of stocking" is of considerable importance.

Lovengreen (1950) also demonstrated the validity of this concept concerning basal area.

The worldwide acceptance of this hypothesis, also known as Langsaeter-Moller, is based on the work of Moller (1947, 1954) with two species, Fagus sylvatica (common beech) and Picea excelsa (Norway spruce), in Northern Europe.

It has been claimed (Heiberg, 1954) that this hypothesis holds true for other species in other climates and continents although further evidence is necessary.

Not many studies have been developed, to test this hypothesis for Douglas-fir.

To base adequate management decisions it is important to analyse and quantitatively describe the growth-stand density hypothesis.

In this study both the relationships of gross basal area growth and gross volume growth to stand density are studied for the successive growth periods, in order to determine the optimum level of density for maximum growth in young Douglas-fir, at Hoskins Experimental Forest, Oregon.

At the same time three different measures of stand density, Basal area, Relative density index (Drew and Flewelling, 1979) and Stand density index (Reineke, 1933) are used and their relative merit is analyzed.

LITERATURE REVIEW

The Langsaeter-Moller Hypothesis

Langsaeter-Moller hypothesis is expressed graphically in Figure 1.

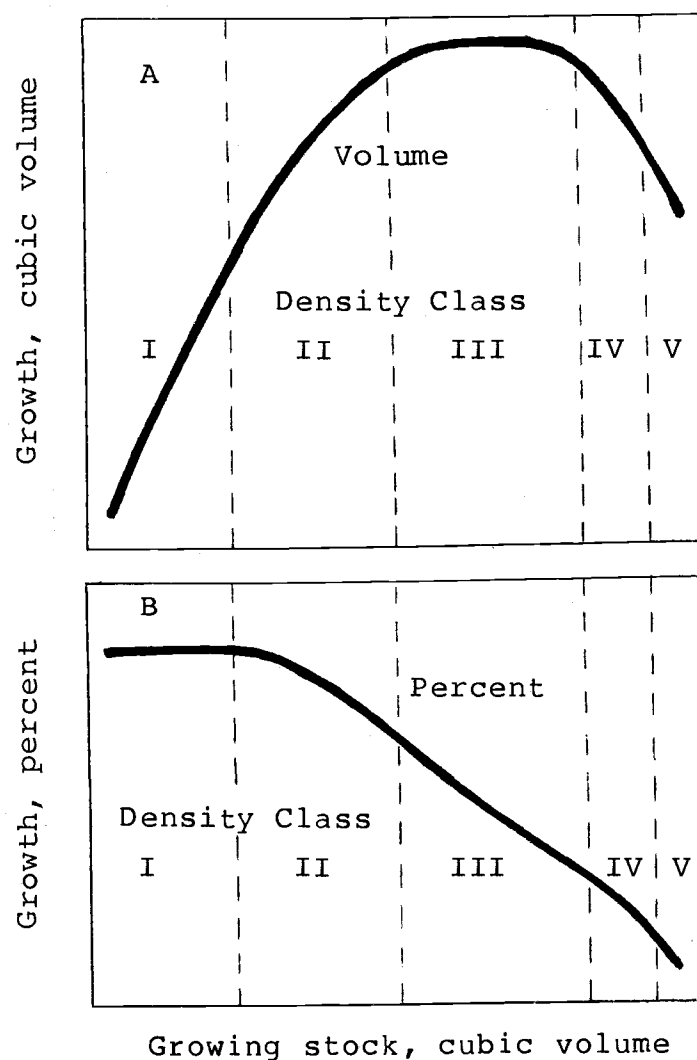


Figure 1. Langsaeter-Moller relationship of growth to growing stock in cubic volume, A, through density classes I to V. The growth percent curve, B, is derived from the relationship. (from Braathe, 1957 in Staebler, 1959).

In density class I, there is no competition among trees, because they are so far apart. The stand volume growth is nearly directly proportional to the growing stock (curve A) at a constant volume growth rate (curve B).

In density class II, the trees start competing for space, light and nutrients. As a consequence stand volume growth is still increasing (curve A) although at a decreasing volume growth rate (curve B).

Density class III, is the zone where the stand volume growth is approximately independent from the growing stock (curve A). The volume growth rate is nearly inversely proportional to the growing stock (curve B). This zone is known as the Langsaeter-Moller plateau.

Density classes IV and V, correspond to the zone of imminent competition-mortality in which both stand volume growth (curve A) and volume growth rate (curve B), very rapidly decline as growing stock increases.

From a management standpoint the critical zone of this curve lies in the border of density classes II and III, since there is no increased gain in volume growth for higher growing stock (Staebler, 1959).

Past Work

Based on a Danish experience, Briegleb (1952),

established the relationship between cubic-foot volume growth and density, measured by the number of trees per acre by average diameter and average height. He found no significant differences between the intensively managed Douglas-fir stands in Denmark and Prussia and stands of the same species in its natural habitat in Washington.

He concluded that the optimum stocking density should lie between 75 and 120 percent of his proposed standard, for most combination of factors.

According to Wortington et al. (1962), the basal area growth and the volume growth showed a tendency to increase with increasing amounts of growing stock (measured by the residual basal area/ac and residual volume/ac) both in thinned and unthinned stands of Douglas-fir, in the Voight Creek Experimental Forest.

The apparent inconsistency with the Langsaeter-Moller hypothesis is possibly a result of the low initial stocking in all plots, below the range of densities which produces the plateau level of increment.

Reukema (1979), in the longest spacing trial (50 years) with Douglas-fir at Wind River, Washington (low site), concluded that both the basal area and the volume growth reached a maximum at a 10*10 feet spacing, 436 trees/ac.

With increasing densities up to 4*4 feet spacing,

2722 trees/ac the growth in both cases declined. This study is particularly interesting, because it confirms the existence of density classes IV and V of Langsaeter-Moller curve which were never shown in previous studies.

This hypothesis was also studied in other species, mainly pines.

While analyzing the relationship of volume growth to stand density (percent full stocking, Stahelin, 1949), in young slash pine stands, Gruschow and Evans (1959), concluded that a maximum volume growth per acre is produced at something less than full stocking particularly in lower sites.

It was also emphasized that young stands required an adjusting period of time to full occupy the growing space, before they reached the zone of Langsaeter-Moller plateau.

Nelson and Brender (1963), while testing the effect of four different measures of stand density (Stahelin's percent of full stocking, Reineke's stand density index, total basal area and initial cubic-foot volume) on the volume growth of Loblolly pine stands, obtained curves with similar shape.

However the use of a direct measure (basal area) rather than an index was suggested.

The optimum density was positively correlated with site quality and age.

Stand Density Measures

One of the most studied problems in forestry has been the measurement of stand density. Several attempts have been made but none of them received general acceptance.

Curtis (1970) presented an excellent discussion about the existing different measures of density and concluded that some of the relative measures of stand density are nuances of the same basic relationship differing in details of algebraic form and methods of calculation of constants.

The stand density measures may be either,

- a) relative measures
- b) direct measures

Relative measures depend on comparison of actual stand characteristics with some normal stand thought to represent a comparable degree of occupancy of site, age, number of trees, diameter, height or some combination of these factors.

Direct measures, on the other hand, are of clear and objective definition and determination.

The following are the stand density measures used in forestry:

A. Relative measures

1. Which compare observed stands with normal

stands of the same age and site.

- Relative basal area (RBA), Curtis (1967).

$$RBA = \frac{\text{observed basal area}}{\text{predicted basal area}}$$

where, predicted basal area is the average basal area of the sample of well stocked stands, expressed as a function of age and site.

2. Which compare observed stands with normal stands of the same diameter.

- Stand density index (SDI), Reineke (1933).

$$SDI = N(QMD/10)^{1.605} \quad (\text{Douglas-fir})$$

where, N is the number of trees per acre.

QMD is the diameter (in.) of the tree of mean basal area.

- Tree area ratio (TAR), Chisman and Schumacher (1940)

$$TAR = .048 + .067D + .027D^2 \quad (\text{Loblolly pine})$$

where, D is the diameter (in.) of a single tree

- Percent of full stocking (PFS), Sthaelin (1949).

$$PFS = \left(\frac{D}{.087 + .071D} \right)^2 \quad (\text{Loblolly pine})$$

where, PFS is expressed in ft²/ac.

D is the diameter in inches.

- Relative density (RD), Curtis (1982)

$$RD = BA/QMD \cdot 5 \quad (\text{Douglas-fir})$$

where, BA is the basal area in ft^2/ac .

QMD is the diameter (in.) of the tree of mean basal area.

3. Which compare observed stands with open-grown trees of the same diameter.

- Crown competition factor (CCF), Krajicek et al. (1961)

$$CCF = \frac{1}{A} a (\sum D_i^2 N_i) + b (\sum D_i N_i) + c (\sum N_i)$$

where, A is the area in acres.

D_i is the d.b.h. class.

N_i is the number of trees in d.b.h. class.

4. Which compare observed stands with normal stands of the same height.

- Spacing factor (F), Wilson (1946)

$$F = 209 / (H * N \cdot 5)$$

where, H is the height of the stand.

N is the number of trees per acre.

5. Which compare observed stands with normal stands of the same diameter and height.

- Stand density (SD), Briegleb (1952).

$$SD = -34.62 + 1.25 (SH) \quad (\text{Douglas-fir})$$

where, SD is the number of trees as percent
of normal for average d.b.h.

SH is the stand height as percent of
normal height for average d.b.h.

- Relative density index (RDI), Drew and
Flewelling (1979)

$$\ln \rho = (12.644 - \ln v)/1.5 \text{ and,}$$

$$\text{RDI} = \text{TPA}(\text{obs.})/\rho \quad (\text{Douglas-fir})$$

where, ρ is the stand density (trees/ac)
for a given v .

v is the mean individual tree volume
(ft³)

TPA is trees/ac.

B. Direct measures

1. Number of trees per acre, TPA.
2. Basal area per acre, ft²/ac.
3. Volume per acre, ft³/ac.

METHODS

Levels-of-Growing-Stock Study in Douglas-fir

A regional cooperative study was initiated in 1962. This study was designed to "determine how the amount of growing stock retained in repeatedly thinned stands of Douglas-fir affects cumulative wood production, tree size and growth growing stock ratios" (Williamson and Staebler, 1962).

The Study Area

Hoskins installation is located 22 miles west of Corvallis (Figure 2) in an area owned by T. J. Starker and Bruce Starker, now Starker Forests Inc. It was established in 1963, and has been maintained since then by the College of Forestry, Oregon State University, in an even-aged, uniform, 20-year-old natural stand, immediately east of the summit of the Coast Ranges.

Average annual precipitation is 70 inches and average annual temperature is 50°F. The aspect is south and the slope range from about 15 to 55 percent. Elevation is about 1000 feet. Soils are classified as deep silty clay loams (Williamson and Staebler, 1971).

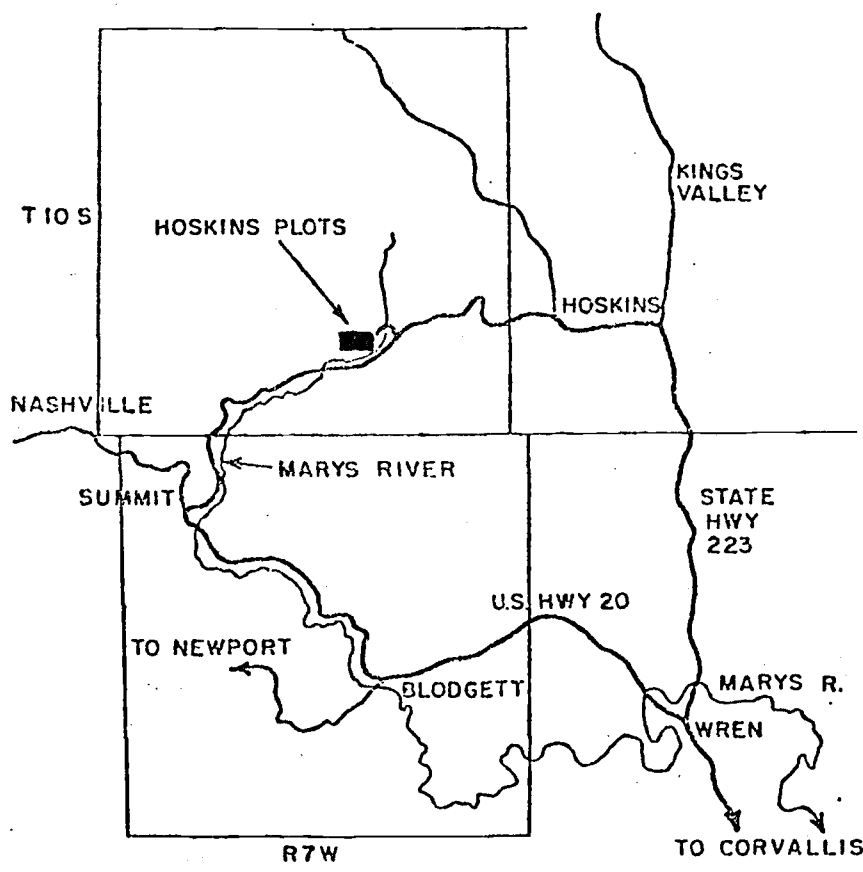


Figure 2. Location of the hoskins study.
(from Berg and Bell, 1979)

At the beginning of the study in 1963, the average stand statistics (control plots) were as follows:

Age	20 years
Trees per acre	1727
Diameter breast height	3.8 inches
Basal area per acre	138.1 ft ² /ac.
Volume per acre	1982 ft ³ /ac.
Site index (King, 1966)	133
Site class	II

The last available (1979) average stand statistics (control plots) are as follows:

Age	36 years
Trees per acre	767
Diameter breath height	8.2 inches
Basal area per acre	278.0 ft ² /ac
Volume per acre	9312 ft ³ /ac.
Site index (King, 1966)	136
Site class	II

Treatments

Eight different thinning regimes were tested. Each one differs in the amount of basal area allowed to accumulate in the growing stock in order to provide a broad range of densities.

The amount of basal area retained after any thinning (BA_n) is a predetermined percentage (P) of the gross basal area growth in the unthinned plots (GBAG) during the previous treatment period, and can be expressed by the equation (Tappeiner et al., 1982):

$$BA_n = BA_{n-1} + GBAG(P) \quad (1)$$

where, BA_{n-1} is the basal area at the beginning of the preceding period.

In Table I, the percentages (P) for each treatment at each given period are indicated.

Table I

Levels-of-growing-stock study schedule. Percent (P) of gross basal area of control plots to be retained in the growing stock by treatment, period and stand age.

Period and Stand Age	TREATMENT							
	1	2	3	4	5	6	7	8
First (23)	10	10	30	30	50	50	70	70
Second (27)	10	20	30	40	50	40	70	60
Third (30)	10	30	30	50	50	30	70	50
Fourth (32)	10	40	30	60	50	20	70	40
Fifth (36)	10	50	30	70	50	10	70	30

A calibration thinning was made at the beginning of the study in all treated plots, a) to reduce the variability in density between them and, b) to achieve a stand as uniform and evenly spaced as possible. As a consequence all treated plots were reduced to about 50 ft²/ac.

Thinnings were made whenever average height growth of crop trees following the year of calibration comes closest to each multiple of 10 feet, until the stand reaches 60 feet in height growth after the calibration (Williamson and Staebler, 1971).

Design of the Study

The experimental design in the study area is a completely randomized split plot experiment.

Each of the nine treatments (eight different thinning treatments plus one unthinned or control) is represented by three plots (replications) which were randomly assigned to the 27 one-fifth acre plots (Williamson and Staebler, 1962).

Although in the long term studies at Wind River and Voight Creek already mentioned the randomized block design was used, in this study the completely randomized design was preferred, because the analysis can better accommodate the expected loss of plots (fortunately not observed so far) in an experiment projected to last a period of time longer than 20 years.

As the values for the response variables to the treatments applied are measured at each treatment period (10 feet of crop trees height growth), these periods may be considered as subplots in a split plot experiment.

The shape of these plots is square because the experiment is placed as close as possible in a single contiguous area (Figure 3) (Williamson and Staebler, 1962).

Measurements and Data

The basic information on each tree includes the diameter at breast height, measured to the nearest 1/10 inch, height (when measured) to the nearest foot, age at breast height by counting the years in increment cores for trees used in stand age determination, and condition class (damage and mortality). Age was measured at the beginning of the study. The other measurements were taken at the start and at the end of each treatment period (Williamson and Staebler, 1962).

Diameter at breast height, basal area and cubic-foot volume are summed by condition class within each plot. These sums are averaged for the three plots within each treatment. Plot and treatment values of number of trees, basal area and cubic volume are expressed on a per acre basis (Williamson and Staebler, 1971).

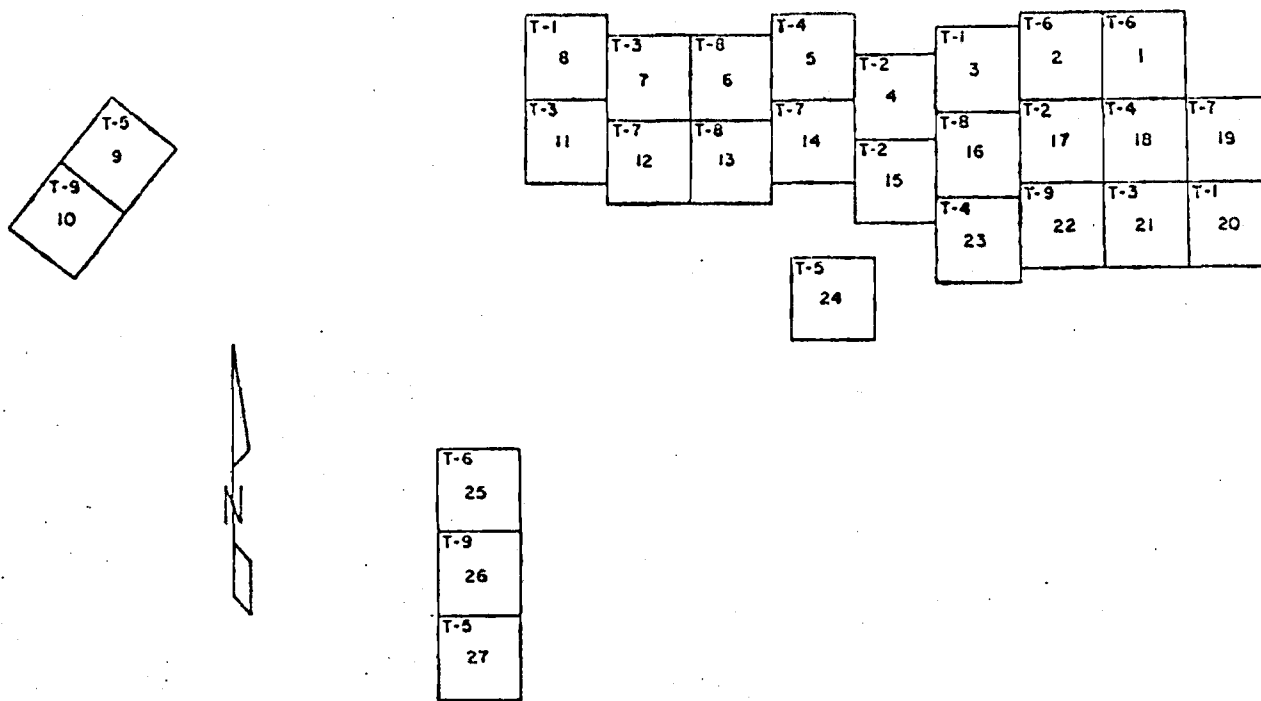


Figure 3. Layout of Hoskins levels-of-growing-stock study. The plots are one fifth acre in size. (from Berg and Bell, 1979)

The College of Forestry, Oregon State University, has kept records of these data since the beginning of the study.

Mortality

The mortality on the thinned plots was completely captured by the thinnings, once it was less than 1 percent of gross volume growth, except for two cases, in which it was less than 5 percent.

On the unthinned plots, the mortality was about 10 percent in the first and second treatment periods, 40 percent in the third and 22 percent in the fourth.

Staebler (1953) classified the natural mortality in two groups: a) normal mortality due to the decrease in number of trees as the stand grows older, a consequence of the fact that as the trees grow larger more growing space is necessary, b) irregular mortality, due to catastrophic events like fire, insects, diseases or climatic extremes causing abnormally high mortality in short periods. This second type is difficult to predict.

In this study normal mortality occurred in the control plots in all the periods, except in the third one.

Analysis of Growth Density Relationships

Langsaeter (1941) cited by Daniel et al. (1979),

specifies that his curves are applicable only to a particular age and site.

In this study, the age is referred to the stand age at the middle of each growth period.

Site index was found to be uniformly distributed across all plots around a mean value of 130 (King, 1966). This uniformity in site was expected because it was one of the conditions for selecting the study area.

Choice of Density Measures

The basal area (ft^2/ac), the relative density index (Drew and Flewelling, 1979) and the stand density index (Reineke, 1933) were selected because each one represents a different category of density measure.

They are probably also the most adequate density measures for Douglas-fir growth studies.

Basal Area Growth and Volume Growth Curves

The following figures, 4 to 9, represent the scatter plots (treatment means) of gross basal area increment and gross volume increment, versus each of the three selected measures of stand density.

From these figures one can observe that:

- a) All plots in this study cover a broad range of the horizontal axis.

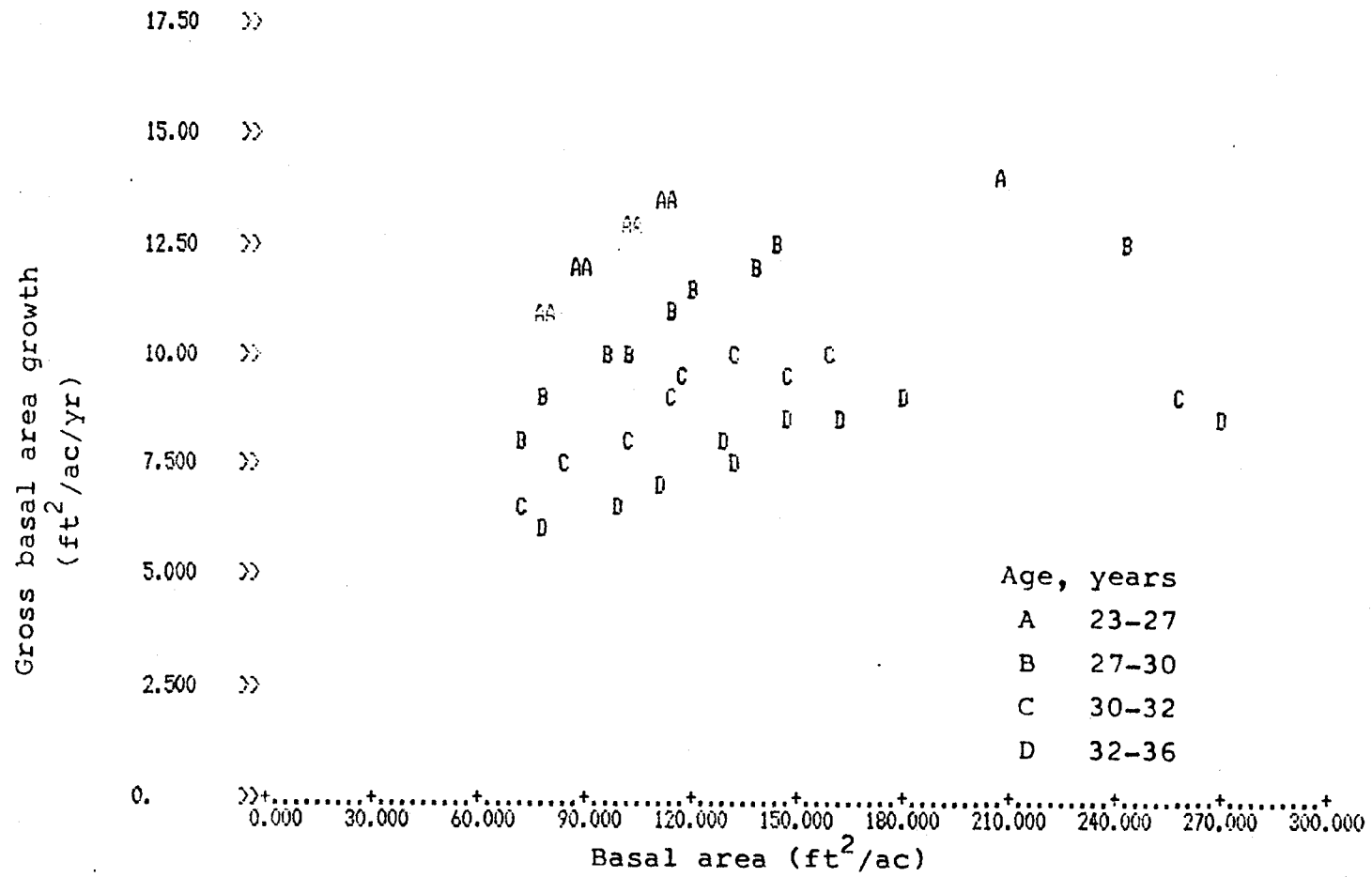


Figure 4. Stand density (basal area) versus gross basal area growth, for the four treatment periods.

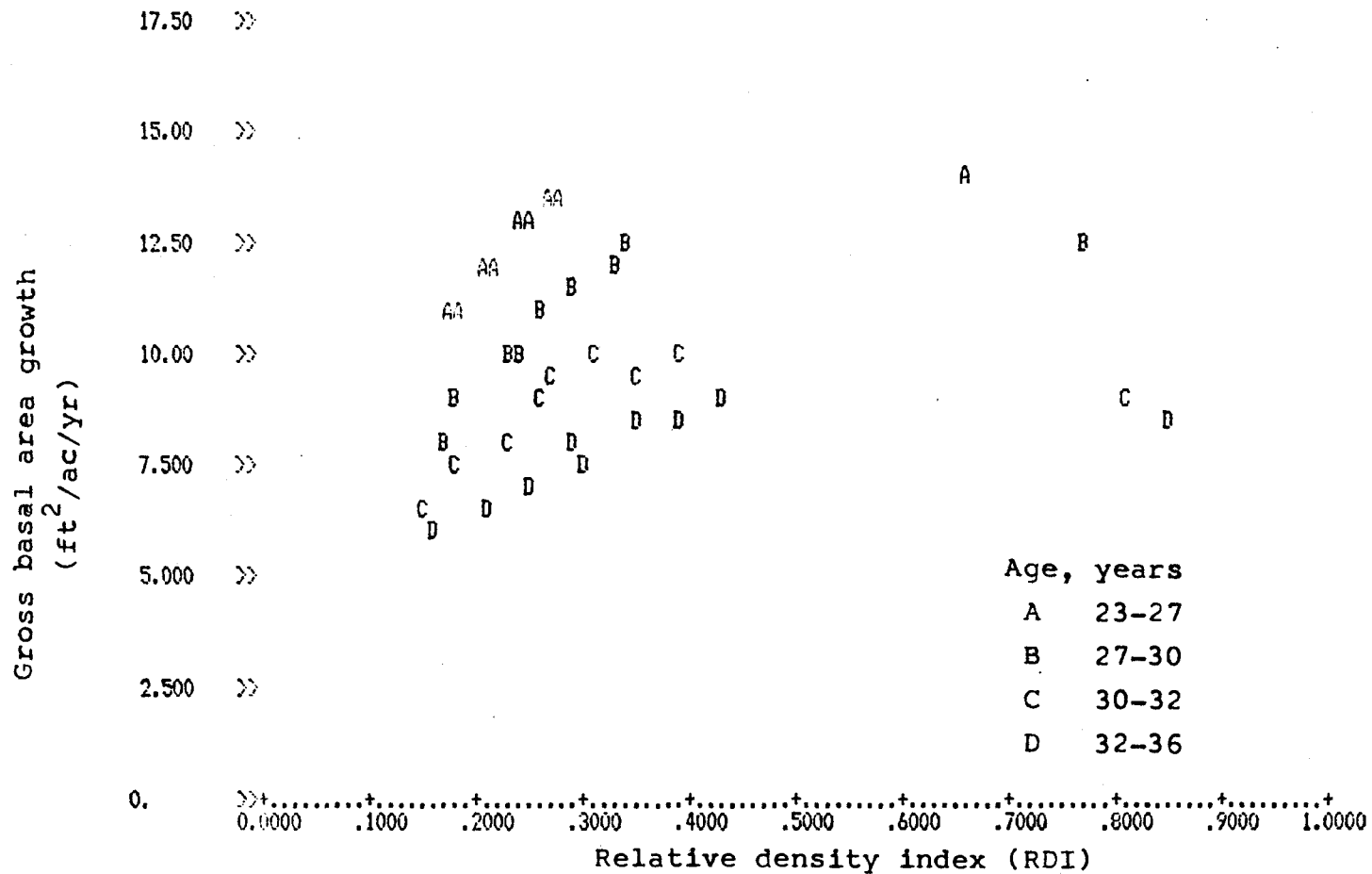


Figure 5. Stand density (RDI) versus gross basal area growth, for the four treatment periods.

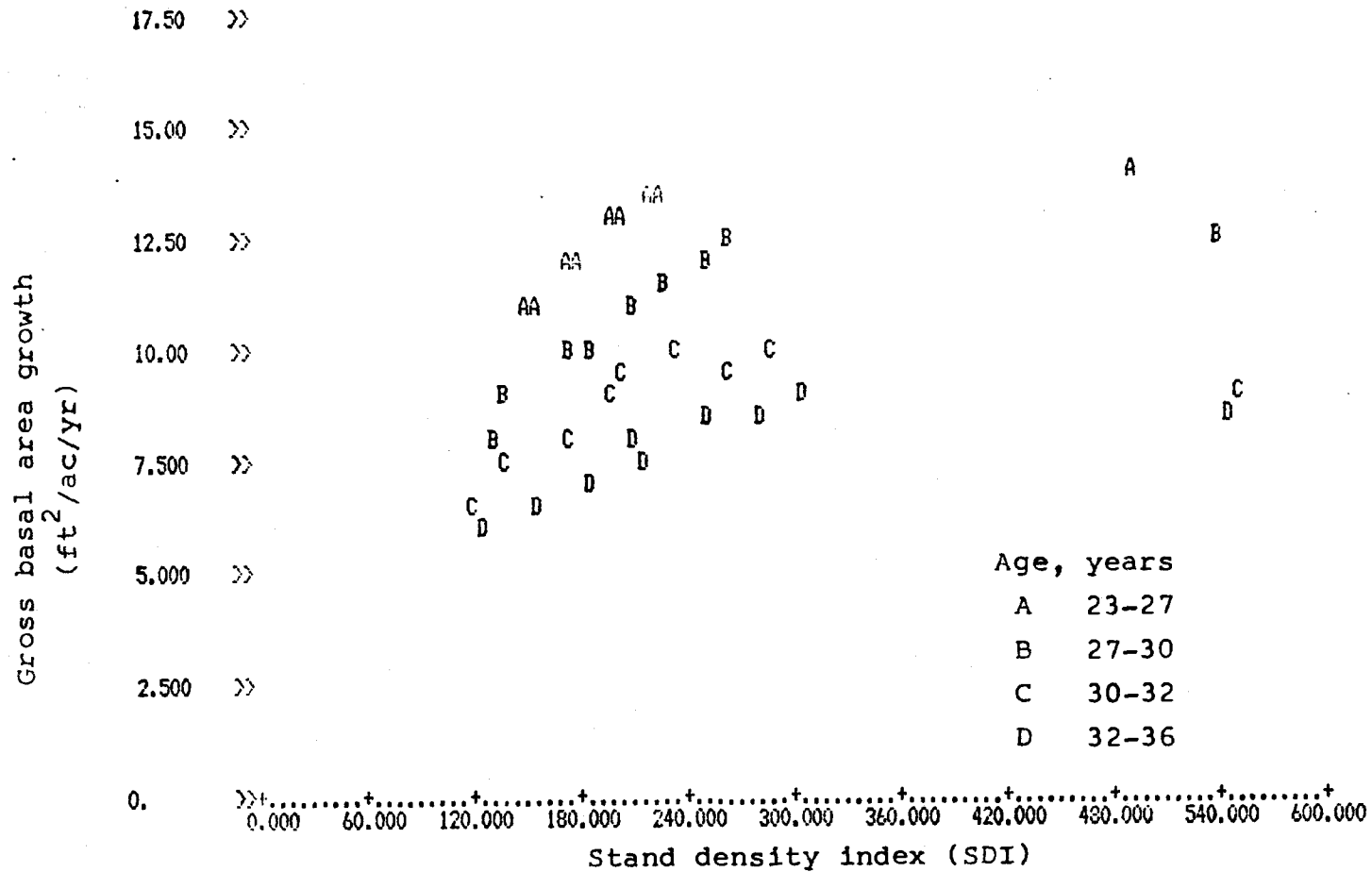


Figure 6. Stand density (SDI) versus gross basal area growth, for the four treatment periods.

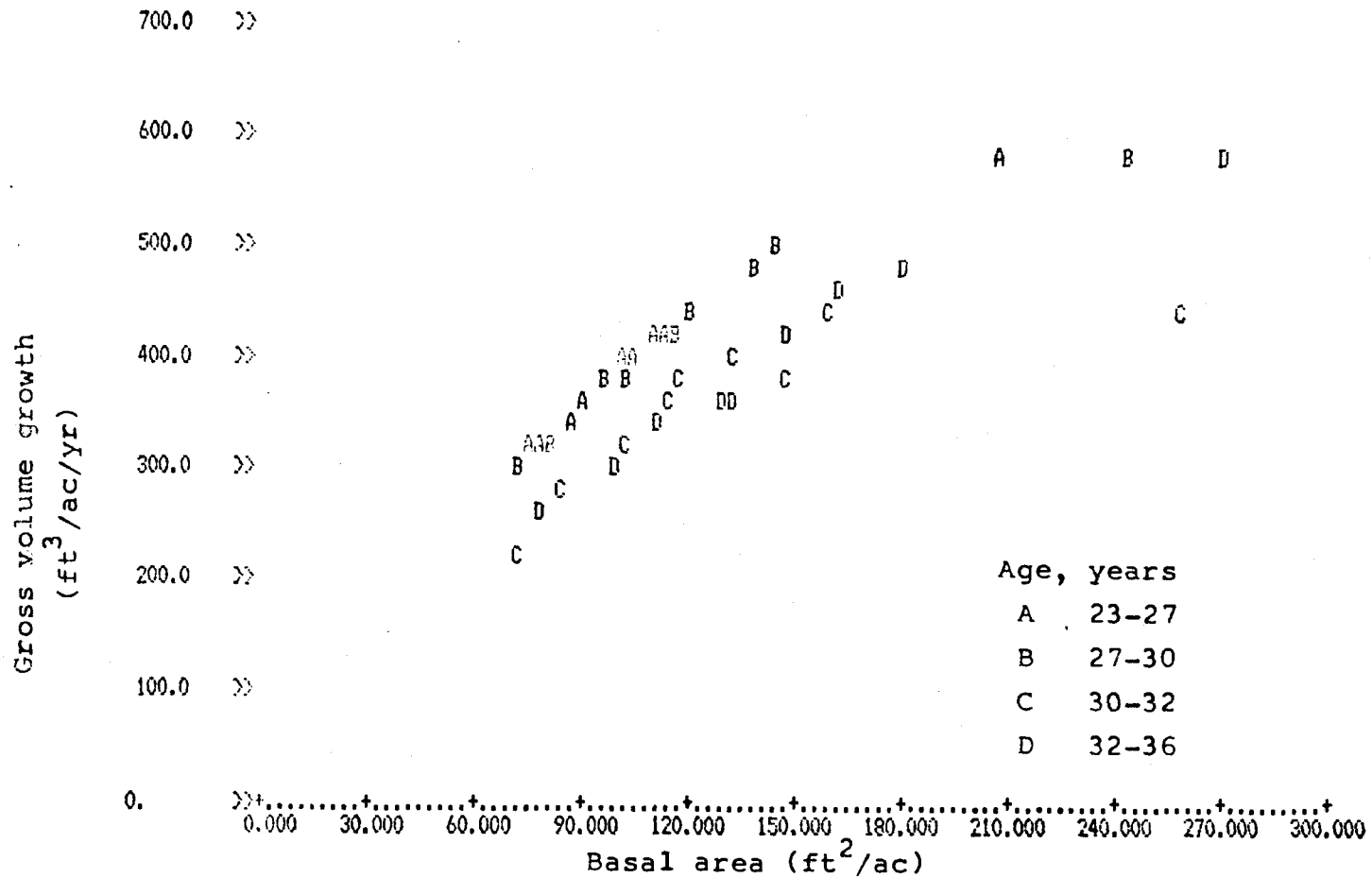


Figure 7. Stand density (basal area) versus gross volume growth, for the four treatment periods.

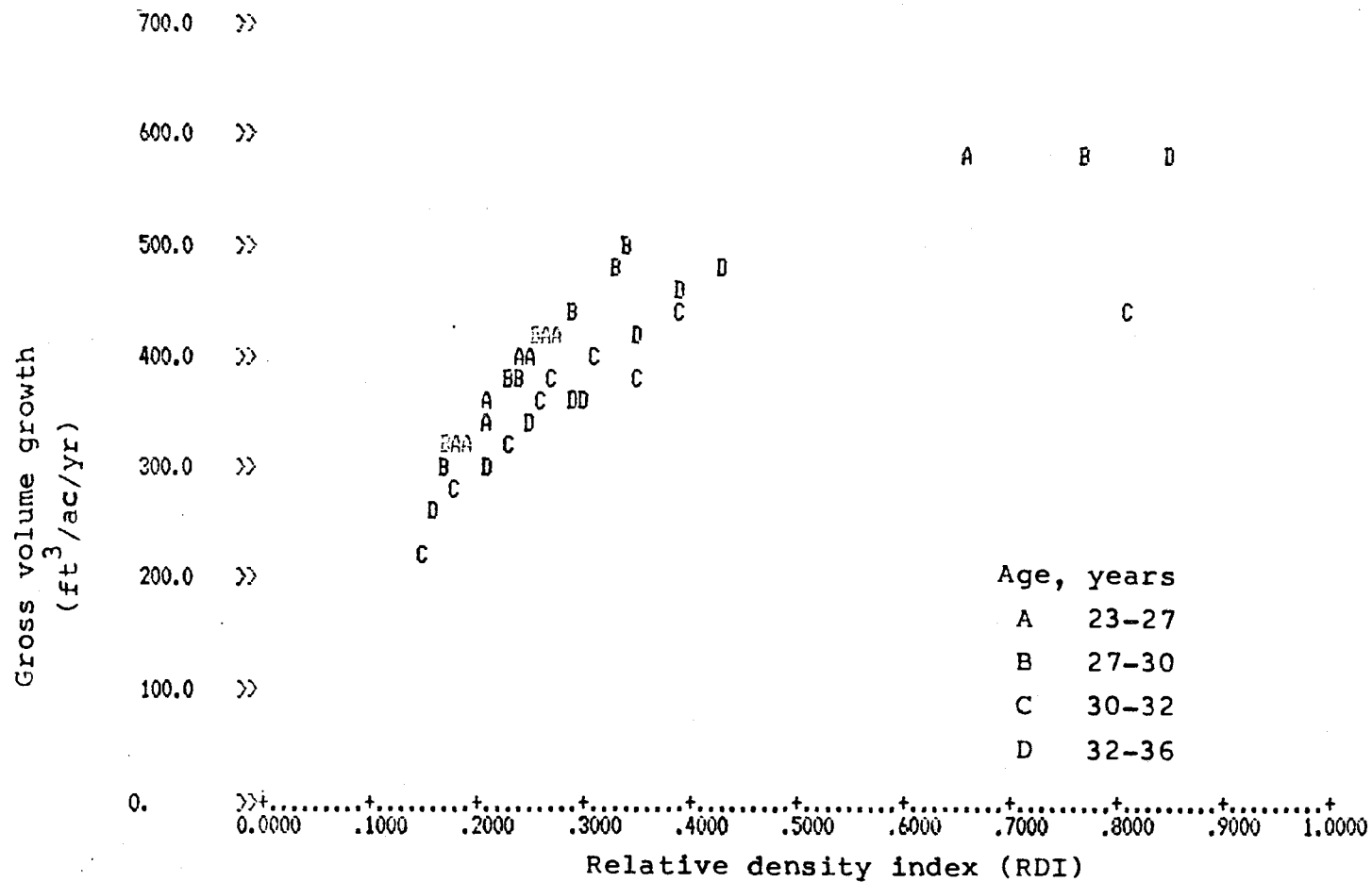


Figure 8. Stand density (RDI) versus gross volume growth, for the four treatment periods.

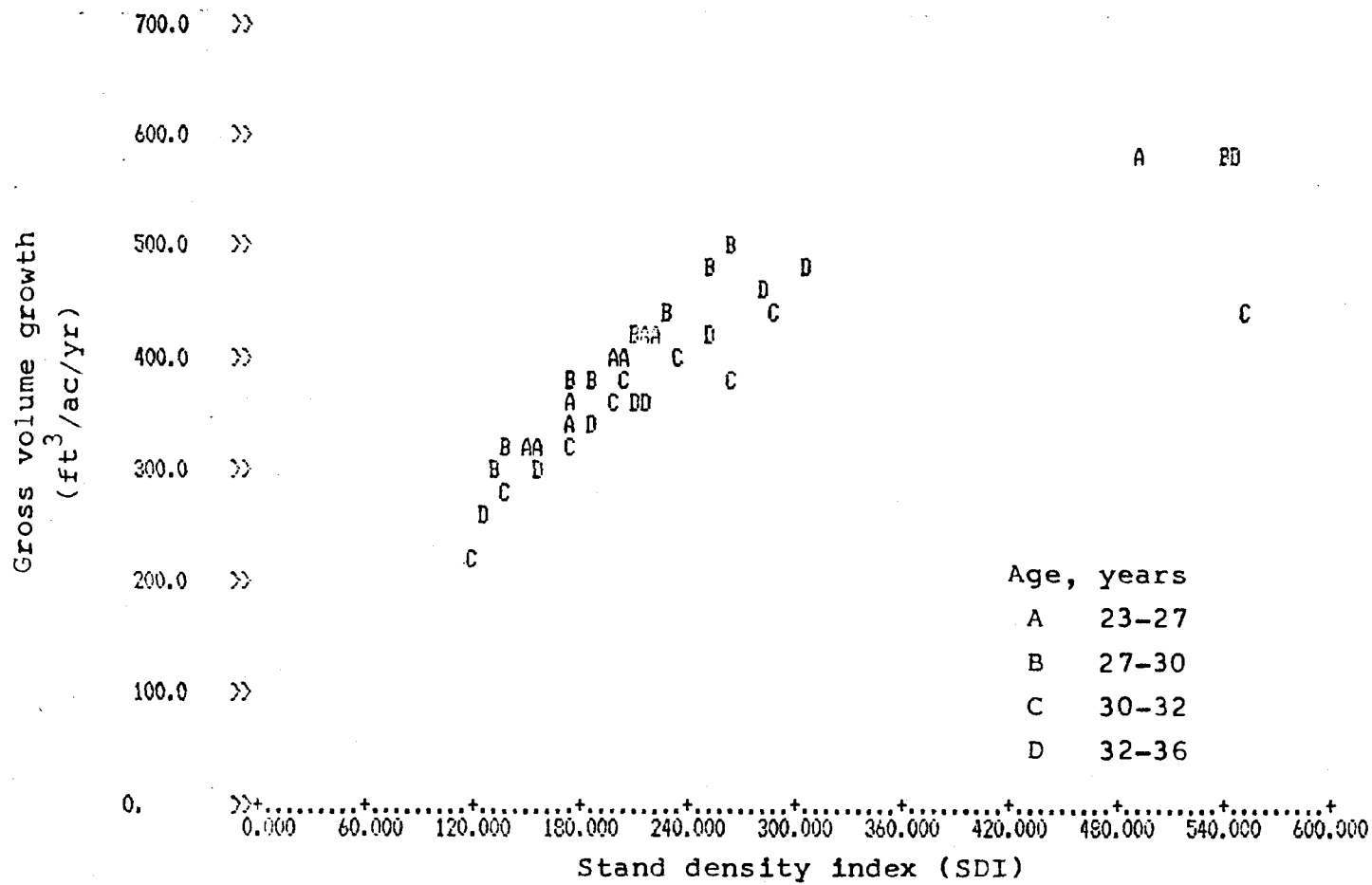


Figure 9. Stand density (SDI) versus gross volume growth, for the four treatment periods.

- b) The unthinned plots correspond to the higher values on the stand density axis and the thinned plots tend to be distributed on the intermediate zone of that axis.
- c) It is apparent that both indices compress the observations in the horizontal axis, when they are compared with the basal area.
- d) The development of the third treatment period (C) is inconsistent with the other periods. This is especially clear in the volume increment plots.
- e) It is apparent that for the basal area growth a maximum density may lie between the thinned and control plots. But for the volume growth this is not so clear and a maximum density may lie beyond the control plots.

Form of Growth Equation Used

The equation suggested by Curtis (1983) for the expression of the growth rates (dY/dT) as a function of stand density was used in this study,

$$dY/dT = a(k)^D (D)^C \quad (2)$$

where, Y is basal area or volume

k is e^b

D is stand density

T is time

If we take logarithms for the dual purpose of converting to linear form and obtaining approximate homogeneity of variance,

$$\ln(dY/dT) = \ln(a) + b(D) + c(\ln(D)) \quad (3)$$

This equation is flexible, depending on the values of the parameters b and c and is similar to the equation that models the growth rate/age curves or increment curves (Avery and Burkhart, 1983).

The Third Growth Period

As pointed out in d), page 26, there is inconsistency in the development of the third growth period relatively to the others.

A scatter plot of gross volume increment versus relative density index is shown in Figure 10. It indicates that the observation of control plot number 10 may be responsible for that inconsistency.

Gross volume increment was regressed on relative density index for the data of the third period using equation (3), both with and without plot number 10.

The following results were observed:

- a) with plot number 10 $R^2 = .64$
- without plot number 10 $R^2 = .84$

There is an increase in the coefficient of multiple determination of 20 points when plot 10 is omitted.

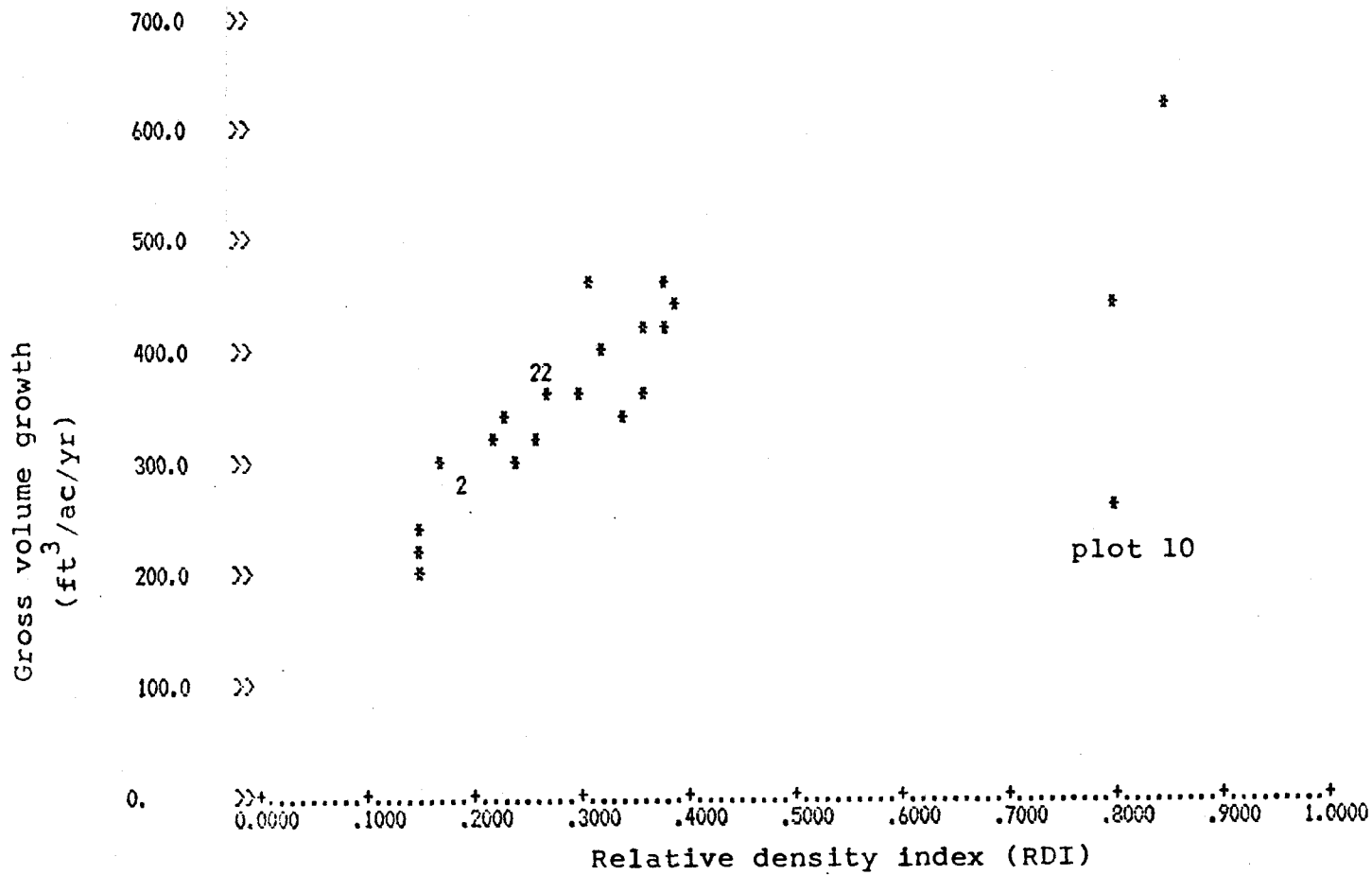


Figure 10. Relative density index versus gross volume growth, for the third growth period, showing plot 10.

b) the studentized deleted residual, $d^*_i = - 5.549$ is highly significant, $t(.99;23) = -2.807$, which means that plot number 10 is an outlying observation.

c) according to Neter et al. (1983), p. 409,

$$\frac{Y(i) - Y_i}{Y_i} = \frac{520-423}{423} = 23\%$$

This means that the omission of plot number 10 observation increases the fitted value by 23 percent.

d) The residual for this observation lies at -3.2 standard deviations from zero.

e) It is suspected that measurement errors in the heights could have occurred (John F. Bell, personal communication).

f) This period is also shorter (2 years), which would magnify the effect of any measurement error.

For all these reasons, plot number 10 was deleted from the data of the third growth period. The scatter plots (treatment means) of gross basal area increment and gross volume increment, versus each of the three measures of stand density, became as shown in Figure 11 to 16.

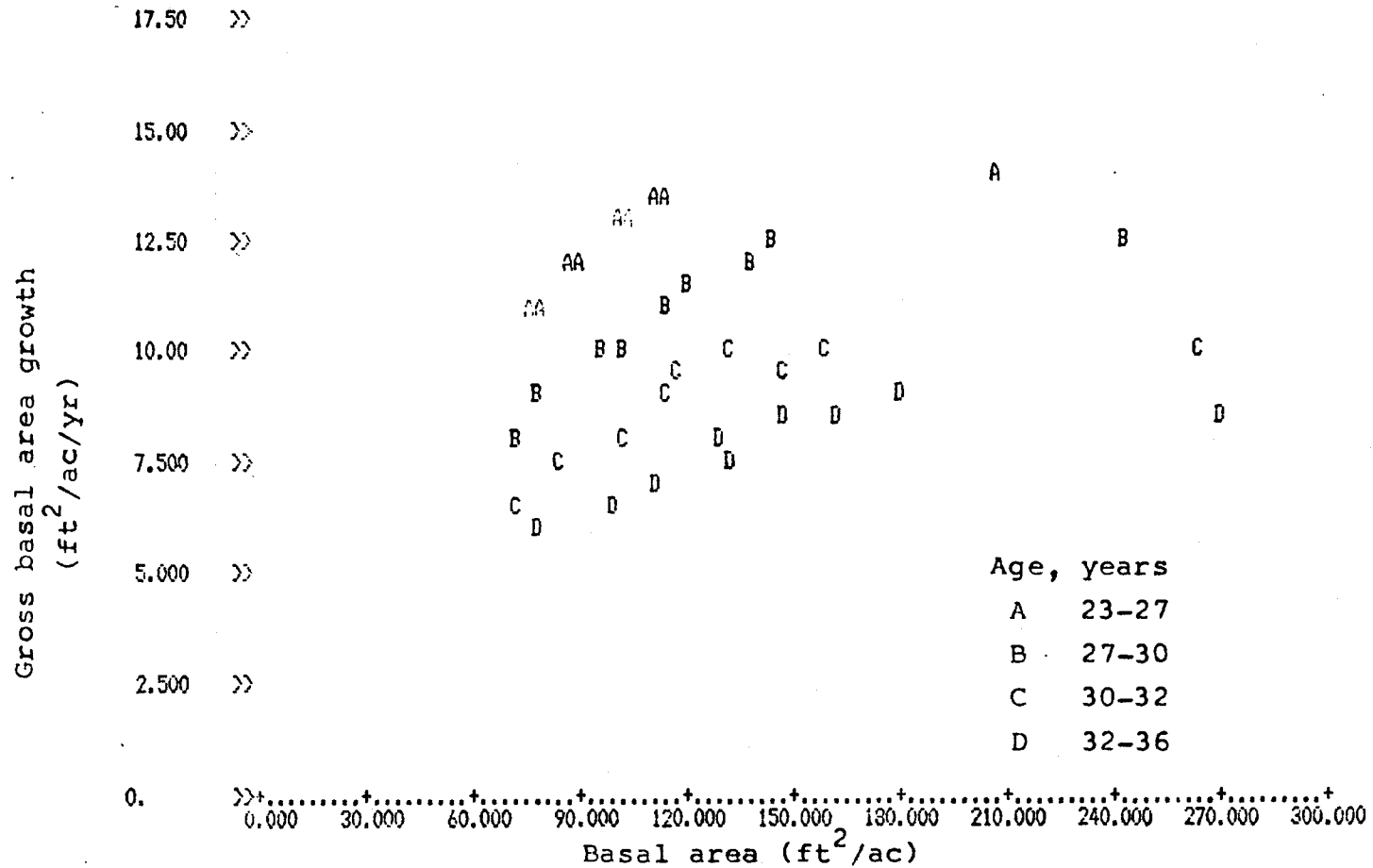


Figure 11. Stand density (basal area) versus gross basal area growth, for the four treatment periods, without plot 10 in the third one.

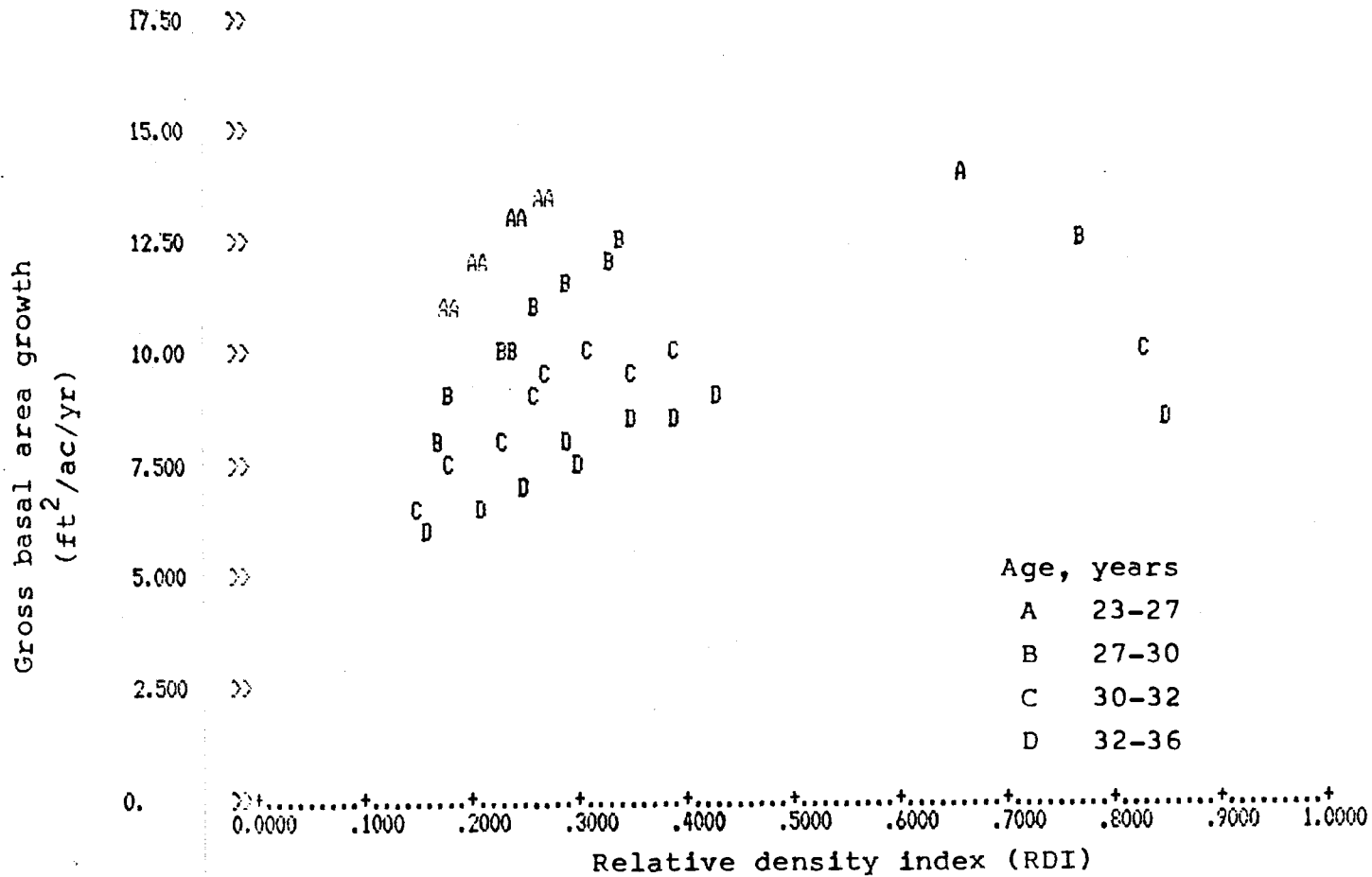


Figure 12. Stand density (RDI) versus gross basal area growth, for the four treatment periods, without plot 10 in the thrid one.

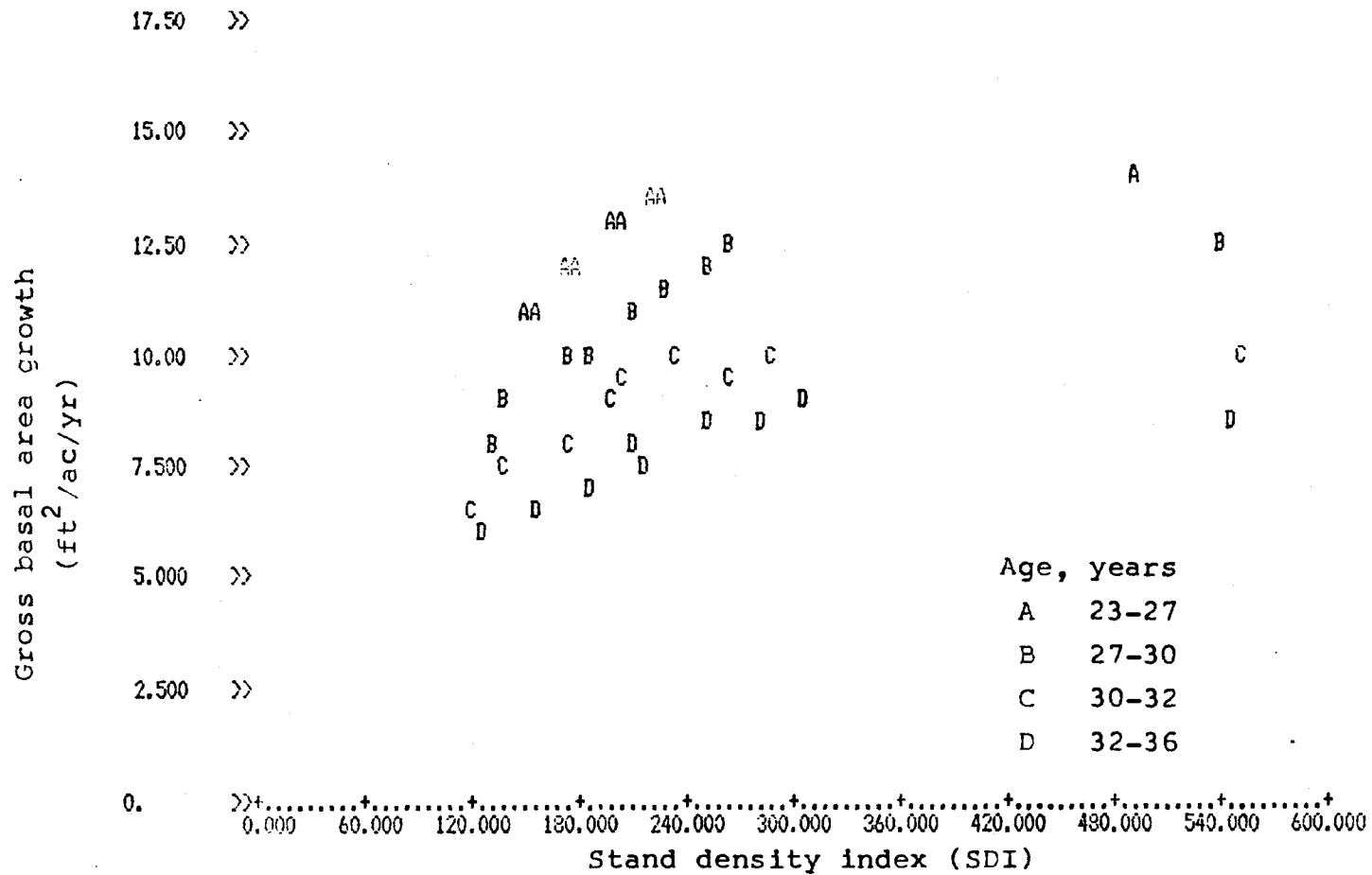


Figure 13. Stand density (SDI) versus gross basal area growth, for the four treatment periods, without plot 10 in the third one.

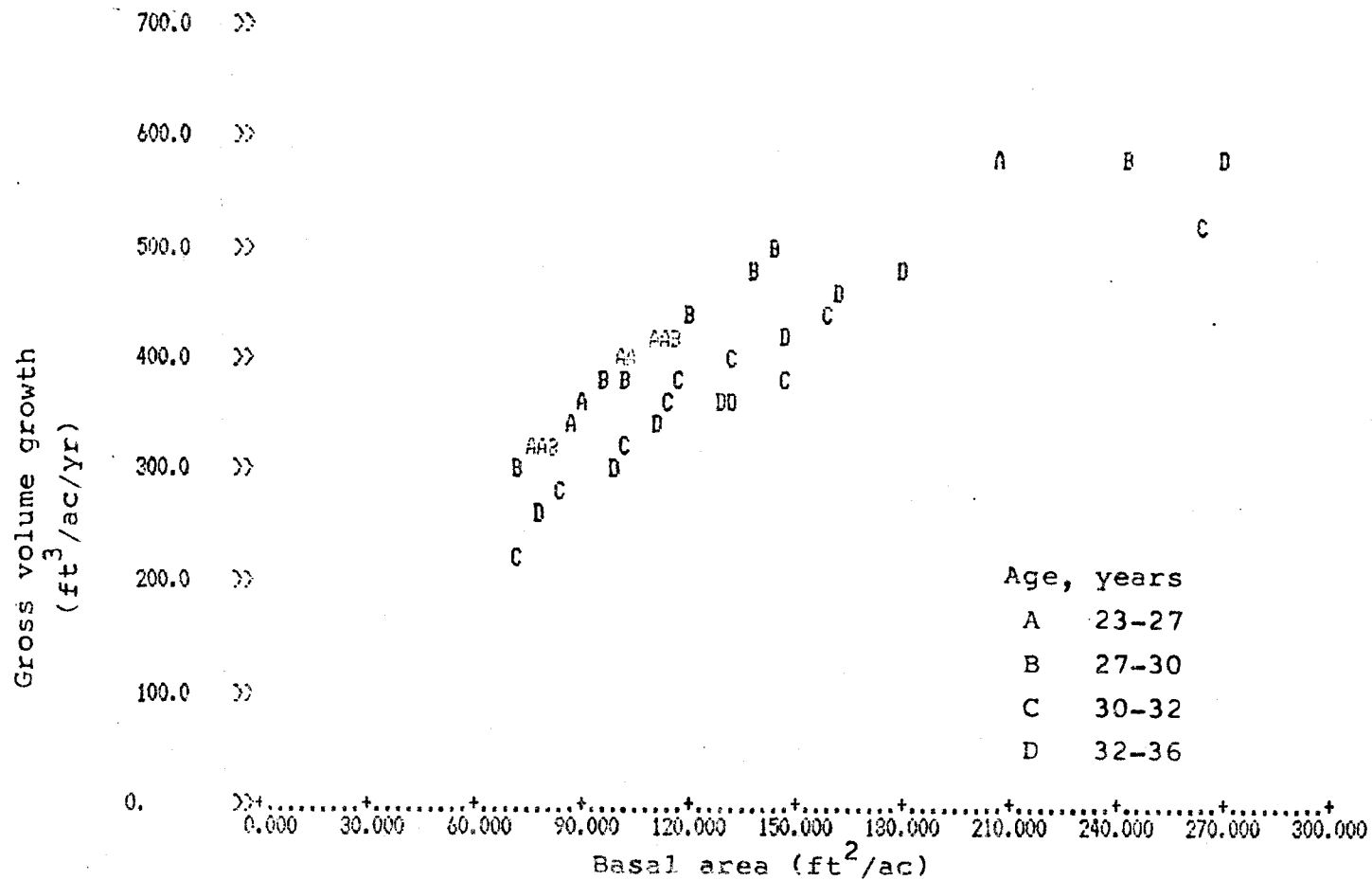


Figure 14. Stand density (basal area) versus gross volume growth, for the four treatment periods, without plot 10 in the third one.

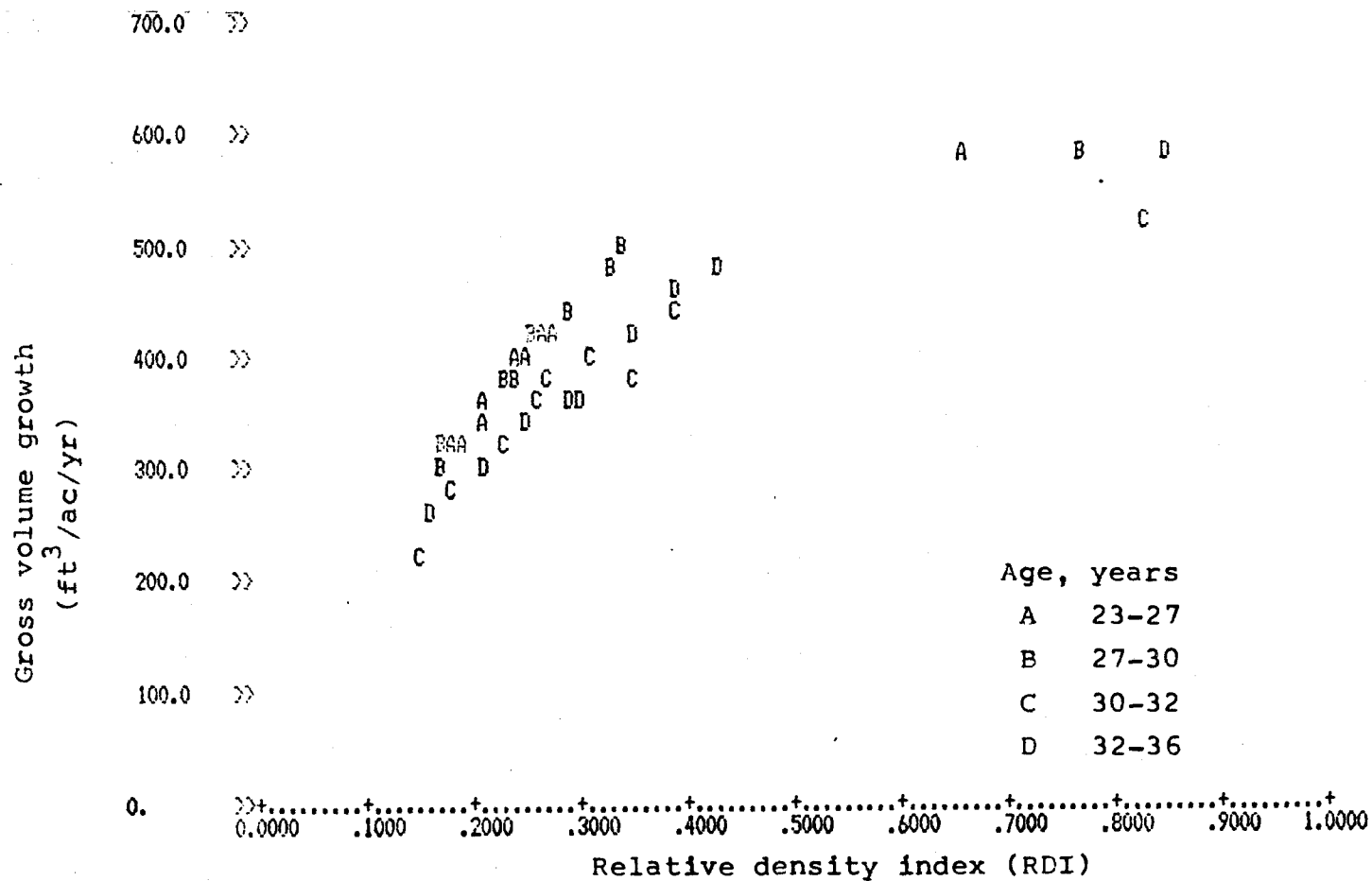


Figure 15. Stand density (RDI) versus gross volume growth, for the four treatment periods, without plot 10 in the third one.

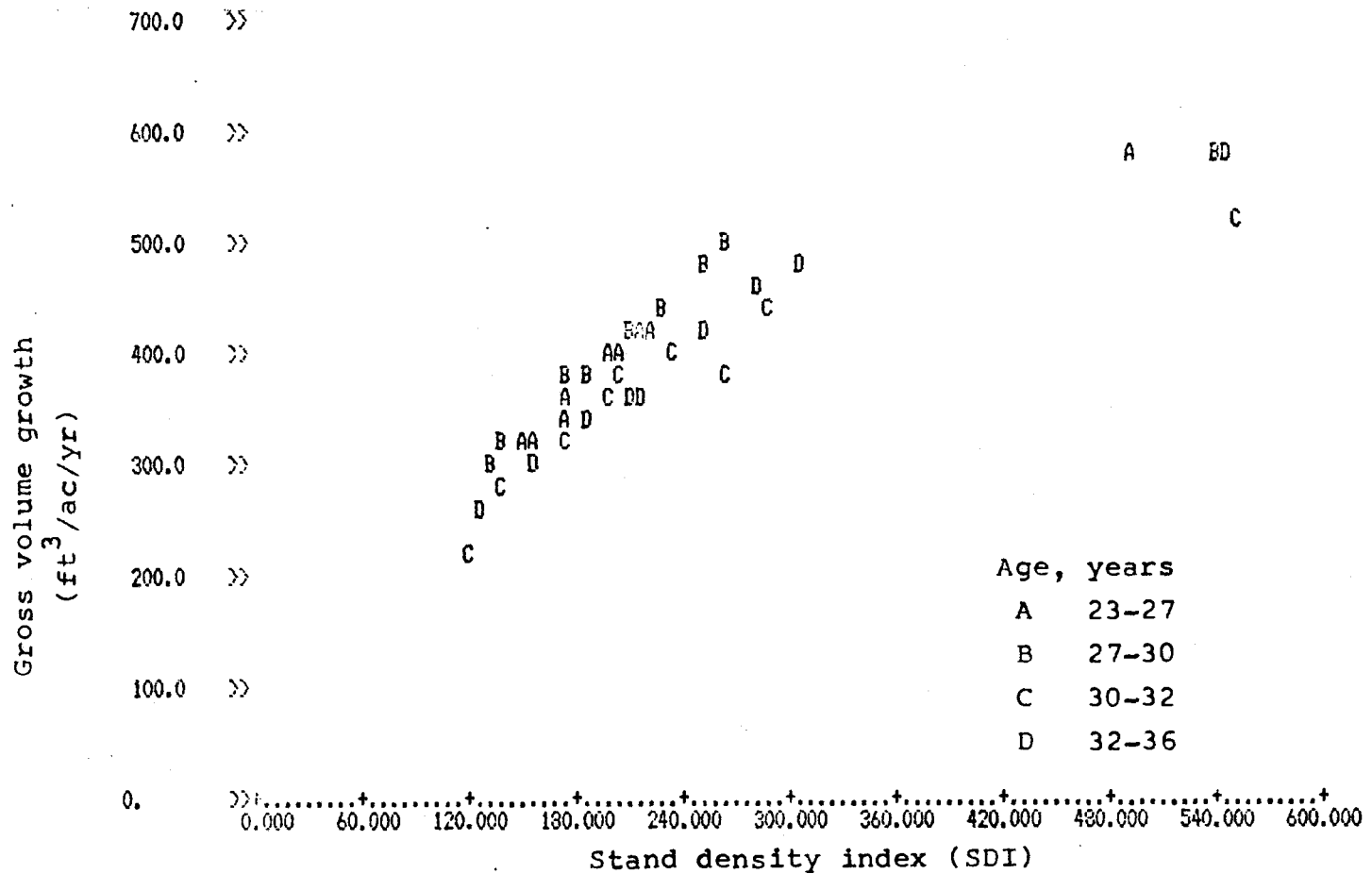


Figure 16. Stand density (SDI) versus gross volume growth, for the four treatment periods, without plot 10 in the third one.

F Test for Lack of Fit

To ascertain the adequacy of the proposed linear model (3) to the data, a F test for lack of fit was performed (Neter et al., 1983). See Appendix A.

It was only developed for the case of relative density index (Drew and Flewelling, 1979) because it was the only measure of stand density studied that provided enough replicate groups in order to conduct the test.

The results proved the adequacy of the model (3) to the relative density index data, both for basal area growth and volume growth. On the other hand, having in mind the similarity of the trends for the three density measures showed in Figures 11 to 16, it is assumed that the same conclusion holds true for the data of the other two remaining density measures.

RESULTS AND DISCUSSION

Gross Basal Area Growth

Table II summarizes the equations developed by density measure and growth period.

For each measure of stand density, both variables, density and $\ln(\text{density})$ were always significant at the 0.01 level.

The coefficients of multiple determination obtained for the different density measures by growth period are always high, suggesting that reasonable estimating equations were obtained.

The analysis of residuals versus the fitted values indicates that the transformation was effective in homogenizing the variance of the error term. Table III summarizes the optimum stand density level for gross basal area growth by density measure and growth period.

Solutions of these equations are shown graphically in Figures 17, 18 and 19.

a) Basal area (Figure 17)

They are relatively flat topped with an apparent maximum for the first period at 175 ft² (14.7 ft²/ac/yr), showing a plateau for the second period between 175-200 ft² (12.8 ft²/ac/yr), for the third period between 200-225

Table II.

Regression Equations for Estimating the Annual Gross Basal Area Growth
(dGBA/dT) Per Acre, Based on Stand Density
at the Middle of the Growth Period.

Density Measure	Growth Period	ln(dGBA/dT) = ln(a) + b(D) + c(ln((D)))			R ²	MSE
		regression coefficients				
		a	b	c		
Basal area (ft ² /ac)	1	.0921	-.0073	1.2303	.86	.0014
	2	.0686	-.0062	1.2219	.94	.0015
	3	.0630	-.5798	1.1848	.87	.0036
	4	.0835	-.0047	1.0561	.84	.0037
RDI (Drew and Flewelling, (1979))	1	85.9263	-2.1150	.9804	.84	.0015
	2	61.9632	-1.7680	.9526	.92	.0018
	3	40.5177	-1.4941	.8421	.84	.0043
	4	27.4784	-1.2182	.7461	.84	.0035
SDI (Reineke, (1933))	1	.0726	-.0030	1.0905	.87	.0012
	2	.0742	-.0026	1.0388	.94	.0013
	3	.0848	-.0025	.9716	.86	.0038
	4	.1020	-.0022	.8939	.85	.0033

Table III

Optimum Stand Density Level for Gross Basal Area
Growth by Density Measure and Growth Period.

Density Measure	Growth Period	Optimum Stand Density Level	Basal Area Growth (ft ² /ac/yr)
Basal area (ft ² /ac)	1	175 (a)	14.7
	2	175 - 200 (b)	12.8
	3	200 - 225 (b)	10.5
	4	225 - 250 (b)	8.9
RDI	1	.45 (a)	15.2
	2	.55 (a)	13.3
	3	.55 - .60 (b)	10.8
	4	.60 (a)	9.1
SDI	1	350 - 375 (b)	15.0
	2	375 - 400 (b)	13.1
	3	375 - 425 (b)	10.7
	4	375 - 450 (b)	9.0

(a) maximum

(b) plateau

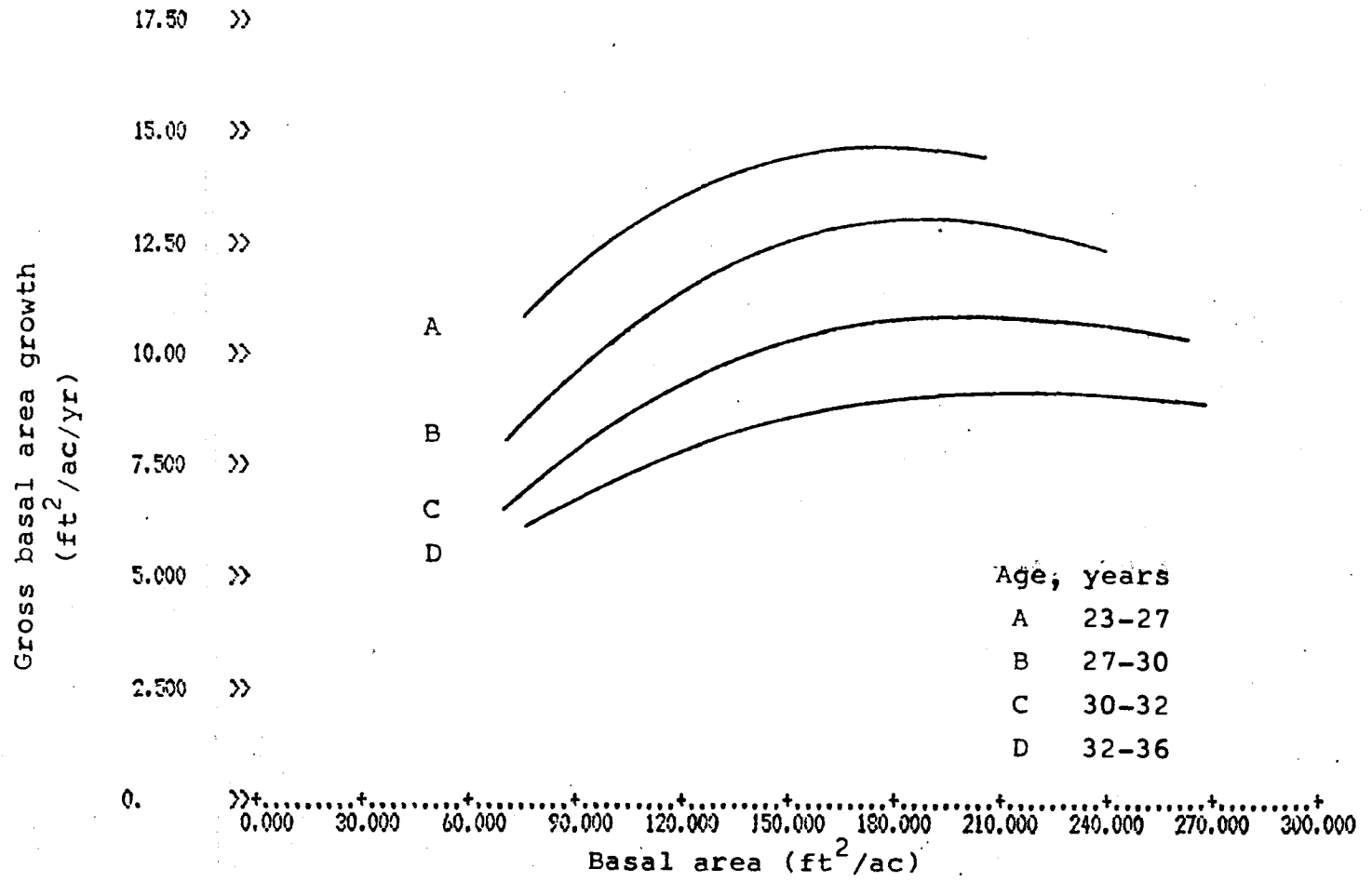


Figure 17. Relation of gross basal area growth to basal area, for the four growth periods.

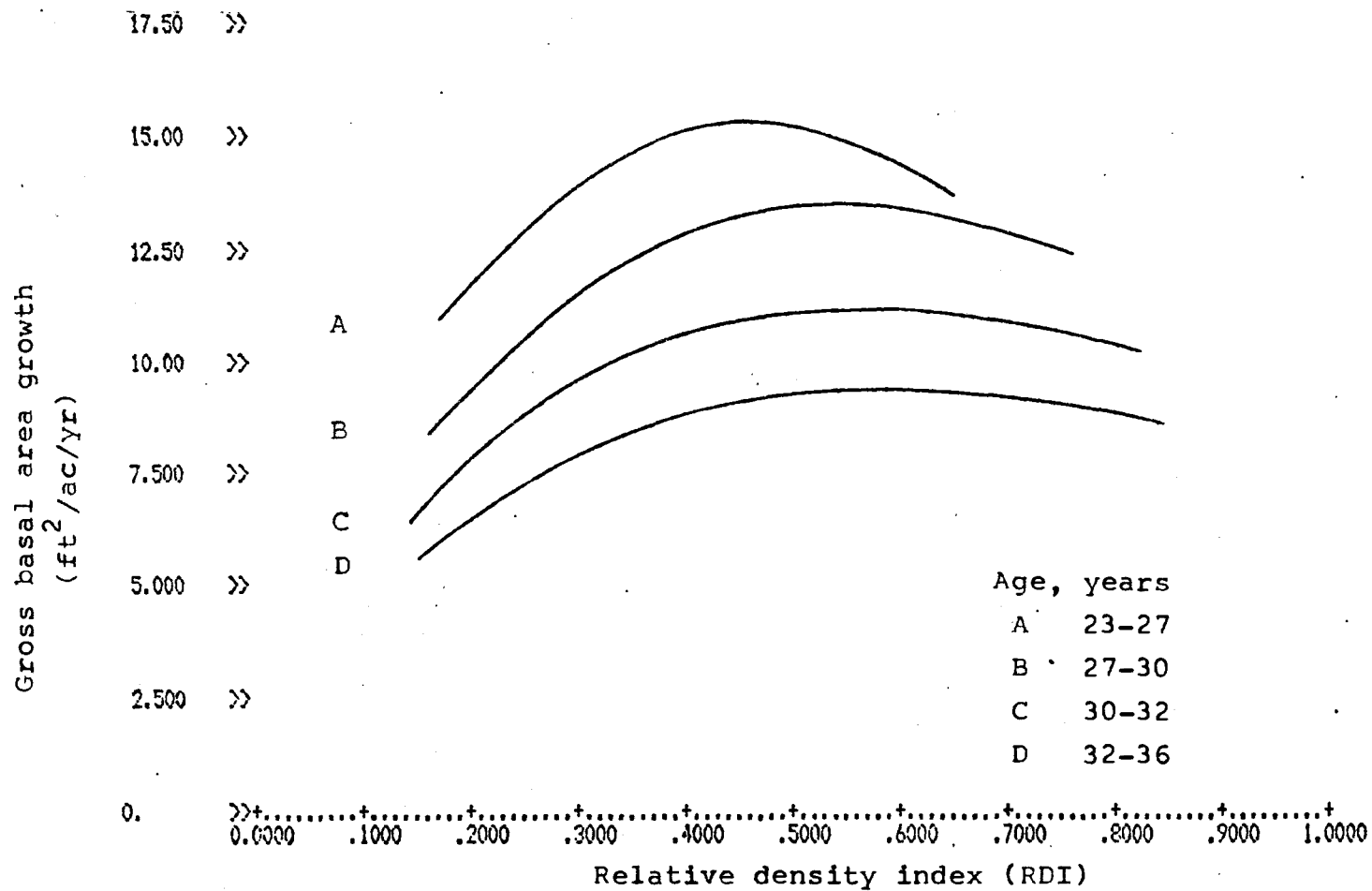


Figure 18. Relation of gross basal area growth to RDI, for the four growth periods.

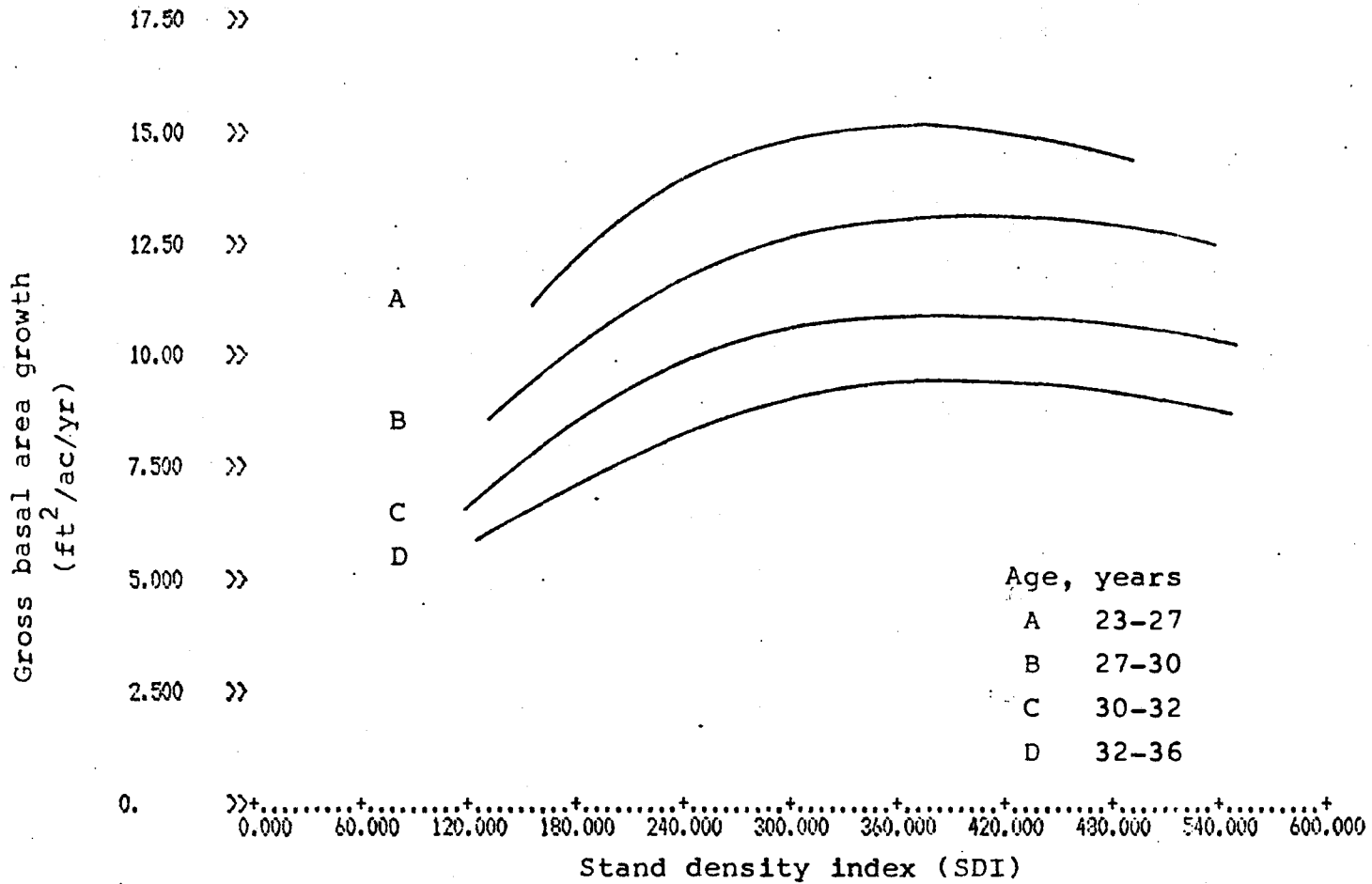


Figure 19. Relation of gross basal area growth to SDI, for the four growth periods.

ft² (10.5 ft²/ac/yr) and for the fourth period between 225-250 ft² (8.9 ft²/ac/yr).

b) Relative density index (Figure 18)

They are relatively flat topped with an apparent maximum for the first period at .45 (15.2 ft²/ac/yr), for the second period at .55 (13.3 ft²/ac/yr), showing a plateau in the third period between .55-.60 (10.8 ft²/ac/yr), and again a maximum in the fourth period at .60 (9.1 ft²/ac/yr).

c) Stand density index (Figure 19)

They are flat topped, showing a plateau, for the first period between 350-375 (15.0 ft²/ac/yr), for the second period between 375-400 (13.1 ft²/ac/yr), for the third period between 375-425 (10.7 ft²/ac/yr) and for the fourth period between 375-450 (9.0 ft²/ac/yr).

It is interesting to note that both Buckman (1962), working with Red pine and Clutter (1963) with Loblolly pine obtained curves for the Basal area growth which, although possessing a maximum were relatively flat, and were similar in their general development to the ones developed in this study.

A visual comparison of the scatter plots in this study (figures 4,5 and 6) with some of the other level-of-growing-stock installations (not shown) indicates that most likely the basal area growth curves will show a

similar development. The differences in growth among these installations would be essentially site class dependent.

Gross Volume Growth

Table IV summarizes the equations developed by density measure and growth period.

For each measure of stand density, both variables, density and $\ln(\text{density})$ were always significant at the 0.01 level, except for two cases: density (basal area) in the first and third periods in which they were significant at the 0.05 level (P values of 0.04 and 0.02, respectively).

The coefficients of multiple determination obtained for the different density measures by growth period are always high. Compared at the period within density measure level, they are generally equal and in some cases higher than the correspondent in the basal area growth equations, suggesting in the same way that reasonable estimating equations were obtained.

The transformation also proved to be effective in homogenizing the variance of the error term. Table V summarizes the optimum stand density level for gross volume growth by density measure and growth period.

Table IV.

Regression Equations for Estimating the Annual Gross Volume Growth
(dGV/dT) per acre, Based on Stand Density at
the Middle of the Growth Period.

Density Measure	Growth Period	ln(dGV/dT) = ln(a) + b(D) + c(ln(D))			R ²	MSE
		a	regression coefficients			
			b	c		
Basal area (ft ² /ac)	1	4.0322	-.3653	1.0722	.92	.0026
	2	2.4568	-.4444	1.1948	.94	.0028
	3	1.8651	-.4074	1.2034	.86	.0093
	4	4.1861	-.1975	.9771	.97	.0020
RDI (Drew and Flewelling, (1979))	1	2546.0540	-1.6056	1.0445	.92	.0029
	2	2522.2333	-1.5525	1.0410	.92	.0036
	3	1739.2165	-1.2377	.9548	.84	.0101
	4	1284.3397	-.7745	.8232	.97	.0019
SDI (Reineke, (1933))	1	2.1398	-.1961	1.0573	.91	.0031
	2	1.9682	-.2166	1.0906	.93	.0035
	3	1.1915	-.1937	1.0525	.84	.0105
	4	3.2987	-.1328	.9349	.97	.0019

Table V.

Optimum Stand Density Level for Gross
Volume Growth by Density Measure
and Growth Period.

Density Measure	Growth Period	Optimum Stand Density Level		Volume Growth (ft ³ /ac/yr)
Basal area (ft ² /ac)	1	207	(c)	577
	2	243	(c)	592
	3	265	(c)	525
	4	270	(c)	584
RDI	1	.65	(a)	572
	2	.65 - .70	(b)	591
	3	.75	(a)	525
	4	.85	(c)	582
SDI	1	491	(c)	573
	2	500	(a)	587
	3	550	(c)	521
	4	553	(c)	580

(a) maximum

(b) plateau

(c) increasing

Solutions of these equations are shown graphically in Figures 20, 21 and 22.

a) Basal area (Figure 20)

They show an increasing trend in all the growth periods, with no indication of a maximum within the range of densities represented in this study.

Adjacent periods (first and second) and (third and fourth) show a similar development.

b) Relative density index (Figure 21)

The first and third growth periods level off as they approach the higher limit of densities at .65 (572 ft³/ac/yr) and at .75 (525 ft³/ac/yr), respectively.

The second growth period shows a plateau between .65-.70 (591 ft³/ac/yr) and the fourth growth period shows no sign of a maximum within the range of densities represented in this study.

c) Stand density index (Figure 22)

The first, third and fourth growth periods show no sign of a maximum within the range of densities represented in this study.

The second period shows a maximum at 500 (587 ft³/ac/yr).

Buckman (1962) and Clutter (1963) working with pines also developed curves for the volume growth, showing an increasing tendency with increasing basal area throughout

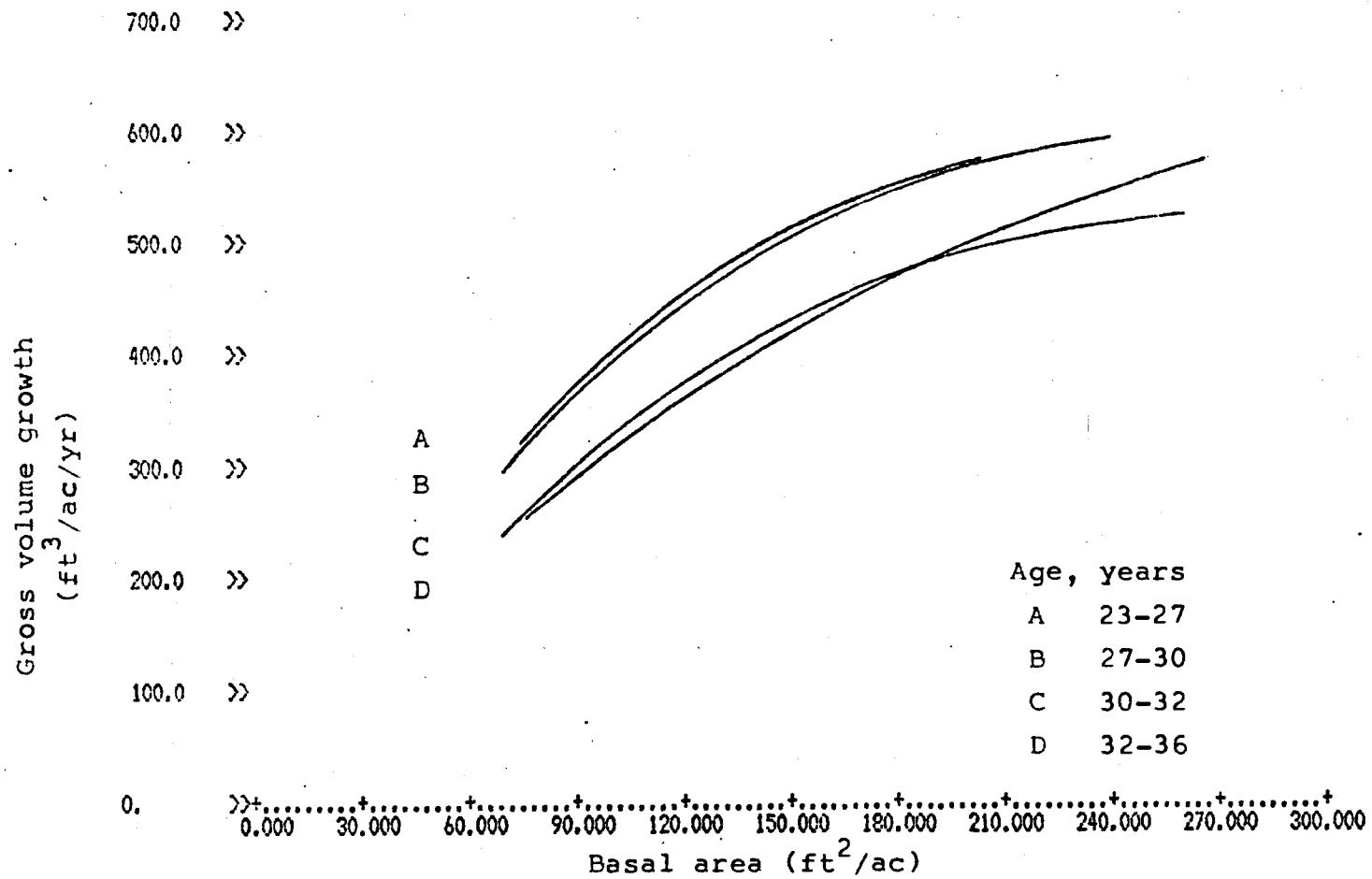


Figure 20. Relation of gross volume growth to basal area, for the four growth periods.

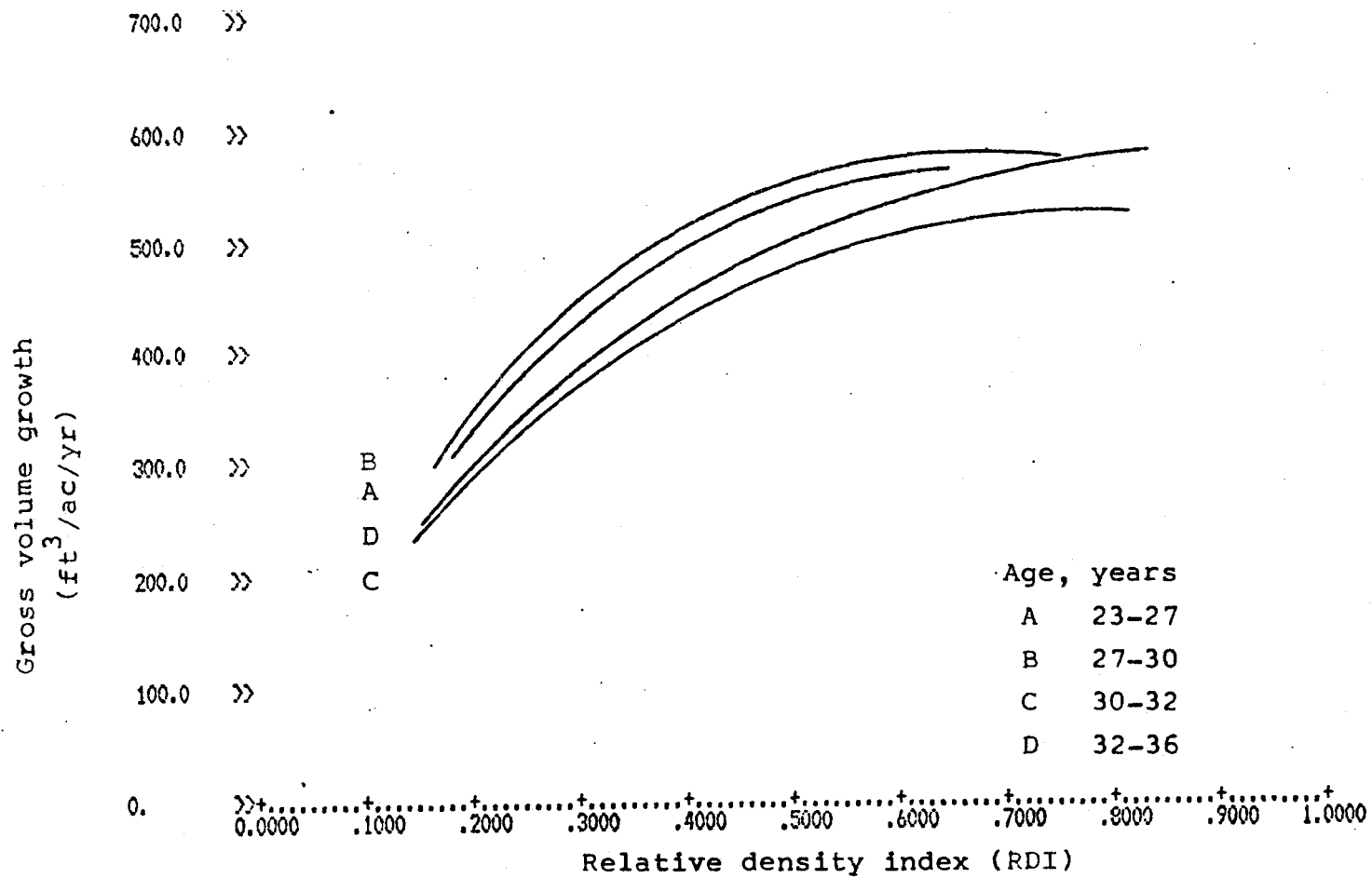


Figure 21. Relation of gross volume growth to RDI, for the four growth periods.

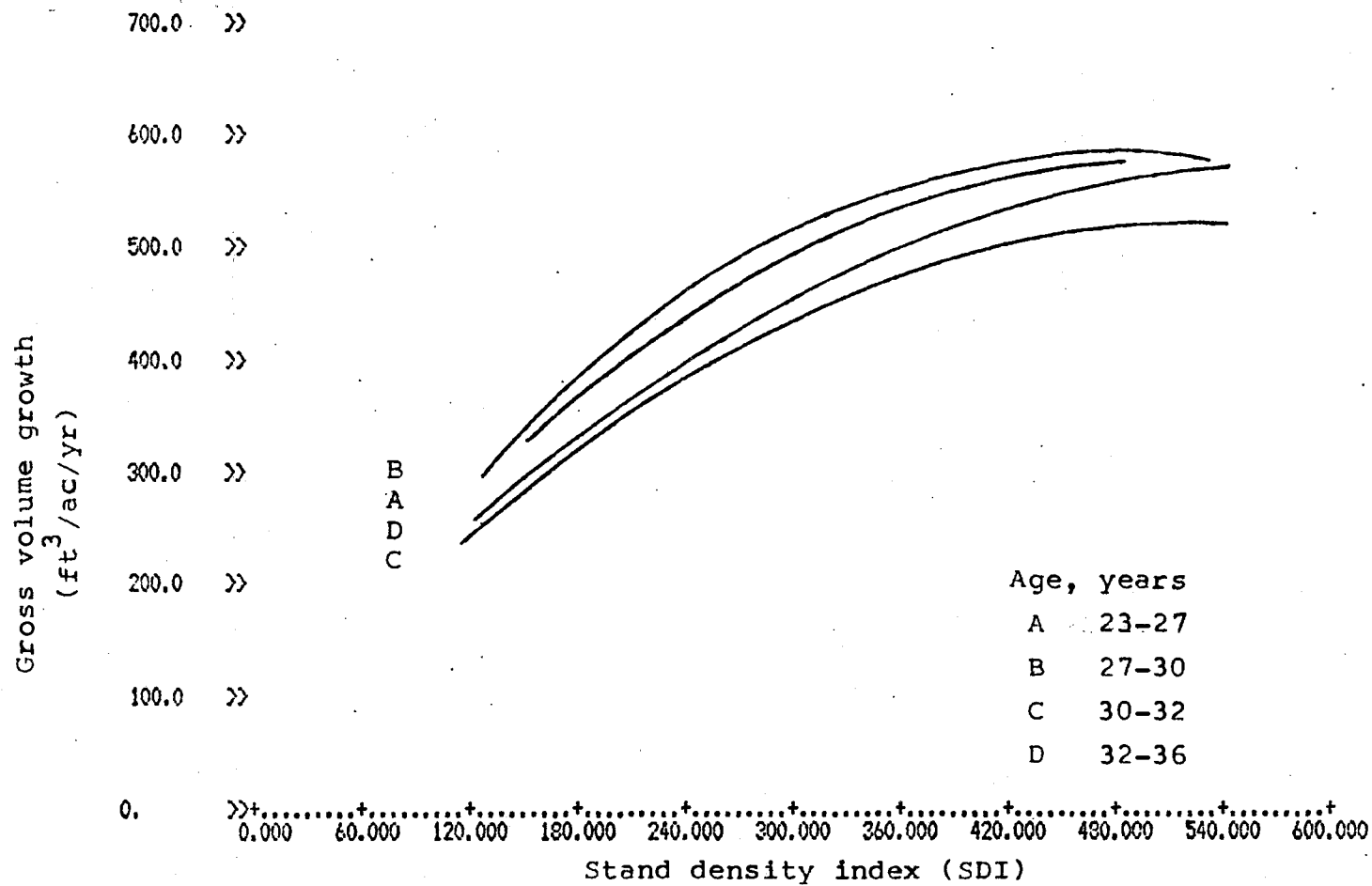


Figure 22. Relation of gross volume growth to SDI, for the four growth periods.

the range of the data. This type of curve development is similar to the general development of the volume curves in this study.

Buckman (1962) argued that a maximum constant volume growth over a broad range of densities will only take place after the initial period of rapid height growth has passed.

On the other hand Evert (1964) in discussing the components of stand volume increment stated that

"the amount of volume resulting from a given increase in basal area is proportional to the height to which this increment is applied, and the amount of volume produced by a given increment in height is proportional to the basal area to which this increment is applied."

This can be expressed by,

$$dV/dT = FB(dH/dT) + FH(dB/dT) \quad (4)$$

where V is stand volume; F is form factor, assumed constant; B is basal area; H is height and T is time.

Evaluation of Density Measures

From the three density measures studies, Basal Area (BA) and Stand Density Index (SDI) are diameter based; the Relative Density Index (RDI) is volume based. RDI and SDI are relative measures; the basal area is a direct measure.

The coefficients of multiple determination (R^2) for the regression equations by growth period for the basal

area, RDI and SDI are used to compare these density measures.

Table VI shows these values for the three density measures by growth period, both for basal area growth and volume growth.

In the case of basal area growth, one can observe that SDI performed better in all growth periods except in the third one, although the difference for the second measure of stand density (the basal area) is quite small (.01).

On the other hand, basal area performed equally or better than RDI in all growth periods.

The density measure, SDI, was selected.

In the case of the volume growth, one can observe that BA performed better in all growth periods.

None of the indices performed clearly better than the other throughout all the growth periods.

The use of basal area as the most adequate expression of stand density in volume growth relationships has been suggested by Dahms (1963) and Nelson and Brender (1966).

On the other hand from Figures 21 and 22, the shift in the relative position of adjacent growth periods (first and second) and (third and fourth) is apparent.

Table VI.

Coefficients of Multiple Determination (R^2) for Basal Area (BA),
Relative Density Index (RDI) and Stand Density Index (SDI),
by Growth Period for the Basal Area Growth and Volume Growth.

	Growth Period	Density Measure		
		BA	RDI	SDI
Basal area growth (ft ² /ac/yr)	1	.86	.84	.87
	2	.94	.92	.94
	3	.87	.84	.86
	4	.84	.84	.85
Volume Growth (ft ³ /ac/yr)	1	.92	.92	.91
	2	.94	.92	.93
	3	.86	.84	.84
	4	.97	.97	.97

This is partially due to the compression of the observations in the horizontal axis, which were already noticed in c), page 26.

It would be expected that those curves would progress according to the age as in the basal area growth curves, (Figures 17 to 19).

The density measure selected, basal area, (Figure 20) is close to this pattern.

Both selected measures, stand density index for the basal area growth and basal area for the volume growth are diameter based.

The maximum basal area growth and volume growth in each growth period shows some fluctuation, which is a result of the existing variability in the plots of each treatment.

CONCLUSIONS

The gross basal area growth curves, especially the one of stand density index (Figure 19) agree with the Lansaeter-Moller hypothesis, which advocates a maximum constant growth over a certain range of stocking densities.

The curve of relative density index (Figure 18) supports the quantitative growth concept, in which the stand growth is at a maximum in the .40 to .55 range (Drew and Flewelling, 1979).

As the stand grows older, the optimum density tends to increase, indicating that the thinning intensity should decrease with the age, for a maximum basal area growth.

From a management standpoint one can say that within the age interval studied (23 to 36 years), the maximum gross basal area growth can be obtained at about 175 to 225 ft²/ac of basal area.

The shape of the gross volume growth curves diverge from the correspondent gross basal area ones.

Contrary to the Langsaeter-Moller hypothesis one observes, in the case of basal area (Figure 20), the absence of a plateau at the higher stocking densities.

The different shape of the curves for the basal area growth and volume growth, reflects the fact that these

young stands are maintaining a fairly rapid and consistent height growth rate, which is a characteristic of this species (Isaac et al., 1958, and Williams, 1968).

The effect of density control (by thinnings) on the basal area growth is not a reliable indicator of its effect on the volume growth.

The reason is that the basal area component of the volume growth can not be ignored because we know that the height growth is not negligible.

The difference between the optimum average density in this study (high site), 8 x 8 feet spacing (approximately) for the basal area growth ($10 \text{ ft}^2/\text{ac}/\text{yr}$) and 6 x 6 feet spacing (approximately) for the volume growth ($450 \text{ ft}^3/\text{ac}/\text{yr}$), and the one for the Wind River, Washington (low site), 10 x 10 feet spacing both for basal area growth ($4.17 \text{ ft}^2/\text{ac}/\text{yr}$) and volume growth ($208 \text{ ft}^3/\text{ac}/\text{yr}$) is due to differences in site class between the two areas.

The application of the results of this study to stands which are similar in site, age and type of treatment, may serve essentially as a guide to potential stand growth, having in mind the differences between growth and yield on small research plots and on large managed forests (Bradley et al., 1966, and Bruce, 1977).

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APPENDIX

APPENDIX A

F TEST FOR LACK OF FIT

To ascertain the adequacy of the proposed model $\ln(dY/dA) = \ln(a) + b(D) + c(\ln(D))$, to the data,

a) the hypothesis to be tested are,

$$H_0: \ln(dY/dT) = \ln(a) + b(D) + c(\ln(D))$$

$$H_a: \ln(dY/dT) \neq \ln(a) + b(D) + c(\ln(D))$$

b) the test statistic is,

$$F^* = \frac{SSE - SSPE}{(n-p) - (n-c)} / \frac{SSPE}{(n-c)} = \frac{SSLF}{(c-p)} / \frac{SSPE}{(n-c)} = \frac{MSLF}{MSPE}$$

c) the decision rule to control the risk of a type I error at 0.01 level of significance is,

if $F^* \leq F(.99; c-p, n-c)$, conclude H_0

if $F^* > F(.99; c-p, n-c)$, conclude H_a

The test is performed for both basal area growth and volume growth as a function of Relative Density Index for each period.

Basal Area Growth

- first period,

$$F^* = \frac{.03638 - .01222}{14 - 3} / \frac{.01222}{27 - 14} = 2.34$$

$$F^* < F(.99; 11, 13)$$

$$2.34 < 4.02$$

Conclusion, the model is adequate

- second period,

$$F^* = \frac{.04255 - .01445}{18 - 3} / \frac{.01445}{27 - 18} = 1.17$$

$$F^* < F (.99; 15, 9)$$

$$1.17 < 4.96$$

Conclusion, the model is adequate

- third period,

$$F^* = \frac{.0978493 - .02706}{17 - 3} / \frac{.02706}{26 - 17} = 1.68$$

$$F^* < F (.99; 14, 9)$$

$$1.68 < 5.00$$

Conclusion, the model is adequate

- fourth period,

$$F^* = \frac{.0841266 - .0203309}{19 - 3} / \frac{.0203309}{27 - 19} = 1.57$$

$$F^* < F (.99; 16, 8)$$

$$1.57 < 5.48$$

Conclusion, the model is adequate

Volume growth

- first period

$$F^* = \frac{.0684614 - .0206698}{14 - 3} / \frac{.0206698}{27 - 14} = 2.73$$

$$F^* < F (.99; 11, 13)$$

$$2.75 < 4.02$$

Conclusion, the model is adequate

- second period,

$$F^* = \frac{.0873962 - .0255193}{18 - 3} / \frac{.0255193}{27 - 18} = 1.45$$

$$F^* < F (.99; 15, 9)$$

$$1.45 < 4.96$$

- third period,

$$F^* = \frac{.231203 - .0545288}{17 - 3} / \frac{.0545288}{26 - 17} = 2.08$$

$$F^* < F (.99; 14, 9)$$

$$2.08 < 5.00$$

Conclusion, the model is adequate

- fourth period,

$$F^* = \frac{.04503 - .009476}{19 - 3} / \frac{.009476}{27 - 19} = 1.88$$

$$F^* < F (.99; 16, 8)$$

$$1.88 < 5.48$$

Conclusion, the model is adequate