## bUCKLING LOADS OF FLAT SANDWICH PANELS IN COMPRESSION

## Buckling of Flat Sandwich Panels

## with All Edges Clamped

## M Original report dated September 1947

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\text { No. 1525-D }
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## Buckling of Flat Sandwich Panels With All

Edges Clamped ${ }^{2}$
(Cores of End-grain Balsa or Cellular Cellulose Acetate and Facings of Aluminum or Glass Cloth Laminate)

## By

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## Summary and Conclusions

This report presents the results of edgewise compression tests to determine the critical loads of flat sandwich panels having all four edges clamped. This investigation was conducted to confirm equations (19) and (20) of the theoretical analysis presented in Forest Products Laboratory Report No. 1525, 4 and modifications introduced subsequently to take into account the effect of transverse shear and of stresses greater than the proportional limit.

IThis report is one of a series of progress reports prepared by the Forest Products Laboratory relating to aircraft. Results here reported are preliminary and may be revised as additional data become available. Original report dated September 1947.
$\underline{2}$ This report is the fifth of a series of reports dealing with the buckling of flat sandwich panels in compression--tested with various types of edge conditions.
${ }^{3}$ Maintained at Madison, Wis., in cooperation with the University of Wisconsin.
${ }^{4}$ March, H. W., and Smith, C. B. Buckling Loads of Flat Sandwich Panels in Compression--Various Types of Edge Conditions, Forest Products Lab. Rpt. 1525. March 1945.

Report No, $1525-\mathrm{D}$

The critical buckling loads, as determined by the tests, were about 82 percent of the computed values. It is believed that the experimental values were lower than the computed values because (1) the testing apparatus at the unloaded edges of the test panels did not rigidly clamp the panels and (2) the formula, which was derived by the energy method, may have yielded buckling loads that were as much as 8 percent too great.

## Introduction

This report is one of a series prepared to confirm the mathematical analysis previously developed ${ }^{4}$ at the Forest Products Laboratory for determining the critical loads of flat sandwich panels supported by various types of edge conditions and modifications to that analysis introduced to take into account the effect of transverse shear and of stresses greater than the proportional limit. It presents the experimental data of edgewise compression tests on the buckling of flat sandwich panels having clamped edges. Information relative to materials, testing procedures, and results is arranged according to the outline used to present similar information in Report No. 1525-A. 5 Duplications of descriptions have been avoided by omitting them and referring to previous reports. $\frac{2}{}$ Where variations from the previous reports were necessary, they are noted and fully described. The preparation and manufacture of specimens, the description of the test panels and coupons, and the procedure for determining the critical loads are identical to those described in Report No. 1525-B. 6 The scope of the work covered by this report differs from that of the work covered by Report No. 1525-B only as the edge conditions differ.

## Methods of Tests

Figure $39{ }^{7}$ - shows the apparatus used to test panels clamped at all four edges. The panels are supported along the loaded edges by clamps (fig. 33 in Report
${ }^{5}$ Boller, K. H. Buckling Loads of Flat Sandwich Panels in Compression-The buckling of flat sandwich panels with edges simply supported, Forest Products Lab. Rpt. 1525-A.
${ }^{6}$ Boller, K. H. Buckling Loads of Flat Sandwich Panels in Compression-The buckling of flat sandwich panels with loaded edges simply supported and the remaining edges clamped, Forest Products Lab. Rpt. 1525-B.
${ }^{7}$ The figures and tables in this report are numbered consecutively with those of Report No. 1525 and its supplements 1525-A, 1525-B, and 1525-C.

No. $1525-C^{8}$ ) and are supported along the vertical edges by guide plates that provide a clamped edge condition (fig. 25 in Report No. 1525-B $\frac{6}{6}$ ). The clamps along the loaded edges support the total width of the panel; the other clamps, however, were made shorter than the length of the panel to provide room for compressive deformation. The distance between the ends of these clamps and the others was about twice the thickness of the panel.

The shear, flexure, and compression tests, made on coupons for the determination of the mechanical properties, were conducted according to methods described in Report No. 1525-B.

## Presentation of Data

Experimental data and the results of computations are presented in tables 18 through 21 and figures 40 through 43. The tables are identical in form and have the same column headings as those of Report No. 1525-B. The descriptions of the columns are the same as presented in that report, except that some of the values were obtained from formulas or by methods that were different from those previously used. The variations from the methods used in Report No. 1525-B were as follows:

Column 19.--The values in column 19 were computed as follows:
(1) For panels having glass cloth facings

$$
P_{\text {cr comp }}=\frac{4}{3} \frac{\pi^{2}}{a^{2}}\left[3 D_{1} \frac{b^{2}}{a^{2}}+3 D_{2} \frac{a^{2}}{b^{2}}+2\left(\frac{E \sigma}{\lambda}\right)_{1,2} I+4 \mu_{1,2} I\right](1)
$$

(2) For panels having aluminum facings

$$
\begin{equation*}
P_{c r \text { comp }}=\frac{4}{3} \frac{\pi^{2}}{a^{2}} \frac{D_{1}+D_{2}}{2}\left[\frac{3 b^{2}}{a^{2}}+\frac{3 a^{2}}{b^{2}}+2\right] \tag{2}
\end{equation*}
$$

The notation employed is that of Report No. 1525 and supplements A, B, and C.

[^0]It should be noted that formulas (1) and (2) are applicable only to panels that are dimensionally proportioned so that they will buckle in a single half wave in the direction of loading. All of the panels in these tests buckled in a single half wave, and in addition several large square panels buckled into two haif waves subsequent to buckling into one half wave. These square panels were border-line cases in which an increase of 10 percent in height would have caused the formation of either one or two half waves at the same load (fig. 5 of Report No. 1525). Panels that buckled in this manner are indicated in the tables by footnotes. After the second half wave developed, these panels continued to support increasing loads until failure occurred.
Column 20. -- In the shear reduction factor, $\frac{1}{1+\eta_{4}}, \eta_{4}$ was computed as follows: -

$$
\begin{equation*}
\eta_{4}=\frac{6 f c P_{c r \operatorname{comp}}}{\left(h^{3}-c^{3}\right)\left[\left(\mu_{y z}\right)_{c}+\left(\mu_{x z}\right)_{c} \frac{b^{2}}{a^{2}}\right]} \tag{3}
\end{equation*}
$$

The values of the shearing moduli that were substituted in the formula were 19, 000 pounds per square inch for balsa and 3,500 pounds per square inch for cellular cellulose acetate.

Columns 21 and 22. - The values of M or N were obtained according to the definitions established in previous reports except that $P_{c r}$ comp and the constant, $\eta_{4}$, in the respective formulas, were computed in accordance with equations (1) or (2) and (3), respectively.

$$
\begin{gather*}
M=\frac{P_{\mathrm{C} \cdot} \operatorname{comp}}{2 \mathrm{f} \mathrm{E}_{\mathrm{f}}}  \tag{4}\\
\mathrm{~N}=\frac{\eta_{4}}{\mathrm{E}_{\mathrm{f}}} \tag{5}
\end{gather*}
$$

Column 23. -- This column presents the values for the critical load obtained by the methods described in Report No. $1525-\mathrm{B}$ but using values of $\eta_{4}, \mathrm{M}$, and $N$ as determined by formulas (3), (4), and (5) of the present report.
$\underline{9}$ This approximate formula was developed by H. W. March at the Forest Products Laboratory.

Figures 40 through 43. --Ratios of observed and computed values (cols. 24, 25 , and 26 of the tables) are presented graphically in figures 40 through 43. Figures 40 and 41 present the data obtained from panels having aluminum facings, and figures 42 and 43 present data obtained from panels having glass cloth facings. Figures 40 and 42 show the relationship of observed values to the values computed by use of formulas (2) and (3) ( $P_{c r}$ comp), and figures 41 and 43 show this relationship when the proper modifications of those formulas are employed. The ordinates and abscissas are the same as those used in Report No. 1525-A.

## Analysis of Results

The critical loads, $P_{c r}$ comp, of the sandwich panels tested are given by equations (19) and (20) of Report No. 1525 under the assumption that the stresses in the facings are less than the proportional limit and that the effect of transverse shear can be neglected. In the present report these equations are modified to apply when these assumptions are not warranted. The application of these modifications resulted in computed values of critical loads that are from 0 to 22 percent greater than the observed values.

From equations (19) and (20) of Report No. 1525 the value of the critical buckling load for stresses in the facings below the proportional limit, when no allowance is made for shear deformations, is given by equation (1) for panels having glass cloth facings and by equation (2) for aluminum facings. These equations apply to panels whose dimensions are such that they will buckle in a single half wave in the direction of the loading.

Values of the elastic properties obtained from tests of coupons of each plate were substituted in equation (1) or (2). . The critical loads thus calculated are presented in columns 19 of the tables. Comparisons of these values with those obtained by tests are shown in figures 40 and 42. The plotted points fall to the right of the 45 -degree line indicating that the computed values are greater than the observed values and that the modifications of formulas (1) and (2), subsequently described, are necessary.

Modification for shear. -- The factor employed to modify formulas (1) and (2) that takes into account the effect of transverse shearing deformations in the core is $\frac{1}{1+\eta_{4}}$, in which values of $\eta_{4}$ are obtained from equations (3) and $P_{\text {cr comp }}$ is obtained from equation (1) or (2).

The values of this factor for the panels tested are given in columns 20 of the tables for those panels that buckled below the proportional limit. The effect of this reduction factor on the theoretical values is shown by comparing the plotted points on figures 42 with those on figure 43. The buckling stresses of these panels were all below the proportional limit. All of the computed values in figure 43 are closer to the 45 -degree line than those in figure 42. The application of the factor reduces the computed values by amounts ranging from 1 to 25 percent. Most of the observed values are lower than the computed values even after this factor is applied.

A somewhat similar effect is shown in figures 40 and 41, which are for panels having aluminum facings. The shear reduction factor was applied to values below the proportional limit (coordinates of about 0.4 on figure 40) and the result of this application is shown on figure 41. These modified values are more nearly equal to the observed values.

Modification for shear and for stresses above the proportional limit. -- The three preceding reports in this series have shown the procedure for the simultaneous application of these two modifications to the formulas given in Report No. 1525. The values of $M$ and $N$ are given by equations (4) and (5).

The values obtained are presented in columns 23 of the tables. A comparison of figure 40 with figure 41 shows the effect of these two modifications on the values obtained by use of equation (2). The computed values shown on figure 40 are reduced but are still greater than the observed values.

The observed values are between 80 and 98 percent of the values computed by use of the equations of Report No. 1525 with the modifications described in the present report. It is believed that the observed values are lower than the computed values because (1) the clamped edges of the test panels were not secured as firmly by the testing apparatus as had been assumed in the derivation of the formulas and (2) the formulas were derived by the energy method and may sometimes lead to buckling loads that are as much as 8 percent too great.



[^1]Z M 74252 F



[^2]


[^3][^4]Tx


[^5]

Figure 39.--Apparatus for testing sandwich panels with all



Figure 41.--Observed critical load plotted against the computed critical load
(modified for reduced modulus and shear); both expressed as ratios of the com-
pressive strength of the sandwich construction. Gritical buckling of panels
faced with aluminum and clamped on all four edges.
 Figure 42--Observed critical load plotted against the computed critical load (no modifications); both expressed as ratios of the compressive strength of the laminate and clamped on all edges.

Figure 43.--Observed critical load plotted against computed critical load (modi-
fied for shear); both expressed as ratios of the sandwich construction. Critical buckling of panels faced with glass cloth laminate and clamped on all edges.
Z M 74259 F

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[^0]:    $\underline{8}_{\text {Boller, K }}$ K. H. Buckling Loads of Flat Sandwich Panels in Compression-The buckling of flat sandwich panels with loaded edges clamped and the remaining edges simply supported, Forest Products Lab. Rpt. 1525-C.

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