Supplement to
EFFECTS OF SHEAR DEFORMATION IN THE CORE OF A FLAT RECTANGULAR SANDWICH PANEL

DEFLECTION UNDER UNIFORM LOAD OF SANDWICH PANELS HAVING FACINGS OF MODERATE THICKNESS

December 1951

INFORMATION REVIEWED
AND REAFFIRMED
1958

**This Report is One of a Series
Issued in Cooperation with
AIR FORCE-NAVY-CIVIL SUBCOMMITTEE
on
AIRCRAFT DESIGN CRITERIA
Under the Supervision of the
AIRCRAFT COMMITTEE
of the
MUNITIONS BOARD**

No. 1583-D

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
FOREST PRODUCTS LABORATORY
Madison 5, Wisconsin
In Cooperation with the University of Wisconsin

Supplement to

EFFECTS OF SHEAR DEFORMATION IN THE CORE OF A FLAT RECTANGULAR SANDWICH PANEL

Deflection Under Uniform Load of Sandwich Panels Having Facings of Moderate Thickness[1]

By

WILHELM S. ERICKSEN, Mathematician

Forest Products Laboratory,[2] Forest Service
U. S. Department of Agriculture

Introduction

This report supplements the mathematical analysis of the deflection of a simply supported rectangular sandwich panel under uniform transverse load given in U. S. Forest Products Laboratory Report No. 1583-C.[3] Formulas are derived and curves presented for determining the central deflection and the maximum stresses and strains in the facings and core of a panel having isotropic core and facing materials. These formulas and curves incorporate the effects of the transverse shear deformations in the core. They also take into account the effects of the bending of the facings about their

---

[1]This progress report is one of a series prepared and distributed by the Forest Products Laboratory under U. S. Navy Bureau of Aeronautics Order No. NAer 01019, 01077, and 00938, Amendments Nos. 1 and 2, and U. S. Air Force No. USAF-(33-038)(51-4066-E and 51-4326-E). Results here reported are preliminary and may be revised as additional data becomes available.

[2]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3]Ericksen, W. S., Deflection Under Uniform Load of Sandwich Panels Having Facings of Unequal Thickness. Forest Products Laboratory Report No. 1583-C, 1950.

own middle surfaces and therefore extend into the range of moderately thick facings. In the analysis the facings are considered as thin plates in the sense that transverse shear deformations within them are neglected.

The analysis in Report No. 1583-C led to a double Fourier expansion for the deflection. This expansion was transformed to the M. Levy form, which is more suitable for use in computations, under the assumptions that the facing and core materials were isotropic, and the facings could be considered as membranes. In the present report, a similar transformation is applied to the double Fourier series derived under the assumption that the facings bend about their own middle surfaces.

Specific formulas for the components of stress and strain in the facings and core were not derived in Report No. 1583-C. Such formulas, however, are readily derived from other formulas developed in that report and are given in the present report in double Fourier form for a sandwich panel having orthotropic core and facing materials. These Fourier expansions, reduced to the case of isotropic core and facing materials, are then transformed into forms more suitable for computations.

The mathematical analysis in Report No. 1583-C, and consequently that in the present report, are based on a number of simplifying assumptions that include the following: (a) the effect of the transverse shear deformations in the facings is negligible; (b) the effect of the bending of the core is negligible; (c) the transverse shear strains in the core are constant over the thickness of the core; and (d) the core and facing materials are not stressed beyond their proportional limits. These assumptions have been widely used in sandwich-plate theory and have been proved applicable to various types of problems by comparisons with results obtained in actual tests. The possibility that one or more of these assumptions may not be applicable to a particular construction and load should be kept in mind, however, in the application of results derived under such assumptions.

In the present analysis, a number of operations are performed on single and double series. These operations have been proved valid by considerations not included in this report.

Notation

a, b	dimensions of the panel
c	thickness of the core
E_f	Young's modulus of isotropic facings
$e_{xx}^{(i)}$, $e_{yy}^{(i)}$	components of strain in facings i, $i = 1, 2$
$e_{zx}^{(c)}$, $e_{yz}^{(c)}$	components of shear strain in the core
f_1, f_2	thicknesses of facings

p	uniform load per unit area
x, y	coordinates with axes shown in figure 1
w	normal deflection of panel
$Z_1 \quad Z_2$	coordinates with axes shown in figure 2
$\lambda_f = 1 - \sigma^2$	
μ'_{zx}, μ'_{yz}	shear moduli of orthotropic core
μ'	shear modulus of isotropic core
σ	Poisson's ratio of isotropic facing material

Results and Discussion

Formulas for determining the central deflection and the components of stress and strain in the facings and core of a uniformly loaded sandwich panel are given in this section. These formulas, which are derived in this report, apply to a rectangular panel having simply supported edges, isotropic facings, and either a core material that is isotropic or one in which $\mu'_{zx} = \mu'_{yz}$.

The following parameters, in addition to those defined in the notation, determine the deflection and components of stress and strain:

$$I_f = \frac{f_1^3 + f_2^3}{12} \tag{1}$$

$$I = \frac{f_1 f_2}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right)^2 \tag{2}$$

$$S = \frac{c f_1 f_2 \pi^2 E_f}{(f_1 + f_2) a^2 \lambda_f \mu'} \tag{3}$$

$$\delta = \sqrt{\frac{S I_f}{I + I_f}} \tag{4}$$

The parameters have the following significance: I_f is the sum of the moments of inertia of the facings about their respective middle surfaces; I is the moment of inertia of the spaced facings about the centroidal axis with the stiffness of the individual facings neglected; S is the shear-flexibility co-efficient of a core, and δ is a parameter that determines mainly the effect of the bending of the facings about their own middle surfaces.

The central deflection is given by the formula

$$w_{max} = \frac{a^4 p \lambda_f}{E_f(I + I_f)} \left[A + \frac{ISA_2}{(I + I_f)} \right] \tag{5}$$

in which parameters A_1 and A_2 are determined from figures 3 and 4, respectively.

The strain components in facing i, i = 1 or 2 according as the thickness of the facing is f_1 or f_2 as shown in figure 2, are maximum with respect to x and y at the center of the panel. Reference to (3.4), (3.10), and (3.13) indicates that these components can be expressed in the forms

$$e_{xx}(i)_{max} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \left[M_i B_1 + S N_i B_2 \right] \tag{6}$$

$$e_{yy}(i)_{max} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \left[M_i C_1 + S N_i C_2 \right] \tag{7}$$

in which parameters B_1, B_2, C_1, and C_2 are determined from figures 5, 6, 7, and 8 respectively, and

$$\left.\begin{aligned}
M_1 &= Z_1 - \frac{f_2}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right) \\
N_1 &= \left\{ Z_1 I + \frac{I_f f_2}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right) \right\} \frac{1}{(I + I_f)} \\
M_2 &= Z_2 + \frac{f_1}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right) \\
N_2 &= \left\{ Z_2 I - \frac{I_f f_1}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right) \right\} \frac{1}{(I + I_f)}
\end{aligned}\right\} \tag{8}$$

Figure 2 indicates the origins of the coordinates Z_1 and Z_2. The corresponding components of stress are given in terms of the same quantities by the formulas

$$X_{xi\ max} = \frac{a^2 p}{(I + I_f)} \left[M_i (B_1 + \sigma C_1) + SN_i \, (B_2 + \sigma C_2) \right] \tag{9}$$

and

$$Y_{yi\ max} = \frac{a^2 p}{(I + I_f)} \left[M_i (\sigma B_1 + C_1) + SN_i (\sigma B_2 + C_2) \right] \tag{10}$$

The maximum shear-strain component in the core occurs at the center of the longer side, length b, and is given by the formula

$$e^{(c)}_{zx\ max} = \frac{a^3 p \lambda_f}{E_f(I + I_f)} \left(1 + \frac{f_1 + f_2}{2c} \right) SD \tag{11}$$

in which parameter D is determined from figure 9. The strain component e_{yz} evaluated at $x = \frac{a}{2}$, $y = 0$, is given by the formula

$$e^{(c)}_{yz\ max} = \frac{a^3 p \lambda_f}{E_f(I + I_f)} \left(1 + \frac{f_1 + f_2}{2c} \right) SF \tag{12}$$

in which the parameter F is determined from figure 10.

1. The M. Levy Form of the Expansion for the Deflection Under Uniform Transverse Load.

The double Fourier form of the expansion for the deflection under uniform load of a simply supported panel having isotropic facing and core materials can be obtained from formulas (A38), (A41), and (A42) of Report No. 1583-C in the form

$$w = \frac{16 a^4 p \lambda_f}{\pi^6 I E_f} \sum_m \sum_n \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn \left[\frac{I_f}{I} (m^2 + n^2\rho^2)^2 + \frac{(m^2 + n^2\rho^2)^2}{1 + S\,(m^2 + n^2\rho^2)} \right]} \tag{1.1}$$

where

$$\rho = \frac{a}{b} \qquad (1.2)$$

$$I = \frac{f_1 f_2}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right)^2 \qquad (1.3)$$

$$I_f = \frac{f_1^3 + f_2^3}{12} \qquad (1.4)$$

$$S = \frac{c f_1 f_2 \, \pi^2 E_f}{(f_1 + f_2) a^2 \lambda_f \mu'} \qquad (1.5)$$

$\underline{x}$ and $\underline{y}$ are coordinates with axes as shown in figure 1, and the symbols $\sum_m$ and $\sum_n$ denote summation over all positive odd integers $\underline{m}$ and $\underline{n}$, respectively. Formula (1.1) can be written

$$w = K \sum_m \sum_n \frac{\{1 + S(m^2 + n^2\rho^2)\} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}}{mn\,(m^2 + n^2\rho^2)^2 \{1 + \delta^2 (m^2 + n^2\rho^2)\}} \qquad (1.6)$$

where

$$K = \frac{16 a^4 p \lambda_f}{\pi^6 E_f (I + I_f)} \qquad (1.7)$$

and

$$\delta = \sqrt{\frac{S I_f}{I + I_f}} \qquad (1.8)$$

The M. Levy form of the expansion for $\underline{w}$ is obtained from (1.6) by summing with respect to $\underline{n}$. In this summation, it is convenient to consider $\underline{w}$ as the sum

$$w = w_1 + w_2 \tag{1.9}$$

where

$$w_1 = K \sum_m \sum_n \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn(m^2 + n^2\rho^2)^2\{1 + \delta^2(m^2 + n^2\rho^2)\}} \tag{1.10}$$

and

$$w_2 = SK \sum_m \sum_n \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn(m^2 + n^2\rho^2)\{1 + \delta^2(m^2 + n^2\rho^2)\}} \tag{1.11}$$

Consider series (1.10) and let

$$W_m = \sum_n \frac{\sin \frac{n\pi y}{b}}{\delta^2 mn(m^2 + n^2\rho^2)^2(\epsilon_m^2 + n^2\rho^2)} \tag{1.12}$$

where

$$\epsilon_m^2 = \frac{1}{\delta^2} + m^2 \tag{1.13}$$

Evidently

$$w_1 = K \sum_m W_m \sin \frac{m\pi x}{a} \tag{1.14}$$

With the use of (1.13), series (1.12) can be written

$$W_m = \sum_n \left[\frac{1}{mn(m^2 + n^2\rho^2)^2} + \frac{\delta^2}{mn(\epsilon_m^2 + n^2\rho^2)} - \frac{\delta^2}{mn(m^2 + n^2\rho^2)} \right] \sin \frac{n\pi y}{b} \tag{1.15}$$

From equation (A49), in Report No. 1583-C

$$\sum_n \frac{\sin \frac{n\pi y}{b}}{mn(m^2 + n^2 \rho^2)^2} =$$

$$\frac{\pi}{4m^5}\left[1 - \frac{\left\{2 + \alpha_m \tanh \alpha_m\right\}\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right) - \frac{m\pi}{a}\left(y - \frac{b}{2}\right)\sinh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{2 \cosh \alpha_m}\right] \qquad (1.16)$$

where

$$\alpha_m = \frac{m\pi}{2\rho} \qquad (1.17)$$

and from (A 48) of the same report,

$$\sum_n \frac{\sin \frac{n\pi y}{b}}{mn(m^2 + n^2 \rho^2)} = \frac{\pi}{4m^3}\left[1 - \frac{\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{\cosh \alpha_m}\right] \qquad (1.18)$$

From the last expression, it is evident that

$$\sum_n \frac{\sin \frac{n\pi y}{b}}{mn(\epsilon_m^2 + n^2 \rho^2)} = \frac{\pi}{4m\epsilon_m^2}\left[1 - \frac{\cosh \frac{\pi\epsilon_m}{a}\left(y - \frac{b}{2}\right)}{\cosh \beta_m}\right] \qquad (1.19)$$

where

$$\beta_m = \frac{\pi\epsilon_m}{2\rho} \qquad (1.20)$$

Substituting formulas (1.16), (1.18), and (1.19) into formula (1.15) gives

$$W_m = \frac{\pi}{4m^5}\left[1 - \frac{\left(2 + \alpha_m \tanh \alpha_m\right)\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right) - \frac{m\pi}{a}\left(y - \frac{b}{2}\right)\sinh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{2 \cosh \alpha_m}\right]$$

$$+ \frac{\pi\delta^2}{4m\epsilon_m^2}\left[1 - \frac{\cosh \frac{\pi\epsilon_m}{a}\left(y - \frac{b}{2}\right)}{\cosh \beta_m}\right] - \frac{\pi\delta^2}{4m^3}\left[1 - \frac{\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{\cosh \alpha_m}\right] \tag{1.21}$$

If this expression is substituted into (1.14), the terms that are independent of hyperbolic function can be summed. The required summations are:

$$\sum_m \frac{\sin \frac{m\pi x}{a}}{m^5} = \frac{\pi^5}{96a^4}\left(x^4 - 2\,ax^3 + a^3x\right) \tag{1.22}$$

$$\sum_m \frac{\sin \frac{m\pi x}{a}}{m^3} = -\frac{\pi^3}{8a^2}\left(x^2 - ax\right) \tag{1.23}$$

and

$$\sum_m \frac{\sin \frac{m\pi x}{a}}{m\,\epsilon_m^2} = \sum_m \frac{\sin \frac{m\pi x}{a}}{m\left(\frac{1}{\delta^2} + m^2\right)} = \frac{\pi\delta^2}{4}\left(1 - \frac{\cosh \frac{\pi}{a\delta}\left(x - \frac{a}{2}\right)}{\cosh \frac{\pi}{2\delta}}\right) \tag{1.24}$$

Formula (1.24) was obtained from (1.18) by interpreting $\frac{m^2}{\rho^2}$ as $\frac{1}{\delta^2}$. The substitution of (1.21) into (1.14) and the use of (1.22), (1.23), and (1.24) gives

$$w_1 = \frac{a^4 p \lambda f}{E_f(I + I_f)} \left[\frac{x^4 - 2ax^3 + a^3x}{24a^4} + \frac{\delta^2(x^2 - ax)}{2\pi^2 a^2} + \frac{\delta^4}{\pi^4}\left(1 - \frac{\cosh \frac{\pi}{a\delta}\left(x - \frac{a}{2}\right)}{\cosh \frac{\pi}{2\delta}}\right) \right.$$

$$- \frac{4}{\pi^5} \sum_m \left\{ \frac{\left(2 + \alpha_m \tanh \alpha_m\right) \cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right) - \frac{m\pi}{a}\left(y - \frac{b}{2}\right) \sinh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{2m^5 \cosh \alpha_m} \right.$$

$$\left. \left. + \frac{\delta^2 \cosh \frac{\pi\epsilon_m}{a}\left(y - \frac{b}{2}\right)}{m\,\epsilon_m^2 \cosh \beta_m} - \frac{\delta^2 \cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{m^3 \cosh \alpha_m} \right\} \sin \frac{m\pi x}{a} \right] \qquad (1.25)$$

The formula for $\underline{w_1}$ for a plate of width $\underline{a}$ and of infinite length consists of the terms in this expression that have been summed with respect to $\underline{m}$.

The summation of the series (1.11) with respect to $\underline{n}$ and partially with respect to $\underline{m}$ can be accomplished similarly.

Let

$$U_m = \sum_n \frac{\sin \frac{n\pi y}{b}}{\delta^2 mn(m^2 + n^2\rho^2)(\epsilon_m^2 + n^2\rho^2)} \qquad (1.26)$$

so that

$$w_2 = KS \sum_m U_m \sin \frac{m\pi x}{a} \qquad (1.27)$$

With the use of (1.13),

$$U_m = \sum_n \left[\frac{1}{mn(m^2 + n^2\rho^2)} - \frac{1}{mn(\epsilon_m^2 + n^2\rho^2)} \right] \sin \frac{n\pi y}{b} \qquad (1.28)$$

and, therefore, with the use of (1.18) and (1.19),

$$U_m = \frac{\pi}{4m^3}\left[1 - \frac{\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{\cosh \alpha_m}\right] - \frac{\pi}{4m\,\epsilon_m^2}\left[1 - \frac{\cosh \frac{\pi\epsilon_m}{a}\left(y - \frac{b}{2}\right)}{\cosh \beta_m}\right] \qquad (1.29)$$

The substitution of formula (1.29) into (1.27) and the use of (1.23) and (1.24), then gives

$$w_2 = \frac{a^4 p \lambda_f S}{E_f(I+I_f)}\left[-\frac{(x^2 - ax)}{2\pi^2 a^2} - \frac{\delta^2}{\pi^4}\left(1 - \frac{\cosh \frac{\pi}{a\delta}\left(x - \frac{a}{2}\right)}{\cosh \frac{\pi}{2\delta}}\right) \right.$$

$$\left. - \frac{4}{\pi^5} \sum_m \left\{ \frac{\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{m^3 \cosh \alpha_m} - \frac{\cosh \frac{\pi\epsilon_m}{a}\left(y - \frac{b}{2}\right)}{m\epsilon_m^2 \cosh \beta_m} \right\} \sin \frac{m\pi x}{a} \right] \tag{1.30}$$

The formula for w_2 for a plate of infinite length consists of the terms in this expression that have been summed with respect to m.

The central deflection, w_{max}, is obtained by evaluating w at $x = \frac{a}{2}$, $y = \frac{b}{2}$. From (1.25), (1.30), and (1.8),

$$w_{max} = \frac{a^4 p \lambda f}{E_f(I + I_f)} \left[A_1 + \frac{ISA_2}{I + I_f} \right] \tag{1.31}$$

where

$$A_1 = \frac{5}{384} - \frac{2}{\pi^5} \sum_m \left\{ \frac{2 + \alpha_m \tanh \alpha_m}{m^5 \cosh \alpha_m} \right\} (-1)^{\frac{m-1}{2}} \tag{1.32}$$

$$A_2 = \frac{1}{8\pi^2} - \frac{\delta^2}{\pi^4}\left(1 - \frac{1}{\cosh \frac{\pi}{2\delta}}\right) - \frac{4}{\pi^5} \sum_m \left\{ \frac{1}{m^3 \cosh \alpha_m} - \frac{1}{m\epsilon_m^2 \cosh \beta_m} \right\} (-1)^{\frac{m-1}{2}} \tag{1.33}$$

Plots of A_1 and A_2 as functions of $\frac{a}{b}$ for various values of δ^2 are given in figures 3 and 4, respectively.

2. The Components of Strain in the Facings and Core of Sandwich Panels Having Orthotropic Core and Facing Materials

The stain components in the facings of a sandwich panel having orthotropic core and facing materials are obtained from equation (A4) and (A5) of Report No. 1583-C in the forms:

$$e_{xx}^{(1)} = \sum_m \sum_n \left(k_{mn}q_{mn} + \frac{f_1}{2} - Z_1\right)\frac{\partial^2 w_{mn}}{\partial x^2} \tag{2.1}$$

$$e_{yy}^{(1)} = \sum_m \sum_n \left(h_{mn}r_{mn} + \frac{f_1}{2} - Z_1\right)\frac{\partial^2 w_{mn}}{\partial y^2} \tag{2.2}$$

$$e_{xx}^{(2)} = -\sum_m \sum_n \left\{k_{mn}\,(c - q_{mn}) + \frac{f_2}{2} + Z_2\right\}\frac{\partial^2 w_{mn}}{\partial x^2} \tag{2.3}$$

and

$$e_{yy}^{(2)} = -\sum_m \sum_n \left\{h_{mn}\,(c - r_{mn}) + \frac{f_2}{2} + Z_2\right\}\frac{\partial^2 w_{mn}}{\partial y^2} \tag{2.4}$$

In these expressions, the superscripts 1 and 2 refer to the facings 1 and 2, respectively, $\underline{Z_1}$ and $\underline{Z_2}$ are coordinates with axes and origins as indicated in figure 2, $\underline{w_{mn}}$ is a term of series (A38) of Report No. 1583-C, and the parameters $\underline{k_{mn}}$, $\underline{h_{mn}}$, $\underline{q_{mn}}$, and $\underline{r_{mn}}$ are determined by (A24) of Report No. 1583-C.

The components of shear strain in the core,

$$e_{zx}^{(c)} = \sum_m \sum_n (1 - k_{mn})\frac{\partial w_{mn}}{\partial x} \tag{2.5}$$

and

$$e_{yz}^{(c)} = \sum_m \sum_n (1 - h_{mn})\frac{\partial w_{mn}}{\partial y} \tag{2.6}$$

are obtained from equation (A6) of Report No. 1583-C.

Combinations of the parameters $\underline{k_{mn}}$, $\underline{h_{mn}}$, $\underline{q_{mn}}$, and $\underline{r_{mn}}$ that are required for the evaluation of the above strain components are determined in the following forms from (A24) of Report No. 1583-C:

$$k_{mn}q_{mn} = -\left[\frac{(f_1^2 - f_2^2 - 2cf_2)}{2\,(f_1 + f_2)}\left\{1 + S_y\left(\frac{n^2\rho^2}{\alpha} + \gamma m^2\right) - S_x\,(\beta-\gamma)\,n^2\rho^2\right\}\right.$$

$$\left. + \frac{f_1}{2}\left\{S_x(\alpha\, m^2 + \gamma n^2\rho^2) + S_x\,(\beta-\gamma)\,n^2\rho^2 + S_xS_yF_{mn}\right\}\right]$$

$$\div\left[1 + S_x(\alpha m^2 + \gamma n^2\rho^2) + S_y\left(\frac{n^2\rho^2}{\alpha} + \gamma m^2\right) + S_xS_yF_{mn}\right] \qquad (2.7)$$

where

$$F_{mn} = (1 - \beta^2)\, m^2n^2\rho^2 + \gamma\left(\alpha m^4 + 2\beta m^2n^2\rho^2 + \frac{n^4\rho^4}{\alpha}\right) \qquad (2.8)$$

and

$$k_{mn} = \frac{\left[1 + S_y\left(\frac{n^2\rho^2}{\alpha} + \gamma m^2\right) - S_x(\beta-\gamma)n^2\rho^2 - \frac{(f_1+f_2)}{2\,c}\left\{S_x(\alpha\, m^2 + \gamma n^2\rho^2) + S_x(\beta-\gamma)n^2\rho^2 + S_xS_yF_{mn}\right\}\right]}{1 + S_x(\alpha\, m^2 + \gamma n^2\rho^2) + S_y\left(\frac{n^2\rho^2}{\alpha} + \gamma m^2\right) + S_xS_yF_{mn}} \qquad (2.9)$$

The expressions for $\underline{h_{mn}r_{mn}}$ and $\underline{h_{mn}}$ are obtained by substituting $\underline{S_x}$ for $\underline{S_y}$, $\underline{m^2}$ for $\underline{n^2\rho^2}$, $\frac{1}{\alpha}$ for $\underline{\alpha}$, and vice versa in (2.7) and (2.9), respectively. Finally, $\underline{k_{mn}(c - q_m)}$ and $\underline{h_{mn}(c - q_{mn})}$ are obtained from $\underline{k_{mn}q_m}$ and $\underline{h_{mn}r_{mn}}$, respectively by substituting $\underline{f_1}$ for $\underline{f_2}$ and $\underline{f_2}$ for $\underline{f_1}$ in the latter expressions. Definitions of the symbols $\underline{S_x}$, $\underline{S_y}$, γ, β, and α that appear in formulas (2.7), (2.8), and (2.9) are given in formulas (A32) of Report No. 1583-C.

3. The Components of Strain in the Facings of Sandwich Panels Having Isotropic Core and Facing Materials

If the core and facing materials are isotropic,

$$\left.\begin{aligned} &\alpha = \beta = 1 \\ &\gamma = \frac{1-\sigma}{2} \\ &\text{and} \\ &S_x = S_y = S \end{aligned}\right\} \qquad (3.1)$$

where σ is the Poisson's ratio of the facing material and S is given by formula (1.5). Equation (2.7) then reduces to

$$k_{mn}q_{mn} = - \frac{\frac{f_1^2 - f_2^2 - 2cf_2}{2(f_1 + f_2)} + \frac{f_1 S}{2}(m^2 + n^2\rho^2)}{1 + S(m^2 + n^2\rho^2)} \tag{3.2}$$

The expression for $k_{mn}(c - q_{mn})$ is obtained from equation (3.2) by substituting f_1 for f_2 and f_2 for f_1. In the case under consideration, w_{mn} is a term in the series (1.6). From formulas (2.1) and (2.3) therefore, the strain components $e_{xx}^{(i)}$, i = 1, 2, may be written as

$$e_{xx}^{(i)} = \frac{K\pi^2}{a^2} \sum_m \sum_n \frac{\left\{M_i + N_i' S(m^2 + n^2\rho^2)\right\} m \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}}{n(m^2 + n^2\rho^2)^2 \left\{1 + \delta^2(m^2 + n^2\rho^2)\right\}} \tag{3.3}$$

where

$$\left.\begin{aligned} M_1 &= Z_1 - \frac{f_2}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right) \\ N_1' &= Z_1 \\ M_2 &= + Z_2 + \frac{f_1}{f_1 + f_2}\left(c + \frac{f_1 + f_2}{2}\right) \\ N_2' &= + Z_2 \end{aligned}\right\} \tag{3.4}$$

Similarly, from formulas (2.2) and (2.3),

$$e_{yy}^{(i)} = \frac{K\pi^2}{a^2} \sum_m \sum_n \frac{\left\{M_i + N_i' S(m^2 + n^2\rho^2)\right\} n\rho^2 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}}{m(m^2 + n^2\rho^2)^2 \left\{1 + \delta^2(m^2 + n^2\rho^2)\right\}} \tag{3.5}$$

where i = 1, 2, and M_i and N_i' are defined by (3.4).

Series (3.3) and (3.5) can be summed with respect to n by methods similar to those used in section 1. It is observed from series (1.10), (1.11), and (3.3) however, that

$$e_{xx}^{(i)} = - M_i \frac{\partial^2 w_1}{\partial x^2} - N_i' \frac{\partial^2 w_2}{\partial x^2} \tag{3.6}$$

and from (1.10), (1.11), and (3.5) that

$$e_{yy}^{(i)} = - \frac{M_i \, \partial^2 w_1}{\partial y^2} - \frac{N_i' \, \partial^2 w_2}{\partial y^2} \tag{3.7}$$

Therefore, the representations of $\underline{e_{xx}^{(i)}}$ and $\underline{e_{yy}^{(i)}}$ that are summed with respect to $\underline{n}$ and partially with respect to $\underline{m}$ can be obtained by using formulas (1.25) and (1.30) in formulas (3.6) and (3.7). Results obtained in this manner are given as follows:

$$e_{xx}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \Bigg[- M_i \Bigg\{ \frac{x^2 - ax}{2a^2} + \frac{\delta^2}{\pi^2}\left(1 - \frac{\cosh \frac{\pi}{a\delta}\left(x - \frac{a}{2}\right)}{\cosh \frac{\pi}{2\delta}}\right)$$

$$+ \frac{4}{\pi^3} \sum_m \Bigg(\frac{(2 + \alpha_m \tanh \alpha_m) \cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right) - \frac{m\pi}{a}\left(y - \frac{b}{2}\right) \sinh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{2 m^3 \cosh \alpha_m}$$

$$+ \frac{m \delta^2 \cosh \frac{\pi \epsilon_m}{a}\left(y - \frac{b}{2}\right)}{\epsilon_m^2 \cosh \beta_m} - \frac{\delta^2 \cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{m \cosh \alpha_m} \Bigg) \sin \frac{m\pi x}{a} \Bigg\}$$

$$+ N_i' S \Bigg\{ \frac{1}{\pi^2}\left(1 - \frac{\cosh \frac{\pi}{a\delta}\left(x - \frac{a}{2}\right)}{\cosh \frac{\pi}{2\delta}}\right) - \frac{4}{\pi^3} \sum_m \left(\frac{\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{m \cosh \alpha_m} - \frac{m \cosh \frac{\pi\epsilon_m}{a}\left(y - \frac{b}{2}\right)}{\epsilon_m^2 \cosh \beta_m} \right) \sin \frac{m\pi x}{a} \Bigg\} \Bigg] \tag{3.8}$$

$$e_{yy}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \Bigg[\frac{4}{\pi^3} M_i \sum_m \Bigg\{ \frac{\alpha_m \tanh \alpha_m \cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right) - \frac{m\pi}{a}\left(y - \frac{b}{2}\right) \sinh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{2 m^3 \cosh \alpha_m}$$

$$+ \frac{\delta^2 \cosh \frac{\pi\epsilon_m}{a^2}\left(y - \frac{b}{2}\right)}{m \cosh \beta_m} - \frac{\delta^2 \cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{m \cosh \alpha_m} \Bigg\} \sin \frac{m\pi x}{a} + \frac{4S}{\pi^3} N_i' \sum_m \Bigg\{ \frac{\cosh \frac{\pi\epsilon_m}{a^2}\left(y - \frac{b}{2}\right)}{m \cosh \alpha_m}$$

$$- \frac{\cosh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{m \cosh \beta_m} \Bigg\} \sin \frac{m\pi x}{a} \Bigg] \tag{3.9}$$

The terms in formula (3.8) that have been summed with respect to m represent the strain component of a plate of width a and of infinite length.

The maximum values of the strain components with respect to x and y occur at the center of the plate where $x = \frac{a}{2}$, $y = \frac{b}{2}$. Formulas for these components can be written:

$$e_{xx\,max}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \left[M_i B_1 + (SN_i' - \delta^2 M_i) B_2 \right] \tag{3.10}$$

where

$$B_1 = \frac{1}{8} - \frac{2}{\pi^3} \sum_m \left(\frac{2 + \alpha_m \tanh \alpha_m}{m^3 \cosh \alpha_m} \right) (-1)^{\frac{m-1}{2}} \tag{3.11}$$

$$B_2 = \frac{1}{\pi^2} \left(1 - \frac{1}{\cosh \frac{\pi}{2\delta}} \right) - \frac{4}{\pi^3} \sum_m \left\{ \frac{1}{m \cosh \alpha_m} - \frac{m}{\epsilon_m^2 \cosh \beta_m} \right\} (-1)^{\frac{m-1}{2}} \tag{3.12}$$

and

$$e_{yy\,max}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \left[M_i C_1 + (SN_i' - \delta^2 M_i) C_2 \right] \tag{3.13}$$

where

$$C_1 = \frac{2}{\pi^3} \sum_m \frac{\alpha_m \tanh \alpha_m}{m^3 \cosh \alpha_m} (-1)^{\frac{m-1}{2}} \tag{3.14}$$

$$C_2 = \frac{4}{\pi^3} \sum_m \left\{ \frac{1}{m \cosh \alpha_m} - \frac{1}{m \cosh \beta_m} \right\} (-1)^{\frac{m-1}{2}} \tag{3.15}$$

The parameters B_1, B_2, C_1, and C_2 are plotted as functions of $\frac{a}{b}$ for various values of δ^2 in figures 5, 6, 7, and 8 respectively.

The stresses in the facings can be obtained from the strain components by use of the formulas

$$X_x = \frac{E_f}{\lambda} \left[e_{xx} + \sigma e_{yy} \right] \tag{3.16}$$

and

$$Y_y = \frac{E_f}{\lambda} \left[\sigma e_{xx} + e_{yy} \right] \tag{3.17}$$

If (3.10) and (3.13) are substituted in expressions (3.16) and (3.17), the maximum stresses in the facing (with respect to $\underline{x}$ and $\underline{y}$) are obtained in the forms

$$X_{xi} = \frac{a^2 p}{(I + I_f)} \left[M_i (B_1 + \sigma C_1) + (SN_i' - \delta^2 M_i)(B_2 + \sigma C_2) \right] \tag{3.18}$$

$$Y_{yi} = \frac{a^2 p}{(I + I_f)} \left[M_i (\sigma B_1 + C_1) + (SN_i' - \delta^2 M_i)(\sigma B_2 + C_2) \right] \tag{3.19}$$

4. The Components of Shear Strain in the Core of a Sandwich Panel Having Isotropic Core and Facing Materials

The relations (3.1), which exist in the isotropic case, reduce formula (2.9) to the form

$$k_{mn} = \frac{1 - \left(\frac{f_1 + f_2}{2\,c} \right) S(m^2 + n^2 \rho^2)}{1 + S(m^2 + n^2 \rho^2)} \tag{4.1}$$

With the use of this expression and with $\underline{w_{mn}}$ a term of series (1.6), series (2.5) takes the form

$$e_{zx}^{(c)} = \left(1 + \frac{f_1 + f_2}{2\,c} \right) \frac{SK\pi}{a} \sum_m \sum_n \frac{\cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{n(m^2 + n^2\rho^2)\left\{1 + \delta^2 (m^2 + n^2 \rho^2)\right\}} \tag{4.2}$$

This series can be summed with respect to $\underline{n}$ and partially with respect to $\underline{m}$ by the method used in section 1. It is observed from (1.11), however, that

$$e_{zx}^{(c)} = \left(1 + \frac{f_1 + f_2}{2\,c} \right) \frac{\partial w_2}{\partial x} \tag{4.3}$$

and, therefore, from (1.30),

$$e_{zx}^{(c)} = \frac{a^3 p \lambda_f}{E_f (I + I_f)} \left(1 + \frac{f_1 + f_2}{2\,c} \right) S \left[- \frac{(2x - a)}{2\,a\pi^2} + \frac{\delta}{\pi^3} \frac{\sinh \frac{\pi}{a\delta} \left(x - \frac{a}{2} \right)}{\cosh \frac{\pi}{2\delta}} \right.$$

$$\left. - \frac{4}{\pi^4} \sum_m \left\{ \frac{\cosh \frac{m\pi}{a} \left(y - \frac{b}{2} \right)}{m^2 \cosh \alpha_m} - \frac{\cosh \frac{\pi \epsilon_m}{a} \left(y - \frac{b}{2} \right)}{\epsilon_m^2 \cosh \beta_m} \right\} \cos \frac{m\pi x}{a} \right] \tag{4.4}$$

The maximum shear strain occurs at the center of one of the edges $x = o$ or $x = a$. At $x = o$, $y = \frac{b}{2}$,

$$e_{zx\,max}^{(c)} = \frac{a^3 p \lambda_f}{E_f(I + I_f)} \left(1 + \frac{f_1 + f_2}{2c}\right) SD \tag{4.5}$$

where

$$D = \frac{1}{2\pi^2} - \frac{\delta}{\pi^3} \tanh \frac{\pi}{2\delta} - \frac{4}{\pi^4} \sum_m \left\{\frac{1}{m^2 \cosh \alpha_m} - \frac{1}{\epsilon_m^2 \cosh \beta_m}\right\} \tag{4.6}$$

This parameter is plotted as a function of $\frac{a}{b}$ for various values of δ^2 in figure 9.

Similarly, it is found from (2.6) and (4.1) that

$$e_{yz}^{(c)} = \left\{1 + \frac{f_1 + f_2}{2c}\right\} \frac{\partial w_2}{\partial y} \tag{4.7}$$

and, therefore, from (1.30),

$$e_{yz}^{(c)} = -\frac{4a^3 p \lambda_f}{\pi^4 E_f(I + I_f)} \left(1 + \frac{f_1 + f_2}{2c}\right) S \sum_m \left\{\frac{\sinh \frac{m\pi}{a}\left(y - \frac{b}{2}\right)}{m^2 \cosh \alpha_m} - \frac{\sinh \frac{\pi \epsilon_m}{a}\left(y - \frac{b}{2}\right)}{m \epsilon_m \cosh \beta_m}\right\} \sin \frac{m\pi x}{a} \tag{4.8}$$

The maximum of this strain component occurs at the center of the shorter sides of the panel. At $y = o$, $x = \frac{a}{2}$,

$$e_{yz\,max}^{(c)} = \frac{a^3 p \lambda_f}{E_f(I + I_f)} \left(1 + \frac{f_1 + f_2}{2c}\right) S\,F \tag{4.9}$$

where

$$F = \frac{4}{\pi^4} \sum_m \left\{\frac{\tanh \alpha_m}{m^2} - \frac{\tanh \beta_m}{m \epsilon_m}\right\} (-1)^{\frac{m-1}{2}} \tag{4.10}$$

The parameter F is plotted in figure 10.

5. The Reaction at the Edges of a Sandwich Panel Having Isotropic Core and Facing Materials

In the present analysis, it is assumed that the effect of the bending of the core is negligible and, therefore, that bending and twisting moments are carried entirely by the facings. These moments are given by the formulas:

$$M_x = \int_{f_1} zY_x^{(1)} dz + \int_{f_2} zX_x^{(2)} dz \tag{5.1}$$

$$M_y = \int_{f_1} zY_y^{(1)} dz + \int_{f_2} zY_y^{(2)} dz \tag{5.2}$$

$$M_{xy} = -\int_{f_1} zX_y^{(1)} dz - \int_{f_2} zX_y^{(2)} dz \tag{5.3}$$

where $\int_{f_i}$ indicates integration over the facing of thickness f_i, and the origin of z may be chosen at any convenient point. The stresses $X_x^{(i)}$ and $Y_y^{(i)}$, $i = 1, 2$, are obtained from (3.16) and (3.17), respectively, with the use of (3.6) and (3.7). The shear stresses $X_y^{(i)}$, $i = 1, 2$, are given by the formula

$$X_y^{(i)} = -2\mu_{xy} \left\{ Mi \frac{\partial^2 w_1}{\partial x\, \partial y} + Ni' \frac{\partial^2 w_2}{\partial x\, \partial y} \right\} \tag{5.4}$$

where Mi and Ni' are defined by (3.4). This formula is derived from equation (A4) of Report No. 1583-C with the use of (3.2).

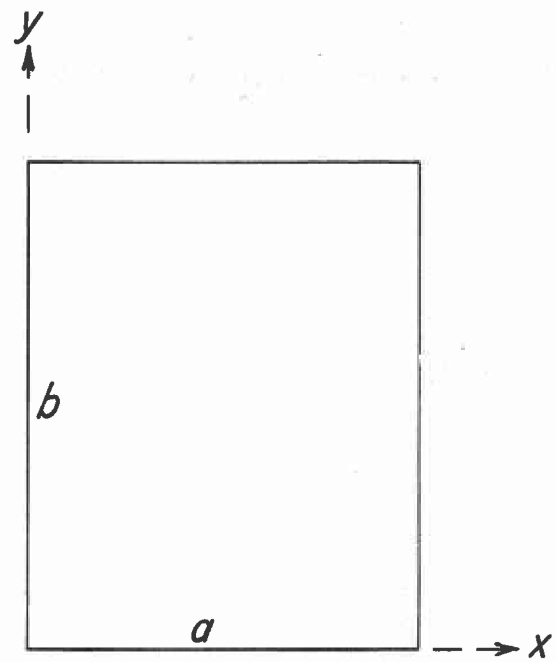
The reactions at the edges of the panel are computed from (5.1), (5.2), and (5.3) by the usual formulas. When these evaluations are made, it is found that the reactions are independent of the parameters S and δ and are the same as those of an ordinary plate. Numerical values of these reactions may be obtained from Timoshenko[1], table 5.

---

[1] Timoshenko, S. Theory of Plates and Shells, New York, 1940.

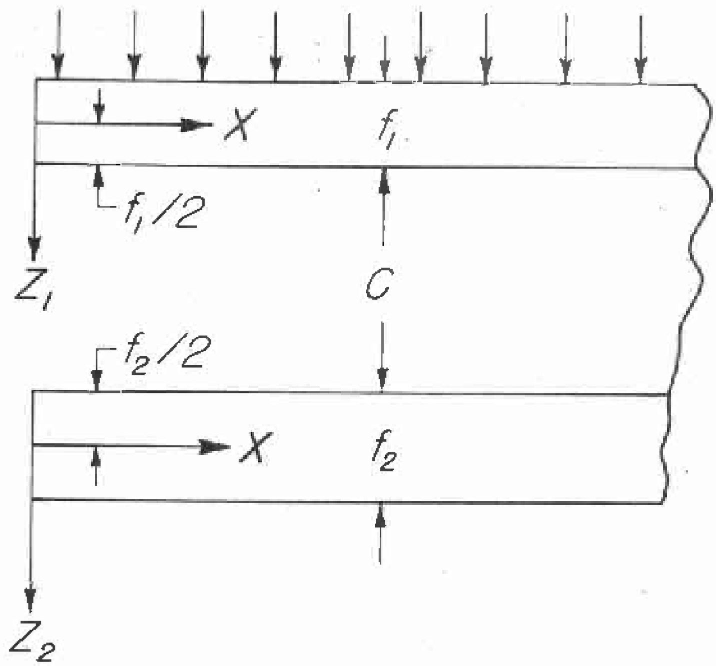
Figure 1.—Section of panel parallel to facings.

(ZM 85798 F)



Z M 85798 F

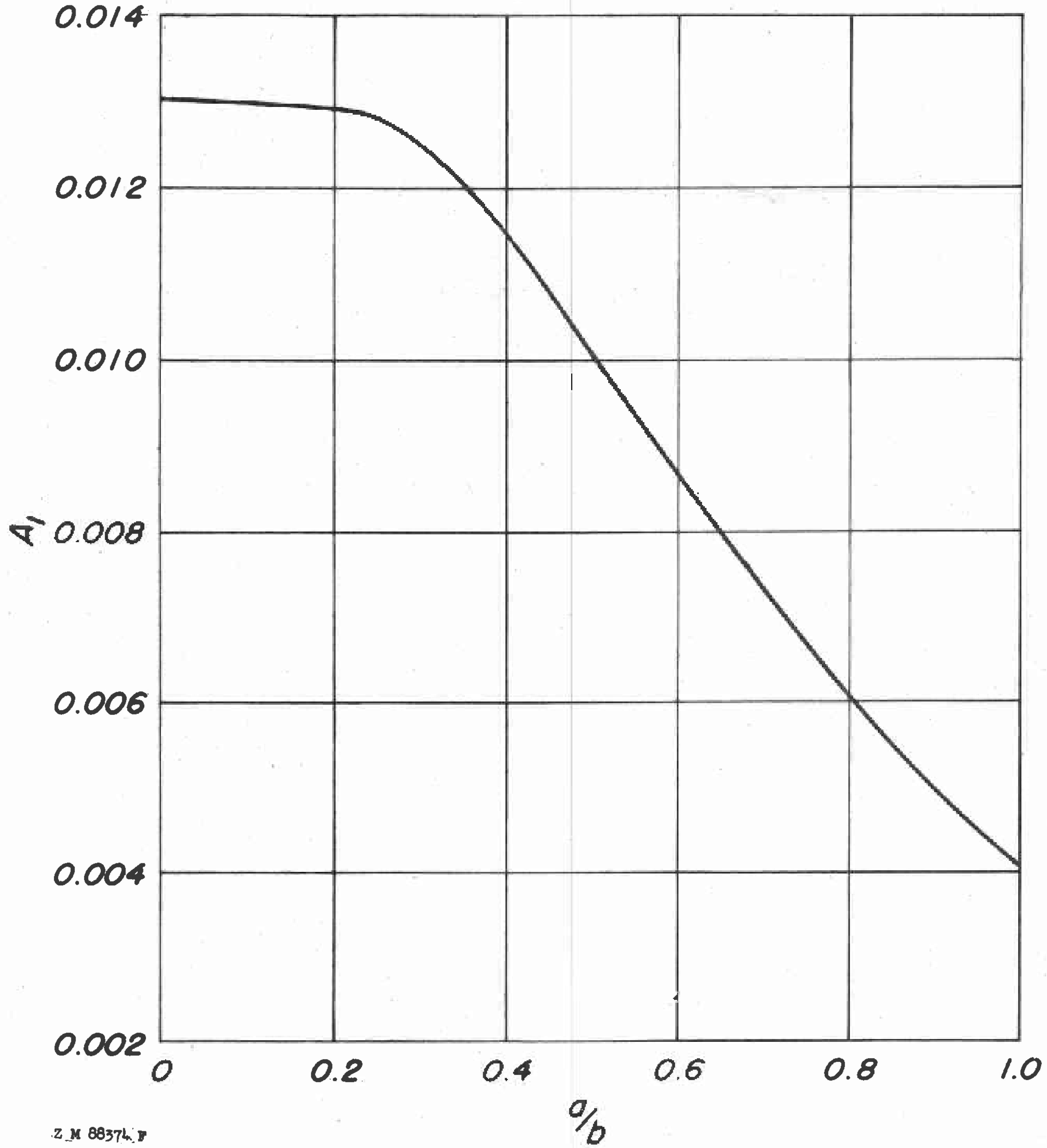
Figure 2.--Cross section of loaded sandwich panel.



Z M 88373 F

Figure 3.--The parameter $\underline{A_1}$ in formula (5),

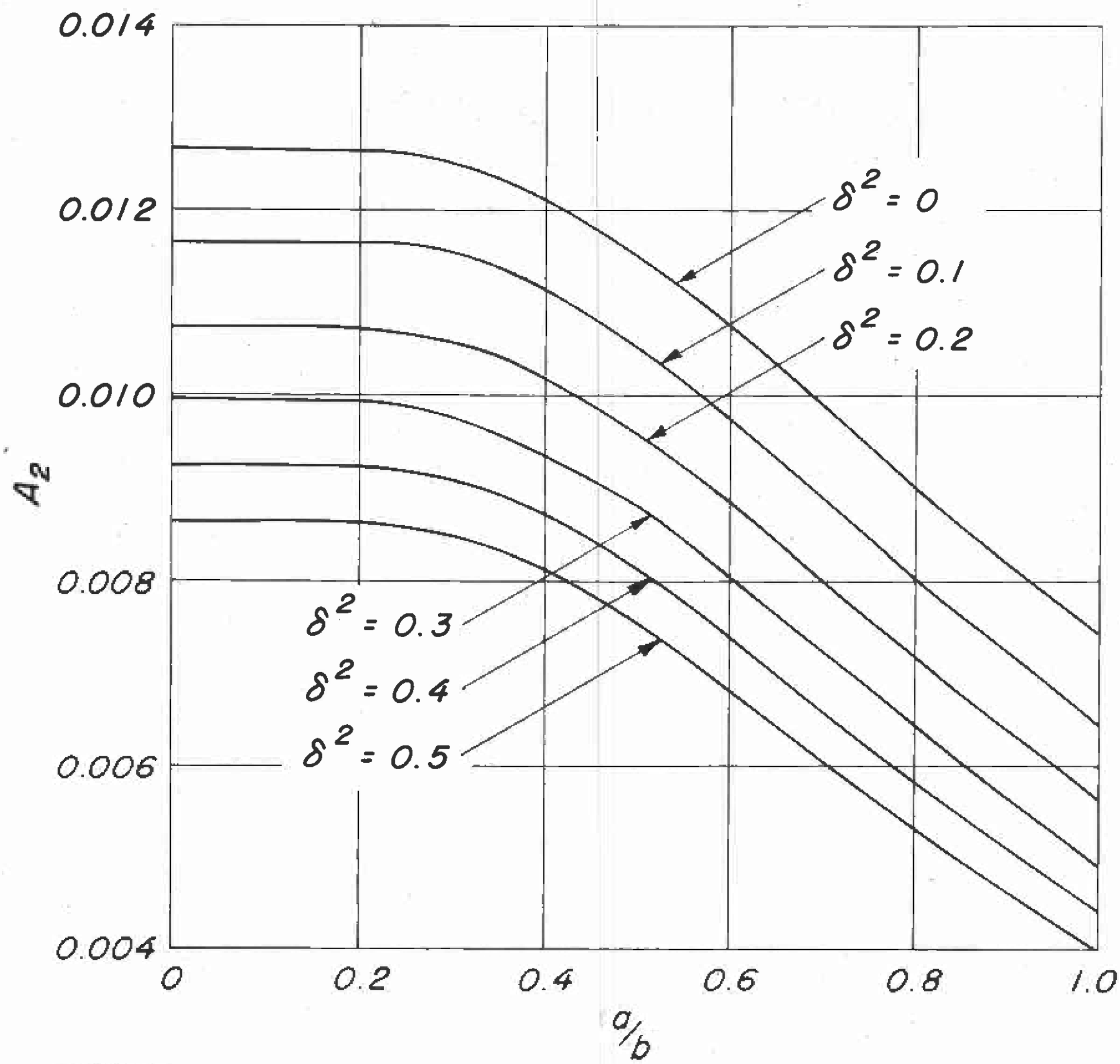
$$w_{max} = \frac{a^4 p \lambda_f}{E_f(I + I_f)} \left(A_1 + \frac{ISA_2}{I + I_f}\right)$$



Z M 88374 F

Figure 4.--The parameter $\underline{A_2}$ in formula (5),

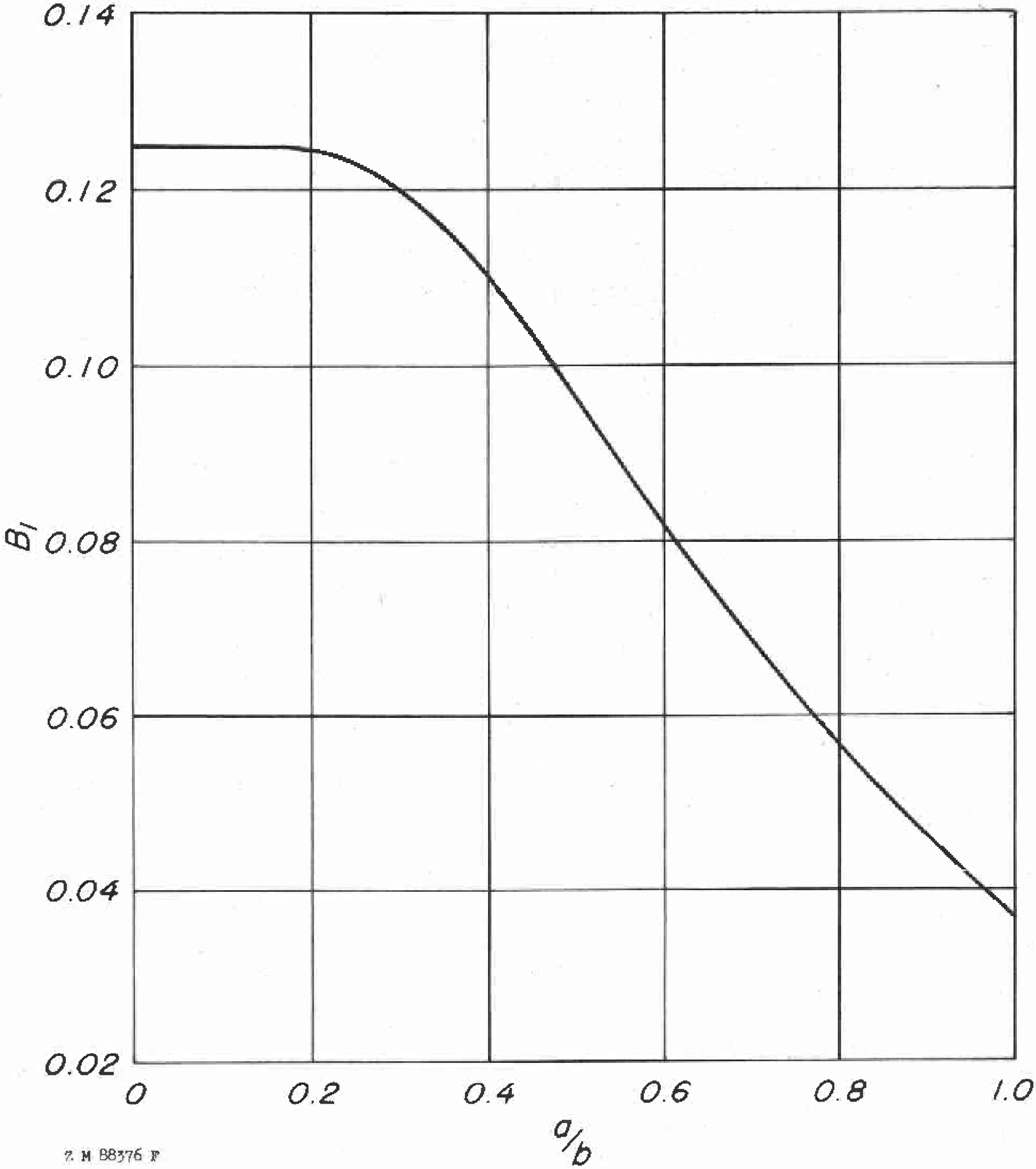
$$w_{max} = \frac{a^4 p \lambda_f}{E_f(I + I_f)} \left(A_1 + \frac{ISA_2}{I + I_f}\right)$$


0.014
0.012
0.010
0.008
0.006
0.004
A_2
0
0.2
0.4
0.6
0.8
1.0
a/b
$\delta^2 = 0$
$\delta^2 = 0.1$
$\delta^2 = 0.2$
$\delta^2 = 0.3$
$\delta^2 = 0.4$
$\delta^2 = 0.5$

Z M 88375 F

Figure 5.--The parameter $\underline{B_1}$ in formula (6),

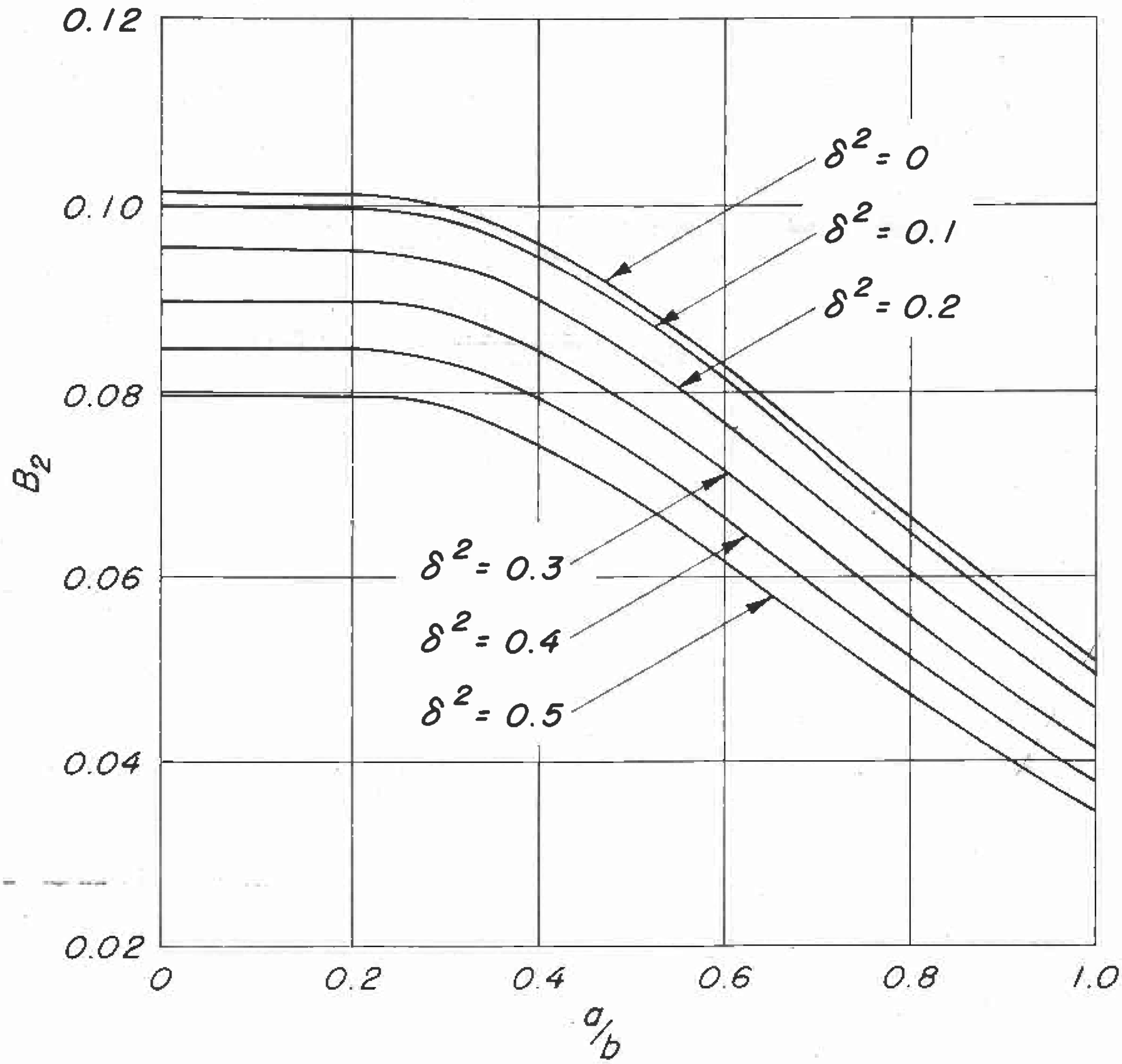
$$e_{xx\,max}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \left[M_i B_1 + S N_i B_2 \right]$$



Z M B8376 F

Figure 6.--The parameter $\underline{B_2}$ in formula (6),

$$e_{xx\,max}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \left[M_i B_1 + S N_i B_2\right]$$

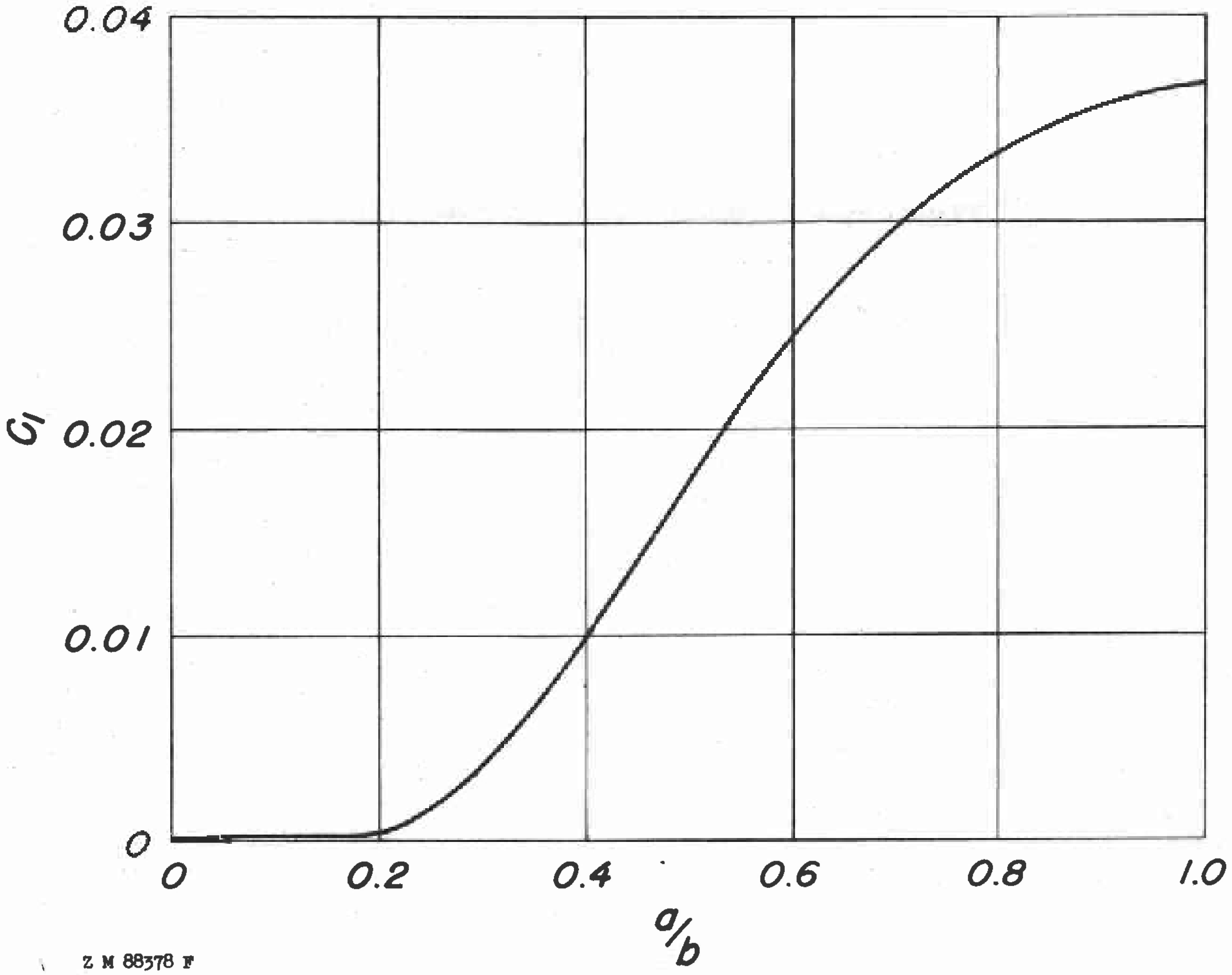


0.12
0.10
0.08
0.06
0.04
0.02
B_2
0
0.2
0.4
0.6
0.8
1.0
a/b
$\delta^2 = 0$
$\delta^2 = 0.1$
$\delta^2 = 0.2$
$\delta^2 = 0.3$
$\delta^2 = 0.4$
$\delta^2 = 0.5$

Z M 88377 F

Figure 7.--The parameter $\underline{C_1}$ in formula (7),

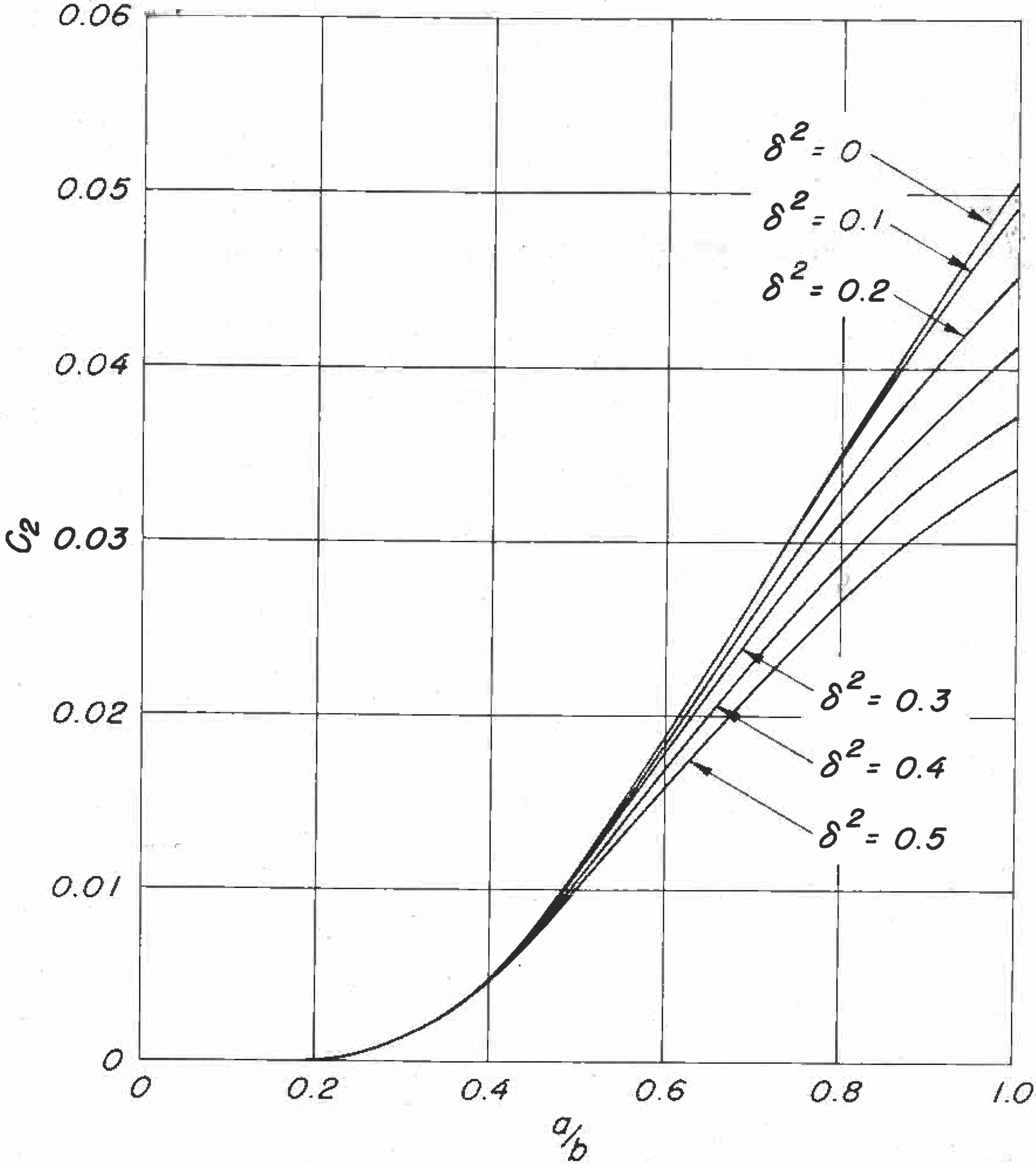
$$e_{yy\,max}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)} \left[M_i C_1 + S N_i C_2 \right]$$



Z M 88378 F

Figure 8.--The parameter $\underline{C_2}$ in formula (7).

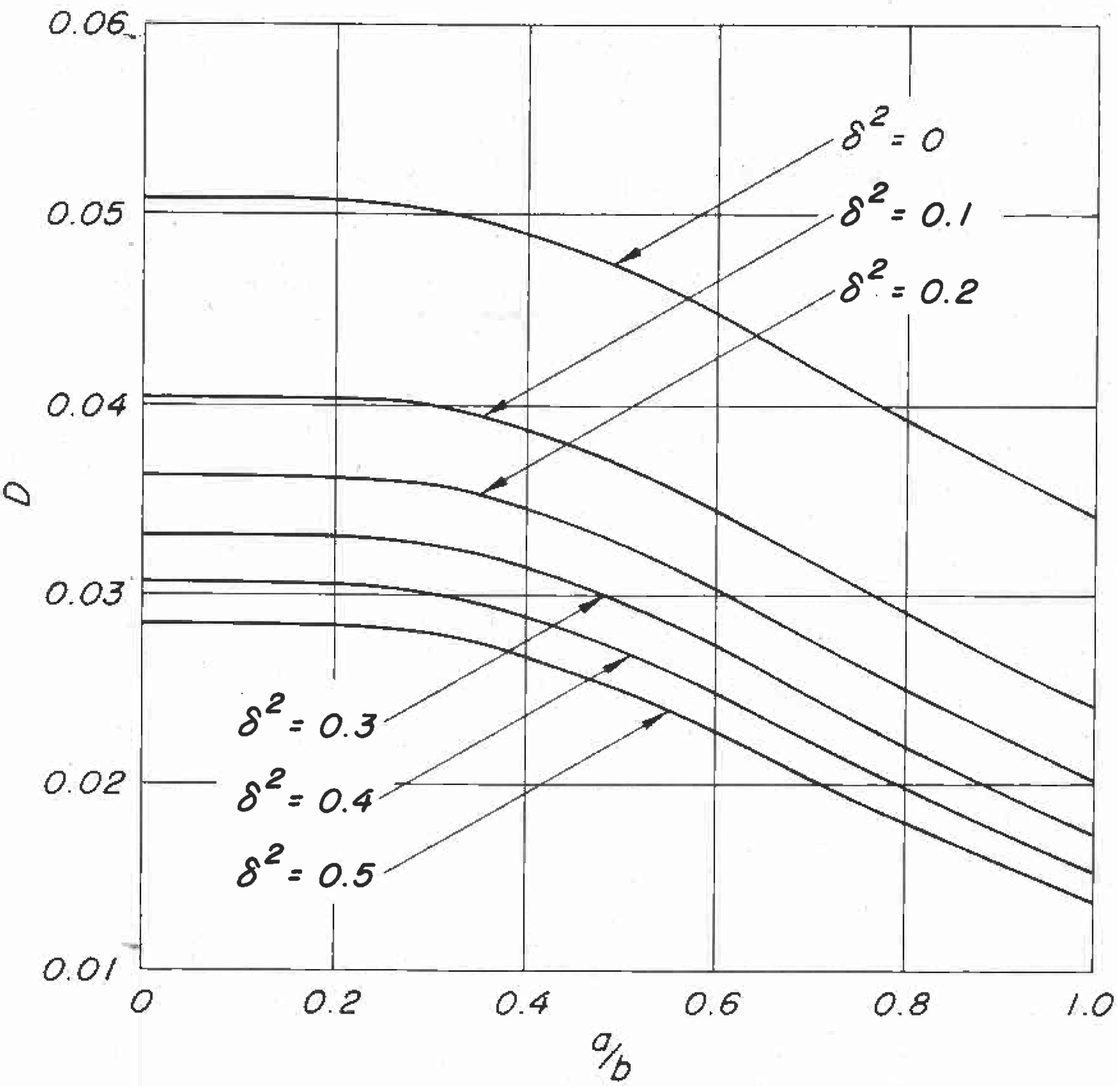
$$e_{yy\,max}^{(i)} = \frac{a^2 p \lambda_f}{E_f(I + I_f)}\left[M_i C_1 + S N_i C_2\right]$$


0.06
0.05
0.04
0.03
0.02
0.01
0
C_2
0
0.2
0.4
0.6
0.8
1.0
a/b
$\delta^2 = 0$
$\delta^2 = 0.1$
$\delta^2 = 0.2$
$\delta^2 = 0.3$
$\delta^2 = 0.4$
$\delta^2 = 0.5$

Z M 88379 F

Figure 9.—The parameter D in formula (11),

$$e_{zx}{}^{(c)}_{max} = \frac{a^3 p \lambda_f}{E_f(I + I_f)} \left(1 + \frac{f_1 + f_2}{2c}\right) SD$$

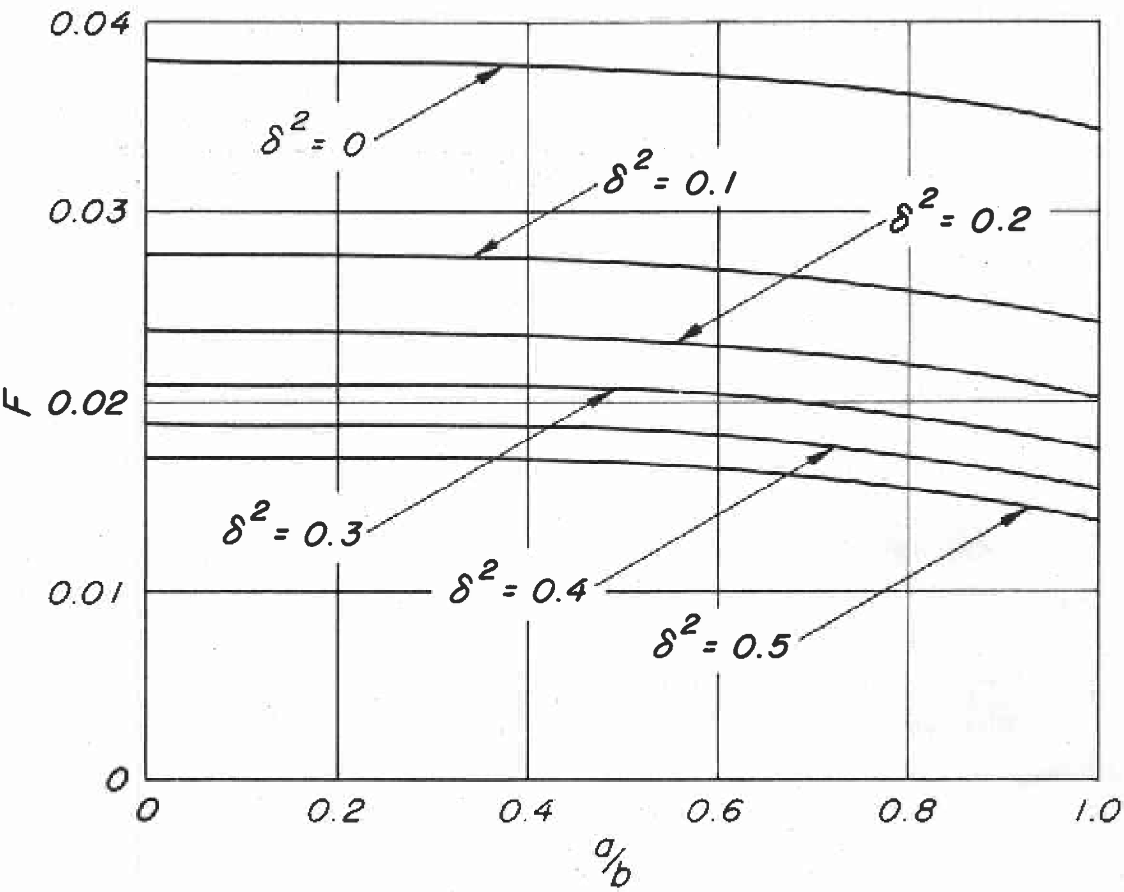


0.06
0.05
0.04
0.03
0.02
0.01
D
0
0.2
0.4
0.6
0.8
1.0
a/b
δ² = 0
δ² = 0.1
δ² = 0.2
δ² = 0.3
δ² = 0.4
δ² = 0.5

Z M 88380 F

Figure 10.—The parameter F in formula (11),

$$e_{yz}{}^{(c)}_{max} = \frac{a^3 p \lambda_f}{E_f(I + I_f)} \left(1 + \frac{f_1 + f_2}{2\,c}\right) SF$$


0.04
0.03
F 0.02
0.01
0
0
0.2
0.4
0.6
0.8
1.0
a/b
δ² = 0
δ² = 0.1
δ² = 0.2
δ² = 0.3
δ² = 0.4
δ² = 0.5

Z M 88381 F