AN ABSTRACT OF THE THESIS OF

Younghoon Kwak for the degree of Master of Science in Mechanical Engineering presented on November 4, 2003.

Title: Particle Image Velocimetry Studies of Low Reynolds Number Flow in Branching Flow Networks.

Abstract approved:

Deborah V. Pence

A particle image velocimetry system was established to determine flow fields in low Reynolds number flows through a branching flow network. Due to the fabrication procedure of the test device, flows were limited to a Reynolds number of 1.5 in the largest channel. Data was acquired at each channel inlet, midstream down the channel, at the channel exit, and at the bifurcation. Velocity magnitudes and vorticity are presented as profiles and as contour plots. Experimental results agree very well with three-dimensional computational fluid dynamics results and fully developed analytical solutions, except for situations encountering developing flow, flow through tapered channels, and flow at a channel exit. Discrepancies in experimental and computational results for the latter two flow conditions are due to the interrogation window in which experimental data were analyzed. For the rectangular window employed, decelerating flow at a channel exit or accelerating
flow in a tapered channel downstream of the location under investigation was averaged into the data. Finally, data was not resolved near the walls due to low seed density, which was attributed to particle settling at the low flow conditions and to asymmetry in the flow network, which tended to yield a non-uniform distribution of particles in the channels.
Particle Image Velocimetry Studies of Low Reynolds Number Flow
in Branching Flow Networks

by

Younghoon Kwak

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APPROVED:

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Dean of the Graduate School

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Younghoon Kwak, Author
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<td>Hydraulic diameter of k branch, ( \frac{4W_k H_k}{2(W_k + H_k)} )</td>
</tr>
<tr>
<td>$e$</td>
<td>Pixel size of CCD camera</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
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<tr>
<td>$H_k$</td>
<td>Height of k branch</td>
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<tr>
<td>$k$</td>
<td>Branch level</td>
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<td>$L_k$</td>
<td>Length of k branch</td>
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<tr>
<td>$L_{k (IW)}$</td>
<td>Length of inner wall of k branch</td>
</tr>
<tr>
<td>$M$</td>
<td>Magnification</td>
</tr>
<tr>
<td>$n$</td>
<td>Index of refraction of medium</td>
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<tr>
<td>$NA$</td>
<td>Numerical aperture</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number, ( \frac{U d_{H,k}}{v} )</td>
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<td>$U$</td>
<td>Mean velocity</td>
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<td>$u$</td>
<td>Instantaneous velocity in x (streamwise) direction</td>
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<tr>
<td>$u_{max}$</td>
<td>Maximum u component of velocity</td>
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<td>$u'$</td>
<td>Non-dimensional u component of velocity, ( \frac{u}{u_{max}} )</td>
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<td>$W_k$</td>
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<td>Streamwise location relative to local coordinate system (( \mu m ))</td>
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<td>$y$</td>
<td>Spanwise location relative to local coordinate system (( \mu m ))</td>
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\( y' \)  
Non-dimensional \( y \)  
\[
\left( y - \left( \frac{1}{2} \frac{W}{L} \right) \right)
\]

\( z \)  
Depth relative to local coordinate system

\( \alpha^* \)  
Aspect ratio  
\[
\left( \frac{H}{W} \right)
\]

\( \delta z \)  
Depth of field

\( \lambda \)  
Wavelength

\( \tau \)  
Time between two laser pulses

\( \omega \)  
Mean vorticity  
\[
\frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)
\]

\( \omega' \)  
Non-dimensional mean vorticity  
\[
\frac{1}{2} \left( -\frac{\partial u'}{\partial y} \right)
\]
CHAPTER 1

INTRODUCTION

With the development of smaller and faster electronic devices, heat dissipation becomes increasingly more important. The development of micro- and nano-scale fluidic devices has helped in effectively cooling high performance electronics. Heat sinks with many small diameter microchannels occupying the same total volume as a single large channel increase the convective surface area for heat transfer. In addition, the heat transfer coefficient through the microchannels is increased, making microchannel heat sinks a highly effective cooling method for electronics. In 1981 the microchannel cooling device for electronic chips was first introduced and studied by Tuckerman and Peace. Since the introduction of their device, numerous investigators have studied thermal and flow characteristics in microchannels.

Although a heat sink with straight, parallel microchannels is considered an effective cooling device for electronics, it has two major disadvantages. The first disadvantage is that as the diameter of the microchannel decreases, although the heat transfer coefficient increases, the pressure drop between the inlet and outlet increases, requiring a higher pumping power for a fixed flow rate. The second disadvantage is a non-uniform axial temperature distribution. A non-uniform
temperature distribution on a heat sink can cause severe damage to the electronics by allowing overheating in certain areas.

To overcome the disadvantages of the straight, parallel channels for cooling of electronics, a fractal-like branching network was developed by Pence (2000). This fractal-like branching network design is based on transport systems found in nature (see West, et al., 1997). Natural systems drive the working fluid through larger diameter and longer branches first, then through consecutively smaller branches. By copying natural systems, the fractal-like branching network device exploits two different heat transfer mechanisms at the large and small diameter branches. For the large diameter initial branches the temperature difference between the channel wall and the working fluid drives heat transfer. In contrast, in the smaller branches heat transfer is driven by the higher convective heat transfer coefficient.

The major advantages of the fractal-like branching channels when compared to parallel channels with the same applied heat flux, convective surface area, channel length and terminal hydraulic diameter are lower pressure drop, lower pumping power and lower wall temperature (Pence, 2002). Use of larger diameters for some of the network reduces the total pressure drop because the pressure drop is inversely proportional to the diameter to the fourth power for a fixed mass flow rate. The pumping power is a directly proportional to the pressure drop squared. Finally,
redeveloping thermal boundary layers following a bifurcation acts to lower the maximum wall temperature.

The fractal-like branching channel has also been used as a passive micro-mixer. Two different fluids are injected in alternate inlets at the smallest channels of the passive micro-mixer of the fractal-like branching channel. Fluid flows from the small channels merging into one large main channel. The short diffusion distance and the large diffusion surface area between each fluid layer created by the passive micro-mixer are able to achieve the mixing passively without a mechanical force to actively mix the two different fluids.

The purpose this study is to validate a three-dimensional computational fluid dynamics (3-D CFD) model for the fractal-like branching channel networks using micro-scale Particle Image Velocimetry (micro-PIV). The 3-D CFD results were used by Alharbi, et al. (2003a, 2003b) to validate a 1-D predictive model. The test section of the fractal-like branching channel was created by micro laser machining and thermal diffusion bonding polycarbonate layers. Mid-depth velocity fields in the fractal-like branching networks were acquired in four of the 5 branching levels of Pence (2002). Deionized (DI) water was chosen for the working fluid and the data was acquired with a Reynolds number of 1.46 in the largest channel.
2.1 Micro-scale Flow for Thermal Applications

Tuckerman and Pease (1981) first introduced and investigated a high performance water cooled heat sink for Very–Large Scaled Integrated (VLSI) circuits. Micro-scale channels with high aspect ratios were designed to decrease the convective resistance and increase the surface area per unit volume. In their theory, thermal resistance was separated into three parts. These include (1) the conductive resistance from the circuit through the heat sink, (2) the convective resistance from the channel wall to the working fluid and (3) the advective resistance from fluid flow through the channels. They found that the most considerable thermal resistance is the convective thermal resistance, because the other thermal resistances are reducible. Conductive resistance can be reduced by changing the material type and thickness. Advective resistance can be reduced by using a high heat capacity fluid at a high flow rate.

High performance heat sinks were designed with straight parallel high aspect ratio microchannels. The channel width, channel depth and wall thickness were varied between 50 and 56 μm, 287 and 320 μm, and 44 and 50 μm, respectively. A maximum temperature rise of 71°C above the inlet water
temperature was measured at the exit with a power density of 790 W/cm² applied at the surface of the heat sink. They reported that theory and experimental data had a good agreement and the great enhancement of the heat sink for VLSI circuit was achieved.

Knight, et al. (1992) performed a theoretical optimization study of heat sink performance while relaxing the strict geometry conditions imposed by the optimization theory of Tuckerman and Peace (1981). The heat sink was optimized for turbulent flow as well as laminar flow. The size of the heat sink, working fluid, pressure drop, and fin material were the same as Tuckerman and Pease (1981) used. They relaxed the ratio of fin thickness to channel width and lifted the restriction on laminar flow to find optimum channel and fin dimensions. When turbulent flow was supplied to the heat sink optimized for these flow conditions, the total thermal resistance was reduced by 35% from that of Tuckerman and Pease (1981).

Micro-scale forced convective heat transfer and flow characteristics were studied experimentally under both laminar and turbulent single-phase flow conditions by Peng and Peterson (1996). Water was selected for the working fluid and 12 different geometric microchannels, which varied from 0.133 to 0.367 mm in hydraulic diameter, were tested. For laminar flow, the aspect ratio and the ratio of the hydraulic diameter over the center-to-center distance between microchannels effected the convective heat transfer. For turbulent flow analysis, a non-dimensional value denoted by Z, and defined as the ratio of the minimum value to the maximum value of the channel aspect, was considered. A value of 0.5 for Z was determined as
the optimum configuration of the microchannel because, for this value of \(Z\), the measured flow resistance reached a minimum. The Reynolds number for the transition to turbulence for microchannel flows was also reported to be much smaller than that for conventional or macro-scale channel flows.

Weilin, et al (2000) experimentally investigated flow characteristics of water in 6 different trapezoidal silicon microchannels which varied from 51 to 169 \(\mu m\) in hydraulic diameter. They found that the pressure gradient and friction factor were higher than those predicted from conventional theory in laminar flow. In fact, the friction factor was not constant, rather it varied with Reynolds number. The authors concluded that the higher pressure gradient and friction factor were caused by roughness in the microchannel.

Numerous papers on micro-scale fluid flow characteristics have been published. Obot (2000) reviewed and presented many of the published results on pressure drop and heat and mass transfer in single-phase microchannel flows. In many of the papers reviewed it was reported that conventional relationships, including the accepted Reynolds number transition to turbulence, were not applicable to microchannels. However, Obot (2000) concluded that due to errors in data acquisition and reduction of the original data, there is insufficient evidence to believe that the transition to turbulence in smooth microchannels occurs below a Reynolds number of 1000. He also concludes that estimates of heat and mass transfer coefficients are within the experimental errors.
Sobhan and Garimella (2001) reviewed over 70 papers in which characteristics of micro-scale fluid flow had been discussed. They emphasized comparing experimental results from the reviewed papers with existing micro-scale theory. Because of the discrepancy between results from the reviewed papers, they determined that there was insufficient evidence to conclude that microchannel flow characteristics clearly differ from macrochannel flow characteristics. They concluded that discrepancies in experimental results were caused by differences in surface roughness, nonuniformity of channel dimensions, and uncertainties in instrumentations and measurement locations.

In a recent, carefully conducted study, Garimella and Singhal (2003) showed that macroscale correlations for laminar flow. Nusselt number and friction factors are applicable to microchannel flows up to Reynolds numbers of 1200 and 1800, respectively.

2.2 Network Flow

Bejan (1997a and 1997b) developed a constructal theory from an optimization process for a flow network between a point source and a volume. Two flow networks between one point and many points were constructed. One was optimized to maximize conductive heat transfer through a body. The second flow network was optimized to minimize flow resistance. In constructal theory, the
progression of a tree-like network construction begins with the smallest volume with a goal of filling an entire volume with many constructs.

Three years later, Bejan and Errera (2000) investigated volumetric convective cooling using constructal theory. A final optimized network was constructed based on the superposition of the optimum construction for fluid flow resistance (1997a) and conductive heat transfer (1997b). A convection component was included and determined by assessing how much energy was dissipated from the channel wall to the flow.

Wechsatol, et al. (2002) studied an optimal tree network system for a disk-shaped body introduced by Pence (2000). The flow network originates from one point at the center of the disk and ends at many points at the perimeter of the disk. The flow network was developed using constructal theory while assuming fully developed laminar flow. Several different designs were considered. It was concluded that the more complex the geometry the better the performance, because the flow resistance was decreased as the number of branching levels increased. For the disk shaped heat sink, channels that branch into two new channels and into three new channels were considered. Using three branches, the flow resistance was decreased by only 3.2% from the resistance using two branches, when the size of the disk and the number of the channel outlets were fixed.

A fractal-like model for mammalian circulatory system was proposed by West, et al. (1997) with a fixed scaling ratio between the branch lengths and diameters across each level of the branching flow network. Using these scaling
relations, Pence (2000) introduced fractal-like branching channel networks in heat sinks, suggesting advantages over parallel channel heat sinks such as a more uniform surface temperatures and lower pressure drops.

Pence (2002) continued investigation of the fractal-like branching channel network and developed a one-dimensional model for predicting the wall surface temperature and pressure distributions along the flow network. Results from the one-dimensional model of the fractal-like branching channel were compared with results from an array of straight parallel channels. A 30% lower pressure drop and 30°C lower wall temperature were reported for a fractal flow network than for a parallel array of straight channels. The comparison was made with each flow network having the same channel length, convective surface area, heat flux, and pumping power. The hydraulic diameter of the straight channels, which were square in cross-section, was the same as the hydraulic diameter of the terminal branch in the fractal-like flow network.

Alharbi, et al. (2003) developed a three-dimensional CFD model of the fractal-like branching flow network to validate the one-dimensional model of Pence (2002) channel. The one-dimensional model overestimated the pressure drop along the fractal-like branching channel when compared with results from the three-dimensional model. However, in general, a similarity in trend in the pressure distributions between the one- and three-dimensional models was observed. However, the three-dimensional model predicted an advantageous pressure recovery at the bifurcation. Using three-dimensional CFD model results, spanwise
and transverse velocity profiles at the inlet and outlet of each branch segment were reported. A prototype of the fractal-like branching channel network was fabricated in two heat sinks. The pressure drop across of the prototypes was experimentally measured as a function of flow rate. The experimental results agreed exceptionally well with the CFD model. However, advanced micro-scale flow field measurements are necessary to validate the 3-D simulated flow fields.

2.3 Experimental Micro-Scale Flow Measurement

Brody, et al. (1996) tested low Reynolds number flows between $10^3$ and $10^4$ in a high aspect ratio channel, which was approximately 6.5, using the particle streak technique. For flow visualization, they used a fluorescent microscope, 0.9 μm diameter fluorescent beads, and a CCD camera capable of 30 frames per second. The rectangular channel had a depth of 11 μm and 72 μm for a width. Instantaneous velocity was determined by measuring the length of the axial distance of a particle streak and the exposure time (1/30 seconds) of the CCD camera. To collect 1000 particle streaks, 10 different images were taken. Each image captured approximately 100 particle streaks. This study showed good agreement between measurement and theory.

Santiago, et al. (1998) first introduced Particle Image Velocimetry (PIV) measurements for microfluidics. They investigated the Hele-Shaw flow field around
a 30 μm diameter cylinder in a Reynolds number flow much less than unity. The micro-PIV system utilized an epi-fluorescent microscope, 100-300 nm spherical fluorescent particles, a continuous Hg-arc lamp, and a CCD camera to get high resolution of the particle field. They introduced the use of the focal depth of the epi-fluorescent microscope instead of a light sheet, as is typically used in macro-scale PIV. The system was limited to low Reynolds number flows due to the long duration of exposure of the CCD camera.

A year later, Meinhart, et al. (1999a and 1999b) improved the micro-PIV capabilities of Santiago, et al. (1998) to a 1 μm spatial resolution capable of measuring a velocity of approximately 8 mm/s. They used an inverted epi-fluorescent microscope, a 5 ns pulsed Nd:YAG laser, 200 nm spherical diameter fluorescent particles, and a cooled interline-transfer CCD camera with 500 ns interval time between two exposures. The test section consisted of a rectangular glass channel with a depth of 30 μm. Several suggestions were made for achieving a 1 μm spatial resolution in micro-PIV. The first was that low seed density can minimize background noise in the flow field. The second was that applying an average correlation method can overcome problems with low seed density in the flow field by providing sufficiently reliable particle signals. Using the average correlation method, the micro-PIV measurement was well matched, within 2%, with the analytical solution for a rectangular channel flow.
Using micro-PIV, Oak, et al. (2000) studied diffusion and flow development of two co-flowing streams in a micro-channel with a high aspect ratio of 20. The microchannel had a depth of 50 μm a width of 1 mm and a splitter plate located between the two co-flowing streams. The two independently controlled inlet streams merged in a mixing chamber. The range of flow rate ratios between the two streams was varied from one to nine. For micro-PIV measurements, they used a Nd:YAG pulsed laser, a dual Q-switch, an air-cooled CCD camera, an epi-fluorescent filter cube, and 2 μm spherical diameter fluorescent particles. They concluded that increasing the flow rate ratio caused an imbalanced in pressure at the end of the splitter plate, and that the fast moving stream was recirculated into the slow moving stream forming a stagnation flow region.

Lee, et al. (2002) measured velocity profiles in rectangular microchannels and in a micro-nozzle using micro-PIV techniques. Two rectangular microchannels with aspect ratios of 1 and 6 and a fixed channel depth of 50μm were tested. The micro-nozzle had a contraction ratio of 5:1. Average microchannel velocities were varied from 0.05 to 0.20 m/s. At a maximum velocity of 0.25 m/s, the vector resolution was 6.8μm×6.8μm.

Lee, et al. (2002) measured hydrodynamic development length and transition to turbulence using micro-PIV measurements. They created a 260 μm wide, 690 μm deep, and 120 mm long microchannel and tested it at five different Reynolds numbers, between 250 and 2900. They concluded that the entrance length
was about 45% shorter than entrance lengths estimated from a macro-scale correlation. Transition to turbulence was reported at a Reynolds number of 2900.

Three-dimensional micro-PIV measurements were made by Klank, et al. (2002) by adjusting the focal plane of the microscope lens by 10 μm intervals to measure the flow field through the depth of the structure. They employed the micro-PIV techniques of Meinhart, et al. (1999a), but without the average correlation method because their interrogation window volume contained a sufficient number of fluorescent particles. Also, a three-dimensional micro-PIV measurement was performed by applying stereoscopic PIV techniques, which uses two CCD cameras to determine three velocity components.
CHAPTER 3

EXPERIMENTAL FACILITY

The purpose of this experiment is to validate the flow field simulated by a three-dimensional Computational Fluid Dynamics (CFD) analysis using micro-PIV. The 3-D CFD model was previous used by Alharbi, et al. (2003) to validate a one-dimensional design model for predicting the pressure distribution in a fractal-like branching channel network. In this chapter, details of the instrumentation, the test devices, and the experimental setup are discussed.

3.1 Test Device

Three requirements of the test device were considered for the micro-PIV measurements. The first was that it had to be transparent for observing fluorescent particles in a flow field. The second was that the test device had to have leak-proof connections to the flow loop. The last requirement was that the test device had to be easily accessible by a laser beam and a microscope objective to illuminate and observe the flow field, respectively. To satisfy the above requirements, a polycarbonate film was chosen to form the channel side walls. The flow pattern was cut into this film using micro laser machining. Three different polycarbonate layers
were laminated and sealed using a thermal diffusion bonding technique to make the test section. Using this lamination process results in constant depth channels. A manifold was designed for leak-proof interconnects between the test section and the flow loop.

3.1.1 Fractal-Like Branching Flow Network

The geometry of the micro-scale fractal-like branching flow network is shown in Figure 3.1. Flow enters the widest channel, which bifurcates into two smaller diameter channels, and is collected after 4th bifurcation. Channel dimensions for each level are discussed in Chapter 4. Detailed drawings of the top and bottom layers are provided in Appendix A.

![Figure 3.1: Geometry of micro-scale fractal-like branching flow network](image-url)
3.1.2 Test Section

For the material of the test section, a polycarbonate film was selected because it is transparent and easily laminated. A 250 μm thick film provided the same depth as the fractal-like branching network modeled for 3-D CFD simulations. The polycarbonate film has a low melting point, 310 °F, which is good for sealing by thermal diffusion bonding. The flow network was cut into the 250 μm thick polycarbonate film using an UV (266 nm) laser, ESI micro Machining System 4420. The laser cutting process required 10 passes of the laser which was set to a DC current of 19.6 Amps, a laser pulse frequency of 4000 Hz and a cutting speed of 1.5 mm/s. To create one test section, three different polycarbonate film layers, shown in Figure 3.2a, were required. The top and bottom layers formed the top and bottom walls of the fractal-like branching flow network, respectively. Holes forming the exit plenums, coincident with the exit of the terminal branches, were also laser micro-machined.

To bond the three polycarbonate laminate layers together, a fixture was designed to align and clamp three layers for the thermal diffusion bonding process. Two pins were used for aligning the fine details of the three polycarbonate layers, and two aluminum panels held the three layers in place. To maintain scratch free surfaces on the test section, two microscope slides were placed between the top and bottom polycarbonate layers and the aluminum panels shown in Figure 3.2b. The reason for using two microscope slides was to prevent imprinting scratches inherent
on the aluminum clamping device onto the surface of the polycarbonate test section during the thermal diffusion bonding process, which occurs at approximately 290 °F. Test devices fabricated without the glass sides resulted in scratches on the surface of the test section that affected the micro-PIV measurements by scattering the incoming laser beam and making it difficult to capture the signal from the fluorescent particles.

Figure 3.2: Fractal-like branching channel flow network. (a) Three polycarbonate layers (top, channel and bottom), (b) three polycarbonate layers with slides and aluminum panels, (c) polycarbonate, glass, and aluminum panels in an aluminum clamp, (d) fractal-like branching network device shown with a penny and an injected green dyed solution.
The three polycarbonate layers, two glass panels, and two aluminum panels were assembled with the aid of the alignment pins and fixed in an aluminum clamp shown in Figure 3.2c. The center bolt noted in the figure was tightened to approximately 3 in-lbs as measured by a Proto 6169A dial torque wrench.

For thermal diffusion bonding, the optimum temperature, time, and pressure were found by trial and error to be 285–290 °F, 40 minutes and 112 psi, respectively. A fractal-like branching network test piece is shown in Figure 3.2d. Figure 3.3 provides a cross-sectional view of the widest channel, which shows that the channels were not deformed by the thermal diffusion bonding process.

![Figure 3.3: Cross-sectional view of k=0 branch of flow network](image-url)
3.1.3 Manifold

The purpose of the manifold is to provide a leak-proof interconnect between the test section and the flow loop. In addition, access of the laser beam and the microscope objective were necessary. The manifold was machined in aluminum with an access allowing the microscope objective to be close to the test section, because the working distance of the microscope objective was 9 mm. A single inlet was located at one end of the manifold, and two outlets were located at the other end of the manifold. The device served a dual purpose for passive mixing studies for Enfield (2003) and the present study. Figures 3.4a and 3.4b show the disassembled and assembled manifold with the test sections, respectively. Figure 3.5 shows a schematic cross-sectional view of the manifold to illustrate how the test section was interconnected to the flow loop via the manifold.
3.2 Flow Loop

A photograph and schematic diagram of the entire assembly are provided in Figure 3.6a and 3.6b, respectively. The flow loop is indicated by the dashed rectangular area in Figure 3.6b. A Cole-Parmer 74900 syringe pump was used to control the flow rate into the test device. The accuracy of the syringe pump, given by the manufacturer, is ±0.5% full scale with ±0.2% reproducibility. A Hamilton GasTight 10 ml gas syringe was used and connected by 1/8\textsuperscript{th} inch Tygon tubing to the inlet of the manifold. The two outlets of the manifold were connected individually to Tygon tubing that terminated in a collection reservoir.
Figure 3.6: (a) Photograph of experimental setup and (b) schematic diagram of experimental setup.
3.3 Micro-PIV Setup

Particle Image Velocimetry (PIV) provides a popular and accurate means for measuring velocity fields in macro-scale flows. Using two sequentially acquired images of particles in the same flow field, instantaneous velocities can be computed by taking the distance that each particle moves in a particular direction and dividing by the time interval between the two sequential images. In general, PIV systems require a pulsed laser, optics, a high speed CCD camera with a full field image and a data acquisition system. The frame rate of the CCD camera and the frequency of a laser pulse limit the maximum velocity the system can resolve.

The basic theory of micro-PIV is the same as PIV except for the manner in which lighting is introduced to create a depth of field. Micro-PIV uses front lighting, which is oriented parallel to the CCD camera, as opposed to a laser sheet created perpendicular to the CCD camera. The other difference between PIV and micro-PIV is use of the optical measurement system in micro-PIV to create a depth of the flow field for investigation compared with PIV in which the thickness of the laser sheet defines the depth of the flow field investigated. For micro-PIV a microscope objective and a dichroic mirror are needed to sufficiently magnify the flow field and direct the laser beam toward the test section, respectively.

For the current experiment setup, a continuous argon ion laser with an optical chopper and a National Television System Committee (NTSC) formatted CCD camera were used instead of using a pulsed laser and a high-speed PIV CCD
camera with full field image. The following components of the micro-PIV system are discussed: argon ion laser, fluorescent particles, optics, optical chopper, and CCD camera. A photograph of the micro-PIV instrumentation and a schematic diagram are shown in Figures 3.6a and 3.6b, respectively.

3.3.1 Argon Ion Laser

A Lexel continuous beam, 0.75 W argon ion laser was used for illuminating the fluorescent particles used to seed the flow field. The major wavelengths of the laser are 488nm and 514nm with 35% and 40% of the total power, respectively. In this micro-PIV experiment, the 514 nm wavelength was used. To pulse the continuous argon laser beam, an optical chopper was used. Details of the optical chopper and a special chopper blade design are discussed in section 3.3.4.

3.3.2 Fluorescent Particles

The flow was seeded with 3.0 μm diameter fluorescent particles from Duke Scientific Co., catalog number R0300. From the manufacturer literature, the density of the fluorescent particle was 1.05 g/ml, which was very close to the 1 g/ml density of water at 20 °C. The fluorescent particles came from the manufacturer diluted in a
solution with $6.7 \times 10^8$ beads per milliliter of water. The maximum excitation and emission of the fluorescent particles were reported to be 542 nm and 612 nm, respectively. Although the wavelength at the maximum excitation of the fluorescent particles is not identical to the 514 nm major wavelength of the argon ion laser, the particle luminance at 514 nm was still sufficient for this experiment. The luminance at 514 nm was approximately 83% of the maximum luminance at the 542 nm wavelength. The wavelength spectra of the micro particles are shown in Appendix B.

According to Meinhart, et al. (1999b), for more accurate micro-PIV measurement, low seed density is recommended. If the particle density is too high the background noise created from unfocused particles can dominate the image. However, low seed density generally does not give sufficient particle signals to generate a velocity vector field. In order to collect a sufficient number of particle signals, an averaged correlation method was recommend. A second recommendation was to employ an average image method. In this study, the second recommendation was used to increase flow seed density.

The fluorescent particles came from the manufacturer diluted in a solution of $6.7 \times 10^8$ beads per milliliter of water. For the present experiment, approximately 0.6 ml of the particle solution was mixed with 40 ml of deionized water. The final ratio of the particle volume to the interrogation window volume at the k=0 branch level is 0.014.
3.3.3 Optics

Two different lenses were positioned between the argon ion laser and the microscope objective. The beam was brought through a double convex lens then through the convex side of a plano convex lens. The second lens is positioned beyond the focal point of the double convex lens. The purpose of this configuration is to expand the argon ion laser beam from approximately 2 mm to 8 mm in diameter to evenly illuminate the flow field. This lens configuration ensures that the laser focal plane is behind the focal plane of the microscope objective.

A Nikon wide green excitation G-2A epi-fluorescent filter cube allowed for illumination of the fluorescent particles by front lighting. Front lighting was necessary due to the manifold design, but also provided much brighter illumination than any other ways of lighting for micro-PIV. The dichroic mirror reflected the incident 514 nm wavelength laser beam at a 90-degree angle toward the test device to illuminate the flow field. The wavelengths above 550 nm and between 400 and 500 nm from the flow field passed through the dichroic mirror. Only the emitted light with wavelengths above 590 nm could pass through the emission filter to be imaged by a CCD camera. Schematic diagrams of the dichroic mirror and transmission spectrum of the emission filter are shown in Figure 3.7 and Figure 3.8, respectively. Note that the epi-fluorescent cube originally came with an excitation filter, in addition to the dichroic mirror and emission filter. Because the excitation filter has a very limited wavelength range, from 510 to 560 nm, the filtered intensity
of the incoming argon ion laser beam was not sufficient to illuminate the fluorescent particles; therefore the excitation filter was removed.

Figure 3.7: Schematic diagram of dichroic mirror

Figure 3.8: Transmission for Nikon G-2A epi-fluorescent cube. Used with permission from Nikon Co.
A Nikon 10×plan apochromat microscope objective lens was used to magnify the image 10-fold. In addition, the high numerical aperture of this objective, 0.3, minimized distortion of the image.

Meinhart, et al. (1999) discussed the total depth of field, δz, which is the visible range that gives a clear particle image through the optical measurement system. The equation used to assess the total depth of field is

\[ \delta z = \frac{n\lambda}{NA^2} + \frac{ne}{M \cdot NA} \]  

(3-1)

For the present experiment setup, n, the index of refraction of the air between the micro-fluidic device and the objective lens was 1.003. From the fluorescent particles, the emitted light wavelength, \( \lambda \), was 612 nm, the numerical aperture, NA, of the microscope objective was 0.3, the average pixel space of the CCD camera, e, was approximately 7 μm, and the total magnification for the optical measurement system, M was 10. Therefore, the total depth of field was calculated to be approximately 9 μm.

3.3.4 Optical Chopper and Blade Design

To pulse the Lexel continuous argon laser beam, a Stanford Research System 540 optical chopper was used. This optical chopper system consisted of a step motor station with a blade and control unit. Two chopper blades, a 6 slot blade
and a 30 slot blade capable of chopping over a frequency range from 4 Hz to 3.7 kHz, were provided by the manufacturer. The motor station has a photo sensor that could detect the light permitted through the spinning slots and provide the frequency to the control unit. Using the external port in the control unit, the light transmission signal was output to a Tektronix TDS 340A oscilloscope to measure frequency.

![Figure 3.9: Schematic chopper blade design.](image)

Because the current experimental setup did not have a means to synchronize the laser pulse with the CCD camera, a specially designed chopper blade was needed to pulse the continuous laser beam. The designed of this chopper blade is shown in Figure 3.9. Note that there are two large slots at a radius coincident with the laser beam and one small slot near the edge of the chopper
blade. The center-to-center angular spacing of these two large slots is 40-degrees, and each slot has a 5-degree angle opening. The small outer slot was designed to allow light detection, from which the frequency was measured by a photo sensor of the optical chopper. The signal from the photo sensor was directly sent to the oscilloscope. Using the measured frequency from the oscilloscope, \( f \), the actual time between two laser pulses, \( \tau \), can be calculated from

\[
\tau = \frac{40° \times 1}{360° \times f}
\]

where \( \tau \) is in seconds and \( f \) in Hz.

### 3.3.5 3-CCD Camera

A Panasonic GP-US522 3-CCD camera head and a control unit with National Television System Committee (NTSC) format were used. The 1/3" interline transfer CCD camera has a maximum resolution of 768×494 pixels, resulting in a pixel size of 6.4×7.4 \( \mu \text{m} \). The fixed frame rate of the CCD camera was 30 fps. A square pixel size of the CCD camera was necessary to compute the total depth of field. Horizontal and vertical pixel sizes were averaged to provide a square pixel size of approximately 7.0 \( \mu \text{m} \).

An NTSC formatted CCD camera has two fields, odd and even, in a frame. The odd field is composed of the odd lines of the camera sensor. At 1/60 of a
second later, an even field, composed of the even lines of the camera sensor, is acquired. Odd and even fields for a stationary particle are combined, as shown Figure 3.10, to create a single image. Figure 3.11 demonstrates the odd and even fields and the combined image for a particle moving from left to right. The time between the odd and even field is $1/60$ of a second. Note that the even and odd line widths are equal to a pixel size. The procedure of how these odd and even camera fields are integrated with laser pulsing will be discussed in Chapter 5.
Figure 3.10: A stationary circular object image captured by NTSC formatted CCD camera.

Figure 3.11: A moving circular object image captured by NTSC formatted CCD camera.
CHAPTER 4

TEST PLAN

A flow rate of 35 μl/min was maintained by a syringe pump, yielding a Reynolds number of 1.46 in the k=0 branch. Each level is numbered starting with 0 for the original branch. Each segment or branch in a level is assigned a letter as noted in Figure 4.1.

Figure 4.1: Label of each branch with the channel geometry

The original plan was to collect flow images at each streamwise location in half of the fractal-like branching channels due to symmetry in the design. However, the end of each k=3 branch and all k=4 branches were not accessible by the microscope objective. Tick marks, identified with numbers, were micro laser machined in the walls of the channels at 600 μm intervals. Each branch in the k=0,
k=1, k=2 and k=3 level was divided into 11, 7, 5 and 3 intervals, respectively. The purpose of these tick marks was to reposition the field of view.

All flow field images were collected mid-depth of the fractal-like branching channel, i.e. 125 μm from the top wall forming the channel. The number of images required to create one good composite flow field image was determined to be 30. This number was the optimum number of images to yield sufficient velocity vector signals. One composite image was used to generate one velocity vector field. Six composite images at the same location were collected and the velocity vector fields averaged. Therefore, the total number of images acquired at the same flow field was 180. The reason why only six composite images were averaged was due to limited data acquired prior to the laser failing.

The dimensions, flow conditions, and data acquisition conditions for each branch level are listed in Table 4.1. The height, width, length, inner wall length, and hydraulic diameter, based on the original design, are presented in the dimension category in Table 4.1. These dimensions are identical to those of CFD model. Mean velocity and Reynolds number for each branch are also listed. In the location and setting category in Table 4.1, the depth at which images were acquired, and field of view and interrogation window sizes are noted, as is the conversion factor from physical space to pixels. This latter quantity is labeled ratio. Note that the chopper frequency value changed from 50 Hz to 35 Hz as the level of branch changed from k=2 to k=3. This setting was necessary to maintain an average distance of particle
movement of approximately 6-7 pixels in a flow field image. Note that the flow speed through the k=3 branch was much slower than through other branches.

Table 4.1 Test Plan

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Level of Branch (k)</th>
<th>k=0</th>
<th>k=1</th>
<th>k=2 (a &amp; b)</th>
<th>k=3 (a, b, c &amp; d)</th>
<th>k=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_k (Height) (mm)</td>
<td></td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
</tr>
<tr>
<td>W_k (Channel Design Width) (mm)</td>
<td></td>
<td>0.543</td>
<td>0.297</td>
<td>0.190</td>
<td>0.130</td>
<td>0.093</td>
</tr>
<tr>
<td>L_k (Channel Design Length) (mm)</td>
<td></td>
<td>5.80</td>
<td>4.10</td>
<td>2.90</td>
<td>2.05</td>
<td>1.45</td>
</tr>
<tr>
<td>L_k (Inner Wall Length) (mm)</td>
<td></td>
<td>6.14</td>
<td>3.93</td>
<td>2.76</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td>d_H,k (Hydraulic diameter) (mm)</td>
<td></td>
<td>0.343</td>
<td>0.273</td>
<td>0.216</td>
<td>0.171</td>
<td>0.136</td>
</tr>
<tr>
<td>Mean Velocity (mm/sec)</td>
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<td>4.29</td>
<td>3.89</td>
<td>3.07</td>
<td>2.24</td>
<td>1.57</td>
</tr>
<tr>
<td>Re</td>
<td></td>
<td>1.46</td>
<td>1.06</td>
<td>0.66</td>
<td>0.38</td>
<td>0.21</td>
</tr>
<tr>
<td>Location of mid-plane (mm)</td>
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<td>0.125</td>
<td>0.125</td>
<td>0.125</td>
<td>0.125</td>
<td>N/A</td>
</tr>
<tr>
<td>Each Field of View (H×L) (Pix×Pix)</td>
<td></td>
<td>448×600</td>
<td>256×600</td>
<td>160×600</td>
<td>128×600</td>
<td>N/A</td>
</tr>
<tr>
<td>Interrogation Window(H×L) (Pix×Pix)</td>
<td></td>
<td>32×128</td>
<td>32×128</td>
<td>32×128</td>
<td>32×128</td>
<td></td>
</tr>
<tr>
<td>Ratio (μm/Pix)</td>
<td></td>
<td>1.214</td>
<td>2.272</td>
<td>1.250</td>
<td>1.200</td>
<td>N/A</td>
</tr>
<tr>
<td>Chopper Frequency (Hz)</td>
<td></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>35</td>
<td>N/A</td>
</tr>
<tr>
<td>Actual Laser Pulse (Hz)</td>
<td></td>
<td>450</td>
<td>450</td>
<td>450</td>
<td>315</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of Intervals</td>
<td></td>
<td>11</td>
<td>7</td>
<td>2×5=10</td>
<td>4×3=12</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of images for one velocity field</td>
<td></td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>N/A</td>
</tr>
<tr>
<td>Number of the velocity fields for average</td>
<td></td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>N/A</td>
</tr>
<tr>
<td>Total number of image needs for each level</td>
<td></td>
<td>1980</td>
<td>1260</td>
<td>1800</td>
<td>2160</td>
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</tr>
</tbody>
</table>
CHAPTER 5
EXPERIMENTAL PROCEDURE AND DATA ANALYSIS

Details of the procedure for collecting the images and how to analyze the data are provided in this chapter.

5.1 Experiment Procedure

The laser was turned on and operated for 30 minutes to reach a steady state condition. The fluorescent particle solution was diluted by deionized water with a solution to water ratio of 1:67. The seeded solution was pumped into the fractal-like branching channel network by the syringe pump at a flow rate of 35 μl/min. This was the highest flow rate that could be tested using our micro-PIV measurement. The specially designed chopper blade was used to generate the necessary double laser pulse. The frequency from the photo-sensor of the optical chopper was recorded, and the time between the two laser pulses was calculated using Equation 3-2. A National Instrument image acquisition (NI-IMAQ) board was installed in a IBM comparable computer with an Intel Pentium II processor (560MHz) and used for acquiring images from the CCD camera. The IMAQ software was used to control the image properties, including image size, position, brightness, contrast, etc.
The manifold, including the test device, was mounted on a translation stage which allowed for changes in the translation position of the test device in x-y-z directions. For accurate positioning, an 80-pitch actuator, i.e. 80 turns per inch of lateral movement, was used. To find the mid-depth location of the channel, it was necessary to locate the top surface of the flow network. This was accomplished by focusing on the edge of the tick marks in the wall of the branching network. The test device was then moved 125 μm toward the camera. The micro-actuator had to be turned 142.0 degrees to achieve a translation of 125 μm.

Rotating the chopper blade at 50 Hz, the time between double laser pulses was 2.22 ms. This value was used for the k=0, k=1 and k=2 branches. To maintain an approximate 6-7 pixel distance in the k=3 branches, the chopper speed was decreased to 35 Hz to yield a pulse spacing of 3.18 ms. Ideally, the first laser pulse would be exposed in the odd field, with the second laser pulse exposed in the even field. However, in the absence of an external trigger port in the CCD camera, the laser pulses and fields of the CCD camera were not synchronizable. To overcome this problem, a large number of flow field images, on the order of 2000, were collected. Statistically about 10% of these images were good, where a good image was defined as one in which one laser pulse coincided with an odd field exposure and the other laser pulse coincided with an even field exposure. Good and bad cases are shown in the schematic diagram of synchronization in Figure 5.1. Each frame is 33.33 ms, and odd and even field require half of the frame time. Between odd and even fields, a 1.3 ms interval exists. Case 1 represents good synchronization.
between the CCD camera and both laser pulses. Case 2, 3, 4 and 5 are not considered to be synchronized. Case 2 and 5 show that only one of the laser pulses coincide with frame 2, which is the image of interest. In case 2, the first laser pulse occurred when the camera was taking its first image, i.e. frame 1, hence no particle pairs show up on frame 2. Likewise case 5 shows the second pulse occurring on a different image or frame. For cases 3 and 4 both laser pulses occur in the same field of the frame. A “sequential image capturing program”, written in LabView®, was capable of collecting 2000 images at each flow field and automatically saving them as TIFF files.
Figure 5.1: Good and bad examples of synchronization between the CCD camera and laser pulse. Case 1 represents a camera laser synchronization yielding a good image. Cases 2-5 yield useless data.
5.2 Data Analysis

To sort out good images from the 2000 images collected, a special program was required. An “image sorting program” written in Matlab® 6.1 was devised to identify images with good pairing, as was defined earlier. The program, after identifying the good images, saves them automatically into a designated folder. Approximately 10% of the total collected images were found to be the good images. From each good image of the flow field, two time sequenced images of the flow field were created by separating the odd and even fields then using a linear interpolation scheme to fill in the even and odd lines, respectively. This image separation process was achieved by automatically using another program in Matlab®. An example of an original flow field image and its separate odd and even field images are shown in Figure 5.2a and Figure 5.2b. Note that Figure 5.2a and 5.2b represent a group of 30 images, and that we are looking at a single image at the top of the stack. Also, note the low seed density in these figures. To increase the seed density, the 30 odd and 30 even field images were combined to generate a single pair of time sequenced images. This was accomplished using a function called “Sum Sequence” in Visflow® PIV analysis software, version 6.11. Figure 5.2c shows a pair of sequential images created from a composite of 30 odd and 30 even fields.
Figure 5.2: Process for creating two sequential images with high seed density. (a) 30 original flow field images acquired at the 6th section of k=0 branch, (b) decomposition of each of the 30 original flow field images into 30 odd and 30 even field images, (c) two sequential images with the high seed density created by a composite of 30 images for each field.
Two sequential images were analyzed by Visiflow® using a cross-correlation function to create a velocity vector field. The interrogation window size was set to 128×32 pixels with 75% overlap for all channels. The interrogation window was made rectangular in order to improve the resolution across the channel width. For each interrogation window, a cross-correlation between particles on two sequential images was performed. The velocities for all pairs of particles in the interrogation window were averaged, and a single velocity vector was reported for each interrogation window. The result is a 2-D vector flow field. The 2-D representation includes the streamwise and spanwise velocity components. Data was smoothed using the smoothing filter, shown in Table 5.1 and Figure 5.3, provided by Visiflow® software. The filter is shifted grid by grid over the entire flow field to generate smooth data.

Table 5.1
Inverse exponential function filter: 5×5 grid of scaling factors

<table>
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<th>0.10687</th>
<th>0.13533</th>
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<td>0.36787</td>
<td>0.24311</td>
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<tr>
<td>0.059106</td>
<td>0.10687</td>
<td>0.13533</td>
<td>0.10687</td>
<td>0.059106</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.3
3D plot of the smoothing filter
Because the size of the flow field images depends upon the width of the channel at each branch level, the number of spanwise velocity vectors varies across the branch levels. The data point distribution (row×column) for each branch level is 53×17, 29×17, 17×17 and 13×17, for branch levels 0, 1, 2, and 3, respectively. Note that the two rows of velocity data near the top and bottom walls of the channel are eliminated because of low seed density near the wall.

The velocity vector data, which includes both x and y position and velocity magnitude and direction, were exported to ASCII file format from Visiflow®. ASCII files were converted to text files, and imported into Matlab® for further analyses and for creating two-dimensional velocity vector plots, and velocity and mean vorticity contour plots and profiles. All Matlab® codes developed for analysis purposes are provided in APPENDIX C.

Mean vorticity, \( \omega \), which is associated with \( u \) and \( v \) components of velocity is discussed in Chapter 6.3. Mean vorticity is calculated as follow

\[
\omega = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)
\]  

(5-1)

where \( v \) is velocity in the spanwise direction and \( u \) is velocity in the streamwise direction. A backward finite difference routine was used to calculate the mean vorticity using \( u \) component average velocity and \( v \) component average velocity.
Experimental velocity profiles in the fractal-like branching flow network were compared to an analytical solution of a fully developed velocity profile in a rectangular channel. The analytical results are based on a non-dimensional velocity profile equation from Shah and London (1978). This equation, used for fully developed flow in rectangular channel with a no-slip boundary condition, is given as

\[ u' = \frac{u}{u_{max}} = \left[ 1 - \left( \frac{z}{b} \right)^n \right] \left[ 1 - \left( \frac{y}{a} \right)^m \right] \]  

(5-2)

where \( u' \) is a non-dimensional streamwise component of velocity, \( u \) is the local streamwise component of velocity, \( u_{max} \) is the maximum centerline velocity, \( z \) is the local channel depth and \( y \) is the channel span location in the rectangular channel. Variables \( a \) and \( b \) represent half of the channel width and channel depth, respectively. Constants \( n \) and \( m \) are given as

\[ m = 1.7 + 0.5 \left( \alpha^* \right)^{-1.4} \]

(5-3)

\[ n = \begin{cases} 
2 & \text{for } \alpha^* \leq 1/3 \\
2 + 0.3 \left( \alpha^* - 1/3 \right) & \text{for } \alpha^* \geq 1/3 
\end{cases} \]

where \( \alpha^* \) is the aspect ratio of the rectangular channel defined as \( 2b/2a \). The coordinate system and variables of the rectangular channel are shown in Figure 5.4.
For all branch channels, \( z \) is equal to zero because experimental data are collected mid-depth in the channel. Therefore, the first bracket in the equation 5.2 becomes unity; hence, only the term in the second bracket needs to be evaluated. For the \( k=0 \) branch, the half width of the channel, \( a \), is 272 \( \mu \)m and the aspect ratio of the channel is approximately 0.46. From Equation 5.3, \( m \) and \( n \) are found to be 2.31 and 2.04, respectively.

![Figure 5.4: Rectangular geometry with coordinate system and parameter identification.](image)

Experimental velocity profiles at different streamwise locations in each channel are normalized by the maximum velocity and plotted with analytical solution. Velocity profiles in the \( k=0 \), 1, and 2b channels are shown Figures 5.5, 5.6, and 5.7, respectively. Seven different streamwise locations, i.e. \( x=870, 1470, 2670, \)
3270, 3870, 4470 and 5070 μm, in the fully developed region along the k=0 channel are presented in Figure 5.5. The vertical axis of Figure 5.5 represents the non-dimensional spanwise position in the channel, varied from -1 to 1. Similarly, Figure 5.6 and 5.7 present a comparison of the analytical solution with the experimental velocity profiles at five and four different streamwise locations in the k=1 and k=2b channels, respectively. All experimental results show excellent agreement with the analytical solution. Therefore, it was concluded that the experimental velocity data is valid and is also fully developed.

Figure 5.5: Experimental and analytical velocity profile at the k=0 branch.
Figure 5.6: Experimental and analytical velocity profile at the k=1 branch.

Figure 5.7: Experimental and analytical velocity profile at the k=2b branch.
CHAPTER 6
RESULTS AND DISCUSSION

In this chapter experimental flow fields in the fractal-like branching channel network are provided and discussed in relation to CFD results. Results are presented in three sections: (1) velocity profiles, (2) velocity contours at bifurcations, and (3) vorticity. In the first section, velocity profiles at the inlet, mid-stream and exit of each branch are discussed in comparison with CFD results. The objective of the second section is to compare experimental data from micro-PIV with CFD results at several bifurcations in the fractal-like flow network. Presented in the third section are vorticity at the bifurcation and inlet region for several locations in the fractal-like channel network.

6.1 Velocity Profiles

Using velocity profiles from micro-PIV measurements, the flow development length, in principle, can be determined, and asymmetry in flow within the fractal-like branching network can be identified. Also, experimental results are used to validate a 3-D CFD model for the fractal-like branching network developed by Alharbi, et al. (2003).
Figure 6.1: Experimental velocity profiles near inlet and at mid-stream of k=0 branch. (Total channel length is 5800 µm).

Figure 6.2: Local coordinate system for k=3c branch.
Figure 6.1 shows spanwise velocity profiles near the inlet of the k=0 branch. The streamwise velocity component is plotted as a function of spanwise distance across the channel. Note that the local coordinate system is always positioned at the point where the inner walls of the two newly formed channels intersect, and the x coordinate is aligned with the inner wall. Figure 6.2 shows where the local coordinate system for the k=3c branch is positioned in the fractal-like branching channel network. Also, this figure provides the channel configuration at the bifurcation between k=2b and k=3c and k=3d with identification of inner and outer walls and a global coordinate system.

The total length of the k=0 branch, based on design, is 5800 μm. In Figure 6.1, the streamwise location closest to the inlet for which data was available for this branch is 235 μm. The mid-stream data was acquired at a streamwise position of 3060 μm. Development of the velocity profiles along the channel near the inlet region is not observable. This is believed to be a consequence of the interrogation window size which was 155 μm wide (streamwise) × 39 μm tall (spanwise). Basically, the interrogation window is much wider in the streamwise direction than in the spanwise direction. Data in each interrogation window is used to generate one average velocity value; hence, each interrogation averages more particles pairs in the streamwise direction than in the spanwise direction. As a consequence, the u component velocity is not as sensitive in the streamwise direction as in the spanwise direction. In addition, the maximum uncertainty at the centerline is estimated to be ±0.2 mm/s, and the centerline velocities in Figure 6.1 are within this
band of uncertainty. In summary, velocity profiles in this study do not provide an adequate means for determining the development length of flow.

Figures 6.3a and 6.3b show u component velocity profiles across the span of the channel acquired from PIV and CFD, respectively. Profiles are taken near the inlet, mid-stream and exit of the k=0 branch. There is excellent agreement between the experimental and computational velocity profiles in terms of shape and magnitude. Note, however, that the magnitudes between experiments and CFD vary by as much as 0.5 mm/s. Part of this discrepancy is attributed to the ±0.2 mm/s uncertainty in experiment data. Also influencing the difference between experimental and CFD results is the difference in the channel width resulting from fabrication of the experimental device in comparison with the CFD model.

There is clearly a symmetry to the flow. A streamwise location closer to the inlet than 235 μm was not obtainable because of a small scratch at the surface of the test section near the inlet, which caused difficulty in viewing fluorescent particles. A flat region in the profile near the middle of channel is evident in both the PIV and CFD profiles and is consistent with the fully developed analytical profile in Figure 5.5. At a streamwise distance 3060 μm downstream of the inlet, the u component velocity profiles appear parabolic with the maximum velocity located at the center of the profile. The maximum values in both the 235 and 3060 μm profiles are within the estimated uncertainty. From the velocity profile near the channel exit, at x=6155 μm, it is noted that the bifurcation affects the flow stream near the exit of the k=0 branch. The velocities noted at spanwise locations of 150 and 400 μm are
Figure 6.3: (a) Experimental and (b) computational velocity profiles near inlet, mid-stream and exit of $k=0$ branch. (Total channel length is 5800 $\mu$m).
Figure 6.4: (a) Experimental and (b) computational velocity profiles near inlet, mid-stream and exit of k=1 branch. (Total channel length is 4100 µm).
greater than near the center of the channel where there exists a velocity deficit (Alharbi et al., 2003).

Figures 6.4a and 6.4b show spanwise profiles of u velocity near the inlet, mid-stream and exit of the k=1 branch from PIV and CFD, respectively. Again, similar trends in flow are observed between these two figures. In contrast with Figure 6.3, the experimental velocity near the inner wall, i.e. y=0, near the channel inlet (x=75 μm) is slower than that near the outer wall. However, for this same streamwise location, the CFD results show symmetry about the channel center line. There are two possible explanations for this observation. The first is low seed density near the outer wall of the k=1 branch. Low seed density at the outer wall may be a result of momentum of the flow at the bifurcation causing a majority of particles to flow toward the inner wall. A second explanation might be error introduced from repositioning the field of view. A positioning error might have caused an offset in the field of view toward the negative y direction relative to the local coordinate system. The velocity profile near the mid-stream location in Figure 6.4a is symmetric and is very similar in shape to the velocity profile at the same streamwise position in Figure 6.4b. In the exit profile, which is noted by x=3900 μm, the maximum velocity is shifted slightly from the center of the channel toward the inner wall. This is attributed to an asymmetry in the bifurcation that causes flow to accelerate around the corner.

Experimental and computational velocity profiles near the inlet, mid-stream and exit of the k=2a branch are shown in Figures 6.5a and 6.5b, respectively.
Figure 6.5: (a) Experimental and (b) computational velocity profiles near inlet, mid-stream and exit of k=2a branch (Total channel length is 2900μm).
Evident from the figures is that the velocity profiles near the mid-stream and exit of the channel are similar in shape between PIV and CFD. Note that the velocity profile near the inlet region in Figure 6.4b is wider in span than the other velocity profiles shown in the figure. This is not evident from Figure 6.5a because of the inability to resolve experimental data to within 40 μm of either wall. In Figure 6.5b, the channel width at x=75 μm is over 200 μm. However, the width of the channel at the other two downstream locations is clearly observed to be 190 μm. The cause for this discrepancy in channel width is noted in Figure 6.6, which shows that the intersection of the outer walls of the k=1 and k=2a branches is downstream of the k=2a coordinate axis origin.

![Figure 6.6: Schematic diagram of asymmetry at bifurcation between branches k=1 and k=2a and k=2b.](image)
As a final note, the experimental channel width for \( x > 80 \ \mu m \) is approximately 10 \( \mu m \) wider than that in the CFD model. This 5% larger flow area for the experimental devices results in a 4.8% lower in velocity compared with CFD. Also contributing to the difference between CFD and PIV velocity is the experimental uncertainty, which is estimated to be \( \pm 0.2 \ \text{mm/s} \).

Comparing Figures 6.7a and 6.7b, the velocity profiles near the inlet and at midstream of the \( k=2b \) channel are very similar. However, a significant difference between the experimental profile in 6.7a and the computational profile in Figure 6.7b exists for the profile near the exit at \( x=2760 \ \mu m \). Although the trend is similar, the magnitude of the maximum experimental velocity is smaller than that from CFD at the same streamwise location. Experimental velocities are collected and averaged 80 \( \mu m \) upstream and 80 \( \mu m \) downstream of the 2760 \( \mu m \) location. Including the upstream data in determining the average is not anticipated to alter the exit velocity profile because the upstream flow is fully developed and the \( u \) component velocity does not change along the \( x \) direction. However, the \( u \) component velocity 80 \( \mu m \) downstream of \( x=2760 \ \mu m \) is decelerating due to an increase in flow area. Therefore, an average velocity in the interrogation window centered at \( x=2760 \ \mu m \) will be artificially low.

Figure 6.8 and Figure 6.9 show experimental and computational \( u \) component velocity profiles near the inlet, mid-stream and exit locations of the \( k=3c \) and \( k=3d \) branches, respectively. In general, the velocity profiles between the PIV and CFD show very similar trends for both \( k=3c \) and \( k=3d \) branches. The major
Figure 6.7: (a) Experimental and (b) computational velocity profiles near inlet, midstream and exit of k=2b branch. (Total channel length is 2900 μm).
Figure 6.8: (a) Experimental and (b) computational velocity profiles near inlet, mid-stream and exit of k=3c branch. (Total channel length is 2050 μm).
Figure 6.9: (a) Experimental and (b) computational velocity profiles near inlet, mid-stream and exit of k=3d branch. (Total channel length is 2050 μm).
difference between CFD and PIV profiles is the following. In Figure 6.8a the x=1780 μm profile intersects the upstream profiles in a different spanwise location than that observed in Figure 6.8b. Again this is believed to be a consequence of the interrogation window size reaching downstream of the channel exit and altering the true experimental profile at x=1780 μm.

Note in both Figures 6.8a and 6.8b an increase in maximum velocity between the midstream location and the channel exit. As a consequence of the design, the k=3c branch segment side walls are parallel for approximately the first half of channel length then taper beyond x=900 μm, as shown schematically in Figure 6.10. The x=1060 μm streamwise location in the k=3c branch is immediately downstream of the location where the taper begins, hence does not show a noticeable change in maximum velocity. However, the velocity profile near the exit of the k=3c branch shows a noticeable increase in velocity magnitude.

In Figure 6.9, the intersection of the x=1060 μm profile with the x=80 μm profile differ between experiment and CFD. In addition the shape of the x=1740 μm profiles differ strongly between 95 μm ≤ y ≤ 115 μm. An explanation for the differences observed between Figures 6.9a and 6.9b lies in the fact that the k=3d branch is tapered such that the channel width decreases in the streamwise direction. Because of tapering, the maximum velocity along the k=3d branch increases with x due to conservation of mass. The averaging process during experimental data reduction, which occurs in a 154 μm ×38 μm interrogation window used for this branch, is expected to be highly influenced by the tapered nature of the channel.
Figure 6.10: Schematic diagram of the bifurcation between $k=2b$ and $k=3c$ and $k=3d$ branch and measurement location at the inlet and 1060 $\mu$m downstream of the inlet.

Figure 6.11: Schematic diagram of the tapered geometry of the $k=3d$ channel and the interrogation windows positioned at the inlet and mid-stream.
The shape of the experimental velocity profiles in Figure 6.9a changes from symmetric to asymmetric in the streamwise direction. However, the profiles in Figure 6.9b are symmetric. The difference between CFD and experiment is caused by uncertainty in the experimental data acquisition and reduction. As noted in Figure 6.11, the local coordinate system originates at the inner wall of the bifurcation, with the x-axis coincident with the inner wall and the y-axis oriented orthogonal. The orthogonal y-axis is shown as a solid line. However, based on the centerline of the channel, a non-orthogonal y axis is shown as a dashed line. Note from Figure 6.11 that the interrogation window is rectangular and is oriented with the inner wall and orthogonal y axis. This yields an interrogation window with the inner edge aligned with the inner wall, but the outer edge not aligned with the outer wall. A schematic representation of a fully developed velocity profile near the tapered channel inlet is provided in Figure 6.11. Although the velocity relative to the dashed line is symmetric about the centerline, the same velocity relative to the orthogonal x-y coordinate axis, i.e. relative to the interrogation window, appears greater near the outer wall than the inner wall. Ideally, a different interrogation window size, one with a higher resolution (smaller pixel size) in the streamwise direction, would be used to evaluate velocity in a tapered channel to minimize the influence of upstream and downstream velocities in the averaging process.

Overall, u component velocity profiles from PIV and 3-D CFD show very good agreement. However, use of a long axial interrogation window results in a low sensitivity in the u component of velocity. Therefore, use of micro-PIV makes
difficult determination of the flow development length and introduces unreasonable errors at the channel exit and along tapered channels.

6.2 Velocity Contours at Bifurcation

Results from PIV are used to validate 3-D CFD and to investigate asymmetry in the flow field. This is further accomplished using contour plots and 2-D vector plots at the bifurcations of the fractal-like branching network. Contour plots of in-plane velocity components, from PIV measurements including combined u-v magnitudes and u and v component magnitudes, are provided and discussed at the following intersections; k=0 and k=1, k=1 and k=2a and k=2b, and k=2b and k=3c and k=3d. The w component of velocity, the out-of-plane velocity component, cannot be resolved by the current PIV system. Velocity vectors and velocity magnitude contours are presented relative to the local coordinate system, which was discussed in the previous section and shown in Figure 6.2. All data are presented at mid-depth, 125 μm from the top surface defining the flow network.

Figure 6.12 contains four different plots. Figure 6.12a is a 2-D vector flow field at the k=0 and k=1 bifurcation. The two components of velocity are in x and y directions. Note that the axes are labeled as x and y relative to the coordinate system aligned with the k=0 branch. Figure 6.12b shows the u-v velocity magnitude contours. Note that in agreement with Figure 6.12a, contours are provided between
Figure 6.12: Experimental 2-D velocity vectors and contours at the bifurcation between $k=0$ and $k=1$. (a) $u$-$v$ component vector plot, (b) $u$-$v$ component contour plot, (c) $u$ component contour plot, (d) $v$ component contour plot.
approximately 40 μm ≤ y ≤ 500 μm, i.e. do not span to inner and outer walls. Also, note the white dash-dot lines in Figure 6.12b. They represent the center lines of the k=0 branch and the two k=1 level branches into which k=0 splits. The intersection of the center of each of these channels occurs at x=5870 μm. This is within about 70 μm of the design length of 5800 μm listed in Table 4.1. Also note that the length of the k=0 branch in Figure 6.12b and Table 4.1, defined by the inner wall length, is 6140 μm. The reason for the discrepancy between inner wall length and the design length is that the fractal-like branching network was designed by a line construction. To create the inner and outer walls, one half of each channel width was offset from the line construction. The result is a wall length longer than the k=0 branch design length reported in Table 4.1. Before presenting the individual u and v component of velocity, which are provided in Figures 6.12c and 6.12d, respectively, the CFD results are compared to the results in Figure 6.12b. The u-v magnitude velocity of the 3-D CFD at mid-depth is shown as contours in Figure 6.13. Comparing Figure 6.12b and Figure 6.13, the PIV flow trend is very similar to that of CFD at the bifurcation. Unlike Figure 6.12b, the CFD results provide information all the way to the walls. Note, however, that due to a smoothing process velocities at the walls are not identically zero, as would be expected. The maximum velocities from PIV and CFD data are approximately 8.4 mm/s and 8.1 mm/s, respectively. The variation in maximum velocity between PIV and CFD is a result of uncertainty in the experimental data and due to variations in channel dimensions resulting from the fabrication process.
Figures 6.12c and 6.12d show u component and v component contours of the velocity, respectively. The u component velocity contours are very similar to the u-v component velocity because the u-component velocity dominates the flow compared with the v-component velocity. The maximum velocities of the u-v contours and u component contours upstream of 6100 μm are essentially identical because the v component of velocity is negligible in this region, as is noted from Figure 6.12d. Downstream of 6100 μm, as expected, the flow has a stronger v component relative to the k=0 coordinate system. The maximum velocities of the u-v contours and u contours downstream of 6100 μm are approximately 8.1 and 7.6
mm/s, respectively, differing by no more than 0.5 mm/s. This explains the similarity in trend and magnitude observed between Figure 6.12b and Figure 6.12c.

One final observation regarding the contour plots in Figure 6.12 is that they exhibit a non-smooth color distribution along the inner wall of the bifurcation. This is a consequence of data reduction. The contour option in Matlab®, a matrix based code, uses an interpolation function to create color shading. Because no data exists in the regions creating the inner walls of the newly formed \( k=1 \) branch segments in these figures, data interpolated near this region exhibits a non-smooth behavior. This non-smooth coloring in contour plots along both inclined and declined walls is noted in other figures in this section as well.

Figure 6.14 shows the same flow plots as in Figure 6.12, but at the bifurcation between \( k=1 \) and \( k=2a \) and \( k=2b \). Recall that because the current PIV system can only measure 2-D flow fields, data presented are limited to \( u \) and \( v \) components of velocity. The local coordinate system is defined by the \( k=1 \) branch. Also, data are only available to within 40 \( \mu m \) of either wall, i.e. \( 40 \mu m \leq y \leq 260 \mu m \).

Figure 6.14a shows the velocity vectors from \( x=3700 \mu m \) into the two new channels formed from the bifurcation. At \( x=3700 \mu m \), the velocity distribution across the channel is symmetric. However, downstream of 3900 \( \mu m \) the magnitude of the velocity vectors along the bottom row, corresponding to \( y \approx 40 \mu m \), increase whereas velocity vectors along the top row, \( y \approx 260 \mu m \), decrease. This trend was
Figure 6.14: Experimental 2-D velocity vectors and contours at the bifurcation between $k=1$ and $k=2a$ and $k=2b$. (a) $u-v$ component vector plot, (b) $u-v$ component contour plot, (c) $u$ component contour plot, (d) $v$ component contour plot.
identified earlier from velocity profiles, in particular from Figure 6.5 and Figure 6.7. Also, Alharbi (2003) notes an acceleration of flow toward the \(k=2b\) branch.

![Local coordinate system for \(k=2a\) and \(k=2b\)](image)

Figure 6.15: Three dimensional CFD \(u-v\) component velocity contours at the bifurcation between \(k=1\) and \(k=2a\) and \(k=2b\).

Figure 6.14b from PIV and Figure 6.15 from 3-D CFD show very good agreement. In Figure 6.14b, the position of maximum velocity moves from a mid-span location at \(x=3850\ \mu m\) more toward the \(k=2b\) channel, i.e. toward smaller \(y\) values, at \(x=3950\ \mu m\). This trend is also observed from the vector plot in Figure 6.14a. Figure 6.14c shows a \(u\)-component velocity contour plot. The flow pattern and the magnitude of the velocity distribution are very similar to Figure 6.14b except near the entrance of the \(k=2b\) branch, which experiences a strong \(v\) component as noted in Figure 6.14d. The magnitude of the \(v\) component of velocity
in Figure 6.14d is negligible for approximately 50% of the image. A non-smooth color distribution is observed along the inner wall of the k=2b branch in Figure 6.14c and 6.14d. The location is identified by a dashed circle in these two figures. Note that the k=2a branch, which is aligned more with the k=1 branch, than is the k=2b branch, does not experience this non-smooth patterning. The reason for non-smooth coloring was noted previously to be a consequence of the interpolation program used in generating contours.

Figure 6.16 shows u-v components of velocity at the bifurcation between k=2b and k=3c and k=3d. The coordinate system is that of the k=2b branch and originates at the inner wall of the k=2b branch. Coordinate y is in the spanwise direction and y=0 is coincident with the inner wall. Unlike the previous plots in Figures 6.12 and 6.14, where the inner wall is at the bottom of the ordinate, the inner wall for Figure 6.16 happens to be at the top of the plot. Refer to Figure 4.1, the fractal flow network schematic diagram, for a physical perspective. Because data acquisition within 40 µm of the walls was not possible, the physical locations of the inner and outer walls are approximately 40 µm above the top layer of velocity data and below the bottom layer of velocity data, respectively.

The velocity vector plot in Figure 6.16a is magnified near the bifurcation to observe changes in the direction and magnitude of the velocity vectors. Comparing Figure 6.16b from PIV and Figure 6.17 from CFD reveals that the u-v magnitudes of velocity from experiments and from 3-D CFD are very similar, with maximum velocities of 5.8 mm/s and 6 mm/s, respectively. The u component and v component
Figure 6.16: Experimental 2-D velocity vectors and contours at the bifurcation between $k=2b$ and $k=3c$ and $k=3d$. (a) $u-v$ component vector plot, (b) $u-v$ component contour plot, (c) $u$ component contour plot, (d) $v$ component contour plot.
velocity contours are provided in Figures 6.16c and 6.16d, respectively. A strong transverse flow toward the k=3c branch is observed in Figure 6.16d.

![Figure 6.17: Three dimensional CFD u-v component velocity contours at the bifurcation between k=2b and k=3c and k=3d.](image)

6.3 Vorticity

Flow rotation, the spanwise distribution of shear stress and the streamwise location where flow becomes fully developed can be determined, in principle, from vorticity. In this section, mean vorticity contours based on u-v velocity data and vorticity profiles across the width of the channel are provided.
Figure 6.18 shows the vorticity at the bifurcation between the k=0 and k=1 branches. Because of the manner in which Matlab® performs interpolation to generate contour plots, the coloring of vorticity near the inner walls in Figure 6.18a is not smooth. By reorienting the image for the k=1 branch such that the x axis of local coordinate system is horizontal, a smooth contour can be generated. Such a smooth contour was generated and subsequently rotated as noted in Figure 6.18b. The local coordinate system for the k=1 branch originates at the apex created by the two newly formed walls. These walls, recall, are referred to as inner walls. The coordinate system for k=1 is sketched by white lines in Figure 6.18a. The same coordinate system is shown in black in Figure 6.18b. The dashed line around the angled branch provides an aid to assist the reader in understanding the relative location between the lower level and higher level branches. Note that the data in Figure 6.18 does not extend to the walls, the reason for which was previously discussed. Note also that the data shown in Figure 6.18b starts approximately 100 μm downstream of the coordinate origin. The direction of vorticity is noted in Figure 6.18a. A counter-clockwise flow rotation is denoted as positive. The strength of vorticity is denoted by color, with red strongly positive and blue strongly negative. The high positive and negative vorticity near the inner walls is a consequence of high shear stress.
Figure 6.18: Mean vorticity at (a) the bifurcation between the k=0 and k=1 channels, and (b) the inlet region of the k=1 channel.

Figure 6.19 shows the mean vorticity across the span of the k=0 channel. The vorticity near the inlet region has more fluctuations compared with vorticity mid-stream. In general, the shape of the vorticity profiles at the various positions along the streamwise are consistent. The profiles are mirror symmetric about the mid-span of the channel, i.e. $y=250 \mu m$ with a region of zero vorticity that extends about $\pm 40 \mu m$ on either side of the mid-span location.

The vorticity trend is very similar to that in Figure 6.20 which was generated using the fully developed analytical velocity profile shown in Figure 5.5.
Figure 6.19: Mean vorticity across width of k=0 channel.

Figure 6.20: Theoretical non-dimensional vorticity plot across k=0 channel.
The vorticity increases slowly away from the mid-span region of the channel with the highest vorticity near the walls. Note that the abscissa of Figure 6.20 is not the same as in Figure 6.19. This is because data in Figure 5.5, from which Figure 6.20 was generated, is normalized. Obvious from Figure 6.19, it is difficult to use the vorticity profile to determine at which streamwise location the flow becomes fully developed.

![Figure 6.21: Mean vorticity across width of k=1 channel.](image)

Figure 6.21 shows the mean vorticity across the width of the k=1 channel. Vorticity near the walls is highest because that is where the shear stress is highest. Vorticity near the outer wall does not change as much as that near the inner wall, because the outer wall boundary layer continues to develop rather than redevelop.
follow a bifurcation as it does near the inner wall (see Alharbi, et al. 2003). For the streamwise location of $x=110 \, \mu m$, the vorticity near the inner wall is higher than vorticity at any other streamwise $x$ locations shown. The vorticity near this inner wall decreases in the streamwise direction. When the inner walls form, a hydrodynamic boundary layer initiates at the apex. As the boundary layer grows in the streamwise direction, the shear stress decreases, as does the vorticity. The vorticity profile at mid-stream is, for all practical purposes, linear, suggesting fully developed flow. The vorticity profiles near the inner wall are bent slightly toward a lower vorticity value for $x \leq 600 \, \mu m$. One possible reason could be that particles are struck on the tip of the bifurcation as a result of static forces. The other plausible explanation could be the uncertainty in vorticity, which from an uncertainty analysis provided in Appendix D is expected to be ±2.

Figure 6.22 shows vorticity contours at the bifurcation between the $k=1$ and $k=2a$ and $k=2b$ branches. Coloring in the vorticity contour plot is not smooth in the lower right corner of Figure 6.22a. The cause for this non-smooth contour is the interpolation function in Matlab®, which was discussed in a previous section. Figures 6.22b and 6.22c are vorticity contour images for the $k=2a$ and $k=2b$, respectively. The local coordinate systems for both the $k=2a$ and $k=2b$ channels are shown as white lines in Figure 6.22a. The same local coordinate systems for the $k=2a$ and $k=2b$ channels are also sketched in black in Figures 6.22b and 6.22c, respectively. The data for both the $k=2a$ and $k=2b$ start approximately 100 $\mu m$ downstream of the local coordinate system. The direction of vorticity is noted in
Figure 6.22: Mean vorticity (a) at the bifurcation between the $k=1$ and $k=2a$ and $k=2b$ channels, (b) the inlet region of the $k=2a$ channel, and (c) at the inlet region of the $k=2b$ channel.

Figure 6.22a. A counter-clockwise and a clockwise flow rotation are denoted as positive and negative, respectively. The strength of vorticity is also shown by color. In Figure 6.22b, positive vorticity is represented as red. In contrast, in Figure 6.22c, the color red corresponds to a negative vorticity. The high positive and negative
vorticity near the inner walls represents high shear stress. The color distribution in the contours for k=2a and k=2b branches is fairly consistent along the streamwise direction. This can be noted from Equation 5-1, because the change of v in the streamwise direction is much smaller than the change of u in the spanwise direction.

Figures 6.23 and 6.24 show the spanwise distribution of vorticity in the form of mean vorticity profiles for the k=2a and k=2b branches, respectively. Near the outer wall in Figure 6.23, the vorticity increases noticeably from the inlet to the streamwise position of x=400 μm. This increase, which is more dramatic than for profiles for x>400 μm, is due to the tapered nature of the wall, as was noted in Figure 6.6. Near mid-stream, the vorticity profile becomes essentially linear. In Figure 6.24, most of the vorticity profiles are linear. At x=120 μm, the edges of the vorticity profile are bent slightly toward a lower vorticity value. Possible explanations for the observation were discussed in a previous section.

Figure 6.25 shows u-v vorticity contours at the bifurcation between the k=2b and k=3c and k=3d branches. The local coordinate systems for k=3c and k=3d are shown together in Figure 6.25a and individually in Figure 6.25b and Figure6.25c, respectively. The rotation of the flow near the inner wall holds the same definition as before. Due to the configuration of this particular bifurcation, the magnitude of vorticity at the inner wall of the k=3c branch is about 50% stronger than along the inner wall of the k=3d branch. This is attributed to flow acceleration from the k=2b to k=3c branch, as was noted by Alharbi, et al. (2003), which would cause a higher shear at the inner wall of the k=3c branch.
Figure 6.23: Mean vorticity across width of k=2a channel.

Figure 6.24: Mean vorticity across width of k=2b channel.
Evident from this vorticity analysis is that mean vorticity across a channel width can not be used to assess the flow development length, due to the magnitude of uncertainty. However, it can be used to sense the aspect ratio of the channel. For example, in the k=0 channel, which has a \( W_0/H_0 \) aspect ratio of 2.2, there is a region
of zero vorticity near the midspan region. Channel segment $k=2b$, which has a $W_{2b}/H_{2b}$ aspect ratio of 0.8, has a linear spanwise vorticity profile. This suggests that channels with higher aspect ratios are more likely to have a zero vorticity region near the center of the channel.
CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

In this study a low-speed, inexpensive micro-PIV visualization technique was developed and applied to investigate two-dimensional flow fields and validate a three-dimensional CFD model of a fractal-like branching flow network. Analytical solutions of fully developed internal duct flow were also provided. Velocity profiles, two-dimensional velocity vector fields, velocity magnitude contour plots, and vorticity contours and profiles were provided. A fixed low flow rate, corresponding to a Reynolds number of 1.46 in the largest channel, was used for this study.

From velocity profiles, the development length and flow patterns were investigated. The flow development length was not obtainable because velocity profile variations in the streamwise direction were within the level of uncertainty. Experimental profiles well matched computational results, except for at the channel exit and in tapered channels. The cause was attributed to the rectangular interrogation window, which averaged upstream and downstream data to determine a local velocity. In general experimental velocity contour plots and computational contour plots were compared and had good agreement.

In the investigation of vorticity, flow rotation, locations where high shear stress exists and vorticity profiles in the fractal-like branching channel network were provided. Several bifurcations and inlet regions were investigated. Higher
vorticity was found to occur near the inner and outer walls. Vorticity profiles in low aspect ratio channels were linear across the spanwise direction. However, in the high aspect ratio channel the spanwise vorticity profile deviated from linear. Region which experience newly formed hydrodynamic boundary layers exhibit higher vorticity values than at other streamwise locations along the inner wall.

In order to improve results for purposes of investigating flow development length, the interrogation window should be changed from a rectangular to a square shape. Otherwise, use of a rectangular interrogation window proved sufficient for investigating vorticity and fully developed velocity profiles. For the current PIV setup, however, changing the shape of the interrogation window may not provide sufficient improvement because of low resolution of the flow field image. The most influential factors in PIV measurements are image resolution and frame speed of CCD camera. A high performance CCD camera for PIV can capture a higher speed of moving particles, hence resolve a higher velocity more clearly. In addition, zooming in on particular regions, such as the inner wall and outer wall regions, would provide a better range of data.


APPENDICES
Figure A.1: Top, channel and bottom layer drawings of micro-scale fractal-like branching flow network.
APPENDIX B

FLUORESCENT PARTICLE WAVELENGTH SPECTRA
Duke Scientific Corporation Red Fluorescent Microspheres
Spectra in DI Water  Stokes Shift = 70nm

Figure A.2: Fluorescent particle wavelength spectra.
APPENDIX C
MATLAB CODES

C.1 Image sorting program

clear all
close all
c

n=1;
currentdir=cd;

imnum=input('Enter the number of images.----> ');
initial=input('Enter the initial number----> ');
base_name=input('Enter the base name of the images(in apostrophes).----> ');

for k=initial:imnum+initial-1
    str=num2str(k);
    callname=[base_name,str];
    p=[callname,'.tif'];
    a=imread(p,'tif');
    b=double(a);
    sz=size(b);

    num_bad=20;
    count=0;
    for i=1:sz(1);
        for j=1:sz(2)-21;

            if(b(i,j)> 200)
                if ((i<round(0.1*sz(1))) ||(i>round(0.9*sz(1)))))
                    space=6;
                    break;
scan=12;
elseif ((i<round(0.25*sz(1))) | (i>round(0.75*sz(1))))
    space=8;
    scan=17;
else
    space=10;
    scan=20;
end
jj=j+space;
while ((jj < j+scan) & (b(i,jj) <200))
    jj=jj+1;
    if b(i,jj)>200
        count=count+1;
    end
end
end
end
end

if (count<num_bad)
    cd good;
    dir1=cd;
    goodstr=num2str(n);
    changename=[basename,goodstr];
    name=[changenametf];
    imwrite(a,name,'tif','compression','none');
    n=n+1;
    cd(currentdir);
else
    delete(p,'tif');
end
C.2 Separation of odd and even field images

%******************************************************************************
%Program for separating odd and even field images using linear interpolation
%pictures have to be taken by NTSC CCD camera
%this program automatically read the input image and separate it into two pictures(odd and even).
%then save those picture as compressed 'tif' file (8bit)
%
%Name: Younghoon Kwak
%Date: Nov. 25, 2002
%******************************************************************************

%clear and close opened file on MATLAB
clear all
close all
clc

imnum=input('Enter the number of images.-----> ');%enter the number of images
initial=input('Enter the initial number-----> ');%enter the initial number
name=input('Enter the base name of the images(in apostrophes).-----> ');%enter the base name of the images

for k=initial:imnum+initial-1
str=num2str(k);

%read the image and expand double size (MATLAB only read 16 bit image)

callname=[name,str,'.tif'];%read the image and expand double size (MATLAB only read 16 bit image)
a=imread(p,'tif');%read the image and expand double size (MATLAB only read 16 bit image)
b=double(a);%read the image and expand double size (MATLAB only read 16 bit image)
sz=size(b);%read the image and expand double size (MATLAB only read 16 bit image)

oddf=zeros(sz(1), sz(2));%read the image and expand double size (MATLAB only read 16 bit image)
evenf=zeros(sz(1),sz(2));%read the image and expand double size (MATLAB only read 16 bit image)

%separate the input image to odd and even images using linear interpolation

even_num = 1;

for i=1:sz(1);
    num=i/2;
    if (num==0)
        even_num = even_num+1;
        if (i==sz(1))
            oddf(i,:)= b(i-1,:);%read the image and expand double size (MATLAB only read 16 bit image)
evenf(i,:)= b(i,:);%read the image and expand double size (MATLAB only read 16 bit image)
        else
            oddf(i,:=(b(i-1,:)+b(i+1,:))/2;
            evenf(i,:=(b(i-1,:)+b(i+1,:))/2;

    else
        oddf(i,:)= b(i,:);%read the image and expand double size (MATLAB only read 16 bit image)
evenf(i,:)= b(i,:);%read the image and expand double size (MATLAB only read 16 bit image)
end
end

%save the images

for i=1:sz(1);
    if (num==0)
        oddf(i,:)= b(i,:);%read the image and expand double size (MATLAB only read 16 bit image)
evenf(i,:)= b(i,:);%read the image and expand double size (MATLAB only read 16 bit image)
    else
        oddf(i,:)= b(i,:);%read the image and expand double size (MATLAB only read 16 bit image)
evenf(i,:)= b(i,:);%read the image and expand double size (MATLAB only read 16 bit image)
end
end

%close and clear opened file on MATLAB
close all
clc
evenf(i,:) = b(i,:);
end
else
if(i==1)
    oddf(i,:) = b(i,:);
    evenf(i,:) = b(i+1,:);
elseif (i==sz(1))
    oddf(i,:) = b(i,:);
    evenf(i,:) = b(i-1,:);
else
    evenf(i,:) = (b(i-1,:) + b(i+1,:))/2;
    oddf(i,:) = b(i,:);
end
end
end

% Gaussian filter
n1=3; n2=3; sigmal = 0.5; sigmax = 0.5; theta = 0;
filter1 = d2gauss(n1,sigmal,n2,sigmax,theta);
evenf = conv2(evenf,filter1,'same');
oddf = conv2(oddf,filter1,'same');

% back to 8bit format of picture
evenf = uint8(evenf);
oddf = uint8(oddf);

name = [callname,'odd','.tif'];
imwrite(evenf,name,tif,'compression','none')

namo = [callname,'even','.tif'];
imwrite(oddf,namo,tif,'compression','none')
end
C.3 Average of velocity data from Visiflow®

%******************************************************************************
%AVEVERAGE DATA AND REARRANGE DATA FORMAT
%This program average the number of data sets and output in txt format.
%1. enter the number of data for averaging ('txt' file format)
%2. enter the base name of the data file
%example--> data file name: image1.txt, image2.txt, image3.txt, image4.txt --> base name is 'image'
%3. enter the z-location along the depth of the channel.
%
%Author: Younghoon Kwak
%Date: Sept. 25, 2002
%Udapted: March 03, 2003
%******************************************************************************

clear all;
close all;
clc

imnum=input('Enter the number of files.-----> ');
base_name=input('Enter the base name of the txt file (in apostrophes).-----> ');
location=input('enter the z-location
');
xlocation=input('enter the initial x-location-----> ');
ylocation=input('enter the initial y-location-----> ');
delx=input('enter the x-increment
');
dely=input('enter the y-increment
');

%Loading data file
for k=1:imnum
    str=num2str(k);
callname=[base_name,str];

    filein=[callname,'.txt'];
p(k,:,:)=load (filein,'txt');
end

%Size of the impoted data and find initial and final values for x and y
picsize=size(p(1,:));
final=picsize(1,2);
initx=p(1,1,1);
inity=p(1,1,2);
finalx=p(1,final,1);
finaly=p(1,final,2);
xincrement=p(1,2,1)-p(1,1,1);

io=1;
while p(1,io,2) == inity
    io=io+1;
end
yincrement = p(1,io,2)-p(1,1,2);
xo=initx;
yo=inity;
zo=zlocation;

numx=(finalx-initx+xincrement)/x increment;
umy=(finaly-inity+yincrement)/yincrement;

%averaging process
l=1;
yynew=ylocation;
for ii=1:numy
    xx=xo;
    yy=yo;
    xxnew=xlocation;
    for jj=1:numx
        xd2(ii,jj)=xx;
        yd2(ii,jj)=yy;
        xnew(ii,jj)=xxnew;
        ynew(ii,jj)=yynew;

        if (xx == p(1,1,1))
            sum_vmag =0;
            sum_u =0;
            sum_v =0;

            for kk=1:imnum
                v_magd2(kk,ii,jj)=p(kk,1,3);
                sum_vmag=sum_vmag+v_magd2(kk,ii,jj);

                ud2(kk,ii,jj)=p(kk,1,3)*cos(p(kk,1,4));
                sum_u=sum_u+ud2(kk,ii,jj);

                vd2(kk,ii,jj)=p(kk,1,3)*sin(p(kk,1,4));
                sum_v=sum_v+vd2(kk,ii,jj);

            end

            v_magnitude(ii,jj)=sum_vmag/imnum;
            uvelocity(ii,jj)=sum_u/imnum;
            vvelocity(ii,jj)=sum_v/imnum;
            zd2(ii,jj)=zo;
            wvelocity(ii,jj)=0;
            l=l+1;
        else
            for kk=1:imnum
                v_magd2(kk,ii,jj)=0;

        end

end
ud2(kk,ii,jj)=0;
v_d2(kk,ii,jj)=0;
end
v_magnitude(ii,jj)=0;
uvelocity(ii,jj)=0;
vvelocity(ii,jj)=0;
wvelocity(ii,jj)=0;
z_d2(ii,jj)=zo;

xxnew=xxnew+delx;
ex=xx+xincrement;
end
yo=yo+yincrement;
yynew=yynew+dely;
end

%%%%%%%%%%% OUTPUT %%%%%%%%%%%%%
filename=input('please type the filename for writing----->');
fileout=[filename, '.txt'];

fid=fopen(fileout,'w');
m=1;
for i=1:numx
    for j=1:numy+2
        if (j==1)
            fprintf(fid,'%8.1f %8.1f %9.7f %9.7f %9.7f %f %f %f
', xnew(j,i),ynew(j)-ylocation,zlocation,U,0,0,0);
        elseif(j==numy+2)
            fprintf(fid,'%8.1f %8.1f %9.7f %9.7f %9.7f %f %f %f
', xnew(j-2,i),ynew(j-2)+ylocation,zlocation,U,0,0,0);
        else
            fprintf(fid,'%8.1f %8.1f %9.7f %9.7f %9.7f %f %f %f
', xnew(j-1,i),ynew(j-1,i),zd2(j-1,i),uvelocity(j-1,i),vvelocity(j-1,i),v_magnitude(j-1,i));
        end
        m=m+1;
    end
end
status=fclose(fid);
C.4 Velocity analysis

clear all
close all

n=input('enter name -----> '); numy=input('enter number of y data space----> ');
ch_width=input('enter channel width---> '); 
filein=[n,.txt'];
p=load (filein,.txt');
ch_depth=250; %fixed channel depth

initialy=p(1,2);
picsize=size(p);
totalnum=picsize(1,1);
numx=totalnum/numy;

count=0;
for i=1:numx
    for j=1:numy
        count=count+1;
        x(j,i)=p(count,1);
        y(j,i)=p(count,2);
        z(j,i)=p(count,3);
        u(j,i)=p(count,4);
        v(j,i)=p(count,5);
        w(j,i)=0;
        v_mag(j,i)=p(count,6);
    end
end

% Figure 1 (2D- Quiver plot) %

figure(1); %2d quiver plot
axes('box','on','LineWidth',factor);
qq=quiver(x,y,u*1000,v*1000);
set(qq,'LineWidth',factor);
PBASPECT([2.24 1 1]) %<<<<<<<<<<<<<<
AXIS([5400 6800 0 ch_width])%<<<<<<<<<<<<<<
set(gca,'FontSize',18,'FontWeight','bold','color','w');

xlabel('x (\mu m)');
ylabel('y (\mu m)');

%%%%%% Figure 2 (Color plot V_mag) %%%%% Figure(2);
axes('box','on','LineWidth',factor);

color(x,y,v_mag*1000);
shading interp;
PBASEP((2.24 1 1))%<<<<<<
H=colorbar('horiz');
set(H,'FontSize',18,'FontWeight','bold');

set(gca,'FontSize',18,'FontWeight','bold','color','w');

xlabel('x (\mu m)');
ylabel('y (\mu m)');

%%%%%% Figure 3 (Color plot U) %%%%% Figure(3);
axes('box','on','LineWidth',factor);

pcolor(x,y,u*1000);
shading interp;
PBASEP((2.24 1 1))%<<<<<<
H=colorbar('horiz');
set(H,'FontSize',18,'FontWeight','bold');

set(gca,'FontSize',18,'FontWeight','bold','color','w');

xlabel('x (\mu m)');
ylabel('y (\mu m)');

%%%%%% Figure 4 (Color plot V) %%%%% Figure(4);
axes('box','on','LineWidth',factor);

pcolor(x,y,abs(v)*1000);
shading interp;
PBASEP((2.24 1 1))%<<<<<<
H=colorbar('horiz');
set(H,'FontSize',18,'FontWeight','bold');

set(gca,'FontSize',18,'FontWeight','bold','color','w');

xlabel('x (\mu m)');
ylabel('y (\mu m)');
figure(5); % contour plot

axes('box','on','LineWidth',factor);
contour(x,y,v_mag*1000,20);
PBAPECT([2.24 1])

H=colorbar('horiz');
set(H,'FontSize',18,'FontWeight','bold');
set(gca,'FontSize',18,'FontWeight','bold')
xlabel('x (um)');
ylabel('y (um)');

figure(6); % velocity profile along the x-location

axes('box','on','LineWidth',factor);

for i=1:2:numx
    v_shift=x(1,i);
    for j=1:numy
        v_magcontour(j,i)=0;
        else
            v_magcontour(j,i)=u(j,i);
        end
        v_contour(j) = v_magcontour(j,i)*6000+v_shift;
        v_contour(j) = u(j,i)*25000+v_shift;
        y_contour(j) = y(j,i);
    end
    hold on
    HH=plot(v_contour,y_contour,'b-');
    set(HH,'LineWidth',2);
end

PBASPECT([2.24 1])

AXIS([5400 6800 0 ch_width])
set(gca,'FontSize',18,'FontWeight','bold','LineWidth',0.5);
xlabel('x (um)');
ylabel('y (um)');
C.5 Mean vorticity calculation

```matlab
%******************************************************************************
%Program for calculating mean vorticity
%
%
%Name: Younghoon Kwak
%Date: March. 20, 2003
%******************************************************************************

clear all
close all

n=input('enter name ------> ');
numy=input('enter number of y data space----> ');
ch_width=input('enter channel width---> ');
filein=[n,'.txt'];
p=load (filein,'txt');
ch_depth=250;  %fixed channel depth
initialy=p(1,2);
picsize=size(p);
totalnum=picsize(1,1);
umx=totalnum/numy;
count=0;

for i=1:numx
    for j=1:numy
        count=count+1;
x(j,i)=p(count,1);
y(j,i)=p(count,2);
z(j,i)=p(count,3);
u(j,i)=p(count,4);
v(j,i)=p(count,5);
w(j,i)=0;
v_mag(j,i)=p(count,6);
end
end

%Mean Vorticity Calculation
for i= numx:-1:1
    for j=numy:-1:1
        if(i==1 | j==1)
            break;
        else
            vorticity(j-1,i-1)=(( 1000000*( (v(j,i-1)-v(j,i)) / (x(j,i-1)-x(j,i)) - ((u(j-1,i)-u(j,i))/(y(j-1,i)-y(j,i)) )) )/2;
x_vort(j-1,i-1)=x(j,i);
y_vort(j-1,i-1)=y(j,i);
end
end
```

% end

end

% GOUGUE ==-------------------------- FIGURE ==--------------------------

figure(1);

% Figure 1 (mean vorticity)
figure(1);

axes('box','on','LineWidth',1);

pcolor(x_vort,y_vort,vorticity);
shading interp;
PBASPECT([3 1 1])
H=colorbar('horiz');
set(H,'FontSize',8,'FontWeight','bold');
set(gcf,'color','w','units','inches','position',[0 0 8 5.17]);
set(gcf,'PaperPosition',[0 0 20.30 13.13]);% Figure size for exporting(cm)

set(gca,'FontSize',18,'FontWeight','bold','color','w');
xlabel('x (\mu m)');
ylabel('y (\mu m)');</ref>
APPENDIX D
UNCERTAINTY ANALYSIS

Uncertainty analysis for the velocity measurements acquired in this experimental study is presented in this appendix. The uncertainty analysis presented by Figliola and Beasley (1999) was followed.

Uncertainty of Velocity

Velocity vectors, which were used in this study, were determined by the displacement of particles during a known time interval. The velocity formula is

\[ V = \frac{\Delta s}{\Delta t} \]  \hspace{1cm} (A-1)

where \( \Delta s \) is the displacement of a particle and \( \Delta t \) is the time interval between two images of the particle. The uncertainty in velocity is associated with three elemental velocity errors which result from PIV velocity measurements, the syringe pump, and the fabrication process. The total uncertainty relation with the three elemental errors is

\[ U_{w,\text{total}} = \sqrt{U_{v,\text{PIV}}^2 + U_{v,\text{pump}}^2 + U_{v,\text{fabrication}}^2} \]  \hspace{1cm} (A-2)
where $U_{v,\text{PIV}}$ is the velocity uncertainty associated with PIV measurements, $U_{v,\text{pump}}$ is the velocity uncertainty attributed to the syringe pump and $U_{v,\text{fabrication}}$ is the uncertainty of velocity resulting from channel fabrication.

### D.1 Uncertainty of velocity from PIV measurements

Using the Kline-McClintock relationship, the uncertainty in velocity from PIV measurements is

$$U_{v,PIV} = \sqrt{\left(\frac{\partial V}{\partial S} U_{\Delta s}\right)^2 + \left(\frac{\partial V}{\partial t} U_{\Delta t}\right)^2}$$

(A-3)

where $V$ is velocity, $s$ is displacement, $U_{\Delta s}$ is the uncertainty of displacement, $t$ is time and $U_{\Delta t}$ is the uncertainty of time measurement. This velocity uncertainty from PIV measurement is function of the uncertainties in displacement and time measurements. Values for uncertainty are reported in these variables are reported in subsequent sections.

#### D.1.1 Displacement uncertainty
For the uncertainty of the displacement ($U_{\Delta}$), the bias ($U_{B\Delta}$) and precision ($U_{P\Delta}$) errors are involved and related by

$$U_{\Delta} = \sqrt{U_{B\Delta}^2 + U_{P\Delta}^2}$$  \hspace{1cm} (A-4)

Bias errors occurred from the resolution of the image ($B_{\Delta(1)}$) and image diffraction ($B_{\Delta(2)}$). The minimum measurement unit in an image is a pixel and in each branch level a pixel corresponds with a different scale in $\mu$m. The uncertainty of displacement is half of the spatial resolution in the image.

The other bias error occurs from image diffraction. For micro-PIV, the spatial resolution is limited by the effective diameter of particle images and this is discussed in Meinhart, et al. (1999). The effective diameter of a particle image is

$$d_e = \left[ M^2 d_p^2 + d_s^2 \right]^{1/2}$$  \hspace{1cm} (A-5)

For the present experiments, the magnification, $M$, was 10, and particle diameter, $d_p$, was 3$\mu$m. The diameter of the diffraction-limited point function is

$$d_s = \left[ 2.44M \frac{\lambda}{2NA} \right]$$  \hspace{1cm} (A-6)

For the present experiment, the wavelength, $\lambda$, of the emission from particles was 590 nm, and the numerical aperture, NA, was 0.3. If a particle image diameter is resolved by 3-4 pixels, the location of a particle image correlation peak can be located to with one-tenth of the effective diameter. Using the effective particle diameter, the measurement uncertainty is calculated by
Precision error which is associated with particle displacement is calculated from the standard deviation with 95% confidence. Visflow® software calculates the velocity data using a time separation input given by the user. Therefore, the particle displacement can be extracted from the velocity data provided by Visflow® software. Six different data sets were used to find the standard deviation of the particle displacement. Because of low number of the samples, the student-t test was used. The precision uncertainties in particle displacement for each branch were calculated. Bias and precision uncertainties are individually reported, as is the total displacement uncertainty calculated from Equation A-4, in Table A.1.

### Table A.1: Displacement uncertainty

<table>
<thead>
<tr>
<th>Branches</th>
<th>Bias (mm)</th>
<th>Precision at the mid-span (mm)</th>
<th>Precision near the wall (mm)</th>
<th>Total displacement uncertainty at the mid-span (mm)</th>
<th>Total displacement uncertainty near the wall (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k=0</td>
<td>0.0007</td>
<td>0.0018</td>
<td>0.0008</td>
<td>0.0019</td>
<td>0.0011</td>
</tr>
<tr>
<td>k=1</td>
<td>0.0011</td>
<td>0.0017</td>
<td>0.0011</td>
<td>0.0021</td>
<td>0.0016</td>
</tr>
<tr>
<td>k=2a</td>
<td>0.0007</td>
<td>0.0020</td>
<td>0.0014</td>
<td>0.0021</td>
<td>0.0016</td>
</tr>
<tr>
<td>k=2b</td>
<td>0.0007</td>
<td>0.0012</td>
<td>0.0009</td>
<td>0.0014</td>
<td>0.0011</td>
</tr>
<tr>
<td>k=3c</td>
<td>0.0007</td>
<td>0.0018</td>
<td>0.0018</td>
<td>0.0019</td>
<td>0.0019</td>
</tr>
<tr>
<td>k=3d</td>
<td>0.0007</td>
<td>0.0003</td>
<td>0.0007</td>
<td>0.0008</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

### D.1.2 Time uncertainty
The time uncertainty resulted from the chopper frequency and the angle between the two lager slots in the designed chopper. The chopper frequency uncertainty is ±0.12 and ±0.09 Hz for \( k \leq 2 \) and \( k=3 \), respectively. The uncertainty of the angle between two slots is ±0.5°, which apply for all branches. The total time uncertainties are 0.028 ms and 0.041 ms for \( k \leq 2 \) and \( k=3 \), respectively.

D.2 Fabrication and pump uncertainty

Fabrication errors occurred from the laser machining process. The machined channel dimensions were measured by a microscope with approximately ±2 μm uncertainty. Given \( Q=AV \), and assuming no errors in the pump readout for \( Q \) at this point, the velocity uncertainties due to fabrication errors are 0.24 and 0.35 mm/s in the \( k=2 \) and \( k=3 \) branches. There are compared with 0.08 mm/s and 0.09 mm/s for \( k=0 \) and \( k=1 \).

The pump accuracy was given by manufacturer as 0.5% of readings and the uncertainties of velocity associated with error of the flow rate with 0.5% accuracy were approximately 0.02 and 0.01 mm/s for \( k \leq 2 \) and \( k=3 \), respectively, assuming no uncertainties in the flow area due to fabrication.

These uncertainties were employed in Equation A-2. Total velocity uncertainties are reported in Table A.2 and Table A.3. Generally, maximum relative uncertainty in velocity occurs near the walls. Near the walls, the velocity
uncertainty is varies between 12 and 20% compared with 8-16% at the channel centerline. The relative uncertainties for near walls are about 3% higher than the centerline uncertainties.

Table A.2: Uncertainties for velocity at mid-span

<table>
<thead>
<tr>
<th>Branches</th>
<th>Mean Velocity (mm/s)</th>
<th>Absolute Velocity Uncertainty (mm/s)</th>
<th>Relative Velocity Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k=0</td>
<td>8.32</td>
<td>0.89</td>
<td>10.7</td>
</tr>
<tr>
<td>k=1</td>
<td>7.87</td>
<td>0.96</td>
<td>12.2</td>
</tr>
<tr>
<td>k=2a</td>
<td>6.00</td>
<td>1.00</td>
<td>16.7</td>
</tr>
<tr>
<td>k=2b</td>
<td>5.82</td>
<td>0.69</td>
<td>11.8</td>
</tr>
<tr>
<td>k=3c</td>
<td>6.00</td>
<td>0.70</td>
<td>11.7</td>
</tr>
<tr>
<td>k=3d</td>
<td>5.82</td>
<td>0.43</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Table A.3: Uncertainties for average velocity near wall

<table>
<thead>
<tr>
<th>Branches</th>
<th>Mean Velocity (mm/s)</th>
<th>Absolute Velocity Uncertainty (mm/s)</th>
<th>Relative Velocity Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k=0</td>
<td>3.32</td>
<td>0.51</td>
<td>15.3</td>
</tr>
<tr>
<td>k=1</td>
<td>3.90</td>
<td>0.75</td>
<td>19.1</td>
</tr>
<tr>
<td>k=2a</td>
<td>3.78</td>
<td>0.76</td>
<td>20.0</td>
</tr>
<tr>
<td>k=2b</td>
<td>4.00</td>
<td>0.57</td>
<td>14.3</td>
</tr>
<tr>
<td>k=3c</td>
<td>4.45</td>
<td>0.70</td>
<td>15.7</td>
</tr>
<tr>
<td>k=3d</td>
<td>3.90</td>
<td>0.47</td>
<td>11.9</td>
</tr>
</tbody>
</table>