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ABSTRACT

The distribution of agricultural production for continuous consumption requires the capacity to maintain inventories which are large relative to the rate of utilization. As a result, the demand for inventories is a critical element in the pricing of agricultural commodities. A component of inventory demand, referred to here as reservation demand, is sensitive to expectations for future prices and may be a significant source of price instability in agricultural markets. Price expectations are, in part, dependent on current price information. This creates a potentially complex and volatile pricing structure.

A cusp catastrophe with slow feedback is used to model the interrelationship between the formation of expectations and price determination. A catastrophe model allows the use of relatively unrestrictive assumptions concerning the formation of expectations and provides a picture of how unstable price behavior can exist within a structurally stable pricing system.

Some of the concepts underlying catastrophe theory are presented in the familiar context of a competitive market. This is followed by a demonstration of how speculative changes in the demand for inventory may disrupt the pricing structure of a competitive market. A cusp catastrophe model of a competitive market for a stored commodity is developed.

Simulation experiments are conducted to demonstrate how alternative forms of price behavior can be represented within the model. Prices generated by the model are compared to actual patterns of price adjustment observed for wheat. An empirical investigation of the pricing structure of the wheat market is then conducted. It is concluded that the relationship between price expectations and the demand for inventory is, at times, a source of price instability for stored commodities. Speculative or transitory changes in the demand for inventory tend to exacerbate existing variability in agricultural prices. This appears to be an important consideration in analysing production and marketing alternatives and in evaluating public policy. A cusp catastrophe with slow feedback provides a good conceptual framework for taking the interdependence of expectations and prices into account.

EXPECTATIONS, RESERVATION DEMAND
AND THE STABILITY OF AGRICULTURAL MARKETS

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EXPECTATIONS, RESERVATION DEMAND AND THE STABILITY OF AGRICULTURAL MARKETS

CHAPTER I

INTRODUCTION

Expectations and Reservation Demand in Agricultural Markets

Inventories are a major component of the agricultural production and marketing system. Most agricultural crops are produced seasonally and must be stored to distribute supplies for consumption throughout the year. Grains and other nonperishable crops held in raw inventories, are commonly referred to as storable commodities. Perishable crops, while not generally referred to as stored commodities, are held in inventories in a variety of processed forms. In livestock industries, animal inventories are a source of current supplies for consumption and future production capacity. Thus, the demand for inventories is an important part of the total demand for agricultural commodities.

Inventories are held at every level of the agricultural production and marketing system. Consequently, inventory demand may reflect a diverse set of economic conditions. Inventory demand may be broadly classified into two categories; facilitative demand and reservation demand. Inventories are held to accomplish or facilitate production and distribution. Facilitative inventory demand is derived from some productive or marketing efficiency gained by holding stocks. This efficiency must be weighed against the costs and risks associated with maintaining inventories. Reservation demand may be defined as a demand to withhold supplies, currently available for use or consumption, in anticipation of higher prices in the future. Reservation demand may be viewed as an investment in inventories for capital gains. Facilitative demand and reservation demand are not mutually exclusive. The capital value of any stocks may be effected by expectations for future price changes. Inventory demand, in general, may be altered in response to expectations for capital gains or losses. Hence, the term reservation demand may be applied to the relationship between price expectations and inventory demand. Changes in inventory demand, which result from a shift in the distribution of expectations held by individuals, may be referred to as a change in the level of reservation demand.

Inventories link price determination to the formation of expectations within a given market. This link is not limited to the effect of changing expectations on prices through reservation demand. Current prices and price trends may influence expectation. The interrelationship between prices and expectations creates a potential for a very complex pricing structure. There are three factors which point to the significance of this interrelationship in agricultural markets. First, as noted earlier, inventory demand may be a large component of total market demand at a given point in time. Second, market supplies are highly price inelastic. A small change in the level of demand may result in a relatively large change in price. Third, agricultural prices are characteristically unstable. This may be reflected in price expectations which move with changes in price. In other words, unstable prices may give rise to unstable expectations which in turn, adds to the variability in prices.

Expectations and Market Prices

Price expectations for a stored commodity may be temporarily self-fulfilling. Expectations for increasing prices, leading to increased reservation demand, places upward pressure on prices. Expectations for declining prices, leading to a reduction in reservation demand, places downward pressure on prices. Inconsistencies between past expectations and current market conditions may bring about a readjustment of prices. However, as long as inventories are retained, expectations may continue to influence current prices, as price expectations are revised in response to new information. A good examples of this type of market behavior is the response in prices in information on an upcoming harvest. A projection of a poor harvest may generate expectations of higher prices in the coming year. An increase in reservation demand may result in a price advance prior to harvest. Actual market conditions at harvest may force a readjustment of prices. At the same time, information on harvest production and prices may bring about a revision in expectations, changes in inventory demand and further adjustment of prices. It is the revision of expectations with current market information that present the greatest difficulty in attempting to model this type of market behavior.

How individuals incorporate market information into the formulation of expectations lies more in the domain of psychology than in economics. Whether an individual discounts current information, projects the present into the future or employs economic logic to information on supply and demand, may depend on a rationality divorced from economic constraints. Expectations may simply be a feeling about the direction prices will take in the future, not an explicit prediction of prices at some future date. Clearly, some assumptions must be made to reduce the formulation of expectations to a level appropriate for economic study. A common approach is to make use of hypothetical expectations formulas. The most frequently used models are adaptive and rational expectations formulas.[1] With an adaptive formula, expectations are based on a lagged distribution of past prices. With a rational expectations formula, expectations are based on economic analysis of projected supply and demand. The problem with defining how expectations are formed is that the behavioral assumptions tend to exclude more implications than they can elicit. Another approach, one taken in this paper, is to attempt to classify expectations with respect to their impact on market prices and stability.[2] In other words, we may consider how alternative forms of expectations may affect pricing structure within a market.

Expectations and Market Stability

For a given set of expectations, held by market participants, an inventory demand curve is downward sloping, as an increase in price increases the cost of acquiring, (or the opportunity cost of holding) inventories. If expectations for a price change are independent of current market prices, then the response in the quantity of inventory demanded to a change in price may be viewed as a movement along an inventory demand schedule. If expectations vary with current market prices, a change in price results in a shift in the expected capital value of inventories. This shift in demand may offset or augment the change in the quantity of inventory demanded attributed to costs. A movement along an inventory demand schedule is accompanied by a shift in the demand schedule. The quantities demanded under changing prices trace out a new effective demand schedule (Figure 1.1).[3] The impact of expectations on effective demand provides a useful means of classifying expectations.

In considering the effect of current prices and price trends on expectations for future price changes, there are three possibilities. First, expectations are uneffected by price movements and the demand for inventory is unchanged. Second, expectations are negatively related to price changes and effective inventory demand becomes more inelastic. Third, expectations move with price changes. In the third case, a movement along a demand schedule is

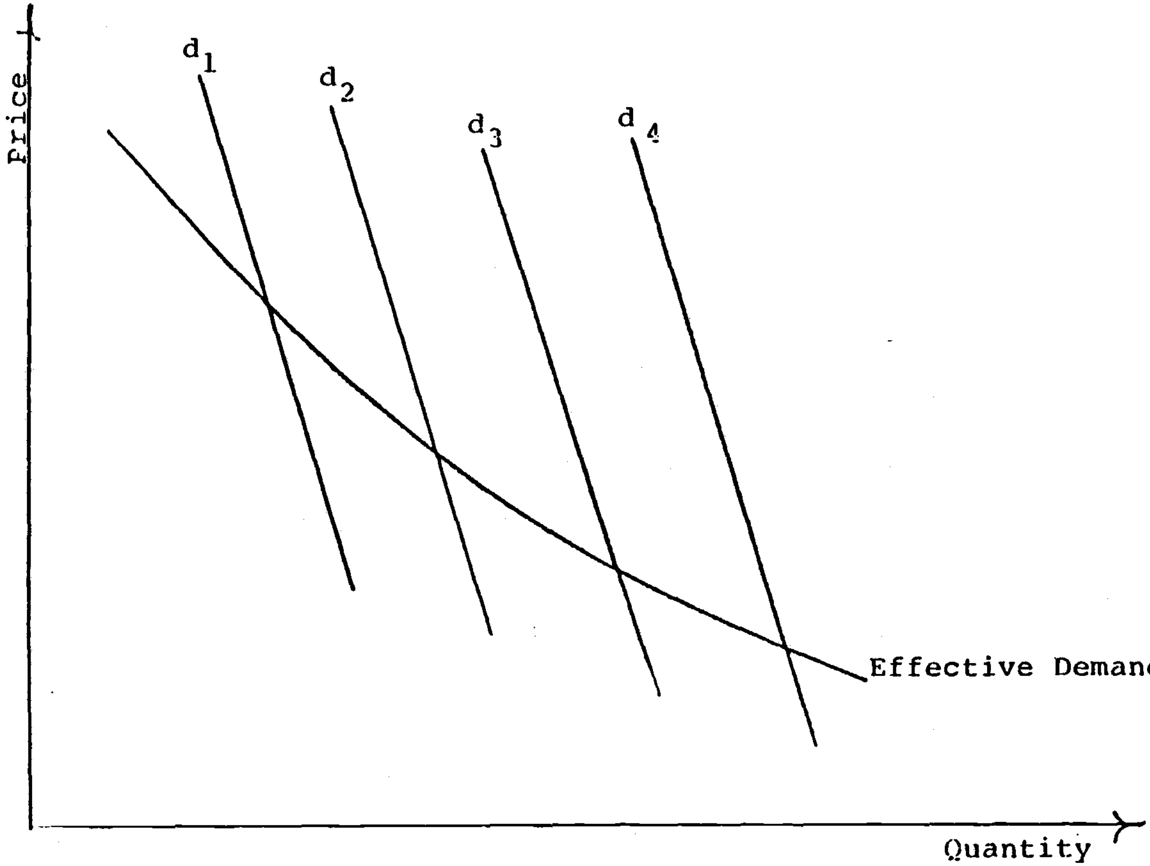

Figure 1.1. An example of short-run and effective inventory demand schedules under price dependent expectations.

offset by a shift in demand. If the shift in expectations partially offsets the change in the cost of holding inventories, then, effective demand is downward sloping and more elastic. If expectations exactly offset costs, an unlikely situation, effective demand is perfectly elastic. If a shift in expectations more than compensates for a change in cost, effective demand is upward sloping. Here, an upward price trend may attract increasing levels of demand, tending to sustain the trend. Falling prices may lead to a reduction in demand, extending the decline in prices. An upward sloping reservation demand relationship is referred to here as speculative demand.

Initially, it may appear that speculative demand is simply an unlikely analytical possibility. However, there is evidence to suggest that speculative inventory investment and disinvestment is relatively common. Livestock inventories are often built up on an upswing in prices and liquidated as prices decline. Within a crop year we can often observe periods in which prices are falling and utilization is increasing. Given that supplies are limited to existing stocks which are declining, any downward pressure on prices can be attributed to a reduction in demand. A reduction in consumption demand is associated with a decline in utilization. A reduction in inventory demand is associated with an increase in utilization. Therefore, a decline in both prices and utilization may correspond to a period on decreasing inventory demand under falling prices. A period of increasing prices and declining utilization may also be an indication of speculative demand. However, this may also be due to declining supplies and an upward movement along the consumption demand schedule.

To address questions concerning the impact of speculation of commodity prices, we need to develop a better understanding of how speculative demand may affect price behavior. Consider an example in which prices begin to rise in response to an external change in the exchange environment. If rising prices attract speculative demand, upward pressure on prices is increased which may sustain the trend. An established trend may further stimulate expectations and attract more speculation. this internal reinforcement of prices and expectations may extend a price trend beyond what was dictated by the external change which initiated an adjustment in prices. Clearly this is a temporary state; expectations can not sustain a trend indefinitely. Prices will eventually turn the corner. The imbalance existing between current prices and the exchange environment may be eliminated. Alternatively, the interaction between prices and expectations may continue to drive prices in the opposite direction. Declining prices may lead to pessimistic expectations and a speculative disinvestment in inventories. This places more downward pressure on prices which may reinforce expectations and sustain a prolonged downward trend.

Thesis Objectives

The potential impact of speculative reservation demand is not a new topic in economics. Samuelson's investigation of the subject led him to state:

> "...any speculative bidding of prices at a rate equal to carrying costs can't last forever. The market literally lives on its own dreams and each individual at every moment is perfectly rational...But I have long been struck by the fact and puzzled by it too, that in all the arsenal of economic theory we have absolutely no way of predicating how long such a phenomenon will last."4

Samuelson points to the types of questions which are of greatest interest to individuals in the purchase and sale of agricultural commodities. When will a market turn the corner? How severely will prices change? Are current prices over or undervalued? We may view our inability to answer these questions as a need for better models of human behavior.[5] We may hope to discover a better formula to represent the formation of an individual's expectations. Alternatively, we may view unpredictability as an important piece of information about the economic system in question. Information which may force us to accept that unconstrained and unpredictable human behavior is an essential aspect of the pricing process. A new set of questions must be formulated. How can the concept of a market be expanded to allow for speculative behavior? Will such a model return any useful insights and applications in the analysis of agricultural prices? Can we identify conditions under which speculative changes are more or less likely to occur? Can we better assess marketing risks or the impact of public policies? The objective of this thesis is to explore these questions.

Theoretical Approach

Speculative behavior cannot be adequately represented within the traditional static or dynamic model of a competitive market. The interaction between formation of expectations on current market information and price determination creates a strange loop in the pricing process. "A strange loop" is a term used by Hofstadter (1979) to describe a system of feedback which perpetuates itself within a given structure. A market, as a conceptual model of exchange, represents a stable structure. Exchange is an ongoing process. Speculation does not bring about a collapse of trade. Yet, the loop between expectations and prices introduces an internal form of structural instability. Price trends may be self-sustaining. This internal instability must somehow be embedded into the overall structure of a market in a stable fashion. This can not be accomplished within a traditional market model without imposing some set of artificial constraints on the process of price adjustment.[6] What is really required is an expanded picture of a market, one allowing a more complete synthesis of the interaction between expectations and price determination.

In 1974, E.C. Zeeman published the first economic application of catastrophe theory in a paper entitled <u>On the Unstable Behavior of Stock Exchanges</u>. Zeeman postulated a dynamic model of a stock exchange based on the structure of a cusp catastrophe with slow feedback. A cusp catastrophe with slow feedback offers a radically different picture of price determination. An illustration provides the best introduction to a cusp market structure, Figure 1.2. The state of the market is represented by the time-rate of change in price, $\dot{P}$, in the vertical dimension. The control variables, representing nonspeculative excess demand and the level of speculation (speculative content), are the horizontal coordinates of the control plane. For a given set of control values, the rate of change in price is represented by a point on the equilibrium surface of the cusp, directly above the corresponding point in the control plane. The effect of changing prices on the control variables is represented by the slow feedback flow along the equilibrium surface.

A wide range of behavior can be depicted within the model. Stable dynamic adjustment can be represented by flows within the single sheeted region of the equilibrium and control surfaces. A price trend which is sustained by increasing speculation, carries the dynamic flow over the double sheeted region of the cusp. The eventual collapse of the trend is depicted by a sudden crash or jump between sheets of equilibrium surface. Structurally unstable behavior is embedded into the stable structure of a cusp catastrophe. Zeeman's model of a cusp catastrophe with slow feedback, extended to a competitive market for a

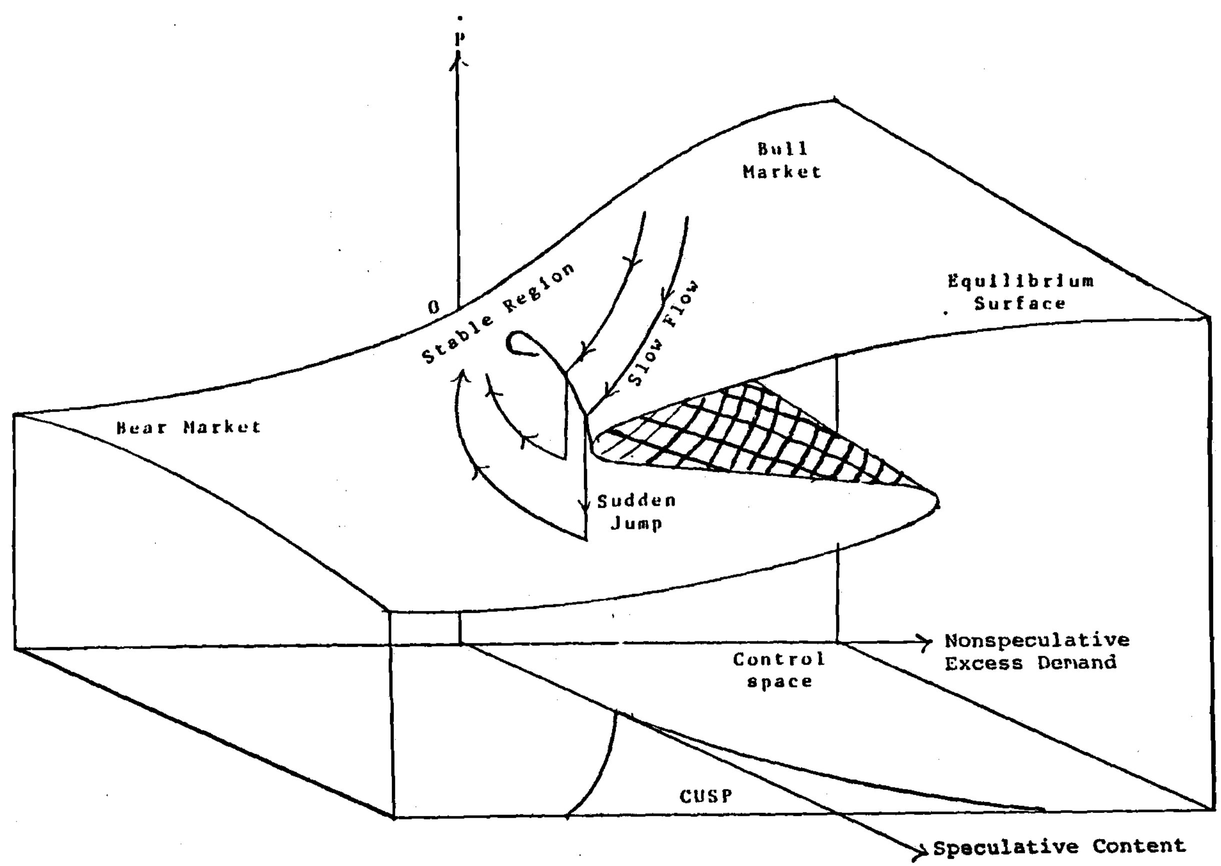

Figure 1.2. A dynamic model of a market based on the structure of a cusp catastrophe with slow feedback: adapted from Zeeman, <u>On the Unstable Behavior of Stock Exchanges</u>, (1974).

stored commodity, forms the theoretical core of this thesis.

Applied Methodology

The impact of prices on the formation of expectations can be traced out along the equilibrium and control surfaces of a cusp catastrophe. The implications of alternative behavioral assumptions can easily be explored. To address the types of questions put forth under the objectives of this thesis, we need to develop testable hypotheses which characterize speculative price instability. This presents a number of methodological problems.

Catastrophe theory is a qualitative mathematical theory. It is not the qualitative nature of catastrophe that presents any new difficulties. Economic theory is qualitative and the methods of catastrophe theory are well suited to economic analysis. A distinction can be made between an economic application and applied economics which points directly to the problems inherent in attempting to apply catastrophe theory. In using the framework of a market to model exchange, we may obtain a simple and understandable picture of a system which is much too complex to analyze in detail. This picture of exchange allows us to describe a set of interacting forces which act to determine exchange rates. An application of a cusp catastrophe model simply provides a new picture, one which allows us to describe an expanded set of interactions. Applied economics enters into the domain of exchange in hopes of being able to verify or disprove inferences drawn from the model through statistical tests. Establishing correspondences between entities of a market model and conditions of exchange requires a better understanding of the model and the system it represents. In applied catastrophe theory, both the model and the system it is intended to represent are more complex. Establishing correspondences between the two can be extremely difficult.

Since the introduction of catastrophe theory by Rene Thom in 1972, applications of catastrophe theory in economics have been very limited. This is partly due to the considerations outlined above. It may also be attributed to the fact that the language, concepts and techniques of catastrophe theory are not a part of the mathematical training of most economists. Generally, applications have been limited to the descriptive use of catastrophe theory (Wilson and Clarke (1979), Varian (1979), Wilson (1980), Madden (1982)). Presently, there does not appear to be any established methods for the empirical evaluation of catastrophe models. An investigation of empirical methods forms a large part of the applications section of this paper.

Thesis Design

Chapter I has served to introduce the nature and potential significance of the relationship between expectations, reservation demand and the pricing structure of agricultural markets. The general analytical problems presented by the interdependence of expectations and price determination were explored. These problems led to a statement of the objective of this thesis. This was followed by a brief overview of the theoretical and applied methodology employed to meet these objectives.

The mathematical structure of a competitive market is examined in detail in Chapter II. There are two objectives of this investigation: one, to introduce the language and underlying concepts of catastrophe in a familiar context and two, to make use of the methods of catastrophe theory to gain a better understanding of competitive market structure. In the first section of Chapter II, the concepts of structural and qualitative equivalence are treated on a intuitive level with the aid of geometric interpretations. In the second part of Chapter II, these same ideas are presented in a more rigorous form using coordinate transformation and perturbation theory. In the final section of Chapter II, a geometric synthesis of competitive market structure is presented.

Chapter III is devoted to the development of catastrophe market structures. A simple profit maximizing inventory investment model is used to demonstrate how expectations may disrupt the internal stability of a competitive market. This is followed by a geometrically oriented development of alternative catastrophe structures as approximations to a competitive market dynamic. In the second part of Chapter III, a more formal treatment of the application of catastrophe theory in economics is presented. This is followed by a detailed examination of the geometry of the cusp catastrophe. In the third section of Chapter III, a critical review of Zeeman's model of a stock exchange is presented. An extension of Zeeman's model of a cusp catastrophe with slow feedback, to a competitive market for a stored commodity, is developed at the conclusion of Chapter III.

Chapter IV is devoted to applications of a cusp market structure in an investigation of the impact of speculative inventory demand on wheat prices and price stability. The problems of identifying catastrophic behavior in an economic system are examined. A market simulation is constructed using the dynamic structure of a cusp catastrophe with slow feedback. The simulation model is designed to meet two objectives. The first is to develop a better picture of a cusp catastrophe as a pricing structure. The second is to use a quantitative model of a cusp catastrophe to generate pricing patterns similar to those observed for stored commodities. A graphical analysis of wheat prices is then presented for comparison. The second part of Chapter IV provides an overview of the domestic wheat market. An emphasis is placed on the primary sources of variability in wheat prices. The remainder of Chapter IV is given designing and testing of hypotheses, in an attempt to determine the significance of speculative reservation demand as a source of price instability in domestic wheat markets.

The implications and conclusions drawn for this research are presented in Chapter V. The results from the study of wheat price stability are evaluated. An emphasis is placed on questions stated in the objectives of this thesis. In the second part of Chapter V, the role of catastrophe theory in economics is evaluated. The use of catastrophe models in market analysis is explored.

General Comments

It is the author's belief that the methods of catastrophe are of greater use to economists than catastrophe models. The concepts of catastrophe theory, which are based on differential topology, are expansive and powerful. A sincere effort has been made to make these ideas affordable to economists with only a moderate interest in mathematics. Wherever possible, verbal interpretations of mathematical properties and results are provided. Verbalizations are not sufficiently precise for the needs of mathematicians but they can be essential in economic applications. Mathematical proofs are not included in this paper. However, specific references are provided for mathematical results in the end notes of each chapter. Mathematical demonstrations are included where needed or when the exercise is informative. In making extensive use of mathematics in economics it is important to retain a sense of perspective. Albert Einstein, in Sidelights on Relativity (1921), wrote:

> "As far as the propositions of mathematics refer to reality they are not certain; and as far as they are certain they do not refer to reality."

Endnotes

1 In an adaptive expectations model expected prices are formulated as a weighted average of current and past prices. Adapted expectations lag behind current price trends and converge with market prices when price levels are steady. The rational expectations model was introduced by John Muth, (1961). The essential property of rational expectations is that expectations are assumed to be unbiased estimators of future price levels.

2 A classification scheme, similar to the one developed here, was proposed by Heiner, (1983).

3 This situation is analogous to the shifting of a firms marginal cost curve as an industry expands, tracing out an effective market supply.

4 Paul Anthony Samuelson, "Intertemporal Price Equilibrium: A Prologue to the Theory of Speculation." Collected Scientific Papers, vol. a, (Cambridge: MIT press, 1966).

5 Current research on the stability of stored commodity prices is engaged in this approach. Recent applications within the rational expectations framework include Masahiro, (1983) and Sarris, (1984).

6 An interesting treatment of markets with bounded price variation is provided by Maddala, (1983). Bounds are assumed to be imposed by institutional constraints.

CHAPTER II

COMPETITIVE MARKET STRUCTURE AND THE DETERMINATION OF EXCHANGE RATES: A MATHEMATICAL OVERVIEW

Motivation

In this chapter we will examine the mathematical structure of a competitive market as a model of exchange. The purpose of this investigation is twofold. First, many of the concepts underlying catastrophe theory can be presented in the more familiar context of traditional market theory. At the same time, we may make use of the methods of catastrophe theory to develop a better understanding of competitive market structure and the behavior of exchange rates. Gilmore (1981) defines the general program of catastrophe theory as the study of how the equilibria of a system change as the parameters controlling the system change. Equilibrium is the central concept about which market theory is organized. Catastrophe theory provides an ideal framework for examining how behavior is represented within a market.

Market Structure

A market is a conceptual model of exchange based on the doctrine of classical physics. Classical physics is a mathematically oriented view in which cause and effect are reduced to actions of forces within a mechanical system. The descriptive framework of a market is not a mechanical model of price determination. The forces acting within a market and a mechanism describing how exchange rates are determined are not expressly defined. The adjustment of prices and quantities of exchange to changing market conditions is represented within an equilibrium system. Hence, the existence of an underlying mechanical process may be considered an implicit assumption of market theory. However, the working hypothesis provided by a market and what may best characterize market structure is our definition of market equilibrium.

There are two aspects of competitive market structure reflected in the concept of equilibrium. First, market equilibrium may be defined as a price equating the quantity supplied with the quantity demanded. Factors such as production costs, consumer income and preferences, establishing specific levels of supply and demand, determine an equilibrium price and quantity distinct from other rates of exchange. The relationship between equilibrium and factors influencing supply and demand may be referred to as external market structure. Second, market equilibrium organizes the flow of behavior within a market. The adjustment of exchange rates is represented by a flow toward or between stable equilibria. This is a form of internal or dynamic structure.

An organizational point of equilibrium, determined by the intersection of a supply and a demand schedule, is a limited picture of a stable market structure. Our perspective can easily be expanded. The external factors affecting supply and demand may be treated as a set of smooth variables defining a family of demand schedules and a family of supply schedules. The intersection of these families define a set of market equilibria corresponding to the values taken by the external variables. An individual equilibrium point may act as a stable attracter, directing dynamic flows for a given set of external values. However, the behavior of exchange rates in response to changes in the exchange environment depends on the overall organization of the set or family of market equilibria. This organization is the level of market structure we will explore with the tools of catastrophe theory.

Formalizing a description of a market in the mathematical form of an equilibrium system requires three assumptions.[1] First, we will assume that the state of a market can be completely specified by a set of internal state or behavioral variables. Here, we will limit our consideration to price, quantity and their respective rates of change over time. Second, we will assume that exchange is under the control of a set of external or control variables which determine the values of the state variables. More specifically, the controls are the variable parameters which define the families of supply and demand schedules, as for example, the slope and intercept of a linear supply or demand curve. The values of the control variables are established by the external factors influencing change. Third, we will assume that the action of forces within a market can be described by a smooth function or potential, representing an attraction towards market equilibrium.

Mathematical relationships describing a physical system are usually asserted to reflect physical laws. Physical laws express quantitative invariants; for example, the attractive force between two bodies is inversely proportional to the square of the distance between them. As a result, the mathematical structure of a physical theory is quantitative. Economic laws express qualitative invariants; for example, the greater the price the less the quantity demanded. A mathematical theory asserted to reflect qualitative laws possesses a corresponding qualitative structure. The distinction between qualitative and quantitative is evident in the language of economics, with the use of terms such as greater or less than, increasing or decreasing, as opposed to equal or proportional to. The mathematics of this distinction will be discussed briefly in the following section.

Mathematical Structure

While there are many levels of mathematical structure, Isnard and Zeeman (1976) in defining the term qualitative state:

> "The whole of mathematics rests on three types of structure, (i) order, (ii) topological, and (iii) algebraic. ... Roughly speaking, in mathematics, those properties which depend upon the order and the differential-topological structures are called qualitative, while those that depend upon the algebraic structure are called quantitative."[2]

We may consider how these three levels of mathematical structure relate to market theory.

Ordering structure pertains to the arrangement of elements within a collection or set. Order is a principal link between economics and mathematics. We may suppose that a supply or demand schedule is a collection of price and quantity pairs which may be represented as a set of points with price-quantity coordinates. The laws of supply and demand impose an ordering relationship upon these sets. A higher price is associated with a smaller quantity demanded and a greater quantity supplied. Extending our consideration to families of supply and demand schedules we may define a set of equilibrium exchange rates. Each element of this set contains an equilibrium price and quantity, along with corresponding values for the variable parameters of the supply and demand families. Given that the laws of supply and demand hold for the respective families as a whole, an ordering structure can be inferred for the equilibrium set. For controls positively affecting quantities demanded at each price (a demand shift), increasing control values are associated with higher equilibrium prices and quantities. For a supply shift control, increasing control values are associated with lower prices and higher quantities of exchange. Properties established at an ordering level shape higher levels of mathematical structure.

Topological structure arises when we assume that the proximity of points comprising a relationship is measurable on a smooth scale.[3] Tangible scales, such as units of trade, or intangible scales, such as utility, are equally valid. In other words, we are assuming that a given relationship is continuous. Curves and surfaces, and the space in which they are embedded, are topological structures. A supply and a demand curve embedded in a two dimensional price-quantity plane is a topological representation of a market. When we assume that a curve or surface is smooth, the tools of differential calculus become meaningful. Differentiation at a point yields information on the local geometry of a curve or surface which can be used to characterize qualitative properties of a topological structure.

Points on a surface with a well defined first derivative or tangent function (regular points) are qualitatively distinct from critical or stationary points with a degenerate derivative or tangent. The qualitative difference between regular and critical points may be used to convey the difference between disequilibrium and equilibrium rates of exchange. Second order derivatives provide information about the curvature of a surface, allowing us to identify the qualitative properties of a critical point. A stable equilibrium or attractor may be represented by a minimum. Unstable or repeller equilibria may be represented by maxima or saddle points. Critical points (or a lack of critical points) organize the qualitative character of a curve or surface. There are critical points which cannot be classified by second order information. These are called degenerate or singular points. Singular points (or a lack of singular points) organize the qualitative character of families of curves or surfaces. The classification of singular points, with respect to their qualitative properties, is the focus of catastrophe theory. It is important to note that we are not equating qualitative properties with topological or ordering structure. Qualitative properties may be imposed on a topological or ordering structure.

Moving from a definition of market equilibrium, as an intersection of a supply and demand curve, to a solution for a system of corresponding market equations, requires a higher level of mathematical structure. Solving a system of supply and demand equations for an equilibrium exchange rate entails the use of algebraic operations such as addition and multiplication. Equally important, the supply and demand equations must initially be defined in a form allowing us to make use of these operations in a meaningful way. In summary, we need an algebraic structure.

It is useful to think of an algebraic structure as a system of rules governing operations on elements of a set, such that any relationship expressed by the operations defined for the set can be solved by the rules governing the operation. The specific properties which a given system must possess define specific algebraic structure. For example, an additive relationship defined on the integers can be solved by addition. Consider the following illustration:

$3 + x$	$= 7$	given
$-3 + (3 + x)$	$= -3 + 7$	addition
$(-3 + 3) + x$	$= -3 + 7$	associative law
$0 + x$	$= -3 + 7$	additive inverse
x	$= -3 + 7$	additive identity
X	$= 4$	computation.

The solution, in general, requires that addition is associative and the set of integers under addition contains an identity element and an inverse for each element in the set. These are the properties of an algebraic structure referred to as a group. Addition on the positive integers does not exhibit a group structure (there are no inverse elements). An expression such as:

$$7 + x = 3$$

cannot be solved. Multiplication on the integers is associative with an identity element equal to one. Again, the set lacks inverse elements and multiplication on the integers does not define a group structure.

Multiplication on the rational numbers does possess an algebraic structure. We may solve the general form of a multiplicative relationship expressed for rational numbers in a manner completely analogous to addition on the integers:

$$
\begin{array}{rll}
a * x &= b & \text{given} \\
1/a * (a * x) &= 1/a * b & \text{multiplication} \\
(1/a * a) * x &= 1/a * b & \text{associative law} \\
1 * x &= 1/a * b & \text{multiplicative inverse} \\
x &= 1/a * b & \text{multiplicative identity} \\
x &= a/b & \text{computation.}
\end{array}
$$

Our interest here does not lie with the particular algebraic structure required to solve a given set of market equations, though it is clear that a quantitative solution does depend on this structure. What is of importance is the method employed to analyze algebraic systems; defining structure by the conditions necessary for determining a solution. This line of reasoning may be applied to the distinction between qualitative and quantitative structure in marketing analysis.

<u>Qualitative Versus Quantitative: Choices of Scale</u>

To draw quantitative conclusions, we must eventually carry out the calculations needed to arrive at a numeric answer. The relationships in question must be cast in a scaler form. Observed prices and quantities of exchange are a natural choice of scaler coordinates, units of measure, in market analysis. However, the supply and demand relationships must also be specified as scaler functions. The problem lies with a choice of functional form. In economics, there are no quantitative laws (or invariants) from which we may assert a specific functional form. By assuming, for example, a linear form for a demand function, a quantitative structure is imposed; a change in price results in a proportional change in the quantity demanded. Hence, quantitative results possess only a sense of qualitative validity.

A choice of scale (coordinates) and a choice of functional form are closely related. Generally, a change of functional form is equivalent to a change in scale.[4] Again, citing Isnard and Zeeman:

> "In most mathematical models of the social sciences, if one uses a scale X for the convenience of making experimental measurements or displaying data, then any qualitatively related scale X' is as valid. Therefore, any conclusion based on the use of the particular scale X is only valid provided the same conclusion also holds using X'. Such a

> conclusion is called qualitatively invariant or, more briefly, a qualitative conclusion."[5]

In econometrically estimating a market model, a change in functional form equivalent to a qualitative change in scale will, in general, alter quantitative results. We do expect qualitative agreement, that the qualitative properties of the estimates are consistent. In comparing, for example, a linear and a loglinear demand function, we expect the slope to be negative for each, a substitute commodity to remain a substitute, and so forth. We can make use of qualitative transformations of coordinates (changes of scale) to identify the simplest algebraic representation of a market structure. The transformations and the rules which govern them can provide insight into how behavior is represented within a model. Before taking up coordinate transformations, we will take a more intuitive approach to the idea of qualitative equivalence.

Structurally Equivalent Linear Approximation

Consider a supply curve continuous and smooth about an arbitrary regular point $A = (P_o, Q_o)$. The best linear approximation to the curve at A is the tangent. In a sufficiently small neighborhood of A, the tangent is a good quantitative estimate of the curve. In other words, for small enough variations from (P_o, Q_o) we can ignore curvature. However, the greater the curvature the more quickly the accuracy of the linear estimate declines. This is not the case for the qualitative properties of a linear approximation. For points on the supply curve in the neighborhood of A the law of supply imposes a strong ordering relationship which is:

I) Asymmetric; either,

i) $P > P_o$ and $Q > Q_o$ or

ii) $P < P_o$ and $Q < Q_o$ or

iii) $P = P_o$ and $Q = Q_o$

II) Transitive; if,

i) $P > P_1 > P_o$ then $P > P_o$ and $Q > Q_o$

ii) $P < P_1 < P_o$ then $P < P_o$ and $Q < Q_o$

Exactly the same relationship is expressed by the tangent at A. The qualitative neighborhood of validity may extend indefinitely so long as the supply curve exhibits strong ordering.

A supply or demand curve and a tangential approximation are isomorphic (structurally equivalent) if there exists between them a transformation which is: one, order preserving and two, point to point (reversible).[6] These conditions imply that we may move from a curve to its tangent and back without any loss of relevant information. The projection of a supply curve onto a tangent along lines parallel to the normal of the tangent satisfies these conditions and establishes the isomorphism (Figure 2.1).

If we interpret the laws of supply and demand as weak ordering relationships, we allow for perfectly elastic or inelastic sections of the curves. Conditions for structural equivalence under these conditions further illustrate information preserving transformations. Weakly ordered points on a supply curve satisfy the conditions of:

I) Antisymmetry; either,

i) $P > P_o$ and $Q > Q_o$ or

ii) $P < P_o$ and $Q < Q_o$ or

iii) $P = P_o$ and Q unrestricted or

iv) P unrestricted and $Q = Q_o$

II) Transitivity.

Sections of a supply curve which are strongly ordered satisfy antisymmetry conditions i and ii. These sections may be represented by the tangent at a regular point. However, the neighborhood of validity does not extend to any degenerate part of a curve. A degenerate section of a

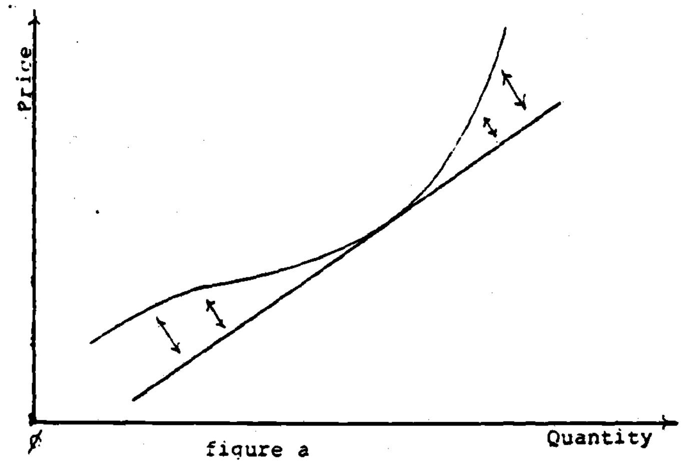

figure a

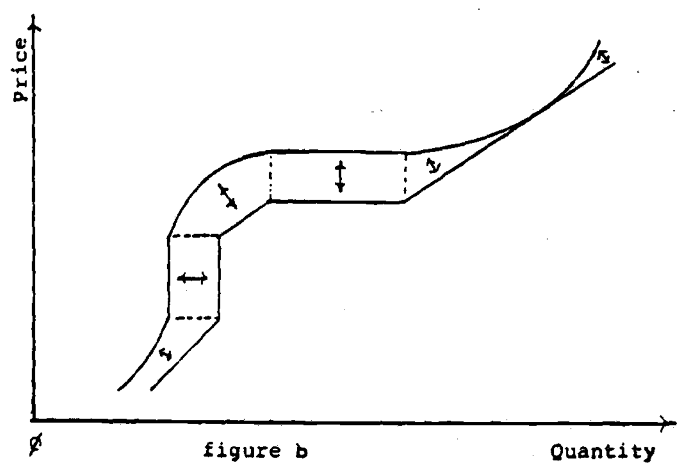

figure b

Figure 2.1. Transformation of a supply curve: Figure a) projection of a smooth supply curve onto a tangential approximation, Figure b) projection of a degenerate supply curve onto a piecewise continuous linear approximation.

supply curve is either vertical, with unrestricted prices for a given quantity, or horizontal, with unrestricted quantities for a given price. An isomorphism requires that unrestricted elements be mapped into unrestricted elements of the same type.[7] Geometrically, vertical lines must be mapped into vertical lines, horizontals into horizontals. Qualitatively different types of points convey different information or behavioral implications. This information is preserved by requiring that the transformation maintains any relevant distinction. A demand curve with both regular and perfectly elastic sections implies that prices may or may not change with shift in supply. This type of behavior cannot be represented as smooth and linear. Figure 2.1b illustrates an isomorphic linear representation of a degenerate supply curve which is piece-wise continuous. The figure shows that tangency is not a required condition; there is simply a correspondence in the number and type of ordering relationships portrayed.

Under strong ordering, the tangent at any regular point on a supply or demand curve is structurally equivalent. This suggests that by choosing tangents to the supply and demand curves at a point of equilibrium an equivalent linear representation of a general static market model can be obtained (Figure 2.2). There appears to be an isomorphic relationship. The ordering of the supply and demand schedules is preserved. Equilibrium is maintained being mapped into itself. However, an appropriate transformation is not evident. A single point to point mapping is required. This is difficult to visualize in a two dimensional market model. A direct consideration of coordinate transformations is a means of approach to the problem. Expanding the dimensions of the model is a less exacting approach but one which exposes some interesting market geometry.

Dimensions and Codimension

In a typical geometric representation of a market there are two dimensions, price and quantity. Supply and demand are one dimensional curves. The difference between the dimension of an object and the space in which it is embedded is the object's codimension. The supply and demand curves are of codimension one. Equilibrium, a single point representing both quantities supplied and demanded, is of codimension two: a zero dimensional object in a two dimensional space.

In general, when the dimensions of a problem are altered and the codimension is preserved the relevant properties of a structure are preserved.[8] Poston and

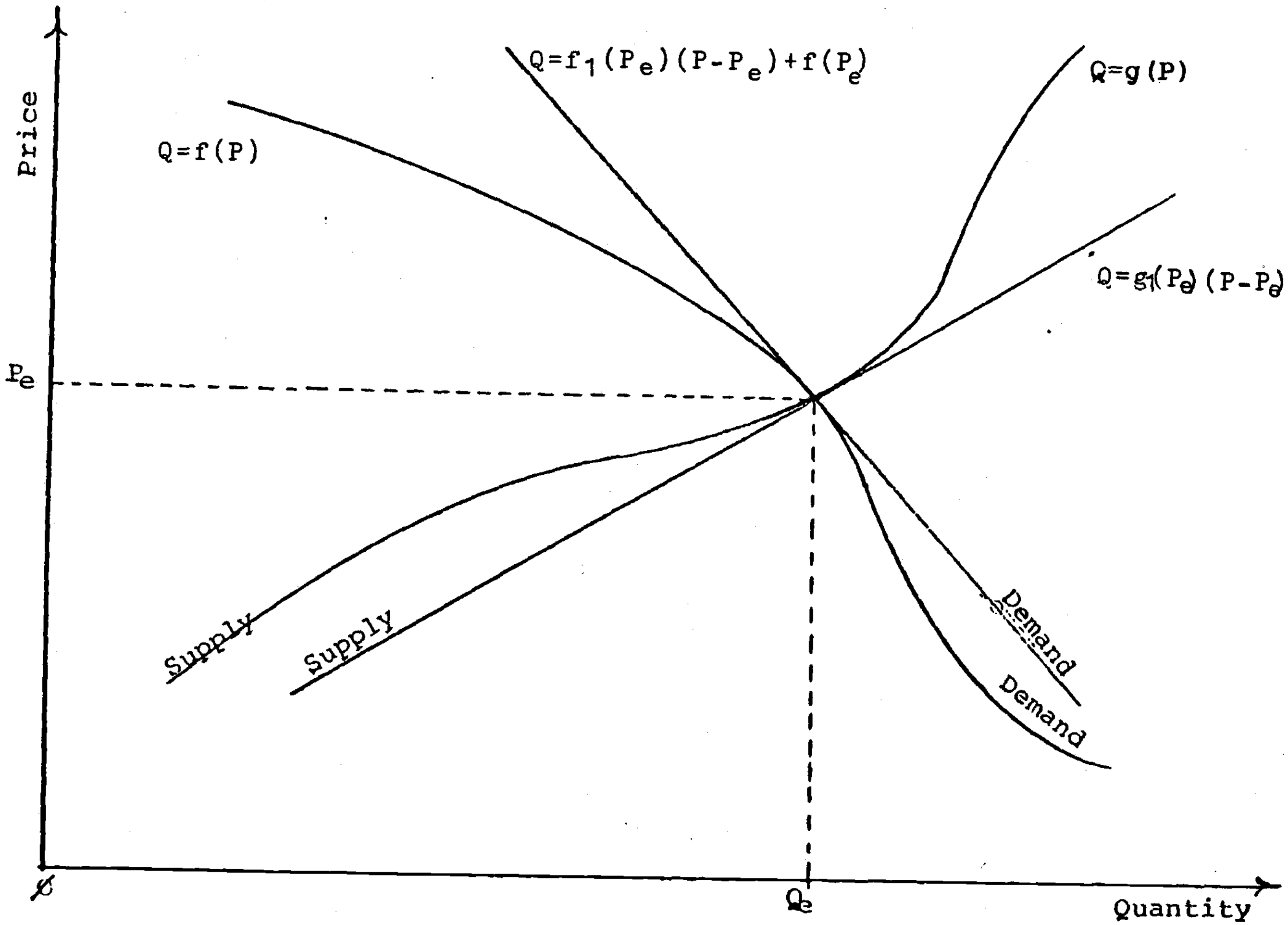

Figure 2.2. A linear approximation of static market equilibrium; tangents to the supply and demand curve at equilibrium.

Stewart illustrate this principle with an example of a geographic border.[9] On a two dimensional map a border is a one dimensional curve with codimension one. In three dimensional space a border is a two dimensional plane or surface; its codimension is again one. The concept of a border is better defined in terms of its codimension than its dimensions in that it divides a space into two regions. If the codimension of an object changes, its properties change. A circle in a plane, codimension one, separates points inside and outside. A circle in three dimensions, codimension two, makes no such separation.

In three dimensions the supply and demand relationships can be combined into a single set. Each element of the set is represented by a single point with price, quantity demanded, and quantity supplied coordinates. The supply-demand relationship may be represented by a one-dimensional curve embedded in a three-dimensional space; an object of codimension two. There is an apparent change in the codimension of the supply and demand relationships (from one to two) which implies some form of structural change. The supply-demand relationship no longer divides the price-quantity space into separate regions. However, this is not a relevant property of the supply and demand relationships, hence; it need not be preserved. In a two-dimensional representation of a market, the equilibrium point is an element of the combined supply-demand set and an object of codimension two. This codimension is preserved in three dimensions. This brings out an important aspect of a market model. The distinction between equilibrium and disequilibrium exchange rates is not a property of the supply and demand relationships. It is imposed upon these relationships. The supply and demand curves simply convey information on the quantities buyer and sellers are willing to exchange at given prices. This information is preserved when we add or ignore inessential coordiante dimensions. The concept of equilibrium must be accounted for in its own context. This can be illustrated by examining the geometric relationships between our two and three dimensional market models.

A three dimensional market model is presented in Figure 2.3. The combined supply and demand relationship is represented by the space curve SD. In reducing the model to two dimensions the curve is projected onto a plane along perpendiculars to the quantity planes (double arrowed lines). An effective choice of coordinates must be made in the reduction. If we simply ignore the demand quantity coordinate Q_d, the supply curve is projected onto

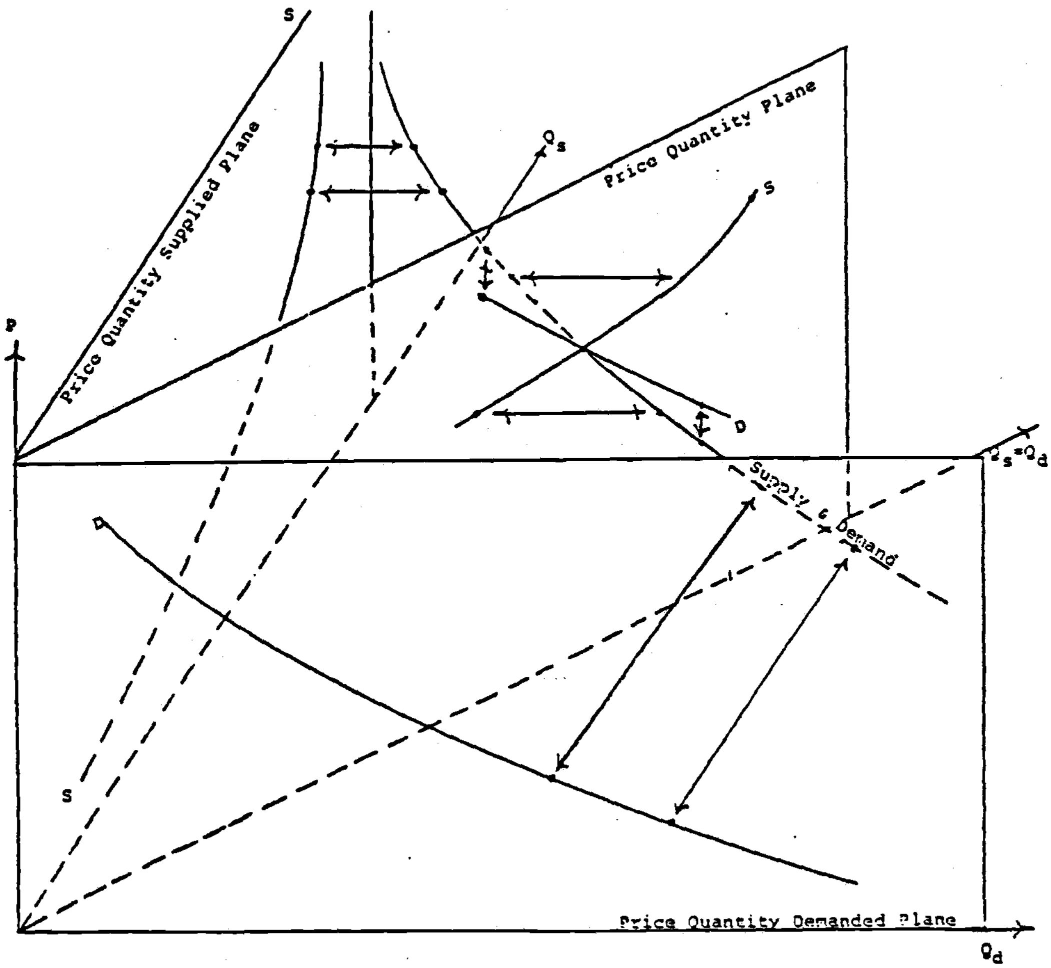

Figure 2.3. A supply-demand curve in three dimensions with two-dimensional projections of supply and demand.

its corresponding plane and the demand relationship is lost. The opposite occurs if the supply coordinate Q_s is ignored. By projecting supply and demand onto the $Q_s=Q_d$ plane (ignoring the perpendicular direction given by $-Q_s=Q_d$), there is no loss of relevant information with the reduction in dimension.

Returning to the problem of a linear transformation, the tangent to a three dimensional curve is a straight line.[10] Its direction is now a two component vector. The demand component, the change in quantity demanded with respect to price, is negative. The supply component, the change in quantity supplied with respect to price, is positive. This directional orientation is maintained throughout the curve by a tangent at any regular point. The ordering imposed by the laws of supply and demand is preserved. The curve may again be projected onto a tangent by means of parallel lines. The parallels may no longer lie in a plane owing to the twisting of the curve, but they may still be drawn perpendicular to the tangent. Our choice of points for the linear representation is no longer arbitrary if we require a correspondence between equilibrium points. In choosing a tangent as the approximation we must also choose its point of coincidence at equilibrium. Projecting the curve and equilibrium tangent onto the price-quantity plane reveals a two dimensional representation of the equivalent market structures (Figure 2.4).

Comments on Structural Equivalence

The term isomorphism refers specifically to the ordering or algebraic structure of sets. In speaking of a supply or demand curve as a topological structure, there is a more appropriate definition of equivalence. In a topological space, points are near enough together to think of transformations as being continuous. Two topological structures are structurally equivalent if there exists between them a continuous order preserving transformation with a continuous inverse. Saunders (1980) states:

> "It is sometimes useful (though not strictly accurate) to think of two geometric objects as being topologically equivalent, or homeomorphic, if one can be continuously deformed into the other without any tearing or pasting together."11

Fraleigh (1976) gives the following intuitive interpretation:

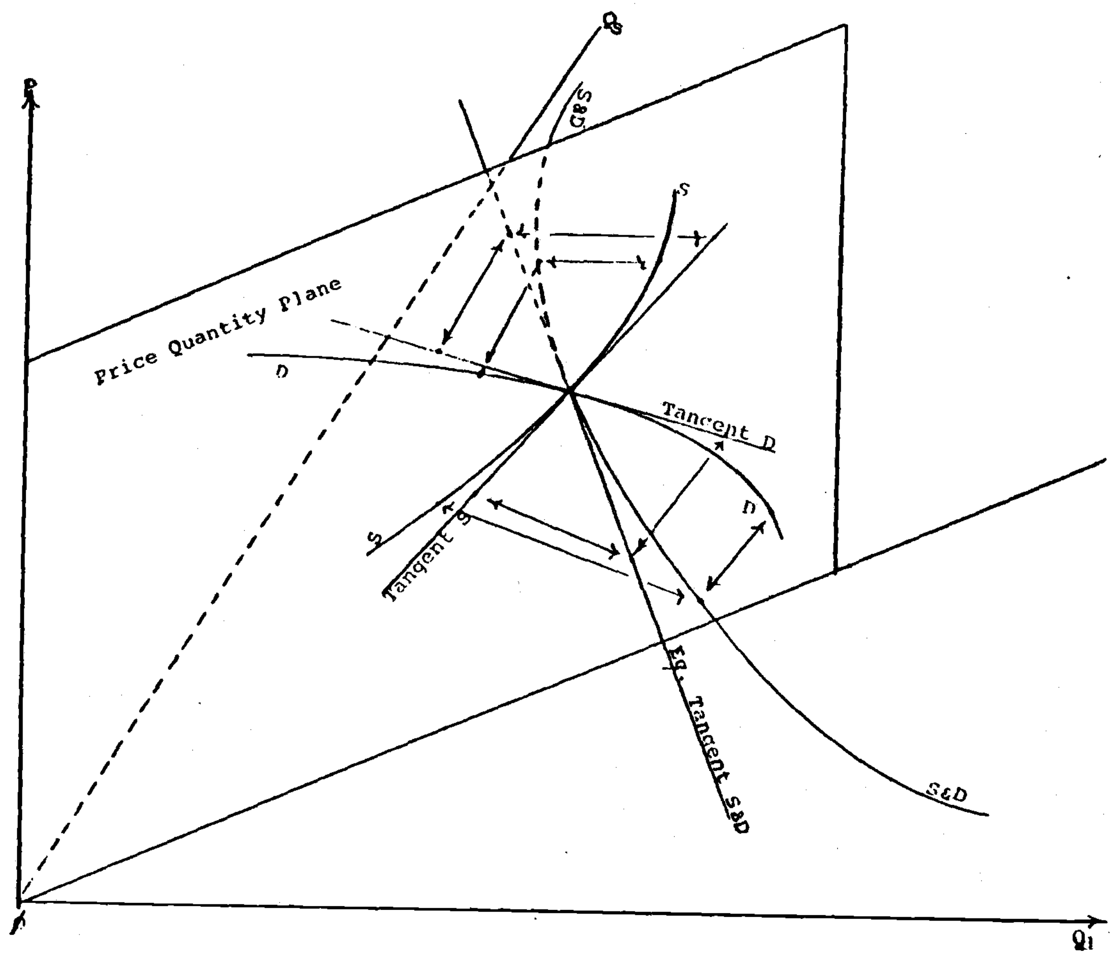

Figure 2.4. A two-dimensional projection of a linear approximation to a supply-demand curve at equilibrium.

> "Vaguely, a continuous transformation with a continuous inverse is one that we can achieve by bending, stretching and twisting the space without tearing or cutting it. ... the boundary of a circle has the same structure as the boundary of a square ... Two spaces which are structurally the same in this sense are homeomorphic. ... The concept of a homeomorphism is to topology as the concept of isomorphism (where sets have the same algebraic structure) is to algebra."12

With respect to topological structure, we may say a supply curve and its tangent at a regular point are homeomorphic and note that the ordering structures of their underlying sets are isomorphic.

The concept of structural equivalence has been used in a restrictive manner to accomplish a few specific objectives. The full meaning of these ideas has not and will not be explored. However, as suggested by the definition of a homeomorphism, structural equivalence does not directly pertain to the orientation of an object to a specific set of coordinates. We work within a framework of downward and upward sloping curves because orientation to the ideal price-quantity coordinates of exchange is meaningful. The choice of linear approximations which maintain this orientation is valid but not required under the mathematical definitions of structural equivalence.

The linear models were termed approximations in the sense that one set of points was substituted for the other. By substituting relationships we retain our original coordinates, relying on the existence of a transformation to establish equivalence. By conducting the transformation, the approximation becomes exact. In bending and stretching a supply or demand curve to linear form, we are deforming the space which contains the curve or our scales of measurement.

Transformation of Coordinates: Motivation

The objective of coordinate transformations, or changes in variables, is simplification. By an appropriate choice of coordinates we may be able to reduce an unrecognized or ill-defined structure to a readily identificable form with known properties. Coordinate transformations may be viewed in two ways: one, as a change from one coordinate system to another in a fixed space, or two, as the displacement of space about a fixed coordinate system.[13] The former, analogous to a change in perspective, facilitates a discussion of the transformations. The latter, owing to our interest in price-quantity coordinates as a point of reference, facilitates the discussion of results.

We can choose scaler coordinates for a market corresponding to units of trade. The exact form of market demand, supply and the underlying dynamic are unknown. To study market structure it would be helpful to reverse this situation by transforming to an unknown set of coordinates to obtain an exact structure in its simplest form. In an absolute sense, information is neither gained nor lost; a change of variables may allow an advantageous reorganization of information. However, we must restrict our use of transformations to those which preserve meaningful properties attributed to a model.

Quantitatively Equivalent Transformations

A rigid displacement of coordinates, not involving any bending, stretching or twisting, preserves algebraic structure. References to location and direction are changed, but the relationship between points is retained. Parallel lines are mapped into parallel lines. Quantities used to measure spactial relationships, such as distance and curvature, are unaltered. Any rigid movement may be obtained by translation and rotation of axes.[14] A translation or displacement of the origin is an inhomogeneous linear transformation, written:

$$x_i \rightarrow x_i' = x_i + b_i \tag{2.10a}$$

A rotation about a fixed origin is a special homogeneous linear transformation, written:

$$x_i \rightarrow x_i' = A_{ij}x_j \qquad (2.10b)$$

where A_{ij} is a rotation matrix;

i) $AA' = I$

ii) $/A/ = 1$

The general linear transformation, written:

$$x_i \rightarrow x_i' = A_{ij}x_j + b_i \qquad (2.10c)$$

where $/A/ = 0$

translates, rotates and stretches the coordinate axes. With the stretching of the coordinate axes a linear transformation is no longer rigid. Space is deformed as parallel lines are not mapped into parallel lines.[15] Quantities used to measure distance and curvature are subject to change. However, the algebraic characteristics of a structure are invariant under a linear transformation. A linear relationship remains linear, a curvilinear relationship does not gain any new bends or straight sections. In other words, the "basic shape" of an object is retained.

If we quantify a physical or economic process using different systems of coordinates related by a linear transformation, we may expect to obtain quantitatively equivalent results. Isnard and Zeeman offer an example of Boyle's law.[16] Investigating the relationship between temperature and pressure under fixed volume, we may select two different temperature scales, centigrade and fahrenheit. These scales have different sized units and origins, but they are linearly related. In both cases, we observe a straight line relationship between temperature and pressure.

If we quantify different processes, or the same process at different points in time, using the same system of coordinates and obtaining different results which are related by a linear transformation, the processes are similar but not quantitatively equivalent. For example, if we estimate a demand function at different points in time we might discover that, about the sample mean, demand has shifted outward and become more elastic. The two curves are related by a translation and rotation, but there has been a quantitative change in relative value as perceived by consumers. The two relationships are qualitatively similar; all the qualitative properties of one are the same as the other. In restricting our attention to qualitative properties, we may consider equivalent nonlinear transformations of coordinates which, in general, do not preserve quantitative structure.

Qualitatively Equivalent Transformations

Earlier, a structurally equivalent transformation, a homeomorphism, was defined as being continuous with a continuous inverse. This obviously includes nonlinear transformations, but our definition of qualitative equivalence requires an additional restriction. Qualitative properties of an equilibrium system, relating to behavioral and structural stability, depend on the local geometry near a critical point of a curve or surface. To within a homeomorphic transformation all local geometry appears the same. A stable minimum may be deformed into an unstable saddle point or a point on a plane. We can retain the character of critical points by requiring smooth transformations.

A qualitatively equivalent transformation of coordinates, a diffeomorphism, is one-to-one, continuous and differentiable. Saunders (1980) provides a useful (though not strictly accurate) definition of a smooth and smoothly invertible transformation:

> "We may think of two geometric objects as being diffeomorphic if they are homeomorphic and if, in addition, the deformation involves no creasing or flattening of creases. Thus, a sphere, an ellipsoid and a cube are all homeomorphic but only the first two are diffeomorphic."[17]

The transformation:

$$x \rightarrow x' = x^2 \quad \text{for } x > 0 \tag{2.11}$$

is a diffeomorphism from and to the positive real numbers. If we include the negative reals (2.11) is no longer a diffeomorphism since its inverse:

$$x' \rightarrow x = \sqrt{x'}$$

has no real values for x' less than zero. In a second example the transformation:

$$x \rightarrow x' = x^3 \quad \text{for } x' \text{ real} \tag{2.12}$$

has an inverse:

$$x' \rightarrow x = \sqrt[3]{x'}$$

However, (2.12) is not a global diffeomorphism since its inverse is not smooth at the origin. The slope of the inverse:

$$\frac{dx'}{dx} = \frac{1}{3\sqrt[3]{x^2}}$$

is undefined at the origin.

The geometric effect of a diffeomorphism is a smooth bending of the coordinate system. The linear transformations considered earlier are diffeomorphisms. Linear transformations are easily inverted nonlinear transformations are not. For two variables the general homogeneous nonlinear transformation may be written:

$$x_i \rightarrow x_i = a_{i1}x_1 + a_{i2}x_2 + a_{i11}1x_1^2 + a_{i22}x_2^2 + a_{i12}x_1x_2 + \cdots \qquad (2.13)$$

Note that at the origin:

$$\frac{\partial x'_i}{\partial x_i} = a_{i1}$$

The transformation is axes preserving.[18] Sufficiently close to the origin the transformation is linear; there is no change in the algebraic character of the surface. By moving the origin to a critical point and performing a homogeneous nonlinear transformation the character of the critical point, its local geometry, is unchanged.

In econometrically estimating economic relationships, using scales and functional forms qualitatively related by a diffeomorphism, we may expect qualitatively equivalent results. Qualitatively related scales abound in economics. In a demand study, we might use nominal prices or prices deflated by a number of different indices of inflation. Quantitative results will vary, but these scales are likely to be qualitatively related. In estimating an aggregate supply function for beef cattle, our choice of prices may vary in location, weight class and grade. We might measure production costs with farm level prices of grains or the price of corn in Chicago. We expect qualitatively equivalent results because these scales are likely to be qualitatively equivalent.[19] If we are interested in quantitative results, then we require quantitative criteria for comparing alternative scales and functional forms. If we are interested in qualitative results or establishing basic criteria for theoretical and empirical consistency, we are justified in using the simplest means available. This is an area where the techniques of coordinate transformations are of value.

Before undertaking the explicit use of variable changes we need an initial set of coordinates. Our state variables are easily defined, but we have yet to consider the relationship between the external variables of the exchange environment and the control variables of a market.

<u>The Control Space Coordinates</u>

Consider a demand or supply function of the form:

$$Q = f(P;X_n) \tag{2.20}$$

where X_n is a fixed vector of values corresponding to the external variables of the exchange environment, other market prices, and so forth. The function is presumed to be smooth at any arbitrary point $A = (P_o, Q_o)$. Translating the origin to A with a transformation (of the type 2.10a):

$$P \rightarrow P' = P - P_o$$

$$Q \rightarrow Q' = Q - Q_o$$

where now:

$$Q' = f'(P';X_n)$$

Dropping primes and expanding the function as a Taylor's series about the new origin:

$$F(P) = f(0;X_n) + f^1(0;X_n)P + 1/2f^2(0;X_n)P^2 + \ldots + 1/k!\ f^k(0;X_n)P^k + \ldots$$

$$= \sum_{k=0} 1/k!\ f^k(0;X_n)P^k$$

$$= \sum_{k=0} a_k P^k \tag{2.21}$$

The Taylor's expansion coefficients, a_i, may be regarded as functions of the exogenous variables X_n. The number of dimensions in the control space is determined by the number of functions which are independent. At most, $m<n$ functions are independent, forming an orthogonal set spanning a control space of m dimensions.[20] We may define a set of m independent control variables, c_i, as functions of the exogenous variables. If the first m Taylor's coefficients are defined by independent functions of the external variables then we may write the controls:

$$
\begin{aligned}
c_o &= a_o = h_o(X_n) \\
c_1 &= a_1 = h_1(X_n) \\
c_2 &= a_2 = h_2(X_n) \\
&\vdots \\
c_{m-1} &= a_{m-1} = h_{m-1}(X_n)
\end{aligned}
\tag{2.22}
$$

The remaining Taylor's coefficients are either dependent on the controls C_m or fixed parameters. The expansion may be written:

$$
\begin{aligned}
F(P) = {} & c_o + c_1P + c_2P^2 + \ . \ . \ . \ c_{m-1}P^{m-1} \\
& + \text{higher terms dependent on } C_m
\end{aligned}
\tag{2.23}
$$

Since the number of controls may range from one to the number of original external variables, we can say little about the dimensions of the control space. This proves to be an important point in the following chapter. Establishing canonical forms of the supply and demand curves, along with their respective controls, is the objective of the following two sections.

<u>The Implicit Function Theorem: Canonical Forms</u>

Given a supply or demand function of the form:

$$Q = f(P;C_o) \tag{2.30}$$

where C_o is a fixed vector of controls,

such that for any admissible control values:

$$\frac{dQ}{dP} \neq 0$$

we may find a diffeomorphism which reduces the curve to linear form:

$$Q' = P' \tag{2.31}$$

Consider the change of variables:

$$P \rightarrow P' = \pm f(P;C_o) \tag{2.32a}$$

$$Q \rightarrow Q' = Q \tag{2.32b}$$

The transformation is smooth since, by assumption, the supply or demand curve is smooth. Since at any point on the curve the tangent is a function of price and quantity: we may appeal to the implicit function theorem or its corollary, the inverse function theorem, for the existence of a smooth inverse:

$$P' \to P = f^{-1}(P';C_o)$$

A supply or demand function may be written in linear form:[21]

$$Q' \simeq \pm P' \tag{2.33}$$

where:

$\simeq$ means equivalent after a diffeomorphism.

We may interpret this smooth bending and displacement of coordinates as a deformation of the curve itself, in our original system of coordinates. An arbitrary point on the curve has been displaced to the origin, but this may be reversed by a simple translation of axes, yielding a canonical form for a supply or demand curve relative to the initial origin:

$$Q'' = \pm P'' + u \tag{2.34}$$

We can examine the underlying procedure with the use of a Taylor's expansion. Translating the origin to an arbitrary point on a given curve, we may expand a supply (or demand) function as a Taylor's series:

$$F(P';C_o) = a_o + a_1P' + a_2P'^2 + \ldots$$

where:

$$a_i = f_i'(0;C_o)(1/i!)$$

Terms of degree two and higher may be eliminated with an axes preserving transformation (2.13), leaving:

$$F'(P'';C_o) = a_o' + a_1'P''$$

The coefficients a_o' and a_1' can be eliminated with a linear transformation to obtain:

$$F''(P''';C_o) = P'''$$

Dropping primes and translating back to the initial origin yields a canonical form:

$$F(P;C_o) = P + u.$$

The implicit function theorem establishes a local result. A curve may be transformed into a qualitatively equivalent linear form at a point if the tangent to the curve at that point is a function of price and quantity.

The neighborhood of qualitative validity extends indefinitely so long as the supply or demand curve remains smooth. This is the same as the neighborhood of validity for linear approximations treated earlier.

A backward bending supply curve, sometimes postulated for labor, is an example of a relationship which is qualitatively nonlinear. At the fold point (Figure 2.5), the tangent is vertical, no longer a function of price and quantity. At the fold, the implicit function theorem is invalid. Linearization destroys the qualitative nature of the curve. A diffeomorphic transformation must retain the fold. Given a supply curve, g(P), and a diffeomorphism:

$$G(g(P))$$

where:

$$\frac{dG(P)}{dg(P)} = G_1 \neq 0$$

we may apply the chain rule for a composite function to obtain:

$$\frac{dG}{dP} = (G_1)(g_1(P))$$

$$= 0 \quad \text{if } g_1(P) = 0$$

$$\neq 0 \quad \text{if } g_1(P) \neq 0$$

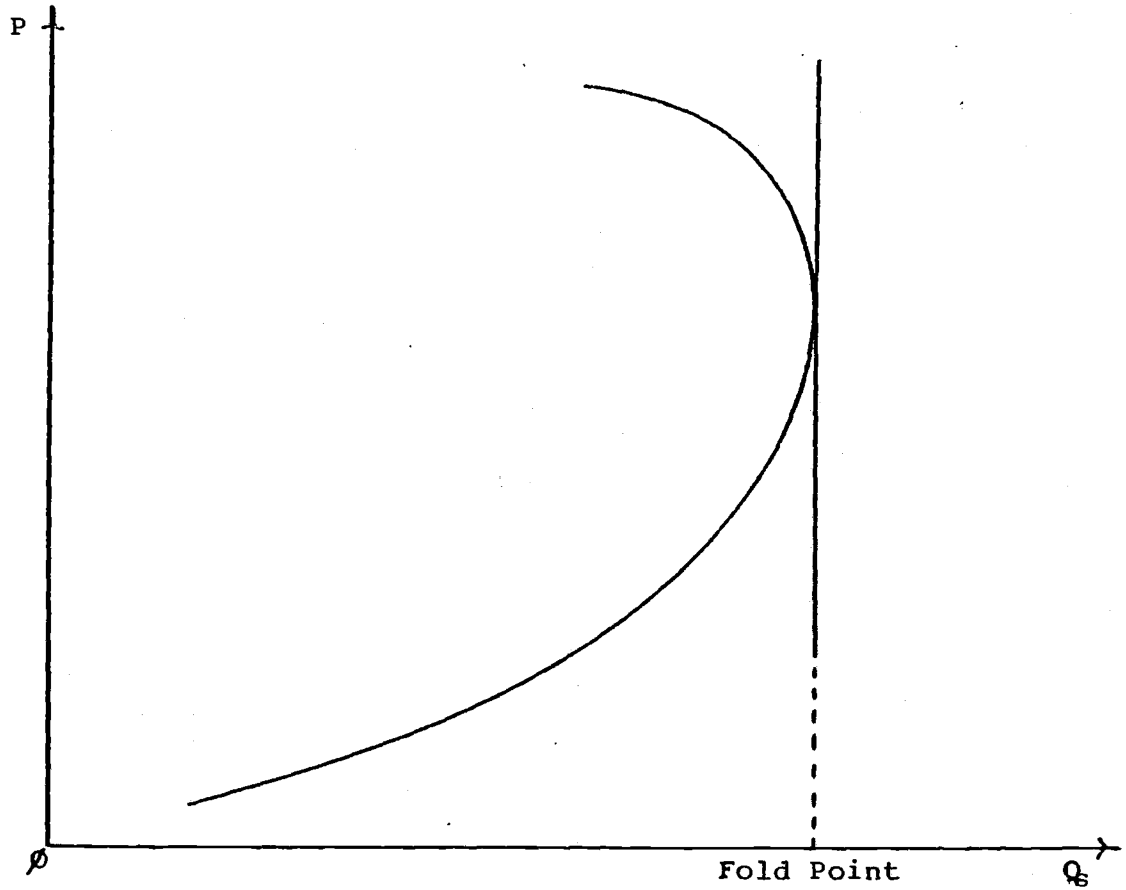

Figure 2.5. A backward bending supply curve with a vertical tangent at the fold point.

Regular points are mapped into regular points. Critical points, such as a fold, are mapped into critical points. The fold, and therefore the double-valuedness of the supply curve, is maintained through a qualitatively equivalent transformation.

The general concept of qualitative equivalence for graphs is illustrated in Figures 2.6a, b and c. The first two figures are qualitatively equivalent. Vertical lines are mapped into vertical lines, thus preserving fold points and multi-valued regions of the curves. The third figure is nonequivalent as the correspondence is lost.[22]

The canonical linear models for smooth supply and demand functions were derived for fixed values of the controls. As the controls change, so do the coordinates of the linear form. We would like to establish a canonical form for families of supply and demand curves defined by variable control parameters. This result can be developed from a consideration of perturbations.

Perturbation of Supply and Demand Functions: Canonical Forms With Variable Controls

In the neighborhood of a given point on a smooth supply or demand function, a small change in the controls may be viewed as a perturbation. A perturbation function may be written:

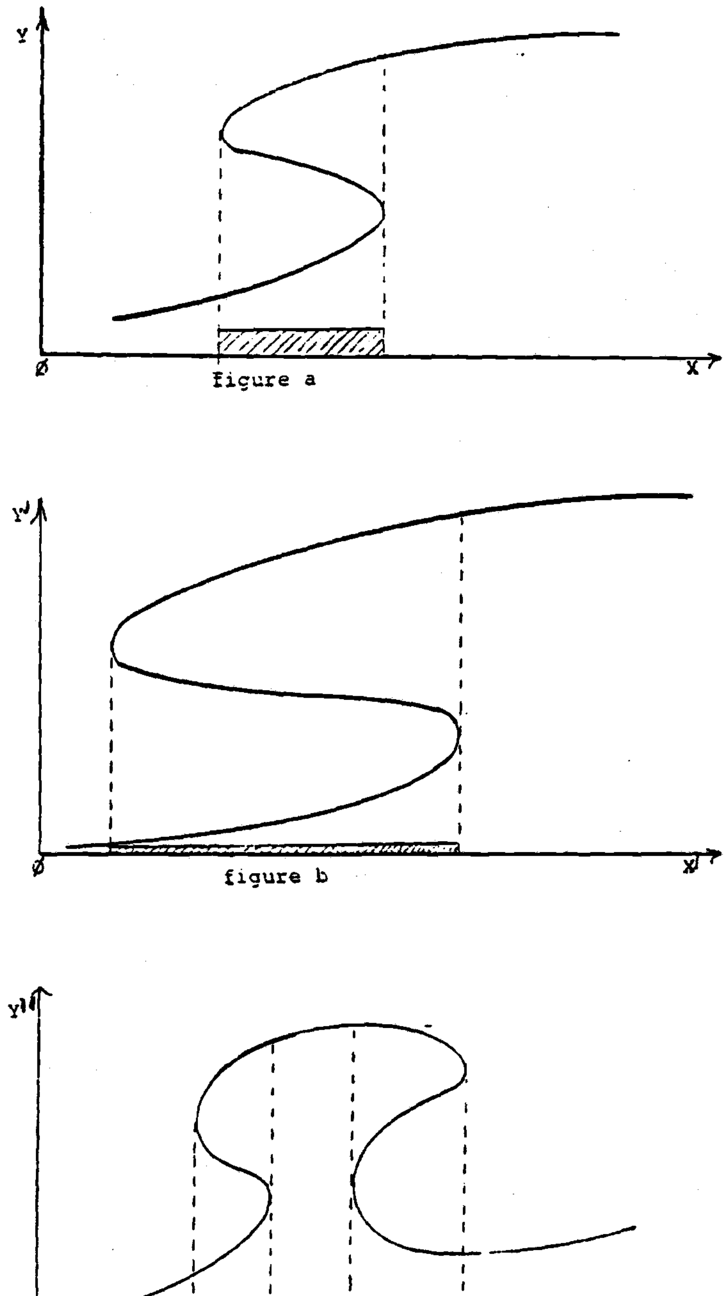

Figure 2.6. Examples of qualitative equivalence for graphs: Figures a and b illustrate qualitatively equivalent graphs, Figure c is not qualitatively equivalent to either a or b. Adapted from Isnard and Zeeman.

$$z(P;C) = f(P;C) - f(P;C_o) \qquad (2.40)$$

where f(P;C) is a family of functions. The perturbed function may be written:

$$F(P;C) = f(P;C_o) + z(P;C) \qquad (2.41a)$$

Given that the laws of supply and demand are maintained, respectively, for every member of the supply and demand families, then:

$$\frac{dF(P;C)}{dP} \neq 0$$

We may appeal to the implicit function theorem for the existence of a diffeomorphism which reduces the perturbed function to a linear form:[23]

$$F'(P';C) = P' + u \qquad (2.41b)$$

A local representation of a family of supply or demand functions requires a minimum of one canonical control, u. To see why, we can examine the dimensions and codimensions of the problem.[24] Consider a point A on a demand curve. The curve, embedded in the price-quantity plane, is a one-dimensional object of codimension one. To construct a family of curves so that every point in the neighborhood of A lies on a member of the family, the object generated by the family must be two-dimensional or of codimension zero. In general, a one parameter family of objects of dimension r is an object of r+1 dimensions. A one parameter continuous family of curves in the price-quantity plane is a two-dimensional section of the plane of codimension zero.

The linear canonical form is a local representation. The neighborhood of qualitative validity extends until a nongeneric (nonregular) point is encountered in either the control space or on a given curve.

The canonical control parameter u varies continuously with changes in the control variables C. This may be seen by expanding the perturbed function as a Taylor's series. First, we may place $f(P;C_o)$ in canonical form (equation 2.33) and rewrite the perturbed function (without primes):

$$F(P;C) = P + z(P;C) \tag{2.41c}$$

Expanding the perturbed function about the new origin as a Taylor's series yields:

$$F = z(0;C) + (1 + z_1(0;C))P + 1/2z_2(0;C)P^2 + \dots \tag{2.42a}$$

Since the linear term is nonvanishing, second and higher degree terms may be eliminated with an axes-preserving nonlinear transformation:

$$F' = z'(0;C) + (d + z_1'(0;C))P' \tag{2.42b}$$

A linear transformation ($P'' = (d + z_1'(0;C))^{-1}P'$) stretches the coordinate axis to absorb the linear coefficient, yielding:

$$F'' = z'(0;C) + P'' \tag{2.42c}$$

The constant term $z'(0;C)$ varies continuously with the control values C.

The behavioral implications of the canonical forms are not surprising. A small change in the controls results in a slight change in the quantity supplied or demanded at a given price. A smooth change in the exchange environment results in a smooth response in supply or demand. There is no qualitative change in the nature of the relationships. Supply and demand are structurally stable.

To consider the impact of changing control values on market equilibrium, we may seek a change of coordinates which linearizes both the supply and demand curves at equilibrium. We can obtain this result from the Thom isotopy theorem.

<u>Transversatility of Supply and Demand: Thom's Isotopy Theorem</u>

Poston and Stewart state:

> "Transversatility is often called general position (because 'nothing special happens')."[25]

The supply and demand curves are transverse if: one, they intersect in exactly one point (if at all); and two, the tangents to the curves at the point of intersection do not coincide.[26] A transverse crossing is illustrated in Figure 2.7a. A non-transverse crossing is illustrated in Figure 2.7b. Non-transverse crossings occur at isolated prices or quantities. Control values corresponding to non-transverse intersections are isolated points in the control space. In the same sense that a real number chosen at random is infinitely unlikely to be equal to , it is unlikely that the control will take on exact values corresponding to a non-transverse crossing. We presume that non-transverse conditions, such as multiple-equilibria, do not occur.

There are two important results of Thom's isotopy theorem for equilibria defined by a transverse

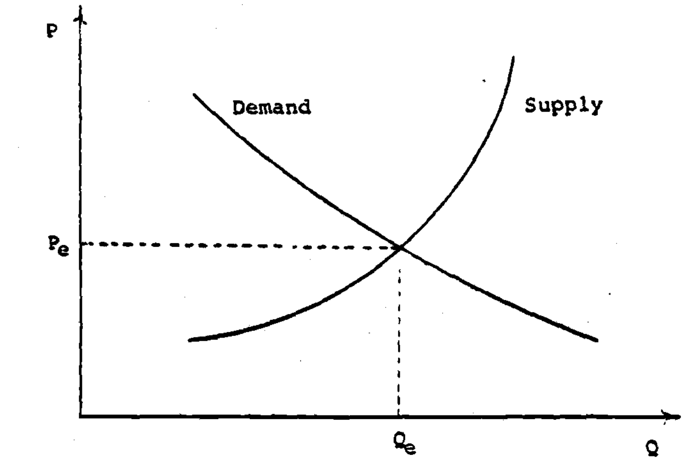

Figure a

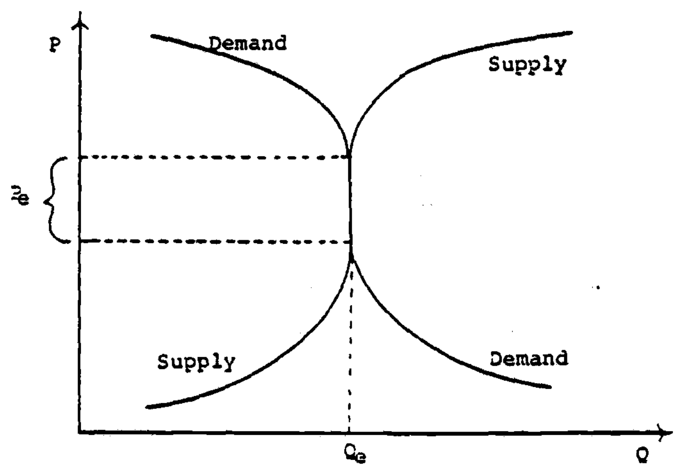

Figure b

Figure 2.7. Transverse and non-transverse intersections of a supply and demand curve: Figure a) a typical transverse intersection, Figure b) a non-transverse intersection. Two conditions of transversality are violated: one, the intersection is of the same codimension as the intersecting lines, and two, the tangents of the intersection are coincident.

intersection. First, any transverse intersection of the supply and demand curves is qualitatively equivalent to Figure 2.8, the intersection of two perpendicular lines. This is consistent with our linear canonical forms for supply and demand at equilibrium. Second, transverse crossings are structurally stable. A small change in the values of the controls results in a slight change in the location of equilibrium. Smooth changes in the exchange environment result in smooth changes in the location of market equilibrium. The qualitative character of equilibrium, stable or unstable, is unaltered.[27]

Transversality implies structural stability of equilibria. It does not imply that equilibrium can and will be achieved. To examine this aspect of market behavior, we must consider the market dynamic. Viewing market structure in the context of a gradient based equilibrium system provides a relatively complete framework for analyzing market adjustment and the determination of exchange rates.

A Gradient Based Market Model: Description

Market models are, in general, derivable from a simple first order gradient system. Stable market equilibria are represented as a minimum of a potential or market objective function. Maxima or saddle points of a

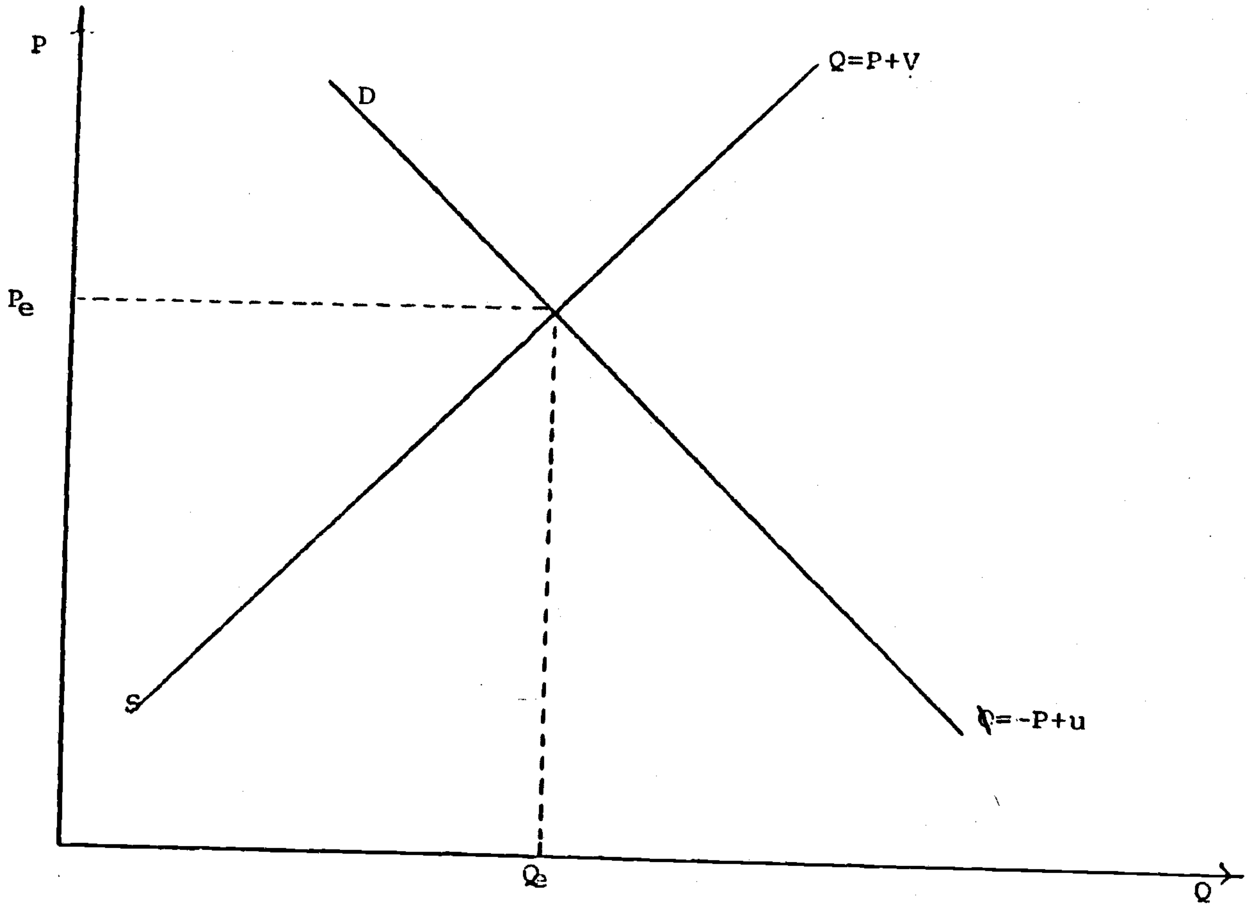

Figure 2.8. A canonical form of a transverse intersection of a supply and demand curve.

potential function represent unstable equilibria. A given potential, from which the market dynamic is derived, corresponds to a given set of control values. For variable controls we may think of the controls as parameters of a family of potentials.

The way in which equilibria organize market structure is reflected by the way in which critical points organize a potential or family of potentials.[28] The force acting to adjust exchange rates is represented by the negative gradient of a potential. The negative gradient is a vector, perpendicular to the contours of a potential: directed downward towards a stable minimum (Figure 2.9a) and outward from an unstable maximum (Figure 2.9b) or in both directions for a saddle point (Figure 2.9c). The components of the market adjustment force are given in the price direction by the partial derivative of the potential with respect to price, and, in the quantity direction, by the partial derivative of the potential with respect to quantity. At an equilibrium or critical point, the price and quantity components of the force are zero.

The slope of the potential vanishes at equilibrium; hence, the tangent to the potential is no longer a function of price and quantity. The implicit function theorem is no longer applicable. A linear approximation

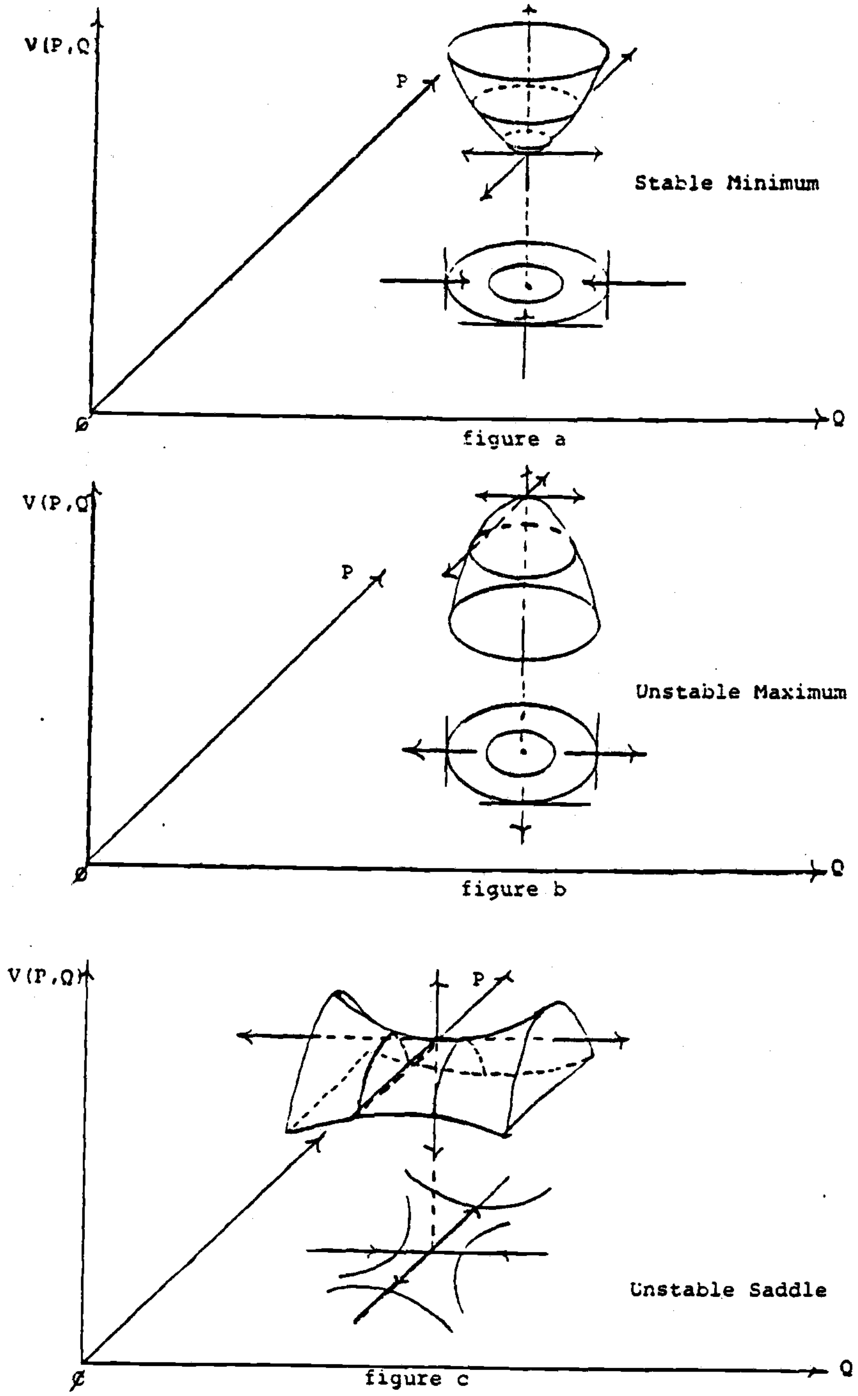

Figure 2.9. The structure of a non-degenerate equilibrium surface with projections of contours and negative gradients along principle directions of curvature: Figure a) a stable minimum, Figure b) an unstable maximum, Figure c) an unstable saddle.

or transformation is not qualitatively equivalent to the potential at this point. We may consider the qualitative properties of a quadratic approximation.

Quadratic Approximation: Equilibrium

In a sufficiently small neighborhood of a nondegenerate equilibrium point (a minimum, maximum or saddle), the potential may be approximated with a quadratic surface, an upward (downward) opening elliptic paraboloid for a minimum (maximum) and a hyperbolic paraboloid for a saddle (Figures 2.9a,b,c).[28] The neighborhood of validity for the quantitative properties of the approximation may be large or extremely small. The curvature of a quadratic surface is constant, and the more quickly the curvature changes the faster quantitative accuracy declines.

The neighborhood of validity for the qualitative properties of the approximation is analogous to linear approximations at regular points. For a stable minimum, all regular points of the approximating surface lie above the tangent plane of the critical point (positive upward curvature). For a maximum, all regular points lie below the tangent plane (positive downward curvature). For a saddle, regular points lie on both sides of the tangent plane (negative curvature). The qualitative validity of the approximation is maintained until another critical point of the potential is encountered.[29] This is illustrated for a single variable potential in Figure 2.10. If the critical point is unique, the validity of the approximation is global.

A quadratic approximation of a potential at a critical point is easily obtained form a Taylor's expansion. Given a market potential expressed in terms of price and quantity, $V(P,Q)$, with a critical point at (P_o,Q_o), we can expand the potential as a Taylor's series about the stationary point:

$$\begin{aligned}\bar{V}(P,Q) = {} & V(P_o,Q_o) + V_1(P_o,Q_o)(P-P_o) + V_2(P_o,Q_o)(Q-Q_o) \\ & + 1/2V_{11}(P_o,Q_o)(P-P_o)^2 + 1/2V_{22}(P_o,Q_o)(Q-Q_o)^2 \\ & + V_{12}(P_o,Q_o)(P-P_o)(Q-Q_o) + \text{higher} \\ & \text{terms}\end{aligned} \tag{2.50}$$

The Taylor's series may be truncated beyond second degree terms and noting that the linear terms vanish at a stationary point:

$$\begin{aligned}V(P,Q) \approx {} & V(P_o,Q_o) + 1/2V_{11}(P_o,Q_o)(P-P_o)^2 + \\ & 1/2V_{22}(P_o,Q_o)(Q-Q_o)^2 + V_{12}(P_o,Q_o) \\ & V_{12}(P_o,Q_o)(P-P_o)(Q-Q_o)\end{aligned} \tag{2.51a}$$

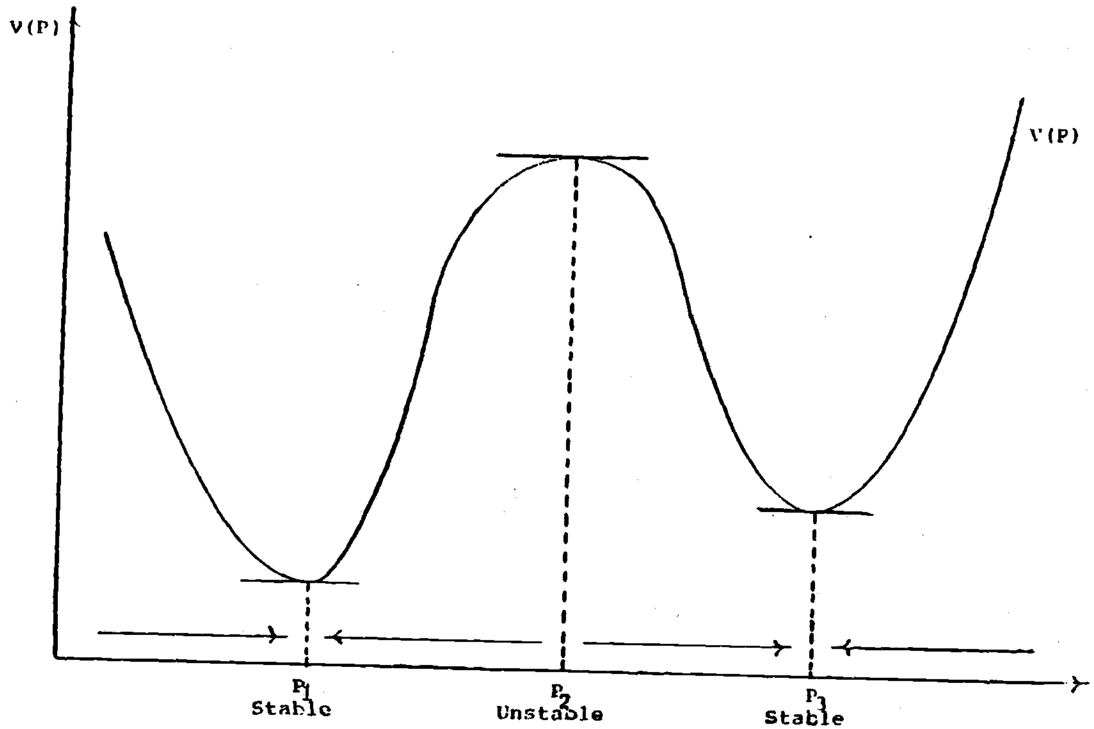

Figure 2.10. The qualitative organization of a potential about its critical points.

The quadratic terms can be diagonalized (cross products eliminated) with a rotation of axes to the principal directions of curvature:

$$V'(P',Q') \approx V'(P_o,Q_o) + V'_{11}(P_o,Q_o)(P-P_o)'^2 + V'_{22}(P_o,Q_o)(Q-Q_o)'^2 \qquad (2.51b)$$

Finally, the origin may be translated to the critical point, yielding:

$$V''(P'',Q'') \approx V'_{11}(P_o,Q_o)P''^2 + V'_{22}(P_o,Q_o)Q''^2 \qquad (2.51c)$$

With the steps outlined in 2.51a,b,c we can obtain a quadratic approximation for a nondegenerate critical point in standard form, written (without primes):

$$V(P,Q) \approx \lambda_1 P^2 + \lambda_2 Q^2 \qquad (2.52)$$

where:

i) $\lambda_1, \lambda_2 > 0$ for a minimum

ii) $\lambda_1, \lambda_2 < 0$ for a maximum

iii) $\lambda_i < 0 < \lambda_j$ for a saddle.

If one or both of the λ_i are zero, the critical point is degenerate, and the approximation may break down. If only one of the λ_i is zero, the approximating surface is a parabolic cylinder; if both the λ_i are zero, the approximating surface is a plane. In either case, the approximation may not correspond qualitatively to the critical point of the potential.[30]

The λ_i are the eigenvalues of the stability matrix (Hessian), V_{ij}, written:

$$V_{ij} = \begin{pmatrix} \dfrac{\partial^2 V}{\partial P^2} & \dfrac{\partial^2 V}{\partial P\, \partial Q} \\ \dfrac{\partial^2 V}{\partial Q\, \partial P} & \dfrac{\partial^2 V}{\partial Q^2} \end{pmatrix} \tag{2.53}$$

By stretching the length of the coordinates axes with the transformation:

$$P \to P' = |\lambda_1|^{1/2} P \tag{2.54a}$$

$$Q \to Q' = |\lambda_2|^{1/2} Q \tag{2.54b}$$

we can write 2.52 in Morse canonical form:

$$V'(P',Q') \approx \pm P'^2 + \pm Q'^2 \tag{2.55}$$

Morse canonical form for a nondegenerate critical point is the counterpart to a linear canonical form for a regular point. The Morse lemma states that in the neighborhood of a nondegenerate critical point, a potential may be placed in Morse canonical form with a smooth reversible change in coordinates.[31] If we are able to successfully approximate the local properties of a critical point with a quadratic function, then we can make the approximation exact with a qualitative change in scale. Before considering the implication of the Morse lemma, we need a working definition of a market potential.

Market Potentials

A market potential is a mathematical description of the response of market prices and quantities to changes in supply and demand. To develop a market potential, an explicit mechanism of market adjustment is required, the market dynamic. There are two components of a market dynamic: one, Walrasian price adjustment specifying the rate of change in prices over time, $\dot{P}$, as a function of excess quantity demanded:

$$\dot{P} = \frac{dP}{dt} = \Theta(Q_d - Q_s)$$

$$= \Theta(f(P;C_d) - g(P;C_s)) \qquad (2.60a)$$

and, two, Marshallian quantity adjustment, specifying the rate of change in quantity over time as a function of excess demand price:

$$\dot{Q} = \frac{dQ}{dt} = \Psi(P_d - P_s)$$

$$= \Psi(f^{-1}(Q;C_d) - g^{-1}(Q;C_s)) \qquad (2.60b)$$

To ensure that equilibrium corresponds to the intersection of the supply and demand curves, we require the functions Θ and Ψ to be homogeneous:

$$\Theta(0) = 0$$
$$\Psi(0) = 0$$

We presume that market equilibrium is stable under the laws of supply and demand. Therefore, excess quantities demanded must decline as prices increase, and:

$$\Theta^1(\ ) > 0$$

and excess demand price must decline as market quantities increase, and:[32]

$$\Psi^1(\ ) > 0$$

The functions of the dynamic, Θ and Ψ, may be viewed as components of a market adjustment force.[33] Assuming that this force may be derived as the negative gradient of a potential, equations 2.60a and 2.60b may be rewritten:

$$\dot{P} = \frac{\partial V(P,Q;C)}{\partial P} = \Theta(f(P;C_d)-g(P;C_s)) \qquad (2.61a)$$

$$\dot{Q} = \frac{\partial V(P,Q;C)}{\partial Q} = \Psi(f^{-1}(Q;C_d)-g^{-1}(Q;C_s)) \qquad (2.61b)$$

Equilibrium conditions are given by zero time derivatives:

$$\dot{P} = 0$$
$$\dot{Q} = 0$$

which clearly correspond to a critical point of the potential V(P,Q;kC):

$$\frac{\partial V(P,Q;C)}{\partial P} = -(f(P;C_d)-g(P;C_s)) = 0 \qquad (2.62a)$$

$$\frac{\partial V(P,Q;C)}{\partial Q} = -(f^{-1}(Q;C_d)-g^{-1}(Q;C_s)) = 0 \qquad (2.62b)$$

The stability matrix for the potential is diagonal; its elements are the eigen values of the matrix, written:

$$V_{ij} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \qquad (2.63)$$

where;

$$\lambda_1 = -\ \Theta_1(\ )(f_1(P;C_d)-g_1(P;C_s)) > 0$$
$$\lambda_2 = -\ \Psi_1(\ )(f^{-1}_1(Q;C_d)-g^{-1}_1(Q;C_s)) > 0$$

The eigenvalues, λ_1 and λ_2, are strictly positive since, one, the first derivatives of Θ and Ψ are positive by assumption and two, the slopes of the supply and demand curves are, respectively, positive and negative for all values of the controls. A point of equilibrium corresponds to a stable minimum of a potential. The set of market equilibria correspond to the minima of a family of potentials specified by the control parameters. We may write a quadratic approximation of the potential at equilibrium:

$$V(P,Q) \approx V(P_e,Q_e) + \lambda_1(P-P_e)^2 + \lambda_2(Q-Q_e)^2 \qquad (2.64)$$

The approximation is simply a truncated Taylor's expansion with the values of λ_1 and λ_2 evaluated at P_e and Q_e, respectively. By translating the origin to the equilibrium point and stretching the coordinate axes to absorb the eigen values (transformations 2.54a,b), the potential may be written in Morse canonical form:

$$V'(P',Q') \approx P'^2 + Q'^2 \qquad (2.65)$$

The Morse lemma guarantees the approximation can be made exact with a diffeomorphic transformation of coordinates.

The Morse Lemma

A demonstration of the Morse lemma, adapted from Gilmore, provides some insight into the process of coordinate transformations.[34] We have and will continue to appeal to the fact that higher terms of a Taylor's expansion may be conveniently eliminated with a smooth change of variables. Here we will make a limited attempt to show why this is so.

To simplify the development of the Morse lemma for a market potential, we can make the following observation. Equations 2.61a and 2.61b contain no cross product terms. The price and quantity components are separable:

$$V(P,Q) = V^1(P) + V^2(Q) \qquad (2.70)$$

We may treat the price and quantity components individually. Focusing on the price component, we may write a Taylor's expansion of the price potential with the origin at P_e:

$$V^1 = \lambda_1 P^2 + cP^3 + dP^4 + eP^5 + \ldots \qquad (2.71)$$

We may define a nonlinear axes preserving transformation:

$$P \rightarrow P' = P + A_2P^2 + A_3P^3 + A_4P^4 + \ldots \qquad (2.72)$$

The A_i are disposable coefficients. These may be chosen to eliminate third and higher degree terms. First, we equate the Morse form with equaiton 2.71 using the nonlinear transformation (equation 2.72):

$$\begin{aligned} V^{1'} &= \lambda_1 P'^2 \\ &= \lambda_1 (P + \sum_{i=2}^{n} A_i P^i)^2 \qquad (2.73) \end{aligned}$$

This places constraints on the disposable coefficients. To obtain these constraints the square of 2.73 is expanded and the disposable coefficients are equated with those of the corresponding monomial P_i of 2.71. For the coefficients shown in 2.71, the constraints are:

$$
\begin{aligned}
P^3 &: \lambda_1 2A_2 &&= c \\
P^4 &: \lambda_1 A_2^2 + \lambda_1 2A_3 &&= d \\
P^5 &: \lambda_1 A_2 A_3 + \lambda_1 2A_4 &&= e
\end{aligned}
\qquad (2.74)
$$

The details for higher degree terms are messier, but there is one new disposable coefficient for each new term of the expansion. The constraints can be solved to eliminate all but the first nonvanishing term of the Taylor's expansion.

A similar nonlinear transformation can be used to reduce the quantity component of the potential to Morse form. The first derivatives of the nonlinear transformations are equal to one at equilibrium (derivative of 3.72 where $P = 0$). Therefore, we may appeal to the inverse function theorem; the transformations are invertible and smooth in the neighborhood of equilibrium. We may write a stable market potential in Morse form after a qualitative change in scale:

$$V(P,Q) \simeq \lambda_1 P'^2 + \lambda_2 Q'^2 \qquad (2.75)$$

The transformation to Morse form is for fixed values of the controls. As the controls change, the coordinates of equation 2.75 change. To treat the controls as variables, we need to consider a family of market potentials. The parameters of this family of potentials are the canonical controls. The canonical controls reflect how equilibrium changes in response to changes in the environment of exchange. As with the supply and demand functions considered earlier, we can determine the form of a family of stable potentials in the neighborhood of equilibrium by studying the effects of perturbations of a given market potential.

Perturbations: Market Equilibrium

Given a market potential, $V(P,Q;C^o)$, with an equilibrium point (P_e,Q_e) and a family of potentials, $V(P,Q;C)$ with stable equilibria, the difference:[35]

$$z(P,Q;C) = V(P,Q;C) - V(P,Q;C^o) \tag{2.80}$$

may be regarded as a perturbation of $V(P,Q;C^o)$ in the neighborhood of $(P_e,Q_e;C^o)$. The perturbed function may be written:

$$V(P,Q;C) = V(P,Q;C^o) + z(P,Q;C) \tag{2.81a}$$

Choosing the origin at equilibrium and placing $V(P,Q'C^o)$ in Morse form, we may rewrite the perturbed function (dropping primes):

$$V(P,Q;C) = \lambda_1 P^2 + \lambda_2 Q^2 + z(P,Q;C) \tag{2.81b}$$

The perturbed function may be expanded as a Taylor's series about the origin:

$$\begin{aligned} V = {} & z(0;C) + z_{11}(0;C)P + z_{12}(0;C)Q + \\ & (\lambda_1 + z_{21}(0;C))P^2 + (\lambda_2 + z_{22}(0;C))Q^2 + \\ & z_{31}(0;C)P^3 + z_{32}(0;C)Q^3 + \ldots \end{aligned} \tag{2.82a}$$

$$\begin{aligned} = {} & z_o + z_{11}P + z_{12}Q + (\lambda_1 + z_{21})P^2 + (\lambda_2 + z_{22})Q^2 \\ & z_{31}P^3 + z_{32}Q^3 + \ldots \end{aligned} \tag{2.82b}$$

The terms of the expansion of degree greater than two may be eliminated with a nonlinear axes preserving transformation:

$$\begin{aligned} V' = {} & z'_o + z'_{11}P' + z'_{12}Q' + (\lambda'_1 + z'_{21})P'^2 + \\ & (\lambda'_2 + z'_{22})Q^2 \end{aligned} \tag{2.83}$$

A linear transformation of the form:

$$P' \rightarrow P'' = P'(\lambda'_1 + z'_{21})^{1/2}$$
$$Q' \rightarrow Q'' = Q'(\lambda'_2 + z'_{22})^{1/2}$$

absorbs the second-degree coefficients into the length of scale:

$$V'' = z'_o = z''_{11}P'' + z''_{12}Q'' \tag{2.84a}$$

We are not concerned with the value of the potential; the origin may be translated in the $\bar{V}''$ direction to eliminate the constant term:

$$V''' = z''_{11}P'' + z''_{12}Q'' + P''^2 + Q''^2 \tag{2.84b}$$

The linear coefficients, z''_{1i}, are continuous functions of the controls C. They may be regarded as a qualitative transformation of the original control variables:

$$V''' = c'_1P'' + c'_2Q'' + P''^2 + Q''^2 \tag{2.85a}$$

Rescaling P" and Q" for convenience and dropping primes, the potential may be written:

$$V = c_1P + c_2Q + 1/2P^2 + 1/2Q^2 \tag{2.85b}$$

Differentiating equation 2.85b with respect to price and quantity, we can obtain a canonical form of the market dynamic:

$$\frac{\partial V}{\partial P} = \dot{P} = -P - c_1 \tag{2.86a}$$

$$\frac{\partial V}{\partial Q} = \dot{Q} = -Q - c_2 \tag{2.86b}$$

Equilibrium conditions, $\dot{P} = \dot{Q} = 0$, yield canonical expressions for equilibrium price and quantity as linear functions of the control variables:

$$P_e = -c_1 \tag{2.87a}$$

$$Q_e = -c_2 \tag{2.87b}$$

In the neighborhood of an initial point of equilibrium, $(0,0;C_o)$, a small change in the control values results in a slight shift in the location of equilibrium to $(-c_1, -c_2)$. Since the canonical controls, c_1 and c_2 vary continuously with changes in C, the location of equilibrium moves smoothly with smooth changes in the exchange environment. This is simply a restatement of the implications of Thom's isotopy theorem for a stable transverse equilibrium. The result is based on local properties but the neighborhood of qualitative validity extends until a nontransverse equilibrium is encountered. Under a strict interpretation of the laws of supply and demand isolated nontransverse crossings can occur and the preceding analysis breaks down. However, we have excluded nontransversality on the basis that it is extremely unlikely to occur.

Two canonical control parameters are required to define the family of market equilibrium points. Again, we can examine the codimensons of the problem to see why. An equilibrium point embedded in the price-quantity plane is an object of codimension two. To construct a family of equilibrium points so that in a given neighborhood every point is an equilibrium point of the family, the codimension of the object generated by the family must be equal to zero. A two-parameter continuous family of zero-dimensional points is an object of two dimensions, a section of the price-quantity plane with codimension zero.

The stability of individual equilibria and the structural stability of the family of market equilibria imply that market prices and quantities can adjust smoothly to a new point of equilibrium in response to changes in the market environment. Whether or not this will, in fact, happen depends upon the temporal structure of a market. Temporal structure, taking into account the relative rates at which state and control variables change, is the final key element of qualitative market structure.

Temporal Market Structure

The canonical equilibrium equations (2.87a,b) are related to our canonical supply and demand equations:

$$Q = -P + u \qquad \text{demand}$$

$$Q = P + v \qquad \text{supply}$$

Solving the equations for equilibrium price and quantity yields:

$$P_e = (u-v)/2 \tag{2.90a}$$
$$= -c_1$$

$$Q_e = (u+v)/2 \tag{2.90b}$$
$$= -c_2$$

The two sets of controls are related by a simple linear transformation. Separating the equilibrium controls into supply and demand shift parameters is informative so the distinction will be maintained.

If we assume the flow of the dynamic is fast in comparison to the rate of change in the controls, we may equate market prices and quantities with equilibrium prices and quantities:[36]

$$P = (u-v)/2 \tag{2.91a}$$

$$Q = (u+v)/2 \tag{2.91b}$$

The adjustment of exchange rates is represented as a continuous transition between alternate states of equilibrium. We may represent this graphically by treating the price and quantity components separately.

The state variable price and the control variables, u and v, may be viewed as coordinates of a three-dimensional phase space; a one-dimensional state space and a two-dimensional control space. The family of equilibrium prices is a surface; drawn in canonical form in Figure 2.11. The control surface corresponds to a section of the u-v plane over which prices are positive. The fast flow of the dynamic is assumed to be a near vertical flow between the control and equilibrium surfaces.[37] The slow flow of market response to changing control values is represented by a movement along the equilibrium surface. Figure 2.12 illustrates the quantity component of the model.

The figures drawn for the equilibrium model are a relatively complete synthesis of a competitive market under the assumptions of fast and slow flow. A delayed adjustment of supplies to current market prices can be represented as a slow feedback of prices on the controls. A gradual growth in demand with oscillations introduced by lagged supply adjustment is illustrated by the slow flow in Figure 2.11. The model reflects qualitative market structure, those aspects of price determination which are independent of our choice of functional forms or coordinates. In general, we may bend, stretch or twist

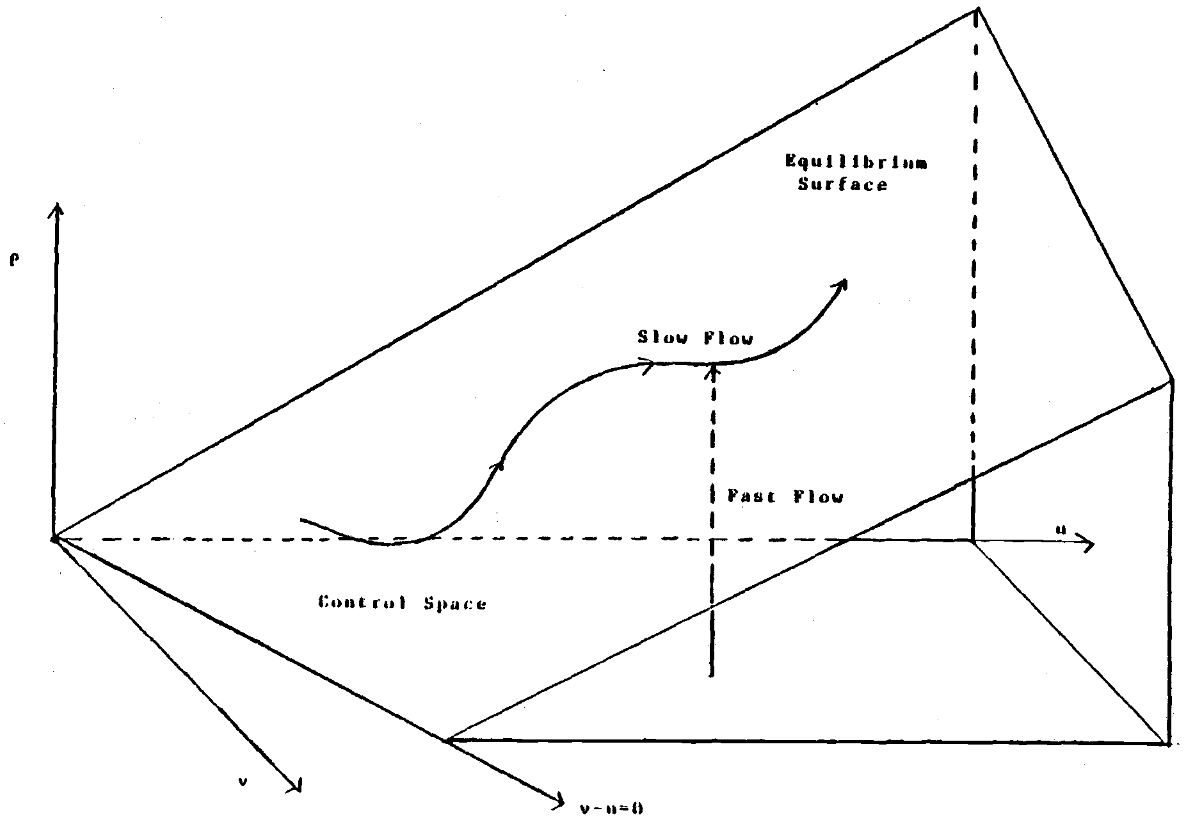

Figure 2.11. Price as a linear function of the demand shift variable u and the supply shift variable v, with fast and slow flows.

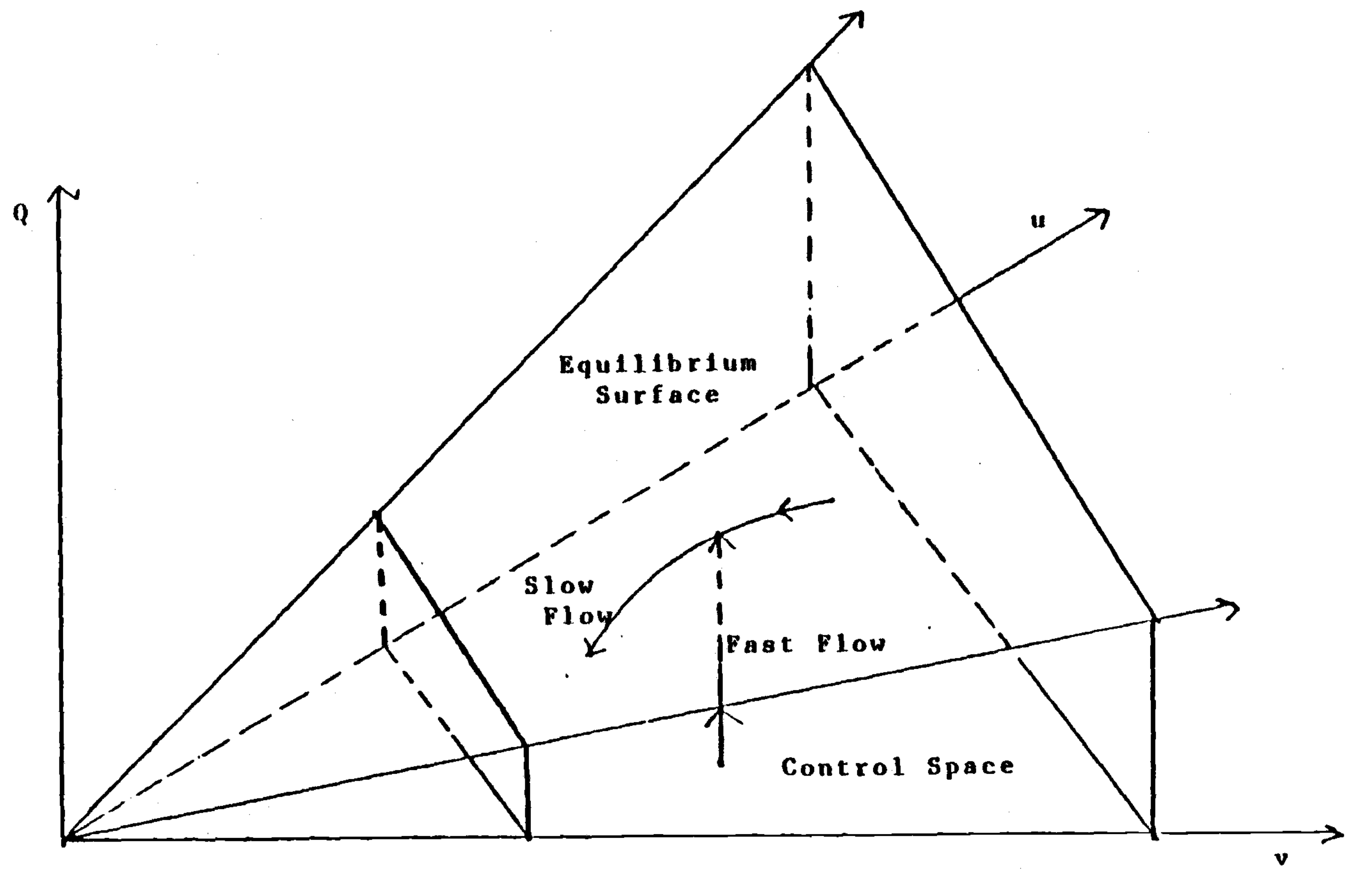

Figure 2.12. Market quantity as a linear function of the supply and demand shift variables.

the equilibrium and control surfaces, so long as we do not tear, crease or fold them, without altering qualitative structure. Figures 2.11 and 2.13 are illustrations of models with a very different quantitative structure but an equivalent qualitative structures.

We can introduce quantitative structure if we attach significance to a particular set of coordinates. For agricultural products we might assert that the slope of the price surface is very steep and the quantity surface quite flat, owing to the inelasticity of supply and demand. Relatively how steep or how flat are quantitative properties. We need to choose scales of measurement to obtain an estimate. Since these choices are to some extent arbitrary, the estimate is to some extent arbitrary. This does not exclude the value of quantitative results in helping to confirm or deny our explanation of why agricultural prices are subject to large variations. It should temper our assertions drawn from a given estimate.

If the control values change rapidly, so that we may no longer think of the dynamic as a vertical flow, the static equilibrium structure of a market may be obscured. A market may exist in a near constant state of disequilibrium. Our focus must shift to the market

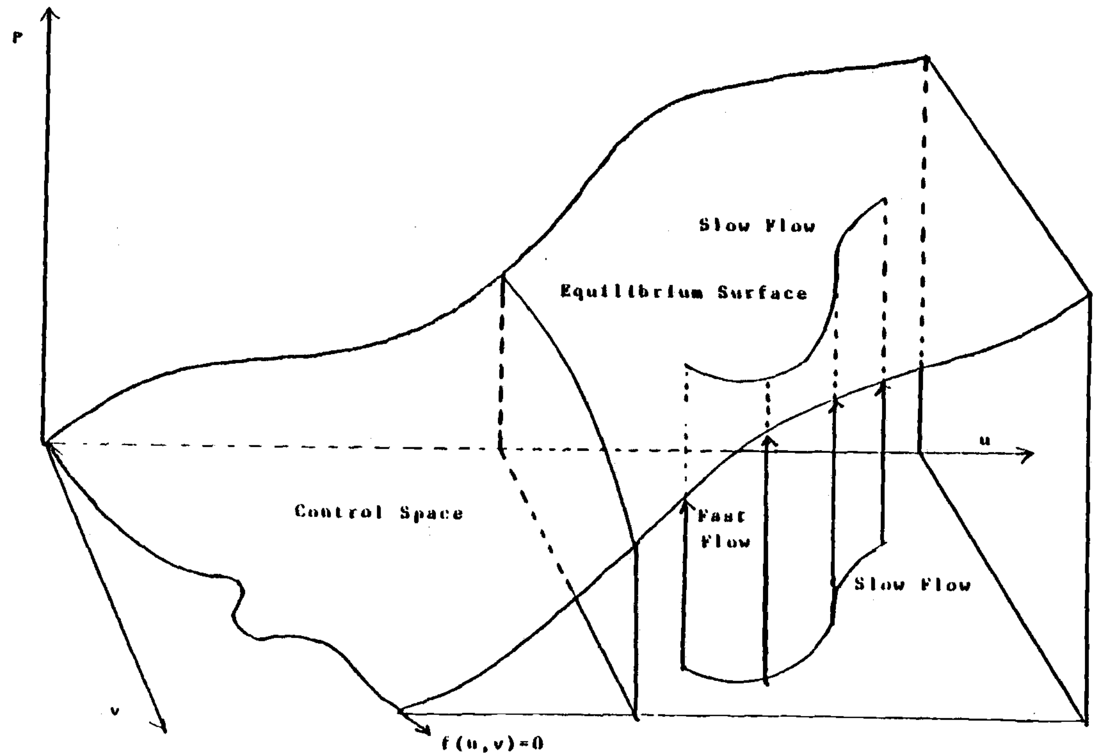

Figure 2.13. Price as a non-linear function of the supply and demand shift variables. This figure may be obtained from Figure 2.11 by a smooth reversible change of coordinates. Figures 2.11 and 2.13 are qualitatively equivalent.

dynamic, illustrated for the price and quantity canonical forms in Figure 2.14. The dynamic model may also be perceived as an equilibrium system.[38] Dynamic equilibria are rates of change in price and quantity, given for the canonical forms by:

$$\dot{P} + P - (u-v)/2 = 0 \tag{2.93a}$$

$$\dot{Q} + Q - (u+v)/2 = 0 \tag{2.93b}$$

The families of dynamic equilibria correspond to the price and quantity trajectory curves toward static equilibrium. The fast flow, towards a dynamic equilibrium rate of change, may be represented by the negative gradient of a dynamic potential:

$$\frac{-\partial U(\dot{P},\dot{Q})}{\partial \dot{P}} = \frac{d\dot{P}}{dt} = -\dot{P} - P + (u-v)/2 \tag{2.94a}$$

$$\frac{-\partial U(\dot{P},\dot{Q})}{\partial \dot{Q}} = \frac{d\dot{Q}}{dt} = -\dot{Q} - Q + (u+v)/2 \tag{2.94b}$$

The potential is structurally stable as the determinant of the stability matrix, U_{ij}:

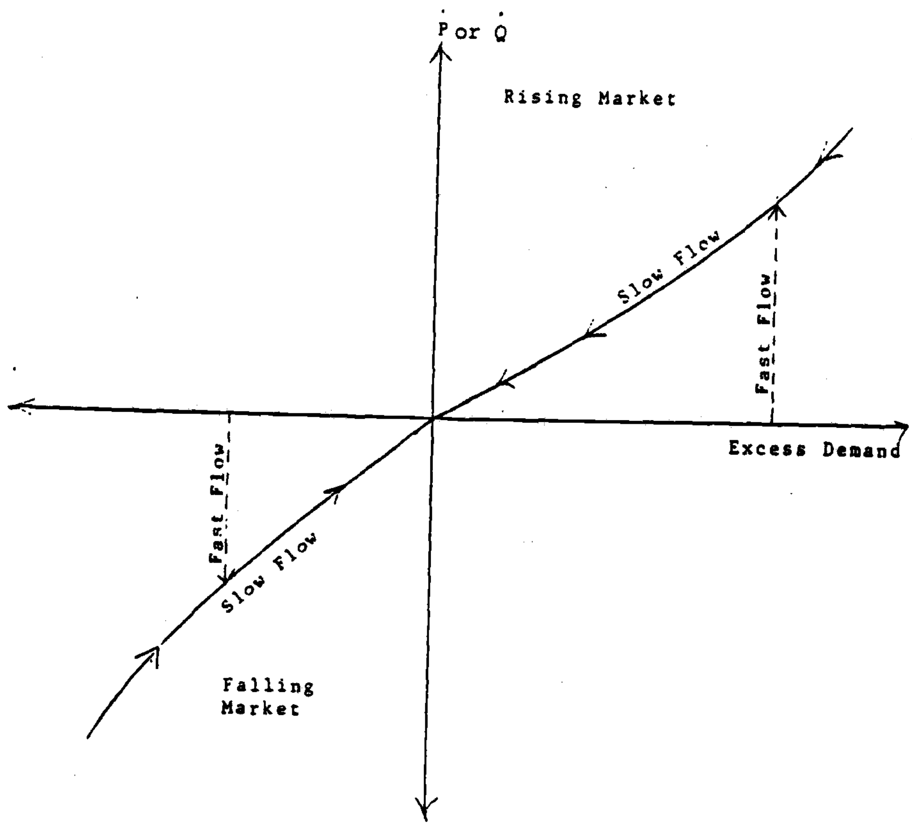

Figure 2.14. Rate of change in price of quantity as a function of excess demand; fast flow towards a stable trajectory and slow flow along a normal Walrasian or Marshallian dynamic.

$$U_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

is nonvanishing (equal to one for all control values). The positive eigen values of the matrix indicate that the dynamic equilibria are stable. The potential may be written:

$$U(\dot{P},\dot{Q}) = 1/2\dot{P}^2 + 1/2\dot{Q}^2 + b_1\dot{P} + b_2\dot{Q} \qquad (2.95)$$

where the controls:

$$b_1 = -P + (u-v)/2$$

$$b_2 = -Q + (u+v)/2$$

The fast flow generated by the dynamic potential is assumed to be vertical. Changes in the controls result in a slow flow along the price and quantity trajectories. Given that factors outside the market stabilize, a slow feedback effect of changing prices and quantities on the controls brings the market to a point of static equilibrium. This feedback is simply the effect of increasing or decreasing prices on excess demand:

$$b_1 = 1/2((-P + u) - (P + v))$$
$$= 1/2(Q_d - Q_s)$$

and the effect of increasing or decreasing quantities on excess demand price:

$$b_2 = 1/2((-Q + u) - (Q - v))$$
$$= 1/2(P_d - P_s)$$

In a dynamic formulation of a market, exchange rates are given as functions of time. For specified values of the external controls, u and v, market prices and quantities follow a time dependent pathway, starting at an initial boundary point (P_o,Q_o) and terminating at a point of static equilibrium. The time paths are given by the solution to the differential equations 2.93a and 1.92b for $P=P_o$ and $Q=Q_o$ at $t=0$.[39]

$$P = (P_o - (u-v)/2)e^{-t} + (u-v)/2$$
$$= (P_o - P_e)e^{-t} + P_e \qquad (2.96a)$$

$$Q = (Q_o - (u+v)/2)e^{-t} + (u+v)/2$$
$$= (Q_o - Q_e)e^{-t} + Q_e \qquad (2.96b)$$

With a few exceptions, the analysis conducted for static equilibria can be applied to the dynamic model. Since dynamic equilibria will be treated at length in the following chapter the details will not be presented here. In Chapter III the methodology presented here will be applied to the more interesting possibilities of catastrophic market behavior. The behavioral implications of normal competitive market structure are not surprising and may be summarized briefly.

Summary

Within the structure of a normal competitive market, the way in which exchange rates are determined is a stable process. The source of price instability is the exchange environment. The response in exchange rates to a change in the market environment may differ owing to the quantitative structure of different markets. However, the qualitative nature of change within a market reflects the qualitative change in external variables. Smooth, sudden, or cyclical patterns of change within a market are attributable to corresponding changes in the predetermined factors which influence supply and demand. Approximately the same external conditions result in approximately the same market conditions.

The qualitative structure of a normal competitive market is stable by design. Under the laws of supply and demand we have defined a stable mechanism of market adjustment. If we wish to consider an internal form of price instability, either through a violation of these laws or an unstable dynamic, we are presented with a problem. Without the existence of a stable attracter state, a market lacks sufficient organization to support exchange. In traditional models of unstable markets, prices either fall to zero or continually increase. In both cases, a collapse of trade is implied. Since this is seldom observed, these models are of little use. In some manner internal instability must be bounded or embedded in a stable system. This is precisely the structure of a cusp catastrophe.

The implications of a cusp structure are strikingly different. The actions of buyers and sellers within a market can generate sudden changes, bull or bear markets, price cycles and very different market states under nearly the same external conditions. In the first part of the next chapter we will examine how expectations acting through reservation demand can alter normal market structure. A market with a dynamic cusp structure is then postulated as an alternative.

Endnotes

1 These assumptions are outlined by P.T. Saunders, An Introduction to Catastrophe Theory (Cambridge: Cambridge University Press, 1980), p. 2. An alternative development of the use of equilibrium systems in economics is presented by Paul Anthony Samuelson, Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947; reprint eg., New York: Atheneum, 1979, Chapter 2.

2 C.A. Isnard and E.C. Zeeman. "Some Models from Catastrophe Theory in the Social Sciences." The Use of Models in the Social Sciences, edited by L. Collins (London: Travistock Publications, 1976). Published in original form in E.O. Zeeman, Catastrophe Theory, Selected Papers 1972-1977 (Reading, Mass.: Addison-Wesley Publishing Company, Advanced book Program, 1977) pp. 319-20.

3 We are assuming that the points comprising a relationship are sufficiently close to think of the relationship as continuous.

4 This will become clearer in the following section on coordinate transformations.

5 Isnard and Zeeman, p. 321.

6 This meets and exceed the requirements for an isomorphism for ordered sets. The definition of an isomorphism for an ordered set is less restrictive; the type of ordering (strong, weak, partial, etc.) must be preserved, but the specific ordering relationship (greater than as opposed to less than) may change. The definition used here is given by Joong Fang, Theory and Problems of Abstract Algebra (New York: Schaum Publishing Co., 1963) p. 49.

7 Ibid, p. 49.

8 Saunders, p. 26.

9 Tim Poston and Ian Stewart, Catastrophe Theory and Its Applications (London: The Pitman Press, 1978) pp. 72-4.

10 An excellent, visually oriented, treatment of the differential geometry of three dimensional curves and surfaces is made by D. Hilbert and S. CohnVossen, Geometry and the Imagination, trans. P. Nemenyi (New York: Chelsea Publishing Company 1952), Chapter IV, pp. 172-221.

11 Saunders, p. 30.

12 John B. Fraleigh, A First Course in Abstract Algebra, 2nd ed. (Reading, Mass: Addison-Wesley Publishing Company 1976) p. 88 and 157.

13 John M.H. Olmsted, Solid Analytic Geometry (New York: 1947) Section 136, p. 162.

14 Ibid.

15 Straight lines through the origin are maintained mapping squares into parallelograms: Poston and Stewart, pp. 14-15.

16 Isnard and Zeeman, p. 322.

17 Saunders, p. 30.

18 Robert Gilmore, Catastrophe Theory for Scientists and Engineers (New York: John Wiley and Sons, 1981) pp. 16-17.

19 Probably the most common application of qualitative scale changes is in the use of composite commodities and price indices to reduce the number of variables considered. The problem was first treated by Leotief (1936) and later by Hicks (1939). Composite commodities and price indices, as well as several other theorems based on qualitatively equivalent scales, are discussed by Samuelson, Chapter VI (on transformations) pp. 125-171 and Chapter VII (on measures of utility) pp. 172-183.

20 This is an extension of the ideas of linear independent and independent functions to the more general nonlinear case. In the neighborhood of a given point, the number of independent functions is equal to the rank of the Jacobian:

$$\left(\frac{\partial a_i}{\partial c_i}\right)$$

W.H. Harlow, Mathematics for Operations Research (New York: John Wiley and Sons, 1978) pp. 22-36.

21 This development is adapted from Gilmore, p. 20.

22 These figures are based on a discussion by Isnard and Zeeman, pp. 324-26.

23 This discussion and result is based on a treatment of perturbations by Gilmore, pp. 33-5.

24 The following is adapted from a discussion of codimension by Saunders, pp. 26-8.

25 Poston and Stewart, p. 67.

26 This meets the definition of transversality for manifolds. Over an open interval (without endpoints), a smooth supply or demand curve is a one-dimenstional manifold embedded in two dimensions. See Poston and Stewart, pp. 65-69.

27 The details of the Thom Isotopy theorem are well beyond the scope of this paper. The interpretation used here is provided by Poston and Stewart, pp. 69-71.

28 Saunders, pp. 21-3.

29 This is what Gilmore refers to as the "Crowbar Principle" for potentials. The number, locations, and type of critical points completely determine the qualitative nature of a potential. The direction of force, represented by the negative gradient, changes only upon passing through a critical point of the potential. See Gilmore, pp. 52-4.

30 The signs of the eigenvalues correspond to the directions of Gaussian curvature at the critical point of the surface. Zero eigen values correspond to zero curvature. A parabolic cylinder has zero curvature in one direction. Using these forms to approximate a degenerate critical point the neighborhood of validity may not extend beyond the

point in question. See Poston and Stewart, pp. 52-54. For a geometric explanation see Hilbert and CohnVossen, pp. 183-192.

31 See Gilmore, p. 9.

32 Walrasian and Marshallian market adjustment and their respective conditions for stability are standard topics of market theory: see, Eugene Silbergerg, The Structure of Economics, A Mathematical Analysis (New York: McGraw-Hill Book Company, 1978) Chap. 16, pp. 510-16: or James M. henderson and Richard E. Quandt, Micro-economic Theory A Mathematical Approach, 3rd ed., (New York: McGraw-Hill Book Company, 1980) Chap. 6, pp. 159-66: or Samuelson, Chap. IX, pp. 260-65.

33 This development of a gradient based system is adapted from Gilmore, Chap. 1, pp. 3-5.

34 Gilmore, pp. 20-23.

35 This discussion of perturbations is based on gilmore, Chap. 4, pp. 36-40.

36 The terms fast and slow are mathematically imprecise. In the limiting case this assumption may be interpreted as instantaneous market adjustment to changes in the controls. However, we have encountered other seemingly vague terms, such as sufficiently small, which can be made mathematically precise. A precise treatment, which is beyond the scope of this paper, is made by E.C. Zeeman, "On the Unstable Behavior of Stock Exchanges." Journal of Mathematical Economics, Vol. 1 (1974) pp. 39-49. Published in original form in: E.C. Zeeman, "Catastrophe Theory, Selected Papers 1972-1977 (Reading, Mass.: Addison-Wesley Publishing Company, Advanced Book Program, 1977) pp. 363-4.

37 Zeeman, "On the Unstable Behavior of Stock Exchanges." p. 363.

38 More precisely, the dynamic is a component of a second order equilibrium system.

39 For a more detailed solution see Henderson and Quandt, pp. 164-66.

CHAPTER III

CATASTROPHE MODELS OF A COMPETITIVE MARKET

Motivation

In this chapter we will consider a competitive market with an internal structure of a cusp catastrophe. The term catastrophe holds a common connotation of disaster. Here, a catastrophe refers to a sudden change in the state of a system in response to a smooth change in its environment. The occurrence of sudden transitions between rising and falling price trends is only one property associated with a cusp market structure. Other behavior characteristics include patterns of over and under valuation, price cycles and the occurrence of radically different market states under similar external conditions.

In the traditional competitive market model, considered in the previous chapter, an internal dynamic was postulated to be a stable process. Under the laws of supply and demand the market moves smoothly toward a position of static equilibrium. When the willingness of some individuals to buy or sell a commodity depends upon expectations for future prices, the stability of the market adjustment process may be disrupted. In the first part of this chapter a very simple profit maximizing reservation demand model is used to demonstrate that a smooth adjustment to static equilibrium may become impossible.

Expectations, Reservation Demand and Market Prices

Market prices have a twofold impact on reservation demand. First, current prices are a variable cost of acquiring (or an opportunity cost of holding inventories) and the marginal revenue of liquidation. Second, current prices are a substantial part of the information available for formulating future expectations upon which the capital value of inventories is based. An increase in price, reflected as an increase in cost, may lead to a reduction in reservation demand. However, if an increase in price gives rise to higher price expectations, sufficient to offset increased costs, reservation demand may increase. If for a decline in price, a change in expectations offsets lower costs, reservation demand may decline as prices fall. Expectations of this type will be called speculative. In essence, speculative expectations project a current price trend into the future which is sufficient to offset changes in carrying costs.

We may develop a simple model of a reservation demand function from a single period investment opportunity in which expected profit, defined by the expression:

$$E(\pi) = E(P)Q - P(1+i)Q - C(Q) \qquad (3.10)$$

where:

E( ) denotes expectations

π = profit

i = interest rate

C(Q) = other costs as a function of quantity

is maximized.[1] First order conditions for a maximum require that marginal revenue equals marginal cost:

$$\frac{dE(\pi)}{dQ} = E(P) - P(1+i) - C_1(Q) = 0$$

$$E(P) = P(1+i) + C_1(Q) \qquad (3.11)$$

$$MR = MC$$

Second order conditions for a maximum require that marginal costs are an increasing function of quantity:

$$\frac{d^2E(\pi)}{dQ^2} = -C_2(Q) < 0 \qquad (3.12)$$

Given second order conditions are met, we may appeal to the inverse function theorem to solve equation 3.11 for a reservation demand function:

$$Q^{*} = C_1^{-1}\ [E(P) - P(1+i)] \tag{3.13a}$$

Assuming we can write an expected price:

$$E(P) = P + E(\Delta P)$$

we may rewrite the reservation demand function:

$$Q^{*} = C_1^{-1}\ [E(\Delta P) - Pi] \tag{3.13b}$$

For completeness, we may identify phases of inventory acquisition and liquidation. For a given initial inventory level I_o; acquisition occurs when $Q^{*} > I_o$ and liquidation when $Q^{*} < I_o$. However, an increase in the rate of inventory acquisition and a decline in the rate of liquidation are essentially equivalent conditions of increased reservation demand. Declining rates of inventory acquisition and increasing rates of liquidation are equivalent conditions of decreasing reservation demand.[2] With respect to market structure and stability, the issue of concern is the response in reservation demand to changes in price.

The slope of the reservation demand curve (equation 3.13b) is given by:

$$\frac{dQ^{*}}{dP} = \left\{\frac{dE(\Delta P)}{dP} - i\right\}\left\{C_2^{-1}\ [E(\Delta P) - Pi]\right\} \tag{3.14}$$

Since marginal costs are assumed to be an incresing function of quantity, the slope of the demand curve is negative if:

$$\frac{dE(\Delta P)}{dP} < i \tag{3.15a}$$

and positive if:

$$\frac{dE(\Delta P)}{dP} > i \tag{3.15b}$$

The slope of the reservation demand curve is dependent upon the psychological impact of changing prices on expectations. The rationality of this impact is not in question. We are presented with alternative hypotheses concerning the formulation of expectations and we must evaluate their consequences. This can be done more clearly by viewing the role of expectations in another way.

We may treat an expected change in price as a demand shift parameter. For a given level of expectations the reservation demand curve is negatively sloped, reflecting only the effect of changes in capital costs. If expectations for future price changes are based on external information (such as normal seasonal price trends, anticipated changes in production or consumption) and:

$$\frac{dE(\Delta P)}{dP} = 0$$

then this short run (fixed expectations) relationship is the effective demand curve. If expectations change as market prices change then a movement along the demand curve is accompanied by a shift in reservation demand. The effective demand curve is traced out by the family of shifting demand curves at given prices. Where:

$$\frac{dE(\Delta P)}{dP} < 0$$

effective demand is negatively sloped and more inelastic than short run reservation demand (Figure 3.1 a). Adaptive expectations are an example of this type of formulation. Where:

$$0 < \frac{dE(\Delta P)}{dP} < i$$

effective demand is negatively sloped and more elastic than short-run demand (Figure 3.1 b). Expectations of this type extrapolate current price trends into the

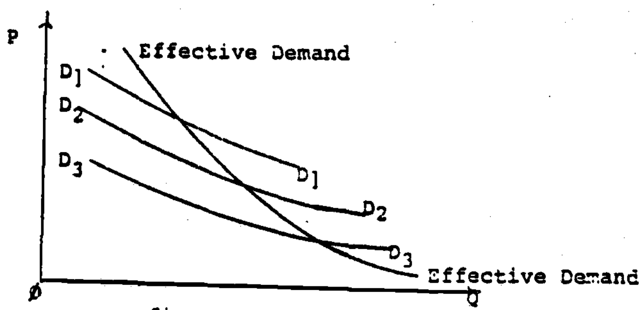

figure a

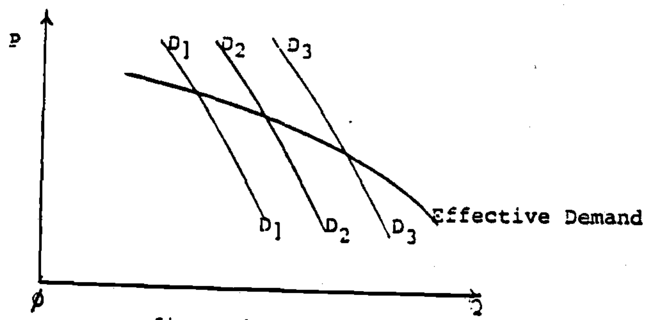

figure b

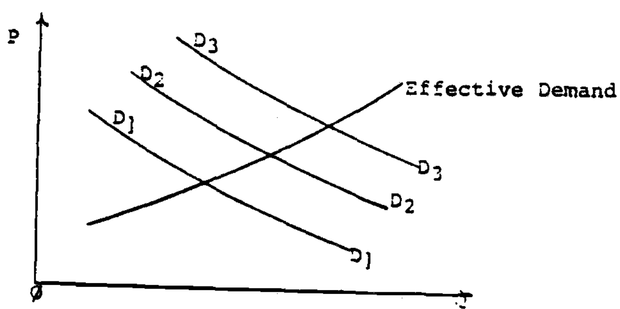

figure c

Figure 3.1. Short run and effective reservation demand under price dependent expectations: Figure a) lagging expectations, Figure b) projective expectations, Figure c) speculative expectations.

future, but the expected return does not cover the immediate change in cost. Under speculative expectations (equation 3.15b), a shift in demand more than compensates for a movement along a demand curve. Effective demand is positively sloped (Figure 3.1c).[3]

Market demand is defined as the summation of quantities demanded by individuals at given prices. Thus, we may classify aggregate expectations with respect to the effective slope of a market demand curve. Aggregate expectations are speculative if the slope of the effective inventory demand curve is positive. However, this ignores the distribution of expectations among individuals. Effective demand may be downward sloping, in accord with the law of demand, while a number of individuals hold speculative expectations. This speculative component of market demand may have a significant effect on market stability. Before reducing expectations to an aggregate form, we need to know what constitutes a significant level of speculative demand.

Speculative Reservation Demand and Market Stability

In a Walrasian model of a competitive market, prices adjust toward a stable equilibrium if as prices rise, excess demand falls (Figure 3.2). We can attempt to apply this criterion for dynamic stability to a market with a given level of speculative demand in addition to a normal set of supply and demand relationships. Consider an initial point of disequilibrium where excess demand is positive and prices are rising. With rising prices there is a corresponding increase in the quantity supplied and a decrease in the quantity demanded. Increasing prices also stimulate speculative reservation demand. If the increase in speculative demand is less than the cumulative response in normal supply and demand then excess demand falls as prices rise and the market appears to converge towards equilibrium. If the increase in speculative demand is sufficient to offset the cumulative response in normal supply and demand then excess demand rises with increasing prices and equilibrium is, at least temporarily, unstable. We may consider the unstable case in greater detail.

As long as increasing prices continue to attract a sufficient level of speculative demand total excess demand and prices may continue to increase. Normal excess demand is falling and may become negative (Figure 3.2b). This cannot go on indefinitely. Marginal costs of holding stocks are presumably increasing. After a prolonged price trend, individuals may begin to anticipate a turn in prices. Normal supply and demand response ultimately

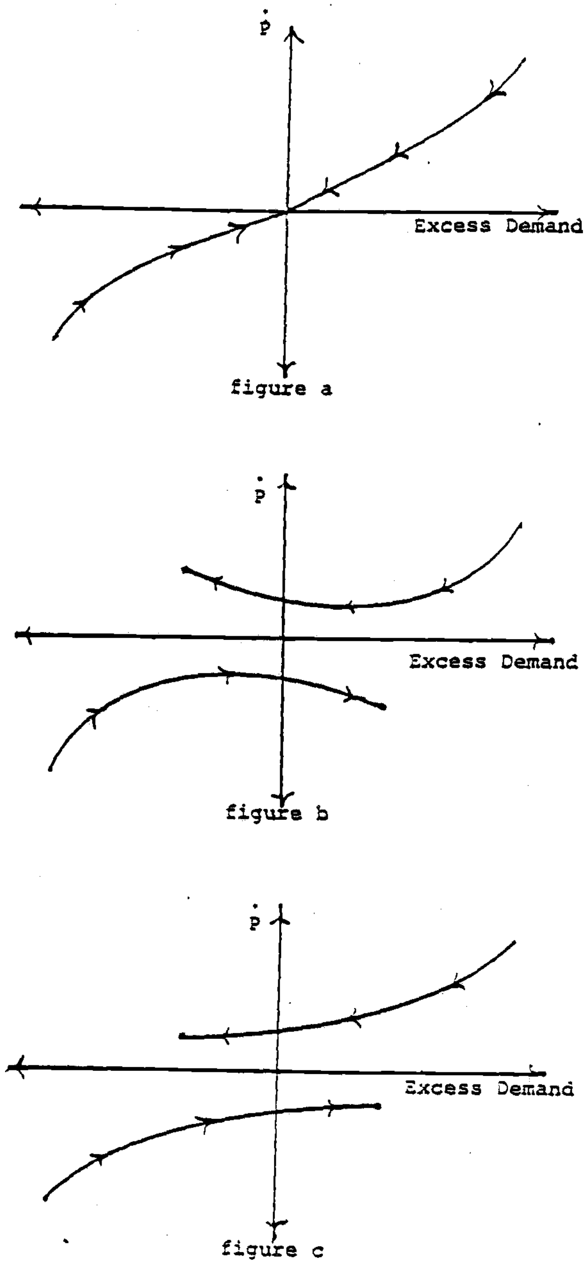

Figure 3.2. Price trajectories as a function of excess demand: Figure a) normal Walrasian dynamic, Figure b) dynamic dominated by speculative demand, Figure c) dynamic sustained by speculative demand.

dominate the market; excess demand begins to fall and the rate of increase in price begins to slow. However, as prices approach a stable equlibrium, the expectations under which speculative inventories are held are no longer consistent with current market conditions. Speculative demand may quickly terminate leaving an imbalance between normal demand and supply. Total excess demand may suddenly become negative, and prices may abruptly begin to fall. As prices fall, speculative expectations may again form. Declining inventory demand may offset the normal response in supply and demand to falling prices. Normal excess demand may become positive as prices continue to fall. An imbalance is created which cannot be maintained. A speculative decline in inventory demand must end. Total excess demand may suddenly become positive and prices may abruptly begin to rise.

When speculative demand offsets the normal response in supply and demand market equilibrium becomes unstable. If equilibrium were to remain unstable the process of exchange represented within a market would collapse as price trends continue indefinitely. Such a model is structurally unstable. Since we do not observe unbounded price trends and the collapse of exchange, we are justified in requiring a market model to be structurally stable. Thus, the occurrence of unstable equilibria must be isolated, surrounded by a structurally stable system. However, as with the previous example, it is the organization not the obtainment of equilibrium, which determines behavior. A stable state of equilibrium need not be obtainable. The return of a stable dynamic acts to terminate a price trend. The imbalance between normal demand and supply may prevent the obtainment of a stable state in the short run. Continuing speculation may result in an ongoing cycle of price adjustment. This type of behavior cannot be represented within the traditional structure of a competitive market.

The conditions under which speculative expectations drive a market may appear unlikely to occur. However, a similar problem arises even when the response in speculative demand is not sufficient to offset the normal response in supply and demand. Consider a market in an initial position of disequilibrium where normal excess demand is positive and prices are rising. If the price trend attracts speculative demand, the rate of decline in total excess demand is slowed. The price trend is sustained over a longer period of time. Normal excess demand may become negative while the trend is artificially maintained by expectations (Figure 3.2c). Again, the phenomenon cannot last. The question is how will the trend terminate.

Several alternative transitions from a point A on an artificially rising trend are illustrated in Figure 3.3. Path a, showing a smooth transition to a point of equilibrium where normal excess demand is zero, is inaccessible. Normal excess demand cannot increase as prices increase. Path b, showing a smooth transition to a falling market and an eventual return to equilibrium, is also unobtainable. As the positive trend terminates and prices begin to fall, the expectations under which speculative inventories are held are no longer consistent with prevailing market conditions. Expectations for higher prices may change quickly. The buildup of inventories may now be perceived as a liability for capital losses. Path c, shows a sudden transition between rising and falling market states. This picture may best reflect impact of transitory speculative demand on market stability. The upward pressure placed on prices by speculative demand simply evaporates as the market no longer sustains expectations. Excess demand suddenly becomes negative and prices abruptly begin to fall. As prices decline, speculative expectations for lower prices may sustain a falling market. Normal excess demand may

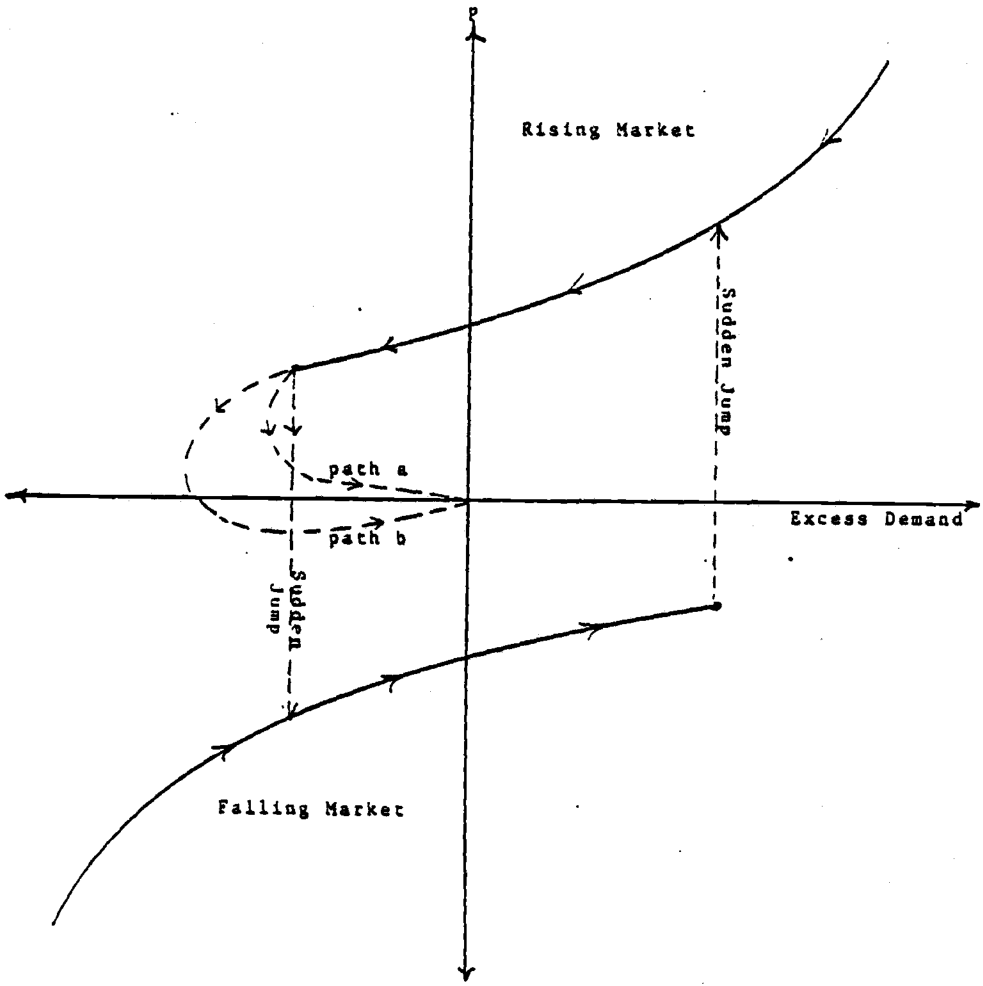

Figure 3.3. A market split into rising and falling states. Transition between states is a sudden jump as smooth paths towards a stable equilibrium are inaccessible, paths a and b.

become increasingly positive until the trend terminates with a sudden jump to a rising market.

When the presence of a price trend attracts inventory investment or disinvestment, an imbalance is created between existing prices and the normal determinants of exchange rates. This imbalance grows as a price trend extends beyond what may be called the physical conditions of the market. It is tempting to draw an analogy to physical structures and say that a price trend sustained by expectations collapses under its own weight. However, this is inaccurate. There is not an apparent mechanism through which the imbalance acts upon a market. Given the full range of possible interactions between market prices and the formulation of expectations, it is unlikely that we may derive an explicit set of deterministic relationships. We can consider a system which exhibits the type of behavior we have discussed; a cusp catastrophe. A market with a dynamic structure of a cusp provides a more complete picture of the interactions between price determination and the formation of expectations.

A Quartic Approximation to a Cusp Catastrophe

While we do not intend to derive a cusp structure from a set of equations governing market behavior, a cusp catastrophe is suggested by the figures drawn in the previous section.[4] In general, we may derive a Walrasian market dynamic from a family of smooth potentials. Each point on the price trajectory is a stable dynamic equilibria, corresponding to a critical point of a potential in the family. For a given level of excess demand, a point along the price trajectory satisfies:

$$\frac{dV(\dot{P};C_o)}{d\dot{P}} = 0$$

By assumption, the dynamic equilibria are stable:

$$\frac{d^2V(P;C)}{d\dot{P}^2} > 0$$

Hence, the dynamic equilibria are nondegenerate, and we may write a qualitatively equivalent quadratic approximation of a potential about any given equilibrium point:[5]

$$V(\dot{P};C_o) \approx V(\dot{P}_o;C_o) + V_2(\dot{P}_o;C_o)\dot{P}^2 \qquad (3.20)$$

Figures 3.2 and 3.3 illustrate price trajectories which are disconnected. Upon casual inspection, it may not appear that this form of a market dynamic may be derived from a smooth family of potentials. However, we may redraw these figures as illustrated in Figure 3.4. The solid portion of the curve corresponds to the disconnected set of stable equilibria. These sections of the curve attract the fast flow of second-order market adjustment. The slow flow of first-order market adjustment follows these stable trajectories. The dashed line is a set of unstable or repeller equilibria. These points are inaccessible. The presence of a repeller sheet between the two stable trajectories prevents a sudden transition between rising and falling markets from occurring except at the two critical points of the curve, A and B.

The critical points of the price trajectories are degenerate equilibria, satisfying the singularity condition:

$$\frac{d^2V(\dot{P};C)}{d\dot{P}^2} = 0$$

The family of dynamic market potentials contains members which cannot be successfully approximated with a quadratic function. Degenerate or singular points organize a family of potentials in a manner similar to which nondegenerate critical points organize the qualitative character of a single potential. A sudden transition between market states occurs only upon passing through a singular point. On either side of a degenerate equilibria, the market is smoothly rising or falling. Since singular points determine qualitative behavior, we are interested in the nature of the potential at these points.

A quadratic approximation to a potential about a degenerate critical point reduces to:

$$V(\dot{P};C) \approx V(\dot{P}_o;C) + (0)\dot{P} \quad = \quad \text{a constant}$$

The next simplest approximating function is a cubic. However, this proves inadequate. A cubic approximation to a potential at a degenerate critical point is of the form:[6]

$$V(\dot{P};C_o) \approx V(\dot{P}_o;C) + V_3(\dot{P}_o;C)\dot{P}^3 \qquad (3.21)$$

To construct a family of potentials based on this approximation we may consider the general form of a cubic equation with variable control parameters:

$$V(\dot{P};C) = a_o + a_1\dot{P} + a_2\dot{P}^2 + a_3\dot{P}^3 \qquad (3.22a)$$

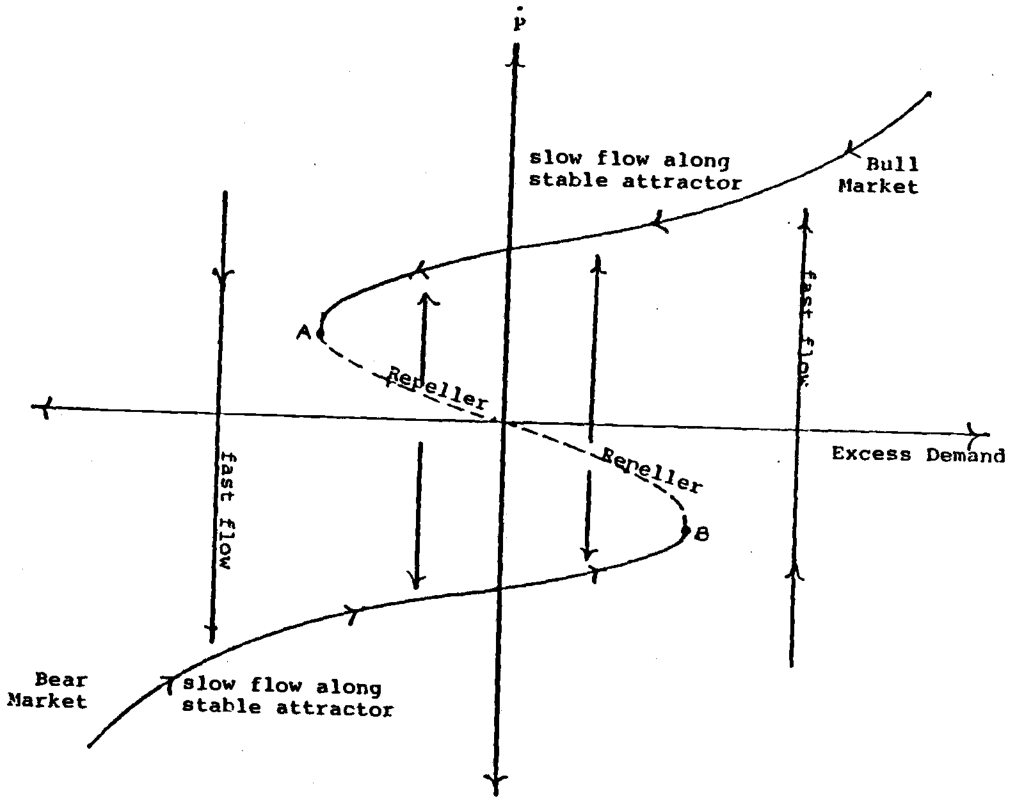

Figure 3.4. A smooth representation of a split market dynamic; stable price trajectories are separated by a repeller surface of unstable dynamic equilibria.

The second degree term may be eliminated with a linear transformation:

$$\dot{P} \rightarrow \dot{P}' = \dot{P} + a_2/3a_3$$

The reduced form of the cubic equation may be written:[7]

$$V(\dot{P}';C) = c_o + c_1\dot{P}' + \dot{P}'^3 \qquad (3.22b)$$

where:

$$c_o = (2a_2^3 - 9a_1a_2a_3 + 27a_3^2a_o)/27a_3^3$$

$$c_1 = (3a_3a_1 - a_2^2)/3a_3^2$$

In the neighborhood of the degenerate equilibrium point, the price trajectory is given by the equilibrium condition:

$$\frac{dV(\dot{P}';C)}{d\dot{P}'} = c_1 + 3\dot{P}'^2 = 0 \qquad (3.23)$$

At the singular point (c_1=0), the equilibrium equation has one repeated root, P'=0. For $c_1 < 0$, the equilibrium equation has two real roots; one stable attractor and one unstable repeller. For $c_1 > 0$, the equilibrium equation has no real roots.[8] The curve is shown as a function of the control value c_1 in Figure 3.5.

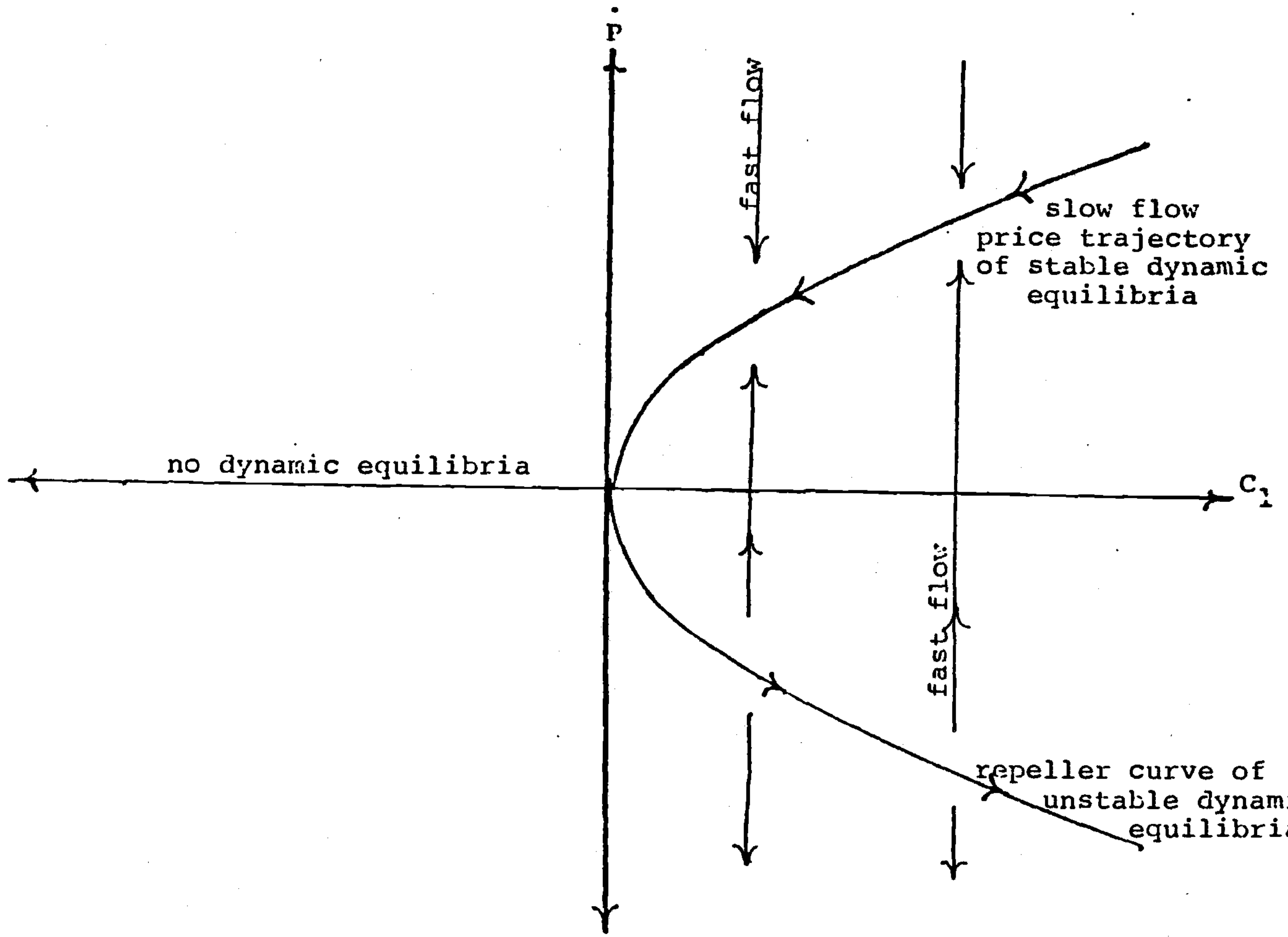

Figure 3.5. Equilibrium rates of change in price as a function of the canonical control in a cubic approximation to a dynamic market potential.

The singularity illustrated in Figure 3.5 is a fold point. A family of potentials with a cubic singularity exhibits a fold catastrophe.[9] The point in the control space corresponding to the fold is called the bifurcation set. The bifurcation set divides the control space into two regions: in this case, the positive and negative c_1 axis. Along the positive control axis, where the equilibrium equation has no real roots, the potentials of the family look like Figure 3.6a. There are no critical points to organize behavior. Within the bifurcation set, the potential has a point of inflection (Figure 3.6b). Along the negative control axis, the potentials of the family look like Figure 3.6c. There are two critical points, a stable minimum and an unstable maximum.

For a market to continually support exchange, it must be structurally stable. This property is not retained with a fold catastrophe. Upon passing through the bifurcation set, a market which is internally and structurally stable becomes both internally and structurally unstable. A fold catastrophe or cubic approximation to a degenerate equilibrium is an inadequate description of the behavior in question.

The next simplest approximation to a degenerate point of equilibrium is a quartic equation:

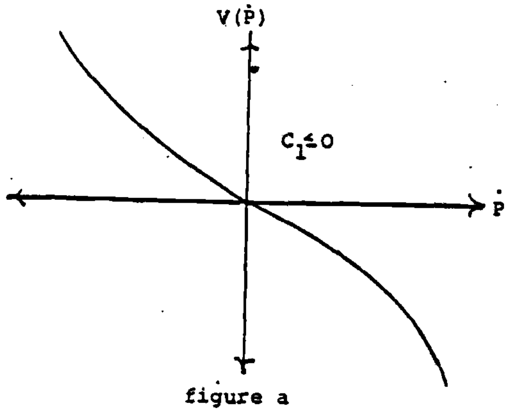

figure a

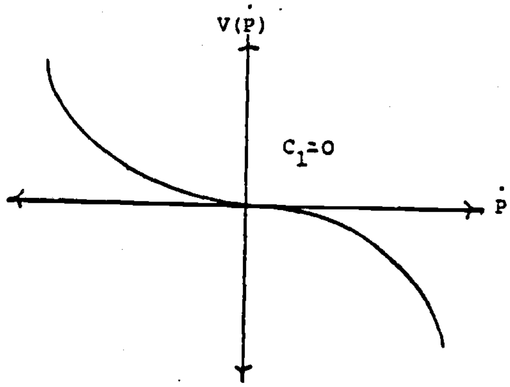

figure b

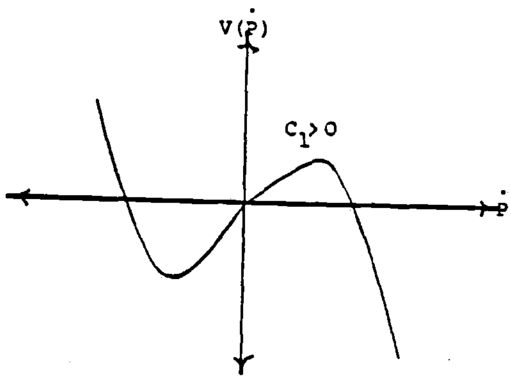

figure c

Figure 3.6. Forms of a cubic potential for different values of c: Figure a) C<Ø, Figure b) C=Ø, Figure c) C>Ø.

$$V(\dot{P}) \approx a_o + a_1\dot{P} + a_2\dot{P}^2 + a_3\dot{P}^3 + a_4\dot{P}^4 \qquad (3.24a)$$

where:

$$a_i = (1/i!)V_i(P_o;C_o)$$

We may construct a family of potentials from this approximation by taking the coefficients, a_i, to be continuous functions of the controls C. By first absorbing the quartic coefficient, a_4, into the scale of $V(\dot{P})$ and then making the linear displacement:

$$\dot{P} \rightarrow \dot{P}' = \dot{P} + a_3/4a_4$$

the quartic equation may be placed in reduced form (dropping primes):

$$V(\dot{P}) = c_o + c_1\dot{P} + c_2\dot{P}^2 + \dot{P}^4 \qquad (3.24b)$$

where:

$$c_o = (-3a_3^4 + 16a_2a_3^2 - 64a_1a_3a_4^2 + 256a_4^3)/(256a_4^4)$$

$$c_1 = (2a_3^3 - 8a_2a_3a_4 + 16a_1a_4^2)/(16a_4^3)$$

$$c_2 = (-3a_3^2 + 8a_2a_4)/(8a_4^2)$$

The equilibrium equation is given by:

$$\frac{dV(\dot{P})}{d\dot{P}} = c_1 + 2c_2\dot{P} + 4\dot{P}^3 = 0 \qquad (3.25)$$

The roots of the equilibrium equation, for given values of the controls, correspond to dynamic equilibria. We may determine the qualitative character of these equilibria by computing the discriminant D of the cubic equation 3.25:[10]

$$D = 8c_2^3 + 27c_1^2 \qquad (3.26)$$

From a theorem on the discriminant of a cubic equation, we may draw the following conclusions. For positive values of the control c_2 the discriminant is strictly positive and the equilibrium equation has exactly one real root. The price trajectory is single valued as in Figure 3.2a. The corresponding potentials look like the one illustrated in Figure 3.7a. For negative values of the control c_2 the discriminant may be greater than, less than, or equal to zero. If D is positive, the preceding results apply. If D is equal to zero, equilibrium is degenerate. The equilibrium equation has a real repeated root and the corresponding potentials look like the one illustrated in Figure 3.7b. If D is negative, the equilibrium equation has three real roots; two stable attractors and one

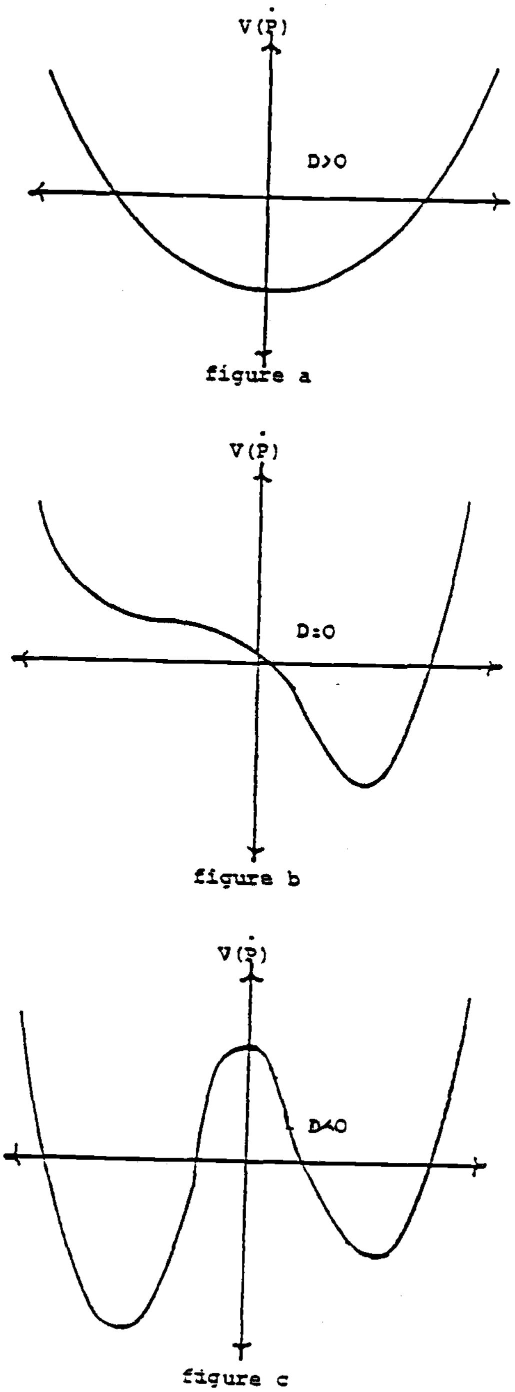

Figure 3.7. Forms of a quartic potential for different values of the discriminant: Figure a) D>0, Figure b) D=Ø, Figure c) D<Ø.

unstable repeller. The corresponding potential, illustrated in Figure 3.7c, has two minima separated by a maximum. The price trajectory, graphed as a function of c_1, has two stable attractor sections separated by a repeller, as illustrated in Figure 3.4.

If we interpret the control parameter c_1 as a qualitative measure of normal excess demand and the control parameter c_2 as a qualitative measure of speculative demand, a model derived from a quartic potential holds a promising description of market behavior. When the speculative content of a market is low, the value of c_2 is small. The dynamic adjustment of exchange rates moves toward a point of static equilibrium. When the speculative content of a market is high, the value of c_2 is large. The market is split into a rising or bull phase, and a falling or bear phase. The transition between a bull and bear market is a sudden crash or jump. We can construct a more detailed picture of this sudden transition from the changing shape of the dynamic potential. The general form of the dynamic potential at points along the price trajectories is illustrated in Figure 3.8. Along single valued sections of the trajectory the potential has a single minimum. Crossing into the double valued section a second minimum

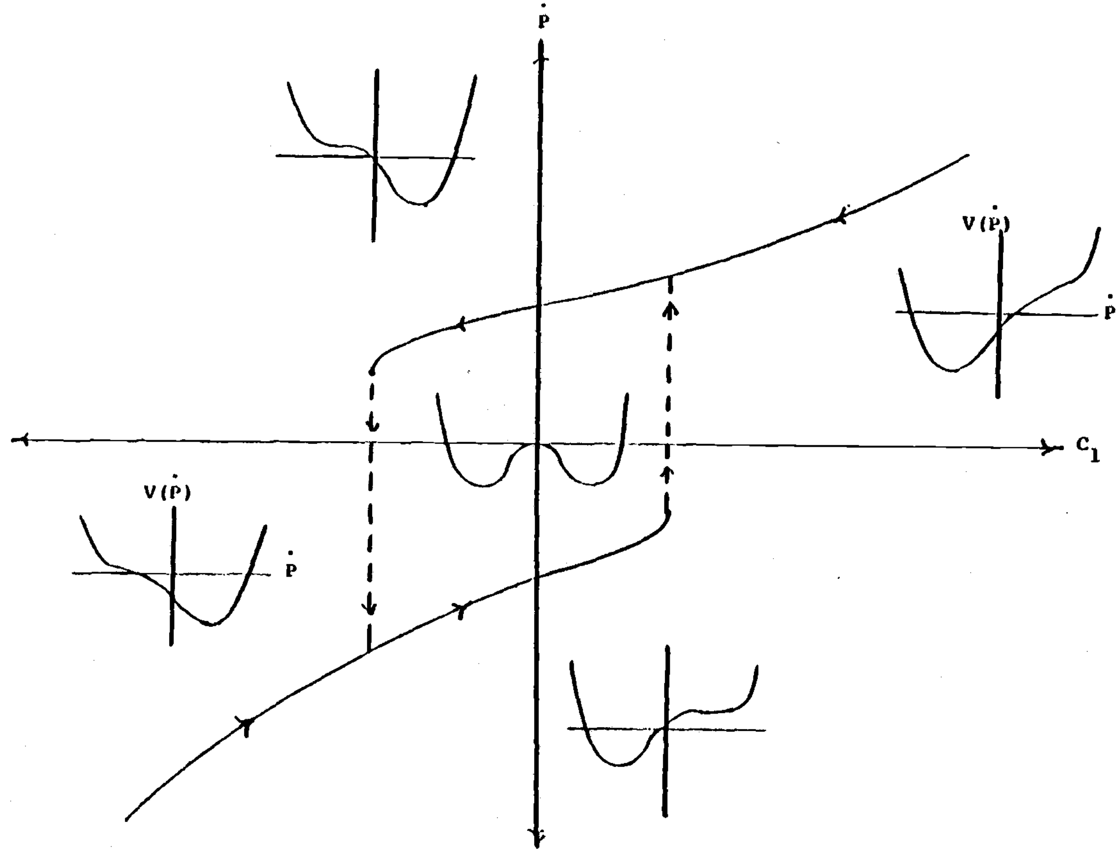

Figure 3.8. Forms of the potential along various regions of the price trajectory.

develops but the market remains organized about the initial local minimum. As the boundary of the double-valued section is reached, the first minimum vanishes. The market is carried by a fast flow to the opposing trajectory of the new minimum. This sequence of events is illustrated with a mechanical analogy in Figure 3.9. The position of a small ball represents a market state. The location of the ball changes suddenly as the basin in which it rests disappears.

A family of potentials with a quartic singularity exhibits the structure of a cusp catastrophe. The reduced form of a cubic equation is, in fact, a canonical form for the equilibrium surface of a cusp catastrophe. Before considering the properties and geometry of a cusp catastrophe in detail, we should consider two theorems which are important in application of catastrophe theory. The first is the splitting lemma. Up to this point, we have ignored the adjustment in quantities of exchange. We would like to know if a discontinuity in a pricing structure is likely to affect, or be effected by, the quantity side of the market. The splitting lemma may be used to address this question. The second is Thom's classification theorem which describes the types of elementary catastrophes that can occur given the number of

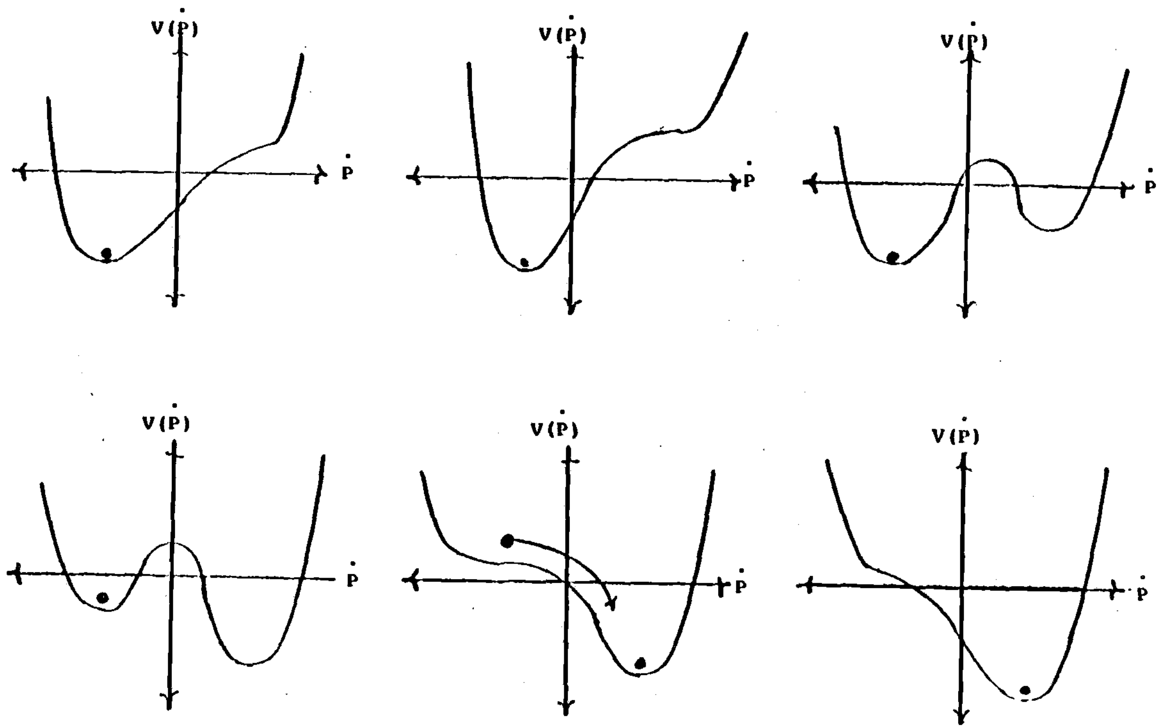

Figure 3.9. A simple mechanical analogy to a sudden jump resulting from smooth changes of a potential; a small ball moving along a curved track. Adapted from Saunders p. 10.

state and control variables. Unfortunately, as demonstrated in the previous chapter, the number of independent controls in an economic system is not generally known. Hence, the classification theorem cannot be applied directly. Another approach is taken to the problem of classification which helps to clarify the assumptions underlying the use of catastrophe models in economics. A mathematical demonstration of the splitting lemma and Thom's classification theorem is beyond the scope of this paper. We will limit our treatment of these theorems to a discussion of results.

The Splitting Lemma

The splitting lemma allows us to split state variable into two classes: one, essential state variables which are involved in a discontinuity; and two, nonessential variables which are not involved in structural instability and may be ignored.[11] Given a smooth potential, written:

$$V = V(x_n) \tag{3.30}$$

equilibrium conditions are given by:

$$\frac{\partial V}{\partial x_i} = 0 \qquad i = 1,2, \ldots n \tag{3.31}$$

The stability matrix:

$$V_{ij} = \left(\frac{\partial^2 V}{\partial x_i \partial x_j} \right) \tag{3.32}$$

at a degenerate equilibrium is singular. The rank of the stability matrix, equal to the number of linearly independent rows of the matrix, is less than the dimensions of the matrix. The difference between the dimensions of matrix (n) and its rank (r) is the co-rank of the matrix (n-r = k). The number of degenerate variables involved in the discontinuity can be reduced to the co-rank of the stability matrix with a diffeomorphic transformation of coordinates. After a smooth reversible transformation of coordinates, the potential may be written with a Morse component and a catastrophe component:[12]

$$V \simeq e_1 x'^2_1 + e_2 x'^2_2 + \ldots e_{n-k} x'^2_{n-k} + \mathrm{Cat}(x'_{n-k+1}, \ldots x'_n) \tag{3.33}$$

where:

$$e_i = \pm 1$$

The stability matrix of second order partial derivatives provides information on the curvature of the potential, at a point of equilibrium. We may align our coordinate system with the principal directions of curvature of the potential at equilibrium, with a rigid linear transformation of coordinates. This transformation eliminates second order cross-product terms, diagonalizing the stability matrix:

$$V_{i,j} = \begin{pmatrix} \lambda_1 & 0 & \cdots\cdots\cdots & & 0 \\ 0 & \lambda_2 & 0 & \cdots\cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \lambda_{k-1} & \cdots\cdots & 0 \\ 0 & \cdots\cdots\cdots & & 0 \; \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots\cdots\cdots\cdots & & & 0 \end{pmatrix}$$

The rank of the matrix is given by the number of nonzero diagonal elements. These are directions in which the curvature of the potential is well defined, either positive or negative. The co-rank of the matrix is given by the number of vanishing or zero elements of the diagonal. These are directions in which the curvature of the potential is degenerate (linear). The splitting lemma states, in essence; the nature of the potential in nondegenerate directions is known, we need only look in degenerate directions to determine the properties of the singularity.

We may apply the splitting lemma to identify the conditions under which structural market instability is confined to prices, and quantities of exchange may be ignored. Given a general form of a smooth, dynamic market potential:

$$V = V(\dot{P},\dot{Q}) \tag{3.34}$$

equilibrium conditions are given by:

$$\frac{\partial V}{\partial \dot{P}} = \frac{\partial V}{\partial \dot{Q}} = 0 \tag{3.35}$$

Assuming the price and quantity dynamics are independent (no cross-product terms in the potential), as was the case for our traditional market model, we may write the stability matrix:

$$V_{ij} = \begin{pmatrix} V_{11}(\dot{P},\dot{Q}) & 0 \\ 0 & V_{22}(\dot{P},\dot{Q}) \end{pmatrix} \tag{3.36}$$

The matrix is assumed to be singular, with a degenerate price component. The stability matrix is doubly degenerate, only if the quantity component also vanishes. If the quantity component is nonvanishing, structural stability is confined to prices and the potential may be written, after a diffeomorphism:

$$V'(\dot{P}',\dot{Q}') = \dot{Q}'^{2} + \mathrm{Cat}(\dot{P}') \tag{3.37}$$

We should note that it is the assumption of independence between price and quantity adjustment that allows us to transform the potential into recognizable price and quantity components. Under this assumption, the structural stability of the quantity side of the market neither effects nor is affected by market prices. The splitting lemma simply states the conditions under which structurally unstable behavior may occur in both price and quantity adjustment. There is a reason why structural instability may be confined to prices. Information on prices is readily available to market participants while information on market quantities of exchange is not. Expectations may influence the offer and acceptance prices of buyers and sellers and therefore, quantities of exchange. However, the return flow of quantity information in the formation of price expectations is presumably weak. This does not preclude a response in expectations when quantity information is made available. However, quantity side information is unlikely to sustain a trend.

If we eliminate the assumption of independence, the splitting lemma may become a very powerful tool. We may still be able to reduce the problem from two state variables to one essential state variable. However, this variable may now be defined in terms of both prices and quantities. What is important is that the number and types of structurally unstable behavior that can occur is determined by the number of essential state variables, not the total number of state variables. This brings us to Thom's classification theorem.

Thom's Classification Theorem

Adequate preparation has and will not be made to make a formal statement of Thom's classification theorem for the seven elementary catastrophes. What the theorem states is a description of the most complex thing that can occur in a smooth system, with no more than two state (or behavioral) variables and no more than four control (or explanatory) variables. To try and gain some insight into the content of Thom's theorem, we may consider a few specific implications.[14]

Given a 1-dimensional state space and a 1-dimensional control space, with a smooth potential of the state variable parameterized by the control variable, the only singularities that can occur in the set of equilibrium values are fold catastrophes. Our example of a cubic potential, discussed earlier, is the most complex situation that can happen. If we add a control, making a

2-dimensional control space, then the only singularities that can occur are fold curves and cusp catastrophes. A new and qualitatively distinct type of structure is added to our list. Our example of a quartic potential again describes the most complex thing that can occur. If we add a third control a new singularity may appear, a swallowtail catastrophe. By adding one more state and one more control variable all seven elementary catastrophes may occur.

In a system where the number of state and control variables is known, Thom's theorem may be directly applicable. However, when the number of controls is unknown, the classification theorem has no real methodological value. With one essential state variable, it is relatively easy to approach the problem of classification directly with the tools we have previously developed.

The Elementary Catastrophes of One Variable

From Thom's classification theorem we can construct the following table for elementary catastrophes with one essential state variable.

Table 3.1. Elementary catastrophes with one essential state variable.

Singularity	Canonical Form	Name
x^3	$x^3 + c_1x$	Fold
x^4	$x^4 + c_2x^2 + c_1x$	Cusp
x^5	$x^5 + c_3x^3 + c_2x^2 + c_1x$	Swallowtail
x^6	$x^6 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x$	Butterfly

We have briefly combined the forms of the fold and cusp catastrophes. The swallowtail and butterfly catastrophes are analogously based on fifth and sixth degree Taylor's expansions with one dependent variable. We can develop a general canonical formula for one variable catastrophe by considering perturbations of a Taylor's expansion of degree k. While our interest in higher degree catastrophes is limited, a derivation of this formula provides some useful insights into catastrophe structures.

The qualitative character of a potential about a given point is determined by the first non-vanishing term of a Taylor's expansion constructed about that point. We may write a local approximation, qualitatively equivalent to the potential, by truncating the Taylor's series beyond the first non-vanishing term. A Taylor's series truncated beyond terms of degree k is called the k-jet of a function. A potential is said to be k-determined if, at every point, the qualitative properties of the potential are determined by its k-jet.[15] This idea extends to families of functions in a natural way. The qualitative character of a family of functions is determined by the first non-vanishing expansion term for the family as a whole. A family of functions is said to be k-determined if, for each family member, the qualitative properties of the potential are determined by its k-jet at each point. A potential or a family of potentials which is k-determined is obviously also (k+1) determined. However, determinancy refers to the lowest degree. We may draw several examples from families of functions we have considered previously.

A supply or demand curve is 1-determined if, at each point, its tangent is a well defined function of price and quantity. A first degree Taylor's approximation or 1-jet, establishes the qualitative properties of the curve. From the implicit function theorem, there exists a smooth reversible transformation of coordinates which reduces the supply or demand curve to a linear form. A family of supply or demand curves is 1-determined if, for all control values, the corresponding curves are 1-determined.

We may construct a local linear representation of the family.

A market potential with a non-degenerate critical point is no longer 1-determined. At a non-degenerate critical point the linear expansion term is zero, but the second degree term is non-vanishing.[16] About a nondegenerate critical point, a potential is 2-determined. From the Morse lemma, there exists a smooth reversible transformation which reduces the potential to Morse form. A family of potentials is 2-determined if all its equilibria are non-degenerate. We may construct a local representation of the family in Morse form.

A potential, or family of potentials, with one or more degenerate critical points is no longer 2-determined. The cubic approximation of the fold catastrophe is an example of a 3-determined family. The structure of a cusp catastrophe is introduced with a 4-determined family. the general condition for k-determinancy is given by:

$$\frac{d^k V(x;C)}{dx_k} = V_{k(x;C)} \neq 0 \qquad (3.40a)$$

where for some control values C^o:

$$V^j(x;C^o) = 0 \quad \text{for } j = 1,2, \ldots k-1 \qquad (3.40b)$$

It must be acknowledged that all functions and families of functions are not determined by some finite k.[17] Furthermore, we may be able to attach meaning to the determinancy conditions. However, as we consider higher order determinancy, new behavioral implications evolve. The stable behavior of the market structure discussed in Chapter II is based on an assumption of 2-determinancy. A brief examination of a market dynamic under expectations revealed conditions which violate 2-determinancy. Now we propose to examine behavior within higher order structures.

Consider a potential of a k-determined family, expanded as a Taylor's series about the origin:

$$\bar{V}(x) = a_1 x + a_2 x^2 + \ldots + a_k x^k + a_{k+1} x^{k+1} + \ldots \qquad (3.41a)$$

where:

$$a_i = (1/i!)V_i(0;C)$$

Since the family is k-determined we can eliminate the first k-1 terms by a choice of control coordinates.[18] A nonlinear axis preserving transformation can be used to eliminate terms of degree greater than k without altering the qualitative properties of the function.[19] Finally, the coefficient a_k may be absorbed into a length of scale, yielding the singularity:

$$\bar{V}'(x') = x'^k \tag{3.42b}$$

If k is even, the potential has a single minimum at the origin. If k is odd, the potential has a point of inflection at the origin. However, if we perturb this singularity slightly the number and type of critical points is subject to change. The potential (not necessarily the family) is structurally unstable.

To classify the singularity we need to reveal all the possibilities. This is called unfolding a singularity. The control parameters which are required to unfold a singularity are the canonical controls.

We may define a perturbation function:[20]

$$z(x;C) = V(x;C) - V(0;C_o) \tag{3.42}$$

The perturbed function may be written:

$$V(x;C) = V(0;C_o) + z(x;C) \tag{3.43}$$

The most general form of the perturbed singularity, written as a Taylor's expansion about the origin, is:

$$\begin{aligned}\bar{V}(0;C) = z_o + z_1 x + z_{k-1} x^{k-1} + (1+z_k) x^k \\ + z_{k+1} x^{k+1} + \ldots\end{aligned} \tag{3.44a}$$

where:

$$z_i = (1/i!)z_i(0;C)$$

The constant term is of no consequence and may be eliminated with a displacement in the value of the perturbed function. The perturbed function is still a member of the family of k-determined potentials. Therefore, we may again eliminate terms of degree greater than k with a nonlinear transformation. Finally, we can rescale the function to eliminate the coefficient of x^k to yield:

$$\bar{V}'(x';C) = z_1'x' + z_2'x'^2 + \ldots + x'^k \qquad (3.44b)$$

By collecting coefficients and translating the origin, we can eliminate one additional term, chosen as x'^{k-1}:

$$\bar{V}''(x'';C) = z_1''x'' + z_2''x^2 + \ldots + z_{k-2}x''^{k-2} + x''^k \qquad (3.44c)$$

The perturbation parameters, z_i'', are continuous functions of the controls C and may be treated as transformed controls C'. Dropping primes we may write the canonical form of a k-determined family:

$$V(x) = c_1x_1 + c_2x^2 + \ldots + c_{k-1}x^{k-1} + x^k \quad (3.45)$$

The canonical form of a k-determined family of functions is equivalent to the canonical forms of the elementary catastrophes of one variable listed in Table 3.1. The canonical equations are arbitrary to a choice of coefficient scales and the lower degree term which is eliminated. More important than the exact form of an equation is the number of canonical controls or the codimension of a catastrophe. The minimum number of dimensions, in which a given catastrophe structure can be represented, is equal to the dimensions of the state space (one) plus the codimension of the structure. The dimensions of the equilibrium set are equal to the codimension of the family of potentials.

The canonical forms of the elementary catastrophes can be obtained by another method. In this context, the canonical controls are referred to as universal unfolding parameters. The term unfolding parameter is suggested by a geometric interpretation. For example, if we disturb the singularity x^4, which has a single minimum at the origin, by allowing the controls to vary slightly about the origin, the singularity unfolds into a complete array of qualitatively distinct potentials as illustrated in Figure 3.10.

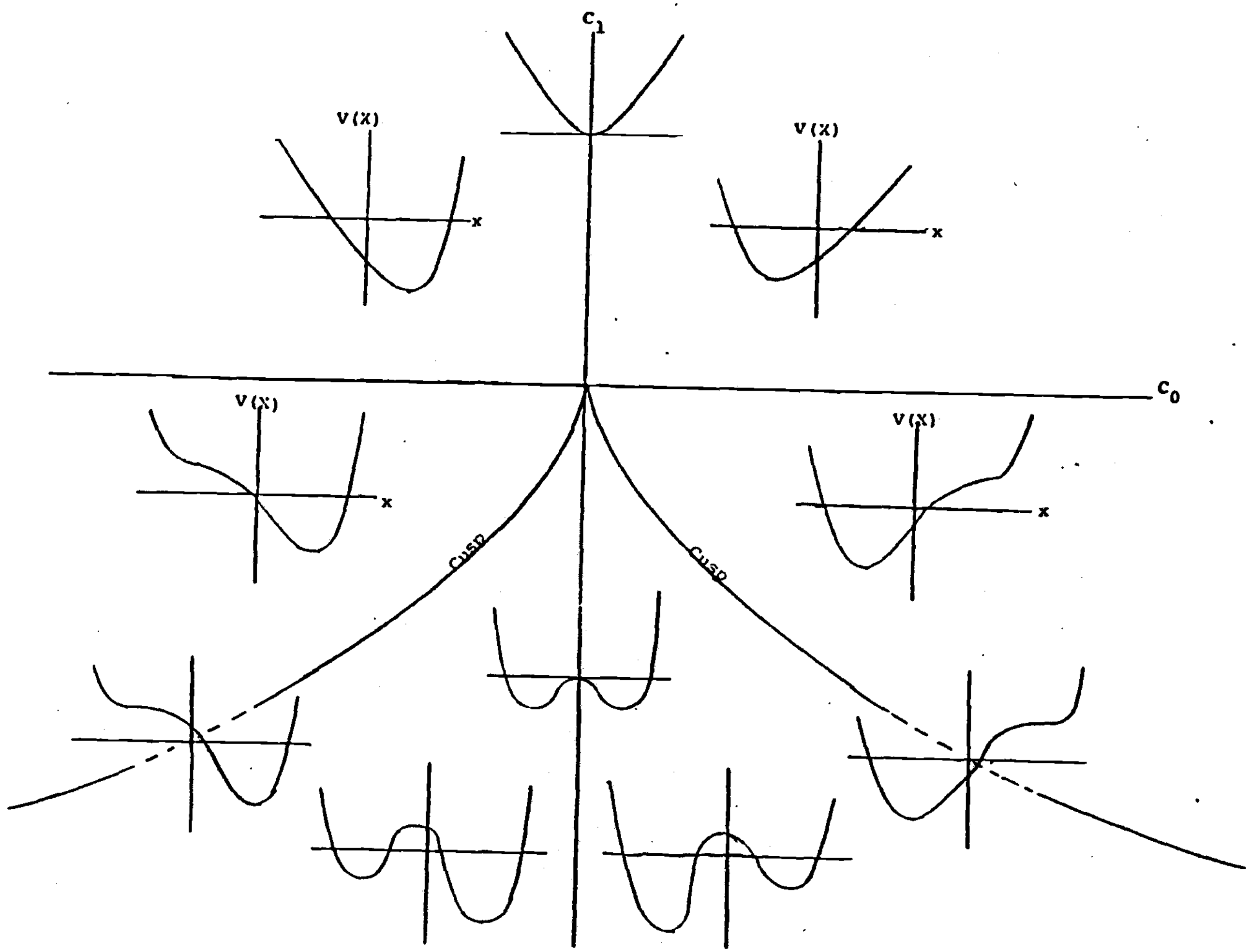

Figure 3.10. The unfolding of the cusp catastrophe; shapes of the potentials in various regions of the control space. Adapted from Saunders, p. 9.

Earlier it was stated that degenerate equilibria organize the qualitative properties of a family of potentials. More precisely, the qualitative character of a family is organized locally about the lowest nonvanishing singularity of the family. The bifurcation set of this singularity divides the control space into different regions. Along a trajectory through the control space which crosses the bifurcation set, behavior may change suddenly. The cubic singularity of the fold catastrophe divides the control space into one, a stable region in which the price trajectory corresponds to a set of dynamic attractors and two, an unstable region in which there are no equilibria. Upon crossing the bifurcation set into the unstable region the structure cannot persist as a stable system. This division between stable and unstable regions exists for any k-determined family for which k is odd.[21] It is not that the families of potentials are mathematically unstable. The structure of an odd degree catastrophe as a whole does not represent an ordered and ongoing process.[22]

We will require that the structural instability introduced by degenerate critical points is embedded into a family of potentials in a stable way. In other words, we require that the system as a whole retains an overall sense of organization. This becomes clearer with a study of the geometry of the cusp catastrophe.

The Geometry of the Cusp Catastrophe

The canonical form of the cusp catastrophe is given by the equation:[23]

$$V(x;C) = x^4 + c_2x^2 + c_1x \qquad (3.50)$$

The controls (c_1, c_2) are variable parameters of a family of functions. The critical points of this family define a two-dimensional surface given by the equation:

$$4x^3 + 2c_2x + c_1 = 0 \qquad (3.51)$$

The equilibrium surface is embedded in three dimensions: a one-dimensional state space and a two-dimensional control space (Figure 3.11). The singularity set of degenerate equilibria is given by the equation:

$$12x^2 + 2C_2 = 0 \qquad (3.52)$$

The state variable, x, can be eliminated from equations 3.51 and 3.52 yielding an expression for the bifurcation set in terms of the controls:

$$8c_2^3 + 27c_1^2 = 0 \qquad (3.53)$$

The cuspoid shape of the bifurcation set is illustrated in Figure 3.11. Fold points of the equilibrium surface correspond to the lines of the cusp in the control space.

We may recognize that the equations for the bifurcation set are also an expression for the determinant of a cubic equation. Outside the cusp, the determinant is strictly negative and the potential has a single minimum. Inside the cusp the determinant is strictly positive. Potentials have three distinct critical points: two minima and one maximum. At the vertex of the cusp the potential has a single minimum corresponding to the singularity x^4. Along the remainder of the cusp potentials have two critical points, a minimum and an inflection point. The cusp and corresponding potentials are illustrated in Figure 3.10.

It is clear that every potential in a 4-determined family possesses at least one stable attractor. Structural instability is confined to the cusp where one of two stable equilibria disappears or, conversely, a new equilibria forms. If a system is organized about a unique minimum outside the cusp, the transition into the cusp is smooth. At the cusp a new equilibrium forms, but the system remains organized by the first attractor.[24] Upon leaving the cusp, this initial equilibrium disappears.

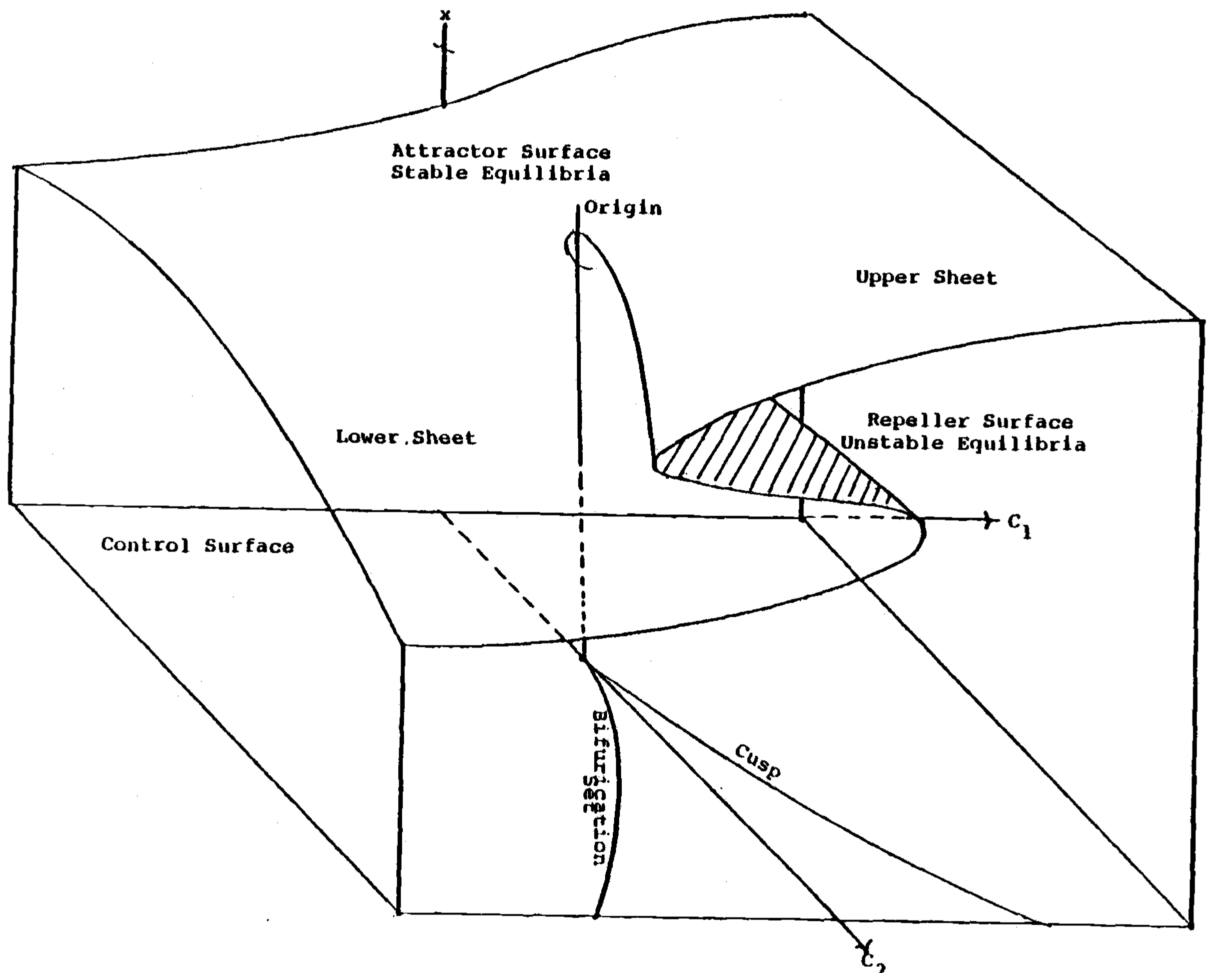

Figure 3.11. The equilibrium surface of the cusp catastrophe projection over the control surface. Adapted from Zeeman Catastrophe Theory. p. 21.

The system is suddenly reorganized about a new set of equilibria. A path through the cusp is illustrated in Figure 3.12 for the control and equilibrium surfaces. While behavior may suddenly change, the overall structure of the system is stable.

The canonical coefficients, c_1 and c_2, are referred to as normal and splitting factors, respectively. The names reflect that when c_2 is positive the system responds smoothly to changes in c_1. As c_2 becomes negative, the equilibrium surface splits into two stable attractors separated by a repeller surface. Cross-sections of the equilibrium surface are illustrated for different values of c_1 and c_2 in Figure 3.13.

A careful examination of the geometry of the cusp catastrophe reveals an important aspect of its construction. Outside the cusp, where the equilibrium surface is single sheeted, the system responds smoothly to changes in the controls. Here, the cusp exhibits the properties of a 2-determined family of potentials. The double sheeted section of the stable equilibrium surface is constructed from a series of folds. Fold points form the boundaries of the equilibrium surface along the lines of the cusp. The apex of the cusp corresponds to the singularity x^4. This type of construction extends to

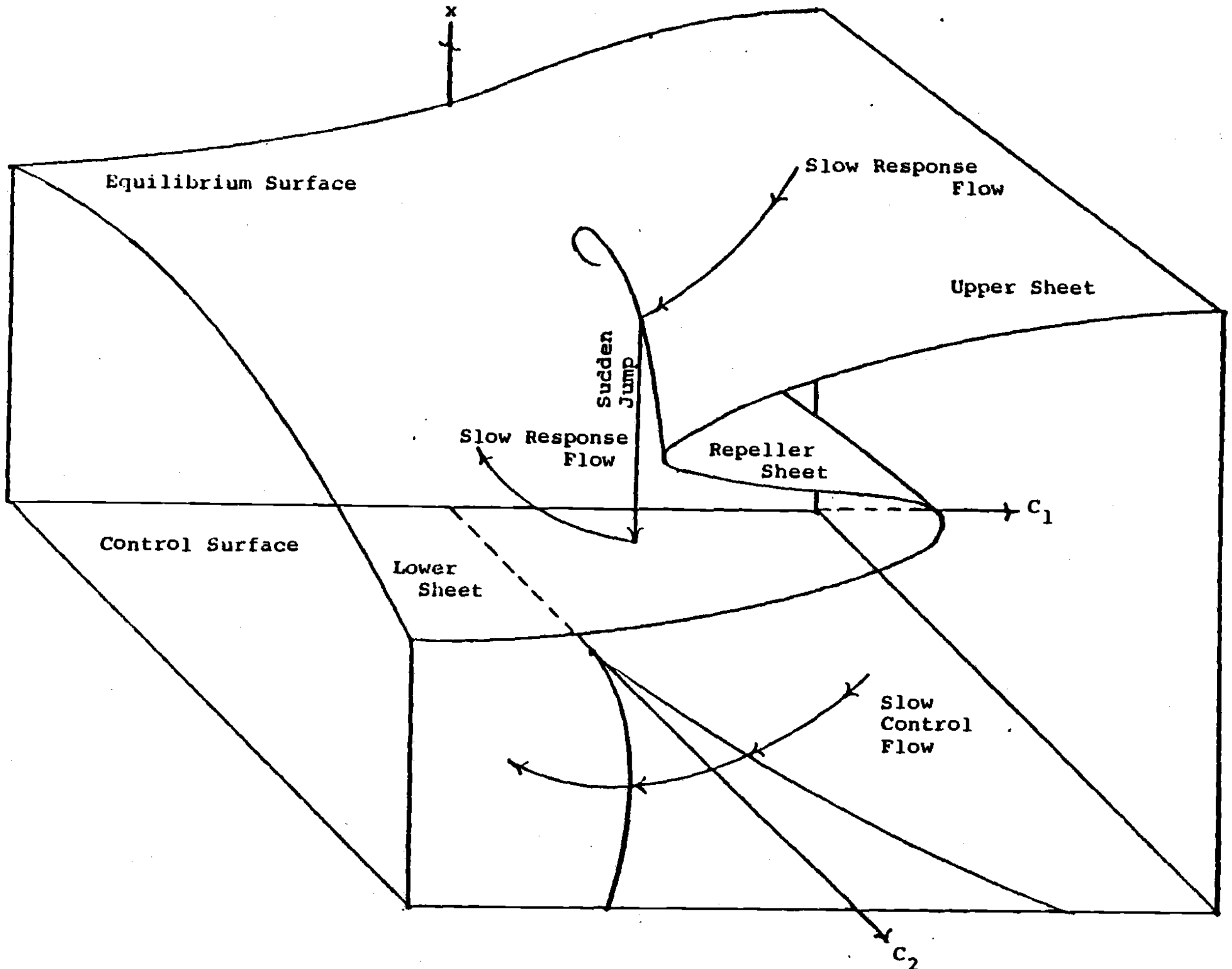

Figure 3.12. A sudden jump in equilibrium along a pathway corresponding to a smooth control trajectory crossing the bifurcation set.

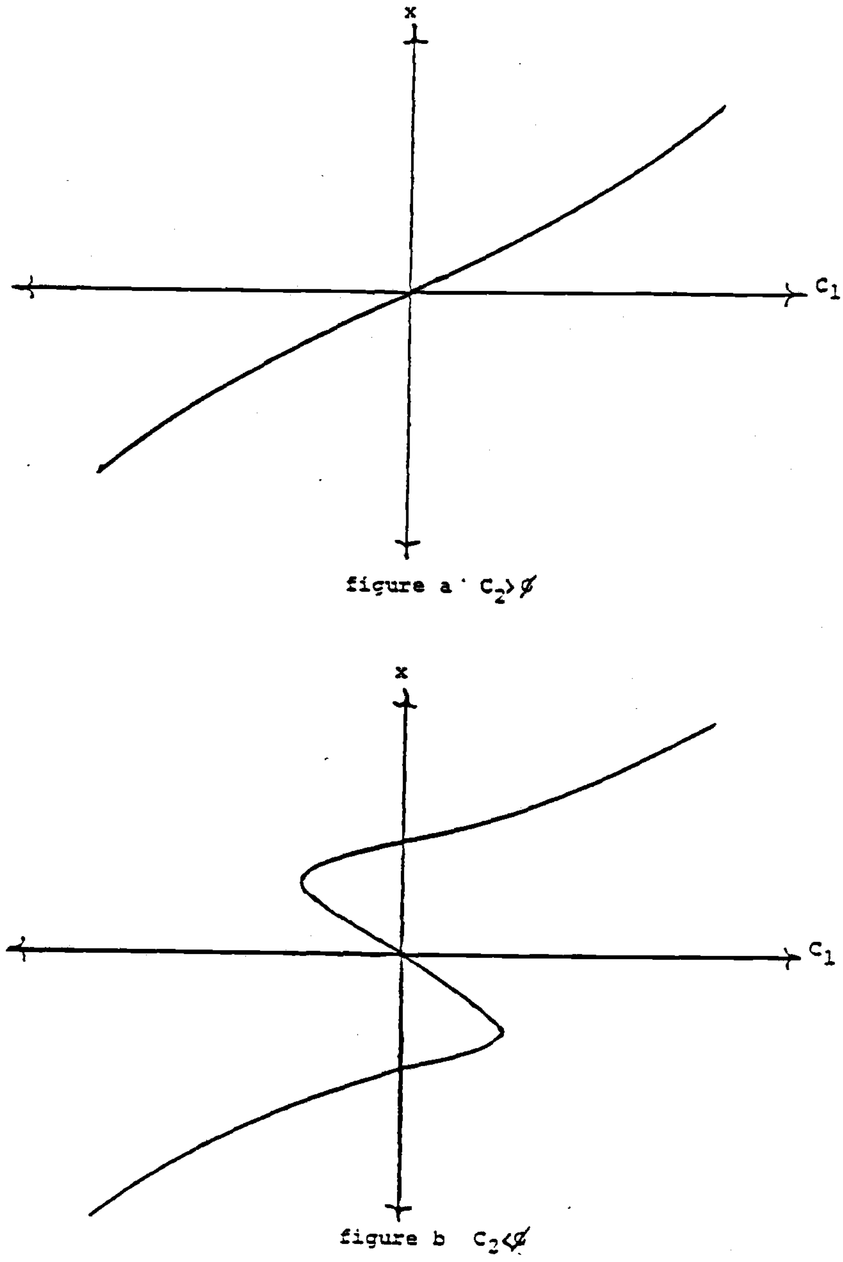

Figure 3.13. Cross sections of the cusp catastrophe for given values of C_2: Figure a) $C_2 > \emptyset$, Figure b) $C_2 < \emptyset$.

higher degree catastrophes whose geometry we will not consider here.[25] A swallowtail catastrophe contains a series of cusps organized about a new singularity x^5. With the addition of each new singularity a new element of qualitative behavior is added. The fold catastrophe introduces a boundary point. Boundary points are aligned with a cusp point to form the separation of the equilibrium surface in a cusp catastrophe. In assuming a market dynamic possesses a cusp structure we can allow for sudden as well as smooth transitions (Figure 3.14). At the same time, we must acknowledge that an endless number of behavioral alternatives, associated with higher degree singularities, are being excluded.

The flow lines representing behavior in Figure 3.14 imply some underlying hypotheses or assumptions about the interrelationships between the state and control variables. Zeeman calls these interrelationships slow feedback. They play a critical role in the application of a cusp structure in economics, which we will consider in Zeeman's model of a stock exchange.

Zeeman's Model of a Stock Exchange

Zeeman proposed a model of a cusp catastrophe with slow feedback to explain the unstable behavior of stock

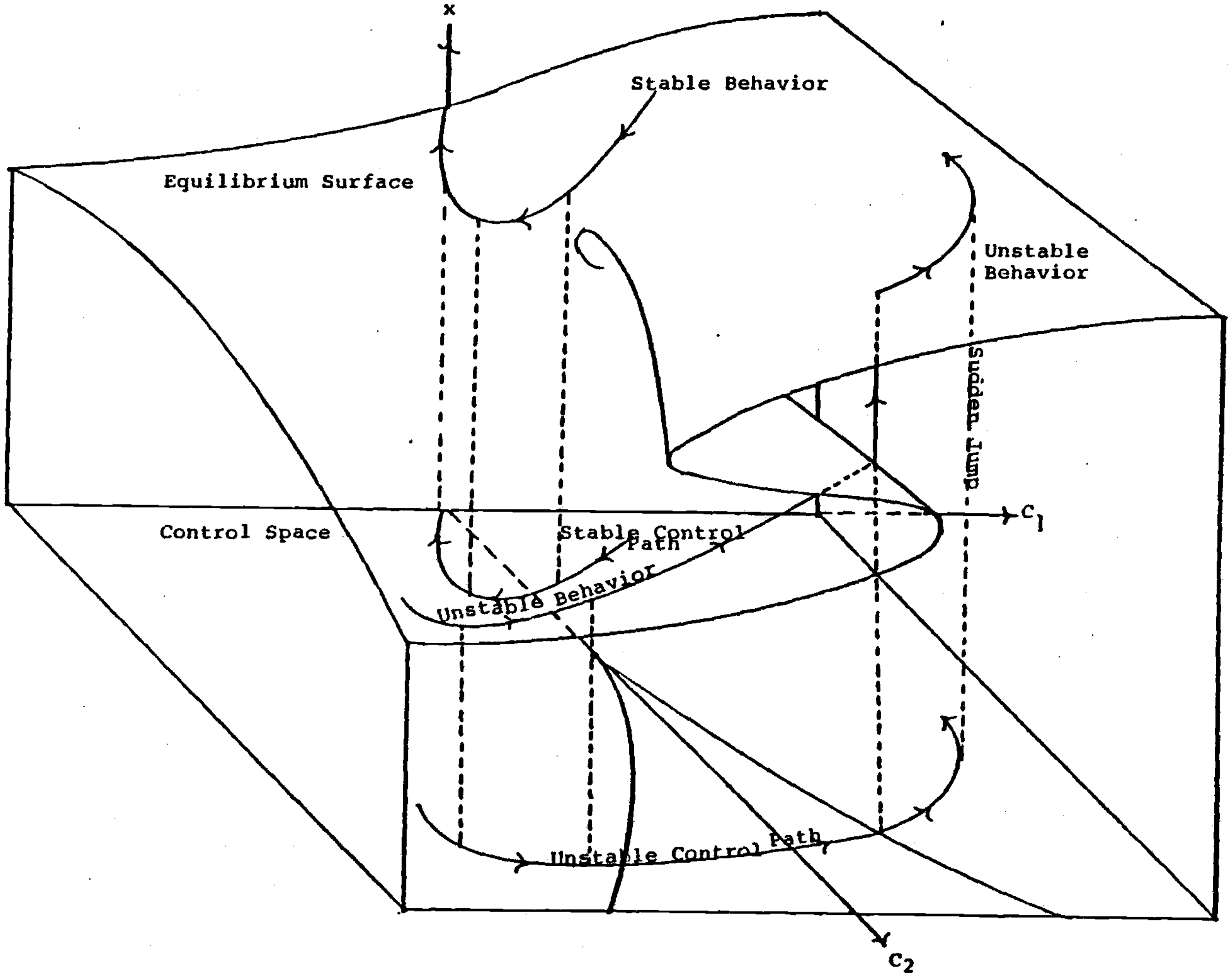

Figure 3.14. Stable and unstable behavior along paths through the control space of a cusp catastrophe.

exchanges.[26] With some modifications, this model can be applied to any market subject to speculative investment. Zeeman's apparent motive in developing a model of a stock market was to demonstrate an application of catastrophe theory in economics. As a result, the economic and mathematical development of the model is limited and presented with little explanation. An effort has been made to provide a mathematical background so that we may now focus upon the economic aspects of Zeeman's application.

In a general description of the operation of a stock exchange, an index, I, is chosen to measure the state of the market. The rate of change in the index over time, J:

$$J = \dot{I} = \frac{dI}{dt} \tag{3.60}$$

is regarded as a dependent variable. The variable J depends on the rate at which investors are willing to buy and sell stocks. At the same time, there is a feedback effect of information about J on the behavior of investors.

Zeeman restricts attention to two broad classes of investors, fundamentalists and chartists. Fundamentalists are assumed to invest on the basis of factors such as yield, long term growth potential, and so forth. In other words, the demand schedule of fundamentalists is independent of the current state of the market. Chartists base their investment strategies upon the behavior of the market itself. As the name implies, chartists may use recent information on the state of the market to predict future market behavior. The demand schedules of chartists are dependent upon the state of the market.

The activities of fundamentalist and chartist investors are represented by two variables, F and C. The variable F represents the excess demand of fundamentalists. The variable C is defined as the proportion of the market held by chartists. Zeeman calls C the speculative content of the market since speculators tend to be chartists. A separate variable for the excess demand of chartists is not introduced. Zeeman offers two reasons for this. First is simplicity; he argues that total excess demand can probably be represented as a function of J and chartist excess demand is simply the difference between total and fundamentalist demand. Second, chartist excess demand can be treated as an internal market mechanism as opposed to fundamentalist demand which is an external driving force.[27] These reasons are cause for some concern and the subject will be taken up in greater detail in the following section.

Zeeman derives the dynamic flow of the model from seven hypotheses. These hypotheses or assumptions rest between the formal mathematics and the economic content of the model. The first three assumptions are used to establish the structure of a cusp catastrophe. The remaining four assumptions are used to generate flows within this structure.

The first hypothesis states that J respond to C and F much faster than C and F respond to J. This is an assumption concerning temporal market structure. The market is assumed to respond very quickly to changes in excess demand in comparison to the rate at which market participants move along their demand schedules. Zeeman states:

> "The main purpose of a stock exchange or money market is to act as a nerve centre so that prices ... can respond as swiftly and as sensitively as possible to supply and demand. Changes in C and F can cause changes in J within minutes, whereas changes in J have a much slower feedback on C and F. The response time for C may be a matter of hours, but is more likely to be days or weeks, while the response time for F can be months, due to the research involved."[28]

As a general model of exchange, the process of price adjustment may represent a diffuse as opposed to a centralized organization. The rate at which the system approaches equilibrium may be much slower. Furthermore, Zeeman's description of the response time for buyers and sellers confuses a movement along a schedule with a shift in supply and demand. The collective response to changes in price could be faster. However, fast and slow flow assumptions are necessary if we are to make use of some form of equilibrium analysis. To describe the state of a system by equilibrium values, we are in fact assuming that equilibrium is achieved sufficiently fast to ignore the process of adjustment.[29]

Zeeman's second and third hypotheses give the conditions under which a discontinuity in the market dynamic occur. Two: When C is small then J is a smooth function of F passing through the origin. If a market is dominated by fundamentalists, then the price trajectory moves toward a point of static equilibrium (Figure 3.15a). There are no singularities along the dynamic. Three: When C is large, a discontinuity is introduced into the dynamic, and static equilibrium is no longer stable. A slight disturbance of the index will be sustained by chartists creating an extended bull or bear market. Zeeman offers a mathematical argument for why the graph of the dynamic is now disconnected (Figure 3.15b). The argument is complex, and it is based on Thom's classification theorem which is, in turn, used with this

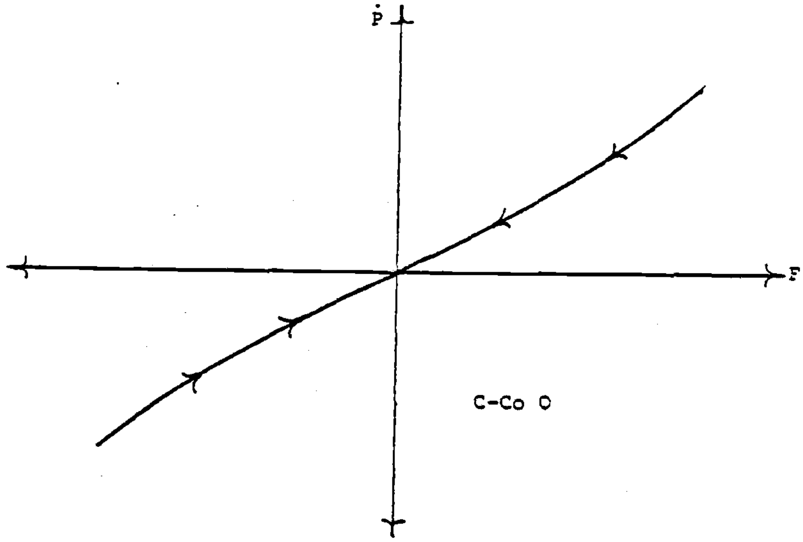

figure a

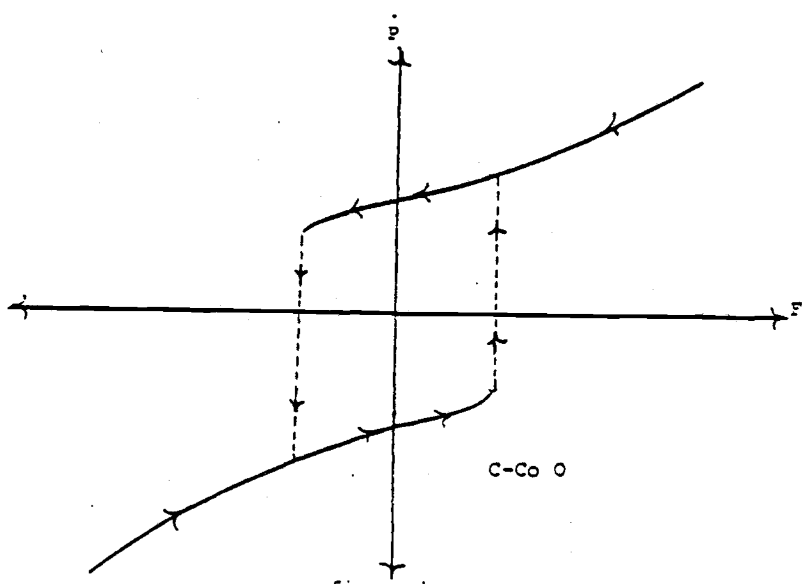

figure b

Figure 3.15. Price trajectories along cross sections of a cusp market structure: Figure a) $C-C_0<\emptyset$, Figure b) $C-C_0>\emptyset$. Adapted from Zeeman <u>On the Unstable Behavior of Stock Exchanges</u>, p. 365.

assumption to generate the model. However, economic reasons for the discontinuity were considered in the second section of this chapter.

A simplified statement of a part of Thom's theorem may help clarify Zeeman's synthesis of a cusp structure: for a family of smooth potentials of a single state variable parameterized by two controls, the equilibria of this family form a smooth surface, and the only singularities are fold curves and cusp catastrophes.[30] Under the assumption of smoothness and by reducing the problem to one state and two control variables, the most complex thing that can occur is a cusp catastrophe. Whether or not this framework provides an adequate description of a system is not a question addressed by Thom's theorem. Given the description is adequate and the system is structurally stable, then the existence of a discontinuity (hypothesis three) can only be attributed to a cusp catastrophe. From hypothesis two, we may infer that the cusp is not located at the origin. Unstable behavior occurs only after some critical level of speculation is reached. Finally, by hypothesis one, behavior is constrained to be near the equilibrium surface. Any control trajectory which crosses through the cusp will result in a sudden change in the state of the market. This synthesis is summarized in Figure 3.16.

The canonical form of cusp is given by the equation:

$$J^3 - (C-C_o)J - F = 0 \qquad (3.61)$$

The critical level of speculation at which the discontinuity begins is given by $C=C_o$. The surface of stable equilibria satisfy the inequality:

$$3J^2 + C_o \geq C \qquad (3.62)$$

The complement of the attractor surface is the repeller surface consisting of unstable equilibria, given by the equation:

$$3J^2 + C_o < C \qquad (3.63)$$

The boundary of the equilibrium surface is the fold curve given by the equation:

$$3J^2 + C_o = C \qquad (3.64)$$

The projection of the fold curve onto the control the plane is the cusp and has the equation:

$$4(C-C_o)^3 = 27F^2 \qquad (3.65)$$

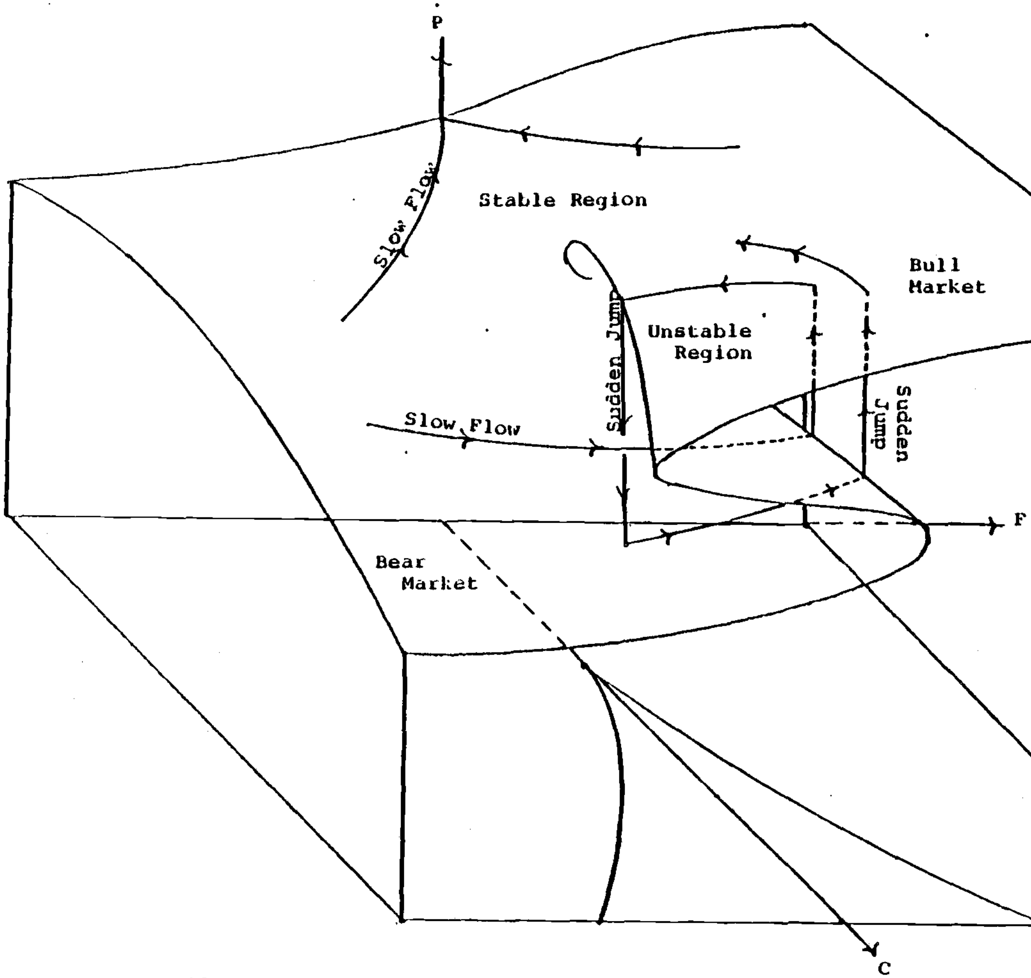

Figure 3.16. A cusp catastrophe model of a dynamic market structure. Adapted from Zeeman _On the Unstable Behavior of Stock Exchanges_. p. 368.

In general, these equations are a variant of the canonical forms, presented in the previous section, for a cusp catastrophe located away from the origin. They are intended to represent the model, illustrated graphically in Figure 3.16, only to within a diffeomorphism. We can construct any number of equivalent models through a smooth reversible transformation of coordinates. We may bend, stretch and twist Figure 3.16 without altering its qualitative properties so long as we do not cut, tear, add or eliminate any folds.

Zeeman's remaining four hypotheses are used to generate the global dynamics of the model. These hypotheses deal with the slow flow impact of the changing index on the controls. Four: C has the same sign as J. A bull or rising market attracts speculation by chartists. When J is positive, the proportion of speculative money in the market increases. We encountered a similar proposal earlier in this chapter as the presence of a price trend gave rise to speculative expectations and increased reservation demand. However, on the other side of Zeeman's hypothesis, a bear or falling market repels chartists and the proportion of speculative money in the market declines. In our treatment of reservation demand falling prices may also lead to speculative expectations and increasing speculative content in the market. For the present, we will hold to Zeeman's assumption.

Five: The rate of change in fundamentalist excess demand is negative after a large rise in the index, even though the index may still be rising. Zeeman contends that as a bull market extends, fundamentalists begin to view the price of a stock as overvalued and start to cash in. Thus, excess demand falls as the index continues to rise. This argument is somewhat contrary to the idea that fundamentalist demand is independent of the state of the market and again confuses a demand shift with a movement along a schedule. However, the hypothesis is consistent with a basic economic assumption concerning excess demand: as prices rise the quantities suppliers are willing to offer increase, while the quantities consumers are willing to purchase decline. Applying this assumption to the purchase and sale of stocks by fundamentalists, excess demand falls as the index rises.

Six: Fundamentalist excess demand is declining after a short fall in the index. Zeeman's argument here is that fundamentalists trade on a margin and opt to sell after a fall in price greater than the margin. In other words, a decline in the index which exceeds an investor's margin leads to a policy of cutting losses through liquidation.

As a result, fundamentalist excess demand falls after an initial decline in the index. This assumption is contrary to the laws of supply and demand. The concept of a buying or selling margin based on price changes is more appropriate as a speculator's strategy. While this hypothesis may be unacceptable, it does not greatly alter the slow dynamic flow due to the following hypothesis.

Seven: Fundamentalist excess demand declines if the index has been falling for some time and is beginning to flatten out. A recovering market is an attractive investment to fundamentalists as they perceive stocks to be undervalued. Short-run losses (in capital value) are likely to be offset by greater gains in the long-run. Thus, excess demand increases as prices continue to fall. This hypothesis is consistent with the laws of supply and demand. However, the logic of this and the previous two hypotheses is questionable. This is not to say we would not expect to observe investment under these strategies. They are based on expectations for capital gains or losses under prevailing market conditions and, therefore, might be termed speculative. Since these assumptions will be disposed of in our treatment of competitive markets, a detailed discussion is not warranted. We will simply examine the dynamic flows implied by hypotheses four through seven.

The changes in the control values are shown as trajectories through the control space and as slow flow lines along the equilibrium surface in Figure 3.17. The direction of the slow flow vectors implied by the four hypotheses are the sum of components, in the directions of the control coordinates, for values of J on the equilibrium surface.

We may use Figure 3.17 to illustrate the global dynamics of the model. Starting from an initial point of static equilibrium, a small disturbance, in the absence of other external forces, may not disrupt the stability of the market. If the proportion of speculative investment remains small, the market will return to a position of static equilibrium. If an external disturbance, resulting in an increasing index, attracts a sufficient level of speculative investment a smooth return to static equlibrium is no longer possible. The bull market is sustained as the trend continues to attract chartists. Fundamentalist demand is declining and eventually becomes negative. After a delay, the bull market crashes and a recession begins. Finally, the market begins to move

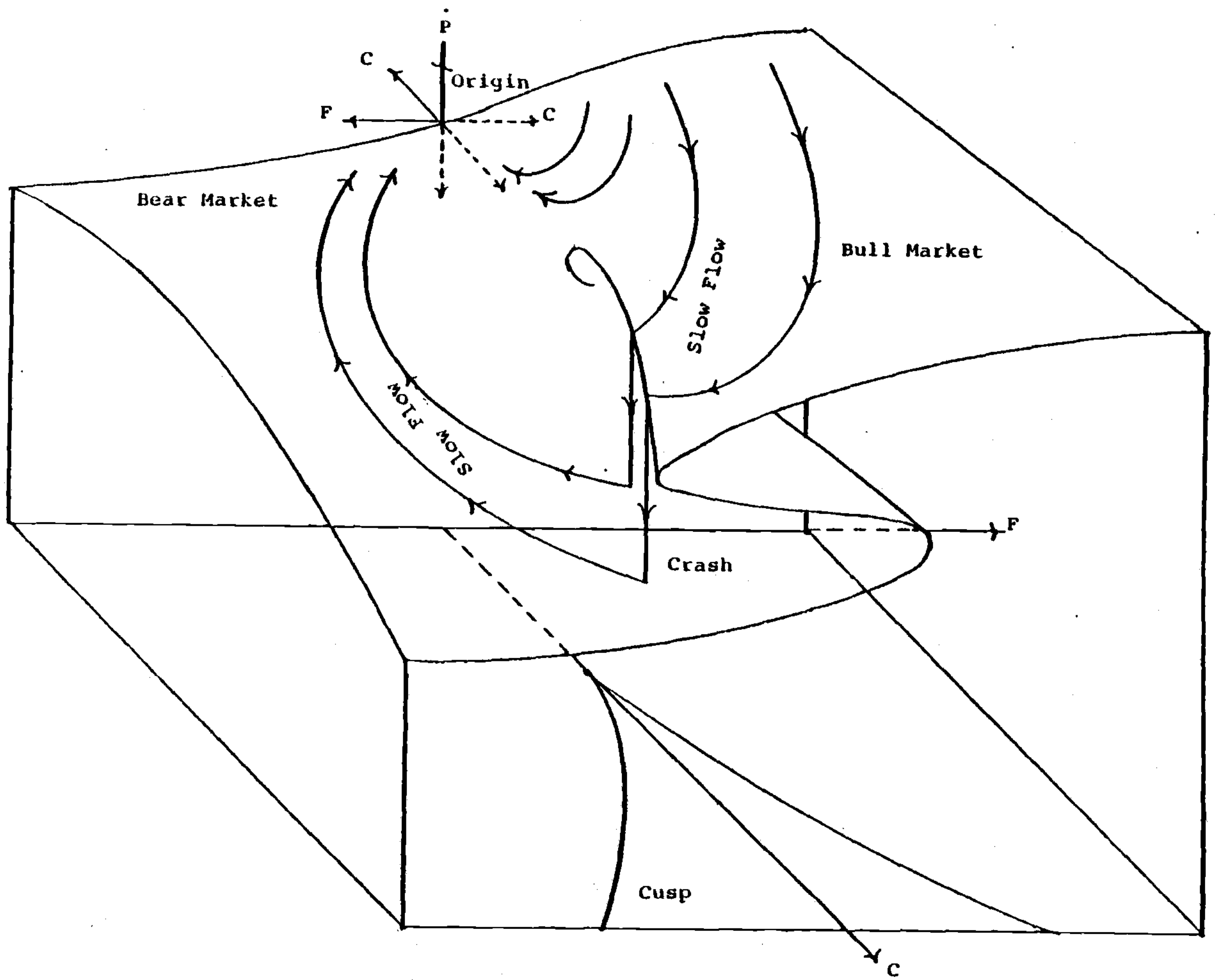

Figure 3.17. Zeeman's model of a stock exchange; a cusp catastrophe with slow feedback. Adapted from Zeeman <u>On Unstable Behavior of Stock Exchanges</u>, p. 368.

toward a point of static equilibrium as fundamentalist demand increases and chartists withdraw.

Zeeman draws two qualitative conclusions based on a cusp market structure. First, the greater the proportion of speculative investment the greater the slope of the recession (rate of decline in prices). Second, the greater the proportion of speculative investment the longer the delay before the market crashes (measured in terms of fundamentalist excess demand). As a corollary of Thom's theorem for $C \geq C_o$, Zeeman gives the following formulae:

$$\text{slope of the recession} = J^* = L(C-C_o)^{1/2} \quad (3.66a)$$

$$\text{delay} = F^* = K(C-C_o)^{3/2} \quad (3.66b)$$

where L and K are smooth non-vanishing functions of C.

For the canonical form of the cusp catastrophe L and K are constants. A recession begins on the lower sheet at a point corresponding to the cusp. Therefore, we may utilize equation 3.64 to determine the slope of the recession. Equation 3.64 may be solved for the terminal slope on the upper sheet:

$$J_t = ((C-C_o)/3)^{1/2} \quad (3.67a)$$

Zeeman reports an identical formula for the slope of the recession which is incorrect. There is no derivation but later conclusions are based on a correct formula. Since we have one root of the equilibrium equation (3.61), we can perform the following synthetic division:

$$\begin{array}{lllll} 1 & +\ 0 & -\ (C-C_o) & -\ F & \underline{\quad}/((C-C_o)/3)^{1/2} \\ & ((C-C_o)/3)^{1/2} & (C-C_o)/3 & -\ 2(C-C_o)^{3/2}/3\sqrt{3} & \\ \hline 1 & (C-C_o)/3)^{1/2} & (-2/3)(C-C_o) & 0 & \end{array}$$

yielding the quadratic equation:

$$J^2 + ((C-C_o)/3)^{1/2}J - (2/3)(C-C_o) = 0$$

The roots of this equation are the remaining roots of the equilibrium equation which can be obtained from the quadratic formula as:

$$r_2 = ((C-C_o)/3)^{1/2}$$

$$r_3 = -2(C-C_o)/3)^{1/2}$$

The second root is repetitious, corresponding to the fold point on the upper sheet. The third root, r_3, is the initial point of the recession on the lower sheet.

Therefore, the slope of the recession is given by the formula:

$$J_r = -2(C-C_o)/3)^{1/2} \qquad (3.67b)$$

Comparing equations 3.67a and 3.67b shows that for the canonical form of a cusp catastrophe, the initial slope of the recession is twice as steep as the rate of increase in price before the crash.

Within the stable region of the model the transition between rising and falling markets is smooth, as illustrated in Figure 3.18a. When the proportion of speculative investment is large the transition is sudden, creating a sharp maximum as in Figure 3.18b. For a canonical cusp catastrophe, the angle of descent (β) is twice the angle of incline (α):

$$\cot(\beta) = 2\cot(\alpha) \qquad (3.68)$$

In general, the equilibrium surface is only equivalent to, not the same as, a canonical form. However, Zeeman states that this relationship provides an approximate form for empirically evaluating an implication of the general form of a cusp catastrophe; the greater the rate of price increase in a bull market, the greater the rate of decline in a recession.[31]

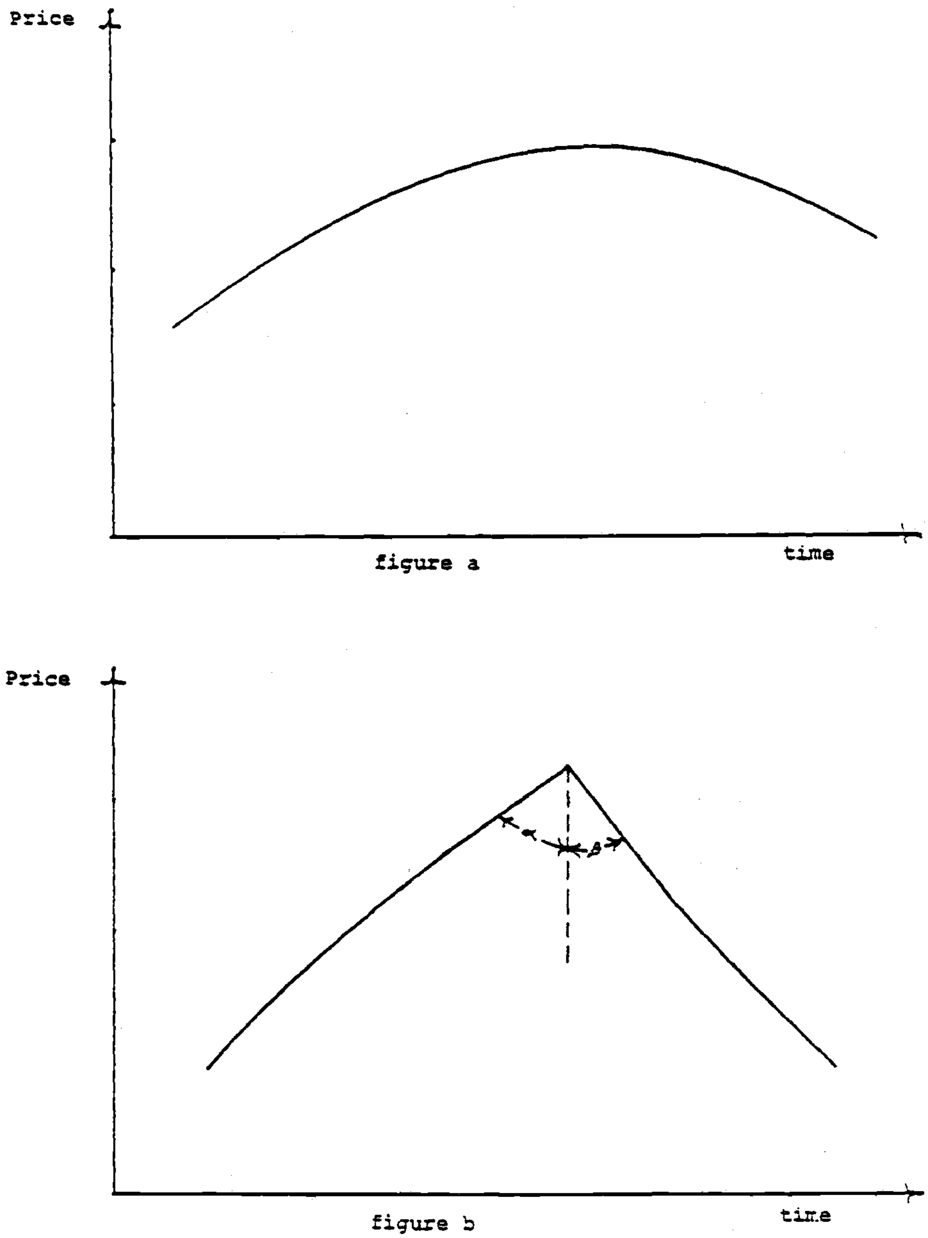

Figure 3.18. Changes in price over time: Figure a) smooth transition, Figure b) sudden transition. Adapted from Zeeman <u>On the Unstable Behavior of Stock Exchanges</u>, p. 366.

Zeeman notes that the dynamic behavior of a cusp structure can be accelerated or slowed by the actions of external forces. Sufficiently large and frequent external disturbances could completely obscure the underlying dynamic. Zeeman fails to consider a second reason why the potential for catastrophic behavior may not be realized. The response of investors to market information, such as a price trend, is unlikely to reflect deterministic relationships. Attitudes of investors are subject to change for reasons outside the domain of economic rationale. A price trend may or may not attract significant levels of speculative investment. In contrast to the mechanical relationships postulated between excess demand and price movements, the slow flow hypotheses, relating prices to speculative demand, represent behavioral alternatives. In other words, we may use dynamic flow hypotheses to try and capture tendencies among market participants. Hopefully, we can identify conditions under which speculative behavior is more or less likely to occur.

The dynamic flows illustrated on the equilibrium surface of a cusp catastrophe show the potential impact of speculative demand for short-term capital gains. The behavior of chartists can be viewed as a form of speculative reservation demand. Chartist demand for stocks may increase with increasing prices when expectations for a continuing trend are projected to yield a return greater than the opportunity cost of the investment.[32] In a more general model of a competitive market for an inventoried commodity, expectations for a continuing trend may lead to speculative disinvestment as well as investment. Expectations for capital losses based on declining prices may induce the mirror image of the behavior outlined in Zeeman's model. Still, it appears that a model of a cusp catastrophe with slow feedback can be readily extended to a competitive market for a stored commodity.

A Dynamic Model of a Competitive Market for a Stored Commodity

Before attempting to recast Zeeman's model of a stock exchange into the framework of a competitive market for a stored commodity, the assumptions underlying the development of the model should be reconsidered. Our motivation here is two fold. First, the application of a catastrophe theory to market structure presents some methodological problems which need to be addressed.

Second, to apply a cusp market structure as a model of exchange, we need a better understanding of the economic implications and limitations of the model.

Zeeman's main hypothesis is simply a definition of variables. The state of a market is measured by the rate of change in an index, which we may now equate with the rate of change in price, $\dot{P}$. Prices are determined, relative to some initial value, by the time path of $\dot{P}$. In defining two types of investors, fundamentalists and chartists, Zeeman distinguishes between two types of supply and demand relationships. Fundamentalist demand or supply schedules are not dependent on the current state of the market. Quantities demanded or supplied are determined at each price by conditions external to the market. A change in price results in a movement along a given schedule. Market relationships which are independent of the state of the market are referred to here as nontransitory. Chartist demand or supply schedules are determined, at least in part, by current market conditions. Changing prices generate future price expectations which shift market demand or supply. Market relationships which are dependent on the current state of a market are referred to here as transitory. The activities of fundamentalist and chartist investors, in determining stock prices, are represented by two variables; fundamentalist excess demand and the proportion of the market held by chartists. Fundamentalist excess demand may be equated with nontransitory excess demand in a competitive market. However, the proportion of a market held by chartists, intended as a measure of speculative content, may not adequately represent the impact of transitory market relationships. We should try to treat transitory supply and demand relationships explicitly before assuming they are an implicit component of the model.

Zeeman offered two arguments for not defining an explicit variable representing transitory excess demand. First, he states that total excess demand can probably be represented as a function of the rate of change in price. Transitory demand would then simply be the difference between total and nontransitory excess demand. If we begin with the assumption that the rate of change in price, at a given point in time, may be written as a function of total excess demand (Walrasian adjustment):

$$\dot{P}(t) = f(ED_N(t) + ED_T(t)) \tag{3.69}$$

where;

$ED_N(t)$ = nontransitory excess demand at time t

$ED_T(t)$ = transitory excess demand at time t then, to write total excess demand as a function of $\dot{P}$ we must appeal to the inverse function theorem. The derivative of $\dot{P}$ with respect to excess demand must be nonvanishing at every point (1-determined). As a result, we may expect the relationship between prices and excess demand to be structurally stable. Hence, any isolated structural instability must involve both excess demand and prices. This leads to an intuitively appealing conclusion. A sudden change in prices is attributable to a sudden change in transitory excess demand. In a traditional market model changes in supply and demand are generated outside the market. Here, transitory excess demand and its eventual collapse is generated internally. This meets with Zeeman's second argument that the response in transitory demand or supply to changing prices is an internal driving force.

The most direct way to incorporate the interrelationship between changing prices and transitory excess demand into a model may be to include transitory excess demand as a state variable as opposed to a control. This helps to explain why discontinuities or sudden changes may occur. Suppose an upward price trend gives rise to speculative demand, sustaining the trend and allowing nontransitory excess demand to become negative.

As the rate of increase in prices slows, transitory demand eventually begins to decline. As a result, the trend begins to terminate more quickly. A more rapid decline in $\dot{P}$ brings about a faster decline in transitory demand and consequently, a faster decline in $\dot{P}$. The interaction between transitory excess demand and prices culminates in a nearly vertical (fast) flow to a declining market. There is a sudden change in both transitory demand and the rate of change in price. Introducing transitory excess demand as a state variable alters two aspects of the model's dynamic structure. First, the temporal flow assumptions are altered. The rate of change in price and transitory excess demand are adjusting at nearly the same rate. Second, the model is not structurally stable over some potentially relevant control values.

Given that we may regard a market as a smoothly responding equilibrium system, we may represent the process of price adjustment as a qualitatively linear function of total excess demand. Given the occurrence of discontinuities in an otherwise smoothly responding equilibrium system, the simplest structurally stable representation of the pricing process is a cusp catastrophe. In considering two independent state variable, the next simplest structurally stable model is a double cusp.[33] With a total of eight canonical controls, a double cusp is very difficult to describe and its geometry is beyond interpretation. To reduce the model back to a single cusp, we may make the assumption that the rate of change in price and transitory excess demand are functionally dependent. Unfortunately, this does not bring us closer to a consistent interpretation of the control variables.

Zeeman's canonical equation for a cusp catastrophe with slow feedback may be rewritten and relabeled in the form:

$$\dot{P}^3 + S_o\dot{P} = S\dot{P} + N \qquad (3.70)$$

where;

N = nontransitory excess demand
S = speculative content
S_o = the location of the cusp point, >0.

If we interpret the term $S\dot{P}$ to be transitory excess demand the equation may be rewritten:

$$\dot{P}^3 + S_o\dot{P} = \text{Total Excess Demand} \qquad (3.71)$$

Since S_o is taken to be greater than zero, the rate of change in price is expressed as a qualitatively linear function of total excess demand.[34] The linear equation for transitory demand as a function of price may be written:

$$ED_T = S\dot{P} \qquad (3.72)$$

The splitting factor may be interpreted as the slope of the nontransitory excess demand curve. While no real argument can be made for the validity of this interpretation, it does provide a reasonably well defined market model with a cusp structure.

Assuming nontransitory excess demand is a function of price, the cusp market model may be written:

$$\dot{P}^3 - (S-S_o)\dot{P} - g(P;C) = 0 \qquad (3.73)$$

where;

$g(P;C)$ = nontransitory excess demand
C = a vector of exogenous controls.

Holding the exogenous controls constant, the slow flow of the normal factor is determined by the slope of the nontransitory excess demand curve. Given that the slope of the excess demand curve is negative, the feedback flow of prices on nontransitory demand is directed toward a state of equilibrium. The feedback flow of price information on the slope of the transitory demand curve has not been established. A large number of factors may influence speculation: the cost of holding inventories, the magnitude of a price trend, the duration of a trend and the perception of risk are at least four price dependent factors that may effect the willingness of individuals to speculate on current price changes. The slow feedback of price information on the slope of the transitory excess demand curve may be complex and varied. If we do not constrain the way in which market information is incorporated into expectations we can not predict the flow of speculation. We can consider a wide range of behavioral alternatives. We can examine a few such alternatives as a means to demonstrate the five properties, or flags, of a cusp catastrophe.

Catastrophe Flags

Three sets of alternative slow flow assumption are illustrated in Figures 3.19, 3.20 and 3.21. In the first figure, slow flows generating stable and unstable behavior are shown. If changing prices fail to stimulate a sufficient level of speculation, the slow flow remains on the single sheeted section of the equilibrium surface. There is a smooth return to a stable equilibrium. If price changes generate a sufficient level of speculative content, the slow flow is carried onto the double sheeted section of the equilbirium surface. As the slow flow crosses the cusp there is a sudden jump between market states. Sudden jumps are one characteristic of catastrophic behavior. In the second figure, highly speculative feedback flows illustrate a second property of a cusp catastrophe, divergence. A relatively small external disturbance leads to divergent paths of dynamic adjustment. The third illustration shows a more complex pattern of speculative flow. Increasing prices generate sufficient speculative content to extend the trend over the upper sheet of a bull market. The eventual collapse of the trend onto the lower sheet initiates a bear market. Falling prices stimulate a speculative reduction in inventory demand which carries the slow flow back through the cusp. The bear market terminates with a sudden jump to a rising state. Increasing prices carry the market through another cycle. The three remaining flags or behavoral implicaitons, of a cusp catastrophe are illustrated in this example. The third flag is inaccesibility. A stable state of equilibrium is inaccessible from within the cusp as the attractor sheets of the equilibrium surface are separated by a repeller sheet of unstable dynamic equilibria. The forth catastrophe flag is bimodality. Given that price

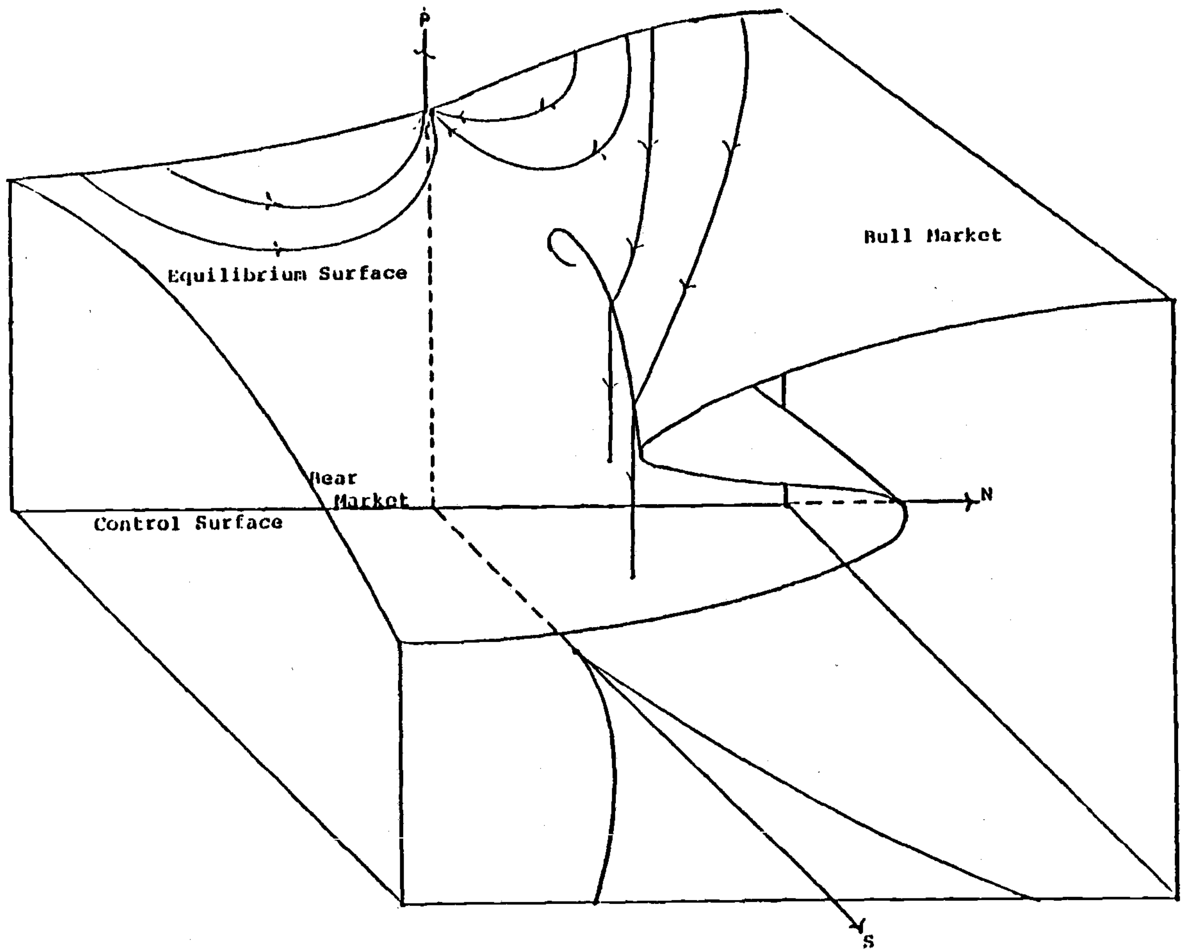

Figure 3.19. Alternative slow flow assumptions for the splitting variable: stable and unstable behavior.

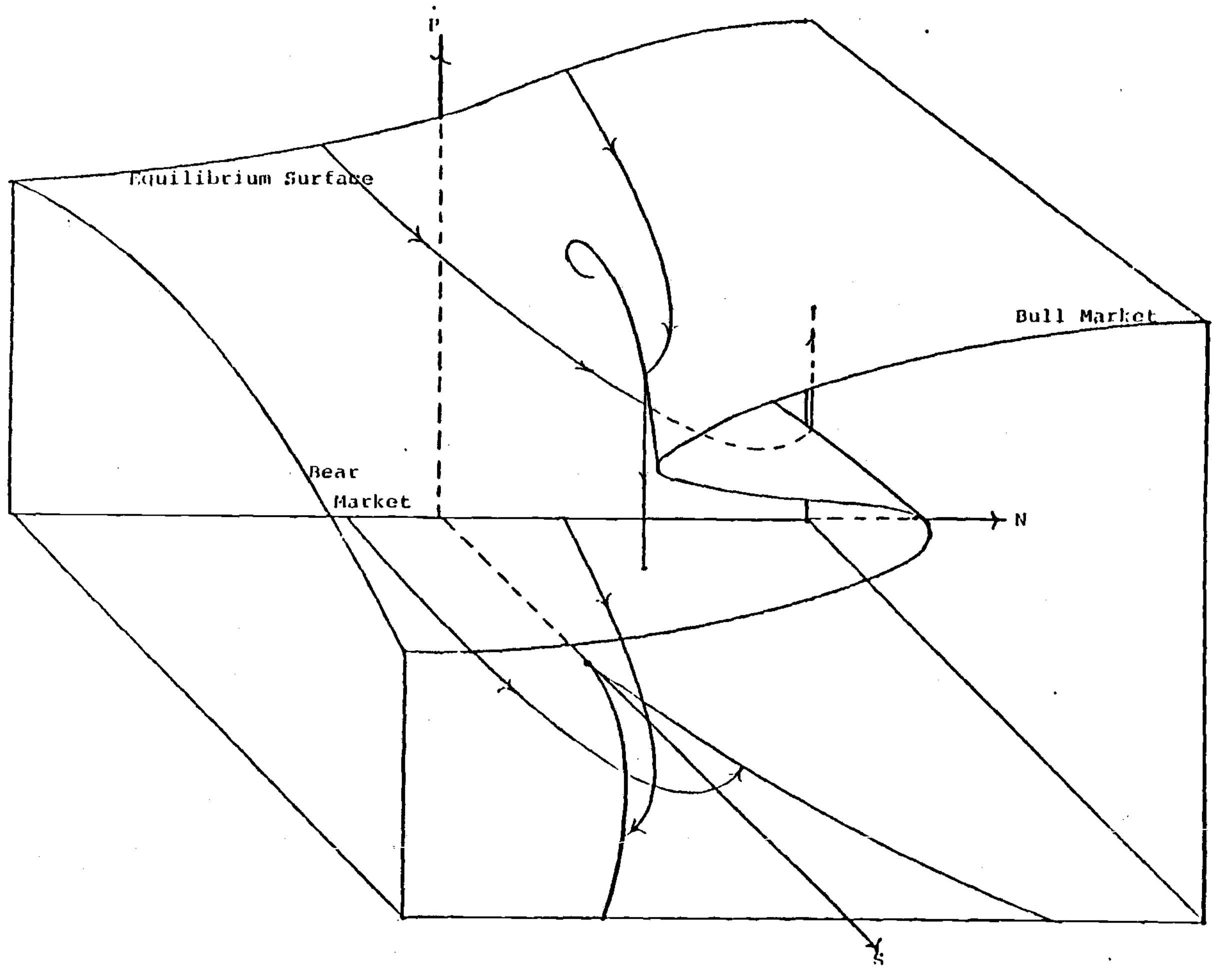

Figure 3.20. Alternative slow flow assumptions for the splitting variable: divergent response with small variations.

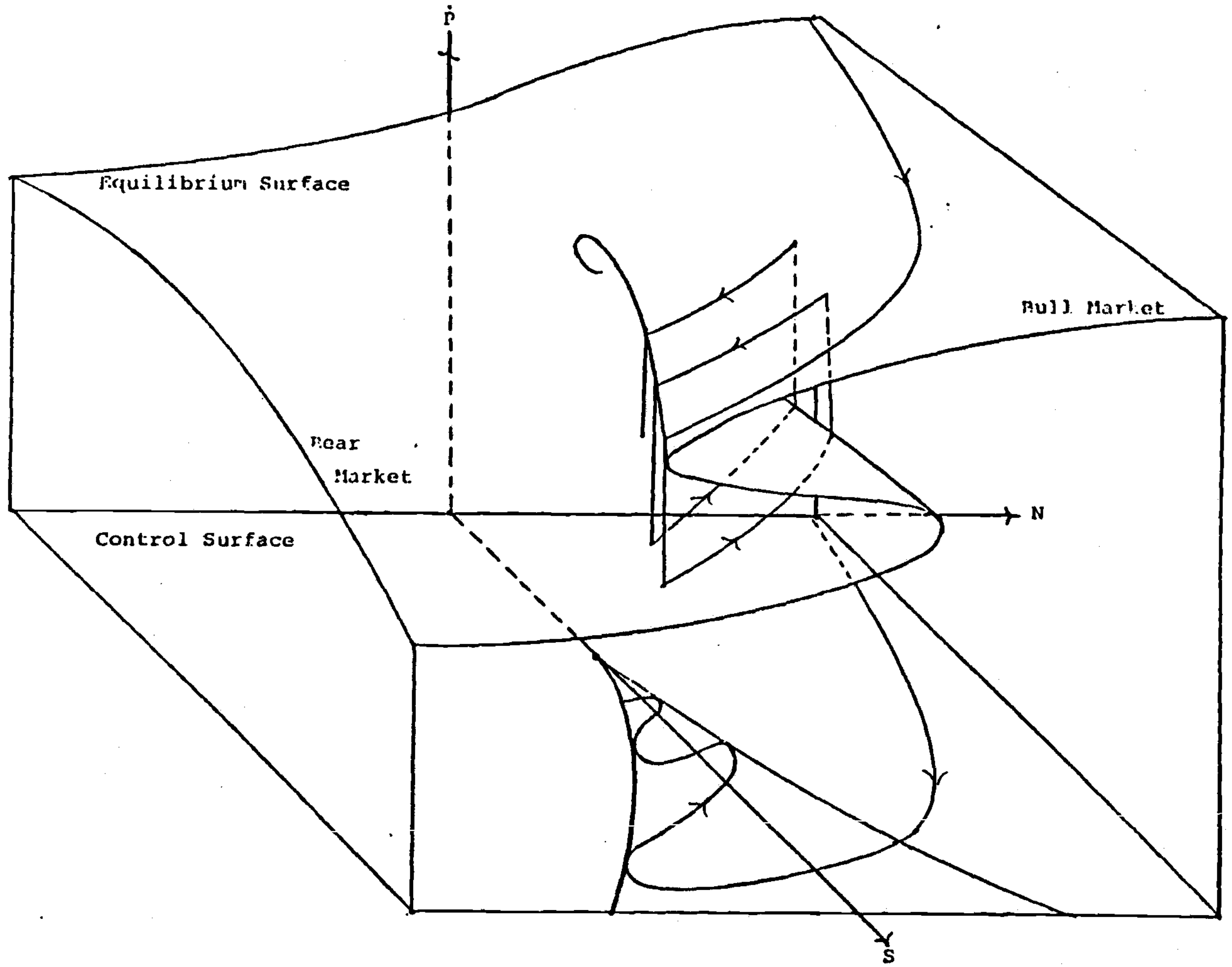

Figure 3.21 Alternative slow flow assumptions for the splitting variable: cycical price oscillations.

movements attract sufficient speculative inventory changes, the flow of the controls continually crosses the cusp. The market tends to exist in either a rising or falling state. The fifth catastrophe flag is hysteresis, the occurrence of irregular cycles. Control flows are unlikely to follow identical paths through the cusp. Hence, repeated crossing of the cusp generates irregular price cycles.

Catastrophe flags are often cited as evidence of an underlying catastrophe structure. In a closed system this may be true. In an open system such as a market, catastrophe flags become very subjective criteria. Changes in the exchange environment could readily mimic the properties of catastrophic behavior. This is an empirical problem and a topic considered in greater detail in the following chapter.

Summary

In the first part of this chapter, a simple profit maximization model of a single period inventory investment opportunity was developed. The model was used to classify the impact of current price information on price expectations. It was then shown that a stable market dynamic is disrupted by a sufficient level of speculative demand.

The speculative dynamic was examined through higher degree approximations of a market potential. A quartic approximation was shown to be the lowest degree approximation which yielded an appropriate dynamic structure: the structure of a cusp catastrophe.

Some of the more important theorems of catastrophe theory were surveyed. Generally, the implications of these theorems for economic applications, appeared to be very limited. The more general methods developed in Chapter Two proved to be a greater value in this regard.

The geometry of a cusp catastrophe was examined. This was followed by a detailed review of Zeeman's model of a stock exchange based on a cusp catastrophe with slow feedback. The economic content of the model development presented by Zeeman was inconsistent. However, the model did appear to offer an adequate framework for the analysis of speculative behavior. A cusp model was readily extended to a competitive market for a stored commodity. An attempt was made to address some of the problems identified in Zeeman's development of cusp market structure. The effort was not overly rewarding. The limitations of the model tended to touch on the limits of economic theory.

Endnotes

1 Models based on optimizing behavior have been subject to substantial criticism. Much of this criticism centers on the degree to which individuals are able to optimize over time. A survey of this problem and an alternative behavioral model are presented by Ronald A. Heiner, "On the Origins of Predictable Behavior." American Economic Review, Vol. 73(4):560-595. In general, an adequate analytical framework for non-optimizing behavior does not exist and optimizing behavior remains a central assumption in economic theory. Our interest in a profit maximizing demand model is limited to showing how expectations may easily disrupt market stability.

2 Rather than treating inventory and supplies separately, available inventories are included in total supplies.

3 Heiner suggests that a crossover mechanism is an alternative to catastrophe modeling. While this point is not pursued in detail, it is of interest to note that he makes an analogous classification of expectations. See Heiner, pp. 582-83.

4 Figures 3.2 and 3.3 may be viewed as cross sections of the cusp catastrophe.

5 The point of equilibrium is chosen as the coordinate origin.

6 Ibid.

7 The cubic coefficient is absorbed into a length of scale transformation.

8 For a detailed treatment of cubic equations see a reference on the theory of equation. The source of this material is from Nelson Bush Conkwright, Introduction to the Theory of Equations (New York: Ginn and Company 1957) pp. 68-77.

9 This discussion of a fold catastrophe is drawn from Saunders, pp. 40-1.

10 For a more detailed treatment of quartic equations see Conkwright, pp. 78-86, or a similar reference on the theory of equations.

11 Since we are assuming there are no price-quantity interactions, this is a somewhat trivial application of the splitting lemma. For a better example, see Saunders, pp. 21-5. For a more rigorous discussion see Gilmore, pp. 23-6 and pp. 41-2.

12 The relation implies equivalence after a diffeomorphism.

13 Here structural instability refers to unstable behavior attributable to internal market interactions. The term structural stability or structural instability may have a number of meanings depending on context in which it is used. This may seem unfortunate, but definitions must be completed to suit the problem at hand (Saunders, pp. 17-8).

14 A more detailed discussion is given by Zeeman, "Catastrophe Theory," pp. 22-27.

15 A simple introduction to jets and determinancy is given by Saunders, pp. 30-5. This material is covered more thoroughly by Poston and Stewart, pp. 123-129, and Gilmore, pp. 615-20.

16 This was developed in Chapter II, pp. 68-77.

17 An example of a function which is not finitely determined is given by Saunders:

$$f(x) = \exp(-1/x^2)$$

18 This can be seen from the determinancy conditions given by equations 3.40a and 3.40b.

19 Removal of terms higher than degree k (by the method shown in Chapter II) does not alter any qualitative properties of the family since each potential in the family is determined by a term of degree less than or equal to k.

20 This discussion of perturbations is based on a derivation of a general form for catastrophes of one variable by Gilmore, pp. 42-5.

21 A mathematically unstable family of functions contains members that, when perturbed slightly, they are excluded from the family. By defining the parameters of a determinant family to be the coefficients of its unfolding terms, all qualitative types of functions are included in the family. Therefore, the family is structurally stable. The family may contain isolated potentials which are structurally unstable, isolated in the sense that we are unlikely to encounter consecutive potentials with degenerate equilibria (they are nowhere dense, comprising geometric objects of a degree lower than the family) and unstable in the sense that nearby potentials are not of the same type. The way in which these structurally unstable potentials are embedded into the family determines the structural character of the family as it represents a given process.

22 For an odd degree catastrophe, the degenerate critical points of the potential divide the control space into at least one empty regime (a region with no stable equilibria). The highest time of the equilibrium equation is of an even degree. For some control values the equilibrium equation may be written:

$$X^{even} = k \quad \text{where } k < 0$$

which has no real roots.

23 This is standard treatment of the cusp catastrophe. For a brief discussion see Saunders, pp. 42-4. For an extended treatment see Gilmore, pp. 97-106.

24 We are assuming that the system remains organized about the nearby local minimum until it disappears. This is called the perfect delay convention. A system which seeks an overall global minimum obeys what is called the Maxwell convention. These conventions represent extremes. There are an endless number of intermediate or barrier conventions. For a detailed discussion of conventions, see Gilmore, pp. 141-146. For a brief review see Saunders, pp. 84-6.

25 See E.C. Zeeman, "Catastrophe Theory." *Scientific American* 234(April 1976):65-83. Published in extended form in E.C. Zeeman, *Catastrophe Theory, Selected Papers 1972-1977*, pp. 1-77, p. 25.

26 Zeeman, "On the Unstable Behaviour of Stock Exchanges."

27 Zeeman, "On the Unstable Behaviour of Stock Exchanges," p. 362.

28 Zeeman, "On the Unstable Behaviour of Stock Exchanges," p. 363.

29 The assumption is not obviously reasonable or unreasonable. It is necessary in that we lack an adequate analytical framework for pure disequilibrium analysis. See Poston and Stewart, pp. 423-5.

30 See Zeeman, "Catastrophe Theory," p. 23.

31 See Zeeman, "On the Unstable Behaviour of Stock Exchanges," p. 370.

32 An admittedly naive approach in that it does not take relative risks into account.

33 Zeeman, "The Umbilic Bracelet and the Double-Cusp Catastrophe." pp. 563-565.

34 The slope of $\dot{P}$ with respect to total excess demand is nonvanishing as the discriminant of the modified cubic equaiton is nonvanishing. Therefore the relationship is structurally stable.

CHAPTER IV

APPLICATIONS

Motivation

We have developed a theoretical framework to examine the impact of price expectations for a stored commodity in a competitive market. In this chapter we will consider quantitative applications of this framework. The methodology explored is directed toward answering three questions. First, is the interrelationship between prices and the formation of expectations a significant force of price instability in agricultural markets? Second, can this interrelationship be adequately represented within a dynamic model of a cusp catastrophe with slow feedback? Third, what implications does a cusp structure hold toward price analysis in the evaluation of marketing alternatives or public policy?

The concepts and methods of catastrophe theory are qualitative. Quantitative applications of a catastrophe model require a number of facilitative assumptions. Some of these assumptions are commonly encountered in quantitative economics, as economic theory is qualitative. However, the intrinsically nonlinear behavior represented by a cusp catastrophe presents several special problems. In the first part of this chapter some of the methodological problems of identifying and modeling catastrophic market behavior are discussed.

Simulation experiments provide a means to investigate qualitatively nonlinear market structures. Simulations can be used to explore quantitative properties of a model and to demonstrate how alternative types of behavior can be represented within a model. Results from simulation experiments can be subjectively compared to observed prices for a stored commodity.

In the remaining sections of the chapter the pricing structure of a specific commodity is investigated. An attempt is made to evaluate the significance of future price expectations in the determination of wheat prices. Generally, hypotheses are designed under the premise of an underlying cusp structure. The implications of these hypotheses are tested using price data from selected cash markets and aggregate information on domestic supplies and disappearances.

Catastrophic Market Behavior: Empirical Methodology

In discussing the application of catastrophe theory in social sciences, Saunders states:

> "If we observe in a system some or all of the features which we recognize as characteristics of catastrophes - sudden jumps, hysteresis, bimodality, inaccessibility and divergence - we may suppose, at least as a working hypothesis, that the underlying dynamic is such that catastrophe theory applies. We then choose what appears to be appropriate state and control variables and attempt to fit a catastrophe model to the observations."[1]

The features noted by Saunders are the properties or flags of a cusp catastrophe presented in the previous chapter. Unfortunately, these characteristics can not be uniquely attributed to an underlying catastrophe structure in an economic system.

There are two reasons why catastrophe flags may not be indicative of a catastrophe structure. First, economic data are discrete. Reported changes in prices and other economic variables are discontinuous. What may constitute a sudden change is a very subjective evaluation. Second, an economic system, such as a market, is open to outside forces. The action of external forces could comprise an alternative explanation for any observed catastrophe flags. If the exchange environment is constantly changing then static equilibrium may appear inaccessible. Oscillations in nontransitory supply and demand may account for observed price cycles (hysteresis) and bimodality. Divergence may be the result of an unaccounted relevant external variable. At the same time, our inability to explain the occurrence of this type of behavior with standard econometric techniques does suggest the possibility of an underlying catastrophe dynamic.

Given the possibility of a catastrophe dynamic, one alternative is to use nonlinear estimation techniques to construct a quantitative model and apply goodness of fit tests. However, there are two reasons why this approach is unlikely to be successful. First, it is very difficult to obtain measures of the control variables. Rene Thom, in discussing quantitative applications of catastrophe theory, states that where explicit observable interpretations are given to the unfolding parameters (controls), many, if not all, of these interpretations will break down.[2] Second, the shape of catastrophe surface (its qualitative topology) is poorly suited to nonlinear estimation. Nonlinear estimation techniques, such as Marquardt's method, utilize local second order information.[3] The geometry of a catastrophe surface is not determined by second order information. Simulations and indirect hypothesis tests provide alternatives to direct estimation. However, Thom's statement points to a general problem in quantitative analysis.

In our theoretical model of a cusp market dynamic the normal factor was defined as nontransitory excess demand. Under ideal conditions it may be possible to obtain good econometric estimates of total excess demand (the change in price is assumed to be a qualitatively linear function of total excess demand). However, there is no plausible means to differentiate between transitory and nontransitory excess demand since the composition of expectations is unknown. Furthermore, under practical conditions, econometric estimates of excess demand are likely to be unreliable as they must be obtained indirectly from partial adjustment models. The splitting factor was only tentatively defined as the slope of the transitory excess demand curve. This simply represents a possible measure of speculative content or the willingness of individuals to speculate on current price trends. We may assume that the splitting factor is dependent on current market information, such as the magnitude and duration of a price trend, which is observable. However, there is no way to determine how market information can be incorporated into an appropriate quantitative measure of speculative content.

Catastrophe theory classifies structures into analogous groups, based on their qualitative characteristics. Quantitative analysis requires explicit definition of the relationships under investigation. To the extent that we can design quantitative models and hypotheses that reflect the qualitative properties of a given market structure, quantitative results may hold some insight into the nature of price determination. However, we should temper our desire for experimental confirmation of an underlying catastrophe or any other market structure. Quantitative economic analysis rests on structural assumptions which are designed to generate a given set of quantitative results. These results, in turn, cannot be offered as evidence for the existence of a given structure. For example, we can not confirm that prices respond to excess demand with a Walrasian price adjustment model.[4] The model used prices to construct a measure of excess demand. We can not confirm that prices are responding to transitory demand if we define transitory demand as a response to changing prices. The value of an economic model, whether qualitative or quantitative, is in the way it may be used to present an organized picture of a very complex system.

Simulations: Model Development

Simulation experiments were conducted to develop a quantitative model of a competitive market based on a cusp catastrophe with slow feedback. The objective of constructing a simulation model is to make a comparative examination of a cusp price dynamic: first, to determine if a cusp structure can be used to generate pricing patterns similar to those observed for a stored commodity, and second, to identify any observable characteristics of a cusp market structure.

The general design of the simulation model is a Walrasian price adjustment system. A canonical form of a cusp catastrophe is utilized as a dynamic fast flow equation. Slow flow equations are developed for the normal and splitting factors. The complete model is a first order differential equation which is solved using an iterative approximation procedure. The final design of the model reflects an extensive trial and error effort. Representative results are presented for three simulation scenarios. First is a closed system adjustment to an initial condition of market disequilibrium. Second is an open system adjustment to continuous external changes. Here the model is used to generate seasonal pricing patterns. Third is an open system adjustment to continuous external changes and random disturbances. Random disturbances are used to represent nonseasonal changes in the exchange environment.

The theoretical market model of cusp catastrophe with slow feedback is based on a rather unrestrictive set of assumptions. Consequently, the model was specified only to within a diffeomorphism of the state and control variables. A number of facilitative assumptions were required to specify the scaler equations required for the simulation. Simplicity was a primary criterion for developing the model. This is reflected in a choice of canonical equations wherever possible. A second criterion was stability. In the absence of external disturbances, the model should converge to a stable point of static equilibrium. It is important to note that the structural stability of the cusp catastrophe does not imply convergence to equilibrium. The convergence properties of the model are determined by the slow flow equations and must be established experimentally. A third criterion is flexibility. The choice of model parameters should allow a wide range of behavior to be represented.

The canonical form of a cusp catastrophe selected for the fast flow equation is given by:

$$\dot{P}^3 - (S - S_o)\dot{P} - N = 0 \tag{4.20}$$

where;

N = a normal factor
S = a splitting factor
S_o = a fixed parameter locating the cusp.

The normal factor is taken to represent nontransitory excess demand:

$$N = Q_{ND} - Q_{NS} \tag{4.21a}$$

where;

Q_{ND} = nontransitory demand
Q_{NS} = nontransitory supply

The supply and demand equation were assumed to be linear. The normal factor equation was respecified as a function of price and the external supply and demand parameters:

$$N = (b_1 - b_2)(P - (u-v)/(b_1 - b_2) \tag{4.21b}$$

where;

b_1 = slope of the demand curve
b_2 = slope of the supply curve
u = demand shift parameter
v = supply shift parameter

A more convenient form of the equation was used for the simulation, written:

$$N = \delta(P - P_e) \tag{4.21c}$$

where;

δ = a fixed adjustment rate
P_e = static equilibrium price determined by the nontransitory supply and demand parameters.

The splitting factor, which may represent the slope of the transitory demand curve, is taken to be a function of the absolute magnitude and duration of the current price trend. The magnitude of a trend is assumed to be positively related to the splitting factor while the duration of a trend is assumed to be negatively related. It was required that the controls vary continuously with time. Both the magnitude and duration of the current trend are discontinuous parameters. Breaks occur at the termination of a trend. This presents some problems which should be considered in detail.

One way in which the splitting factor may be specified as a continuous function of time is to assume that the rate of change in S ($\dot{S}$) is a function of the absolute magnitude and duration of the current price trend:[5]

$$\dot{S} - f(|\Delta P|, t_d) \qquad (4.22)$$

where;

ΔP = the magnitude of the current trend
t_d = the duration of the current trend.

The solution to this open form equation may be approximated with a discrete procedure. This was the initial approach taken. A number of linear and nonlinear forms were investigated. In general, these efforts proved unsatisfactory. It was difficult to scale prices and price changes to desired levels. Model performance and stability were affected by relatively small parameter changes. A closed form equation was needed.

It was helpful to examine a number of known solutions to differential equations with parameters which are discontinuous with respect to time.[6] Generally, an explicit solution to a discontinuous differential equation requires prior knowledge of the magnitude and location of any discontinuities. The solution commonly involves a series of time dependent functions, corresponding with the intervals between discontinuous points, which together vary continuously with time. Here, discontinuities occur endogenously. Prior information is unavailable. However, using a similar approach a continuous transition between endogenously determined price trends can be specified. The equation selected for the splitting factor is written:

$$S = S(t^*)e^{-\lambda_1 t_d} + \alpha|\Delta P|e^{-\lambda_2 t_d} \qquad (4.23)$$

where;

$S(t^*)$ = the value of the splitting factor at a discontinuity
λ_i = dampening parameter
α = driving parameter.

At the termination of a price trend the value of the splitting factor is $S(t^*)$. At the initial point of a new trend the magnitude and duration of the current trend are zero. Hence:

$$S = S(t^*)e^{0} = S(t^*)$$

The transition between price trends is continuous.

The endogenous parameters for the splitting factor are consistent with the initial assumptions. The impact of the absolute magnitude of a price trend is positive:

$$\frac{\partial S}{\partial \Delta P} = \alpha e^{-\lambda_2 t_d} > 0$$

and the impact of the trend duration is negative:

$$\frac{\partial S}{\partial t_d} = -\lambda_1 S(t^*)e^{-\lambda_1 t_d} - \lambda_2 \alpha |\Delta P| e^{-\lambda_2 t_d} \leq 0$$

The fixed driving parameter (α) is intended to modify the effect of the trend magnitude. The fixed dampening parameters (λ_i) are intended to modify the effect of the trend duration. However, an inspection of the above two equations shows that the parameters are interdependent. The equation for the splitting factor is linear in price and exponential in time. Hence, for a sufficiently long trend the splitting factor will approach zero. This does not imply that S will converge to zero in the absence of external forces or disturbances. Static equilibrium is potentially unstable. The behavior of S is dependent upon the overall structure of the model.

The complete dynamic equation for the simulation model is given by the equation:

$$\dot{P}^3 - [S(t^*)e^{-\lambda_1 t_d} + \alpha|\Delta P|e^{-\lambda_2 t_d} - S_o] - \delta(P - P_e) = 0 \tag{4.24}$$

There are a total of ten explicit parameters; four endogenous (P, $|\Delta P|$, t_d, $S(t^*)$), one external (P_e) and five internal constants (λ_1, λ_2, α, S_o, δ). In solving the model for market prices, time becomes an explicit parameter. Time within the simulation must be made to correspond with time periods the model is intended to represent. This can be achieved by scaling simulation time so that price changes generated by the model correspond to the general magnitude of observed price movements over a given period. Initial values must be selected for the endogenous parameters. Typically these are zero for the magnitude and duration of the current price trend. An initial price may be selected which is representative of a given commodity. The model operates on relative rather than absolute price levels. Selection of the fixed parameters was again a trial and error process.

In conducting simulation experiments, some information concerning the sensitivity of the model to parameter changes was discovered. The performance of the model was not greatly affected by changes in two of the fixed parameters, S_o and λ_1. The parameter S_o locates the cusp point. At values of S greater than S_o prices may oscillate between rising and falling markets. S_o may be selected to eliminate insignificant levels of speculative price variation. The parameter λ_1 determines the rate at which past levels of speculative content is dissipated. The dampening parameter λ_1 may be chosen to be relatively large in order to effectively eliminate the endogenous parameter S(t*). The model is sensitive to the remaining fixed parameters.

Using the rules for differentiating implicit functions, the derivatives of $\dot{P}$ with respect to the parameters δ, α and λ_2 may be written:

$$\frac{\partial \dot{P}}{\partial \delta} = (P-P_e)/(3\dot{P}^2 - S + S_o) \tag{4.25a}$$

$$\frac{\partial \dot{P}}{\partial \alpha} = (\dot{P}\alpha|\Delta P|e^{-\lambda_2 t_d})/(3\dot{P}^2 - S + S_o) \tag{4.25b}$$

$$\frac{\partial \dot{P}}{\partial \lambda_2} = -(\dot{P}\lambda_2\alpha|\Delta P|e^{-\lambda_2 t_d})/(3\dot{P}^2 - S + S_o) \tag{4.25c}$$

For the parameter δ, which may be used to control the rate of adjustment in nontransitory excess demand:

$$\text{Sign}\left(\frac{\partial \dot{P}}{\partial \delta}\right) = -\text{Sign}(N) \tag{4.26a}$$

As the rate of adjustment in nontransitory excess demand is increased (δ becomes more negative) two things occur: one, the rate at which prices converge toward equilibrium is increased and two, the rate at which prices diverge from equilibrium is reduced. The behavior of prices becomes more stable. This may be interpreted as follows: if nontransitory supply and demand dominate a market then the market adjustment should remain relatively stable. The directional effects of the splitting factor parameters on prices are:

$$\text{Sign}\left(\frac{\partial \dot{P}}{\partial \alpha}\right) = \text{Sign}(\dot{P}) \tag{4.26b}$$

$$\mathrm{Sign}\left(\frac{\partial \dot{P}}{\partial \lambda_2}\right) = -\,\mathrm{Sign}(\dot{P}) \tag{4.26c}$$

These effects correspond to the impact of the parameters on the splitting factor. Increasing the driving parameter (α) increases the splitting factor and the level of price variation. Increasing the dampening parameter decreases the splitting factor and reduces the level of price variation.

The external parameter P_e, specifying the location of static equilibrium, may be used to represent the impact of external forces. Discontinuous changes in P_e may be interpreted as a sudden shock. Continuous changes in P_e may be used to reflect seasonal changes in supply or other continuous changes in the exchange environment. In general, equilibrium price levels may be selected to reflect a specific commodity.

Simulations: Solution Procedure

A computer program was designed to solve the simulation equation for market prices. The technical details of the simulation program are presented in Appendix A. The program has three major components: one, a master control component with utilizes an iterative Runge-Kutta procedure to solve the differential equation for prices; two, a subroutine which solves the cubic cusp equation for $\dot{P}$; three, a subroutine which computes the normal and splitting factors. A flowchart for the simulation program is presented in Figure 4.1.[7]

Runge-Kutta procedures are treated in standard texts on differential equations (Braun, 1978) and numerical analysis (Hildebrand, 1956). The Runge-Kutta method for solving differential equations is widely used because of its accuracy and simplicity. A third order method is utilized in the simulation model, written:

$$P_{t+1} = P_t + \frac{\Delta t}{6}(I_0 + 4I_1 + I_2) \qquad (4.30)$$

where;

$$I_0 = \dot{P}:[t,\ P_t]$$
$$I_1 = \dot{P}:[t+\ 1/2\Delta t,\ P_t+1/2\Delta t I_0]$$
$$I_2 = \dot{P}:[t+\Delta t,\ P_t+2\Delta t I_1-\Delta t I_0]$$

and,

$\dot{P}$:[] denotes the evaluation of the implicit equation defining $\dot{P}$ at a specified time and price.

The iteration increment chosen for all simulation runs is Δt=.005. Observations on all variables are recorded at five iteration intervals ($T=5\Delta t$). Each iteration is

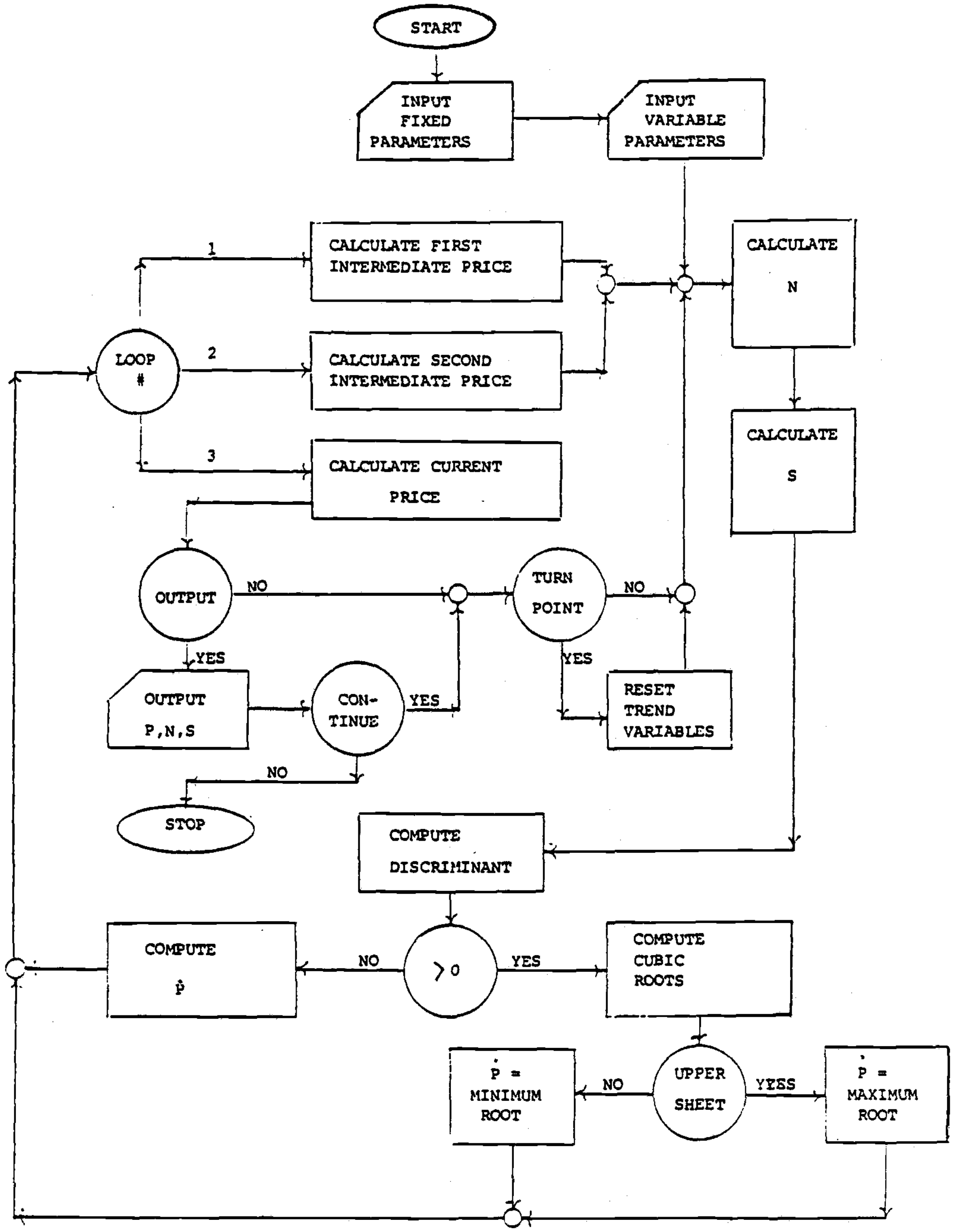

Figure 4.1. Flow chart for the cusp market simulation.

checked for a turning point. At turning points variables are recorded and the trend parameters are reset.

For a third order Runge-Kutta procedure the levels of local and global errors are of an order:

$$\text{Local Error} = O(\Delta t)^4 = O(6.25\text{x}10^{-10}) \qquad (4.31a)$$

$$\text{Global Error} = O(\Delta t)^3 = O(1.25\text{x}10^{-7}) \qquad (4.31b)$$

where;

$O(\ )$ = order; a linear function of an unknown constant.

While these formulas may not be completely accurate, they do suggest that errors should be acceptably small.[8] Test runs conducted with a smaller iteration increment (Δt=.001) did not alter prices to within two decimal places.

The cubic equation of the price dynamic must be solved three times for each iteration of the Runge-Kutta procedure. Since the dynamic is a variant of a reduced form cubic equation direct solution methods are most efficient.[9] The discriminant for the equation for given values of the normal and splitting factor is computed. If the discriminant is greater than or equal to zero, Cardan's formula may be applied to find a single real root:

$$r = \left[\frac{N + \sqrt{N^2 - (4/27)S^3}}{2}\right]^{1/3} + \left[\frac{N - \sqrt{N^2 - (4/27)S^3}}{2}\right]^{1/3} \qquad (4.32)$$

If the discrininant is less than zero a trigonometric solution, known as the method of cosines, may be applied to find three real roots:

$$r_i = 2\sqrt{S/3}\,\cos\frac{\theta + 2i\pi}{3} \qquad i=0,\ 1,\ 2 \qquad (4.33)$$

where;

$$\theta = \cos^{-1}\left[N/(2\sqrt{S^3/27})\right]$$

If the cusp is entered from the upper (positive) sheet, the largest positive root is selected. If the cusp is entered from the lower (negative) sheet, the most negative root is selected. This allows discontinuities to occur only after completely passing through to the other side of the cusp. This is the perfect delay convention for a cusp catastrophe.

Computational procedures for the normal and splitting factors is straight forward and needn't be elaborated in detail. The normal factor is computed using equation 4.21c. External changes and disturbances are incorporated through incremental changes in the static equilibrium price. Continuous changes are approximated by small step changes in P_e within each iteration. The splitting factor is computed using equation (4.23). The endogenous parameters are calculated using intermediate prices from the Runge-Kutta procedure and variable values recorded at turning points.

Simulation Results

The simulations presented here were designed to reflect observed prices and price variation for wheat over the last twelve years. Wheat prices were selected for three reasons. First, on a worldwide basis, wheat is perhaps the single most important agricultural commodity. Second, wheat prices have exhibited a high degree of variability since the beginning of the 1972/73 crop year. Third, wheat production and marketing is fairly representative of grain industries in general. Results generated for wheat prices are readily extended to corn and other coarse grain prices.

In the first set of simulation runs, the model is placed in an initial disequilibrium condition and allowed to adjust in the absence of external disturbances. While this might be interpreted as a simulated response to an external shock, the primary purpose of these runs is to illustrate model behavior under different choices of the model parameters. Results are presented graphically here and in tabular form in Appendix A.

Three groups of simulations were run under pure disequilibrium adjustment conditions. A different set of fixed parameters was selected for each group. In the first two groups of simulation runs, three different initial and equilibrium price conditions were specified. The third group consists of a single run in which the parameters were set to produce unending speculative price cycles. The parameter values selected for the simulation runs are summarized in Table 4.1.

In the first group of simulations, there is only one major price cycle in the adjustment path toward static equilibrium (Figure 4.2a). The severity of the cycle is dependent upon the difference between the initial and equilibrium price. The trajectories of the control variables through the control space are illustrated in Figure 4.2b. Each trajectory enters the cusp from outside. Cyclical variations are contained within the cusp. As the magnitude of the price cycles decline, the trajectories approach the cusp point.

In the second group of simulation runs, the frequency and severity of the price cycles were increased by increasing the driving and dampening parameters of the splitting factor equation (Figure 4.3). The parameter changes effectively increased the rate at which

Table 4.1. Parameter values and initial conditions for the pure disequilibrium adjustment simulations runs; cusp market model, run set I.[a]

Run Group	Fixed Parameters					Initial Run Conditions					
						a		b		c	
	δ	λ_1	λ_2	α	S_o	P_o	P_e	P_o	P_e	Po	P_e
1	100	45	3	50	2	3.75	4.00	4.00	3.60	3.60	3.75
2	100	90	12	120	2	3.75	4.00	4.00	3.60	3.60	3.75
3	100	120	9	120	2	3.75	4.00	--	--	--	--

[a]Initial values for all endogenous trend parameters are set to zero for each simulation run.

speculative demand enters and exits the market. In the final simulation run of the first set, the primary dampening parameter, λ_2, was reduced to produce an unending series of price cycles (Figure 4.4). The cycles are bounded, owing to the exponential dampening parameters, but static equilibrium is unobtainable. The overall structure of the model might be considered stable in a mathematical sense. However, the market is not stable in an economic sense if we assume that pure speculation can not sustain price movements indefinitely.

Figure a

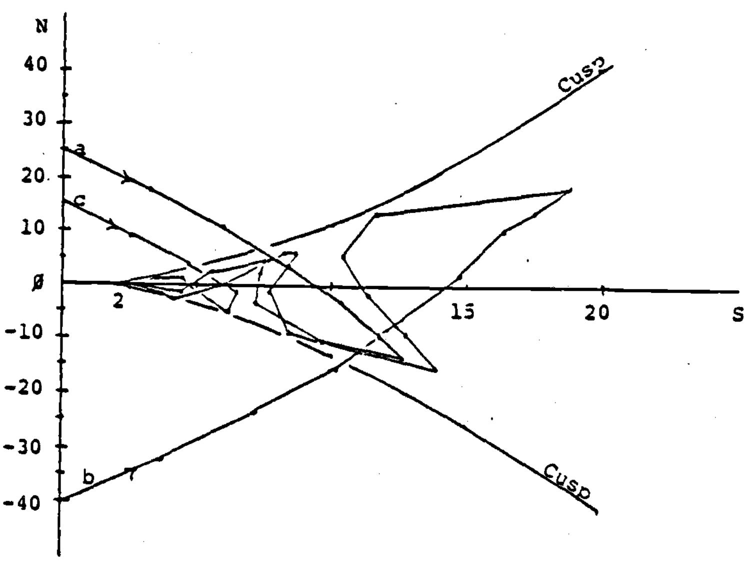

Figure b

Figure 4.2. Simulated wheat prices, pure disequilibrium adjustment; group 1: Figure a) prices over time, Figure b) control trajectories.

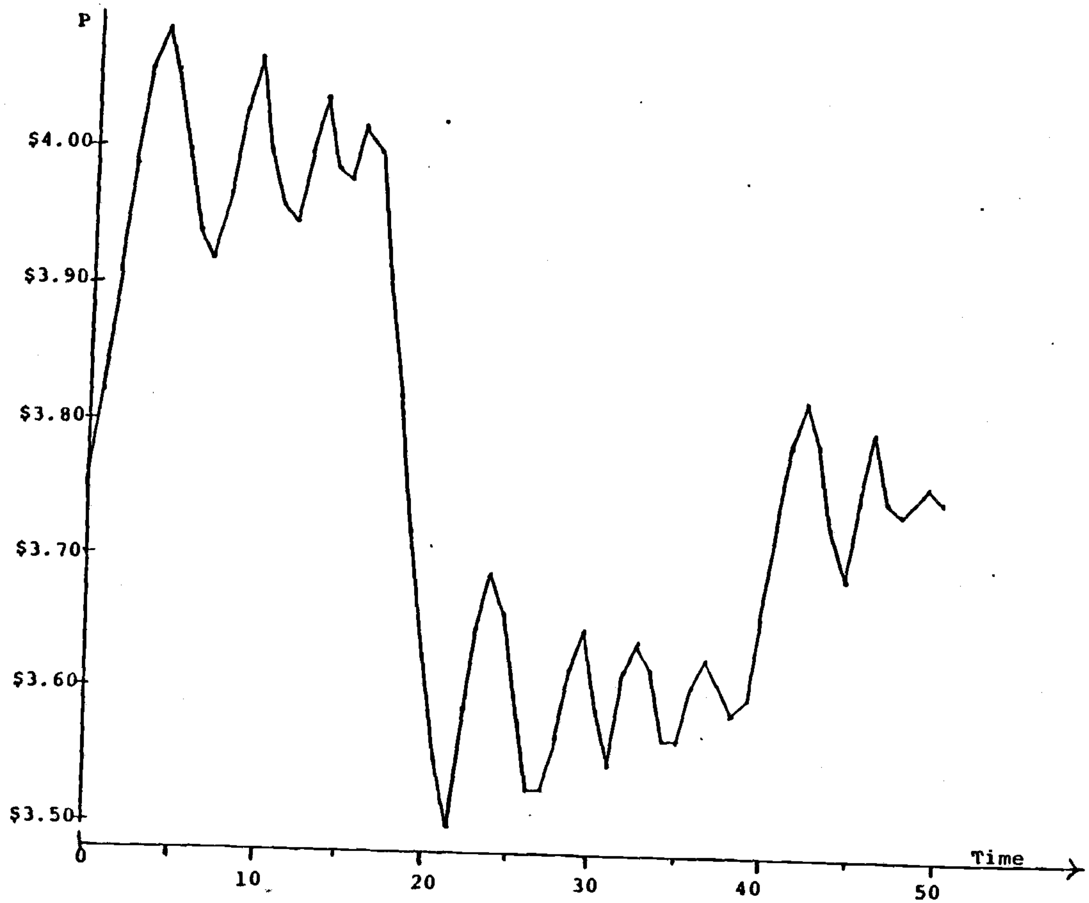

Figure 4.3. Simulated wheat prices over time, pure disequilibrium adjustment; group 2.

Table 4.2. Parameter values, initial conditions and external adjustment rates for the continuous adjustment simulations.

Run Group	Fixed Parameters				Initial State			External Adjustment Parameters Rate and Time											
	δ	λ_1	λ_2	α	S_o	P_o	P_e	P_e	T	P_e	T	P_e	T	P_e	T	P_e	T	P_e	T
1.1	100	45	3	60	2	3.50	3.50	.025	20	0	5								
1.2	100	60	3	65	2	3.50	3.50	.025	20	0	11								
2.1	100	120	9	120	2	3.00	3.00	.050	8	.025	16								
2.2	100	120	9	120	2	4.00	4.00	−.050	8	.025	16								
2.3	200	90	9	140	3	4.00	4.00	−.075	8	.025	16								
3.1	150	120	9	120	3	4.00	4.00	−.075	8	.025	16	.050	8	.025	16				
3.2	200	90	9	140	3	4.00	4.00	−.075	8	.025	16	.075	8	.025	16	−.075	8	.025	16

The structural stability of a cusp catastrophe does not guarantee a stable market structure. For a given model and parameter choices, this must be established by experimentation.[10] However, the stability of static equilibrium under large external shocks may not be relevant if the exchange environment is assumed to adjust continuously.

In the second set of simulation runs the market model is placed in an initial state of equilibrium. External changes in nontransitory excess demand are introduced through changes in the equilibrium price. The level of change in the equilibrium price is specified for each observation interval. Actual changes in the equilibrium price are made in small equal increments at each iteration. Three groups of simulations are presented. In the first group of two simulation runs, the equilibrium price was increased gradually from $3.50 to $4.00 per bushel. In the second group of simulation runs, normal seasonal patterns of crop year price adjustment were explored. In the third group of simulation runs, seasonal patterns of price variation were extended between crop years. Graphical results are presented here, tabular results are presented in Appendix A. Parameter values for the second set of simulations are summarized in Table 4.2.

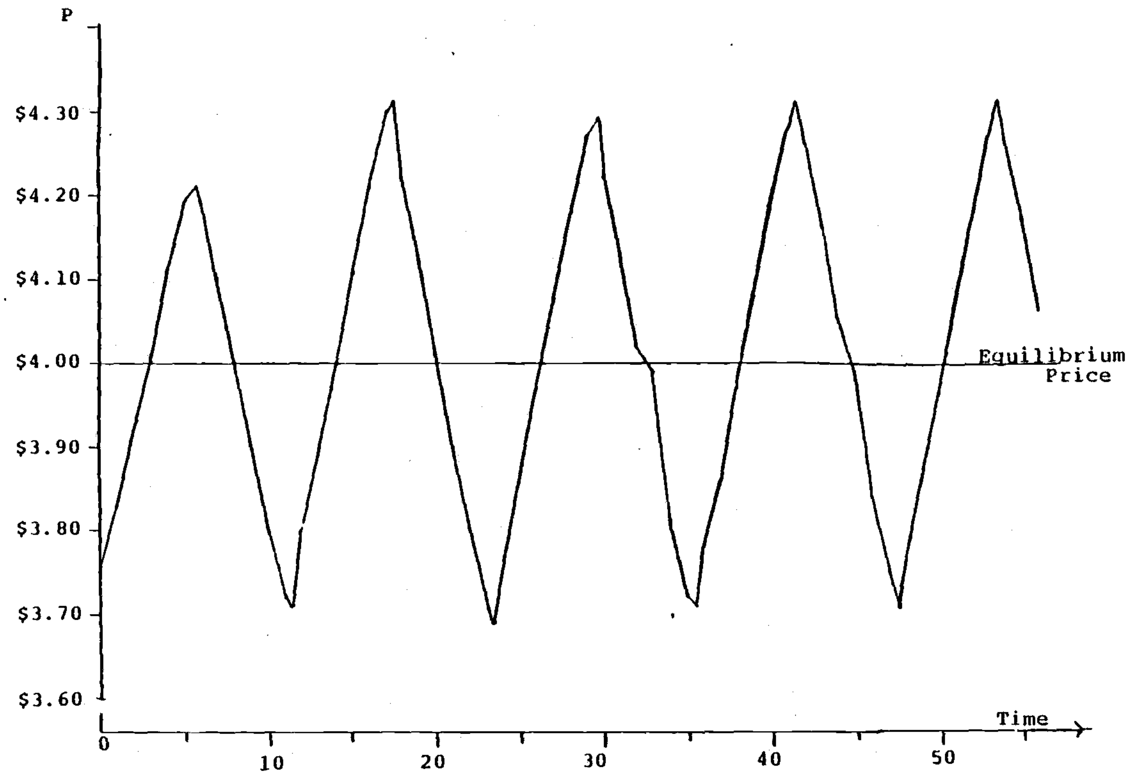

Figure 4.4. Simulated wheat prices over time, pure disequilibrium adjustment bound cycles; group 3.

In the first group of simulation runs, the model was started at an initial equilibrium point of $3.50. The equilibrium price was then increased to $4.00 at a rate of 2.5 cents per observation interval. After the equilibrium price reached $4.00, the model was allowed to stabilize without any further external disturbances. Prices over time for the first run are plotted in Figure 4.5a. The control trajectories for the first run are plotted in Figure 4.5b. An interesting pattern is evident in the price graph. Prices cycle about the trend in external market conditions. An extended period of overestimation is followed by a short period of underestimation. This type of pattern is commonly observed in actual commodity prices which are tending to trend sharply upward or downward. The driving parameter for the splitting factor is increased slightly in the second run to amplify the price cycles. Prices over time are plotted in Figure 4.6a. Price changes over time are plotted in Figure 4.6b. The graph of price changes reveals a second interesting feature. Periods of relative smooth price adjustment are separated by large sudden jumps. The distribution of price changes along the dynamic may tend to cluster about a mean near zero with the sudden jumps at the tails of the distribution.[11]

In the second group of simulations, annual patterns of price adjustment are generated. Observations are intended to represent two week periods. Equilibrium prices were allowed to adjust relatively quickly over the first eight observations representing the harvest period. In the first two simulations, equilibrium prices were increased and decreased at a rate of 10 cents per month over harvest (Figures 4.7a and 4.7b respectively). For the remaining 16 observations representing the balance of the crop year, equlibrium prices were increased relatively slowly at 5 cents per month, to reflect the gradual decline in available supplies. In the last month of the crop year, speculative demand was artificially eliminated by setting the driving parameter for the splitting factor equal to zero. This was done to reflect the possibility that expectations for a continuing trend may give way in anticipation of price changes over the harvest period. A similar set of conditions were specified for the third simulation run. In the third run, the rate of change in prices over harvest was increased and the rate at which the model responded to price changes was increased (Figures 4.8a and 4.8b).

The general seasonal pattern of price adjustment exhibited be the second group of simulation runs is

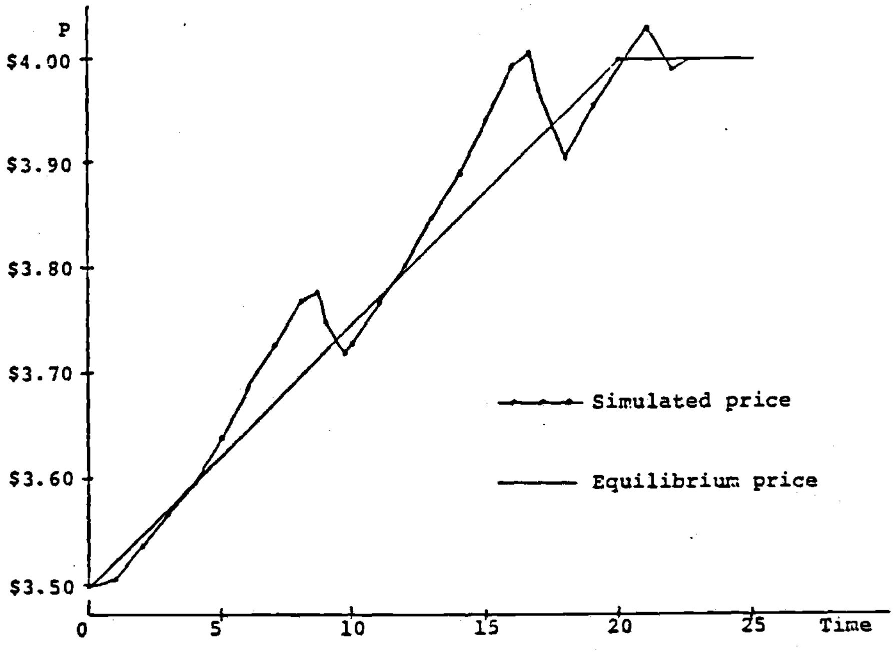

figure a

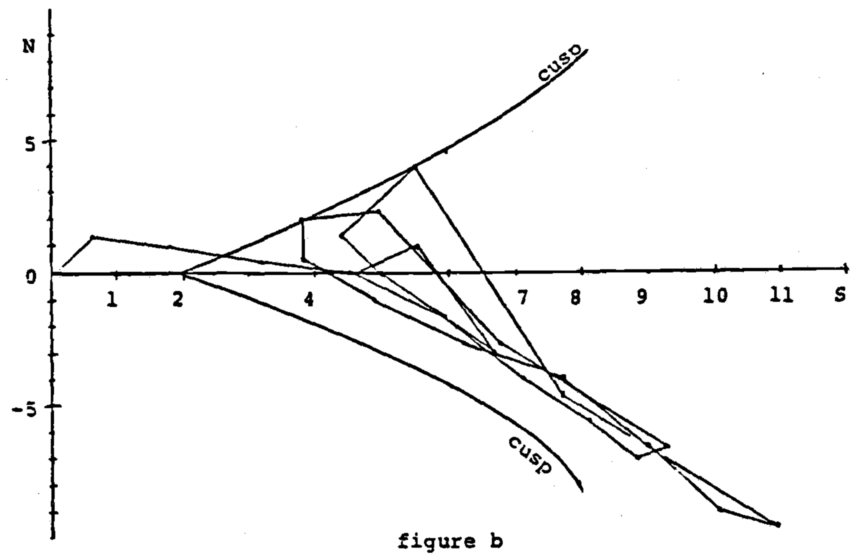

figure b

Figure 4.5. Simulated wheat prices, continuous adjustment; group 1, run 1: Figure a) prices over time, Figure b) control trajectory.

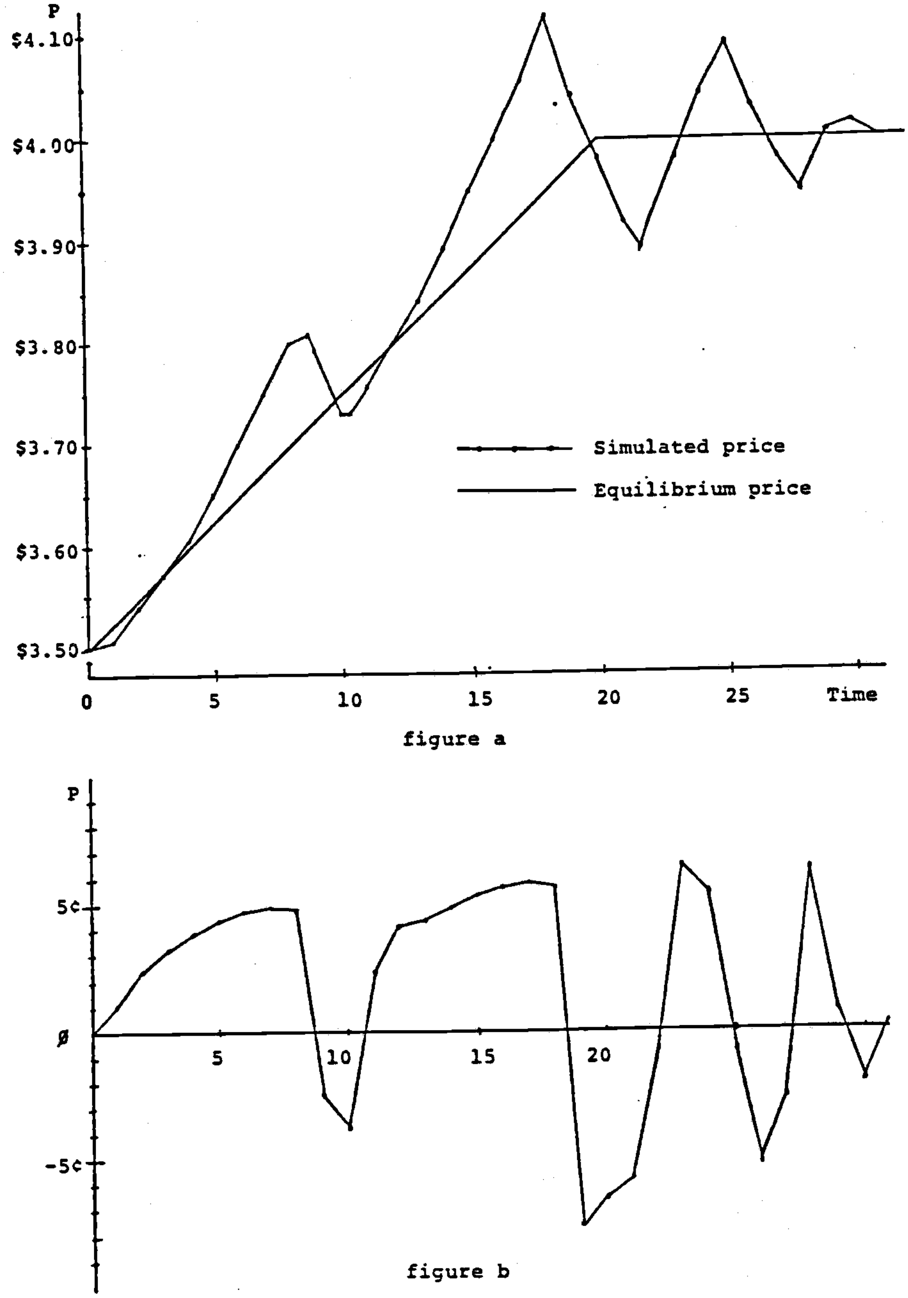

Figure 4.6. Simulated wheat prices, continuous adjustment; group 1, run 2: Figure a) prices over time, Figure b) price change over time.

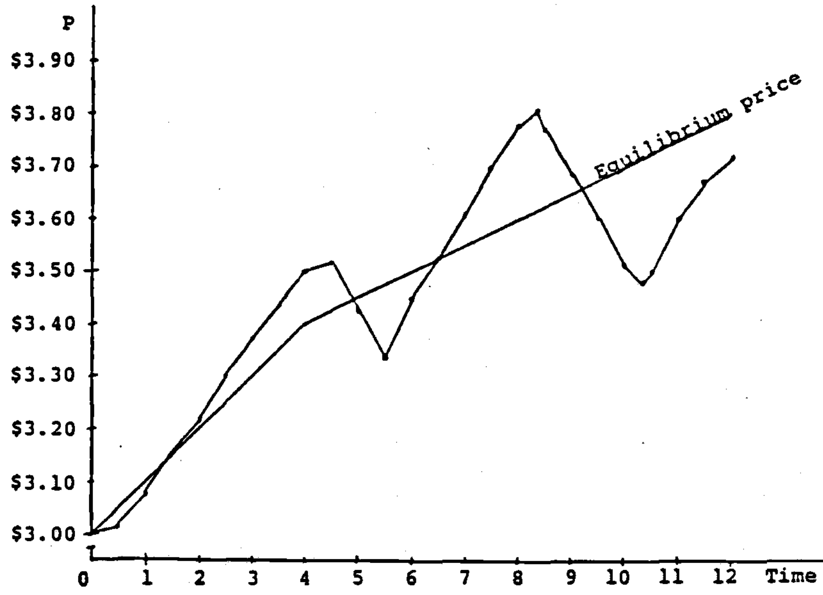

figure a

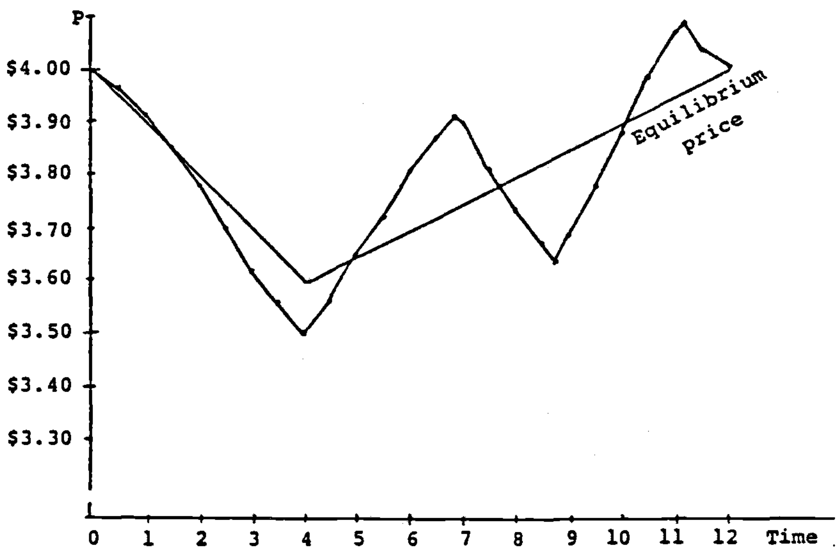

figure b

Figure 4.7. Simulated wheat prices, continuous external adjustment over a crop year: Figure a) run 1, Figure b) run 2.

striking. The harvest period trend is overestimated. Prices then cycle about the upward trend in equilibrium prices over the remainder of the crop year. Price cycles through crop years are commonly observed in wheat and other grain prices. By increasing the rate at which the model responds to price changes, the degree to which the harvest trend is overestimated is not greatly affected. However, the number of price cycles in the remainder of the crop year is increased. In all three runs, the magnitude of the price cycles tends to decline. The variation introduced by catastrophic behavior is more clearly seen in the graph of price changes over time for run three (Figure 4.8b).

In the third group of continuous external adjustment simulations, the artificial elimination of speculative content is removed, and prices are generated over a two to three year period. In the first run, a two year period is simulated. There is very little change in the seasonal pattern of adjustment generated by the model (Figure 4.9). The first year exhibits relatively high speculative content while the second year exhibits relatively low speculative content. In the second run, a three year period is simulated. The rate at which the model responds to external changes was increased. However, there is no

figure a

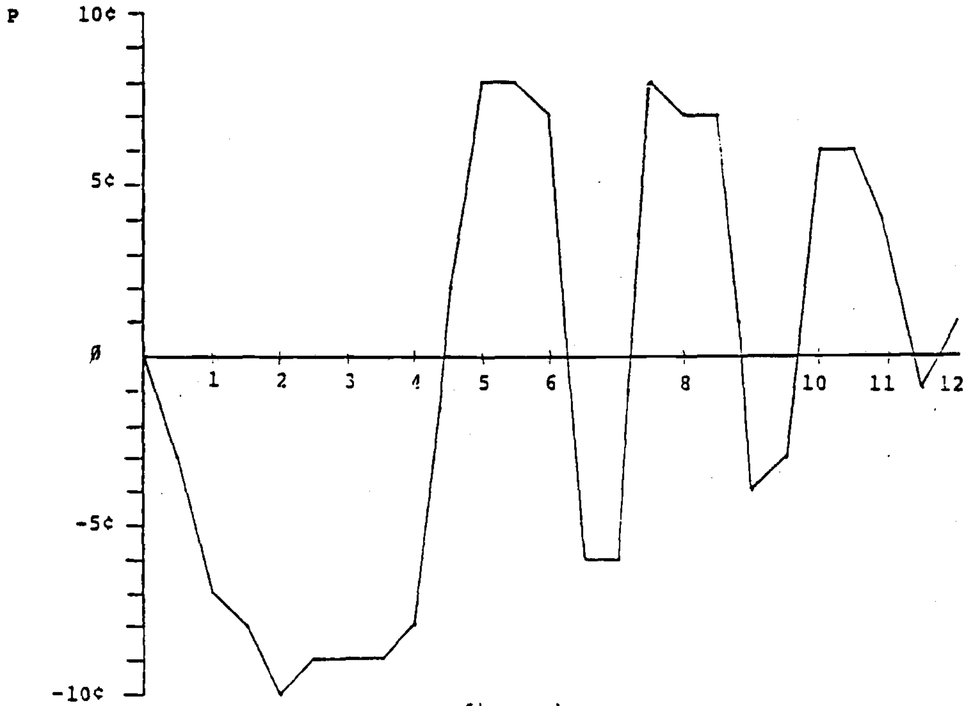

figure b

Figure 4.8. Simulated wheat prices, continuous external adjustment over a crop year; run 3: Figure a) prices over time, Figure b) price changes over time.

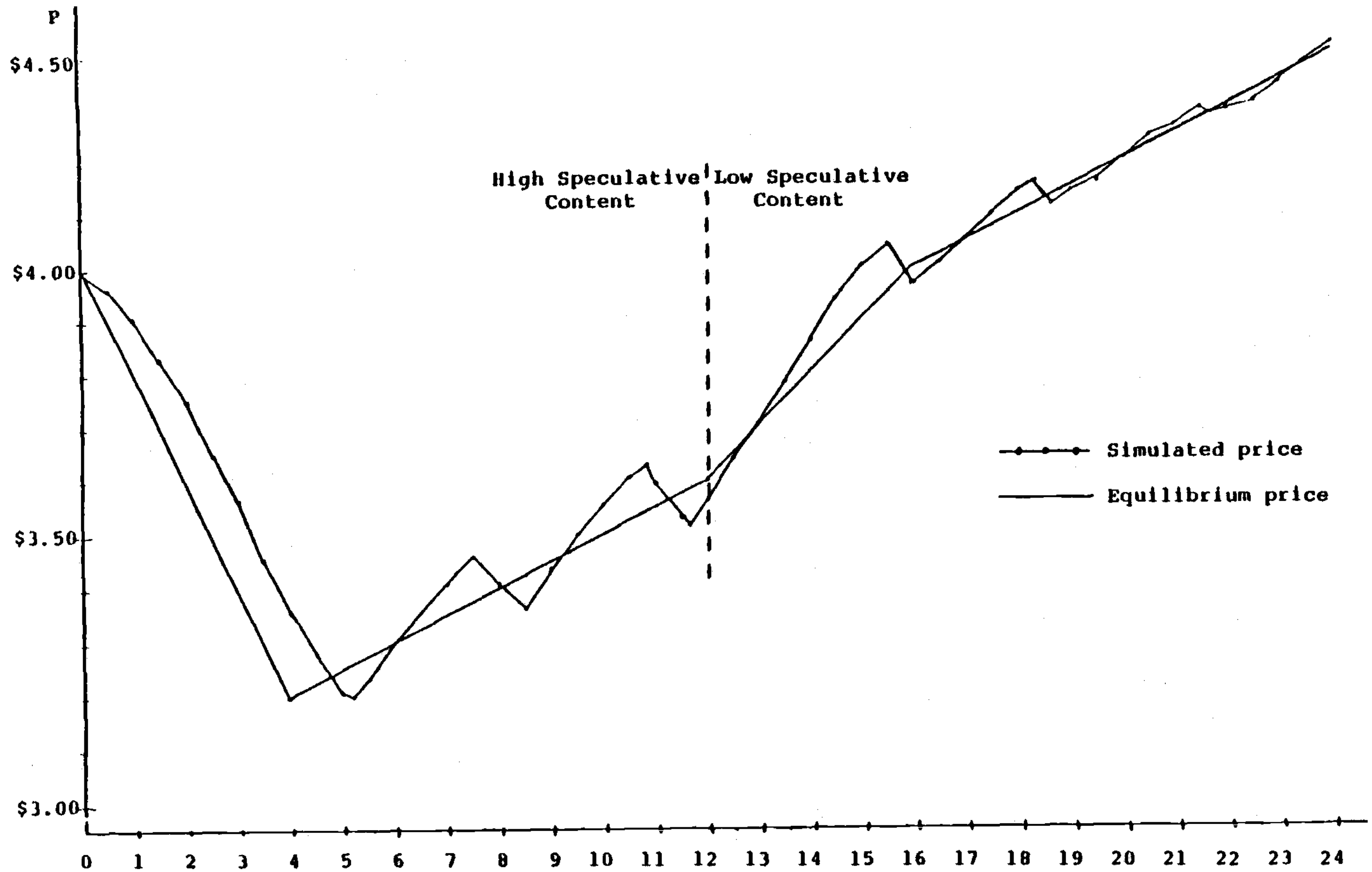

Figure 4.9. Simulated wheat price, continuous external adjustment over two crop years, group 3, run 1.

significant change in the pattern of seasonal adjustment; harvest trends are overestimated and prices cycle about the upward trend in equilibrium prices over the remainder of the crop years. Simulated prices for years one and three exhibit relatively high speculative content while speculative content in the second year is relatively low (Figure 4.10).

In the third and final set of simulations, random disturbances are introduced into the continuous external adjustment of equilibrium prices. Two runs from the seasonal price adjustment simulations were selected as base lines for the introduction of random disturbances; the third run of the single crop year simulation (group 2, run 3) and the three year price simulation (group 3, run 2). A random disturbance was added to the seasonal change in the equilibrium price at each observation. The net change in the equilibrium price was entered in small increments at each iteration. The random disturbances were taken from a normal distribution with a mean of zero and a standard deviation of $0.05 per bushel. The actual generation of random disturbances was accomplished with a pseudo random number generator; details of the process are presented in Appendix A, along with tabular results.

In the first random disturbance run, parameter values and the seasonal pattern of adjustment in equilibrium prices correspond to the third run of the single crop year simulations (Figure 4.8). Simulated prices and price changes over time for the first run are presented in Figure 4.11. The seasonal decline in prices over harvest is still in evidence. However, the gradual increase in the equilibrium price due to declining stocks is almost completely obscured by speculative price cycles and random disturbances. The sudden jumps along the price dynamic can be clearly seen in the graph of price changes over time. However, the pattern of relatively smooth price movements followed by a sudden change is somewhat obscured by the random disturbances. In the second random disturbance run, parameter values and the seasonal pattern of adjustment in equilibrium prices correspond to the three year price simulation (Figure 4.10). Simulated prices over time are presented in Figure 4.12. In comparing the two three year simulations, the introduction of random disturbances increased the variability of prices. The impact of random disturbances is sometimes moderated by the level of speculative demand. At other times, the impact of random disturbances is amplified by speculative demand.

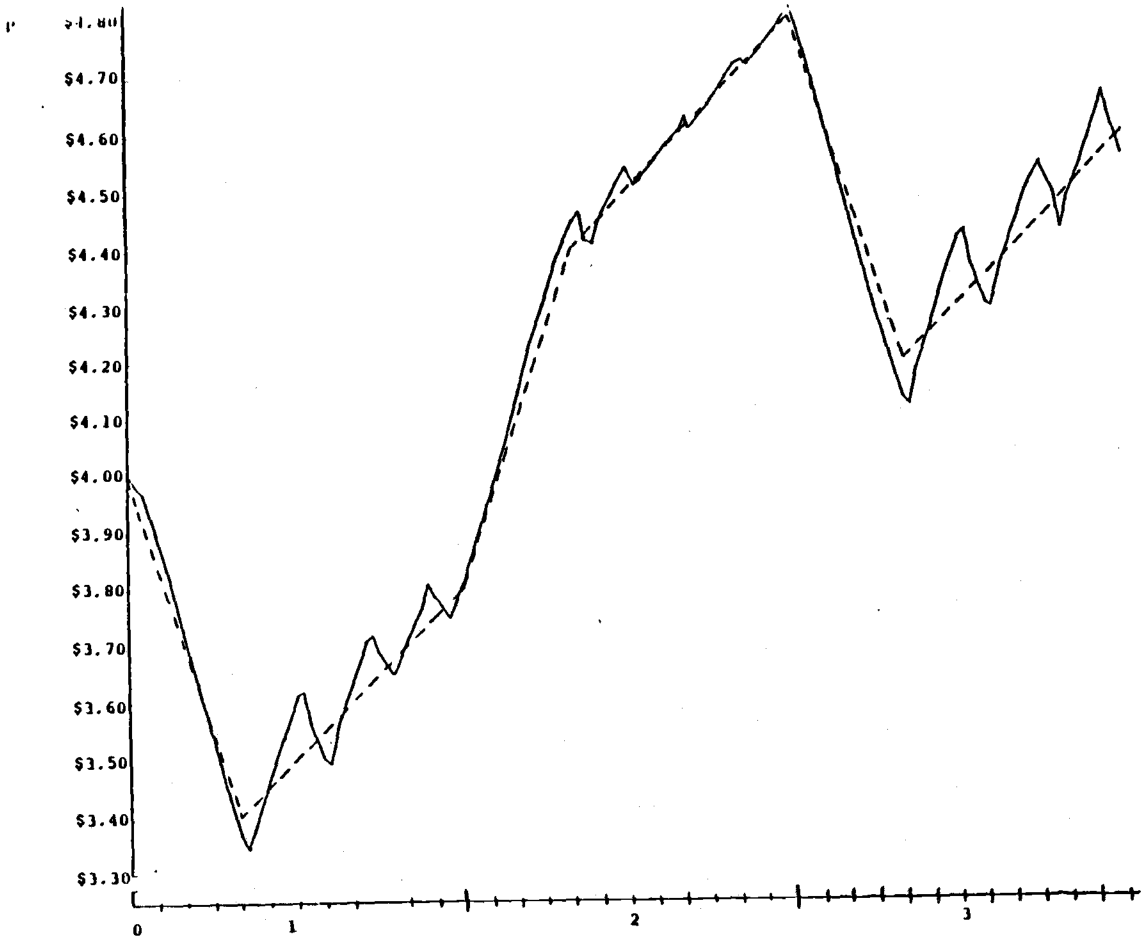

Figure 4.10. Simulated wheat prices, continuous external adjustment three year seasonal adjustment pattern; group 3, run 2.

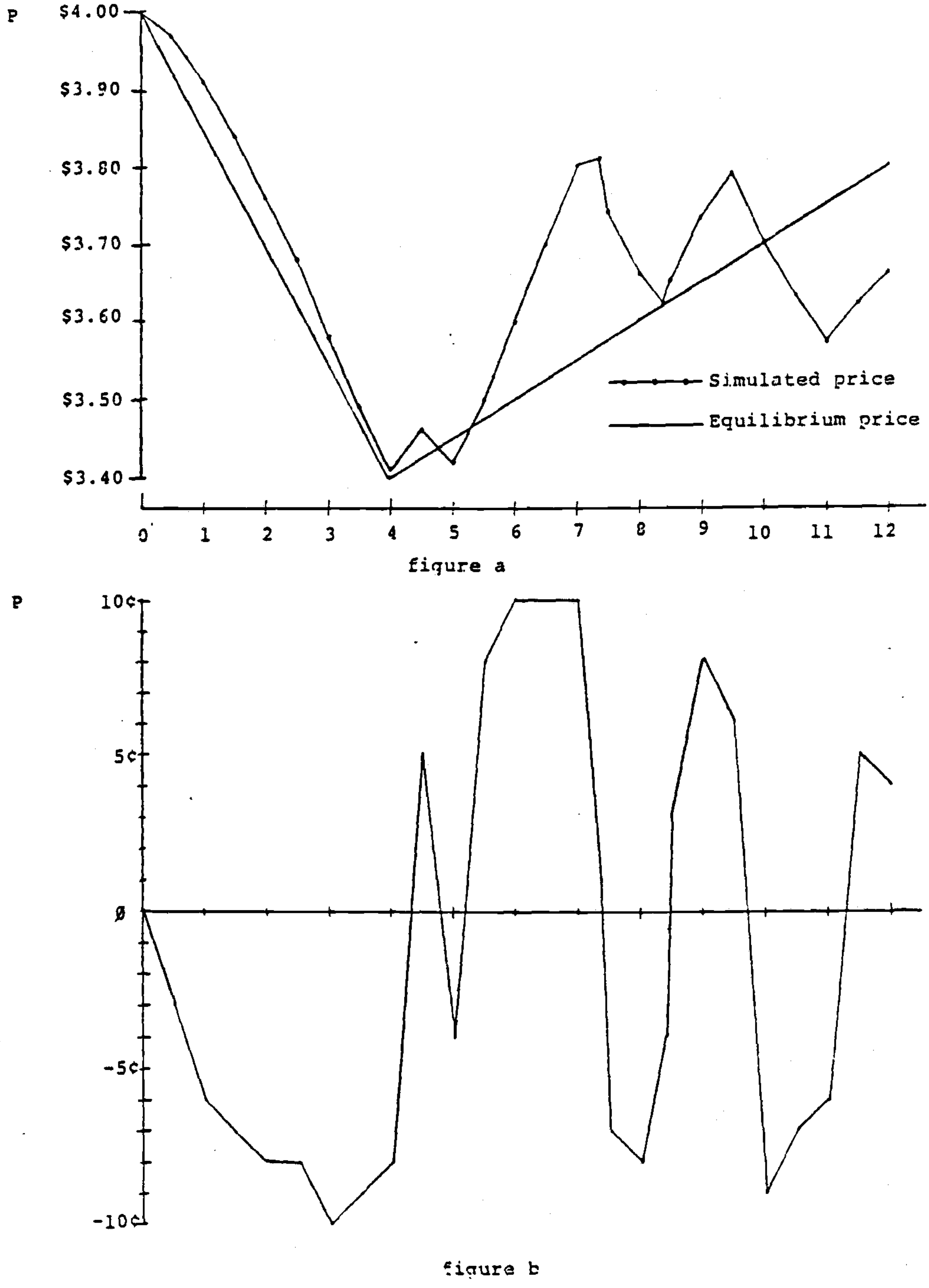

Figure 4.11. Simulated wheat prices continuous seasonal adjustment with random disturbances over a crop year: Figure a: prices over time, Figure b) price changes over time.

The simulations presented here are by no means an exhaustive examination of the possible types of behavior that can be represented by the model. The effects of different parameter choices on model stability and performance have not been fullly explored. However, the model can be used to generate pricing patterns similar to those observed for stored commodities. Without prior knowledge, it would be difficult to distinguish the prices generated from the simulation model from actual prices. This contention is put up for subjective evaluation in the following section, in which a graphical analysis of actual wheat prices is presented. The simulation results also show that pricing patterns indicative of catastrophic behavior are clearer in the pattern of price changes as opposed to absolute price levels. Thus, most of the empirical investigation of wheat prices in the remainder of this chapter is directed toward relative price movements.

Wheat Prices: A Graphical Analysis

A graphical examination of wheat prices is a good start to an empirical analysis of wheat markets. There are two initial considerations. First, a representative

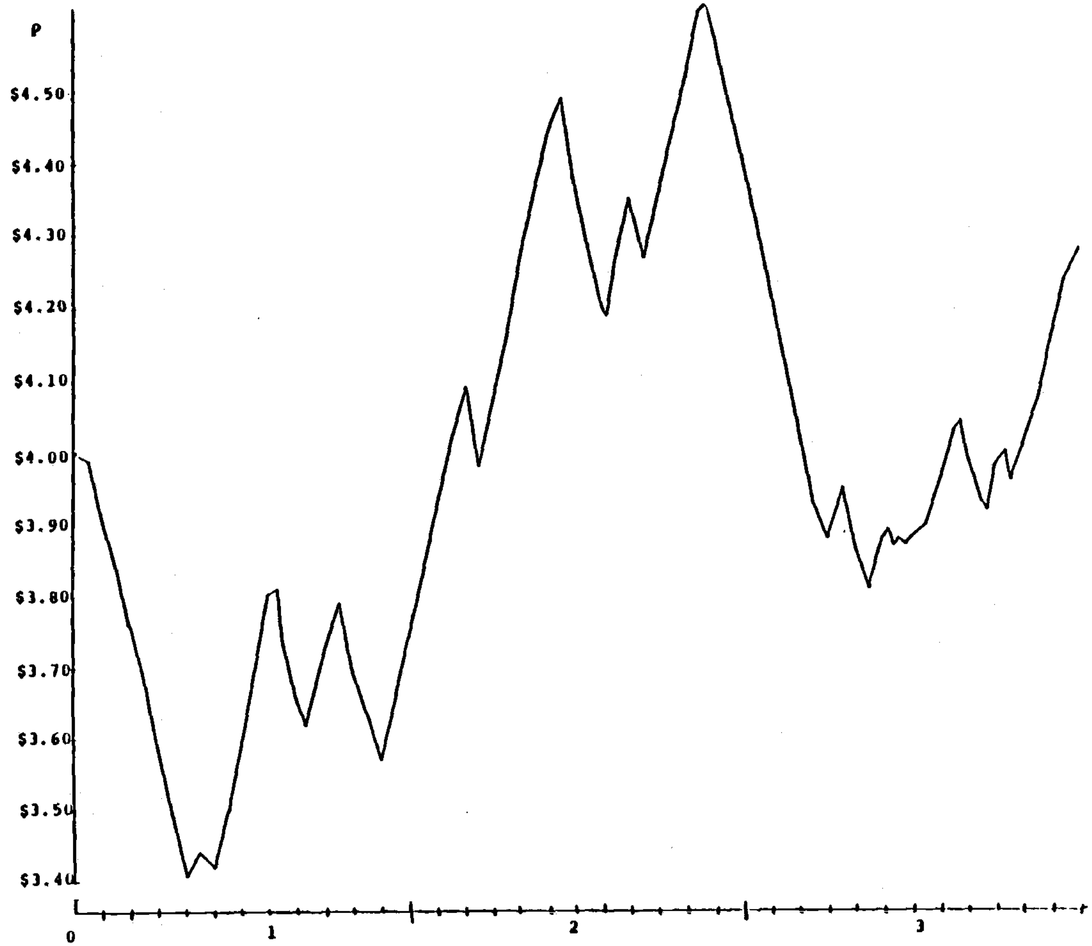

Figure 4.12. Simulated wheat prices, continuous seasonal adjustment with random disturbances over three crop years.

set of prices must be selected. Second, an observation interval must be chosen.

Wheat prices are reported at a large number of cash markets. Prices are differentiated by class, grade and protein content. Generally, strong relationships exist between prices for wheat at different locations and for wheat of different classes and grades. However, given prices may reflect specific conditions. For example, country elevator prices may reflect local supplies or the relative buying power of the elevator operator. One alternative, in attempting to represent general price levels for wheat in storage, is to construct a composite price based on weighted averages. A second alternative, the one chosen here, is to use prices at a principal intermediate market. Prices at major cash markets may more clearly reflect the impact of speculation. Hard Red Winter wheat prices at Kansas City were selected (No. 1 ordinary protein). Hard Red Winter is the dominant class produced in the U.S. and Kansas City is the principal cash market in the central plains states where winter wheat is grown.

It is reasonable to expect that speculation is a relatively short-term form of market behavior. Frequent price observations may be required to investigate the effects of speculative or transitory demand. At the same time, to consider external sources of price variation, observations on prices may have to be matched with less frequent observations on variables such as consumption, exports and supplies. A monthly observation period was selected as a compromise in this regard.

Average monthly wheat prices from 1972 through 1983 are presented in Figure 4.13. The graph reveals some interesting aspects of the variation in wheat prices over this period. Prices can be divided into two distinct periods; period I from 1972 to 1978 and period II from 1979 to 1983. In period I prices tended either to be rapidly rising or falling. Price swings were relatively large and turning points were sharp. In period II there were intervals of relative stability. Price swings were smaller and the transition between rising and falling markets tended to be smoother. The general characteristics of catastrophic behavior were exhibited to a much greater degree in period I. Strong price trends in period I may have attracted greater speculation. Interest rates were lower in comparison to period II. Hence, the costs of speculating was lower. Prices tended to be above government support levels in period I and government controlled stocks were at a minimum. A greater proportion

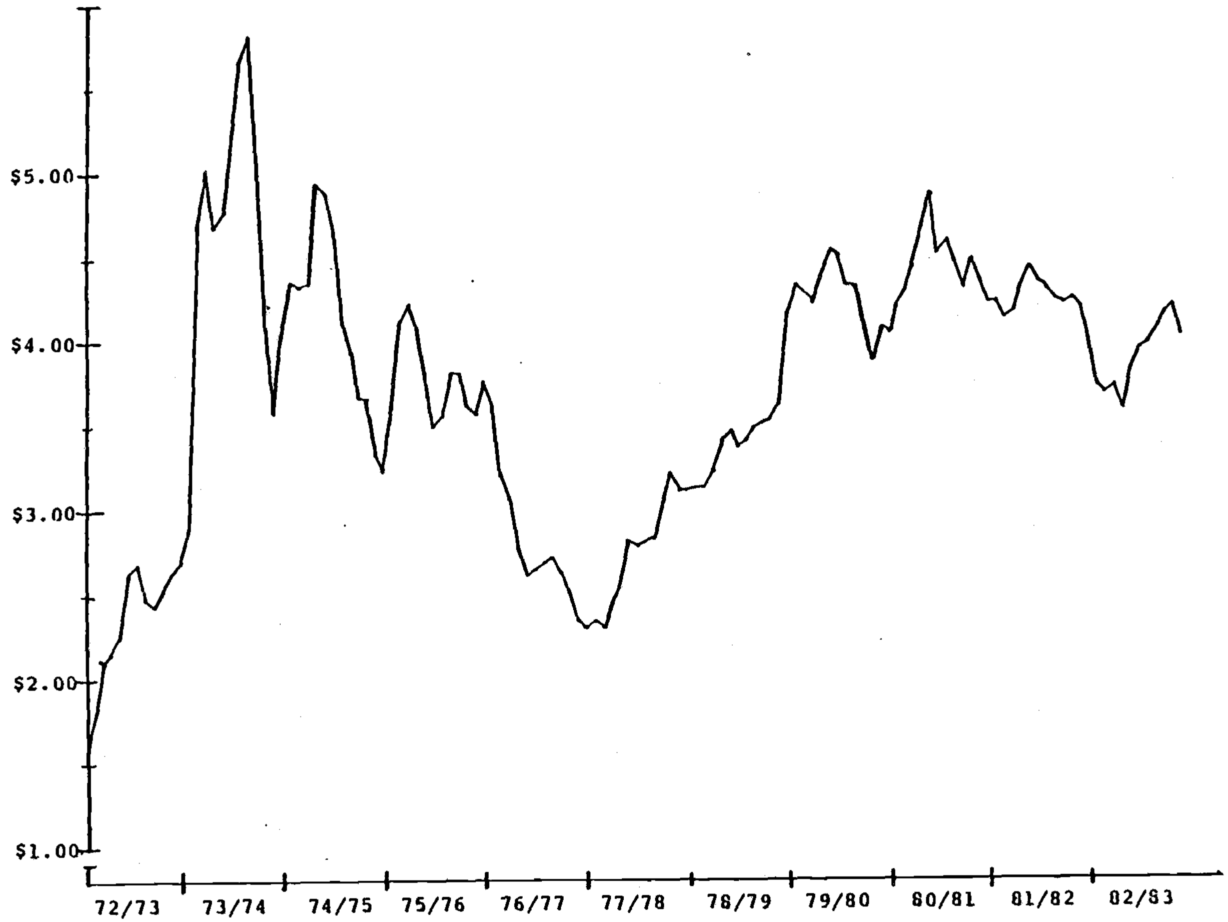

Figure 4.13. Wheat prices over time; Hard Red Winter wheat (No. 1 ordinary protein) at Kansas City 1972/73 to 1982/83; source, wheat situation.

of available stocks were subject to speculative changes in expectations.

Wheat prices for period I and II are plotted separately in Figure 4.14 and 4.15 respectively. Actual prices are plotted with exponentially smoothed prices in each graph. The exponential smoothing procedure is intended to filter out what may be short-term speculative variation in order to reveal underlying external trends. The formula for the exponential filter is given by the equation:

$$\hat{P}t = 0.4(\hat{P}_{t-1}) + 0.6(P_t) \quad (4.50)$$

where;

$\hat{P}$ = exponentially smoothed price

The graph for period I compares well with the crop year price simulation with relatively high speculative content. The graph for period II compares with crop year simulations with relatively low speculative content. In general, cyclical patterns of price adjustment occur within crop years for both periods I and II. These cycles correspond with the cylical patterns generated in the cusp simulations. This is in contrast with what is often called a normal seasonal pattern of increasing prices predicted on the basis of a decline in available stocks

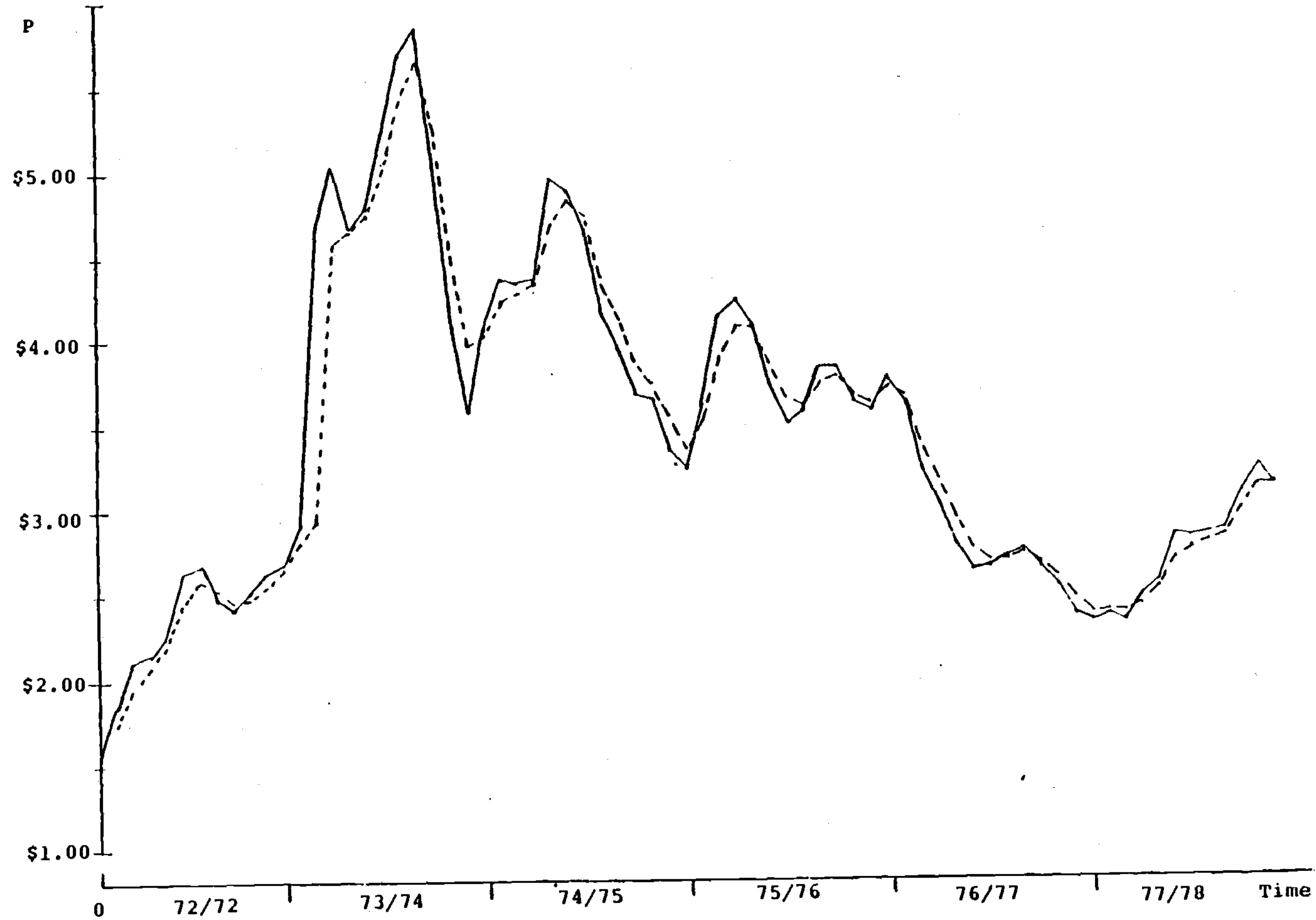

Figure 4.14. Actual and exponentially smoothed wheat prices; Hard Red Winter wheat at Kansas City 1972/73 to 1977/78.

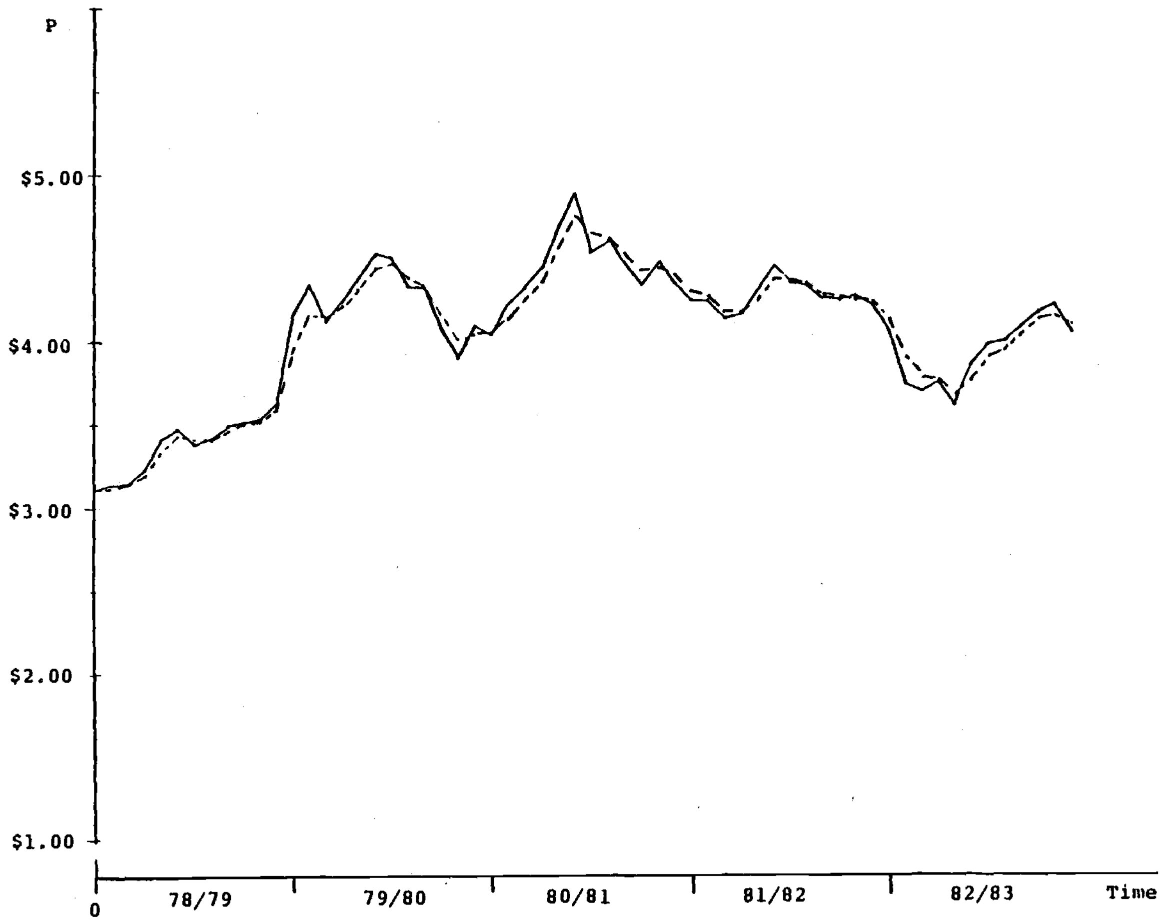

Figure 4.15. Actual and exponentially smoothed wheat prices; Hard Red Winter wheat at Kansas City 1978/79 to 1982/83.

through a crop year. Over the sample period there is no apparent normal return to storage.

Price changes over the sample period are plotted in Figure 4.16. The graph brings out the difference between periods of relative price instability and stability. There are definite times when wheat prices exhibit sudden switching between rapidly rising and falling states. These parts of the graph compare well with simulated price changes with random disturbances (Figure 4.11b).

While a graphical analysis may suggest the possibility of speculative behavior, we need to consider other potential sources of price variation. Between 1972 and 1983 there were significant changes in levels of domestic production, export demand and public farm policy. To investigate the effect of changes in the exchange environment we must rely on the tools of aggregate supply and demand analysis. However, we are interested in a form of internal price variation which can not be accounted for in traditional market analysis. Hence, we should remain aware of other nonspeculative sources of unexplained price variation. This requires a fairly good understanding of the exchange environment and the limitations of aggregate supply and demand analysis. One area that is easily

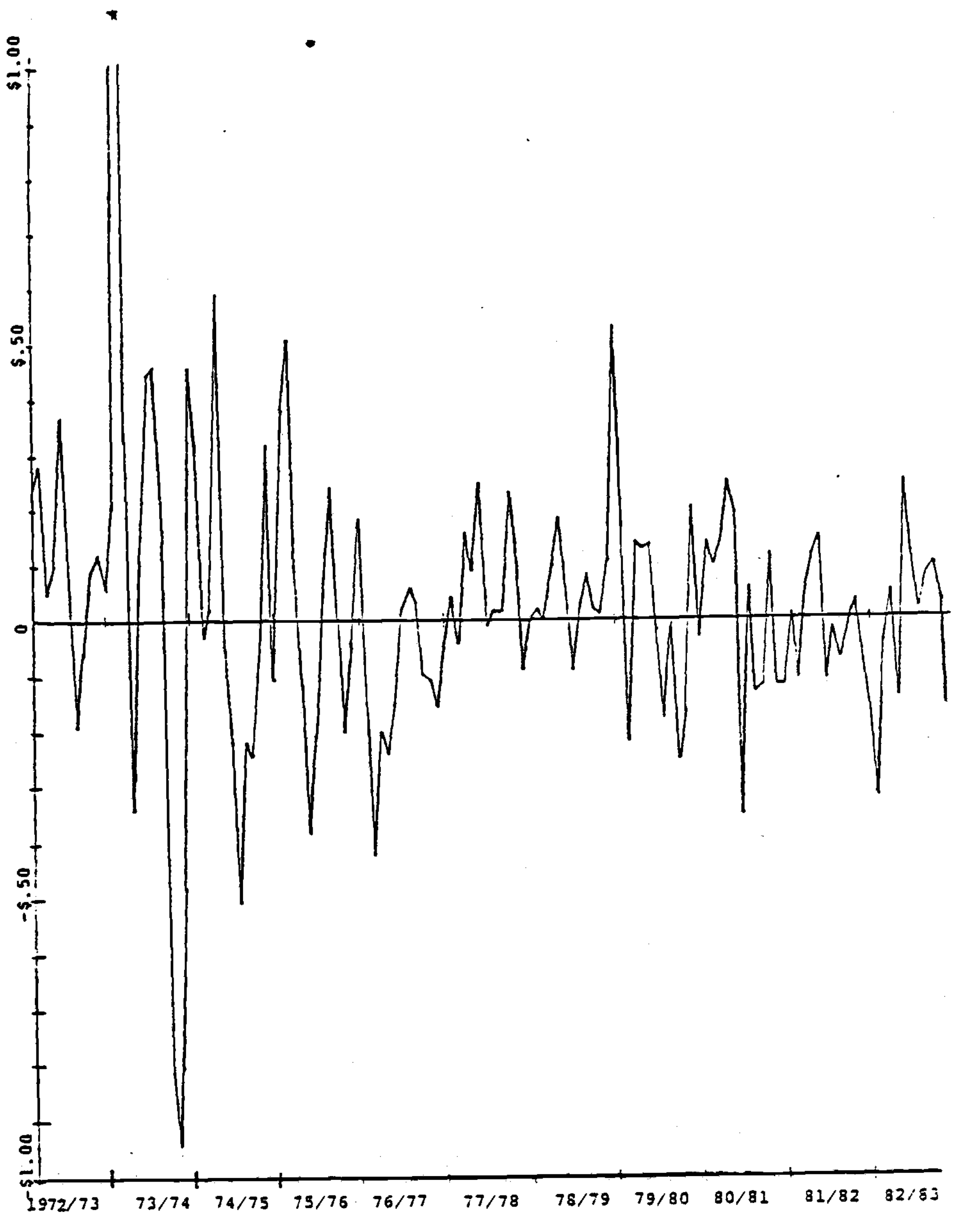

Figure 4.16. Changes in wheat prices over time; Hard Red Winter wheat at Kansas City 1972/73 to 1982/83.

overlooked is the assumption that wheat may be treated as an aggregate commodity.

Wheat as an Aggregate Commodity

Wheat may be classified by different production and use characteristics. Under U.S. grade standards wheat may be divided into five major classes. These are listed in Table 4.3 along with approximate percentage figures for production, domestic use and exports for the sample period. The distribution of wheat production in the U.S. reflects the adaptation of varieties and cultural practices to growing conditions. Winter wheats, Hard Red, Soft Red and White, are usually planted in the fall and harvested between May and September of the following year. Spring wheats, Hard Red and Durum, are planted in the spring and harvested between July and September.
Hard Red Winter and Hard Red Spring varieties, grown in the central and northern plains states respectively, are high protein wheats primarily used to produce bread flour. Soft Red Winter varieties grown in the northeast and White varieties grown in the northwest and north central states, are lower in protein and are more suitable for pastry flour and animal feed. Durum varieties, grown in the

northern plains states, are a very hard high protein wheat used to make semolina flour for pastas.[12]

Table 4.3. Wheat classes and the approximated composition of production, domestic use and exports by class.

Class	Production	Domestic Use	Exports
Hard Red Winter	49%	49%	50%
Hard Red Spring	15%	17%	14%
Soft Red Winter	16%	19%	14%
White	14%	8%	18%
Durum	6%	7%	4%
	100%	100%	100%

Source: U.S. Department of Agriculture, Wheat Situation (Washington, D.C.) various issues 1977 to 1984.

In grouping heterogeneous classes of wheat into a single aggregate commodity we are ignoring relative changes in the composition of supply and demand as a source of price variation in a given market. From another perspective, we are assuming that prices for different classes of wheat are equivalent scales in measuring aggregate value. The concept of a composite commodity is useful only to the extent that prices are related through aggregate supply and demand. If prices tend to vary independently, then, prices for different classes of wheat do not reflect equivalent measures of value and aggregation becomes meaningless. We can evaluate this problem, to a limited extent, by estimating linear relationships between market prices for wheat. A good linear fit, indicating that prices tend to change proportionately, supports the idea of an underlying aggregate relationship. A poor linear fit, while not excluding the possibility of a nonlinear aggregate relationship, suggests a need for disaggregate analysis.

Five market prices were selected to represent the wheat classes under consideration. Market locations were chosen to correspond with principal regions of production. Hard Red Winter at Kansas City (HRW), representing nearly 50 percent of total production, was selected as the independent price variable. Separate least squares regression estimates were made for the four dependent price variables: Hard Red Spring at Minneapolis (HRS), Soft Red Winter at Chicago (SRW), Soft White at Portland (SW) and Amber Durum at Minneapolis (AD). Results are summarized in Table 4.4. The regression relationships are all highly significant as indicated by the regression F statistics. For HRS and HRW the linear fit is very good; the proportion of unexplained variation is less than five percent. This is consistent with the fact that hard wheats are good substitutes in producing bread flour. Hard and soft wheats are poorer substitutes which may explain the increase in independent variation for the classes. Nonetheless, over 77 percent of the variation in soft wheat prices is accounted for by a linear transformation of HRW prices. This percentage drops to 62 percent for Durum wheat. However, Durum wheat accounts for only about five percent of domestic production. While we can not conclude that relative price variation between categories of wheat is insignificant, an aggregate approach does appear to be a useful simplification.

Table 4.4. Linear price estimates between wheat classes; average monthly prices 1977/78 through 1982/83.

Independent Variable: P_{HRW}; No. 1 Hard Red Winter at Kansas City (ordinary protein) cents/bu.

Dependent Variable: P_{HRS}; No. 1 Dark Northern Spring at Minneapolis (ordinary protein) cents/bu.

$$P_{HRS} = -9.56 + 1.006 P_{HRW} \qquad r^2 = .952$$

$$(10.35) \quad (0.027) \qquad MSE = 207.9$$

$$F^* = 1428.0$$

Table 4.4. (continued)

Dependent Variable: P_{SRW}; No. 2 Soft Red Winter at Chicago, cents/bu.

$$P_{SRW} = \underset{(22.11)}{24.91} + \underset{(0.057)}{0.880}P_{HRW}$$

$r^2 = .774$
MSE = 948.4
F* = 239.5

Dependent Variable: P_{SW}; No. 1 Soft White at Portland, cents/bu.

$$P_{SW} = \underset{(15.60)}{127.90} + \underset{(0.040)}{0.709}P_{HRW}$$

$r^2 = .774$
MSE = 472.0
F* = 312.4

Dependent Variable: P_{AD}; No. 1 Hard Amber Durum at Minneapolis, cents/bu.

$$P_{AD} = \underset{(51.85)}{-94.09} + \underset{(0.133)}{1.453}P_{HRW}$$

$r^2 = .629$
MSE = 5216.3
F* = 118.7

a. All estimates made using ordinary least squares.
b. Standard errors of the coefficients are in parentheses.
c. MSE is mean squared error, F* = F statistic.
d. Degrees of freedom = 70.
e. Data source: U.S. Department of Agriculture, Wheat Situation (Washington, D.C.) various issues 1977 to 1984.

The World Wheat Market: An Overview

Before focusing on domestic wheat production and marketing, a brief consideration of the world wheat market and the U.S. role in the international wheat trade is appropriate. The relative importance of wheat as a world food source has been increasing. World wheat production has nearly doubled in the last 25 years. World production has expanded from about 240 million metric tons in 1961/62, to over 450 million metric tons in 1981/82. Over the same period U.S. production increased from approximately 35 to 76 million metric tons. The U.S. currently accounts for between 16 and 17 percent of world production.[13]

As of 1982, about 22 percent of world production enters into international trade. The worlds' leading wheat exporting countries (listed in decreasing order of market share); the United States, Canada, Western Europe, Australia and Argentina account for approximately 96 percent of all exports. U.S. exports rose from about 32 million metric tons in 1975/76 to about 50 million metric tons in 1981/82. This represents an increase in the U.S. market share from 43 to nearly 50 percent of total exports.[14] The concentrated structure of international wheat trade has led a number of researchers to model world wheat trade as an oligopolist market with the U.S. acting as a dominant firm (MacGregor and Kulshreshtha, 1980; Bredhal and Green, 1983). However, there is far from a consensus view as to the design of effective policies for the U.S. grain trade.

The U.S. Wheat Market: Production, Consumption and Exports

Wheat prices are influenced by a wide range of factors effecting the exchange environment. These include physical conditions such as climate, microeconomic factors such as production costs and other commodity prices, and macroeconomic factors such as inflation, interest rates and exchange rates. External sources of price variation may be broadly classified with respect to their effect on production, consumption, export demand and inventory demand. Inventory demand will be considered in the following section. The remaining categories are reviewed here.

Between the 1972/73 and 1982/83 crop years, U.S. wheat production averaged 2117 million bushels. Production ranged from a low of 1545 million bushels in 1972/73 to a high of 2812 million bushels in 1982/83. Over this period production increased an average of 6.6

percent. The largest percentage increase between crop years was 19.5 percent in 1980. The largest decline was 11.7 percent in 1978. The average absolute change in production was 10 percent. Given that the demand for wheat is relatively price inelastic, changes in production levels would account for substantial variations in wheat prices.

Variations in production might be anticipated to be reflected in price changes over the harvest period. However, the average absolute change in prices over harvest was only 11.1 percent for the sample period. This can be explained, in part, by changes in expectations and reservation demand. Information is provided by the U.S.D.A. prior to harvest which allows for the anticipation of changes in supply. For example, prior information on a decline in production may lead to expectations for higher prices and increased reservation demand. This would lead to an anticipatory price increase before harvest.

Domestic consumption demand may be broken down into four categories. Between 1970 and 1980; 31 percent of total U.S. production was utilized for food, 5 percent for feed, 1 percent for seed and less than 1 percent for alcoholic beverages.[15] In contrast to corn, the dominant

U.S. grain, the impact of changes in the livestock industry on wheat prices in comparatively small. Presumably, shifts in domestic consumption demand occur slowly with changes in the population, income and tastes. Between the 1972/73 and 1982/83 crop years, domestic consumption averaged 798 million bushels. Consumption ranged from a low of 686 to a high of 934 million bushels. The average absolute change in consumption was 6.8 percent. These figures do not differentiate between movements along a given demand schedule and shifts in demand.

An estimate of the elasticity of domestic demand can be used to decompose changes in domestic utilization into demand shift and schedule components. Sarris and Freebairn (1983) report an elasticity of domestic demand equal to -0.15, based on estimates of a world grain-oilseed-livestock model developed by the U.S. Department of Agriculture. This elasticity estimate can be incorporated into the following adjustment formula:

$$\%CHQ_s = \%CHQ - (-0.15)\%CHP \qquad (4.80)$$

where;

$\%CHQ_s$ = percentage change in quantity attributed to demand shifts
$\%CHQ$ = observed percentage change in quantity consumed
$\%CHP$ = observed percentage change in price.

Using this formula, the average absolute change in domestic utilization attributed to shifts in demand is 6.0 percent. The average absolute change in domestic utilization attributed to prices is 0.8 percent. Variations in consumption attributed to shifts in demand are approximately four times smaller than supply variations for the sample period.

Exports accounted for over 61 percent of U.S. wheat production between 1970 and 1980. This is the other side of the dominant role of the U.S. in world wheat trade. Domestic prices are very sensitive to changes in world demand. A sharp increase in exports during the world food shortage in the early 1970's was accompanied by a threefold increase in domestic prices. As the shortage eased, exports remained high but domestic prices declined over 50 percent.

U.S. exports averaged 1270 million bushels between 1972/73 and 1982/83. Exports ranged from a low of 950 million bushels in 1976/77 to a high of 1771 million bushels in 1981/82. The average absolute annual change in exports was 14 percent. An estimate of an export demand elasticity may again be used to decompose export variation into demand shift and price schedule components. However, the economic and physical relationships that determine foreign demand for U.S. wheat are complex and difficult to model. As a result there is no consensus on the price elasticity of export demand. Gallagher, Lancaster, Bredhal and Ryan (1981) obtained an inelastic estimate of -0.413. Konandreas and Schmitz (1978) obtained an elastic estimate of -3.13. An even more elastic estimate of -6.72 was reported by Johnson et al., (1977).

The ratio of the average annual absolute percentage change in exports, divided by the average annual absolute percentage change in price, is equal to 0.7 for the sample period. If the effects of simultaneous movements along a demand schedule and shifts in demand are roughly offsetting, then this ratio provides a crude lower bound on the elasticity of export demand at -0.7. This is consistent with the idea that the demand for staple commodities is inelastic. Using the elasticity estimate of -0.413 and the adjustment formula in equation (4.80), the variation in export attributed to shifts in export demand is greater that the observed variation. The approximate absolute annual percentage change in export due to shifts in demand is 18.2 percent or about 230 million bushels per year. This is slightly greater than the average annual change in production, equal to about 212 million bushels.

The figures presented are intended to give an idea of the relative importance of changes in production, domestic consumption demand and exports on wheat prices. Changes in production and export demand appear to be the primary sources of external variation in wheat markets. Presumably, production affects general price levels within a crop year. However, wheat prices are subject to relatively continuous variation. Export demand varies throughout crop years. At the same time, export and consumption demand is generally small in relation to the total volume of available supplies. This suggests that inventory demand is a substantial source of price variability within crop years.

Inventory Demand and Storage

There is an important distinction between the demand for storage and the demand for inventories held in storage. The former reflects the function of storage in the grain marketing system. The latter reflects the returns to individuals and firms willing to carry out this function. The capacity to store large volumes of wheat and other grains is required to distribute seasonal production for continuous consumption. The demand for storage is an implicit component of grain production, marketing and consumption. The willingness of firms to supply storage capacity and to hold inventories is explicitly dependent on the returns to storage.

Returns to storage may be grouped into two general categories: one, an expected return in anticipation of a change in prices and two, a facilitative return to acquiring or holding inventories. Changes in inventory demand arising from expectations for future price changes has been termed reservation demand. Short-term expectations may be based on current market information or price signals. Intermediate range expectations may be formed on the basis of information concerning production and inventory carry-over into the following year. Long-term expectations, as a basis for supplying storage capacity, would require a well established pattern of seasonal price increases within crop years.

Facilitative returns to storage include a wide range of economic incentives for holding inventories. At each stage of the grain marketing system inventories are needed to facilitate smooth production and distribution over time. This is commonly referred to as pipeline storage. On farm storage capacity may expedite harvesting and allow flexibility in choosing marketing alternatives. Elevator operators may exploit their bargaining position as both a buyer and seller to obtain favorable price differentials. In addition, elevators have facilities to clean, dry, and blend grains, allowing operators to take advantage of price differences between grades. The motivation behind government storage programs is more political than economic.

Empirical evidence on the relative importance of facilitative versus reservation demand for grain inventories is indirect. The lack of consistent seasonal price increases through the harvest year would tend to support the importance of facilitative demand. Information on seasonal price variation in prices of No. 1 Hard Red Winter Wheat at Kansas City (ordinary protein) is presented in Table 4.5. The only period over which prices tended to show a significant upward trend was from October through December (an 88 percent confidence level for the mean greater than zero based upon cumulative frequencies of the t distribution). Over the remainder of the harvest year prices tended to decline. The probabilities of a price increase (based upon an assumption of normally distributed price changes) do not appear sufficient to support the idea of a normal price return to storage.[16] Recent studies on the demand for inventories focus on the role of price expectations in a rational expectations

Table 4.5. Summary statistics for seasonal price changes in No. 1 Hard Red Winter Wheat Prices at Kansas City 1972/73 to 1982/83.[a]

Statistic	Harvest to 4th Quarter	4th Quarter 1st Quarter	1st Quarter (April-May)
mean[b]	18.3	-1.9	-21.3
standard deviation	48.7	38.3	51.9
coefficient of variation	266	-2005	-244
probability of a price increase[c]	64.6%	49.8%	34.1%

[a]Source: Wheat Situation, [b]Units: cents/bu.
[c]From cumulative frequency of a normal distribution.

framework (Helmberger and Weaver, 1977; Helmberger, Weaver and Haygood, 1982; Sarris, 1984). While seasonal price increases may be rational on the basis of economic theory, those involved in the storage of grains are unlikely to ignore market history. From an analytical perspective it is difficult to ignore the fact that grains are stored in large quantities while prices exhibit strong tendencies to decline.[17]

In general, returns to storage may tend to follow the marketing channel from producer to consumer rather than temporal pricing patterns. However, this does not imply that changing expectations for future prices do not affect inventory demand. While firms are holding stocks they are faced with at least some degree of price uncertainty which results in capital gains or losses.

Expectations and Inventory Demand

Given that a firm is actively acquiring or holding inventories in anticipation of a normal or facilitative return, expectations for a price change may alter expected returns and inventory demand. These expectations may not be well defined. An individual may not have a specific price or time interval in mind. Expectations may reduce to a feeling of optimism or pessimism in the evaluation of marketing alternatives. An emphasis has been placed on the role of current price information in the formation of expectations. At a given point in time, other sources of information may be equally or more significant. However, it is difficult to disregard current price information at any time. For example, a decision based on expectations for increasing wheat exports can be quickly undermined if prices fail to begin rising. On the other hand, an upturn in the wheat market can reinforce expectations and strengthen the posture of a decision maker. The impact of current price information may not be explicit. Current prices and price trends may act to moderate other sources of information.

Individuals may or may not view their expectations with sufficient confidence to affect a decision. They may simply accept windfall gains or losses. An individual's response to his or her expectations may only result in an attempt to transfer risks through participation in futures trading, foward contracting, or government programs. Thus, we should relax the contention that changes in expectations necessarily effect inventory demand. However, when the marginal return to inventories above costs is small, even a slight change in expectations may significantly alter inventory demand.

Since we can not restrict the way or degree to which information enters into the formulation of expectations, we can only attempt to classify expectations. Any classification scheme to some extent may fail because individuals are free to ignore the parameters which are used to make divisions. Three categories were established in the previous chapter. The first are lagged or regressive expectations: current prices and price trends are discounted and expected prices tend to lag behind current prices. Adaptive expectation models fit within this framework. Adaptive expectation models have come under recent criticism because adaptation is not an optimization process (Fisher, 1982). However, human behavior is often characterized as adaptive.[18] For an individual weighting the past in assessing current market conditions may represent the best use of information given his or her own experience. The second are neutral expectations: expectations for future price changes are not based on current market information. Rational expectations may fit within this framework. Expected future price movements may be based on anticipated changes in supply and demand. Information on supply and demand conditions may provide a logical basis for formulating expectations. However, our ability to predict price changes, with available information on factors affecting supply and demand, is very limited.[19] Equating current prices with future prices is a second form of neutral expectations which often compares favorably with econometric forecasts. The third are projective or speculative expectations: current price trends are projected into the future. Expected prices tend to extend beyond current prices. Taken collectively, this classification scheme covers a wide spectrum of humanistic response, adaptive behavior, rational or deductive behavior and speculative behavior.

In aggregate, it is more appropriate to consider a distribution rather than a single form of expectations. The distribution of lagged, neutral and projective expectations among individuals is subject to change over time. For example, when the Department of Agriculture releases information on wheat plantings, rational expectations may become dominant. When prices are moving erratically up and down, adaptive expectations may become increasingly important. When a market has been exhibiting strong trends the proportion of speculative expectations may increase. It is important to note that significant levels of transitory demand or supply may arise even when nonspeculative forms dominate the distribution of expectations.[20]

The composition of expectations and reservation demand may tend to exhibit seasonal patterns within a crop year. During harvest there is a rapid forced buildup in inventories. The potential market interactions are complex. Anticipated production and carryover are gradually replaced with information on available supplies. However, this type of information is relatively difficult to analyze. Current price adjustments may provide the best information on general price levels for the crop year. This is an unstable condition since expectations may in turn influence prices. There may be a tendency for harvest price trends to be artificially extended, resulting in an under or over valuation of current stocks. If price trends over harvest are carried into the crop year, there should be a relatively strong positive correlation between price changes over harvest and price changes in the first quarter of the remaining crop year. If stocks tend to be over or under valued in the first part of a crop year, a negative correlation may exist between price changes over the first and second halves of the crop year. These hypotheses are readily tested.

Correlation coefficients for price trends in successive periods of the wheat crop year are presented in Table 4.6. A relatively strong positive correlation between harvest and fourth quarter price trends is evident in the sample period. A relatively strong negative correlation between price trends in the first and second halves of the crop year are also evident in the sample period. The square of the correlation coefficients is the goodness of fit measure, r^2, for the simple linear regression prediction equations:

$$P_{t4} = 11.92 + 0.521\ P_{tH}$$
$$(10.22)\quad (0.207)\qquad r^2 = .518$$

where;

P_{t4} = fourth quarter trend
P_{tH} = harvest price trend;

$$P_{JD} = -9.24 + -0.428\ P_{JM}$$
$$(15.76)\ (.167)\qquad r^2 = .449$$

where;

P_{JD} = June to December Price trend
P_{JM} = January to May price trend.

Table 4.6. Correlation coefficients and significance levels for wheat price trends in successive intervals of the crop year. No. 1 Hard Red Winter wheat at Kansas City, 1972/73 to 1982/83.[a]

Periods From	Periods To	Correlation Coefficient	Significance Level
June Sept.	Oct. Dec.	0.72	99.4%
Oct. Dec.	Jan. March	0.21	73.2%
Jan. March	April May	-0.27	78.9%
April May	June Sept.	-0.15	67.0%
June Dec.	Jan. May	-0.67	98.3%

[a] Source: Wheat Situation
[b] Based on cumulative t distribution; $t^* = r\ n-2\ /\ 1-r^2$

A very hypothetical consideration of expectations and speculation leads to a surprisingly good set of predictions.

It was noted earlier that an individual response to uncertain prices may be to transfer risk. Futures market and forward contracts are a means of transferring risk. An individual holding pessimistic expectations may not choose to liquidate inventories on the open market. He or she may decide to participate in a government reserve or other price support program. Institutional arrangements and public policy, on the farm and national level, have a significant effect on wheat and other grain prices.

Institutional Arrangements and Government Policy

Institutional arrangements affect a wide range of grain marketing activities and their impact on prices and price stability is varied. Changes in institutional arrangements, such as transport regulations, may affect prices but they are an unlikely source of general price instability. Arrangements, such as grades and standards, are weakly linked to price determination through inventory demand. Price differential between grades may be exploited by elevator operators with the capacity to clean, dry and blend grain. If these price differentials are affected by relative supplies and demand, returns to inventories may be affected, resulting in shifts in total inventory demand. Legal contracts are institutional arrangements which are more closely tied to prices and price variability.

There are essentially two types of grain contracts.[21] First, there are foward contracts in which terms are negotiated by the buyer and seller. Second, there are futures market contracts in which terms are established by a board of trade. Forward contracts eliminate price risks, though not the risk of being able to sell at a higher or buy at a lower price in the future. Since forward contracts are negotiable, contract prices may reflect the relative position of buyers and sellers under current or expected market conditions. Futures contracts are used to hedge a future purchase or sale. Price risks are reduced to the degree futures and cash market prices tend to move together. Contracting is usually viewed as a response to price variability or expected price changes. The effects of contracting on market prices is not totally clear.[22] If we view contract prices as market prices modified by perceived risks and expectations, forward and futures contracting may be a source of price variation. Changes in expectations or perceived risks may alter contract prices, affecting inventory demand and in turn, market prices. On the other hand, if contracts reduce risks of price changes, inventory demand may become more stable and price variations may be reduced. We may draw at least one relevant conclusion: contracts may be used to reduce risk but prices may remain sensitive to changes in expectations.

Public policy, on an international, domestic or farm level, may greatly affect grain production and marketing.[23] The scope of government economic policies is too broad to consider more than a few examples. On an international level, the U.S. moved to a market determined exchange rate beginning in 1972. The value of U.S. currency relative to the currencies of other countries participating in grain trading affects U.S. export demand. With variable exchange rates grain prices are sensitive to factors ranging from inflation to world confidence in a given economy. On the domestic level; Federal Reserve policy, designed to meet objectives through control of the money supply, affect interest rates. In 1980 interest rates rose from near 12 to over 20 percent as the rate of growth in the money supply was slowed to curb inflation. At prevailing prices (about $4.50/bu.) this is a monthly increase in storage costs of about 3 cents per bushel. To cover interest costs on six months storage would require an increase of 45 cents per bushel. The effect of interest rates on speculative reservation demand have been considered previously. High interest rates should lead to decreased speculative holding of inventories. The marginal change in costs for a price change, which must be offset by an expected price change, is equal to the interest rate. The impact of interest rates is not limited to inventory demand. Grain production is capital intensive. Changes in current interest rates are likely to affect future production and prices. Agricultural policies are of special interest in that a number of programs have been designed to affect grain prices.

Since 1973, commodity price support programs have incorporated three policy tools: a government controlled reserve, direct payments and acreage set aside provisions. The basic design of U.S. agricultural policy has remained unchanged but objectives have changed from price support to stabilization and recently, the elimination of excess inventories. Target prices and loan rates are established for wheat and coarse grains. When prices are at or below the loan rate, producers tend to transfer ownership of stocks to the government as payment for nonrecourse loans. When prices exceed the target price stocks are released on the open market and no direct payments are made. The government may also release inventories to reduce inventories and/or stabilize prices. When prices have fallen between target prices and loan rates, different policies have been followed. Between 1973 and 1977, producers were allowed to sell on the open market and received payment for the difference between market and target prices. After 1977 producers were required to hold grain in reserve for three to five years while receiving a nonrecourse loan and storage payments. At the end of the initial storage period stocks could either be sold on the open market and the loan repaid or ownership could be transferred to the government as payment for the loan. If deemed necessary, participants in the program could be required to set aside acreage out of production. In 1983 the Payment in Kind program (PIK) was introduced to reduce government stocks and to decrease production. Participants were required to set aside acreage out of production to qualify for payments in grain which could be stored privately or sold on the open market.

Under ideal conditions the policy instruments of the commodity price support program could constrain prices to near target levels. Direct payments and acreage controls could be used to try to match production with anticipated demand. Unanticipated changes in production or demand could be offset by acquisition or release of government controlled stocks. The effectiveness of farm policy instruments is determined by a large number of factors including; participation levels and the size of the reserve, prevailing conditions in grain markets and the general economy, other public policies and the ability of public decision makers to implement a given policy. In general, the price support program has maintained open market prices at or above the loan rate. This reflects both the willingness of the government to acquire large inventories and high rates of participation by individuals when prices are low. The release of government stocks when prices are above target levels has not effectively controlled high prices. The size of government reserves has not been sufficient to offset the major increases in export demand which have occurred in the last twelve years.

A government reserve program, based on a decision rule to acquire stocks when prices are low and to release stocks when prices are high, promotes price stability through inventory demand. However, a policy decision rule does not affect inventory demand in a smooth way. If, for example, the government is willing to purchase all available supplies when prices reach the loan rate and is willing to sell all existing stocks when prices reach some specified level above the target price, inventory demand may resemble the segmented curve drawn in Figure 4.17a. At the loan rate or release price, demand is perfectly elastic. No price-quantity combinations exist below the loan rate since the government is willing to purchase all free market stocks. Price-quantity combination above the release price exist only after government stocks are exhausted. If market equilibrium falls along a perfectly elastic section of the demand curve then a small disturbance in supplies, consumption, or exports, will not alter prices. If market equilibrium falls along a downward sloping section of the demand curve then a slight disturbance will produce a change in price.[24] While this model may be oversimplified, this public inventory demand model provides some insight into the effects of the price support program on speculative reservation demand.

The divergence of a market from external conditions, under speculation requires two interactions; the formation of expectations on current trends and a reaction in prices to transitory demand. An existing policy the response in expectations to a price trend may be moderated by if individuals believe a program effects market prices. If,

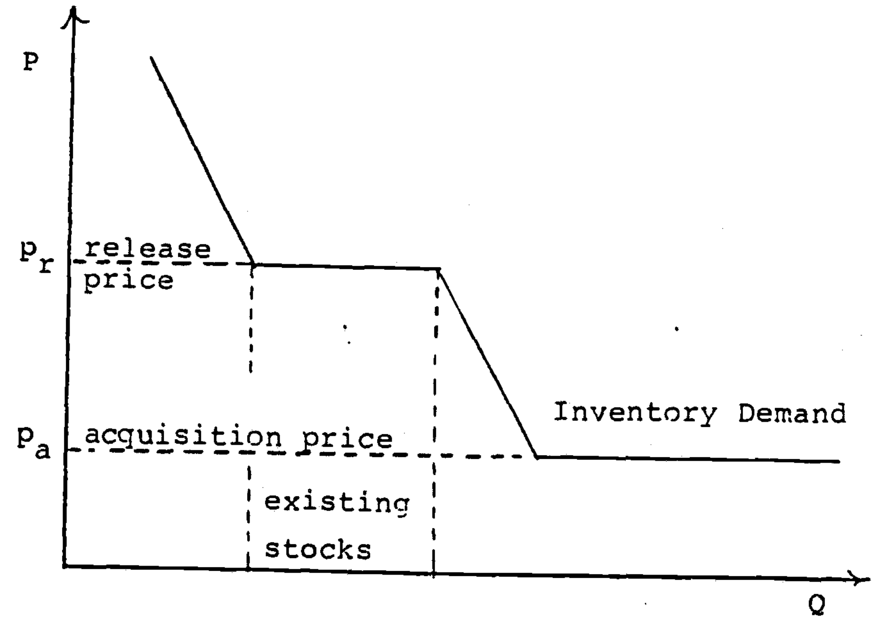

figure a

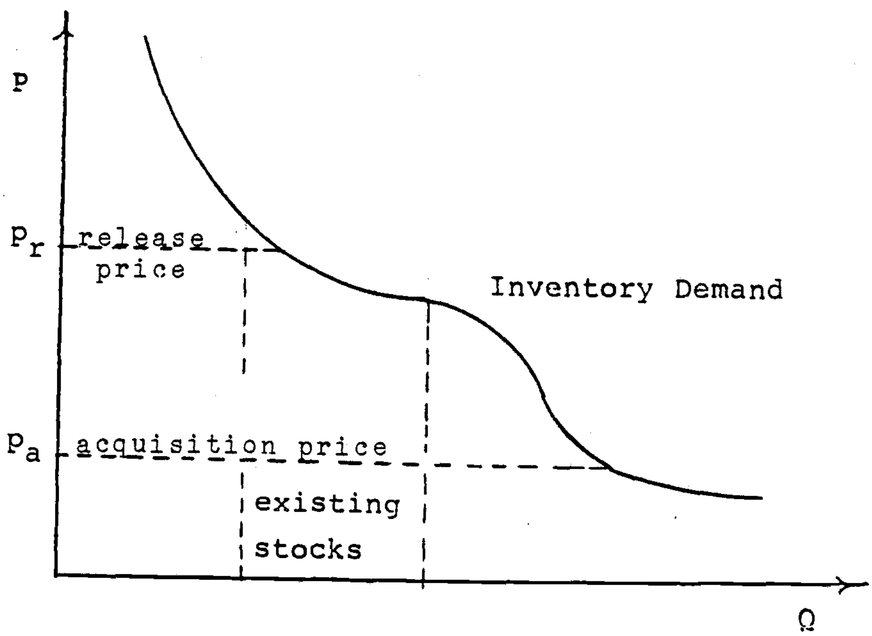

figure b

Figure 4.17. The demand for inventory under a government reserve program: Figure a) perfect implementation, Figure b) imperfect implementation.

for example, the loan rate is believed to be a price floor, speculative disinvestment may be eliminated as prices approach the loan rate. Speculative investment may be dampened if individuals believe the release of government controlled stocks may lessen upward pressure on prices. A government reserve program may or may not moderate the impact of speculative reservation demand on prices. So far as the government is willing and able to acquire or release stocks at or near targeted price levels, inventory demand becomes more elastic over a limited range (Figure 4.17b). Over this range, the response in prices to a shift in supply or demand is dampened. Thus, a government reserve may inhibit both levels of dynamic flow in the interaction between expectations and price formation. When prices are below target levels, we may expect to observe more stable prices. From 1977 through 1979 prices were below support levels. The relative stability of prices over this period is quite evident in the graphs of wheat prices and price changes over time (Figures 4.13 and 4.16). However, how much this may be attributed to a reduction in speculative demand is unknown, as external changes are moderated as well.

There were two different farm policy programs in effect over the sample period. Between 1973 and 1977, the open market program was implemented. Government subsidies reduced downside risk, tending to reduce speculative liquidation. However, by allowing participants to sell on the open market, expectations for prices greater than subsidy levels may have lead to speculative inventory demand. This may have created an upward speculative bias; inducing greater instability when prices were increasing. We can compare the monthly variation in price increases and declines over this period (period I):

$$V(\Delta P^{+}) = 899.8\ (¢/\text{bu.})^2$$
$$V(\Delta P^{-}) = 482.0\ (¢/\text{bu.})^2$$

Construction of an "F" test:

$$F^*_{(35,34)} = (898.9)/(482.0) = 1.865$$

indicates that the variation in price increases is significantly greater than the variation in price declines, at the 95 percent confidence level. In 1978 the farmer-owned reserve was implemented. To obtain a secure price, a participant was required to set stocks aside for a three to five year period. Expectations for prices above subsidy levels must be weighed against a secure return, as open market and program participation were temporarily exclusive. This may have tended to dampen speculative demand and supply in a relatively unbiased manner. We can compare the monthly variation in price increases and declines over this period (period II):

$$V(\Delta P^{+}) = 97.4\ (\text{¢/bu.})^2$$

$$V(\Delta P^{-}) = 97.1\ (\text{¢/bu.})^2$$

Constructing a corresponding "F" test:

$$F^*_{(34,23)} = (97.4)/(97.1) = 1.00$$

leads us to conclude that the variation in price increases and price declines is equal. The fact that participation in the farmer-owned reserve grew steadily over period II may explain, in part, the general reduction in price variability between 1978 and 1983.

If price stability is a policy objective of a grain reserve program; the extent to which a reserve system inhibits speculative reservation demand represents a positive contribution. The extent to which policies should be designed to inhibit speculation is another question. The evidence presented here is not sufficient to claim speculative behavior is a major cause of price instability. However, it does appear that the implications of speculative behavior should not be ignored in the evaluation of alternative policy implements.

Aggregate Supply and Demand Analysis: Modeling Considerations

In attempting to determine the impact of speculative behavior on wheat prices, it would be extremely helpful to know the effects of other sources of price variation. By estimating the variation due to the external determinant of supply and demand, the remaining variation can be examined for evidence of speculative price movements. An econometric model can be constructed to control for external sources of change within wheat markets.

Generally, researchers have taken two approaches to modeling price determination in wheat markets. One, is to specify and estimate an equilibrium system of supply and demand equations.[25] The second, is to specify and estimate a lagged adjustment price equation.[26] The choice of approaches is largely dependent upon research needs and objectives. Equilibrium system models are commonly used to investigate characteristics of supply and demand, such as price elasticities, and to evaluate policy alternatives. Examples of this type of modeling for wheat may be found in Shei and Thompson (1977), Johnson et al.

(1977), Greenes, Johnson and Thursby (1978), Konandreas and Schmitz (1978), Sarris and Freebairn (1983). Lagged adjustment equations are often used when accuracy and simplicity are of primary importance. Lagged price equations are commonly found in price forecasting models and large scale sector models.

Equilibrium system models for wheat and other grains are estimated, almost exclusively, on an annual basis. This, in part, reflects the seasonal aspect of grain production. The availability of information is an important limiting factor. To empirically estimate price movements within a crop year a lagged adjustment model is generally necessary, simply because the information needed to estimate a simultaneous set of supply and demand equations is unavailable. The primary difficulty lies with estimating foreign demand. Quarterly price predictions are common forecasting and sector model applications. A model of the livestock and feed grain sectors by Arzac and Wilkinson (1979) provides a typical example of a lagged price prediction equation:

$$P_t = b_o + b_1 P_{t-1} = b_2 I_t + b_3 E_t + d_1 Q_2 + d_2 Q_2 + d_3 Q_3 \qquad (4.12.0)$$

where;

P = grain price
I = inventories
E = exports
Q_i = seasonal dummy for quarter i
b_i = estimated variable coefficient
d_i = estimated dummy variable coefficient

A slightly more general formulation might include domestic consumption. By subtracting P_{t-1} from both sides of the above equation reveals the underlying disequilibrium partial adjustment form of the model:

$$\Delta P_t = -(1-b_1)P_{t-1} + b_o + b_2 I_t + b_3 E_t + d_1 Q_2 + d_2 Q_3 + d_4 Q_4 \qquad (4.12.1a)$$

By rearranging terms the model can be written:

$$\Delta P_t = (1-b_1)[(1/1-b_1)(b_o+b_2 I_t+b_3 E_t+d_1 Q_2+d_2 Q_3+d_3 Q_4) - P_{t-1}] = (1-b_1)[Pe_t - P_{t-1}] \qquad (4.12.1b)$$

where;

Pe_t = imputed equilibrium price at time t.

If we interpret the model to be a Walrasian price adjustment model, then; the term in brackets, $[Pe_t-P_{t-1}]$, may be viewed as a qualitative measure of excess demand.

In applications regarding inventory demand, the disequilibrium form of the price adjustment equation yields a better picture of the predictive accuracy of the model.[27] Our interest is in price changes over time. The tendency for P and P_{t-1} to be of nearly the same magnitude is not reflected in measures of the disequilibrium models explanatory power. Generally, R^2 values for the disequilibrium form are much lower. Errors for the two models are identical while the variation in prices is usually much larger than the variation in price changes.[28]

Data limitations present an additional problem in attempting to estimate a disequilibrium lagged adjustment model to control for external sources of price variation. Data on wheat inventories, domestic usage and exports is reported by the U.S.D.A. at only four irregular intervals within a crop year. There are two approaches to incorporating this information into a monthly model: one, seasonal dummy variables can be introduced for the months between reporting periods; two, monthly observations can be interpolated from the reported information. Dummy variables adjust the mean response in prices for the sample period. Interpolation allows changes in the exogenous variable to be projected over monthly intervals. Interpolation would appear to have a better potential for explaining external price variation.

A modified form of the disequilibrium adjustment model was used to estimate changes in wheat prices over the sample period. The estimating equation can be written:

$$\Delta P_t = b_o + b_1 P_{t-1} + b_2 MD\%I \qquad (4.12.2)$$

where;

MD%I = total monthly disappearances (domestic use + exports) as a percentage of inventories.

This form of the model is based on the hypothesis that the ratio of consumption to inventories provides a better measure of relative scarcity than a linear combination of consumption and inventories. However, this does not eliminate the fundamental methodological problem of using lagged adjustment models. Levels of inventories, domestic consumption, and exports are not qualitatively equivalent measures of the external determinants of inventory, domestic and foreign demand. First, changes in inventories and consumption may be due to movements along a demand schedule as well as shifts in demand. A change in, for example, exports may not be associated with any change in the level of export demand. Second, within a crop year inventories and consumption are in a fixed relationship. An increase in consumption is matched by a greater rate of inventory reduction. However, the source of this change may be due to either, or both, a reduction in inventory demand or an increase in consumption demand. While this problem may be unavoidable, estimations based on a lagged adjustment models are likely to suffer from specification bias. Results should be examined carefully and conclusions should be tempered.

Estimation of a linear Price Adjustment Model

Monthly changes in Hard Red Winter wheat prices at Kansas City were estimated using a disequilibrium lagged price model. To quantify the model, aggregate figures on wheat supplies and disappearances were interpolated to monthly data points. Supply and disappearance figures are reported by the USDA for four irregular intervals; June through September (harvest), October through December, January through March and April through May. The simplest procedure is to use a linear interpolation. Monthly averages are computed for domestic use and exports over each observation period. Inventory levels over the period may be accounted for by using the average rate of disappearance. There are two disadvantages of using a linear interpolation. First, no additional information on disappearance is provided. Monthly exports and domestic usage are constant within a reporting period. Second, transitions between reporting periods tend to be overly abrupt. Both these problems can be reduced by incorporating more information in a nonlinear interpolation.

The interpolation problem is illustrated in Figure 4.18. The interval to be interpolated is from two to four months. The points bounding an interval are cumulative disappearance as of the end of the reporting period (or equivalently the beginning of the next period). Two data points are required to define the interpolation interval. Two more data points, a total of four, are needed to include the preceding and following intervals. A third degree Lagrangian interpolating polynomial may be constructed to pass through these four points with the formula:[29]

$$L(t) = \sum_{i=1}^{3} l_i(t)y \qquad (4.13.0)$$

where;

$$l_i(t) = \prod_{\substack{j=0 \\ i \neq j}} \frac{(t - t_j)}{(t_i - t_j)} \quad i = 0,1,2,3,$$

and;

t = cumulative time in months
y = cumulative disappearance
i,j = subscripts denoting data points

Interpolations are made by substituting in time and disappearance values into the formula for appropriate data points and specifying time at the point to be interpolated. Total disappearances between 1972/73 and

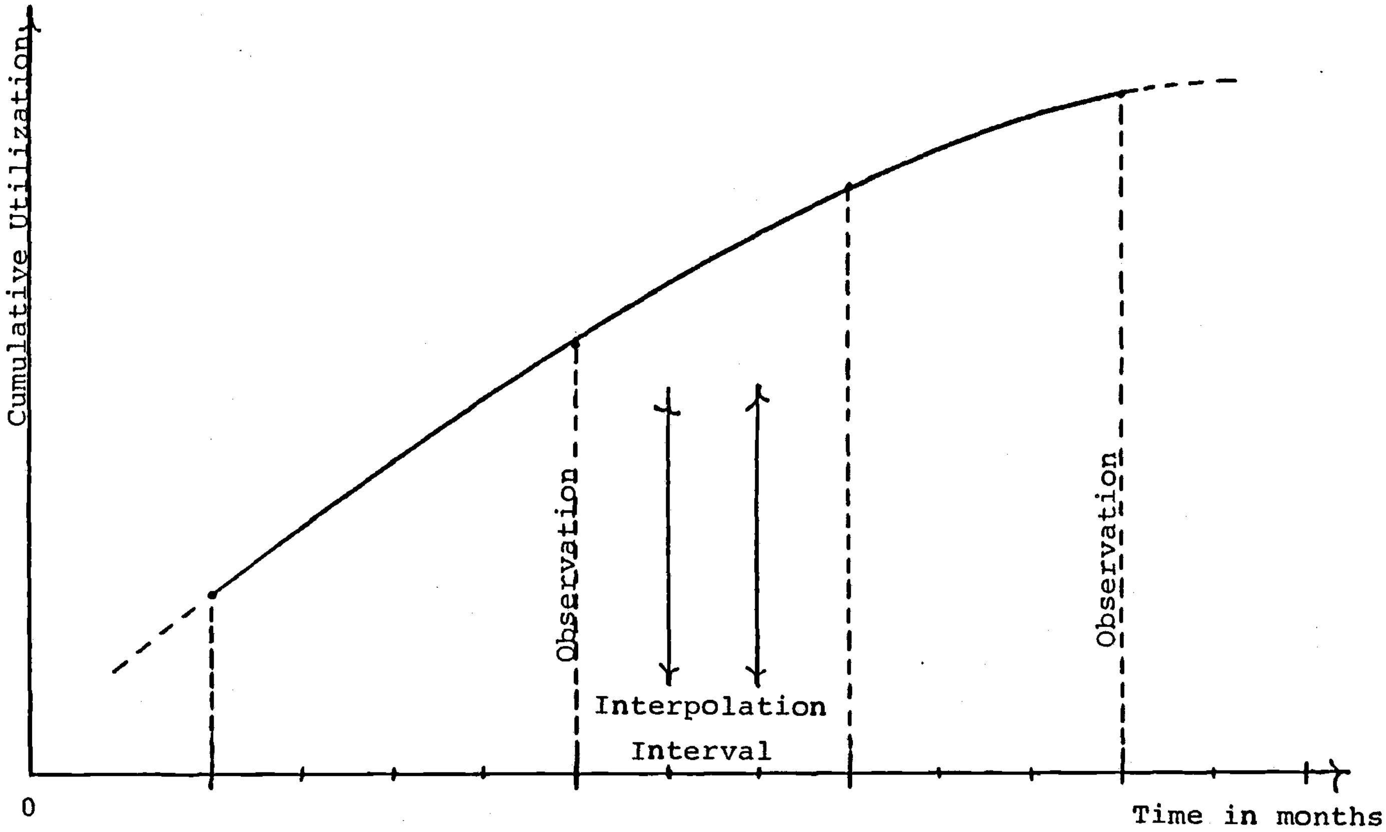

Figure 4.18. Interpolation of monthly observations from aggregated observations reportated at irregular intervals.

1982/83 (domestic consumption plus exports), were interpolated to cumulative monthly data. These data were then transformed into monthly disappearances. The results are presented in tabular form in Appendix B.

There are at least two ways to compute the relative rate of disappearance. First, monthly disappearances may be expressed as a percentage of currently available supplies Second, monthly disappearances may be expressed as a percentage of beginning stocks (production plus carryover). The first measure tends to exhibit a strong seasonal pattern, following the sudden increase in inventories over harvest and the decline in stocks over the remainder of the crop year. This would fit a pattern of increasing prices through a crop year with a sharp decline over harvest. This pattern is not evident in the sample period. The second measure, selected here, follows current consumption relative to annual production and carryover. It ignores the cumulative effect of consumption through a crop year. Monthly disappearance as a percentage of beginning inventories, MD%BI, is plotted with prices in Figure 4.19. The figure illustrates that prices tend to follow changes in the rate of disappearance. However, a number of exceptions can be seen. It is of interest to note that the variation in

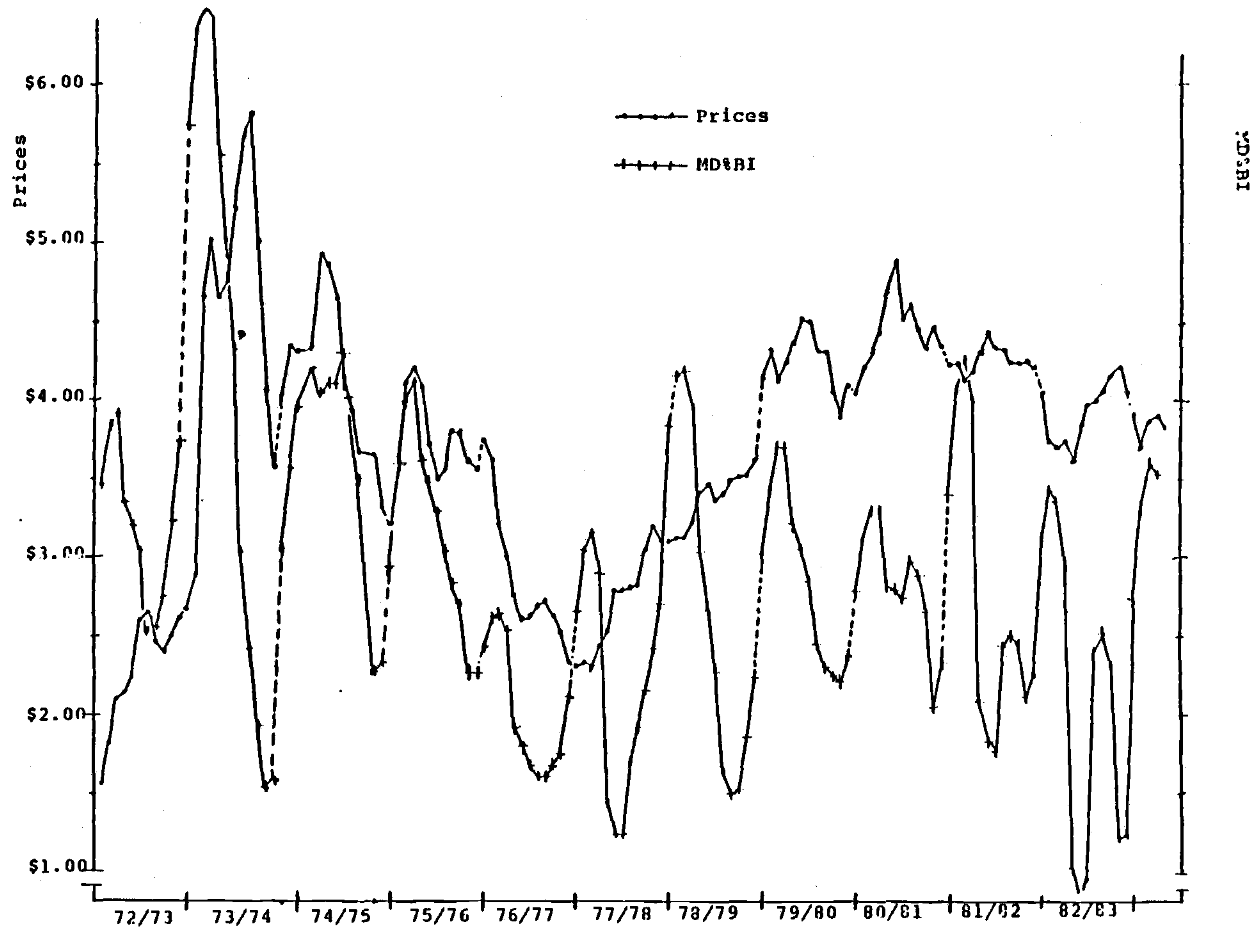

Figure 4.19. Monthly prices and disappearances as a percentage of beginning inventories (MD%BI) over the sample period.

MD%BI is relatively constant throughout the sample period. The variation in prices between 1972/73 and 1977/78, period I, is substantially greater than price variation between 1978/79 and 1982/83, period II. This does support an earlier contention regarding the relative degree of speculation in the two periods. Conditions were more favorable for speculation in period I.

The model was estimated using Ordinary Least Squares (OLS). The results are presented in Table 4.7. One statistic that stands out is the low coefficient of determination, $R^2 = .169$. The poor fit is a good indication of how well econometric models predict short-term price changes in general. By adding P_{t-1} to both sides of the regression equation the corresponding price prediction model is obtained. The coefficient of determination for the price model, $R^2 = .906$, indicates a good fit with the sample data. However, the explanatory power of the price model is due to the fact that prices are relatively large in comparison to monthly price changes. The lagged adjustment model explains only about 17 percent of monthly variation in wheat prices. It is reasonable to assume that the percentage of price variation due to changes in the exchange environment is considerably higher.

Table 4.7. Estimation of wheat price changes and prices: No. 1 Hard Red Winter wheat at Kansas City 1972/73 to 1982/83.[a,b]

Linear Disequilibrium Price Adjustment Model

$$\Delta P = -15.67 - 0.089P_{t-1} + 0.085MD\%BI$$
$$(15.18) \quad (.027) \qquad (.021)$$

R^2 = .169 D.W. = 1.400
MSE = 639.06 d.f. = 127

Implicit Price Prediction Model

$$P = -15.67 + 0.911P_{t-1} + 0.085MD\%BI$$
$$(15.18) \quad (.027) \qquad (.021)$$

R^2 = .906 D.W. = 1.400
MSE = 639.06 d.f. = 127

[a]Source: Wheat Situation, various issues.
[b]Standard errors in parentheses.

The signs of the estimated coefficients for P_{t-1} and the rate of consumption are appropriate. With the exception of the constant term, the estimated coefficients are significant at the 99 percent confidence level. Despite the lack of predictive ability, which may be an unavoidable limitation, the model appears to be reasonably well structured. The level of positive serial correlation indicated by the Durbin-Watson test statistic is significant at the 99 percent confidence level. This may be an indication of specification bias but this was anticipated.

The variance in monthly price changes in each of the two sample subperiods, periods I and II, can be compared using an F test:

$$F^*_{(69,59)} = \frac{s^2(\Delta P)_I}{s^2(\Delta P)_{II}} = 5.392$$

The gross difference in variation is significant at the 99.9% confidence level. An equivalent test for the variance in model errors can be constructed:

$$F^*_{(67,57)} = \frac{s^2(e)_I}{s^2(e)_{II}} = 3.428$$

The net (unexplained) variation in period I is significantly greater than in period II, at the 99.9% confidence level. If the model adequately explained the impact of external sources of price variation, we might conclude that speculative reservation demand may have been a significant source of price variability. However, given the inadequacies of the model, these results simply fail to reject this hypothesis. Unfortunately, this condition would appear to hold true for any hypothesis tests that are explicitly dependent on controlling for the external determinants of supply and demand.[30]

Before exploring some alternative hypotheses, there are a few interesting questions which can be investigated within the general framework of a lagged adjustment model. It is reasonable to assume that individuals would show a greater tendency to speculate on continuing trends if past price changes are a relatively good predictor of future price changes. At the same time, if individuals tend to speculate on price movements and artificially extend trend, then there should be a greater tendency for price changes to be correlated over time.

Current price changes can be estimated as a linear function of lagged prices and price changes. In addition, the impact of past price movements can be estimated individually for periods I and II by using binary interaction variables.[31] The estimation equation may be written:

$$\Delta P = b_o + b_1 P_{t-1} + b_2[x_1(\Delta P)] + b_3[x_2(\Delta P_{t-1})] \qquad (4.13.1)$$

where;

$$x_1 = \begin{cases} 1 \text{ in in period I; 1972/73-1977/78} \\ 0 \text{ if in period II; 1978/79-1982/83} \end{cases}$$

$$x_2 = \begin{cases} 0 \text{ if in period I} \\ 1 \text{ if in period II} \end{cases}$$

The model is estimated using OLS and the results are summarized in Table 4.8. The model explains a slightly greater proportion of the variation in price changes, nearly 20 percent. A comparison of the lagged price change variable in each period is of greater interest. About 40 percent of the last period change is projected forward in period I. The period I coefficient is very significant (t* = 4.674). Only 20 percent of last period

Table 4.8. Estimation of monthly wheat price changes using lagged and price lagged price changes: No. 1 Hard Red Winter wheat at Kansas City.[a,b]

$$\Delta P = 33.58 + -0.088P_{t-1} + 0.402(x_1\Delta P_{t-1}) + 0.212(x_2\Delta P_{t-1})$$

(10.38) (.027) (.086) (.217)

R^2 = .199
MSE = 620.94
df = 126

[a] Source: Wheat Situation, various issues.
[b] Standard errors are in parentheses.

change is projected forward in period II. The period II coefficient is not significant at the 90 percent confidence level ($t^* = 0.997$).

Lagged price changes may be incorporated into the disequilibrium adjustment model developed earlier. The estimation equation can be written:

$$\Delta P = b_o + b_1 P_{t-1} + b_2 MD\%BI + b_3 \Delta P_{t-1} \qquad (4.13.2)$$

The model is estimated using OLS for both the complete sample period and period I. The results are summarized in Table 4.9. There is an improvement in the coefficient of determination, $R^2 = .250$, over the full sample. This represents about a 10 percent reduction in unexplained variation (a 48 percent increase in explained variation), in comparison to the initial disequilibrium adjustment model estimate. The model coefficients are of appropriate sign. With the exception of the constant term, the coefficients are significant at the 99 percent confidence level. For the period I subsample, 35 percent of the monthly variation in price changes is explained by the model, $R^2 = .353$. The coefficients remain of appropriate sign and are significant at the 99 percent confidence level. There is a reduction in the magnitude and significance of the lagged price change coefficient in period I. This is somewhat surprising since there appeared to be a greater correlation between price changes in period I as opposed to period II. However, this may reflect the fact that the initial form of the adjustment model performs relatively poorly in period II. Relative to the other dependent variables in the equation, lagged price changes may be a better predictor of current price changes in period II.

The attempt to control for external source of variation in monthly wheat prices using a lagged adjustment model was not particularly successful. This greatly reduces our capacity to objectively examine prices and price movements for evidence of catastrophic behavior. The results did suggest that current market information on prices and price trends are perhaps the best predictors of future prices and price changes. This points to the potential link between the formation of expectations and price determination. We can consider some alternative hypotheses concerning the impact of expectations and speculative behavior, which may be evaluated on a more subjective basis.

Table 4.9. Estimated changes in monthly wheat prices, disequilibrium adjustment with lagged price changes; No. 1 Hard Red Winter at Kansas City 1972/73 to 1982/83.[a,b]

Full Sample

$$\Delta P = -1.50 + -0.095P_{t-1} + 0.064MD\%BI + 0.308\Delta P_{t-1}$$

(14.97) (0.026) (0.020) (0.080))))

R^2 = .250
MSE = 591.24
df = 125

Period I: 1972/73 to 1977/78

$$\Delta P = -20.67 + -0.136P_{t-1} + 0.113MD\%BI + 0.272\Delta P_{t-1}$$

(20.52) (0.037) (0.031) (0.107)

R^2 = .353
MSE = 831.92
df = 65

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[a] Source: Wheat Situation, various issues
[b] Standard errors in parentheses

Wheat Prices: Turning Point Changes

The single most important indication of catastrophic behavior is the occurrence of sudden jumps in the state of a system, which is otherwise responding smoothly. Prices and price changes, which describe the state of exchange for a given commodity, are reported at discrete intervals.

With, for example, average monthly wheat prices, the distinction between smooth and sudden change becomes very subjective. We can use descriptive statistics to help analyze turning point changes in wheat prices over the sample period. In addition, Zeeman's hypothesis concerning turning points can be modified slightly and evaluated.

Turning points for wheat prices between 1972/73 and 1982/83 are presented in Table 4.10. Included in the table are the preceding and following rates of change in price about a turning point, and the total change in price over a turning point. The average absolute change over a turning point was 27.0 cents per bushel. Excluding turns occurring during harvest, the average absolute change was slightly greater at 28.6 cents per bushel. The average absolute price change represents about a 7.8 percent change in the average price for the sample period ($3.70/bu.). The greatest turning point increase for the sample period was 94 cents per bushel. The greatest turning point decline was 95 cents per bushel. The standard deviation for absolute turning point changes was 21.2 cents per bushel for the complete sample and 20.3 cents per bushel excluding harvest periods.

Table 4.10. Wheat prices; turning point changes: No. 1 Hard Red Winter at Kansas City.[a]

Dates		ΔP_{t-1}	ΔP_t	Total Change	Dates		ΔP_{t-1}	ΔP_t	Total Change
Jan/Feb	73	5	-19	-24	Sept/Oct	79	-22	14	36
Mar/Apr	73	-6	9	15	Nov/Dec	79	14	-2	-16
Sept/Oct	73	34	-34	-68	Apr/May	80	-17	20	37
Oct/Nov	73	-34	11	45	May/Jun*	80	20	-3	-23
Feb/Mar	74	14	-81	-95	Jun/Jul*	80	-3	14	17
May/Jun*	74	-48	46	94	Nov/Dec	80	19	-35	-54
Jun/Jul*	74	31	-3	-34	Dec/Jan	81	-35	6	41
Jul/Aug*	74	-3	2	5	Jan/Feb	81	6	-13	-19
Oct/Nov	74	59	-6	-65	Mar/Apr	81	-12	13	25
Jun/Jul*	75	-11	38	49	Apr/May	81	13	-12	-25
Sept/Oct	75	9	-12	-21	Jun/Jul*	81	-12	13	25
Dec/Jan	76	-21	7	28	Jul/Aug*	81	1	-11	-12
Mar/Apr	76	0	-20	-20	Aug/Sept*	81	-11	5	16
Nov/Dec	76	-15	2	17	Nov/Dec	81	15	-11	-26
Feb/Mar	77	3	-10	-13	Mar/Apr	82	-1	3	4
Jun/Jul*	77	-5	4	9	Apr/May	82	3	-6	-9
Jul/Aug*	77	4	-4	-8	Aug/Sept*	82	-4	5	9
Aug/Sept*	77	-4	16	20	Sept/Oct	82	5	-14	-19
Nov/Dec	77	25	-1	-26	Oct/Nov	82	-14	25	39
Dec/Jan	78	-1	2	3	Apr/May	83	3	-16	-19
Apr/May	78	14	-9	-23					
May/Jun*	78	-9	0	9					
Nov/Dec	78	6	-9	-15					
Dec/Jan	79	-9	3	12					
Aug/Sept*	79	17	-22	-39					

* denotes harvest period

[a]Source Wheat Situation: various issues.

It is evident that wheat prices do tend to turn sharply. In 40 percent of the observations, turning point changes were greater than or equal to 25 cents per bushel, 22 percent were greater than or equal to 35 cents per bushel and 16 percent exceeded 45 cents per bushel. It would not appear inappropriate to refer to changes of this magnitude as being sudden or abrupt. However, these changes are not necessarily linked to speculative behavior.

Zeeman derived a hypothesis based on an underlying dynamic structure of a cusp catastrophe with slow feedback; at a turning point, the greater the rate of increase in price the greater the rate of decline.[32] We can extend this hypothesis to a jump to a rising market; the greater the rate of decline the greater the rate of increase. If speculative reservation demand gives rise to a cusp dynamic then we should expect to observe a significant correlation between price changes preceding and following a turning point. For the sample period, the correlation between preceding and following price changes is -.588, which is significantly greater than zero at the 99 percent confidence level. The square of the correlation coefficient is the coefficient of the determination for the simple regression prediction equation:

$$\Delta P_t = -1.17 + -0.620\ \Delta P_{t-1} \qquad R^2 = .345$$
$$(0.458) \quad (0.130) \qquad MSE = 265.00$$

The predictive accuracy of the regression equation compares favorably with the lagged adjustment models developed earlier.

<u>Wheat Prices</u>: <u>Stability I</u>

A large amount of attention has been focused on market stability. For the most part, stability has been defined mathematically and has been used in reference to market structure. Here, we will consider the common concept of stability. Interestingly, this does not lead to a more concrete definition. The terms stable and unstable relate to the variability of prices. However, there is no definitive way to measure stability. We can use statistical measures to describe the apparent distribution of prices and price changes for an arbitrarily specified observation interval. If these distributions appear to be normal, then we may make statistical inferences concerning relative price variation. Nevertheless, to describe a market as stable or unstable is purely a subjective evaluation.

A primary hypothesis of this paper is that speculative behavior introduces greater instability into market prices. This should be reflected in the distribution of price changes for a given stored commodity. This might be measured by an increase in the variance of price change. Earlier we compared the variance in monthly price changes between two periods; period I, 1972/73 to 1977/78 and period II, 1978/79 to 1982/83. The variance in period I was found to be significantly greater than in period II:

$$V(\Delta P)_{I} = 1198.64 \quad (¢/bu)^2$$

$$V(\Delta P)_{II} = 222.65 \quad (¢/bu)^2$$

The extent to which this difference can be attributed to higher levels of speculative demand in period I is unknown. Both speculative and external changes in supply and demand may have an equivalent impact on the variance of price movements.

If greater price variability in period I was due to a corresponding increase in the variance of nontransitory supply and demand, then, the only difference we would expect to observe between the distribution of price changes for the two periods is a higher variance. If the greater price variability in period I was due, in part, to higher levels of speculative demand, we should observe a difference in the shape of the price change distributions. As speculative content increases, sudden jumps occur with increasing frequency and magnitude. The increase in variation is introduced at the tail ends of the price change distribution. The distribution of price changes may exhibit positive kurtosis (leptokurtic); with a large number of observations concentrated near the mean and with the remaining observations tending towards the extreme ends of the distribution. In contrast to a normal distribution (Figure 4.20a), a leptokurtic distribution (Figure 4.20b) tends to be more peaked and have elevated tails.

Kurtosis coefficients can be estimated from the second and fourth moments of the observed price changes in periods I and II. The formula for the kurtosis coefficient is given by:

$$g_2 = (u_4/u_2^2) - 3$$

where;

g_2 = coefficient of kurtosis
u_2 = second sample moment
u_4 = fourth sample moment

$$u_i = (1/n) \sum_{j=1}^{n} (\Delta P - \overline{\Delta P})^i$$

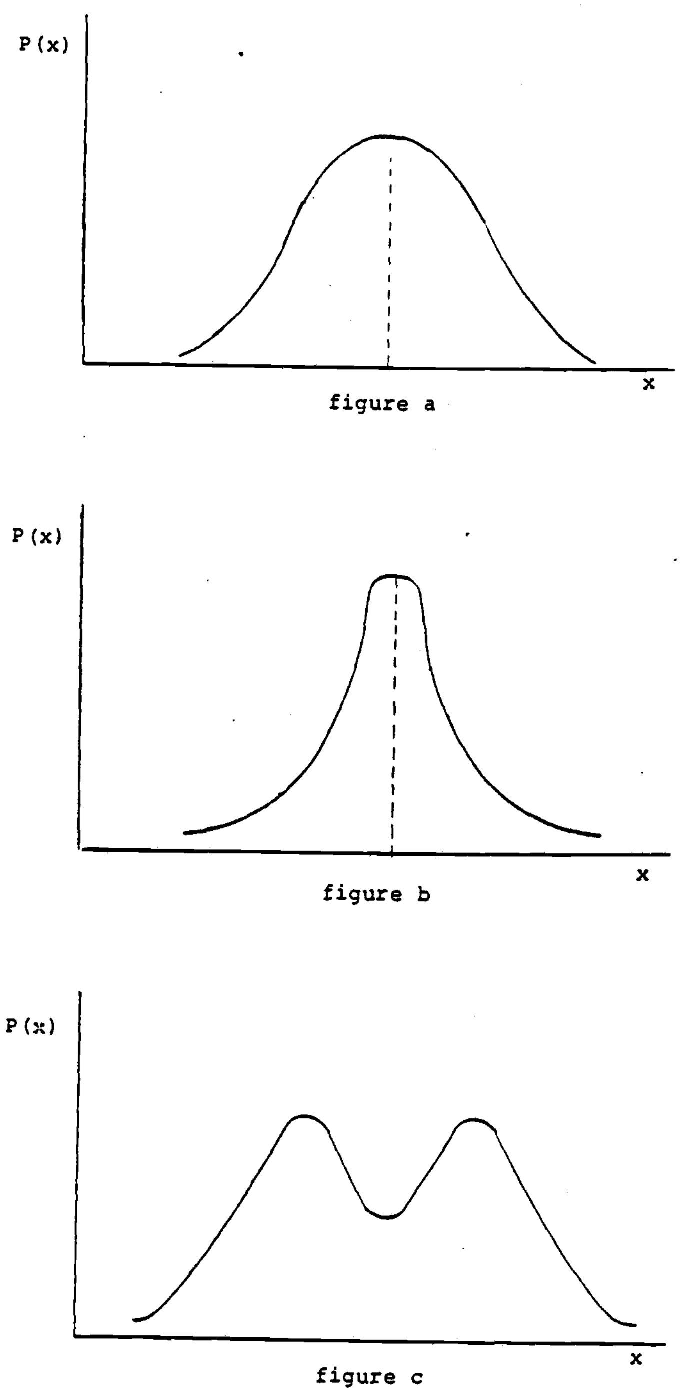

Figure 4.20. Probability distributions: a) normal, b) leptokurtic (positive kurtosis), c) bimodal.

The estimated standard error for the coefficient of kurtosis is given by the formula:

$$SE_{g_2} = 2(6/n)^{1/2}$$

The coefficients of kurtosis and their respective standard errors for periods I and II are:

$$(g_2)_I = 8.057 \qquad (SE_{g_2})_I = 0.586$$
$$(g_2)_{II} = 1.288 \qquad (SE_{g_2})_{II} = 0.632$$

Assuming that the periods can be treated as independent samples and the sample coefficients are distributed normally, the significance of the difference between the coefficients of kurtosis can be tested by computing a standard normal variate:[33]

$$z^* = (8.057-1.288)/(.586^2+.632^2)^{1/2}$$
$$= 7.854$$

The degree of kurtosis is significantly greater in period I at the 99.9 percent confidence level.

The preceding test for kurtosis was applied directly to monthly price changes. In considering market adjustment and movements along a price dynamic, it is more appropriate to evaluate the distribution of first differences in price changes (second differences in price). In other words, the contrast between a smooth movement along a dynamic versus a sudden jump between alternative dynamics is more evident in the rate of change in price adjustment, as opposed to the magnitude of price adjustment. First differences in price changes for periods I and II were calculated. The variances for the distributions of first differences in the two periods are:

$$V(\Delta_2 P)_I = 39.350$$
$$V(\Delta_2 P)_{II} = 19.525$$

The coefficients of kurtosis and their respective standard errors for the two periods are:

$$(g_2)_I = 4.814 \qquad (SE_{g_2})_I = 0.590$$
$$(g_2)_{II} = 0.361 \qquad (SE_{g_2})_{II} = 0.638$$

The standard normal deviate for the difference between the sample period coefficients is:

$$z^* = (4.453)/(0.869) = 5.126$$

The coefficient of kurtosis for the distribution of first differences in price changes is significantly greater in period I at the 99.9 percent confidence level. The distribution is apparently normal in period II and leptokurtotic in period I. This is some indirect evidence that one: speculative behavior is, in part, responsible for the greater variation in prices in period I; and two: lower levels of speculative reservation demand are associated with market conditions in period II, i.e., weaker price trends, higher interest rates and an active government reserve program.

The potential for speculative behavior to increase the variance in prices may appear much more important than the introduction of kurtosis into the distribution of price changes. To a large extent this is true, kurtosis was introduced as a test for speculative behavior. However, the presence of positive kurtosis holds some interesting implications towards the assessment of price risk. An evaluation of price risk is commonly based on the mean and variance of price distribution which is assumed to be normal. A significant level of kurtosis may create serious errors in the estimation of confidence intervals and probabilities for price changes. The positive kurtosis exhibited in the distribution of wheat price changes would lead to an under estimation of both the probability of small and very large price movements. Using the standard deviation of wheat price change over the sample period, 27.5, would overstate nominal price risk as too much weight is given to the middle of the distribution. At the same time, catastrophic risk is understated as the tail is discounted. This is an area that warrants future research.

Bimodality, the tendency for a market to exist in either a rising or a falling state, is another characteristic or flag of catastrophic behavior. Bimodality is closely related to inaccessibility. If a stable state of equilibrium is inaccessible, then a market must be either rising or falling. We might expect that the distribution of monthly price changes should exhibit bimodality, if there exists an underlying cusp structure. A bimodal distribution is illustrated in Figure 4.20c. Histograms for the monthly distribution of price changes and first differences in prices changes are illustrated in Figures 4.21 and 4.22 respectively. Clearly, these distribution are unimodal (the figures also show the kurtosis of the distributions). The level of speculative reservation demand in the wheat market is not sufficient to sustain alternating states of rapidly rising and falling prices. Nontransitory supply and demand are the primary determinants of prices and price movements. However, the distribution of monthly price movements provides a myopic view of market stability. Monthly price movements give no indication whether if prices tend to be

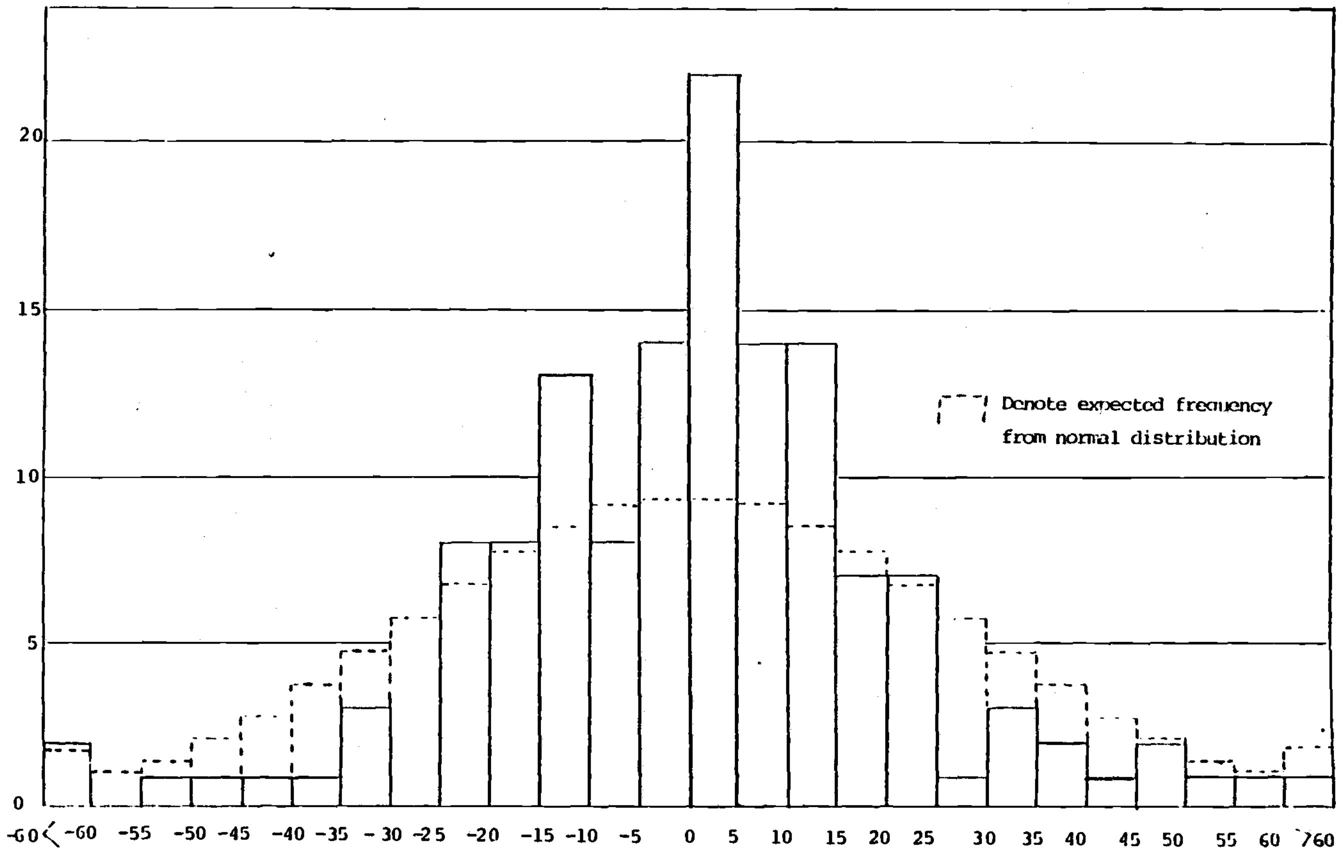

Figure 4.21. Histogram: Distribution of monthly wheat price changes; Hard Red Winter wheat at Kansas City, 1972/73 to 1982/83.

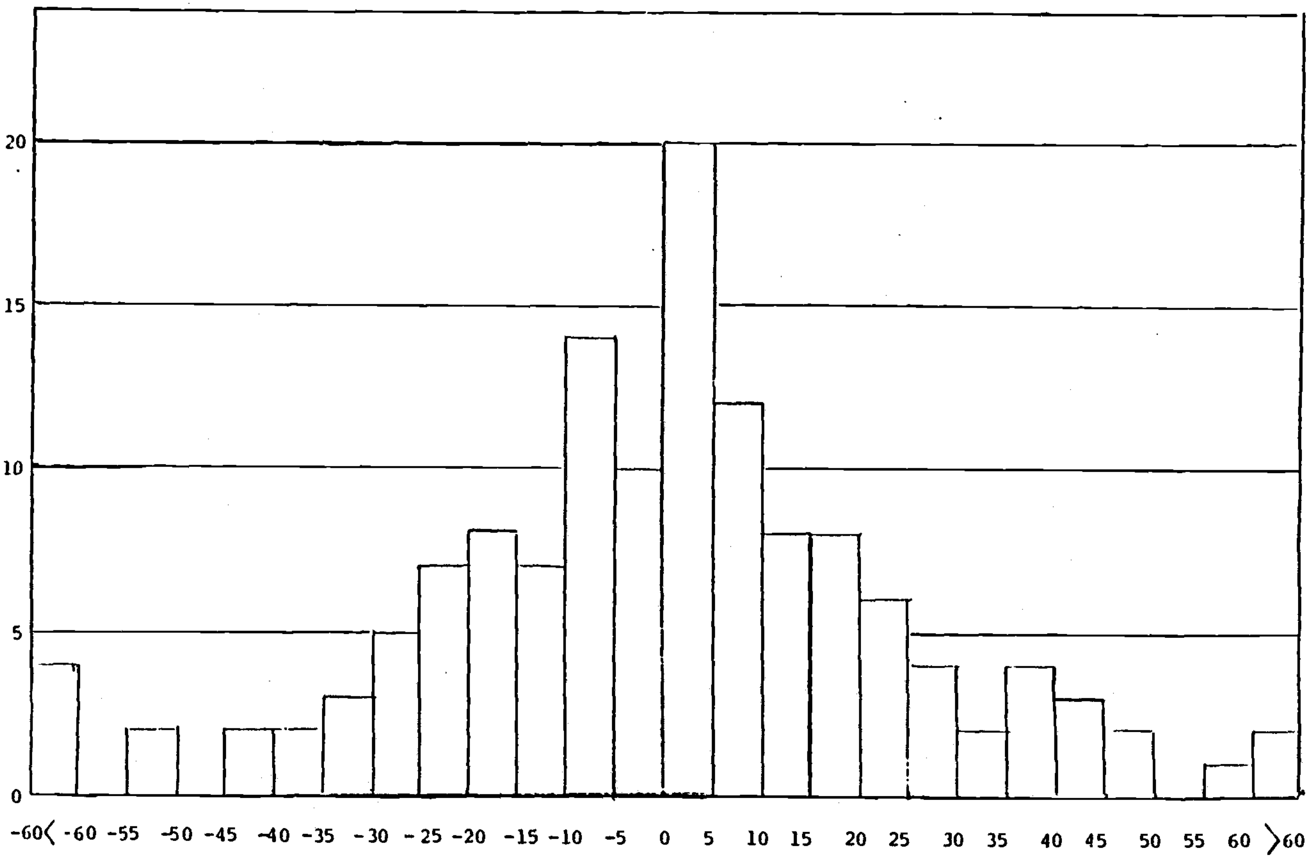

Figure 4.22. Histogram: Distribution of second differences in monthly wheat prices; Hard Red Winter wheat at Kansas City, 1972/73 to 1982/83.

rising or falling over any extended period of time. The plot of price changes over the sample period, Figure 4.16, provides a better picture of the pattern of market adjustment. This pattern is consistent with the concept of a bimodal market. An examination of pricing patterns over time, price trends, may yield a broader perspective of market stability.

Wheat Price Stability: II

The most tangible, and perhaps the most important, hypothesis that can be drawn from a cusp market model of speculative behavior is that speculation artificially sustains price trends. The simulation model was used, in part, to demonstrate the irregular cycles of overevaluation and underevaluation (relative to an exogenously defined state of equilibrium) created by a positive link between current price trends and transitory demand. Aside from simply attempting to find some empirical verification of this hypothesis, it would be desirable to find a measure of speculative content which gives a prior indication of the risk of catastrophic price changes.

Given that individuals are willing to speculate on current price trends, an existing trend is some measure of speculative content. Hence, we should observe positive relationships between the magnitude of successive price trends. We may also anticipate that price trends are a poor measure of speculative content, as trends vary with changes in nontransitory supply and demand. In previous efforts to account for nontransitory supply and demand, the rate of inventory consumption was used as an indirect measure of relative value. Generally, there may exist a qualitative correspondence between relative value and the rate of consumption. Prices and the rate of consumption should rise (fall) with a decline (increase) in supplies. Prices and the rate of consumption should rise (fall) with an increase (decrease) in the level of final demand. An inversion of this relationship may occur with rising prices and increasing inventory demand; the rate of consumption may fall with an upward movement along the utilization demand schedule. An inversion may also occur with falling prices and declining reservation demand; the rate of consumption may increase with an upward movement along the utilization demand schedule.

An inverted relationship between price movements and the rate of inventory consumption may be a fairly good indication of ongoing speculation. This inversion may not yield a qualitatively equivalent measure of speculative content but it may lead to a reasonable approximation. Current price trends can be decomposed into two components: one, price changes accompanied by an equivalent directional change in the rate of consumption and two, price changes accompanied by an inverse directional change in the rate of consumption. The results of this decomposition for the sample period are presented in Table 4.11. Included in the table are the dates and magnitudes of price trends over the sample period, the average rate of change in price and the change in the monthly rate of consumption as a percentage of beginning inventories. The final two entries of the table are the decomposed price changes (computed on a cumulative month to month basis over a trend).

As a basis for comparison, the following estimation equation was estimated using OLS:

$$(\Delta_T P)_t = b_o + b_1 (\Delta MD\%BI)_t + b_2 (\Delta_T P)_{t-1}$$

where;

$\Delta_T P$ = the price trend

$\Delta MD\%BI$ = change in the rate of monthly disappearance as a percentage of beginning inventories

t = time over successive price trends.

Table 4.11. Wheat price trends; Hard Red Winter at Kansas City 1972/73 to 1982/83.[a]

n	Dates		$\Delta_T P$	T	$\overline{\Delta P}$	ΔMD%BI	$\Delta_T P'$	$\Delta_T P^*$
1	Jul/Jan	73	109	6	18.2	-95	52	57
2	Feb/Mar	73	-25	2	-12.5	24	0	-25
3	Apr/Sep	73	259	6	43.2	355	225	34
4	Oct	73	-34	1	-34.0	-87	-34	0
5	Nov/Feb	74	115	4	28.8	-316	0	115
6	Mar/May	74	-223	3	-74.3	-83	-175	-48
7	Jun/Jul	74*	77	2	38.5	200	77	0
8	Aug	74*	-3	1	-3	38	0	-3
9	Sep/Oct	74	61	2	30.5	9	2	59
10	Nov/Jun	75	-171	8	-21.4	-111	-71	-100
11	Jul/Sep	75*	98	3	32.7	117	98	0
12	Oct/Dec	75	-71	3	-23.7	-82	-71	0
13	Jan/Mar	76	31	3	10.3	-70	0	31
14	Apr/May		24	2	-12.0	-31	-24	0
15	Jun	76*	18	1	18.0	15	18	0
16	Jul/Nov	76	-113	5	-22.6	-67	-59	-54
17	Dec/Feb	77	11	3	3.7	-18	0	11
18	Mar/Jun	77	-42	4	-10.5	106	0	-42
19	Jul	77*	4	1	4.0	38	4	0
20	Aug	77*	-4	1	-4.0	10	0	-4
21	Sep/Nov	77	50	3	16.7	-191	50	50
22	Dec	77	-1	1	-1.0	0	-1	0
23	Jan/Apr	78	41	4	10.3	118	41	0
24	May	78	-9	1	-9.0	28	0	-9
25	Jun	78*	0	1	0	-86	0	0
26	Jul	78*	2	1	2.0	31	2	0
27	Aug	78*	0	1	0	3	0	0
28	Sep/Nov	78	34	3	11.3	-151	0	34
29	Dec	78	-9	1	-9.0	-40	-9	0
30	Jan/Jul	79	95	7	13.6	117	84	11
31	Aug	79*	-22	1	-22.0	26	0	-22
32	Sep/Nov	79	41	3	13.7	-65	0	41
33	Dec/Apr	80	-63	5	-12.6	-85	-63	0
34	May	80	20	1	20.0	17	20	0
35	Jun	80*	-3	1	-3.0	41	0	-3
36	Jul/Nov	80	82	5	16.4	0	-38	44
37	Dec	80	-35	1	-35.0	-3	-35	0
38	Jan	81	6	1	6.0	22	6	0
39	Feb/Mar	81	-25	2	-12.5	-31	-25	0
40	Mar	81	13	1	13.0	-61	0	13

Table 4.11. Continued.

n	Dates	$\Delta_T P$	T	$\overline{\Delta P}$	ΔMD%BI	$\Delta_T P'$	$\Delta_T P*$
41	May/Jun 81	-24	2	-12.0	135	0	-24
42	Jul 81*	1	1	1.0	66	1	0
43	Aug 81*	-11	1	-11.0	21	0	-11
44	Sep/Nov 81	32	3	10.7	-245	0	32
45	Dec/Mar 82	-21	4	-5.3	63	-12	-9
46	Apr 82	3	1	3.0	-34	0	3
47	May/Aug 82	-56	6	-9.3	126	-2	-54
48	Sep 82*	5	1	5.0	-43	0	5
49	Oct 82	-14	1	-14.0	-193	-14	0
50	Nov/Apr 83	60	6	10.0	-80	22	38
51	May/Jul 83	-50	3	-16.7	110	0	-50

[a] Variable Codes:

$\Delta_T P$ = price trend

T = trend durration in months

$\overline{\Delta P}$ = average rate of change in price

ΔMD%BI = change in rate of monthly dissappearance as a precentage of beginning inventories

$\Delta_T P'$ = corresponding change in price

$\Delta_T P*$ = inverted change in price

Results are summarized in Table 4.12. The results are similar to those obtained for estimated monthly price changes, in terms of relative predictive accuracy and the significance levels of the variables. The model accounts for roughly 23 percent of the variation in price trends. Trend variation is substantial; the sample standard deviation for price trends is over 70 cents per bushel. Within the model, the contribution of information by the nearby trend is more significant than current information on changes in the rate of consumption. If this is purely due to that fact that the rate of inventory utilization is a very poor measure of relative value, then decomposing a price trend on the basis of a qualitative change in the rate of consumption would not tend to improve the explanatory power of the model. If the unexplained variation in price trends is, to a significant extent, due to speculation, then, decomposing trends into a speculative component should significantly improve the explanatory power of the model.

The estimation equation for the speculative decomposition model was estimated using OLS:

$$(\Delta_T P)_t = b_o + b_1(\Delta MD\%BI)_t + b_2(\Delta_T P^*)_{t-1}$$

Table 4.12. Results for wheat price trend estimations; Hard Red Winter wheat at Kansas City, 1972/73 to 1982/83.

Base Model

$$(\Delta_T P)_t = 5.33 + 0.113(\Delta MD\%BI)_t + -0.462(\Delta_T P)_{t-1}$$

(8.972) (0.082) (0.129)

t*=0.59 t*=1.37 t*=3.58

R^2 = .228
MSE = 3994
RMSE = 63.2
df = 47

Speculative Decomposition Model

$$(\Delta_T P)_t = 7.03 + 0.098(\Delta MD\%BA)_t + -1.250(\Delta_T P^*)_{t-1}$$

(7.880) (0.072) (0.129)

t*=0.89 t*=1.35 t*=5.11

R^2 = .407
MSE = 3068
RMSE = 55.4
df = 47

Comparison of Model Error Variances

$F^*_{(47,47)}$ = (3994)/(3068) = 1.30

Significance level = 85% (one-tailed test)

The speculative decomposition model yields a surprisingly better estimate (Table 4.12). There is a 23 percent reduction in the unexplained variation and an increase in the significance of the nearby trend variable intended to measure speculative content. The model by no means explains the variation in wheat price trends: 60 percent of the variance in price trends remains unexplained. However, the reduction in error variance in the speculative decomposition model is significant at the 85 percent confidence level.

These results do suggest that wheat prices are subject to speculative fluctuations. Furthermore, an inversion in the relationship between prices and the rate of utilization appears to indicate that prices are tending to be over or undervalued. Certainly, wheat prices exhibit wide erratic price swings which are, for the most part, unpredictable. This too, is an expected consequence of speculative behavior.

Summary

In this chapter we have explored a cusp dynamic as a pricing structure through simulation experiments. The simulated link between the formation of expectations and price determination yielded pricing patterns that closely resembled actual price movements. A graphical presentation of wheat prices and price changes over time was made for comparison. While this comparison could not validate the hypothesis of an underlying cusp structure in the wheat market, it did suggest that the model offers a plausible picture of the dynamics of price adjustment for stored commodities.

The wheat production and marketing system was examined to determine potential sources of variability in wheat prices. Aggregate changes in production and export demand appear to be the principal determinants of general price movements. In investigating the impacts of inventory demand and government farm policies, a consideration of speculative behavior led to a pair of interesting hypotheses. These hypotheses were evaluated and the results were found to be consistent with the occurrence of speculative price cycles in the wheat market.

To attempt to develop more conclusive tests for speculative behavior, a linear pricing model was estimated to control for external sources of price variation. The model's predictive accuracy, in estimating monthly changes in wheat prices, was judged inadequate for this purpose.

However, the model did provide some indirect evidence of speculative behavior.

In the remainder of this chapter, a direct examination of wheat prices was conducted. Turning points, the distribution of price changes, and market trends were explored. Hypotheses based on a postulated cusp dynamic were developed and tested. In general, the results were consistent with these hypotheses. As with the other quantitative tests conducted to evaluate the significance of speculative behavior, no truly conclusive evidence was found. Considerable evidence was found to suggest that speculative reservation demand is a potential source of price instability in the wheat market. Whether or not this compilation of indirect evidence supports a specific set of conclusions is taken up in the following chapter.

Endnotes

1 Saunders, An Introduction to Catastrophe Theory, p. 83.

2 Thom, "Catastrophe Theory: Its Present State and Future Perspectives." Published in E.C. Zeeman, Catastrophe theory Selected Papers 1972-1977, pp. 635-38.

3 This conclusion is based on a review of unconstrained nonlinear minimization techniques. Source material was found in Himmelblau (1972). Marquardt's method is a frequently used algorithm for nonlinear least squares estimation.

4 This is a fundamental criticism of the equilibrium versus disequilibrium hypothesis tests developed by Fair and Jaffe (1972), Bowden (1978), Quandt (1978) and Ziemer and White (1982).

5 The magnitude and duration of the current trend are discontinuous but they are piecewise continuous with respect to time. The integral value, S, of a piece wise continuous function is continuous. See Braun, Differential Equations and Their Applications, p. 221.

6 The simplest example of a discontinuous differential equation is an exogenously determined jump in the time dependent parameter of the equation. See Braun, pp. 214-19.

7 An excellent text on the subject of system simulation by Gordon (1978), provided much of the information used in the design and construction of the simulation.

8 The occurrence of discontinuous jumps may increase the order of error.

9 A development of the formulas used to solve the cubic equation may be found in: Conkwright, Introduction to the Theory of Equations, pp. 68-78.

10 Conditions required for determining the stability of nonlinear differential equation systems are not met by the cusp dynamic equation. The stability matrix is degenerate. The only conclusion we may draw from the qualitative theory of differential equations is that the stability of the static equilibrium solution is unknown: See Braun, 361-62.

11 This idea is examined in greater detail and subjected to statistical evaluation later in this chapter.

12 The general material presented here on wheat production and utilization was gathered from Leonard and Martin (1963).

13 Figures presented here are cited from Michael Martin, "United States and World Grain Production." In: Grain Marketing Economics, ed. Gail Cramer and Walter Heid (New York: John Wiley and Sons, 1983) pp. 2-21.

14 Figures presented here are cited from Gail Cramer, "World Grain Trade." In: Grain Marketing Economics, ed. Gail Cramer and Walter Heid (New York: John Wiley and Sons, 1983) pp. 238-262.

15 Figures presented here are cited from Walter Heid, "Grain Supply and Utilization." In: Grain Marketing Economics, ed. Gail Cramer and Walter Heid (New York: John Wiley and Sons, 1983) pp. 24-59

16 The assumption of normally distributed price changes is examined in greater detail in a following section. The assumption proves to be poor.

17 This is not a criticism of the rational expectations hypotheses. From the simple predictive model introduced by Muth (1961), current rational expectations models now include an adaptive learning process based on past performance (Bray, 1982 and Blume and Easley 1982). The criticism here extends to any model which equates storage demand with expectations for increasing prices over a crop year.

18 As noted earlier, adaptive learning has been incorporated into rational expectations models; see endnote 17.

19 Rational expectations models have received a vast amount of attention in economic literature over the past several years. Theoretical interest has focused on the existence of rational expectations equilibrium when expectations are conditioned by current market information: Radner (1978), Jordan and Radner (1982), Hellwig (1982) are just a few prominent examples. I have two criticisms of this work. First, the existence of a stable market equilibrium in a rapidly changing market environment is of less interest than the dynamic flows of adjustment, and how these flows exist within a structurally stable system. We do not observe stable economic states nor do we observe the collapse of exchange. Second, to assume that market participants run about adjusting an explicit set of economic forecasting models to new information, represents an irrational assessment of economics as a forecasting tool. Economic models are not good unbiased predictors of the future, (though we may desire these properties in our statistical estimates we know they elude us). Combining the problem of utility maximization or profit maximization with assigning probabilities to possible future outcomes represents an extremely complex task, which further removes economic behavioral assumptions from reality. One might argue that this still leads to more realistic economic models in which the importance of uncertainty is accounted for. However, this realism reflects the fact that the level of complexity in the model more closely resembles the level of complexity in the system being modeled, not a correspondence between the two. More is to be gained by relaxing behavioral assumptions and attempting to cope with the analytical problems this presents.

20 This was shown in the first section of Chapter Three.

21 For a more complete discussion of grain contracting see Dahl (1983).

22 This is an area of active research interest. Some recent publications on the subject include Kawai (1983) and Sarris (1984).

23 A specific reference for the material presented here is Bob Jones, "Government Policy." In: Grain Marketing Economics, ed. Gail Cramer and Walter Heid

24 (New York: John Wiley and sons, 1983) pp. 266-298. These conclusions are based on a residual supplier model of coarse grain trade developed by Bredahl and Green (1983).

25 Equilibrium system models include both static equilibrium adjustment models (adjusting through a continuous set of alternative equilibria) and disequilibrium adjustment models specifying an equilibrium rate of adjustment to excess demand.

26 Explicit supply and demand relationships are not specified in a lagged price adjustment equation. The exogenous variables are intended to approximate the current equilibrium price directly. Predicted prices reflect a partial adjustment toward an imputed equilibrium price. If the exogenous variables properly specify an implicit set of supply and demand relationships, the predicted price change may be interpreted as an equilibrium rate of adjustment.

27 For a complete consideration of predictive accuracy, relative to predicted values versus predicted changes in values, see Henri Theil, Applied Economic Forecasting, (Chicago: Rand McNally, 1966) pp. 15-40.

28 Two perfectly correlated variables are being added to both sides of the regression equation, hence, there is no new source of unexplained variation. The impact of the added explanatory variable has already been accounted for, hence, there is no new source of explained variation. The errors remain unchanged. The reduction in the coefficient of determination given a decrease in the variance of the dependent variable, errors held constant, is clear from a computational formula for R^2:

$$R^2 = 1 - \frac{\mathrm{Var(Error)}}{\mathrm{Var(D.Var)}}$$

29 A Lagrangian interpolating polynomial is used in place of the more standard differencing polynomial interpolations of Gauss and Newton because of the uneven spacing of the intervals.

30 This would seem to preclude developing meaningful tests for two catastrophe flags, inaccessibility and divergence.

31 Binary interaction variables are used to estimate separate slopes for the lagged price change variable in the two periods. For a detailed treatment of binary interaction variable see John Neter and William Wasserman, Applied Linear Statistical Models, (Homewood, Illinois: Richard D. Irwin, Inc., 1974) pp. 304-309.

32 This hypothesis was discussed in detail in the review of Zeeman's model of a stock exchange in Chapter Three.

33 Procedures for testing for kurtosis were adapted from McNemar (1962).

CHAPTER V

IMPLICATIONS AND CONCLUSIONS

An Evaluation of the Empirical Results

In considering all of the empirical evidence gathered in the previous chapter, one must conclude that the results do not validate the hypothesis of an underlying catastrophe structure in competitive markets for stored commodities. The results do not prove, in any formal sense, that speculative inventory demand is a significant source of price instability. The results do suggest that a model of a cusp catastrophe with slow feedback is a useful framework for market analysis. In a descriptive sense, the empirical results support the hypothesis that speculative behavior exacerbates price instability in the wheat market.

A market based on a cusp catastrophe or any other structure is a frame of reference. It shapes how we define and measure observable characteristics of the system it is intended to represent. A frame of reference can not be validated. Consistency can be demonstrated but not completeness. Godel's incompleteness theorem demonstrates that incompleteness is inherent in any categorical system of thought.[1] Given this limitation, we may be willing to accept a sufficiently large number of statistical tests of consistency as validation for an economic model.

The hypothesis tested in Chapter Four demonstrates a reasonable degree of consistency. However, consistency is not unique to a given interpretation of observed events or statistical tendencies. The Loenheim-Skolem theorem demonstrates that within a categorical system of thought, there exist radically different explanations which are consistent with a given set of observations.[2] Fortunately, as observers of economic systems, we are not constrained to view economic activity within the framework of a given theory. We can, to a limited extent, transcend the frame of reference and make use of intuition. Subjective evaluations can be made which are more meaningful than refusals to accept or deny a hypotheses. Intuition reflects conditioning as well as insight but the pure application of the scientific method in economics is often sterile.

In evaluating the application of a cusp catastrophe as a model of the pricing process for a stored commodity, the questions presented under the objective of this thesis should be addressed. Is the model useful in the analysis of wheat prices? Can we identify conditions under which speculation is more likely to occur? Can we better assess marketing risks and public policy alternatives?

A cusp model of speculative market behavior does offer a plausible explanation for the irregular price cycles which commonly occur in grain prices during a crop year. There does appear to be a tendency for commodities to become over or undervalued as price trends are sustained by speculative changes in inventory demand. Harvest trends are frequently extended into a crop year and a period of readjustment often follows. The relationship between expectations and price determination does establish a link between past and present price movements. This link can and often is exploited in attempts to forecast commodity prices. However, the most relevant implication that the model offers is that speculative inventory demand leads to very unpredictable short-term price behavior. The psychological impact of market conditions on expectations is not readily measurable. Hence, the effect of expectations on prices can not be accounted for in a forecasting model. This explains, in part, why econometric models are not generally regarded as good short-term forecasting tools.

Speculation has not been considered as a primary source of price variation in agricultural markets. Price movements are postulated to generate speculative inventory demand. Tests conducted on the distribution of price changes indicated that the level of speculation increased with existing variation in wheat prices. In other words, speculative reservation demand tended to increase under more unstable market conditions. The simulation models provided a clear demonstration of how a positive flow between prices and expectations over a cusp catastrophe surface may generate speculative variation about an externally induced trend.

An attempt was made to approximate the level of speculative content in the wheat market by decomposing recent price trends. Price changes which varied inversely with the rate of inventory consumption were used as a measure of speculative content. Current price trends were then estimated using current changes in the rate of inventory utilization and nearby estimates of speculative content as dependent variables. In the relative context of trying to explain the variation in wheat prices, decomposed price trends appear to be a relatively good measure of speculative content. In the absolute terms of a predictive tool, this measure of speculative content was moderately significant.

The model was not used as a tool for risk analysis in the wheat market. The perception of risk should have a significant impact on the flow of information between prices and the formation of expectations. Results did indicate that speculative demand increases with a reduction in downside risk. Specific tests of the distribution of price changes indicated a significant level of positive kutosis. As a result, normative measures of risk tend to understate the risk of both relatively small and large price changes, while tending to overstate the risk of a moderate change.

The two major price support policies in effect between 1973 and 1983 roughly divided the sample period in half. An open market policy, tending to eliminate downside risk, was in effect through the 1977/78 crop year. This was postulated to result in an upward speculative bias in inventory demand. Upward price movements showed a significantly greater variation than downward price movements over this period. The farmer owned reserve system was placed into effect in 1978. Participation required that grain be held for a minimum period of three years. This was postulated to temporarily eliminate any speculative bias as the gain of a secure price eliminated upward speculation on the open market. The variances for upward and downward price changes over the subsample period were nearly identical. The total variances in price changes for the subsample periods was significantly lower under the farmer owned reserve. It is tempting to conclude that the farmer owned reserve was a better policy implement. However, at the end of three years, farmers held mature grain which could be sold on the open market or be kept in reserve for up to two more years while receiving subsidy storage costs. This amounted to free speculation. The policy self-destructed into the Payment in Kind program in 1983.

An Evaluation of the Theoretical Perspective

A market was defined as a categorical system of thought in the previous section. Every possible influence in the exchange environment is classified as either a determinant of supply or a determinant of demand. Competitive markets, monopolistic markets and speculative markets classify exchange into a few simple analogous structures. Catastrophe theory classifies mathematically defined systems into analogous structures; qualitatively linear families of functions (one-determined), qualitatively quadratic families of function (two-determined), qualitatively cubic and so forth. There is a natural correspondence between the two systems of thought. Hence, the claim that catastrophe theory is well suited to economic theory.

Catastrophe theory, as an independent body of mathematics, is being combined with bifurcation theory and perturbation theory to form a broader system of differential topology. It is of interest to note that differential topology is the mathematical basis for many of the proofs of rational expectations equilibrium theory. There is also the subject of potential abuse. It is easy to become lost in the abstractions one can create with the tools of qualitative mathematics. As a model becomes irreversibly abstract, information is lost. There is a constant need for effective verbalization.

A market model based on a cusp catastrophe with slow feedback provided a basis for examining the relationship between the formation of expectations and price determination. The model synthesized two critical elements. One, the interaction between price information and expectations could be represented by simply describing tendencies in the flow of information along the catastrophe surface. Second, the concept of transitory demand could be incorporated into a structurally stable model of price determination.

The principal advantage of a cusp over higher catastrophe structure is that it may be visualized. Higher catastrophes may hold any number of interesting implications. However, they can not be applied with the flexibility that geometric intuition provides.

Endnotes

1 This simple interpretation of Godel's theorem was given by Morris Kline, Mathematics the Loss of Certainty, (Oxford: Oxford University Press, 1980) p. 272.

2 This is not a correct statement of the Loenheim-Skolem theorem. The terms referring to observation have been freely substituted for the axioms of a categorical system. The interpretation of the theorem on which this intuitive generalization is based was given by Kline, p. 272.

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APPENDIX

APPENDIX A

Simulation Program Listing and Tabular Results

Program Listing and Operating Procedures: Hewlett-Packard 67 Programmable Calculator.

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBL A	31 25 11	Start Observation		LBL d	32 25 14	Computation of $\dot{P}$
	RCL 6	34 06			P⇄S	31 42	
	ST I	35 33			RCL 0	34 00	
	LBL 1	31 25 01	Master Loop	060	RCL 6	34 06	
	RCL A	34 11	Begin R-K Proc.		-	51	
	STO C	33 13			3	03	
	GSB b	32 22 12	Call for N & S		÷	81	
	GSB d	32 22 14	Call for $\dot{P}$		STO 9	33 09	
	STO 9	33 09			3	03	
010	RCL 0	34 00			y^x	35 63	
	x	71			RCL E	34 15	
	2	02			2	02	
	÷	81			÷	81	
	RCL A	34 11		070	STO 8	33 08	
	+	61			x^2	35 63	Discriminant
	STO C	33 13			+	61	Check
	GSB b	32 22 12	Call for N & S		x<0	31 71	
	GSB d	32 22 14	Call for $\dot{P}$		GTO 3	22 03	
	STO 8	33 08			$\sqrt{x}$	31 54	Cardan's Method
020	RCL 0	34 00			STO 7	33 07	
	2	02			RCL 8	34 08	
	x	71			+	61	
	x	71			GSB e	32 22 15	
	RCL 0	34 00		080	RCL 8	34 08	
	RCL 9	34 09			RCL 7	34 07	
	x	71			-	51	
	-	51			GSB e	32 22 15	
	RCL A	34 11			+	61	
	+	61			P⇄S	31 42	
030	STO C	33 13			RTN	35 22	
	GSB b	32 22 12	Call for N & S		GSB e	32 22 15	
	GSB d	32 22 14	Call for $\dot{P}$		x<0	31 71	
	STO 7	33 07			SF 2	35 51 02	
	RCL 8	34 08		090	ABS	35 64	
	4	04			3	03	
	x	71			$1/x$	35 62	
	+	61			y^x	35 63	
	RCL 9	34 09			CF 2	35 71 02	
	+	61			CHS	42	
040	RCL 0	34 00			RTN	35 22	
	x	71			LBL 3	31 25 03	Method of
	6	06			RCL E	34 15	Cosines
	÷	81			RCL 9	34 09	
	RCL B	34 12		100	3	03	
	RTN	35 22			y^x	35 63	
	STO B	33 12	Turning Point		CHS	42	
	x	71	Test		$\sqrt{x}$	31 54	
	x<0	31 71			2	02	
	GSB c	32 22 13			x	71	
050	RCL B	34 12			÷	81	
	RLC A	34 11			SIN^{-1}	32 63	
	+	61	End R-K Proc.		STO 6	33 06	
	STO A	33 11	Loop Control		RCL 9	34 09	
	DSZ	31 33		110	CHS	42	
	GTO 1	22 01			$\sqrt{x}$	31 54	
	RTN	35 22	End Master Loop		2	02	

REGISTERS

0	1	2	3	4	5	6	7	8	9
Δt	ΔP_e	P_e	$S(t^*)$	$P(t^*)$	t_d	T_c	IP_{III}	IP_{II}	IP_I
S0	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**
S_o	root 1	root 2	root 3	Used	Used	Used	Discriminant	Used	Used

A	B	C	D	E	I
S	ΔP	P_I	S	N	Loop Control

Program Listing (continued).

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
	x	71	
	STO 5	33 05	
	RCI	35 34	
	STO 4	33 04	
	4	04	
	STI	35 33	
	LBL 4	31 25 04	
120	DSZ	31 33	
	GTO 7	22 07	
	RCL 4	34 07	
	STI	35 33	
	CF Ø	35 61 00	
	LBL 5	31 25 05	
	RCL 3	34 03	Root Sorting
	RCL 2	34 02	
	x>y	32 81	
	x⇄y	35 52	
130	STO 2	33 02	
	R↓	35 53	
	STO 3	33 03	
	F? Ø	35 71 00	
	GTO 6	22 06	
	RCL 2	34 02	
	RCL 1	34 01	
	x>y	32 81	
	x⇄y	35 52	
	STO 1	33 01	
140	R↓	35 53	
	STO 2	33 02	
	SF Ø	35 51 00	
	GTO 5	22 05	
	LBL 6	31 25 06	
	RCL B	34 12	
	x 0	31 71	Root Selection
	RCL 1	34 01	
	x<0	31 81	
	RLC 3	34 03	
150	P⇄S	31 42	
	RTN	35 22	
	LBL 7	31 25 07	
	RCL 6	34 06	
	3	03	
	÷	81	
	sin	31 63	
	RCL 5	34 05	
	x	71	
	STO (i)	33 24	
160	3	03	
	6	06	
	0	00	
	STO+ 6	33 61 06	
	GTO 4	22 04	
	LBL b	32 25 12	
	RCL 1	34 01	Compute N
	STO+2	33 61 02	
	RCL 2	34 02	
	RCL C	34 13	
170	−	51	
	2	02	
	0	00	δ
	0	00	
	x	71	
	STO E	33 15	
	RCL 0	34 00	Compute S
	3	03	
	÷	81	
	STO+5	33 61 05	
180	RCL 3	34 03	
	RCL 5	34 05	
	9	09	λ_1
	0	00	
	x	71	
	CHS	42	
	e^x	32 52	
	x	71	
	RCL C	34 13	
	RCL 4	34 04	
190	−	51	
	ABS	35 64	
	RCL 5	34 05	
	9	09	— λ_2
	x	71	
	CHS	42	
	e^x	35 52	
	x	71	
	1	01	
	4	04	α
200	0	00	
	x	71	
	+	61	
	STO D	33 14	
	RTN	35 22	
	LBL C	32 25 13	
	RCL 5	34 05	Output Trend Duration
	R/S	84	
	0	00	
	STO 5	33 05	
210	RCL D	34 14	
	STO 3	33 03	
	RCL A	34 11	Output turning Point Price
	STO 4	33 04	
	R/S	84	
	RTN	35 22	
	LBL B	31 25 12	Input P_e
	RCL 6	34 06	
	3	03	
	x	71	
220	÷	81	
	STO 1	33 01	
	RTN	35 22	

LABELS

					FLAGS
A P	B ΔP_e	C	D	E	0 Sort Control
a	b N & S	c Reset	d $\dot{P}$	e $\sqrt[3]{x}$	1
0	1 Master Loop	2	3 Cosines	4 Used	2 Sign test
5 Used	6 Used	7 Used	8	9	3

SET STATUS

FLAGS	TRIG	DISP
ON OFF	DEG	FIX
0	GRAD	SCI
1	RAD	ENG
2		n 3
3		

Program Listing (continued).

User Instructions

Cusp Market Simulation

P ΔP_e

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Initialize Simulation		f	CL REG	
2	Input S_e	S_e	STO	Ø	
3			f	P/S	
4	Input Δ_t	Δ_t	STO	Ø	
5	Input P_e	P_e	STO	2	
6	Input P_o	P_o	STO	A	
7			STO	4	
8	Input T_c	T_c	STO	6	
9	Input ΔP_e	ΔP_e		B	
10	Initiate Observation			A	
	To generate next observation go to step 10.				
	To change ΔP_e go to step 9.				

Program Listing (continued).

Program Description

Program Title Cusp Market Simulation

Name Stephen Carroll Beare Date April 1985

Address

City State Zip Code

Program Description, Equations, Variables, etc.

The following paramebers must be loaded directly into program storage: σ, λ_1, λ_2, α. Locations are noted in program listing.

Psuedo-rendom normal variables were obtained from Hewlett-Packard statistical library program (# St-04A). These were added at each observation by utilizing LBL B.

Operating Limits and Warnings

Selected Tabular Results

A) Variable Identification Code: Model Variable

T = observation

P = price

N = normal factor; speculative content

P_e = equilibrium price

ΔP = current price trend

t_d = trend duration in simulation time

T_d = trend duration in observation time

u = random disturbance of equilibrium price

P_o = initial price

B) Variable Identification Code: Model Parameters

δ = rate of normal factor adjustment parameter

λ_1 = splitting factor dampening parameter

λ_2 = splitting factor dampening parameter

α = splitting factor acceleration parameter

S_o = cusp point location parameter

P_e = rate of adjustment in equilibrium price per observation period

C) Initialization Conditions: All Simulations

Δ_t = 0.005 (iteration increment)

T_C = 5 (iterations per observation)

ΔP_t = 0

t_d = 0

$S(t^*)$ = 0 (value of splitting factor at a turning point)

Table A1. Tabular results for pure disequilibrium adjustment simulations; group one.

Parameter values:

$\delta = 100$ $\alpha = 50$ $P_o = (3.75,\ 4.00,\ 3.60)$
$\lambda_1 = 45$ $S_o = 2$
$\lambda_2 = 3$ $P_e = (4.00,\ 3.60,\ 3.75)$

T	P	N	S	ΔP	t_d	T_d
0	3.75	25	0	0	0	0
1	3.82	18	3.18	.07	.025	1
2	3.89	11	5.96	.14	.050	2
3	3.96	4	8.33	.21	.075	3
4	4.03	-3	10.27	.28	.100	4
5	4.09	-9	11.75	.34	.125	5
5.8	(4.13)	-13	12.64	.38	.145	5.8
6	4.10	-10	9.75	-.03	.010	0.2
7	4.03	-3	7.16	-.10	.035	1.2
8	3.97	3	7.36	-.16	.060	2.2
8.8	(3.95)	5	7.48	-.18	.075	3.0
9	3.97	3	5.67	.02	.010	0.2
10	(4.01)	-1	4.33	.04	.035	1.2
11	4.00	0	2.12	-.01	.025	1.0
12	4.00	0	0.85	.00	---	---
*	4.00	-40	0.00	.00	.000	0
13	3.92	-32	3.82	-.08	.025	1
14	3.83	-23	7.21	-.17	.050	2
15	3.75	-15	10.17	-.25	.075	3
16	3.66	-6	12.68	-.34	.100	4
17	3.57	3	14.76	-.43	.125	5
18	3.49	11	16.38	-.51	.150	6
19	3.41	19	17.55	-.59	.175	7
20	(3.36)	24	18.93	-.64	.200	8
21	3.46	14	11.60	.10	.025	1
22	3.54	6	10.39	.18	.050	2
23	3.62	-2	11.39	.26	.075	3
24	3.69	-9	12.80	.33	.100	4
25	(3.75)	-15	13.94	.39	.125	5

Table A1. (continued).

T	P	N	S	ΔP	t_d	T_d
26	3.68	-8	8.43	-.07	.025	1
27	3.61	-1	7.85	-.14	.050	2
28	3.53	7	8.66	-.22	.075	3
29	(3.53)	7	8.12	-.22	.100	4
30	3.60	0	5.92	.09	.025	1
30.8	(3.62)	-2	4.06	.09	.045	1.8
31	3.61	-1	3.89	-.01	.010	0.2
32	3.60	0	2.30	-.02	.035	1.2
33	3.60	0	1.29	.00	---	---
*	3.60	15	0.00	.00	.000	0
34	3.66	9	2.56	.06	.025	1
35	3.71	4	4.72	.11	.050	2
36	3.76	-1	6.44	.16	.075	3
37	(3.80)	-5	6.03	.20	.100	4
38	3.74	1	4.44	-.06	.025	1
38.6	(3.74)	1	3.47	-.06	.040	1.6
39	3.75	0	2.97	.01	.010	0.4
40	3.75	0	1.54	.00	---	---
41	3.75	0	0.52	---	---	---
42	3.75	0	0.18	---	---	---

* Endogenous parameters reset
() denotes turning point

Table A2. Tabular results for continuous external adjustment simulation: group 2, run 1.

Parameter Values:

λ_1 = 90 α = (140.0) δ = 200
λ_2 = 9 S_o = 3 = (-0.075, 0.025)

T	P	N	S	P_e	ΔP	t_d	T_d
0	4.00	0	0	4.00	0	0	0
1	3.97	-9.0	3.98	3.925	-.03	.025	1
2	3.90	-10.0	9.10	3.850	-.10	.050	2
3	3.82	-9.0	13.18	3.775	-.18	.075	3
4	3.72	-4.0	15.72	3.700	-.28	.100	4
5	3.63	-1.0	16.83	3.625	-.37	.125	5
6	3.54	2.0	16.79	3.550	-.46	.150	6
7	3.45	5.0	15.94	3.475	-.55	.175	7
8	3.37	6.0	14.61	3.400	-.63	.200	8
8.8	(3.34)	16.0	12.39	3.420	-.66	.220	8.8
9	3.38	9.0	12.70	3.425	.04	.005	0.2
10	3.46	-2.0	13.22	3.450	.12	.030	1.2
11	3.54	-13.0	16.64	3.475	.20	.055	2.2
12	3.61	-22.0	18.29	3.500	.27	.080	3.2
12.4	(3.62)	-22.0	19.89	3.510	.28	.105	3.6
13	3.56	-7.0	12.04	3.525	-.08	.015	0.6
14	3.50	10.0	12.59	3.550	-.12	.040	1.6
14.2	(3.49)	13.0	10.51	3.555	-.13	.045	1.8
15	3.57	-1.0	10.43	3.575	.08	.020	0.8
16	3.64	-8.0	16.66	3.600	.15	.045	1.8
17	3.71	-18.0	16.75	3.625	.22	.070	2.8
17.6	(3.72)	-16.0	17.18	3.640	.23	.085	3.4
18	3.68	-6.0	11.54	3.650	-.04	.010	0.4
19	(3.65)	5.0	8.26	3.675	-.07	.035	1.4
20	3.71	-2.0	9.58	3.700	.06	.025	1.0
21	3.77	-9.0	12.36	3.725	.12	.050	2.0
21.8	(3.81)	-13.0	9.25	3.745	.16	.070	3.8
22	3.78	-6.0	8.98	3.750	-.03	.010	0.2
23	(3.77)	1.0	0.97	3.775	-.04	.035	1.2
24	3.78	-4.0	0.10	3.400	.01	.025	1.0

() denotes turning point

Table A3. Tabular results for continuous external adjustment simulation with randon disturbances: run 1.

Parameter Values:

$\lambda_1 = 90$ $\alpha = (140,0)$ $\delta = 200$ $u \sim N(0,0.05)$
$\lambda_2 = 9$ $S_o = 3$ $P_e = (-0.75,\ 0.25)$

T	P	N	S	u	P_e	ΔP	t_d	T_d
0	4.00	0	0	-	4.00	0	0	0
1	3.97	-9	3.98	.06	3.925	-0.3	.025	1
2	3.91	0	8.11	.01	3.910	-0.9	.050	2
3	3.84	-1	11.12	-.04	3.845	-.16	.075	3
4	3.76	-6	13.50	.00	3.730	-.24	.100	4
5	3.68	-3	14.83	-.02	3.655	-.32	.125	5
5	3.58	-4	15.14	-.01	3.560	-.42	.150	6
7	3.49	-3	14.71	.00	3.475	-.51	.175	7
8	(3.41)	-2	13.75	.00	3.400	-.59	.200	8
9	(3.46)	-7	10.75	.00	3.425	.05	.025	1
10	(3.42)	6	5.08	.08	3.450	-.04	.025	1
11	3.50	20	9.87	.05	3.600	.08	.025	1
12	3.60	15	16.01	-.01	3.675	.18	.050	2
13	3.70	-2	20.00	-.04	3.690	.28	.075	3
14	3.80	-25	21.26	-.04	3.675	.38	.100	4
.8	(3.81)	-28	16.98	-	3.669	.39	.125	5
15	3.74	-16	13.17	.05	3.660	-.07	.145	5.8
16	3.66	15	15.22	-0.6	3.735	-.15	.005	0.2
.8	(3.62)	18	11.06	-	3.707	-.19	.030	1.2
17	3.65	10	10.60	.03	3.700	.03	.050	2
18	3.73	5	12.03	-.07	3.755	.11	.005	0.2
19	(3.79)	-16	9.70	-.07	3.710	.17	.030	1.2
20	3.70	-7	10.88	.00	3.665	-.09	.025	1
21	3.63	12	14.42	-.06	3.690	-.16	.050	2
22	(3.57)	18	15.73	-.01	3.665	-.22	.075	3
23	3.62	10	1.07	.01	3.670	.05	.025	1
24	3.66	9	0.31		3.705	.09	.050	2
$\bar{x}$				-.01				
S(x)				.04				

APPENDIX B

Transformed Data for Graphical and Regression Ananlysis

Table B1. Actual and smoothed monthly wheat prices; No. 1 Hard Red Winter at Kansas city, 1972/73 to 1983/84.[a]

Crop	P^b	P_s^c	Crop	P^b	P_s^c	Crop	P^b	P_s^c
72/73			75/76	323	336	78/79	312	312
	158			361	351		314	313
	182	172		412	388		314	314
	210	195		421	408		324	320
	215	207		409	408		342	333
	225	218		371	386		348	342
	262	244		350	364		339	340
	267	258		357	360		342	341
	248	252		381	373		350	347
	242	246		381	378		352	350
	251	249		361	368		353	352
	263	257		357	361		364	359
73/73	269	264	76/77	375	370	79/80	417	394
	290	280		363	366		434	418
	467	392		321	339		412	414
	501	457		301	316		426	421
	467	463		277	293		439	432
	478	472		262	274		453	445
	522	502		264	268		451	448
	568	542		270	269		433	439
	582	566		273	271		432	435
	501	527		263	266		407	418
	407	455		252	258		390	401
	359	397		236	245		410	407
74/75	405	402	77/78	231	236	80/81	407	407
	436	422		235	236		421	415
	439	429		231	233		431	425
	435	433		247	241		445	437
	494	469		256	250		470	457
	488	481		281	269		489	476
	466	472		280	275		454	463
	415	438		282	279		460	461
	393	411		284	282		447	453
	369	386		307	297		435	442
	366	374		321	311		448	446
	334	356		312	312		436	440

Table B1. (continued).

Crop	P^b	$P_s{}^c$
81/82	424	430
	425	427
	414	419
	419	419
	431	426
	446	438
	435	436
	433	434
	426	429
	425	427
	428	427
	422	424
82/83	406	413
	374	390
	370	378
	375	376
	361	367
	386	378
	398	390
	400	396
	408	403
	418	412
	421	417
	405	410
83/84	392	399
	371	382
	388	386
	390	388
	384	386
	382	383
	385	384

[a]Source; U.S.D.A., Wheat Situation, (Washington, D.C.) various issues 1978-1984.
[b]Average Monthly Price
[c] Smoothed Price; $P_{s_t} = .6(P_t) + .4(P_{s_{t-1}})$

Table B2. Monthly wheat data for regression analysis: Prices, interpolated consumption (Lagranigian), and consumption as a percentage of beginning inventories;[a] Prices[b] No. 1 Hard Red Winter at Kansas City.

Crop Year	P	P	MD[c]	MD%BI[d]	Crop Year	P	P	MD[c]	MD%BI[d]
72/73			149	589	75/76	323	-11	153	595
	158		164	648		361	38	170	661
	182	24	173	684		412	51	180	700
	210	28	175	691		421	9	183	712
	215	5	161	636		409	-12	170	661
	225	10	157	620		371	-38	167	679
	262	37	153	605		350	-21	162	630
	267	5	140	553		357	7	155	603
	248	-19	141	557		381	24	150	585
	242	-6	146	577		381	0	144	560
	251	9	158	624		361	-20	136	529
	263	12	171	676		357	-4	136	529
73/74	269	6	202	876	76/77	375	18	153	544
	290	21	216	936		363	-12	158	562
	467	177	221	958		321	-42	159	565
	501	34	218	945		301	-20	156	555
	467	-34	198	858		277	-24	139	494
	478	11	185	802		262	-15	135	480
	522	44	169	733		264	2	132	469
	568	46	140	607		270	6	130	462
	582	14	125	542		273	3	130	462
	501	-81	114	494		263	-10	132	469
	407	-94	105	455		252	-11	134	477
	359	-48	106	459		236	-16	144	512
74/75	405	46	130	608	77/78	231	-5	179	568
	436	31	141	659		235	4	191	606
	433	-3	149	697		231	-4	194	616
	435	2	154	720		247	16	186	590
	494	59	151	706		256	9	140	444
	488	-6	152	711		281	25	134	425
	466	-22	152	711		280	-1	134	425
	415	-51	156	730		282	2	148	470
	393	-22	150	702		284	2	155	492
	369	-24	139	650		307	23	163	517
	366	-3	113	529		321	14	171	543
	334	32	114	533		312	-9	180	571

Table B2. (continued).

Crop Year	P	P	MD[c]	MD%BI[d]	Crop Year	P	P	MD[c]	MD%BI[d]
78/79	312	0	204	685	81/82	424	-12	243	641
	314	2	213	716		425	1	268	707
	314	0	214	719		414	-11	276	728
	324	10	207	696		419	5	266	702
	342	18	180	605		431	12	193	509
	348	6	169	568		446	15	183	483
	339	-9	157	528		435	-11	181	478
	342	3	138	464		433	-2	206	544
	350	8	134	450		426	-7	209	551
	352	2	135	454		425	-1	207	546
	353	1	145	487		428	3	194	512
	364	11	156	524		422	-6	200	528
79/80	417	53	185	603	82/83	406	-16	244	612
	434	17	198	645		374	-32	256	643
	412	-22	206	671		370	-4	254	638
	426	14	206	671		375	5	237	595
	439	13	191	622		361	-14	160	402
	453	14	186	606		386	25	152	382
	451	-2	180	587		398	12	158	397
	433	-18	168	547		400	2	215	540
	432	-1	163	531		408	8	220	552
	407	-25	161	525		418	10	211	530
	390	-17	160	521		421	3	168	422
	410	20	165	538		405	-16	169	424
80/81	407	-3	190	579	83/84	393	-13	228	574
	421	14	201	613		371	-21	251	632
	431	10	207	631		388	17	262	660
	445	14	207	631		390	2	260	655
	470	25	191	582		384	-6		
	489	19	190	579		383	-2		
	454	-35	189	576		385	3		
	460	6	196	598					
	447	-13	193	589					
	435	-12	186	567					
	448	13	166	506					
	436	-12	175	534					

[a]Source: Wheat Statistics
[b]interpolation formula, see text page
[c]monthly disappearance, million bushels
[d]monthly disappearance as a percent beginning inventory x 100