AN ABSTRACT OF THE THESIS OF

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Title: Biased and Unbiased Estimation: An Econometric Application in the Tuna Industry

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An econometric model of the canned tuna market is used to evaluate biased and unbiased estimators. Four methods for improving mean square error when multicollinearity is present in a regression equation are examined and compared with the results of ordinary least squares (OLS). Exact and inexact prior information methods are used to improve regression estimates when information is available. Ridge regression and principal components methods are used when accurate prior information is unavailable.

Results of this study indicate that when the degree of multicollinearity is low, only ridge regression achieves improved mean square error estimates. In general, when the degree of multicollinearity is moderate, both principal components and ridge regressions improve mean square error estimates. However, consistent expected signs of the coefficients were achieved only by two of the three ridge estimators. When the degree of multicollinearity is high,
both methods achieve significant improvement in mean square error estimation, although ridge regression produces slightly more accurate and reasonable results than does principal components. Both the ridge regression method and the principal components method are effective in producing low mean square error estimates, reduced variance, and expected signs on all coefficients. Either method appears to offer a viable alternative to the full model estimated by OLS.

Exact and inexact prior information are introduced to improve regression estimates in the presence of multicollinearity. These methods reveal that if prior information is available and accurate and/or consistent, it should be incorporated directly into the estimation. If prior information is inconsistent and/or inaccurate, it is better to incorporate imprecise information approximately through other methods, such as ridge regression rather than to insist on formulating prior information with imprecise information.

Price and quantity relationships are estimated for price levels of the U.S. canned tuna market. Retail level demand is found to be price inelastic. Wholesale supply price is positively correlated with import prices. Wholesale demand is price elastic for chunk light tuna, but inelastic for solid white tuna. Ex-vessel demand is most significantly influenced by world landings. Import demand is price elastic for albacore, price inelastic for yellowfin.
Biased and Unbiased Estimation: An Econometric Application in the Tuna Industry

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1. Introduction

There are two basic ingredients in any econometric study: theory and facts. The theory (economic theory and mathematical economics) is concerned solely with pure deductive implications of certain postulate systems involving economic phenomena, such as explaining the relationships among economic variables and using that information with a general theory of choice to explain production, allocation and distribution decisions for a system that must operate within the implications of scarcity. For example, the quantity of commodity \( Q_t \) demand in the market is regarded as a function of its price \( P_t \)

\[
Q_t = f(P_t).
\]

The cost of producing a product \( C_t \) is assumed to be a function of the amount produced \( Q_t \)

\[
C_t = f(Q_t).
\]

These are examples of two theoretical variable relationships. However, more realistic formulations require the specification of several variables in such theoretical relationships. Therefore, quantity demanded \( Q_t \) may be regarded as a function of price \( P_t \), disposable income \( Y \), and price related commodities \( P_r \),

\[
Q_t = f(P_t, Y, P_r).
\]

Production cost \( C_t \) may be a function of the rate of production \( R_t \), factor price \( P_t \) and the change of production rate \( \Delta R_t \),

\[
C_t = f(R_t, P_t, \Delta R_t).
\]
Facts are the other basic ingredients in an econometric study. Facts are events in the real world relating to the phenomena under investigation. In general, facts have to be refined or massaged in a variety of ways to make them suitable for use in an econometric study. This refinement includes various adjustments such as seasonal or cyclical adjustment, extrapolation, interpolation, merging of different data sources and, in general, the use of other information to adjust the facts.

The third basic ingredient in an econometric study is statistical inference, which is concerned with drawing conclusions from limited facts. Since facts are almost always limited it has been necessary to develop a general theory for dealing with this.

Thus, econometrics applies economic theory, mathematical economics, and statistical inference to facts (economic data). Two of the most common econometric techniques include basic linear regression and simultaneous equations.

In this study, we will focus on linear regression and more specifically on one of the more common problems with linear regression models, multicollinearity. In econometrics, interrelation among explanatory economic variables is common. Economic variables tend to move in the same pattern over time. This is due to use of aggregate data over time or over some geographical location. In other words, the explanatory variables are highly correlated and it is difficult to identify their separate effects on the dependent variable. This phenomenon is called multicollinearity. When multicollinearity is severe, the ordinary least square (OLS) estimate, although it is the best linear unbiased estimate (BLUE), is unreliable and imprecise because of large
sampling variances. Under these circumstances, the high value of the standard error of the individual coefficient may result in the wrong sign, even though the overall regression measured by $R^2$ may be highly significant. One common solution to overcome the problem of multicollinearity is to drop highly intercorrelated variables from the regression model. Unfortunately, this may result in serious specification bias, and may be a poor method for solving problems of multicollinearity (Tek-Wei-Hu, 1973; Sass, 1979; Rahuma, 1981).

Another common problem is autocorrelation or serial correlation. This occurs when the error associated with observations in a given time period carries over into the future time periods and/or when the model is misspecified, particularly due to the exclusion of the relevant variables from the model. The presence of serial correlation will not affect the unbiasedness or consistency of the OLS estimate. However, its efficiency is affected. For example, in the event of positive serial correlation, the loss of efficiency will be masked by the fact that the estimates of standard errors obtained from OLS will be smaller than the true standard errors. This will lead to the conclusion that the parameter estimates are more precise than is actually the case. There will be a tendency to reject the null hypothesis when, in fact, it should not be rejected (Johnston, 1972; Intriligator, 1978; Pindyck and Rubinfield, 1981).

Marine economic data are characteristically poor as a result of bulkiness of the information and the difficulty in handling each individual factor. This is because marine economic data are very often
based on limited available sources such as the Current Fishery Statistics (CFS) or the National Marine Fisheries Service (NMFS) census. Moreover, in the past, marine fishery researchers have faced difficulties in attempting to estimate regression coefficients precisely due to lack of independence among the variables in the models' set of explanatory variables. For example, Waugh and Norton (1969) have difficulty in precisely estimating the explanatory variable coefficients which have explained the price variation of marine sardines.

Brown, Singh, and Castle (1973) have problems estimating income and distance travel; both of these are the explanatory variables for variation of travel cost for fishing salmon and steelhead in Oregon. Abraham (1978) also indicates the problem caused by the interrelationship among explanatory variables in determining the volume of exports of chum salmon.

However, there are a number of new techniques recently recommended which may be used to deal with the multicollinearity problem. They are as follows:

1. collection of additional data
2. disaggregation of existing data
3. incorporation of prior information
   a) exact linear restriction
   b) inexact prior information
   c) inequality constraints
4. biased linear estimation
   a) variable detection
   b) use of principle components
   c) use of ridge regressions
In general, the purpose of this study is to find and evaluate the techniques to see if the estimations can be improved and under what conditions the estimations will work. A secondary objective is to gain some knowledge and understanding of the tuna industry in the United States. An econometric model (biased and unbiased) for the tuna industry will be constructed and estimated. The relationships between price and quantity will be determined and quantitatively evaluated.

The specific objectives are:

1. to find alternative methods of estimation, both biased and unbiased, for explaining the determination of tuna prices and quantities;
2. to evaluate alternative methods of estimation, both biased and unbiased, for explaining the relationship between tuna prices and quantities;
3. to establish a framework for analyzing U.S. tuna demand;
4. to identify and select relevant theoretical variables associated with U.S. tuna and with the world market.

In this study, tuna data were chosen for analysis because tuna data have useful characteristics for experimenting with techniques and because it is an important product for consumers and producers in the U.S. and a source of protein throughout the world.

During the past fifty years, the demand for tuna products has increased considerably in connection with the growth of the world population and increasing per capita income in many geographical areas such as the U.S., Japan, and Western Europe. This growing demand led to an enlarged international market which, in turn, stimulated the
development of the world tuna fishery to its present state. Tuna is an internationally utilized species, consumed worldwide. However, the greatest percentage of consumption is restricted to high income nations -- mainly because tuna has a relatively high value. The United States, which has the largest consumption of tuna, accounts for almost half of the world's consumption of the major market species, followed by Japan and Western Europe (France, Spain, Italy, and West Germany). Exploitation by the less developed countries is undertaken primarily to acquire badly needed foreign exchange (Broderick, 1973; Saila, and Norton, 1974).
CHAPTER 2

History of Estimation Methods

The Ordinary Least Square and Gauss-Markov Theorem

The ordinary least squares (OLS) estimators are unbiased and consistent. One of the nice properties of ordinary least squares estimators is that they are all linear unbiased estimators, i.e., estimators which are linear in the independent variable and which yield unbiased estimators.

Estimators resulting from the OLS estimator have the minimum variance, i.e., are "best." This is the basis of the Gauss-Markov Theorem (1938) which was discussed in the 1938 article "Extension of the Markov Theorem on Least Square" by David and Neyman. The Gauss-Markov Theorem is summarized below.

Given assumptions:

1. The relation between \( Y \) and \( X \) is linear

\[
Y = XB + u
\]

where

- \( Y \) is a random variable
- \( X \) is fixed or nonstochastic
- \( B \) is a vector of the true parameter
- \( u \) is a random error term.

2. The \( X \)'s are nonstochastic and are fixed. The variance of \( X \) is nonzero; it is finite for any sample size.

3. a) The error term has zero expected value and consistent variance for all observations; that is, \( E(u_1) = 0 \) and \( E(u_1^2) = \sigma^2 \).

b) The random variables \( u_1 \) are uncorrelated in a statistical
sense, i.e., errors corresponding to different observations have zero correlation. Thus, $E(u_i u_j) = 0$ for $i \neq j$.

The estimators $\hat{B}$ are the best (most efficient) linear unbiased estimators of $B$ in the sense that they have the minimum variance of all linear unbiased estimators (Pindyck and Rubinfeld, 1981).

$$\hat{B} = (X'X)^{-1}X'Y$$ and

$$E(\hat{B}) = B$$ and

$$\text{var}(\hat{B}) = \sigma^2 (X'X)^{-1}$$ is minimum (for more detail, see Appendix 1).

**Suggested Solutions to Multicollinearity**

As mentioned before, when the problem of multicollinearity occurs, the ordinary least square (OLS) estimate is unreliable and imprecise, although it is the best linear unbiased estimate (BLUE). There are several suggested solutions to multicollinearity in the literature. They are as follows.

**Data Disaggregation**

Brown et al. (1973) found that data disaggregation tends to reduce the degree of multicollinearity. However, this solution might not be possible in most situations since the data are subjected to confidentiality restrictions or are secondary data and therefore not available for disaggregation (e.g., time series, census data, etc.).

**Additional Sample Information**

The most obvious solution in reducing the degree of multicollinearity is to obtain additional sample data. However, there is no guarantee that some pattern of intercorrelation will not also exist
in the additional observations (Sass, 1979). Moreover, Silvey (1969) indicated in his article "Multicollinearity and Imprecise Estimation" that there is a problem of determining the optimal values of the explanatory variables in a new observation. Suppose in the model \( Y = XB + u \), that there is one exact linear dependency. Then there is one vector \( p \) such that \( X'Xp = 0 \) which means that \( p'B \) is not estimable, nor is any linear function \( c'B \) for which \( c \) has a nonzero component in the direction \( p \). Since \( X'Xp = 0 \) if, and only if, \( Xp = 0 \), the multicollinearity problem exists if every row of \( X \) is orthogonal to \( p \). Thus, to reduce multicollinearity, a new observation must be chosen that is not orthogonal to \( p \). An obvious choice is to choose an observation in the direction of \( p \) itself, say \( X_{t+1} = lp \) where \( l \) is a scalar. However, there are two more points to consider. First, how is the precision of estimation affected by taking another observation at \( X_{t+1} \) not necessary in the direction of the characteristic vector of \( X'X \)? Second, how should \( X_{t+1} \) be chosen to improve as much as possible the estimation of a specific linear function \( c'B \)? Silvey responds to the first question by letting \( b_t \) and \( b_{t+1} \) be the least square estimates of \( B \) based on \( T \) and \( T + 1 \) observation, respectively. Then

\[
\text{var}[cb_t] - \text{var}[cb_{t+1}] = \frac{\sigma^2 a'\Lambda^{-1}Z'\Lambda^{-1}a}{1 + Z'\Lambda^{-1}Z}
\]

where \( a = P'c \), \( Z = P'X_{t+1} \) and \( \Lambda = \text{diagonal} (X_1, \ldots, X_k) \).

This expression improves estimator precision for any linear combination.

A second question arises when one is interested in certain linear combinations of parameters, or when one is interested in making a
prediction for a particular set of the explanatory value $c$. To answer the second question, Silvey went ahead to prove the following theorem.

The theorem gives the usual linear model and the condition $X'_{t+1} X_{t+1} = g^2$ which is satisfied by a new set of observations $X_{t+1}$ for improving the estimation precision of $c'B$. "$c'B" is the vector $H$, where $H = (I + g^{-2}X'X)^{-1}c$. In other words, the theorem demonstrates that the new observation should be proportionate to $H$. It is assumed that the error associated with the new observation is not correlated with errors in the original model and has the same variance as each of them. No condition is imposed on the rank of $X$.

The constraint $X'_{t+1} X_{t+1} = H^2$ is placed on the values of the explanatory variables because the greater the length of $X_{t+1}$, the greater the improvement in the precision of estimation of $c'B$. Thus, even if $c'B$ cannot initially be estimated, the optimum direction for a new observation is still provided by the theorem.

However, the disadvantage of using additional observations is that it is often impossible to obtain additional observations when using published data. Even though this suggestion often is not practical, the possibility should not be overlooked.

Use of Prior Information

Incorporation of prior information about the coefficient of independent variables can also be used to break the pattern of multicollinearity. There are three methods of using prior information as follows.

Exact Linear Restriction. There are many instances in applied work when the investigator has exact information on a particular parameter or linear combination of parameters. For example, in estimating
a log-linear production function, information may be available indicating that the firm is operating under the condition of constant returns to scale. Alternatively, in estimating a demand relation, information may be available from consumer theory on the homogeneity condition or an estimate of, say, the income response coefficient available from empirical work. In other words, prior information about some coefficient in a regression model might be obtained from similar studies or from theoretical and practical considerations. For example, assume the original regression model is

\[ Y = B_0 + B_1X_1 + B_2X_2 + B_3X_3 + u \]

where \(X_2\) and \(X_3\) are highly correlated. If we have some prior estimate of \(B_3\), then we can estimate the model.

\[ Y - B_3X_3 = B_0 + B_1X_1 + B_2X_2 + u \]

The accuracy of the estimates of \(B_0\), \(B_1\), and \(B_2\) depend on the accuracy of \(B_3\). The estimates obtained by this method may be biased and perhaps should be included in the class of biased estimators, even though they are not usually considered to be biased.

Use of prior information is encouraged by Smith and Kempbell (1980) in the article "A Critique of Some Ridge Regression Methods" which argued that if prior information is available, then it should be incorporated directly into the estimation procedure, and that this incorporation should be done "accurately and efficiently." However, the limitation of using prior information is that it is sometimes biased and imprecise.

Inexact Prior Information (Theil and Goldberger Mixed Model).

One specific method of incorporating prior information into a
regression model is the method introduced by Theil and Goldberger (1960). The Theil and Goldberger Mixed Model Theorem states that:

"Given inexact but unbiased prior information \( R_{g1} \) and unbiased \( R_{gk} \) where \( E(\sigma_{g1}^2) = 0 \) and \( E(\sigma_{g1}^2 \sigma'_{1g}) = \Psi_{gg} \) is positive definite and there are \( n \) sample observations. \( Y = XB + e \), \( E(u) = 0 \), \( E(\sigma_{u}^4) = \sigma^2 I_n \), \( \rho(X_{nk}) = K \), etc., and \( u \) and \( v \) are independent, then the GLS estimate considering both prior and sample information is

\[
B = (X'X + R'R)^{-1} (X'Y + R'\psi^{-1}r) \quad \text{and} \quad \text{var}(B) = (\lambda X'X + R\psi^{-1}R)^{-1}
\]

where \( \lambda = 1/\sigma^2 I_n \).

(for more detail, see Appendix I).

The limitation of the Theil and Goldberger Mixed Model is that the sample and prior information can be inconsistent and biased.

Inequality Restrictions. An alternative approach to exact and inexact prior information has been developed by Judge and Takayama (1966) in "Inequality Restrictions in Regression Analysis" which was based on the initial approach developed by Chipman and Roa (1964) in "The Treatment of Linear Restrictions in Regression Analysis" and by Zellner (1961) in "Linear Regression with Inequality Constraints on the Coefficients: An Application of Quadratic Programming and Linear Decision Rules." They deal with the case where prior knowledge of the coefficients comes in the form of inequality restrictions. Such restrictions could, of course, be handled with the Theil-Goldberger method by choosing a mean and variance for the prior distribution of the parameter which will only yield values outside the postulated upper and lower limits with arbitrarily low probabilities. According
to Judge et al. (1980), using linear inequality restrictions can improve the precision of estimation when near extreme multicollinearity is present, but cannot in the extreme multicollinearity situation.

**Deletion of Variables**

Variable deletion is usually exercised when the regression results have been affected by multicollinearity and no other satisfactory solution appears to be found. Johnston (1972) notes that deleting a relevant variable may be undesirable because of the resulting specification bias. Suppose that

\[ Y = XB + Zv + u \]

is the true model, but we fit

\[ B = (X'X)^{-1}X'Y \]

then,

\[ E(B) = E(X'X)^{-1}X'(X'B + Zv + u) \]

since

\[ Y = XB + Zv + u \]

\[ = B + (X'X)^{-1}X'Zv \]

where

\[ (X'X)^{-1}X'Zv \]

is the bias.

This bias could be large depending on the correlation between omitted and retained variables and depending on coefficient \( v \).

**Principal Components**

Principal components regression has received considerable attention in recent years as a method for dealing with multicollinearity. See, for example, Fomby and Hill (1978) in "Multicollinearity and the Value of a Prior Information": Greenberg (1975) in "Minimum Variance Properties of Principal Components"; Johnson et al. (1973) in "Principal Components and the Problem of Multicollinearity"; and

Principal components considers the transformation of explanatory variables (X's) to a new set of variables which will be uncorrelated pairwise. These variables are the principal components (Z_i) that are linear combinations of X variables (Z_i = XQ_i). The first variable will have the maximum possible variance, the second the maximum possible variance among those uncorrelated with the first and so forth. The transformation is the orthogonal transformation where the model Y = XB + u is transformed to Y = Zα + U. That means that Y = XB + U

\[ \implies Y = XIB + U \implies Y = XQQ'B + U \]

since Q is an orthogonal matrix such that Q'Q = QQ' = I and Q'X'XQ = Λ where Λ is a diagonal matrix of the eigenvalues (characteristic roots) of X'X, then Q will always exist if X'X is a positive definite matrix (Johnston, 1972).

Thus, Y = (XQ) (Q'B) + U by the associative law

\[ = Zα + U \]

where z = XQ and α = Q'B.

An OLS regression of the dependent variable on the principal components yields estimated coefficients, \( \hat{α}_i \), which can be transformed to the OLS coefficients of the X variable (\( \hat{B} \)).

Since,

\[ \hat{α} = Q'\hat{B} \implies \hat{B} = Q\hat{α} \] because Q'Q = QQ' = I

and \[ \text{var}(\hat{α}) = σ^2Λ^{-1} \] since \[ \text{var}(\hat{B}) = σ^2 a'a \]

(for more detail see Appendix I).

Principal components is a method of biased estimation which can be effective in mitigating the effect of multicollinearity. Normally, the operation is done to reduce the information demands on the data by
deleting some principal components. Principal components regression is similar to variable deletion; however, it involves deleting components rather than deleting the whole variable.

Reducing the number of principal components can result in more stable estimates than OLS since, in terms of the principal components, the total variation in the X variables is

$$\text{tr} (X'X) = \text{tr} (Z'Z) \text{ since } Q'Q = QQ' = I$$

and

$$Q'(X'X)Q = \Lambda$$

$$= \sum_{i=1}^{k} Z_i^2$$

$$= \sum_{i=1}^{p} \lambda_i \text{ since } Z'Z = \Lambda$$

$$= k \text{ (number of explanatory variables).}$$

For the case where some \( \lambda_i \) are near zero, a smaller number of \( \lambda_i \) will usually account for most of the variation in the explanatory variables (X's); equivalently, the corresponding smaller number of component \( Z_i \) will usually account for most of the variation in the explanatory variables.

The deleting of principal components creates the new matrix, \( Q_i^* \), which is used in a transformation of the coefficients into estimates of the coefficients \( B \) of the X variables.

$$B^* = Q\alpha^* \implies B^* = Q\alpha^*$$

and

$$\text{var}(B^*) = \sigma^2 \Lambda^*^{-1}$$

However, variance of \( B^* \) is smaller than the variance of \( \hat{b} \) since one or more of the diagonal elements is associated with the smallest \( \Lambda^{-1} \) values and has been set to zero in \( \Lambda^*^{-1} \).
There are several principal component estimators which can be employed, each differentiated by the method used in selecting principal components:

1. Components corresponding to the smallest eigenvalues are chosen for deletion.

2. Deciding whether a component should be deleted, according to the t or F statistical significance of each component coefficient.

3. Elimination of principal components based on the mean square error criterion, which is a method considered by Sass (1979) and Rahuma (1982). In other words, it is desirable to delete a component only if it reduced the estimated square error loss of the estimated parameter vector, $\beta^*_k$.

**Ridge Regression**

Hoerl and Kennard (1970a, 1970b) published the original papers on ridge regression. They recommend ridge regression as a procedure for (1) investigating the sensitivity of least square estimates based on data showing near extreme multicollinearity, and (2) where a small change in the data may cause a big change in the estimated coefficient. There have been a large number of papers written on the subject. So far, only Monte Carlo experiments have been employed to investigate the properties of the resulting estimators in the literature. This is due to the difficulty of obtaining analytical results for ridge estimators (Judge et al., 1980).

Ridge regression is a method of biased linear estimation. Ridge regression reduces multicollinearity by adding positive $K$ to the main
diagonal element of $X'X$, the correlation matrix. Consider the model as follows:

$$Y = XB + U.$$  

The ridge estimator is defined as:

$$B = (X'X + KI)^{-1}X'Y$$

The variance-covariance of $\hat{\beta}^*$ is:

$$\text{var}(\hat{\beta}^*) = (X'X + KI)^{-1}X'X(X'X + KI)^{-1}$$

(for more detail, see Appendix I).

When $K = 0$, the ridge estimator and its variance-covariance are equal to the OLS estimator and its variance-covariance. Moreover, ridge regression is the method of biased linear estimation because

$$E(\hat{\beta}^*) = B - K(X'X + KI)^{-1}IB$$

(for more detail, see Appendix I).

The value of $K$ can be determined by considering an orthogonal transformation of the regression model. $Y = XB + u$ is transformed to $Y = Z\alpha + u$ where $Z = XQ$ and $\alpha = Q'B$. For this transformed model, the OLS estimator is

$$\hat{\alpha} = (Z'Z)^{-1}Z'Y$$

and variance-covariance $\hat{\alpha}$ is

$$\text{var}(\hat{\alpha}) = \sigma^2 (Z'Z)$$

where $Z'Z = \Lambda$.

The ridge estimator for the transformed model is

$$\hat{\alpha}^* = (Z'Z + KI)^{-1}Z'Y$$

and the variance-covariance of $\hat{\alpha}$ is

$$\text{var}(\hat{\alpha}) = (Z'Z + KI) Z'Z (Z'Z + KI)$$
and variance of $\alpha$ is

$$V(\hat{\alpha}^*) = \sigma^2 \sum_{i=1}^{k} \frac{\lambda}{(\lambda + K)^2}$$

(for more detail, see Appendix 1).

Choice of $K_i$, optimum values for $K_i$ for generalized ridge regressions are those $K_i$'s which minimize $\text{MSE}(\hat{\alpha}^*)$, which also minimize $\text{MSE}(\hat{\beta}^*)$. It is shown in Appendix 1, that the optimum $K_i = \sigma^2/\alpha_i^2$. However, in most empirical analyses, $\sigma^2$ and the values of $\alpha$ are not known. As a result, the value of $K$ usually must be chosen based on estimates of the true values, or some other criterion for selecting $K$. 
CHAPTER 3

The Tuna Industry

Major Tuna Market Species

According to Joseph (1973), current tuna fisheries are based primarily on six species:

- Albacore (Thunnus alalunga)
- Bigeye (Thunnus obesus)
- Bluefin, Northern (Thunnus thynnus)
- Bluefin, Southern (Thunnus maccocyii)
- Skipjack (Katsuwonus pelamis)
- Yellowfin (Thunnus albacores)

These six major market species presently comprise about 75 percent of the world catch and nearly 100 percent of international trade of tuna and tuna-like species such as Bonito (Sarda spp), frigate mackerel (Auxis thazard), little tunas (Euthynnus spp), and blackfin (Thunnus atlanticus and Thunnus tongolii). In this study, we are concerned primarily with these six species because consumers and processors in the United States utilize them.

These six species are found in varying quantities through all tropical and temperate oceanic and coastal waters of the world's major oceans, primarily between 30 degrees north and 30 degrees south. Bigeye, skipjack, and yellowfin are called tropical tuna species since they appear to be nearly continuous within the limits of their northern and southern ranges throughout the Pacific, Atlantic, and Indian Oceans. Bigeye and yellowfin tuna grow to a large size, up to 200 or more pounds. They are relatively short lived, with a total life span of five years or less.
Figure 3-1. World fishing for tuna is between 30° N and 30° S.
Skipjack which have approximately the same life cycle, are the smallest, and tend to live in the cooler water of subtropical or more temperate latitudes. Albacore and bluefin tend to grow more slowly and live longer than the tropical species. Their life cycle is approximately 15 years or more. Albacore reach a maximum weight of about 80 pounds, while bluefin attain over 300 pounds.

The fact that tuna are wide ranging in their migrations adds to the complexity of appropriate management schemes. Bluefin and skipjack are reported to migrate extensively. Yellowfin and bigeye tuna migrate less extensively than the other tuna species. All tuna seem to have the same general depth range in the high seas as well as over the continental shelves. Bluefin, albacore, yellowfin and bigeye tend to live in the upper layers of the ocean. Even the deepest swimming species are found in the uppermost 490 feet. Skipjack, however, are not regarded as commonly occurring below 230 feet (Blackburn, 1965; Joseph, 1973; ICCAT, 1971; Kasahara, 1964; Hayasi, 1971).

The major share of all tuna taken throughout the world is captured using one of three methods: 1) long-lining; 2) pole and line, live-bait fishing; and 3) purse-seining. Long-lining and purse-seining account for over 90 percent of the total world catch of tuna since the mid-1970s. Tuna fishing can be further broken down into two basic types of operation: 1) fishing by small local vessels, and 2) fishing by larger, long-range vessels. The small local vessels travel from their home ports to the fishing grounds nearly every day. This type of operation is used primarily by less developed countries which lack the capacity to operate in the more distant off-shore fishing grounds.
Figure 3-2. Leading countries in catching tuna for years 1965, 1970, 1975 and 1980.
Figure 3-2. (continued)
The large, long-range vessels using mechanical refrigeration systems to preserve their catches may remain on the fishing grounds for many days, or even months. Operations are generally limited to the more advanced countries or less developed nations which can obtain the technical and financial assistance necessary to enable them to operate in the off-shore fisheries. There have been at least forty nations involved in tuna fishing, but Japan, U.S., Taiwan, Republic of Korea, France, and Spain account for over 70 percent of the world catch during the last two decades. During the last decade, the Philippines, Indonesia, Maldives, Ecuador, Sri Lanka, Solomon Islands, Mexico, Ghana, Senegal, and the Ivory Coast have been improving their catch in skipjack and yellowfin significantly.

The world catch of tuna species for the years 1965, 1970, 1975, and 1980 is shown in figure 3-3. Among the major commercial species, only yellowfin and skipjack have shown significant increases in production. The increase in yellowfin harvests are mainly due to greater surface fishing effort in both the eastern Pacific and eastern Atlantic fishing grounds. Much of the increase in skipjack catches in the early 1960s came from the eastern Pacific, while in recent years the increase came primarily from the expansion of the western Pacific fishery. Albacore catches have been relatively stable over the years. Catches of bigeye have increased slightly, but catches of bluefin have decreased slightly over the years.

The United States accounts for fifty percent of the world tuna consumption. However, most of the United States consumption is of canned tuna, which is distributed in three basic styles: 1) solid
Figure 3-3. World catch of tuna by species for years 1965, 1970, 1975, and 1980.
Figure 3-3. (continued)
Albacore is normally canned in solid pack as "white meat tuna" in the U.S. and Western European markets. The U.S. consumer pays the highest prices for white solid meat canned tuna. Consumers apparently prefer the white color and mild taste of albacore. According to Broadhead (1971), the U.S. market accounts for about 70 percent of world consumption of albacore. The remainder is consumed mostly by France and Spain.

Yellowfin, bigeye, skipjack and bluefin are canned and sold as "light meat tuna." This light meat tuna is normally packed in chunk style, while flaked and grated is a residual and thus the lowest priced canned product. Yellowfin is ranked second to albacore in unit value and is the preferred species. Bigeye is often sold as yellowfin in the U.S., and normally brings the same price as yellowfin. Skipjack is an almost perfect substitute for yellowfin but it brings the lowest price of the major market species since it is smaller and yields less raw material per pound of whole fish. Bluefin also brings a lower price than yellowfin since it has darker meat and a stronger taste (Broderick, 1973; Saila and Norton, 1974).

According to Saila and Norton (1974), the U.S. fishing fleet provides less than half of the U.S. tuna consumed. U.S. processors depend heavily upon imports, mostly from Japan and Taiwan, in the form of frozen raw tuna or tuna canned in brine. However, most imported raw tuna is canned in plants owned by U.S. companies in Puerto Rico and American Samoa.

The world's second largest consumer of tuna is Japan. However, Japanese consumption is entirely different from the American consumption
since the Japanese consume only a small portion of canned tuna. According to Broderick (1973), tuna is eaten raw as sashimi (marinated), thin-sliced raw, steaks, smoked as katsuo-bushi, and dried as namori-bushi. Prices for these products are generally very high. For example, bluefin -- which is the desired species for sashimi and steaks -- sometimes brings $10 per pound (Kask, 1969). According to Saila and Norton (1974), Japan exports both raw and canned forms. Japan exports almost 30 percent of its total tuna catch, with more than half of these exports going to the U.S. Japan also exports large quantities of raw tuna to Italy and canned tuna to West Germany.

Western European countries represent the third largest market for tuna. Almost 90 percent of the tuna used in this market is consumed in canned form. Only Italy and Spain are largely self-sufficient and their fleets dominate the eastern Atlantic Ocean, while the other countries -- especially West Germany and Italy -- must depend on imports (Broderick, 1973; Saila and Norton, 1974).

According to Saila and Norton (1974), Japan is the major exporting nation, but the Republic of Korea, Taiwan, Spain, Portugal, Norway, France, Peru, and Canada also export significant quantities of tuna. The U.S., Italy, West Germany, Canada, the United Kingdom, Switzerland, and Yugoslavia are primary importing nations.
CHAPTER 4

Theoretical Framework and Econometric Model

Theoretical Framework

Considering Marshall's emphasis on supply and demand as the "engine of analysis," the forces of supply and demand interact to determine the price of goods which consumers would buy and also to determine the price of services used in the production of these goods.

The demand for the final product directly reflects the "utility" attached to them because the ultimate consumer is the one who determines the shape and position of the demand functions. For this reason, consumer demand relationships are often referred to as "primary demand."

The demand for factors of production (services) is derived from primary demand. Normally, the demand for factors of products (derived demand) differs from primary demand only in the marketing and processing costs per unit of product. The tuna market does not appear to fit this perfectly competitive model. An imperfect market such as oligopoly, oligopsony, monopoly, monopsony may exist.

Four types of market solution are likely to occur in an oligopoly. They are the nonprice competitive system, the quota system, the price leadership by lower cost firm system, and the price leadership by dominant firm system. For the nonprice competitive system, the cartel sets a uniform price. All firms then price at or above this uniform price. For quota system, there is no uniform principle by which quotas can be determined. However, in practice the
bargaining ability of a firm, its share of the market, its size and financial situation are likely to be the most important elements in determining a quota. Under this quota system, two popular methods are usually exercised. The first of these has a quantitative base; either the relative sales of the firm in some precartel period or the productive capacity of the firm. The second popular basis for a quota system is geographical division of the market by agreement among firms in the cartel. Another type of oligopoly market solution is price leadership by one or a few firms. This solution does not require open collusion, but the firms must tacitly agree to the solution. Under price leadership by the lower cost firm system, the lower cost firm tolerates a competitor because of the antitrust law. Therefore, while it does not share the market equally, the lower cost firm nevertheless sets a price high enough for the higher cost firm to remain in the market. Price leadership by a dominant firm which exists in several American industries is discussed in detail later in this chapter.

Empirically, derived demand relationships can be estimated either directly, using pricing and quantity data which apply to approximate stages of marketing, or indirectly by subtracting appropriate margins from the primary demand schedule (Friedman, 1976; Tan, 1976; Tomek, 1972).

Retail Market

Primary retail demand is determined by the response of the ultimate consumers. Therefore, the retail demand curve for canned tuna indicates the quantities of canned tuna consumers will purchase at
the various price levels in given time periods. Thus, the slope of the demand curve indicates the impact of change of price on quantities sold and vice versa. The most common of these relationships is the concept of own price elasticity and own price flexibility of demand. Own price elasticity is simply a ratio which expresses the percentage change in quantity associated with a given percentage change in price. Own price flexibility is simply a ratio which expresses the percentage change in price associated with a given percentage change in quantity.

The curvature and slope of the retail demand curve for canned tuna determines, in part, the curvature and slope of the processor demand curve and, in turn, that of the ex-vessel demand curve. Both fishermen and processors have an interest in an estimate of how much price will change as production changes. Whether the quantity change or price change is proportionately greater determines whether total revenues will rise or fall as sales vary. Of equal interest is the position of the demand curve determined by such factors as 1) the population, 2) consumers' disposable income, and 3) the availability and price of substitutes for tuna.

As population increases, the quantity of canned tuna consumed is expected to increase proportionately, assuming all other things remain constant. Increases in consumer demand for canned tuna will result in a greater quantity demand for canned tuna at the same price. This process results in an outward shift of the demand curve for canned tuna.

Consumer buying habits are greatly influenced by income. Income places a limit on the ability to buy and consume. As income
increases, a greater total amount of food is purchased. Normally, the more desirable superior or normal types of foods are purchased in larger amounts while basic or necessary foods remain relatively the same or decline as income rises. Thus, if a particular good, such as tuna, is considered a normal good, then increasing income would have a positive effect on quantity demand, other things remaining constant. That is, the consumer demand curve would shift up, thereby raising the quantity demanded at any given price, or consumers would be willing to pay a higher price for the same quantity.

Income elasticity is a measure of quantity change response to income change, other factors remaining the same. Income elasticity of demand is simply a ratio which expresses the percentage change in quantity associated with a given percentage change in income. In most cases, the coefficient is positive. This is consistent with the idea that, as income increases, a consumer buys more of most products and when income decreases, the opposite occurs.

Almost every good can be replaced in some or in all of its uses. Fish, meat, and poultry are rarely consumed in the same meal. Bell (1969), Waugh and Norton (1969) found that fish competed in the market with meat, tuna competed with salmon, and imported fish competed with domestically harvested fish. Therefore, the price of tuna is determined not only by the quantities of tuna available in the market but also by the quantities and prices of meat, poultry, salmon, and other kinds of fish. Normally, measures of how the quantity purchased of one good (A) responds to changes in the price of another good (B) -- other things remaining the same -- is the concept of
of cross price elasticity of demand. Cross price elasticity of demand is defined as the proportional change in demand of "A" to the proportional change in the price of "B", other things remaining constant. In general, there are three types of cross relationships classified as substitutes, complements or independents, based on the substitution effect of the price change of "B". Generally, substitute goods have positive cross elasticities; complementary goods have negative cross elasticities; and independent goods have zero cross elasticities. However, from a technical point of view, these generalizations need not be true. The income effect may "outweigh" the substitution effect, resulting in a net reduction in the demand good "A" when the price of good "B" increases. Therefore, the net effect may be negative even though the two goods are substitutes. Nevertheless, the size of expenditure on the good relative to total expenditure indicates the magnitude of the income effect. Naturally, if the expenditure on one good is a small fraction of the total expenditures, the income effect does not outweigh the substitution effect. Thus, the generalization about size of cross elasticities usually holds. However, the concepts of substitutes, complements and independents between pairs of commodities are not symmetric. In other words, reversing the goods in the cross elasticity equation does not necessarily give the same coefficient. That means "A" may be a substitute for "B", but "B" may not be a substitute for "A". Similarly, "A" may be a complement of "B", but "B" may not be a complement of "A". This also applies to the independent relationship. Even if the substitution effect (for substitutes or complements) is
symmetric, the income effect is not. The exact relationships among elasticities are explained by the homogeneity condition, the Slutsky condition, and the Engel aggregation conditions (Tomek, 1972; Silberberg, 1978; Layard and Walters, 1978; Deaton and Muellbauer, 1980).

**Wholesale Market**

In this study, canners are their own wholesalers. Through sales offices and branch warehouses, they contact and service retailers directly. Thus, canners act as wholesalers by distributing their product directly to retailers. That means, that in the wholesale market, canners are the suppliers and retailers represent the demands.

In figure 4-1, $D_w$ is the wholesale demand curve derived by subtracting the ordinate (vertical) distance between the supply curve of retail marketing services ($S_{rs}$) and the abscissa (horizontal) axis from the retail demand curve ($D_r$). The derived demand curve, $D_w$, indicates the highest price which retailers would pay in order to get the various quantities of canned tuna supplied at the wholesale level.

Increasing (decreasing) consumer demand ($D_r$), as well as decreasing (increasing) retail market service ($S_{rs}$), will cause the wholesale demand curve ($D_w$) to shift upward (downward); this is due to retail market service or cost of marketing service such as labor cost and transportation, etc. Increases in costs of marketing services will shift the supply curve of marketing services upward. Thus, they tend to shift the wholesale demand curve in the opposite direction.

In short, the wholesale demand can be represented as related to the price of canned tuna at the wholesale level, consumers' income, population, prices of related products, marketing cost, time, tastes and preferences.
Figure 4-1. Relationship between wholesale demand and retail demand for canned tuna, either white or light canned tuna.
Ex-vessel Market

Theoretically, price paid to fishermen for raw tuna are determined by the interaction of the demand for and supply of raw tuna at the first point of sale, which is at the ex-vessel level. The ex-vessel demand for raw tuna is likened to a demand for inputs -- raw tuna being the resource used to produce white and light canned tuna for sale to retailers. This input demand is derived from the retailers' demand for either white or light canned tuna. Normally, the input used to produce white canned tuna is albacore, and inputs used to produce light canned tuna are yellowfin, skipjack, bluefin, and bigeye. These demand curves are discussed above in a similar manner, i.e., as the vertical difference between the wholesale demand curve and the supply curve of canners' services. The resulting demand curves are indicated by $D_{ex}^a$ and $D_{ex}^{ysb}$ in figure 4-2. The aggregate ex-vessel demand $D_{ex}^T$ is also illustrated in figure 4-2 -- the horizontal summation of $D_{ex}^{ysb}$ and $D_{ex}^a$.

Given the aggregate ex-vessel supply function $S_{ex}^T$ in figure 4-2, and assuming competitive conditions, equilibrium in this market would be established at the point of intersection of $D_{ex}^T$ and $S_{ex}^T$. The ex-vessel price would be $P_{ex}^T$ and total quantity of tuna sold would be $Q_{ex}^{ysb}$ of which $Q_{ex}^T$ would be purchased by canners for producing light canned tuna and $Q_{ex}^a$ for white canned tuna. Regardless of the nature of the ex-vessel supply curve, the traditional bioeconomic model suggests that biological production is the constraining factor limiting supply. Therefore, the supply of tuna available to canners at the wholesale level is fixed once the equilibrium is established at the
Figure 4-2. Price determination in white and light canned tuna markets and allocation of ex-vessel sales between albacore and other species of tuna.
ex-vessel level. These curves are indicated as $S^w_w$ and $S^l_w$ in figure 4-2. The price of white canned tuna received by canners is $P^w_w$, equal to the price of tuna paid to fishermen plus the per unit cost of handling $Q$ units of tuna ($P^w_w = p^T_{ex} + C^a_{ex}$). The price of light tuna received by canners from sale is $P(P^l_w = p^T_{ex} + C^{ysb}_{ex})$.

**Import Market**

One might ask why the United States imports a large percentage of her total fish supply. One obvious answer is that it is cheaper to import than to expand domestic production. In other words, the marginal cost of importation is less than the marginal cost of domestic production. But as to why this happens, the answer to the problem frequently addresses institutional constraints, technology and ecology.

Whitaker (1972) and Batie (1974) identified nine such constraints: a) high initial cost of vessels; b) high cost of insurance; c) competition for resources of the environment; d) selective demand for a handful of species; e) structure of industry; f) government owned and subsidized foreign competitors; g) nonregulation of common property; h) technological obsolescence; and i) the physical inability of the fishery resource to respond to increased production. Therefore, the institutional constraints, technology and ecology are the constraints responsible for this situation. However, much of the increase in imports may result from the U.S. fish trade firms building markets in the U.S. and then reaching out to foreign suppliers for raw material with which to supply them. In the case of tuna, for instance, note the following quote from Chapman (1969, p. 37):
The U.S. tuna canner ... has created a complex and very extensive global producing and collecting system for tuna raw material ... The Ralston Purina Company (Van Camp Division), for example, use more than 10 percent of all tuna caught in the world, and Heinz Company (Starkist Foods) is not far behind ... They have assisted in financing the construction of tuna fishing fleets ... they have assisted suppliers in various ways in securing and financing supply bases, cold storages, transshipments of suppliers ..."

"Quite substantial operations involve numerous U.S. owned vessels, partially manned by U.S. crews, delivering to U.S. owned freezing and storage plants in foreign ports for shipment to the U.S. market."

These tuna are landed in other countries and exported to the U.S. Therefore, they appear in the statistics as imports rather than landings. In essence, these imports are really "repatriation" of American capital.

To consider the determination of quantity and price of the imports of raw tuna is to consider import demand as a residual. The rest of the world will sell tuna only to the extent that its supplies exceed its own demand. Referring to figure 4-3, if $D^{US}$ is the total demand in the U.S. and $S^{US}$ is the total supply in the U.S., then $ED^{US}$ is the difference between the two schedules and is the excess demand schedule. The determination of the distribution of quantities to domestic and import markets, and the determination of prices for tuna can be conveniently summarized by incorporating 1) the rest of the world demand, 2) the rest of the world supply, and 3) excess demand in import markets into a model such as that illustrated in figure 4-3. Referring to figure 4-3, the rest of the world demand and supply schedules are represented by $D^{Rw}$ and $S^{Rw}$, respectively. The introduction of an excess demand by U.S. ($ED^{US}$) causes total demand for the world produced tuna to increase. The world prices
Figure 4-3. The determination of price and quantities sold in domestic and import markets for a specific species of tuna.
increase from $P_o^w$ to $P_1^w$, total production increases from $q_o^{RW}$ to $q^w$, quantity supplied to the rest of the world decreases from $q_o^{RW}$ to $q_1^{RW}$ and the quantity imported is $q^{TUS} = (q^w - q_1^{RW})$.

The theoretical framework which has been outlined in this section suggests a basis for an analysis of imports of U.S. tuna. Specifically, in order to explain and predict quantities and prices of U.S. imported tuna, it is necessary to gain an understanding of those factors which influence the rest of the world's demand and supply of tuna, and those factors which affect the import demand for tuna. The groups of factors which may influence the import demand for tuna can be identified as 1) the quantity of tuna supplied by U.S. and the rest of the world other than U.S.; 2) price of tuna in the world market, and other factors affecting market demand in consuming countries (income, population); 3) the cold storage holding fresh and frozen tuna; 4) transportation costs, tariff and nontariff barriers and the exchange rate between the U.S. and other countries.

Econometric Model of U.S. Canned Tuna

Retail Market

Retail Supply. Assume that the retailers of canned tuna simply pass along all quantities obtained from canners to consumers without holding stock or inventories for price speculation. Therefore, we would hypothesize that the yearly supply of canned tuna at the retail level is independent of the price of canned tuna and any market influence in a given year. The functions representing the supply for canned tuna are expressed as:
\[ \frac{Q_t}{N_t} = \frac{H_t}{N_t} \]  

(4-1)

where

\[ Q_t = \text{quantity of tuna supply in year } t \]
\[ H_t = \text{quantity of tuna obtained from canner in year } t \]
\[ N_t = \text{population in year } t \]
\[ \frac{Q_t}{N_t} = \text{tuna supply per capita in year } t \]
\[ \frac{H_t}{N_t} = \text{quantity of tuna obtained from canners on} \]
\[ \text{a per capita basis in year } t \]

Retail Demand. The demand equation for canned tuna at the retail level is developed based on the previous discussion. For the individual consumer, it may be reasonable to assume the quantity demanded is dependent on own price, the prices of related goods and income are explanatory variables.

The function representing the demand for canned tuna is formulated in the following manner.

\[ \text{Tuna}_t = \beta_0 + \beta_1 \text{CPIF}_t + \beta_2 \text{Income}_t + \beta_3 \text{PM}_t + \beta_4 \text{PPO}_t + \beta_5 \text{YR}_t \]
\[ + \beta_6 \text{PY}_t + u_t \]  

(4-2)

where

\[ \text{Tuna}_t = \text{Per capita tuna consumption in pounds in a given year} \]
\[ \text{CPIF}_t = \text{Average price per pound of canned tuna at retail level} \]
\[ \text{deflated by consumer price index for all fish in a given year (1967=100)} \]
\[ \text{INCOME}_t = \text{per capita disposable income deflated by consumer price index of all items in a given year (1967=100)} \]
\[ \text{PM}_t = \text{consumer price index for meat deflated by consumer price index of all items in a given year (1967=100)} \]
\[ PPO_t = \text{consumer price index for poultry deflated by consumer price index of all items in a given year (1967=100)} \]
\[ YR = \text{time in year (1970/41=1)} \]
\[ PY_t = \text{interrelationship between deflated price of canned tuna per pound and deflated per capita disposable income.} \]

Econometric models are estimated using the data for fiscal years 1953 through 1979. Consumer disposable income is used in the analysis to reflect shift in demand as a result of changes in consumers' purchasing power. The per capita poultry and meat consumption indexes are used to reflect the possibilities of substitution and/or independence for canned tuna as the price of canned tuna changes. A time variable is used to measure continuous and systematic shifts in demand due to factors not included in the analysis. Interrelationships between price and income are included in the equation:

1) to show that the demand curve becomes less elastic as income levels increase from $0 to $Z and that price changes have negative impacts on demand. However, beyond an income of $Z, any change in price will have a positive effect on demand.

2) to show that, as price ranges between $0 and $X income changes affect demand negatively. However, when price exceeds $X, an increase in income increases demand.

Therefore,

\[
\frac{\partial Tuna}{\partial CPIF} = \beta_1 + \beta_6 \text{INCOME} < 0 \\
\text{as INCOME (Y) } < Z
\]

\[
\frac{\partial Tuna}{\partial Income} = 2 + \beta CPIF > 0 \\
\text{as CPIF (P) } > X.
\]
The relationships reveal that as long as consumers' income remains at $Z or less and the retail price does not fall below $X, canned tuna will be consumed as normal goods. However, once these conditions are violated, canned tuna will be classified as an inferior or Giffen good. Moreover, prices of canned tuna, meat, poultry, and per capita income are inflated by the consumer price indices for all fish and for all items, respectively, to adjust for the effect of changing price levels and to convert the data to a relative basis.

**Wholesale Market**

The wholesale market is characterized by high concentration on the selling side. According to Onnley (1982), there are six canners but only two major canners are the leaders in sales. In this analysis, prices and quantities of white and light tuna will be used for analysis instead of aggregate price and quantity of canned tuna. Chunk light accounts for 75 percent of all canned tuna available in the market. Solid white and chunk white canned tuna account for only 15 and 5 percent, respectively, of all tuna existing in the market.

**Wholesale Supply.** Most American packs of tuna use the oil additive (vegetable broth) although the Japanese can vast amounts of tuna in brine. Something like 50 percent of the brine pack is exported annually to the U.S. and is sold at lower prices than the higher quality American packs of tuna. With a moderately high price relative to the import price plus quota on imported supply, the major canners could out-price the small competitors. The smaller canners would face higher costs in domestic operation compared with the major canners. On the other hand, if the domestic price were set
too high, the major canners would receive challenges from the small canners in the domestic market. The major canners would consequently lose ground to the smaller canners. Theoretically, the major canners must be able to keep small canners from domestic operation. Given the antitrust laws in the U.S., the major canners could not drive the smaller canners out of the market because the major canners would face legal problems. A better solution, from the viewpoint of major canners, would be to tolerate competitors. Thus, while not sharing the market equally, the major canner would set the price high enough for the small canners to remain in the market.

The problem confronting the major canners is to determine the price that will maximize its profit while allowing the small canners to sell all they wish at that price. To do this, it is necessary to find the demand curve for the major canners, e.g., figure 4-4, $P'abd$ and its marginal revenue ($MR^m$). The major canners set the price at $OP^*$ and sell $OQ^m$ units. At this price, the small canners sell $OQ^s$ since the small canners realize the fact that small canners behave just as perfectly competitive firms. That is, they will regard their demand curve as a horizontal line at the existing price and sell that amount for which marginal cost ($MC^s$) equals price ($OP^*$).

The range of prices that can be set is determined by the shift in $MC^s$ and $MC^m$ of small canners and major canners, respectively. There are two variables believed to be responsible for major shifts in $MC^s$ and $MC^m$: a) the import volume of canned tuna, and b) the ex-vessel and/or imported price of raw tuna.
Figure 4-4. The determination of price and quantities sold by major and smaller canners.
The ex-vessel and/or imported price of raw tuna is a key factor input cost. Changes in ex-vessel price and/or imported price directly affect the marginal cost, and therefore, the supply at wholesale. Thus, the quantity of canned tuna supplied may be positively related to the ex-vessel and/or imported price of raw tuna. Since a decrease (increase) in ex-vessel and/or imported price decreases marginal cost and consequently, shifts the supply curve to the right (left).

Also, the quantity of canned tuna supplied is positively related to the import volume of canned tuna since the larger the import volume, the larger the quantity supplied will be. Thus, quantity supplied is hypothesized to be a linear function of quantity imported volume, the ex-vessel price, and domestic wholesale price. The functions representing the wholesale price for canned tuna are expressed as:

\[ \frac{p_W}{p_{WF}} = \alpha_0 + \alpha_1 Q^W_t + \alpha_2 Q^I_t + \alpha_3 \frac{p^{ex}}{p_t} + \alpha_4 \frac{p^I}{p_t} \]  

(4-3)

where

- \( p^W_t \) = wholesale price of canned tuna in a given year
- \( Q^W_t \) = quantity of canned tuna available for domestic consumption (pack + inventory + import) in a given year
- \( Q^I_t \) = canned import volume in a given year
- \( p^{ex}_t \) = ex-vessel price of raw tuna in a given year
- \( p^I_t \) = imported price of raw tuna in a given year
- \( p^{WF}_t \) = wholesale price index of fish in a given year (1967=100)
Wholesale Demand. At this level, the larger retailers of canned tuna, such as chain stores, seek their supply of canned tuna directly from canners' sales offices and brokers. Small customers such as restaurants and grocers obtain their supplies through independent wholesalers.

Since most of the purchases are made by the chain stores, it is reasonable to assume that these retailers make the most demands for canned tuna at the wholesale level. The demand of retailers for canned tuna is a derived demand faced by canners, being derived directly from the consumption demand, dependent upon a) the wholesale price which retailers must pay for canned tuna; b) the price which retailers receive from the consumers for canned tuna; c) the retail price of other inputs (i.e., the wages of retail workers); and d) the price of substitute goods.

Thus, wholesale demand is specified as a linear function of the retail price, the wholesale price, the wage rate paid by retailers, and the wholesale price of a substitute for canned tuna. The functions which represent the wholesale demand for canned tuna are expressed as:

\[ Q_w^t = \beta_0 + \beta_1 \frac{P^r_t}{P_{wt}} + \beta_2 \frac{P^w_t}{P_{wt}} + \beta_3 \frac{P^r_t}{P_{wt}} + \beta_4 \frac{P^s_t}{P_{wt}} \]  

(4-4)

where

\( Q_w^t \) = quantity sold at wholesale in a given year

\( P^r_t \) = retail price of canned tuna in a given year

\( P^w_t \) = wholesale price of canned tuna in a given year

\( W^r_t \) = wage rates paid by retailers in a given year
\( p^S_t \) = wholesale price of substitute canned tuna in a given year

\( p^{wt}_t \) = wholesale price index of fish in a given year (1967=100).

Ex-vessel Market

Ex-vessel Supply. The fluctuating biological supply in tuna populations and, in turn, the impact on ex-vessel supply is frustrating not only to suppliers planning future production, but also to economists who would like to model a supply function for research and policy purposes.

The theoretical production function for the output of landings can be expressed as:

\[
Q = f(X_1, X_2, X_3)
\]  

(4-5)

where

- \( Q \) = landing per day
- \( X_1 \) = capital (vessels and years)
- \( X_2 \) = labor expended per day
- \( X_3 \) = the density of catchable tuna per volume of water.

\( X_3 \), density of tuna, is itself a function of time of the year, water temperature, numbers of predators, food supply and other factors. Any attempt to estimate a production function must consider these environmental factors in some manner. Thus, the supply response model for estimation can be:

\[
Q'_{sT} = f(X_4, X_5, X_6, X_7, X_8)
\]  

(4-6)

where
\[ Q_{AT} = \text{aggregate yearly landing of albacore, yellowfin, skipjack, bluefin, bigeye in the U.S.} \]

\[ X_4 = \text{number of trips completed by suppliers} \]

\[ X_5 = \text{number of boats in tuna fishery} \]

\[ X_6 = \text{number of troll in tuna fisher} \]

\[ X_7 = \text{number of purse-seining in tuna fishery} \]

\[ X_8 = \text{ex-vessel price} \]

Unfortunately, we do not have access to these data. Therefore, we must assume that the annual supply of raw tuna at ex-vessel level is independent of any factors in a given year. This can be expressed as:

\[ Q_t^T = Q_t^F \]  \hspace{1cm} (4-7)

where

\[ Q_t^T = \text{raw tuna landing in U.S. in year } t \]

\[ Q_t^F = \text{quantity obtained from fishermen in year } t \]

\textbf{Ex-vessel Demand.} In designing the model for ex-vessel tuna demand, it is helpful to examine the factors affecting the individual canner's purchase decision. Given a quantity of tuna landings the canner's decision will be affected by: 1) production process, 2) the quantity of tuna the canner is currently processing and storing, 3) the relative difference between the price he might pay for tuna, and the price he expects to receive for canned tuna, 4) knowledge about the quantity of tuna in frozen storage, 5) the way tuna are moving in local, national, and imports markets, and 6) the response of other buyers to his actions. Thus, the functions representing the ex-vessel demand for tuna may be expressed as:
\[ Q_{EX}^t = C_0 + C_1 S_t + C_2 P_{ex}^t + C_3 P_i^t + C_4 W_t^w + C_5 RW_t \] (4-8)

where

- \( Q_{EX}^t \) = quantity of tuna available for canniners (U.S. landings plus imports) in a given year
- \( S_t \) = quantity of tuna held in the frozen storage in a given year
- \( P_{ex}^t \) = ex-vessel price of raw tuna in a given year
- \( P_i^t \) = imported price of raw tuna in a given year
- \( W_t^w \) = wage rate paid by the canniers in a given year
- \( RW_t \) = the rest of the world landing of tuna in a given year.

**Import Demand**

Imports of frozen tuna represent approximately 60 percent of the total supply in the U.S. Japan is the major exporting nation, in either raw or canned form, exporting almost one-third of its total tuna catch, with about 60 percent of these exports going to the U.S. The Republic of Korea, Taiwan, Spain, Portugal, Norway, France, Peru, and Canada export the remaining U.S. tuna imports.

The international supply of tuna is more elastic than the domestic supply because numerous and diverse fishery grounds provide a larger international source for the world market than for the domestic supply. Also, import demand is considered to be residual. Therefore, imports of tuna are the closest substitutes to domestic production in the sense that they "materialize only where American capacity is not sufficient to satisfy demand at the corresponding normal price" (Neisser, 1953).
Import demand is assumed, therefore, to be formulated so that

import demand \((I)\) is the function of

\[
I_t = \alpha_1 + \alpha_2 P_t^T + \alpha_3 Q_t^W + \alpha_4 S_t + \alpha_5 Q_t^D + \alpha_6 EX_t
\]

where

- \(I_t\) = the quantity of tuna import in a given year \(t\)
- \(P_t^T\) = the price of tuna import in a given year \(t\)
- \(Q_t^W\) = the quantity of tuna supply in the rest of the world in a given year \(t\)
- \(S_t\) = the cold storage holding fresh and frozen tuna in a given year \(t\)
- \(Q_t^D\) = the quantity of tuna in the domestic supply in a given year
- \(EX_t\) = the exchange rate between the U.S. and other countries in a given year.
CHAPTER 5

Methodology and Data

In order to find and evaluate alternative methods of estimation both biased and unbiased for prediction of tuna price and quantity, the ordinary least square (OLS) estimate is compared with the principal components and ridge regression methods.

Exact and inexact prior information will be employed if the information is available and at the same time will be used to incorporate with principal components and ridge regression in order to evaluate and justify these techniques.

Several principal component methods of estimation can be employed, each differentiated by the methods used in selecting principal components; these are:

1. the components corresponding to the smallest eigenvalues are deleted (smallest criteria);
2. the component is deleted or not depending upon the t or F statistic significance of each component coefficient, $\alpha_i$ (t-criteria);
3. deleting principal components based on the estimated squared error loss criteria (MSE-criteria).

Ridge regression can be classified according to the method used to select K. Three ridge estimates were used on estimators proposed by:

1. Hoerl, Baldwin, and Kennard ridge estimator ($K_{JBK}$);
2. Lawless and Wang ridge estimator ($K_{LW}$);
3. Dempster, Schotzoff and Wermath ridge estimator ($K_{DSW}$).
Detection of Multicollinearity

When multicollinearity is severe, the variance of the regression coefficient may be so greatly increased that no coefficient is significant even though the overall regression may be highly significant. In other words, it is a situation where low t statistics and high F statistic for the overall regression indicate severe multicollinearity. However, this does not provide an index for measuring the degree of multicollinearity.

Measuring for the degree of multicollinearity can be done by:

1. A simple two variable model. A simple correlation coefficient $r_{ij} = 0$ indicates no multicollinearity; $r_{ij} = ±1$ indicates perfect collinearity. However, when more than two variables are in the model, the simple interpretation is not valid.

2. Farra and Glauber (1967) Method. Their method involves only the calculation of the correlation matrix $(X'X)$ and its inverse $(X'X)^{-1}$. The variance inflation factor (VIF) of the diagonal element of the inverted matrix is a good measure of the degree and location of multicollinearity.


For the first alternative, two items corresponding to explanatory variables ($X_i$) are needed.

a) The corrected sum of square $\Sigma X_i^2$ where,

$$\Sigma X_i^2 = (X_i - \bar{X}_i)^2 \quad \text{or} \quad \Sigma X_i^2 = (\text{standard deviation})^2/n-1$$
b) \( C_{ii} \) where,

\[
C_{ii} = \frac{(\hat{\sigma}_{\beta_i}^2)}{\sigma^2}
\]

where

\[
\sigma_{\beta_i}^2 = \text{square of the standard error of estimated coefficient of } X_i
\]

\( \hat{\sigma} \) = estimate of mean square error.

Then the variance inflation factor (VIF) caused by multicollinearity is

\[
VIF = C_{ii} \sum X_{ii}^2
\]

For the second alternative, the value of \( R_{ii}^2 \) of each independent (explanatory) variable \( i \) regressed on all the other explanatory variables are needed. The variance inflation factor (VIF) caused by multicollinearity is

\[
VIF = \frac{1}{1 - R_{ii}^2}
\]

4. Thisted Method. Thisted (1976, 1978) and Thisted and Morris (1979) pointed out the inadequacy of (VIF) measure of multicollinearity and introduced an index based on MSE and predictive mean square error (PMSE). These indices are calculated for overall model estimation (Mci) and for model predictability (Pmci).

\[
Mci = \sum_{i=1}^{k} \frac{\lambda_i^2}{n_i^2}
\]

\[
Pmci = \sum_{i=1}^{k} \frac{\lambda_i n_i}{\lambda_i^2}
\]
where $\lambda_n$ is the smallest eigenvalue of $(X'X)$, values of $M_{ci}$ and $PM_{ci}$ close to one indicates a high multicollinearity problem and a value greater than two indicates relatively low multicollinearity.

### The Loss Function and Estimation of the Loss Function for Biased Linear Estimation

One method of judging the accuracy of the estimates of parameters in a regression model is through the loss function. It is defined as the sum of squared deviation between the estimate ($\hat{\beta}_i$) and its true parameter $\beta$.

$$\sum_{i=1}^{k} [\hat{\beta}_i - \beta]^2$$

The loss function can be minimized, for example, if one knows the contribution of each estimated coefficient. If $\hat{\beta}_i$ is relatively unstable, the loss function might be minimized by setting $\hat{\beta}_i = 0$. Then the loss function is equal to $\beta_i^2$. On the other hand, without deleting $X_i$ there would be a net gain if $(\hat{\beta}_i - \beta_i)^2 < \beta_i^2$. Unfortunately, the preceding loss function and deletion criterion are of little help in empirical analysis, since the true parameters are unknown. Practical use of the loss function is only possible by estimation. That means it is conceivable to estimate the loss function when its expected value, the mean square error, is expressed as the sum of variance and the square of bias.

$$E[MSE(\hat{\alpha}^*)] = V(\hat{\alpha}^*) + BIAS^2 \hat{\alpha}^*$$
\[
\sigma^2 = \frac{k}{\sum_{i=1}^{k} \frac{\lambda_i}{[\lambda + K]^2}} + \frac{k^2}{\sum_{i=1}^{k} \frac{\alpha_i^2}{[\lambda + K]^2}}
\]

\[
= \frac{k}{\sum_{i=1}^{k} \left[m_i^2 \left(\sigma^2 / \lambda_i\right) + (m_i - 1)^2 \alpha_i^2\right]}
\]

if \( K = 0 \) the ridge estimator is equal to the OLS estimator.

Thus, \( V(\hat{\alpha}^*) = V(\hat{\alpha}) = \text{then MSE}(\hat{\alpha}^*) = \text{MSE}(\hat{\alpha}) \)

Although the variance component of MSE (\( \hat{\alpha}^* \)) can be estimated using the OLS estimate of \( \sigma^2 \), however, the bias square component depends upon the unknown \( \alpha_i \) parameter. According to Sass (1979), in order to estimate the bias square component, assume that \( \alpha_i \) is distributed about zero and \( \alpha_i \)'s variance is equal and finite. This implies that:

1) \( E(\alpha_i) = 0 \)

2) The variance of \( \alpha \) becomes

\[
E[\alpha_i - E(\alpha_i)]^2 = E(\alpha_i^2)
\]

3) \( E(\alpha_i^2) = E(\alpha_1^2) = \ldots = E(\alpha_k^2) = c^2 \) and from the orthogonal transformation, we know that

\[
\frac{k}{\sum_{i=1}^{k} \lambda_i} = k \Rightarrow 1/k \sum_{i=1}^{k} \lambda_i = 1
\]

multiplying both sides by \( E(\alpha_1^2) = c^2 \), we have

\[
c^2 = E(\alpha_1^2) = 1/k \sum_{i=1}^{k} \lambda_i \alpha_i^2 = 1/k \sum_{i=1}^{k} \lambda_i \alpha_i^2
\]

Therefore, \( \text{Bias}^2 (\hat{\alpha}^*) = \sum_{i=1}^{k} (m_i - 1)^2 c^2 \)
However, $C^2$ itself must be estimated since

$$C^2 = \frac{1}{k} \sum_{i=1}^{k} \lambda_i \alpha_i^2.$$ 

This can be done by using the OLS estimates, $\hat{\alpha}_i^2$, to estimate $C^2$.

Therefore,

$$\hat{C}^2 = \frac{1}{k} \sum_{i=1}^{k} \lambda_i \hat{\alpha}_i^2.$$ 

Thus, the formula for estimating the mean square error of $\hat{\alpha}^*$ is

$$\text{est MSE}(\hat{\alpha}^*) = \sum_{i=1}^{k} \left[ \frac{m_i^2 \sigma^2 / \lambda_i}{m_i - 1} C^2 \right]$$

Rahuma (1982) estimated that the loss function by $\alpha'\alpha = E(\alpha'\alpha)$ - $\sigma^2 \Lambda^{-1}$ is an unbiased estimate of $\alpha'\alpha$, to be used instead of $\hat{C}^2$.

Since the transformation of the general regression model where

$$Y = X \beta + U \implies Y = XQ'Q'B + u \implies Y = Z + u.$$ 

The OLS estimate of $\alpha$ is $\hat{\alpha}_1$ given by

$$\hat{\alpha} = (Z'Z)^{-1} Z'Y$$

$$= (Z'Z)^{-1} Z'(Z\hat{\alpha} + u) \quad \text{since} \quad Y = Z\hat{\alpha} + u$$

$$= (Z'Z)^{-1} Z'Z\hat{\alpha} + (Z'Z) Z'\hat{u}$$

$$= \alpha + (Z'Z)^{-1} Z\hat{u} \quad \text{since} \quad (Z'Z)^{-1} Z'Z = I$$

Thus,

$$\hat{\alpha}'\hat{\alpha} = (\alpha + (Z'Z)^{-1} Z'\hat{u}) (\alpha + (Z'Z)^{-1} Z'\hat{u})$$

$$= (\alpha + u'Z(Z'Z)^{-1}) (\alpha + (Z'Z)^{-1} z'u)$$

since

$$(AB) = B' A'$$

$$= \alpha'\alpha + \alpha' (Z'Z)^{-1} Z'\hat{u} + u'Z(Z'Z)^{-1} \alpha + u'Z(Z'Z)^{-1} Z'\hat{u}$$

Therefore,

$$E(\hat{\alpha}'\hat{\alpha}) = \alpha'\alpha + E(u'Z(Z'Z)^{-1} (Z'Z)^{-1} Z'\hat{u})$$

since $Z$ is fixed and $Eu = 0$
\[
\begin{align*}
\alpha'\alpha &= E[(u'u) Z'Z(Z'Z)^{-1} (Z'Z)^{-1}] \\
&= \alpha'\alpha + \sigma^2 I(Z'Z)^{-1} \quad \text{since} \quad E(u'u) = \sigma^2 I \\
\text{and} \quad (Z'Z) (Z'Z)^{-1} &= I \\
&= \alpha'\alpha + \sigma^2 \Lambda^{-1}
\end{align*}
\]

Thus,
\[
\alpha'\alpha = E(\hat{\alpha}'\hat{\alpha}) - \sigma^2 \Lambda^{-1}
\]

where \( \Lambda \) is the \((k \times k)\) matrix of eigenvalues and the individual coefficient \( \alpha_i^2 \) is
\[
\hat{\alpha}_i^2 = [E(\hat{\alpha}_i^2) - \sigma^2/\lambda_i]
\]

The mean square error expression could be estimated by substitution \( \hat{\delta} = (\hat{\alpha}_i^2 - \sigma^2/\lambda_i) \), restricting \( \hat{\delta} \geq 0 \) in place of \( C \) to become
\[
\text{est MSE} (\hat{\alpha}_i^*) = \sum_{i=1}^{k} \left[ m_i^2 \sigma^2/\lambda_i + m_i - 1 \right]^2 \hat{\delta}
\]

In short, the decision criterion could be to delete components \( Z_i \) if \( \text{var}(\hat{\alpha}_i) \geq \hat{\delta} \) and use for computing the loss function as used by Rahuma. This criterion is similar to the principal which is employed by Brown (1978) and Sass (1979) where the criteria were to delete the component \( Z_i \) if \( \text{var}(\hat{\alpha}_i) \geq \hat{C}^2 = (\sum_{i=1}^{k} \lambda_i \alpha_i^2)/k \) and use \( \hat{C}^2 \) for computing the loss function.

**Accuracy of the Ridge Regression**

The accuracy of the ridge estimator, \( B = (X'X + KI)^{-1} X'Y \) depends on the value of \( K \) chosen. Since the true \( B \) parameters and the error term variance, \( \sigma^2 \), are unknown, \( K \) must be chosen based on estimation of the true values or some other criterion for selecting
K. The ridge estimator can be classified according to the method used to select K. Three ridge estimators were used: one proposed by Hoerl, Baldwin and Kennard (1975) \( K_{HBK} \); one proposed by Lawless and Wang (1976) \( K_{LW} \); and one proposed by Dempster, Schatyoff and Wermuth (1977) \( K_{DSW} \).

The Hoerl, Baldwin and Kennard value of K is defined as:

\[
K_{HBK} = \frac{k\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}}
\]

where,

- \( k \) is the number of explanatory variables
- \( \hat{\sigma}^2 \) is the OLS estimate of \( \sigma^2 \) (variance)
- \( \hat{\beta} \) is the parameter of OLS estimates, \( Y = X\beta + U \)

The Lawless and Wang value of k is defined as:

\[
K_{LW} = \frac{k\hat{\sigma}^2}{k \sum_{i=1}^{k} \lambda_i \hat{\alpha}_i^2}
\]

where

- \( k \) is the number of explanatory variables
- \( \hat{\sigma}^2 \) is the OLS estimate of variance
- \( \lambda_i \) is the \( i^{th} \) eigenvalue of the \( X'X \) correlation matrix
- \( \hat{\alpha}_i \) is the OLS estimate for the orthogonalized model, \( Y = Z\alpha + u \).

The \( K_{LW} \) is different from \( K_{HBK} \) by the fact that the \( K_{LW} \) coefficients are weighted according to the eigen values of \( (X'X) \).

The Dempster, Schatzoff and Wermuth value of K is defined as:

\[
K_{DSW} = \frac{k\hat{\sigma}^2}{\sum_{i=1}^{k} \lambda_i \hat{\alpha}_i^2 - k\hat{\sigma}^2}
\]

where
k is the number of explanatory variables
\( \hat{\sigma}^2 \) is the OLS estimate of variance
\( \lambda_i \) is the eigenvalue of \( X'X \) correlation matrix
\( \hat{\alpha}_i \) is the OLS estimate for the orthogonalized model, \( Y = Z\alpha + u \)

Lawless (1978) concluded that there is actually very little gain over \( K_{LW} \) by using \( K_{DSW} \).

A Review of Data Used in the Study

The availability of data inevitably places a number of constraints upon the specification of the empirical model. The econometrics model which includes three market levels: retail, wholesale, and ex-vessel. In the import market, import demand as the residual of ex-vessel demand and supply is considered. Notation for the variables is in Table 5-1.

Retail Market. Regarding the quantity of canned tuna at the retail level, annual aggregate data from 1953-1979 is used. This data is found in Current Fishery Statistics (CFS), National Marine Fisheries Service (NMFS) under the title of "Fisheries of the United States," and by National Fisherman's "Pacific Packers Report." These data show the aggregate consumption levels which include all white and light canned tuna produced in the domestic industry plus imports.

Data regarding per capita consumption of meat and poultry were made available by the United States Department of Agriculture (USDA) under the titles of "U.S. Food Consumption: Source of Data and Trends, 1909-63," "Food Consumption Price and Expenditure 1960-80," and "Agricultural Statistics." However, these data needed to be
adjusted to the same standard measurement. We used the publication of USDA entitled "Conversion Factors and Weights and Measures: For Agricultural Commodities and Their Products" to adjust our data.

For prices of canned tuna at the retail level, the average annual reports by NMFS from 1953-79 are used. ("Basic Economic Indicator Tuna 1947-72" and "Food Fish: Market Review and Outlook.") The year 1953 is used as the beginning year because the data were available back to this date.

There were concerns about prices of meat and poultry. Consumer price indices of meat and poultry provided for the public by the USDA is used ("U.S. Food Consumption: Source of Data and Trends, 1909-63" and "Food Consumption Price and Expenditure 1960-80") where 1967=100 as the base year.

Population and disposable income are provided by the United States Department of Commerce (USDC) in the volume "Historical Statistics of United States: Colonial Times to 1970" and in the periodical "Statistical Abstract of the United States."

The consumer price index is used for all items to deflate the price indices of meat and poultry as well as to adjust income. Also used was the consumer price index for fish to deflate the price of canned tuna at the retail level. Both indices designated 1967 as the base year. The consumer price index for all items can be found in "Historical Statistics of the United States: Colonial Times to 1970" and Statistic Abstract of the United States," both of which are reported by the USDC. The consumer price index for fish given and calculated by the USDA is in the "Food Consumption Price and

**Wholesale Market.** As mentioned previously, canned tuna can be distinguished into two different types: white and light canned tuna. However, chunk light canned tuna accounts for approximately 75 percent of canned tuna available in the market. Solid white canned tuna and chunk white canned tuna account for 15 and 5 percent, respectively, of all canned tuna available in the market. The amounts of white and light canned tuna are available from NMFS in CFS "Canned Fisheries Products" and in the "Pacific Packers Report."

The prices of white and light canned tuna used in this analysis will be the annual average price per standard case from the publication published by NMFS in CFS "Canned Fisheries Product" and in "Pacific Packers Report." The prices of both white and light canned tuna are deflated by the producer price index for fish reported in "U. S. Food Consumption: Source of Data and Trends, 1909-63" and "Food Consumption Price and Expenditure, 1960-80" and in Statistical Abstract of the United States."

Most imported tuna canned in brine comes from Japan. Although the U.S. has a quota on canned tuna imports, rarely will the import volume exceed the quota. Canned tuna imports can be found in CFS "Imports and Exports of Fishery Product" and also in "Pacific Packers Report" and in the U. S. Bureau of Census (USBC), "United States Imports of Merchandise for Consumption."

Data on wages are available from the USDC in "Statistical Abstract of the United States."
Canned tuna inventories in standard cases are not normally available to the public. However, personal contact with the NMFS at Terminal Island, California, has enabled us to acquire some information about inventories for white meat and light meat canned tuna for the years 1976-1981, total inventory data for canned tuna are available for the years 1965-1981.

**Ex-vessel and Foreign Market.** Quantities and prices of tuna at the ex-vessel level can be divided into two different categories depending on the type of tuna used in producing canned tuna. Albacore is the main ingredient in producing white canned tuna. To produce light canned tuna, at least four species of tuna are used: yellowfin, skipjack, bluefin, and bigeye. However, the bluefin and skipjack are the main input in producing light canned tuna.

Prices and quantities of albacore, yellowfin, skipjack, bluefin, and bigeye are reported by CFS, NMFS in "Fisheries of the United States" and also reported in "Food Fish: Situation and Outlook" by the U.S. Department of Commerce.


The quantity of tuna supplied by the rest of the world is available in FAO's "Yearbook of Fishery Statistics."

Inventory data of frozen tuna from 1947-1970 were obtained from the U.S. Department of Interior in "Frozen Fishery Products."
Inventory figures are not available after 1970 as responsibility was transferred to the U.S. Department of Commerce which did not continue collecting the data so crucial to any econometric market study.
<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definitions and Measurements</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWT</td>
<td>Total amount of solid white canned tuna produced in the United States (million standard cases)</td>
<td>U.S. Department of Commerce, &quot;Canned Fisheries Product.&quot;</td>
</tr>
<tr>
<td>PSWT</td>
<td>Total value of solid white canned tuna produced in the United States (million U.S. dollars)</td>
<td>U.S. Department of Commerce, &quot;Fisheries of the United States.&quot;</td>
</tr>
<tr>
<td>WCT</td>
<td>Total amount of chunk white canned tuna produced in the United States (million standard cases)</td>
<td>Pacific Packer Report</td>
</tr>
<tr>
<td>PWCT</td>
<td>Total value of chunk white canned tuna produced in the United States (million U.S. dollars)</td>
<td></td>
</tr>
<tr>
<td>FGWT</td>
<td>Total amount of flakes and grated white canned tuna produced in the United States (million standard cases)</td>
<td></td>
</tr>
<tr>
<td>PFGWT</td>
<td>Total value of flakes and grated white canned tuna produced in the United States (million U.S. dollars)</td>
<td></td>
</tr>
<tr>
<td>SLT</td>
<td>Total amount of solid light canned tuna produced in the United States (million standard cases)</td>
<td></td>
</tr>
<tr>
<td>PSLT</td>
<td>Total value of solid light canned tuna produced in the United States (million U.S. dollars)</td>
<td></td>
</tr>
<tr>
<td>LCT</td>
<td>Total amount of chunk light canned tuna produced in the United States (million standard cases)</td>
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</tr>
<tr>
<td>PLC2</td>
<td>Total value of chunk light canned tuna produced in the United States (million U.S. dollars)</td>
<td></td>
</tr>
<tr>
<td>FGLT</td>
<td>Total amount of flakes and grated light canned tuna produced in the United States (million standard cases)</td>
<td></td>
</tr>
<tr>
<td>PFGLT</td>
<td>Total value of flakes and grated light canned tuna produced in the United States (million U.S. dollars)</td>
<td></td>
</tr>
<tr>
<td>ICQ</td>
<td>Total amount of imported all canned tuna (thousand pounds)</td>
<td>U.S. Department of Commerce, &quot;Fisheries of the United States.&quot;</td>
</tr>
<tr>
<td>ICD</td>
<td>Total value of imported all canned tuna (thousand U.S. dollars)</td>
<td></td>
</tr>
<tr>
<td>OCQ</td>
<td>Total amount of imported light canned tuna (thousand pounds)</td>
<td></td>
</tr>
<tr>
<td>OCD</td>
<td>Total value of imported light canned tuna (thousand pounds)</td>
<td></td>
</tr>
<tr>
<td>ALUSQ</td>
<td>Total amount of albacore landed in the United States (thousand pounds)</td>
<td></td>
</tr>
</tbody>
</table>

(continued)
Table 5-1. (continued)

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definitions and Measurements</th>
<th>Source</th>
</tr>
</thead>
</table>
| ALUSD | Total value of albacore landed in the United States (thousand U.S. dollars) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| SKUSDQ | Total amount of skipjack landed in the United States (thousand pounds) | U.S. Bureau of the Census, "United States Imports of Merchandise for Consumption."
| YPUSDQ | Total value of yellowfin landed in the United States (thousand U.S. dollars) | Food and Agriculture Organization (FAO) "Yearbook of Fishery Statistics."
| BFUSDQ | Total amount of bluefin landed in the United States (thousand pounds) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| USL | Total amount of all tuna imported in the United States (thousand pounds) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| IMUL | Total value of all tuna imported in the United States (thousand U.S. dollars) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| ALIQ | Total amount of imported albacore (thousand pounds) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| ALID | Total value of imported albacore (thousand U.S. dollars) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| SKIQ | Total amount of imported skipjack (thousand pounds) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| SKID | Total value of imported skipjack (thousand U.S. dollars) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| YFIQ | Total amount of imported yellowfin (thousand pounds) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| YFW | Total value of imported yellowfin (thousand U.S. dollars) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| ALW | World catch of albacore (thousand metric tons (m.t.)) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| SKW | World catch of skipjack (thousand m.t.) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| YFW | World catch of yellowfin (thousand m.t.) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| BFH | World catch of bluefin (thousand m.t.) | U.S. Department of Commerce, "Imports and Exports of Fishery Products."
| PIA | United States consumer price index (1967=100) | U.S. Department of Commerce, "Basic Economic Indicators from 1942-72."
| WIC | United States producer price index (1967=100) | U.S. Department of Commerce, "Food, Fish Situation and Outlook."
| PIT | Average canned price of canned tuna at the retail level per pound (U.S. cents) | U.S. Department of Commerce, "Food, Fish Situation and Outlook."

(continued)
<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definitions and Measurements</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1PO</td>
<td>Consumer price index for poultry (1967=100)</td>
<td>U.S. Department of Agriculture, &quot;Food Consumption Price and Expenditure.&quot;</td>
</tr>
<tr>
<td>P1M</td>
<td>Consumer price index for meat (1967=100)</td>
<td>U.S. Department of Agriculture, &quot;Food Consumption Price and Expenditure, 1960-80.&quot;</td>
</tr>
<tr>
<td>P1F</td>
<td>Consumer price index for all fish (1967=100)</td>
<td>U.S. Department of Agriculture, &quot;Food Consumption Price and Expenditure.&quot;</td>
</tr>
<tr>
<td>W1F</td>
<td>Wholesale price index for all fish (1967=100)</td>
<td>U.S. Department of Agriculture, &quot;Food Consumption Price and Expenditure.&quot;</td>
</tr>
<tr>
<td>POULTRY</td>
<td>Per capita food consumption index for poultry (1967=100)</td>
<td>U.S. Department of Agriculture, &quot;Conversion Factors and Weight and Measures for Agricultural Commodities and Their Products.&quot;</td>
</tr>
<tr>
<td>PORK</td>
<td>Per capita food consumption index for pork (1967=100)</td>
<td>U.S. Department of Agriculture, &quot;Conversion Factors and Weight and Measures for Agricultural Commodities and Their Products.&quot;</td>
</tr>
<tr>
<td>MEAT</td>
<td>Per capita food consumption index for meat (1967=100)</td>
<td>U.S. Department of Agriculture, &quot;Conversion Factors and Weight and Measures for Agricultural Commodities and Their Products.&quot;</td>
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</tbody>
</table>
CHAPTER 6

Estimating Price and Quantity Relationships in the Tuna Industry

The performance of biased and unbiased estimations are compared and evaluated where there is multicollinearity in the explanatory variables of the model and some prior information about the true regression coefficient. In order to accomplish this, an econometric model was applied to the multi-level tuna industry involving its retail demand, wholesale supply and demand, ex-vessel demand, and import demand. The summary in Table 6-A shows which econometric procedures are applied to each model of the multi-level tuna industry. Included are the number of models at various levels of the tuna market, both biased and unbiased estimation procedures, and table numbers which compare results in standardized OLS, ridge regression, and principal components methods. Note that procedures of biased and unbiased estimation used in the retail demand are laid out in detail. The same can be applied to the rest of the analysis at all levels of the tuna industry. However, in order to avoid repetition and save space, the analytical results of the wholesale supply and demand, ex-vessel demand, and import demand are reported in a short and specific fashion.

The Retail Demand

This analysis is based on time series data for the period of 1953-79. Retail supply for canned tuna is not estimated because it is specified as identity as mentioned in Chapter IV. The retail demand for canned tuna is specified as equation 4-2.
<table>
<thead>
<tr>
<th>An Econometric Model Applied To:</th>
<th>OLS$^a$</th>
<th>BM$^b$</th>
<th>EPI$^c$</th>
<th>IEPI$^d$</th>
<th>PC$^e$/MSE</th>
<th>PC$^f$/t</th>
<th>PC$^g$/sm</th>
<th>K$^h$/LM</th>
<th>K$^i$/DSW</th>
<th>K$^j$/HBLK</th>
<th>Table numbers where results of standardized OLS, principal components &amp; regression are compared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Demand</td>
<td>3 0</td>
<td>1 1</td>
<td>3 3 3</td>
<td>3 3 3</td>
<td>3 3 3</td>
<td>3 3 3</td>
<td>1, 2, 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wholesale Supply</td>
<td>2 0</td>
<td>0 0</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>4, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wholesale Demand</td>
<td>2 0</td>
<td>0 0</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>5, 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ex-vessel Demand</td>
<td>3 0</td>
<td>0 0</td>
<td>3 3 3</td>
<td>3 3 3</td>
<td>3 3 3</td>
<td>3 3 3</td>
<td>8, 9, 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Import Demand</td>
<td>3 1</td>
<td>0 0</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>2 2 2</td>
<td>11, 12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$/ OLS: The ordinary least square.

$^b$/ BM: Beach and MacKinnon procedure.

$^c$/ EPI: Exact price information.

$^d$/ IEPI: Inexact prior information.

$^e$/ PC$^e$/MSE: Deleting principal components based on mean square error criteria (MSE criteria).

$^f$/ PC$^f$/t: Deleting principal component based on t-criteria (t-criteria).

$^g$/ PC$^g$/sm: Deleting principal components based on smallest eigenvalue-criteria (smallest eigenvalue criteria).

$^h$/ K$^h$/LM: Lawless and Wang ridge estimator.

$^i$/ K$^i$/DSW: Dempster, Shatoff, and Wermath ridge estimator.

$^j$/ K$^j$/HBLK: Hoerl, Baldwin and Kennard ridge estimator.
Use of the Ordinary Least Square (OLS)

The equations for the retail demand for canned tuna is shown as:

1. Linear Demand Equation

\[
TUNA = -0.97659 + 0.01425(CPIF) + 0.00157(INCOME) - 0.09938(PPO) \\
+ 2.56276(PM) - 0.27183(YR) - 0.000007(PY) \\
(0.3440) (0.4591) (0.9545) (1.137) \\
(2.9585) (0.4709) (0.6542)
\]

\[R^2 = 0.9188 \quad D-W = 2.4136 \quad F = 38.2581 \quad \text{adj } R^2 = 0.8958\]

\[\beta_L = 1.01 \quad \delta_u = 1.86 \quad 4-\beta_L = 3.99 \quad 4-\delta_u = 2.14 \quad K = 6 \quad (6-1)\]

The pattern of residuals indicates that the variance of the error term is not constant but increases with the dependent variable.

The variance inflation factors also measure the degree and location of multicollinearity. For example, VIF(cpi) = 149.9925; VIF(income) = 433.0879; VIF(PPO) = 29.3049; VIF(PM) = 2.0234; VIF(t) = 227.2727; and VIF(PY) = 20.2339.

2. Double Logarithmic Demand Equation

\[
\log(TUNA) = -0.90163 - 0.74686 \log(CPIF) - 0.48987 \log(PPO) \\
+ 1.12382 \log(PM) + 0.41521 \log(PY) \\
(0.3027) (2.9109)* (2.5856)* \\
(3.7007)* (1.8152)
\]

*significant at 1% level,

\[R^2 = 0.9465 \quad D-W = 2.1537 \quad F = 97.3242 \quad \text{adj } R^2 = 0.9367\]

\[\beta_L = 0.166 \quad \delta_u = 1.65 \quad 4-\beta_L = 2.84 \quad 4-\delta_u = 2.35 \quad K = 4\]

The log(INCOME) and the log(YR) variables are eliminated from the equation because the F or tolerance level is insufficient for computation in the stepwise method.
According to the pattern of residuals the variance of the error term is not constant but increases with the dependent variable. The variance inflation factors are as follows:

\[
\text{VIF log(CPIF)} = \infty; \quad \text{VIF log(INCOME)} = \infty; \quad \text{VIF log(PPO)} = 14.5990; \\
\text{VIF log(PM)} = 2.1861; \quad \text{VIF log(t)} = 189.3939; \quad \text{VIF log(PY)} = \infty.
\]

3. Semilogarithmic Demand Equation

\[
\log(TUNA) = -0.721784 - 0.002191\log(CPIF) + 0.00234\log(INCOME) \\
(0.9762) \quad (0.5043) \quad (1.0479) \\
- 0.431938\log(PPO) + 1.076954\log(PM) - 0.002394\log(YR) \\
(2.4412) \quad (3.7802)\ast \quad (1.1327). 
\]

\ast \text{significant at 1\% level}

D-W=2.3548 \quad F=80.7623 \quad R^{2}=0.9505 \quad \text{adj } R^{2}=0.9388

3L=1.08 \quad 3u=1.86 \quad 4-3L=2.92 \quad 4-3u=2.24 \quad K=5

PY has been eliminated from the equation because the F or tolerance level is insufficient for computation by the stepwise method.

The pattern of residuals again reveals that the variance of the error terms increases with the dependent variable.

The variance inflation factors in this case are:

\[
\text{VIF(CPIF)} = 149.9925; \quad \text{VIF(INCOME)} = 2.3308; \quad \text{VIF(PPO)} = 29.3049; \\
\text{VIF(PM)} = 2.0234; \quad \text{VIF(t)} = 227.2727; \quad \text{VIF(PY)} = 20.2339; \\
TUNA = -6.59469 + 1.841711\log(INCOME) - 0.477341\log(PPO) \\
(0.5291) \quad (0.7037) \quad (0.6885) \\
+ 2.546999\log(PM) + 0.752356\log(YR) - 0.706628\log(PY) \\
(2.7018)\ast \quad (1.1718). 
\]

\ast \text{significant at 1\% level}

\[R^{2}=0.91568 \quad D-W=2.4959 \quad F=45.61945 \quad \text{adj } R^{2}=0.89560
\]

3L=1.08 \quad 3u=1.76 \quad 4-3L=2.92 \quad 4-3u=2.24 \quad K=5

Log(CPIF) has been eliminated from the equation again due to
insufficient F or tolerance for the stepwise method. Similarly, error term variance increases with the dependent variable.

The variance inflation factors from these equations are:

\[
\begin{align*}
VIF \log(CPIF) &= \infty; \quad VIF \log(INCOME) = \infty; \quad VIF \log(PPO) = 14.599; \\
VIF \log(PM) &= 2.1861; \quad VIF \log(YR) = 189.3939; \quad VIF \log(PY) = \infty.
\end{align*}
\]

All functional equations have three common major problems:

I. All three functional equations demonstrated a Durbin-Watson value in the "inconclusive" range of 5 percent. In this range, it is possible that the appearance of the error term correlation is due to the autocorrelation of the dependent variable and/or misspecification of the model.\(^1\) This violates one of the assumptions of OLS which states that the explanatory variables have a finite mean variance and are uncorrelated with the error term in the model.

---

\(^1\) According to Pindyck and Rubinfeld (1981), there are three possible situations in which independent variables may be correlated with the error term.

a) One or more of the independent variables is/are measured with error. When this situation occurs, the OLS process loses some of its desirable properties. In general, it leads to inconsistent ordinary least square parameter estimates. However, some of these lost properties can be regained if new variables, called instruments, replace the variables measured with error and a new instrumental variables technique is used to replace ordinary least square.

b) One or more of the independent variables is/are determined through one or more separate equations by the dependent variable. This situation occurs when variables in one equation "feed back" into variables in another equation, the error terms are correlated with those variables, and OLS leads to both biased and inconsistent parameter estimates. Some of these lost properties can be regained by employing a simultaneous system of equations. However, a serious specification error in one equation can affect the parameter estimates in all equations of the model. Thus, the decision to use system estimation involved a trade off between the gain in efficiency and the potential costs of specification errors.

c) One or more of the independent variables is a lagged dependent variable in a model in which the error term is autocorrelated. This can be solved by using modified instrumental variables or maximum likelihood techniques.
II. All three functional equations demonstrate a pattern of residuals indicating that the variance of the error term is not constant. This is a violation of one of the assumptions of OLS which states that the errors are independently distributed from a normal population with expected value and constant variances.

III. From inspection of variance inflation factors, as mentioned in Chapter IV, all three functional equations indicate collinearity between two or more explanatory variables. This is a violation of one of the OLS assumptions which states that no linear relationship exists between one or more of the explanatory independent variables.

After inspecting the model, one can conclude that 1) the model has problems of error specification; 2) there is evidence of a possible serial correlation problem; 3) multicollinearity is present.

The model is modified by eliminating the problems of error specification and possible serial correlation, and we obtain three unique equations. These three equations can be used to explain and evaluate the biased and unbiased estimation and at the same time can be used to address the relationship among the variables in the model in the economic sense. These are as follows.

---

2 By inspecting the pattern of residuals which indicate that the variance of error term is not constant, two possible problems may occur: heteroscedasticity, Pindyck and Rubinfield (1981) suggest that the test of Park (1966), Glejser (1969) as an appropriate statistical procedure. Moreover, Glejser (1969) points out that if the performed test finds that the residuals are homoscedastic, then error specification rather than heteroscedastic problems are possible. One should keep in mind that the heteroscedasticity or unequal variances does not usually occur in time series studies, because changes in the dependent variables and changes in one or more of the independent variables are likely to be the same order of magnitude (Prais and Houthakker, 1955).
In double logarithmic form:
\[ \log\text{TUNA} = f \log(\text{INCOME}, \text{PM}, \text{CPIF}) \]
\[ \log\text{TUNA} = f \log(\text{INCOME}, \text{YR}, \text{PM}, \text{CPIF}) \]

In linear form:
\[ \text{TUNA} = f(\text{CPIF}, \text{INCOME}, \text{YR}, \text{PY}) \]

The empirical result is shown as equation (5) using OLS regression.
\[ \log(\text{TUNA}) = 1.252845 + .467145 \log(\text{INCOME}) + .642636 \log(\text{PM}) \]
\[ - .92440 \log(\text{CPIF}) \]

\[ (0.3193) \quad (1.8357)^* \quad (2.3999)^** \]

\[ - .92440 \log(\text{CPIF}) \]
\[ (3.3330)^** \]

* significant at 5% level, D-W = 2.1202
** significant at 1% level, \( R^2 = 0.9302 \)

\( R^2 \) is 0.93, indicating that 93 percent of the variation in the quantity demand of canned tuna \( \log(\text{TUNA}) \) at the retail market level is associated with the variations in average price per pound of canned tuna \( \log(\text{CPIF}) \), in price of meat \( \log(\text{PM}) \) and in per capita disposable personal income \( \log(\text{INCOME}) \). D-W stands for the Durbin-Watson statistic. The calculated D-W value is 2.12 which suggests that one can reject the hypothesis, at the five percent level, that autocorrelation is present in the error terms. The \( t \)-statistic is listed in parentheses directly below the individual coefficients. All coefficients are significant at the five percent level, except for the constant terms, and have the expected sign.

The negative sign on the retail price coefficient \( \log(\text{CPIF}) \) of canned tuna is consistent with the theoretical demand relationship that the higher the retail price, the lower the quantity that consumers are willing to purchase, \textit{ceteris paribus}. The own price elasticity
measured at the mean values is -0.924. This means that, other things
remaining constant, a one percent increase (decrease) in the retail
price from the average of 1953-1979 value would result in a 0.924
percent decrease (increase) in the quantity sold and correspondingly
would result in an increase (decrease) in sales revenues for the
retailers.

Thus, U.S. retailers would experience an increase in canned tuna
sales revenues if they increased the average price of canned tuna at
the retail level, ceteris paribus. The positive relationships between
[log(TUNA)] and [log(PM)] signifies that canned tuna and meat are sub-
stituted for each other by U.S. consumers. This indicates that, other
things being constant, the demand for canned tuna tends to increase as
the retail price of meat rises. The cross price elasticity is 0.643,
implying that a one percent rise (fall) in the retail price of meat
away from its mean value for the 1953-1979 period would lead to a
0.643 percent rise (fall) in the quantity demanded of canned tuna.

The relationship between logTUNA and logINCOME is positive and
its income elasticity is 0.467, implying that canned tuna is a normal
good since a one percent increase (decrease) in consumer's income
would lead to a 0.467 percent increase (decrease) in quantity demanded
by consumers of canned tuna.

The sum of VIF is 21.5, which is considered evidence of low to
moderate multicollinearity. In this case, the price of tuna (logCPIF)
and income per capita (logINCOME) are highly correlated since \( r_{13} = r_{31} = -0.040 \).
However, a time variable may be needed in the equation (logYR) in order to measure continuous and systematic shifts in demand due to factors not included in the analysis. However when logYR is introduced into the equation it also increases the degree of multicollinearity. This is because log(YR) and log(INCOME) tend to move in the same pattern over time and moreover, log(CPIF) is the average aggregate price of canned tuna. Here, a time variable [log(YR)] was highly correlated with the income variable [log(INCOME)] ($r_{21} = r_{12} = 0.987$) and with the price of tuna [log(CPIF) ($r_{24} = r_{42} = -0.978$), respectively. The main diagonal elements of the inverted correlation matrix, which are useful measures of location and degree of multicollinearity, are shown as follows.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Variance Inflation Factors (VIF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log INCOME</td>
<td>54.2</td>
</tr>
<tr>
<td>log YR</td>
<td>122.4</td>
</tr>
<tr>
<td>log PM</td>
<td>1.2</td>
</tr>
<tr>
<td>log CPIF</td>
<td>31.1</td>
</tr>
</tbody>
</table>

The preceding VIF show exactly how much the variances of the regression coefficients for the variables have been increased because of multicollinearity. For example, the variance of the coefficient for log(INCOME) is 54.2 times larger than it would have been if log(INCOME) were uncorrelated with log(YR), log(PM), and log(CPIF).

These diagonal elements show that log(YR) is most likely to be affected by multicollinearity, log(INCOME) is the second most likely, followed by log(CPIF). Log(INCOME) and log(YR) are highly correlated with the other variables, since

$$ R_{\log YR: \log \text{INCOME}, \log \text{PM}, \log \text{CPIF}}^2 = 1 - (1/122.4) = 0.9911; $$
logYR: logINCOME, logPM, logCPIF

if log(YR) is regressed on the other explanatory variables. Similarly,

$$R^2_{logINCOME} = 1 - (1/54.2) = 0.9815;$$

And the sum of VIF, when log(YR) is added to the equation is 208.9, indicates a high degree of multicollinearity.

Adding a time variable log(YR), the model was estimated using OLS with the following results.

$$\log(TUNA) = -3.36598 - 0.276336 \log(INCOME) + 1.893174 \log(YR)$$

$$+ 0.687534 \log(PM) - 0.354271 \log(CPIF)$$

$$D-W = 2.1726 \quad R^2 = 0.9363$$

None of the coefficients except that of log(PM) is significant at the 5 percent level even though the overall regression is significant. Note that, in addition to being insignificant, the per capita income \([\log(INCOME)]\) variable has an unexpected sign.

In order to fully consider the relationship between demand price and income, a third demand equation is introduced. An interrelation term (PY), is included in this equation. The empirical result is shown in equation (6.7) using OLS estimation.

$$TUNA = 0.446546 + 0.331479E-02 \log(YR) + 0.116726E-02 \log(INCOME)$$

$$- 0.712716E-03 \log(CPIF) - 0.603798E-05 \log(PY)$$

$$D-W = 1.9733 \quad R^2 = 0.8838$$
The sum of VIP is 313.3, indicating a high degree of multicollinearity. The income variable is the variable most likely to be affected by multicollinearity, followed by YR and CPIF, since VIF_{INCOME} = 144.5, VIF_{YR} = 118.8, and VIF_{INCOME} = 41.9, respectively. None of the coefficients are significant at 5 percent level, even though the overall regression is significant. In addition to being insignificant, the PY takes the wrong sign. The Durbin-Watson statistic (D-W=1.97) rejects the hypothesis that autocorrelation is present in the residuals.

Faced with nonsensical results, one might consider principal components analysis and ridge regression as possible alternative methods. Given the high multicollinearity and moderate information to noise ratio, these two biased estimation methods are likely to produce more accurate results than OLS estimates (BLUE) of the model.

**Principal Components Analysis**

As the first step in principal components analysis, the quantity demanded in the retail model in (6.5, 6.6, 6.7) was transformed to the orthogonal form, \( Y = Z \alpha + \epsilon \), where, \( Z = XQ \), \( \alpha = Q' \beta \), and \( Z'Z = \Lambda \). The \( Z \) variables are the principal components. The transformation model was estimated, using OLS for these three demand equations (6.5, 6.6, 6.7), with the following results.

\[
\begin{align*}
\log(TUNA) & = 1.17049Z_1 - 0.03958Z_2 - 0.04783Z_3 \\
& \quad \quad \quad \quad \quad (0.0105) \quad (0.0171) \quad (0.0650) \\
\log(TUNA) & = 0.14416Z_1 + 0.00716Z_2 + 0.05625Z_3 + 0.19168Z_4 \\
& \quad \quad \quad \quad \quad (0.0078) \quad (0.0164) \quad (0.0631) \quad (0.1916) \\
TUNA & = 0.32614Z_1 - 0.06487Z_2 + 0.20950Z_3 + 0.41774Z_4 \\
& \quad \quad \quad \quad \quad (0.0252) \quad (0.0443) \quad (0.3703) \quad (0.6776)
\end{align*}
\]
(Note all coefficients of $Z_i$ are in nonstandardized form and standard deviations are listed in parentheses directly below the individual coefficients.)

As explained in Chapter II, the $a_i$ coefficients in these regression equations (6.8, 6.9, 6.10) are linear combinations of the $\beta$ coefficients in (6.5, 6.6, 6.7), respectively.

**Deletion of the Smallest Eigenvalue Criteria.** The smallest eigenvalue is deleted from the corresponding components.

$$Z'Z = Q'(X'X)Q = \Lambda = \begin{pmatrix} 
\lambda_1 & \cdots & 0 \\
0 & \lambda_2 & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \lambda_n 
\end{pmatrix}$$

where

- $Z =$ the principal components
- $Q =$ eigenvectors
- $\lambda_i =$ eigenvalues
- $X'X =$ correlation matrix of X's

From equation (6.8) the smallest eigenvalue component is 0.0507 ($\lambda_3$). Therefore, $\lambda_3 = 0.0507$ would be deleted from the corresponding components. From equations (6.9, 6.10) the smallest eigenvalues are 0.00532 and 0.00412, respectively. Therefore, $\lambda_4 = 0.00532$ in equation (6.9) and $\lambda_4 = 0.00412$ in equations 6.10 are deleted from the corresponding components in equation (6.9, 6.10), respectively.

In order to delete the eigenvalue ($\lambda_i$), the corresponding $\lambda_i$ can be set equal to zero. In other words, deletion of the eigenvalue can be accomplished by replacing the corresponding eigenvectors of matrix $Q$ with null vectors. This new matrix $Q^*$ can be used in transformation
of the $\alpha$ coefficients into estimated coefficients, $\beta^*$ from the standardized $X$ variables since $\beta^* = Q\alpha^*$ or $\beta^*(Sy/SX)$ in terms of non-standardized forms.

The result of transformation to $\beta^*$ after deletion of the smallest eigenvalue from the corresponding components is as follows (in nonstandardized terms):$^3$

\[
\log TUNA = -1.0910 + 0.65131 \log(INCOME) + 0.62094 \log(PM) \\
- 0.72412 \log(CPIF) \\
(6.5a)
\]

\[
\log TUNA = 1.19560 + 0.23418 \log(INCOME) + 0.62089 \log(YR) \\
+ 0.65259 \log(PM) - 0.72838 \log(CPIF) \\
(6.6a)
\]

\[
TUNA = -0.76758 + 0.03758 + 0.3735 \, YR + 0.00052 \, INCOME \\
- 0.00244 \, PCIF - 2.38639E-06 \, PY \\
(6.7a)
\]

The deletion of the smallest eigenvalue in this case reduces (decreases) the MSB($\beta^*$) in equations (6.5a, 6.6a, and 6.7a) by 50 percent, 85 percent and 78 percent, respectively, when the results are compared with OLS estimates in equations (6.5, 6.6, and 6.7), respectively. However, the variable representing the interrelationship between price and income ($\log PY$) in equation (6.7a) shows the unexpected sign.

Deletion of the Principal Components by the $t$ Criteria. A critical $t$ value corresponding to a significance level of 0.10 is chosen for deleting principal components. The calculated $t$ value from the principal components in equations (6.8, 6.9, and 6.10) are as follows.

$^3$ Also reported in the standardized form in Table 6.1, 6.2, 6.3.
From Equation 6.8

<table>
<thead>
<tr>
<th>Z_1</th>
<th>t*</th>
<th>Z_1</th>
<th>t*</th>
<th>Z_1</th>
<th>t*</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.160</td>
<td></td>
<td>18.341</td>
<td></td>
<td>12.906</td>
<td></td>
</tr>
</tbody>
</table>

From Equation 6.9

<table>
<thead>
<tr>
<th>Z_1</th>
<th>t*</th>
<th>Z_1</th>
<th>t*</th>
<th>Z_1</th>
<th>t*</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.315</td>
<td></td>
<td>0.436</td>
<td></td>
<td>1.463</td>
<td></td>
</tr>
<tr>
<td>0.735</td>
<td></td>
<td>0.891</td>
<td></td>
<td>0.566</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Equation 6.10

<table>
<thead>
<tr>
<th>Z_1</th>
<th>t*</th>
<th>Z_1</th>
<th>t*</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.906</td>
<td></td>
<td>1.463</td>
<td></td>
</tr>
</tbody>
</table>

where \( t_{a/2} = 4.303 \) where \( t_{a/2} = 3.182 \) where \( t_{a/2} = 3.182 \)

When \( t^* = \hat{\alpha}/\sqrt{\hat{\sigma}} \). A component is deleted if \( t^* < t_{a/2} \). Consequently, the \( t \)-criterion requires the deletion of \( Z_2, Z_3 \) in equation (6.8); \( Z_2, Z_3, Z_4 \) in equation (6.10). Note that, in order to delete a principal component the corresponding \( \hat{\alpha}_1 \) can be set equal to zero. Let \( \hat{\alpha}^* \) denote the resulting vector of coefficients when some coefficients are set equal to zero. The \( \hat{\alpha}^*_1 \) can be transformed into the principal component estimates, \( \beta_1 \), the coefficients of the standardized and/or nonstandardized \( X \) values, since \( \beta^* = Q \) standardized or \( \beta^* \) (SY/SX) (nonstandardized).

Deleting \( Z_1 \) by the \( t \) criterion discussed above and then transforming to the \( \beta^* \) (SY/SX) (nonstandardized) values, we have the following results:

1) Eliminating \( Z_2, Z_3 \) from equation (6.8) and transforming to \( \beta^* \) (SY/SX) we have

\[
\log TUNA = -0.96713 + 0.59064 \log INCOME + 1.2103 \log PM \\
- 0.6474 \log CPIF
\]

(6.6b)

2) Deleting \( Z_2, Z_3, Z_4 \) from equation (6.9) and transforming to \( \beta^* \) (SY/SX), we have

\[\text{Also reported in the standardized form in Tables 6.1, 6.2, 6.3.}\]
\[ \log TUNA = -3.2144 + 0.4275 \log \text{INCOME} + 0.6555 \log \text{YR} \]
\[ + 0.7416 \log \text{PM} - 0.4691 \log \text{CPIF} \]  \hspace{1cm} (6.6b)

3) Deleting \( Z_2, Z_3, Z_4 \) from the equation (6.10) and transforming to \( \beta^*(\text{SY}/\text{SX}) \), we have

\[ TUNA = -9.5709 + 0.0234 \text{YR} + 0.0003 \text{INCOME} - 0.01154 \text{CPIF} \]
\[ + 0.5914E^{-05} \text{PY}. \]  \hspace{1cm} (6.7b)

Note that elimination of the principal components by t criterion in this case reduced (decreases) the \( \text{MSE}(\beta^*) \) in equation (6.5b, 6.6b, and 6.7b) by 15 percent, 84 percent, and 94 percent, respectively, when compared to the result with OLS estimates in equations (6.5, 6.6, and 6.7), respectively. The per capita income (\( \log \text{INCOME} \)) in equation (6.6b) and the variable representing the interrelationship between price and income (\( \log \text{PY} \)) in equation (6.7b) show the expected signs.

The Proposed Loss Function Related Criterion (MSE criteria).

Deleting principal components based on the mean square error criterion or the proposed loss function related criterion requires a comparison of \( V(\hat{\alpha}_i) \) with \( C^2 = (1/k) \sum_{i=1}^{k} \lambda_i \alpha_i^2 \). If \( V(\hat{\alpha}_i) > C^2 \), then \( Z_i \) is deleted. In this case, \( C^2 \) has been calculated from the eigenvalues and coefficient of the principal components (standardized) reported in equations (6.8, 6.9, and 6.10), respectively. The \( C^2 \) values are as follows:

<table>
<thead>
<tr>
<th>( C^2 ) Value</th>
<th>Calculated from Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C^2 = 0.3101 )</td>
<td>(6.8)</td>
</tr>
<tr>
<td>( C^2 = 0.2341 )</td>
<td>(6.9)</td>
</tr>
<tr>
<td>( C^2 = 0.2210 )</td>
<td>(6.10)</td>
</tr>
</tbody>
</table>
\[ C^2 = \frac{1}{k} \sum_{i=1}^{k} \lambda_i \hat{\alpha}_i^2 \] estimated variance of \( \hat{\alpha}_i \) in equations (6.8, 6.9, and 6.10) are as follows:

<table>
<thead>
<tr>
<th>Equation 6.8</th>
<th>Equation 6.9</th>
<th>Equation 6.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\alpha}_1 )</td>
<td>( V(\hat{\alpha}_1) )</td>
<td>( V(\hat{\alpha}_1) )</td>
</tr>
<tr>
<td>0.0157</td>
<td>0.0067</td>
<td>0.0127</td>
</tr>
<tr>
<td>( \hat{\alpha}_2 )</td>
<td>0.0483</td>
<td>0.0277</td>
</tr>
<tr>
<td>( \hat{\alpha}_3 )</td>
<td>0.6878</td>
<td>0.4126</td>
</tr>
<tr>
<td>( \hat{\alpha}_4 )</td>
<td>2.9855</td>
<td>9.3312</td>
</tr>
<tr>
<td>Sum ( V(\hat{\alpha}_i) )</td>
<td>0.7518</td>
<td>3.4325</td>
</tr>
</tbody>
</table>

where

\[ V(\hat{\alpha}_1) = \frac{\sigma^2}{\lambda_1} \]

A component is deleted if \( V(\hat{\alpha}_1) > C^2 \). Consequently, the proposed loss function related criterion indicated that \( Z_3 \) in equation (6.8), \( Z_3 \) and \( Z_4 \) in equation (6.9), and \( Z_3 \) and \( Z_4 \) in equation (6.10) should be deleted. Thus, deleting principal components based on the mean square error criterion would provide the following results.

1) Deleting \( Z_3 \) from equation (6.8) and transforming to \( \beta^*(SY/SX) \) (nonstandardized), we have

\[
\log TUNA = -1.0910 + 0.6513 \log INCOME + 0.6209 \log PM \\
- 0.7241 \log CPIF; \\
(6.5c)
\]

2) Eliminating \( Z_3 \) and \( Z_4 \) from equation (6.9) and transforming to \( \beta^*(SY/SX) \) (nonstandardized), we have

\[
\log TUNA = -3.264 + 0.4336 \log INCOME + 0.6666 \log YR \\
+ 0.6293 \log PM - 0.4773 \log CPIF; \\
(6.6c)
\]
3) Deleting $Z_3$ and $Z_4$ from equation (6.10) and transforming $\beta^*$ (SY/SX), we have

$$\text{TUNA} = 0.5529 + 0.2422 \text{ YR} + 0.0003 \text{ INCOME} - 0.0130 \text{ CPIF} + 0.1360 \times 10^{-5} \text{PY};$$

(6.7c)

Deleting $Z_i$ by the proposed loss function related criterion, allows estimation of the mean square error as discussed in Chapter IV, as follows:

<table>
<thead>
<tr>
<th>Equation 6.5c</th>
<th>Equation 6.6c</th>
<th>Equation 6.7c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{MSE}(\hat{\alpha}^<em>_1) = \hat{V}(\hat{\alpha}^</em>_1) = 0.0157$</td>
<td>$\text{MSE}(\hat{\alpha}^<em>_1) = \hat{V}(\hat{\alpha}^</em>_1) = 0.0067$</td>
<td>$\text{MSE}(\hat{\alpha}^<em>_1) = \hat{V}(\hat{\alpha}^</em>_1) = 0.0127$</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\alpha}^<em>_2) = \hat{V}(\hat{\alpha}^</em>_2) = 0.0483$</td>
<td>$\text{MSE}(\hat{\alpha}^<em>_2) = \hat{V}(\hat{\alpha}^</em>_2) = 0.0277$</td>
<td>$\text{MSE}(\hat{\alpha}^<em>_2) = \hat{V}(\hat{\alpha}^</em>_2) = 0.0410$</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\alpha}^*_3) = C^2 = 0.5101$</td>
<td>$\text{MSE}(\hat{\alpha}^*_3) = C^2 = 0.2341$</td>
<td>$\text{MSE}(\hat{\alpha}^*_3) = C^2 = 0.2210$</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\alpha}^*_4) = C^2 = 0.2341$</td>
<td>$\text{MSE}(\hat{\alpha}^*_4) = C^2 = 0.2341$</td>
<td>$\text{MSE}(\hat{\alpha}^*_4) = C^2 = 0.2210$</td>
</tr>
<tr>
<td>$\text{MSE}(\alpha^<em>) = \text{MSE}(\beta^</em>) = 0.3741$</td>
<td>$\text{MSE}(\alpha^<em>) = \text{MSE}(\sigma^</em>) = 0.5026$</td>
<td>$\text{MSE}(\alpha^<em>) = \text{MSE}(\sigma^</em>) = 0.4957$</td>
</tr>
</tbody>
</table>

Recall that $C^2$ is an estimate of the true bias squared component which is added to MSE when $Z_i$ is deleted.

In this case, $\text{MSE}(\beta^*)$ in equations (6.5c, 6.6c, and 6.7c) is estimated to be 0.3741, 0.5026, and 0.4957, respectively, which is much less than 0.7518, 3.4325, and 12.0648, the estimated variance of OLS estimates in equations (6.8, 6.9, and 6.10). The application of principal components analysis to this highly intercorrelated and aggregated data set has apparently produced more accurate estimates as measured by $\text{MSE}(\beta^*)$, than OLS.
Ridge Regression

Three ridge estimators are used to estimate the following models.

\[ \log TUNA = f \log(\text{INCOME}, \text{PM}, \text{CPIF}) \]

\[ \log TUNA = f \log(\text{INCOME}, \text{YR}, \text{PM}, \text{CPIF}) \]

\[ TUNA = f (\text{YR}, \text{INCOME}, \text{CPIF}, \text{PY}) \]

with the highly intercorrelated and aggregated data set. The three estimators are:

Model I:

1) Lawless Wang, the Lawless Wang value of \( K \) was computed as

\[
K_{LW} = \frac{k \sigma^2}{k} = \frac{k \sigma^2}{\sum \lambda_i \alpha_i^2} = \frac{k \sigma^2}{\alpha'Z'Z\alpha} = \frac{k \sigma^2}{\beta'X'X\beta} = \frac{k \sigma^2}{C}
\]

2) Dempster, Shatzoff, and Wermuth; the RIDGM value of \( k \) was computed as

\[
K_{DSW} = \frac{k \sigma^2}{k} \sum \lambda_i \alpha_i^2 - k \sigma^2 = \frac{k \sigma^2}{\alpha'Z'Z\alpha} k \sigma^2 = \frac{k \sigma^2}{\beta'X'X\beta} - k \sigma^2
\]

3) Hoerl, Baldwin and Kennard; the Hoerl, Baldwin and Kennard value of \( K \) was calculated as

\[
K_{HBK} = \frac{k \sigma^2}{\alpha'\alpha} = \frac{k \sigma^2}{\beta'\beta}
\]

<table>
<thead>
<tr>
<th></th>
<th>Equation 6.11</th>
<th>Equation 6.12</th>
<th>Equation 6.13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{LW} )</td>
<td>0.11125</td>
<td>0.0906</td>
<td>0.1753</td>
</tr>
<tr>
<td>( K_{DSW} )</td>
<td>0.1266</td>
<td>0.0996</td>
<td>0.2125</td>
</tr>
<tr>
<td>( K_{HBK} )</td>
<td>0.2254</td>
<td>0.0990</td>
<td>0.1745</td>
</tr>
</tbody>
</table>

The empirical results for each of the three models using the three methods above are shown as follows:
\[
\text{logTUNA} = -0.2604 + 0.5575 \text{logINCOME} + 0.6488 \text{logPM} - 0.7448 \text{logCPIF}
\]
\[\text{(6.11a)}\]

where \( K = 0.1125; \)

\[
\text{logTUNA} = -0.3069 + 0.5578 \text{logINCOME} + 0.6499 \text{logPM} - 0.7350 \text{logCPIF}
\]
\[\text{(6.11b)}\]

where \( K = 0.1266; \)

\[
\text{logTUNA} = -0.4678 + 0.5487 \text{logINCOME} + 0.6538 \text{logPM} - 0.6313 \text{logCPIF}
\]
\[\text{(6.11c)}\]

where \( K = 0.2254 \)

All coefficients in (6.11a, 6.11b, and 6.11c) have the expected sign.

Model II:

\[
\text{logTUNA} = -2.2737 + 0.3205 \text{logINCOME} + 0.7013 \text{logYR} + 0.6309 \text{logPM} - 0.5336 \text{logCPIF}
\]
\[\text{(6.12a)}\]

where \( K = 0.0906; \)

\[
\text{logTUNA} = -2.3105 + 0.3261 \text{logINCOME} + 0.6944 \text{logYR} + 0.6293 \text{logPM} - 0.5286 \text{logCPIF}
\]
\[\text{(6.12b)}\]

where \( K = 0.0996; \)

\[
\text{logTUNA} = -2.3084 + 0.3258 \text{logINCOME} + 0.6948 \text{logYR} + 0.6294 \text{logPM} - 0.5289 \text{logCPIF}
\]
\[\text{(6.12c)}\]

All coefficients have the expected sign for the three ridge estimators. In particular, income per capita (logINCOME) variables when estimated by OLS demonstrated a negative sign. However, they switched to the positive sign, as expected, when ridge estimators were applied.
Model III:

\[ TUNA = 0.4619 + 0.0230 \, YR + 0.0003 \, \text{INCOME} - 0.0112 \, \text{CPIF} \]
\[ + 0.1382 \times 10^{-5} \, \text{PY} \]  
(6.13a)

where \( K = 0.1753; \)

\[ TUNA = 0.4766 + 0.0227 \, YR + 0.0003 \, \text{INCOME} - 0.0112 \, \text{CPIF} \]
\[ + 0.1503 \times 10^{-5} \, \text{PY} \]  
(6.13b)

where \( K = 0.2125; \)

\[ TUNA = 0.4616 + 0.0230 \, YR + 0.0003 \, \text{INCOME} - 0.0112 \, \text{CPIF} \]
\[ + 0.1379 \times 10^{-5} \, \text{PY} \]  
(6.13c)

where \( K = 0.1745. \)

All coefficients have the expected sign. The PY variable, when estimated by OLS, has a negative coefficient. However, the PY variable has the expected positive sign when ridge estimators are applied.

In summary, none of the OLS estimated coefficients for equations (6.12 and 6.13), are statistically significant. Moreover, the per capita income variable (\( \log \text{INCOME} \)) in equation (6.12) and the price income relationship variable (PY) in equation (6.13) have taken unexpected signs. These results are symptoms of the severe multicollinearity present in the highly interrelated and aggregated data. In contrast, the ridge estimators are effective in coping with the multicollinearity since the per capita income variable (\( \log \text{INCOME} \)) in equations (6.12a, 6.12b, and 6.12c) and the price income relationship (PY) variable in equations (6.13a, 6.13b, and 6.13c) have the expected sign. In addition, the variance of the estimates is greatly reduced by the use of ridge estimators as compared to OLS (see Tables 6.1, 6.2, and 6.3).
<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>Ridge Regression</th>
<th>Principal Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K=0</td>
<td>KLW</td>
<td>KDSW</td>
</tr>
<tr>
<td>( \hat{\beta} ) Log INCOME</td>
<td>0.32470</td>
<td>0.38751</td>
<td>0.38772</td>
</tr>
<tr>
<td>( \hat{\beta} ) Log INCOME</td>
<td>0.35975</td>
<td>0.04256</td>
<td>0.03712</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) Log INCOME)</td>
<td>0.35975</td>
<td>0.11809</td>
<td>0.10927</td>
</tr>
<tr>
<td>( \hat{\beta} ) Log PM</td>
<td>0.14619</td>
<td>0.14762</td>
<td>0.14786</td>
</tr>
<tr>
<td>( \hat{\beta} ) Log PM</td>
<td>0.04274</td>
<td>0.03211</td>
<td>0.03109</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) Log PM)</td>
<td>0.04274</td>
<td>0.03694</td>
<td>0.03634</td>
</tr>
<tr>
<td>( \hat{\beta} ) Log CPIF</td>
<td>-0.58078</td>
<td>-0.46800</td>
<td>-0.46182</td>
</tr>
<tr>
<td>( \hat{\beta} ) Log CPIF</td>
<td>0.34916</td>
<td>0.04222</td>
<td>0.03692</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) Log CPIF)</td>
<td>0.34916</td>
<td>0.11538</td>
<td>0.10684</td>
</tr>
<tr>
<td>MSE(( \hat{\alpha}^{<em>} ))=MSE(( \hat{\beta}^{</em>} ))</td>
<td>0.75165</td>
<td>0.27041</td>
<td>0.25245</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(-64.02)</td>
<td>(-66.41)</td>
</tr>
</tbody>
</table>

**NOTE:** Percent gain (decreasing) or loss (increasing) of MSE(\( \hat{\beta}^{*} \)) are parentheses.
Table 6.2. Estimated Standardized Coefficients, Estimated Variances and Estimated MSE of OLS, Ridge Estimators, Principal Components for Four Explanatory Variables, Retail Demand for Tuna Model

<table>
<thead>
<tr>
<th></th>
<th>OLS K=0</th>
<th>Ridge Regression</th>
<th>Principal Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(K_{LW})</td>
<td>(K_{DSW})</td>
</tr>
<tr>
<td>(\hat{\beta}) Log INCOME</td>
<td>-0.19250</td>
<td>0.22281</td>
<td>0.22667</td>
</tr>
<tr>
<td>(\hat{V}(\hat{\beta}) Log INCOME)</td>
<td>1.14828</td>
<td>0.02872</td>
<td>0.02551</td>
</tr>
<tr>
<td>MSE((\hat{\beta}) Log INCOME)</td>
<td>1.14828</td>
<td>0.12065</td>
<td>0.11218</td>
</tr>
<tr>
<td>(\hat{\beta}) Log Yr</td>
<td>0.86339</td>
<td>0.31986</td>
<td>0.31669</td>
</tr>
<tr>
<td>(\hat{V}(\hat{\beta}) Log Yr)</td>
<td>2.59490</td>
<td>0.11212</td>
<td>0.00981</td>
</tr>
<tr>
<td>MSE((\hat{\beta}) Log Yr)</td>
<td>2.59490</td>
<td>0.14801</td>
<td>0.13555</td>
</tr>
<tr>
<td>(\hat{\beta}) Log PM</td>
<td>0.15641</td>
<td>0.14354</td>
<td>0.14317</td>
</tr>
<tr>
<td>(\hat{V}(\hat{\beta}) Log PM)</td>
<td>0.02635</td>
<td>0.02061</td>
<td>0.02019</td>
</tr>
<tr>
<td>MSE((\hat{\beta}) Log PM)</td>
<td>0.02635</td>
<td>0.02307</td>
<td>0.02283</td>
</tr>
<tr>
<td>(\hat{\beta}) Log CPIF</td>
<td>-0.22260</td>
<td>-0.33530</td>
<td>-0.33217</td>
</tr>
<tr>
<td>(\hat{V}(\hat{\beta}) Log CPIF)</td>
<td>0.65913</td>
<td>0.03424</td>
<td>0.03049</td>
</tr>
<tr>
<td>MSE((\hat{\beta}) Log CPIF)</td>
<td>0.65913</td>
<td>0.10996</td>
<td>0.10292</td>
</tr>
<tr>
<td>MSE((\hat{\alpha}*)=(MSE((\hat{\beta})*)</td>
<td>4.42966</td>
<td>0.40169</td>
<td>0.37348</td>
</tr>
<tr>
<td>(0)</td>
<td>(-90.93)</td>
<td>(-91.93)</td>
<td>(-91.53)</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of MSE (\(\hat{\beta}\)*) are in parentheses.
Table 6.3. Estimated Standardized Coefficients, Estimated Variances, and Estimated MSE of OLS, Ridge Estimators, Principal Components for Four Explanatory Variable Retail Demand Canned Tuna Model

<table>
<thead>
<tr>
<th></th>
<th>OLS K=0</th>
<th>Ridge Regression K</th>
<th>Ridge Regression K$_{HKB}$</th>
<th>Principal Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}$ Yr</td>
<td>$\hat{\beta}$ Yr</td>
<td>$\hat{\beta}$ Yr</td>
<td>$\hat{\beta}$ Yr</td>
</tr>
<tr>
<td>$\hat{\beta}$ Yr</td>
<td>0.04176</td>
<td>0.29988</td>
<td>0.29591</td>
<td>0.2996</td>
</tr>
<tr>
<td>$\hat{V}(\hat{\beta}$ Yr)</td>
<td>4.59987</td>
<td>0.00323</td>
<td>0.00261</td>
<td>0.00325</td>
</tr>
<tr>
<td>$\hat{MSE}(\hat{\beta}$ Yr)</td>
<td>4.59987</td>
<td>0.14504</td>
<td>0.12117</td>
<td>0.14564</td>
</tr>
<tr>
<td>$\hat{\beta}$ INCOME</td>
<td>0.93182</td>
<td>0.30734</td>
<td>0.39966</td>
<td>0.30749</td>
</tr>
<tr>
<td>$\hat{V}(\hat{\beta}$ INCOME)</td>
<td>5.59406</td>
<td>0.00269</td>
<td>0.00221</td>
<td>0.00270</td>
</tr>
<tr>
<td>$\hat{MSE}(\hat{\beta}$ INCOME)</td>
<td>5.59406</td>
<td>0.14768</td>
<td>0.12310</td>
<td>0.14831</td>
</tr>
<tr>
<td>$\hat{\beta}$ CPIF</td>
<td>-0.017935</td>
<td>-0.28201</td>
<td>-0.28137</td>
<td>-0.28201</td>
</tr>
<tr>
<td>$\hat{V}(\hat{\beta}$ CPIF)</td>
<td>1.62337</td>
<td>0.00160</td>
<td>0.00420</td>
<td>0.00522</td>
</tr>
<tr>
<td>$\hat{MSE}(\hat{\beta}$ CPIF)</td>
<td>1.62337</td>
<td>0.12861</td>
<td>0.10356</td>
<td>0.12911</td>
</tr>
<tr>
<td>$\hat{\beta}$ Py</td>
<td>-0.13189</td>
<td>0.03016</td>
<td>0.32797</td>
<td>0.03010</td>
</tr>
<tr>
<td>$\hat{V}(\hat{\beta}$ Py)</td>
<td>0.31501</td>
<td>0.00884</td>
<td>0.00822</td>
<td>0.00886</td>
</tr>
<tr>
<td>$\hat{MSE}(\hat{\beta}$ Py)</td>
<td>0.31501</td>
<td>0.04534</td>
<td>0.04197</td>
<td>0.04542</td>
</tr>
<tr>
<td>$\hat{MSE}(\hat{\beta})$=MSE($\hat{\beta}$)</td>
<td>12.13231</td>
<td>0.46667</td>
<td>0.39480</td>
<td>0.46848</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(-96.15)</td>
<td>(-96.75)</td>
<td>(-96.14)</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of $\hat{\beta}$ are in parentheses.
Use of Exact Prior Information

Previous research in demand for tuna has been undertaken by Bell (1969), Waugh and Norton (1969), and NMFS (1974, whose results are presented below.

Bell (1969) estimated the demand for canned tuna over the period 1947-1967 as:

\[
\log C/N = -2.4034 - 0.9900 \log (Pt/Pw) + 1.4055 \log (Y/Pc)/N
\]

\[
+ 0.1493 \log (Ps/Pw) - 0.0222 \log (Pm, F, P/C)
\]

\[
R^2 = 0.969 \quad D-W = 1.28
\]

where:

- \( C \) = total aggregate consumption of tuna
- \( N \) = population
- \( Y \) = current aggregate income
- \( Pt \) = price of tuna (wholesale level)
- \( Pw \) = wholesale price index (all commodities)
- \( Ps \) = price of salmon (wholesale)
- \( Pc \) = consumer price index (all commodities)
- \( Pm, F, P \) = consumer price index for meat, fish, and poultry.

Waugh and Norton (1969) estimated the demand for canned tuna for the period 1936-1967:

\[
\log Q_t = 0.305 + 1.295 \log INC + 0.504 \log Ps - 1.121 \log Pt
\]

\[
R^2 = 0.97 \quad D-W = 1.40
\]

where

- \( Q \) = per capita consumption of tuna (pounds)
- \( Ps \) = deflated price of salmon (dols/case)
- \( Pt \) = deflated price of tuna (dols/case)
INC = deflated consumer disposable income per capita (thousands of dolrs/year).

NMFS(1974) estimated demand for tuna for the period 1947-1971 as:

\[
\log(C/N) = 2.9671 - 0.6966 \log P + 1.2091 \log(Y/N)
\]  

\( R^2 = 0.92 \quad D-W = 2.03 \)  

(4.55) \quad (6.60)  

where

\[ C = \text{tuna consumption} \]
\[ P = \text{real ex-vessel price of tuna} \]
\[ Y = \text{real disposable personal income} \]
\[ N = \text{United States population}. \]

From our estimation of demand for canned tuna in equation (6.6), we have

\[
\log\text{TUNA} = -3.3660 - 0.2763 \log\text{INCOME} + 1.8932 \log\text{YR}
\]  

\(-0.48\) \quad (1.45)  

\[ + 0.6875 \log\text{PM} - 0.3543 \log\text{CPIF} \]  

\(2.61\) \quad (-0.74)  

(6.6)

Note that t statistics are listed in parentheses directly below the individual coefficients.

As mentioned earlier, this equation (6.6) faces a problem of multicollinearity. Information available from the previous research can be used, but all three previous research results understandably yielded different estimates. What kind of information is needed? Do we need the coefficients of certain variables, such as price and/or income (own price elasticity and/or income elasticity or both)? How can we select the necessary information from the previous research? To answer these questions, we need to refer back to equation (6.6) and select variables which are likely to be affected by multicollinearity. It was found that \log\text{YR}, \log\text{INCOME}, and \log\text{CPIF}, in order of
severity, are indeed affected by multicollinearity. In addition, three previous demand functions are calculated in order to select the most likely representation of the information for breaking the pattern of multicollinearity. The NMFS estimated demand function for tuna is preferred, due to the significance of the t statistics and the reasonable Durbin-Watson statistic.

Given the information which we selected from the NMFS estimated coefficients such as own price elasticity ($\lambda_{1y} = \lambda_{1y}$ = -0.6966) and income elasticity ($\lambda_{1y}$ = 1.2091), the model can be estimated:

$$\log TUNA + 0.6966 \log CPIF = \beta + \beta_1 \log INCOME + \beta_2 \log YR + \beta_3 \log PM$$

or

$$\log TUNA - 1.2091 \log INCOME = \alpha_0 + \alpha_1 \log YR + \alpha_2 \log PM + \alpha_3 \log CPIF$$

or

$$\log TUNA + 0.6966 \log CPIF - 1.2091 \log INCOME = \tau_0 + \tau_1 \log YR + \tau_2 \log PM$$

The empirical results are as follows:

1. **logTUNA + 0.6966 logCPIF**

   $$= -0.2460 - 0.0687 \log INCOME + 1.1226 \log YR + 0.6722 \log PM$$

   $$(0.256) \quad (-0.141) \quad (1.531) \quad (2.586)^*$$

   $$R^2 = 0.82 \quad D-W \ 2.16$$

   (6.17a)

2. **logTUNA - 1.2091 logINCOME**

   $$= 0.5864 - 1.1689 \log YR + 0.5631 \log PM - 0.9869 \log CPIF$$

   $$(0.125) \quad (-0.834) \quad (1.941) \quad (-2.1455)^*$$

   $$R^2 = 0.35 \quad D-W = 1.91$$

   (6.18a)

3. **logTUNA + 0.6961 logCPIF - 1.2091 logINCOME**

   $$= -2.3499 - 0.7715 \log YR + 0.5607 \log PM$$

   $$(-3.944)^* (-5.439) \quad (1.957)$$

   $$R^2 = 0.55 \quad D-W = 1.82$$

   (6.19a)

*Significant at the 5% level.*
Equations (6.17a) and (6.18a) indicated symptoms of multicollinearity as most of their coefficients are insignificant and some of the coefficients took the wrong sign. This implies that the use of either own price elasticity or income elasticity alone cannot break the pattern of multicollinearity in this particular case (equation 6.6).

For equation (6.19a), all of the coefficients are significant at the 5 percent level. However, time demonstrates an unexpected negative. This raises the issue that the information we are using may be inconsistent and/or inaccurate in the estimation of this particular case.

Now, suppose we assume that the estimation of equation (6.5) and equation (6.11c) are more accurate than the NMFS estimates. Consequently, the use of information from these two equations yields the following results. Using information from equation (6.5), we have:

\[
\log TUNA + 0.0244 \log CPIF - 0.4671 \log INCOME \\
= 1.1753 + 0.1858 \log YR + 0.6273 \log PM \\
(2.176) (0.1344) (2.415) \\
R^2 = 0.23 \quad D-W = 2.11
\] (6.21)

* significant at the 5% level

With information from equation (5.11c), we have:

\[
\log TUNA + 0.6831 \log CPIF - 0.5487 \log INCOME \\
= -1.3983 + 0.2227 \log YR + 0.6182 \log PM \\
(-2.589) (2.467) (2.379) \\
R^2 = 0.54 \quad D-W = 2.01
\] (6.22)

* significant at the 5% level

The results from equations (6.21) and (6.22) are more satisfying than those of equation (6.20) because the time variable has the expected sign in both equations (6.21 and 6.22). Moreover, equation (6.22)
improves the estimate of the coefficient of the tuna variable since the MSE of $\hat{\beta}^*$ of (6.22) is less than the MSE $\hat{\beta}^*$ of (6.6).

Use of Inexact Prior Information

From NMFS (1972) estimates of tuna demand functions, useful information is obtained in order to break the pattern of multicollinearity in equation (6.6). To do this "Mixed Model," Theil and Goldburger, is employed as discussed in Chapter II. When the estimate of $\beta$ is

$$\hat{\beta} = [\lambda(X'X) + R'\psi^{-1}r]^{-1} (\lambda X'Y + R'\psi^{-1}r)$$

and variance-covariance is

$$\text{Var}(\hat{\beta}) = [\lambda(X'X) + R'\psi r]^{-1}$$

where

$$\lambda = 1/\sigma^2 I$$

$$X'X = \text{sum square corrected of own and cross products matrix}$$

$$R = \text{prior information design matrix that expresses the structure of the information on the individual parameter}$$

$$\psi = \text{prior information consisting of unbiased estimates of variances of parameter}$$

$$r - \text{vector of known of parameter}$$

To estimate $\hat{\beta}$ we need, 1) prior information from equation (6.6) about the corrected sum of squares of own and crossed product of the variables, and the variance of the regression ($\sigma^2$).

Prior information from equation (6.6) is as follows. Corrected sum of square of own and cross products:
\[
\begin{array}{c|c|c|c|c}
\text{logTUNA} & \text{logINCOME} & \text{logYR} & \text{logPM} & \text{logCPIF} \\
1.8443 & 1.2028 & 0.8037 & 0.1834 & -1.0974 \\
1.2028 & 0.8911 & 0.5771 & 0.1076 & -0.7641 \\
0.8037 & 0.5771 & 0.3836 & 0.3836 & 0.0651 \\
0.1833 & 0.1076 & 0.0651 & 0.0987 & -0.0815 \\
-1.0973 & -0.7641 & -0.5168 & -0.0815 & 0.7280 \\
\end{array}
\]

\[ \sigma^2 = 0.00534 \implies \lambda = 1/\sigma^2 \ln = 187.26591 \]

2) Prior information from the NMFS estimates in equation (6.16) about the unbiased estimates of parameter \( \beta_1 \) and their variance are as follows:

- \( \beta_1 \) (income per capita) = 1.2091 and standard deviation of \( \beta \) is 0.15310.
- \( \beta_2 \) (price of tuna) = -0.6966 and its standard deviation is 0.18319.

From this prior information we can transfer to

\[
R = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
42.66212 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 29.7074 & 0 \\
\end{pmatrix}
\]

\[
\psi^{-1} = \begin{pmatrix}
1.2091 \\
-0.6966 \\
\end{pmatrix}
\]

The result

\[
\hat{\beta} = \begin{pmatrix}
1.0962 \\
-0.6441 \\
0.4679 \\
-0.7499 \\
\end{pmatrix}
\]
From these results it can be seen that the time variable has the unexpected sign. This again raises the issue of whether the information we have acquired might be inaccurate and inconsistent.

If we assume that the estimates in equation (6.5) are correct, use of this information to estimate \( \hat{\beta} \) yields:

\[
\psi^{-1} = \begin{pmatrix}
15.4437 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 13.0032
\end{pmatrix}
\]

\[
r = \begin{pmatrix}
0.46715 \\
-0.92441
\end{pmatrix}
\]

The result is as follows:

\[
\hat{\beta} = \begin{pmatrix}
0.36518 \\
0.29028 \\
0.55693 \\
-0.86170
\end{pmatrix}
\]

Accordingly, all coefficients have their expected signs, indicating that the prior information from NMFS estimation might indeed be inaccurate and/or inconsistent.

The Wholesale Market

As mentioned in Chapter IV, two different and important types of canned tuna — solid white and chunk light — were selected for analysis of the wholesale market level. The wholesale supply equations are price-setting equations and are hypothesized to be perfectly price elastic with respect to quantities supplied, as discussed in Chapter III. The wholesale supply for chunk light and solid white canned tuna are specified as:
\[ PPCL = \alpha_0 + \alpha_1 LC + \alpha_2 OCIDW + \alpha_3 YFUSDW + \alpha_4 SKIDQW + \mu \quad (6.23) \]

where

\[ PPCL = \text{average annual price of chunk light canned tuna deflated by the wholesale price index of fish (1967=100)}. \]

\[ LC = \text{per capita consumption of chunk light tuna in a given year (pounds)}. \]

\[ OCIDW = \text{average annual imported price in dollars per pound of light canned tuna deflated by the wholesale price index of fish (1967=100)}. \]

\[ YFUSDW = \text{average annual price of yellowfin landing in the United States in dollars per pound deflated by the wholesale price index of fish (1967=100)}. \]

\[ SKIDQW = \text{average annual price of imported skipjack in dollars per pound deflated by the wholesale price index of fish (1967=100)}. \]

and

\[ PPSW = \beta_0 + \beta_1 SW + \beta_2 ICALDW + \beta_3 ALUSDW + \mu \quad (6.24) \]

\[ PPSW = \text{average annual price of solid white canned tuna in dollars per standard case deflated by the wholesale price index of fish (1967=100)}. \]

\[ SW = \text{per capita consumption of solid white canned tuna in a given year}. \]

\[ ICALDW = \text{average annual imported price of white canned tuna inflated by the wholesale price index of fish (1967=100)}. \]
ALUSDW = average annual price of albacore landed in the United States in dollars per pound deflated by the wholesale price index for fish (1967=100).

Use of Ordinary Least Square (OLS)

The empirical equations for supply of chunk light and solid white canned tuna equations is shown as

\[
PPCL = 4.7099 - 1.6318 \text{ LC} + 6.7762 \text{ OCIDW} + 47.2426 \text{ YFUSDW} \\
\quad (3.7038) \quad (-5.7195)** \quad (2.2354)* \quad (5.6984)** \\
+ 4.9207 \text{ SKIDQW} \\
\quad (1.5759)
\]

\[ R^2 = 0.9654 \quad D-W = 1.9456 \]

* significant at the 1% level

** significant at the 5% level

All coefficients except SKIDQW variables are significant at the 5 percent level and in addition to being significant, all variables have an expected sign. The sum of VIF is 9.30, indicating a low degree of multicollinearity.

\[
PPSW = 5.1104 - 9.2418 \text{ SW} + 23.0104 \text{ ICALDW} + 11.993 \text{ ALUSDW} \\
\quad (2.7915) \quad (-6.129)** \quad (5.2155)** \quad (1.2804) \quad (6.26)
\]

\[ R^2 = 0.885 \quad D-W = 1.95 \]

** significant at the 1% level

The only ALUSDW variable is insignificant at the 5 percent level. However, all variables have the expected sign. A low degree of multicollinearity is indicated by the VIF sum of 5.95.

The demand for solid white and chunk light canned tuna at the wholesale level is a derived demand of the retailers, as discussed
in Chapter III. Wholesale demand for chunk light and solid white canned tuna is specified as follows:

\[ LC = \alpha_0 + \alpha_1 \text{PPCLS} + \alpha_2 \text{RD1} + \alpha_3 \text{PITD} + \mu \]  

(6.27)

where

\text{PPCLS} = \text{predicted average annual price of chunk light canned tuna in dollars per standard case.}

\text{RD1} = \text{average wage rates paid by retailers deflated by the consumer price index (1967=100) at dollars per hour in a given year.}

\text{PITD} = \text{aggregate average retail price of canned tuna deflated by consumer price index (1967=100) – dollars per pound in a given year.}

and

\[ SW = \beta_0 + \beta_1 \text{PPSWS} + \beta_2 \text{RD1} + \beta_3 \text{PM} + \beta_4 \text{PITD} + \beta_5 \text{PPCL} + \mu \]  

(6.28)

where

\text{SW} = \text{per capita consumption of solid white canned tuna in a given year.}

\text{PPSWS} = \text{predicted average annual price of solid white canned tuna in dollars per standard case.}

\text{RD1} = \text{average rates paid by retailers deflated by the consumer price index (1967=100) in dollars per hour for a given year.}

\text{PITD} = \text{aggregate average retail price of canned tuna deflated by consumer price index (1967=100) in dollars per pound for a given year.}
PPCL = average annual price of chunk light canned tuna in dollars per case deflated by wholesale price index for fish (1967=100).

Two stage least squares is employed to eliminate the simultaneous equation bias. According to Koutsoyiannis (1977), the sources of bias are the endogenous variables in the set of explanatory variables of the function. Such endogenous variables have a systematic component — determined by the predetermined variables of the model — and random components. The latter creates the dependence of the relative variable on the random term $\mu$ of the structured equation. Moreover, the method of two-stage least squares seems appropriate for estimating demand and supply at the wholesale level because the system has overidentified equations.

The empirical demand equation for chunk light and solid white canned tuna equations is shown as

$$\text{LC} = -1.0193 - 0.14944 \text{ PPCLS} + 0.81761 \text{ RD1} + 2.66034 \text{ PITD}$$

$$R^2 = 0.9572 \quad \text{D-W} = 1.7272$$

** significant at the 1% level

All the coefficients are significant the the 5 percent level. However, the RDI variable shows the unexpected sign. The sum of VIF is 12.33, indicating the low degree of multicollinearity.

$$\text{SW} = -0.91973 - 0.00547 \text{ PPSWS} + 0.37156 \text{ RD1} + 0.5816 \text{ PM}$$

$$- 0.3479 \text{ PITD} + 0.0234 \text{ PPCL}$$

$$R^2 = 0.9572 \quad \text{D-W} = 1.7272$$

** significant at the 1% level

All the coefficients are significant the the 5 percent level. However, the RDI variable shows the unexpected sign. The sum of VIF is 12.33, indicating the low degree of multicollinearity.
The sum of VIF is 26.96, indicating a moderate degree of multicollinearity. Only two variables (RDI, PM) are significant at the 5 percent level even though the overall regression is significant. Moreover, RDI and PPCL variables have the unexpected sign. The hypothesis of correlation in the residuals is discounted by the Durbin-Watson statistic (1.9803).

When the problem of multicollinearity is severe, principal components and ridge regression methods seem to work well, according to the previously examined consumer demand equation. However, it is important to know how well the principal components and ridge regression methods for wholesale demand and supply relationships will apply where multicollinearity seems to be minimal.

Principal Components Analysis

Three differentiated methods of principal components are again employed. There are: 1) deletion of smallest eigenvalue-criteria (smallest eigenvalue criteria); 2) elimination of principal components by t or F criteria (t-criteria); 3) the proposed loss function related criterion (MSE criteria).

We performed the three different principal components methods on the following model where

\[ PPCL = f (\text{LC, OCIDW, YFUSDW, SKIDQW}) \] \hspace{1cm} (6.31)
\[ \text{LC} = f (\text{PPCLS, RD1, PITD}) \] \hspace{1cm} (6.32)
\[ PPSW = f (\text{SW, ICALDW, ALUSDW}) \] \hspace{1cm} (6.33)
\[
SW = f (\text{PPWS}, \text{DRI}, \text{PM}, \text{PITD}, \text{PPCL}), \text{respectively,}
\]
and then compared the result with the OLS estimators which are reported in Tables 6.4, 6.5, and 6.7, respectively.

In Table 6.4, where equation (6.31) is evaluated, and where the sum of VIF is 9.30, the results are summarized as follows: 1) deletion by smallest eigenvalue criteria and/or elimination of principal components by t or F criteria, increases MSE(\(\hat{\beta}^*\)) by 87 percent instead of decreasing its MSE(\(\hat{\beta}^*\)) when compared with the MSE(\(\hat{\beta}^*\)) of OLS; 2) use of the proposed loss function related criteria gives the same result as the OLS estimate.

Table 6.5 results, where equation (6.32) is evaluated and where the sum of VIF is 12.33, are summarized as follows: 1) results for the proposed loss function related criteria are the same as for the OLS estimator; 2) principal components are excluded by t or F criteria and by smallest eigenvalue criteria. Instead of reducing MSE (\(\hat{\beta}^*\)), MSE(\(\hat{\beta}^*\)) is increased by 37 percent when compared with the MSE(\(\hat{\beta}^*\)) of OLS.

In Table 6.5, where equation (6.33) is evaluated and where the sum of VIF is 15.92, the result is summarized as follows: 1) elimination by the smallest eigenvalue criterion increases MSE (\(\hat{\beta}^*\)) by 16 percent while exclusion of principal component by t or F criteria increases MSE (\(\hat{\beta}^*\)) by 82 percent when compared with the MSE (\(\hat{\beta}^*\)) of OLS; 2) the proposed loss function related criteria had the same results as the OLS estimator.

In Table 6.7, where equation (6.34) is evaluated, where the sum of VIF is 26.96, the results are summarized as follows: 1) MSE(\(\hat{\beta}^*\))
decreases by 46 percent in response to exclusion of principal components by smallest eigenvalue criteria, when compared with the MSE ($\hat{\beta}^*$) of OLS; 2) deleting the principal components by t or F criteria decreases $\text{MSE}(\hat{\beta}^*)$ by 62 percent when compared with the $\text{MSE}(\hat{\beta}^*)$ of OLS; 3) use of the proposed loss function related criteria decreases $\text{MSE}(\hat{\beta}^*)$ by 68 percent when compared with $\text{MSE}(\hat{\beta}^*)$ of OLS.

Ridge Regressions

The three ridge estimators employed are

1) Lawless Wary estimator ($K_{LW}$).
2) Dampster, Schotzoff and Wermouth or RIDGM estimator ($K_{DSW}$).
3) Hoerl, Baldwin and Kennard estimator ($K_{HKB}$).

We employed the three ridge estimators on the function model equations (6.31, 6.32, 6.33, and 6.34), respectively. The results were compared with the OLS estimate which is reported in Tables 6.4, 6.5, 6.6, and 6.7, respectively.

Results:

The Lawless Wary estimator ($K_{LW}$), RIDGM estimator ($K_{DSW}$) and Hoerl, Baldwin and Kennard estimator ($K_{HKB}$) were used in Table 6.4, where equation (6.31) is evaluated and the sum of VIP is 9.30. MSE($\hat{\beta}^*$) is decreased by 25 percent, 26 percent, and 37 percent, respectively, when compared with the MSE($\hat{\beta}^*$) of the OLS.

In Table 6.5, equation (6.32) is evaluated and the sum of VIF is 12.33. $K_{LW}$, $K_{DSW}$, and $K_{HKB}$ were used to reduce the MSE($\hat{\beta}^*$) by 35 percent, 37 percent, and 36 percent, respectively, when compared with the MSE($\hat{\beta}^*$) of the OLS.
Table 6.4. Estimated Standardized Coefficients, Estimated Variances and Estimated MSE of OLS, Ridge Estimators, Principal Components for Four Explanatory Variables, Wholesale Supply for Chunk Light Canned Tuna Model

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>Ridge Regression</th>
<th>Principal Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K=0</td>
<td>K_{LW}</td>
<td>K_{DSW}</td>
</tr>
<tr>
<td>( \hat{\beta} ) LC</td>
<td>-0.34767</td>
<td>-0.34921</td>
<td>-0.34910</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) LC)</td>
<td>0.02586</td>
<td>0.01920</td>
<td>0.01905</td>
</tr>
<tr>
<td>( \hat{MSE}(\hat{\beta} ) LC)</td>
<td>0.02586</td>
<td>0.02222</td>
<td>0.02207</td>
</tr>
<tr>
<td>( \hat{\beta} ) OCIDW</td>
<td>0.19585</td>
<td>0.23385</td>
<td>0.02830</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) OCIDW)</td>
<td>0.05373</td>
<td>0.02902</td>
<td>0.02830</td>
</tr>
<tr>
<td>( \hat{MSE}(\hat{\beta} ) OCIDW)</td>
<td>0.05373</td>
<td>0.03917</td>
<td>0.03865</td>
</tr>
<tr>
<td>( \hat{\beta} ) YFUSDW</td>
<td>0.58687</td>
<td>0.50408</td>
<td>0.50101</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) YFUSDW)</td>
<td>0.07424</td>
<td>0.03557</td>
<td>0.03448</td>
</tr>
<tr>
<td>( \hat{MSE}(\hat{\beta} ) YFUSDW)</td>
<td>0.07424</td>
<td>0.05122</td>
<td>0.05041</td>
</tr>
<tr>
<td>( \hat{\beta} ) SKIDQW</td>
<td>0.09067</td>
<td>0.05675</td>
<td>0.05223</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) SKIDQW)</td>
<td>0.02317</td>
<td>0.01857</td>
<td>0.01837</td>
</tr>
<tr>
<td>( \hat{MSE}(\hat{\beta} ) SKIDQW)</td>
<td>0.02317</td>
<td>0.02073</td>
<td>0.02062</td>
</tr>
<tr>
<td>MSE((\alpha^<em>))=MSE((\hat{\beta}^</em>))</td>
<td>0.17702</td>
<td>0.13336</td>
<td>0.13177</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(-24.66)</td>
<td>(-25.56)</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of MSE(\(\hat{\beta}^*\)) are in parentheses.
Table 6.5. Estimated Standardized Coefficients, Estimated Variances and Estimated MSE of OLS, Ridge Estimators, Principal Components for Three Explanatory Variable Wholesale Demand for Chunk Light Canned Tuna Model

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>Ridge Regression</th>
<th>Principal Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(K=0)</td>
<td>(K_{\text{LW}})</td>
<td>(K_{\text{DSW}})</td>
</tr>
<tr>
<td>(\hat{\beta} \text{ PPCLS})</td>
<td>-0.68868</td>
<td>-0.57777</td>
<td>-0.57258</td>
</tr>
<tr>
<td>(\hat{V}(\hat{\beta} \text{ PPCLS}))</td>
<td>0.12767</td>
<td>0.04776</td>
<td>0.04531</td>
</tr>
<tr>
<td>(\hat{\text{MSE}}(\hat{\beta} \text{ PPCLS}))</td>
<td>0.12767</td>
<td>0.07756</td>
<td>0.75481</td>
</tr>
<tr>
<td>(\hat{\beta} \text{ RD1})</td>
<td>0.49535</td>
<td>0.52322</td>
<td>0.52329</td>
</tr>
<tr>
<td>(\hat{V}(\hat{\beta} \text{ RD1}))</td>
<td>0.10011</td>
<td>0.04112</td>
<td>0.03927</td>
</tr>
<tr>
<td>(\hat{\text{MSE}}(\hat{\beta} \text{ RD1}))</td>
<td>0.10011</td>
<td>0.06327</td>
<td>0.06172</td>
</tr>
<tr>
<td>(\hat{\beta} \text{ PITD})</td>
<td>0.43381</td>
<td>0.35507</td>
<td>0.35057</td>
</tr>
<tr>
<td>(\hat{V}(\hat{\beta} \text{ PITD}))</td>
<td>0.03596</td>
<td>0.02566</td>
<td>0.02520</td>
</tr>
<tr>
<td>(\hat{\text{MSE}}(\hat{\beta} \text{ PITD}))</td>
<td>0.03596</td>
<td>0.03000</td>
<td>0.02970</td>
</tr>
<tr>
<td>(\text{MSE}(\alpha^<em>)=\text{MSE}(\hat{\beta}^</em>))</td>
<td>0.26375</td>
<td>0.17084</td>
<td>0.16690</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(-35.23)</td>
<td>(-36.72)</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of \(\text{MSE}(\hat{\beta}^*)\) are in parentheses.
Table 6.6. Estimated Standardized Coefficients, Estimated Variances and Estimated MSE of OLS, Ridge Estimators, Principal Components for Three Explanatory Variables, Wholesale Supply for Solid White Canned Tuna Model

<table>
<thead>
<tr>
<th></th>
<th>OLS K=0</th>
<th>Ridge Regression</th>
<th>Principal Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K=0</td>
<td>K LW</td>
<td>K DSW</td>
</tr>
<tr>
<td>( \hat{\beta} ) SW</td>
<td>-0.47102</td>
<td>-0.39946</td>
<td>-0.38631</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) SW)</td>
<td>0.06496</td>
<td>0.04251</td>
<td>0.03894</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) SW)</td>
<td>0.06496</td>
<td>0.05244</td>
<td>0.05015</td>
</tr>
<tr>
<td>( \hat{\beta} ) ICALDW</td>
<td>0.57557</td>
<td>0.44815</td>
<td>0.43009</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) ICALDW)</td>
<td>0.13396</td>
<td>0.04997</td>
<td>0.04264</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) ICALDW)</td>
<td>0.13396</td>
<td>0.08033</td>
<td>0.07366</td>
</tr>
<tr>
<td>( \hat{\beta} ) ALUSDW</td>
<td>0.14522</td>
<td>0.22236</td>
<td>0.22847</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) ALUSDW)</td>
<td>0.14150</td>
<td>0.05078</td>
<td>0.04304</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) ALUSDW)</td>
<td>0.14150</td>
<td>0.08338</td>
<td>0.07623</td>
</tr>
<tr>
<td>MSE(( \alpha^* )) = MSE(( \hat{\beta}^* ))</td>
<td>0.34043</td>
<td>0.21616</td>
<td>0.20005</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(-36.50)</td>
<td>(-41.23)</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of MSE(\( \hat{\beta}^* \)) are in parentheses.
<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>Ridge Regression</th>
<th>Principal Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K=0$</td>
<td>$K_{LW}$</td>
<td>$K_{DSW}$</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPSWS</td>
<td>-0.10098</td>
<td>-0.13973</td>
<td>-0.14275</td>
</tr>
<tr>
<td>$\hat{V}$ (PSW)</td>
<td>0.58411</td>
<td>0.01900</td>
<td>0.00984</td>
</tr>
<tr>
<td>MSE (PSWS)</td>
<td>0.58411</td>
<td>0.09595</td>
<td>0.06249</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RD1</td>
<td>0.90362</td>
<td>0.38294</td>
<td>0.31095</td>
</tr>
<tr>
<td>$\hat{V}$ (RD1)</td>
<td>0.25247</td>
<td>0.02818</td>
<td>0.01563</td>
</tr>
<tr>
<td>MSE (RD1)</td>
<td>0.25247</td>
<td>0.07701</td>
<td>0.05459</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PM</td>
<td>0.28706</td>
<td>0.18673</td>
<td>0.16576</td>
</tr>
<tr>
<td>$\hat{V}$ (PM)</td>
<td>0.11147</td>
<td>0.03075</td>
<td>0.02014</td>
</tr>
<tr>
<td>MSE (PM)</td>
<td>0.11147</td>
<td>0.05369</td>
<td>0.04216</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PITD</td>
<td>-0.22774</td>
<td>-0.08715</td>
<td>-0.07839</td>
</tr>
<tr>
<td>$\hat{V}$ (PITD)</td>
<td>0.19234</td>
<td>0.02893</td>
<td>0.01794</td>
</tr>
<tr>
<td>MSE (PITD)</td>
<td>0.19234</td>
<td>0.06381</td>
<td>0.04738</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPCL</td>
<td>0.44211</td>
<td>-0.07136</td>
<td>-0.10377</td>
</tr>
<tr>
<td>$\hat{V}$ (PPCL)</td>
<td>0.49273</td>
<td>0.02113</td>
<td>0.01094</td>
</tr>
<tr>
<td>MSE (PPCL)</td>
<td>0.49273</td>
<td>0.09200</td>
<td>0.06127</td>
</tr>
<tr>
<td>MSE($\hat{\alpha}$)=MSE($\hat{\beta}$)</td>
<td>1.63314</td>
<td>0.38159</td>
<td>0.26791</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(-76.63)</td>
<td>(-83.60)</td>
</tr>
</tbody>
</table>

**NOTE:** Percent gain (decreasing) or loss (increasing) of MSE ($\hat{\beta}$) are in parentheses.
In equation (6.33) in Table 6.6, the sum of VIF is 5.92. $K_{\text{KLW}}$, $K_{\text{DSW}}$, and $K_{\text{HBK}}$ estimators decrease the MSE($\hat{\beta}$) by 37 percent, 41 percent, and 46 percent, respectively, when compared with the MSE ($\hat{\beta}$) of the OLS.

Table 6.7, equation (6.34), where the sum of VIF is 26.96, demonstrates that use of $K_{\text{KLW}}$, $K_{\text{DSW}}$, and $K_{\text{HBK}}$ estimators decreases the MSE($\hat{\beta}$) dramatically by 77 percent, 84 percent, and 69 percent, respectively, when compared with the MSE($\hat{\beta}$) of the OLS.

The Ex-vessel Market

Canners and consumers in the U.S. tuna industry use six primary species, as mentioned in Chapter III. In this analysis, we are concerned with the three species which account for at least 90 percent of the total raw tuna used in this industry—albacore, skipjack, and yellowfin.

The Raw Tuna Supply

Fluctuation in tuna populations and, in turn, in ex-vessel supply, impacts not only suppliers planning future production, but economists seeking to model a supply function for research and policy purposes. Theoretically, production models can be built as expressed in equation (4.6), but since the data are not accessible, the annual supply of raw tuna at ex-vessel level is assumed to be independent of any factors in a given year (see equation (4.7)).

Demand for Raw Tuna

Factors relevant to individual canners' purchasing processes are important for planning ex-vessel raw tuna demand. The ex-vessel
demand for albacore, skipjack and yellowfin are specified as:

\[
ALQ = \alpha_0 + \alpha_1 ALIDQW + \alpha_2 RWAL + \alpha_3 WHDW + \alpha_4 TWHI + \mu \tag{6.35}
\]

where

\[
ALQ = \text{quantity of albacore tuna available for canners (U.S. landing plus imports) in thousand pounds per given year}
\]

\[
ALIDQW = \text{average annual price of imported albacore tuna in dollars per pound deflated by the wholesale price index for fish (1967=100)}
\]

\[
RWAL = \text{quantity of albacore tuna landed by the rest of the world in thousands of pounds per given year}
\]

\[
WHDW = \text{average wage rate paid by canners in dollars per hour deflated by wholesale price index for fish (1967=100) at a given year}
\]

\[
TWHI = \text{a year lagged of total aggregation of white canned tuna in pounds per capita}
\]

\[
SKQ = \beta_0 + \beta_1 SKIDQW + \beta_2 WHDW + \beta_3 RWSK + \beta_4 TLHI + \beta_5 AYO + \mu \tag{6.36}
\]

where

\[
SKQ = \text{quantity of skipjack tuna available for canners (U.S. landing plus import) in thousands of pounds per given year}
\]

\[
SKIDQW = \text{average annual price of imported skipjack tuna in dollars per pound deflated by wholesale price index for fish (1967=100)}
\]
\[ WHDW = \text{average wage rate paid by canners in dollars per hour deflated by wholesale price index for fish (1967=100) at a given year.} \]

\[ RWSK = \text{quantity of skipjack tuna landed by the rest of the world in thousands of pounds per given year} \]

\[ TLH1 = \text{a year lagged of total aggregation of light canned tuna in pounds per capita} \]

\[ AYQ = \text{the quantity aggregation of albacore and skipjack that is available for canners (U.S. landing plus import) in thousand pounds for a given year} \]

and

\[ YFO = \gamma_0 + \gamma_1 \text{SKQ} + \gamma_2 \text{ALQ} + \gamma_3 \text{RWYF} + \gamma_4 \text{WHDW} + \mu \quad (6.37) \]

where

\[ YFO = \text{quantity of yellowfin tuna available for canners (U.S. landing plus imports) in thousands of pounds per given year} \]

\[ SKQ = \text{quantity of skipjack tuna available for canners (U.S. landing plus imports) in thousands of pounds per given year} \]

\[ ALQ = \text{quantity of albacore tuna available for canners (U.S. landing plus imports) in thousands of pounds per given year} \]

\[ RWYF = \text{quantity of the rest of the world landing of yellowfin tuna in thousands of pounds per given year} \]

\[ WHDW = \text{average wage rate paid by canners in dollars per hour deflated by wholesale price index for fish (1967=100) per given year.} \]
Use of the Ordinary Least Square (OLS)

The empirical ex-vessel demand for albacore, skipjack and yellowfin is shown:

\[
ALQ = -0.2842 + 0.12403E + 0.07 ALIDQW + 0.32347 RWAL
\]
\[
- 9705.96 WHDW + 24233 TWH1
\]
\[
R^2 = 0.8102 \quad D-W = 2.0301
\]

** significant at the 1% level

The sum of VIF is 5.82, indicating a low degree of multicollinearity. RWAL and WHDW are insignificant at the 5 percent level. However, none of the explanatory variables have an unexpected sign.

\[
SKQ = 38193 - 66684 SKIDQW - 66092.8 WHDW
\]
\[
+ 0.38634 RWSK - 64643.3 TLH1 - 0.25456 AYQ
\]
\[
R^2 = 0.9072 \quad D-W = 1.9005
\]

** significant at the 1% level

Only the RWSK variable is significant at a 5 percent level even though the overall regression is significant. In addition to being insignificant, SKIDQW and TLH1 variables have an unexpected sign. The sum of VIF is 25.84, indicating moderate degree of multicollinearity.

The Durbin-Watson statistic (1.90) is in the indeterminate range at the 5 percent level, indicating the possibility that the apparent correlation of error is due to the autocorrelation of independent
variables and/or misspecification of the model. However, inspection of the pattern of residuals or inspection with the Glejster test demonstrates that in this case, it is possible that our models are misspecified, since — owing to the lack of complete data (as mentioned in Chapter IV) — important variables like frozen storage for raw tuna were not included in the model.

\[ YFQ = 53706 - 0.33595 \text{SKQ} + 0.60956 \text{ALQ} + 0.08188 \text{RWYF} \]
\[ \quad (2.6697) (-2.7835) (3.6297)** (1.1016) \]
\[ - 15530 \text{WHDW} \]
\[ (-2.0566) \]

\[ R^2 = 0.7267 \quad \text{D-W} = 1.9712 \]

** significant at the 1% level

The sum of VIF is 17.38, indicating a moderate degree of multicollinearity. SKQ and ALQ are significant at the 1 percent level and all coefficients have the expected sign. The Durbin-Watson statistic (1.9712) rejects the hypothesis that autocorrelation is present in the residuals.

Again, the principal components and ridge regression methods are employed. It would be interesting to see the result of principal components and ridge regression method response in the case of equation (6.39) where the Durbin-Watson statistic is in the "inconclusive" range and the degree of multicollinearity is moderate.

Principal Components Analysis

Three principal components methods, as mentioned earlier, are employed to evaluate equations (6.35, 6.36, and 6.37), respectively.
In Table 6.8, equation (6.35) is evaluated and the sum of VIF is 5.82. Use of the proposed estimate of mean square error criteria gives the same result as the OLS estimate. However, exclusion of principal components by t-criteria and deleting the smallest eigenvalue criteria increases its MSE(\(\hat{\beta}^*\)) by 74 percent and 4 percent, respectively, when compared with the MSE(\(\hat{\beta}^*\)) of OLS.

The sum of VIF in Table 6.9 (equation (6.36)) is 25.84. The proposed estimate of mean square error-criteria, elimination of principal components by t-criteria and of the smallest eigenvalue criteria decreases its MSE(\(\hat{\beta}^*\)) by 29 percent, 4 percent, and 21 percent, respectively, when compared with the MSE(\(\hat{\beta}^*\)) of OLS. Note that SKIDQW and TLHI variables, when estimated by OLS, demonstrate a negative sign, but switch to the positive sign, as expected, when the proposed estimate of mean square error and deleting of principal components by t methods are applied.

In Table 6.10, equation (6.37) is evaluated, and the sum of VIF is 17.38. The proposed estimate of mean square error is used, as is deletion of principal components by t and of the smallest eigenvalue methods. MSE(\(\hat{\beta}^*\)) is reduced by 68 percent, 43 percent, and 47 percent when compared with the MSE(\(\hat{\beta}^*\)) of OLS. Note that all coefficients have the expected sign.

Ridge Regressions

The ridge estimators, as mentioned earlier, are employed to evaluate equations (6.35, 6.36, and 6.37) respectively. The results are summarized as follows.
Table 6.8. Estimated Standardized Coefficients, Estimated Variances, and Estimated MSE of OLS, Ridge Estimators, Principal Components for Four Explanatory Variables, Ex-vessel Demand for Albacore Tuna Model

<table>
<thead>
<tr>
<th>OLS</th>
<th>K=0</th>
<th>K_LW</th>
<th>K_DSW</th>
<th>K_HKB</th>
<th>PC_MSE</th>
<th>PC_t</th>
<th>PC_sm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} ) ALIDQW</td>
<td>0.37964</td>
<td>0.30013</td>
<td>0.27458</td>
<td>0.27196</td>
<td>0.37964</td>
<td>0.13734</td>
<td>0.37955</td>
</tr>
<tr>
<td>( \hat{\beta} ) ALIDQW</td>
<td>0.06518</td>
<td>0.03707</td>
<td>0.03003</td>
<td>0.04270</td>
<td>0.13734</td>
<td>0.00187</td>
<td>0.06490</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) ALIDQW)</td>
<td>0.06518</td>
<td>0.04913</td>
<td>0.04420</td>
<td>0.04371</td>
<td>0.06518</td>
<td>0.10953</td>
<td>0.10079</td>
</tr>
<tr>
<td>( \hat{\beta} ) RWAL</td>
<td>0.32628</td>
<td>0.27997</td>
<td>0.26403</td>
<td>0.26234</td>
<td>0.03268</td>
<td>0.36746</td>
<td>0.32631</td>
</tr>
<tr>
<td>( \hat{\beta} ) RWAL</td>
<td>0.12166</td>
<td>0.03956</td>
<td>0.02886</td>
<td>0.03675</td>
<td>0.12166</td>
<td>0.01337</td>
<td>0.01530</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) RWAL)</td>
<td>0.12166</td>
<td>0.06809</td>
<td>0.05736</td>
<td>0.05637</td>
<td>0.12166</td>
<td>0.21431</td>
<td>0.08228</td>
</tr>
<tr>
<td>( \hat{\beta} ) WHDW</td>
<td>-0.04738</td>
<td>-0.08392</td>
<td>-0.08999</td>
<td>-0.09047</td>
<td>-0.04737</td>
<td>-0.52374</td>
<td>-0.04737</td>
</tr>
<tr>
<td>( \hat{\beta} ) WHDW</td>
<td>0.10131</td>
<td>0.03893</td>
<td>0.02944</td>
<td>0.03876</td>
<td>0.10131</td>
<td>0.01307</td>
<td>0.03785</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) WHDW)</td>
<td>0.10131</td>
<td>0.06158</td>
<td>0.05290</td>
<td>0.05208</td>
<td>0.10131</td>
<td>0.17700</td>
<td>0.09363</td>
</tr>
<tr>
<td>( \hat{\beta} ) TWH1</td>
<td>0.53414</td>
<td>0.42261</td>
<td>0.38711</td>
<td>0.38348</td>
<td>0.53414</td>
<td>0.26935</td>
<td>0.53413</td>
</tr>
<tr>
<td>( \hat{\beta} ) TWH1</td>
<td>0.07967</td>
<td>0.03826</td>
<td>0.03005</td>
<td>0.04090</td>
<td>0.07967</td>
<td>0.00718</td>
<td>0.06178</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) TWH1)</td>
<td>0.07967</td>
<td>0.05465</td>
<td>0.04816</td>
<td>0.04753</td>
<td>0.07967</td>
<td>0.13893</td>
<td>0.10570</td>
</tr>
<tr>
<td>MSE(( \alpha^* ))=MSE(( \hat{\beta}^* ))</td>
<td>0.36782</td>
<td>0.23344</td>
<td>0.20263</td>
<td>0.19968</td>
<td>0.36782</td>
<td>0.64047</td>
<td>0.38239</td>
</tr>
<tr>
<td>(0)</td>
<td>(-36.54)</td>
<td>(-44.91)</td>
<td>(-45.71)</td>
<td>(0)</td>
<td>(74.13)</td>
<td>(3.96)</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of MSE(\( \hat{\beta}^* \)) are in parentheses.
Table 6.9. Estimated Standardized Coefficients, Estimated Variances, and Estimated MSE of OLS, Ridge Estimators, Principal Components for Five Explanatory Variables, Ex-vessel Demand for Skipjack Tuna Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS K=0</th>
<th>Ridge Regression K _ LW</th>
<th>Ridge Regression K _ DSW</th>
<th>Ridge Regression K _ HBK</th>
<th>Principal Components PC _ MSE</th>
<th>Principal Components PC _ t</th>
<th>Principal Components PC _ sm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} ) SKIDQW</td>
<td>-0.06016</td>
<td>-0.02990</td>
<td>-0.02945</td>
<td>-0.03240</td>
<td>0.01234</td>
<td>-0.13612</td>
<td>0.02996</td>
</tr>
<tr>
<td>( \hat{\gamma} ) (( \hat{\beta} ) SKIDQW)</td>
<td>0.03082</td>
<td>0.01973</td>
<td>0.01887</td>
<td>0.02209</td>
<td>0.02540</td>
<td>0.01554</td>
<td>0.02835</td>
</tr>
<tr>
<td>M( \hat{\sigma}^2 ) (( \hat{\beta} ) SKIDQW)</td>
<td>0.03082</td>
<td>0.02406</td>
<td>0.02345</td>
<td>0.02567</td>
<td>0.04408</td>
<td>0.04356</td>
<td>0.03769</td>
</tr>
<tr>
<td>( \hat{\beta} ) WHDW</td>
<td>-0.13512</td>
<td>-0.22030</td>
<td>-0.22365</td>
<td>-0.20824</td>
<td>-0.32357</td>
<td>-0.19744</td>
<td>-0.20196</td>
</tr>
<tr>
<td>( \hat{\gamma} ) (( \hat{\beta} ) WHDW)</td>
<td>0.14663</td>
<td>0.03335</td>
<td>0.02902</td>
<td>0.04833</td>
<td>0.00465</td>
<td>0.00208</td>
<td>0.14826</td>
</tr>
<tr>
<td>M( \hat{\sigma}^2 ) (( \hat{\beta} ) WHDW)</td>
<td>0.14663</td>
<td>0.06909</td>
<td>0.06424</td>
<td>0.08368</td>
<td>0.09352</td>
<td>0.13539</td>
<td>0.18970</td>
</tr>
<tr>
<td>( \hat{\beta} ) RWSK</td>
<td>1.12124</td>
<td>0.59436</td>
<td>0.56647</td>
<td>0.68061</td>
<td>0.30806</td>
<td>1.03202</td>
<td>0.35852</td>
</tr>
<tr>
<td>( \hat{\gamma} ) (( \hat{\beta} ) RWSK)</td>
<td>0.20450</td>
<td>0.03188</td>
<td>0.02719</td>
<td>0.04921</td>
<td>0.00319</td>
<td>0.00210</td>
<td>0.02739</td>
</tr>
<tr>
<td>M( \hat{\sigma}^2 ) (( \hat{\beta} ) RWSK)</td>
<td>0.20450</td>
<td>0.07940</td>
<td>0.07302</td>
<td>0.09946</td>
<td>0.12713</td>
<td>0.18803</td>
<td>0.08936</td>
</tr>
<tr>
<td>( \hat{\beta} ) TLHI</td>
<td>-0.21677</td>
<td>0.11851</td>
<td>0.13263</td>
<td>0.07100</td>
<td>0.34761</td>
<td>-0.37585</td>
<td>0.41675</td>
</tr>
<tr>
<td>( \hat{\gamma} ) (( \hat{\beta} ) TLHI)</td>
<td>0.17421</td>
<td>0.03113</td>
<td>0.02690</td>
<td>0.04641</td>
<td>0.00660</td>
<td>0.00291</td>
<td>0.05204</td>
</tr>
<tr>
<td>M( \hat{\sigma}^2 ) (( \hat{\beta} ) TLHI)</td>
<td>0.17421</td>
<td>0.07164</td>
<td>0.06618</td>
<td>0.08860</td>
<td>0.11218</td>
<td>0.16130</td>
<td>0.19484</td>
</tr>
<tr>
<td>( \hat{\beta} ) AYQ</td>
<td>-0.19466</td>
<td>-0.06485</td>
<td>-0.05611</td>
<td>-0.08997</td>
<td>-0.05328</td>
<td>0.04137</td>
<td>-0.04510</td>
</tr>
<tr>
<td>( \hat{\gamma} ) (( \hat{\beta} ) AYQ)</td>
<td>0.04263</td>
<td>0.02419</td>
<td>0.02281</td>
<td>0.02801</td>
<td>0.03519</td>
<td>0.00492</td>
<td>0.03582</td>
</tr>
<tr>
<td>M( \hat{\sigma}^2 ) (( \hat{\beta} ) AYQ)</td>
<td>0.04263</td>
<td>0.03129</td>
<td>0.03030</td>
<td>0.03395</td>
<td>0.06103</td>
<td>0.43690</td>
<td>0.04874</td>
</tr>
<tr>
<td>M( \hat{\sigma}^2 ) (( \hat{\beta} ) AYQ)</td>
<td>0.59880</td>
<td>0.27548</td>
<td>0.25718</td>
<td>0.33138</td>
<td>0.43794</td>
<td>0.57197</td>
<td>0.47033</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of MSE (\( \hat{\beta}^* \)) are in parentheses.
Table 6.10. Estimated Standardized Coefficients, Estimated Variances, and Estimated MSE of OLS, Ridge Estimators, Principal Components for Four Explanatory Variables, Ex-vessel Demand for Yellowfin Tuna Model

<table>
<thead>
<tr>
<th></th>
<th>OLS K=0</th>
<th>Ridge Regression K_LW</th>
<th>Ridge Regression K_DSW</th>
<th>Ridge Regression K_HKB</th>
<th>Principal Components PC_MSE</th>
<th>Principal Components PC_t</th>
<th>Principal Components PC_sm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} ) SKQ</td>
<td>-0.79141</td>
<td>-0.09157</td>
<td>-0.00399</td>
<td>-0.55804</td>
<td>-0.17488</td>
<td>-0.73992</td>
<td>-0.76277</td>
</tr>
<tr>
<td>V(( \hat{\beta} ) SKQ)</td>
<td>0.37723</td>
<td>0.03108</td>
<td>0.01316</td>
<td>0.22030</td>
<td>0.00982</td>
<td>0.00124</td>
<td>0.37005</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) SKQ)</td>
<td>0.37723</td>
<td>0.10222</td>
<td>0.06091</td>
<td>0.28790</td>
<td>0.09538</td>
<td>0.21547</td>
<td>0.41284</td>
</tr>
<tr>
<td>( \hat{\beta} ) ALQ</td>
<td>0.59788</td>
<td>0.35211</td>
<td>0.26343</td>
<td>0.5463</td>
<td>0.46308</td>
<td>0.29027</td>
<td>0.63896</td>
</tr>
<tr>
<td>V(( \hat{\beta} ) ALQ)</td>
<td>0.12661</td>
<td>0.04033</td>
<td>0.02137</td>
<td>0.19965</td>
<td>0.09916</td>
<td>0.00326</td>
<td>0.11183</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) ALQ)</td>
<td>0.12661</td>
<td>0.06780</td>
<td>0.04849</td>
<td>0.11211</td>
<td>0.12788</td>
<td>0.07516</td>
<td>0.12619</td>
</tr>
<tr>
<td>( \hat{\beta} ) RWYF</td>
<td>0.36022</td>
<td>0.18029</td>
<td>0.14123</td>
<td>0.31839</td>
<td>0.06336</td>
<td>0.69593</td>
<td>0.56660</td>
</tr>
<tr>
<td>V(( \hat{\beta} ) RWYF)</td>
<td>0.49892</td>
<td>0.02829</td>
<td>0.01273</td>
<td>0.23247</td>
<td>0.02209</td>
<td>0.00179</td>
<td>0.12586</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) RWYF)</td>
<td>0.49892</td>
<td>0.10317</td>
<td>0.06075</td>
<td>0.33839</td>
<td>0.13526</td>
<td>0.28533</td>
<td>0.18246</td>
</tr>
<tr>
<td>( \hat{\beta} ) WHDW</td>
<td>-0.73693</td>
<td>-0.26745</td>
<td>-0.19845</td>
<td>-0.5654</td>
<td>-0.14943</td>
<td>-0.54081</td>
<td>-0.48853</td>
</tr>
<tr>
<td>V(( \hat{\beta} ) WHDW)</td>
<td>0.59918</td>
<td>0.02537</td>
<td>0.01108</td>
<td>0.25919</td>
<td>0.01195</td>
<td>0.00220</td>
<td>0.05906</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) WHDW)</td>
<td>0.59918</td>
<td>0.10968</td>
<td>0.06287</td>
<td>0.39266</td>
<td>0.14785</td>
<td>0.34247</td>
<td>0.12703</td>
</tr>
<tr>
<td>MSE(( \hat{\alpha} ))=MSE(( \hat{\beta} )*)</td>
<td>1.60193</td>
<td>0.38287</td>
<td>0.23303</td>
<td>1.13106</td>
<td>0.50637</td>
<td>0.91844</td>
<td>0.84853</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of MSE(\( \hat{\beta} \)*) are in parentheses.
In Table 6.8, where equation (6.35) is evaluated, the sum of VIF is 5.82. $K_{LW}$, $K_{DSW}$ and $K_{HBK}$ estimators reduce its $MSE(\hat{\beta}^*)$ by 37 percent, 45 percent, and 46 percent respectively, when compared with the $MSE(\hat{\beta}^*)$ of the OLS. All the coefficients also have the expected sign.

In Table 6.9 (equation (6.36)) the sum of VIF is 25.84. $K_{LW}$, $K_{DSW}$ and $K_{HBK}$ estimators decrease its $MSE(\hat{\beta}^*)$ by 54 percent, 57 percent, and 45 percent, respectively, when compared with $MSE(\hat{\beta}^*)$ of OLS. Note that the TLHI variable, when estimated by OLS, demonstrates a negative sign, but it switches to the positive sign as we expected when the $K_{LW}$, $K_{DSW}$, and $K_{HBK}$ estimators were applied. However, SKIDQW variables still have the unexpected sign.

The sum of VIF is 17.38 in Table 6.10, where equation (6.37) is evaluated. $K_{LW}$, $K_{DSW}$ and $K_{HBK}$ estimators reduce its $MSE(\hat{\beta}^*)$ by 76 percent, 85 percent, and 29 percent, respectively, when compared with the $MSE(\hat{\beta}^*)$ of OLS. All coefficients have the expected signs.

**Import Demand**

Raw tuna imports represent approximately 60 percent of the total U.S. tuna supply. Japan is the major exporting nation, exporting one-third of its catch to the U.S., and the Republic of Korea, Taiwan, Spain, Portugal, Norway, France, Peru, and Canada provide the remaining tuna imported by the U.S.

The residual nature of import demand must be considered in determining quantity and price, since the U.S. will purchase tuna only to the extent that its demand exceeds its own suppliers.
Accordingly, the theoretical framework mentioned in Chapter III is outlined in this analysis. Import demand for albacore, skipjack, and yellowfin is specified as follows:

\[ ALIQ = \alpha_0 + \alpha_1 ALIDQW + \alpha_2 RWAL + \alpha_3 TSYB + \alpha_4 TWH1 + \mu \] (6.41)

where

\[ ALIQ = \text{quantity of imported albacore tuna in thousands of pounds for a given year} \]
\[ ALIDQW = \text{average annual price of imported albacore tuna in dollars per pound deflated by wholesale price index for fish (1967=100)} \]
\[ RWAL = \text{quantity of albacore tuna landed by the rest of the world in thousands of pounds for a given year} \]
\[ TSYB = \text{the aggregate quantity of skipjack, yellowfin and bluefin landed in the U.S. for a given year} \]
\[ TWH1 = \text{a year lagged of total aggregation of white canned tuna in pounds per capita.} \]

\[ SKIQ = \beta_0 + \beta_1 SKUSA + \beta_2 RWSK + \beta_3 TYBA + \beta_4 TLH1 + \mu \] (6.42)

where

\[ SKIQ = \text{quantity of imported skipjack tuna in thousands of pounds for a given year} \]
\[ SKUSA = \text{quantity of skipjack tuna landed in U.S. in thousands of pounds per given year} \]
\[ RWSK = \text{quantity of skipjack tuna landed by the rest of the world in thousands of pounds per given year} \]
\[ TYBA = \text{quantity aggregation of yellowfin, bluefin and albacore in thousands of pounds for a given year} \]
\( TLH_1 = \) a year lagged of total aggregation of light canned tuna in pounds per capita;

and

\[
YFIQ = \gamma_0 + \gamma_1 YFIDQW + \gamma_2 SKIQ + \gamma_3 TSB + \gamma_4 TLH_1 + \mu \quad (6.43)
\]

where

\( YFIQ \) = quantity of imported yellowfin tuna in thousands of pounds per given year

\( YFIDQW \) = average annual price of imported yellowfin tuna in dollars per pound deflated by wholesale price index for fish (1967=100)

\( SKIQ \) = quantity of imported skipjack tuna in thousands of pounds per given year

\( TSB \) = quantity aggregation of skipjack and bluefin in thousands of pounds per given year

\( TLH_1 \) = a year lagged of total aggregation of light canned tuna in pounds per capita.

Use of Ordinary Least Square (OLS)

The empirical import demand equations for albacore, skipjack and yellowfin equation are shown as follows:

\[
ALIQ = -46276 + 0.12180 + 0.7 ALIDQW + 0.33219 RWAL
\]

\[ (-4.7706) \quad (3.4332)** \quad (2.6302)** \]

\[ + 0.33854 TSYB + 24711 TWH_1 \]

\[ (1.8771) \quad (4.4993)** \]

\[ R^2 = 0.8437 \quad D-W = 1.9383 \quad (6.44) \]

** significant at the 1% level
All coefficients except the TSYB variable are significant at the 1 percent level and in addition to being significant, all variables have the expected sign. The Durbin-Watson statistic \( (D-W = 1.94) \) suggests that the hypothesis can be rejected at the 5 percent level and that autocorrelation is present in the residuals. The sum of VIF is 4.99, indicating the low degree of multicollinearity.

\[
\text{SKIQ} = 55382.2 - 0.56461 \text{SKUSA} + 0.34492 \text{RWSK} \\
+ (-0.46846) \text{TYBA} + 795.019 \text{TLH1} \\
R^2 = 0.9353 \quad D-W = 1.9697
\]

** significant at the 1% level

The sum of VIF is 14.41, indicating the moderate degree of multicollinearity. None of the coefficients except RWSK is significant at the 5 percent level and the Durbin-Watson (1.9697) rejects autocorrelation in the residuals.

\[
\text{YFIQ} = 98083.2 + 42914 \text{YFIDQW} + 0.17653 \text{SKIQ} + 0.26626 \text{TSB} \\
+ (-14606.6) \text{TLH1} \\
R^2 = 0.6026 \quad D-W = 2.5495
\]

** significant at the 1% level

Only SKIQ and TSB variables are significant at the 1 percent level. However, all of the variables except the TLH1 variable have the expected sign. The Durbin-Watson statistic \( (2.55) \) exhibits negative first order autocorrelation in error terms.
Though the disturbances are autoregressive, the least squares estimators of the regression coefficients are still unbiased and consistent, but no longer efficient (Pindyck and Rubinfeld, 1980). The statistical tests are not valid in this case, since the variances of the estimates are biasedly estimated. There are several methods available for coping with the autocorrelation problem: the Cochrane-Graitt (1949), the Hildreth-Lu (1960), the Durbin (1960), and the Beach-MacKinnon (1978) procedures.

Moreover, in an article dealing with positive correlation, Kramer (1980, p. 1005) states:

"In the standard linear regression model $Y = \lambda + \rho u$, with error following a first-order stationary autoregressive process, it shows that the relative efficiency of the ordinary least square (OLS) as compared with Gauss-Markov estimator defends to a great extent of $\lambda$ matrix observation ..."

In other words, the relative efficiency of the OLS as compared with the prominent methods such as Prais-Winston (PW) and Durbin (D) estimators which approximate Gauss-Markov estimator by estimating $\rho$ in a first stage, can be reviewed as follows:

Where rho ($\rho$) approaches 1 and a constant dummy variable as the first column of $X$ is present, the OLS shows little inferiority to PW and D methods, though in cases where $\rho$ approaches 1 and a constant dummy variable as the first column of $X$ is deleted, the PW and D methods outperform OLS by far. In short, it means that where no constant dummy variables are present, OLS should definitely be avoided as soon as one suspects correlation among the error terms.

However, in this case, we are faced with the problem of negative serial correlation. We selected the Beach-MacKinnon (BM)
(Maximum Likelihood-Iterative Technique) procedure to solve this problem because it treats the first observation in a special way. Rather than dropping it, the stationarity requirement is imposed on the estimate of the serial correlation parameter by requiring \( p \) to be between -1.0 and 1.0. However, the BM has some disadvantage in cases where the regression is weighted or where there are gaps in the sample.

As noted, three empirical equations of (6.43) have been estimated by OLS and BM methods. First, when no constant dummy variable is present, the OLS is used.

\[
\begin{align*}
YFIQ &= 77203 \times YFIDQW + 0.17014 \times KSIQ - 0.21595 \times TSB \\
&\quad \text{[standard errors]} (7.0941) \quad (2.7357) \quad (-2.0340) \\
&\quad + 12103.4 \times TLH1 \\
&\quad \text{[standard error]} (0.8749) \\
D-W &= 2.2678
\end{align*}
\]

** significant at the 1% level

All coefficients except the TLH1 variable have the expected sign. Again, the Durbin-Watson statistic (2.27) indicates negative first-order autocorrelation in error terms.

The Beach-MacKinnon (BM) Procedure

The BM procedure was used to correct negative serial correlation where a constant dummy variable was present.

\[
\begin{align*}
YFIQ &= 13354 + 25008 \times YFIDQW + 0.16557 \times KSIQ - 0.24110 \times TSB \\
&\quad \text{[standard errors]} (2.201) \quad (1.0951) \quad (2.9507) \quad (-2.71) \\
&\quad - 21552.1 \times TLH1 \\
&\quad \text{[standard error]} (-0.9021) \\
R^2 &= 0.7986 \\
D-W &= 2.0444
\end{align*}
\]

** significant at the 1% level
All variables except TLH1 have the expected sign. However, only SKIQ and TSB are significant at the 5 percent level. The Durbin-Watson statistic (2.04) demonstrates that there is no longer negative serial correlation present in the error terms.

The BM procedure was employed to correct negative serial correlation where no constant dummy variable is present.

\[
YFIQ = 73977 \ FIDQW + 0.14166 \ SKIQ - 0.22536 \ TSB \\
(7.4277) \quad (2.3590)^* \quad (2.1909)^*
\]

\[
+ 18769.3 \ TLH1 \\
(1.3875)
\]

(6.49)

\[ R^2 = 0.9902 \quad D-W = 1.9884 \]

* significant at the 5% level

All coefficients except the TLHI variable are significant at the 5 percent level and all of the variables have the expected sign. The Durbin-Watson statistic (1.99) indicated the absence of autocorrelation in the error terms.

A comparison of equations (6.46, 6.47, 6.48, and 6.49) demonstrates that equation (6.49) employs the BM procedure where no constant dummy variable is present. Two main goals are thus accomplished: 1) correction of the problem of negative serial correlation; 2) switching of the TLH1 variable to the positive sign as expected.

**Principal Components Analysis**

Now equations (6.41) and (6.42) are used to evaluate biased and unbiased estimations. Three principal components and three
ridge regression estimators (biased) are employed to compare with the OLS estimator (unbiased). Results of the analysis are summarized in Tables 6.11 and 6.12. In Table 6.11, the smallest eigenvalue criteria and the principal component were deleted by t criteria, increasing its MSE($\hat{\beta}^*$) by 34 percent and 156 percent instead of decreasing its MSE($\hat{\beta}^*$) when compared with the MSE($\hat{\beta}^*$) of OLS. With use of the proposed loss-function, related criteria provided the same result as the OLS estimate. $K_{LW}$, $K_{DSW}$, and $K_{HBK}$ estimators decrease the MSE($\hat{\beta}^*$) by 38 percent, 33 percent, and 34 percent when compared with the MSE($\hat{\beta}^*$) of OLS.

In Table 6.12, elimination of the smallest eigenvalue criteria and of the principal component by t-criteria increases its MSE($\hat{\beta}^*$) by 1 percent and 129 percent instead of decreasing its MSE($\hat{\beta}^*$) when compared with the MSE($\hat{\beta}^*$) of OLS.

The proposed loss-function related criteria offers the same result with the OLS estimate.

**Ridge Regressions**

$K_{LW}$, $K_{DSW}$, and $K_{HBK}$ estimators method decreases its MSE($\hat{\beta}^*$) by 41 percent, 43 percent, and 39 percent when compared with the MSE($\hat{\beta}^*$) of the OLS.
Table 6.11. Estimated Standardized Coefficients, Estimated Variances, and Estimated MSE of OLS, Ridge Estimators, Principal Components for Four Explanatory Variables, Import Demand for Albacore Tuna Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS $\hat{\beta}$</th>
<th>Ridge Regression $\hat{\beta}$</th>
<th>Principal Components $\text{PC}_{MSE}$</th>
<th>$\text{PC}_{t}$</th>
<th>$\text{PC}_{c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K=0$</td>
<td>$K_{K}$</td>
<td>$K_{DSW}$</td>
<td>$K_{HKB}$</td>
<td>$K_{PC}$</td>
</tr>
<tr>
<td>$\hat{\beta}$ ALIDQW</td>
<td>0.36325</td>
<td>0.30373</td>
<td>0.28695</td>
<td>0.28270</td>
<td>0.37326</td>
</tr>
<tr>
<td>$\hat{\nu}(\hat{\beta}$ ALIDQW)</td>
<td>0.55160</td>
<td>0.03402</td>
<td>0.02978</td>
<td>0.02876</td>
<td>0.05516</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\beta}$ ALIDQW)</td>
<td>0.05516</td>
<td>0.04323</td>
<td>0.04041</td>
<td>0.03970</td>
<td>0.05516</td>
</tr>
<tr>
<td>$\hat{\beta}$ RNAL</td>
<td>0.33548</td>
<td>0.32631</td>
<td>0.31877</td>
<td>0.31658</td>
<td>0.33598</td>
</tr>
<tr>
<td>$\hat{\nu}(\hat{\beta}$ RNAL)</td>
<td>0.07592</td>
<td>0.03659</td>
<td>0.03072</td>
<td>0.02939</td>
<td>0.07592</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\beta}$ RNAL)</td>
<td>0.07592</td>
<td>0.05220</td>
<td>0.01478</td>
<td>0.04649</td>
<td>0.07592</td>
</tr>
<tr>
<td>$\hat{\beta}$ TSYB</td>
<td>0.21369</td>
<td>0.16103</td>
<td>0.14996</td>
<td>0.14724</td>
<td>0.21390</td>
</tr>
<tr>
<td>$\hat{\nu}(\hat{\beta}$ TSYB)</td>
<td>0.06047</td>
<td>0.03448</td>
<td>0.02986</td>
<td>0.02877</td>
<td>0.06047</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\beta}$ TSYB)</td>
<td>0.06047</td>
<td>0.04534</td>
<td>0.04207</td>
<td>0.04125</td>
<td>0.06047</td>
</tr>
<tr>
<td>$\hat{\beta}$ TWH1</td>
<td>0.54535</td>
<td>0.44305</td>
<td>0.41920</td>
<td>0.41319</td>
<td>0.5453</td>
</tr>
<tr>
<td>$\hat{\nu}(\hat{\beta}$ TWH1)</td>
<td>0.06856</td>
<td>0.03572</td>
<td>0.03042</td>
<td>0.02920</td>
<td>0.6856</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\beta}$ TWH1)</td>
<td>0.06856</td>
<td>0.04905</td>
<td>0.04508</td>
<td>0.04410</td>
<td>0.0685</td>
</tr>
<tr>
<td>$\text{MSE}(\hat{\alpha})$</td>
<td>0.26011</td>
<td>0.18982</td>
<td>0.17513</td>
<td>0.17153</td>
<td>0.26011</td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of $\text{MSE}(\hat{\beta})$ are in parentheses.
Table 6.12. Estimated Standardized Coefficients, Estimated Variances, and Estimated MSE of OLS, Ridge Estimators, Principal Components for Four Explanatory Variables, Import Demand for Skipjack Tuna Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS</th>
<th>KLW</th>
<th>DSW</th>
<th>HKB</th>
<th>PC_MSE</th>
<th>PC_t</th>
<th>PC_sm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} ) SKUSQ</td>
<td>-0.15349</td>
<td>-0.09451</td>
<td>-0.09148</td>
<td>-0.09706</td>
<td>-0.15349</td>
<td>0.03865</td>
<td>-0.06436</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) SKUSQ)</td>
<td>0.03495</td>
<td>0.02334</td>
<td>0.02262</td>
<td>0.02394</td>
<td>0.03495</td>
<td>0.00124</td>
<td>0.03041</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) SKUSQ)</td>
<td>0.03495</td>
<td>0.02821</td>
<td>0.02773</td>
<td>0.02860</td>
<td>0.34948</td>
<td>0.08017</td>
<td>0.05672</td>
</tr>
<tr>
<td>( \hat{\beta} ) RWSK</td>
<td>0.98535</td>
<td>0.70446</td>
<td>0.68982</td>
<td>0.71671</td>
<td>0.98535</td>
<td>0.47462</td>
<td>0.99478</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) RWSK)</td>
<td>0.12215</td>
<td>0.03466</td>
<td>0.03186</td>
<td>0.04713</td>
<td>0.12215</td>
<td>0.00326</td>
<td>0.00626</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) RWSK)</td>
<td>0.12215</td>
<td>0.06397</td>
<td>0.06113</td>
<td>0.06636</td>
<td>0.12215</td>
<td>0.27914</td>
<td>0.09821</td>
</tr>
<tr>
<td>( \hat{\beta} ) TYBA</td>
<td>-0.14349</td>
<td>-0.06637</td>
<td>-0.06197</td>
<td>-0.07002</td>
<td>-0.14349</td>
<td>0.06833</td>
<td>-0.04607</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) TYBA)</td>
<td>0.03818</td>
<td>0.02471</td>
<td>0.02389</td>
<td>0.02540</td>
<td>0.03818</td>
<td>0.00199</td>
<td>0.03258</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) TYBA)</td>
<td>0.03818</td>
<td>0.03032</td>
<td>0.02976</td>
<td>0.03077</td>
<td>0.03818</td>
<td>0.08822</td>
<td>0.06132</td>
</tr>
<tr>
<td>( \hat{\beta} ) TLH1</td>
<td>0.00262</td>
<td>0.22320</td>
<td>0.23262</td>
<td>0.21507</td>
<td>0.14349</td>
<td>0.04709</td>
<td>-0.02833</td>
</tr>
<tr>
<td>( \hat{V}(\hat{\beta} ) TLH1)</td>
<td>0.11532</td>
<td>0.03391</td>
<td>0.03126</td>
<td>0.03623</td>
<td>0.11532</td>
<td>0.00320</td>
<td>0.00857</td>
</tr>
<tr>
<td>MSE(( \hat{\beta} ) TLH1)</td>
<td>0.11532</td>
<td>0.06128</td>
<td>0.05863</td>
<td>0.06351</td>
<td>0.11532</td>
<td>0.26365</td>
<td>0.09538</td>
</tr>
<tr>
<td>MSE(( \hat{\alpha} )<em>) = MSE(( \hat{\beta} )</em>)</td>
<td>0.31060</td>
<td>0.18378</td>
<td>0.17726</td>
<td>0.18925</td>
<td>0.31060</td>
<td>0.71118</td>
<td>0.31163</td>
</tr>
<tr>
<td>(0)</td>
<td>(-40.83)</td>
<td>(-42.93)</td>
<td>(-39.07)</td>
<td>(0)</td>
<td>(128.97)</td>
<td>(0.33)</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Percent gain (decreasing) or loss (increasing) of MSE(\( \hat{\beta} \)*) are in parentheses.
CHAPTER 7

Summary and Conclusions

The tuna industry was chosen for this study because tuna data have useful characteristics for experimenting with biased and unbiased estimation methods, i.e., multicollinearity exists in the explanatory variables in the model and there is some prior information about the true regression coefficients. Moreover, it is an important product for consumers and producers in the U.S., Japan and Western Europe. These countries consume about 90 percent of world tuna products. The U.S. is the largest consumer of tuna products, accounting for almost 50 percent of the world consumption.

The conclusions are divided into two parts. The first summarizes price and quantity relationships at various levels in the U.S. tuna industry. The second summarizes the biased and unbiased estimation methods which are the main objective of the study. New material like the summary of price and quantity relationships in this chapter is not conventionally introduced in the conclusion. However, though these relationships can be interpreted from Chapter VI, they are reported here to serve a secondary objective — learning something about tuna.

Summary and Conclusion About Price and Quantity Relationships in the Tuna Industry

The Retail Demand

Data for 1953 to 1979 indicate that retail demand or demand for canned tuna at the consumption level was price inelastic (-0.53),
thus during that period U.S. retailers would experience an increase in canned tuna sale revenues if the average price of canned tuna at the retail level was increased, all things remaining constant. The cross price elasticity was positive between canned tuna and meat, and equal to 0.63. Therefore, canned tuna and meat were substituted for each other by U.S. consumers and a one percent rise (fall) in the retail price of meat away from its mean value for the 1953-1979 period led to a 0.63 percent rise (fall) in the quantity demand of canned tuna. In short — other things being constant — the demand for canned tuna tended to increase as the retail price of meat rose. The positive relationship between the quantity of tuna bought and time (0.70) suggested that the quantity of canned tuna at retail level would increase at about 0.70 percent per year over the year. The interrelationship between demand, price and income revealed that, as long as the consumers' real income remained at $8187 or less, and the retail price of canned tuna did not fall below zero, canned tuna was consumed as a normal good. However, once these conditions were violated, canned tuna would be classified as an inferior good.

The Wholesale Supply and Demand

The wholesale price of solid white and chunk light canned tuna, set by canners was positively related to the import price of canned tuna, the ex-vessel price, and the import price of raw tuna, according to data from 1953 to 1979. Thus, these prices had a significant influence on the wholesale price of U.S. canned tuna. The supply price appeared to be correlated negatively with the quantity available for the canners' sales, implying the increases in availability
of canned tuna for the domestic market would reduce the price set by the major canners.

The demand for chunk light tuna at the domestic wholesale level was relatively price elastic (-1.54), implying that -- other things being constant -- a one percent fall (rise) in the wholesale price would result in a 1.5 percent rise (fall) in the quantity sold to the retailer and would result correspondingly in a rise (fall) in sale revenues for the wholesale supplier. However, the demand for solid white canned tuna at the domestic wholesale level was relatively price inelastic (0.22), indicating that wholesale suppliers would experience a rise in sale revenues if they increased the average price of solid white canned tuna per standard case.

The Ex-vessel Demand

Data for 1960 to 1979 indicated that the ex-vessel quantity demand for albacore and demand for skipjack tuna by canners at the ex-vessel level were positively related to their import prices, the amount of albacore and skipjack landing in the rest of the world, and the amount of solid and light canned tuna production from the previous year. These variables had a significant influence on the quantity demand for albacore and skipjack at the ex-vessel level. The coefficients of the variables representing the ex-vessel quantity of albacore and skipjack had a negative relationship with the wage rate paid to cannery workers, with the expected sign. The cross price elasticities of albacore and skipjack were equal to 1.08 and 0.12 with respect to changes in the import price of albacore and skipjack.
The ex-vessel quantity demand for yellowfin was positively related to the amount of albacore available to the canners and the amount of yellowfin landed in the rest of the world. However, it was negatively related to the amount of skipjack available to canners and wage rates paid by the canners to their workers.

The Import Demand

The import quantity demand for albacore was positively related to its import price, the amount of the rest of the world's landing of albacore, the amount of skipjack yellowfin, and bluefin available to canners, and the amount of solid canned tuna production from the previous year (data for 1960-1979). However, import demand for skipjack was positively related to the amount of skipjack landed in the rest of the world and the amount of light canned tuna production from the previous year, but was negatively related to the amount of skipjack landed in the U.S. (domestic) and the amount of yellowfin, bluefin and albacore caught by the U.S. The yellowfin import demand was positively related to its import price, the amount of skipjack imported to the U.S., and the amount of light canned tuna production from the previous year, but was negatively related to the amount of skipjack and bluefin caught by the U.S. The import price elasticities of albacore and yellowfin were equal to 1.32 and .82, respectively.

Summary and Conclusions About Unbiased and Biased Estimation in the Tuna Industry

Data from 1960 and 1979 indicate that when the degree of multicollinearity was low to moderately low, only ridge regression
achieved improvement in estimating the mean square error, ranging from 25 percent to 46 percent from one equation to the other when compared with the MSE (\(\hat{\beta}^*\)) of the OLS. In this particular case, it was undesirable to choose which ridge estimators would be preferable, because each was not much different in precision percent gain MSE(\(\hat{\beta}^*\)). Of the principal components methods, the MSE criteria (PC MSE) gave the same result as the OLS estimate, though the other two principal components methods (t-criteria and smallest eigenvalue criteria) increased the MSE(\(\hat{\beta}^*\)) instead of reducing it, ranging from one percent to 156 percent from one equation to the other, and even obtained the wrong (unexpected) sign for the variable in the model.

When the degree of multicollinearity was moderate, the data from 1953 to 1977 indicated that both principal components and ridge regression achieved improvement in estimating the mean square error. The ridge estimators and principal components methods — when they were applied — reduced its MSE(\(\hat{\beta}^*\)) range from 30 percent to 85 percent and 15 percent to 69 percent, respectively, from one equation to the other when compared with the MSE(\(\hat{\beta}^*\)) of the OLS. Consequently, the \(K_{DSW}\) and \(K_{LW}\) estimators produced more accurate and better results of estimation than other biased methods. However, the \(K_{HBK}\) estimator showed an inconsistent result from estimation of the MSE(\(\hat{\beta}^*\)). Of the principal components, the MSE criteria achieved improvement in estimated mean square error when compared with the other two principal components methods. However, when MSE criteria was applied, the unexpected sign of the variable was obtained in some equations. In this particular case, the ridge estimator (\(K_{LW}\) and \(K_{DSW}\)) appeared to
offer a viable alternative to the full model estimated by the OLS and principal components.

Again based on tuna data for 1953 to 1979, both principal components methods and ridge regression achieved remarkable improvement in estimating the mean square error when the degree of multicollinearity was high. The ridge estimators and principal components method, when they were applied, reduced its $\text{MSE}(\hat{\beta}^*)$ range from 91 percent to 97 percent, and 77 percent to 96 percent, from one equation to the other when compared with the $\text{MSE}(\hat{\beta}^*)$ of the OLS. In this particular case, ridge regression had produced slightly more accurate and reasonable results than principal components methods. Of the principal components methods, the smallest eigenvalue criteria in one of the equations obtained the unexpected sign for the variable in the model.

What the ridge regression and principal components (especially t-criteria and MSE-criteria) did in this particular case was quite remarkable; low estimate MSE, variance was reduced to a fraction of OLS, and all coefficients had the (expected) right sign. Even though the estimates obtained by ridge regression and principal components methods (t-criteria and MSE-criteria) were biased, they appeared to have desirable characteristics, were stable, had the correct sign, and lacked symptoms of nonsense regression when multicollinearity was severe. So, when the multicollinearity was high, ridge regression, t-criteria and MSE-criteria of the principal components appeared to offer viable alternatives to the full model estimated by OLS.

As discussed in Chapter V, the Durbin-Watson statistic ($D-W=1.90$) for imported skipjack tuna demand was in the indeterminate range at
the 5 percent level. From the test of the residual pattern and
Glejser test we concluded that the model was misspecified since an
important variable — cold storage for tuna — was not incorporated
into the model for lack of data. In this particular case, when we
employed ridge regression — even though ridge estimators achieved
remarkable improvement in estimating the mean square error — they
did not give the right sign for some of the variables in the model.
At the same time, the principal components methods were employed.
These principal components methods also improved the estimation of
the mean square error. The MSE criteria and smallest eigenvalue
criteria consistently gave the right sign for the variables in this
particular model.

In Chapter V, the imported demand for yellowfin (model) faced
negative serial correlation. The Beach and MacKinnon procedure (BM)
was employed to correct the negative serial correlation. The model
used in the analysis was both included and excluded from the constant
dummy variable as the first column of $X$ by OLS and ML. We found that
using BM with a nonconstant dummy variable solved the problem of nega-
tive serial correlation and at the same time obtained the right sign
for the variable in the model.

Also in Chapter V, the other unbiased estimate of exact and in-
exact prior information was introduced for the improvement of regres-
sion estimates in the presence of multicollinearity. From the analy-
sis it was found that the prior information on hand was inaccurate
and/or inconsistent. Therefore, if prior information was available
and appeared to be accurate and/or consistent, it would have to be
incorporated directly into the estimation. However, if prior information was inconsistent and/or inaccurate, the researcher would do better to incorporate imprecise information approximately through other methods such as ridge regression, rather than to insist on formulating prior information with imprecise information.
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APPENDIX 1

Theorems, Lemma and Assumptions of Biased and Unbiased Estimations Related in the Study
Ordinary Least Squares (OLS) 5

Let us assume that a linear relationship exists between a dependent variable $Y$ and $k-1$ explanatory variables $X_2, X_3, \ldots, X_k$. If we have a sample of $n$ observations on $Y$ and $X$'s, we can write

$$Y_i = \beta_1 + \beta_2 X_{2i} + \ldots + \beta_k X_{ki} + \mu_i$$  \hspace{1cm} (1)

where $i = 1, 2, \ldots, n$

In matrix form, we can write as

$$Y_n = X_n \beta_k + \mu_n$$  \hspace{1cm} (2)

A simple set of assumptions follows in which the ordinary least square (OLS) will give the best linear unbiased estimate (BLUE).

1) $E(\mu_{n1}) = 0$
2) $E(\mu_{n1}' \mu_{n1}) = \sigma^2 I_n$
3) $X_{nk}$ is a set of fixed numbers (nonstochastic variable)
4) $X_{nk}$ has rank $k < n \implies (X'X)$ is nonsingular.

Assumption (1) and (2) imply that the error term has zero expected mean value and constant variance (homoscedasticity) for all observations. The error term is normally distributed and uncorrelated. Assumption (3) means that in repeated sampling the only source of variation in the dependent variable is variation in the error term. Assumption (4) indicates that the number of observations exceeds the number of parameters to be estimated and no exact linear relationship exists between any of the $X$ variables. Since $(X'X)$ is a symmetric matrix of order $k$, this would mean that inverse $(X'X)$ does exist.

*This section is drawn heavily from Brown 1978, 1980; Pindyck and Rubinfeld, 1980; and Intriligator, 1978.*
Least Square Estimation

To find the "good" estimators of the regression parameter $B_{kl}$, the method of least square is employed. Least square estimation considers the deviation of $Y_{n1}$ from its expected value. This deviation is the set of residuals $e_{n1}$.

$$e_{n1} = Y_{n1} - X_{nk} \hat{B}_{kl}$$

(3)

Therefore, the sum of squared residuals is

$$\sum_{i=1}^{n} e_{i}^2 = e'e = \begin{pmatrix} Y - X\hat{\beta} \end{pmatrix}'(Y - X\hat{\beta})$$

(4)

$$= (Y'Y - 2\hat{\beta}'X'Y + \hat{\beta}'X'XY)$$

To find the value of $\hat{\beta}$ which minimized the square residuals, we differentiate (4)

$$\frac{\partial e'e}{\partial \hat{\beta}} = -2X'Y + 2X'X \hat{\beta} = 0$$

(5)

finally we get

$$\hat{\beta} = (X'X)^{-1} XY$$

since $(X'X)^{-1} (X'X)_{kk} = I_{kk}$

(6)

According to Assumption (1) and (3), one may find the true value of parameter $\beta$ is the average of value $\hat{\beta}$ that one would obtain over repeated samples.

That means

$$E(\hat{\beta}) = \beta$$

(7)

since

$$E(\hat{\beta}) = E((X'X)^{-1} X'Y)$$

$$= E((X'X)^{-1} X'(X\beta + \mu))$$

$$= (X'X)^{-1} X'X E(\beta) + (X'X)^{-1} X'E(\mu)$$
\[ \beta + (X'X)^{-1} X'E(\mu) \quad \text{since} \quad (X'X) = I \]

\[ = \beta \quad \text{since} \quad E(\mu) = 0 \quad \text{Assumption (1) and} \quad X \text{ is fixed by} \]

Assumption (3). Therefore, the least square estimators are unbiased.

The variance-covariance of \( \hat{\beta} = \text{var}(\hat{\beta}) \). Since \( E(\beta_i) = \beta_i \) for \( i = 1, 2, \ldots, k \). It follows that \( E[(\hat{\beta}_i) - E(\beta_i)]^2 \) is the variance of \( \beta_i \) and \( E[(\hat{\beta}_i) - E(\beta_i)] [\hat{\beta}_j - E(\beta_j)] \) is the covariance of \( \beta_i \) and \( \beta_j \).

Thus,

\[ \text{Var}(\hat{\beta}) = E[\hat{\beta} - E(\hat{\beta})] [\hat{\beta} - E(\hat{\beta})]' \quad (8) \]

for

\[ [\hat{\beta} - E(\hat{\beta})] = [(X'X)^{-1}X'Y - \beta] \quad \text{since} \quad \hat{\beta} = (X'X)^{-1}X'Y \quad \text{and} \]

\[ E(\hat{\beta}) = \beta \quad \text{for the previous proof}. \]

\[ = [(X'X)^{-1}X'(X\beta + \mu) - \beta] \]

\[ = \beta + (X'X)^{-1}X'\mu - \beta \quad \text{since} \quad (X'X)^{-1}X'X = I \quad (9) \]

\[ = (X'X)^{-1}X'\mu \]

Therefore

\[ \text{Var}(\hat{\beta}) = E[(X'X)^{-1}X'\mu] [(X'X)^{-1}X'\mu]' \quad \text{by substitute (9) in (8)} \]

\[ = (X'X)^{-1}X'E(\mu\mu')X(X'X)^{-1} \quad \text{since} \quad (ABC)' = C'B'A' \]

and

\[ [(X'X)^{-1}]' = (X'X)^{-1} \quad \text{if} \quad (X'X) \quad \text{is symmetric}. \]

\[ = (X'X)X'\sigma^2\mu X(X'X)^{-1} \quad \text{since} \quad E\mu\mu' = \sigma^2I \]

from Assumption (2)

\[ = \sigma^2 \quad (X'X)^{-1} \quad \text{since} \quad (X'X)^{-1}X'X = I \quad (10) \]

Consider a linear parametric function of \( e'\beta \) where \( c \) is \( k\times1 \) column vector of known constant. A possible estimator of \( c'\beta \) is \( c'\hat{\beta} \) where \( \hat{\beta} = (X'X)^{-1}X'Y \).

This estimator is unbiased for

\[ E(c'\hat{\beta}) = c' \quad \text{since} \quad c'\beta \quad \text{is constant and} \quad E(\hat{\beta}) = \beta. \quad (11) \]
And its variance is

\[ \text{Var}(c'\beta) = E \left[ (c'(\beta' - E(\beta)) \right]^2 \]

\[ = E \left[ c'(\hat{\beta} - \beta) \right] \left[ c'(\hat{\beta} - \beta) \right] \] since \( E(\hat{\beta}) = \beta \)

\[ = E \left[ c'(\beta - \beta) \right] \left[ (\beta - \beta)' \right] \] since \( (\beta \beta)' = \beta' A' \)

\[ = c' \left[ E (\hat{\beta} - \beta) (\hat{\beta} - \beta)' \right] c \]

\[ = c' X'(X'X)^{-1} c \] since \( \text{Var} \ \hat{\beta} = (\hat{\beta} - \beta) (\hat{\beta} - \beta)' \)

for the previous proof

\[ = \sigma^2 c'(X'X)^{-1} c \] (12)

Let \( b = a'Y \) be any other linear unbiased estimator of \( c'\beta \).

Therefore

\[ E(b) = E(a'X\beta + a'\mu) \] since \( Y = X\beta + \mu \)

\[ = a'X\beta \] since \( E(\mu) = 0 \)

\[ = c'\beta \] if and only if \( a'X = c' \) (13)

\[ \Rightarrow b = (c'\beta + a'\mu) \] (14)

Thus

\[ \text{Var}(b) = E \left[ (b - c'\beta)^2 \right] \]

\[ = E \left[ (b - E(b)) (b - E(b))' \right] \] since \( c'\beta = E(b) \) (15)

\[ (b - E(b)) = a'Y - E(a'Y) \]

\[ = a' (X\beta + \mu) - a' E(X\beta + \mu) \] since \( Y = X\beta + \mu \)

\[ = a'X\beta + a'\mu - a'X\beta + a'E\mu \]

\[ = a'\mu \] since \( E(\mu) = 0 \) (16)

Therefore

\[ \text{Var}(b) = E \left[ (a'\mu Xa'\mu)' \right] \] by substitute (16) in (15)

\[ = a'E\mu a \] since \( (AB)' = B'A' \)

\[ = a'\sigma^2 I a \] since \( E(\mu\mu') = \sigma^2 I \)

\[ = \sigma^2 a'a \] (17)
By the same manner, we can also prove that
\[ \text{Var}(c'b) = \sigma^2 c'a'ac \] by the earlier proof (18)

Therefore, in the general linear model as we assume that

1) \( Y = X\beta + \mu \)
2) \( E(\mu) = 0 \)
3) \( E(\mu\mu') = \sigma^2 I \)
4) \( X \) is set of fixed number
5) \( \rho(X)_{nk} = k \quad k<n \implies (X'X) \) is nonsingular and given that

\[ E(c'\hat{\beta}) = c'\hat{\beta} \quad \text{and} \quad \text{Var}(c'\hat{\beta}) = \sigma^2 c'(X'X)^{-1} c \]

then

\[ E(c'b) - E(c'\beta) \geq 0 \quad \text{for any} \quad c'\hat{\beta} \quad \text{is the BLUE of} \quad c'\beta. \]

Since

\[ E(c'b) - E(c'\hat{\beta}) = \text{tr} \left[ \text{Var}(c'b) - \text{Var}(c'\hat{\beta}) \right] \]
\[ = \text{tr} \left[ \sigma^2 c'a'ac - \sigma^2 c'(X'X)c \right] \quad \text{since} \]
\[ \text{Var}(c'\beta) = \sigma^2 c'a'ac \quad \text{and} \quad \text{Var}(c'\beta) = \sigma^2 c'(X'X)c \]
from the previous proof.

Therefore

\[ E(c'b) - E(c'\hat{\beta}) = \sigma^2 \text{tr} \left[ c'a'ac - c'(X'X)^{-1} c \right]. \]

As \( b \) is BLUE of \( \beta \implies E(b) = E(\beta) \)

\[ \implies E(c'b) = E(c'\beta) \quad \text{since} \quad E(c'b) = c'E(b) = c'\beta \]

but also

\[ E(c'b) = E(c'a'Y) \]
\[ = E(c'a' (X\beta + \mu)) \quad \text{since} \quad Y = X\beta + \mu \]
\[ = c'a'X\beta \quad \text{since} \quad X \text{ is fixed and} \quad E(\mu) = 0 \]

\[ \implies c'a'X\beta = c'\beta \]
let
\[ z' = c'a \quad \text{or} \quad z = ac \implies z'X\beta = c'\beta; \]
\[ z'X = c' \quad \text{pr} \ X'z = c. \]

Then
\[ E(c'b) - E(c'\beta) = \sigma^2 \text{tr}(z'z - z'X(X'X)^{-1} X'z) \]
by substitution
\[ = \sigma^2 \text{tr} [z' (I - X(X'X)^{-1} X) z] \]
\[ = \sigma^2 \text{tr} [z'Mz] \quad \text{since} \quad M = [I - X(X'X)^{-1} X'] \]
\[ = \sigma^2 \text{tr} [z'MMz] \quad \text{because} \ M \text{ is idempotent} \]
\[ \implies M = MM. \]
\[ = \sigma^2 \text{tr} [z'M'mz] \quad \text{because} \ M \text{ is symmetric} \]
\[ = \sigma^2 \text{tr} [(Mz)' (Mz)] \quad \text{because} \ (AB)' = B'A' \]
\[ = \sigma^2 \text{tr} [(Mz)' (Mz)] \geq 0 \quad \text{since} \ \text{tr}(A'A) = \text{tr} (AA') \]
\[ = \text{sum of all element square of} \ A. \quad (19) \]

Turning now to the residual sum of square as we know
\[ e = Y - X\hat{\beta} \]
\[ = [X\beta + \mu] - X[(X'X)^{-1} (X\beta + \mu)] \quad \text{since} \ Y = X\beta + \mu \]
and
\[ \hat{\beta} = (X'X)^{-1} X'Y. \]
\[ = X\beta + \mu - X\beta - X(X'X)^{-1} X'\mu \quad \text{since} \ (X'X)^{-1} X'X = I \]
\[ = [I - X(X'X)^{-1} X'] \mu \]
\[ = M\mu \quad \text{since} \quad M = (I - X(X'X)^{-1} X') \quad (20) \]

Therefore, the residual sum square
\[ e'e = (M\mu)' (M\mu) \]
\[ = \mu'M'M\mu \quad \text{since} \quad (AB)' = B'A' \]
\[ = \mu'M\mu \quad \text{because} \ M \text{ is symmetric and idempotent} \quad (21) \]
Taking expected value of both sides

\[ E(e'e) = E(\mu'M\mu) \]

\[ = \sigma^2 \text{tr} [I - X(X'X)^{-1}X'] \quad \text{since} \quad E(\mu\mu') = \sigma^2 I_n \]

and

\[ M = I - X(X'X)^{-1}X' \]

\[ = \sigma^2 [\text{tr} I - \text{tr} X(X'X)^{-1}X] \]

\[ = 2 [n - \text{tr} (X'X)^{-1} (X'X)] \quad \text{since} \quad I_n \text{ is the order of } n \]

and

\[(AB)' = B'A' \]

\[ = 2[n - k] \quad \text{since} \quad (X'X) \text{ is the order of } k \text{ so that} \]

\[(X'X)^{-1}X'X = I_k \quad \text{(22)} \]

Thus

\[ s^2 = \frac{e'e}{n-k} \quad \text{(23)} \]

which provides us with an unbiased estimator of the disturbance variance.
Specification Bias\(^6\)

Deletion of variables and omitted variable-created specification bias means \(E(\hat{\beta}) = \beta + d_{k1}\) and \(\text{MSE}(b) = V(b) + \text{Bias}^2(b)\).

Suppose that 
\[ Y_{n1} = X_{nk} \beta_{k1} + Z_{nr} \nu_{r1} + \mu_{n1} \] is the true model, but that we fit 
\[ \hat{\beta} = (X'X)^{-1}X'Y \] then 
\[ E(\hat{\beta}) = E[(X'X)^{-1}X'Y] \] 
\[ = E[(X'X)^{-1}X' (X\beta + Z\nu + \mu)] \] 
\[ = (X'X)^{-1}X'X E(\beta) + E[(X'X)^{-1}X'Z\nu] + E[(X'X)^{-1}X'\mu] \] since \(X\) is fixed 
\[ = \beta_{k1} + E[(X'X)^{-1}X'Z\nu] + E[(X'X)^{-1}X'\mu] \] by distribution law and since \((XX)^{-1}X'X = I\) and \(E(\beta) = \beta\) by the previous proof.

\[ = \beta_{k1} + (X'X)^{-1}X'Z\nu \quad \text{since } E(\mu) = 0 \]
\[ = \beta_{k1} + d \quad \text{(22)} \]

where

\[ d_{k1} = (X'X)^{-1} \begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_k' \end{pmatrix} (Z_1 Z_2 \ldots Z_{r1}) V_{r1} \]

\[ = (X'X)^{-1} (X'Z_1 X'Z_2 \ldots X'Z_r) V_{r1} \]

\[ = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kr} \end{pmatrix} \begin{pmatrix} V_1 \\ \vdots \\ \vdots \\ V_r \end{pmatrix} \]

\(^6\) This section is drawn heavily from Brown, 1978, 1980.
\[ \hat{Z}_1 = \hat{p}_{11}X_1 + \hat{p}_{21}X_2 + \cdots + \hat{p}_{k1}X_k \]
\[ \hat{Z}_2 = \hat{p}_{12}X_1 + \hat{p}_{22}X_2 + \cdots + \hat{p}_{k2}X_k \]
\[ \vdots \]
\[ \hat{Z}_k = \hat{p}_{1k}X_1 + \hat{p}_{2k}X_2 + \cdots + \hat{p}_{kk}X_k \]

and for any linear estimation, \( b_{k1} \), \( \text{MSE}(b) = V(b) + \text{bias}^2(b) \).

By definition
\[ \text{MSE}(b) = \sum_{i=1}^{k} \mathbb{E}[(b_i - \beta_i)^2] \]
\[ = \mathbb{E}[(b_i - \beta_i)'(b_i - \beta_i)] \]
\[ = \mathbb{E}[(b_i - Eb) + (Eb - \beta)'](b_i - Eb) + (Eb - \beta)] \]
\[ = \mathbb{E}[(b_i - Eb)'(b_i - Eb)] + \mathbb{E}[(b_i - Eb)'(Eb - \beta)] + \mathbb{E}[(Eb - \beta)'(b_i - Eb)] + \mathbb{E}[(Eb - \beta)'(Eb - \beta)] \]
\[ = V(b) + \text{Bias}^2(b) + 2\mathbb{E}[(b - Eb)'(Eb - \beta)] \] since \( \lambda_i' = \lambda_{i1} \)

and
\[ V(b) = \mathbb{E}[(b_i - Eb)'(b_i - Eb)] \] and
\[ \text{Bias}^2(b) = \mathbb{E}[(Eb - \beta)'(Eb - \beta)] \] by definition

also
\[ 2\mathbb{E}[(b - Eb)'(Eb - \beta)] = 2\mathbb{E}[(a'X\beta - \beta)'(b - Eb)] \]

since
\[ Eb = a'X\beta \] from the previous proof
\[ = 2\mathbb{E}[(a'X\beta - \beta)'(a'\mu)] \]

since
\[ (b - Eb) = a'\mu \] from the previous proof
\[ = 2\mathbb{E}[(a'X\beta - \beta)'(E a\mu)] \]
\[ = 0 \] since \( \mathbb{E}(\mu) = 0 \).
The Eigenvalues and Vectors (Characteristic Roots and Vectors)

The eigenvalues and vectors problem (the characteristic value problem) is defined as that of finding values of a scalar $\lambda$ and an associated vector $q \neq 0$ which satisfy

$$A \frac{\mathbf{q}}{n} = \lambda \frac{\mathbf{q}}{n} \text{ or } (A - \lambda \mathbf{I}) \frac{\mathbf{q}}{n} = 0$$

where $A$ is some $n \times n$ matrix. $\lambda$ is called an eigenvalue (characteristic root of $A$ and $q$, an eigenvector (characteristic vector).

$(A - \lambda \mathbf{I}) \mathbf{q} = 0$ only has a non-trivial solution, $q \neq 0$, if $(A - \lambda \mathbf{I})$ is singular that is if $|A - \lambda \mathbf{I}| = 0$

which yields a polynomial in the unknown $\lambda$, which may be solved for $\lambda$ and then the characteristic vectors obtained, for example, in the $2 \times 2$ case.

Suppose $(X'X)_{2\times2} = \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}$

let

$|A - \lambda \mathbf{I}| = \begin{vmatrix} 1 & r \\ r & 1 \end{vmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{vmatrix} 1-\lambda & r \\ r & 1-\lambda \end{vmatrix}$

$= (1-\lambda)^2 - r^2$

$= \lambda^2 - 2\lambda + (1-r)^2$

from $\alpha x^2 + bx + c \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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7 This section draws heavily from Johnston, 1972, and Brown 1978, 1980.
we have
\[ \lambda_i = 1 \pm r \implies \lambda_1 = (1+r), \lambda_2 = (1-r) \]

Compute the \( \hat{q}_i \) (eigenvectors or characteristic vectors) corresponding to the \( \lambda_i \) (eigenvalues or characteristic roots).

For \( \lambda_1 = (1+r) \)
we have
\[
(A - \lambda_1 I)\hat{q}_1 = \begin{pmatrix} 1 - (1+r) & r \\ r & 1 - (1+r) \end{pmatrix} \begin{pmatrix} q_{11} \\ q_{21} \end{pmatrix} = 0
\]
\[
= \begin{pmatrix} -r & r \\ r & -r \end{pmatrix} \begin{pmatrix} q_{11} \\ q_{21} \end{pmatrix} = 0
\]
\[
\implies -rq_{11} + rq_{21} = 0 \implies \hat{q}_{11} = \hat{q}_{21}
\]
or \[
\hat{q}_{11} + (-r\hat{q}_{21}) = 0 \implies \hat{q}_{11} = \hat{q}_{21}
\]

We may normalize the vector by setting its length at unity, that is, by making
\[ q_{11}^2 + q_{21}^2 = 1.0 \]
\[ 2\hat{q}_{1} = 1.0 \] since \( q_{11} = q_{21} \) from the above proof.

Therefore,
\[ \hat{q}_1 = \pm 1/\sqrt{2} \implies q_{11} = 1/\sqrt{2} \text{ and } q_{21} = 1/\sqrt{2} \]

So the eigenvectors associated with \( \lambda_1 = (1+r) \) are
\[ \hat{q}_1 = (1/\sqrt{2}, 1/\sqrt{2}) \]

Similarly it may be shown that the eigenvectors associated with
\[ \lambda_2 = (1-r) \text{ is } \hat{q}_2 = 1/\sqrt{2}, -1/\sqrt{2} \]
By the definition, the two vectors $x$ and $y$ are said to be orthogonal if $X'Y = Y'X = 0$ and $X'X \neq 0, y'y \neq 0$, Hadley, 1961, pp. 240-241.

That means

$$q_1^t q_2 = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = 0 \Rightarrow \text{the eigenvectors of this symmetric are orthogonal.}$$

In general $n^{\text{th}}$ order symmetric matrix $A$ has eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Corresponding to these eigenvalues are a set of orthogonal eigenvectors $q_1, q_2, \ldots, q_n$ such that

$$q_i^t q_j = 0 \quad i \neq j, i, j = 1, 2, \ldots, n$$

and we can normalize the vectors so that $q_i^t q_i = 1$ for all $i$. So we can then write the conditions on the $q$ vector as

$$q_i^t q_j = \delta_{ij} \quad \delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

where $\delta_{ij}$ is the Kronecker delta.

If $Q$ denotes the $n^{\text{th}}$ order matrix whose columns are the vectors $q_1, q_2, \ldots, q_n$, then

$$Q'Q = QQ' = I_n$$
and so that

\[ Q' = Q^{-1} \]

That is, the transposition of Q is equal to its inverse. Such a matrix is said to be an orthogonal matrix.

\[ \implies Q'(X'X)Q = \Lambda \]

\[
\begin{pmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \lambda_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_n
\end{pmatrix}
\]

That is, the product \( Q'(X'X)Q \) has produced a diagonal matrix with the eigenvalues of \( (X'X) \) or \( \Lambda \) matrix displayed on the principal diagonal.

Principal Components \(^8\)

Principal components analyses begin by transforming the explanatory variables into another set of variables which are pairwise uncorrelated. These variables are the principal components. The transformation used is the orthogonal transformation, where

\[ Y = X\beta + u \implies Y = Z\alpha + u \]

\[ Z = XQ, \alpha = Q'\beta \]

Proof:

\[ Y_{n1} = \sum_{k=1}^{n} X_{nk} \beta_{k1} + u_{n1} \]

\[ = \sum_{h=1}^{n} X_{nh} (I_{k} \beta_{k1}) + u_{n1} \]

\[ = \sum_{k=1}^{n} \sum_{h=1}^{n} X_{nk} (QQ')_{kh} \beta_{k1} + u_{n1} \]

where \( Q(X'X)Q = \Lambda \)

\[ = (XQ) (Q'\beta) + u \]

by associated law

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\(^8\) This section is drawn heavily from Johnston, 1972, and Brown 1978, 1980.
and

\[ \text{Var}(\hat{\alpha}) = \sigma^2 \Lambda^{-1} \quad \text{since} \]

\[ \text{Var}(\hat{\alpha}) = \sigma' a' a \text{ be earlier proof } a' = \alpha \mathbf{Y} \quad \text{where} \]

\[ a' = (Z'Z)^{-1}Z \implies \hat{\alpha} = (Z'Z)^{-1}Z'\gamma \]

Therefore

\[ \text{Var}(\hat{\alpha}) = \sigma^2[(Z'Z)^{-1}][(Z'Z)^{-1}Z'] \]

\[ = \sigma^2[(Z'Z)^{-1}A'] [Z(Z'Z)^{-1}] \quad \text{since } (A\beta)' = \beta'A' \]

\[ = \sigma^2(Z'Z)^{-1}I_k \]

\[ = \sigma^2(Z'Z)^{-1} \]

\[ = \sigma^2[(XQ)'(XQ)]^{-1} \]

\[ = \sigma^2[Q'(X'X)Q]^{-1} \]

\[ = \sigma^2 \Lambda^{-1} \]

\[ = \sigma^2 \begin{pmatrix} 1/\lambda_1^0 & 0 \\ 0 & \ddots \\ \ddots & \ddots & 0 \\ 0 & \ldots & 1/\lambda_k \end{pmatrix} \]

and

\[ \text{Var}(\hat{\alpha}) = V(\hat{\beta}) \quad \text{if the full model is fitted} \quad \text{since} \]

\[ \text{Var}(\hat{\beta}) = \text{tr} \left( \text{Var}(\hat{\alpha}) \right) \]

\[ = \text{tr} \left( \sigma^2(Z'Z)^{-1} \right) \]

\[ = \sigma^2 \text{tr} [(XQ)'(XQ)]^{-1} \]

\[ = \sigma^2 \text{tr} [Q'(X'X)Q]^{-1} \]

\[ = \sigma^2 \text{tr} [(X'X)Q'Q]^{-1} \]

\[ = \sigma^2 \text{tr} [(X'X)_{kk}I_k^{-1}]^{-1} \quad \text{since } Q'Q = QQ' = I \]

\[ = \sigma^2 \text{tr} (X'X)^{-1} \]

\[ = V(\hat{\beta}) \]
Generalized Least Square (GLS)

Lemma and Theorem

GLS lemma if \( \Omega \) is a real symmetric positive definite, then \( P \) exist such that \( PP' = \Omega \) and \( (P^{-1})'P^{-1} = \Omega^{-1} \)

let \( P = Q^{-\frac{1}{2}} \)

where \( \Lambda^{-\frac{1}{2}} = \begin{pmatrix} \sqrt{\lambda_1} & 0 & \cdots & 0 \\ 0 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \sqrt{\lambda_n} \end{pmatrix} \)

Since \( \Omega \) is real symmetric positive definite \( \Rightarrow \) \( Q \) exist such that \( Q'\Omega Q = \Lambda \) and \( Q'Q = QQ' = I_n \) from the previous proof.

Therefore

\[
P'P = (QA^{-\frac{1}{2}}) (QA^{-\frac{1}{2}})',
\]

\[
= QA^{-\frac{1}{2}}A^{-\frac{1}{2}}Q'
\]

\[
= QAQ'
\]

\[
= Q(Q'\Omega Q)Q' \text{ since } \Lambda = (Q'\Omega Q)
\]

\[
= \Omega \text{ since } Q'Q = QQ' = I
\]

\[
\Rightarrow \Omega = QQ'
\]

Therefore

\[
P'P = \Omega \quad (3)
\]

and

\[
(P^{-1})'P^{-1} = (\Lambda^{-\frac{1}{2}}Q')^{-1} (\Lambda^{-\frac{1}{2}}Q')
\]

where

\[
p^{-1}(QA^{-\frac{1}{2}})^{-1} = (\Lambda^{-\frac{1}{2}}Q') \text{ since } (AB)^{-1} = B^{-1}A^{-1}
\]

if \( A, B \) are nonsingular

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\(^9\) This section is drawn heavily from Johnston, 1972, and Brown 1978, 1980.
\[ = QA^{-1}Q' \]
\[ = \Omega^{-1} \text{ since } Q(Q'\Omega Q)' = QAQ' \] \hspace{1cm} (5)
\[ \Rightarrow \Omega = QAQ' \]
\[ \Rightarrow \Omega^{-1} = (Q')^{-1}A^{-1}Q'^{-1} \text{ since } (ABC)^{-1} = C^{-1}B^{-1}A^{-1} \text{ if } A, B, C \text{ are nonsingular} \]
\[ \Rightarrow \Omega^{-1} = QA^{-1}Q' \] \hspace{1cm} (6)

**GLS Theorem**

If \( E \mu = 0, \rho(X_{nk}) = k, \) \( X_{nk} \) is fixed but \( E\mu' = \sigma^2\Omega, \text{ and } \Omega \text{ is known and positive definite then BLUE of } \beta \text{ in } Y = X\beta + \mu \text{ is} \)

\[ b = [X'\Omega^{-1}X]^{-1}X'\Omega^{-1}Y \] \hspace{1cm} (7)

First show that

\[ b = [X'\Omega^{-1}X]^{-1}X'\Omega^{-1}Y \]

let premultiple \( Y = X\beta + \mu \) by \( p^{-1} \)

where

\[ p = QA^{-0.5} \Rightarrow p^{-1} = \Lambda^{0.5} \]

let

\[ p^{-1}Y = Y_*, \quad p^{-1}X = X_*, \quad p^{-1}\mu = \mu_* \]

Therefore

\[ Y_* = X_*\beta + \mu_* \] \hspace{1cm} (8)

From the OLS earlier results

\[ b = (X_*X_*)^{-1}X_*Y_* \] \hspace{1cm} (9)

\[ = [(P^{-1}X)^{-1} (P^{-1}X)]^{-1} (P^{-1}X)' (P^{-1}Y) \]

\[ = [X' (P^{-1}p^{-1}) X]^{-1} [X' (P^{-1}p^{-1}) Y] \]

\[ = [X'\Omega^{-1}X]^{-1}X'\Omega^{-1}Y \text{ since } p^{-1}p^{-1} = \Omega^{-1} \] \hspace{1cm} (10)

Second, show that \( b \) is the BLUE of \( \beta \)

\[ E(\mu_*\mu_*') = E[(P^{-1}\mu) (P^{-1}\mu)'] \]
\[ \begin{align*}
&= E[p^{-1}(\mu \mu)', p^{-1}'] \\
&= p^{-1}E(\mu \mu') p^{-1}' \\
&= \sigma^2 p^{-1}p^{-1}' \quad \text{since } \Omega \text{ and therefore } P \text{ and } p^{-1}
\end{align*} \]

are assumed to be known by definition

\[ \begin{align*}
&= \sigma^2 [p^{-1}(PP')p^{-1}'] \quad \text{since } PP' = \Omega \text{ from GLS Lemma} \\
&= \sigma^2 [I_n P'P^{-1}'] \quad \text{since } P^{-1} = I \Rightarrow P^{-1}P = I \\
&= \sigma^2 [I_n (\Lambda^{-5}Q') (Q\Lambda^{-5})] \quad \text{since } P = QA^{-5}
\end{align*} \]

and \( p^{-1} = \Lambda^{-5}Q' \)

\[ \begin{align*}
&= \sigma^2 \Lambda^{-5} \Lambda^{-5} \\
&= \sigma^2 I_n
\end{align*} \]

Therefore \( b \) is the BLUE from the earlier OLS results.

GLS \( \text{Var}(b) = \sigma^2 (X'\Omega^{-1}X)^{-1} \) \( (12) \)

Proof.

From

\[ Y_* = X_* \beta + \mu_* \]

where

\[ Y_* = p^{-1}Y, \quad X_* = p^{-1}X, \quad \mu_* = p^{-1}\mu \]

and from the OLS results earlier

\[ \text{Var}(b) = \sigma^2 (X_*X'_*)^{-1} \]

\[ = \sigma^2 [(P^{-1}X)' (P^{-1}X)]^{-1} \quad \text{since } X_* = P^{-1}X \]

\[ = \sigma^2 [X'p^{-1}P^{-1}X]^{-1} \quad \text{since } (AB)' = B'A' \]

\[ = \sigma^2 (X'\Omega^{-1}X)^{-1} \quad \text{since from the GLS lemma} \]

\[ P = QA^{-5} \Rightarrow p^{-1} = \Lambda^{-5}Q' \]

\[ \Rightarrow p^{-1}'p^{-1} = (QA^{-5})(\Lambda^{-5}Q') = QA^{-1}Q' \]

also \( Q'\Omega Q = \Lambda \Rightarrow \Lambda^{-1} = Q'\Omega^{-1}Q \Rightarrow p^{-1}'p^{-1} = QA^{-1}QQ' \)

\[ = \Omega^{-1} \quad \text{since } QQ' = QQ' = I \]
The generalized least squares model is sometimes in the form
\[ Y = X\beta + \mu \]  \hspace{1cm} (15)
with
\[ E(\mu) = 0 \]  \hspace{1cm} (16)
and
\[ E(\mu\mu') = V \]  \hspace{1cm} (17)
where \( V \) is assumed to be a known symmetric positive definite matrix.

The contrast between \( E(\mu\mu') = \sigma^2 \Omega \) and \( E(\mu\mu') = V \) is that \( \sigma^2 \Omega \) has been replaced by \( V \).

Therefore
\[ \hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}Y \]  \hspace{1cm} (18)
and
\[ \text{Var}\, \hat{\beta} = (X'V^{-1}X)^{-1} \]  \hspace{1cm} (19)
as the corresponding expressions for the GLS estimator and its variance-covariance matrix.

Proof.

Since
\[ E\mu\mu' = \sigma^2 \Omega = V \]
then
\[ V^{-1} = (1/\sigma^2 \Omega^{-1}) \]
since
\[ (K_{11} \beta_{nn})^{-1} = (1/k_{11})B^{-1} \]
if
\[ |B| \neq 0 \quad \text{and} \quad |K| \neq 0 \]
\[ \hat{\beta} = [X'(1/\sigma^2 \Omega^{-1})X] \left(1/\sigma^2 \Omega^{-1}\right)Y \] by substitution
\[ V^{-1} = (1/\sigma^2 \Omega^{-1}) \] in (18)
\[ = [1/\sigma^2 X' \Omega^{-1} X]^{-1}X' \left(1/\sigma^2 \Omega^{-1}\right)Y \]
\[ = (X' \Omega^{-1} X)^{-1} \left(1/\sigma^2\right)^{-1} 1/\sigma^2 X' \Omega^{-1} Y \]
\[ = (X' \Omega^{-1} X)^{-1} \sigma^2 1/\sigma^2 X' \Omega^{-1} Y \]
\[ = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} Y \]
\[ = \text{GLS} \, \hat{\beta}. \]
Similarly,

\[
\text{Var} \hat{\beta} = (X'(1/\Omega^2 -1)X)^{-1} \\
= (1/\sigma^2)^{-1}(X'\Omega^{-1}X) \\
= \sigma^2(X'\Omega^{-1}X) \\
= \text{GLS Var}(b).
\]

Inexact Prior Information: Theil and Goldberger Mixed Model\(^\text{10}\)

The Theil and Goldberger mixed model theorem state that:

"Given inexact but unbiased prior information \(r_{gl}R_{gk} \beta_{k1} + V_{gl}\) where \(E(V_{gl}) = 0_{gl}\) and \(E(V_{gl}'V_{gl}) = \Psi_{gg}\) is positive definite and there are \(n\) sample observation." \(Y = X\beta + \mu\), \(E(\mu) = 0\), \(E(\mu\mu') = \sigma^2I\), \(\rho(X_{nk}) = k\), etc., and \(\mu\) and \(V\) are independent, the GLS estimate considering both prior and sample information is

\[
\hat{\beta} = [\ell'X'X + R'\Psi^{-1}R]^{-1} [\ell'X'Y + R'\Psi^{-1}r] \\
(1)
\]

and

\[
\text{Var} (\hat{\beta}) = [\ell'X'X + R'\Psi^{-1}r] \\
\text{where} \quad \rho = 1/\sigma^2 \\
(2)
\]

This means the general model with prior information becomes

\[
\left(\begin{array}{c}
Y \text{ml} \\
\ell' \\
\end{array}\right)_{\text{n+g,1}} = \left(\begin{array}{c}
X \text{nk} \\
R_{gk} \\
\end{array}\right)_{\text{n+g,k}} \beta_{k1} + \left(\begin{array}{c}
U \text{n1} \\
V_{gl} \\
\end{array}\right)
\]

or letting

\[
\eta = \left(\begin{array}{c}
Y \\
\ell \\
\end{array}\right), \quad Z = \left(\begin{array}{c}
X \\
R \\
\end{array}\right), \quad \varepsilon = \left(\begin{array}{c}
\mu \\
V \\
\end{array}\right) \\
(3)
\]

then

\[
\eta_{\text{n+g,1}} = Z_{\text{n+g,k}} \beta_{k1} + \varepsilon_{\text{n+g,1}} \\
(4)
\]

From the earlier work we know that

\[
\text{GLS} \ \hat{\beta} = [Z'V^{-1}Z]^{-1}Z'V^{-1}\eta \\
(5)
\]

\(^{\text{10}}\) This section is drawn heavily from Brown 1978, 1980.
but

\[ V = E(\varepsilon \varepsilon') \quad \text{since} \quad 1/\sigma^2 \Omega^{-1} = V^{-1} \implies V = \sigma^2 \Omega = E(\mu \mu') \]

\[ = E\left( \begin{bmatrix} \mu_{11} \\ \vdots \\ \mu_{n1} \\ V_{1g} \end{bmatrix} \begin{bmatrix} \mu'_{11} & \cdots & \mu'_{1g} \\ V'_{1g} \end{bmatrix} \right) \]

\[ = E\left( \begin{bmatrix} \mu_{11} & \mu'_{11} \\ \vdots & \mu_{1g} & \mu'_{1g} \\ V_{1g} & V'_{1g} \end{bmatrix} \begin{bmatrix} \mu'_{11} & \cdots & \mu'_{1g} \\ V'_{1g} \end{bmatrix} \right) \]

\[ = \begin{bmatrix} \sigma^2 I & 0 \\ 0 & \Psi \end{bmatrix} \quad \text{since} \quad E(\mu \mu') = \sigma^2 I \quad \text{and} \quad E(\varepsilon \varepsilon') = \Psi \quad (6) \]

Therefore

\[ V^{-1} = \begin{bmatrix} 1/\sigma^2 & 0 \\ 0 & -\Psi \end{bmatrix} \quad (7) \]

let \( \lambda = 1/\sigma^2 I_n \)

Therefore

\[ V^{-1} = \begin{bmatrix} \lambda & 0 \\ 0 & \Psi^{-1} \end{bmatrix} \]

Therefore

\[ \text{GLS Var}(\hat{\beta}) = (Z'V^{-1}Z)^{-1} \]

and also from the earlier work we know that

\[ \text{GLS Var}(\hat{\beta}) = (Z'V^{-1}Z)^{-1} \]

Therefore

\[ \text{GLS Var}(\hat{\beta}) = (\lambda'X' + R'\Psi^{-1}R)^{-1} \text{ for earlier results on GLS b.} \]
Ridge Regression

Consider the model as follows

\[ Y = X\beta + \mu \]  \hfill (1)

The ridge estimator is defined as

\[ \hat{\beta}^* = (X'X + KI)^{-1} X'Y \]  \hfill (2)

The variance-covariance of \( \hat{\beta}^* \) is

\[ \text{Var}(\hat{\beta}^*) = \sigma^2 (X'X + KI)^{-1} X'X (X'X + KI)^{-1} \]  \hfill (3)

since

\[ \hat{\beta}^* = (X'X + KI)^{-1} X'Y = \alpha'Y = \hat{\beta}^* \text{ is a linear estimator} \]

but

\[ \hat{\beta}^* = \alpha'Y \]  \hfill (4)

\[ \Rightarrow \text{Var}(\hat{\beta}^*) = \sigma^2 \alpha' \alpha \]

\[ = \sigma^2 [(X'X + I)^{-1} ]' [(X'X + KI)^{-1} X'] \text{ since} \]

\[ \alpha' = (X'X + KI)^{-1} X' \]

\[ = \sigma^2 [(X'X + KI)^{-1} X'] [X(X'X + KI)^{-1}] \text{ since} \]

\[ [(X'X) + KI]^{-1} \text{ is symmetric.} \]  \hfill (5)

Therefore

\[ \text{Var}(\hat{\beta}^*) = \sigma^2 (X'X + KI)^{-1} X'X (X'X + KI)^{-1} \]  \hfill (6)

Ridge regression is the method of biased linear estimation because

\[ E(\hat{\beta}^*) = \beta - K(X'X + KI)^{-1} I\beta \]  \hfill (7)

as we know

\[ E(\hat{\beta}^*) = E[(XX' + KI)^{-1} X' (X\beta + \mu)] \text{ since } Y = X\beta + \mu \]

\[ = E[(X'X + KI)^{-1} X'X\beta + 0 \text{ since } E(\mu) = 0] \]

\[ = E[(X'X + KI)^{-1} (X'X \beta + KIJ\beta - KI\beta)] \]

11 This section is drawn heavily from Brown 1978, 1980; Rahuma 1982; and Sass 1979.
\[ E[(X'X + KI)^{-1}] [(X'X + KI) \beta - KI\beta] \]
\[ = E[(X'X + KI)^{-1} (X'X + KI) \beta - E[(X'X + KI)^{-1}K\beta] \]
\[ = \beta - K(X'X + KI)^{-1} I\beta \quad \text{since} \quad X, K, \beta \text{ are fixed} \]
and
\[ E(\beta) = \beta \quad \text{since} \quad A^{-1}A = AA^{-1} = I. \]

The Value of \( K \) and Choice of \( K_i \)

The value of \( k \) can be determined by considering an orthogonal transformation of the regression model. Again consider the model

\[ Y = X\beta + \mu \text{ is transformed to } Y = Z\alpha = + \mu \text{ where } Z = X\delta, \]

and \( \alpha = Q'\beta. \)

For this transformed model, the OLS estimator is

\[ \hat{\alpha} = (Z'Z)^{-1}Z'Y \quad (8) \]

and

\[ \text{Var}(\hat{\alpha}) = \sigma^2 (Z'Z)^{-1} \quad (9) \]

and

\[ \text{Var}(\hat{\alpha}) = \sigma^2 \text{tr} (Z'Z)^{-1} \]

\[ = \sigma^2 \Lambda^{-1} \quad \text{since} \quad (Z'Z) = \Lambda \]

\[ = \sigma^2 \sum_{i=1}^{k} 1/\lambda_i \quad (10) \]

Therefore, the ridge estimator for the transformed model is

\[ \hat{\alpha}^* = (Z'Z) + KI)^{-1}Z'Y \quad (11) \]

and the variance-covariance of \( \hat{\alpha}^* \) is

\[ \text{Var}(\hat{\alpha}^*) = \sigma^2 (Z'Z) + KI)^{-1} (Z'Z) (Z'Z + KI)^{-1} \quad (12) \]

and

\[ \text{Var}(\hat{\alpha}^*) = \sigma^2 \sum_{i=1}^{k} \frac{\lambda_i}{(\lambda_i + K)^2} \quad (13) \]

since
\[ \text{Var}(\beta^*) = 2\text{tr} \left( (Z'Z + \mathbf{K})^{-1} Z'Z (Z'Z + \mathbf{K})^{-1} \right) = 2\text{tr} \left[ (Z'Z + \mathbf{K})^{-2} \right] \text{ since } (Z'Z + \mathbf{K})^{-1} \text{ is symmetric.} \]

\[ = \sigma^2 \sum_{i=1}^{k} \frac{\lambda_i}{(\lambda_i + \mathbf{K})^2} \]

and from the transformation we know that

\[ \hat{\alpha} = (Z'Z)^{-1} Z'Y \text{ for OLS} \]

\[ = \mathbf{A}^{-1} Z'Y \]

\[ = \frac{Z'Y}{\lambda_i} \quad \text{(14)} \]

and

\[ \hat{\alpha}^* = (Z'Z + \mathbf{K})^{-1} Z'Y \text{ for ridge regression} \]

\[ = \mathbf{A}^{-1} Z'Y \]

\[ = \frac{Z'Y}{\lambda_i + \mathbf{K}} \quad \text{(15)} \]

multiply both sides by \( \frac{\lambda_i}{\lambda_i} \)

\[ \frac{\lambda_i}{\lambda_i} \hat{\alpha}^* = \frac{\lambda_i}{\lambda_i} \cdot \frac{Z'Y}{\lambda_i + \mathbf{K}} \]

\[ = \frac{\lambda_i}{\lambda_i + \mathbf{K}} \cdot \frac{Z'Y}{\lambda_i} \]

\[ = m_i \cdot \hat{\alpha} \text{ since let } \frac{\lambda_i}{\lambda_i + \mathbf{K}} = m_i \text{ and } \frac{Z'Y}{\lambda_i} = \hat{\alpha} \]

Therefore

\[ \hat{\alpha}^* = m_i \hat{\alpha} \quad \text{(16)} \]

Optimum values for \( \mathbf{K_i} \) for generalized ridge regressions are those \( \mathbf{K_i} \)'s which minimize \( \text{MSE}(\hat{\alpha}^*) \), which also minimize \( \text{MSE}(\hat{\beta}^*) \).
Since
\[ \text{MSE}(\hat{\alpha}^*) = \sum_{i=1}^{k} (\hat{\alpha}^* - \alpha)^2 \]

\[ = (\hat{\alpha}^* - \alpha)' (\hat{\alpha}^* - \alpha) \]

\[ = (Q'\hat{\beta}^* - Q'\beta)' (Q\hat{\beta}^* - Q\beta) \quad \text{since} \quad \hat{\alpha}^* = Q'\beta^* \]

and
\[ \hat{\alpha} = Q'\beta \quad \text{from the transformation} \]
\[ = (\hat{\beta}^* - \beta)' QQ'(\hat{\beta}^* - \beta) \]
\[ = (\hat{\beta}^* - \beta)I (\hat{\beta}^* - \beta) \]
\[ = \text{MSE} (\hat{\beta}^*) \quad (17) \]

and we know that
\[ E(\text{MSE}(\alpha_i^*)) = \gamma(\hat{\alpha}_i^*) + \text{bias}^2 \hat{\alpha}^* \quad \text{from the earlier work} \]
\[ = m_i \gamma(\hat{\alpha}_i) + (m_i - 1)^2 \alpha_i^2 \quad \text{since} \quad \hat{\alpha}^* = m_i \hat{\alpha} \]
\[ \implies \gamma(\hat{\alpha}^*) = m_i^2 \gamma(\hat{\alpha}) \quad (18) \]

and
\[ \text{Bias} \hat{\alpha}_i^* = E(\hat{\alpha}_i^* - \alpha_i) \]
\[ = E(m_i E\hat{\alpha}_i - \alpha_i) \quad \text{since} \quad m \text{ is fixed} \]
\[ = m_i \alpha_i - \alpha_i \quad \text{since} \quad E(\alpha) = \alpha_i \]
\[ = (m_i - 1) \alpha_i \quad (19) \]

Therefore
\[ E(\text{MSE}(\alpha_i^*)) = \frac{\lambda_i}{(\lambda_i + K)^2} \frac{\sigma^2}{\lambda_i} + \frac{\lambda_i}{(\lambda_i + K)^2} (\frac{\alpha_i^2}{\lambda_i - 1}) \quad \text{since} \]
\[ m_i = \frac{\lambda_i}{\lambda_i + K} \quad (20) \]
\[ = \frac{\lambda_i}{(\lambda_i + K)^2} \sigma^2 + \frac{\lambda_i}{(\lambda_i + K)^2} \frac{\alpha_i^2}{(\lambda_i + K)^2} \]

let
\[ E(\text{MSE}(\hat{\alpha}^*)) = \phi \]

\[ \frac{\partial \phi}{\partial K} = -\frac{2\lambda_1 \sigma_i^2}{(\lambda_1 + K)} + \frac{2K \lambda_i \sigma_i^2}{(\lambda_1 + K)^3} = 0 \]

\[ -\sigma^2 + K \alpha_i^2 = 0 \]

\[ K = \frac{\sigma_i^2}{\alpha_i^2} \quad (21) \]

Second order condition

\[ \frac{\partial^2 \phi}{\partial^2 K} = \frac{\partial}{\partial K} \left( -\frac{2\lambda_1 \sigma^2 + 2K \lambda_i \sigma_i^2}{(\lambda_1 + K)^2} \right) \]

\[ = \frac{2\lambda_1 \sigma_i^2}{(\lambda_1 + K)^3} > 0 \quad \text{since} \quad X'X \text{ is positive definite} \]

\[ \Rightarrow \lambda_i > 0, \; K > 0, \; \alpha_i^2 > 0 \quad (22) \]
APPENDIX 2

Selected Statistics Used to Calculate Tables 1-12, in the Text
### Table A2-1. Selected Statistics Used to Calculate Table Retail Demand

#### Means and Standard Deviations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTUNA</td>
<td>0.80782</td>
<td>0.26633</td>
</tr>
<tr>
<td>LINCOME</td>
<td>7.86654</td>
<td>0.18512</td>
</tr>
<tr>
<td>LPM</td>
<td>0.01837</td>
<td>0.06059</td>
</tr>
<tr>
<td>LCPIF</td>
<td>4.46951</td>
<td>0.16730</td>
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</tbody>
</table>

The error term variances = 0.034871

#### The Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>LTUNA</th>
<th>LINCOME</th>
<th>LPM</th>
<th>LCPIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTUNA</td>
<td>1.00000</td>
<td>0.93832</td>
<td>0.51667</td>
<td>0.94708</td>
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<tr>
<td>LINCOME</td>
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<tr>
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<td>-0.93871</td>
<td>-0.39841</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

#### The Diagonal Elements of the Inverted Z'Z Correlation Matrix

\[
r_{11} = 10.31523 \quad r_{22} = 1.22574 \quad r_{33} = 10.01117
\]

#### The Eigenvalues of X'X Correlation Matrix

\[
\lambda_1 = 2.22725 \quad \lambda_2 = .72203 \quad \lambda_3 = .05070
\]

#### The Eigenvector of X'X Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>LINCOME</td>
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<td>LCPIF</td>
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</table>
Table A2-2. Selected Statistics Used to Calculate Table Retail Demand

<table>
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<th>Standard Deviations</th>
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<tbody>
<tr>
<td>LTUNA</td>
<td>0.80781</td>
<td>0.26633</td>
</tr>
<tr>
<td>LINCOME</td>
<td>7.86654</td>
<td>0.18512</td>
</tr>
<tr>
<td>Lyr</td>
<td>4.18260</td>
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<tr>
<td>LCPIF</td>
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<td>0.16733</td>
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</table>

The error term variance - .021202

The Correlation Matrix

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<th>Lyr</th>
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<td>Lyr</td>
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<tr>
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<td>-0.97871</td>
<td>-0.39807</td>
<td>-0.39841</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

The Diagonal Elements of the Inverted X'X Correlation Matrix

\[ r_{11} = 54.15709 \quad r_{22} = 122.38492 \]
\[ r_{33} = 1.24279 \quad r_{44} = 31.08712 \]

The Eigenvalues of X'X Correlation Matrix

\[ \lambda_1 = 3.17689 \quad \lambda_2 = .76640 \]
\[ \lambda_3 = .051390 \quad \lambda_4 = .00532 \]

The Eigenvectors of X'X Correlation Matrix

<table>
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<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>LINCOME</td>
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<td>-0.1945</td>
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Table A2-3. Select Statistics Used to Calculate Table Retail Demand

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<tr>
<td>PY</td>
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The error term variance = .012656

The Correlation Matrix

<table>
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<th></th>
<th>TUNA</th>
<th>YR</th>
<th>INCOME</th>
<th>CPIF</th>
<th>PY</th>
</tr>
</thead>
<tbody>
<tr>
<td>TUNA</td>
<td>1.00000</td>
<td>0.93627</td>
<td>0.93113</td>
<td>-0.90556</td>
<td>0.29681</td>
</tr>
<tr>
<td>YR</td>
<td>0.93627</td>
<td>1.00000</td>
<td>0.98778</td>
<td>-0.95879</td>
<td>0.32698</td>
</tr>
<tr>
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<td>0.93113</td>
<td>0.98778</td>
<td>1.00000</td>
<td>-0.92314</td>
<td>0.44357</td>
</tr>
<tr>
<td>CPIF</td>
<td>-0.90556</td>
<td>-0.95879</td>
<td>-0.92314</td>
<td>1.00000</td>
<td>-0.09569</td>
</tr>
<tr>
<td>PY</td>
<td>0.29681</td>
<td>0.32698</td>
<td>0.44357</td>
<td>-0.95690</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

The Diagonal Element of the Inverted X'X Correlation Matrix

\[ r_{11} = 118.78507 \]
\[ r_{22} = 144.45868 \]
\[ r_{33} = 41.92129 \]
\[ r_{44} = 8.13474 \]

The Eigenvalues of X'X Correlation Matrix

\[ \lambda_1 = 3.03806 \]
\[ \lambda_2 = 0.94337 \]
\[ \lambda_3 = 0.01445 \]
\[ \lambda_4 = 0.00412 \]

The Eigenvectors of X'X Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>YR</td>
<td>0.5694</td>
<td>-0.1009</td>
<td>0.4973</td>
<td>0.6467</td>
</tr>
<tr>
<td>INCOME</td>
<td>0.5724</td>
<td>0.0291</td>
<td>0.5274</td>
<td>-0.7512</td>
</tr>
<tr>
<td>CPIF</td>
<td>-0.5385</td>
<td>0.3424</td>
<td>0.7674</td>
<td>-0.0626</td>
</tr>
<tr>
<td>PY</td>
<td>0.2412</td>
<td>0.9337</td>
<td>-0.2379</td>
<td>0.1162</td>
</tr>
</tbody>
</table>
### Table A2-4. Selected Statistics Used to Calculate Table Wholesale Supply

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPCL</td>
<td>13.06540</td>
<td>2.44916</td>
</tr>
<tr>
<td>LC</td>
<td>1.26658</td>
<td>0.52179</td>
</tr>
<tr>
<td>OCIDW</td>
<td>0.41465</td>
<td>0.07079</td>
</tr>
<tr>
<td>YFUSDW</td>
<td>0.15301</td>
<td>0.03042</td>
</tr>
<tr>
<td>SKIDQW</td>
<td>0.07849</td>
<td>0.04513</td>
</tr>
</tbody>
</table>

The error term variance = 0.011989

#### The Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>PPCL</th>
<th>LC</th>
<th>OCIDW</th>
<th>YFUSDW</th>
<th>SKIDQW</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPCL</td>
<td>1.00000</td>
<td>-0.84573</td>
<td>0.86387</td>
<td>0.94969</td>
<td>-0.62372</td>
</tr>
<tr>
<td>LC</td>
<td>-0.84573</td>
<td>1.00000</td>
<td>-0.60965</td>
<td>-0.72765</td>
<td>0.53359</td>
</tr>
<tr>
<td>OCIDW</td>
<td>0.86387</td>
<td>-0.60965</td>
<td>1.00000</td>
<td>0.87752</td>
<td>-0.65002</td>
</tr>
<tr>
<td>YFUSDW</td>
<td>0.94969</td>
<td>-0.72765</td>
<td>0.87752</td>
<td>1.00000</td>
<td>-0.68424</td>
</tr>
<tr>
<td>SKIDQW</td>
<td>-0.62372</td>
<td>0.53359</td>
<td>-0.65002</td>
<td>-0.68424</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

#### The Diagonal Elements of the Inverted X'X Correlation Matrix

\[ r_{11} = 2.15744 \]
\[ r_{22} = 4.48217 \]
\[ r_{33} = 6.19329 \]
\[ r_{44} = 1.93292 \]

#### The Eigenvalue of X'X Correlation Matrix

\[ \lambda_1 = 3.23590 \]
\[ \lambda_2 = 0.46599 \]
\[ \lambda_3 = 0.20113 \]
\[ \lambda_4 = 0.09699 \]

#### The Eigenvector of X'X Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC</td>
<td>0.4484</td>
<td>0.8528</td>
<td>0.1852</td>
<td>0.1935</td>
</tr>
<tr>
<td>OCIDW</td>
<td>-0.5184</td>
<td>0.3617</td>
<td>0.3249</td>
<td>-0.7034</td>
</tr>
<tr>
<td>YFUSDW</td>
<td>-0.5269</td>
<td>0.0106</td>
<td>0.5500</td>
<td>0.6479</td>
</tr>
<tr>
<td>SKIDQW</td>
<td>-0.5025</td>
<td>0.3767</td>
<td>-0.7467</td>
<td>0.2191</td>
</tr>
</tbody>
</table>
Table A2-5. Selected Statistics Used to Calculate Table Wholesale Demand

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC</td>
<td>1.26658</td>
<td>0.52179</td>
</tr>
<tr>
<td>PPCLS</td>
<td>13.06540</td>
<td>2.40471</td>
</tr>
<tr>
<td>RD1</td>
<td>2.18558</td>
<td>0.31613</td>
</tr>
<tr>
<td>PITD</td>
<td>0.92161</td>
<td>0.08508</td>
</tr>
</tbody>
</table>

The error term variance = 0.021391

The Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>LC</th>
<th>PPCLS</th>
<th>RD1</th>
<th>PITD</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC</td>
<td>0.100000</td>
<td>-0.86137</td>
<td>0.91039</td>
<td>-0.20046</td>
</tr>
<tr>
<td>PPCLS</td>
<td>-0.86137</td>
<td>1.00000</td>
<td>-0.87891</td>
<td>0.60554</td>
</tr>
<tr>
<td>RD1</td>
<td>0.91039</td>
<td>-0.87891</td>
<td>1.00000</td>
<td>-0.43857</td>
</tr>
<tr>
<td>PITD</td>
<td>-0.20046</td>
<td>0.60554</td>
<td>-0.43857</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

The Diagonal Element of Inverted X'X Correlation Matrix

\[ r_{11} = 4.96863 \quad r_{22} = 4.68031 \]
\[ r_{33} = 1.68130 \]

The Eigenvalue of X'X Correlation Matrix

\[ \lambda_1 = 2.30016 \quad \lambda_2 = 0.60203 \]
\[ \lambda_3 = 0.09781 \]

The Eigenvector of X'X Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPCLS</td>
<td>0.6329</td>
<td>-0.1992</td>
<td>-0.7483</td>
</tr>
<tr>
<td>RD1</td>
<td>-0.5949</td>
<td>0.4933</td>
<td>-0.6346</td>
</tr>
<tr>
<td>PITD</td>
<td>0.4955</td>
<td>0.9467</td>
<td>0.1937</td>
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</tbody>
</table>

Table A2-6. Selected Statistics to Calculate Table Wholesale Supply

Means and Standard Deviation

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPSW</td>
<td>15.8888</td>
<td>2.5505300</td>
</tr>
<tr>
<td>SW</td>
<td>0.39163</td>
<td>0.1299900</td>
</tr>
<tr>
<td>ICALDW</td>
<td>0.52109</td>
<td>0.0637985</td>
</tr>
<tr>
<td>ALUSDW</td>
<td>0.20172</td>
<td>0.0310413</td>
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</tbody>
</table>

The error term variance = 0.057518

The Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>PPSW</th>
<th>SW</th>
<th>ICALDW</th>
<th>ALUSDW</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPSW</td>
<td>1.00000</td>
<td>-0.666795</td>
<td>0.805254</td>
<td>0.739527</td>
</tr>
<tr>
<td>SW</td>
<td>-0.666795</td>
<td>1.000000</td>
<td>-0.254692</td>
<td>-0.338612</td>
</tr>
<tr>
<td>ICALDW</td>
<td>0.805254</td>
<td>-0.254692</td>
<td>1.000000</td>
<td>0.755420</td>
</tr>
<tr>
<td>ALUSDW</td>
<td>0.739527</td>
<td>-0.338612</td>
<td>0.755420</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

The Diagonal Element of Inverted X'X Correlation Matrix

\[ r_{11} = 1.129511 \]
\[ r_{22} = 2.329160 \]
\[ r_{33} = 2.460148 \]

The Eigenvalue of X'X Correlation Matrix

\[ \lambda_1 = 1.94250 \]
\[ \lambda_2 = 0.81832 \]
\[ \lambda_3 = 0.23912 \]

The Eigenvector of X'X Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW</td>
<td>0.4073</td>
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<td>0.9160</td>
</tr>
<tr>
<td>ICALDW</td>
<td>-0.6356</td>
<td>0.3541</td>
<td>-0.6860</td>
</tr>
<tr>
<td>ALUSDW</td>
<td>-0.6558</td>
<td>0.2212</td>
<td>0.7218</td>
</tr>
</tbody>
</table>
Table A2-7. Selected Statistics Used to Calculate Table Wholesale Demand

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW</td>
<td>0.39163</td>
<td>0.12999</td>
</tr>
<tr>
<td>PPSWS</td>
<td>15.88880</td>
<td>2.39935</td>
</tr>
<tr>
<td>RD1</td>
<td>2.18558</td>
<td>0.31613</td>
</tr>
<tr>
<td>PM</td>
<td>1.01918</td>
<td>0.06432</td>
</tr>
<tr>
<td>PITD</td>
<td>0.92161</td>
<td>0.08508</td>
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</tbody>
</table>

The error term variance = 0.060521

The Correlation Matrix

<table>
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<tr>
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<th>RD1</th>
<th>PM</th>
<th>PITD</th>
<th>PPCL</th>
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</thead>
<tbody>
<tr>
<td>SW</td>
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<td>0.82763</td>
<td>0.51554</td>
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<tr>
<td>PPSWS</td>
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<td>-0.79969</td>
<td>-0.43909</td>
<td>0.70668</td>
<td>0.90880</td>
</tr>
<tr>
<td>RD1</td>
<td>0.82763</td>
<td>-0.79969</td>
<td>1.00000</td>
<td>0.43418</td>
<td>-0.43857</td>
<td>-0.86238</td>
</tr>
<tr>
<td>PM</td>
<td>0.51554</td>
<td>-0.43909</td>
<td>0.43418</td>
<td>1.00000</td>
<td>0.05290</td>
<td>-0.43372</td>
</tr>
<tr>
<td>PITD</td>
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<td>0.70668</td>
<td>-0.43857</td>
<td>0.05280</td>
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<tr>
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<td>0.90880</td>
<td>-0.86238</td>
<td>-0.44372</td>
<td>-0.58849</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

The Diagonal Element of Inverted X'X Correlation Matrix

\[
\begin{align*}
r_{11} &= 9.65137 \\
r_{22} &= 4.17168 \\
r_{33} &= 1.84184 \\
r_{44} &= 3.17807 \\
r_{55} &= 8.14153
\end{align*}
\]

The Eigenvalue X'X Correlation Matrix

\[
\lambda_1 = 3.37046 \\
\lambda_2 = 1.06634 \\
\lambda_3 = 0.37544 \\
\lambda_4 = 0.12014 \\
\lambda_5 = 0.06762
\]

The Eigenvector of X'X Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPSWS</td>
<td>0.5261</td>
<td>0.0772</td>
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<td>0.3517</td>
<td>-0.7581</td>
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<tr>
<td>RD1</td>
<td>-0.4867</td>
<td>0.1108</td>
<td>-0.6217</td>
<td>0.6002</td>
<td>0.9646</td>
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<tr>
<td>PITD</td>
<td>0.3704</td>
<td>0.6230</td>
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<td>-0.3812</td>
<td>0.2383</td>
</tr>
<tr>
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<td>0.2083</td>
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<td>0.5917</td>
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</table>
Table A2-8. Selected Statistics to Calculate Table Ex-vessel Demand

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
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<tbody>
<tr>
<td>ALQ</td>
<td>222528</td>
<td>57842</td>
</tr>
<tr>
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<td>0.19383</td>
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</tr>
<tr>
<td>RWAL</td>
<td>401812</td>
<td>58344</td>
</tr>
<tr>
<td>WHDW</td>
<td>1.94398</td>
<td>0.28232</td>
</tr>
<tr>
<td>TWH1</td>
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<td>0.08735</td>
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</tbody>
</table>

The Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>ALQ</th>
<th>ALIDQW</th>
<th>RWAL</th>
<th>WHDW</th>
<th>TWH1</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALQ</td>
<td>1.00000</td>
<td>0.49241</td>
<td>0.65860</td>
<td>-0.41146</td>
<td>0.72815</td>
</tr>
<tr>
<td>ALIDQW</td>
<td>0.49241</td>
<td>1.00000</td>
<td>0.16797</td>
<td>-0.12228</td>
<td>-0.09768</td>
</tr>
<tr>
<td>RWAL</td>
<td>0.85860</td>
<td>0.16797</td>
<td>1.00000</td>
<td>-0.60986</td>
<td>0.44869</td>
</tr>
<tr>
<td>WHDW</td>
<td>-0.41146</td>
<td>-0.12228</td>
<td>-0.60986</td>
<td>1.00000</td>
<td>-0.22219</td>
</tr>
<tr>
<td>TWH1</td>
<td>0.72814</td>
<td>0.09768</td>
<td>0.44869</td>
<td>-0.22219</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

The Diagonal Element of Inverted X'X Correlation Matrix

\[ r_{11} = 1.030462 \quad r_{22} = 1.601821 \]
\[ r_{33} = 1.923572 \quad r_{44} = 1.259650 \]

The Eigenvalue of X'X Correlation Matrix

\[ \lambda_1 = 1.92889 \quad \lambda_2 = 0.91589 \]
\[ \lambda_3 = 0.78950 \quad \lambda_4 = 0.33643 \]

The Eigenvector of X'X Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALIDQW</td>
<td>0.2387</td>
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<tr>
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<td>-0.1066</td>
<td>0.7522</td>
</tr>
<tr>
<td>WHDW</td>
<td>-0.5625</td>
<td>0.1374</td>
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<td>0.5810</td>
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<tr>
<td>TWH1</td>
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<td>0.8123</td>
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</table>
Table A2-9. Selected Statistics Used to Calculate Table 9, Ex-vessel Demand

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Standard Deviation</th>
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</thead>
<tbody>
<tr>
<td>SKQ</td>
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<td>138099</td>
</tr>
<tr>
<td>SKIDQW</td>
<td>0.10019</td>
<td>0.01245</td>
</tr>
<tr>
<td>WHDW</td>
<td>1.10019</td>
<td>0.28231</td>
</tr>
<tr>
<td>RWSK</td>
<td>955058</td>
<td>400791</td>
</tr>
<tr>
<td>TLH1</td>
<td>1.60801</td>
<td>0.46309</td>
</tr>
<tr>
<td>AYQ</td>
<td>556451</td>
<td>105603</td>
</tr>
</tbody>
</table>

The error term of variance = 0.023176

The Correlation Matrix

<table>
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<tr>
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<th>SKIDQW</th>
<th>WHDW</th>
<th>RWSK</th>
<th>TLH1</th>
<th>AYQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>SKQ</td>
<td>1.00000</td>
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<td>0.83556</td>
<td>0.45848</td>
</tr>
<tr>
<td>SKIDQW</td>
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<td>1.00000</td>
<td>0.07778</td>
<td>-0.11572</td>
<td>-0.11572</td>
<td>-0.11572</td>
</tr>
<tr>
<td>WHDW</td>
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<td>0.07778</td>
<td>1.00000</td>
<td>-0.90116</td>
<td>-0.90116</td>
<td>-0.90116</td>
</tr>
<tr>
<td>RWSK</td>
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<td>-0.11572</td>
<td>-0.90116</td>
<td>1.00000</td>
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</tr>
<tr>
<td>TLH1</td>
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<td>-0.86290</td>
<td>-0.86290</td>
<td>0.89282</td>
<td>0.42733</td>
</tr>
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The Diagonal of the Inverted X'X Correlation Matrix

\[ r^{11} = 1.32978 \]
\[ r^{22} = 6.32691 \]
\[ r^{33} = 8.82371 \]
\[ r^{44} = 7.51694 \]
\[ r^{55} = 1.83939 \]

The Eigenvalue of the X'X Correlation Matrix

\[ \lambda_1 = 3.18748 \]
\[ \lambda_2 = 1.14072 \]
\[ \lambda_3 = 0.48868 \]
\[ \lambda_4 = 0.10838 \]
\[ \lambda_5 = 0.07478 \]

The Eigenvectors of the X'X Correlation Matrix

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Table A2-10. Selected Statistics Used to Calculate Table 10, Ex-vessel Demand

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<td>SKQ</td>
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<td>ALQ</td>
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The error term variance = 0.091112

The Correlation Matrix

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The Diagonal Element of Inverted X'X Correlation Matrix

\[
\begin{align*}
    r_{11}^1 &= 4.14027 \\
    r_{22}^2 &= 1.38961 \\
    r_{33}^3 &= 5.47586 \\
    r_{44}^4 &= 6.37627
\end{align*}
\]

The Eigenvalues of X'X Correlation Matrix

\[
\lambda_1 = 2.91330 \\
\lambda_2 = 0.81531 \\
\lambda_3 = 0.17395 \\
\lambda_4 = 0.09743
\]

The Eigenvectors of X'X Correlation Matrix

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The error term variance = 0.052101

The Correlation Matrix

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The Diagonal Element of Inverted X'X Correlation Matrix

\[
r_{11}^2 = 1.05871 \\
r_{22}^2 = 1.45719 \\
r_{33}^2 = 1.16068 \\
r_{44}^2 = 1.31583
\]

The Eigenvalue of X'X Correlation Matrix

\[
\lambda_1 = 1.55623 \\
\lambda_2 = 1.15324 \\
\lambda_3 = 0.88369 \\
\lambda_4 = 0.42685
\]

The Eigenvector of X'X Correlation Matrix

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The error term variance = 0.021560

The Correlation Matrix

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The Diagonal Element of the Inverted X'X Correlation

\[ r_{11}^{11} = 1.62096 \]
\[ r_{22}^{22} = 5.66542 \]
\[ r_{33}^{33} = 1.77110 \]
\[ r_{44}^{44} = 5.34892 \]

The Eigenvalues of X'X Correlation Matrix

\[ \lambda_1 = 2.22587 \]
\[ \lambda_2 = 1.25873 \]
\[ \lambda_3 = 0.42279 \]
\[ \lambda_4 = 0.09261 \]

The Eigenvectors of X'X Correlation Matrix

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APPENDIX 3
Data Used in the Analysis
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*See definitions, measurements, and source in Table 5-1.*