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Title Photo-elastic Analysis of a Pier Section of the Bonneville Dam

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Photo-elasticity was investigated as a method of stress analysis for a gravity dam, using in particular the pier section of the spillway dam at Bonneville, Oregon, on the Columbia River. Certain definite limitations were found, the most important being the approximations necessary in applying a gravity load to a small model.

The section analyzed was cut from clear bakelite with the masonry and foundation monolithic. Loads were applied to simulate (1) water load only, (2) dead weight correct at the base only, and (3) the complete load correct at the base only. The complete analysis, including the magnitudes of the principal stresses and their directions, was made for the condition of water load only. The magnitudes and directions of the maximum shearing stresses at two planes in the foundation and the principal stress directions near the base were determined for the three loading conditions.

An effort was made to determine the effect of a weak foundation material on the stresses in the dam itself by constructing models made of two different kinds of bakelite joined along approximate construction contours. Due to the type of joint, irregular boundary, and complicated loading, no quantitative results were attempted.

The theory of photo-elasticity was discussed in its general relation to the field of stress analysis and more particularly as to the underlying principles of the method itself. The determination of the separate principal stresses by various methods was outlined and a new method applicable to some problems was given in detail.

PHOTO-ELASTIC ANALYSIS OF A PIER SECTION OF THE BONNEVILLE DAM

by

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PREFACE

This study is the result of experimentation and research suggested by Dr. D. C. Henny, Consulting Engineer of Portland, in connection with certain problems arising in the analysis and design of the spillway dam an the Columbia River at Bonneville, Oregon. Throughout the work he has continually offered both encouragement and constructive criticism.

Mr. C. I. Grimm, Chief Civilian Engineer, and the Engineers of the Bonneville Section assisted materially by their cooperation and financial support. In addition the Bureau of Reclamation, Denver, Colorado, has made available copies of recent unpublished reports of analysis and research on masonry dams.

This report is in the nature of a joint thesis between the departments of Civil and Mechanical Engineering. The cooperation between the two departments has been excellent. Much credit is due Professor S. H. Graf, Head of the Department of Mechanical Engineering, and Professor C. A. Mockmore, Head of the Department of Civil Engineering, for without their help this study would not have been possible.

H. D. E.

Attention must be called to the color photographs. They are the work of K. R. Eldredge of the Photography Department and are actual reproductions of the color patterns as they appeared when the model was subjected to a load. The printing was done by hand from three-color plates made of dyed gelatine. Four of these photographs have been included, the same pictures also appearing as black and white prints.

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INTRODUCTION

PHOTO-ELASTIC ANALYSIS OF A PIER SECTION OF BONNEVILLE DAM

PART I. INTRODUCTION

1. Status of Masonry Dam Analysis. For many years engineers have been aware that the stresses near the base of gravity dams differed considerably from an assumed straight line variation. The effect of reentrant corners at the heel and toe and the restraining action of the foundation were uncertain. As the size of proposed dams increased, so did the necessity of a more exact stress analysis.

Dams were formerly designed as vertical cantilever beams. In the past few years investigations have been made by different methods, both theoretical and experimental, for particularly shaped sections under their individual loading conditions. These have shown that the influence of the restraining action of the base on the stress distribution extended approximately to one third the height of the dam.

In investigating this condition experimentally, much ingenuity has been used and interesting results have been obtained. (30,39)* No one method has been accepted, however, as outstanding in its adaptability and exact duplication of conditions. In model studies where stresses were determined from measurements of strain or displacements, approximations had to be made either because of the elastic constants of available material, or the ratio of dead load

*Numbers in parenthesis refer to bibliography.

to water load. The conditions of similitude present difficult problems.

For a mathematical analysis, it has generally been necessary to make certain simplifying assumptions before a stress function could be found to satisfy the boundary conditions of shape and loading. The case of a triangular section with hydrostatic load on only the upstream side has been treated in this way. (9, 21) Advances are continually being made along mathematical lines. But, in general, the elastic theory has its definite limitations in addition to its difficulty of application.

It is natural, then, that as new methods of stress analysis are developed, an attempt should be made to apply them to the solution of this problem. With this thought in mind, the present investigation of a pier section of Bonneville spillway dam was undertaken by the method of photoelasticity.

After this work had been under way for some time, it was discovered that a similar investigation had been made by Dr. J.H.A. Brahtz, of the Bureau of Reclamation on a section of the Morris dam being built for the city of Pasadena, California. To date, his investigations have not been published, but he has kindly made his manuscript copy available. (6) As the contours and loading conditions are entirely different in the two dams the results can be compared only in a general way.

2. <u>Statement of Problem.</u> The purpose of this investigation was as follows:

(a) To inquire as to the adaptability of the photoelastic method to gravity dam analysis;

(b) To analyze as far as possible the stresses in a pier section of the Bonneville spillway dam;

(c) To attempt a study of the effect of a weak foundation material on the stresses in the dam itself.

Three types of loading were used: (Fig. 13)

(1) Water load only, including the gate load taken by the upper part of the pier, reservoir level at elevation + 72, and tail water level at elevation + 5;

(2) Masonry load only, applied as three distributed loads to the top of the model section to be correct at the base only (elevation -45);

(3) Both (1) and (2) corresponding to full load, taken to be correct at the base only.

In attempting to determine the effect of lower modulus material on stresses in the dam, the following sections were made and tested:, (Fig. 1)

(1) Section I, pier section and foundation slab monolithic, made from quarter inch clear bakelite, E = 613,000 lb./sq.in.;

(2) Section II, constructed from the same material as Section I, to the same outline with a slight modification at the toe, but with the pier section cut apart from

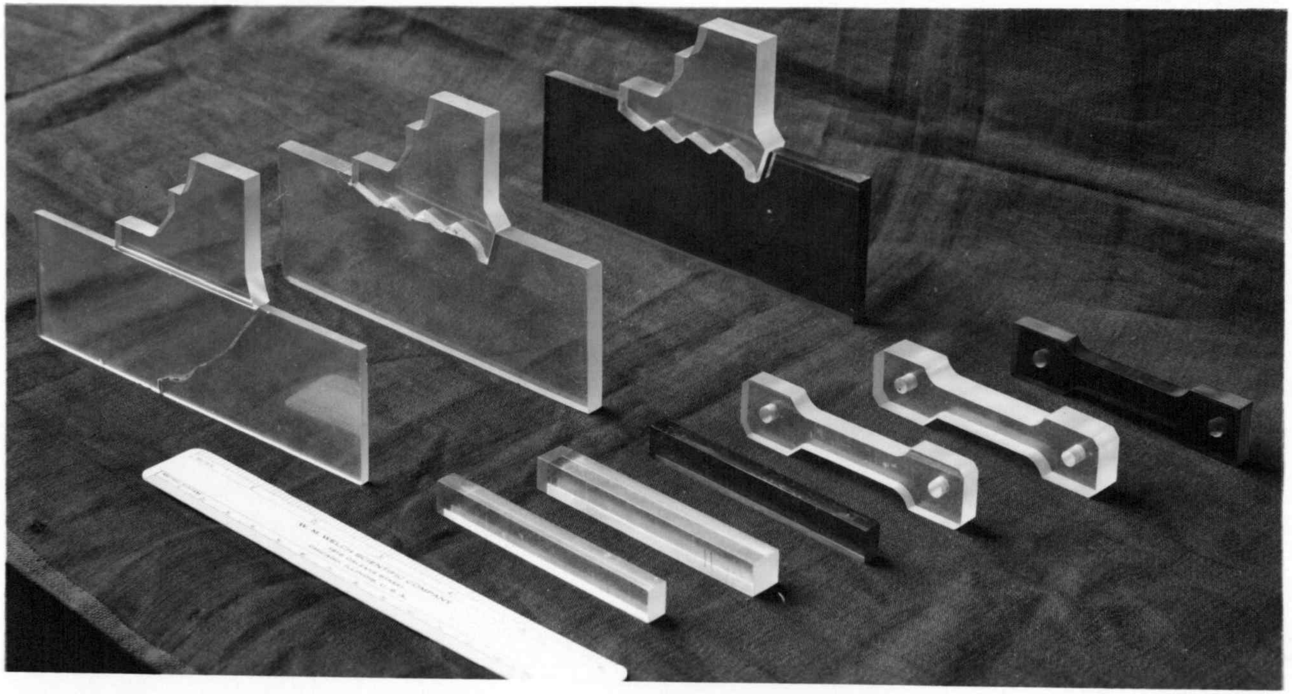


Fig. 1.-Pier Section Models, Calibration Beams, and Tension Members

the foundation along approximate construction lines;

(3) Section III, with the identical outline and construction joint as Section II but with the pier section made of 3/8 inch clear bakelite, E = 700,000 lb./sq.in. The effective ratio of Young's modulus is therefore 1 to 3, approximately.

3. Stress Analysis in General. One of the big problems in engineering is to determine the critical unit stress to which any material in use in a machine or a structure may be subjected. By critical stress is meant that stress which will cause the machine or structure to be no longer capable of carrying on the function for which it was designed.

It follows, therefore, that stress analysis is only one of three things that make for an ultimately successful performance. In the first place, the loads or forces acting must be knows; second, the stresses produced by these loads or forces must be determined; and third, the knowledge of materials must be sufficient to insure the product to be both safe and economical. An error in any one of these may cause either failure or gross extravagance. Since both are wasteful, it is customary to strive for the middle ground of efficiency and economy.

So it comes about that forcasting is a primary function of an engineer. He must say what the demands will be, based on previous experience or his ability to look ahead. He must analyze those demands to see their effect on each individual part, and tell how the individual parts will function together as a whole. And finally he must choose materials strong enough and large enough to efficiently take care of their respective duties.

It has been said concerning this last point, the knowledge of the properties of materials, that if the exact stresses to be taken were known, the most efficient and economical material could be chosen. This is probably true for some materials whose characteristics and properties are consistent. After experience and information has aided in the judgment as to the loads to design for, the part remaining is to determine the stresses induced by those loads.

When a load or force is applied to an elastic material, it resists any attempt to change its size or shape. The customary way to measure the intensity of the deforming force is in units of force or weight acting over an area, as poinds per square inch. This intensity of force being resisted by a material is known as stress, and may be applied to a point as well as a finite area. Quantities of this sort defy direct measurement. There is no scale or mechanism known to measure pounds per squre inch. Resort must be made to indirect methods, by measuring either those things that cause the stress and the dimensions of the stressed area or the results of that stress acting in the form of strain, displacement, or other changed properties, and finally by using these quantities and computing the stress.

First and foremost among the methods of stress analysis must be placed that of mathematical solution. This includes both the ordinary formulas as well as the more exact elastic theory. All of these expressions are based on the assumption that there is continuity of elastic action throughout the members and therefore that the stress distribution on any section can be expressed by some mathematical law or equation. One example is a tension member in which the stress is assumed to be uniformly distributed over each cross-section. Another is a beam in which the stress increases directly as the distance from the neutral axis.

But the assumption that there are no discontinuities in the stress distribution may be erroneous and the results obtained not even a rough approximation of the real stress at a point. Some conditions that cause high concentration are: abrupt changes in section; discontinuities in the material itself, as air holes in concrete, pitch pockets and knots in timber; initial stresses in a member due, for example, to shrinkage in setting, heat treatment, or cold working; and excessive pressures at points where external loads are applied. Stresses due to actions such as these are called localiz ed stress and sometimes prove to be the controlling factor.

Problems of this kind continually must be dealt with and since the mathematical solution is either too limited or becomes too involved other methods have had to be developed. Some of these are very ingenious and unique for certain types of problems.

Special properties of brittle, ductile, or plastic material have been used to determine stresses in elastic material. (29)

Another group depends upon the use of scale models where, in general, the model and the prototype have essentially the same form. The extent of the similarity must in all cases be based upon a strict dimensional analysis. This often leads to unsurmountable difficulties, particularly where constants of gravitational acceleration, elasticity, etc., must be reduced or increased to conform with the requirements of dimensional homogeniety between the model and prototype.

Problems in indeterminate structures have been solved by the use of dynamically similar models where deflections could be measured and the resulting stress computed.

Still another group depends upon the similarity of the differential equations describing elastic, electric, hydrodynamic and thermodynamic phenomena. These are commonly termed analogies. Seldom do the analogous systems have even the slightest physical resemblance.

4. <u>Photo-elasticity in the Field of Stress Analysis.</u> Photo-elasticity is the name given to the science of measuring stress by utilizing the effect of the changed properties in the material itself on the transmission of light. The idea of measuring pounds per square inch by matching colors in order to determine the relative difference of the length of lightwaves is unique. The principles of the method are not new, being based on discoveries by Sir David Brewster in 1816, enlarged and treated mathematically by James Clerk Maxwell about 1850, and used probably for the first time in analyzing an engineering structure by Professor Mesnager of Paris in 1913. Until recently progress has been held back by the lack of suitable material for constructing models. Glass was then the only material used. Professor E. G. Coker (10) of Cambridge, discovered that celluloid and bakelite were both optically suitable and very easy to machine. At about the same time Professor L. N. G. Filon, an eminent mathematician, also of Cambridge, simplified Maxwell's method of stress determination from optical observations. Due mainly to the work of these two men, photo-elasticity has increased in its use and popularity until it is now accepted as one of the most powerful tools in stress analysis.

5. <u>Limitations and Advantages of Photo-elasticity.</u> As all methods of stress analysis in use today have their limitations, so does photo-elasticity. In the first place

the apparatus necessary represents a large investment. Secondly, the operation of that apparatus requires considerable skill and technique. It is therefore possible for the human element to enter somewhat into the solution although this may be minimized by the use of photography. Much time is required for the construction of models and loading devices, but this is negligible when the fact is considered that problems may be solved that were impossible before.

The principal limitation to date is that two dimensional stress systems only may be solved. That is, systems in which the forces all act in one plane producing stress constant through any thickness of the model. Many problems in three dimensions may be broken up and analyzed as coplaner systems. But there are still many remaining to be solved that may be considered as two dimensional.

In the past, solutions have only been obtained for problems where the body forces or dead weights could be neglected. In the present investigation the weight of the entire masonry was applied at the top of the dam model to obtain correct values at the base only. Dr. Brahtz has suggested a method (6) that has since been tried at Columbia University and an article recently published showing examples. (34) This gives interesting results by replacing the gravitational pull on models of large structures by a centrifugal force obtained by rotating the model at a constant velocity. The method gives promise of further enlarging the scope of photo-elasticity.

Among the advantages of photo-elasticity may be listed the following: indeterminate structures may be analyzed as well as local stress concentrations; the maximum shear at all points, the magnitude of any stress concentration, and the tangential stress along any unloaded boundary may be determined directly from observations; the direction of the principal stresses may be obtained from observations and a small amount of drafting; and the quantitative value of the principal stresses may be obtained by methods to be outlined later.

6. Similarity Conditions. Many problems in plane stress analysis are completely independent of the elastic constants. This is true of problems where the stress on the boundary is everywhere given. It is shown in treatises on elasticity (22:p.145) (10:p.129) that the stresses in any two dimensional problem may be expressed in terms of a function known as Airy's stress function. This function itself is independent of the elastic constants of the material. For the case where the boundary conditions are all stress conditions the stresses in terms of this stress function also do not involve these constants and the distribution is entirely independent of the material used.

The same is not true, however, when the boundary condition involves either displacements or partial displacements. (10:p.414) But if stress conditions can be substituted for these displacement conditions the problem may still be solved.

As long as the material of the model is homogeneous, isotropic, and obeys Hooke's law of proportionality of stress to strain, results obtained from a model of, say, bakelite may be applied to the usual engineering materials.

PART II

THEORY

PART II. THEORY

1. Definition of Principal Stresses and Shear. In any elastic material under any kind of load there are possible only three different kinds of stress: tension, compression, and shear. These may be considered as acting in any direction and in any combination on the faces of a small element of the material. When the exterior forces are all in one plane the stress normal to that plane is zero or at least negligible in a thin plate. With the remaining forces constant over any cross-section the element of volume, the cube, may be thought of as a square with unit stresses acting on the four faces. The small square must be in equilibrium so the stresses on opposite sides, when resolved in one direction will be equal and opposite and the square may be rotated so that all the stresses are perpendicular to the four sides. These normal stresses, acting on planes along which the shear is zero, are called the principal stresses. They are the maximum and minimum acting at any point, and are at right angles to each other. If they are known, the stresses in any direction may be obtained most conveniently in terms of normal and tangential components. This tangential component is called shear.

The maximum value of the shear at any point is important because some materials are weakest in shear. It is easily shown that this maximum value is equal to one half the difference of the principal stresses.

Consider a small element AD, Fig. 2, with principal unit stresses P and Q acting on opposite faces. As is customary, take a tensile force as positive and a compression as negative. Pass a plane through a corner, B, making an angle θ with the side AB. Then with triangle BAC as a free body, resolve all the forces parallel to the plane BC. Since P, Q, and the shear S, are all unit forces they must be multiplied by the area of the face upon which they act. As the thickness is unity, this is equal to the length. This gives:

$$BC\ S = AB\ P \sin\theta - AC\ Q \cos\theta$$

or $$S = \frac{AB}{BC} P \sin\theta - \frac{AC}{BC} Q \cos\theta$$

but $$\frac{AB}{BC} = \cos\theta \quad \text{and} \quad \frac{AC}{BC} = \sin\theta$$

so that $$S = P \sin\theta \cos\theta - Q \cos\theta \sin\theta$$

$$S = (P-Q) \sin\theta \cos\theta$$

or $$S = 1/2\ (P-Q) \sin 2\theta$$

The sin 2θ is a maximum and equal to 1 when $2\theta = 90°$ or $\theta = 45°$. The maximum shear is therefore equal to $1/2(P-Q)$ and acts in a direction making 45° angles with the principal stresses.

2. <u>General Theory of Photo-elasticity.</u> An external load on a transparent thim plate is responsible for the existence at every point within of two principal stresses, usually called P and Q. The plate at once assumes the property of double refraction, causing a beam of plane

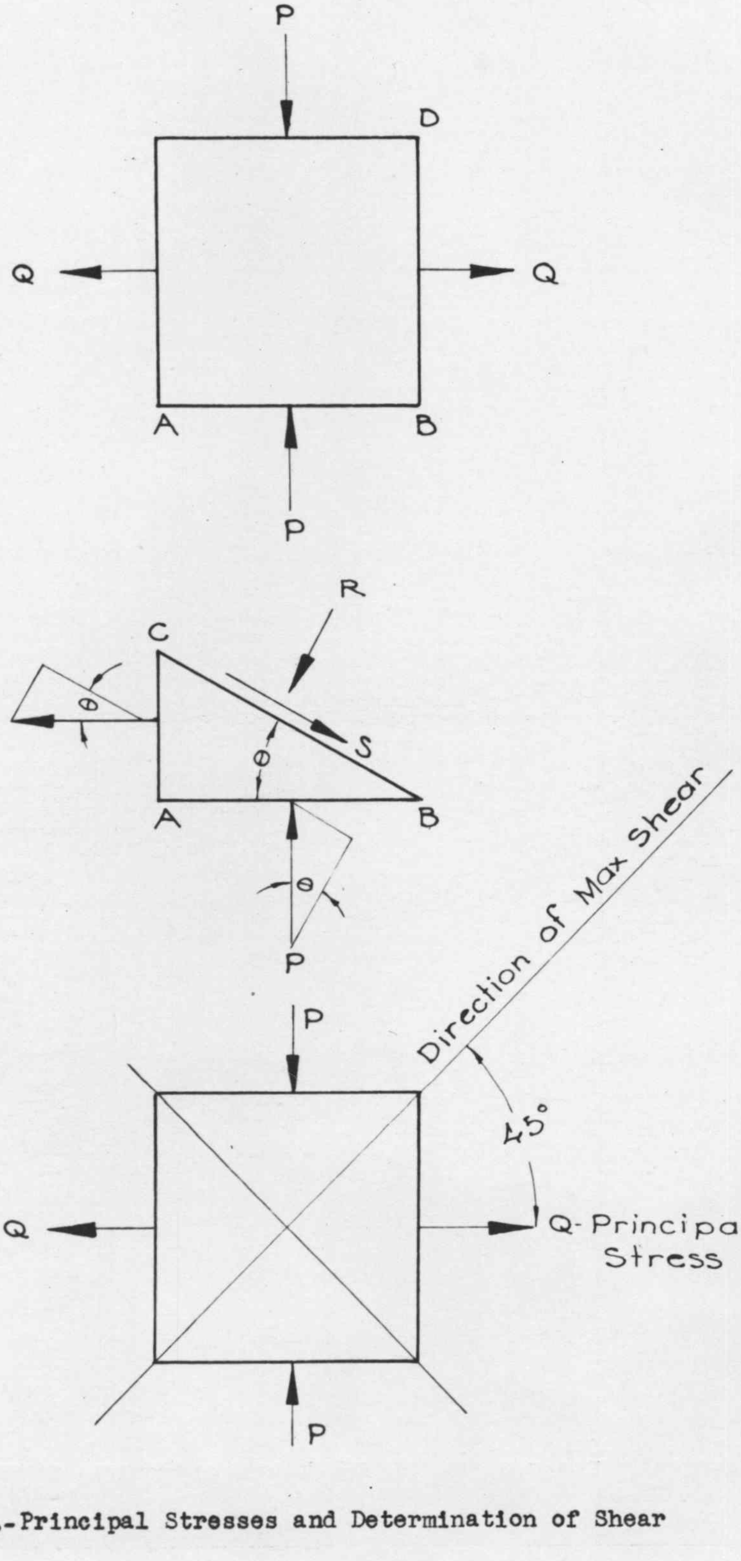


Fig. 2.-Principal Stresses and Determination of Shear

polarized light to break up into two orthogonal beams vibrating in planes which contain the directions of P and Q as elements. Of equal importance is the fact that the two beams pass through the plate at varying velocities, each beam being retarded in direct proportion to the magnitudes of the principal stresses. It is upon these facts that the whole science of photo-elasticity is based.

Fig. 3 shows a ray of plane polarized light, OA, from a polarizing prism passing perpendicularly through a loaded plate at a point where the principal stress directions are P and Q as shown. The ray OA is broken up into its two components, OB and OC, parallel to P and Q. If the principal stresses at this point are not of equal magnitudes, OB and OC will be out of phase, since a relative retardation will exist. Only the components of these two rays parallel to a particular plane will be transmitted by the analyzing prism and the remainder absorbed. The light leaving the analyzer is also plane polarized, but it is out of phase by the same amount as the two rays which left the stressed point. The phase difference existing in the analyzed beam gives rise to interference colors which cover the image of the strained specimen. These bands, therefore, are a direct measure of the difference of the principal stress. Since the maximum value of the shear at any point is one half the difference of the principal stresses, a direct measurement is obtained.

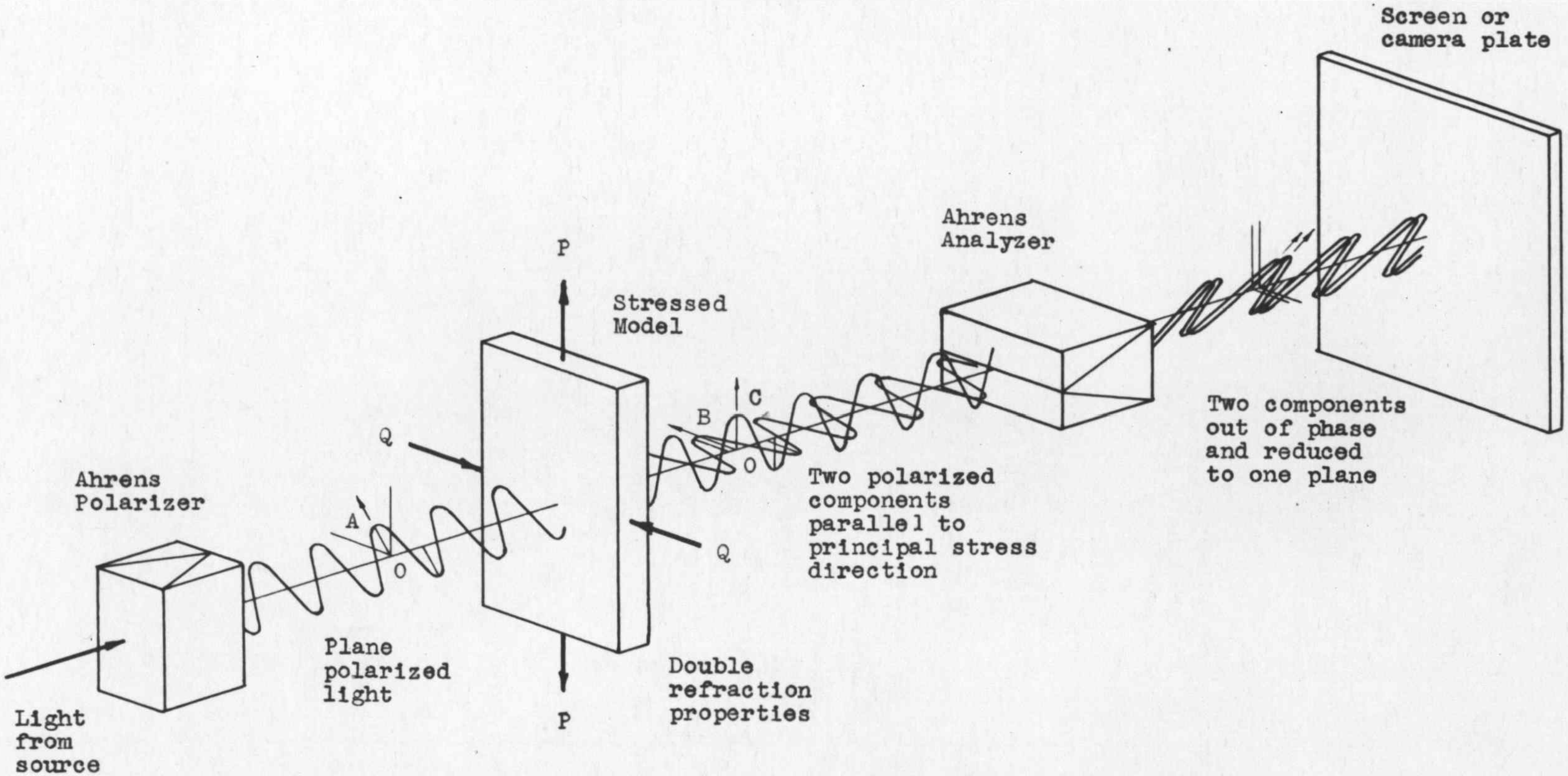


Fig. 3.-Schematic Diagram of the Action of One Light Ray

Using a source of white light the interference fringes are color bands, called "isochromatics", that increase in regular sequence so that the same color appearing several times will have a different value or be of different "order". If a monochromatic source is used interference causes a change of intensity so that light and dark bands or "fringes" are formed.

If at any point the principal stresses are equal there will be no relative retardation, the two components emerging from the analyzer will be in phase, and complete eclipse will take place causing a dark point or line to appear.

Also, if the principal stress direction coincides with the plane of polarization only one ray will emerge from the loaded plate. It will be in a plane at right angles to the plane of light transmitted by the analyzing prism and therefore will be completely absorbed causing a dark band. This dark band is called an "isoclinic" since it is the locus of all points whose principal stress direction makes a constant angle with a reference axis. Fig. 5 shows an isoclinic for one angular setting of the crossed prisms. If a map is made showing the isoclinic lines for different positions of the plane of polarization, sufficient information will be obtained to develop a complete network of the stress trajectories. (See Fig. 17)

Using plane polarized light, then a dark band may have either of two meanings: that the stress difference (shear) is zero or that the stress direction coincides with

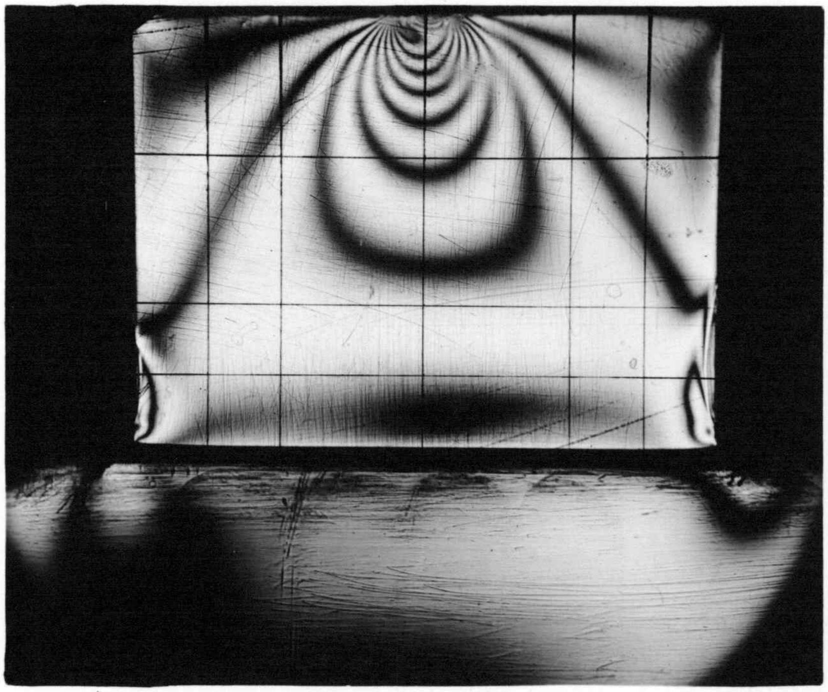


Fig. 4.-Quarter-Wave Plates in Place
Showing Fringes Only

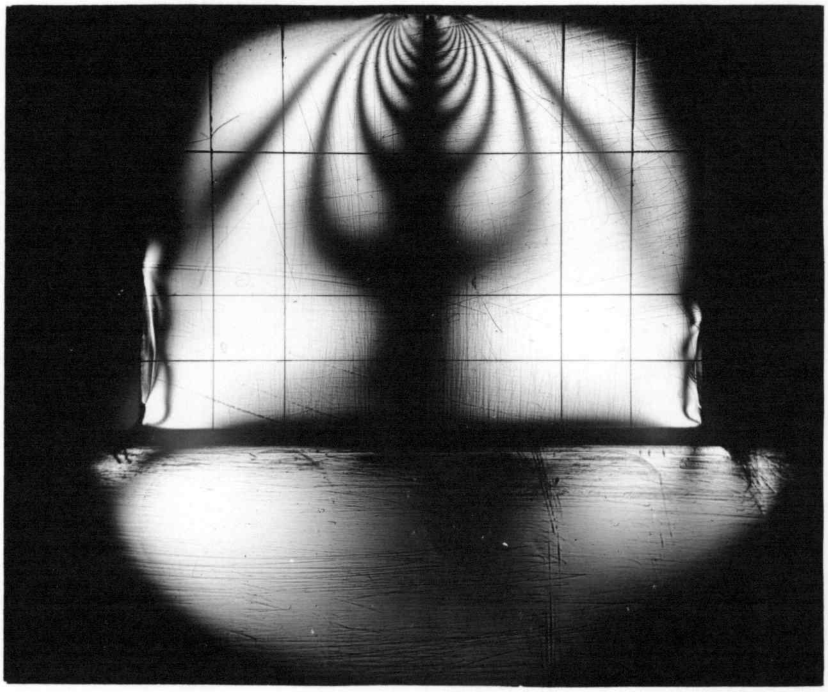


Fig. 5.-Quarter-Wave Plates Removed, Showing Fringes
with Isoclinics for Vertical Stress Direction

the orientation of the plane of polarization. To distinquish between the two, so-called quarter-wave plates are inserted on each side of the stressed plate to remove the directional effect. The first quarter-wave plate has the power to circularly polarize light; this is, it produces an effect equivalent to spinning the beam of plane polarized light about its axis of propagation. When this light strikes the specimen it will have no directional properties capable of producing isoclinics. The second quarter-wave plate stops the "spinning" of the beam so that, as before, only double refracted plane polarized light will strike the analyzer. Now, if dark bands are present they can mean only that the stress difference is zero. Since all the isoclinics are removed, an uninterrupted view is obtained of the isochromatics. Another way to distinquish between the two is to rotate simultaneously the crossed prisms using only plane polarized light. The isoclinics will rotate and the (P-Q) lines will remain stationary.

A quarter-wave plate will produce circularly polarized light only for the light of the wave length for which it was designed. When white light, composed of vibrations of different wave lengths is used all the wave lengths except one will be elliptically polarized, which has somewhat undesirable directional properties. For this reason a monochromatic source is sometimes used, obtaining light and dark fringes instead of color bands, and in addition making it possible to obtain much clearer photographs.

Perhaps the most satisfactory method of determining the magnitude of the fringe or color order in pounds per square inch is to put a known load on a specimen in such a way that one of the principal stresses is zero and the other may be calculated using the common engineering formulas. Such a specimen is a simple tension member where the stress is all uniform and longitudinal. The amount of stress required to change the color through a complete cycle is a measure of the fringe order. A more common method is to use a simple beam with equal loads applied at the one third points (Fig. 11) so that no vertical shear is present between the loads and the fringe magnitude increases from the neutral axis to the outside fiber. (Fig. 12) The transverse stress being zero, the color bands represent directly the longitudinal fiber stress which may be calculated knowing the bending moment applied and the dimensions of the beam. This is commonly known as a calibration beam since it calibrates fringe orders in models under test in pounds per square inch.

<ins>Summary.</ins> Information obtained optically and its interpretation.

Color bands--isochromatics or fringes which are lines of constant shear or (P-Q) throughout the model.

Isoclinics, or the locus of points whose principal stress direction makes a constant angle with a reference axis.

Concentration of stress is indicated by the distance between isochromatics.

At unloaded boundaries or other places where one principal stress is known to be equal to zero the isochromatic is a direct measure of the other principal stress.

3. Determination of P and Q separately. The problem presented is one of finding two unknowns with one equation. Knowing (P-Q), the direction of P and Q at any point, and the stress at the boundary, it is possible to determine P and Q separately or the quantity (P+Q) which is the second equation needed.

(A) Method of Graphical Integration.

This method is due to L. N. G. Filon. It is thoroughly derived in his book "Photo-elasticity" (10), in an article written by him and published in "Engineering" in 1923, and in several other articles to which reference is given. (12),(24). The derivation must be understood to properly apply the equations but since they were not used in this report they will only be stated and explained.

Progress is made from a boundary where the stress is known, along a stress trajectory by adding or subtracting the change from the previously determined stress. It amounts to finding the value of (P-Q) for points along the trajectory, measuring certain angles made with the isoclinics crossed, plotting the product of (P-Q) and the cotangent of that angle as ordinates and the increment of the isoclinic parameter as abscissas and determining the area underneath the curve to the point where the stress is desired. The equations are:

$$P = P_o + \int (P-Q) \cot \psi \, do$$

$$\text{and} \quad Q = Q_o + \int (Q-P) \cot \psi_1 \, do,$$

where P_o and Q_o are known boundary stresses, ψ and ψ_1, are angles made by the isoclinic line and the direction respectively of P and Q at the point in question, and $d\phi$ is the difference in degrees between two adjacent isoclinics, usually kept constant. Fig. 6 is a diagram illustrating these symbols. Where the angle ψ becomes small or large the cot ψ changes rapidly and any error in the angle measurement causes a large error in the final value of the stress. Resort is then made to the geometry of the figure and cot ψ determined in terms of the intercept in the P or Q directions between the isoclinic through the point and a neighborhing isoclinic and the distance along the trajectory from the last isoclinic. These equations are:

$$P = P_o - \Delta\phi \int (P-Q)\frac{ds_1}{\Delta y}$$

$$Q = Q_o - \Delta\phi \int (P-Q)\frac{ds_2}{\Delta x}$$

where the only change is ds_1 and ds_2 which are the distances along the trajectory in question and Δy and Δx which are intercepts made by the isoclinics.

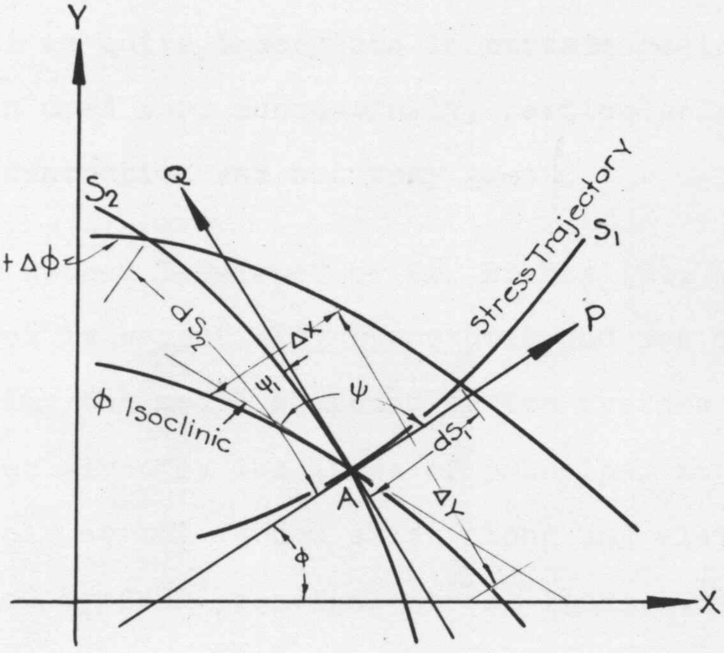
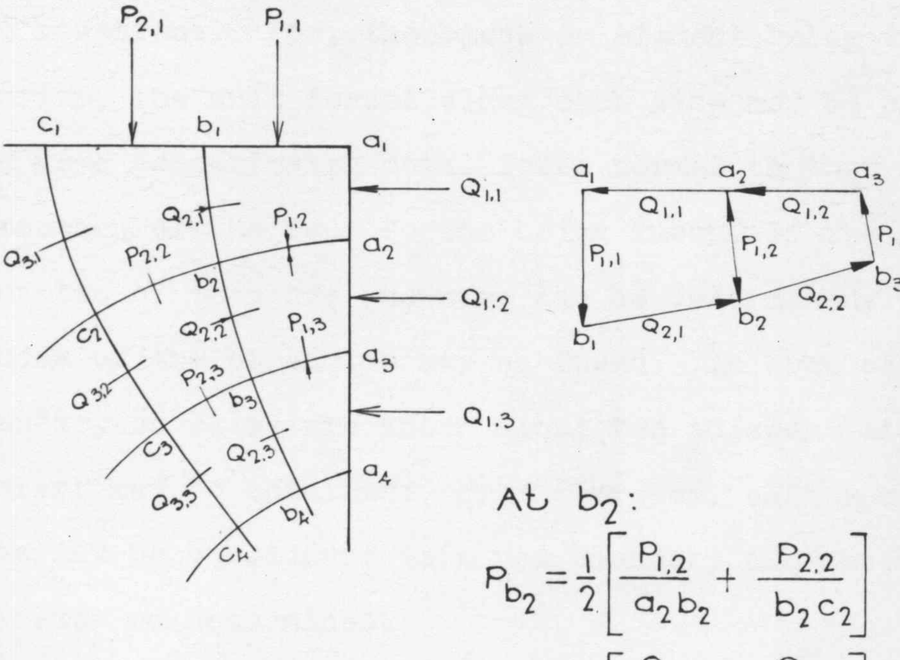


Fig. 6.-Symbols Used in the Graphical Integration Method



At b_2:

$$P_{b_2} = \frac{1}{2}\left[\frac{P_{1,2}}{a_2 b_2} + \frac{P_{2,2}}{b_2 c_2}\right]$$

$$Q_{b_2} = \frac{1}{2}\left[\frac{Q_{2,1}}{b_1 b_2} + \frac{Q_{2,2}}{b_2 b_3}\right]$$

Fig. 7.-Graphical Method of Obtaining Principal Stresses

As may be seen the whole method is rather long and tedious as well as quite inaccurate in certain regions. But it has been used very successfully, particularly where the stress concentration was not very great.

(B) Graphical Method Developed by Dr. Brahtz (6).

This method is essentially structural and was developed by considering the model composed of two systems of orthognal arches given by the lines of principal stress. No shear but only normal forces exist along any element or block bounded by four principal stress lines, two from each system. The problem is then reduced to determining the force polygon having given the funicular polygon and certain reactions. For, the block or element being in equilibrium, the unit forces along each side may be considered as a concentrated total force normal to that side. The direction of the four forces being known, if the magnitudes of two of them are known or can be determined, the magnitudes of the other two may be found. In some cases the boundary stresses are known along two adjacent sides and a start may be obtained. From there on, each row of elements may be considered as a new boundary and the forces on the next row determined.

Consider a system of P and Q stress trajectories (Fig. 7) intersecting at the points a_1, a_2, a_3, etc. in equilibrium with normal forces acting on each face. If two of the forces as $P_{1,1}$, $Q_{1,1}$, are known the other two may be determined by constructing the force polygon. Proceed next to the element $A_2\ A_3\ B_2\ B_3$, where the force $Q_{1,2}$ is known and $P_{1,2}$ taken from the previous element. It is necessary to start the construction at the boundary $a_1\ a_2\ a_3$ --- where the forces are known or can be determined from the isochromatics and boundary forces. When all the forces $Q_{2,n}$ are found, consider $b_1\ b_2\ b_3$ --- as a new boundary, apply the forces $Q_{2,n}$ and determine the forces $Q_{3,n}$ as before. Since these are total forces, the average stresses over the elementary sides are found by dividing the forces by the corresponding sides. Finally the principal stresses at any point, b_2 say, may be found by adding the average stress from the faces on each side together and dividing by two. If the adjoining faces differ much as to size, account must be taken and the ratio adjusted accordingly.

Only one force, $P_{n,1}$, will have to be computed for each strip by the use of the isochromatics so a continual check is obtained in the construction. The construction is easily arranged in a force diagram so that duplication of lines is avoided.

This method has the advantage that it is fast, and everything is in plain view with many opportunities for continual check and adjustment. The size of the elementary divisions would depend on the sharpness of the curvature of the stress line. It is an approximation to the extent that the elements are considered as straight on each side with the stress varying uniformly along the sides. However, the error induced is undoubtedly negligible when, as was the case in the tests of the dam reported here, the acting unit forces on the model are eight times the actual unit forces on the prototype, making it necessary to divide the stresses obtained in the model by eight in applying the results to the actual structure.

(C) Other Methods.

1. A graphical method for constructing lines having a constant value for the sum of the principal stresses (P+Q) called "isopachic lines" has been reported from the Technishe Hochschule, Munich, Germany, by Dr. H. P. Neuber (25). From the two networks, (P+Q) and (P-Q), the individual stresses at any point can be determined. The method is quite involved and has not as yet been accepted in the form published in this country.

2. A method proposed by Prof. Coker (10) and used very extensively by him is that of measuring the change in thickness of a model under load. By means of this measurement, together with a knowledge of Poisson's ratio and Young's modulus, the value of (P+Q) can be determined at any point. The disadvantage of this method is that the deformations to be measured are in the order of millionths of an inch and reliable results are very improbable.

3. Purely optical determinations are being developed at the present time. One method by M. M. Frocht (14) consists of photographing interference fringes that show lines of constant (P+Q).

Another method used by Favre in Zurich consists of measuring, by the use of an interferometer, the retardation of a ray of plane polarized light when the plane of polarization coincides with the principal stress direction. This retardation is proportional to the stress. Although the method is possible it is not of great practical importance as it requires very sensitive precision apparatus, a large amount of time and a very high degree of technical skill for its operation.

4. Membrane Analogy. The equation of the small ordinates of a membrane, stressed with a constant tension and having the same pressure on both sides, is the same as the equation defining the distribution of (P+Q). A membrane may be stretched over a form whose height from a datum plane is equal to the sum of the principal stresses at the boundary of a loaded model. The value of these stresses may be obtained from photo-elasticity. The ordinates of the membrane from the datum plane will then give the value of (P+Q) at any point.

As may be seen, this final step to a complete quantitative analysis requires considerable skill, technique and experience. Sometimes, under complex loading conditions and irregular contours the solution by any method would be subject to question. Most methods require the boundary stresses to be known making the final results dependent on the accuracy of the boundary determination, which, due to the application of loads and the possibility of initial stress, are the least accurate.

Fortunately this complete analysis is not the strongest point of photo-elasticity. The determination of the direction of stress at any point by itself is very important. The value of the maximum shear and the principal stress at unloaded boundaries is likewise of value. Often the maximum fiber stress is at a boundary or shear may be the controlling stress of the material. Even a glance at a loaded model which shows where the stress concentrations occur and their relative amounts is highly instructive. With the development of technique in model construction, loading apparatus design, and experience in testing, much valuable information may be obtained in a short time.

PART III

PROCEDURE

PART III. PROCEDURE

1. General.

Providing the optical apparatus is at hand the following is an outline of the procedure for the solution of any problem in photo-elastic stress determination:

(1) The material used for a model must be free from initial stress due to manufacturing processes and temperature changes. If it is not, annealing should be done before the models are machined to scale, by heating to a temperature of 70° C. to 80° C. and cooling at about 3° C. per hour to room temperature.

(2) The model should be accurately cut to the proper size, the final cuts with sharp tools and small bites. A calibration beam should be cut from the same part of the stock as the model and undergo similar treatment.

(3) Place the model in a loading frame and in a parallel beam of circularly or elliptically polarized light.

(4) Apply known loads accurately.

(5) Photograph or sketch the isochromatics or fringes.

(6) Using plane polarized light, sketch the isoclinics for various angular settings of the crossed prisms.

(7) Determine fringe or order value in pounds per square inch by using a calibration beam subjected to a constant bending moment over part of its length.

(8) Interpret the results.

(a) From the isoclinics derive the stress trajectories or the maximum shear directions.

(b) Determine the maximum shear at any point by noting the fringe order.

(c) The fringe order at unloaded boundaries gives the magnitude of the principal stress at that point which is tangential.

(d) Determine P and Q separately at any point by the methods previously outlined.

2. Optical Apparatus Used.

The photo-elastic apparatus in the Dept. of Mechanical Engineering at Oregon State College is shown in Fig. 8 and in diagrammatic form in Fig. 9. It does not differ from the ordinary apparatus except for the addition of chain gears to make possible simultaneous rotation of the crossed prisms. This has proved a great labor saving device and made it possible to obtain greater accuracy in sketching isoclinics in models under complicated loads. The polarizer and analyzer were 15 mm. Ahrens (Zeiss) prisms, making the size of the field obtainable limited only by size of the lenses. The quarter-wave plates (Zeiss) were of mica and fastened to each prism case by a pivot so they could be swung out of the way when using plane polarized light. The axes of the mica plates were always kept at an angle of 45 degrees with the axes of the Ahrens prisms.

3. Recording Data.

The vertical drawing board may be seen in Fig. 8 as

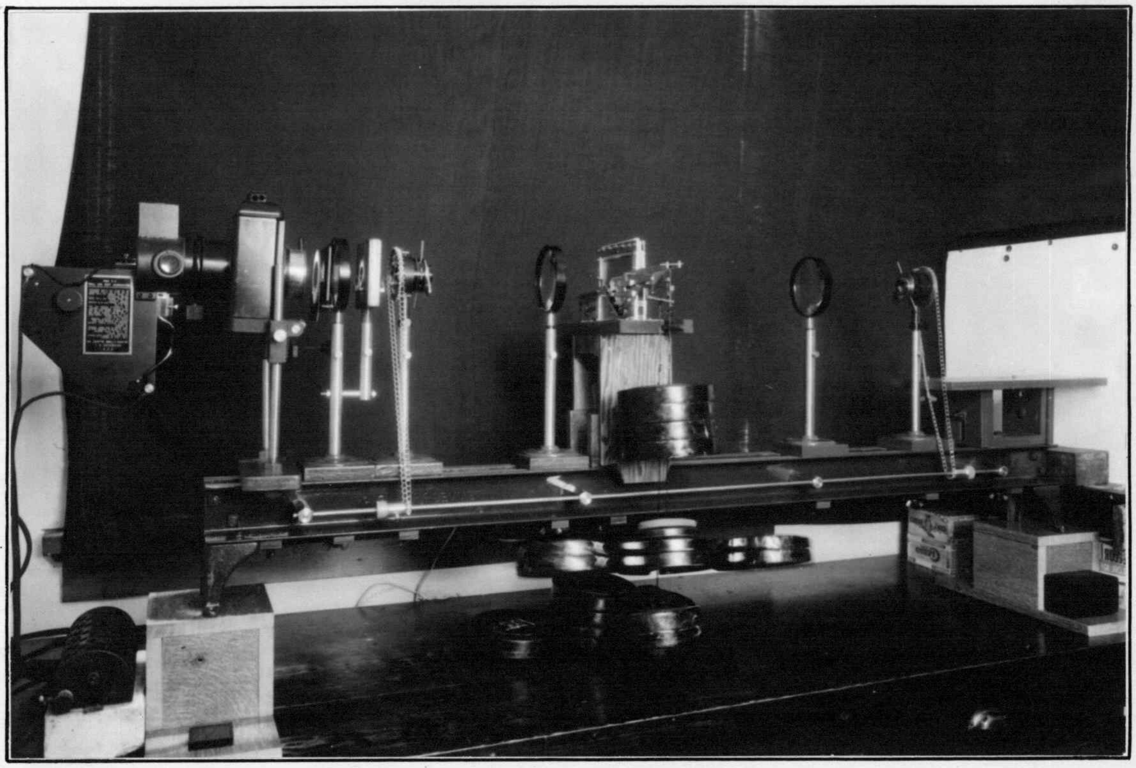


Fig. 8.-Photo-elastic Apparatus

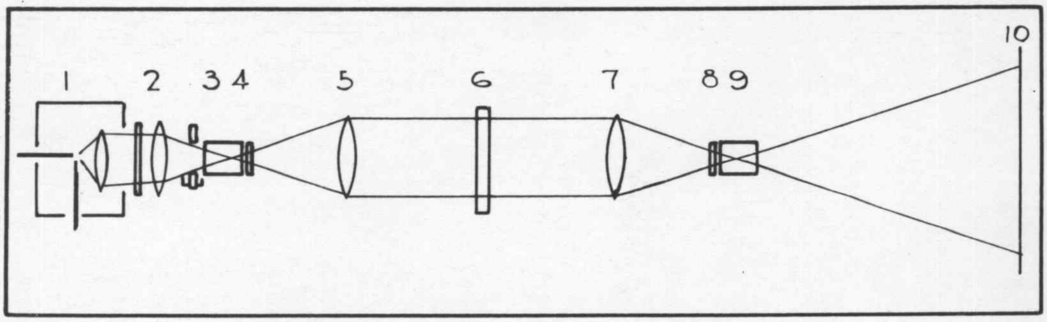


Fig. 9.-Diagram of Apparatus

1. Carbon-arc source with lens and heat filter
2. Converging lens followed by color filter holder
3. Ahrens polorizing prisim
4. Quarter-wave plate
5. Lens at focal length to obtain parallel beam
6. Loaded model.
7. Converging lens
8. Quarter-wave plate
9. Ahrens analyzing prism
10. Vertical drawing board or camera plate

well as the camera support. For taking photographs Wratten filters No. G-15 and H-45 were used to obtain as narrow a light band as practicable and approach monochromatic light. This combination gave the densest fringe pattern and the sharpest lines. The same filters were used for photographs taken throughout the testing.

As soon as the model was in position and loaded a photograph was taken with the quarter-wave plates in place. To check the black and white photograph and make it possible to tell, by the sequence of colors, whether the shear was increasing or decreasing a quick color sketch was made directly under the image as it was projected on the vertical drawing board. A colored sketch also distinguishes the black band denoting zero shear or P=Q.

Removing the quarter-wave plates, the isoclinic lines were sketched for different settings of the prisms. For complicated loadings this proved to be difficult but with patience and continual use of simultaneous rotation of the prisms good results were obtained.

To complete the record, loading conditions and amounts of loads were recorded along with the photograph plate number.

4. Preparation of Models.

The size of the models was limited by the diameter of the available beam of light. The section of the dam finally used was 1.47 inches high (elevation +72' to -45') with a base width of 2.22 inches (177'-6"). The foundation slab was 1.75 inches by 5.25 inches. The scale used was 1 inch to 80 feet.

The material used was the new water white bakelite, BT-61-893, manufactured by the Bakelite Corporation especially for photo-elastic work. A lower modulus bakelite, BT-41001, was obtained from the same company for use in model section III, giving an effective modulus ratio of about 1 to 3.

At the beginning of the work it was thought necessary to cut out the model, polish it and then anneal it to remove the initial stress or residual stress due to cutting. Lately, it was discovered that better results could be obtained by polishing the material first to see if initial stresses were present, annealing if necessary, and finally machining accurately to size. The last model constructed proved to be without initial stress and very little was induced in machining. This method also prevented the edges from being rounded in the polishing operation.

The annealing was done in a water bath with the model between oiled plate glass. A large quantity of water was used to more equalize any temperature change and a motor driven propellor used to insure uniform temperature throughout.

The temperature was raised to about 75° C. and held constant by thermostat for between one and two hours. It was then lowered to room temperature at about 3°C. per hour. The temperature was controlled by two electric elements working through a thermostat with a relay. This method of annealing proved fairly successful but not entirely so. Evidence of edge stress may be seen in some of the accompanying photographs. It was found that an annealed model acquired stresses along the edges very soon after it was removed from the bath, so that the testing had to be done at once.

5. Loading Frame.

The problem of applying seven known loads to a small model in a specified manner proved to be difficult. Fig. 10 is a picture of the loading frame finally used. The model was inverted so that vertical forces could be obtained by simple levers. It had the added advantage of causing the image to be right side up on the screen. Lever fulcrums used were all drill rods with the bottom side filed to a sharp edge. Forces were transferred from the levers to steel bearing plates by using the diagonal corners of square steel rods as knife edges. Horizontal water forces were applied through bell crank levers. Steel bearing pieces were made to fit the surfaces acted upon by working the two materials together with fine grain alundum. Effort had continually to be made to avoid inducing any shear. The base slab was clamped between padded brass plates.

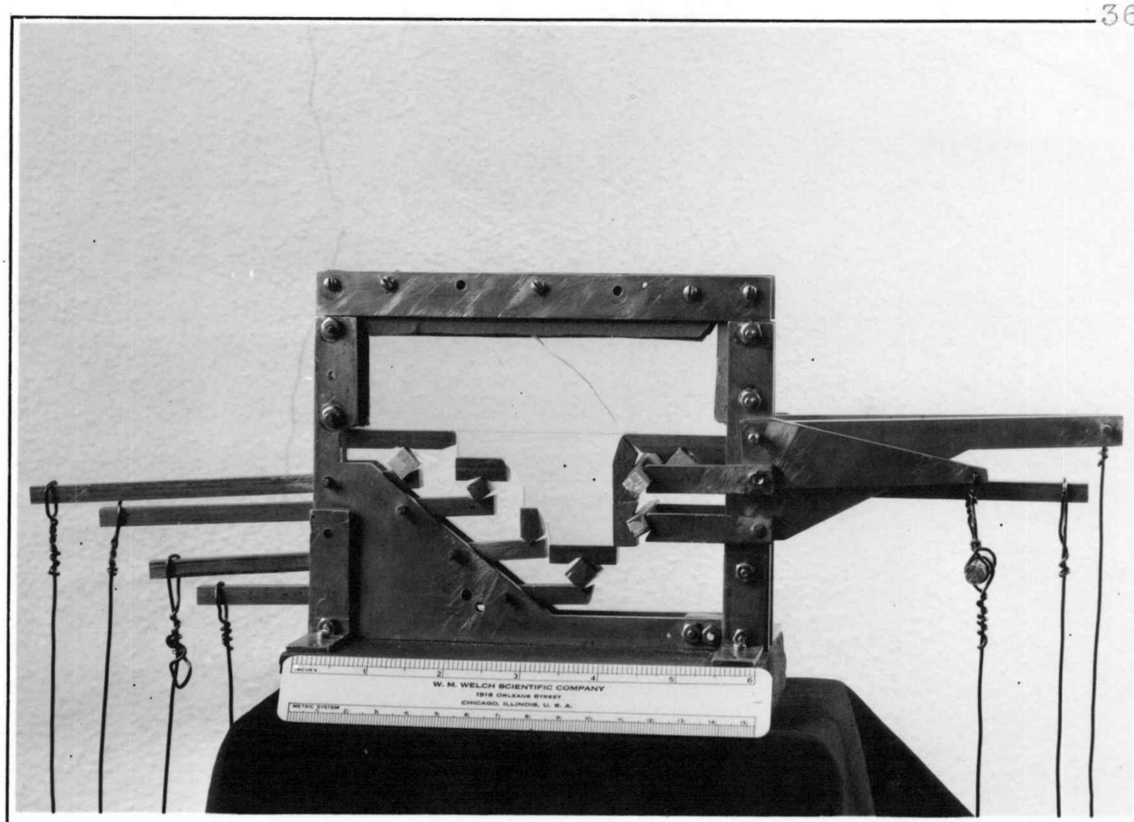


Fig. 10.-Loading Frame with Section I in Place

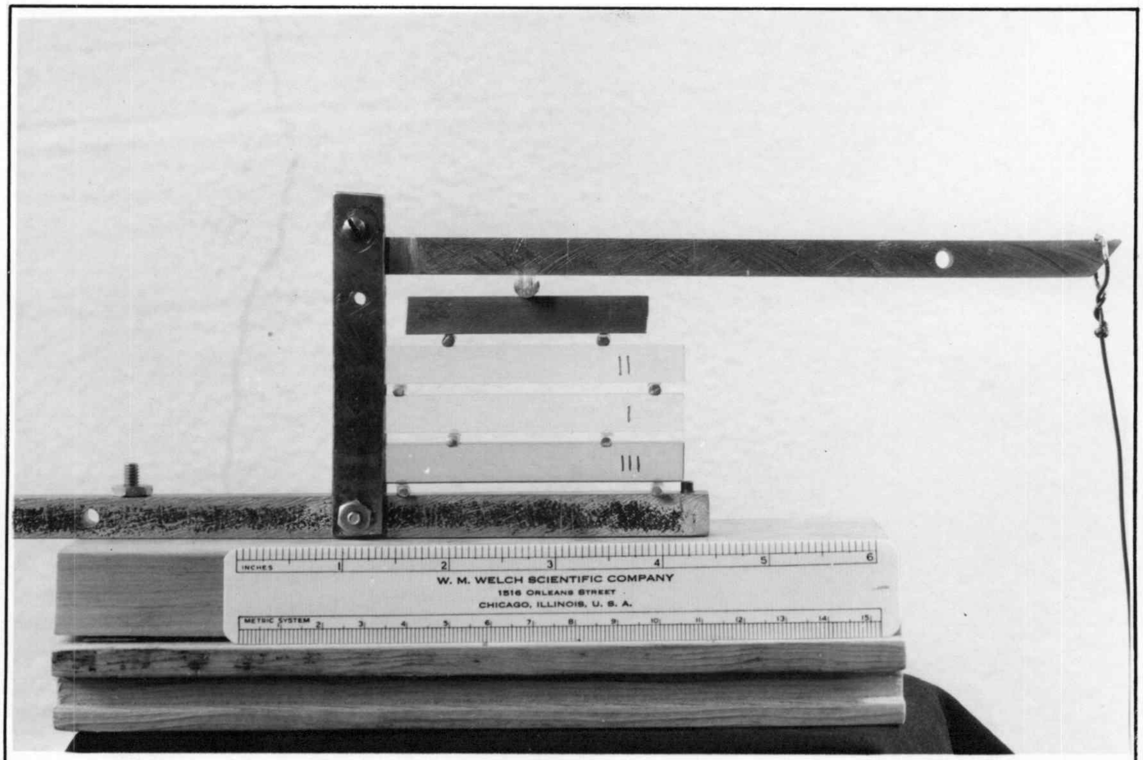


Fig. 11.-Method of Loading Calibration Beams

6. Calibration of Fringe Orders.

The apparatus used for determining the magnitude of the fringe orders in the different materials and the calibration beams for these materials are shown in Fig. 11. The lever worked through a knife edge onto a rigid member which transferred the load to the different beams by means of round rod supports. Besides having small bearing surfaces they tended to relieve any horizontal motion induced by the curvature of bending. A direct comparison of the meanings of the fringes in the different materials is obtained since the bending moment on each beam is the same.

Fig. 12 shows the result of a weight of 10 pounds on the end of the lever. The resulting force on the rigid member was 49.08 pounds which caused a bending moment of 12.04 inch pounds in the middle part of each beam. Beam No. 1 was cut from the same material as model sections I and II. Beam No. 2 refers to the masonry section and No. 3 to the foundation section of model No. III.

Tables I and II give the data used in determining the magnitudes of the fringe orders in the different materials. It will be noted that the extreme fiber stress was not used since, from the photograph of the fringes, considerable edge stress is indicated by the unsymmetrical number of fringes on opposite sides of the neutral axis. In each case the neutral axis was at the center of the beam. Points on each side and equidistant from the neutral

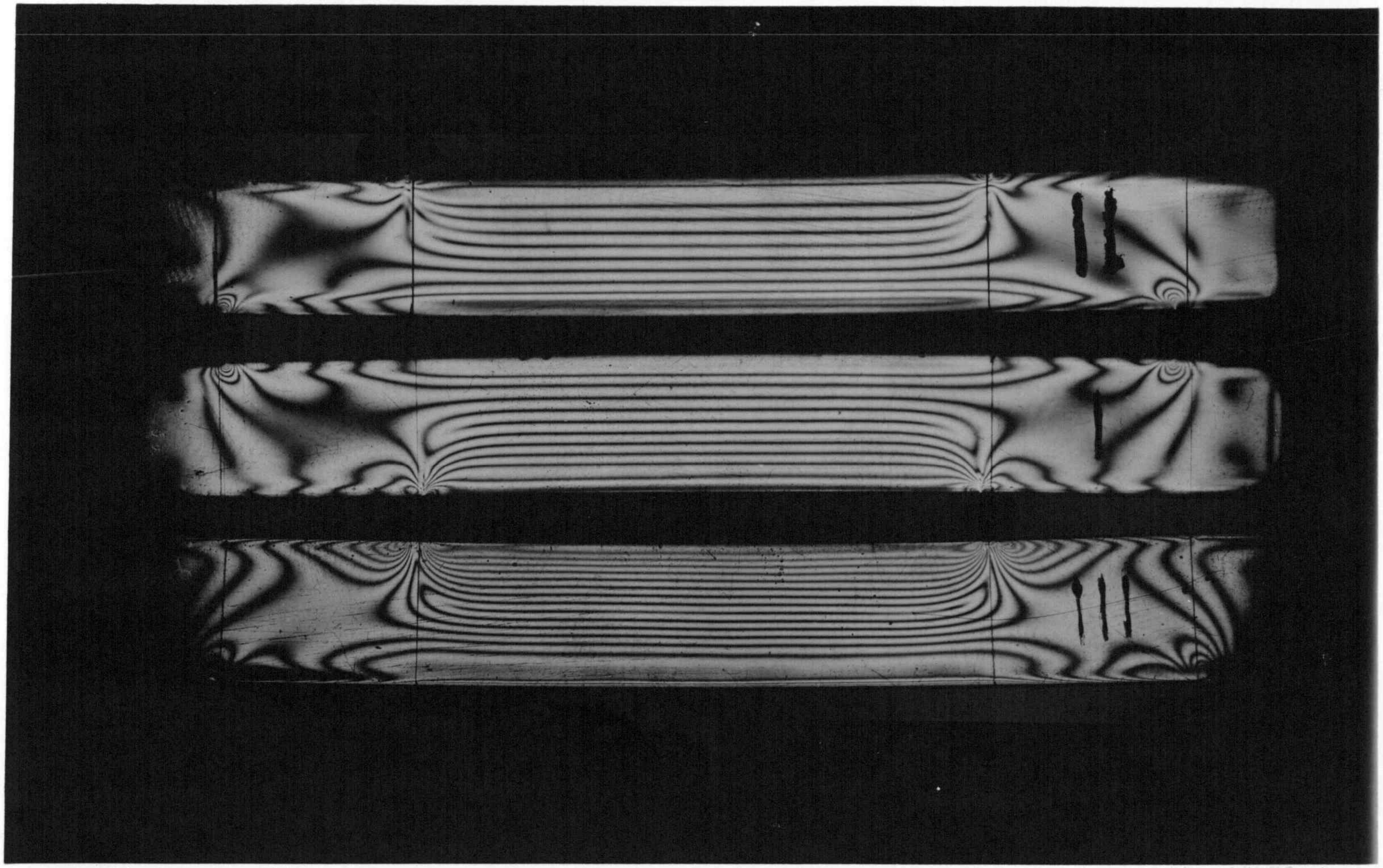


Fig. 12.-Fringe Photograph of the Three Calibration Beams Under Equal Bending Moment

TABLE I

CALIBRATION BEAM DIMENSIONS

Beam No.	Length inches	Thick-ness inches	Depth inches	Section Modulus $\frac{I}{C}$ in.^{3}	Young's Modulus #/sq.in.
I	3.00	0.256	0.378	0.0061	613,000
II	3.01	0.389	0.380	0.0091	700,000
III	3.02	0.234	0.390	0.0059	400,000

TABLE II

DETERMINATION OF FRINGE MAGNITUDE

Beam No.	Moment in.lb.	Max. Fiber Stress #/sq in	Dist. from Center to Point Calcu-lated inches	Fiber Stress at Cal-culated Point #/sq in	No. Fringes at Cal-culated Point	Fringe Magni-tude #/sq in
I	12.04	1,970	0.117	1,220	4	305
II	12.04	1,322	0.102	710	3	236
III	12.04	2,036	0.109	1,163	5	232

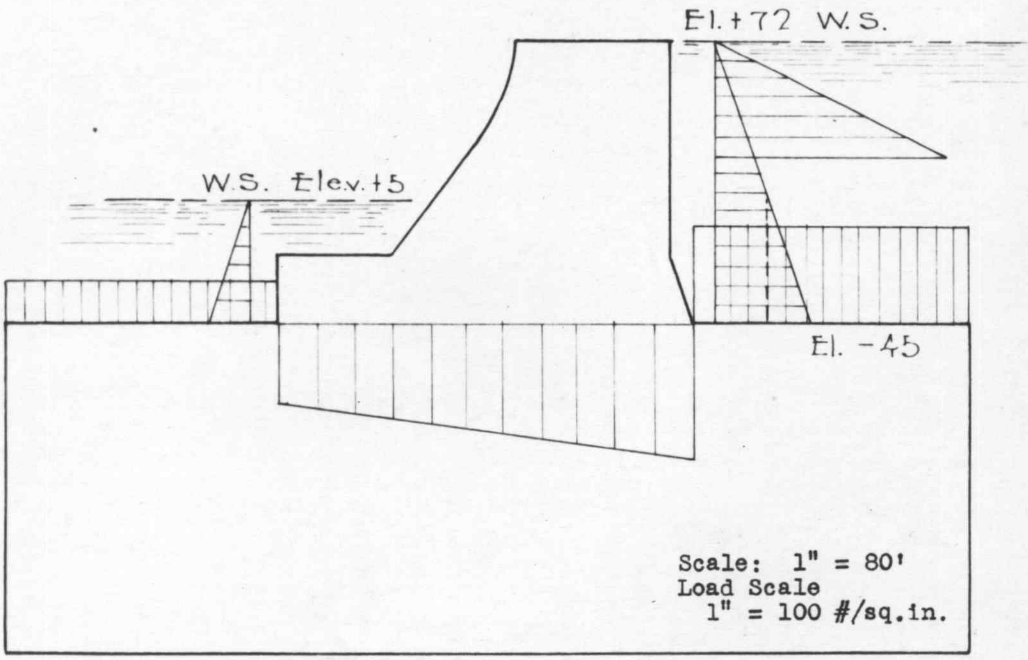
neutral axis were chosen where the fringes were still symmetrical. The fiber stress was calculated by the formula $S = \frac{My}{I}$ and divided by the number of fringes between the point and the center. This value was then taken as the magnitude of the fringe order. That is, if the fringe order were 236 lb./sq.in., a fringe of the fourth order would represent 4 x 236 lb./sq.in. or 944 lb./sq.in.

The modulus of elasticity of the materials used in model construction was determined by testing the tension members shown in Fig. 1, using two Huggenberger tensometers on a gage length of one half inch.

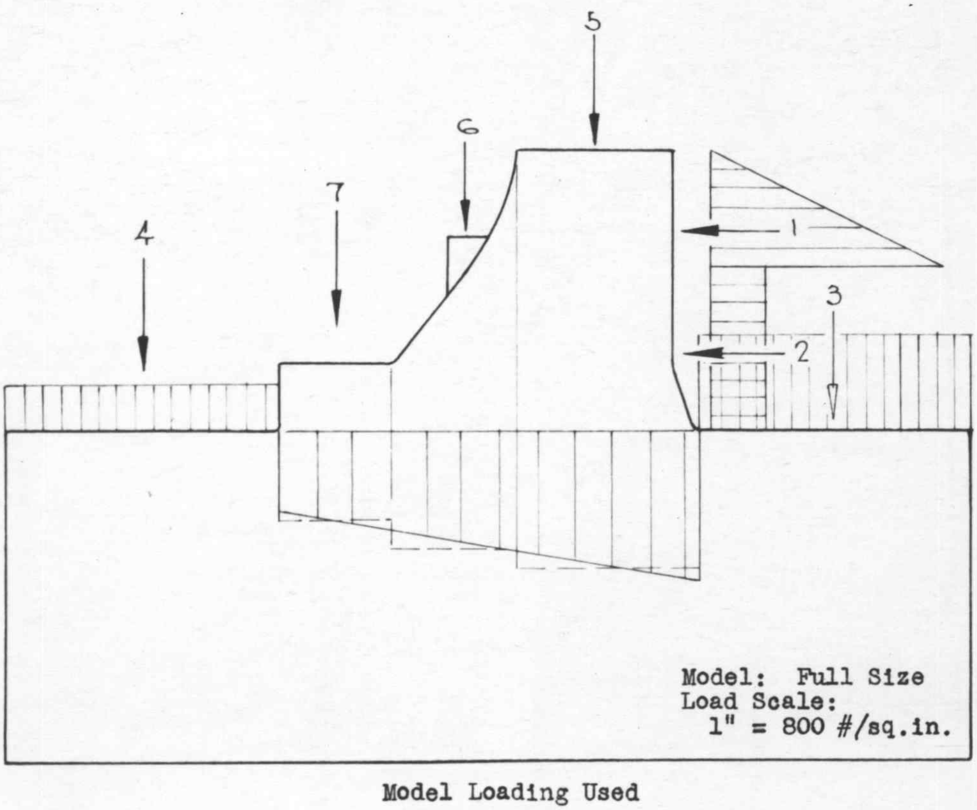
7. Determination of Loads to Use on Model.

The actual unit masonry load of the pier section on the foundation as calculated by the Bonneville Dam Section of the Office of the District Engineer was taken from blueprints furnished by them. Unit water loads were calculated from the conditions assumed.

The pier is 10 feet wide with 50 foot gates on the overflow sections between gates as shown in Fig. 14. It is assumed that since the horizontal force on the gate must be taken by the pier that each foot width of the pier takes the force against a 5 foot section of the gate. Each linear foot, then, of the pier takes the water load of six linear feet as far down as the bottom of the gate. This assumed actual loading is shown by diagram Fig. 13.

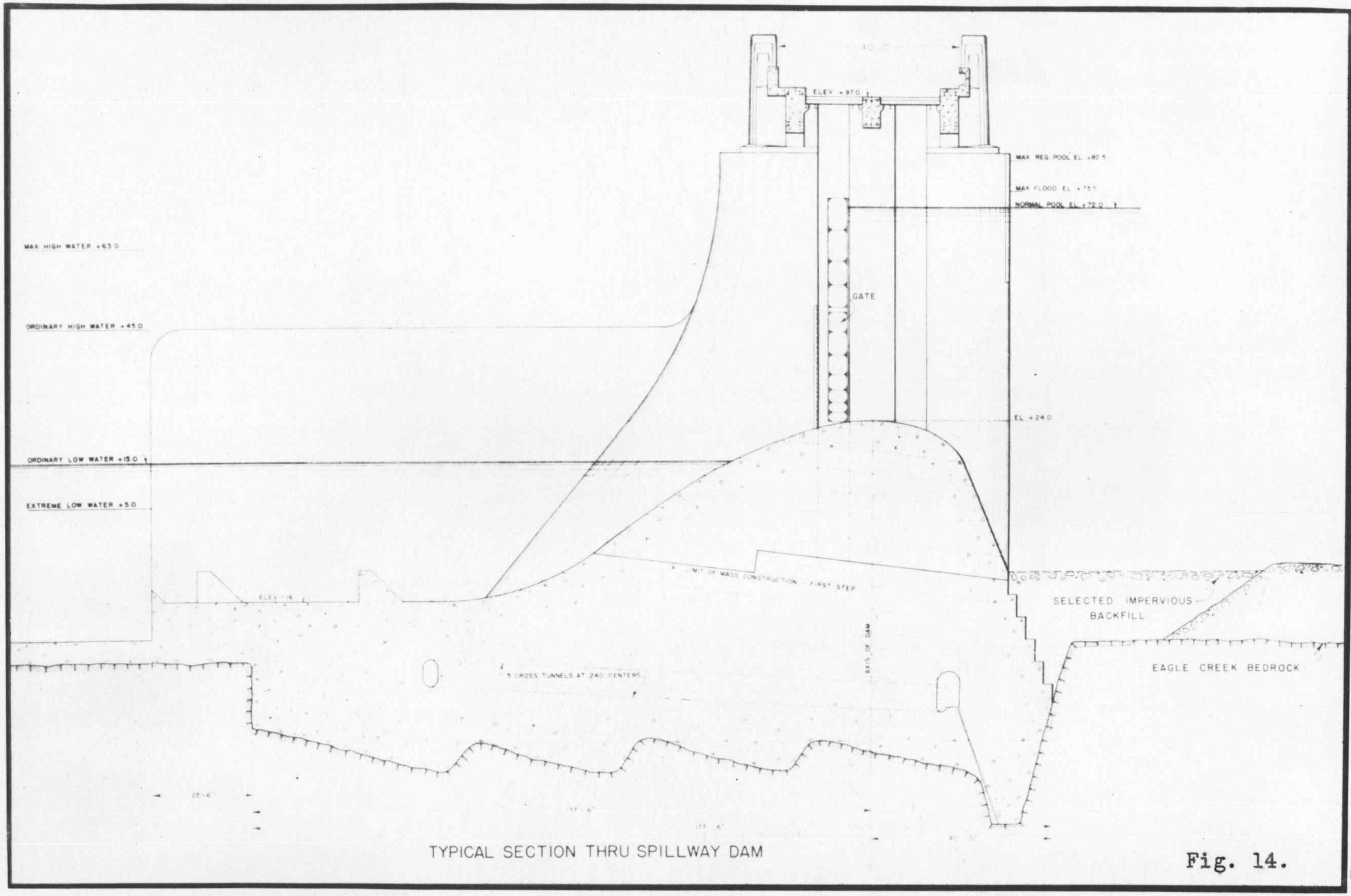


Actual Pier Loading



Model Loading Used

Fig. 13.-Loading Diagrams



TYPICAL SECTION THRU SPILLWAY DAM

Fig. 14.

TABLE III

WEIGHT TO USE FOR ACTUAL UNIT LOADS

Load No.	Actual Unit Load #/sq.in.	Area Model Loaded sq.in.	Total Load lbs.	Lever Arm Ratio	Coef.	Weight for Total Load lbs.
°1	0.0- 20.8	0.1462	1.52	1.23:4.50	3.66	0.415
1	0.0-124.8	0.1462	9.12	1.23:4.50	3.66	2.49
2	20.8- 29.1	0.2102	5.87	0.58:2.75	4.75	1.23
3	50.8	0.376	19.10	0.70:4.20	6.00	3.19
4	21.7	0.359	7.80	1:4	4.00	1.95
5	70.0	0.234	16.40	1:4	4.00	4.10
6	62.0	0.159	9.86	1:4	4.00	2.47
7	43.2	0.151	6.52	1:4	4.00	1.63

° Water Load against 1 foot Width of Dam

TABLE IV

WEIGHTS USED FOR UNIT LOADS 8 TIMES ACTUAL

Load No.	8 Times Actual Unit Load lbs./sq.in.	Total Load on Model lbs.	Weight to Use lbs.
1	0 to 1,002	73.25	20.0
2	226	47.50	10.0
3	408	153.8	25.0
4	174	62.4	15.6
5	578	135.2	32.8
6	503	80.0	20.0
7	339	51.2	12.8

These loads were simplified somewhat in order to make it possible to apply them to a model. The horizontal tail-water load was combined with the horizontal load against the face of the dam and that still further simplified into nearly a uniform load over the lower section. The remaining water loads were not changed.

The masonry load presented the greatest difficulty. An attempt was made to obtain a unit load correct at the base only by applying uniform forces along the top parts of the section. For that purpose a shoulder was left on the model to take part of the load.

The actual unit loads assumed to act on the dam itself, the corresponding areas of the model where these loads acted, lever arm ratios, etc. are shown in Table III. The unit loads finally used to obtain sufficient fringes were 8 times the actual unit loads, corresponding to a dam 936 ft. high (Bonneville being 117 ft. from water level at elevation +72 to foundation elevation -45). Consequently, the stresses obtained on the models have all been divided by 8 to give actual results on the prototype.

PART IV

RESULTS AND INTERPRETATION

PART IV. RESULTS AND INTERPRETATION

The data and results obtained from experiment are shown in the following photographs and diagrams. Fringe photographs and isoclinic lines for each model and loading condition are followed by maps of stress trajectories and those quantitative results which are of significance and at the same time practical to obtain.

1. Model Section I.

Fig. 15 shows the fringes or lines of constant shear for water load only. It is apparent, from the crowded fringes, that a high concentration of stress exists at the upstream corner. Failure of the model would undoubtedly have occurred if the corner had not been machined with a finite radius. Nine fringes may be distinguished denoting a shearing stress of nearly 1,400 pounds per square inch in the model, or 175 pounds per square inch in the prototype.

Fig. 16 is a reproduction of the sketched isoclinic lines for different settings of the crossed prisms. The reference axis makes an angle of 45 degrees with the upstream face of the dam so that a zero isoclinic is the locus of points whose direction of stress is 45 degrees from the vertical. The prisms were given a clockwise rotation and the angle measured from the zero isoclinic. A 15 degree isoclinic, therefore, denotes points where the direction of stress is 15 degrees clockwise from the zero isoclinic or 30 degrees from the vertical. Proceeding in this manner, the short lines were drawn over the isoclinics with Fig. 17 as the result. Starting from any convenient point, as a point on the face of the dam, lines may be drawn in the direction indicated by the short lines. This determines one system of stress directions. The other system is obtained by drawing a second set normal to the first or by starting from the isoclinics with short tangent lines at right angles to those first used. The result is shown in Fig. 18.

Fig. 20, the direction of maximum shear, was obtained in a similar manner, except that, since the maximum shear occurs at an angle of 45 degrees with the principal stresses, the degree of the isoclinic indicates the angle the maximum shear makes from the vertical, measured clockwise.

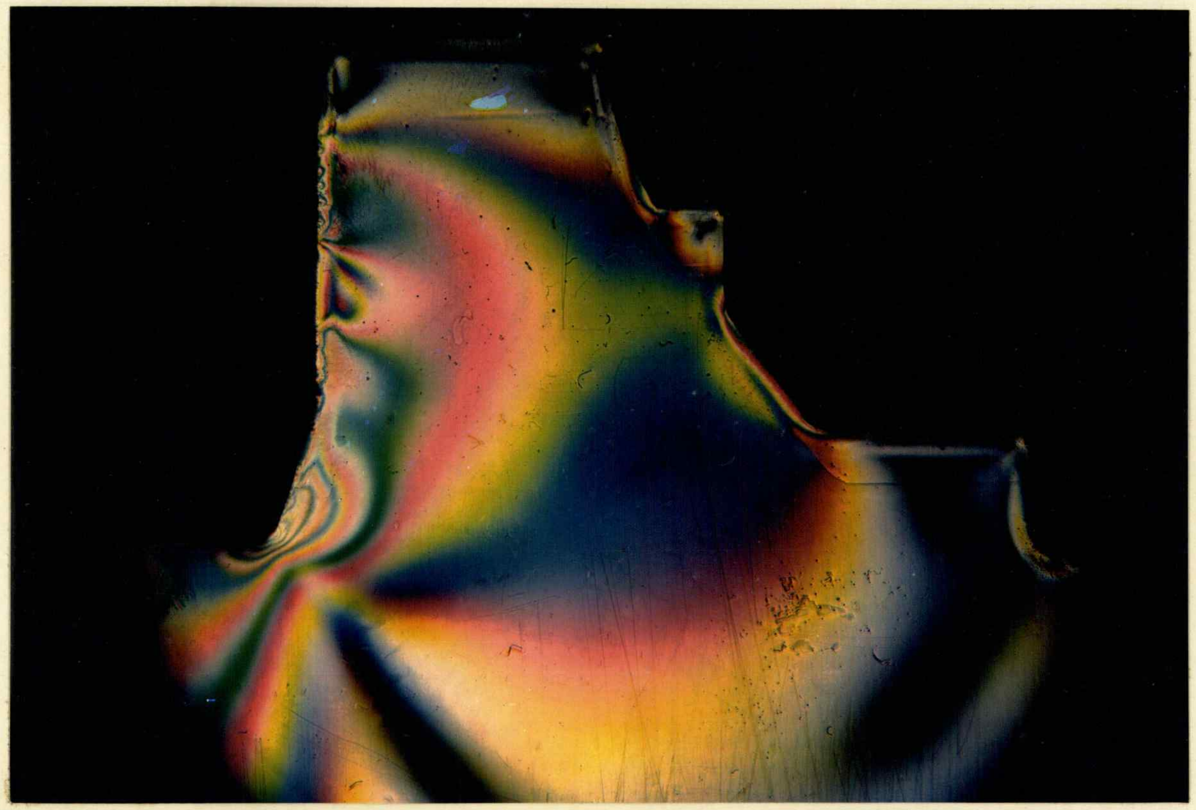
The magnitudes of the principal stresses shown in Fig. 19 were obtained by the graphical method suggested by Dr. Brahtz and outlined on page 25. The values are in pounds per square inch for the dam itself. As is customary, compression is given as negative and tension as positive.

This constitutes a complete quantitative analysis for this particular loading. It is not of much practical value in itself other than as an illustration of the method of analysis and for possible use in combination with the stresses due to masonry loads determined by other means.

Due to the fact that the masonry load had to be applied to the upper part of the section to obtain an approximately correct distribution for the base only, a similar complete analysis was not made. For the conditions of dead load only and complete load the fringe orders at the boundaries were very difficult to determine with any degree of accuracy. Figs. 21 and 23 record the fringes due to dead load and full load respectively, and Figs. 22 and 24 show the corresponding isoclinics and stress trajectories. It may be seen that the principal stress direction for full load coincides fairly well with the proposed foundation contour shown in Fig. 14.

Fig. 25 is a comparison of the maximum shear values at elevation -45 and -60 for the different loading conditions. These values were taken directly from the fringe photographs. It must be noticed that these values are for maximum shear acting at points, and not necessarily in the direction of the horizontal plane. The values plotted as ordinates are all to the same scale as given in the top diagram. The arrows along the origin line merely indicate clockwise or counter-clockwise shear forces, the former being plotted above the line, the latter below. The direction that these maximum shears act are shown in Fig. 26.

These were derived from the isoclinic lines.



Pier Section of Bonneville Spillway Dam, Model No. I
Showing Isochromatics or Lines of Constant Shear
for Water Load Only

Fig. 15.-Fringes, Sec. I, Water Load Only

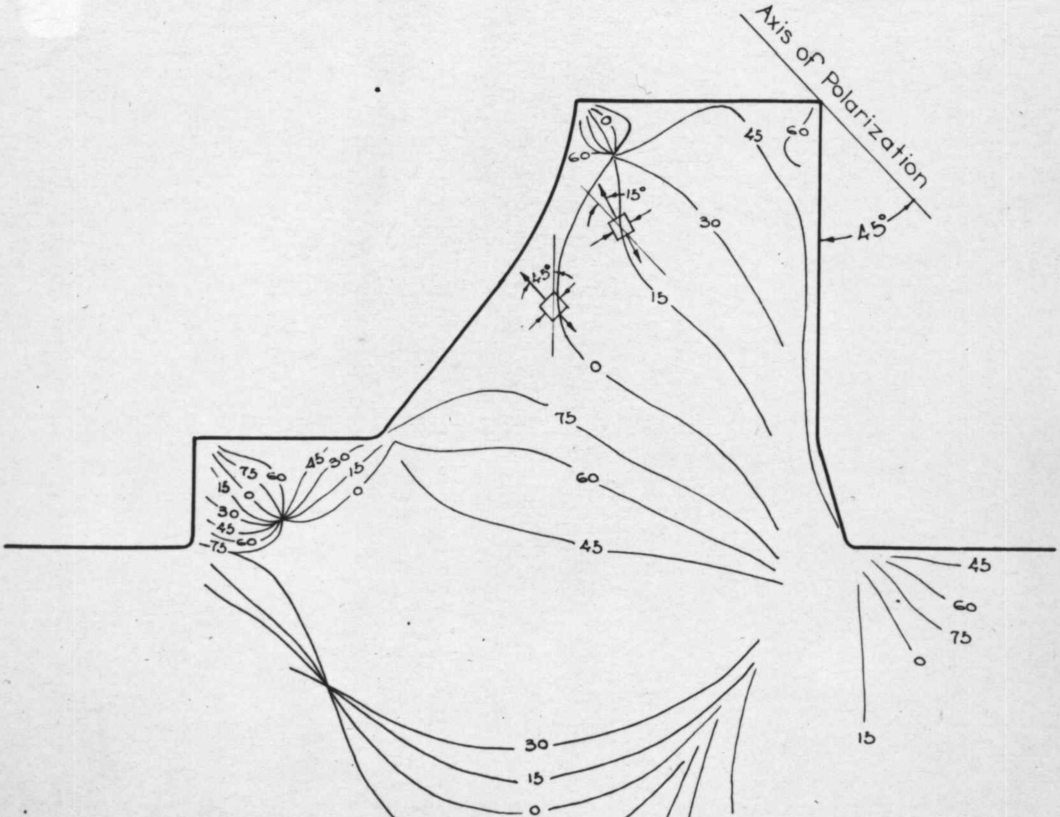


Fig. 16.-Isoclinic Lines, Sec. I, Water Load Only

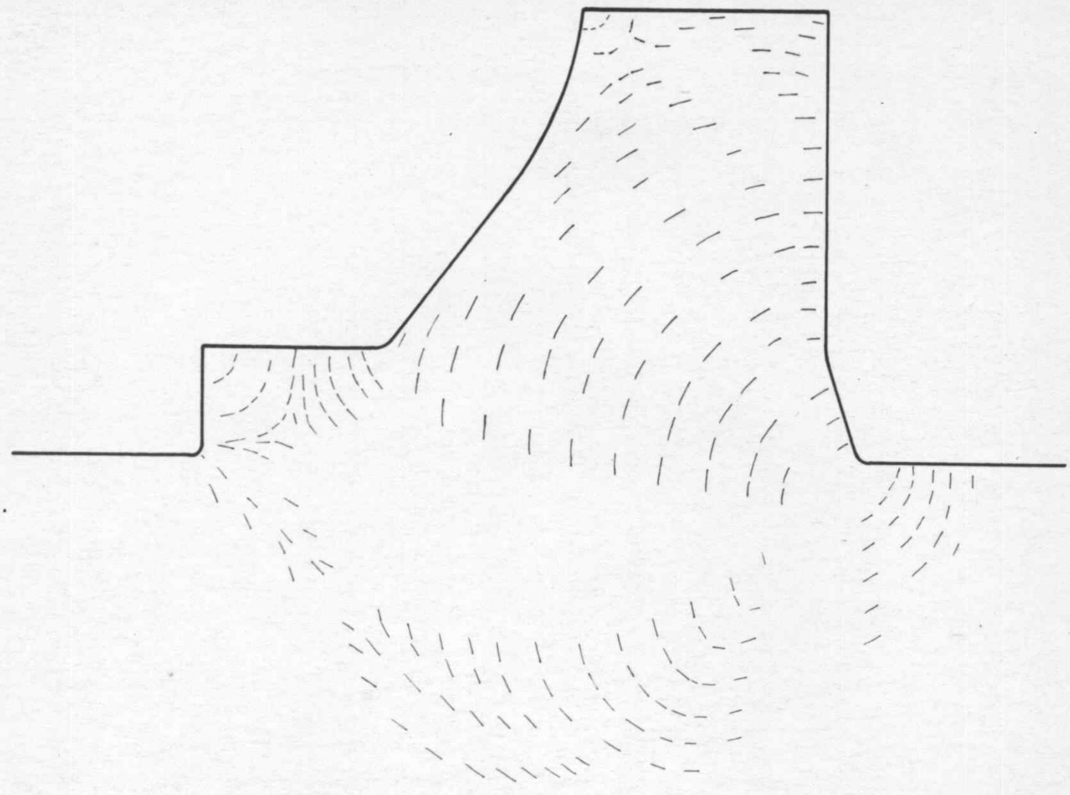


Fig. 17.-Method of Determining Stress Trajectories from Isoclinic Lines

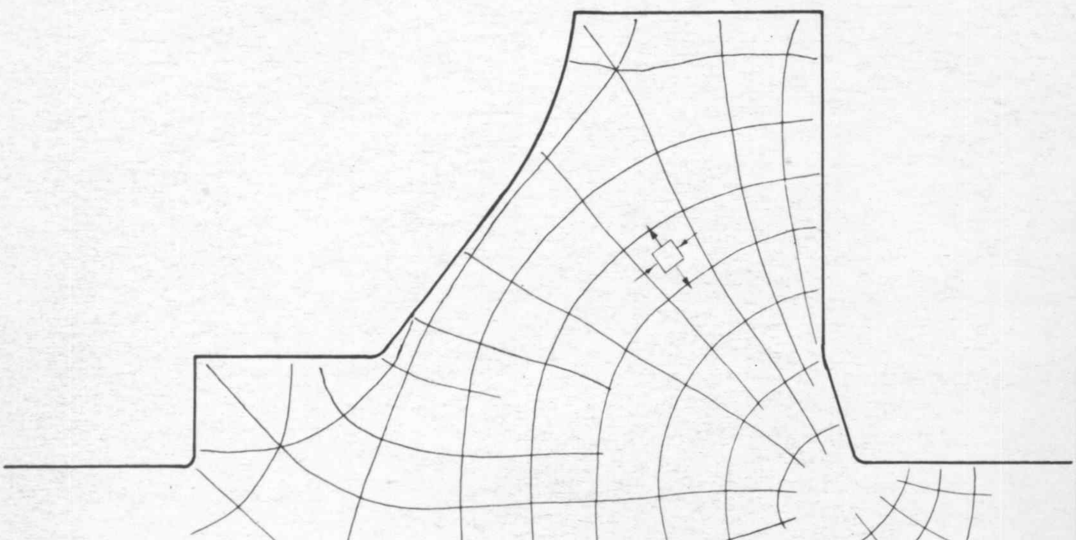


Fig. 18.-Stress Trajectories, Sec. I, Water Load Only

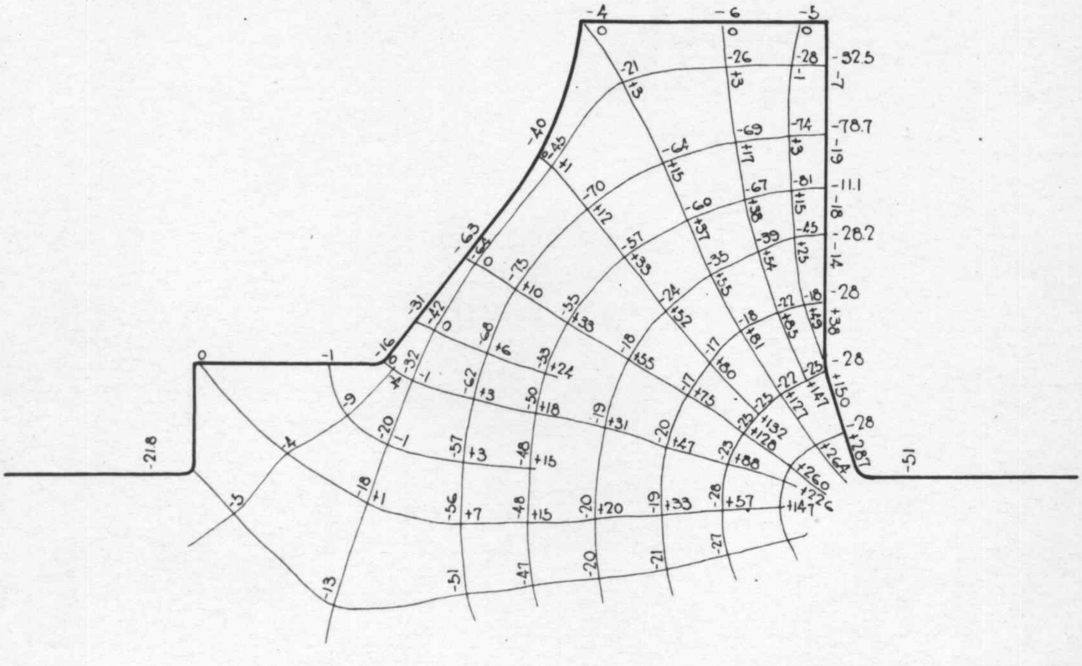


Fig. 19.-Principal Stress Values, lbs./sq.in., Sec. I, Water Load Only

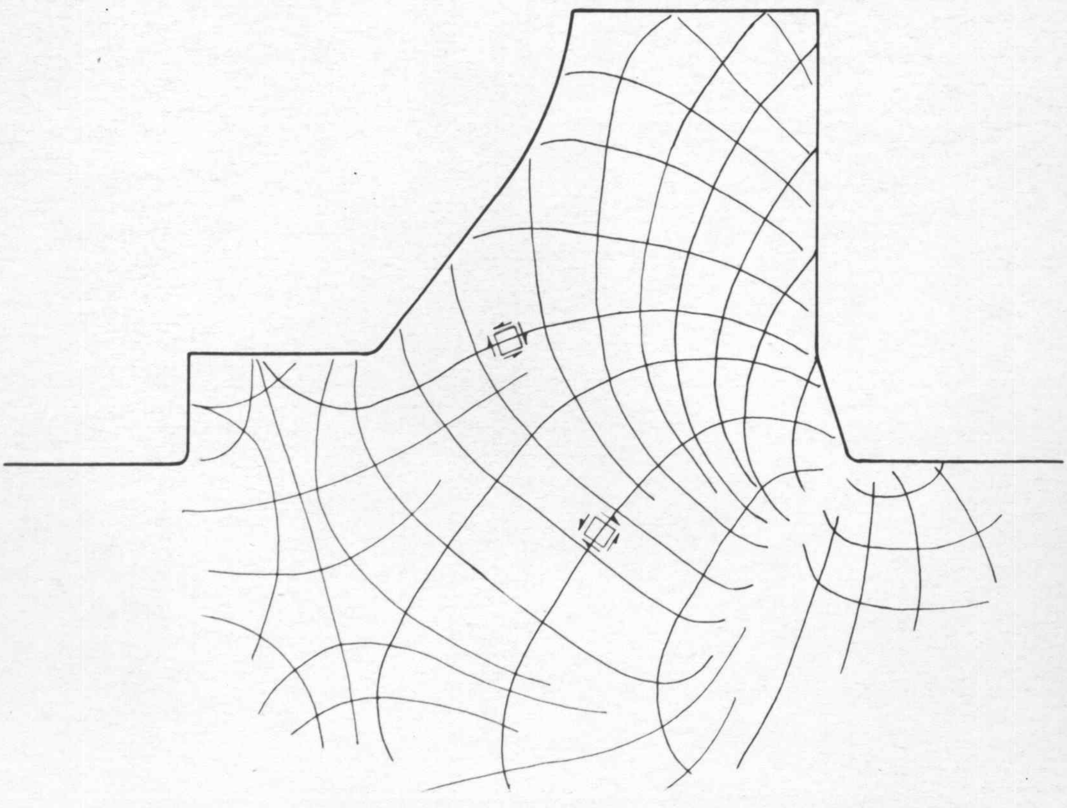


Fig. 20.-Direction of Maximum Shear, Sec. I, Water Load Only

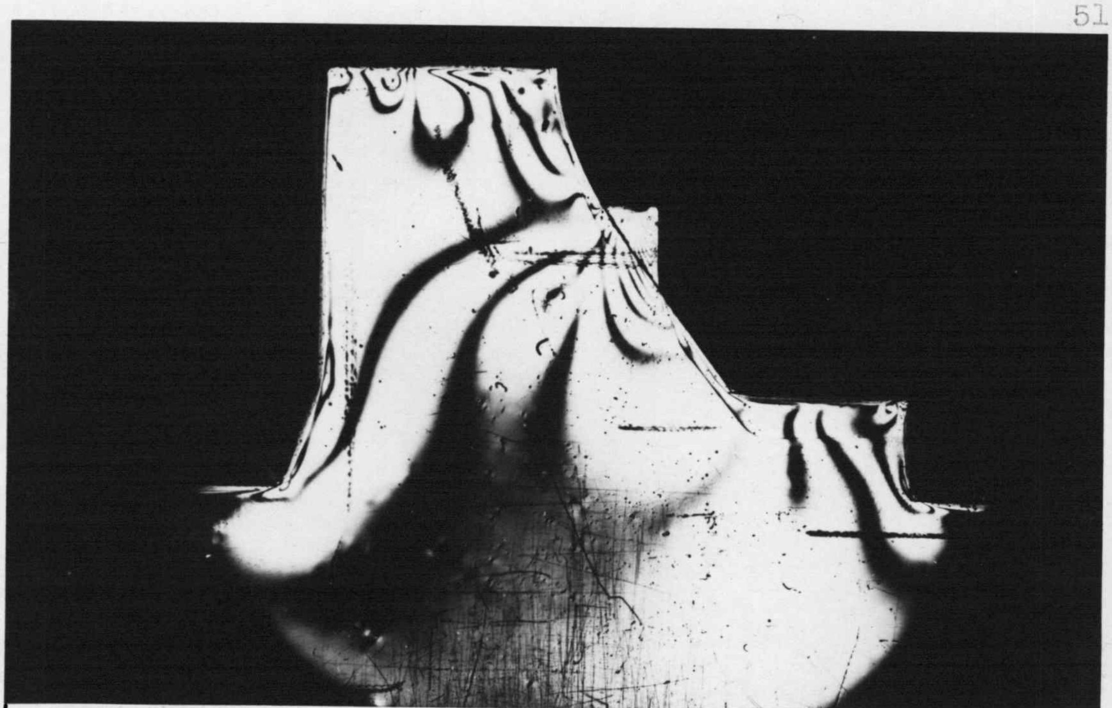


Fig. 21.-Fringes, Sec. I, Dead Load, Correct near Base Only

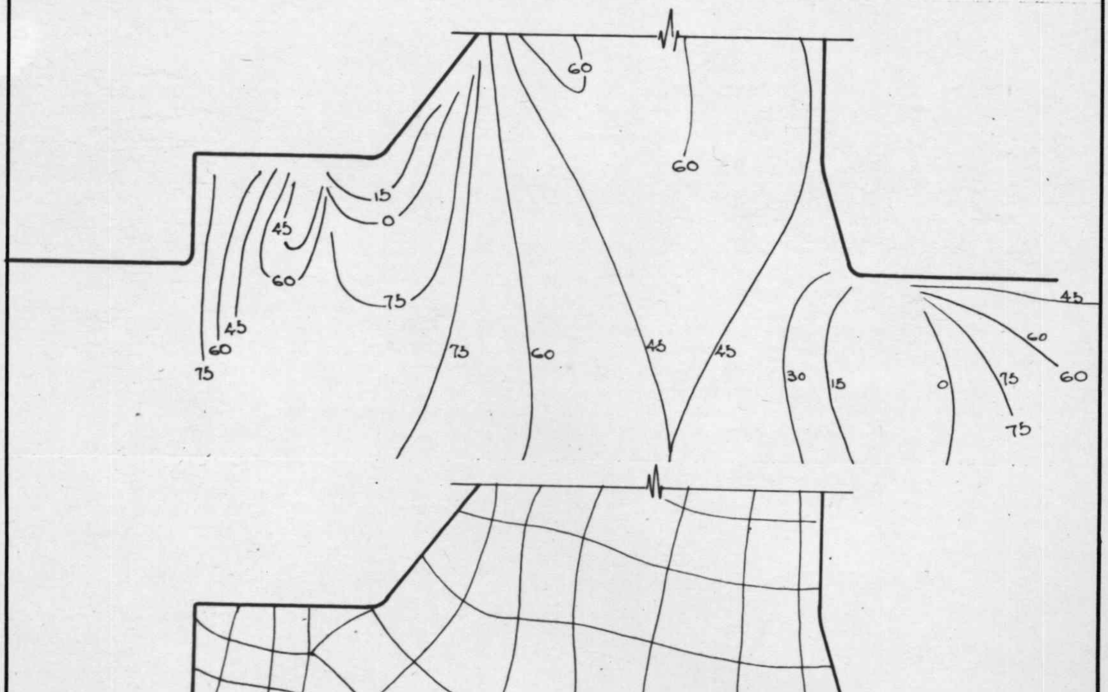
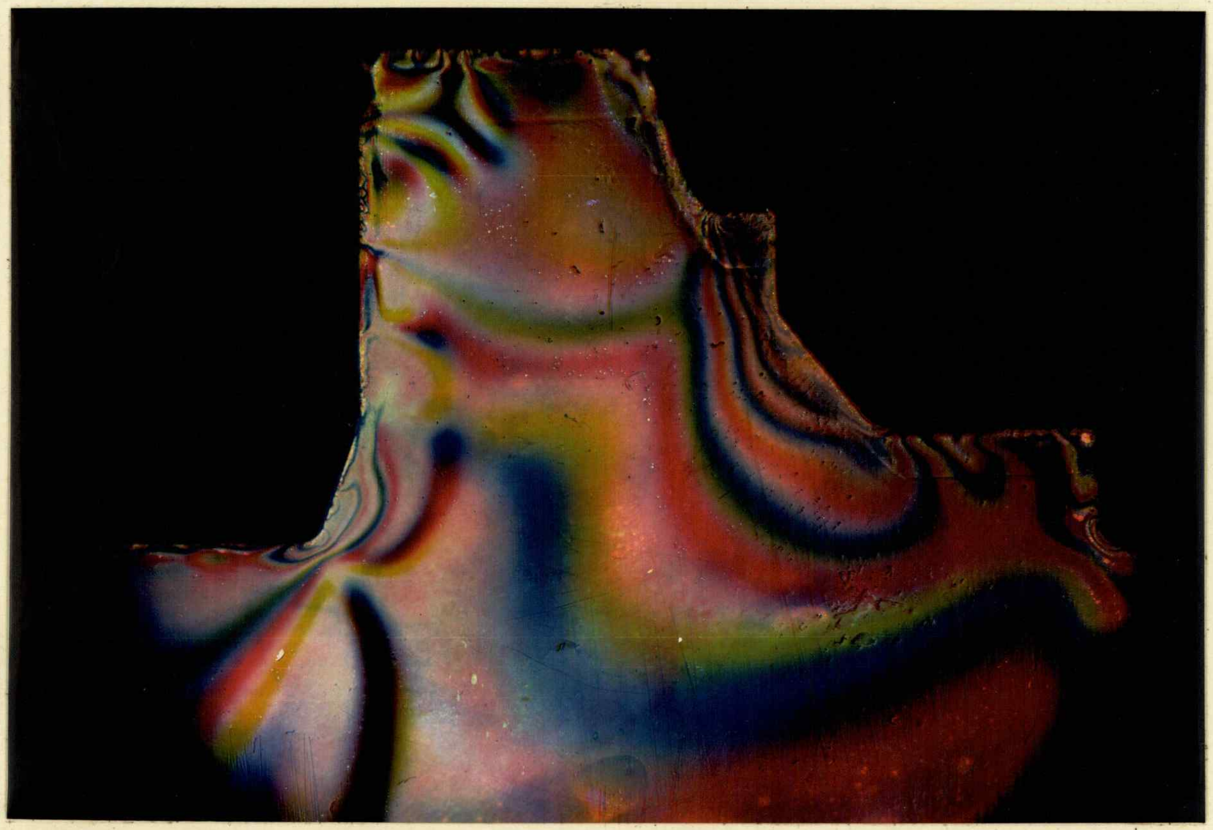


Fig. 22.-Isoclinic Lines and Principal Stress Trajectories, Sec. I, Dead Load



Isochromatic Lines - Constant Shear or (P-Q) - for
Model No. I, Full Load, Correct at Base Only

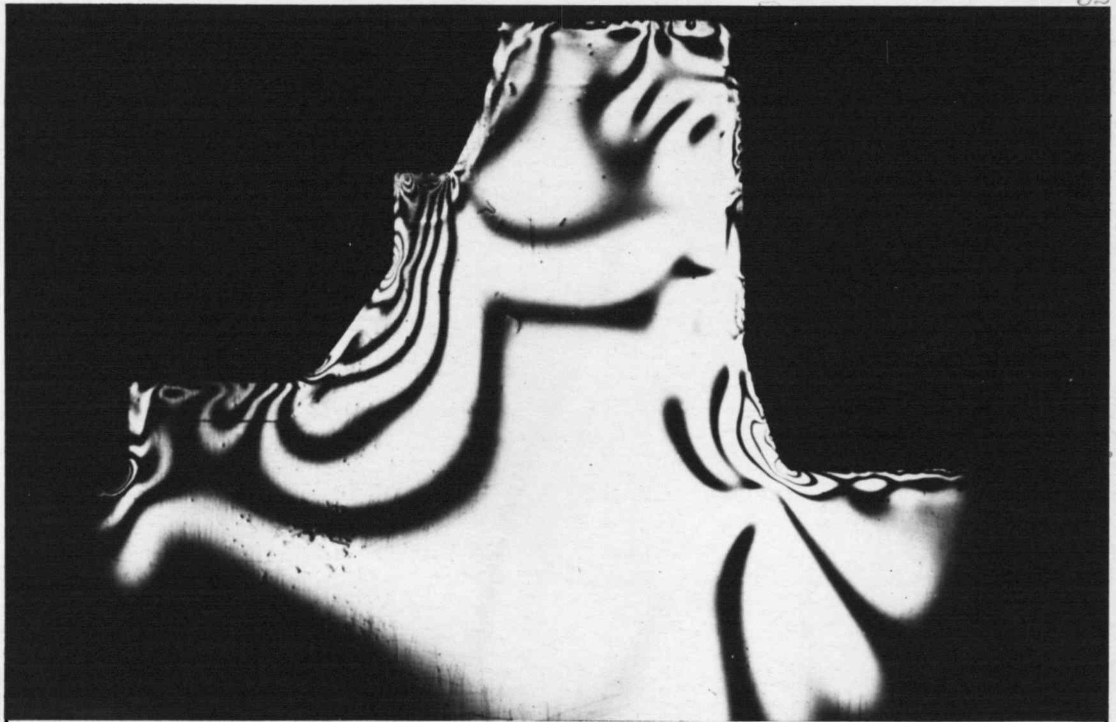


Fig. 23.-Fringes, Sec. I, Full Load, Correct near Base Only

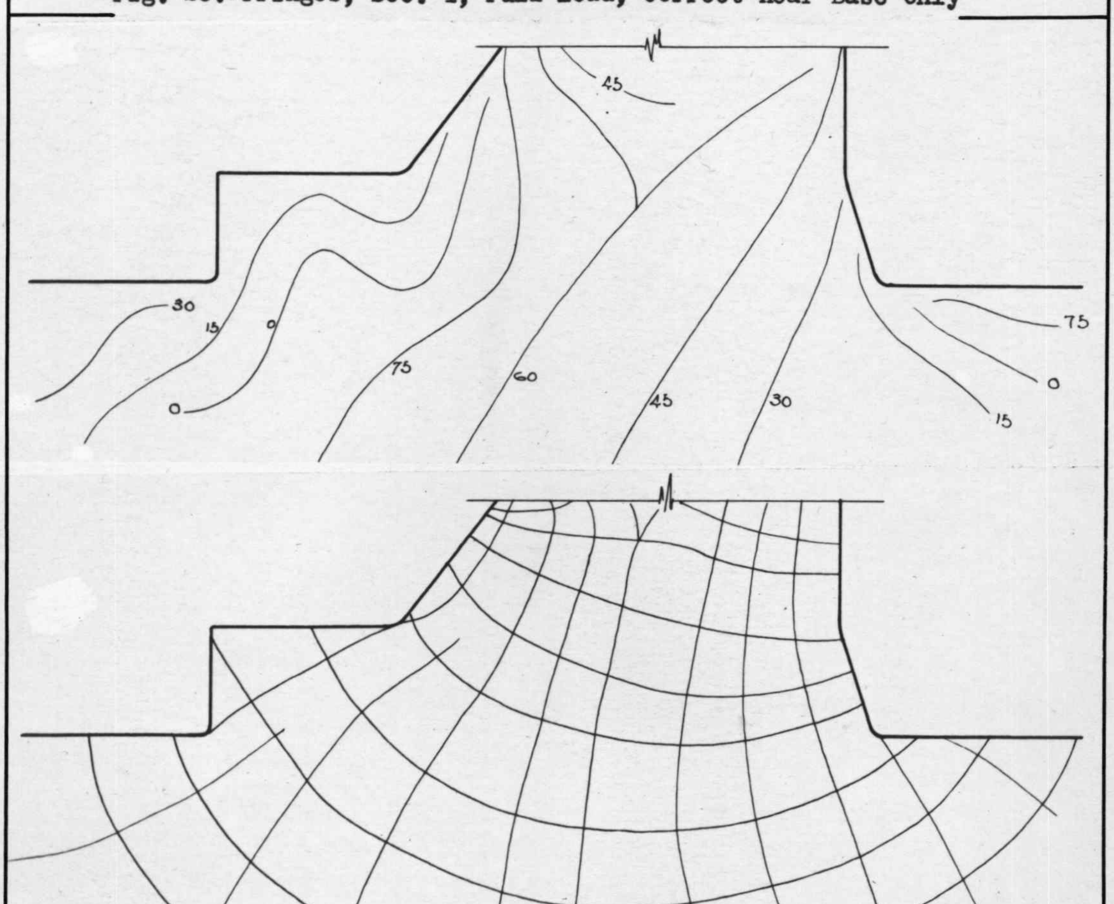


Fig. 24.-Isoclinic Lines and Principal Stress Trajectories, Sec. I, Full Load

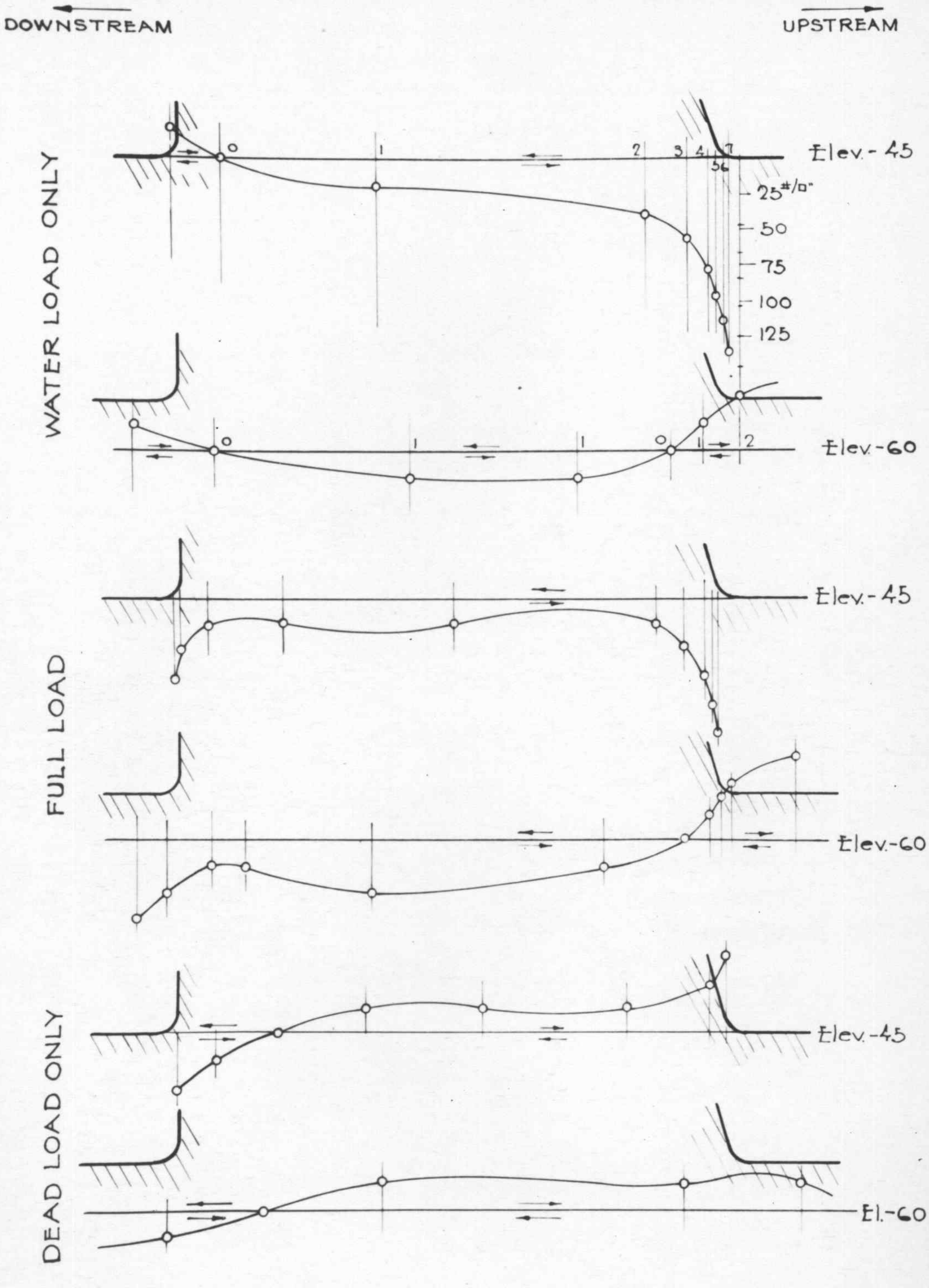


Fig. 25.-Magnitudes of Maximum Shear Acting on Points in the Foundation

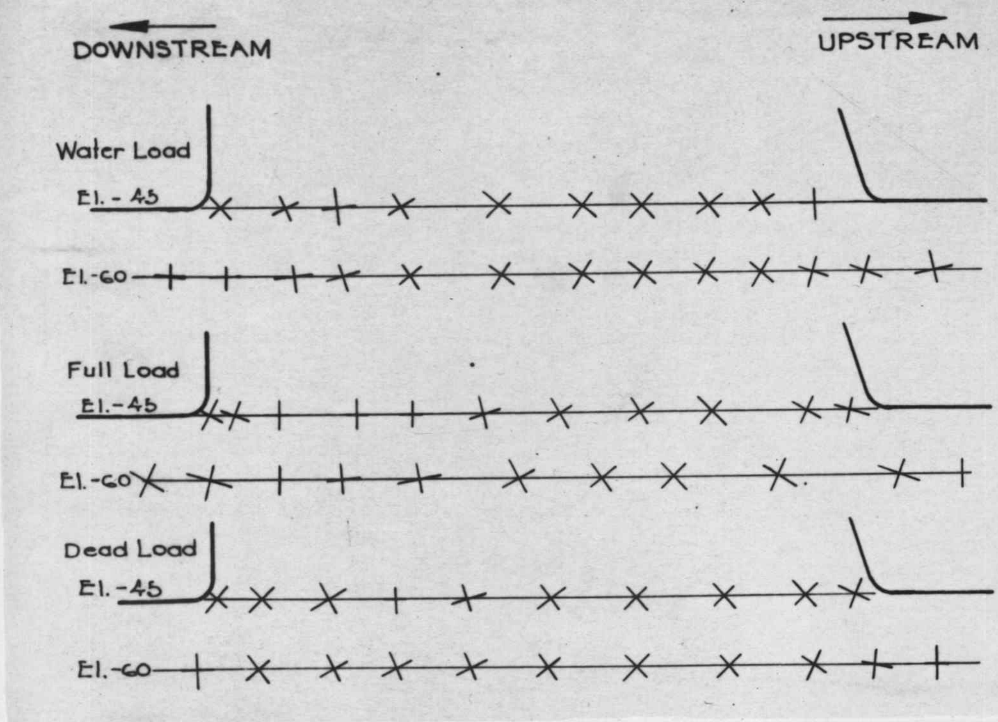


Fig. 26.-Direction of Maximum Shear at Points in the Foundation

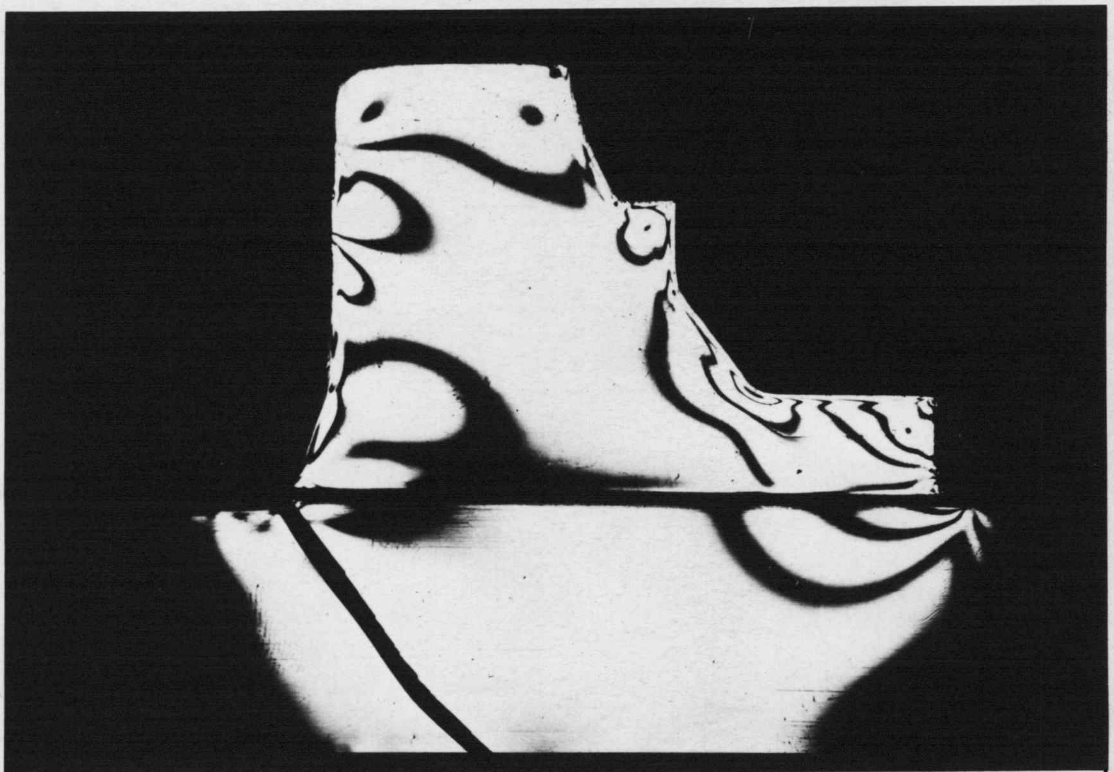


Fig. 27.-Fringe Photograph of Sec. I with Machined Base under Water Load Only. Model Broke under Load

2. Sections II and III.

In an attempt to determine the effect on the stress distribution of a base of weaker material models II and III were constructed. Both models were made of two parts and joined along the approximate construction line of the concrete dam and the rock foundation as shown in Fig. 14. The joint was made with great care but a perfect fit was not obtained. Irregularities invariably appeared when placed in the path of polarized light. Under load, these small irregularities became points of high stress concentration as may be seen in Figs. 34 and 35. Something of this sort probably occurs in nature, especially where the properties of the foundation material vary from one place to another. But the loading was so complex and the outline so irregular that no attempt will be made here to make any predictions or draw any conclusions.

For the purposes of comparison similar pictures or diagrams for the two models were placed on the same page. Model II was made of two pieces of the same material as that used in Model I while the two parts of Model III had an effective Young's Modulus ratio of one to three.

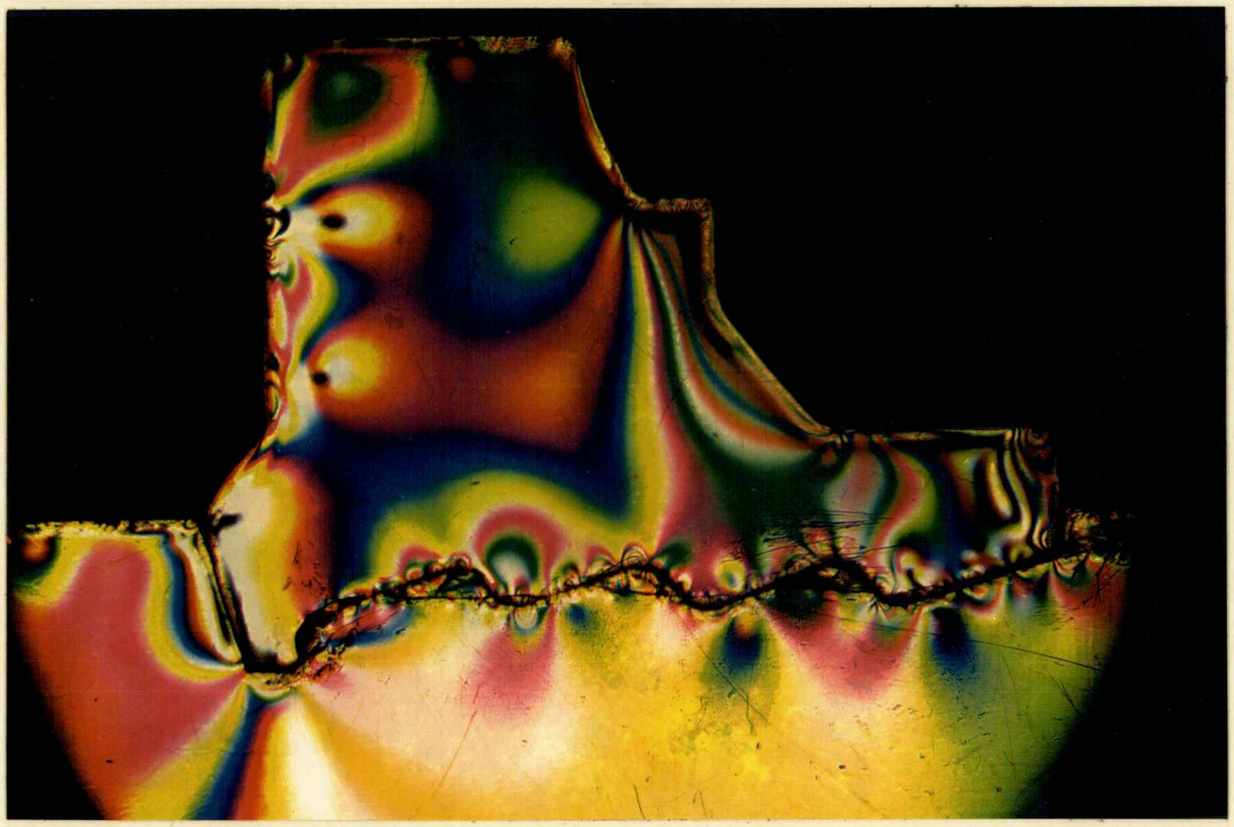
The isoclinic lines near the joint were very difficult to obtain. Farther away, in the foundation especially, they were readily visible. The stress trajectories obtained for the two models differ somewhat but no conclusions can be formulated.

It is interesting to note the lack of stress concentration at the upstream corner that characterized the monolithic section. It is also interesting to compare the stress trajectories in the foundation material for the dead load only and for the full load.

In a further attempt to simulate a lower modulus material and also to get away from irregularities in joining two different materials together, the base of the monolithic section was machined to half its thickness and the model placed under a load. The first load applied was the water load, the last two pound weight of which caused failure from the upstream corner as shown in Fig. 27. At the change of section the corners were nearly square and some buckling action undoubtedly caused extreme stress. This, together with the large tension previously seen in the monolithic section under water load, was more than the material could stand even though it had an ultimate strength of 9,000 pounds per square inch.

Since that time a series of tests have been started on a model from which the photographs shown in Figs. 4 and 5 were made. A piece of 3/8 inch bakelite was used, the base of which was machined after a set of loads had been used and the data recorded. The tests are still in the process of being completed so that results can not be stated at this time.

In reality, the stress system close to the change in section, is one of three dimensions but at a short distance away the stress becomes nearly uniform across the plate as may be determined by a model taken in that plane.



Pier Section of Bonneville Dam, Model No. II, Showing
Isochromatics or Lines of Constant Shear for
Full Load, Correct at Base Only

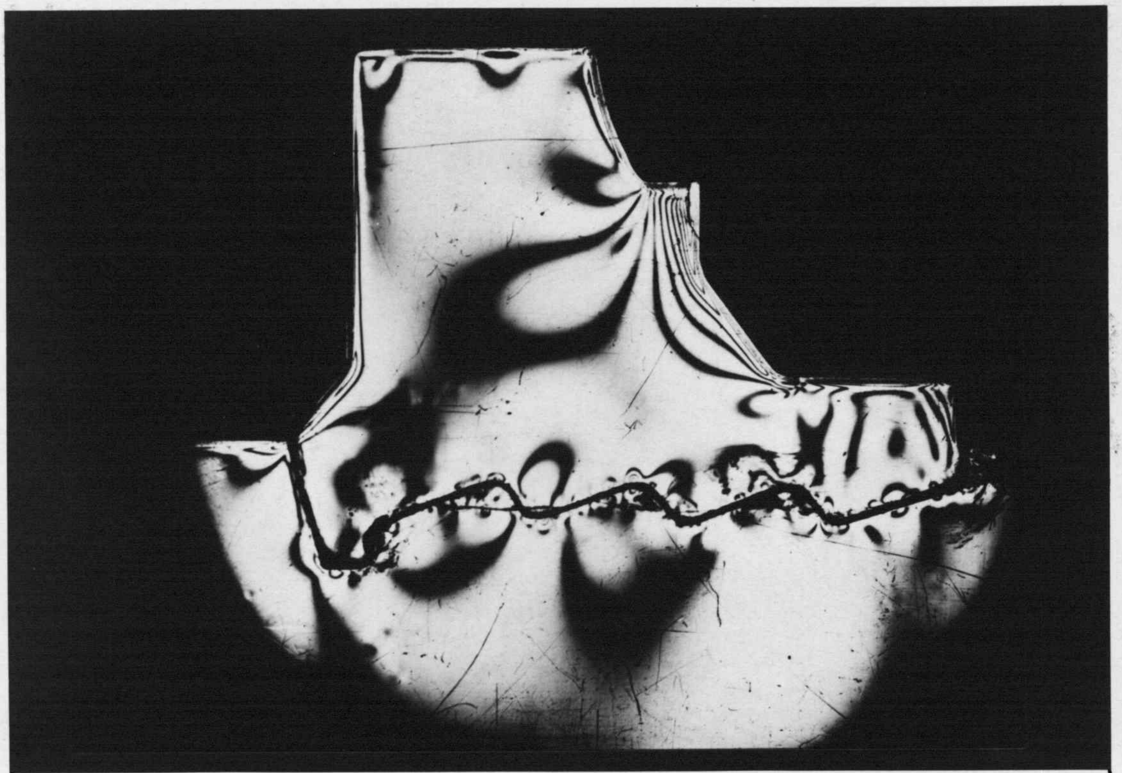


Fig. 28.-Fringes, Sec. II, Dead Load Correct near Base Only

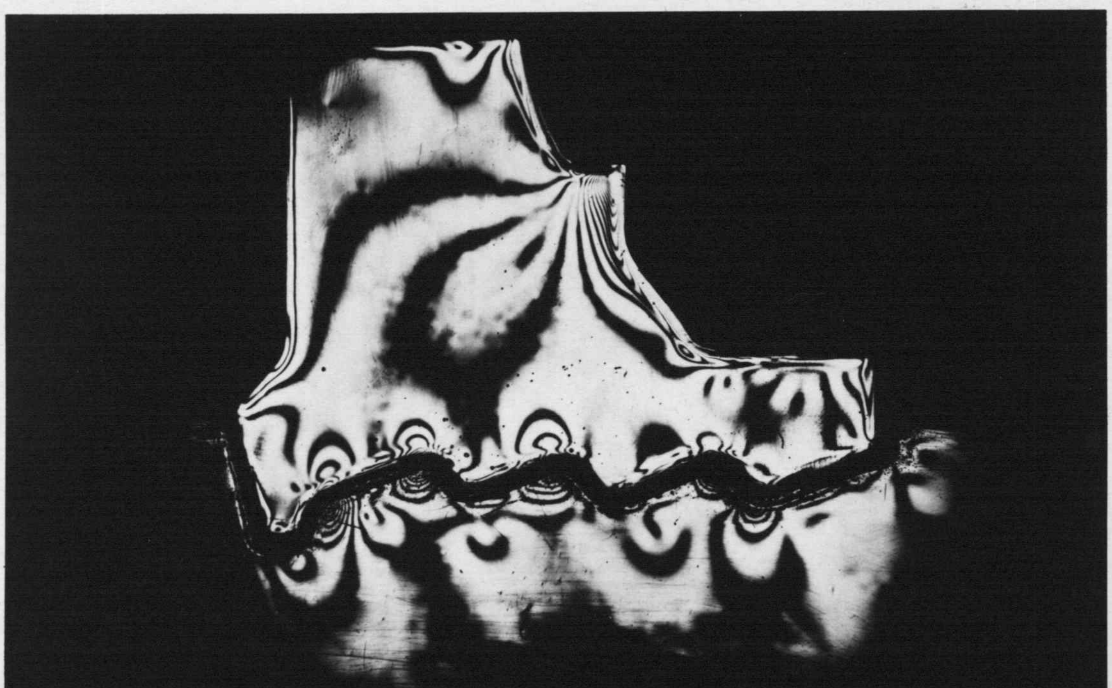


Fig. 29.-Fringes, Sec. III, Dead Load Correct near Base Only

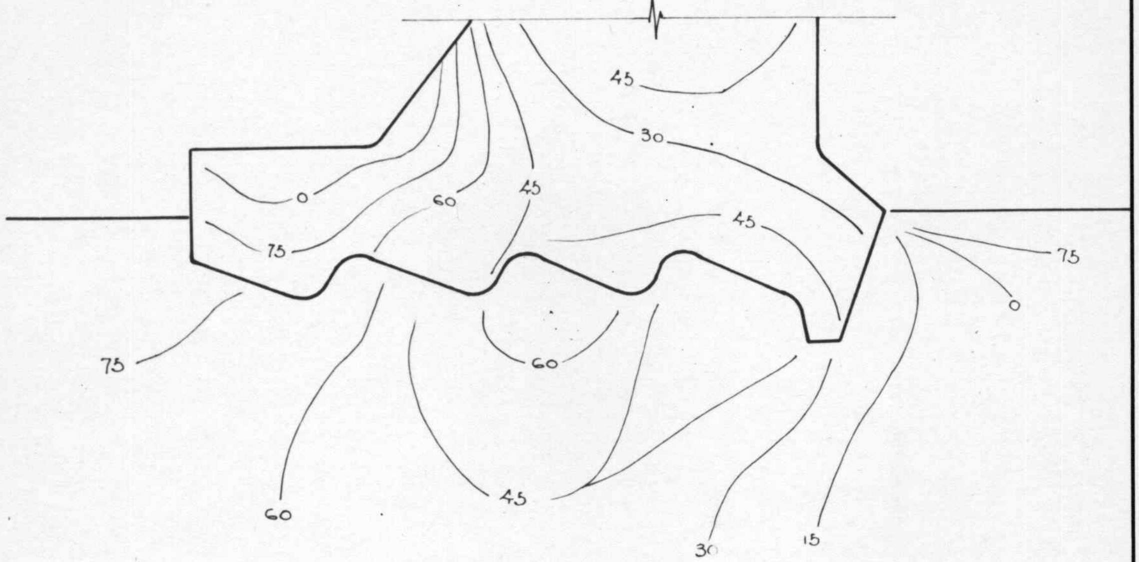


Fig. 30.-Isoclinic Lines, Sec. II, Dead Load

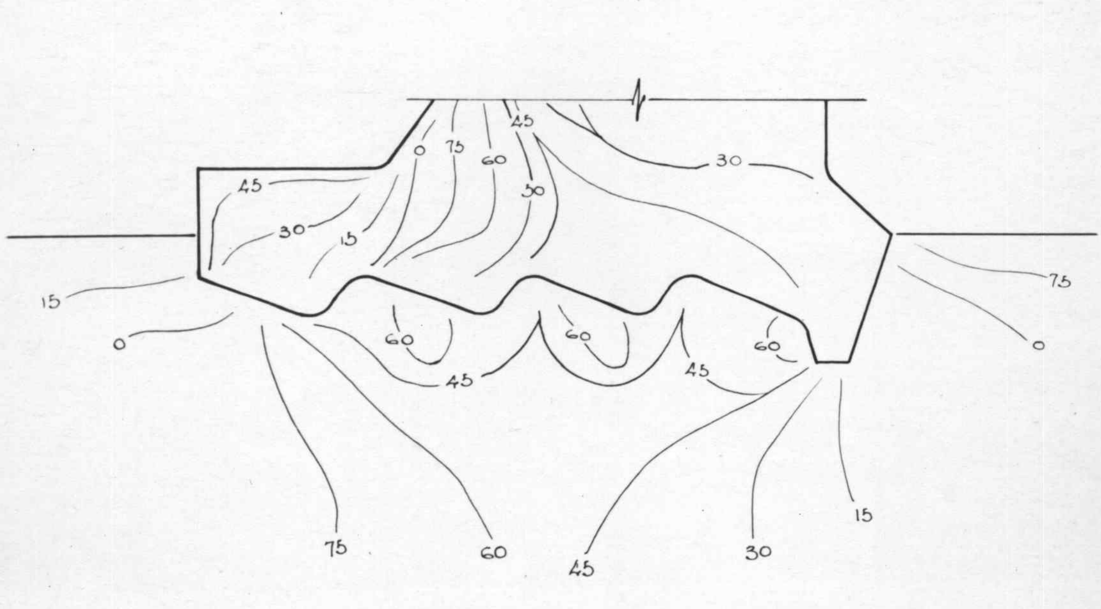


Fig. 31.-Isoclinic Lines, Sec. III, Dead Load

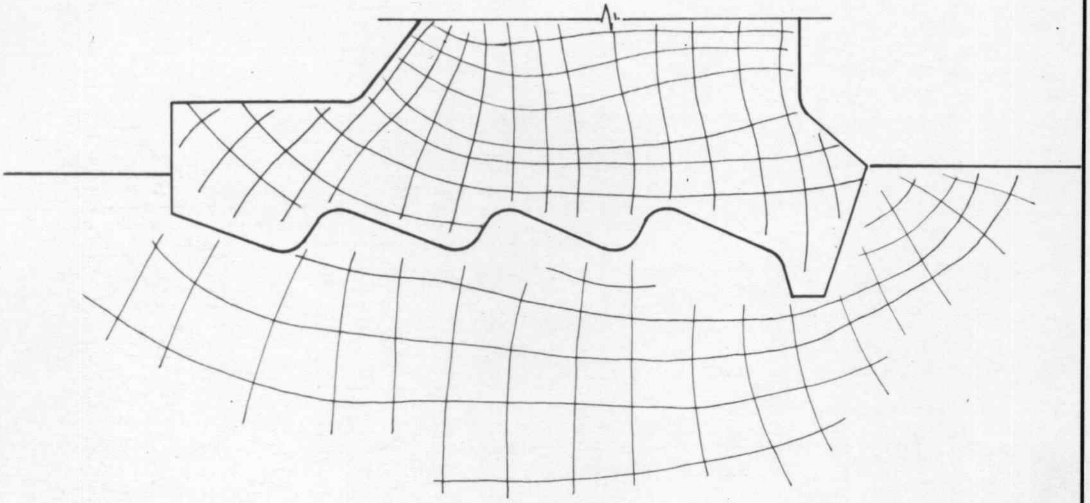


Fig. 32.-Stress Trajectories, Sec. II, Dead Load

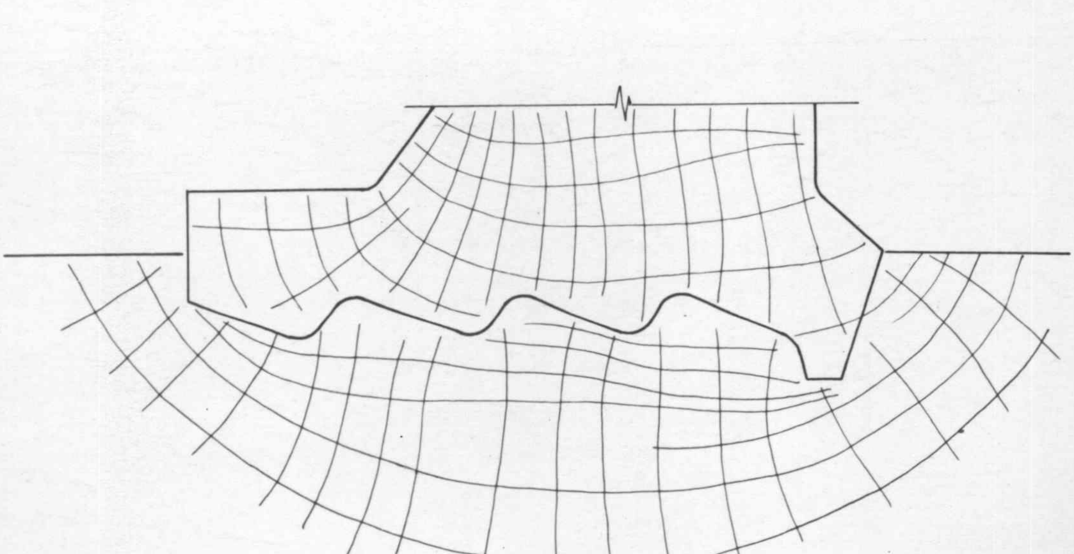


Fig. 33.-Stress Trajectories, Sec. III, Dead Load

Fig. 34.-Fringes, Sec. II, Full Load Correct near Base Only

Fig. 35.-Fringes, Sec. III, Full Load Correct near Base Only

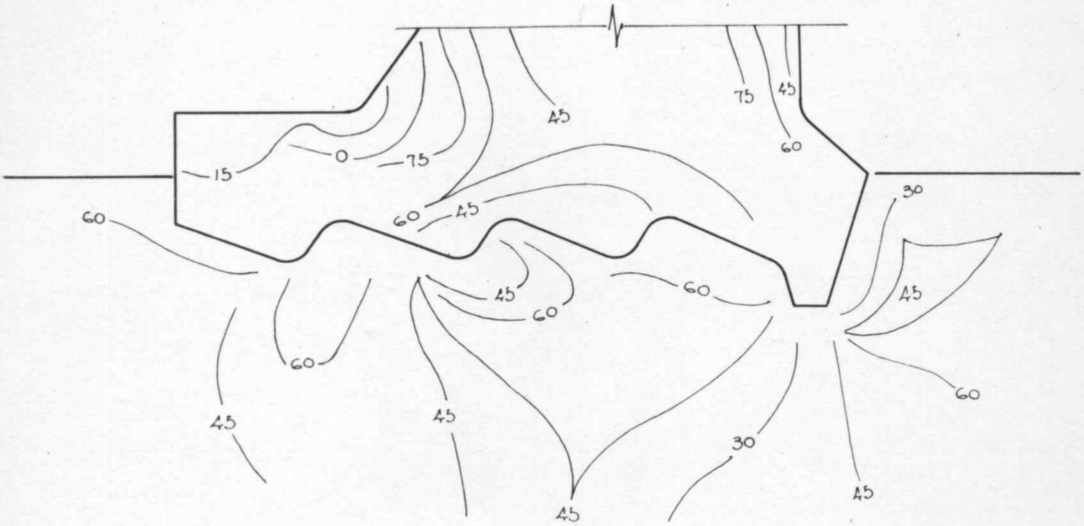


Fig. 36.-Isoclinic Lines, Sec. II, Full Load

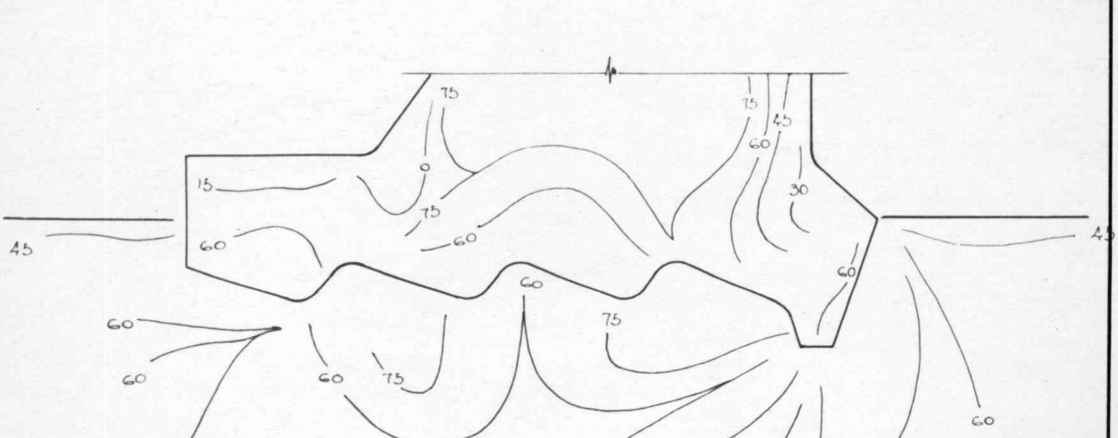


Fig. 37.-Isoclinic Lines, Sec. III, Full Load

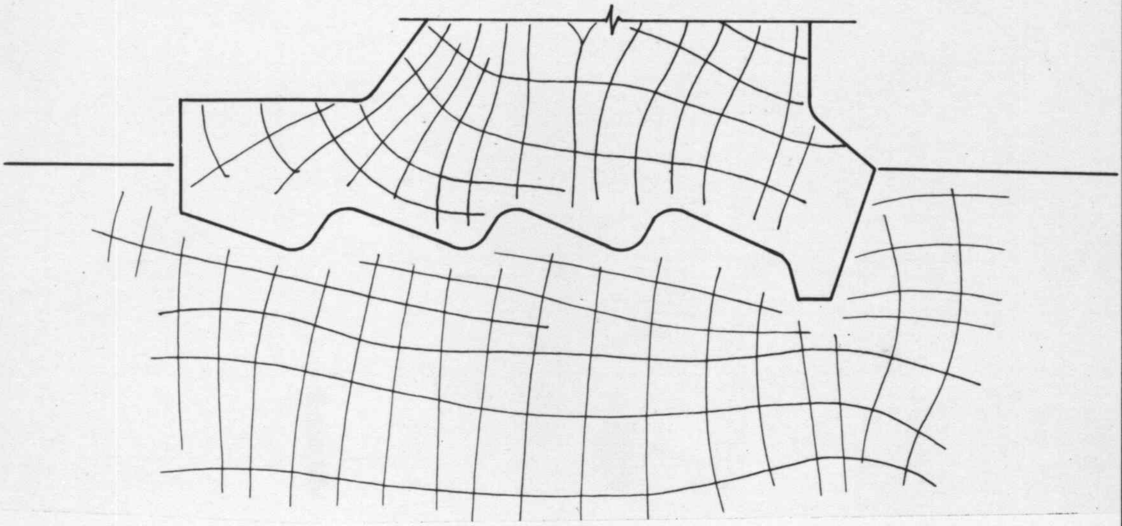


Fig. 38.-Stress Trajectories, Sec. II, Full Load

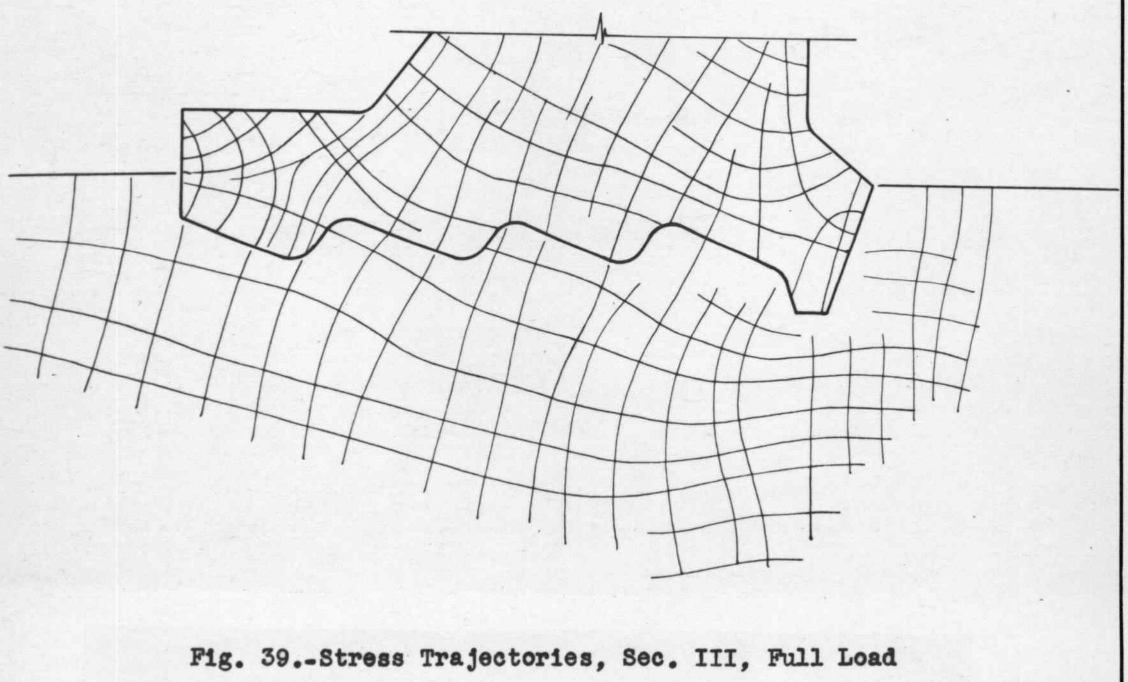


Fig. 39.-Stress Trajectories, Sec. III, Full Load

PART V

CONCLUSIONS

PART V. CONCLUSIONS

Any attempt to analyze the stresses in a structure by means of a small model when body forces of the prototype, such as those due to gravity, can not be neglected has proved very unsatisfactory. By substituting a centrifugal field for the gravitational field and increasing the centrifugal force as the model scale is decreased the stresses due to dead weight could be obtained.

It has been possible to obtain certain results of some value. Verification has been made of the principal stress direction for full load at the base of the dam. A complete analysis has been obtained for water load only to show the application of the method of photoelasticity when the loading is not too complex.

The value of the maximum shear has been determined for points in the foundation. These were found to be of small magnitude at every point except the heel and toe for the water load only and for the full load. Since these values were obtained from the monolithic section where much tension may be developed and no readjustment take place, they are not alarming.

It has been impossible to determine the effect on stresses in the dam itself of a foundation material of lower modulus of elasticity by the method used here. Further experimentation is under way and it is hoped that a result, either positive or negative, may be obtained in the near future. It may be stated, however, that the relationship is a very complex one, but that it seems possible to obtain results by the photo-elastic method for particular cases.

PART VI

APPENDIX

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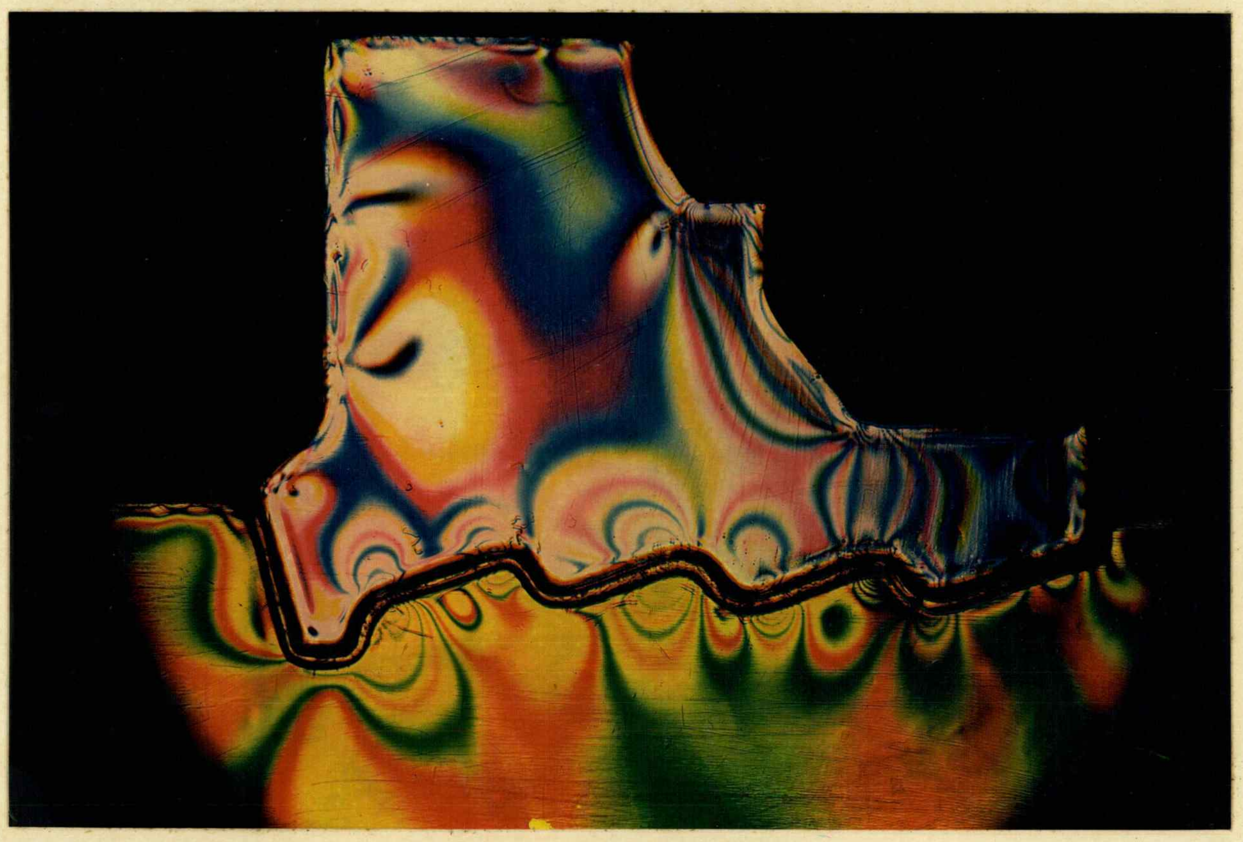

Isochromatic Lines - Constant Shear or (P-Q) - for
Model No. III, Full Load, Correct at Base Only