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Title: Calculation of the Reactivity Change from Void-

ing the Heavy Water Coolant of the Plutonium Recycle

Test Reactor

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Abstract approved: ___
　　　　　　　　　　　　Dr. John C. Ringle

　　　A method is developed for calculating the reac-

tivity worth of the heavy water coolant of the Plutonium

Recycle Test Reactor (PRTR). The coolant reactivity

worth is determined by calculating the effective multi-

plication constant of the reactor core with and without

the coolant in given process tubes. The calculational

method includes a method for calculating the resonance

integrals for PRTR circular 19-rod cluster fuel ele-

ments based on experimental resonance integral data for

single UO_2 fuel rods and hexagonal 19-rod clusters of

UO_2 fuel rods.

　　　The calculational method is evaluated by comparing

the theoretical results with experimental coolant reac-

tivity measurements previously obtained at the PRTR.

The accuracy of the calculational method is shown to

be within the experimental accuracy of the cross

section data by calculating the reactivity change pro-
duced by the change in each individual multigroup
cross section.

Calculation of the Reactivity Change
from Voiding the Heavy Water
Coolant of the Plutonium Recycle
Test Reactor

by

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I. INTRODUCTION

One important parameter of a water-cooled power
reactor from a nuclear safety standpoint is the reac-
tivity worth of the coolant. This parameter is impor-
tant because under certain accident conditions, such
as flow blockage, rapid steam formation may expel the
coolant from a fuel element channel in a very short
period of time.

Because of the need to predict the reactivity
worth of the coolant in advance and because of the
reactor outage time required to make experimental
coolant reactivity worth measurements, it is desirable
to be able to calculate the reactivity worth of the
coolant using analytical methods.

In this work a method is developed and evaluated
for calculating the reactivity worth of the coolant of
the Plutonium Recycle Test Reactor (PRTR). The calcu-
lational method is evaluated by comparing the theoreti-
cal results with experimental coolant reactivity mea-
surements previously obtained at the PRTR.

The reactivity worth of the coolant is calculated
by performing diffusion theory calculations of the
effective multiplication constant of the reactor core

with and without the coolant in given process tubes.
Special attention is given to developing a method for
calculating the geometry-dependent resonance integrals
for the PRTR fuel elements.

II. EXPERIMENTAL COOLANT REACTIVITY WORTH DATA

Description of the PRTR Core

The PRTR is a heavy water moderated and cooled, vertical tube type reactor. The unpressurized heavy water moderator is contained in an aluminum calandria. Passing vertically through the calandria are 85 Zircaloy-2 process tubes, containing the fuel elements which are cooled with pressurized heavy water. The process tubes are arranged on an eight inch hexagonal lattice.

The primary mode of reactor control is by varying the height of the moderator (moderator level) in the calandria. The calandria can contain up to 18 shim assemblies for gross adjustment of the core reactivity.

The standard PRTR fuel element is a 19-rod cluster of Zircaloy-2 clad urania (UO_2) or urania-plutonia (UO_2-PuO_2) fuel. Additional information on the PRTR is given in the Final Safeguards Analysis [17].

Reactivity Worth Measurements

Experimental coolant reactivity worth data have previously been measured at the PRTR. This establishes a basis of comparison with the results of the calculational method. The experimental coolant reactivity worth measurements that are used in this work were made

by R. I. Smith [14] at the PRTR in June 1964. The
reactor core configuration at the time of the experi-
mental measurements is shown on Figure 1. All fuel
elements are 19-rod clusters with a fuel length of
223.5 cm (88 inches). The top of the fuel elements
are at a moderator level of 248.90 cm (98 inches).
Experimental measurements were made of the reactivity
worth of the coolant in: (a.) two process tubes con-
taining unirradiated UO_2 fuel, (b.) three process tubes
containing unirradiated UO_2-0.48 wt% PuO_2 fuel, and
(c.) three process tubes containing unirradiated
UO_2-1.0 wt% PuO_2 fuel.

The reactivity worth of the coolant was measured
in terms of the difference in the critical moderator
level with coolant and without coolant in the process
tubes.[1] The critical moderator level was measured by
making inverse multiplication plots of the neutron flux
as a function of the moderator level.

The critical moderator level with all process
tubes containing coolant was measured to be 250.50 cm.
The measured differences in the critical moderator
levels with the coolant removed from the process tubes

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1.   Measurement of the reactivity worth of the coolant
     by making period measurements is usually not possi-
     ble for a heavy water moderated reactor containing
     irradiated fuel because the (γ, n) flux causes an
     excessive temperature increase in the uncooled fuel.
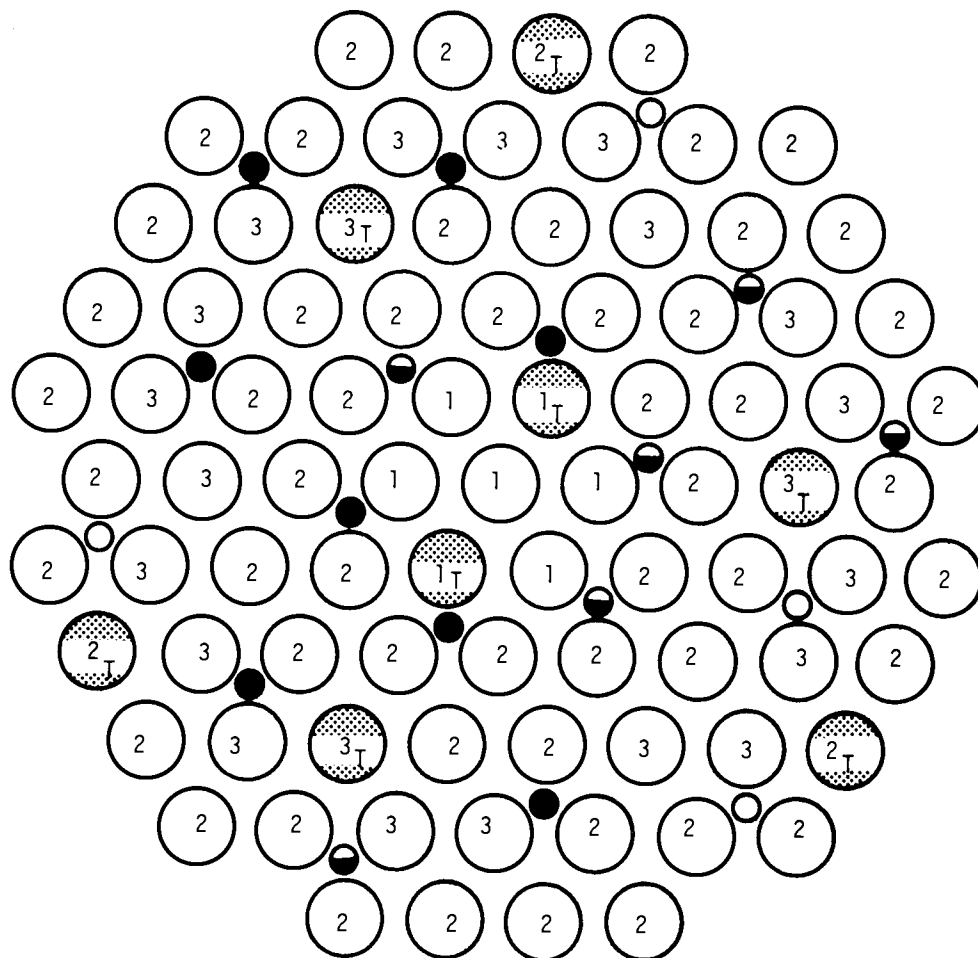
Figure 1.   Reactor Core Configuration

containing the unirradiated test fuel elements are listed in Table I. The possible errors listed in Table I are the standard deviations of the measurements.

TABLE I. MEASURED CRITICAL MODERATOR LEVEL DIFFERENCES

Fuel Type	Number of Fuel Elements	Moderator Level Change from Removing Coolant, cm
UO_2	2	-0.38 ± 0.04
UO_2-0.48% PuO_2	3	$+0.49 \pm 0.06$
UO_2-1.0% PuO_2	3	$+0.09 \pm 0.05$

As shown in Table I the critical moderator level decreased when the coolant was removed from the UO_2 fuel elements, increased when the coolant was removed from the UO_2-0.48% PuO_2 fuel elements, and changed very little (slight increase) when the coolant was removed from the UO_2-1.0% PuO_2 fuel elements. The small change in the critical moderator level for the UO_2-1.0% PuO_2 fuel was verified with two measurements.

III. CROSS SECTION CALCULATIONS

The reactivity worth of the coolant is calculated by making a diffusion theory calculation of the reactor multiplication constant with coolant and without coolant in given process tubes. For the diffusion theory calculations, the reactor core at the time of the experimental coolant reactivity worth measurements (see Figure 1) is converted into the 17 cylindrical regions listed in Table II. The average few-group cross sections required for the diffusion theory calculations are calculated for each of these cylindrical regions from available nuclear cross section and resonance integral data.

Thermal Cross Sections

The spectrum and volume averaged thermal cross sections for the fueled regions of the core are calculated with the computer code THERMOS [7,18] The integral transport equation is solved by the THERMOS code to determine the thermal neutron spectrum as a function of position in cylindrical geometry.

The neutron energy range for the THERMOS calculations is from 0 to 0.683 ev. The basic cross section data is from the BNW Master Library [16].

TABLE II. CYLINDRICAL CORE REGIONS

Region	Number of Fuel Elements in Region	Type of Fuel in Region	Outer Radius of Region, cm	Average Exposure of Fuel in Region, MWd/MTM	Shims per Region
1	1	UO_2	10.67	4,600	0
2	2	UO_2	18.48	4,600	1/2
3	2	UO_2	23.85	0	2/3
4	2	UO_2	28.23	4,600	1/3
5	12	UO_2-0.48% PuO_2	46.50	5,000	3
6	12	UO_2-0.48% PuO_2	59.40	5,000	1 1/2
7	2	UO_2-1.0% PuO_2	61.29	1,500	0
8	3	UO_2-1.0% PuO_2	64.01	0	0
9	7	UO_2-1.0% PuO_2	69.96	1,500	1 1/2
10	10	UO_2-1.0% PuO_2	77.67	1,500	1 5/6
11	8	UO_2-0.48% PuO_2	83.33	3,200	1
12	4	UO_2-0.48% PuO_2	86.01	3,200	0
13	3	UO_2-0.48% PuO_2	87.97	0	0
14	17	UO_2-0.48% PuO_2	98.36	3,200	2/3
15	-	Moderator-D_2O	106.00	-	-
16	-	Calanderia-Al	106.81	-	-
17	-	Reflector-D_2O	166.20	-	-

For the THERMOS calculations, the PRTR elementary hexagonal lattice cell is converted to a cylindrical

cell of the same area, and the 19-rod-cluster fuel ele-
ment is converted to the three fuel regions shown in
Figure 2. In converting the PRTR 19-rod-cluster to a
center fuel rod surrounded by two rings of fuel, the
coolant areas inside the center line radius r_b, the
coolant areas between the radii r_b and r_c, and the cool-
ant outside the radius r_c are conserved. The cladding
is homogenized with the fuel, and the helium gas gap
between the process tube and the shroud tube is homo-
genized with the shroud tube to reduce the number of
cell regions.

The shim rods adjacent to the two unirradiated
UO_2 test fuel elements are homogenized with part of
the moderator as shown on Figure 2. No shims are adja-
cent to the unirradiated UO_2-0.48% PuO_2 and UO_2-1.0%
PuO_2 test fuel elements.

The dimensions of the radii shown on Figure 2 for
the unirradiated test fuel elements used in the experi-
mental coolant worth measurements are listed in
Table III.

The isotopic compositions and the density of the
unirradiated test fuel elements are listed in Table IV.

The isotopic composition of the primary coolant
at the time of the coolant worth measurements was
97.25 molecular percent D_2O and 2.75 molecular percent
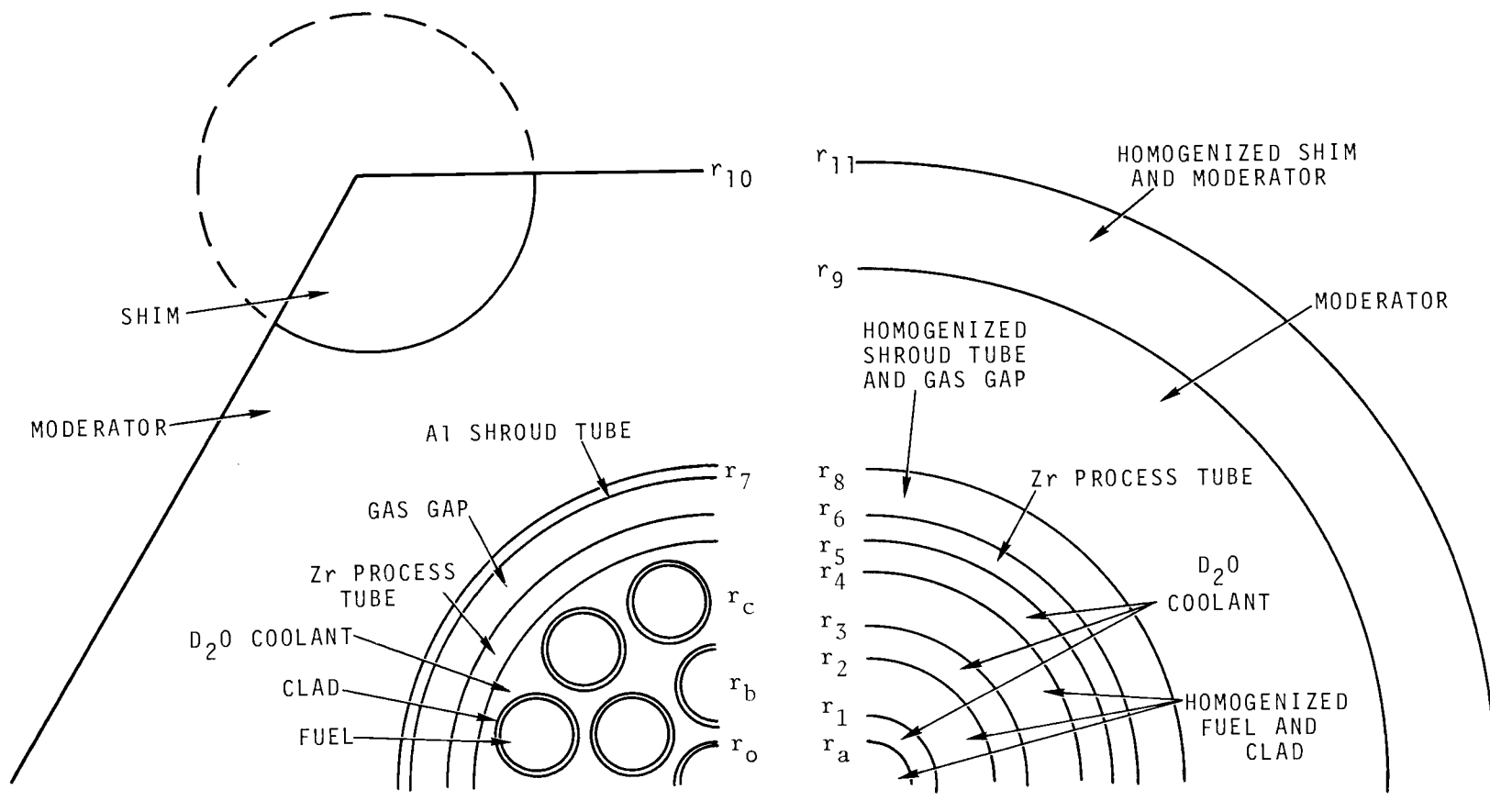
Figure 2.   PRTR Hexagonal Cell and Cylindrical Cell for the THERMOS
Calculations

TABLE III. FUEL AND CELL DIMENSIONS

Radii (Shown on Figure 2)	Dimensions of UO_2 fuel, cm	Dimensions of UO_2-0.48% PuO_2 and UO_2-1.0% PuO_2 Fuel, cm
r_0	0.6350	0.6443
r_a	0.7137	0.7268
r_b	1.632	1.637
r_e	3.155	3.162
r_1	1.133	1.116
r_2	2.083	2.101
r_3	2.654	2.643
r_4	3.627	3.650
r_5	4.13	4.13
r_6	4.52	4.52
r_7	5.25	5.25
r_8	5.40	5.40
r_9	8.83	-
r_{10}	10.16	10.16
r_{11}	10.67	10.67

H_2O. The isotopic composition of the moderator was 99.7 molecular percent D_2O and 0.3 molecular percent H_2O.

The thermal cell-averaged absorption and fission cross sections calculated by THERMOS for the three kinds of unirradiated test fuel elements are listed on Table V, along with the other cell-averaged few group parameters.

TABLE IV. FUEL DENSITY AND ISOTOPIC COMPOSITIONS

	UO_2 Fuel	UO_2-0.48% PuO_2 Fuel	UO_2-1.0% PuO_2 Fuel
Fuel Density, gm/cm^3	9.42	9.76	9.69
Weight Fraction of ^{239}Pu in Pu	-	0.857	0.909
Weight Fraction of ^{240}Pu in Pu	-	0.13	0.083
Weight Fraction of ^{241}Pu in Pu	-	0.013	0.008
Uranium	Natural	Natural	Natural

The isotopic compositions of the irradiated fuel elements are obtained from burnup calculations previously performed at the PRTR [13]. The cell-averaged cross sections calculated for the irradiated fuel elements are similar to the cross sections for the unirradiated test fuel elements that are listed in Table V.

The thermal diffusion coefficients for all regions and the thermal absorption cross sections for the non-fuel regions are calculated with the nondimensional computer code TEMPEST [12]. The Wigner-Wilkins light mass moderator equation is solved with the TEMPEST code to determine the thermal neutron flux spectrum. The energy-dependent cross sections are then averaged over the calculated thermal neutron spectrum to obtain average thermal parameters for the diffusion theory calculations.

TABLE V.   CELL AVERAGE FEW GROUP PARAMETERS

Parameter*	UO_2 Fuel		UO_2-0.48% PuO_2 Fuel		UO_2-1.0% PuO_2 Fuel		Moderator
	Not Void	Void	Not Void	Void	Not Void	Void	
D_1	2.3438+00	2.5110+00	2.3455+00	2.5036+00	2.3467+00	2.5049+00	1.973
D_2	1.2671+00	1.3595+00	1.2614+00	1.3481+00	1.2611+00	1.3479+00	1.2309
D_3	0.92269	1.0015	0.92387	0.99749	0.9214	0.9934	8.52-01
Σ_{r_1}	8.2953-02	7.6917-02	8.2465-02	7.6765-02	8.2435-02	7.6736-02	9.523-02
Σ_{r_2}	1.0682-02	9.7038-03	1.0740-02	9.8084-03	1.0790-02	9.8570-03	1.277-02
Σ_{r_3}	1.0705-02	1.0648-02	8.3380-03	8.5959-03	1.0067-02	1.0462-02	8.332-05
$\nu\Sigma_{f_1}$	1.9893-03	1.9881-03	2.1645-03	2.1633-03	2.1851-03	2.1839-03	0.0
$\nu\Sigma_{f_2}$	3.6533-04	3.6111-04	6.5570-04	6.4641-04	9.5147-04	9.3181-04	0.0
$\nu\Sigma_{f_3}$	6.8667-03	6.8671-03	1.2322-02	1.2769-02	1.6302-02	1.7015-02	0.0
Σ_{1-2}	8.1731-02	7.5731-02	8.1163-02	7.5498-02	8.1129-02	7.5464-02	9.441-02
Σ_{1-3}	1.2174-10	1.2012-10	1.2357-10	1.2197-10	1.2358-10	1.2197-10	0.0
Σ_{2-3}	8.9861-03	8.1140-03	8.6467-03	7.8380-03	8.5234-03	7.7218-03	1.276-02

*   1 fast energy group (1.35 Mev to 10 Mev)

    2 epithermal energy group
    (0.683 ev to 1.35 Mev)

    3 thermal energy group (0 to 0.683 ev)

Note: $1.000-03 = 1.000 \times 10^{-3}$

D_i diffusion coefficient for energy group i

Σ_{r_i} removal cross section for energy group i

Σ_{f_i} fission cross section for energy group i

Σ_{i-j} cross section for scattering from energy group i to energy group j

The thermal diffusion coefficients and the thermal absorption cross sections for the moderator are listed in Table V.

Resonance Integrals for Uranium-238

The uranium-238 resonance integrals for single UO_2 fuel rods and some clusters of UO_2 fuel rods have been experimentally measured using activation techniques. Hellstrand [5,6] found that the measured resonance integral data for UO_2 fuel rods fits the equation

$$RI = 4.15 + 26.6 \sqrt{S/M}, \tag{3.1}$$

where RI is the resonance integral of uranium-238 in barns excluding 1/v capture, S is the surface area of · the fuel per unit length in cm^2/cm, and M is the mass of the UO_2 fuel per unit length in grams/cm.

Carlvik and Pershagen, as cited by Hellstrand [5] developed a method based on the experimental data of Hellstrand to calculate an effective surface area for hexagonal clusters. The uranium-238 resonance integrals for hexagonal clusters could then be calculated from Equation (3.1). Later, Pershagen [11] showed that the calculational methods developed by Carlvik and Pershagen did not adequately calculate the resonance integrals that were measured by Bernander and Jirlow as cited by Pershagen [11]. The discrepancy occurred for hexagonal

clusters with water-to-fuel ratios larger than the water-to-fuel ratio used in the resonance integral measurements of Hellstrand.

In this work a different method is developed for calculating the effective surfaces of clusters. These effective surfaces can be used to calculate the resonance integrals of PRTR-type circular clusters as well as hexagonal clusters. Diagrams of the circular and hexagonal 19-rod clusters are shown on Figure 3.

For calculating the effective surface of a fuel element cluster with no coolant between the fuel rods, the cluster is assumed to be a low density fuel rod with the same outer surface as the cluster. The effective surface (S_{eff}) used in Equation (3.1) for calculating the resonance integral of a cluster with no coolant between the fuel rods is

$$S_{eff} = 1.08 S_{rb}, \qquad (3.2)$$

where S_{rb} is the surface that would be formed if a rubber band were stretched around the cluster with no cladding on the fuel rods (rubber band surface). The equation for calculating the rubber band surface for either the circular 19-rod cluster or the hexagonal 19-rod cluster is

$$S_{rb} = 12[\ell + 30\pi r_o/180] \qquad (3.3)$$

where ℓ is the distance between the centers of the fuel rods, and r_o is the radius of the bare fuel rods.

Circular Cluster (PRTR)
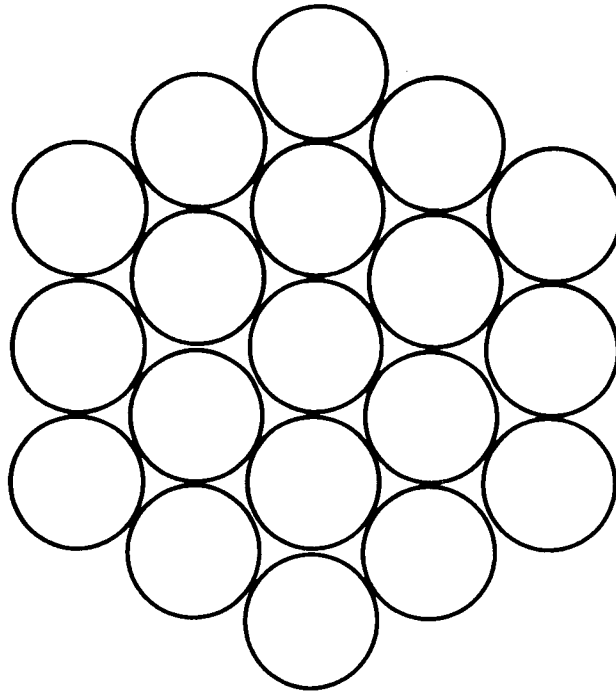
Hexagonal Cluster

Figure 3. 19-Rod Clusters

The constant of 1.08 in Equation (3.2) is obtained by fitting the Equations (3.1) and (3.3) to the experimental data of Hellstrand.
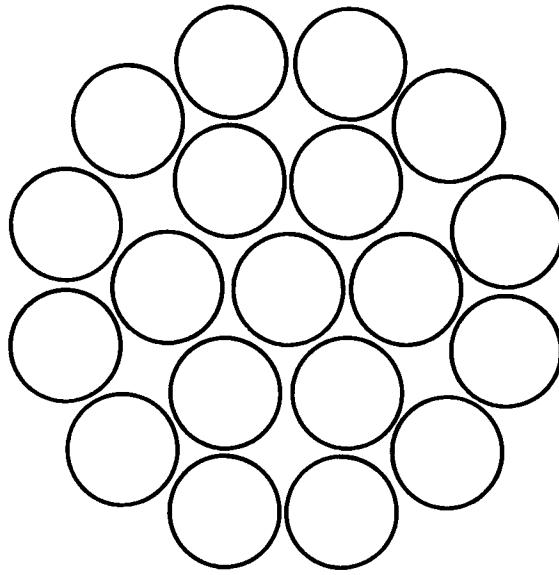
For calculating the effective surface of a fuel element cluster with coolant between the fuel rods, the cluster is assumed to be a single fuel rod with a larger surface than the outside of the cluster. The larger surface accounts for the additional neutrons scattered into the resonance energy range by the coolant inside the cluster. The effective fuel surface (S_{eff}) used in Equation (3.1) for calculating the resonance integral of clusters with coolant between the fuel rods is

$$S_{eff} = S_{rb} + 4\Sigma_M \sum_i V_{M_i} \, {}^A F_i \, P_{o_i} , \qquad (3.4)$$

where S_{rb} is the rubber band surface, Σ_M is the macroscopic scattering cross section of the coolant, V_{M_i} is the volume of the ith coolant region inside the cluster, $^A F_i \, P_{o_i}$ is the probability that a neutron scattered into the resonance energy range in the ith coolant region reaches the fuel without additional scattering, and the sum is over all coolant regions inside the cluster. A coolant region is defined as the volume of coolant bounded by three or four fuel rods inside the cluster. The derivation of Equation (3.4) is given in the Appendix.

The nonscattering probability in Equation (3.4) is expressed in terms of a geometrical parameter A_{F_i} and a moderator noncollision probability P_{o_i}. Each of these parameters is based in part on the measured resonance integral data.

The geometrical parameter A_{F_i} accounts for the scattering of neutrons between fuel rods. For a coolant region surrounded by three equally spaced fuel rods, the equation for A_{F_i} is

$$A_{F_i} = 1 - \frac{6}{360} \tan^{-1} [(\ell - 2r_o)\sqrt{3}/\ell], \qquad (3.5)$$

where ℓ is the distance between fuel rod centers, r_o is the radius of the bare fuel rods, and the inverse tangent is in degrees. For a coolant region surrounded by four equally spaced fuel rods, the equation for A_{F_i} is

$$A_{F_i} = 1 - \frac{8}{360} \tan^{-1} [(\ell - 2r_o)/\ell], \qquad (3.6)$$

where ℓ and r_o are the same as in Equation (3.5). The derivations of Equations (3.5) and (3.6) are given in the Appendix.

The moderator noncollision probability P_{o_i} accounts for the neutrons that are scattered out of the resonance energy range before they reach the fuel. The P_{o_i} is calculated by assuming that each coolant region inside the cluster is an infinite cylinder with a volume equal to the volume of the coolant plus the

volume of the cladding bordering the coolant region.
The collision probability $(1 - P_o)$ for an infinite
cylinder is given by Case, de Hoffmann, and Placzek [3]
as a function of cylinder radius and scattering cross
section. In this work, the macroscopic scattering cross
section of the PRTR coolant (97.25 percent heavy water)
is taken as 0.38 cm^{-1}. The scattering of the coolant
used in the resonance integral measurements is taken
as 0.35 cm^{-1}.

The experimentally measured resonance integrals,
the resonance integrals calculated from Equations (3.1),
(3.2) and (3.4), and the parameters used in the calcu-
lations are listed in Table VI.

As shown in Table VI there is a reasonable agree-
ment between the measured and the calculated resonance
integrals for the hexagonal clusters over a wide range
of rod pitches (ℓ) and interior coolant volumes (V_M).

Resonance Integrals for Uranium-235 and Plutonium Isotopes

The resonance integrals for the uranium-235 and
the plutonium isotopes are calculated with the HRG [2,8]
computer code using the methods of Adler, Hinman, and
Nordheim [1]. The calculational method uses the narrow-
resonance (NR) and narrow-resonance-infinite-mass-
absorber (NRIA) approximations to solve the integral
equations for the neutron flux in the absorber.

TABLE VI. MEASURED AND CALCULATED URANIUM-238 RESONANCE INTEGRALS

19-Rod Cluster	ℓ, cm	r_o, cm	V_M,* cm^3/cm	P_o*	A_F*	S_{rb}, cm^2/cm	Calculated RI's, barns		Measured RI's, barns	
							Coolant Not Void	Coolant Void	Coolant Not Void	Coolant Void
hexagonal[a]	1.89	0.74	0.4118	0.8245	0.6567	27.32	12.78	12.1	12.8 ± 0.3	12.1 ± 0.3
hexagonal[b]	2.11	0.85	0.569	0.8137	0.690	30.66	12.16	11.31	12.23 ± 0.23	11.50 ± 0.14
hexagonal[b]	2.60	0.85	1.568	0.7423	0.4842	36.54	13.42	11.97	13.66 ± 0.18	12.41 ± 0.14
circular[c]	1.632	0.635	0.3530	0.8324	0.6497	23.58	14.20	13.07	-	-
			1.063	0.7508	0.7222					
circular[d]	1.637	0.6443	0.3305	0.8342	0.6628	23.69	13.85	12.80		
			1.020	0.7523	0.7330					
circular[e]	1.637	0.6443	0.3305	0.8342	0.6628	23.69	13.89	12.83		
			1.020	0.7523	0.7330					

a   Hellstrand UO_2 fuel

b   Bernander and Jirlon UO_2 fuel

c   PRTR UO_2 fuel

d   PRTR UO_2-0.48% PuO_2 fuel

e   PRTR UO_2-1.0% PuO_2 fuel

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*   The circular 19-rod cluster has two types of coolant regions inside the cluster, as shown in Figure 3.

For the fuel element clusters with no coolant (air) between the fuel rods, the clusters are assumed to be a single, low density fuel rod for the resonance integral calculations. The volume of the low density fuel rod is equal to the volume of the fuel plus the volume of the cladding and void (air) inside the cluster.

For the fuel element clusters with coolant between the fuel rods, the resonance integrals are calculated for a single fuel rod and associated coolant.

A potential scattering cross section of 10.7 barns is used for the uranium and plutonium isotopes and a scattering cross section of 3.7 barns is used for the oxygen in the fuel in the resonance integral calculations.

The geometrical $\bar{\ell}$ (mean cord length in the fuel) parameters required to calculate the resonance integrals for the plutonium and uranium-235 isotopes are determined by finding the $\bar{\ell}$ parameters that will give the uranium-238 resonance integrals calculated from Equations (3.1), (3.2) and (3.4). This method of determining the $\bar{\ell}$ parameters is used because the $\bar{\ell}$ parameters cannot be simply defined for cluster type fuel elements.

The resonance integrals for uranium-238 listed in Table VI include all resonance absorption except the

low energy 1/v absorption. The resonance integral cal-
culation for uranium-238 in HRG considers resonance
absorption up to 2.5 kev excluding the 1/v absorption.
Absorption in uranium-238 above 2.5 kev is treated in
HRG by cross section information on the HRG data tape.
The contribution to the uranium-238 resonance integral
above 2.5 kev is estimated to be 2.5 barns and is
assumed to be independent of the coolant in the cluster.
This estimate is based on contributions of 1.30 barns
between 1.0 kev and 30 kev and 1.7 barns above 30 kev
given by Nordheim [10], and a contribution of 0.5 barns
between 1.0 kev and 2.5 kev calculated by HRG. This
contribution of 2.5 barns above 2.5 kev is subtracted
from the resonance integrals listed in Table VI to
determine the uranium-238 resonance integrals required
to determine the $\bar{l}$ parameters.

The calculated $\bar{l}$ parameters and the calculated
resonance integrals for absorption and fission in the
uranium-235 and plutonium isotopes are listed in
Table VII for the UO_2, the UO_2-0.48% PuO_2 and the
UO_2-1.0% PuO_2 fuel elements. The same $\bar{l}$ parameters
are used for the UO_2-0.48% PuO_2 and the UO_2-1.0% PuO_2
fuel elements because the geometry is the same and the
calculated uranium-238 resonance integrals (see
Table VI) are approximately the same for both types
of fuel elements.

TABLE VII. CALCULATED MEAN CORD LENGTHS ($\overline{\ell}$) AND RESONANCE INTEGRALS (RI)

	UO_2 Fuel		UO_2-0.48% PuO_2 Fuel		UO_2-1.0% PuO_2 Fuel	
	Coolant Not Void	Coolant Void	Coolant Not Void	Coolant Void	Coolant Not Void	Coolant Void
$\overline{\ell}$, cm	4.25	9.85	4.54	10.42	4.54	10.42
^{235}U RI_a, barns	300.7	296.8	299.6	295.6	299.8	295.8
^{235}U RI_f, barns	186.3	184.2	185.7	183.5	185.9	183.6
^{239}Pu RI_a, barns	-	-	308.5	300.8	275.0	263.8
^{239}Pu RI_f, barns	-	-	180.8	176.8	163.3	157.5
^{240}Pu RI_a, barns	-	-	3706.	3365.	3188.	2893.
^{241}Pu RI_a, barns	-	-	343.4	343.3	343.4	343.3
^{241}Pu RI_f, barns	-	-	285.0	284.9	285.0	284.9

Epithermal and Fast Cross Sections

In addition to calculating the resonance inte-
grals, the HRG code is used to calculate the average
broad group epithermal and fast cross sections for
each of the core regions. The HRG code computes the
energy-dependent neutron flux spectrum necessary for
averaging the cross sections by solving the Boltzmann
transport equation. The B_1 approximation is used to
consider the effect of leakage on the neutron spectrum.
A leakage parameter of 2.5×10^{-2} cm^{-1} (square root of
the reactor buckling) is used in all HRG calculations.

The epithermal cross sections are averaged over the
neutron energies from 0.683 ev to 1.35 Mev and the fast
cross sections are averaged over the neutron energies
from 1.35 Mev to 10 Mev to obtain the average broad
group epithermal and fast cross sections for the diffu-
sion theory calculation. The basic cross section data
is from the BNW Master Library.

The average broad group epithermal and fast cross
section parameters for the three kinds of unirradiated
test fuel elements are listed in Table V along with the
average thermal cross sections. The calculated cross
section parameters for the irradiated fuel are similar
to the cross section parameters for the unirradiated
fuel listed in Table V.

IV. COOLANT REACTIVITY WORTH CALCULATIONS

The reactivity worth of the coolant is calculated
by calculating the effective multiplication constant of
the reactor core with and without the coolant in given
process tubes using the diffusion theory code HFN [9].
The HFN code uses numerical methods to solve the multi-
region, one space-dimensional, multigroup diffusion
equation for the lowest eigenvalue. The diffusion
theory calculations of the coolant reactivity worth are
performed in cylindrical geometry.

The reactor core at the time of the experimental
coolant reactivity worth measurements is divided into
the 17 cylindrical regions listed in Table II. The fuel
elements in each region are of the same enrichment with
approximately the same burnup.

The neutron leakage in the axial direction is
handled in HFN by an axial buckling. The axial buck-
ling is calculated from the equation

$$B_z^2 = \left(\frac{\pi}{H + \delta_1 + \delta_2}\right)^2, \quad (4.1)$$

where

B_z^2 = axial buckling,

H = core height in cm,

δ_1 = top reflector savings in cm, and

δ_2 = bottom reflector savings in cm.

The reflector savings are calculated by finding the
core height with no top and bottom reflector that has

the same multiplication constant as the core with a top
and bottom reflector. The difference in the two core
heights is equal to the sum of top and bottom reflector
savings $(\delta_1 + \delta_2)$. The reflector savings calculations
are performed in slab geometry using the diffusion
theory code HFN. The reactor core is assumed to con-
sist of UO_2-0.48% PuO_2 fuel elements for the
calculations.

The neutron leakage in the radial direction for
the slab calculations is handled by a radial buckling
calculated from the equation

$$B_r^2 = \left(\frac{2.405}{R + \delta}\right)^2, \qquad (4.2)$$

where

B_r^2 = radial buckling,

R = core radius in cm, and

δ = radial reflector savings in cm (assumed
to be 15 cm).

The calculated sum of the top and bottom reflector
savings $(\delta_1 + \delta_2)$ is 30.9 cm. Inserting this reflector
savings into Equation (4.1) gives an axial buckling
(B_z^2) of 1.52×10^{-4} cm^{-2} for the leakage correction in
the diffusion theory calculation of the coolant reac-
tivity worth.

The coolant reactivity worths calculated by HFN
for the three types of unirradiated test fuel elements
are listed in Table VIII.

TABLE VIII. CALCULATED COOLANT REACTIVITY WORTHS

Fuel Type	Number of Elements	Coolant Reactivity Worth, mk
UO_2	2	-0.07
UO_2-0.48% PuO_2	3	+0.14
UO_2-1.0% PuO_2	3	+0.16

V. MODERATOR LEVEL COEFFICIENT CALCULATIONS

As stated in Chapter II, the reactivity worth of
the coolant was measured in terms of a change in the
moderator level. To compare the measured coolant reac-
tivity worth to the calculated coolant reactivity worth
in mk, the moderator level coefficient (mk per cm of
moderator level) must be calculated. The moderator
level coefficient is calculated by performing diffusion
theory calculations in slab geometry. The multiplica-
tion constant is calculated at two moderator levels,
and the moderator level coefficient is equal to the
difference in the two multiplication constants divided
by the difference in the moderator levels. The bottom
reflector region for the slab calculations consists of
the D_2O moderator, the process tubes, the shroud tubes,
and the primary coolant. The top reflector region is
the same with the addition of the stainless steel fuel
hanger rods.

The radial buckling for the slab calculations is
calculated from Equation (4.2). The calculations are
performed for single region cores of UO_2-0.48% PuO_2
and UO_2-1.0% PuO_2 fuel element clusters.

Moderator level coefficients of 0.195 mk/cm and
0.213 mk/cm are calculated at a moderator level of
250.50 cm for the UO_2-0.48% PuO_2 and UO_2-1.0% PuO_2 sin-
gle region reactor cores. These calculated moderator

level coefficients agree well with a moderator level coefficient of 0.21 ± 0.01 mk/cm that was measured at the PRTR in March, 1962, by V. W. Gustafson [4]. The moderator level coefficient was measured at a moderator level of 264 cm by making period measurements.

VI. COMPARISON OF MEASURED AND CALCULATED
COOLANT REACTIVITY WORTHS

The measured changes in the critical moderator
level resulting from removing the coolant from the
unirradiated test fuel elements listed in Table I are
converted to coolant reactivity worths using the average
calculated moderator level coefficient of 0.204 mk/cm.

These measured coolant reactivity worths converted
to mk are listed in Table IX along with the calculated
coolant reactivity worths from Table VIII for the UO_2,
the UO_2-0.48% PuO_2, and the UO_2-1.0% PuO_2 unirradiated
test fuel elements.

As shown in Table IX, the calculated reactivity
worth of the coolant agrees with the measured reactivity
worth of the coolant for the UO_2 and the UO_2-0.48% PuO_2
fuel elements. For the UO_2-1.0% PuO_2 fuel elements,
the calculational method overestimates the coolant
reactivity worth.

TABLE IX. MEASURED AND CALCULATED COOLANT
REACTIVITY WORTHS

Fuel Type	Number of Fuel Elements	Measured Reactivity Worth, mk	Calculated Reactivity Worth, mk
UO_2	2	-0.077 ± 0.008	-0.07
UO_2-0.48% PuO_2	3	$+0.100 \pm 0.012$	$+0.14$
UO_2-1.0% PuO_2	3	$+0.018 \pm 0.010$	$+0.16$

To determine possible reasons for this overestimation, the contribution to the coolant reactivity worth from each cross section parameter is estimated by calculating the change in the multiplication constant which results from a change in each parameter. The calculated contributions to the coolant reactivity worth due to changes in each cross section are listed in Table X for the UO_2-0.48% PuO_2 and the UO_2-1.0% PuO_2 fuel elements.

As shown in Table X, the coolant reactivity worth is primarily determined by changes in the epithermal capture, thermal capture, thermal fission, and epithermal scattering cross sections. Since the contribution to the coolant reactivity worth from changes in each of these cross sections is larger in absolute magnitude than the net coolant worth listed in Table IX, small errors in these cross sections will produce large errors in the calculated coolant reactivity worth.

For example, the change in the epithermal capture cross section depends primarily on the change in the uranium-238 resonance integral. As shown in Table VI, the uranium-238 resonance integral changes by 1.06 barns when the coolant is removed from the UO_2-1.0% PuO_2 fuel cluster. If the possible error in the uranium-238 resonance integral with coolant relative to the resonance integral without coolant is assumed to be 0.3 barns

TABLE X. CALCULATED CONTRIBUTIONS TO THE COOLANT
REACTIVITY WORTH FROM CHANGES IN EACH CROSS SECTION

Cross Section Changed*	Contribution to Coolant Reactivity Worth, mk	
	UO_2-0.48% PuO_2 Fuel	UO_2-1.0% PuO_2 Fuel
Σ_{t_1}	+0.01	-
Σ_{t_2}	-	+0.05
Σ_{t_3}	+0.04	-
Σ_{c_1}	-	-0.01
Σ_{c_2}	-0.27	-0.45
Σ_{c_3}	+0.26	+0.55
Σ_{f_1}	$\sim$0.0	-
Σ_{f_2}	-	+0.04
Σ_{f_3}	-0.51	-1.10
Σ_{s_1}	-	-0.01
Σ_{s_2}	+0.45	+0.92

* 1 fast neutron energy group (1.35 Mev to 10 Mev)

  2 epithermal neutron energy group (0.683 ev to 1.35 Mev)

  3 thermal neutron energy group (0 to 0.683 ev)

Σ_{t_i} transport cross section for energy group i

Σ_{c_i} capture cross section for energy group i

Σ_{f_i} fission cross section for energy group i

Σ_{s_i} scattering cross section for energy group i

(a reasonable assumption considering the possible experimental errors listed in Table VI), the -0.45 mk epithermal capture contribution to the coolant reactivity worth listed in Table X for the UO_2-1.0% PuO_2 fuel could be in error by $(0.3/1.06)(0.45) = 0.13$ mk. This would give a calculated coolant reactivity worth of 0.16 ± 0.13 mk relative to the measured coolant reactivity worth of 0.018 ± 0.010 mk for the UO_2-1.0% PuO_2 fuel. The error for the UO_2-0.48% PuO_2 fuel elements would be 0.076 mk which would give a calculated coolant worth of 0.14 ± 0.076 mk.

Similar possible errors may exist in the epithermal scattering, thermal capture and thermal fission cross sections.

VII. SUMMARY AND CONCLUSIONS

In this work, a method is developed for calculating the reactivity worth of the coolant of the PRTR. The coolant reactivity worth is determined by calculating the effective multiplication constant of the reactor core with and without the coolant in given process tubes. The calculational method includes a method for calculating the resonance integrals for PRTR circular 19-rod cluster fuel elements based on experimental resonance integral data for single UO_2 fuel rods and hexagonal 19-rod clusters of UO_2 fuel rods.

The calculational method is compared with previously measured coolant reactivity worths. The reactivity worth of the coolant for the two UO_2 fuel elements is calculated to be -0.071 mk which agrees with the measured worth of -0.077 mk. The coolant reactivity worth for the three UO_2-0.48% PuO_2 fuel elements is calculated to be 0.14 mk which is in reasonable agreement with the measured worth of 0.10 mk. For the three UO_2-1.0% PuO_2 fuel elements, the reactivity worth of the coolant is calculated to be 0.16 mk, which is not in agreement with the measured worth of 0.018 mk. The coolant worth for the three UO_2-1.0% PuO_2 fuel elements was measured twice, and thus the calculated reactivity worth is most likely in error.

The possible error in the calculated coolant reactivity worth is evaluated by calculating the reactivity change produced by the change in each individual multigroup cross section. For the three UO_2-1.0% PuO_2 fuel elements, the changes in the cell-averaged epithermal capture, thermal capture, thermal fission, and epithermal scattering multigroup cross sections produce calculated reactivity changes of -0.45, +0.55, -1.10, and +0.92 mk respectively which are each greater in absolute magnitude than the net reactivity change of +0.16 mk.

Based on the experimental errors in the uranium-238 resonance integrals, a possible error of 0.3 barns (approximately two percent of the resonance integral) is assumed for the uranium-238 resonance integral with coolant relative to the resonance integral without coolant. This possible error in the uranium-238 resonance integral gives a calculated coolant reactivity worth of 0.16 ± 0.13 mk relative to the measured worth of 0.018 ± 0.010 mk for the three UO_2-1.0% PuO_2 fuel elements, and a calculated coolant worth of 0.14 ± 0.076 mk relative to the measured worth of 0.10 ± 0.012 mk for the three UO_2-0.48% PuO_2 fuel elements. Possible errors in the calculated coolant worth may also exist due to errors in the calculated epithermal scattering, thermal capture and thermal fission multigroup cross sections.

In conclusion, the calculational method developed in this work will calculate the reactivity worth of the coolant of the PRTR within the accuracy of the cross section data.

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APPENDIX

APPENDIX

DERIVATION OF THE EQUATIONS FOR CONVERTING

FUEL ELEMENT CLUSTERS TO SINGLE FUEL RODS

FOR CALCULATING THE URANIUM-238

RESONANCE INTEGRALS

Hellstrand has shown that the uranium-238 resonance integral (RI) in barns for a fuel rod can be expressed by the following equation

$$RI = 4.15 + 26.6 \sqrt{S/M}, \tag{A.1}$$

where S is the surface of the fuel rod per unit length in cm^2/cm and M is the mass of the fuel per unit length in grams/cm.

In this appendix, equations are derived to convert a cluster type fuel element to a single fuel rod such that Equation (A.1) can be used to calculate the uranium-238 resonance integral of clusters.

For a fuel surface in an infinite scattering media where absorption can be neglected, the neutron current (J) into the fuel surface is

$$J = \phi/4, \tag{A.2}$$

where ϕ is the neutron flux in the scattering media. A derivation of Equation (A.2) is given in Stephenson (15) on pages 138 and 139.

If $\phi_1 \Sigma_S dV$ is the number of scattering collisions per unit time in volume element dV that scatter neutrons into resonance energies, the number of resonance energy neutron per second (I_1) that enter a fuel element cluster or a single fuel rod from scattering collision in the moderator outside the fuel cluster or fuel rod is

$$I_1 = \phi_1 S_F L/4, \qquad (A.3)$$

where S_F is the outside surface of the fuel per unit length and L is the length of the fuel. For a cluster, S_F is assumed to be S_{rb} (the surface that would be formed if a rubber band were stretched around the cluster).

For a moderator region i inside a fuel element cluster (the coolant between the fuel rods), the number of scattering collisions that scatter neutrons into resonance energies is $\phi_1 \Sigma_S V_{M_i}$ where V_{M_i} is the volume of the moderator region i, per unit length of fuel and Σ_S is the macroscopic scattering cross section of the moderator.

The number of resonance energy neutrons per second (I_2) that enter the fuel rods from scattering collisions in the moderator (coolant) inside a cluster is

$$I_2 = L \sum_i \phi_1 \Sigma_S V_{M_i} A_{F_i} P_{o_i}, \qquad (A.4)$$

where $\phi_1 \Sigma_S V_{M_i}$ and L are defined above, A_{F_i} is a geometrical parameter that accounts for the scattering of neutrons between the fuel rods, P_{o_i} is a moderator noncollision probability, and the sum is over all moderator regions inside the cluster.

The total number of resonance energy neutrons per second (I_T) that enter the fuel from all scattering collisions is the sum of Equations (A.3) and (A.4) which is

$$I_T = \frac{\phi_1 L}{4} \left(S_{rb} + 4\Sigma_S \sum_i V_{M_i} A_{F_i} P_{o_i} \right),$$ (A.5)

where the neutron flux ϕ_1 in the moderator outside the cluster is assumed to be equal to the neutron flux ϕ_1 in the moderator (coolant) inside the cluster.

If a cluster is assumed to have an effective fuel surface of

$$S_{eff} = S_{rb} + 4\Sigma_S \sum_i V_{M_i} A_{F_i} P_{o_i},$$ (A.6)

the number of resonance energy neutrons per second that enter the cluster is in the same form as Equation (A.3) for a single fuel rod.

The effective fuel surface given by Equation (A.6) is used in Equation (A.1) for calculating the uranium-238 resonance integral for a fuel element cluster with coolant between the fuel rods.

The geometrical parameter A_{F_i} is based in part on the experimental measurements. For a coolant region surrounded by three equally spaced fuel rods, A_{F_i} is calculated from the equation

$$A_{F_i} = 1 - 3\alpha/360 \tag{A.7}$$

where α is the angle in degrees shown in Figure 4. The angle α is given by the equation

$$\alpha = 2 \tan^{-1}\left[(\ell - 2r_0) \sqrt{3}/\ell\right] \tag{A.8}$$

where r_0 is the radius of the bare fuel rod and ℓ is the distance between the fuel rod centers.

For a coolant region surrounded by four equally spaced fuel rods, A_{F_i} is calculated from the equation

$$A_{F_i} = 1 - 4\beta/360 \tag{A.9}$$

where β is the angle in degrees shown in Figure 4. The angle β is given by the equation

$$\beta = 2 \tan^{-1}\left[(\ell - 2r_0)/\ell\right] \tag{A.10}$$

where r_0 and ℓ are the same as for Equation (A.8).

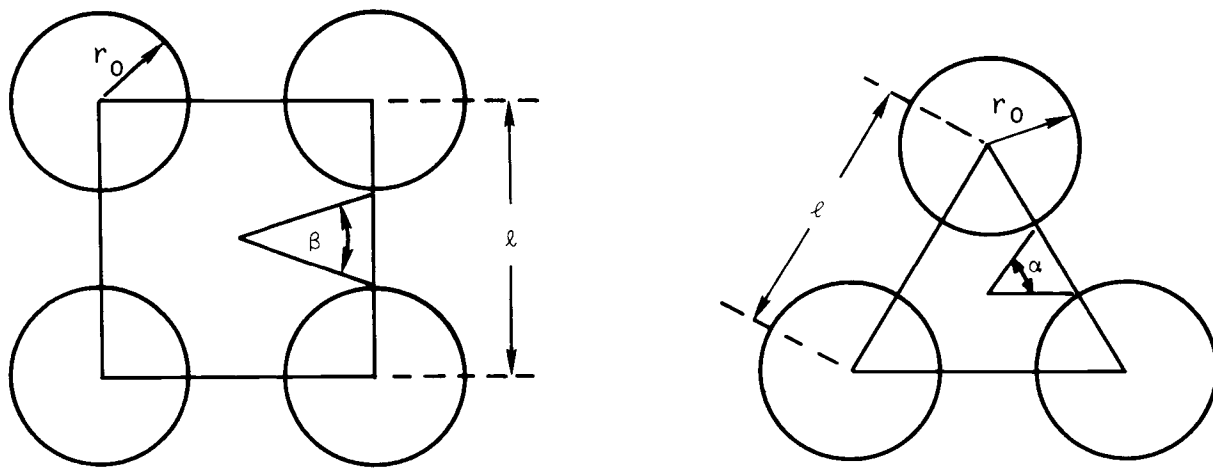The angles $\alpha/2$ and $\beta/2$ are the principal values in degrees of the inverse tangent functions.

Figure 4.   Diagram of Angles α and β