Applicability of Point Sampling to the Forests of Liberia

by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

APPLICABILITY OF POINT SAMPLING TO THE FORESTS OF LIBERIA

INTRODUCTION

Purpose of Paper

The purpose of this paper is to present a cursory review of the theory and principles of point sampling, and to offer some practical proposals on the possibility of its introduction to the forest of Liberia.

Description of Point Sampling

Point sampling is a relatively new method of cruising forest stands as compared to conventional sampling. Point sampling deals with variable plots and conventional sampling deals with fixed plots. In point sampling, the size of the plots will vary according to the size of the trees, while in conventional sampling the size of the plots remain fixed, and tree size exerts no influence on it.

The idea of point sampling is embodied in the concepts of sampling with probability proportional to basal area, and point sampling forests from aerial photographs or maps (Bitterlich 1949a; Grosenbaugh 1952a). From a point within a stand, a 360° sweep is made with an instrument designed to establish an angle. All trees subtending an angle greater than that of the instrument are counted. The total number of counted trees multiplied by a constant angle factor gives an estimate of basal area per acre of the stand (Bitterlich, 1949b; Grosenbaugh; 1952a; Keen, 1950). Volume can be obtained through multiplication of the basal area per acre by the form factor and mean height of the species (Bitterlich, 1948). Using form-height tables by calculating volume-basal area ratios for appropriate height and diameter classes also facilitates volume computations (Grosenbaugh, 1952a). Other pertinent forest statistics can be determined indirectly as suggested by Grosenbaugh (1952a, 1952b).

Development and Modification

The method of angle gauge sampling was developed by Dr. Walter Bitterlich of Austria in 1947, and after whom the technique is named, the "Bitterlich method of sampling." Other descriptive terms for the method are: Winkelzahlprobe (original name given by Bitterlich), plotless sampling method, variable plot radius method, prism cruising, basal area sweep, relaskop sampling. These names will be employed interchangeable in subsequent discussions. Bitterlich (1949b, 1962) continued to develop and improve his idea. He designed an angle gauge instrument, the relaskop, to use in variable plot sampling. In the meantime, Bitterlich's idea was being tested and investigated in North America (Grosenbaugh, 1952a, 1952b, 1958; Grosenbaugh and Stover, 1957; Afanasiev, 1957; Beers and Miller, 1964), the United Kingdom (Keen, 1950; Finch, 1957), Asia (Hirata, 1956, reported by Grosenbaugh, 1958) and probably elsewhere.

Grosenbaugh (1952a) introduced the method of angle gauge sampling to the American foresters, and since then has contributed significantly to the development and application of the idea. He named the method "point sampling," because he explained the principle as an ingenious field application of the known concept of point sampling forests from aerial photographs or maps. It was he who also injected the second important idea of sampling with probability proportional to tree size now associated with angle gauge sampling.

Bruce (1955)[1] in cooperation with the Consulting Foresters of Mason, Bruce and Girard of Portland, Oregon, developed a drastically different instrument, the wedge prism, for point sampling. Other foresters in the United States and Canada have investigated the angle gauge method with satisfactory results.

In Britain, Keen (1950) experimented with a prototype of Bitterlich's relaskop. He reported good results for both basal area and volume determinations, but noted some mechanical and practical limitations. However, Finch (1957) did not think that the method of point sampling would replace the "more efficient conventional fixed plot sampling in Britain in the near future."

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[1]Muller (1953) in Austria had independently developed a similar prism in his article "Das Baumzahlroh."

Hirata (reported by Grosenbaugh, 1958) used the relaskop to determine mean height in coniferous stands in Japan, thus employing a vertical angle instead of the usual horizontal angle.

The principles and techniques of plotless sampling have been rapidly developed in the past decade, especially in North America. This is partly responsible for the large body of literature on the subject. Finch (1957) suggested consolidation of the ideas of point sampling in this regard. Beers and Miller (1964) made comprehensive consolidation when they brought together most of the important ideas and current practices on point sampling in America. Though they limited their discussions exclusively to the thin wedge-shaped prism, the effort is a significant contribution to the standardization of the principles of the angle gauge method of sampling.

Advantages of Point-sampling

The advantages associated with plotless sampling have been largely responsible for its enthusiastic reception and development. Some of these advantages are:

1. Elimination of diameter and plot boundary measurements
2. Determination of basal area per acre through tree count only
3. Relatively more intensive sampling of the larger and more valuable growing stock

4. Efficiency in time, cost and accuracy

In general, it can be stated that the wide acclaim of point sampling is due to its simplicity, general efficiency and great versatility.

Utility of Point-sampling

The utility of point sampling with angle gauge is fairly wide. If it is regarded essentially as a sampling technique, then it theoretically covers many fields in which sampling is used to gather information. Particularly, it was developed to sample forest trees, and so has been most widely used in the field of forestry. Within this limited field, point sampling can be efficiently employed in the following disciplines:

1. Mensuration - This is the subject that employs the method mostly for data collection and research. It is now even proposed in aerial photo-sampling (Paine, 1967).

2. Silviculture - For diagnostic and prescription purposes.

3. Ecology - To determine random distribution of species associations and their frequency.

4. Management - For extensive and intensive inventory to provide data for the preparation of policy statements and working plans.

PRINCIPLES AND THEORY OF POINT SAMPLING

The theory of angle gauge sampling covers the principles of sampling tree parameters of height, diameter and basal area (Bitterlich, 1949a; Grosenbaugh, 1958). Two kinds of angle gauge sampling may be distinguished:

1. Vertical angle sampling - used to estimate height of trees.
   a) Line sampling - probability of tree selection is proportional to the height of the tree or H.
   b) Point sampling - probability is proportional to height squared or H^2.
2. Horizontal angle sampling - used to estimate diameter or basal area of trees.
   a) Line sampling - probability of tree selection is proportional to the diameter of the tree, or D.
   b) Point sampling - probability is directly proportional to the basal area of the tree or D^2.

This paper discusses only the theory and principles of point sampling for basal area.

Principles

There are two important principles embodied in the theory of the plotless method of sampling forest stands:

1. Stand basal area is directly proportional to the total number of trees that will be intercepted by the angle gauge.

2. The probability that any tree will be selected by the angle gauge is proportional to the basal area of the tree, and not to the frequency of the tree as in conventional fixed plot sampling. Accordingly, larger trees will be more likely to be included in the sample than smaller trees (Grosenbaugh, 1958).

The first and basic principle has been mathematically demonstrated by Bitterlich in explaining the operational techniques of the relaskop (Bitterlich, 1948, 1949a). The second, though implicit in the first, was conceived by Grosenbaugh who gave the theoretical justification for it (Grosenbaugh, 1958).

Operational Considerations

From the above principles, four important operational considerations can be derived:

1. The angle gauge projects a constant angle, usually termed the reference or critical angle, regardless of size of object trees and stand density.

2. The sides of the reference angle are tangent only to the "borderline" trees.

3. From a point (plot center) within the stand, a series of imaginary concentric circles are established with the angle gauge.

4. The ratio of the radius of any borderline tree to the radius of the sampling point (distance from the point to tree center) is a constant known as plot radius factor (PRF).

Figure 1 illustrates the above mentioned points diagrammatically.

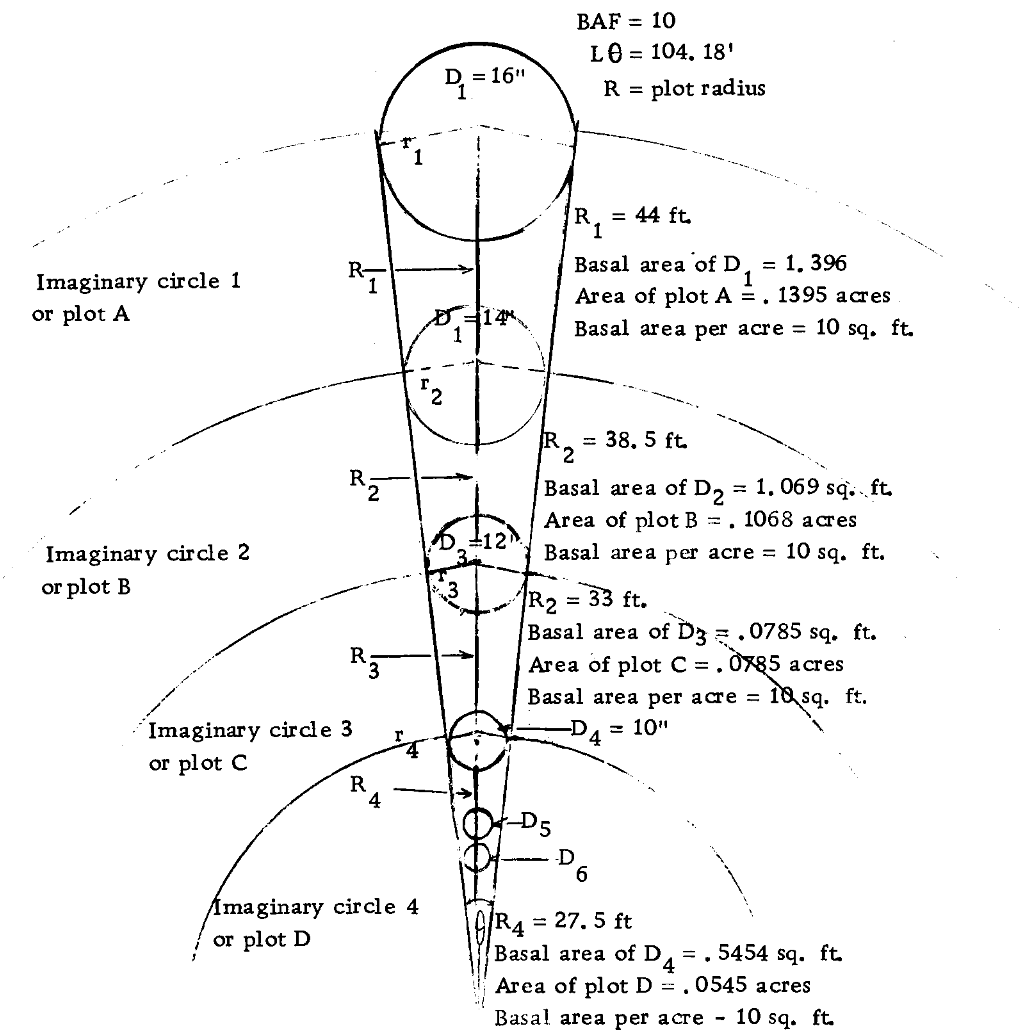

Figure 1. Constant angle, borderline trees, concentric circles, and constant gauge ratio. (Modified from Bell and Alexander, 1957).

The critical angle θ is fixed for trees with different diameters, D_1, D_2, D_3, and D_4. The tree with diameter D_1 has radius r_1 and plot area A, D_2 has r_2 and plot B.

Similarly, D_3 and D_4 have corresponding dimensions. Trees with diameters D_5 and D_6 are small to be sampled by the critical angle. The constant ratio, $\frac{r_i}{R_i}$ = sine α = gauge constant, or

$$\frac{D_1}{R_1} = \frac{D_2}{R_2} = \frac{D_3}{R_3} = \frac{D_4}{R_4} = \sin \theta = k$$

Assumptions

The theory of the method of plotless sampling implies that certain conditions be fulfilled (Grosenbaugh, 1958):

1. The trees which are selected for sampling are cylindrical in form.

2. These trees are standing vertically upright.

3. The sides of the reference angle, whose vertex is at the point of sight, are tangent to the tree at diameter breast height, or at some other point on the tree.

4. The sampling units (trees) are all located on essentially level ground.

5. The sampling points are distributed well within the forest stand.

As in any theoretical or hypothetical elucidation, all of these assumptions are not satisfied in application. Grosenbaugh (1958) has suggested and developed procedures to correct for elliptical and leaning trees which satisfy assumptions one and two. Assumption three is built in within the basic equation of the angle gauge which gives it a degree of significance. Bitterlich (1949a) indicated that errors arising from sighting at the apparent diameter are practically inconsequential. He reported errors of 0.02 and 0.05mm with basal area factors of two and four respectively within forest stands with tree diameters up to 100cm. This relationship suggests that errors would normally increase with the magnitude of the angle, and hence with the size of the basal area factor. Keen (1950) reported an error of 0.01 percent for an instrument ratio of $\frac{1}{50}$ cm. Kendall and Sayn-Wittgenstein (1959) in their investigation of the relaskop stated that the bias introduced in the equation on which the instrument is based is minimal for practical purposes. They showed that the error was not more than half of one percent for angles as large as 8^{o}, corresponding to basal area factors of 210. Since this range includes essentially all factors used, this error could be considered non-functionary and so non-existent.

The basic formula can be modified to exclude the slight error discussed above. This is done by expressing the equation in terms of the tangent or cotangent of the critical angle instead of its sine (Bitterlich, 1962). Bruce (1955), Bell and Alexander (1957), Beers and Miller (1964), Dilworth (1965), and others have used the formula of the cotangent of the critical angle. Bruce (1962) stated that the numerical figure one in the denominator of this equation compensates for not sighting at the true diameter. Beers and Miller (1964) indicated that the cotangent formula adjusts for using a rectangular prism to sight at a circular target, and that this formula will give the "exact" basal area factor. For all practical purposes, assumption three can be regarded as fulfilled.

Bitterlich (1949a, 1950) and others have designed automatic slope correction instruments for sloping terrain. There are other direct and simple slope correction techniques for the prism, such as simple rotation to slope inclination, using an abney level or clinometer with the prism, etc. With Bitterlich's Spiegelrelaskop (Bitterlich, 1962), and Stage's Cruising Computer (Stage, 1959), assumption four is not a problem, since slope is automatically corrected.

Bitterlich did not conceive the idea that the imaginary tree rings could project beyond the stand or forest boundary. Grosenbaugh (1958) thought that if the center of a tree is located near the periphery, it would have less chance to be sampled as its size would indicate. This is because sections of the plot area associated with such a tree would be located outside the treact area, thus reducing the likelihood of a sampling point falling within the plot. Grosenbaugh called this condition "slopover," and suggested techniques to eliminate its effect. When slopover is not eliminated, there is "edge-effect bias." Barret and Allen (1966) developed a procedure to calculate this bias, and the senior author (Barret, 1964) worked out adjustment factors to reduce it. Probably, the more realistic solution to the problem is to anticipate slopover prior to sampling, and to design a sampling scheme that will eliminate it.

Grosenbaugh (1958) suggested classification of all periphery trees according to their relevant locations to tract boundaries, making half or quarter sweeps according to the positions of the trees, and multiplying the results by two or four for half and quarter sweeps respectively to give them their full weights. With this approach, slopover could also be considered insignificant, and assumption five can be regarded as satisfied.

MATHEMATICAL PROOF OF THE THEORY

The theory states that the stand basal area is directly proportional to the total number of trees whose diameters are greater than the width of the reference target or crossarm.

Derivation Based on the Tangent of the Critical Angle

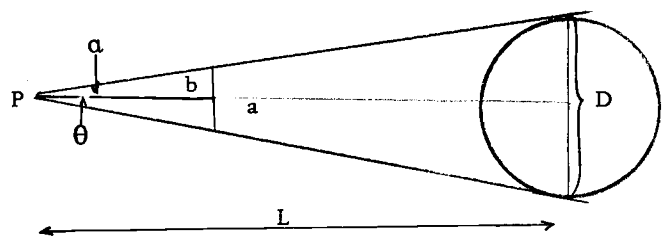

Figure 2. Theory of the angle gauge.

D = diameter of the tree

a = width of instrument crossarm

b = length of instrument rod

P = sampling point or position of observer

θ = critical angle

α = half critical angle

L = distance from sampling point P to tree center D

From Figure 2, tangent $\alpha = \frac{a}{b}$, which defines the instrument angle. For a tree with the diameter D, the distance L is the maximum plot radius within which all such trees will be sampled. The proportional relationship is:

$$\frac{a}{b} = \frac{D}{L}, \quad \text{and}$$

$$L = \frac{Db}{a} = \text{maximum plot radius for } D \text{ sized trees}$$

The basal area of the tree of D diameter is

$$g = \frac{\pi}{4}D^2$$

and, basal area per acre of the stand = G. The area of the sample plot is

$$A = \pi\left(\frac{b}{a}\right)^2 D^2 \quad (\text{from } L = \frac{b}{a}D)$$

From the relationship of ratios

$$\frac{\text{basal area of the stand per acre}}{\text{basal area of D tree}} = \frac{\text{area of one acre}}{\text{area of plot}}$$

$$\frac{G}{\frac{\pi}{4}D^2} = \frac{43,560 \text{ sq. ft.}}{\pi\left(\frac{b}{a}\right)^2 D^2}$$

$$G = \frac{\frac{\pi}{4}D^2 \, 43,560}{\pi\left(\frac{b}{a}\right)^2 D^2}$$

$$G = 10,890\left(\frac{a}{b}\right)^2$$

or

$$G = 10,890(\tan^2 \alpha)$$

(Bitterlich, 1949b; Finch, 1957; Kendall and Wittgenstein, 1959, p. 6; Prodan, 1965, p. 310).

If Σxi trees were counted in a stand, the basal area per acre of the stand will be equal to the product of Σxi and the constant above, thus

$$G = (\Sigma xi)10,890(\frac{a}{b})^2$$

This proves the statement that with an angle gauge, only tree count is necessary to calculate the basal area per acre of the stand.

If specific values are assigned to the dimensions "a" and "b" of the instrument, definite basal area factors (BAFs) can be determined for different angles.

Suppose $a = 1$, and $b = 33$ inches, as suggested by Grosenbaugh, (1952a), then

$$\tan \alpha = \frac{a}{b} = \frac{1}{33}$$

For a tree of D diameter, the relationship is

$$\frac{1}{33} = \frac{D}{L}, \quad \text{and}$$

$$L = \frac{33D}{1}$$

The area of the sample plot is

$$A = \pi 33^2 D^2$$

Previously, the ratio

$$\frac{G}{\frac{\pi}{4}D^2} = \frac{43,560}{\pi(\frac{b}{a})^2 D^2}$$

and

$$\text{basal area of the stand per acre} = \frac{\text{basal area of counted trees}}{\text{sample area}}$$

$$\frac{G}{43,560} = \frac{(\Sigma xi)(\frac{\pi}{4}D^2)}{\pi 33^2 D^2}$$

$$G = \frac{(\Sigma xi)(\frac{\pi}{4}D^2)(43,560)}{\pi 1089 D^2}$$

$$G = (\Sigma xi)(10)$$

Therefore, basal area per acre is equal to the number of trees times the basal area factor (BAF) of 10. If Σxi = eight trees, then basal area per acre will be (8 x 10) 80 square feet per acre. Here, a = one has been chosen, but other values could be selected, keeping b fixed or variable. If b is fixed, the basal area factor will be defined by $a^2 10$, and not by 10 alone. In this case, multiple basal area factors can be computed. The tabulation below illustrates the point.

$$BAF = a^2 10$$

If a =	1.0	1.5	2.0	2.5	3.0	3.5
then $a^2 10$ =	10.0	22.5	40.0	62.5	90.0	122.5

To determine the BAF or critical angle, only the width of the cross-arm "a" is needed here.

Derivation Based on the Sine of the Critical Angle

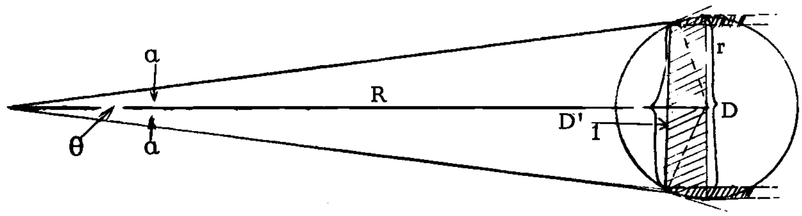

Figure 3. Sine derivation and error

R = plot radius of tree of D diameter

r = radius of the tree D

D = true diameter of the tree

D' = apparent diameter of the tree, which is tangent to the sides of the angle θ.

Other notations are as before.

The sine derivation of the theory of point sampling involves the assumption that the tree is viewed at its true diameter. This assumption incorporates the error discussed previously. Actually, it is the apparent diameter D' that is directly viewed, and not the true diameter D. The apparent diameter D' is generally less than the true diameter D. Figure 3 illustrates the discrepancy involved by the hatched area. With increase in the diameter of the trees, the bias shown by the hatched area is expected to increase.

The critical angle is defined by the ratio

$$\frac{\text{tree radius}}{\text{plot radius}} = \frac{r}{R} = \sin \alpha = k$$

$$R = \frac{r}{\sin \alpha}$$

With trees of differing size D, the maximum distance R within which they will be sampled will vary, but the ratio of their respective radius (r) to that of the corresponding plot (R) will be a constant k. Continuing with the solution:

$$\frac{\text{tree basal area}}{\text{plot area}} = \frac{\pi r^2}{\pi R^2} = \frac{\pi(1/2D)^2}{\pi(1/2\ 2R)^2} = \sin^2 \alpha$$

$$\frac{D^2}{4R^2} = \sin^2 \alpha = k^2$$

When the relationship is extended to the acre as the area unit, the constant ratio becomes:

$$\frac{\text{tree basal area}}{\text{plot area}} = \frac{\text{basal area per acre}}{\text{area of one acre}}$$

$$\sin^2 \alpha = \frac{\text{basal area per acre}}{43,560}$$

$$\text{basal area per acre or } G = \sin^2 \alpha 43,560$$

$$G = \left(\frac{D^2}{4R^2}\right)43,560 = BAF$$

(Bitterlich, 1948; Dilworth, 1965, p. 265).

If D = 12in., and R = 33ft., the BAF can be calculated thus:

$$BAF = \frac{12^2(43,560)}{(4)(33^2)(12^2)} = 10$$

same as before.

The equation, $BAF = \frac{D^2}{4R^2}$, assumes that the sides of the angle are tangent to the true diameter of the tree, a fact which is not valid. The cotangent formula gives exact basal area factors for the actual points of tangency. The error due to the fact that the apparent diameter D' is less than the actual diameter D - as assumed in the sine method - is negligible (Keen, 1950; Kendall and Sayn-Wittgenstein, 1959). The cotangent equation is usually employed for the calibration of the wedge prism.

Derivation Based on the Cotangent of the Critical Angle

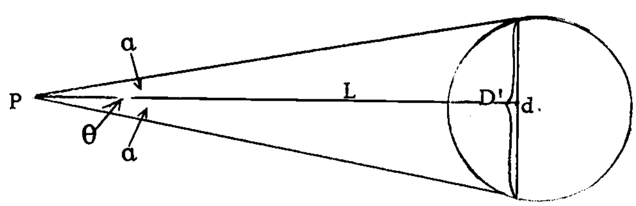

Figure 4. Cotangent derivation.

OX = D' = width of reference target or apparent diameter

$d = \frac{D'}{2}$ = half of target width

PO, PX = tangents to D'

Other notations remain the same. From the diagram, POd may be regarded as a right angle triangle in which

$$L = \text{base}$$

$$d = \text{perpendicular}$$

$$PO = \text{hypothenuse}$$

From trigonometric ratios and identities

$$\frac{d}{PO} = \sin\alpha$$

$$\frac{PO}{d} = \operatorname{cosec}\alpha$$

$$\frac{d}{L} = \tan\alpha$$

and

$$\frac{L}{d} = \cot\alpha$$

but,

$$\sin\alpha = \frac{1}{\operatorname{cosec}\alpha}$$

and

$$\operatorname{cosec}\alpha = 1 + \cot\alpha$$

The basal area factor was derived as

$$BAF = \sin^2\alpha 43,560$$

but

$$\sin^2\alpha = \frac{1}{1 + \cot^2\alpha}$$

therefore,

$$BAF = \frac{43,560}{1 + \cot^2 \alpha}$$

$$\cot^2 \alpha = (\frac{L}{d})^2 = (\frac{L}{1/2D'})^2$$

$$BAF = \frac{43,560}{1 + 4(\frac{L}{D'})^2}$$

If, as before, L = 33ft., and, D' = 1ft., then

$$BAF = \frac{43,560}{1 + 4(\frac{33}{1})^2} = 9.995...$$

(Bruce, 1955; Bell and Alexander, 1957, p. 3; Beers and Miller, 1964, p. 6; Dilworth, 1965, p. 266). From the above prism derivation, it can be observed that the basal area factor (BAF), is slightly less than 10. This is due to three factors:

1. prism calibration using the above formula, is to a flat target
2. the apparent diameter D' is less than 1ft., but was not reduced
3. an average distance of "L" was not taken from few trials

The second principle in the theory of point sampling is, "probability proportional to tree size." Specifically, the probability of sampling a tree is directly proportional to the basal area of the tree of D diameter.

Figure 5 illustrates this principle diagrammatically, and also the condition of "slopover" which reduces the chances of tree selection is portrayed.

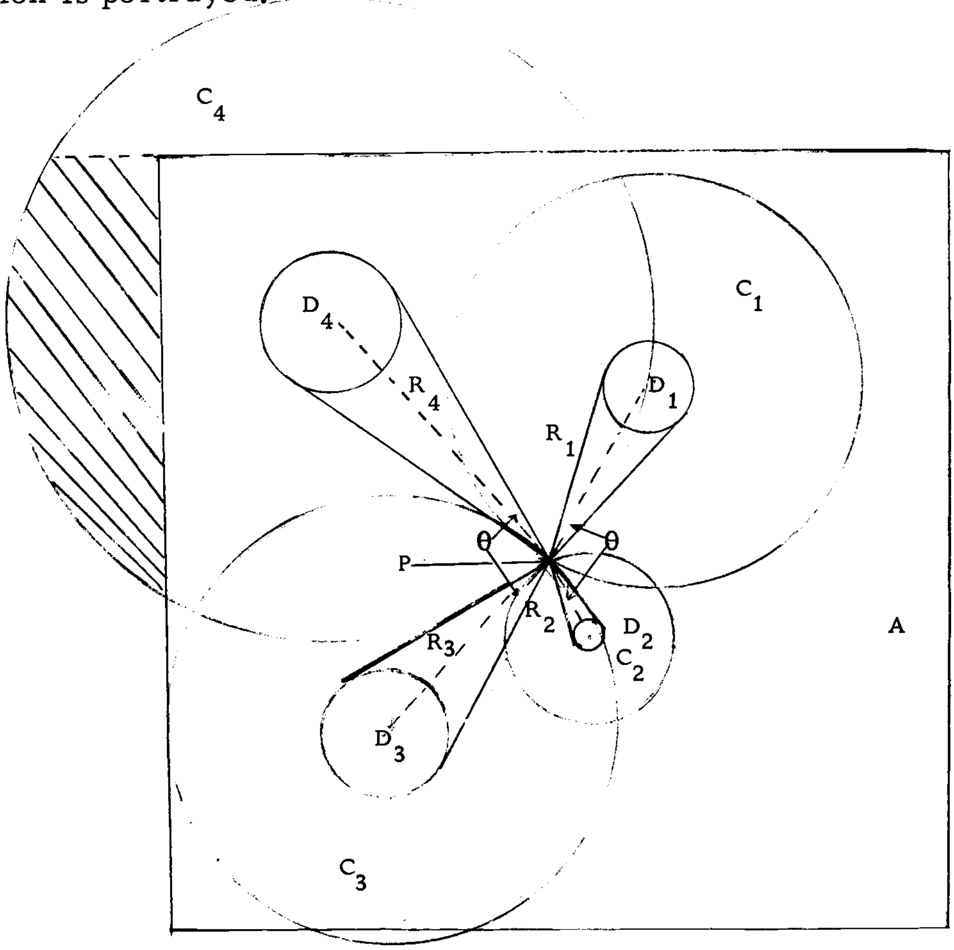

Figure 5. Probability proportional to size and slopover.

$$\frac{D}{R} = \text{sine}\theta = k$$

$$R = \frac{D}{\sin\theta} = \frac{D}{k}$$

C1-4 = imaginary circles or plots of D1-4 trees with P as sampling point.

A = tract area

Sections of C4, for example, plot area of D4, are outside the tract area "A", which demonstrates "slopover" condition.

When each circle represents the plot area associated with the tree of D diameter, the probability that any particular tree will be selected on the tract area A, will be equal to the likelihood of the sampling point P falling within the plot area of that tree (Grosenbaugh, 1958; Beers and Miller, 1964, p. 7). Proof:

$$\frac{\text{plot area}}{\text{tract area}} = \frac{\pi R^2}{A}$$

$$R = \frac{D}{k}$$

$$\frac{\text{plot area}}{\text{tract area}} = \frac{\pi\left(\frac{D}{k}\right)^2}{A}$$

Expressing the above ratio in terms of the basal area of the tree of D diameter, the following relationship is established:

$$\frac{\text{plot area}}{\text{tract area}} = \frac{\frac{\pi}{4} D^2 \frac{1}{k^2}}{\frac{A}{4}} = \frac{\pi D^2}{k^2 A}$$

From the last expression, only the tree basal area πD^2 is a variable statistic, the second term of the expression is a constant. This proves that probability is directly proportional to tree basal area, i.e. $P_i = \frac{\pi D^2}{k^2 A}$.

INSTRUMENTATION

Many angle gauge instruments have been designed, ranging from the simplest stick-type instrument to the more compact and complicated Spiegelrelaskop. Even the thumb, or a penny can be used as an angle gauge instrument for rough estimates (Carow and Stage, 1953). Only five instruments will be discussed briefly. They are:

1. Stick-type gauge
2. Relaskop
3. Panama gauge
4. Wedge prism
5. Spiegelrelaskop (Mirror Relaskop)

Stick-type Angle Gauge

Actually, this instrumentation is the forerunner and basis of Bitterlich's relaskop (Bitterlich, 1948). The main features of this instrument are: a rod, a rectangular blade, and a peephole (Keen, 1950; Grosenbaugh, 1952). Grosenbaugh (1952) recommended a rod length of 33 inches, and a blade width of one inch, thus giving a ratio of $\frac{1}{33}$. These values establish an angle of 104.18 minutes and a basal area factor of 10. Bitterlich (1948) used a ratio of $(\frac{1}{50})$ one to 50 centimeters. The values can be varied to give different angle factors, but shorter rod lengths are not convenient as they give erratic results

(Keen, 1950; Grosenbaugh, 1952). From a point within the stand, the cruiser sights through the peephole and counts all trees that appear larger than the width of the rectangular blade.

The main advantage of the stick-type gauge is its simplicity in construction and use. Its disadvantages are: difficulty in maintaining a constant distance between peephole and the eye, lack of automatic slope correction, hence limited to flat and uniformly sloping terrain, and eye-strain.

Figure 6 illustrates the instrument.

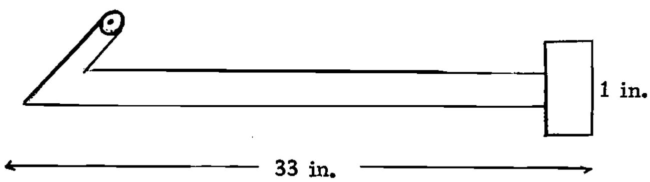

Figure 6. The stick-type angle gauge.

Relaskop

This instrument is based on the ratio of blade width to rod length. It was designed by Bitterlich in 1948. The original relaskop consisted of a rod divided into three different length sections — 100 (39.37in. full length), 70.7, and 66 centimeters. The blade width was also divided into three widths of one, $\sqrt{2}$, and two centimeters respectively (.39in., .555in., and .787in.). Full rod length of 100cm. was used with two centimeters blade width, giving a ratio of $\frac{1}{50}$, for basal area determinations. Rod length of 70.7cm. was used with one and two centimeters blade widths for tree volume determinations. A circular hoop, graduated in direct proportion to $\frac{\pi}{4}$, was attached to aid in the determinations of tree volume. Blade width $\sqrt{2}$' was employed to assess tree form (Keen, 1950). Trees larger than the width of the blade were counted.

The relaskop was cumbersome to use in the field. It gave inconsistent results due to strain on the eye and arm. The rod had the tendency to bend, and there was no automatic slope adjustment.

Panama Angle Gauge

The Panama gauge is a round tube about eight inches long and three-fourth inch in diameter. A small cross-like aperture at one end of the tube defines the critical angle. At the other end is a peephole. Trees appearing larger than the aperture are counted.

The Panama gauge, like both the stick gauge and relaskop, is not used frequently. It is difficult to maintain a constant distance from the eye to the target, also there is no provision for slope adjustment.

It is cumbersome to use in fairly dense stands, because of its small opening through which trees are viewed. However, this instrument is often used in Canada (Kendall and Sayn-Wittgenstein, 1959).

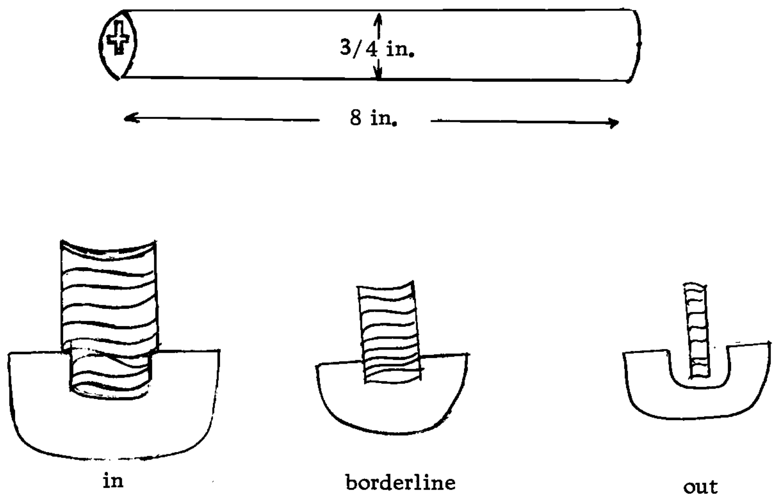

Figure 7. The Panama angle gauge.

Wedge Prism

The prism instrumentation was conceived independently in Europe and America; by Muller in 1953 (as cited by Grosenbaugh, 1958) and by Bruce in 1955 (Bruce, 1955), respectively. However, the development of the prism as a successful point sampling instrument must be ascribed to Bruce.

The wedge prism is a thin glass that bends light rays to establish the critical angle. The rectangular prism is about three-fourth inch wide and one and a half inches long. A prism of one "diopter" strength is equivalent to a "right angled deflection of one unit in one hundred units distance" (Bruce, 1955, p. 163). Trees displaced or deflected less than their diameters are tallied.

The prism is by far the simplest, cheapest, and is reasonably accurate. Thus, it overcomes all the limitations of the other instruments discussed above. Bruce (1955) suggested and constructed decentered and achromatic lenses, automatic slope adjustment instruments, and adjustable strength prisms for better accuracy. The simple instrument can be rotated or used in combination with an abney level or a clinometer for slope correction (Bruce, 1955; Bell and Alexander, 1957; Dilworth, 1965).

The prism is widely used in the United States of America due to its simplicity, inexpensiveness and general accuracy. All discussions on the application of point sampling to the forests of Liberia will be based on the prism.

Figure 9 demonstrates the prism and how it works in the field.

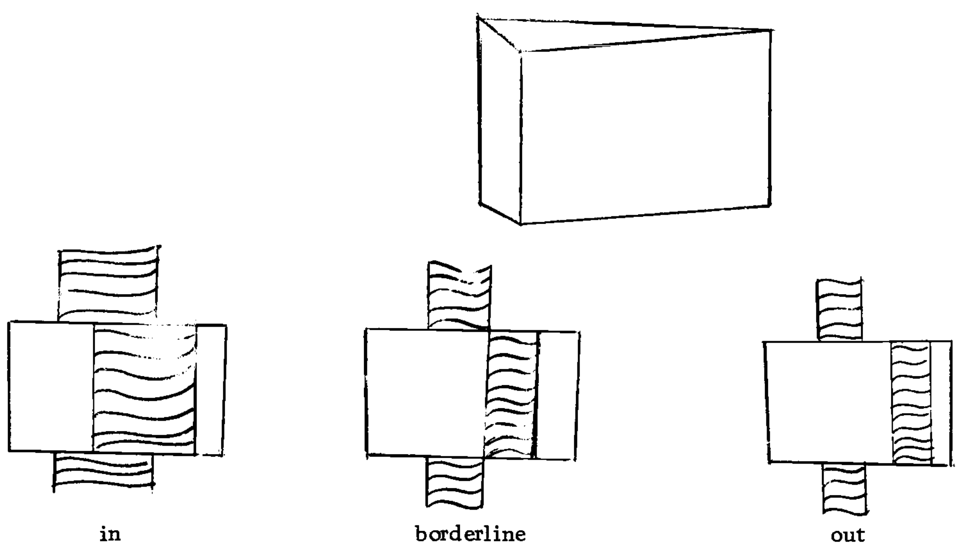

Figure 8. The rectangular wedge prism.

Spiegelrelaskop (Mirror Relaskop)

This is a drastic modification of the original relaskop designed by Bitterlich (Bitterlich, 1962). The Spiegelrelaskop is a compact instrument with windows on the sides to let in light, and a series of scales for basal area, diameter, height, distance and slope measurements. It is a real versatile and an accurate instrument if properly used. This instrument is also provided with wide scale for large diameter trees.

The main limitation of the mirror relaskop is its high cost. With adequate training, it can be used without difficulty.

Kurth and Fischer (1965) investigated the systematic errors associated with the mirror relaskop when used to measure diameters at four, eight, and 12 meters above ground (about 12, 24, and 36 feet), form-height and total height. They reported good accuracy for measurements taken only at four meters (12 feet) above ground. Diameter measurements at 24 and 36 feet gave high systematic errors. This is because there is less control in the accurate use of the instrument, if a tripod is not used and the points of measurements are not marked. Obstructions on the boles of the trees may also shift the points of measurements. These results indicate a need for further tests of the Spiegelrelaskop before it is widely used for vast measurements high up on forest trees.

Comparatively, the mirror relaskop is a more efficient instrument. Kendall and Sayn-Wittgenstein (1959) gave the following conclusions when they used the prism, Panama gauge and the Spiegelrelaskop in tests conducted in Canada:

1. There is little difference among the instruments in basal area determination, but the Spiegelrelaskop was the most efficient.

2. The wedge prism does not glare as the relaskop when used in bright sunlight.

3. The Spiegelrelaskop is preferable in hilly country.

POSSIBLE APPLICATION TO THE FORESTS OF LIBERIA

Review of Some Results

Point sampling techniques have been used in Europe, America, Canada, Asia, and Africa. Many investigators have reported satisfactory results and recommended the use of plotless sampling where feasible. However, due to the dense understory vegetation and the inefficiency of the original relaskop, point sampling has not been introduced to the forests of Liberia. Recent silvicultureal treatments of the forests and establishment of plantations, improvement of the Spiegelrelaskop and the development of the wedge prism, have made the application of point sampling to the forests of Liberia worth investigating. The most significant advantage of this technique is the substantial economy in time and cost effected along with comparative accuracy, which is a very desirable requirement in all sampling schemes.

Keen (1950) reported unbiased estimates for basal area per acre of a coniferous forest in England. He reported a 60 to 64 percent saving in time as compared to fixed plot sampling.

Husch (1955) noted that plotless sampling gave consistent underestimates of tree frequency, basal area, and volume per acre of the stands. Only the BAF 40 gave good results. Husch's unsatisfactory results, like that of many others, may be due to one or a multiple of the following four factors:

1. Making comparison by individual point and plot estimates, instead of by whole population
2. Sampling insufficient forest area
3. Excluding from repeated sampling some potential trees that may belong to two or more sample points
4. Arbitrary application of too small or too large basal area factors to a given stand.

Husch (1955) believes repeated sampling of one or more trees at two or more points is inaccurate, and he suggested how to avoid it.

Grosenbaugh (1958) did not think that repeated sampling of a unit will cause any bias. In fact, he equated it to sampling with replacement, which is an ideal selection method in sampling. Bruce (1961, p. 16) and Dilworth (1965, p. 290) support this contention. Accordingly, there should be no bias in the estimates due to repeated sampling of units as long as the sampling is controlled by chance. Husch's (1955) suggestion may be based on practical expediency, which usually supersedes theory. For example, a forester would like to have an adequate representation of his growing stock in the sample.

Grosenbaugh and Stover (1957) obtained very good results for both point and plot samples when they compared the two methods in southeastern Texas. They showed that generally, more point samples will be required, using BAF 10, to achieve comparable accuracy with a given number of fixed plots. For example, a total of 655 acres consisting of 1,310 points and one-fourth acre plots respectively, were sampled. They found that 20 percent more point samples were needed to achieve the same accuracy as the plot samples. However, 7,012 trees were tallied in the point samples as compared to 28,510 trees in the plots. It is expected that time and cost would be considerably less for the point samples, probably in the order of two to four times less. The corresponding difference in accuracy for point samples as compared to fixed plot samples is practically insignificant, as can be observed from Table 1.

Table 1 below demonstrates results of point and plot samples from Grosenbaugh and Stover (1957).

Table 1. Comparison of basal area and volume per acre for point and plot samples, and their associated errors.

Sampling technique	Basal area per acre square feet			Volume per acre cubic feet			Volume per acre board feet		
	Mean	SE*	SD*	Mean	SE	SD	Mean	SE	SD
Point	53.76	1.49	38.2	692.1	25.9	662.9	3260	150	3848
Plot	53.80	1.25	32.0	691.7	23.6	603.4	3298	143	3663

*SE = standard error
*SD = standard deviation

From the above comparison, it is clear that both methods give about the same mean. Eventhough the standard error of the mean for the plot sample is lower, it should be pointed out that it took 28, 510 tree measurements for plot sampling as compared to only 7, 012 trees for point sampling. This clarifies the efficiency of point sampling.

Kendall and Sayn-Wittgenstein (1959) also concluded that two to three points per one-fifth acre plot were necessary to achieve equal precision in the forest types they sampled, using a basal area factor of ten. They also recorded unbiased results, and a 50 to 66 percent saving in time for point sampling.

Table 2 gives a partial reproduction of their results.

Table 2. Comparison of point and plot results for both uniform and mixed stands.

Stand type and sampling technique	Mean basal area per acre		Mean volume per acre	
	square feet	SD	cubic feet	SD
Uniform Stand:				
Point sample	97. 6	37. 4	2, 279	882
Plot sample	97. 3	21. 3	2, 246	510
Mixed Stand:				
Point sample	115. 3	38. 3	2, 686	847
Plot sample	115. 0	25. 2	2. 669	656

Though table 2 gives high standard errors for point sampling, it should be remembered that more trees were measured in the plots. The mean basal area and volume per acre are about equal in both point and plot samples. This is a significant advantage in favor of point sampling.

Generally, point sampling is more efficient than plot sampling if the proper BAF is chosen, and the sampling carefully conducted.

Determination of Pertinent Factors for Liberian Forests

It is assumed that dense understory or cover, or other factors will not preclude the use of at least the wedge prism in the forests of Liberia. Sights taken at higher points on the tree stem, say at 16 feet, might reduce any brush problem. Before point sampling is undertaken in the field, certain constants need to be calculated to facilitate field work. These are listed below in the order of their importance:

1. The basal area factor or BAF
2. The tree factor or TF
3. The volume factor or VF, and the volume basal area factor or V-BAR
4. The plot radius factor or PRF
5. The slope correction method (optional)

Actually, when the BAF is determined, the other three factors (TF, VF, and PRF) are subsequently derived, because they depend on the BAF.

The BAF represents the basal area in square feet per acre of each counted tree. It is the most important factor. BAF can be determined by approximation through the formula:

$$BAF = \frac{\text{average basal area per acre of the stand}}{\text{average number of expected trees per point}}$$

The number of trees that will be tallied at any point will depend on three things (Howard, 1957):

1. The diameter of trees around the point
2. The density of the stand and
3. the BAF used.

If the average basal area per acre of a given stand (stand density) is 100 square feet per acre, and the expected number of trees per point is four, then

$$BAF = \frac{100}{4} = 25$$

Therefore, each tallied tree will represent 25 square feet per acre.

Prisms calibrated to exact basal area factors (BAFs) are available, and may be ordered from the following company:

Cruise-Master Prisms Inc.
P.O. Box 336
Silverton, Oregon 97381

The tree factor (TF) is the number of trees per acre represented by each tallied tree. If the basal area factor (BAF) is known, TF can be calculated for trees of any diameter. Suppose "D" is the diameter of a tree, and $\frac{\pi D^2}{4}$ is the basal area (BA) in square feet of the tree, then,

$$TF = \frac{BAF}{\frac{\pi D^2}{4}} = \frac{BAF}{BA}$$

If

$$D = 36 \text{ inches,}$$

$$BA = 7.07 \text{ square feet}$$

$$BAF = 25,$$

$$TF = \frac{BAF}{BA} = \frac{25}{7.07} = 3.5$$

Therefore, each 36 inch-tree sampled by a BAF of 25, represents a total of 3.5 trees per acre of 36 inch-trees.

The volume factor (VF) also gives the volume per acre represented by each tallied tree. With a known TF, and volume of any tree, the VF can be derived thus:

$$VF = TF \times V = \frac{BAF}{BA} V = \frac{V}{BA} BAF$$

The ratio $\frac{V}{BA}$ is known as the volume basal area ratio (V-BAR). It represents the volume per square foot of basal area. Tables for V-BAR have been prepared to expedite volume calculations. When V-BAR is multiplied by BAF, the volume per acre is obtained, which is equivalent to the VF above.

Suppose BAF = 25, TF = 3.5, BA = 7.07 square feet, and V = 353.5 cubic feet.

Then,

$$VF = \frac{V}{BA} BAF = \frac{353.5}{7.07} 25 = 50 \text{ cu. ft./sq. ft.} \times 25$$

$$= 1250 \text{ cu. ft./acre}$$

Therefore, each sampled 36 inch tree represents a volume per acre of 1250 cu. ft. The volume basal area ratio, V-BAR, is equal to 50 cubic feet per square foot in the above example. Tables for V-BAR factors are usually constructed from existing volume tables, or from collected data. The V-BAR approach is easier, since it is independent of the BAF.

The PRF (plot radius factor) facilitates decision on borderline and doubtful trees. With this constant, the maximum radius (R) of any tree plot can be calculated. If the distance from the point to the center of the tree is equal to R, it is a borderline tree and it will be tallied as half tree, or as otherwise stipulated. For example, with the ratio 1/33, a tree of diameter D must be situated at a distance of 33D or less to be sampled. If R = 33 ft., and D = 12 in., the tree associated with the above values will be sampled if it is (33 x 12) 33 feet away. The plot radius factor, PRF, is 33/12 = 2.75. With PRF = 2.75, any tree plot radius (R) can be determined from the simple equation: R = PRF x D. An example will illustrate the point.

Suppose

$$D1 = 6 \text{ inches}$$

$$D2 = 10 \text{ inches}$$

and

$$PRF = 2.75$$

The respective distances at which these trees will be tallied are:

$$R1 = PRF \times D1 = 2.75 \times 6 = 16.5 \text{ feet}$$

$$R2 = PRF \times D2 = 2.75 \times 10 = 27.5 \text{ feet}$$

Large diameter trees will be located further away from the sampling point than small trees.

Slope correction factors may or may not be calculated depending on the method of slope correction chosen. If the secant of the angle of slope is used as suggested by Grosenbaugh (1955), slope correction factors will be calculated. Grosenbaugh has given factors to a maximum slope percent of 100.8. For steeper slopes, he recommends the use of the formula: $SCF = \sqrt{1 + \text{slope percent}/100.}$ When an average tally of basal area per acre is made on a sloping ground, this value will be multiplied by the secant of the slope to give it its horizontal value. Bell and Alexander (1957) recommend the use of the prism with the abney level to adjust for slope for each tree. In this case, no correction factors need to be calculated, since each tree at a sampling point will be adjusted. This method can be cumbersome and time consuming. When the relaskop is used, no slope adjustments will be necessary, since the instrument automatically corrects for slope.

Slope correction may not be required in Liberia, since the forest populations in question are distributed on fairly level ground.

There is no indication at the moment that, even the distribution of species in forests situated on sloping terrain is influenced by the degree of elevation.

When the pertinent factors are determined, a sampling scheme can be planned. Usually, a series of tables of the discussed factors are prepared in advance of sampling or after some preliminary sampling, to expedite future computations.

STATISTICAL ASPECTS

Sampling Scheme

Point sampling is a sampling technique, and as such, any of the known selection methods can be used, depending on the nature of the population and accuracy or precision desired (Grosenbaugh, 1958; Dilworth, 1965, p. 290).

Grosenbaugh prefers random sampling. He maintains that a selection system that resamples a tree more than one time is not biased. In fact, this is ideal selection method in the theory of sampling. Bruce (1961, p. 16) also believes repeated sampling will by no means affect the results. However, Husch (1955) stated that such sampling results will be biased, and he suggested a technique to avoid repeated sampling of a tree.

In point sampling, the chances of sampling a tree more than once is highly probable. This probability increases with the size of the trees, proximity of the points, and the basal area factor used (Husch, 1955). To avoid or reduce the chances of counting a tree more than one or two times, the points should not be too close to each other, and the basal area factor should not be small, probably not less than ten. Local conditions should dictate the technique to be used.

Cluster and double sampling schemes will be used in three forest types: old growth, second growth, and plantation forests. Primary sampling units will be selected from the cluster sample. Trees that will be counted by the prism for basal area per acre determination will be considered as primary sampling units. Secondary sampling units will be selected from the second phase or double sample. In the second phase sample, a portion of the clusters will be selected from which trees will be measured for V-BAR. Trees chosen for V-BAR measurements from the already counted trees will be the secondary sampling units. In both old growth and second growth forest types, all commercial species will be measured for volume basal area ratios, due to their small numbers. In the plantation forests, all species are commercial, so only a determined fraction will be measured for V-BAR. For all forest types therefore, the same formula will be applied.

Determination of Sample Size

The infinite population equation will be used to calculate the size of the sample:

$$n = \frac{t^2 (CV)^2}{a^2}$$

where,

n = sample size

t = probability index

CV = coefficient of variation between units

a = allowable error

The points will be located in the clusters according to the method given by Husch (1955). Briefly, the method requires that the points be located at a distance twice the plot radius of the largest tree within the stand. The points will be systematically distributed, but random formulae are used for the error calculations. This is predicated on the assumptions that, there is no trend or periodicity effect, and that calculated errors will be conservative.

Calculation of Means and Standard Errors Per Acre

Mean basal area per acre:

$$\overline{G} = \frac{\sum^{n}\sum^{m} x_i}{nm} BAF$$

where,

n = number of clusters

m = number of points per cluster

x_i = number of counted trees per point

BAF = basal area factor

Mean volume per acre:

$$\overline{V} = \frac{\sum^{h} V\text{-}BAR}{h} \overline{G}$$

where,

h = number of trees measured for V-BAR

Standard error of mean tree count:

$$S\bar{x} = \sqrt{\frac{sc^2}{n} + \frac{sp^2}{m}}$$

where,

$$sc^2 = \frac{\sum^{n}(\bar{x}i - \bar{X})^2}{n - 1}$$

between cluster variance and,

$$sp^2 = \frac{\sum^{m}(xi - \bar{x})^2}{m - 1}$$

within cluster variance

xi = number of counted trees per point

$\bar{x}i$ = average tree count per cluster

$\bar{X}$ = average tree count for the whole sample

$\bar{x}$ = average tree count for cluster

It is better to express the standard error as a percent of the mean, since it gives a relative weighting to the error. It is computed as:

$$S_{\bar{x}}\% = \frac{S_{\bar{x}}}{\bar{x}} 100$$

Standard error of mean V-BAR per acre:

$$S_{\bar{v}\text{-bar}} = \sqrt{\frac{\sum^{h} V\text{-BAR}^2 - \sum^{h}(V\text{-BAR})^2/h}{h - 1}}$$

$$S_{\bar{v}\text{-bar}\%} = \frac{S_{\bar{v}}\text{-bar}}{\sum^{h} V\text{-BAR}/h} 100$$

Standard error of mean volume per acre in percent

$$S_{\overline{v}}\% = \sqrt{S_{\overline{v}\text{-bar}}\%^{2} + S\overline{x}\%^{2}}$$

The error propagation formula suggested by Bruce (1961) has been used above to calculate the standard error of mean volume per acre. This formula has been chosen because of its simplicity. An error equation that would incorporate all factors of cluster and double sampling could be very complicated. Even the approximate volume formula, suggested by Johnson (1961), which takes account of only double sampling is tedious enough for the layman without statistical foundation.

Field Form for Tree Count, Basal Area and Volume

A composite field tally sheet is recommended for easy computation of total and average tree count, basal area, and volume per acre.

The basal area per acre will be based on the equation:

$$\text{BA/acre} = \text{Average tree count per point x BAF}$$

The basis of the volume equation cannot be given now, since some tests are necessary to establish the relevant relationship between diameter, height and volume per species group. Momentarily, the following equation is suggested (Beers and Miller, 1964, p. 9):

$$V = bD^2H$$

where,

V = volume of a tree

b = regression constant

D = diameter at breast height or above

H = height of 16.4 foot logs in a tree

The formula assumes constant form factors for all species within the locality. Age, site and other differences are considered uniform for each species group, locality and size class.

If, it is discovered that V-BAR changes little with diameter within given height classes, time will be saved, when only log heights are measured and diameters roughly estimated or omitted (Dilworth, 1965, p. 277-279; Meyers, 1964, p. 3). However, it is recommended to measure diameters for better accuracy, and construction of stand tables.

With the volume equation decided, volume tables and volume-basal area ratio factors for the commercial species can be prepared. A grading system or table will be constructed later. The form below is a modification of the tally sheet suggested by Beers and Miller (1964, p. 36).

Figure 9. Proposed field tally sheet.

Point No.	Tree factor	DBH class	V-BAR BAF =	Commercial Species Group Group 1*	Group 2*	Basal Area total and per acre	Volume total and per acre	Tree Count total and per acre
Basal area	total					x x x x x		
	per acre					x x x x x		
Volume	total						x x x x x	
	per acre						x x x x x	
Trees Count	total							x x x x x
	per acre							x x x x x

* Group 1 = first grade timber, and Group 2 = second grade timber.

SUMMARY AND CONCLUSION

A review of the development, theory and principles of point sampling has been given. Three derivations of BAF were attempted. However, in practice, the tangent and the cotangent derivations are used. The instruments usually employed in plotless sampling were briefly discussed. From the discussions, the wedge prism was recommended because of its simplicity and general accuracy. However, the Spiegelrelaskop is the most efficient angle gauge instrument now available on the market. For this reason, it should be investigated later. A discussion of some practical results from point sampling was undertaken, and it was demonstrated that point sampling was more efficient than the conventional fixed plot sampling. Factors that would be required to conduct plotless sampling in the forests of Liberia were determined. Cluster and double sampling selection systems were suggested, and formulae to calculate mean basal area and volume per acre, and standard error of the mean volume per acre were given. A composite field form to expedite field computations of collected data was recommended.

Point sampling is an efficient technique, if properly applied. The effective speed and economy with which it attains accurate results, together with its simplicity give it a wide acclaim. The development of the Spiegelrelaskop, the wedge prism, the application of silvicultural treatments to the forests of Liberia and the establishment of plantations should facilitate application of point sampling.

All suggestions contained in this paper are preliminary subject to field trials.

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