

On the Dichotomy in Theoretical Treatments of the Atmospheric Boundary Layer

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ABSTRACT

The two main approaches presently in use for studying the boundary layer are quite dichotomous. It is shown that the Ekman approach which uses an eddy coefficient fixed in height can lead to a serious contradiction; this approach should be avoided if boundary-layer structure is being investigated.

1. Introduction

Two consecutive papers in the July 1981 issue of the *JOURNAL OF THE ATMOSPHERIC SCIENCES* clearly illustrate the dichotomy in approaches toward study of the boundary layer which must bewilder the uninitiated. In the first one, Krishna (1981) adopts the earliest Ekman (1905) approach in which the use of a constant eddy viscosity implies that the turbulence extends to all heights at which any wind shear exists, whatever the Richardson number may be. In the next article, Nieuwstadt and Tennekes (1981) utilize the opposing viewpoint that the boundary layer extends only to the (time-dependent) height to which its turbulence has allowed it to propagate, commensurate with a Richardson number not exceeding a critical value. We shall call this second approach the dh/dt approach. In this note we would like to point out a major contradiction, or inconsistency, with the Ekman approach and additional advantages of the dh/dt approach.

2. Inconsistency of using a fixed eddy coefficient

In the Ekman approach the Reynolds stress $\overline{V'w'}$ is represented by

$$\overline{V'w'} = -K\partial\bar{V}/\partial z, \quad (1)$$

where K is the eddy coefficient, assumed constant above the surface layer by Krishna (1981, p. 1404), and the overbar is a suitable mean. If this constancy in K is assumed to extend above the boundary layer, then (1) indicates that turbulence is implied to exist there too (a contradiction), whenever a mean shear exists and however large the Richardson number may be. In Krishna's study a thermal wind existed, so that the geostrophic and actual winds at and above the boundary layer top contained shear, resulting in the contradiction. Only in situations where turbulence does not exist, as in certain laboratory studies per-

formed at small Reynolds numbers, does the contradiction vanish.

On the other hand, if K is assumed to be constant only up to $z = \bar{h}$ and to be zero above, the equation of motion for the horizontal velocity \bar{V} using (1), i.e.,

$$d\bar{V}/dt = -(1/\rho)\nabla_H\bar{p} - f\bar{k} \times \bar{V} + K\partial^2\bar{V}/\partial z^2 + (\partial K/\partial z)(\partial\bar{V}/\partial z), \quad (2)$$

indicates that unless $\partial\bar{V}/\partial z = 0$ at $z = \bar{h}$ an infinite acceleration would exist there where $-\partial K/\partial z \rightarrow \infty$. The undesired infinite acceleration of course denotes a fallacy, and would certainly violate Krishna's assumed steady state. The upper boundary condition in such a study evidently requires that $\partial K/\partial z \rightarrow 0$ as well as $K \rightarrow 0$ as $z \rightarrow \bar{h}$.

If K is instead allowed to approach zero in a thin layer atop the constant- K layer, one may investigate the additional ageostrophic component that occurs relative to calculations which omit this consideration. Suppose the layer has a thickness of 5% of a representative Ekman depth ($0.3u_*/f$), while $\partial\bar{u}/\partial z$ at the base of this transition layer is given the largest mean value of $\partial u_g/\partial z$ reported by Krishna in Table 3 with $f = 2.46 \times 10^{-3} \text{ s}^{-1}$. Also, let K there be assigned the value $0.002u_*^2/f$ [see Krishna's Eqs. (3.9) with $\Phi = 1$ and (3.13) with $B_i = 0.005$]. Integration of his Eq. (3.1) above the constant- K layer then shows that the \bar{v} component will have an average geostrophic departure of $34u_*$ in this layer alone. Typical values of u_g/u_* range between 20 and 30. Thus the discrepancy between this approach and the constant- K approach is large and increases if the transition layer is shrunk.

3. Removing the inconsistency

The simplest means of removing this inconsistency, which does not exist in most versions of the dh/dt approach, is to allow K to be a continuous function

of wind shear and stability such that it becomes negligible where the Richardson number exceeds some critical value (e.g., see Obukhov, 1946; Lykosov, 1972). In this manner it need not be assumed that the atmosphere containing the boundary layer is barotropic or devoid of wind shear in order to avoid the contradiction. Instead, a distinct time-dependent boundary-layer depth then emerges.

The preceding discussion applies to turbulent boundary layers in general, the exception being a steady neutral boundary layer with no stratification at any height (never observed). The trade-wind boundary-layer observations utilized by Krishna refer to a slightly unstable mixed layer topped by a weak capping inversion at a height of about 600 m (Augstein *et al.*, 1974), and surmounted by the trade-wind cloud layer in which the clouds are not always present. For treating such unstable boundary layers, or mixed layers, the use of K theory or a Richardson-number dependence for K may cause difficulties associated with vanishing mean gradients. These can be avoided with closures in second-moment equations (Yamada and Mellor, 1975) or third-moment equations (André *et al.*, 1978) at the expense of increased complexity and computer time.

4. Other advantages of the dh/dt approach

a. The length scale h

An added advantage of the dh/dt approach is that the important length scale \bar{h} emerges from the time-dependent calculations in a more realistic manner than by directly assuming a constant or diagnosed value. Predictive equations for mean properties within the boundary layer or near the earth's surface require knowledge of \bar{h} , whose value can change over an order of magnitude in one day.

b. Boundary-layer pumping

A further advantage is the more proper treatment of the interaction between the boundary layer and the free atmosphere associated with boundary-layer divergence or convergence, a topic outside the scope of the study of Krishna or Nieuwstadt and Tennekes. The equation for the growth of \bar{h} (e.g., Carson, 1973; Betts, 1976) is approximately

$$\partial\bar{h}/\partial t + \bar{\mathbf{V}}(\bar{h}) \cdot \nabla\bar{h} = \bar{w}(\bar{h}) + w_e - w_c, \quad (3)$$

in which $\bar{w}(\bar{h})$ is the large-scale or mesoscale vertical motion at $z = \bar{h}$ associated with the divergence, w_e is the (positive) entrainment rate of non-turbulent air from above \bar{h} down into the boundary layer, and w_c is (positive) cloud-induced subsidence in cases where convective clouds are venting mass from the boundary layer or sub-cloud layer as it may then be called. This equation shows that the boundary-layer top (as well as any associated thermal structure)

tends to be carried up or down with the mean vertical motion at \bar{h} , not necessarily allowing air to pass through its top as in the concept of Ekman suction [$\bar{w}(\bar{h}) < 0$] or pumping [$\bar{w}(\bar{h}) > 0$] (e.g., Holton, 1979). With that concept, the boundary-layer top is considered to lie at the height $(2K/f)^{1/2}$ and is therefore incapable of moving up or down with the mean vertical flow. In fact, Sarachik (1974) has emphasized the dissimilarity between the Ekman pumping concept and the use of (3). However, (3) allows us to see that in steady, non-advecting situations the concept can be maintained. When positive boundary-layer divergence is present, cloud convection may be suppressed, so that the entrainment rate is equivalent to boundary-layer suction. This steady state may be achieved by thinning of the boundary layer by boundary-layer suction until the subsidence at the boundary-layer top is sufficiently weak to be balanced by entrainment. In the opposite case, after convective cloud-induced subsidence has stabilized \bar{h} , $w_e - w_c$ in (3) is equivalent to boundary-layer pumping. The upward pumping motions occur only within the active convective clouds.

With regard to this topic, Paegle (1979) has mentioned another aspect of the dichotomy. The Ekman-layer approach has emphasized boundary-layer pumping, whereas the mixed-layer-growth approach has emphasized cases with subsidence (suction).

c. Further research

The dh/dt approach is not without uncertain aspects. Among them is the appropriate definition of \bar{h} in those stable cases where the contrast between the weak but continuous turbulence below \bar{h} and intermittent turbulence above is nebulous. Another is the treatment of dh/dt in a boundary layer within which an increasing Richardson number evolves, causing turbulence at higher levels to decay and \bar{h} to shallow. A third is the appropriate definition of \bar{h} when vigorous convective clouds are so numerous that continuous boundary-layer turbulence may be considered to include the cloud layer (e.g., Augstein *et al.*, 1974) as well as the sub-cloud layer. These uncertainties constitute areas in which further research is especially needed.

5. Summary

With regard to the interaction between the boundary layer and a barotropic free atmosphere, the dichotomy in theoretical treatments is not necessarily as great as it may seem. However, future studies intended to further our understanding of boundary-layer structure itself should avoid using an eddy coefficient that is fixed in height.

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