This investigation was designed to determine the general achievement, ability to solve applied problems, and attitude toward the role of mathematics in science of college calculus students grouped in an experimental class according to academic major during their first three terms. The subgroups were biology-business, mathematics, and science-engineering majors. The effects of grouping upon these abilities and attitude were examined by comparing group mean test scores of the grouped students to those of a non-grouped control class. Subgroups between and within the control and experimental groups were compared in order to further assess the effects of grouping on calculus achievement, ability to solve applied problems and attitude toward the role of mathematics in science.

Criterion tests were the Cooperative Calculus Test, Form B, the Applied Problem Test and the Math-Science Attitude Inventory, the latter two constructed by the researcher. Since the study was a
post-test only design, the criterion instruments were administered to the control and experimental groups at the end of their respective Spring terms.

One factor analysis of covariance using **CCT** and **APT** group means was used to statistically test the null hypotheses. Combined overall high school and mathematics grade point averages were applied as covariant control for academic achievement and ability. One factor analysis of variance using **MSAI** group means was used to statistically test the remaining null hypotheses. F ratios were computed and evaluated to determine whether differences in group means on the criterion instruments were significant. The data were further analyzed to determine correlations among several variables.

**Findings**

The following conclusions were drawn from the data analyzed in this investigation:

1. Grouping college calculus students by academic major resulted in significantly (10 and 20 percent levels) greater calculus achievement and ability to solve applied problems by the experimental group.

2. Subgroups of the experimental group did not consistently differ significantly from subgroups of the control group in calculus achievement nor in ability to solve applied problems.

3. Subgroups within the control group exhibited marked
differences in calculus achievement and ability to solve applied problems.

4. Subgroups within the experimental group did not exhibit significant differences in calculus achievement nor ability to solve applied problems.

5. The control and experimental group did not differ significantly in their attitude toward the role of mathematics in science.

6. Mathematics majors in calculus have a more positive attitude toward the role of mathematics in science than do science or biology-business majors.

7. The MSAI effectively measures the attitude of calculus students toward the role of mathematics in science.

8. The APT effectively measures the ability of calculus students to solve applied problems.

9. Team teaching at the college level can effectively be designed and implemented.
Some Effects of Grouping By Subject Matter Major on Student Performance in College Calculus

by

Ronald Rudolph Steffani

A THESIS

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Doctor of Philosophy

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Professor of Science Education
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Chairman of the Department of Science Education

Redacted for Privacy

Dean of Graduate School

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I. INTRODUCTION

Historically mathematics, and in particular the Calculus, was developed from the study of physical phenomena. This relationship between science and mathematics has existed at least since Archimedes used geometry to establish the science of statics (12, p. 536). For many centuries mathematics, perhaps with the exception of geometry, was considered to be a "tool," or a subject that was in the main a means to an end. Expressed more elegantly it was said that "mathematics is the handmaiden to the sciences" (63, p. 702).

In terms of mathematics teaching and curriculum, the view of mathematics as a tool led to what is commonly called "cookbook" mathematics. That is, mathematics was taught by rote, and more as a mental discipline than as a subject useful in itself. The rigorous and formal aspects of the subject were almost completely ignored, not only in the teaching of mathematics, but also in the development of mathematics. This state of affairs persisted from the beginning down to the nineteenth century. Prior to this time the status of servant was the only one mathematics knew (63, p. 702).

During the nineteenth century developments in mathematics led to a new role for mathematics. With the emergence of non-classical mathematics such as non-Euclidean geometry, set theory, symbolic
logic, and analysis, mathematics attained a large degree of independence from the physical world. It achieved a completeness and internal consistency that mathematics had not known before (63, p. 703). Mathematics was no longer dependent on other disciplines, and thus the way was opened for scholars to establish and pursue its own goals.

Concomitant with the growth of mathematics, was the development of new theories in psychology which began to refute the theory of formal discipline and faculty psychology, and which eventually led to the downfall of teaching mathematics as a mental discipline. Pestalozzian and Herbartian ideas led to the inclusion of the inductive approach to mathematics as well as an emphasis on the content of the subject rather than its disciplinary value. However, mathematics education did not move entirely away from the useful aspects of the subject as is evidenced by John Dewey's fundamental thesis that the physical and social environment in which we live presents problems which the human mind solves by number and number relations (61, p. 48).

These parallel developments of the nineteenth century resulted in mathematics no longer being regarded a servant, but rather it emerged as a peer to its mentors of the past, indeed to some it became the queen of the sciences. The study of mathematics could now become an end in itself. Thus 2500 years of mathematical development have
given rise to two extreme views of mathematics and its place in culture. This dichotomy is evidenced on the one hand by those so called "ivory tower" mathematicians that would say "here is to my specialty, may it never have any application" (12, p. 542), while on the other hand there are those who would claim that if mathematics had not been useful, it would long ago have disappeared from our school curriculum as a required study (29, p. 86).

The concept that mathematics is a subject worthy to be studied for its own sake, which originated in the nineteenth century, reached its culmination in the "new mathematics" revolution of the mid-twentieth century. The demise of the classical mathematics curriculum opened the way for a new one emphasising the structure and consistency of mathematics, and at the same time old and new topics were presented more abstractly, rigorously and deductively. These developments are mirrored in the flood of new textbooks, and "alphabet" curricula that have been appearing in recent years.

As a result of the evolution of "new mathematics" the gulf between the two above mentioned views of mathematics has widened. However, in the past few years voices have been raised calling for a mathematics curriculum which would represent a "meeting of the minds" of the two polar views. There has been a growing awareness on the part of mathematicians and mathematics teachers of the need for a mathematics curriculum which will meet the needs of all students.
One of the objectives set forth by the Committee on the Undergraduate Program in Mathematics (CUPM, a committee of the Mathematical Association of America and financed by the National Science Foundation) in its study of the preparation of students for graduate study in mathematics is that the student be made aware of the applicability of mathematics to other disciplines (20, p. 7). On the other hand, the committee studying the undergraduate program for engineers and physicists recommended a strengthening of interest in more algebraic and abstract concepts, and recognized certain areas of mathematics where they would insist on a rigorous development of the subject. In the same study they stated: "A mathematics course for engineers and physicists must involve the full spectrum from motivation and intuition to sophistication and rigor" (21, p. 3).

The remarks of the CUPM committees certainly pertain to the Calculus, since

In addition to adding a whole new branch of mathematics to the students equipment, the study of calculus holds tremendous possibilities for extending and deepening understandings of previously studied subjects. Moreover, by the time the student has come through a study of calculus he should have acquired a feeling for the nature of and the necessity for mathematical rigor as such (15, p. 599).

Statement of the Problem

The problem then is, can a college calculus class be structured, and the curriculum altered so that the needs and interests of all the
students can be met without slighting either the structure and rigor of the subject, or the applications of the Calculus? The objective is to structure the class so that students to whom mathematics is a means to an end will not be overcome by all the rigorous development of the subject, and at the same time be afforded the opportunity of concerning themselves with how calculus is related to their own field of interest. At the same time, for the student who views mathematics as an end in itself, the curriculum should be devoted to a more thorough study of the structure and deductive nature of calculus, yet still allow the student to have an opportunity to appreciate the various applications of calculus to other disciplines. These objectives must be accomplished without either group of students declining in general achievement. At the same time, it would be expected that the two groups of students would increase their understanding of calculus as it relates to their chosen field, as compared to a conventionally taught calculus course.

In addition to the areas of achievement mentioned in the previous paragraph, such a class would provide an opportunity to determine if the application oriented student differs from the mathematics major in his attitude toward the role of mathematics in the sciences. Such an attitude scale could also be administered to a control group to detect attitude differences within the group as well as differences between control and experimental groups.
The purpose of this study was to compare the achievement and attitudes of students taught in a calculus class structured to meet the objectives mentioned in the previous paragraphs to those taught in the traditional lecture-discussion manner.

More specifically, answers were sought to the following questions.

1. Can the calculus taught in a college be structured so as to meet the needs of all students, regardless of subject matter major?

2. To what extent do students who are completing such a program differ in understanding calculus from those in a traditional program?

3. What effect does such a calculus program have on the student's attitude toward the role of mathematics in science as compared with traditionally taught students?

This study was designed to investigate the above questions based on a calculus class, and a curriculum mix structured by the researcher. This class, the experimental group, was team taught two days of the week in a large group session. During these sessions the basic topics, concepts and skills of calculus were presented by three members of the Southern Oregon College mathematics department. On the remaining two days of the week the students met in three small sections (labs) grouped by subject matter major, and taught by one of
the members of the teaching team. One of the smaller classes consisted of mathematics majors, the second of physical science and engineering majors and the third of biological science and business majors. These sessions were devoted to expanding topics presented in the large group sessions, presenting topics, concepts and skills relevant to the particular subject matter major group at hand, relating calculus to the development of the specific fields of interest, and solving problems encountered in assignments. The comparison or control group for this experiment consisted of 47 students who completed a traditionally taught, three quarter calculus sequence at Southern Oregon College the previous year.

**Importance of the Study**

The need for revision of mathematics curriculum has become acute during the past five years. This need has been recognized not only by mathematicians, but also by those in the disciplines which are dependent on mathematics for their advanced study. In an article in *Engineering Education* Cook stated that "Educators must adjust courses and curriculum more often than in the past--courses must be replaced and updated continuously" (18, p. 509). In the same article Cook states, "Mathematics teachers must emphasize structural and abstract aspects of mathematics so that students get better understandings of concrete situations." In *The Mathematics Teacher* of
November, 1967, it was stated that:

As the useful potential of mathematics expands it becomes increasingly incumbert upon us in mathematics education to organize new mixes of subject matter; to tailor the emphasis carefully; to find better and more effective ways of imparting the subject to those who, for their own purposes, rightfully see in mathematics simply a subserviant means to more important ends (63, p. 703).

The need for a revision of the teaching of calculus can be seen not only in light of the statements of the previous paragraph, but also there are many factors within the subject or directly related to it that point out the need for changes. Within the subject itself, concomitant with the "new math" revolution, new topics have been added and the regular classical topics have been reformulated in modern terms. In the early part of this decade calculus textbooks were written with a degree of mathematical rigor heretofore considered too difficult for beginning students. In the preface to the second edition of their calculus textbook, Johnson and Kiokemeister stated:

It is our conviction that college students . . . , are not only able to understand and appreciate rigorous mathematical theory but also are more interested in courses containing both rigorous proofs and applications of the theorems (49, p. v).

Also during this time analytic geometry was integrated with the Calculus, thus adding more material to an already crowded syllabus. This integration resulted in the extension of the calculus sequence from three to four quarters. However, other factors have tended to make even this extension of time inadequate.
Among the indirect factors making changes necessary is the rapidly increasing number of fields in which calculus, indeed all of mathematics, is being used. In the past decade several prominent men in mathematics and mathematics education have reported this trend. In 1963 Bell stated, "Today in almost every field of endeavor one finds mathematics not only useful but necessary" (9, p. 302). In the same year Adler wrote, "Mathematics has always been an indispensible tool for the physicist, statistician, and the engineer, ..., it is rapidly becoming one for the biologist, psychologist and the economist" (2, p. 505). Four years later Leichliter stated that "business and economic progress are linked imutably to advances in mathematics education" (53, p. 449). Regarding calculus, Willoughby wrote,

A well taught course in the calculus contains a great deal of excellent mathematics which is useful not only in the physical sciences and engineering, but also in many recent applications to the social sciences (93, p. 381).

Another fact pointing to the increasingly utilitarian aspect of mathematics is that the number of mathematicians employed in private industry approximately doubled between 1954 and 1960 (9, p. 303).

The rapid expansion of applied mathematics has brought about other trends which make the need for curriculum revision a pressing issue. In order to meet the need for more mathematically trained people, "Colleges are adding requirements for additional mathematical
background work in the fields of economics, sociology, psychology, and business administration" (9, p. 302). As a result, an increasing percentage of college students are pursuing the study of mathematics. Thus,

College mathematics departments are facing tremendous changes in their student population, not only are mathematics students as a group both increasing in number, but they are also becoming more heterogeneous in their abilities, backgrounds and vocational goals (91, p. 642).

This heterogeneity is evident in the 1968-69 calculus class at Southern Oregon College where enrollment in the Fall term class was approximately evenly distributed among three groups, mathematics, physical science and biology-business majors.

Despite the previously mentioned reasons for changing mathematics curriculum in order to meet the needs of the ever increasing number of students, with ever increasing diversity of backgrounds, changes in this direction seem to be slow in coming about. Mathematics curriculum is still being influenced by the momentum of the "new math" revolution, and is being revised in a manner that will benefit those students who are captivated with the internal consistency of mathematics. In 1966, Fehr wrote, "The large experimental programs have so far almost completely ignored application and the relation of the subject to science instruction" (29, p. 88). One need only review the myriad of new textbooks published in the past three years to verify this trend. In fact, it appears that "Most people of
the mathematicians' world are almost totally oblivious of the desires and motivations of those who seek mathematical knowledge as a device, tool or lever" (63, p. 703). Mathematics departments, particularly at larger universities, have turned inward on themselves so much that they lose sight of the fact that "For every student who makes mathematics a career, there are dozens to whom it is only an elegant tool" (12, p. 542). In calculus in particular, "Many of our 'better' colleges and universities have adopted calculus books that stress formalism at the expense of both intuition and application" (93, p. 396).

The tendency to regard applied mathematics as an entity with little if any dignity began at an early stage in our scheme of mathematics education. When a student makes the transition from elementary school to the junior high school, he either proceeds on to the study of algebra or is shunted to consumer, or shop mathematics or something equivalent. Once he has been relegated to this track he is branded for the remainder of his school career as one who is mathematically incompetent. As John Bowen so aptly put it,

We proclaim a value system which says that applied mathematics is for dullards and assume at the same time that the better students will learn the applications by some mysterious osmosis (12, p. 542).

The failure of mathematicians and mathematics educators to adjust the curriculum to meet the needs of the "users" of mathematics, and the resulting disdain in the world of mathematics for applied
mathematics, has resulted in an increasing dissatisfaction with the type of curriculum being offered. The criticism has been voiced in articles in the journals of various disciplines. As far back as 1944, Ayers in The American Mathematical Monthly stated the need for more study of applications in mathematics courses (7, p. 200-205). In 1965, Feynman criticized the content of the "new" mathematics (30, p. 9-14). In calculus in particular, physical scientists interested in the teaching of the subject have felt that the curriculum has not been adjusted to meet their interests (18, p. 510). If these justifiable criticisms are ignored by the world of mathematics, then those teachers and students whose mathematical needs are different from the mathematicians' will move on their own to develop a suitable mix of topics, "Although they would be the first to admit that the product they are able to fabricate on their own will be inferior to what it might have been with our help" (63, p. 708). In November 1967, Fremont, aptly summarized the direction that mathematics curriculum must take, when he stated:

What we need, then are curricula and methods that are designed to meet the important weaknesses that we have known to result from our present curricula for some time now: rote learning, too much abstraction too soon, and meaningless manipulations. The time has come to create the kind of mathematics curricula that will result in our mathematics courses making a contribution to a liberal education. This implies that mathematics must be taught so that its role as a tool for the understandings of our environment is uppermost in the mind of both teacher and student (34, p. 717).
The philosophy of mathematics education that should guide curriculum revision of the kind mentioned above was clearly stated by Howard Fehr when in 1967 he wrote,

"The uses and applications of mathematics, the needs of future scientists and humanists, the understanding of laymen, the coordination of instructions of mathematics with that in the sciences, are all factors to be considered in a new program in mathematics (29, p. 85)."

It was with this philosophy in mind, then, that this study was conceived and carried out.

**Definition of Terms**

**Control Group**

The control group consisted of the 47 students which completed the three quarter calculus sequence, Math 200, 201, and 202, at Southern Oregon College during the 1967-1968 academic year.

**Experimental Group**

The experimental group consisted of the 41 students which completed the three quarter calculus sequence, Math 200, 201, and 202, at Southern Oregon College during the 1968-1969 academic year.

**Large Group**

The term large group refers to the experimental group as it was taught in a single class during the three term sequence.
Team Teaching

Team teaching is a type of instructional organization in which three teachers were given the cooperative responsibility for the instruction of the large group.

Small Group (Lab)

The term small group refers to the subgroups of the experimental group. There were three small groups, the members were determined according to the subject matter major of the students. One group consisted of mathematics majors, a second of biological science and business majors, and a third of physical science and engineering majors.

Attitude

An attitude is an affectivity toned idea or group of ideas predisposing an organism to action, with reference to specific attitude objects.

Criterion Tests

Criterion tests are defined to be those evaluation instruments which were used to test the hypotheses investigated in this study. The criterion tests used were the Cooperative Mathematics Test Calculus, Form B (CCT) published by Educational Testing Services, an Applied Problem Test (APT), and the Math-Science Attitude
Inventory (MSAI) developed by the researcher.

**Basic Assumptions**

In preparation for and the implementation of this study several assumptions concerning the population and measuring instruments were made. The assumptions were:

1. Any statistically significant differences between the control and experimental groups are the result of the differences between the experimental and traditional methods of treatment.
2. There are no differences in the basic concepts of calculus that were presented to the control and experimental groups.
3. The subject matter testing instruments used in the study reliably and validly measure the students general and specific achievement in calculus.
4. The Math-Science Attitude Inventory validly and reliably measures the students attitudes toward the role of mathematics in science.
5. The responses of the students on the attitude inventory reflect their true attitude toward the role of mathematics in science.
6. The sum of the student's overall high school and high school mathematics grade point average \((G_o + G_m)\) provides a valid and reliable index of his academic achievement and ability.
7. Student learning is related to the structure of a class and the
curriculum-mix which he is exposed to.

8. Team teaching and large group instruction are as effective as conventional instruction.

Limitations of the Study

The study is subject to the following limitations:

1. The study is limited to those students who enrolled in the three quarter calculus sequence, Math 200, 201, and 202, at Southern Oregon College during the academic years 1967-1968 and 1968-1969.

2. The subject matter presented in the sequence basically follows the content of the first 16 chapters of College Calculus With Analytic Geometry, by Murray Protter and Charles Morrey.

3. The comparison of general achievement in calculus was made in terms of topics chosen for the Educational Testing Service's Cooperative Mathematics Calculus Test, Form B.

4. The comparison of ability to solve applied problems was made by a nine item test developed by the researcher.

5. The Math-Science Attitude Inventory was developed by the researcher.
Hypotheses to be Tested

In order to determine if there were any significant differences between the control and experimental groups the following null hypotheses were tested:

1. There is no significant difference between the control and experimental groups in general achievement in calculus as measured by the *Cooperative Calculus Test, Form B*.

2. There is no significant difference between the control and experimental groups in ability to solve applied problems as measured by the *Applied Problem Test*.

3. At the end of the respective academic year there is no difference between the control and experimental groups in their attitude toward the role of mathematics in science as measured by the *Math-Science Attitude Inventory*.

In the following null hypotheses the identification of subgroups is indicated in Table 1.

<table>
<thead>
<tr>
<th>Year</th>
<th>Biology Majors</th>
<th>Business Majors</th>
<th>Science Majors</th>
<th>Engineer Majors</th>
<th>All Science Majors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967-1968</td>
<td>Group IB</td>
<td>Group IM</td>
<td>Group IS</td>
<td>Group IB+S</td>
<td>Group IB+S</td>
</tr>
<tr>
<td>1968-1969</td>
<td>Group IIIB</td>
<td>Group IIM</td>
<td>Group IIIS</td>
<td>Group IIIB+S</td>
<td>Group IIIB+S</td>
</tr>
</tbody>
</table>
In order to determine if there were any significant differences between subgroups, the following null hypotheses were tested:

4. There is no significant differences between Group $I_B$ and Group $II_B$ in general achievement in calculus as measured by the Cooperative Calculus Test.

5. There is no significant differences between Group $I_B$ and Group $II_B$ in ability to solve applied problems as measured by the Applied Problems Test.

6. There is no significant difference between Group $I_B$ and Group $II_B$ in their attitude toward the role of mathematics in science as measured by the Math-Science Attitude Inventory at the end of the respective academic year.

Table 2 lists the 15 additional combinations of subgroups for which similar null hypotheses were formed and tested.

<table>
<thead>
<tr>
<th></th>
<th>$I_M$</th>
<th>$I_S$</th>
<th>$I_{B+S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCT</td>
<td>$II_M$</td>
<td>$II_{B+S}$</td>
<td>$II_S$</td>
</tr>
<tr>
<td></td>
<td>7*</td>
<td>19*</td>
<td>10*</td>
</tr>
<tr>
<td>APT</td>
<td>$II_M$</td>
<td>$II_{B+S}$</td>
<td>$II_S$</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>21</td>
<td>12</td>
</tr>
<tr>
<td>MSAI</td>
<td>$II_M$</td>
<td>$II_{B+S}$</td>
<td>$II_S$</td>
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</tbody>
</table>

* Number of hypotheses tested in Chapter IV.
In order to determine if there were any significant differences within the control group the following null hypotheses were tested.

22. There is no significant difference between Group IB and Group IM in general achievement in calculus as measured by the Cooperative Calculus Test.

23. There is no significant difference between Group IB and Group IM in ability to solve applied problems as measured by the Applied Problem Test.

24. There is no significant difference between Group IB and Group IM in their attitude toward the role of mathematics in science as measured by the Math-Science Attitude Inventory at the end of the respective academic year.

Table 3 lists the nine additional combinations of subgroups for which similar null hypotheses were formed and tested.

<p>| Table 3. Combinations for Null Hypotheses Within Control Group |
|----------------------------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th></th>
<th>IS</th>
<th>IM</th>
<th>IB+I S</th>
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<td>IM</td>
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<td>IM</td>
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<td>27</td>
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<td>33</td>
<td></td>
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</tbody>
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* Number of hypotheses as tested in Chapter IV.
In order to determine if there were any significant differences within the experimental group the following null hypotheses were tested.

34. There is no significant difference between Group II<sub>B</sub> and Group II<sub>M</sub> in general achievement in calculus as measured by the Cooperative Calculus Test.

35. There is no significant difference between Group II<sub>B</sub> and Group II<sub>M</sub> in ability to solve applied problems as measured by the Applied Problem Test.

36. There is no significant difference between Group II<sub>B</sub> and Group II<sub>M</sub> in their attitude toward the role of mathematics in science as measured by the Math-Science Attitude Inventory at the end of the respective academic year.

Table 4 lists the nine additional combinations of subgroups for which similar null hypotheses were formed and tested.

Table 4. Combinations for Null Hypotheses Within Experimental Group

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* Number of hypotheses as discussed in Chapter IV.
II. REVIEW OF RELATED LITERATURE

A review of the literature on mathematics education revealed no studies had treated the effects of grouping by subject matter major on college calculus students. In this chapter a series of studies which relate to seven aspects of the investigation under consideration will be reviewed. These are:

1. Recent trends in calculus curricula.
2. Commonly accepted objectives of calculus teaching.
3. Studies concerned with team teaching.
4. Studies concerned with large group instruction.
5. Studies concerned with predicting success in college.
7. Studies concerned with integrated calculus courses.

**Recent Trends in Calculus**

In Chapter I the opposing viewpoints of the study of mathematics for its own sake, and the study of mathematics as a "tool subject" were stated. Both of these extreme viewpoints have had an effect on the topics included in the study of calculus. The addition of a rigorous development of the underlying theory of calculus to the calculus curriculum is a well known result of the "modern mathematics revolution" of the post-sputnik years. At the same time there has been an
increase, although not as dramatic and extensive, in the amount of non-geometric applications included in calculus textbooks.

In a review of the indexes of three popular textbooks published in the mid 1950's (56, p. 526; 82, p. 579; 83, p. 558) it was found that an average of 10 percent of the first 400 pages was devoted to non-geometric applications. In three well known textbooks published in the mid 1960's (50, p. 789; 70, p. 897; 78, p. 1008) an average of 18 percent of the first 500 pages was devoted to non-geometric topics.

In addition to the increased amount of applications included in recent calculus textbooks, there have been a few textbooks published within the last three years especially for non-mathematics majors. These textbooks include non-physical science applications, and are intended for a one or two term introductory course for business and social science majors. "The main impetus for this has been the increasing need for training in this subject (calculus) by students in the social science, biology, and other fields" (40, p. v). Authors of these textbooks recognize the need for a good calculus background for their students rather than the "one chapter approach" of past years. Youse and Stalnaker state,

It is our opinion that if anything is to be gained from requiring the business students to study calculus, his course work should be sufficiently concentrated to insure a thorough understanding of rudimentary techniques. Proofs should be omitted if they do not contribute to an understanding of the basic techniques of calculus (96, p. v).
These changes in calculus textbooks indicate, at least, that mathematicians and mathematics educators are aware of the problems of academic major heterogeneity in today's calculus classes. However, the changes per se do not guarantee that they are being effectively utilized. Fehr stated in the November, 1968 The Mathematics Teacher that,

It is a fact that in spite of some high-sounding changes that have been recently instituted, what we are doing today in mathematics does not differ much from the traditional college entrance program of 40 years ago (28, p. 666).

As further evidence of the failure to recognize the needs of the non-mathematics major,

More than one school of engineering, as well as other divisions of some universities, have found it expedient to offer its own mathematics course because suitable courses were not available in the universities' mathematics department (63, p. 705).

Thus, in spite of the aforementioned changes in calculus textbooks, curriculum development in calculus is in no way complete. There is agreement, both in principle and practice, that curriculum development involves four phases (22, p. 18):

1. Selecting learning outcomes.
2. Choosing subject matter which is best adopted to the realization of those learning outcomes.
3. Selecting the most economical teaching methods that will affect the realization of the designed outcomes.
4. Developing an adequate evaluation program to determine to what extent the outcomes have been attained.

Hence, objectives (outcomes) play a vital role in any consideration of the teaching of calculus.

Objectives of Calculus Teaching

One of the earliest attempts to set forth a list of objectives for the teaching of calculus was that of Rietz in the October 1919 issue of The American Mathematical Monthly (73, p. 341-345). All of his objectives were concerned with the application of differentiation and integration to the interpretation of physical laws.

A decade later Woods (95, p. 28-32) in one of the most comprehensive listings, proposed the following objectives. A study of calculus ought to:

1. Unfold a student's rational facilities.
2. Enable a student to express himself clearly and concisely.
3. Enable a student to recognize forms and acquire fact and techniques.
4. Enable a student to see relationships between things.
5. Unify all previous mathematical learning.
6. Be the key to the study of other fields of mathematics.
7. Enable a student to interpret physical phenomena.
This list of objectives was not appreciably added to nor revised during the ensuing 20 years. However, during this time a series of articles appeared in engineering and mathematical journals which initiated the de-emphasis of the applications of calculus (77, p. 635-640; 41, p. 485-521; 38, p. 205-209; 23, p. 186-193). These articles, spanning the period between 1932 to 1945, called for the added objective of proof and rigor in the study of calculus. Their cry was for proof as well as use.

The post second world war, pre-sputnik era, saw then, additional goals added to the list of objectives of calculus teaching. Parker (66, p. 347-349), Macduffee (57, p. 335-337) and Nowlan (64, p. 73-80), proposed these additional objectives. A study of calculus ought to:

1. Enable a student to understand the fundamental principles and concepts of the subject.
2. Enable a student to understand and master manipulative skills.
3. Develop a student's intuition.
4. Train a student in precise thinking.

The post-sputnik, modern mathematics era has seen the acceleration and increasing emphasis of a rigorous development of calculus, and as noted in Chapter I, a decline in the study of applications of calculus. The apogee of this trend was reflected in the 1963
Cambridge Report. The report of this conference proposes a strong program in calculus in high school that would develop "the foundations upon which applications to the sciences, engineering, and mathematics are built" (16, p. 102).

**Team Teaching**

According to Shaplin, team teaching emerged in American education in 1954, and since then has grown rapidly and had a substantial impact on education (81, p. 10-13). Team teaching has developed in this country at the elementary and secondary level under the supervision of colleges and universities. Since 1957, literature on this subject of team teaching has grown rapidly, however, very little has been written on team teaching at the college level.

In a 1961 review of studies dealing with team taught student achievement, Drummond concluded, "Students do as well or perhaps a little better on standardized tests when taught by teaching teams" (24, p. 162). In 1964 Heathers in his review of team teaching studies found that, "The self-contained classroom held the advantage in language and in arithmetic skills and problem solving" (43, p. 328). Similarly in 1964, Bair found that instructional outcomes were no less in a team teaching situation than they would have been under a traditional system (8, p. 192).

In addition to these reviews of team teaching research, there
have been a few studies reported in the area of team teaching mathematics. Most of these have been carried out at the elementary and secondary levels. In 1959, Carmichael reported on a study conducted at George Peabody College. Freshman mathematics was team taught by a graduate student-professor team. He found that there were no discernible differences in achievement (17, p. 280). In 1966 Stevenson (86) and Paige (65) in two studies conducted at the junior high school level reported no significant difference in achievement between team taught and conventionally taught students.

Similar studies in other disciplines by Zitelli (97) and Aden (1) reported no significant differences in team and conventionally taught students. Shaplin noted in his book Team Teaching that:

The development of team teaching should proceed through four interrelated phases: design, implementation, evaluation, and dissemination. To date, almost all research on team teaching has focused on its evaluation (87, p. 306).

This present study, then, was carried out under the assumption that team teaching has been sufficiently evaluated and that it has been found to be as effective as conventional instruction. Therefore, rather than an evaluation of team teaching this research was conceived in part to be a study of the design and implementation of team teaching at the college level.
**Large Group Instruction**

According to Anderson, "Most of the inquiry into effects of class size between 1911 and 1951 was related to the efforts of administrators to cut costs and increase efficiency" (6, p. 204).

However, since the development of team teaching and the concomitant use of large group instruction when using team teaching, there has been since 1954 a renewed interest in the effects of large group instruction. The studies can be divided into two categories, those in which students were taught only in large groups and those in which students were taught in large groups part of the time and divided into subgroups at other times. In either case research results have not been consistent. In 1964, Anderson reported that: "No studies known to the authors are of sufficient worth to warrant any definite conclusions about class size" (6, p. 205).

Several studies have been made on class size at the secondary school level in mathematics. In 1958, Patterson in a study of the effects of class sizes of 10, 20, 35, and 70 on the achievement of the tenth grade geometry students, reported no significant differences between any of the classes (67, p. 167). In a study conducted in 1968, in which geometry students were lectured to for 30 minutes two days a week and taught in three smaller groups three days, Bhushan, reported no significant differences when the students were compared
to conventionally taught classes (10, p. 774). However, Madden in 1966 reported groups of 70 to 85 students did significantly better in ninth grade general mathematics than students in a class of 25 to 40 (58, p. 631).

Similar studies have been made at the college level with equally inconsistent results. As early as 1928 and 1932 Hudleson (46) and Remmers (71) in studies comparing class sizes found no significant differences in student achievement. In comparing groups of college calculus students of sizes 30 and 90, Stockton in 1960 found no apparent differences in learning (87, p. 1025). In a similar study in 1966 in which calculus students were lectured to for three days and randomly divided into smaller groups for two days, Turner reported no significant differences in achievement (90, p. 769). In a report in 1962, May found that a group of college mathematics students who were lectured to three days a week in a group of 250 and taught in groups of 20 two days a week did not significantly differ in achievement from conventionally taught students (59, p. 433). However, in 1965, Anderson reported that mathematics for elementary teachers when taught to a group of 140 students was more effectively done than when taught to a group of 40 students (5, p. 180).

The diversity of opinion regarding class size is reflected in two statements made by researchers, the first in the report by May and the second in that of Anderson:
"The majority (of teachers) preferred to have a slightly heavier load accompanied by the satisfaction of managing a small class completely" (59, p. 433).

"It is more desirable to sit 100 feet from a competent instructor than to sit 10 feet from a mediocre one" (5, p. 180).

Predicting Success in College

In studies concerned with prediction of academic success in college and the proper placement of incoming students, the variable used to measure these factors was college grade point averages. Prior to the second world war almost all studies examined the correlations between general mental test scores, and scholastic success. In 1934, in summarizing reported studies, Segel reported a median correlation coefficient of .44 between scores on mental tests and success in college as measured by college grades (80, p. 70). Garrett (46, p. 101) in a similar review reported a range in correlation coefficients of .17 to .67, with a median of .47.

Since the 1950's few studies have been reported involving correlations between intelligence test scores and collegiate success. Rather they have turned to investigating other factors such as achievement test scores, college entrance examinations, aptitudes test scores, reading tests, high school rank, placement test scores and high school grade point averages.
Many studies have been made of the relationships between academic success in high school and similar success in college. In 1950, in summarizing reported studies, Frederiksen and Schrader found correlations ranging from .33 to .65, with a median coefficient of .57 (33, p. 255). A decade and a half later Giuisti (37) analyzed the literature and stated that the most significant conclusion resulting from analyses of prediction studies is the superiority and stability of the high school grade point average as a source for predicting college success. Michael (60) and others found that high school grades were better predictors of college grades than either part or total scores on the College Board Scholastic Aptitude Test. They also found that there was only a .04 improvement in the .52 correlation coefficient when high school grade point averages and SAT scores were combined to form a predictor of success.

In a study of 24 factors related to success in college conducted in 1965 at Southern Oregon College, Elle (27, p. 145) found that high school grade point averages had the highest correlation coefficient (.67) than any other computed in the study. His recommendation, as a result of the study, was to base general prediction and placement upon high school grade point averages, rather than upon Scholastic Aptitude Test or School and College Ability Test scores. As a result of this study the mathematics department at Southern Oregon College devised a method of placement of incoming students in mathematics courses
based on high school mathematics grade point average and the amount of mathematics taken in high school. All students in the control and experimental populations were placed in the calculus sequence on this basis.

In addition to these studies of predictive factors in overall college success, several recent studies have been made of predictive factors for success in mathematics. Hassinger (42) in a study of engineering students, found that high school mathematics grade point average was the better predictor of success in calculus (.43), than number of semesters of high school mathematics, SAT-verbal scores, or proficiency in algebra. In similar studies Scott, (79) and Smith (84) found high school grade point averages to be good predictors of success in college mathematics when compared to admission tests and mathematics tests. In 1965 Wick, (91, p. 647) reported in a study of eight factors associated with success in first year college mathematics that high school mathematics grade point average had the highest correlation coefficient (.46). In a placement study in analytic geometry and calculus, Francis (32) found that mathematics placement tests and SCAT-Q test scores had correlation coefficients of .22 and .38 respectively with college mathematics grade point averages.

In a study of achievement of biology students in 1965, Kochersberger (51) used high school grade point averages as a covariable in his statistical analysis.
Measuring Attitudes

Much has been written that suggests the importance of favorable attitudes toward a subject relative to achievement in that subject. Remmers, (72, p. 362) has suggested that unless a person has a favorable attitude toward a set of instructional objectives and sets them up as desirable goals for himself, the educative process will be relatively ineffective. Johnson (48, p. 113) has stated that: "No matter what subject we are teaching or what method we are using, many concomitant learnings and changes in attitudes are taking place."

Several studies have shown that attitudes developed by students are directly related to teacher attitude or experiences with former mathematics teachers. Lerch (54, p. 119) indicated that a student's success in mathematics is more basically dependent upon his teachers' attitudes than it is upon classroom procedure. Proffenberger and Norton (69, p. 171-176) indicated that the teacher plays a significant part in the development of their students attitude toward mathematics.

Other studies have shown that as understanding of the basic concepts of mathematics increases, the student's attitude toward mathematics improves and conversely. In 1957, Johnson (48, p. 116) indicated that attitudes toward mathematics may be enhanced by understanding the basic structure of the subject. Hartung (41, p. 39) found that an increase in interest and appreciation tends to accompany an
increase in understanding of mathematics.

Attitude scales, as such, were developed in the late 1920's and early 1930's. In 1929, Thurstone (89, p. 36-45) at the University of Chicago developed an objective type test in which the participant checks only statements with which he agrees. In 1932, Likert (55) developed an objective type test in which the participant checks each item on a scale ranging from strongly agree to strongly disagree.

Prior to the 1950's almost all attitude scales were constructed to measure social attitudes. However, as early as 1934 Remmers, (72, p. 84-88) constructed a Thurstone type attitude scale to measure attitude toward any school subject. In 1954, Dutton (26, p. 24-31) developed a 22 item, Thurstone-type, scale to measure attitudes toward arithmetic. He reported that students liked arithmetic because it was practical, provided a challenge, was definite and logical, and provided satisfaction when they were successful in working problems. Unfavorable attitudes were generated when the students did not feel secure in the subject. In a similar study conducted at the junior high school level, Dutton (25, p. 18-22) reached similar conclusions.

In 1961 Aiken (3, p. 19-24) developed a 20 item Likert-type attitude scale to measure the attitudes of college students toward mathematics. He reported that attitude toward mathematics is related to final mathematics course grades, to the prediction of final grades in mathematics courses, and to achievement test scores in
mathematics. He also reported that attitude toward mathematics was not related to general personality variables.

Several attitude scales have been developed in recent years to measure student attitudes toward science. In 1957, Allen (4) developed a 95 item Likert-type attitude scale to measure attitudes toward science and scientific careers. He concluded that high school seniors generally had positive attitudes toward science, and the teacher was the most important factor in developing positive attitudes toward science. Studies by Howe (45) and Brown (13) using a modified form of Allen's inventory, reported similar conclusions. In 1954, Wilson (94, p. 160) developed a modified Likert-type science attitude scale which he administered to college freshmen and sophomores. He concluded that science majors have a better attitude toward science than non-science majors.

In this study a Likert-type attitude inventory has been constructed in an attempt to measure students attitude toward the role of mathematics in the study and pursuit of scientific knowledge. In the construction of attitude inventory items Likert (55, p. 49) suggested that:

1. Statements should be of desired behavior and not fact.
2. Statements should be clear and concise.
3. Statements should be worded so that model reaction to it is in the middle of possible responses.
4. In the set of original statements half should express a positive attitude and half a negative attitude.

5. The statements should be directed toward a single attitude variable.

Likert suggests that a split half reliability coefficient should be calculated by correlating the sum of the odd item scores with the sum of the even item scores for each individual. In his 1932 study he found that using $\mu_D = \mu_H - \mu_L$ (See Chapter IV), to determine the best discriminators in the set of original statements was highly correlated (.91) with the phi-coefficient for each item.

**Integrated Courses**

In 1948, Rosenhead (75) in an attempt to better meet the needs of physics majors, outlined a coordinated physics and mathematics sequence. In a similar program outlined by Cook (18), separate but coordinated physics and mathematics courses were taught at Pratt Institute to engineering majors.

In 1961-1962 Sandler (76) and Wilcox (92) proposed completely integrated courses in mathematics and physics at the college level. Their studies revealed that students in integrated classes had significantly better knowledge of physics, but exhibited no significant difference in knowledge of mathematics when compared to conventionally taught students.
III. DESIGN OF THE STUDY

The design of this study was directed principally toward four questions:

1. How can a calculus class and curriculum be structured to meet the needs of the increasingly heterogeneous population found in today's college and university mathematics classes?

2. Can the basic concepts and skills of calculus be learned effectively in such a class?

3. Can the concepts and skills be effectively related to the various subject matter subgroups of a calculus class, and can the related abilities and knowledge of such students be improved?

4. Will such a class have an effect on the student's attitude toward the role of mathematics in science?

The purpose of the analyses used was to determine:

1. Whether there were any significant differences between the control and experimental group in general achievement in calculus.

2. If there were any significant differences within the control and experimental groups in general calculus achievement.

3. If there were any significant differences between and within the control and experimental groups in ability to solve
applied calculus problems.

4. Whether or not there were any significant differences within and between the control and experimental groups relative to student's attitude toward the role of mathematics in science.

Thus the statistical designs employed to test the hypotheses were analyses of covariance, and analysis of variance.

The criterion instrument used to test general calculus achievement was the Cooperative Mathematics Test in Calculus, Form B, published in 1963 by Educational Testing Service, Princeton, New Jersey. The criterion instrument used to test ability to solve applied problems in calculus was an Applied Problem Test constructed by the researcher. The instrument used to measure attitude differences was the Math-Science Attitude Inventory constructed by the researcher.

The covariable used in the analysis of covariance was the sum of a student's overall high school (grades 10 through 12) grade point average, and his high school mathematics grade point average;

\[ G_o + G_m = G \]

\( G_o \) and \( G_m \) were obtained from each student's file at the registrar's office at Southern Oregon College.

Additional personal data were obtained for each student from a Personal Data questionnaire constructed by the researcher and completed by every student in the study (see Appendix C). Data concerning classes and/or students not directly involved in the study were obtained
from the files of the Department of Mathematics at Southern Oregon College.

The Experimental Design

The design chosen for this study contains aspects that differ from the true experimental designs as described by Campbell and Stanley (35, p. 195). The design of this experiment most closely resembles the Posttest-Only Control Group type, described in symbols by Campbell and Stanley as:

\[
\begin{align*}
R_1 & \times 0_1, 0_2, 0_3 \\
R_2 & 0_1, 0_2, 0_3,
\end{align*}
\]

where \( R_1 \) and \( R_2 \) are the randomly selected control and experimental group respectively, \( X \) is the experimental treatment, and \( 0_1, 0_2, 0_3 \) are the criterion measures used. The actual design of this study deviates from this scheme in that the control group consisted of students which completed the Math 200, 201, and 202 calculus sequence during the academic year 1967-1968, while the experimental group consisted of the students completing the sequence during the 1968-1969 academic year. Thus \( R_1 \) and \( R_2 \) were separated by an academic year in time. The students, then, where not randomly assigned to a control and experimental group, rather each group was regarded as a random sample of a population of college students.
Therefore, the control and experimental groups were considered to fulfill the criterion of randomness required in experimental and statistical designs. Thus, with these noted deviations, this study was considered to fulfill the Posttest-Only Control Group Design as described by Campbell and Stanley.

In this study differences in academic ability, and academic achievement were considered to be factors which would affect two of the three criterion test scores, and were adjusted for through the covariance techniques of statistical analysis.

**The Population Samples**

The research plan of this study was to investigate the effects of grouping calculus students by subject matter major on student performance, abilities and attitudes. These abilities, attitudes and achievements were compared with those of calculus students who had been taught in the conventional classroom setting with no grouping other than that brought about by normal class scheduling. The study was carried out over a two academic year period extending from September 1967 to June 1969 at Southern Oregon College. Southern Oregon College is a four year degree granting unit of the Oregon State System of Higher Education. The total consisted of 88 students, 47 in the control group and 41 in the experimental group.
Control Group

The control group consisted of students who completed the three term calculus sequence during the 1967-1968 academic year at Southern Oregon College. Of the 78 students matriculating 49 completed the three term sequence, and of these 49 students the overall high school and mathematics grade point averages were available for 47. The students ranged in age from 18 to 47 with an average age of 21. The group contained 39 men and 8 women. There were 23 freshmen, 19 sophomores, 2 juniors, 2 seniors and 1 graduate student. The group consisted of 23 mathematics majors, 16 physical science majors and 8 biological science and business majors.

Experimental Group

The experimental group consisted of students who completed the three term experimental calculus sequence during the 1968-1969 academic year at Southern Oregon College. Of the 68 students matriculating 42 completed the three term sequence, and the overall high school and mathematics grade point average were available for 41 of these. The students of the experimental group ranged in age from 18 to 24, with an average age of 19.4. The group consisted of 35 men and 6 women. There were 31 freshmen, 6 sophomores, 3 juniors, 1 senior and no graduate students. There were 14
mathematics majors, 22 physical science majors and 5 biological science and business majors.

The following tables (5, 6, and 7) summarize and elaborate on the distribution of the control and experimental groups with respect to size, sex, academic standing and secondary school grade point average.

Table 5. Numbers and Percentages by Sex of the Control and Experimental Groups

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Table 6. Academic Standings of the Control and Experimental Groups

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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Science</td>
<td>22</td>
<td>15</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bio. and Busi.</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 7. Grade Point Average Profile for the Control and Experimental Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>Range Overall GPA</th>
<th>Range Math GPA</th>
<th>Mean Overall GPA</th>
<th>Mean Math GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>1.92--3.86</td>
<td>1.40--4.00</td>
<td>2.89</td>
<td>2.79</td>
</tr>
<tr>
<td>Mathematics</td>
<td>1.92--3.79</td>
<td>1.50--4.00</td>
<td>2.92</td>
<td>2.88</td>
</tr>
<tr>
<td>Science</td>
<td>2.31--3.14</td>
<td>1.50--3.00</td>
<td>2.79</td>
<td>2.62</td>
</tr>
<tr>
<td>Bio. and Busi.</td>
<td>1.92--3.86</td>
<td>1.40--3.87</td>
<td>3.03</td>
<td>2.87</td>
</tr>
<tr>
<td>Experimental</td>
<td>2.08--3.97</td>
<td>1.75--4.00</td>
<td>3.18</td>
<td>3.10</td>
</tr>
<tr>
<td>Mathematics</td>
<td>2.54--3.85</td>
<td>2.54--4.00</td>
<td>3.14</td>
<td>3.13</td>
</tr>
<tr>
<td>Science</td>
<td>2.08--3.90</td>
<td>1.75--3.75</td>
<td>3.14</td>
<td>2.95</td>
</tr>
<tr>
<td>Bio. and Busi.</td>
<td>3.06--3.97</td>
<td>3.50--4.00</td>
<td>3.48</td>
<td>3.70</td>
</tr>
</tbody>
</table>

**Treatment of the Control Group**

The 1967-1968 calculus students were taught in the traditional lecture-discussion manner. Fall quarter there were three sections of approximately 25 students each taught by three members of the mathematics department. Due to usual attrition there were two sections ranging from 25 to 30 students during the Winter and Spring terms. Two of the three teachers continued teaching these sections for the remainder of the academic year. All sections met four; 50 minute, periods per week. No attempt was made to influence which section the students registered for, thus almost all students had more than one instructor during the academic year.

The topics and curricula presented to the control group were those which comprise the first 16 chapters of *College Calculus With*
Analytic Geometry, written by Murray Protter and Charles Morrey, and published by Addison-Wesley in 1963. Although each instructor acted independently in teaching his section, the material to be presented each term was mutually agreed on by the staff prior to the beginning of each term. No special attempt was made to present topics relevant to any particular subject major found in the classes, and no attempt was made by the instructors who taught the sequence to determine the major field of interest of any of the students in the classes.

The instructors of the control group were all regular full-time members of the mathematics department of Southern Oregon College. All of the teachers had at least a Master of Arts or Science degree in mathematics, three years of teaching experience at the college level, and the academic rank of Assistant Professor of Mathematics. Each teacher also had at least three years of teaching experience at the secondary level, and in addition had completed at least 20 quarter hours of graduate level mathematics beyond the master's level.

Treatment of the Experimental Group

The 1968-1969 calculus students were taught using a modified team-teaching approach. All students who enrolled in calculus (Math 200) Fall quarter 1968 at Southern Oregon College were placed in the experimental class for the academic year. The entire matriculating class of 68 students was taught on Monday and Wednesday by a team of
three members of the mathematics department. Because of attrition
due to failure, dropping out of school, completing their mathematics
requirements, etc., the number of students Winter term was reduced
to 56 and to 45 Spring term. On the two remaining days of the week,
Tuesday and Friday, the students were divided into three classes
grouped according to the subject matter major of the students. The
three groups will be referred to as Group II$_B$, which consisted of
biological science, social science and business majors; Group II$_M$
which consisted of mathematics majors; and Group II$_S$, which consisted
of chemistry, physics and engineering majors. Each of the three staff
members of the teaching team were assigned to one of the smaller
(lab) sessions each quarter, but not the same group each term. All
sessions met at 12 o'clock noon and were 50 minutes in length. The
12 o'clock hour was chosen so as to minimize any scheduling conflicts
since the experimental class was the only calculus sequence available
to the students.

The topics and curricula presented to the experimental group
were basically those which comprise the first 16 chapters of *College
Calculus With Analytic Geometry*, by Murray Protter and Charles
Morrey, the same edition as used by the control group. For a more
complete elaboration of the curriculum refer to the headings which
follow.

The instructors of the experimental group were regular
full-time members of the mathematics department. All members of
the team had at least a Master of Arts or Science degree in mathe-
matics, three years of teaching experience at the college level, and
the academic rank of Assistant Professor. Each teacher also had at
least three years of teaching experience at the secondary level and
had completed at least 21 quarter hours of graduate level mathematics
beyond the master's level. One member of the team had also par-
ticipated in the instruction of the control group during the previous
academic year.

Large Group Sessions

The basic topics, concepts, and skills of calculus were
presented by one or more members of the teaching team during the
large group sessions. These classes were basically lecture-
discussion sessions in which new material was presented to the stu-
dents. The students asked questions as the presentation on the over-
head projector or on the board progressed. However, at no time
during these large group sessions were questions pertaining to pre-
vious problem assignments answered by the instructor. This restric-
tion allowed the instructor to present more than one topic and/or
concept during a large session whenever necessary. Assignments
were given to the group by the teaching member of the team and were
handed in at a specified time to be corrected by student aides.
Small Group or Lab Sessions

During the small group meetings, in which students were segregated by their subject matter major, the concepts and topics presented in the previous large group sessions were discussed in further detail, and when possible related to the particular subject major of the group at hand. The latter task was accomplished by discussing relevant applications of the topics and concepts, by stressing different aspects of a given topic or concept to one or more of the sections, by introducing new topics or concepts and by deleting topics and concepts judged to be irrelevant to the subgroup at hand. In addition to the curricula differentiation aspect of the lab sessions, time was given over to problem solving and discussion of any questions or difficulty the students were encountering with the concepts or skills presented in previous class sessions.

Assignments, unique to each group, dealing with any special material presented in the lab sessions were made by the instructors of the respective groups. These were handed in by the students and graded by the instructor. Also several times during the year differentiated assignments were given to the subgroups in the large session. Since a student had his calculus class alternating in two different classrooms during the week, some confusion developed as to when and where assignments were to be handed in. In order to alleviate this
problem the team members, who met once a week, decided to distribute to the students, on Friday, a schedule for the coming week which listed the ratio of large to small group sessions, the room number they were scheduled to meet in, the assignments to be made, and the day they were to be handed in (see Appendix D). This effectively ended all scheduling and assignment problems for the year.

Since all classes were scheduled to meet at 12 o'clock noon, and all classrooms were reserved for this hour daily, it was possible to deviate from the ratio and sequence of large and small group meetings whenever necessary. Deviations in the schedule generally occurred during the first week of a quarter when two or three large sessions were held in succession followed by two or one lab sessions. The schedule was also varied for mid-term examinations, given in the large sessions, so that a review lab session preceded the test.

Testing and Grading

Mid-term and final examinations, cooperatively constructed by the team members, were administered in the large group sessions. These tests were given to determine the student's mastery of the material presented in the large group sessions. Each quarter two 50 minute mid-terms and a two hour final were given. These were corrected by the three team members by having one member correct the same page or pages on every paper. Thus the students who were
taught by each of the team members, had their tests graded by each of the team members. After the mid-term tests were scored, the scores were ordered from high to low and "cut off" scores were cooperatively determined by the team members for letter grades (see Appendix E). The scale was then typed and duplicated for distribution to the students. Therefore, by adding the "cut off" scores for the various letter grades, and determining the sum of his own two mid-term scores, each student was able to determine his standing prior to the final examination.

In addition to the mid-term and final examinations, weekly quizzes were given in the lab sessions by the individual instructors. Also any special lab assignments given during the term were graded by the instructors. These scores in addition to a lab final, usually given in conjunction with the regular final or during the last lab of the term, were used to determine a student's lab score for the term. The final grade for each student was a composite of his lab score and his mid-term--final scores. A weight of 70 was given to the mid-term--final score and a weight of 30 to the lab score. These composite scores were then ranked and "cut off" scores were determined for the final letter grades.
Grouping of Students

The determination of which subject matter majors should be grouped together in the lab sessions was begun with a survey of the control group taken Winter term 1968. The results of the survey are shown in Table 8.

Table 8. Distribution by Subject Matter Major Math 201 Winter Term 1968

<table>
<thead>
<tr>
<th>Major</th>
<th>Number</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biology</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Business</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Chemistry</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Engineering</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>Mathematics</td>
<td>25</td>
<td>41</td>
</tr>
<tr>
<td>Physics</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>58</td>
<td>100</td>
</tr>
</tbody>
</table>

Two definite subgroups were apparent, mathematics majors, and engineering--physics majors. The decision of how to group the remaining majors was based on two criteria:

1. The different expectations each field, depending on calculus for its advanced study, has from the Calculus.

2. A 1964 report from the Committee on the Undergraduate Program in Mathematics (19).

Unlike mathematicians who feel that calculus is the basis for the study
of advanced mathematics, physical scientists and engineers view calculus as a "tool for the construction of idealized mathematical models of physical phenomena" (21, p. 5). Thus it was decided to group the chemistry majors with the physics and engineering majors. In 1964 CUPM reported that biological, management, and social sciences, "need mathematics primarily as a language for scientific reasoning" (19, p. 5). Furthermore, in an article in Scientific American it was stated that:

In everyone of the social sciences it has become increasingly evident that an exclusively verbal description of complex systems, ..., results in generalizations too difficult to analyze, compare and apply (87, p. 168).

Therefore, it was decided that the third group should be composed of biological, social science and business majors.

The actual sectioning of the experimental group was accomplished during the first large group meeting of Fall quarter. Each student completed a personal data sheet (see Appendix) on which he indicated his major field of study. At that time it was stressed that his indication would determine to which small group he was to be assigned, and that he would be obliged to remain in that group throughout the academic year unless some unusual hardship resulted. As a result, Fall term, Group $I_B$ consisted of 20 students, Group $I_M$ of 22 students, and Group $I_S$ consisted of 26 students. No student who finished the three quarter sequence asked to change lab sections.
Team Teaching Structure

The team of three teachers was horizontally structured in the form of a "collegeium" where equals determined the implementation of the program by democratic procedures. Leadership by the researcher was in the realm of administrative detail, such as arrangement of place and time of meetings, scheduling classrooms, typing schedules, etc. The members of the team, other than the researcher, were not specifically selected to participate, rather they were scheduled to teach the sequence during the academic year as a matter of course. However, the experimentor did confer with each of them prior to the start of the academic year to acquaint them with the proposed goals, plans and structure of the program. It was also made clear that they were free not to participate if they so desired.

The first formal team meeting took place during faculty orientation week prior to the beginning of fall quarter. At that time a general outline of the year's work was drawn up, including a matrix assigning each team member to one of the lab groups each quarter. The objectives, materials, methods and procedures for the quarter were discussed and outlined. It was decided that no attempt would be made to equalize the time each member taught the large group sessions, rather the length and number of times each instructor assumed this responsibility would be dependent on the continuity of topic presentation, and
the interest of an instructor in a given topic.

Throughout the academic year the team met formally once each week (Thursday) to discuss and prepare the following week's work. Regardless of which team member was teaching the large group session, the entire team decided which topics were to be included during the coming week's instruction. Also any special topics or concepts to be presented in the lab sessions were discussed. In addition, assignments, quizzes, tests, problems encountered by students and class schedules were discussed. Preliminary planning for two weeks in advance was done and test dates were chosen three weeks in advance. A schedule for the coming week was then typed, duplicated, and distributed to the students the following day (Friday). Since the offices of the team members were located in close proximity to each other spontaneous planning and discussions also took place.

The team members were encouraged to attend every large group session so that they could assume or resume teaching the class at anytime. Also this provided an opportunity for feedback to the teaching team member regarding his methods, visual aids and effectiveness. In the lab sessions each instructor assumed general responsibilities similar to those of a traditional autonomously taught class, however, at all times every team member was aware of what was being done in all lab sessions.

A lab instructor's main objective was to present topics, concepts
and problems which related calculus to the group's subject matter interests. Secondly he answered questions, solved problems and discussed particular areas of difficulty indicated by the students. Those students whose lab instructor was not the large group instructor thus had the opportunity to get two viewpoints or explanations of a given topic, and at the same time could identify with a particular member of the teaching team.

**CORD Research Grant**

In order to derive maximum benefit from time devoted to problem solving in the lab sessions it was imperative that assignments from both the large and lab sections be corrected and returned to the students as soon as possible. To accomplish this goal a College Research and Development Project (CORD) grant of $385 was applied for and received. Fall and Winter term three, and Spring term two senior mathematics majors were employed to correct papers, one for the students in each lab session. During Spring term Group II_M and Group II_B papers were handled by one corrector. One of the correctors also offered help sessions two hours per week to those students who wanted any additional help. Similar help sessions were also available to the control group during the previous academic year.
Objectives and Rationale

The experimental class, was designed to enable the mathematics department of a small (4000 students) college better meet the needs of the various subgroups found in the increasingly heterogeneous calculus classes found on modern day campuses. The modified team teaching approach described in this chapter was designed with the following objectives in mind.

1. To significantly increase the experimental groups achievement in:
   a. their mastery of the basic skills of the Calculus,
   b. their understanding of the basic concepts of the Calculus,
   c. their ability to solve problems related to their field of interest.

2. To affect the experimental groups' attitude toward the role of mathematics in their fields of interest.

3. To more effectively relate the Calculus to a student's major field.

4. To increase the mathematics major's appreciation of and the need for a rigorous development of the Calculus.

5. To increase the physical science major's appreciation of and ability to construct mathematical models of physical phenomena.
6. To increase the biological science, social science and business major's appreciation of and ability to construct descriptions of complex systems too difficult to analyze verbally.

Although, as indicated in Chapter II, research has shown that team taught students do not differ significantly in achievement when compared to conventionally taught students, this mode of teaching was chosen so that:

1. All students in the experimental group would have common instruction in the basic skills and concepts of the Calculus.
2. The instructors of the lab sessions would have a first hand knowledge of and a direct responsibility for the topics presented in the large group sessions.
3. Time could be devoted in the lab sessions to relating concepts introduced in the large group sessions to the major fields of interest.
4. New topics and their application to the major fields could be presented in the lab sessions.

An alternative design to the way in which the experimental class was structured would have been to have three entirely separate grouped classes with individual instructors. This alternative was not selected because:

1. Complete separation would prohibit the valuable interchange
of ideas and viewpoints that take place in a "mixed" class, resulting in students with "tunnel" vision with respect to the role of mathematics in our culture.

2. The students have not at this time in their college career completely determined their choice of subject matter major.

3. The duplication of instruction in basic skills, topics, and concepts could be eliminated, thus providing the opportunity for the members of the team to better prepare for their lab sessions.

4. The influence of a particular teacher on a particular group could not be minimized.

5. There was a need to have some degree of control over and coordination of the different treatments accorded the three groups.

6. Scheduling three distinct classes four days a week would have caused scheduling and teaching load difficulties.

7. It would have been more difficult to measure differences within the experimental group without a common basis from which to measure.

8. There would have been no fair basis for the common mid-terms and final examinations given to the experimental group each quarter.

In this study the control and experimental groups were not only
subjected to different treatments, but, in addition they were not students during the same academic year. The principle reasons for this aspect of the design were:

1. The number of calculus students was not large enough in a given academic year to conduct the study as designed.

2. If the number of calculus students had been large enough to conduct the experiment in one academic year, class scheduling conflicts would have effected grouping.

3. The students of the two groups did not have the opportunity to influence one another by comparing their experiences, and studying together.

4. The awareness of the experimental group of being treated differently was minimized, thereby, reducing bias due to the Hawthorne effect.

The Evaluation Instruments

Cooperative Mathematics Test

In order to measure differences in the cognitive domain between and within the two groups, the Cooperative Mathematics Test Calculus, Form B was administered. The test, published by Educational Testing Service in 1963, was given to all students in the study as a post-test to measure general achievement in calculus. The test was selected
because it was the only nationally standardized calculus test constructed by mathematics educators that was available at the time. It is a multiple choice test consisting of two 30 item parts requiring 40 minutes per part. Achievement is measured in terms of a student's "comprehension of the basic concepts, techniques, and unifying principles in each content area" (Test Manual, 1964). The test handbook suggests that the combined score for the two parts be used to assess the achievement level of a student in relation to specified reference groups, and to appraise average level of achievement for evaluation of instructional program. National norms have been developed by the publisher based on samples of students in all major fields of study, with separate norms established for education, engineering and liberal arts students.

The nine Cooperative Mathematics Tests were constructed by 46 mathematics teachers. The items were then reviewed and edited by the ETS staff, who assembled the items into pre-tests. These pre-tests were reviewed by four mathematics teachers and administered to a national sample of students. The tests results were analyzed, the tests revised and then readministered in order to establish norms. This procedure was followed to assure good content validity. For the Cooperative Calculus Test, Form, B, the Kuder-Richardson reliability coefficient, a measure of internal consistency was determined to be .84. The mean and standard deviation, based on a sample size of 350 was found to be 27.39 and 8.37 respectively. The test scores had
a correlation coefficient of .39 with the Quantitative section of the School and College Ability Test (SCAT-Q).

**Applied Problem Test**

An **Applied Problem Test** was constructed by the researcher to measure the student's ability to solve problems related to their subject matter major. It was found necessary to construct such a test because no published test including such material could be located.

The **Cooperative Mathematics Test** in calculus contained two items related to velocity and acceleration; but were of such a general nature that they were judged inappropriate and too few in number. Items for the test were written by the researcher after a study of applied problems found in 10 calculus textbooks published for use in the college classroom within the past 10 years. In addition six calculus textbooks written for specific subject matter areas were also studied. Eighteen items were written and submitted to three members of the mathematics department of Southern Oregon College for evaluation and comment.

The objective was to include an equal number of items in the test that could be regarded as relevant to each of the three proposed subgroups of the experimental group. Thus the first test constructed consisted of six multiple choice items, two items per subgroup. This form of the test was then evaluated by two members of the mathematics department at Oregon State University. The test was then administered
to 15 fourth quarter calculus students at Southern Oregon College. The tests were corrected and an item analysis made to determine the coefficient of difficulty for each item. The test was then revised and administered to 19 fourth term calculus students at Oregon State University. Again the items were analyzed and revised. As a result of a consideration of the time factor involved in the two administrations of the test, it was decided to increase the length of the test from six to nine items. The lengthened and revised test was then administered to 29 third term calculus students at Oregon College of Education. Again the test was corrected, items analyzed and a few revisions were made. This test, which was to become the final form, was then given to a group of 13 fourth term calculus students at Southern Oregon College. Since this applied problem test contained three specific problems for each of the three subgroups of the experimental group, the problems included were intended to measure different skills and concepts.

According to Guilford,

If a test is heterogeneous, i.e., different parts measure different traits, then one should not expect a high degree of internal consistency. The only meaningful estimate of reliability for a heterogeneous test is of the test-retest reliability (39, p. 178)

Therefore, two weeks after the test was given to the group of students at Southern Oregon College it was again given to them, and the test-retest reliability coefficient was calculated to be .74. The mean calculated for the nine item test was 3.83 with a standard deviation of 1.53.
Attitude Inventory

A Math-Science Attitude Inventory was constructed by the experimenter to use as a measure of differences in the affective domain within and between the experimental and control group. Since no such attitude inventory could be found, a Likert-type survey was constructed following the procedures outlined by Likert (55, p. 52) and Remmers (72, p. 437). The purpose of this inventory was to survey the student's attitude toward the role of mathematics in science. The objective was to construct an inventory that would enable the experimenter to gather evidence to answer questions such as:

1. Do students in one subject matter major perceive the role of mathematics in science differently than those of another?

2. Do students of various disciplines view the study of mathematics as essentially different from the study of science?

As a preliminary step in the construction of the attitude inventory 50 items were written by seven members of a science education doctoral seminar of which the researcher was a member. In order to determine whether the items expressed a positive or a negative attitude toward the role of mathematics in science, the seven members of the seminar individually evaluated each item. Each item was marked with a (+) or a (-), then item by item the marks were compared to determine if any item was ambiguous. On all items except seven
there was unanimous agreement as to the positiveness or negativeness of the item. In the exceptional cases if there were more than two disagreements out of the seven, the item was rewritten so as to obtain unanimous agreement. If there were two or less the item was retained. None of these seven items were included in the final form of the inventory.

The 50 items were used to make up an attitude inventory with five possible responses per item, strongly agree, agree, uncertain, disagree, and strongly disagree. The inventory was administered to a total of 145 college and university students, before the final form was attained. An item considered to express a positive attitude was scored 5, 4, 3, 2, 1 relative to the five choices, and a negative item was scored 1, 2, 3, 4, 5. Thus a neutral response was always scored three. The maximum possible score is five times the number of items, and the minimum possible score is equal to the number of items.

As suggested by Likert (55, p. 49), a "greatest mean difference" method was used to determine the best discriminators among the 50 items. In this method the high 25 percent of the highest scored papers and the low 25 percent of the lowest scored papers are selected, where the score is determined by

\[ \mu_H = \frac{1}{n} \sum_{j=1}^{n} r_j \]

\( \mu \) is the score of the jth item. The high mean response \( \mu_H \) was
computed for each item on the high score set of papers, and the low score set of papers was used to calculate the low mean response $\mu_L$. The mean difference, $\mu_D = \mu_H - \mu_L$, was then determined for each item, the best discriminating items being those with the largest $\mu_D$ value. If $\mu_D < 0$ this is in indication that an item has been scored as a positive (negative) item when it should have been scored as a negative (positive) item.

Forms of the attitude inventory were administered to 145 students. The director of the doctoral seminar suggested, rather than giving the 50 item inventory to one large group, that a better indication of discriminatory power would be obtained by administering the 50 item inventory to two smaller groups, and determining the best discriminators from one set. Having done this, then make up an inventory from these items and administer it to a third group. Then for the final form take the best discriminating items that appeared on any two or all three sets of papers. In this manner it would be possible to obtain a degree of cross validation of $\mu_D$ scores. To this end the 50 item inventory was administered to 78 Southern Oregon College calculus students and 32 Oregon State University mathematics students. The $\mu_D$ scores were calculated from the 78 Southern Oregon College papers and those with a $\mu_D \geq .9$ were selected to comprise a 20 item inventory. This inventory was given to 35 Oregon State University biology students. The $\mu_D$ scores were then calculated from the 32
Oregon State University mathematics papers. A comparison of $\mu_D$'s on the two mathematics subgroups found that 13 of the best discriminators agreed. Using the biology group $\mu_D$ scores as cross validation scores, it was found that eight items were good discriminators on all three inventories, and eight items agreed on two inventories. Tables 1 and 2 in Appendix A illustrate the above relationships. Therefore, the final attitude inventory consisted of 16 items, 10 "negative" and 6 "positive." The odd-even split half reliability coefficient was calculated to be .75.

Procedures Used in Collecting Data

Criterion Test Scores

Educational Testing Services' Cooperative Mathematics Calculus Test, Form B was administered to both the control and experimental group at the end of the respective Spring quarters of 1968 and 1969, during the time normally designated for final examinations. In each case one week before the test was to be given the students were told that their final examination would be a timed standardized calculus test, and they should therefore review the basic concepts, skills, and topics of the calculus sequence. None of the teachers involved except the researcher had access to the test prior to the week of finals. The tests were mailed to the instructors of the control group so that they
received them on the last Friday of regularly scheduled classes. The two 40 minute parts of the test were administered to both groups during the two hours set aside for their final examination. The time was carefully monitored and a five minute break was given to the students between the two parts. The tests were corrected by the teachers of the control and experimental groups and used in the determination of final grades for the respective quarters. The corrected answer sheets and tests of the control group were filed and not referred to or recorded by the experimenter until after the test had been administered to the experimental group. Every attempt was made to keep the circumstances surrounding the administration of this test as similar as possible for the two groups. The only deviation in the administration of this test was that it was given to all the experimental group in one room, while the control group took the test in two rooms. However, in each case that was the normal procedure for examinations.

The Applied Problem Test was given to the control and experimental groups on Friday of the last week of the respective Spring terms. Each of the groups were told, on the previous Monday of that week, that they would be given a test stressing the applications of calculus because Educational Testing Services' test contained no items of this type. The researcher had proposed that no warning be given to the students regarding this test, but was urged to abandon this plan by the other teachers involved. The test was not available to the instructors
of the control or experimental groups until the day before the test was given, thus eliminating the chance of prompting the students prior to the time the test was given. The students were allowed one 50 minute period to complete the test. The experimental group took the examination in their lab sessions. The multiple choice test was scored by the instructor of each group of students and was counted as a "quiz" score when final grades were determined.

The Math-Science Attitude Inventory was given to the control and experimental groups on the Wednesday of the week before final examinations of the respective Spring quarters. The students were not given special warning, advance preparation, or advice regarding this inventory. The only directions given to the students were to mark one of the five spaces for each item and to indicate their college major in the space provided. The inventories were scored by the researcher.

Grade Point Averages

For purposes of statistical analyses, the overall high school grade point average and mathematics grade point average was determined for each student in the two groups. These grade point averages were calculated for the grades 10 through 12, and were obtained from the records of each student in the registrar's office at Southern Oregon College. As noted in Chapter II, several research studies have shown these GPA's to be the best predictor of a student's success in college.
Consequently the mathematics department at Southern Oregon College no longer requires or utilizes SCAT or ACT scores for placement of students in their program, rather placement is done on the basis of a student's overall and secondary school mathematics grade point averages. Hence, the sum of these two averages $G = G_o + G_m$ was used as the covariable in the statistical analyses of the CCT and Applied Problem Test scores.

Statistics Utilized in Analysis of Data

The control and experimental calculus classes were evaluated at the end of the Spring terms of the 1968 and 1969 academic years respectively with respect to general calculus achievement, ability to solve applied problems, and attitudes regarding the role of mathematics in science.

General achievement in calculus was measured by Educational Testing Services' Cooperative Mathematics Series Calculus Test, Form B (CCT). Ability to solve applied problems was measured by the Applied Problem Test (APT) constructed by the researcher. The sum of the overall high school and high school mathematics grade point averages, $G = G_o + G_m$, was computed for each student. This combined grade point average was used as a covariable to adjust for student differences in ability and achievement in tests of the null hypotheses concerning these measuring instruments.
Attitudes toward the role of mathematics in science were measured by the Math-Science Attitude Inventory (MSAI) constructed by the researcher. Data concerning age, sex, academic standing, etc., were obtained from a Personal Data questionnaire filled out by all students in the study at the end of the respective calculus sequences.

Group means were used as the unit of analysis, since it was the groups, not individual students which were compared. Group means were calculated for the criteria measures and for other data obtained from transcripts and questionnaires. In each case means were calculated by totaling raw scores for the group and dividing by the number in the group.

Standard deviations were computed for data obtained. This measure of variability was computed according to the procedure outlined by Roscoe (74, p. 51), and in accordance to the MCF (Mean, Correlation Coefficient) program of the OS-3 computer at Oregon State University. Distributions of grade point averages were compared with expected distributions if the scores were representative of a normal distribution. The procedure described by Roscoe (74, p. 64) was followed in making this comparison. Pearson product-moment coefficients of correlation between each of the criterion scores and grade point averages were calculated for the two groups.

In the tests of the null hypotheses dealing with CCT scores and APT scores, the sum of the overall and mathematics secondary school
grade point averages was used as the covariable. Since such grade point averages are usually correlated with success in college, one factor analysis of covariance was used in the statistical analysis of these hypotheses. According to Roscoe (74, p. 254) "the result is equivalent to matching the various experimental groups with respect to the variable or variables being controlled."

Adjusted sums of squares, degrees of freedom, and mean squares were calculated for the criterion using the method described by Roscoe and contained in the analysis of covariance programs in the OS-3 computer at Oregon State University. Adjusted means were also calculated according to the regression equation \( \bar{y}_1' = \bar{y}_1 + b(\bar{x}_1 - \bar{x}) \)

where \( b = \frac{SS_y}{SS_x} \). The F-ratios were calculated by using the ratio of the adjusted mean square between groups and the adjusted mean square within groups. Tables (31, p. 49-57) giving F-ratios for various levels of significance were consulted in order to determine acceptance or non-acceptance of the null hypotheses.

In the tests of hypotheses dealing with MSAI scores, analysis of variance was the statistical method used to determine whether there were any significant difference between various groups and subgroups. F-ratios were computed using the ratio of mean squares between groups and mean squares within groups. As above, appropriate tables were consulted to determine whether the F-ratios did or did not indicate
significant differences.

Statistical consultant for the statistical design of this study was Professor David Faulkenberry, a member of the statistics department at Oregon State University.

**Processing of the Data**

Data from the various sources were tabulated on data sheets and cards. The data was then transferred to teletype tape and processed by the researcher, to the computer center at Oregon State University. This was made possible by the University's NSF grant which enabled other institutions of higher education in Oregon to be linked by teletype to the OS-3 computer in Corvallis.

Forty-five null hypotheses concerning attitudes and the achievement of students in calculus were tested using the OS-3 conversational statistics program library. The programs used were ANCOV1, ANOVA12, and MCF, analysis of covariance, analysis of variance and mean-correlation coefficients respectively. In order to verify that the data was being processed correctly by the researcher, trial tests were made using data found in statistics textbooks. The computer output was then compared to the data found in the textbooks. Members of the mathematics and business departments at Southern Oregon College also aided the researcher in properly processing the data.
Limitations of the Study

Effects of grouping students by subject matter major on general calculus achievement, applied problem solving ability and attitude toward the role of science were the main concerns of this study. Significance in mean group differences in the first 2 categories, as measured by the criterion instruments, may have resulted from group variations in academic ability and achievement, but these variations were statistically adjusted for using covariance techniques.

Hypotheses concerning factors other than grouping, academic ability, and achievement were not tested in this study. For example, the groups may have been subjected to informal learning situations or teacher influences which were not considered in this study.

The Math-Science Attitude Inventory was constructed to measure attitudes toward the role of mathematics in science and not as a measure of a student's attitude toward mathematics itself.

The study was confined to two relatively small groups of students at a single college, therefore, the extent to which the findings can be generalized is limited.
IV. PRESENTATION AND ANALYSIS OF THE FINDINGS

This study was undertaken in order to determine some of the effects of grouping calculus students according to college major on their general achievement in calculus, ability to solve applied problems, and their attitude toward the role of mathematics in the understanding, study and pursuit of scientific endeavors. Ungrouped calculus students taught in conventionally structured classes were compared in the three above mentioned areas to calculus students who were taught approximately half the time in one large group and the remainder of the time in three smaller classes in which the students were grouped according to their subject matter major. The investigation was conducted at Southern Oregon College during the 1967-1968 and 1968-1969 academic years. The subjects were all students at Southern Oregon College during the periods of the investigation.

Data were obtained through: (1) single administrations of the instruments selected to measure calculus achievement, ability to solve applied problems, and attitudes toward the role of mathematics in science, (2) analyses of students records obtained from the office of the registrar at Southern Oregon College, and (3) from analyses of personal data supplied by the students involved in the study. Data obtained from these sources were tabulated on data sheets and then punched on teletype tape for processing.
The collected data in this study were used in three kinds of statistical analyses: (1) analyses of data in which group means, standard deviations, and correlations between criterion test scores and grade point averages were computed; (2) one factor analysis of covariance in which null hypotheses regarding general achievement and ability to solve applied problems were tested; and (3) one factor analysis of variance in which null hypotheses concerning attitudes toward the role of mathematics in science were tested.

The null hypotheses tested have been stated in Chapter I and will be reviewed in this chapter when tests of the hypotheses are examined. The criterion measures in the tests of the hypotheses were the group means on the Cooperative Calculus Test, Form B, the Applied Problem Test, and the Math-Science Attitude Inventory. The sum of overall and mathematics high school grade point averages was used as covariance control for the Cooperative Calculus Test and the Applied Problem Test group means.

Analyses of the Data

Non-Criterion Variables of the Participating Groups

This section is concerned with the following factors which the researcher hypothesized to be related to group means on the criterion instruments:
1. General academic achievement, \( G_o \), in high school as measured by accumulative grade point averages.

2. Mathematics achievement, \( G_m \), in high school as measured by accumulative mathematics grade point averages.

3. Academic achievement and ability as measured by the sum of the accumulative overall and mathematics grade point averages \( G = G_o + G_m \).

4. Number of hours of science completed by the control and experimental groups.

Accumulative High School Grade Point Averages. The primary concern of this study dealt with the effect of grouping by academic major in a college calculus class, therefore, it was considered necessary to obtain a covariable to control the inherent differences between the control and experimental group. As mentioned in Chapter II, high school grade point average has been found to significantly correlate with success in college; success being defined as the ability to achieve passing college grades. The covariable was used in the comparison of mean scores of mathematics tests. Therefore, the overall accumulative grade point average, \( G_o \), which indicates general academic achievement and ability, and the accumulative mathematics grade point average, \( G_m \), which indicates mathematical achievement and ability, were combined \((G = G_o + G_m)\) for each student and used as a covariance control. Table 9 reveals that the experimental group had higher mean
overall and mathematics high school grade point averages of approximately .3 grade point.

Table 9. Mean Overall, Mathematics and Covariable Grade Point Averages of the Control and Experimental Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>H.S. GPA</th>
<th>S.D.</th>
<th>Math GPA</th>
<th>S.D.</th>
<th>G</th>
<th>S. D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>47</td>
<td>2.89</td>
<td>.48</td>
<td>2.79</td>
<td>.72</td>
<td>5.68</td>
<td>1.15</td>
</tr>
<tr>
<td>I_B</td>
<td>8</td>
<td>3.03</td>
<td>.68</td>
<td>2.87</td>
<td>.87</td>
<td>5.90</td>
<td>1.55</td>
</tr>
<tr>
<td>I_M</td>
<td>23</td>
<td>2.92</td>
<td>.52</td>
<td>2.88</td>
<td>.84</td>
<td>5.80</td>
<td>1.32</td>
</tr>
<tr>
<td>I_S</td>
<td>16</td>
<td>2.79</td>
<td>.23</td>
<td>2.62</td>
<td>.42</td>
<td>5.41</td>
<td>0.61</td>
</tr>
<tr>
<td>Experimental</td>
<td>41</td>
<td>3.18</td>
<td>.43</td>
<td>3.10</td>
<td>.56</td>
<td>6.28</td>
<td>0.96</td>
</tr>
<tr>
<td>II_B</td>
<td>5</td>
<td>3.48</td>
<td>.40</td>
<td>3.70</td>
<td>.27</td>
<td>7.18</td>
<td>0.66</td>
</tr>
<tr>
<td>II_M</td>
<td>14</td>
<td>3.14</td>
<td>.46</td>
<td>3.13</td>
<td>.58</td>
<td>6.28</td>
<td>1.01</td>
</tr>
<tr>
<td>II_S</td>
<td>22</td>
<td>3.14</td>
<td>.42</td>
<td>2.95</td>
<td>.52</td>
<td>6.09</td>
<td>0.84</td>
</tr>
</tbody>
</table>

* All based on grades 10 through 12.

All subgroups of the experimental group also had higher grade point averages than those of the control group. The mathematics subgroups had the smallest differences of approximately .2 grade point in both GPA's. The science subgroups were next with approximately .3 grade point differences. The biology-business subgroups had the largest differences of all subgroups of approximately .45 grade point and .83 grade point respectively.

Group Means on the Covariable $G = G_o + G_m$. Comparisons of mean $G$ scores in Table 9 disclosed that in all cases differences
existed between the control and experimental groups, and between sub-
groups thereof. G scores for all experimental groups except II were
larger than those of corresponding control groups, ranging from
a maximum of 1.3 points between the biology-business subgroups to a
minimum of .48 point between the mathematics subgroups. In both
groups G was smallest for the science subgroups and largest for the
biology-business subgroups. The G standard deviations were all
larger for the control group, indicating that the experimental group
was more homogeneous with respect to G. The largest standard
deviation (1.55) was displayed by the control biology-business sub-
group, while the smallest (0.61) was displayed by the control science
subgroup. Distributions of G scores of the two groups and the com-
bined groups were compared to each other and to the expected fre-
quencies of a normally distributed population. These comparisons,
shown in Table 10, reveal that the G scores of the control group
were skewed slightly to the lower scores. The G scores of the experi-
mental group were skewed toward the higher scores. The distribu-
tion of the G scores of the combined groups was close to the expected
frequencies for a normal curve, with the exception of the void of
scores between +2σ and +3σ.
Table 10. Comparison of Expected Frequencies of a Normal Curve and Distribution of $G^*$ Scores of the Control and Experimental Groups

<table>
<thead>
<tr>
<th></th>
<th>$-3\sigma$</th>
<th>$-2\sigma$</th>
<th>$-\sigma$</th>
<th>$+\sigma$</th>
<th>$+2\sigma$</th>
<th>$+3\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Frequency</td>
<td>2%</td>
<td>14%</td>
<td>34%</td>
<td>34%</td>
<td>14%</td>
<td>2%</td>
</tr>
<tr>
<td>Control Group</td>
<td>2.1%</td>
<td>9.1%</td>
<td>27.7%</td>
<td>27.7%</td>
<td>23.4%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Experimental Group</td>
<td>2.4%</td>
<td>14.6%</td>
<td>26.9%</td>
<td>39.9%</td>
<td>17.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Combined Groups</td>
<td>2.3%</td>
<td>11.3%</td>
<td>34.1%</td>
<td>37.5%</td>
<td>14.8%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

$G = G_o + G_{m}$; Sum of overall and mathematics high school grade point averages.

**College Science Preparation.** Knowledge that college science requirements for the various college majors differ and the fact that this variable could affect scores on the Applied Problem Test suggested to the investigator that the control and experimental groups and subgroups should be compared relative to their college science backgrounds. The data summarized in Table 11 shows that differences did exist in the amount and type of science studied by the various groups that participated in this study. The fact that the mean number of quarter hours of all sciences, when the control group and subgroups were compared to the experimental group and subgroups, was in the ratio of approximately 1:9:1:1:1 was unexpected since the ratio of non-freshmen in the two groups was 2:1. Thus in mean number of quarter hours of all sciences the control and experimental groups displayed a
marked similarity. In both groups the biology-business subgroups had the largest mean number of quarter hours of biological science, and the smallest mean number of quarter hours of physics. In both groups the mathematics subgroups had the smallest mean number of credit hours in science. The science subgroups in each case had the largest concentration in physics and the smallest concentration in biology.

Table 11. Mean Number of Quarter Hours of Science Completed by the Control and Experimental Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Hours in Biological Science</th>
<th>Hours in Chemistry</th>
<th>Hours in Physics</th>
<th>Hours in all Sciences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>47</td>
<td>4.09</td>
<td>3.19</td>
<td>3.64</td>
<td>10.92</td>
</tr>
<tr>
<td>I_B</td>
<td>8</td>
<td>8.00</td>
<td>2.25</td>
<td>1.12</td>
<td>11.37</td>
</tr>
<tr>
<td>I_M</td>
<td>23</td>
<td>3.52</td>
<td>3.13</td>
<td>2.22</td>
<td>9.16</td>
</tr>
<tr>
<td>I_S</td>
<td>16</td>
<td>2.44</td>
<td>3.75</td>
<td>6.94</td>
<td>13.13</td>
</tr>
<tr>
<td>Experimental</td>
<td>41</td>
<td>2.37</td>
<td>5.07</td>
<td>3.88</td>
<td>11.32</td>
</tr>
<tr>
<td>II_B</td>
<td>5</td>
<td>6.00</td>
<td>7.20</td>
<td>0.00</td>
<td>13.20</td>
</tr>
<tr>
<td>II_M</td>
<td>14</td>
<td>1.93</td>
<td>5.79</td>
<td>1.29</td>
<td>9.01</td>
</tr>
<tr>
<td>II_S</td>
<td>22</td>
<td>1.82</td>
<td>4.14</td>
<td>6.41</td>
<td>12.37</td>
</tr>
</tbody>
</table>

Marked differences are evident when subgroups are compared between the two groups. The biology-business subgroups differed greatly in mean number of credit hours of chemistry and physics accumulated, with a combined ratio of approximately 4:9 favoring the experimental group. The mathematics subgroup differed in science
concentration in all three areas with the control group having approximately a 2:1 advantage in biology and physics preparation while that ratio was 1:2 in hours of chemistry. The science subgroups displayed the greatest homogeneity in science preparation with a 1:1 combined ratio in physics and chemistry, and 4:3 ratio favoring the control group in biology.

**Calculus Achievement of the Two Groups**

Test means and standard deviations on the *Cooperative Calculus Test*, published by Educational Testing Service, for each participating group are shown in Table 12. It is evident from this data that the groups differed in calculus achievement as measured by this test. In all cases the experimental groups had mean test scores higher than the corresponding control group, with a difference of 3.9 points for the two groups. The largest difference (13.25 points) was found between the biology-business subgroups. The smallest difference, (1.95 points) was found between the science subgroups, while a difference of 2.89 points was found between the mathematics subgroups. The $G$ score differences for these subgroups were similarly ranked.

Within the control group the biology-business subgroup had a smaller mean *Cooperative Calculus Test* score by about six points when compared to the mathematics and science subgroups. However, the biology-business $G$ score was higher than either of the two latter
subgroups. The mathematics and science subgroups had almost identical mean scores, while their $G$ scores differed by $.39$ point.

Table 12. Comparison of Cooperative Calculus Test Scores for Control and Experimental Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>CCT Mean</th>
<th>S. D.</th>
<th>Variation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>47</td>
<td>27.68</td>
<td>6.17</td>
<td>0.29</td>
</tr>
<tr>
<td>$I_B$</td>
<td>8</td>
<td>22.75</td>
<td>6.54</td>
<td>-4.64</td>
</tr>
<tr>
<td>$I_M$</td>
<td>23</td>
<td>28.82</td>
<td>5.55</td>
<td>1.43</td>
</tr>
<tr>
<td>$I_S$</td>
<td>16</td>
<td>28.50</td>
<td>6.19</td>
<td>1.11</td>
</tr>
<tr>
<td>Experimental</td>
<td>41</td>
<td>31.59</td>
<td>7.59</td>
<td>4.20</td>
</tr>
<tr>
<td>$II_B$</td>
<td>5</td>
<td>36.00</td>
<td>8.72</td>
<td>8.61</td>
</tr>
<tr>
<td>$II_M$</td>
<td>14</td>
<td>31.71</td>
<td>7.77</td>
<td>4.32</td>
</tr>
<tr>
<td>$II_S$</td>
<td>22</td>
<td>30.45</td>
<td>7.39</td>
<td>3.06</td>
</tr>
<tr>
<td>Combined Group</td>
<td>88</td>
<td>29.49</td>
<td>7.14</td>
<td>2.10</td>
</tr>
</tbody>
</table>

*Variation = Variation from CCT national norms, $\mu = 27.39$, $\sigma = 8.37$.

Within the experimental subgroup the biology subgroup had the higher mean CCT score by about five points when compared to the mathematics and science subgroups. Likewise it had a higher $G$ score of about one point. As in the control group the mathematics and science subgroups had similar mean scores, differing by approximately 1.2 points. This similarity was also evident in $G$ scores as
they differed by only .19 point.

A comparison of the standard deviations of the CCT scores revealed that in all cases the experimental subgroups varied more than did the control subgroups. The lowest variation was evident in the control mathematics subgroup, while the highest variation was evident in the experimental biology-business subgroup.

Coefficients of correlation between Cooperative Calculus Test scores and each of several variables, and the slope of regression lines for CCT and G are shown in Table 13. Regression equations are found in Appendix F. A table of r values at the 5 and 10 percent levels of significance was consulted (74, p. 301) in order to determine whether the correlations were statistically significant. If the correlations were significant at either level, it was concluded that the correlation was too large to be due to chance alone.

Correlations were found to be significant between CCT scores and G, G and APT scores in both the control and experimental groups. Correlations between G scores and CCT scores were significant in four of the six subgroups in the study. In both groups the mathematics subgroup was found to have the least number of significant correlations, while the biology-business subgroups had the greatest number.
Table 13. Correlations Between Individual Scores on the Cooperative Calculus Test and Several Variables for the Control and Experimental Group

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Overall H. S. GPA</th>
<th>Math H. S. GPA</th>
<th>G</th>
<th>APT</th>
<th>b***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>47</td>
<td>.24**</td>
<td>.37**</td>
<td>.33**</td>
<td>.41**</td>
<td>1.76</td>
</tr>
<tr>
<td>I</td>
<td>8</td>
<td>.72**</td>
<td>.71**</td>
<td>.72**</td>
<td>.73**</td>
<td>3.02</td>
</tr>
<tr>
<td>II</td>
<td>23</td>
<td>.22</td>
<td>.34*</td>
<td>.30</td>
<td>.10</td>
<td>1.27</td>
</tr>
<tr>
<td>II S</td>
<td>16</td>
<td>.10</td>
<td>.45**</td>
<td>.35*</td>
<td>.66**</td>
<td>3.57</td>
</tr>
<tr>
<td>Experimental</td>
<td>41</td>
<td>.32**</td>
<td>.37**</td>
<td>.38**</td>
<td>.25*</td>
<td>2.99</td>
</tr>
<tr>
<td>II B</td>
<td>5</td>
<td>.87**</td>
<td>.94**</td>
<td>.91**</td>
<td>.44</td>
<td>11.96</td>
</tr>
<tr>
<td>III M</td>
<td>14</td>
<td>.18</td>
<td>.15</td>
<td>.16</td>
<td>.44*</td>
<td>1.19</td>
</tr>
<tr>
<td>III S</td>
<td>22</td>
<td>.24</td>
<td>.35*</td>
<td>.35*</td>
<td>.09</td>
<td>2.99</td>
</tr>
<tr>
<td>Combined Groups</td>
<td>88</td>
<td>.34**</td>
<td>.40**</td>
<td>.39**</td>
<td>.36**</td>
<td></td>
</tr>
</tbody>
</table>

* Indicates significance at 10 percent level.
** Indicates significance at 5 percent level.
*** b = Slope of regression equation.

Summary. In all cases the experimental groups had higher mean scores on the CCT as well as higher standard deviations. Group differences between subgroups as great as 13.4 points and as small as 2 points were observed on CCT mean scores. Biology-business group mean differences were the highest, followed by those of the mathematics subgroups and science subgroups in that order. The control I_B group mean was the lowest of any subgroup, while the
experimental $II_B$ group mean was the highest.

Within both groups the mathematics and science subgroups had nearly the same CCT mean, while the biology-business subgroups had the most varied mean.

Correlations between CCT scores and overall high school grade point average, mathematics grade point average, $G$, and APT scores were significant for both the control and experimental groups. $G$ scores were significantly correlated to CCT scores in four out of the six subgroups of the groups. No negative correlations were observed.

**Ability to Solve Applied Problems**

Test means and standard deviations on the Applied Problem Test for each participating group are shown in Table 14. As in the case of the CCT comparisons, the experimental groups had higher mean test scores than those of the corresponding control groups, with a difference of .66 point between the control and experimental group. The largest difference in mean score (1.12) as with the CCT mean scores and $G$ scores, was found between the biology-business subgroups. The smallest difference (.06 point) was found between subgroups $I_S$ and $II_S$, while a difference of 1.00 point was found between subgroups $I_M$ and $II_M$, the same pattern as displayed in the CCT mean scores.
Table 14. Comparison of Applied Problem Test Scores for the Control and Experimental Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>APT Mean</th>
<th>APT (9 Items) S. D.</th>
<th>Variation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>47</td>
<td>4.12</td>
<td>1.61</td>
<td>0.29</td>
</tr>
<tr>
<td>I_B</td>
<td>8</td>
<td>3.88</td>
<td>1.13</td>
<td>0.05</td>
</tr>
<tr>
<td>I_M</td>
<td>23</td>
<td>3.87</td>
<td>1.42</td>
<td>0.04</td>
</tr>
<tr>
<td>I_S</td>
<td>16</td>
<td>4.62</td>
<td>2.03</td>
<td>0.79</td>
</tr>
<tr>
<td>Experimental</td>
<td>41</td>
<td>4.78</td>
<td>1.62</td>
<td>0.95</td>
</tr>
<tr>
<td>II_B</td>
<td>5</td>
<td>5.00</td>
<td>1.58</td>
<td>1.17</td>
</tr>
<tr>
<td>II_M</td>
<td>14</td>
<td>4.86</td>
<td>1.61</td>
<td>1.03</td>
</tr>
<tr>
<td>II_S</td>
<td>22</td>
<td>4.68</td>
<td>1.73</td>
<td>0.85</td>
</tr>
<tr>
<td>Combined Groups</td>
<td>88</td>
<td>4.43</td>
<td>1.64</td>
<td>0.60</td>
</tr>
</tbody>
</table>

*Variation = variation from APT construction mean, 3.83.

Within the control group subgroup I_S had the largest mean score, while the I_B and I_M mean scores were identical. Subgroup I_S had the largest standard deviation (2.03) and subgroup I_B had the least variation (1.13) in scores.

Within the experimental group the largest mean score difference, .32 point was found between subgroups II_S and II_B, while the difference between I_M and these two groups was .18 and .14 point respectively. The II_S group had the largest standard deviation (1.73), and the II_B had the least variation (1.58) in scores.
The standard deviation of scores in both groups was 1.6. In both groups the standard deviations were ranked in the same order, the biology-business standard deviation being the smallest, and the science groups having the largest.

Coefficients of correlation between Applied Problem Test scores and each of several variables and the slope of the regression equation for APT and G scores are shown in Table 15. Regression equations are found in Appendix F. A table of r values was consulted (74, p. 301) to determine which correlations were significant at the 5 or 10 percent level. If the correlations were significant it was concluded that the correlations were too large to be due to chance alone.

Correlations were significant between APT scores and G_o, G_m, G_s, and CCT scores for the experimental group, and the mathematics subgroup. No other correlations in the experimental subgroups were significant.

The biology-business and science subgroups of the control group had significant correlations for two and four of the four variables, respectively. The mathematics subgroup had the lowest correlation coefficients of any subgroup in the two groups.

A comparison of the control and experimental subgroups revealed that where one subgroup had high correlations, the corresponding subgroups had low correlations. Thus, when the two groups were considered as a whole, all correlations were significant.
Table 15. Correlations Between Individual Scores on the Applied Problem Test and Several Variables for the Control and Experimental Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Overall H. S. GPA</th>
<th>Math H. S. GPA</th>
<th>G</th>
<th>CCT</th>
<th>b***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>47</td>
<td>.18</td>
<td>.16</td>
<td>.17</td>
<td>.41*</td>
<td>.23</td>
</tr>
<tr>
<td>IB</td>
<td>8</td>
<td>.73**</td>
<td>.20</td>
<td>.72**</td>
<td>.73**</td>
<td>.52</td>
</tr>
<tr>
<td>IM</td>
<td>23</td>
<td>-.01</td>
<td>.01</td>
<td>.00</td>
<td>.10</td>
<td>-.0003</td>
</tr>
<tr>
<td>IS</td>
<td>16</td>
<td>.53**</td>
<td>.43*</td>
<td>.50**</td>
<td>.65**</td>
<td>1.69</td>
</tr>
<tr>
<td>Experimental</td>
<td>41</td>
<td>.26*</td>
<td>.33**</td>
<td>.33**</td>
<td>.25*</td>
<td>.56</td>
</tr>
<tr>
<td>II_B</td>
<td>5</td>
<td>.08</td>
<td>.29</td>
<td>.17</td>
<td>.44</td>
<td>.39</td>
</tr>
<tr>
<td>II_M</td>
<td>14</td>
<td>.62**</td>
<td>.47**</td>
<td>.55**</td>
<td>.44*</td>
<td>.88</td>
</tr>
<tr>
<td>II_S</td>
<td>22</td>
<td>.06</td>
<td>.26</td>
<td>.19</td>
<td>.09</td>
<td>.39</td>
</tr>
<tr>
<td>Combined Groups</td>
<td>88</td>
<td>.26**</td>
<td>.26**</td>
<td>.27**</td>
<td>.36**</td>
<td></td>
</tr>
</tbody>
</table>

* Indicates significance at the 10 percent level.
** Indicates significance at the 5 percent level.
*** b = slope of regression equation.

Summary. In all cases the experimental groups had higher mean scores on the APT than the corresponding control groups.

Standard deviations were higher in the experimental subgroups with the exception of the science subgroup. Group differences were as high as 1.12 points and as low as .06 point. Biology-business subgroup mean differences were the highest, followed by those of the mathematics subgroups and science subgroups in that order. The
control $I_B$ and $I_M$ group means were the lowest of any subgroups, while the experimental $II_B$ group mean was the highest.

Within the control group, $I_B$ and $I_M$ group means were nearly identical, while within the experimental group $II_B$ and $II_M$ group means were within .14 point of each other.

Correlations between APT scores and the four variables, $G_o$, $G_m$, $G_s$, and CCT scores were all significant for the experimental group, but only for the CCT scores in the control group. A negative correlation was observed between APT scores and $G_o$ in the mathematics subgroup of the control group.

**Attitude Toward the Role of Mathematics in Science**

Attitude inventory means and standard deviations on the Math-Science Attitude Inventory, constructed by the researcher, are shown for each participating group in Table 16. It is evident that the control and experimental groups did not differ in their attitude toward the role of mathematics in science, as their mean scores differed only by .86 point. The rankings of the subgroups within the two groups were the same. The mathematics subgroups had the highest mean scores followed by the science and biology-business subgroups in that order. Subgroups $I_S$ and $II_S$ differed by about two points. Subgroups $I_B$ and $II_B$ differed by .25 point, and subgroups $I_M$ and $II_M$ by .40 point.
Table 16. Comparisons of Math-Science Attitude Inventory Scores for the Control and Experimental Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>MSAI Mean</th>
<th>MSAI (16 Items) S. D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>47</td>
<td>62.51</td>
<td>6.42</td>
</tr>
<tr>
<td>I_B</td>
<td>8</td>
<td>57.75</td>
<td>5.68</td>
</tr>
<tr>
<td>I_M</td>
<td>23</td>
<td>64.17</td>
<td>5.81</td>
</tr>
<tr>
<td>I_S</td>
<td>16</td>
<td>62.50</td>
<td>6.95</td>
</tr>
<tr>
<td>Experimental</td>
<td>41</td>
<td>61.65</td>
<td>6.88</td>
</tr>
<tr>
<td>II_B</td>
<td>5</td>
<td>64.57</td>
<td>8.39</td>
</tr>
<tr>
<td>II_M</td>
<td>14</td>
<td>64.57</td>
<td>6.20</td>
</tr>
<tr>
<td>II_S</td>
<td>22</td>
<td>60.64</td>
<td>6.74</td>
</tr>
<tr>
<td>Combined Groups</td>
<td>88</td>
<td>62.11</td>
<td>6.65</td>
</tr>
</tbody>
</table>

Within the control group the mean of the biology-business subgroup differed from the mathematics subgroup by 6.42 points. The difference between the mathematics and science subgroup was 1.67 points. The biology-business and science subgroups differed by 4.75 points.

Within the experimental group the mean MSAI scores of the biology-business subgroup differed by 6.57 points, from the mathematics subgroup. The difference between the mathematics and science subgroups was 3.93 points. The biology-business and science subgroups differed by 2.64 points. Thus the differences between the mathematics subgroups and non-mathematics subgroups was larger.
in the experimental group than in the control group.

The standard deviation of the MSAI scores for the two groups differed by less than one-half point. With the exception of the science subgroups, the standard deviations were larger for the experimental subgroups. Subgroup II_B had the largest variation (8.39) in scores of all subgroups, and the I_B subgroup had the smallest variation in scores (5.68).

Summary. Although there was very little difference in MSAI scores between the control and experimental groups and subgroups thereof, there were differences evident within the two groups. In both cases the biology-business subgroups had the lowest mean score, followed by the science and then the mathematics subgroups.

The control groups had the more uniform attitude toward the role of mathematics as indicated by the generally lower standard deviations found in their MSAI scores. The largest divergence in attitude appeared in the II_B scores, while the smallest occurred in the I_B subgroup scores.

Tests of the Hypotheses

In the tests of the hypotheses concerning Cooperative Calculus Test or Applied Problem Test group means, one factor analysis of covariance, as outlined by Roscoe (74, p. 262) was the statistical model employed. The group means of the sum of the overall and
mathematics high school grade point averages were used as the covariable when comparing CCT or APT group means.

The $F$ ratios were computed using the adjusted between mean square as the numerator and the adjusted within mean square as the denominator. $F$ ratios were evaluated by consulting a table of significant values of $F$ (31, p. 51-53).

In the tests of the hypotheses concerning the Math-Science Attitude Inventory group means, one factor analysis of variance, as outlined by Roscoe (74, p. 230) was the statistical model employed. $F$ ratios were computed using the between mean square as the numerator and the within mean square as the denominator. $F$ ratios were evaluated as described in the previous paragraph.

When the $F$ ratio was large enough to indicate statistical significance, the researcher concluded that the difference in group means was so large that they were not caused by chance. It was presumed that the difference could be attributed to the different treatment accorded the control and experimental groups.

Hypotheses Concerning Differences Between the Control and Experimental Group

Null hypotheses one through three asserted that there was no differences between the control and experimental groups in calculus achievement, ability to solve applied problems, and attitude toward
the role of mathematics in science, as measured by the criterion instruments. Table 17 is the analysis of variance table for the Cooperative Calculus Test, Applied Problem Test and the Math-Science Attitude Inventory scores.

The table shows that hypotheses three, which stated that there is no difference in attitude toward the role of mathematics in science between the control and experimental groups, as measured by the MSAI, was not rejected. Hypotheses one and two asserting that there is no difference in calculus achievement and ability to solve applied problems, were rejected at the 10 and 20 percent level of significance respectively.

Summary. The control and experimental groups did not differ significantly in attitude toward the role of mathematics in science, as measured by the criterion instrument. The experimental group's achievement in calculus, as measured by the CCT, was shown to be significantly greater than that of the control group (10 percent level). The ability of the experimental group to solve applied problems, as measured by the APT, was greater than that of the control group (20 percent level of significance).

Hypotheses Concerning Differences Between Subgroups of the Control and Experimental Groups

Null hypotheses 4 through 21 stated that there are no differences between subgroups of the two groups in calculus achievement, ability
Table 17. Relationship of the Control and Experimental Group in Calculus Achievement, Ability to Solve Applied Problems and Attitude

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Group</th>
<th>Mean Score</th>
<th>Adjusted SS</th>
<th>F Ratio</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Control</td>
<td>Experimental</td>
<td>Total</td>
<td>Within</td>
</tr>
<tr>
<td>G</td>
<td>5.68</td>
<td>6.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1* CCT</td>
<td>27.68</td>
<td>31.59</td>
<td>3802.52</td>
<td>3671.80</td>
<td>1.85</td>
</tr>
<tr>
<td>2 APT</td>
<td>4.13</td>
<td>4.78</td>
<td>225.63</td>
<td>220.85</td>
<td>1.85</td>
</tr>
<tr>
<td>3 MSAI</td>
<td>62.51</td>
<td>61.65</td>
<td>3896.86</td>
<td>3880.96</td>
<td>1.86</td>
</tr>
</tbody>
</table>

*Number of the hypothesis being tested
to solve applied problems and attitude toward the role of mathematics in science, as measured by the criterion instruments. Table 18 is the analysis of variance table for the Cooperative Calculus Test, Applied Problem Test and the Math-Science Attitude Inventory scores. The table shows that null hypotheses five and six, which stated that there is no difference between the biology-business subgroups in ability to solve applied problems and attitude toward the role of mathematics in science were not rejected. Hypothesis four which stated that there was no difference between $I_B$ and $II_B$ in calculus achievement was rejected at the five percent level of significance.

Hypotheses seven and nine which stated that there are no differences between the mathematics subgroups in calculus achievement, and attitude toward the role of mathematics in science were not rejected. Hypothesis eight which asserted no difference existed between subgroups $I_M$ and $II_M$ in ability to solve applied problems, was rejected at the 10 percent level of significance.

Hypotheses 10, 11, and 12 which asserted no differences existed between the science subgroups, as measured by the criterion instruments, were not rejected. Similar hypotheses, 13, 14, and 15, which asserted no differences between the non-mathematics subgroups were also not rejected.

Hypothesis 17 which asserted that there is no difference between the non-mathematics control subgroups and the experimental
Table 18. Relationships Between Subgroups of the Control and Experimental Groups in Calculus Achievement, Ability to Solve Applied Problems, and Attitude

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Group Mean Score</th>
<th>Adjusted SS</th>
<th>F Ratio</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I_B</td>
<td>II_B</td>
<td>Total</td>
<td>Within</td>
</tr>
<tr>
<td>G</td>
<td>5.90</td>
<td>7.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4*S CCT</td>
<td>22.75</td>
<td>36.00</td>
<td>493.57</td>
<td>326.20</td>
</tr>
<tr>
<td>5 APT</td>
<td>3.87</td>
<td>5.00</td>
<td>14.62</td>
<td>14.07</td>
</tr>
<tr>
<td>6 MSAI</td>
<td>57.75</td>
<td>58.00</td>
<td>507.69</td>
<td>507.50</td>
</tr>
<tr>
<td></td>
<td>I_M</td>
<td>II_M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>5.80</td>
<td>6.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 CCT</td>
<td>28.83</td>
<td>31.71</td>
<td>1424.58</td>
<td>1380.50</td>
</tr>
<tr>
<td>8 APT</td>
<td>3.86</td>
<td>4.86</td>
<td>82.23</td>
<td>76.72</td>
</tr>
<tr>
<td>9 MSAI</td>
<td>64.17</td>
<td>64.57</td>
<td>1244.11</td>
<td>1242.73</td>
</tr>
<tr>
<td></td>
<td>I_S</td>
<td>II_S</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>5.41</td>
<td>6.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 CCT</td>
<td>28.50</td>
<td>30.45</td>
<td>1523.36</td>
<td>1523.04</td>
</tr>
<tr>
<td>11 APT</td>
<td>4.63</td>
<td>4.68</td>
<td>15.41</td>
<td>15.05</td>
</tr>
<tr>
<td>12 MSAI</td>
<td>62.50</td>
<td>60.64</td>
<td>1709.26</td>
<td>1677.09</td>
</tr>
<tr>
<td></td>
<td>I_B+S</td>
<td>II_B+S</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>5.57</td>
<td>6.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 CCT</td>
<td>26.58</td>
<td>31.48</td>
<td>2235.69</td>
<td>2162.42</td>
</tr>
<tr>
<td>14 APT</td>
<td>4.37</td>
<td>4.74</td>
<td>134.12</td>
<td>134.11</td>
</tr>
<tr>
<td>15 MSAI</td>
<td>60.91</td>
<td>60.14</td>
<td>2340.74</td>
<td>2333.24</td>
</tr>
<tr>
<td>Criterion</td>
<td>Group Mean Score</td>
<td>Adjusted SS</td>
<td>F Ratio</td>
<td>Level of Significance</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------</td>
<td>-------------</td>
<td>---------</td>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
<td>I F+B+S</td>
<td>II M</td>
<td>Total</td>
<td>Within</td>
</tr>
<tr>
<td>G</td>
<td>5.57</td>
<td>6.28</td>
<td>1811.50</td>
<td>1699.17</td>
</tr>
<tr>
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<td>26.58</td>
<td>31.71</td>
<td>1666.66</td>
<td>1.17</td>
</tr>
<tr>
<td>17 APT</td>
<td>4.38</td>
<td>4.88</td>
<td>86.56</td>
<td>66.54</td>
</tr>
<tr>
<td>18 MSAI</td>
<td>60.91</td>
<td>64.57</td>
<td>1687.36</td>
<td>1569.26</td>
</tr>
<tr>
<td></td>
<td>I M</td>
<td>II B+S</td>
<td>Total</td>
<td>Within</td>
</tr>
<tr>
<td>G</td>
<td>5.80</td>
<td>6.29</td>
<td>1960.19</td>
<td>1933.05</td>
</tr>
<tr>
<td>19 CCT</td>
<td>28.83</td>
<td>31.48</td>
<td>124.46</td>
<td>116.76</td>
</tr>
<tr>
<td>20 APT</td>
<td>3.87</td>
<td>4.74</td>
<td>60.15</td>
<td>58.71</td>
</tr>
<tr>
<td>21 MSAI</td>
<td>64.17</td>
<td>60.15</td>
<td>2208.71</td>
<td>2006.71</td>
</tr>
</tbody>
</table>

* Number of the hypothesis being tested
mathematics subgroup in ability to solve applied problems was not rejected. Hypotheses 16 and 18 which asserted no differences existed between the control non-mathematics subgroups and the experimental mathematics subgroup in calculus achievement and attitude toward the role of mathematics in science were rejected at the 20 percent level of significance.

Hypothesis 19 which asserted that there is no difference between the control mathematics subgroup and the non-mathematics experimental subgroups in calculus achievement was not rejected. Hypotheses 20 and 21 which stated that there are no differences between subgroups $I_M$ and $II_{B+S}$ in ability to solve applied problems and attitude toward the role of mathematics in science were rejected at the 10 and 5 percent level of significance respectively.

Summary. Six of the 18 hypotheses concerning differences between subgroups of the two groups were rejected. As measured by the CCT, the experimental biology-business subgroup had a significantly higher level of calculus achievement than the control biology-business subgroup (five percent level). At the 20 percent level of significance the experimental mathematics subgroup had a higher level of calculus achievement when compared to the control non-mathematics subgroups.

The ability to solve applied problems, as measured by the APT, displayed by the experimental mathematics subgroup was significantly
greater (10 percent level) than that of the control mathematics subgroup. The ability of the experimental non-mathematics subgroup, as measured by the criterion instrument, was significantly greater than that of the control mathematics subgroup (10 percent level).

The attitude toward the role of mathematics in science, as measured by the MSAI, was higher for the experimental mathematics subgroup than that of the control non-mathematics subgroup (20 percent level). At the five percent level this same attitude was significantly higher in the control non-science subgroup than it was in the experimental science subgroup.

The biology-business subgroups did not differ significantly in ability to solve applied problems nor in attitude toward the role of mathematics in science. In all tests of hypotheses regarding these subgroups it should be noted that the sample sizes were relatively small.

The mathematics subgroups did not differ significantly in calculus achievement nor in attitude toward the role of mathematics in science.

The science subgroups and the non-mathematics subgroups did not differ significantly in any of the areas, as measured by the three criterion instruments, while as noted above the mathematics subgroups of the two groups differed significantly in four out of six cases when compared to the non-mathematics subgroups.
Hypotheses Concerning Differences Within the Control Group

In order to explore some of the possible effects of conventional calculus classes on the control group, null hypotheses 22 through 33 were tested to determine significant differences between subgroups of the control group. Table 19 is the analysis of variance table for the CCT, APT, and MSAI scores of the subgroups of the control group.

Hypothesis 23 which stated that there is no difference in ability to solve applied problems between the control business-biology and mathematics subgroup was not rejected. Hypotheses 22 and 24 which asserted that there is no significant difference between subgroups \( I_B \) and \( I_M \) in calculus achievement and attitude toward the role of science in mathematics were rejected at the one percent level.

Hypotheses 25, 26, and 27 which asserted that there are no differences between the control biology-business and science subgroup in calculus achievement, ability to solve applied problems, and in attitude toward the role of mathematics in science were rejected at the one, and the latter two at the 20 percent level.

Hypotheses 28 and 30 which asserted that there are no differences between the control mathematics and science subgroups in calculus achievement and attitude toward the role of mathematics in science were not rejected. Hypothesis 29 which asserted that there is no difference between these subgroups in ability to solve applied
Table 19. Relationships Between Subgroups of the Control Group in Calculus Achievement, Ability to Solve Applied Problems and Attitude

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Group Mean Score</th>
<th>Adjusted SS</th>
<th>df</th>
<th>F Ratio</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I&lt;sub&gt;B&lt;/sub&gt;</td>
<td>I&lt;sub&gt;M&lt;/sub&gt;</td>
<td>Total</td>
<td>Within</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>5.90</td>
<td>5.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22* CCT</td>
<td>22.75</td>
<td>28.83</td>
<td>1028.51</td>
<td>796.38</td>
<td>1,28</td>
</tr>
<tr>
<td>23 APT</td>
<td>3.88</td>
<td>3.87</td>
<td>52.11</td>
<td>52.10</td>
<td>1,28</td>
</tr>
<tr>
<td>24 MSAI</td>
<td>57.75</td>
<td>64.17</td>
<td>1213.74</td>
<td>968.80</td>
<td>1,29</td>
</tr>
<tr>
<td>I&lt;sub&gt;B&lt;/sub&gt;</td>
<td>I&lt;sub&gt;S&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>5.90</td>
<td>5.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 CCT</td>
<td>22.75</td>
<td>28.50</td>
<td>922.80</td>
<td>653.32</td>
<td>1,21</td>
</tr>
<tr>
<td>26 APT</td>
<td>3.88</td>
<td>4.63</td>
<td>62.73</td>
<td>56.06</td>
<td>1,21</td>
</tr>
<tr>
<td>27 MSAI</td>
<td>57.75</td>
<td>62.50</td>
<td>1069.83</td>
<td>949.50</td>
<td>1,22</td>
</tr>
<tr>
<td>I&lt;sub&gt;M&lt;/sub&gt;</td>
<td>I&lt;sub&gt;S&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>5.80</td>
<td>5.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28 CCT</td>
<td>28.83</td>
<td>28.50</td>
<td>1146.59</td>
<td>1145.82</td>
<td>1,36</td>
</tr>
<tr>
<td>29 APT</td>
<td>3.87</td>
<td>4.63</td>
<td>110.83</td>
<td>104.41</td>
<td>1,36</td>
</tr>
<tr>
<td>30 MSAI</td>
<td>64.17</td>
<td>62.50</td>
<td>1493.74</td>
<td>1467.30</td>
<td>1,37</td>
</tr>
<tr>
<td>I&lt;sub&gt;B+S&lt;/sub&gt;</td>
<td>I&lt;sub&gt;M&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31 CCT</td>
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<td>1593.97</td>
<td>1553.88</td>
<td>1,44</td>
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<tr>
<td>32 APT</td>
<td>4.38</td>
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<td>33 MSAI</td>
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<td>64.17</td>
<td>1937.74</td>
<td>1813.14</td>
<td>1,45</td>
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</table>

* Number of hypothesis being tested
problems was rejected at the 20 percent level.

Hypotheses 31 and 32 which asserted that there are no differences in calculus achievement and ability to solve applied problems between the control non-mathematics and mathematics subgroups were not rejected. Hypothesis 33 which asserted that there was no difference in attitude toward the role of mathematics in science between these subgroups was rejected at the 10 percent level of significance.

Summary. The control mathematics and business-biology subgroups did not differ significantly in ability to solve applied problems as measured by the criterion instrument. The calculus achievement and attitude toward the role of mathematics in science of the control mathematics subgroup were significantly higher (one percent level) than those of the biology-business subgroup.

The control science subgroup had significantly higher mean scores on all criteria, as measured by the criterion instruments, than those of the biology-business subgroup (1 and 20 percent level).

The control mathematics and science subgroup did not differ in calculus achievement and attitude toward the role of mathematics in science. The control science subgroup had a greater ability to solve applied problems than the mathematics subgroup (20 percent level).

The control non-mathematics subgroup and mathematics subgroup did not differ significantly in ability to solve applied
problems nor in calculus achievement. The attitude toward the role of mathematics of the control mathematics subgroup was significantly higher than that of the non-mathematics subgroup (10 percent level).

**Hypotheses Concerning Differences Within the Experimental Group**

In order to further explore some of the possible effects of the experimental class and curriculum on the experimental group, null hypotheses 34 through 45 were tested to determine significant differences between the subgroups of the experimental group. Table 20 is the analysis of variance table for the CCT, APT, and MSAI scores of the subgroups of the experimental group.

Hypotheses 34 and 35 which asserted that there are no differences between the experimental biology-business subgroups and the mathematics subgroup, as measured by the criterion instruments, in calculus achievement, and ability to solve applied problems were not rejected. Hypothesis 36 which asserted that there is no significant differences between these subgroups in attitude toward the role of mathematics in science was rejected at the 10 percent level.

Hypotheses 37, 38, and 39 which asserted that there are no differences between the experimental biology-business subgroup and science subgroup, as measured by the criterion instruments, in calculus achievement, ability to solve applied problems and attitude toward the role of mathematics in science were not rejected.
Table 20. Relationships Between Subgroups of the Experimental Group in Calculus Achievement, Ability to Solve Applied Problems, and Attitude

<table>
<thead>
<tr>
<th>Criterion</th>
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<th>F Ratio</th>
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<tr>
<td></td>
<td>II B</td>
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<td>998.21</td>
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<td>APT</td>
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</tr>
<tr>
<td>MSAI</td>
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<tbody>
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<td>II B</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>30.45</td>
<td>1200.73</td>
<td>1195.66</td>
</tr>
<tr>
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<td>70.26</td>
</tr>
<tr>
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<td>60.64</td>
<td>1263.41</td>
<td>1235.09</td>
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<th>F Ratio</th>
<th>Level of Significance</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>II M</td>
<td>II S</td>
<td>Total</td>
<td>Within</td>
</tr>
<tr>
<td>G</td>
<td>6.28</td>
<td>6.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCT</td>
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<td>30.45</td>
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<td>APT</td>
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<td>1585.00</td>
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<td>4.85</td>
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<td>64.57</td>
<td>1943.22</td>
<td>1762.84</td>
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* Number of hypothesis being tested
Hypotheses 40 and 41 which asserted that there are no differences between the experimental mathematics subgroup and science subgroup, as measured by the criterion instruments, in calculus achievement and ability to solve applied problems were not rejected. Hypothesis 42 which asserted that there is no difference in attitude toward the role of mathematics in science between these subgroups was rejected at the 10 percent level.

Hypotheses 43 and 44 which asserted that there are no significant differences between the experimental non-mathematics and mathematics subgroup, as measured by the criterion instruments, in calculus achievement and ability to solve applied problems, were not rejected. Hypothesis 45 which asserted that there is no difference between these subgroups in attitude toward the role of mathematics in science was rejected at the 10 percent level of significance.

Summary. In all cases there was no significant difference between the experimental subgroups, as measured by the CCT and APT, in calculus achievement and ability to solve applied problems. The experimental mathematics subgroup had a significantly higher attitude toward the role of mathematics in science than did the biology-business, science and non-mathematics subgroups (10 percent level). The attitude of the science subgroup toward the role of mathematics in science was not significantly higher than that of the biology-business subgroup.
Summary

One factor analysis of covariance using the sum of the overall and high school mathematics grade point average (G) as a covariable was used to statistically test the null hypotheses concerning CCT and APT group means. One factor analysis of variance was used to statistically test the null hypotheses concerning MSAI group means.

The experimental group had significantly higher mean scores on the Cooperative Calculus Test and the Applied Problem Test than did the control group (10 to 20 percent level). The mean scores on the Math-Science Attitude Inventory were not significantly different for the two groups.

Six of the 18 null hypotheses concerning mean score differences between subgroups of the control and experimental groups were rejected (5, 10, and 20 percent level). Subgroups I_B, II_B, and I_B+S and II_M, had statistically significant differences in CCT group means. Subgroups I_M and II_M, and I_M and II_B+S had statistically significant differences in APT group means. Subgroups I_B+S and II_M, and I_M and II_B+S had significantly different MSAI group means. The remaining 12 null hypotheses regarding differences in mean CCT, APT and MSAI scores were not rejected.

Seven of the 12 null hypotheses concerning mean score differences of subgroups within the control group were rejected (1, 10
Subgroup $I_M$ had statistically significant higher mean \textit{CCT} and \textit{MSAI} scores than those of subgroup $I_B$. These subgroups did not differ significantly in ability to solve applied problems. Subgroup $I_S$ had statistically significant higher mean \textit{CCT}, \textit{APT}, and \textit{MSAI} scores, than those of the $I_B$ subgroup. Subgroup $I_S$ had a statistically higher mean \textit{APT} score than that of the $I_M$ subgroup. These same subgroups did not differ significantly in mean \textit{CCT} or \textit{MSAI} scores. Subgroup $I_M$ had a significantly higher mean \textit{MSAI} score than that of the $I_B+S$ subgroup. These two subgroups did not differ significantly in mean \textit{CCT} or \textit{APT} test scores. The remaining five null hypotheses regarding differences in mean \textit{CCT}, \textit{APT}, and \textit{MSAI} scores were not rejected.

Three of the 12 null hypotheses concerning mean score differences of subgroups within the experimental group were rejected (10 percent level). The subgroup $II_M$ had significantly higher \textit{MSAI} scores than those of the $II_B$, $II_S$ and $II_B+S$ subgroups. The remaining nine null hypotheses regarding experimental subgroup differences were not rejected.
V. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

This study was designed to determine some of the effects on college calculus students which resulted from their being grouped according to academic major, and from an adjustment of the calculus curriculum to meet the needs of these groups. The effects of such a class upon calculus achievement, ability to solve applied problems, and attitude toward the role of mathematics in science were examined by comparing group mean test scores of the experimental group to those of a group of students who completed a conventional non-grouped calculus sequence. Corresponding subgroups of the experimental and control groups were compared as well as the two groups. In addition comparisons were made of subgroups within the two groups.

Participants in the study were: a) 47 students who completed a conventional three quarter calculus sequence, and b) 41 students who completed a three quarter experimental calculus sequence in which they were grouped (part of the time) according to academic major. All participants in the study were students at Southern Oregon College during the 1967-1968 or 1968-1969 academic years.

Criterion tests were the Cooperative Calculus Test, Form B, the Applied Problem Test, and the Math-Science Attitude Inventory.
Since the study was a post-test only design, the criterion instruments were administered at the end of the respective Spring quarters.

The sum of the high school overall grade point average and mathematics grade point average, \( G = G_o + G_m \), served as a covariate to adjust for group differences in academic achievement and ability.

Analysis of the data revealed that the experimental group and its subgroups had higher \( G \) scores than did the control group and its corresponding subgroups. Considering the age and academic standing differences of the two groups, the mean number of quarter hours of science completed by the two groups was unexpectedly similar.

**Cooperative Calculus Test Scores.** The experimental group and its subgroups had higher mean \( \text{CCT} \) scores than those of the control group and its subgroups. The difference amounted to more than 3.9 points for the two groups. The \( \text{II}_B \) subgroup obtained the highest \( \text{CCT} \) group mean, and the \( \text{I}_B \) subgroup obtained the lowest \( \text{CCT} \) mean score of the six subgroups. In a majority of the cases (seven out of nine) the mean \( \text{CCT} \) scores were significantly correlated to \( G \) scores.

In three out of seven cases, null hypotheses regarding group differences in \( \text{CCT} \) mean scores between the control and experimental groups were rejected (5, 10, and 20 percent levels). The experimental subgroups \( \text{II}_B \) and \( \text{II}_M \) had significantly higher group means than
did subgroups $I_B$ and $I_{B+S}$ respectively. The remaining four null hypotheses regarding differences in CCT mean scores were not rejected.

Within the control group, with respect to mean CCT scores, the subgroups were ranked $I_M', I_S'$ and $I_B$ in descending order. In two out of four cases, the null hypotheses regarding CCT mean scores of subgroups within the control group were rejected (one percent level). Subgroups $I_M$ and $I_S$ had significantly higher CCT group means than those of subgroup $I_B$.

Within the experimental group, with respect to mean CCT scores, the subgroups ranked $II_B', II_M$ and $II_S$ in descending order. None of the four null hypotheses regarding CCT subgroup differences within the experimental group were rejected.

**Applied Problem Test Scores.** The experimental group and its subgroups had higher mean APT scores than those of the control group and its subgroups. The difference was more than .65 point for the two groups. The $II_B$ subgroup obtained the highest APT subgroup mean, and the $I_B$ and $I_M$ subgroups had the lowest APT subgroup means. In five out of nine cases the APT mean scores were significantly correlated to the G scores. In six out of nine cases the APT scores were significantly correlated with the CCT scores.

In three out of seven cases null hypotheses regarding group differences in APT means were rejected (10 and 20 percent levels).
The experimental group had significantly higher group APT means than did the control group. Subgroups II_M and II_B+S had significantly higher group means than that of subgroup I_M.

Within the control group, the subgroups were ranked I_S, I_B, and I_M in descending order with respect to APT mean scores. In two out of four cases, the null hypotheses regarding APT mean scores of subgroups within the control group were rejected (20 percent level). The I_S subgroup had a significantly higher group APT mean score than did the I_B and I_M subgroups.

Within the experimental group the subgroups were ranked II_B, II_M, and II_S in descending order. However, none of the four null hypotheses regarding subgroup mean APT score differences within the experimental group were rejected.

Math-Science Attitude Inventory Scores. Mean MSAI scores of the control and experimental groups differed by less than one point. This same pattern was found when comparing subgroups of the two groups with the exception of the I_M and II_M subgroups which differed by more than two points. The II_M subgroup obtained the highest mean MSAI score, and the II_S subgroup had the lowest.

Two of the seven null hypotheses regarding mean MSAI scores of the experimental and control group were rejected (5 and 20 percent level). Subgroups II_M and I_M had significantly higher mean MSAI scores than those of the I_B+S and II_B+S subgroups respectively.
Within the control group with respect to mean MSAI scores, the subgroups were ranked $I_M$, $I_S$ and $I_B$ in descending order. In three out of four cases null hypotheses regarding mean MSAI scores within the control group were rejected (1, 10 and 20 percent level). The $I_M$ subgroup had a significantly higher mean score than did either of the $I_B$ or $I_B+S$ subgroups. The $I_S$ subgroup had a significantly higher mean MSAI score than that of the $I_B$ subgroup.

Within the experimental group, with respect to mean MSAI scores, the subgroups were ranked $II_M$, $II_S$ and $II_B$ in descending order. In three out of four cases the null hypotheses regarding mean MSAI scores were rejected (10 percent level). Subgroup $II_M$ had a significantly higher mean MSAI score than those of the $II_B$, $II_S$ and $II_B+S$ subgroups.

Conclusions

The following conclusions were drawn from the data presented in this investigation:

1. Grouping college calculus students by academic major with a concomitant adjustment of the curriculum to meet the needs of the individual groups provided a better learning situation than that found in a conventionally structured class. Evidence for this conclusion was:
   a. The experimental group had a significantly higher (10
percent level) mean \textit{CCT} score than that of the control group.

b. The experimental group had a significantly (20 percent level) higher mean \textit{APT} score than that of the control group.

c. The percentage (61.8) of students completing the experimental calculus sequence was not significantly different \((z = .13)\) than the percentage (62.8) of students completing the control sequence.

d. The percentage (61.9) of the experimental group continuing the calculus sequence during the 1969 Fall term was greater than the percentage (51.0) of the control group that continued the sequence during the 1968 Fall term.

2. The structure and curriculum of the experimental calculus sequence did not consistently improve the achievement in calculus of the subgroups of the experimental group when compared to subgroups of the control group. Evidence for this conclusion was:

a. The acceptance of 10 of the 14 null hypotheses which compared the mean \textit{CCT} or \textit{APT} scores of subgroups of experimental and control groups.

b. The four rejected null hypotheses revealed no consistent
pattern as evidenced by:

i. The experimental biology-business subgroup had a significantly (5 percent level) higher \text{CCT} score than that of the control biology-business subgroup.

ii. The experimental mathematics subgroup had a significantly higher (20 percent level) mean \text{CCT} and a significantly (10 percent level) higher mean \text{APT} test score than the control non-mathematics subgroup and biology-business subgroup respectively.

iii. The experimental non-mathematics subgroup had a significantly (10 percent level) higher mean \text{APT} score than the control mathematics subgroup.

3. Within the conventionally structured college calculus classes the biology-business group of students does not learn calculus as well as the science and/or mathematics oriented student. Evidence for this conclusion was:

a. The rejection of three out of four null hypotheses which compared mean \text{CCT} or \text{APT} scores of subgroups within the control group.

i. The control mathematics subgroup had a significantly (one percent level) higher mean \text{CCT} score than that of the biology-business subgroup.

ii. The control science subgroup had significantly (one
and 20 percent level) higher mean CCT and APT scores than that of the biology-business subgroup.

b. The biology-business subgroup consistently ranked last with respect to CCT, APT, and MSAI scores.

4. Within the conventionally structured college calculus classes the science subgroup's ability to solve applied problems was greater than the non-physical science subgroups. Evidence for this conclusion was:
   a. The science subgroup had a significantly (20 percent level) higher mean APT score than that of the biology-business subgroup.
   b. The science subgroup had a significantly (20 percent level) higher mean APT score than that of the mathematics subgroup.

5. The structure and curriculum of the experimental calculus sequence allowed each subgroup to attain a level of calculus achievement consistent with their ability. Evidence for this conclusion was:
   a. The rankings of the experimental subgroups with respect to mean G, CCT, and APT scores were identical.
   b. The acceptance of all eight null hypotheses which compared mean CCT or APT scores of subgroups within the experimental group.
6. The structure and curriculum of the experimental calculus sequence did not significantly effect, the attitude of the experimental group toward the role of mathematics in science when compared to the control group. Evidence for this conclusion was:

a. The acceptance of the null hypotheses which compared mean MSAI scores of the control and experimental groups.
b. The acceptance of four of six null hypotheses which compared the mean MSAI scores of subgroups of the control and experimental groups.
c. In both groups the mathematics subgroups had significantly (5 and 20 percent levels) higher mean MSAI scores than those of the non-mathematics subgroups.
d. Within the two groups the pattern of rejection of null hypotheses was similar as evidenced by:
   i. The mathematics subgroups had significantly (one and 10 percent levels) higher mean MSAI scores than those of the biology-business subgroup.
   ii. The mathematics subgroups had significantly (10 percent level) higher mean MSAI scores than those of the combined non-mathematics subgroups.

7. Mathematics majors ("nonusers") have a more positive attitude toward the role of mathematics in science than do
science or biology-business majors ("users"). Evidence for this conclusion was:

a. The control (experimental) mathematics subgroup had a significantly (10 percent level) higher mean MSAI score than that of the experimental (control) combined non-mathematics subgroups.

b. Within the control group the mathematics subgroup had significantly (one and 10 percent levels) higher mean MSAI scores than those of the biology-business and combined non-mathematics majors respectively.

c. Within the experimental group the mathematics subgroup had a significantly (10 percent level) higher mean MSAI scores than those of the biology-business and combined non-mathematics majors.

8. The MSAI effectively measures the attitude of calculus students toward the role of mathematics in science. Evidence for this conclusion was:

a. The rankings of the three subgroups within both the control and experimental groups with respect to MSAI scores were identical.

b. The rejection of 8 of 15 null hypotheses which compared the mean MSAI scores of subgroups of the two groups.

c. The consistency (six of the eight null hypotheses) with
which the mathematics subgroups displayed higher mean MSAI scores.

9. The APT effectively measures the ability of calculus students to solve applied problems. Evidence for this conclusion was:
   a. APT scores were significantly correlated (five percent level) with G scores for the combined groups.
   b. APT scores were significantly correlated (five percent level) with CCT scores for the combined group.
   c. Within the experimental group APT scores were ranked identically with CCT and G scores.
   d. Within the control group the science subgroup had a significantly (20 percent level) higher mean APT score than did the mathematics or biology-business subgroups.

10. Team teaching calculus at the college level can be effectively designed and implemented. Evidence for this conclusion was:
   a. The achievement in calculus of the experimental group was greater than that of the control group.
   b. Questionnaires completed by the experimental subjects indicated a majority approved the structure of the experimental sequence.
   c. The instructors involved indicated satisfaction with the program and agreed to continue it during the 1969-1970 academic year.
Recommendations

On the basis of the data presented in this study and the conclusion that the experimental method of teaching calculus resulted in better learning of the concepts and skills of calculus by the experimental group, replicative studies should be carried out to determine if such students consistently score higher on the criterion instruments than do conventionally taught students. In addition, follow-up studies should be carried out to determine if such students continue to excel in achievement during the fourth term of calculus. In order to determine if the experimental treatment not only has an effect on calculus learning, a study should be made to determine if such students differ significantly in achievement from conventionally taught students in science courses taken concurrently with the Calculus.

Since the results of the study indicate that adjusting the curriculum to meet the needs of specific subgroups of the experimental group was beneficial to the students, a basic calculus textbook should be written and supplemented with a series of paperback textbooks containing specific material for the various academic major subgroups.

Even though the experimental group exhibited significantly better CCT and APT scores than those of the control group, conclusion two stated that subgroups of the experimental group did not consistently show significantly better criterion scores than the subgroups of the
control group. Therefore, replicative studies should be made to determine if non-significant differences on the criterion instruments consistently occur.

Although the four rejected null hypotheses concerning mean CCT and APT scores between subgroups of the two groups revealed no consistent pattern, the experimental mathematics group had significantly higher mean CCT scores than did the control combined non-mathematics subgroup. Likewise, it had a significantly higher mean APT than the control mathematics subgroup. This suggests that a study should be made to determine if groups of students subjected to a greater amount of the theory of calculus differ significantly in achievement from those who are not. Also a longitudinal study should be made to determine if mathematics students, grouped in their study of calculus, do significantly better in upper division mathematics courses than ungrouped students.

Subgroups within the control group showed marked differences in calculus achievement while subgroups within the experimental group exhibited no significant differences in criterion scores. These results suggest a replicative study to determine if these patterns are consistently found within similarly treated control and experimental calculus groups. It would also be of interest to conduct a study to determine if complete separation of subgroups would produce similar results.
These same conclusions would indicate that mathematics educators, calculus teachers in particular, should determine:

1. The importance and amount of applications suitable for various academic major subgroups found in contemporary calculus classes.

2. The amount and degree of mathematical theory and rigor needed by academic major subgroups found in contemporary calculus classes.

3. How to provide learning experience specifically designed to show the interrelationships of mathematics and science.

4. How to provide learning situations which demonstrate the relevance of calculus to the advanced study of various subjects.

Based on the conclusion that the experimental treatment did not produce significantly higher MSAI scores between the two groups, nor among subgroups thereof, replicative studies should be made to determine if this condition consistently exists between other groups of calculus students under similar circumstances. Also a study should be made designed to measure changes in attitudes toward the role of mathematics in science of grouped and ungrouped calculus students. Since the mathematics subgroups of both groups consistently had significantly higher mean MSAI scores than the other subgroups, a study should be made to determine the effects of additional upper division
mathematics on the attitude inventory scores. A similar study should be made to determine the effects of the number of science courses completed on attitude inventory scores.

It was also concluded that the MSAI was an effective measure of attitude of the role of mathematics in science. This conclusion suggests that studies should be made to determine if:

1. Similar differences in attitude are exhibited by science and mathematics teachers, since it has been shown that the attitudes of teachers directly influence the attitudes of students.

2. Differences in attitude are evident between groups of students who are not mathematics nor science majors.

3. Differences in attitude toward the role of mathematics in science correlate with attitudes toward mathematics and/or science.

Based on the conclusion that the APT is an effective measure of ability to solve applied calculus problems, replicative studies should be made to determine if groups subjected to the experimental treatment consistently differ from conventionally taught students in ability to solve applied problems. Since APT scores did not always correlate with CCT or G scores studies should be conducted to determine those specific skills associated with solving applied problems.
BIBLIOGRAPHY


55. Likert, R. A technique for the measurement of attitudes. Archives of Psychology, 1932. 52 p. (No. 140).


APPENDICES
APPENDIX A

Table 1. Distribution of Attitude Inventory Scores

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Table 2. Attitude Inventory $\mu_D$ Scores

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APPENDIX B

CALCULUS TEST (APT)

Name ______________________

In each of the following problems circle the letter of the correct answer. Do not circle more than one letter in each problem.

1. If \( \int f(x) \, dx = F(x) \), then \( F'(x) = (\ ?) \)
   
   A. \( \frac{[f(x)]^2}{2} \)
   
   B. \( f'(x) \)
   
   C. \( f(x) \)
   
   D. \( f(x) + c \)
   
   E. None of these

2. The height of an object thrown up from the ground with an initial velocity of 144 ft/sec is given by \( s = 144t - 16t^2 \). At what time \( t \), \( 0 \leq t \leq 9 \), will the velocity of the object be zero?
   
   A. \( t = 4.5 \)
   
   B. \( t = 6 \)
   
   C. \( t = 2.5 \)
   
   D. \( t = 9 \)
   
   E. None of these

3. Suppose an orchard has 30 trees planted per acre, and for \( x \) additional trees planted per acre the yield is given by \( y = (400 - 10x)(30 + x) \). What is the total number of trees that should be planted per acre so that the yield is a maximum?
   
   A. 30
   
   B. 35
   
   C. 40
   
   D. 45
   
   E. None of these
4. In a certain business the total cost of producing \( x \) items is given by \( C = 10 + 15x - 6x^2 + x^3 \). The marginal cost is defined to be \( \frac{dc}{dx} \). For what value of \( x \) will the marginal cost be increasing?

A. \( x < 2 \)
B. \( x < 3.5 \)
C. \( x > 3.5 \)
D. \( x > 2 \)
E. None of these

5. \( \lim_{x \to 3} \frac{2x^2 - x - 15}{x^2 - 2x - 3} = \)

A. 0
B. 2/3
C. 11/4
D. 1
E. None of these

6. A gate in an irrigation ditch is \( d \) feet wide. If the weight of one cubic foot of water is 62 lbs., with what force does water press against the gate when the water is \( b \) feet deep?

A. \( 31db^2 \)
B. \( 62db \)
C. \( 31d^2b \)
D. \( 62d^2b^2 \)
D. None of these

7. If \( y = \begin{cases} x + 1, & 0 \leq x < 1 \\ 3 - x, & 1 \leq x \leq 2 \end{cases} \), then \( \frac{dy}{dx} \) at (1, 2) is (?)

A. 1
B. -1
C. 0
D. \( \infty \)
E. None of these
8. Suppose that the rate of change of the number of people contracting an infectious disease is ten times the number of people who have the disease. If \( N_0 \) is the number of people with the disease at time \( t = 0 \), determine an expression for the number of people who are infected with the disease at any time \( t \).

A. \( N = 10N_0 t \)  
B. \( N = 10e^{N_0 t} \)  
C. \( N = 10N_0 e^t \)  
D. \( N = N_0 e^{10t} \)  
E. None of these

9. Any two electrons repel each other with a force inversely proportional to the square of the distance between them. If the constant of proportionality is 36, find the work done when the electron moves on the x-axis from \((8, 0)\) to \((-2, 0)\) if there are two other electrons held fixed at \((-10, 0)\) and \((10, 0)\).

A. \( 36 \int_{-2}^{8} \frac{1}{(10 + x)^2} - \frac{1}{(10 - x)^2} \, dx \)  
B. \( 36 \int_{-2}^{8} \frac{1}{(-10 + x)^2} - \frac{1}{(10 + x)^2} \, dx \)  
C. \( 36 \int_{-2}^{8} \frac{1}{(10 - x)^2} + \frac{1}{(10 + x)^2} \, dx \)  
D. \( 36 \int_{-2}^{8} \frac{1}{(10 + x)^2 (10 - x)^2} \, dx \)  
E. None of these
### ATTITUDE INVENTORY

**College Major**

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<th>Uncertain</th>
<th>Disagree</th>
<th>Strongly disagree</th>
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<td>1. Scientific developments are based more on practical experience than on the use of mathematical concepts.</td>
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<td>2. A scientist working in a laboratory would not need to know very much mathematics.</td>
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<td>3. Scientists would consider mathematics as a means of describing the real world.</td>
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<td>4. It should be possible for some research scientists to conduct their experiments mathematically instead of in a laboratory.</td>
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<td>5. A scientist should ignore mathematical principles if his data contradicts them.</td>
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<td>6. To appreciate modern science fully, a person should have a good understanding of mathematics.</td>
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<td>7. The use of computers should result in a decrease of the amount of mathematics needed by scientists.</td>
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<td>8. An attempt should be made to develop scientific ideas without the use of mathematics.</td>
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<td>9. Science and mathematics could not exist without each other.</td>
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<td>10. Science and mathematics majors should not take any mathematics courses together.</td>
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<td>11. The application of mathematical ideas is essential to the development of new scientific theories.</td>
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<td>12. Scientific principles should be the important concept, not the mathematics involved.</td>
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<td>13. All scientists should be interested in mathematics.</td>
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<td>14. I wish science did not involve mathematics.</td>
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<td>15. Mathematics has become so abstract that it should not be required for scientific study.</td>
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<td>16. Mathematics should not be considered as a science.</td>
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APPENDIX C

PERSONAL DATA SHEET

INSTRUCTIONS: The purpose of the information requested below is to obtain personal data about students currently taking Mathematic 202. Please check (✓) those items which are true in your case.

1. Name ____________________________ (Last) ____________________________ (First) ____________________________ (Middle)

2. Sex ________ M ________ F Age __________

3. Class
   ____ Freshman
   ____ Sophomore
   ____ Junior
   ____ Senior
   ____ Graduate
   ____ Other (Specify)

4. Major
   ____ Mathematics
   ____ Physics
   ____ Chemistry
   ____ Engineering
   ____ Biology
   ____ Business
   ____ Other (Specify)

5. Teaching ______ Yes ______ No

6. High School Mathematics Background
   ____ None
   ____ General Mathematics
   ____ First-Year Algebra
   ____ Second-Year Algebra
   ____ Geometry
   ____ Trigonometry
   ____ Calculus
   ____ Business Mathematics
   ____ Other (Specify)
7. College Mathematics Background
   _____ None
   _____ Intermediate Algebra
   _____ College Algebra
   _____ Business Mathematics
   _____ Trigonometry
   _____ Calculus
   _____ Others (Specify)
   _____ No. of quarters at SOC

8. High School Science Background
   _____ General Science
   _____ Biology
   _____ Chemistry
   _____ Physics
   _____ Other (Specify)

9. College Science Background
   _____ Physical Science
   _____ Biology
   _____ Zoology
   _____ Botany
   _____ Chemistry
   _____ Physics
   _____ Other (Specify)
   _____ No. of quarters

10. Subject liked most in high school ____________________________
    Subject liked least in high school ____________________________
APPENDIX D

MATH 201 - Calculus

Hill, Krewson, Steffani

Monday, Feb. 10, 1969
Large Section
Section 9.1, 9.2, 9.3, 9.4

Tuesday, Feb. 11, 1969
Laboratory
Assignment Due: P. 256 #'s 1, 2, 6, 9, 10, 13, 16.
P. 259 #'s 2, 6, 10, 14, 18.
Discuss Section 9.5

Wednesday, Feb. 12, 1969
Large Section
Section 9.6 and 9.7
Assignment Due: P. 262 #'s 1, 2, 5, 6, 8, 15,
P. 266 #'s 1, 4, 5, 8, 12.
P. 272 #'s 9, 10.

Friday, Feb. 14, 1969
Laboratory
Assignment Due: P. 277 #'s 1, 4, 7, 10, 13, 16, 17, 20.
P. 280, #'s 1, 2, 4, 10, 16, 17.
Discuss Section 9.8
Assignment Due on Monday, Feb. 17, 1969
P. 285 #'s 1, 2, 4, 5, 6, 8, 12.
## APPENDIX E

### MATH 200 TEST II

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APPENDIX F

Regression Equations for the Criterion Measure Scores and the Covariable G

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<td>$y = 12.23 + 2.99x$</td>
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Mean and Adjusted Mean Criterion and Covariable Scores for the Control and Experimental Groups

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<td>II B+S</td>
<td>31.48</td>
<td>4.74</td>
<td>6.29</td>
<td>30.39</td>
<td>4.56</td>
</tr>
</tbody>
</table>
Graph of the Control and Experimental CCT-G Regression Equations

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- **Control Group**
- **Experimental Group**
Graph of the Control and Experimental APT-G Regression Equations

- - - - - = Control Group

- - - - - - - - - = Experimental Group