HEAT LOSSES IN THE PERMEABLE WALL FURNACE
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HEAT LOSSES IN THE
PERMEABLE WALL FURNACE

INTRODUCTION

The reduction of fuel costs in industrial furnaces has received much attention in the past, and as a result regenerators, recuperators and waste heat boilers are often found as a part of a modern large furnace installation. All of these heat saving devices, however, suffer from certain disadvantages. Regenerators are large and rather inefficient. Recuperators may be leaky if non-metallic, and must be limited in temperature if metallic, while waste heat boilers are necessarily limited in capacity by the steam requirement of the plant.

The permeable wall furnace is an innovation in the design of fuel-fired furnaces which was first proposed in England in 1944 by R.H. Anderson, D.C. Gunn and A.L. Roberts (1, p169). Their proposal was to construct a furnace in which the combustion volume is surrounded by a refractory wall which is more or less permeable to gases. No flue or vent is provided from the combustion volume, so that the products of combustion have no route of escape except through the permeable wall. After passing through the wall the gases are collected in an open channel, backed with refractory as in an ordinary furnace, and vented from this channel to the atmosphere. A schematic drawing of a furnace of this type is shown in Figure 1.
The advantages claimed for this furnace are twofold; first, that some of the heat carried by the products of combustion is deposited within the permeable wall, thereby reducing the temperature gradient and the loss of heat by conduction through the wall; and second, that the flue gases are discharged from the system at a lower temperature than would be the case in a conventional furnace, reducing the sensible heat loss.

An analysis of the heat saving was made by the originators of this design, and was based upon the assumptions that the thermal conductivity of the refractory comprising the permeable wall was unaffected by the flow of gas through it and that the temperature existing in the gas collector zone, that is, at the cold face of the permeable wall, was the same temperature as would exist at the corresponding point in a conventional furnace having a wall of thickness equal to the total thickness of permeable wall plus non-permeable backing wall. Subject to these assumptions these workers presented solutions which indicated that rather considerable heat savings might be obtained in a furnace constructed in this manner.

Several installations have been made by them in England since 1944, ranging from a twelve inch diameter crucible furnace to a large car-bottom furnace. These
furnaces are reported to have performed satisfactorily in regard to refractory life, and to have shown fuel savings of from fifteen to sixty percent of that of the comparable conventional furnace.

The potential of this development seems large and apparently untapped in this country. For this reason it seemed desirable to investigate further the heat transfer conditions in a furnace of this type, in an attempt to evaluate design parameters and to provide a basis for estimating the required heat input. This paper is an attempt to provide some of this information.
FIGURE 1
PERMEABLE WALL
OVEN FURNACE
FRONT ELEVATION
THE STEADY STATE HEAT FLOW EQUATION

The physical condition which exists in the permeable wall of the furnace described is that of a unidirectional flow of gas through a porous permeable body, accompanied by a flow of heat in the same direction, resulting from differences in both pressure and temperature between the parallel faces of the wall.

When no gas flow exists within the porous medium it is supposed that heat flows through it by the following mechanisms:

1. Conduction within the solid from particle to particle. The quantity of heat transferred by this mechanism is proportional to the conductivity of the solid particles, and decreases as the porosity increases.

2. Conduction through the fluid within the pores of the solid. The heat transferred in this manner is proportional to the conductivity of the fluid, and increases as the porosity increases.

3. Radiation across the pores of the solid, varying with the emissivity of the solid particles and the fourth power of the absolute temperature.

4. Heat carried by convection currents in the pores of the solid. Convective heat transfer is generally considered to be negligible for the size of pore
encountered in the average refractory.

The resistance of the porous body to heat flow may be specified by an apparent thermal conductivity \( k \), which, according to Jakob (4, p.87) depends upon the conductivity of the solid and the gas, two characteristic lengths, and upon the emissivity and temperature.

When a pressure difference exists across the body a directed velocity is given to the fluid within the pores with the exception of a stagnant layer close to the solid particles. In this case there must be added to the list of heat transfer mechanisms a fifth item, heat carried by the moving fluid particles in the direction of the principal fluid flow.

In order to derive an equation for the flow of heat in the steady state, it is convenient to make the assumption that the temperature of the solid and the temperature of the fluid are identical at any point. That this cannot be true will be shown later. However, the assumption enables one to speak of a unique temperature at a point, and for this reason it is often made, and will be made here. With a given temperature and temperature gradient at any point then, it seems reasonable to assume that the heat transferred by conduction from particle to particle of the solid, and that transferred by radiation across the pores of the solid would
be independent of any flow of fluid through the body, provided only that the fluid is relatively transparent to radiation. Furthermore, since the fluid velocities to be considered here are small, and since the pressures deviate from atmospheric by only a few inches of water, it may be supposed that the flow of heat through the fluid in the pores of the solid is relatively unaffected by the flow of the fluid. We may then combine items one, two and three in the list of heat transfer mechanisms into an apparent thermal conductivity \( k \), dependent, for a given porous material and for pressures close to atmospheric, only on temperature, and expect this apparent thermal conductivity to change in magnitude not at all or by only a small amount as we pass from the condition of stagnant fluid in the pores to the condition of small directed fluid velocities in the pores.

The heat carried by the particles of fluid in the direction of the principal fluid flow may be evaluated in terms of the rate of fluid flow and the enthalpy of the fluid.

Subject to these assumptions, the heat crossing a plane at a distance \( x \) in the wall is

\[
q_x = -k \frac{A}{x} \frac{dt}{x} + w \frac{A}{x} \frac{H}{x} \tag{1}
\]

and that crossing the plane at \( x+dx \) is
\[ q_{x+dx} = -k_{x+dx} A_{x+dx} \frac{dt}{dx} + wA_{x+dx} H_{x+dx} \]

where \( q \) represents heat flow, Btu per hour, \( k \) represents the apparent thermal conductivity of the porous solid, Btu per hour per square foot per degree Fahrenheit per inch, \( A \) represents area perpendicular to the principal heat flow, square feet, \( t \) represents temperature, degrees Fahrenheit, \( x \) represents distance measured in the direction of the principal heat flow, inches, \( w \) represents mass rate of fluid flow, pounds per square foot per hour, and \( H \) represents enthalpy of the fluid, Btu per pound.

Since, in the steady state, \( q_x = q_{x+dx} \), and for a plane wall \( A_x = A_{x+dx} \), then

\[-k_{x} \frac{dt}{dx} + wH_{x} = -k_{x+dx} \frac{dt}{dx} + wH_{x+dx}\]

from which

\[ k \frac{d^2t}{dx^2} + \frac{dk}{dt} \left( \frac{dt}{dx} \right)^2 - w \frac{dH}{dt} \frac{dt}{dx} = 0 \quad (2)\]

This equation, with \( w \) set equal to zero, is the equation for temperature gradient in a rectangular coordinate system with a variable thermal conductivity. If the conductivity is approximated as a linear function of the temperature, \( k = a + bt \), then, with the boundary conditions \( t = t_0 \) at \( x = 0 \) and \( t_L = t \) at
The temperature gradient at \( x = 0 \) is then

\[
\left( \frac{dt}{dx} \right)_{x=0} = \frac{2a(t_L - t_o) + b(t_{2L}^2 - t_o^2)}{2L (a + bt_o)}
\]

where \( k_m \) denotes the thermal conductivity at the average temperature and \( k_o \) the thermal conductivity at the temperature \( t_o \).

The more general case of equation 2, where the air flow rate is other than zero, was set up on the electric analog computer of the Mechanical Engineering Department for a number of conditions of boundary temperatures, air flow rates, and thermal conductivities of the solid material. The analog computer, the set up of which is described in Appendix A, enables one to obtain directly plots of temperature and temperature gradient versus \( x \) coordinate. The general trend of the temperature - coordinate curves is concave downward, with the degree of concavity increasing with increasing airflow rate. A typical family of these curves is illustrated in Figure 2, where the dimensionless temperature \( \frac{t - t_L}{t_o - t_L} \) is
plotted against the dimensionless coordinate $x/L$, with the parameter being the dimensionless ratio $\frac{w c_p L}{k}$.

Equation 2 may also be solved by the method suggested by Kirchoff (4, p. 192) by writing the equation in the form (where $c_p = dH/dt$):
\[
\frac{d}{dx} \left( k \frac{dt}{dx} \right) - w c_p \frac{dt}{dx} = 0
\]
and making the substitutions
\[
k \frac{dt}{dx} = k_m \frac{d\theta}{dx} \quad \text{and} \quad c_p \frac{dt}{dx} = c_m \frac{d\theta}{dx}
\]
where $k_m$ and $c_m$ are constants to be determined, and the condition that $\theta = t$ at $x = 0$ and at $x = L$ is to be satisfied. These conditions lead to the equations
\[
k_m = \int_{t_o}^{t_L} k \frac{dt}{t_L - t_o} \quad \text{and} \quad c_m = \int_{t_o}^{t_L} c_p \frac{dt}{t_L - t_o}
\]
Representing both $k$ and $c_p$ by linear functions of temperature,
\[
k = a + bt \quad \quad c_p = f + gt
\]
then $k_m = a + \frac{b}{2} (t_L - t_o)$ and $c_m = f + \frac{g}{2} (t_L - t_o)$
or, in each case, the value of the property at the average temperature.

Then equation 2 becomes
\[
k_m \frac{d^2 \theta}{dx^2} - w c_m \frac{d\theta}{dx} = 0
\]
and its solution, subject to the boundary conditions \( \theta = t_o \) at \( x = 0 \) and \( \theta = t_L \) at \( x = L \) is

\[
\theta = t_o e^{-\frac{\omega c_m x}{K_m}} + \frac{t_o e^{-\frac{\omega c_m L}{K_m}} - t_L}{1 - e^{-\frac{\omega c_m L}{K_m}}} \left[ \frac{\omega c_m x}{K_m} - 1 \right]
\]

But \( \theta = t_o + \frac{a (t - t_o) + b/2 (t^2 - t_o^2)}{k_m} \)

Equating the two expressions for \( \theta \), differentiating with respect to \( x \) and setting \( x \) equal to zero,

\[
\left. \frac{d\theta}{dx} \right|_{x=0} = -\frac{w c_m t_o - t_L}{k_m e^{-\frac{\omega c_m L}{K_m}} - 1}
\]

Forming the ratio of this expression with that given by equation 3 for the temperature gradient at \( x = 0 \) with no fluid flow, one obtains

\[
R = \frac{\frac{w c_m L}{k_m}}{e^{-\frac{\omega c_m L}{K_m}} - 1}
\]

where \( R \) denotes the ratio of the temperature gradient at \( x = 0 \) for any rate of fluid flow to that which would exist if the fluid flow were zero but other conditions remained the same. This is the form of the equation which expresses the saving in heat passing by conduction
through the permeable wall, and will be used as the basis for the evaluation of the heat losses from the furnace, together with equation 1. Equation 4 is shown in graphical form in Figure 3.

Mention has been made of the transfer of heat from the fluid to the solid within the pores of the solid. An indication of the rate at which this heat must be transferred may be obtained in the following manner: the total heat carried across any plane perpendicular to the direction of the principal heat flow is constant in the steady state. However, on the plane at \( x \), the heat carried by the fluid as sensible heat is \( w H_x \), while on the plane at \( x + dx \) the fluid carries an amount of heat \( w H_{x+dx} \). Accordingly, the heat transferred from the fluid to the solid in the volume \( A \ dx \) is

\[
dq_t = w (H_x - H_{x+dx}) , \quad \text{or,}
\]

\[
\frac{dq_t}{dx} = -w \ \frac{dH}{dx} = -w \ \frac{dH}{dt} \ \frac{dt}{dx}
\]

Setting this equal to a rate of heat transfer given by

\[
dq_t = h' \ dt \ A \ dx
\]

where \( h' \) is a volume rate of heat transfer, Btu per hour per cubic foot per degree Fahrenheit, and includes all the mechanisms by which heat is transferred from the fluid to the solid, and \( \Delta t \) represents the temperature
difference between the fluid and the solid, degrees Fahrenheit, then

\[ \Delta t = - \frac{w}{h'} \frac{dH}{dt} \frac{dt}{dx} \]

so that the temperature difference \( \Delta t \) exists whenever a temperature gradient \( \frac{dt}{dx} \) exists, and is proportional to the temperature gradient.
FIGURE 3

CONDUCTION REDUCTION FACTOR

(Equation 4)
POROSITY AND PERMEABILITY

Porosity is defined as the fraction of solids per unit volume. Permeability depends not only upon the porosity but also upon the geometry of the pores and upon the degree to which they are interconnected. Permeability is defined (6, p. 551) by the relation

\[ \lambda = \frac{\mu \cdot \frac{w \cdot v}{\Delta p/\Delta x}} \]

in which \( \lambda \) is the permeability, square feet, \( \mu \) is viscosity of the permeating fluid, pound seconds per square foot, \( v \) is the specific volume of the fluid, cubic feet per pound, \( p \) is the pressure, pounds per square foot, and the other symbols are as defined previously.

The permeability of a refractory is generally tested using air at room temperature as the permeating fluid, and permeability is reported as

\[ \lambda' = \frac{w \cdot v}{\Delta p/\Delta x} \]

in which \( \lambda' \) is the apparent permeability, cubic feet per hour per square foot per pound per square inch per inch.

Assuming an ideal gas so that \( v = R T / p \), equation 5 becomes

\[ \lambda = \frac{\mu \cdot \frac{w \cdot R \cdot T}{p \cdot dp/dx}} \]
in which \( R' \) is the gas constant, foot pounds per pound degree Rankine, and \( T \) is the absolute temperature, degrees Rankine.

This equation, upon integration between the limits \( p = p_0 \) at \( x = 0 \) and \( p = p_L \) at \( x = L \) is

\[
\lambda = \frac{2 \mu R'}{p_0^2 - p_L^2} \int_0^L \frac{T}{\mu(T)} \, dx \quad (6)
\]

Clews and Green (3, p.223) have reported that, while for a given pressure difference the volume of air passed through a refractory brick decreases with increasing temperature, this effect is due almost entirely to the increased viscosity of the air. Trinks (9, p.498) mentions flow of air through a refractory, and states that the rate of flow is inversely proportional to the viscosity of the fluid, and proportional to some power of the pressure difference, the power being between one-half and one.
In order to test the equations and solutions proposed, it was necessary to reproduce on an experimental scale the physical conditions found in the permeable wall furnace, that is, to obtain a unidirectional flow of heat and gas.

The material chosen for the test was a commercial insulating firebrick, obtained in the standard nine-inch straight size, two and one-half inches by four and one-half inches by nine inches. The specimen cut from this straight was a cylinder nominally four and one-quarter inches in diameter, and a maximum of two and one-half inches in thickness.

The specimen was prepared for test by cutting the cylinder and drilling one-sixteenth inch diameter holes for thermocouples. These holes were located in two banks; one so that the junctions of the thermocouples would be approximately on the geometrical axis of the cylinder, and one so that the junctions would be on the periphery of the cylinder. Distances of the holes from the base of the cylinder were such that for each central thermocouple there would be a corresponding couple on the periphery, at the same distance from the base. The location of the thermocouples may be seen in the drawing of the test chamber assembly, Figure 4.
Extreme care was used in drilling the holes for the thermocouples, using a slow hand feed and a high spindle speed. However, it was not found possible to completely eliminate "walking" of the drill. Upon completion of the drilling, the specimen was set upon a plane table, the drill successively inserted in each hole and a measurement made of the height above the table at each end of the drill. If these dimensions differed by more than one-thirty second of an inch for any hole the specimen was rejected. Otherwise the average of the two dimensions was recorded as the x coordinate of that thermocouple.

The thermocouples themselves were fourteen gauge chromel-alumel, flash butt welded, the weld ground to the wire diameter, and the couple straightened before inserting into the specimen. The circumference of the specimen was then sealed with a refractory cement and water glass mixture, rammed into place.

The test chamber itself is shown in Figure 4, and consists of a steel shell with affixed pipe nipples for the various connections. The lining was of insulating firebrick in the lower portion of the chamber, and of cast cement - water glass mixture in the upper part. The chamber and the specimen were heated by an electric coil mounted in a position as shown, with lead wires passing
out of the chamber through a mercury seal. The coil itself was a flat disc in shape, the heating element being entirely sealed in refractory, with a number of small holes through the flat faces of the disc to allow air flow.

The calorimeter assembly used to measure the heat passing through the specimen consisted of a fifty mesh bronze screen resting on the top of the specimen, and above this the inner and outer calorimeters, each made up of several turns of copper tubing, ground flat on the underside so as to provide better contact with the screen. The calorimeter was held in good thermal contact with the specimen by means of rubber pressure pads and a clamp, as shown in the figure. The volume above the calorimeter coils was loosely filled with insulation to reduce the heat transfer from the room to the calorimeter. The temperature rise of the water in the inner calorimeter was measured with a ten times multiplying copper constantan differential thermocouple. A constant head water supply was provided to reduce variations in the water flow rate.

Twenty-two gauge chromel and alumel lead wires were welded to the fourteen gauge thermocouple wires, and were carried to a junction point where they were soldered to copper lead wires. This junction point was
FIGURE 4
TEST CHAMBER ASSY
within a region of fluctuating temperature, so precautions were taken that all junctions were at the same temperature. This was done by taping the junctions into a compact bundle and insulating them thermally from the ambient. The copper lead wires lead to a twenty-four point selector switch, and a common cold junction was provided, consisting of a twenty-two gauge thermocouple immersed in kerosene in a glass tube which was in a water-ice mixture in a vacuum flask.

Thermocouple voltages were read with a Leeds and Northrup precision potentiometer. A milliammeter was provided in the thermocouple circuit so that the couple output could be switched to either the potentiometer or to the milliammeter. This was intended as a protection for the potentiometer in case one of the guard heaters should become shorted to a thermocouple. Before making a reading on the potentiometer the thermocouple to be read was switched to the ammeter, to confirm that the output was of the proper order of magnitude.

The thermal guarding of the specimen circumference was accomplished by coils of nichrome wire which were placed around the circumference in various positions along the length of the cylinder. In general a coil was placed between each bank of thermocouples. The current in each coil was independently variable, controlled by a
variable transformer and a bank of resistors.

Air flow through the specimen was provided by a small positive displacement pump, controlled by a pressure regulator and metered by a wet gas meter having a least scale division of one-one thousandth of a cubic foot. The pressure in the chamber was indicated on an inclined-tube manometer which could be read to one-one hundredth of an inch of water.

Test runs were made in the following manner: the test chamber, with the specimen mounted in place and the calorimeter assembly clamped down upon it, was brought up to the temperature of the run. This was done extremely slowly in order to reduce the possibility of cracking refractory sections, about one hundred degrees Fahrenheit being the usual heating rate per hour. While the chamber was heating the air flow was turned on, and the pressure in the chamber adjusted to approximately that desired for the test. At the same time the current in the guard circuit was turned on, and an attempt was made to keep the temperatures of each guard - center line thermocouple pair equal. When the desired chamber temperature had been reached, the chamber pressure was given its final adjustment, and two plots were made of temperature in millivolts versus x coordinate, one for the temperatures at the center of the cylinder, and one for the temperatures
on the periphery of the cylinder. The guard temperatures were then adjusted by resetting the variable transformer and the resistances, until these two curves coincided. A time of about one hour was then allowed to insure that the steady state had been reached. Just before the beginning of the test run the center line temperatures were recorded and the air flow meter reading and corresponding time noted. The start of the test run was the time when the center calorimeter water outlet was switched from the drain to the weigh bucket. The duration of test was twenty minutes or half an hour, and during this time the calorimeter water temperature rise was recorded each minute and the center line temperatures and the air flow meter reading recorded at the halfway point. At the close of the test period the calorimeter water was switched back to the drain, the final readings made of air flow and specimen temperatures, and the calorimeter water weighed. It was found early in the tests that it was unnecessary to record calorimeter water temperature rise each minute, and the interval was changed to two minutes.

Temperature changes in the specimen over the test period were quite small, in general less than five degrees Fahrenheit, because the test runs were made at times of day when line voltage variation was unlikely.
A single test run usually took a day to complete, starting from the first attempt to balance temperatures.

Two specimens were tested. The first was a grade 20 insulating firebrick, suitable for temperatures up to two thousand degrees Fahrenheit. The thickness of this specimen was 1.304 inches, and three pairs of thermocouples were located in it, the center line couples being located at distances of 0.322 inches, 0.681 inches and 1.172 inches from the hot face of the cylinder. The second specimen was a grade 26 insulating firebrick, suitable for temperatures up to twenty-six hundred degrees Fahrenheit, and of higher permeability and thermal conductivity than the first specimen, but of lower porosity. The thickness of this specimen was 2.500 inches, and four pairs of thermocouples were located in it, at distances of 0.300 inches, 0.932 inches, 1.580 inches and 2.230 inches from the hot face of the specimen.
EXPERIMENTAL RESULTS

A total of eighteen runs were made on the two specimens. These runs were made at temperatures at the hot face of the specimen up to approximately eighteen hundred degrees Fahrenheit, and with air flow rates up to a maximum of a little over ten pounds per square foot per hour. The results of these runs are summarized in Table 1.

The calculated heat flow values listed in Table 1 are calculated from the observed temperatures and air flow rate, using equations 1 and 4 and the thermal conductivity values obtained from the runs at no air flow. A comparison of these values with the observed values shows that in all cases with the exception of one the calculated heat flow is within ten percent of the observed, and in the majority of cases is within five percent. Five percent would seem to be an optimistic estimate of the error in the experiment. Probably the single largest source of error was in the thermal guarding of the specimen. The assumption was necessarily made that the specimen and the guards were symmetrical about the geometric axis of the cylinder, so that the temperature at any point was dependent only upon its x coordinate. Certainly this condition was not met exactly, although
### TABLE 1
#### EXPERIMENTAL RESULTS

**Test Series 1 - Grade 20 Insulating Firebrick**

<table>
<thead>
<tr>
<th>Temperature, ( ^\circ F )</th>
<th>( w ), pounds per hour</th>
<th>( \Delta p ), inches of water</th>
<th>Heat flow, Btu per hour per ( ft^2 ) observed</th>
<th>( \lambda x 10^{10} )</th>
<th>( ft^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>at ( x = 0.322'' )</td>
<td>at ( x = 0.681'' )</td>
<td>at ( x = 1.172'' )</td>
<td>per ( ft^2 )</td>
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<td>0</td>
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**Test Series 2 - Grade 26 Insulating Firebrick**

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<tr>
<th>Temperature, ( ^\circ F )</th>
<th>( w ), pounds per hour</th>
<th>( \Delta p ), inches of water</th>
<th>Heat flow, Btu per hour per ( ft^2 ) observed</th>
<th>( \lambda x 10^{10} )</th>
<th>( ft^2 )</th>
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<tr>
<td>at ( x = 0.300'' )</td>
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<td>at ( x = 1.580'' )</td>
<td>at ( x = 2.230'' )</td>
<td>per ( ft^2 )</td>
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every care was taken that the error introduced in this manner be held to a minimum.

The location of the thermocouples within the specimen with respect to their x coordinate is believed to have been determined accurately within 0.015 inches. However, the temperature gradients encountered, particularly at the cold face of the specimen, were quite steep. Assuming an error of 0.015 inches in the location of the thermocouple and a temperature gradient of two thousand degrees Fahrenheit per inch, there is a thirty degree error in the temperature.

The error in the determination of the cross sectional area of the specimen is probably of the order of one percent, and the error in the measured air flow not more than one percent. Error due to lack of uniformity of the air flow across a plane of constant x might arise from two factors; first, a non-uniform temperature across the plane, and second, a resistance to flow at the sealed circumference of the specimen.

Lack of uniformity of temperature over the planes of constant x was probably very small at and close to the cold face of the specimen, where the screen of the calorimeter would tend to distribute heat evenly. The variation might be expected to be greater near the hot face of the specimen, since the thermal conductivity of the
heating element, while higher than that of the specimen, is rather low, and therefore there was more tendency for thermal gradients to be present. In further tests a permeable metal heater plate in contact with the specimen surface might be used to advantage.

The values of the permeability listed in Table 1 are calculated from the data using equation 6, employing a graphical integration to evaluate the integral. There is some uncertainty in the determination of this integral due to the small number of temperatures available over the length of the specimen. However, there seems to be a definite trend toward increasing permeability with increasing temperature. This is not necessarily in contradiction to the results of Clews and Green mentioned earlier, since their results indicated only that viscosity change was the major factor in the variation of flow rate with temperature for a given pressure drop, which is verified by the results presented here. It seems reasonable to suppose that a result of the thermal expansion of the solid material accompanying temperature increase would be to open the pores of the solid, and provide an easier path for gas flow.

The range of air flows tested here, up to a maximum of about ten pounds per square foot per hour, is believed to include those flow rates most likely to be
encountered in furnace practice. For most gaseous fuels a flow rate of ten pounds per square foot per hour corresponds to a heat input of around 13,000 Btu per square foot of permeable wall per hour. While not particularly pertinent to the immediate problem, it would be of interest to experiment with higher air flow rates, where the rate of heat transfer from the gas to the solid becomes higher.

At temperatures higher than those tested, higher than about 1800 degrees Fahrenheit, the transfer of heat by radiation across the pores of the solid becomes more predominant, and as a result it is believed that the assumptions leading to equation 2 would be more nearly fulfilled.
ANALYSIS OF THE FURNACE

In order to apply the foregoing to the problem of the determination of the heat losses from a permeable wall furnace, reference is made to Figure 1, which illustrates a typical oven furnace. It is assumed that the operating temperature is specified, and that trial values of the design parameters such as wall thickness, geometry of the collector, etc. have been assumed. It remains to compute the temperatures at the various points in order to confirm the selection of refractories, and to estimate the required heat input in order to choose burner equipment of the proper capacity.

Of the total heat supplied by the combustion of the fuel in the combustion volume, a portion $q_1$ is lost in a manner having no relation to the permeable wall construction. This heat might be absorbed by the charge, lost by radiation thru openings, absorbed in water-cooled components such as skid rails, carried out by hot material leaving the furnace, etc. The remainder of the heat supplied, $q_2$, passes out thru the permeable wall sections of the furnace. At the colder face of the permeable wall, plane 1 in Figure 1, this heat must leave either in the form of radiation to plane 2, or as
sensible heat in the gases flowing out of the wall at plane 1, or by convection from the wall to the gases existing in the collector at the temperature $t_g$. The total heat which is received at plane 2, both by radiation from plane 1 and by convection from the gases in the collector, must pass by conduction to the furnace casing at plane 3, and thence be dissipated to the ambient by radiation and convection.

A heat balance on the gas mass in the collector indicates that heat is received (a) by an inflow of gas at the temperature $t_1$ and (b) by convective heat transfer from plane 1, while heat is lost by an outflow of gas at the temperature $t_g$ and by convective heat transfer to plane 2, or,

$$w A_1 h_1 + h_2 A_2 = w A_1 h_1 A_1 (t_1 - t_g) = w A_1 H_g + h_2 A_2 (t_g - t_2)$$

where $h$ represents the coefficient of convective heat transfer, Btu per hour per square foot per degree Fahrenheit, and where the subscripts refer to the plane at which the property is evaluated. Then

$$t_g = \frac{w A_1 c_p t_1 + h_1 A_1 t_1 + h_2 A_2 t_2}{w A_1 c_p + h_1 A_1 + h_2 A_2}$$

or, the temperature of the gases in the collector is a weighted mean between the temperatures of the two enclosing walls.
The magnitude of the convection coefficient at the permeable wall, \( h_1 \), might be approximated to be of the same order of magnitude as the convection coefficient would be upon the same wall if the air flow rate were reduced to zero, since the velocities involved are quite small, and since it would seem that the velocity components parallel to the wall would be negligible. However, this assumption is not necessary, since the heat impinging by radiation on plane two is, in most cases, so large compared to the heat transferred from plane 2 to plane 3, that for all practical purposes planes 2 and 3, and therefore the gases in the collector, may be considered to be at the same temperature. The problem reduces, then, to one of finding, by trial, a temperature \( t_1 \) such that the total heat transferred through the permeable wall, \( q_2 \), is balanced by the heat carried out of the system by the flue gases, \( q_3 \), plus the heat transferred by conduction from plane 2 to plane 3, \( q_4 \). The following procedure is suggested:

1. Calculate the heat losses \( q_1 \).
2. Assume a value for the temperature of the colder face of the permeable wall (\( t_1 \)).
3. Estimate the heat loss \( q_2 \).
4. The total heat to be supplied is the sum of \( q_1 \) and \( q_2 \). Calculate the flow rate \( w \) required to supply this.
quantity of heat.

5. Correct the heat loss $q_2$ for this value of $w$.

6. Calculate $q_3$ and $q_4$.

7. Repeat this process, adjusting the temperature $t_1$ until $q_2 = q_3 + q_4$.

A sample calculation is shown in Appendix B.

It is suggested that the error introduced by computing the ratio $\frac{w \cdot c_p \cdot L}{k}$ using the specific heat of air rather than of the products of combustion is small for the usual range of temperatures. In the sample furnace calculation of Appendix B, for example, the specific heat of air at the collector temperature (1300°F) is 0.281 Btu per pound per degree Fahrenheit, while the specific heat of the products of combustion of a stoichiometric natural gas - air mixture at the same temperature is 0.284 Btu per pound per degree Fahrenheit, a difference of about one percent.
SUMMARY

For the case of unidirectional heat and gas flow through a permeable refractory material, the total heat transmitted has been found to be given within an accuracy of about ten percent by an equation

\[ \frac{q}{A} = \omega H_{t_0} = k_m R \frac{\Delta t}{\Delta x} \]

where the subscript \( t_0 \) refers to the temperature at the hot face of the refractory, and where \( R \) is a factor dependent upon the properties and geometry of the solid material, the mass rate of flow and the specific heat of the fluid flowing, and is

\[ R = \frac{\left( \frac{w c_p L}{k} \right)_{avg}}{\left( \frac{\omega c_p L}{k} \right)_{avg} e^{1 - 1}} \]

The permeability of the refractories tested was found to be essentially constant over the range of temperatures and flow rates used, with some indication of increasing permeability with increasing temperature.

Based upon these results, a procedure is proposed for the calculation of the heat losses from a furnace of permeable wall construction.
BIBLIOGRAPHY


APPENDIX A
ANALOG COMPUTER SOLUTIONS

The electric analog computer which was used to obtain solutions of equation 2 in the form of plots of temperature versus x coordinate consists of a number of components which may be wired together so as to perform electrically the operations required for the solution of the equation in question. The computer requires that the independent variable be time and the dependent variable be voltage, so that, in the case of equation 2, it was necessary to transform, with appropriate scale factors, temperature into voltage and x coordinate into time.

In order to program the equation for the components available on the computer it was written in the form

\[ t'' = t' \left( \frac{w c_p - b t'}{k} \right) \]

where the prime and the double prime represent first and second derivative with respect to x respectively, and where \( c_p \) and \( k \) are approximated by linear functions of temperature, so that \( b \) represents the temperature coefficient of the thermal conductivity. The components and circuit used to solve the equation in this form are shown in Figure 5.

The time scales chosen varied from one to three
seconds per inch of \(x\) coordinate, and the temperature versus \(x\) curve was displayed on an oscilloscope, using a sweep of one centimeter per second. The initial condition of temperature at \(x=0\) was fed into the computer, and the initial condition of temperature gradient at \(x=0\) was varied by a trial and error process to obtain the desired temperature at \(x=L\). The output was then fed into a two channel Sanborn recorder where simultaneous plots of temperature and temperature gradient versus \(x\) coordinate were made. It is from these plots that the family of curves shown in Figure 2 was made.
Figure 5
Schematic Analog Computer Circuit
APPENDIX B
SAMPLE COMPUTATION FOR THE FURNACE

Assumed Design Conditions:

Chamber dimensions: 18" wide, 24" long, 12" high
Sidewalls, backwall and arch to be permeable, of $4\frac{1}{2}$" grade 26 insulating firebrick
Door and hearth to be non-permeable, of $4\frac{1}{2}$" grade 26 insulating firebrick, backed with $2\frac{1}{2}$" grade 16 insulating firebrick.
Collector backing to be $4\frac{1}{2}$" grade 16 insulating firebrick.
Operating temperature 1800°F, ambient 80°F.
Heat losses due to radiation through door openings, water-cooled parts, etc. is 10,000 Btu per hour.
Fuel to be natural gas, 1280 Btu per pound of air plus gas.

Fixed heat losses ($q_1$):

Radiation and cooling 10,000 Btu/hr
Door, 1.5ft x 299 Btu/ft² hr¹ 450
Hearth, 3 ft x 299 Btu/ ft² hr 900

$q_1 = 11,350$ Btu/hr

Assume collector temperature ($t_1$) is 1400°F:

¹ Heat loss figures taken from tables in manufacturer's literature (2, p.12-13)
\[
\frac{c_m L}{k_m} = \frac{0.279}{2.88} = 0.436
\]

For no flow through permeable wall,
\[q = -k_m A \frac{\Delta t}{\Delta x} = -2.88 (8.5) \frac{-400}{4.5} = 2180 \text{ Btu/hr}\]

Then \[q_2 = 2180 R + w A (H_{t_0} - H_{t_{\text{ambient}}})\]
\[= 2180 R + w (8.5) (448.84)\]

Total heat input = \(q_1 + q_2\):
\[(8.5 \text{ w}) \text{#/hr} \times 1280 \text{ Btu/#} = 11,350 + 2180 R + 8.5w(448.8)\]
solving, \(w = 1.81\) and \(R = 0.655\) (using Figure 3)

Then \[q_2 = 2180 (0.655) + 1.81 (8.5) (448.84) = 8330\]
\[q_3 = w A (H_{t_1} - H_{t_{\text{amb.}}}) = 1.81 (8.5) (337.35) = 5190\]
\[q_4 = 19.3 \text{ ft} \times 226 \text{ Btu/ft}^2 \text{ hr} = 4360\]
\[q_3 + q_4 = 5190 + 4360 = 9550 > q_2\]

Assume collector temperature of 1300°F:
\[
\frac{c_m L}{k_m} = \frac{0.278}{2.83} = 0.442
\]

For no flow through permeable wall,
\[q = -2.83 (8.5) \frac{-500}{4.5} = 2680 \text{ Btu/hr}\]

Total heat input = \(q_1 + q_2\):
\[8.5 \text{ w} (1280) = 11,350 + 2680 R + 8.5 w(448.84)\]
solving, \(w = 1.84\) and \(R = 0.64\)

Then \[q_2 = 2680 (0.64) + 1.84 (8.5) (448.84) = 8735\]
\[ q_3 = 1.84 \times (8.5) \times (310.04) = 4850 \]
\[ q_4 = 19.3 \, \text{ft} \times 203 \, \text{Btu/ft} \, \text{hr} = 3910 \]
\[ q_3 + q_4 = 4850 + 3910 = 8760 \, \text{Btu/hr} \]

The annulus temperature for the furnace is then approximately 1300°F, and the required heat input, \( q_1 + q_2 \) is 20,100 Btu per hour, or about twenty cubic feet per hour of natural gas.

It is of interest to compute the heat input necessary to maintain at 1800°F a conventional furnace of the same dimensions, with a wall construction which is identical to that of the permeable furnace considered, but with no collector space.

- Radiation and cooling: 10,000 Btu/hr
- Door and hearth: 1,350
- Arch, 3 ft x 223 Btu/ft² hr: 670
- Sidewalls, 2 @ 2 x 223: 890
- Backwall, 1.5 x 223: 340

**Total**: 13,250 Btu/hr

Assuming a flue gas temperature 100°F above the furnace operating temperature, which is probably on the conservative side (8, p.54), then with an available heat above 1900°F for natural gas of 520 Btu/cubic foot (7,p9.) the required gas input is 25.5 cubic feet per hour.

The heat saving resulting from the permeable construction is then of the order of twenty percent. This saving is obtained at the expenditure of pumping power
\[ P = \Delta p \cdot v \cdot A \cdot w \]

Where \( P \) is the pumping power, foot pounds per hour, and the other symbols are as defined previously.

By equation 6

\[ \Delta p = \frac{2 w R'}{\lambda (p_1 + p_2)} \int_0^L T \mu(T) \, dx \approx \frac{w R'}{\lambda p_2} (T' \mu')_{\text{avg}} L \]

\[ = \frac{1.84 (53.3)}{(14 \times 10^{-6})(14.7 \times 144)} \frac{2010(9.4 \times 10^{-7})}{12} \frac{1}{3600} \]

\[ = 6.5 \text{#/ft}^2 \]

Then \( P \approx (6.5)(0.0735)(8.5)(1.84) = 1390 \text{ ft#/hr} \)

It remains to show that the approximation \( t_1 = t_g = t_2 \) is valid. The heat transferred by conduction from plane 2 to plane 3 was 203 Btu/ft hr. Setting this equal to the heat transferred by radiation from plane 1 to plane 2, using, as an approximation, the situation for infinite parallel walls,

\[ \sigma \left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \right) (T_1^4 - T_2^4) = 203 \]

Taking \( \varepsilon_1 = \varepsilon_2 = 0.57 \) (6, p. 502),

\[ 0.173 \times 10^8 \frac{1}{0.57} \frac{1}{0.57} - 1 (T_1^4 - T_2^4) = 203 \]

Then \( t_1 - t_2 = \frac{2.95 \times 10^9}{(T_1^2 + T_2^2)(T_1 + T_2)} \approx \frac{2.95 \times 10^9}{2(1760)^2(2 \times 1760)} = 14^\circ F. \)
APPENDIX C
DATA:

SPECIMEN TEMPERATURES

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WATER FLOW

time...20 minutes, weight of water...15.875#
temperature rise, average of 20 observations,
  0.1124 mv, 2.96°F

AIR FLOW

time    0 min. meter reading 96.648 cubic feet
10 min. 97.469
20 min. 98.291

Barometer 29.96 inches of mercury
Chamber pressure 0.93 inches of water
Pressure at meter 1.2 inches of water
Room temperature 77°F
Meter temperature 78°F
Water temperature 67°F
AIR FLOW RATE

\[
\frac{(98.291 - 96.648) \times 60 \text{ min}}{20 \text{ min}} \times \frac{0.0734 \text{#/ft}^3}{0.0998 \text{ ft}^2} = 3.657 \text{#/ft}^2 \text{hr}
\]

HEAT FLOW

\[
\frac{(15.875 \times 2.96) \times 60 \text{ min}}{20 \text{ min}} \times \frac{0.0581 \text{ ft}^2}{0.0998 \text{ ft}^2} + 3.657 \times 0.24 (113 - 67)
\]

= 2358 Btu/ft\(^2\)hr

PERMEABILITY

\[
\lambda = \frac{2w R'}{P_0^2 - P_L^2} \int L \mu(T) \, dx
\]

By graphical integration,

\[
\int L \mu(T) \, dx = 18.5 \times 10^{-4} \text{#/sec}\,\text{in} \text{ft}^2
\]

\[
\lambda = \frac{(3.657) (53.3) (18.5 \times 10^{-4})}{(29.99 \times 0.495 \times 144) (0.93 \times 5.062)} \times \frac{1}{3600} \times \frac{1}{1728}
\]

= 8.40 \times 10^{-10} \text{ ft}^2

CALCULATED HEAT FLOW

\[
\frac{w c_m L}{k_m} = \frac{(3.657) (0.265) (1.850)}{1.03} = 0.80
\]

From Figure 3, R = 0.65
\[
\frac{q}{A} = -k_m R \frac{\Delta t}{\Delta x} + w (H_{t_0} - H_{t_{cal}})
\]

\[
= -1.03 (0.65) \frac{460 - 1610}{850} + 3.657 (398.58)
\]

\[
= 2368 \text{ Btu/ft}^2 \text{ hr}
\]