

# COMPRESSIVE BUCKLING CURVES FOR SANDWICH CYLINDERS HAVING ORTHOTROPIC FACINGS

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FOREST SERVICE

In Cooperation with the University of Wisconsin

COMPRESSIVE BUCKLING CURVES FOR SANDWICH  
CYLINDERS HAVING ORTHOTROPIC FACINGS<sup>1</sup>

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Abstract

Design curves are presented for determining the axial buckling stress of sandwich cylinders having orthotropic facings and orthotropic cores. They apply particularly to sandwich having glass-fabric laminate facings and honeycomb cores. Sandwich constructions with isotropic cores are included.

Introduction

The derivation of the formulas for the buckling stress of axially loaded circular cylinders of sandwich construction is given in Forest Products Laboratory Report No. 1830.<sup>3</sup> The formulas apply to sandwich

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<sup>2</sup>—Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

<sup>3</sup>—March, H. W., and Kuenzi, Edward W. Buckling of Cylinders of Sandwich Construction in Axial Compression. Forest Products Laboratory Report No. 1830. Revised December 1957.

constructions having similar orthotropic facings of unequal thicknesses and orthotropic cores. The natural axes of the orthotropic materials are parallel to or perpendicular to the axis of the cylinder. These formulas are simplified to apply to sandwich constructions having isotropic facings and orthotropic cores. Design curves are given that apply to isotropic cores and to orthotropic cores for which the moduli of rigidity associated with radial and axial directions and radial and tangential directions are related by the ratio 2.5 or 0.4. These ratios apply approximately to most honeycomb-core materials.

In the present report the formulas of Report No. 1830<sup>3</sup> are put in terms of three groups of the elastic properties of the orthotropic facings. It is shown in Forest Products Laboratory Report No. 1867<sup>4</sup> that two of these groups have about the same values for all of the glass-fabric laminates currently in use and that three values of the third group will suffice for the same laminates. Design curves are presented for sandwich cylinders having glass-fabric laminates of these types and honeycomb or isotropic cores.

### Mathematical Development

The three groups of elastic properties of the facings are:

$$\alpha = \sqrt{\frac{E_x}{E_y}} \quad (1)$$

$$\beta = \alpha \sigma_{yx} + 2\gamma \quad (2)$$

$$\gamma = \frac{\lambda \mu_{xy}}{\sqrt{E_x E_y}} \quad (3)$$

The nomenclature of Report 1830<sup>3</sup> is used so that  $E_x$  and  $E_y$  are the moduli of elasticity of the facings in the axial and tangential directions,

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<sup>4</sup>Norris, Charles B. Compressive Buckling Curves for Simply Supported Sandwich Panels with Glass-Fabric Laminate Facings and Honeycomb Cores. Forest Products Laboratory Report No. 1867. December 1958.

$\mu_{xy}$  is the modulus of rigidity associated with these directions,  $\sigma_{yx}$  is the Poisson's ratio of the contraction in the x direction to extension in the y direction due to a tensile stress in the y direction,  $\sigma_{xy}$  is similarly defined, and  $\lambda = 1 - \sigma_{yx}\sigma_{xy}$ .

It is required to express equation (79) of Report No. 1830<sup>3</sup>

$$K = \frac{64}{3z^2} \left( \frac{\gamma_3}{\eta} - \frac{9\gamma_2^2}{32\gamma_1\eta} + \gamma_4\eta \right) \quad (4)$$

in terms of  $\alpha$ ,  $\beta$ , and  $\gamma$  and to minimize it with respect to  $y$  and  $z$ . To do this,  $\gamma_1$ ,  $\gamma_2$ ,  $\gamma_3$ , and  $\gamma_4$ , according to equations (80), (81), (82), and (86) of Report No. 1830<sup>3</sup>, are expressed in these terms. Substituting equations (1), (2), and (3) into equations (80), (81), and (82) of Report No. 1830<sup>3</sup> and letting

$$\delta = \frac{1-\beta^2}{\gamma} + 2\beta$$

the following expressions are obtained:

$$\begin{aligned} \gamma_1 &= \frac{1}{4096\alpha^2} \left[ \frac{\alpha^2 z^4 + 1 + \frac{8\alpha^2 z^4}{(\alpha z^2 + 96)\alpha z^2 + 81}}{\alpha z^2 + 1} + \frac{8\alpha^2 z^4}{(81\alpha z^2 + 96)\alpha z^2 + 1} + \frac{34\alpha^2 z^4}{(\alpha z^2 + 8)\alpha z^2 + 1} \right] \\ \gamma_2 &= \frac{8}{4096\alpha^2} \left[ 1 + \frac{16\alpha^2 z^4}{(\alpha z^2 + 8)\alpha z^2 + 1} \right] \\ \gamma_3 &= \frac{16}{4096\alpha^2} \left[ 1 + \frac{8\alpha^2 z^4}{(\alpha z^2 + 8)\alpha z^2 + 1} \right] \end{aligned} \quad (5)$$

The expression for  $\gamma_4$  given by equation (86) of Report No. 1830<sup>3</sup> remains unchanged if the proper expressions for  $d_1$ ,  $d_2$ ,  $d_3$ , and  $T$  given by equations (57) and (83) of Report No. 1830<sup>3</sup> are used. These expressions are:

$$d_1 = z^2 E_x \left( 3z^2 + \frac{\gamma}{\alpha} \right)$$

$$d_2 = z^2 E_x \frac{\beta - \gamma}{\alpha}$$

$$d_3 = E_x \frac{1}{\alpha} \left( \frac{3}{\alpha} + \gamma z^2 \right)$$

$$T = \frac{1}{\alpha^2} \left[ (3 \alpha z^2 + 2\beta) (\alpha z^2 + 3) \right]$$

(6)

and from equation (86) of Report No. 1830<sup>3</sup>

$$\gamma_4 = \frac{T}{32 \lambda h^2 (f_1 + f_2)} \times$$

$$\left[ \frac{I + \frac{I (d_1 d_3 - d_2^2)}{T E_x^2} \left( \frac{S_x}{z^2} + S_y \right) \eta}{1 + \frac{\eta d_1 S_x}{E_x z^2} + \frac{\eta d_3 S_y}{E_x} + \frac{\eta^2 (d_1 d_3 - d_2^2) S_x S_y}{E_x^2 z^2}} + I_f \right] \quad (8)$$

in which

$$I = \frac{f_1 f_2}{f_1 + f_2} \left( c + \frac{f_1 + f_2}{2} \right)^2$$

$$I_f = \frac{f_1^3 + f_2^3}{12}$$

$$S_x = \frac{2 c f_1 f_2}{f_1 + f_2} \frac{E_x}{3 \lambda r h \mu'_{zx}}$$

$$S_y = \frac{2 c f_1 f_2}{f_1 + f_2} \frac{E_x}{3 \lambda r h \mu'_{yz}}$$

Where  $f_1$ ,  $f_2$ , and  $c$  are the thicknesses of the two facings and the core,  $h$  is the thickness of the sandwich,  $r$  the radius of the cylinder,  $\mu'_{zx}$  and  $\mu'_{yz}$  the moduli of rigidity of the core associated with the radial and axial and the radial and tangential directions.

Equation (4) can now be written as:

$$K = \frac{M_1}{\eta} + \frac{2I}{3 \lambda h^2 (f_1 + f_2)} \left[ \frac{M_2 \eta + M_3 \eta^2 S_x}{1 + M_4 \eta S_x + M_5 \eta^2 S_x} + \frac{I_f}{I} M_2 \eta \right] \quad (9)$$

in which:

$$M_1 = \frac{64}{3z^2} \left( \gamma_3 - \frac{9\gamma_2^2}{32\gamma_1} \right)$$

$$M_2 = \frac{T}{z^2}$$

$$M_3 = \frac{1}{\alpha^3} \left[ (3\gamma\alpha z^2 + 9 + 2\beta\gamma - \beta^2) \alpha z^2 + 3\gamma \right] \left( \frac{1}{z^2} + \theta \right)$$

$$M_4 = \frac{1}{\alpha^2} \left[ \alpha(3\alpha z^2 + \gamma) + \theta(3 + \gamma\alpha z^2) \right]$$

$$M_5 = M_3 \frac{\theta}{\frac{1}{z^2} + \theta}$$

$$\theta = \frac{\mu'_{zx}}{\mu'_{yz}} = \frac{S_y}{S_x}$$

Equation (9) is exactly the same as equation (93) of Report No. 1830<sup>3</sup> but applies to sandwich having orthotropic facings as well as isotropic facings because of the more general expressions given for  $M_1$ ,  $M_2$ ,  $M_3$ ,  $M_4$ , and  $M_5$ . This equation is to be minimized with respect to both  $y$  and  $z$ . It is convenient to proceed in a manner similar to that used in Report No. 1830<sup>3</sup>. If the core has moduli of rigidity so great that the shear deformations in it may be neglected,  $S_x$  becomes zero and equation (9) becomes

$$K_o = \frac{M_1}{\eta} + \frac{2(I + I_f)}{3\gamma h^2 (f_1 + f_2)} M_2 \eta \quad (10)$$

This equation can readily be minimized with respect to  $\eta$ . Thus,

$$K_o = \frac{L_o}{Q_1}$$

Where  $L_o = \sqrt{\frac{8}{3} M_1 M_2}$  minimized with respect to  $z$

and  $Q_1 = \sqrt{\frac{\lambda (f_1 + f_2) h^2}{I + I_f}}$

Then equation (9) may be written

$$N = \frac{M_1 Q_1}{L_o \eta} + \frac{2\eta}{3L_o Q_1 (1 + Q_2)} \times \left[ \frac{M_2 + M_3 \eta S_x}{1 + M_4 \eta S_x + M_5 \eta^2 S_x^2} + Q_2 M_2 \right] \quad (11)$$

Where  $N = \frac{KQ_1}{L_o}$

and  $Q_2 = \frac{I_f}{I}$

Values of N minimized with respect to  $\eta$  and  $z$  are given by the curve sheets presented.

### Design Curves

The critical buckling stress in the facings is given by:

$$p_{f_1} = \frac{NL_o}{Q_1} E_x \frac{h}{r} \quad (12)$$

in which  $h$  is the thickness of the sandwich,  $r$  is the mean radius of the cylinder, and  $E_x$  is the modulus of elasticity of the facings in the direction of the natural axis directed parallel to the axis of the cylinder. Values of  $Q_1$  are given by equation (119) of Report No. 1830<sup>3</sup> or

$$Q_1 = \sqrt{\frac{12\lambda \left(1 - \frac{c}{h}\right)^2}{\left[1 - \left(\frac{c}{h}\right)^3\right] \left[1 - \frac{c}{h}\right] - \frac{3c(f_1 - f_2)^2}{h^3}}}$$

Or, for facings of equal thickness,

$$Q_1 = 2 \sqrt{\frac{3\lambda}{\left(\frac{c}{h}\right)^2 + \frac{c}{h} + 1}}$$

Where  $c$  is the thickness of the core,  $f_1$ , and  $f_2$  are the thicknesses of the facings, and  $\lambda$  is one minus the product of the two Poisson's ratios of the facings (about 0.99 for the glass-fabric laminates considered in this report<sup>4</sup>).



Values of  $N$  are plotted in figures 1 to 9 against values of  $S_x$  and  $\frac{c}{h}$  for particular values of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\theta$ . These particular values were chosen because they apply to sandwich panels having glass-fabric facings and honeycomb cores.<sup>4</sup> There are three values of  $\alpha$ , 1.5, 1.0, and 0.67; one value of  $\beta$ , 0.6; one value of  $\gamma$ , 0.2; and three values of  $\theta$ , 0.4, 0.1, and 2.5.

Values of the parameter  $S_x$  are given by:

$$S_x = \frac{2cf_1f_2}{f_1 + f_2} \frac{E_x}{3\lambda rh\mu'_{zx}}$$

where  $\mu'_{zx}$  is the modulus of rigidity of the core associated with the radial and axial directions of the cylinder.

Values of the parameter  $\theta$  are given by:

$$\theta = \frac{\mu'_{zx}}{\mu'_{yz}}$$

where  $\mu'_{yz}$  is the modulus of rigidity of the core associated with the radial and tangential directions of the cylinder.

The values of  $L_o$  are associated with the values of  $\alpha$  according to the following tabulation:

$\alpha$	$L_o$
1.50	0.530
1.00	0.795
0.67	1.193

In calculating the values of  $N$ , sandwich having facings of equal thickness was assumed; however, the curves can be used for sandwich having facings of unequal thickness with very little error.

The abscissas of the curves in figures 1 to 9 are plotted to two scales, a large scale to the right and a small scale to the left. The two scales meet at an abscissa of unity and the large one overlaps

the small one so that some of the data from zero to unity are plotted twice, once to the large scale and once to the small scale. This was done so that more accurate values of  $N$  may be obtained in the range of  $S_x$  from zero to unity.

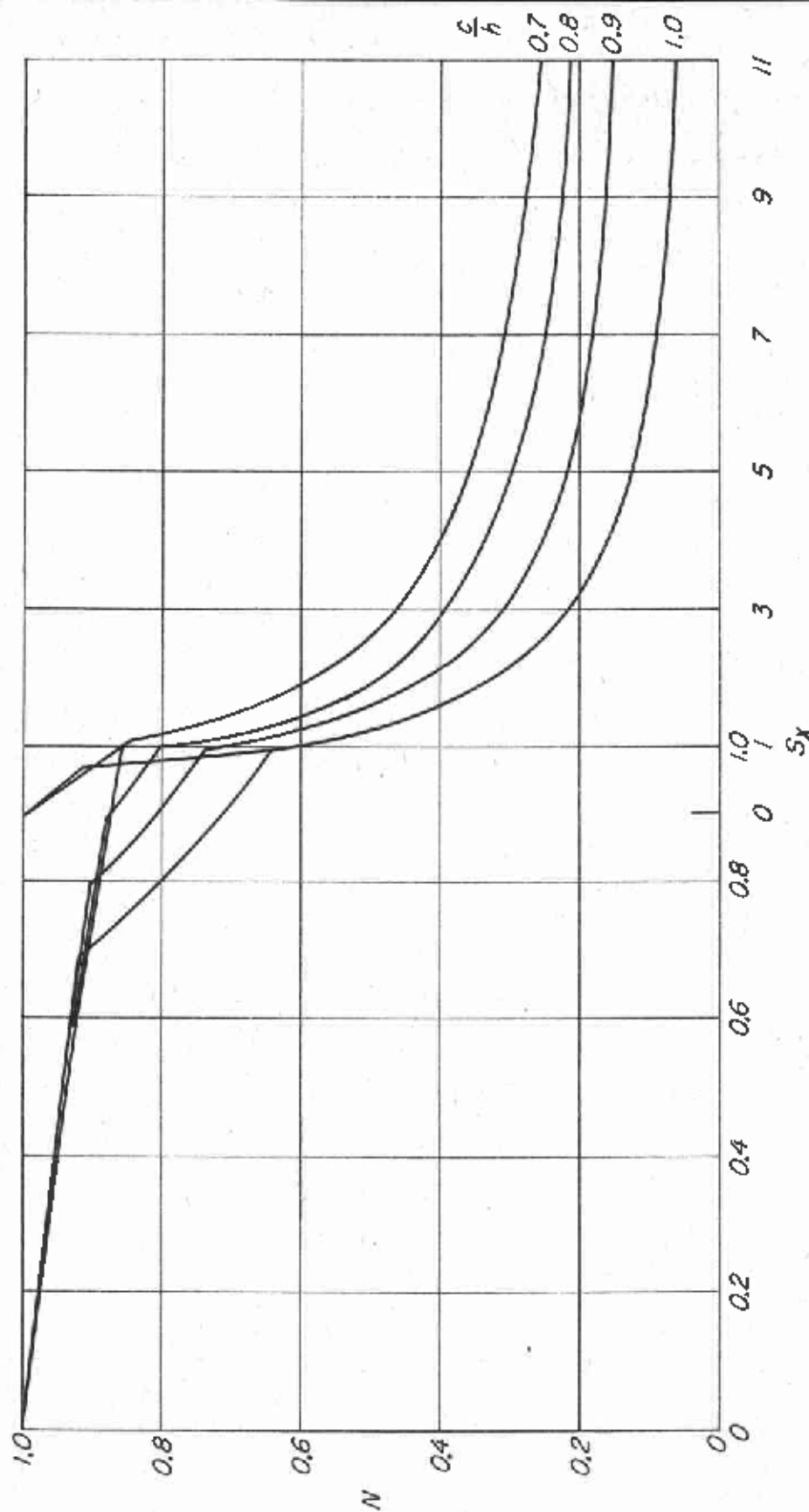


Figure 1.--Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 1.5$ ,  
 $\beta \approx 0.6$ ,  $\gamma = 0.2$ , and  $\theta \approx 0.4$ .

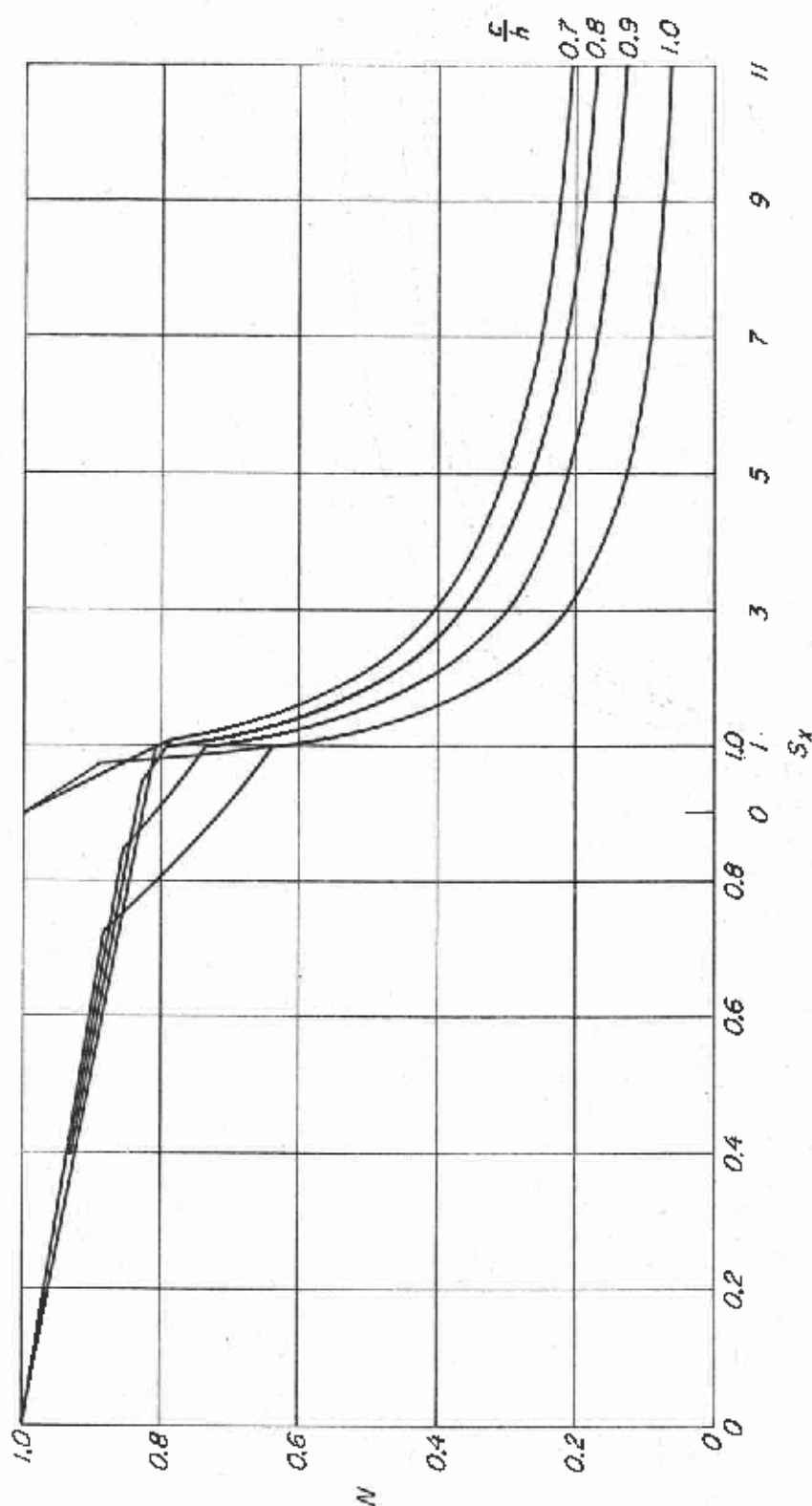


Figure 2. -- Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 1.5$ ,  
 $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\theta = 1.0$ .

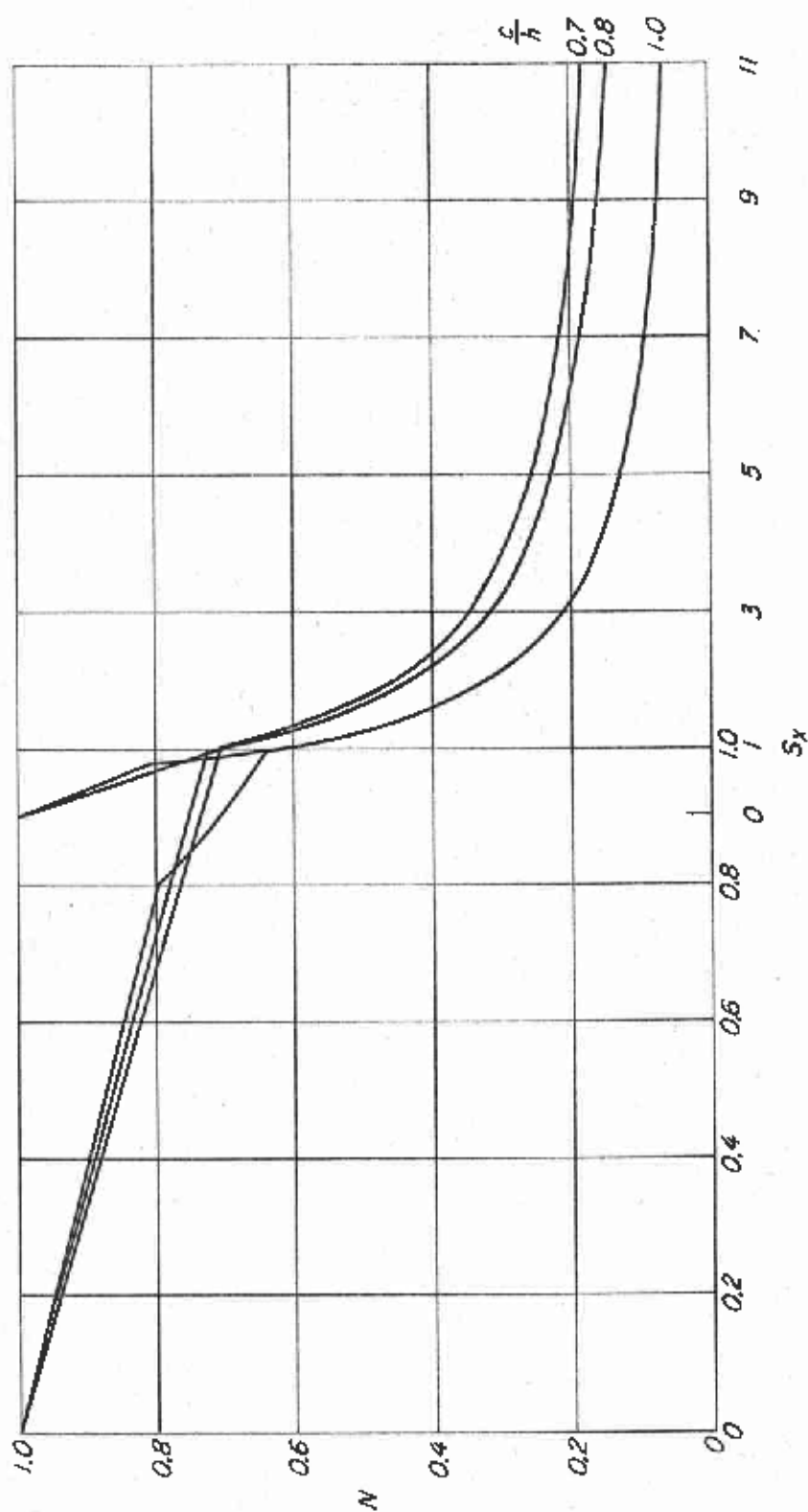


Figure 3.--Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 1.5$ ,  
 $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\phi = 2.5$ .

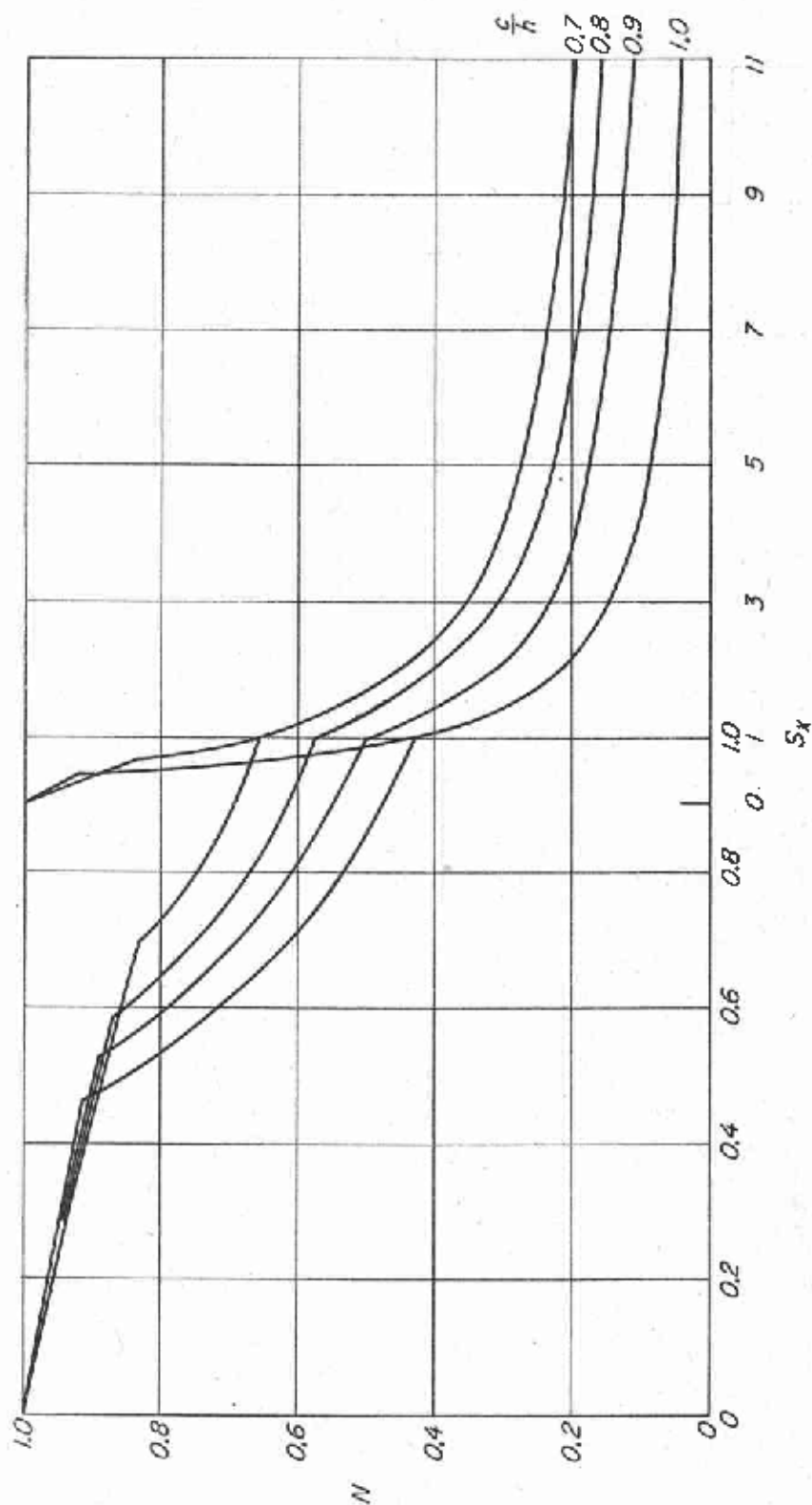


Figure 4.--Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 1.0$ ,  $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\theta = 0.4$ .

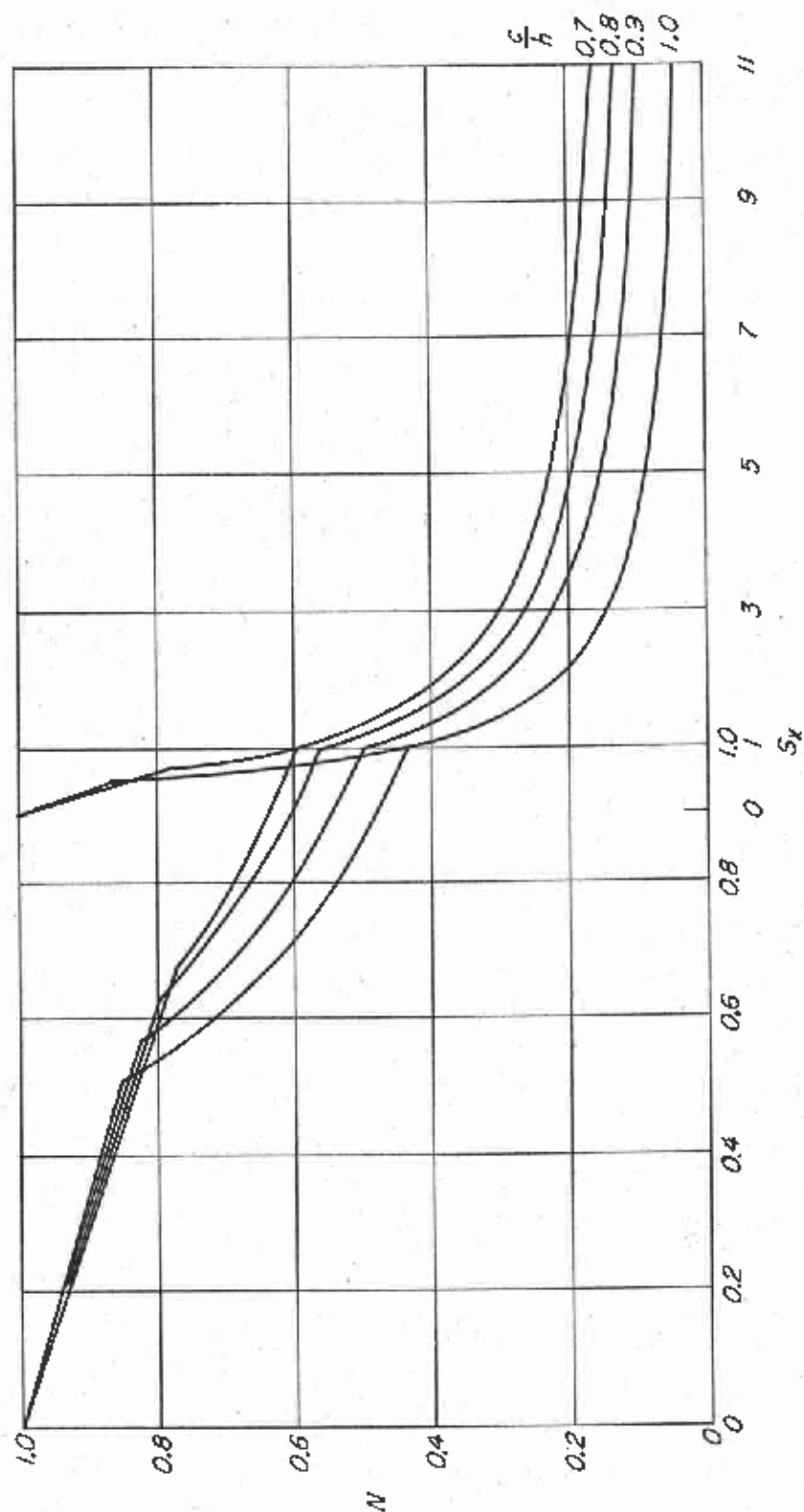


Figure 5. -- Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 1.0$ ,  $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\theta = 1.0$ .

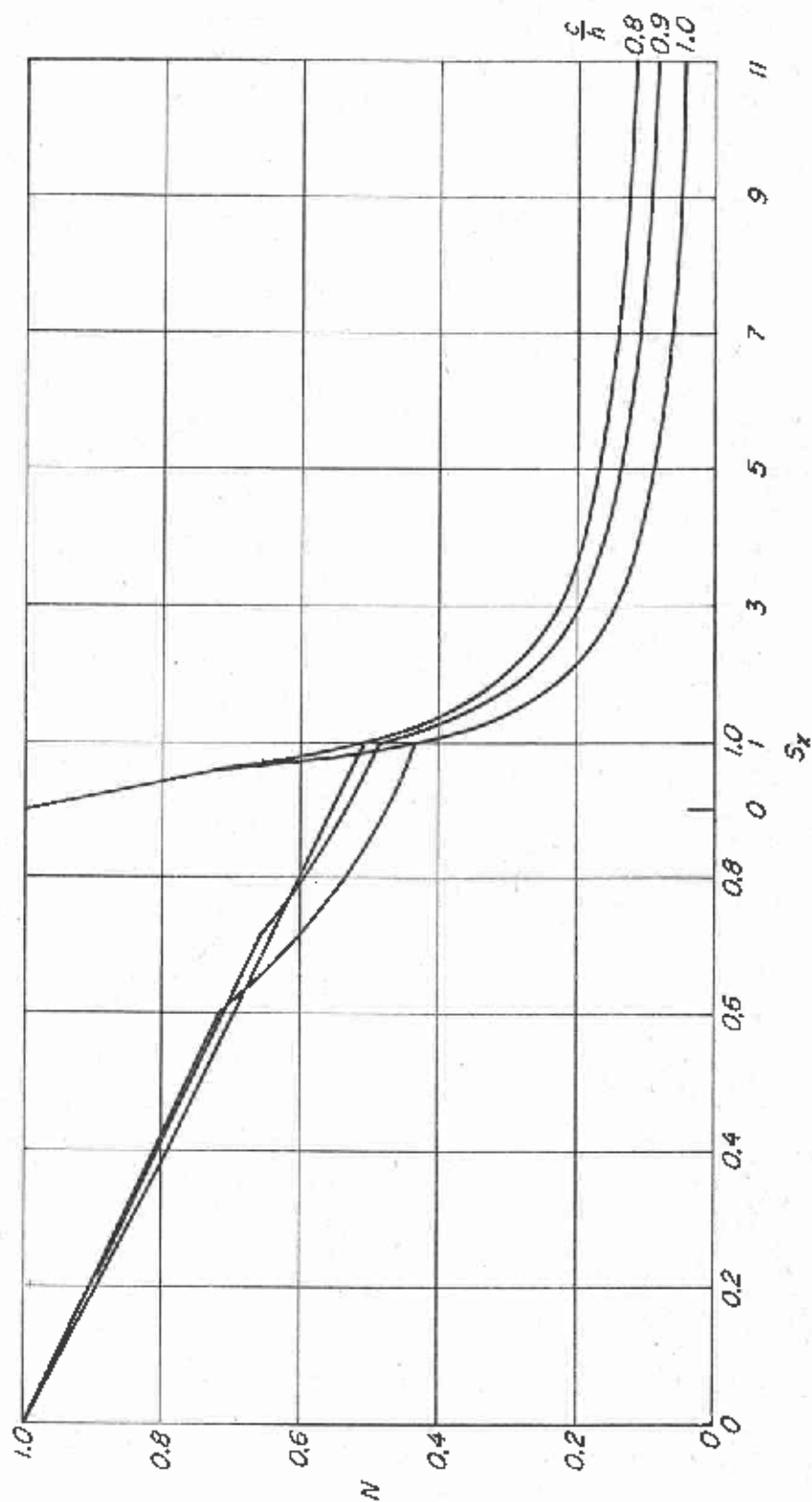


Figure 6. --- Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 1.0$ ,  $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\theta = 2.5$ .



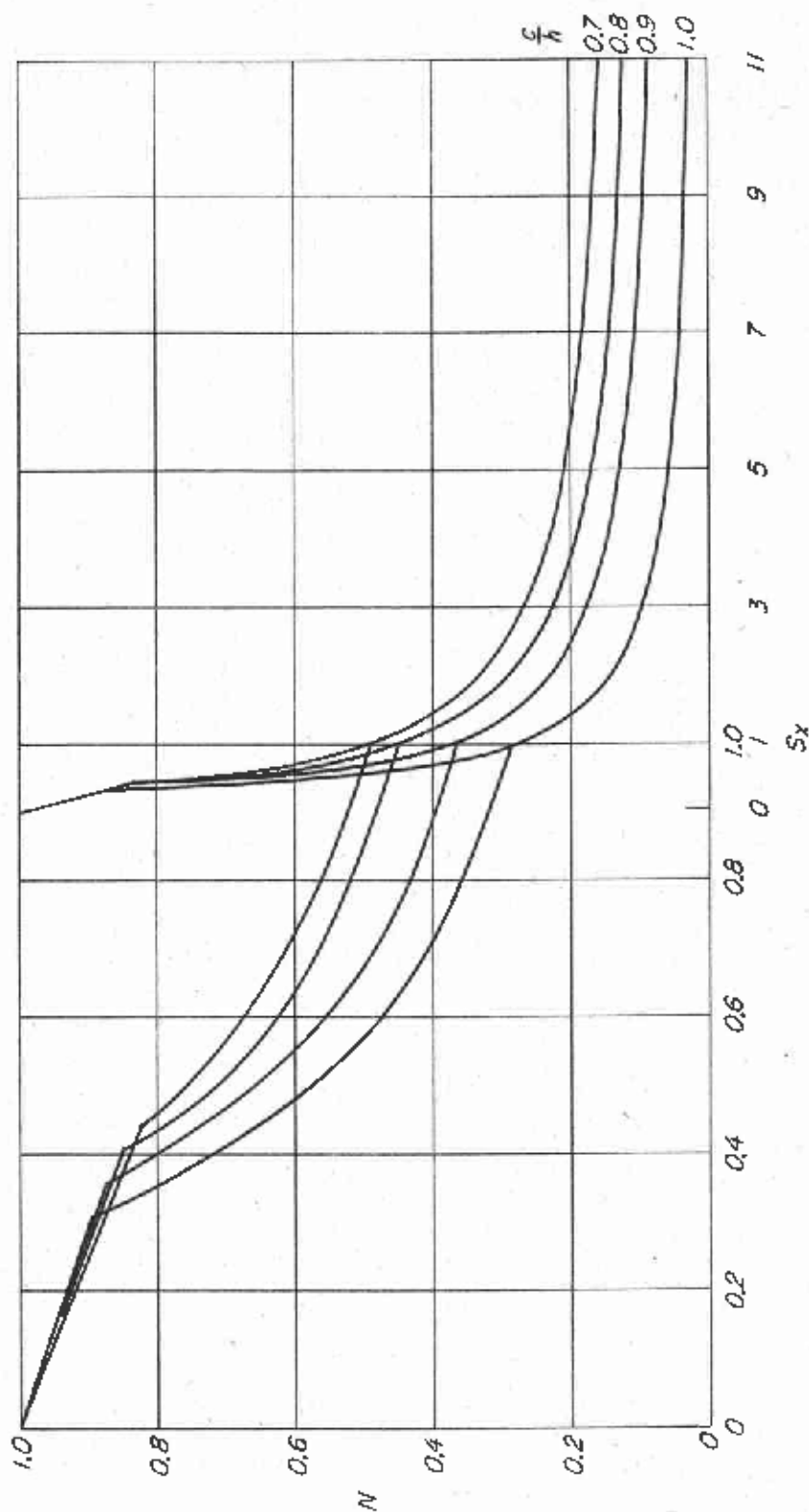


Figure 7. --- Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 0.67$ ,  $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\theta = 0.4$ .

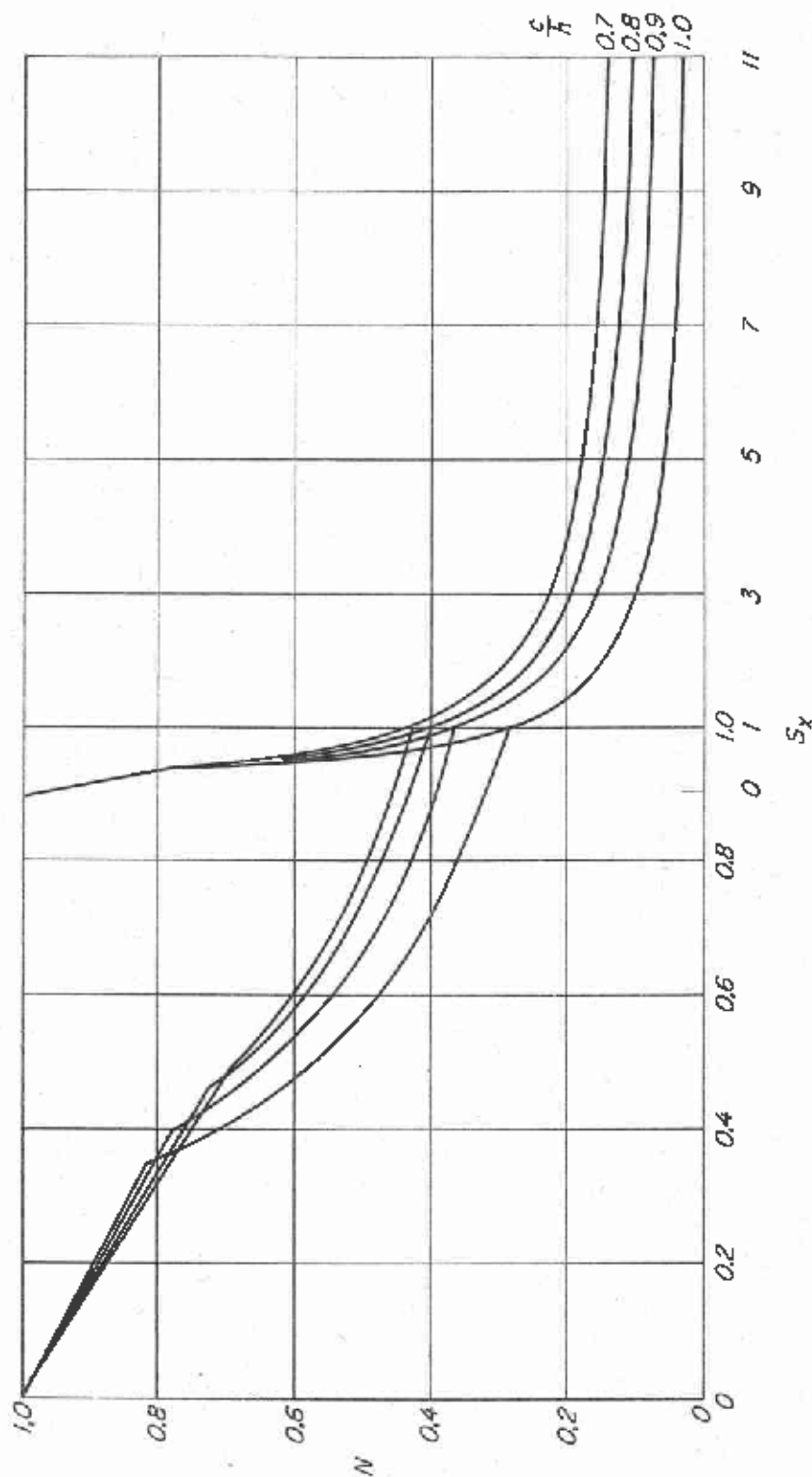


Figure 8. --Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 0.67$ ,  $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\theta = 1.0$ .

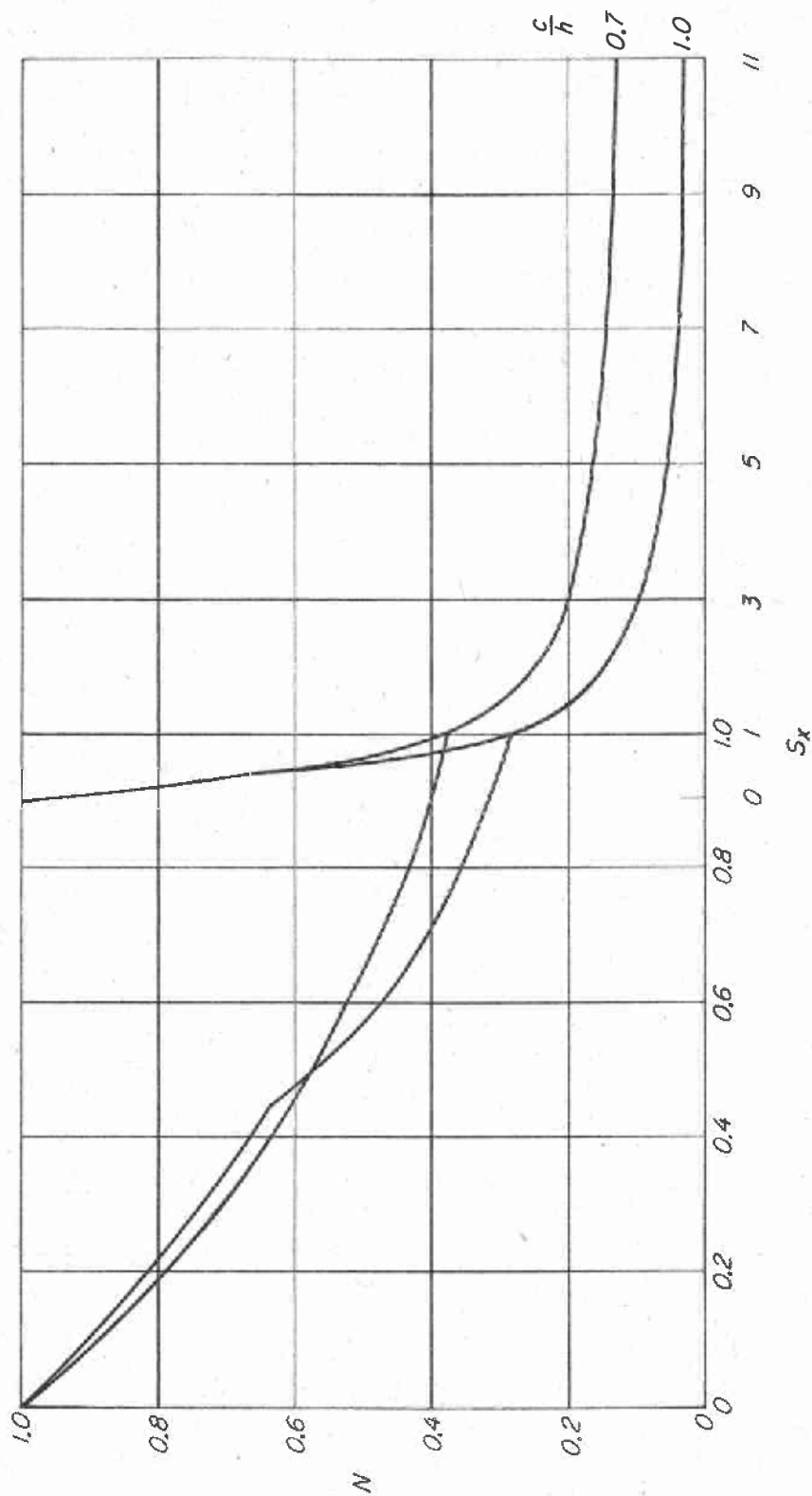


Figure 9. -- Values of  $N$  plotted against values of  $S_x$  and  $\frac{c}{h}$  for  $\alpha = 0.67$ ,  $\beta = 0.6$ ,  $\gamma = 0.2$ , and  $\theta = 2.5$ .