Estimation of Cost Functions of Northwest Beef Feedlots From Expected Marginal Revenue Observations

Technical Bulletin 120

AGRICULTURAL EXPERIMENT STATION
Oregon State University
Corvallis, Oregon

In cooperation with Farm Production Economics Division Economic Research Service U.S. Department of Agriculture

February 1973
AUTHORS: J. B. Johnson is an Agricultural Economist, Farm Production Economics Division, Economic Research Service, United States Department of Agriculture, and Albert N. Halter is a Professor of Agricultural Economics, Oregon State University.

This work was carried out under a cooperative project between the Farm Production Economics Division, Economic Research Service, United States Department of Agriculture, and the Oregon Agricultural Experiment Station, entitled “The Pacific Northwest Beef Industry: Its Characteristics and Potential.”
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J. B. JOHNSON and ALBERT N. HALTER

ABSTRACT

An integration procedure for estimating cost functions from cross-sectional firm data is developed. The procedure is based upon the hypothesis that firms equate expected marginal revenue to marginal costs in deciding how much output to produce. Since the total cost function is the integral of the marginal cost function plus some arbitrary constant considered the total fixed cost, and by hypothesis the expected marginal revenue is equal to marginal cost, then the total cost function is the integral of the expected marginal revenue plus some constant.

A procedure is developed for obtaining the expected marginal revenues from a cross-section of beef feedlot operators. An estimate of the total cost function for a range of beef output is obtained by discrete summation. The estimated total cost function is statistically tested against values of total costs obtained from the ordinary cost equation. Implications for further research are outlined.

Key words: Expected marginal revenue, marginal costs, total cost function, beef feedlot.

Introduction

Families of cost functions derived for groups of firms of different sizes provide a portion of the data needed by entrepreneurs contemplating a change in firm size. These families of cost functions also provide a portion of the data needed by public policy makers to assess the effects of specific price levels on the number of firms that will remain in an industry, on industry supplies of product, and to assess other intra-industry and interregional policy considerations.

The family of cost functions most commonly estimated is the family of short-run average total cost functions. If these families of curves can be assumed to be generated from observations taken from several different sizes of firms, each observed at several levels of output, economic theory specifies the tangential relationships needed for construction of the long-run average cost function, sometimes referred to in the economic literature as the "firm planning curve."

Although a number of long-run average cost functions have been estimated for several types of agricultural firms, a doubt has been cast as to the usefulness of such studies attempting to quantify long-run average
cost curves. Upchurch (1961) contends that despite economists' vast experience with studies of this nature, techniques used in quantifying or defining long-run average cost curves have been particularly fuzzy.

However, Upchurch (1961) makes a plea for more work in the area of defining differences in cost relating to firm size. He suggests a series of studies be conducted for different types of farming and the same techniques of "costing" be used throughout the series. With such a series of cost analyses, both entrepreneurs and public policy makers would have more reliable data on which to base size adjustment decisions. The intention of this study is to present an unconventional technique for doing this "costing."

Statement of Problem

A variety of procedures have been used to estimate cost functions of firms. Data requirements for some of the estimation procedures are burdensome in terms of cost of data acquisition, time, and maintaining rapport with respondents. The purpose of this study was to develop an efficient estimation procedure for firm cost functions that requires a minimum of data.

Objectives of the Study

The objectives of this report are the following:

1. Present those elements of existing economics theory which are relevant to specifying the relation between firm output and the cost function of the firm.

2. Propose a hypothesis which, if not rejected, would provide a method for estimating firm cost functions consistent with economic theory.

3. Develop a statistical model for quantifying the relations specified in the economic model and provide the basis for testing the hypothesis.

4. Test the hypothesis as specified in the statistical model, using cross-sectional survey data obtained from a sample of Pacific Northwest beef feedlot firms.

Order of Research and Presentation

The development of the economic theory underlying the hypothesis to be tested is presented in the next section of this bulletin.

A statistical model was developed which provided the decision rule required to judge whether the hypothesis specified was rejected. This model also specified the form in which the data were to be collected and prepared to perform the statistical test.

Next, the procedures specified for the acquisition of data are presented. Historical data were taken from the secondary sources, and a
questionnaire was designed to collect data from the sample respondents. From the sample data, computations were made in preparation for the statistical test. Then calculations and tests of the statistical hypothesis are presented.

A summary of the conclusions that can be drawn from the tests of the hypothesis, the “risky” nature of the hypothesis test, the implications for use of the methodology developed, and the needs for further research are presented in the last section of the bulletin.

**Underlying Economic Theory of Firm Cost Functions**

Several of the procedures used to estimate cost functions for firms yield results which are inconsistent with outcomes deduced from economic theory.

Presented in this section are those elements of economic theory which are relevant to specifying the derivation of the cost function of the firm. A hypothesis is proposed which, if not rejected, would allow the estimation of firm cost functions which are consistent with outcomes deduced from economic theory. A review of methods commonly used in the estimation of firm cost functions is also presented.

**Definition of Short-Run and Long-Run Concepts**

A firm cost function expresses cost as an explicit function of the quantity of output. A firm can increase its output by intensifying production in a given plant, or by increasing plant size and producing a greater volume in a larger plant (assuming a single-plant firm).

Intensification is generally thought of as a short-run concept. In the short run the entrepreneur has time to increase and vary the use of variable inputs in the production process, but not enough time for modification of the size of fixed plant.

Increasing plant size is a long-run concept. The firm has time to increase all factors of production. That is, the firm has time to adjust the use of inputs which are variable in the short run, and at the same time employ a different and better technique of production, provided that the production technique is already a part of the “state of the arts.” Such change is not viewed as a technological improvement: the change in size is necessary to make use of more elaborate technology. When all factors of production are increased in like proportion, economies (or diseconomies) realized are economies (or diseconomies) of scale. If factors are increased by different proportions, economies (or diseconomies) realized are economies (or diseconomies) of size.
Derivation of Short-Run Cost Functions

To determine the total cost function of a firm analytically, economic theory specifies that the following information is needed:
1. The firm’s production function;
2. The firm’s expansion path; and
3. The firm’s cost equation.

Assume for simplicity the following production function, defined for one time period:

\[ Y = f(X_1, X_2 | X_3, \ldots, X_n), \]

where \( Y \) is the output of the firm, \( X_1, X_2 \) are variable inputs one and two, and \( X_3, \ldots, X_n \) are fixed inputs.

Given the production function \( Y = f(X_1, X_2 | X_3, \ldots, X_n) \), the marginal productivities of the variable inputs may be calculated. Define

\[ f_j = \frac{\partial Y}{\partial X_j}, \quad \text{where} \quad j = 1, 2. \]

For a two-variable input production function, \( f_1 \) and \( f_2 \) are the marginal productivities which can be calculated and \( f_3, \ldots, f_n = 0 \).

Assume that the firm buys its variable inputs in a perfectly competitive input market. That is, the variable input prices to the firm do not change with increased use of the input by the firm. Thus \( r_j, j = 1, 2 \), the per unit variable input prices, are constants. To combine variable inputs in optimum economic proportions, the ratios of the marginal productivities to the input prices must be equal for all variable inputs. Therefore, the condition

\[ \frac{f_1}{r_1} = \frac{f_2}{r_2} \]

must hold. Solving this condition, the firm’s expansion path is then

\[ f_1 r_2 - f_2 r_1 = 0. \]

The firm’s cost equation may be expressed as \( C = r_1 X_1 + r_2 X_2 + b \), where \( b \) is the total cost of the fixed resources for the production period.

Solving the following three equations simultaneously, the firm’s cost function is determined:

\[
\begin{align*}
Y &= f(X_1, X_2 | X_3, \ldots, X_n), \quad \text{production function,} \\
C &= r_1 X_1 + r_2 X_2 + b, \quad \text{cost equation, and} \\
0 &= f_1 r_2 - f_2 r_1, \quad \text{expansion path.}
\end{align*}
\]

Solved simultaneously, the firm’s cost function is expressed as a function of output, \( Y \):

\[ TC = \phi(Y | X_3, \ldots, X_n) + b. \]

One way of obtaining an empirical estimate of the cost function would be to estimate the production function and then follow the above procedure. However, consider the following two-equation system as an alternative formulation leading to a method of estimating the cost function:
\[
\frac{dTC}{dy} = \frac{P_o - w_i}{TC - b} = \Phi'(Y),
\]

where \( P_o \) is the market price of the output, and \( w_i \) is the deviation (positive or negative) of the market price of the output from the firm's expectation of output price.

The cost function is identified in this system, as \( TC \) and \( Y \) are the endogenous variables and \( w_i \) and \( P \) are the exogeneous variables (Klein, 1956). That is, since there are as many exogeneous variables omitted from the cost function as there are endogeneous variables, the cost function is identified (Tinter, 1957). Thus, the condition that expected marginal revenue is equal to marginal cost, \( P_o - w_i = \Phi'(Y) \), identifies the cost function.

**Profit-maximization conditions**

To develop the above condition it is assumed, first, that firms are not uncertain about product price, i.e., \( P_o \) is known without error, or \( w_i = 0 \). Secondly, if it is assumed that the firm has its inputs combined in expansion path proportions for a given size of plant, the firm's profit equation can be written in terms of output and product price as follows:

\[
\pi = P_o Y - TC.
\]

The total cost function, \( TC \), is expressed in terms of output, \( Y \). As total cost is the sum of total variable costs and total fixed costs, total cost can be expressed as \( TC = \Phi(Y) + b \), where \( \Phi(Y) \) are variable costs and \( b \) represents fixed costs.

To determine the profit-maximizing level of output, the first derivative of this function, with respect to \( Y \), is set equal to zero,

\[
\frac{d\pi}{dY} = P_o - \Phi'(Y) = 0 \quad \text{and solved,} \quad P_o = \Phi'(Y).
\]

The first-order condition for profit maximization requires marginal revenue to be equal to marginal cost.

The second-order condition for profit maximization requires that the marginal cost function be increasing at the profit-maximizing output level. That is, \( \Phi''(Y) > 0 \). The second derivative of the profit function is \( d^2\pi/dY^2 \), which for profit maximization must be negative.

Therefore, \( d^2\pi/dY^2 = -\Phi''(Y) < 0 \).

But \( -\Phi''(Y) < 0 \) may be rewritten as \( \Phi''(Y) > 0 \) by multiplication of both sides of the inequality by \((-1)\). In summary, by expressing the profit function in terms of output, the two conditions for an unconstrained profit
maximization are (1) that marginal cost equals marginal revenue (output price) and (2) that marginal cost is increasing at the level of output produced.¹

There is one case where the firm would not operate, given the above two conditions were satisfied. That is where \( P_o < \Phi(Y)/Y \) for a particular output level. If product price will not cover short-run average variable cost, then the firm will choose not to produce.

In some instances firms are not capable of achieving the profit-maximizing output level. Firms may be so restricted in operating capital that the maximum level of output they can achieve is less than that where \( P_o = \Phi'(Y) \), where \( \Phi''(Y) > 0 \). In such cases the firm might be capable of producing at an output level where \( P_o = \Phi'(Y) \), \( \Phi''(Y) < 0 \). However, this is the profit-minimizing level of output. Therefore, a firm so constrained by operating capital will choose to operate at those levels of output where \( P_o > \Phi'(Y) \), where \( \Phi''(Y) \) may be less than, greater than, or equal to zero.

Modification of profit-maximization conditions

The assumption was made that the firm knows the market price of the commodity it is producing at the time the decision to produce is made. This assumption may not deviate far from reality in certain manufacturing industries where the decision to produce and the marketing of the product is separated by only an hour, a few days, or a week. However, in agricultural production, the time interval between the date of production planning and the marketing of the product is usually several weeks, a crop season, or a feeding period. As the time interval lengthens between the decision to produce and the sale of product, one would expect the price of the output to become less certain to the producing firm, as the forces determining price in the market have more time to adjust to conditions both internal and external to the market. Consequently, most agricultural production firms do not make production decisions based on some certain market price, but rather on expectations of the market price formulated at the beginning of the production period.

Therefore, the profit function for the firm could be rewritten as follows:

\[
\pi = E(P) \cdot Y - TC,
\]

where

\[
E(P) = \sum_{i=1}^{m} f(P_i) \cdot P_i,
\]

¹This is assuming that the producer sells in a perfectly competitive product market.
\( i = 1, 2, \ldots, m, \)

- \( P_i \) is the price interval \( i \), and
- \( f(P_i) \) is the frequency with which the \( i \)th price interval occurs

and \( m \) is the number of price intervals in the domain of relevant prices.

Therefore, the firm bases its production decision on the expected value of the distribution of anticipated product prices.

Substituting \( \Phi(Y) + b \) for total cost into the profit equation above will give:

\[
\pi = E(P) \cdot Y - \Phi(Y) - b.
\]

The first-order condition for unconstrained profit maximization is:

\[
E(P) = \Phi'(Y)
\]

and the second-order condition remains unchanged;\(^2\) that is,

\[
\Phi''(Y) > 0.
\]

The firm will choose not to produce if \( E(P) < \Phi(Y)/Y \).

Earlier \( \Phi'(Y) = P_o - w_i \) was seen to identify the cost function.

To show that \( P_o - w_i = E(P) \), let \( z_i = P_o - P_i \), that is, the difference between the actual market price and the \( i \)th price interval from the frequency function \( f(P_i) \). Now substitute for \( P_i \) into:

\[
E(P) = \sum_{i=1}^{m} f(P_i) \cdot P_i
\]

\[
= \sum_{i=1}^{m} f(P_o - z_i) \cdot (P_o - z_i)
\]

\[
= \sum_{i=1}^{m} f(P_o - z_i) \cdot P_o - \sum_{i=1}^{m} f(P_o - z_i) \cdot z_i
\]

\[
E(P) = P_o \cdot E(z_i).
\]

Now only if \( w_i = E(z_i) = 0 \) would \( E(P) = P_o \). Hence, \( E(z_i) \) is what was called \( w_i \) above.

Therefore, \( P_o \cdot E(z_i) = P_o - w_i = E(P) = \Phi'(Y) \) expresses the first-order condition for a profit maximum.

If it is possible to determine empirically \( E(P) \), then \( TC \) can be found by integrating.

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\(^2\) This formulation of the first- and second-order condition assumes that the decision-maker’s utility function in risky situations is linear. When the utility function is nonlinear, then the strict equality may not hold. However, for testing the hypothesis of this research, the equality condition is the natural one to assume. For a discussion of the conditions for profit maximization under risk, see McCall (1971).
Thus, $\frac{\partial C}{\partial q} = \int_{Y}^{r} \phi(Y) \, dY = \int_{Y}^{E(P)} \phi(Y) \, dY$.

The question remains whether a total cost function can be found by integration of the first-order condition for firm profit maximization. Previous attempts to estimate firm cost functions have assumed that the firms from which data were taken had resources combined in expansion path proportions. However, most of these studies did not assume that the firms were operating at profit-maximizing levels of output. Therefore, "cost functions" were derived from cost equation data by either synthesis or regression methods.

Statement of hypothesis

The hypothesis is that if the empirical total cost function is constructed (integrated) from output price expectation data taken from a cross-section of firms of like technology but different output levels, then the values given by the cost equation will be identical to the values obtained from the cost function.3

Review of Methods Used in Estimating Firm Cost Functions

Previous studies which have attempted to estimate cost functions of firms (long-run and/or short-run curves) can be categorized by the methodologies employed for estimation of the cost function. In one group of studies cost functions were fit by regression techniques to cost equation data obtained primarily from cross-sectional surveys of similar firms. The other group of studies includes those commonly referred to in economic literature as "cost synthesis" studies.

Regression cost functions

Numerous studies have been made of the costs of operating various plants in a given industry for a stated time period. Cost equation data are obtained for each firm through cross-sectional surveys. Such a cross-section of costs of operation for a given period must "catch" many of the firms in some sort of maladjustment which in important cases is not readily explained by the usual regression of cost against volume (Erdman, 1944).

Usually a regression line is fitted to cost-output observations of firms grouped as similar on some a priori basis. These cost-volume data are presented as a scatter diagram, with an average regression line fitted to the

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3 TC values will be compared with C values at the same output level. Henceforth, in this analysis C will be referred to as the cost equation of a cross-section of firms.
scatter. This curve shows the average relation between plant output and cost.

However, such a curve combines and confuses cost changes that result from the more complete utilization of a plant of a given size with the changes that accompany changes in size. Attempts to stratify the sample into meaningful subsamples based on size of plant can reduce the effects on cost introduced through the confusion of size with level of utilization.

Due to the nature of the regression technique, an equation fitted to cross-sectional data will indicate costs above the minimum levels for a plant of given size operating at that level of output. The slope of the short-run average variable cost curve for a particular size plant will understate the change in average cost that could be realized by a change in volume of production (Bressler, 1945).

Another major disadvantage of the regression technique for deriving cost curves directly from cross-sectional farm cost survey data is one of statistical measurement often referred to as the “regression fallacy.” That is, individual firms with similar fixed resources are often placed into the same subsample. However, firms with like fixed resources often operate at different levels of output because of limitations on other resources, risk and uncertainty, and related reasons; a regression equation fitted to such a scatter of cost-volume points gives a cost curve which lies above the “true” cost curve (Carter and Dean, 1961).

In summary, there seem to be three major problems in using regression analyses to estimate firm cost functions from cross-sectional firm data:

1. There is no assurance that firms observed are not in some sort of maladjustment.
2. Stratification of firms into subsamples does not eliminate the problem of “regression fallacy.”
3. The properties of regression analysis preclude the possibility of obtaining an estimated cost function which will coincide with the same function as defined in economic theory.

A similar problem exists in the use of regression techniques to estimate production functions.

As Carlson suggests “... if we want the production function to give only one value for the output of a given service combination, the function must be so defined that it expresses the maximum production obtainable from the combination at the existing state of technical knowledge” (1939, page 4).

Consequently, regression estimates of the production function do not yield a function consistent with economic theory. A production function estimated by regression techniques underestimates the theoretical production function.
Synthesized cost functions

Most methods of synthesizing cost functions are designed to obtain firm (or plant) cost functions. The two most common methods are partial budgeting and complete cost synthesis.

Partial budgeting is most commonly used when plant size is given. Costs are then synthesized for various combinations of variable resources and/or for the plant operating at a given percent of total capacity. Where constraints are numerous, e.g., plant capacity defined in terms of several resource constraints, linear programming has been used.

Complete cost synthesis involves the synthesis of both variable and fixed costs. Researchers using complete cost synthesis have allowed the combinations and levels of variable resources to change, and have also changed the technical organization of the plant to assess the changes induced in the firm's cost structure.

Bressler cites the two main problems in the synthesis of cost curves: First, increasing variable costs may be overlooked, although some of the engineering data will provide a clue in this matter. Second, it is frequently held that some costs are forgotten in this process, and the actual costs that will eventually characterize the plant will be higher than the estimates (1945, p. 536).

Cost synthesis techniques have been adopted from the work of engineers and architects. Their estimates of costs are made from known cost data obtained from experimental results and cross-sectional surveys of firms, and tempered by their knowledge of the principles of physics and engineering. They usually assume constant marginal productivities for a variable resource used in conjunction with some fixed facility. This precludes them from recognizing the possibility of increasing variable costs, as Bressler suggests in his first point. Also, these studies have dealt primarily with the synthesis of those inputs which are measurable in quantity and often can be assessed for quality. Consequently, differences in productivity and costs due to management, quality of labor, and so forth, are not explicitly recognized in their cost synthesis (Knutson, 1958).

Specification of the Statistical Model for the Hypothesis Test

The statistical model developed in this section provides a means to test the conjecture (hypothesis) that the derived total cost function constructed (integrated) from output price expectation data taken from firms of like technology but different volumes of production gives values identical to the cost equation of this same set of firms. The statistical model outlines the form in which data will have to be prepared to perform the hypothesis test, and provides a rule for deciding whether to reject the
hypothesis once the values of the data have been determined. Upon formulation of the statistical model, data were taken from respondents, summarized, and the test of hypothesis was performed.

Data Series

From interviews, the following data were obtained from each respondent:

1. \[ E(P)_k = \sum_{i=1}^{m} f(P_i)_k P_i \]
   
   where:
   - \( E(P)_k \) is the product price expectation of the \( k \)th respondent,
   - \( f(P_i)_k \) is the frequency distribution of product price associated with the \( k \)th respondent, and
   - \( P_i \) is price interval \( i \).

2. \[ C_k = \sum_{j=1}^{J} r_{jk} X_{jk} + b_k \]
   
   where:
   - \( C_k \) is the level of cost from the cost equation of the \( k \)th respondent,
   - \( r_{jk} \) is the price of input \( j \) for the \( k \)th respondent,
   - \( X_{jk} \) is the level of input use of the \( j \)th input in the \( k \)th firm, and
   - \( b_k \) is the level of fixed costs of the \( k \)th firm.

3. \( Y_k \) is the output level of the \( k \)th respondent or firm.

As previously derived, \( E(P) = \Phi'(Y) \) at the profit-maximizing output level for the firm. If \( E(P)_k \) has been observed, then \( E(P)_k \) can be taken as an estimate of \( \Phi'(Y_k) \) and henceforth denoted as \( \Phi'(Y_k) \).

The total cost function, \( TC \), was shown to be derived from the integration of the marginal cost function, \( \Phi'(Y) \). However, since \( E(P)_k \) is obtained from each of the firms operating under like technology but different volumes of output, \( Y_k \), \( TC \) is defined as a discrete summation, as \( Y_k \) values are discrete observations. In the limit, that is, by increasing the number of \( Y_k \) observations, the discrete summation will yield an estimate equivalent to that obtained by integration of continuous function.

To carry out the summation, let the output levels of each respondent, \( Y_k \), within like technologies, be arranged in ascending order, i.e., from the lowest \((k = 1)\) to the highest \((k = N\) for technology I and \( k = N_II \) for technology II) output level. The summation is given by:

\[ TC = \Phi'(Y_1)(Y_1) + \Phi'(Y_2)(Y_2-Y_1) + \cdots + \Phi'(Y_N)(Y_N-Y_{N-1}) + \bar{b}. \]

The above expression represents the total cost of producing at output level \( Y_k \), where \( k = 1, 2, \cdots, N \). In a cross-sectional analysis, different firms
of the same technology will have fixed assets of different ages and, hence, different valuations will be placed on fixed facilities. Therefore, it is necessary to obtain a weighted total fixed cost.

The weighted total of fixed cost for firms in a particular technology is defined as:

\[
\bar{b} = \frac{1}{N} \sum_{k=1}^{N} \frac{Y_k b_k}{\sum_{k=1}^{N} Y_k}
\]

Now each \(TC_k\) value, taken from cost function \(TC\) above, can be compared with the \(C'_k\) value for the same output level, \(Y_k\), where \(C'_k\) is defined as follows:

\[
C'_k = \bar{b} + \sum_{j=1}^{I} r_{jk} X_{jk}
\]

**Test Statistic**

Define \(V_k = TC_k - C'_k\), \(k=1, 2, ..., N\)

\[
W_1 = V_1
\]

\[
W_2 = V_2 - V_1
\]

\[
\vdots
\]

\[
W_k = V_k - V_{k-1}
\]

\[
\vdots
\]

\[
W_N = V_N - V_{N-1}
\]

To clarify the notation problem and multiple definitions, a simple diagram may aid in interpretation of the test statistic. In Figure 1 the \(TC\) function is shown after the discrete summation, and \(C'\) is shown relative to its associated output level \(Y\). The \(V_k\) are the deviations between \(TC_k\) and \(C'_k\). The \(W_k\) are defined for each output level but are not shown in Figure 1. They are differences in successive \(V_k\) values, except \(W_1\) which is equal to \(V_1\).

By defining \(W_k\) as the difference between \(V_k\) and \(V_{k-1}\), \(W_k\) does not include those deviations between \(TC\) and \(C'\) at levels of \(Y < Y_k\). The removal of the previous deviations assures that the errors are not compounded due to the summation process.

To calculate the test statistic, the absolute values of \(W_k\) are arranged in ascending order. Each \(W_k\) value is assigned a rank number. The smallest
Figure 1. A hypothetical TC function, hypothetical C' equation, and V_k values for four volume levels.

The absolute value of W_k is assigned the rank of 1, the second smallest value the rank of 2, and so forth, until the largest value of W_k is assigned the rank of N, N being the number of W_k values calculated (Wine, 1964).

Once rank numbers have been assigned to the absolute values of W_k, the W_k values are separated into two subsamples, one subsample consisting of those W_k with negative sign and the other with those of positive sign. The rank numbers of the W_k values in each subsample are then summed.
Let \( S_N = \text{minimum} \) where \( S^+ \) is the sum of rank numbers of all positive \( W_k \) values,
\( S^- \) is the sum of rank numbers of all negative \( W_k \) values, and
\( n_1 + n_2 = N \).

Providing \( n_1 = n_2 \), then \( S_N \) is compared with the tabled critical value of \( S_N \). When \( n_1 \neq n_2 \), a further calculation is required to obtain the test statistic.

First, find the sum of the ranks for the subsample with the smaller number of observations and call the sum \( S_s \). Supposing \( n_1 \) were the smallest subsample, compute \( S_L = n_1 \left( n_1 + n_2 + 1 \right) - S_s \). The value to be compared with the tabled critical value is then
\[
S_N = \text{minimum} \left( S_s, S_L \right)
\]
where \( S_s \) is the subsample with the smallest number of observations and \( S_L \) the subsample with the largest number of observations.

The test described above was developed by Wilcoxon, although it is sometimes referred to as the Mann-Whitney test.\(^5\) It was developed to facilitate the analysis of two-sample problems where sample observations were not paired. In this analysis, the two samples are (1) the observations at given output levels, \( Y_k \), from the \( TC \) function, and (2) the observations at given output levels, \( Y_k \), from \( C' \) equation.

In this test, it is assumed that two random and independent samples are drawn from two distributions which have the same form but possibly different values of the location parameter (e.g., mean or median). Thus under the usual null hypothesis, the random and independent samples are assumed to come from a single population. The alternative hypothesis may be expressed so that the test is either one-sided or two-sided (Wine, 1964).

In this particular analysis, the Wilcoxon rank-sum test provides the appropriate decision rule needed for the test of hypothesis. As the test is nonparametric, no assumption is necessary concerning the distribution of the deviations, the \( W_k \) values. As there were no \textit{a priori} reasons to assume that the \( W_k \) values are normally distributed, this nonparametric rank-sum test is more appropriate than the two-sample \( t \) test, which would be applicable if the \( W_k \) values were assumed to be normally distributed.

Wine (1964) reports that he and other researchers have shown that if all assumptions of the two-sample \( t \) test hold, the rank-sum test is valid and the power of efficiency of the rank-sum test relative to the two-sample \( t \) test is 0.95. Thus, in order to provide the same power, approximately 5 percent more observations are required for the rank-sum test than for the \( t \)

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\(^5\) The test was suggested to the authors by Fred Ramsey, Department of Statistics, Oregon State University.
test. Where applicable, the rank-sum test has one additional advantage—it is more easily computed than alternative parametric tests.

**Critical Values of Wilcoxon Test Statistic**

Wine (1964) has tabled critical values of $S_N$ for both the 0.05 and 0.01 significance levels. If $n_1 \neq n_2$, $n_1$ as designated in the table heading is taken to be the subsample with the smallest number of observations and $n_2$ is taken to be the subsample with the largest number of observations.

Given the level of significance, and $n_1$ and $n_2$, the tabled critical value is that which is common to both the $n_1$ column and $n_2$ row.

If, for a given significance level, $S_N$ (calculated) < $S_N$ (tabled critical value), the hypothesis is rejected.

**Statement of Hypotheses**

Formally, the null and alternative hypotheses given in the previous section may be specified as follows:

$$H_0 : \frac{1}{N} \sum_{k=1}^{N} W_k = 0,$$

and the TC function is not statistically different in location from the cost equation, $C'$.

$$H_A : \frac{1}{N} \sum_{k=1}^{N} W_k \neq 0,$$

and the TC function is statistically different from the cost equation, $C'$ (two-tailed).

**Acquisition of Data for Hypothesis Test**

The Pacific Northwest beef feedlot industry was chosen as the economic sector from which a sample of firms was selected to provide data necessary for the hypothesis test. Presented below are sampling procedures; general characteristics of the feedlot firms; source, derivation, and use of $E(P)h$ values; and the source of data and derivation of the cost equation.

**Sampling Procedures**

Twenty-one beef feedlot firms were selected as sample respondents for this analysis. The respondents were selected because: (1) they were known to have historical records of sufficient detail from which cost of production data could be taken, and (2) an indication of the types of production technology employed by these firms was available from a 1967 survey.

---

6 In this analysis a beef feedlot is defined as a firm which feeds cattle to slaughter weight.
The sample respondents were interviewed during October 1969. Questions asked of the respondents were framed in the context of their 1969 feeding year (see Appendix A). Several questions were asked to update information obtained by previous interviews concerning the 1967 feeding year.

General Characteristics of Sample Feedlot Firms

The sample of 21 feedlot firms was divided into two technology levels. One level includes those feedlot firms with "incomplete" or "no" milling facilities. The second level includes feedlot firms with "complete" milling facilities. Milling facility inventories were used as a proxy measure of technology to specify degree of completeness. Firms with "complete" milling facilities were more specialized firms; that is, either they were single enterprise firms or firms in which the feedlot was the primary enterprise.

As several firms had ceased feeding cattle, only 14 of the 21 respondents intended to feed cattle in 1969. Four of these firms had "incomplete" or "no" milling facilities; the other 10 firms had "complete" milling facilities.

The likeness or similarity among a cross-sectional sample of firms is one prerequisite for the application of this cost estimation method. The sample observations must satisfy those restrictions necessary to assure the existence of a short-run production function, although the explicit estimation of a short-run production function is not required in this cost estimation method.

In essence, those restrictions to assure the existence of a short-run production function are that each firm (1) has time to complete its productive process, (2) does not adjust to a new technology, or (3) does not change the levels of fixed inputs during the period of analysis. Operationally, the first restriction presents only minimal problems in assuring a similarity among sample firms. However, the empirical measurement of the second and third restrictions requires careful consideration. To select a sample of like firms, the researcher must have knowledge of, and recognition of, similarities in the fixed inputs of the firms. To preserve likeness among firms, none can be selected which are changing firm size in the period of observation.

Previous researchers often have attempted to assure likeness in sample firms by selecting firms of similar "size," where size is interpreted strictly as a measure of level of output (Stollsteimer, Bressler, and Boles, 1961). Dean suggests that likeness can be achieved through sampling on firm size (Dean, 1959). However, his interpretation of "size" is more closely related to technology. He suggests that "conceptually we distinguish between changes in scale and technological changes by saying that
an increase in size may employ a different and better technique, provided it is already a 'state of the arts'. This change is not a technological improvement: the increased firm size was necessary to make use of more elaborate technology" (Dean, 1959).

Dean further suggests three alternative criteria for empirically measuring firm size, all requiring an expression of a level of technology in physical or value terms. These alternative criteria are the (1) amount of fixed equipment, (2) output capacity, and (3) input capacity (Dean, 1959). However, if prior information is available to the researchers, sample firms can be selected on likeness in technology. This precludes the need for Dean's suggested transformation of technology characteristics into size equivalents to facilitate the selection of a cross-sectional sample of like firms.

A qualitative assessment of the 10 firms in the "complete mill" technology level revealed the following similarities in technology characteristics:

1. Each of the 10 firms was capable of processing a complete ration in its stationary milling installation.
2. Each of the 10 firms delivers feedstuffs to fence-line bunks in self-unloading trucks.
3. Lots in which cattle are held and fed have no covering shelter. Most lots used by each of the 10 firms have hard-surfaced aprons abutting the fence-line bunk, and a watering facility within each lot.
4. Square footage of lot space per animal was fairly uniform among the 10 firms observed. Adjustments in square footage allotment for animal size and seasonal weather conditions were similar among firms.

Additional variables more amenable to quantitative assessment were evaluated (Table 1). To assure that each of the 10 firms had sufficient time to complete its productive process, a measure of the "days on feed" for animals in each feedlot was made. In-weights and out-weights of the cattle were observed to determine the homogeneity among firms of one of the major variable inputs (feeder cattle) and the homogeneity of the total salable product (fed beef).

The other two restrictions which assure the existence of a short-run production function are that the firm has no opportunity to adjust production technology during the period for which data are taken and has insufficient time to change the level of fixed factors of production. As previously discussed, a failure to satisfy either of these restrictions would imply a change in firm size, employing Dean's interpretation of "firm size." Using one of the three alternative criteria for measuring firm size, which measures technology in physical terms, each of the 10 respondents in the "complete mill" technology level was asked if firm size was determinant and
stable for the period of analysis. The criterion used was that of output capacity. Particular questions asked were the following:

Could you feed more cattle at this time than you have in your lot? If yes, would you give those reasons why you choose not to feed more? If no, what factors in your current operation restrict the feeding of additional cattle?

Each of the five respondents giving a “no” answer to the question gave identical responses to the question on what factors restricted their current operation—their feedlot pens were fully utilized. Such a response suggests that each of these firms has reached its physical maximum output level under its present level of technology.

The other five firms in the “complete mill” technology level reported that they could be feeding more cattle in their feedlots. Four of the five gave an identical reason for their choosing not to feed more—additional feeder cattle were available only at prices which would not allow them to operate in a break-even or profit position. The other respondent was shifting his feeding operation to another region. From the reasons given, it was apparent there would be no immediate incentives for the firms to adapt new technologies—i.e., firm size would remain constant for the analysis period. Therefore, each of the 10 firms selected on \textit{a priori} evidence for inclusion in the “complete mill” technology level satisfied those restrictions necessary to preserve similarity among firms and satisfied those restrictions necessary to insure the existence of a short-run production function, a prerequisite to obtaining a short-run firm cost function. (An assessment of the same qualitative and quantitative variables for firms in the “incomplete mill” or “no mill” technology level also indicated that restrictions were satisfied, which would assure the existence of a short-run firm cost function.)

20
Derivation of $E(P)$ Values

To estimate the total cost function for firms in a given technology level, it was necessary to determine $E(P)$ for each firm within that level. As was stated earlier, $E(P)_k = \overline{P}'(Y_k)$. That is, the expected value of the distribution of expected prices was defined to be an estimate of the marginal cost of firm $k$ producing at output level $Y$ within a given technology level.

The interviews conducted to obtain $E(P)_k$ were completed in two stages. First, through the use of historical frequency distributions, the respondents were asked questions concerning their sales prices on cattle marketed over the 10-year period from 1959 to 1968. Once the respondent was familiar with the concept of price frequency distributions and the interviewing techniques, he was asked to characterize his price expectations for the most recent lot of cattle placed on feed. It was assumed that the respondent had no influence on the selling price of the last lot of cattle placed on feed. (See Appendix B for a discussion of questions and responses which were used to perform an independent test of this assumption.)

Frequency distributions of fed cattle prices

The frequency distributions of fed cattle prices used in the interviews were constructed from monthly price data reported by the Livestock Division, Consumer and Marketing Service, U.S. Department of Agriculture, Portland, Oregon (USDA Livestock Division, 1959-1968). Data summarized were for a 10-year period of operation at the Portland market, starting January 1, 1959, and ending December 31, 1968.

The data are reported by two weight classes for choice grade steers, good grade steers, choice grade heifers, and good grade heifers. For each of the eight weight-grade classes of cattle, price frequency distributions were established as follows:

1. The domain in the monthly average prices over the 10-year period was determined.
2. One-dollar price intervals were specified within the domain.
3. The frequency of monthly prices occurring within each price interval was calculated.
4. The empirical frequency distribution was plotted on an 8½ x 11 inch card.
5. In addition to the historical frequency distribution, hypothetical frequency distributions were constructed over the same domain of prices. Each of these six hypothetical frequency distributions was plotted on an 8½ by 11 inch card.
Use of historical frequency distributions in obtaining \( E(P) \)

The grade-weight class of fed cattle most often sold by the feedlot operator was determined at the time of the interview. The operator was shown the seven cards picturing the historical frequency distributions for that particular grade-weight class of cattle. He was asked to rank these seven distributions by visual inspection, indicating first that plot which most closely approximated the distribution of prices he received for his cattle sales of that grade-weight class over the 10-year period, 1959-1968, and then indicating the one least like his.

There was no a priori reason for expecting the feedlot operator to identify any particular plot, as some feeders may sell continually above the market average in all months sales are made. Others might sell continually below the market average.

The purpose of the question was to acquaint the feedlot operator with price frequency distributions. No use of the historical price distributions was made in the cost of production estimates.

Frequency distributions for feedlot operator’s next sale

Daily price data for the September through December period of 1968 were summarized for each of the eight grade-weight classes of cattle sold through the Portland market and country markets within the state of Oregon (USDA, Livestock Division) to obtain the possible domain of prices from the date of interview to the possible time of sale.

Choice grade fed cattle prices exhibited a four-dollar price domain during the September through December period of 1968. From inspection of data available, seven frequency distributions were constructed for choice grade fed cattle prices (Table 2).

Good grade fed cattle prices exhibited a six-dollar price domain during the September through December period of 1968. From inspection of the data available seven frequency distributions were constructed for good grade fed cattle prices (Table 3). Each of the frequency distribu-

<table>
<thead>
<tr>
<th>Price interval</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.25</td>
<td>.16</td>
<td>.15</td>
<td>.20</td>
<td>.35</td>
<td>.10</td>
<td>.10</td>
</tr>
<tr>
<td>B</td>
<td>.25</td>
<td>.34</td>
<td>.35</td>
<td>.30</td>
<td>.15</td>
<td>.20</td>
<td>.15</td>
</tr>
<tr>
<td>C</td>
<td>.25</td>
<td>.34</td>
<td>.30</td>
<td>.35</td>
<td>.15</td>
<td>.30</td>
<td>.20</td>
</tr>
<tr>
<td>D</td>
<td>.25</td>
<td>.16</td>
<td>.20</td>
<td>.15</td>
<td>.35</td>
<td>.40</td>
<td>.55</td>
</tr>
</tbody>
</table>

1 Using a four-dollar price domain and one-dollar price intervals.
Table 3. Frequency distributions of good grade fed cattle prices

<table>
<thead>
<tr>
<th>Price interval</th>
<th>Distribution</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>.167</td>
<td>.06</td>
</tr>
<tr>
<td>B</td>
<td>.167</td>
<td>.10</td>
</tr>
<tr>
<td>C</td>
<td>.167</td>
<td>.34</td>
</tr>
<tr>
<td>D</td>
<td>.167</td>
<td>.34</td>
</tr>
<tr>
<td>E</td>
<td>.166</td>
<td>.10</td>
</tr>
<tr>
<td>F</td>
<td>.166</td>
<td>.06</td>
</tr>
</tbody>
</table>

1 Using a six-dollar price domain and one-dollar price intervals.

Use of frequency distribution in obtaining E(P)

Each feedlot operator interviewed was asked (1) when he placed his most recent lot of cattle on feed, (2) the grade at which he intended to sell the cattle, (3) the length of time he intended to feed the cattle, (4) the selling weight of the cattle, and (5) a four-dollar domain of prices within which he expected to receive a price for his fed cattle (a six-dollar domain of expected prices was obtained for those selling good grade cattle).

Given the price domain specified by the feedlot operator for the most recent lot of cattle placed on feed, these prices were assigned along the horizontal axes (price scale) of the seven frequency distributions for the grade-class of fed beef specified. Once the prices were assigned to these plots the feedlot operator was asked to rank the frequency distributions, indicating first the one which most closely approximated his expectations of the prices he would receive for the most recent lot of cattle placed on feed, then indicating the second most likely, and so forth.

The frequency distribution that he selected as most likely was used to calculate the E(P) value which provided an estimate of the marginal cost. The calculation was performed as follows:

\[ E(P)_k = \sum_{i=l}^{m} f(P_i) P_i, \]

where

- \( l \) is minimum price expected, plus \$.50,
- \( m \) is maximum price expected, minus \$.50,
\[ f(P_i) \] is the frequency with which the \( i \)th price interval occurs, depending on which of seven frequency distributions were selected by the feedlot operator, and

\[ P_i \] is the midpoint of the price interval \( i \).

**Derivation of Cost Equation**

The total hundredweight of gain, \( Y_k \), produced by each firm in 1969 was estimated. Given the estimate of annual output, information on the total quantities of variable inputs and the prices of inputs was obtained for the 1969 production period. Prices and quantities of variable inputs were assumed observed without error. It was also assumed that each firm was so small in terms of the total market for an input that it could not affect the price it paid for an input. (See Appendix B for a discussion of the questions and responses used to make an independent test of this assumption.)

As each of the sample respondents had been interviewed prior to their 1969 production period, several questions were asked to update information obtained from their 1967 records. Changes in feeding methods and machinery inventories since the 1967 production period were obtained. This information, in conjunction with information on this period, was used to calculate each firm's 1969 level of total fixed costs. Uniform procedures were used to calculate the cost equation for each firm.

**Results**

In this section, values of the required data obtained for firms within each technology level are presented and used to test the TC-C' hypothesis. Discussed first is the construction of the TC function and C' equation for each technology level. Then \( V_k \) and \( W_k \) values for firms in each technology level are presented. Finally, the statistical tests of hypothesis are performed for each technology level.

**Construction of TC Function and C' Equation**

For each level of technology, the sample observations were assembled in ascending order of annual volume for the construction of the TC function and C' equation. Data needed for construction of the TC function and C' equation for firms at the "no mill" or "incomplete mill" technology level are summarized in Table 4.

The weighted annual fixed costs for firms in this technology level are \( b \approx \$4,222 \).

The \( TC_k \) and \( C'_k \) values for each successive volume level of firm in the "no mill" or "incomplete mill" technology level are presented in Table 5.
Table 4. Construction of TC function and $C'$ equation for firms at the "no mill" or "incomplete mill" technology level

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Gain (cwt.)</th>
<th>Total variable cost</th>
<th>Expected price</th>
<th>Fixed cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>206</td>
<td>$ 6,088</td>
<td>$27.00</td>
<td>$ 78</td>
</tr>
<tr>
<td>5</td>
<td>1,548</td>
<td>39,995</td>
<td>30.50</td>
<td>1,336</td>
</tr>
<tr>
<td>6</td>
<td>2,837.5</td>
<td>58,906</td>
<td>26.73</td>
<td>3,950</td>
</tr>
<tr>
<td>10</td>
<td>5,085.5</td>
<td>113,375</td>
<td>28.50</td>
<td>5,418</td>
</tr>
<tr>
<td>Total...</td>
<td>9,677.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 From the cost equation, $C'$.

Table 5. Total cost values for firms at the "no mill" or "incomplete mill" technology level

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Gain (cwt.)</th>
<th>Total cost</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>206</td>
<td>$ 9,784</td>
<td>$ 10,310</td>
</tr>
<tr>
<td>5</td>
<td>1,548</td>
<td>50,715</td>
<td>44,217</td>
</tr>
<tr>
<td>6</td>
<td>2,837.5</td>
<td>85,178</td>
<td>63,128</td>
</tr>
<tr>
<td>10</td>
<td>5,085.5</td>
<td>149,246</td>
<td>117,597</td>
</tr>
</tbody>
</table>

1 From the cost function, $TC_k$.
2 From the cost equation, $C'_k$.

Table 6. Construction of TC function and $C'$ equation for firms at the "complete mill" technology level

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Gain (cwt.)</th>
<th>Total variable cost</th>
<th>Expected price</th>
<th>Fixed cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1,016</td>
<td>$ 24,194</td>
<td>$30.00</td>
<td>$ 4,256</td>
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<tr>
<td>12</td>
<td>10,410</td>
<td>239,659</td>
<td>28.26</td>
<td>20,270</td>
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<tr>
<td>13</td>
<td>14,370</td>
<td>382,639</td>
<td>27.72</td>
<td>23,655</td>
</tr>
<tr>
<td>17</td>
<td>15,503</td>
<td>441,449</td>
<td>27.73</td>
<td>15,348</td>
</tr>
<tr>
<td>16</td>
<td>18,490</td>
<td>633,079</td>
<td>31.05</td>
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</tr>
<tr>
<td>14</td>
<td>19,650</td>
<td>472,971</td>
<td>26.02</td>
<td>17,799</td>
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<td>15</td>
<td>20,875</td>
<td>507,178</td>
<td>27.05</td>
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<td>18</td>
<td>20,898</td>
<td>496,217</td>
<td>30.36</td>
<td>20,112</td>
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<tr>
<td>20</td>
<td>96,238</td>
<td>2,408,772</td>
<td>27.88</td>
<td>56,445</td>
</tr>
<tr>
<td>21</td>
<td>164,450</td>
<td>3,310,743</td>
<td>26.75</td>
<td>80,337</td>
</tr>
<tr>
<td>Total...</td>
<td>381,900</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

1 From the cost equation, $C'$.
and plotted in Figure 2. These values were computed by following the algorithms previously specified in this bulletin.

Data obtained for construction of the $TC$ function and $C'$ equation for firms at the “complete mill” technology level are presented in Table 6. The weighted annual fixed costs for firms in this technology level are $\bar{b} \approx 54,347$.

The $TC_k$ and $C'_k$ values for each successive volume level for firms in the “complete mill” technology level are presented in Table 7 and plotted
Table 7. Total cost values for firms at the “complete mill” technology level

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Gain (cwt.)</th>
<th>Total cost(^1)</th>
<th>Total cost(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1,016</td>
<td>$84,827</td>
<td>$78,541</td>
</tr>
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<td>12</td>
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<td>460,072</td>
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<td>17</td>
<td>15,503</td>
<td>491,490</td>
<td>495,796</td>
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<tr>
<td>16</td>
<td>18,490</td>
<td>584,236</td>
<td>687,426</td>
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<tr>
<td>14</td>
<td>19,650</td>
<td>607,459</td>
<td>527,318</td>
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<tr>
<td>15</td>
<td>20,875</td>
<td>640,595</td>
<td>561,525</td>
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<tr>
<td>18</td>
<td>20,898</td>
<td>641,293</td>
<td>550,564</td>
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<td>96,238</td>
<td>2,741,995</td>
<td>2,463,119</td>
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<td>21</td>
<td>164,450</td>
<td>4,566,666</td>
<td>3,365,090</td>
</tr>
</tbody>
</table>

\(^1\) From the cost function, \(TC_k\).
\(^2\) From the cost equation, \(C'_k\).

in Figure 3. The values were computed by following the algorithms previously specified.

**Calculation of \(V_k\) and \(W_k\) Values**

For each firm \(V_k\) was defined to be the following:

\[
V_k = TC_k - C'_k, \quad k = 1, 2, \ldots, N.
\]

\(N\) = the total number of observations in each technology level.

\(W_k\) values are defined to be the following:

\[
W_1 = V_1,
\]

\[
W_2 = V_2 - V_1,
\]

\[
\vdots
\]

\[
W_k = V_k - V_{k-1}
\]

\[
\vdots
\]

\[
W_N = V_N - V_{N-1}.
\]

\(V_k\) values were calculated for firms in the “no mill” or “incomplete mill” technology level (Table 5). Using the \(V_k\) values calculated, \(W_k\) values were calculated as previously described. Both \(V_k\) and \(W_k\) values for firms in the “no mill” or “incomplete mill” technology level are presented in Table 8.

The \(W_k\) and \(V_k\) values for firms in the “complete mill” technology level are presented in Table 9. The \(V_k\) values were calculated from data in Table 7.
Figure 3. Plot of cost equation values, $C'$, and integral cost function values, $TC$, "complete mill."
Table 8. $V_k$ and $W_k$ values for firms at the "no mill" or "incomplete mill" technology level

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Gain (cwt.)</th>
<th>$V_k$ values</th>
<th>$W_k$ values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>206</td>
<td>$-$526</td>
<td>$-$526</td>
</tr>
<tr>
<td>5</td>
<td>1,548</td>
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</tr>
<tr>
<td>10</td>
<td>5,085.5</td>
<td>31,649</td>
<td>9,599</td>
</tr>
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</table>

Table 9. $V_k$ and $W_k$ values for firms at the "complete mill" technology level

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Gain (cwt.)</th>
<th>$V_k$ values</th>
<th>$W_k$ values</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
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<td>$6,286</td>
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<tr>
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<td>1,201,576</td>
<td>922,700</td>
</tr>
</tbody>
</table>

The $W_k$ values for firms at the "no mill" or "incomplete mill" technology level are plotted in Figure 4. $W_k$ values for firms at the "complete mill" technology level are plotted in Figure 5.

If the $W_k$ values plotted in Figure 4 and Figure 5 oscillate from positive to negative around $W_k = 0$, there would be reason to expect that the null hypothesis would not be rejected. That is, the mean of the $W_k$ values would be expected not to be significantly different from zero, given that the magnitudes of the oscillations above and below $W_k = 0$ were similar.

In Figure 4, three of the four $W_k$ values lie above $W_k = 0$, each by a greater magnitude than the only negative $W_k$ value. In Figure 5, for the first eight $Y_k$ values, it can be seen that the associated $W_k$ values lie both above and below the line by similar magnitudes. The ninth and tenth $W_k$ values lie far above $W_k = 0$. However, these extreme observations have only a small influence on the outcome of the statistical test.

After visual inspection of $W_k$ values for both levels of technology, the statistical test of hypothesis was performed for both levels to determine if the mean of the $W_k$ values were significantly different from zero.
Performance of Statistical Tests

$W_k$ values for firms in the two technology levels were ranked in ascending order of their absolute values.

"No mill" or "incomplete mill" technology level

The absolute values of $W_k$ values for firms in this technology level were assigned the following ranks:

<table>
<thead>
<tr>
<th>$W_k$ values</th>
<th>Absolute values</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-526$</td>
<td>$526$</td>
<td>1</td>
</tr>
<tr>
<td>7,024</td>
<td>7,024</td>
<td>2</td>
</tr>
<tr>
<td>9,599</td>
<td>9,599</td>
<td>3</td>
</tr>
<tr>
<td>15,552</td>
<td>15,552</td>
<td>4</td>
</tr>
</tbody>
</table>

The rank numbers were then separated into two subsamples, one subsample consisting of $W_k$ values with a negative sign and the other with those of positive sign.

Negative sign: (1)
Positive sign: (2, 3, 4).
The rank numbers of the $W_k$ values in each subsample were summed.

$$S'_{n_1} = 9$$
$$S'_{n_2} = 1,$$

As $n_1 \neq n_2$, a further calculation was required to obtain the test statistic.

The subsample of ranks representing $W_k$ values of negative sign was the smaller, $n_2 = 1$. $S_s$, the total of the ranks of this subsample, was 1. To compute $S_L$, the total of the ranks of the larger subsample, the following equation was used:

$$S_L = n_2 (n_1 + n_2 + 1) - S_s$$
$$S_L = 1 (1 + 3 + 1) - 1$$
$$S_L = 3.$$

The $S_N$ value to be compared with the critical value is

$$S_N = \text{minimum } (S_s, S_L).$$
Therefore,
\[ S_N = \text{minimum} \,(1, 3), \text{ and} \]
\[ S_N = 1. \]

Critical values of \( S_N \) at the 0.05 level of significance for two-sided tests where \( n_1 + n_2 \geq 8 \) are available in published tables, given that \( n_1 \) and \( n_2 \) are each \( \geq 4 \). However, for \( n_1 + n_2 < 8 \), the critical values of \( S_N \) must be calculated (Wine, 1964). The calculated critical value of \( S_N \) for the two-tailed test at a significance level of \( \alpha = 0.50 \) was 1.

For rejection of the null hypothesis, it is required that the calculated value of \( S_N \) be less than the tabled critical value of \( S_N \). For the “no mill” or “incomplete mill” technology level the calculated value of \( S_N \) was equal in value to the critical value of \( S_N \). Therefore, the test of hypothesis failed to reject the null hypothesis at the 0.50 level of significance.

"Complete mill" technology level

The absolute values of \( W_k \) values for firms at this technology level were assigned the following ranks:

<table>
<thead>
<tr>
<th>( W_k ) values</th>
<th>Absolute values</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,071</td>
<td>1,071</td>
<td>1</td>
</tr>
<tr>
<td>6,286</td>
<td>6,286</td>
<td>2</td>
</tr>
<tr>
<td>11,659</td>
<td>11,659</td>
<td>3</td>
</tr>
<tr>
<td>-27,392</td>
<td>27,392</td>
<td>4</td>
</tr>
<tr>
<td>-33,209</td>
<td>33,209</td>
<td>5</td>
</tr>
<tr>
<td>50,009</td>
<td>50,009</td>
<td>6</td>
</tr>
<tr>
<td>-98,884</td>
<td>98,884</td>
<td>7</td>
</tr>
<tr>
<td>183,331</td>
<td>183,331</td>
<td>8</td>
</tr>
<tr>
<td>188,147</td>
<td>188,147</td>
<td>9</td>
</tr>
<tr>
<td>922,700</td>
<td>922,700</td>
<td>10</td>
</tr>
</tbody>
</table>

The rank numbers were separated into two subsamples, one consisting of the \( W_k \) values with a negative sign and the other with those of positive sign:

Negative sign: \( (1, 4, 5, 7) \)
Positive sign: \( (2, 3, 6, 8, 9, 10) \)

The rank numbers of the \( W_k \) values in each subsample were summed.

\[ S_{-1}^+ = 38 \]
\[ S_{-2}^- = 17. \]

As \( n_1 \neq n_2 \), a further calculation was required to obtain the test statistic. The subsample of ranks representing \( W_k \) values of negative sign was smaller, \( n_2 = 4 \). The total of the ranks of this subsample, \( S_r \), was 17. To compute \( S_r \), the total of the ranks of the larger subsample, the following equation was used:
In a 1967 review of previous controversies over the appropriateness of the assumption of profit maximization as the effective objective of the firm in theoretical models of firm theory, Machlup outlines several points related to research in this bulletin. In particular, he suggests that “the marginalist solution of price determination under conditions of ‘heavy competition’ is not seriously questioned.”

Independent tests were conducted in this research to determine if (a) the firms sold their product in a perfectly competitive market and (b) factors of production were purchased in perfectly competitive input markets. Results of these tests suggest that beef feedlot firms face “heavy competition” in both the product and factor markets. (Refer to Appendix B for additional information on these tests.)

Under Machlup’s interpretation, the firm is viewed as a theoretical construct and only as an empirical construct under restrictive types of analysis such as when the organization of the firm’s properties and processes are the objects of investigation. It remains a matter of judgment as to whether this analysis of firm cost curves warrants the high degree of correspondence given to the theoretical and real-world firm.

\[
S_L = n_2 (n_1 + n_2 + 1) - S_s \\
S_L = 4 (6 + 4 + 1) - 17 \\
S_L = 27.
\]

The \( S_N \) value to be compared with the critical value of \( S_N \) is:

\[
S_N = \text{minimum} \ (S_L, S_s).
\]

Therefore,

\[
S_N = \text{minimum} \ (27, 17).
\]

The critical value of \( S_N \) for the two-tailed test of hypothesis at \( \alpha = .05 \) significance level is \( S_N = 12 \) (Wine, 1964). As \( S_N = 17 \), the test fails to reject the hypothesis as \( S_N \) (calculated) > \( S_N \) (tabled critical value).

**Discussion of Results and Implications**

The statistical tests of hypothesis failed to reject the \( TC-C' \) hypothesis. A total cost function was estimated which is consistent with that defined by economic theory. The estimating procedure developed uses a minimum of data relative to other estimating techniques to provide a total cost function which is not significantly different from the cost equation constructed from first principles.

Presented in this section are discussions of the risky nature of the hypothesis test, and implications for use of this procedure in estimating cost functions for other agricultural industries and facilitating additional research into other theoretical aspects of firm cost functions.

**Risky Test of Hypothesis**

In a 1967 review of previous controversies over the appropriateness of the assumption of profit maximization as the effective objective of the firm in theoretical models of firm theory, Machlup outlines several points related to research in this bulletin. In particular, he suggests that “the marginalist solution of price determination under conditions of ‘heavy competition’ is not seriously questioned.”

\[
33
\]
This analysis does, however, support Machlup's suggestion that those critics are in error who refute marginal analysis because it fails to yield exact numerical calculations of marginal magnitudes such as cost, revenue, and productivity for a particular real-world firm. Stronger evidence would have been generated to assist Machlup's critics of marginal analysis if the hypothesis tested in this analysis had been rejected. In this analysis, it was assumed that firms were operating at their profit-maximizing levels of output, that is, where expected marginal revenue was equated with marginal cost. The hypothesized estimation technique, based on this assumption, yielded predictions not statistically different than the more conventional cost estimation technique. If the hypothesis tested had been rejected, more could have been said about the real-world applicability of the assumption which equates marginal revenue and marginal cost at profit-maximizing levels.

But what can be said from this analysis is somewhat weaker; that is, that expected marginal revenue is an estimate of marginal cost. To reiterate, the test of the hypothesis for firms in each of the two technology levels failed to reject the null hypothesis that the total cost function constructed from the integral of the $\Phi'(Y)$ function for firms provides total cost values identical to the values of the cost equation constructed from first principles. Therefore, it can be concluded that $E(P)_k$ is an estimate of $\Phi'(Y_k)$.

To show that this is a risky test to perform, other possibilities of the relation of $E(P)_k$ to $\Phi'(Y_k)$ can be considered. The expected value of the distribution of expected prices obtained from each respondent will be equal to, greater than, or less than the marginal cost of output at the level observed for a particular firm. Each of these possibilities is presented for a hypothetical firm in Figure 6.

At output levels $Y_o$ and $Y_e$ there exists an equality between $E(P)_k$ and $\Phi'(Y)$. However, few firms would choose to operate at $Y_o$, as it is the profit-minimizing level of output. If the firm were to produce at $Y_1$, $E(P)_k > \Phi'(Y_1)$. If the firm were to operate at $Y_3$, $E(P)_k < \Phi'(Y_3)$. A firm would operate at $Y_1$ if it were so constrained in variable capital that it could not achieve $Y_2$ output level. A firm would operate at $Y_3$ because of estimation error in its cost and/or $E(P)_k$ calculations.\footnote{Either inequality condition could arise due to the decision-maker's nonlinear utility function in risky situations (McCall, 1971).}

At an output level of $Y_1$, $E(P)_k > \Phi'(Y_1)$. However, as previously defined $E(P)_k$ is taken to be the estimate of $\Phi(Y_k)$. Therefore, $\Phi'(Y_1) > \Phi'(Y_1)$ by the magnitude $P_o C_1$. Calculation of $TC_1$ using $\Phi'(Y_1)$ will yield a $TC_1$ value which is greater than $C'_1$. 


If a firm were observed operating at $Y_3$, $TC_3$ could be less than $C'_3$. At $Y_3$, $V(Y_3) < V'(Y_3)$ by a magnitude of $C_3 P_0$. Calculation of $TC_3$ would yield $TC_3 < C'_3$, given that $V(Y_1) \geq V'(Y)$ and $V(Y_2) \geq V'(Y_2)$.

Therefore, the $TC$ function expressed as an integral of the marginal cost function can only be specified under the assumption that each firm is observed where $E(P)_k = \Phi'(Y)$. For the firm in Figure 6, this occurs at output $Y_2$. At output level $Y_2$, $\Phi'(Y_2) = E(P)_2 = \Phi'(Y_2)$.

To demonstrate that the statistical test of hypothesis would not have had to be performed if another assumption had been made, consider the application of the statistical test under the assumption that $E(P)_k > \Phi'(Y_k)$ for $k = 1, 2, \ldots, N$. (That is, the inequality exists for all observations in a particular technology level.)

Under the assumption of $E(P)_k > \Phi'(Y_k)$ for all $k$, it can be shown that the statistical test used in this study will reject the hypothesis that the
TC function integrated from the $\Phi'(Y)$ is equal to the cost equation for a given set of firms. To show this, five hypothetical firms where $E(P)_k > \Phi'(Y_k)$ will be used. Suppose the firms were observed at increasing levels of output and let $Y_1, Y_2, Y_3, Y_4,$ and $Y_5$ represent equal increments of output.

The following expresses the $TC$ function values for each firm, assuming $\bar{b} = 0$.

$$TC_1 = E(P)_1 Y_1$$
$$TC_2 = E(P)_1 Y_1 + E(P)_2 Y_2$$
$$\vdots$$

$$TC_5 = \sum_{k=1}^{5} E(P)_k Y_k.$$  

Under the same assumption that $\bar{b} = 0$, the $C'$ equation values are expressed by:

$$C'_1 = \Phi'(Y_1) Y_1$$
$$C'_2 = \Phi'(Y_1) Y_1 + \Phi'(Y_2) Y_2$$
$$\vdots$$

$$C'_5 = \sum_{k=1}^{5} \Phi'(Y_k) Y_k.$$  

The successive $V_k$ values are defined in general to be $TC_k - C'_k$. For these five firms they can be expressed as follows:

$$V_1 = E(P)_1 Y_1 - \Phi'(Y_1) Y_1$$
$$V_2 = \sum_{k=1}^{2} E(P)_k Y_k - \sum_{k=1}^{2} \Phi'(Y_k) Y_k$$
$$\vdots$$

$$V_5 = \sum_{k=1}^{5} E(P)_k Y_k - \sum_{k=1}^{5} \Phi'(Y_k) Y_k.$$  

All values of $W_k$, except $W_1 = V_1$, are defined as the difference in successive $V_k$ values, that is $W_k = V_k - V_{k-1}, k = 2, 3, 4, 5.$
The test statistic is calculated from the rank values attached to the absolute values of the $W_k$ values. If $S_N$ calculated is less than the critical value, then the hypothesis is rejected. To prevent the calculated value of $S_N$ from being equal to zero and hence assuring rejection requires that at least one value of $W_k$ be negative. Therefore since

$$W_1 = V_1 > 0$$

assume that

$$W_2 = V_2 - V_1 < 0.$$  

Under the assumption of $E(P)_k > \Phi'(Y_k)$ for all $k$,

$$W_1 = V_1 = E(P)_1Y_1 - \Phi'(Y_1)Y_1$$

$$W_2 = V_2 - V_1 = E(P)_1Y_1 + E(P)_2Y_2 - \Phi'(Y_1)Y_1$$

$$-\Phi'(Y_2)Y_2 - E(P)_1Y_1 + \Phi'(Y_1)Y_1.$$  

The value of $W_2$ expressed in terms of the $V_k$ components reduces to:

$$W_2 = E(P)_2Y_2 - \Phi'(Y_2)Y_2.$$  

Factoring out $Y_2$, the expression becomes

$$W_2 = Y_2 [E(P)_2 - \Phi'(Y_2)].$$  

For $W_2$ to be negative would require $\Phi'(Y_2) > E(P)_2$ which contradicts the previous assumption that $E(P)_k > \Phi'(Y_k)$ for all $k$. Hence $W_2 > 0$.

Now suppose that $W_2 > 0$, $W_3 > 0$, $W_4 > 0$, and $W_5 < 0$.

For $W_5$ to be less than zero, the following conditions would have to exist:

$$W_5 = V_5 - V_4 < 0$$

$$W_5 = E(P)_1Y_1 + E(P)_2Y_2 + E(P)_3Y_3 + E(P)_4Y_4$$

$$+ E(P)_5Y_5 - \Phi'(Y_1)Y_1 - \Phi'(Y_2)Y_2 - \Phi'(Y_3)Y_3$$

$$- \Phi'(Y_4)Y_4 - \Phi'(Y_5)Y_5 - E(P)_1Y_1 - E(P)_2Y_2$$

$$- E(P)_3Y_3 - E(P)_4Y_4 + \Phi'(Y_1)Y_1 + \Phi'(Y_2)Y_2$$

$$+ \Phi'(Y_3)Y_3 + \Phi'(Y_4)Y_4.$$  

The value of $W_5$ expressed in terms of $V_k$ components reduces to

$$W_5 = E(P)_5Y_5 - \Phi'(Y_5)Y_5.$$  

Factoring out $Y_5$, the expression becomes

$$W_5 = Y_5 [E(P)_5 - \Phi'(Y_5)].$$  

For $W_5$ to be negative would require $\Phi'(Y_5) > E(P)_5$, which contradicts the assumption that $E(P)_k > \Phi'(Y_k)$ for all $k$. Hence $W_5 > 0$.

Thus, it has been shown that to get any reversal in the sign of $W_k$ as $k$ increases would require that $E(P)_k < \Phi'(Y_k)$ for some $k$, which is contrary to the assumption that $E(P)_k > \Phi'(Y_k)$. A similar situation would arise if the assumption is made that $E(P)_k < \Phi'(Y_k)$ for all $k$. Therefore,
any other assumption that $E(P)_k = \Phi'(Y_k)$ for all $k$ causes the statistical test to reject the $TC-C'$ hypothesis.

Under the assumption that $E(P)_k = \Phi'(Y_k)$, the error in estimating $TC$ from the integral of $E(P)_k$ provides the alternatives in the sign of $W_k$. The test statistic provides the means for testing the significance of the errors. Thus, given that the test does not reject the hypothesis, it says that $TC$ estimated by integrating $E(P)_k = \Phi'(Y_k)$ is a "good" fit in the statistical sense to $C'$.

**Implications**

Although the sample size was limited and the hypothesis was mainly methodological, the conclusions presented were not contradicted by the data obtained. While several empirical hypotheses could now be formulated and tested using the methodology developed by this study, several implications concerning the nature and future use of the estimating procedure are more immediate.

**Use of Methodology**

The integration of the marginal cost function to obtain the total cost function for a group of firms provides an estimate of the total cost function which is at least not inconsistent with that defined in economic theory. Previous studies which used regression procedures gave a biased estimate of the cost function. Regression procedures gave a best fit to a scatter of points but denied the definition of the cost function given by economic theory.\(^8\)

The discrete summation procedure used in this study to obtain the total cost function is also more efficient than previously used procedures in that empirical observations need be made only of the output level and associated expected price and fixed cost rather than of input levels and input prices, a procedure proven to be costly and time consuming. Easily obtained cross-sectional data can be used to make rapid calculations of the total variable cost functions for a group of firms by the method presented here if one chooses not to measure fixed costs. This procedure also can be used to obtain an estimate of one firm's cost function from time series observations of expected prices assuming no change in technology.

\(^8\) The term "best" denotes that statistic for the parameter of concern which has minimum variance (Hogg and Craig, 1965). There is an additional requirement that the statistic be unbiased. This additional requirement is satisfied by least squares regression estimates. In several of the regression-cost analysis studies referred to earlier in the text, the researchers made model adjustments attempting to reduce the mean square error (variance estimate). Most of these efforts, in reality, were attempting to reduce that portion of residual sum of squares due to lack of fit error (Draper and Smith, 1967).
It has been demonstrated how the methodology applies to firms operating under two different technologies but within the same industry. The beef-feeding industry was used as the testing ground. Previous attempts at estimating cost functions using regression analysis for this industry have been fraught with difficulty. It is hoped that the procedure developed here should be applicable to several other agricultural industries.

Further Research

The estimating procedure developed is readily adaptable to several other agricultural industries comprised of single enterprise firms. With some modification of the procedure, a means could be developed for estimating cost functions for firms which produce outputs through joint production processes or for multiple enterprise firms. Traditional methods of enterprise accounting, in attempting to estimate cost functions for a single product (or enterprise) produced in a multiple product firm, are generally inconsistent with the theoretical concepts of joint production. These methods have not provided a means for obtaining a meaningful or useful joint product cost function. Such a joint product cost function is not meaningful from the standpoint of economic theory and is useless for decision-making. Research should be initiated using the basic methodology of this study to develop a procedure for combining the marginal cost estimates for each product produced into a total cost function. The marginal cost functions developed through the methodology of this study can provide the information to decide on the level of output for a single product firm and, given the level of one product, provide the decision information for the level of output of other product(s) for a two (multiple) enterprise firm. The total cost function for a group (cross-sectional sample) of joint product firms would provide the marginal cost information for the profit-maximization product mix decision.

9 Use of this method of cost estimation by farm management specialists assisting feedlot operators in short-term planning decisions could be as follows: Select a cross-section (a small sample of firms) of feedlot operators with similar technology but different levels of output. Obtain from each an estimate of their expected marginal revenue. These estimates, along with volume data, will allow the construction of a marginal cost curve relevant to all firms of this technology level. Then, if a participant in a beef analysis program, not necessarily one of the sample respondents, notifies the farm management specialist that he has been offered a certain contract price or expects fed cattle to sell at a certain price, the farm management specialist can assist the feedlot operator in determining what volume level he should operate at to maximize profits. That is, the participant's price (contract or expected) is compared with the marginal cost curve to determine the point of intersection. This point of intersection indicates the profit-maximizing level of output. This technique, however, would have some of the same limitations that other cross-sectional studies of farm firms have had, e.g., selection of sample.
In addition to the possible use in firm management decision-making, the methodology should have broader application to such empirical problems as economies of size and estimation of supply functions. With a procedure yielding an estimate of the cost function that is consistent with the fundamentals of economic theory, the issue of economies of size in agricultural production can be readdressed.

The short-run supply function of each firm is defined to be that segment of its marginal cost curve which lies above the minimum point on its average variable cost curve. The firm will provide a given level of output at all product prices greater than the minimum value of the average variable cost curve.

The aggregate supply function is the horizontal summation of the individual firm's supply functions. As the second-order condition for profit maximization requires that the marginal cost curve be increasing, the firm's supply function is therefore also monotonically increasing. The short-run aggregate supply function also has a positive slope, as the horizontal sum of monotonically increasing functions is itself monotonically increasing.

To obtain empirical estimates of industry supply functions from marginal cost functions derived from the method presented in this study, the values of the marginal cost function need only be multiplied by $N$, the number of firms in a particular technology subsample. This would provide an aggregate supply function for a particular subsample. This process could be repeated for each technology subsample. Then the horizontal summation of the aggregate supply functions for each subsample would yield the industry supply function.¹⁰

¹⁰ Theoretically, long-run industry supply functions are derived in a manner similar to those in short-run periods. The first- and second-order relationships are expressed in terms of long-run prices and marginal costs, and all costs are considered variable, i.e., firm size is assumed adjustable. The method of cost estimation developed in this study is not directly applicable to the estimation of long-run firm or industry supply functions, as when cross-sectional data are taken something is fixed—firm size. Time series data would have to be taken to provide long-run cost functions amenable to the derivation of long-run supply functions. The possibility of using data taken over a period of time in the estimation of long-run marginal cost functions using the particular estimation technique needs further attention.
References


These are graphs based on the prices received by feedlot operators selling through Portland, Oregon, during the 10-year period, 1958-1968. Take for example this graph (use +1, 700-900 lb. Choice heifers). It shows that on the average about 8.4% of the prices received were in the $20.00-$21.00 interval, 7.4% of the prices were in the $21.00-$22.00 interval, 11.2% of the prices were in the $22.00-$23.00 interval, 28% were in the $23.00-$24.00 interval, 15% in the $24.00-$25.00 interval, 19.8% in the $25.00-$26.00 interval, 9.3% in the $26.00-$27.00 interval, and 0.9% in the $27.00-$28.00 interval.

From your knowledge of the market and your cattle sales during this period, would you rank these seven graphs, starting with the one which most closely approximates what you recall about cattle prices over this 10-year period?

Heifers
Steers
Quality grade
Weight class

APPENDIX A
Feedlot Interview Sheet

NAME OF OPERATOR

A. Would you please provide the following information on the most recent lot of cattle you placed on feed?

1. When did you place your most recent lot of cattle on feed?
   Date

2. How many head were placed on feed?
   No. of head

3. Were the feeders purchased all steers or heifers—or was it a mixed lot?
   No. of steers
   No. of heifers

4. What was the average purchase weight of feeders?
   Average purchase weight (steers)
   Average purchase weight (heifers)

5. What was the average price per hundredweight paid for these feeders?
   Average purchase price (steers)
   Average purchase price (heifers)

6. How many days do you plan to have these cattle on feed?
   Days on feed (steers)
   Days on feed (heifers)

7. At what average weight do you plan to sell these cattle?
   Average sale weight (steers)
   Average sale weight (heifers)

8. What grade do you expect your fed cattle to reach?
   Grade (steers)
   Grade (heifers)

B. (Use the cards of expected prices to determine $E(P)_k$ for the particular lot of cattle.)

Historical Price Frequency Distributions

These are graphs based on the prices received by feedlot operators selling through Portland, Oregon, during the 10-year period, 1958-1968.

Take for example this graph (use +1, 700-900 lb. Choice heifers). It shows that on the average about 8.4% of the prices received were in the $20.00-$21.00 interval, 7.4% of the prices were in the $21.00-$22.00 interval, 11.2% of the prices were in the $22.00-$23.00 interval, 28% were in the $23.00-$24.00 interval, 15% in the $24.00-$25.00 interval, 19.8% in the $25.00-$26.00 interval, 9.3% in the $26.00-$27.00 interval, and 0.9% in the $27.00-$28.00 interval.

From your knowledge of the market and your cattle sales during this period, would you rank these seven graphs, starting with the one which most closely approximates what you recall about cattle prices over this 10-year period?

Heifers
Steers
Quality grade
Weight class
Would you give a "range" of the prices you might receive for the most recent lot of cattle you placed on feed?

I have placed the set of prices you gave me on seven different graphs similar to those we worked with for prices over the last 10 years.

From your knowledge of the market conditions, would you rank these graphs, starting with the one which most closely approximates your expectations of the prices you might receive for the most recent lot of cattle placed on feed?

"Range" of prices

Heifers
Steers
Weight class

Ranking

Would you please explain your ranking?

Now I would like to ask you the following on the most recent lot of cattle you have placed on feed:

Good Cattle Price Expectation Frequency Distributions

Good cattle prices during the September-December period of each year tend to exhibit about a $6.00 "range."

For example, good steer prices in the Portland market during 1968 varied from $21.01-$27.00 during the September-December period. During the same period good heifer prices varied from $20.00-$26.00.

Would you give a $6.00 "range" of the prices you might receive for the most recent lot of cattle you placed on feed?

I have placed the set of prices you gave me on seven different graphs similar to those we worked with for prices received over the last 10 years.

From your knowledge of the market conditions, would you rank these graphs, starting with the one which most closely approximates your expectations of the prices you might receive for the most recent lot of cattle placed on feed?

"Range" of prices

Heifers
Steers
Weight class

Ranking

Would you please explain your ranking?

Now I would like to ask you the following on the most recent lot of cattle you placed on feed:

Choice Cattle Price Expectation Frequency Distributions

Choice cattle prices during the September-December period of each year tend to exhibit about a $4.00 "range."

For example, choice steer prices in the Portland market during 1968 varied from $25.00 to $29.00 during the September-December period. During the same period choice heifer prices varied from $24.00-$28.00.

Would you give a $4.00 "range" of the prices you might receive for the most recent lot of cattle you placed on feed?

I have placed the set of prices you gave me on seven different graphs similar to those we worked with for prices received over the last 10 years.

From your knowledge of the market conditions, would you rank these graphs, starting with the one which most closely approximates your expectations of the prices you might receive for the most recent lot of cattle placed on feed?

"Range" of prices

Heifers
Steers
Weight class

Ranking

Would you please explain your ranking?
Would you please explain your ranking?

The price "range" you selected as most likely was $________________________
Suppose you had the following choices:
(1) You can pick a ball from a box with 50% red and 50% black balls.
    If you pick a black ball you will win the value of the lot of finished cattle today.
(2) You can wait for the time of sale and receive a price from the $________________________
    interval for the same lot of cattle.
    Which is your choice?  Choose from box________________________
    Wait for sale of cattle________________________

[If the operator chooses from the box, this implies a probability of price interval < .5.
Repeat the question, lowering the number of black balls.]

Suppose you had the following choices:
(1) You can pick a ball from a box with % red and % black balls,
    If you pick a black ball you will win the value of the lot of finished cattle today.
(2) You can wait for the time of sale and receive a price from the $________________________
    interval for the same lot of cattle.
    Which is your choice?  Choose from box________________________
    Wait for sale of cattle________________________

If the question is continuing—
(1) How many black balls would there have to be in the box before you would
    choose to wait for the sale of your cattle at a price in the $________________________
    interval?
    Number of black balls _______________________
    or
    (2) What do you think is the probability of receiving a price in the $________________________
    interval?
    Probability _______________________

C. Would you please give me the following information on your ration on a per animal basis?

<table>
<thead>
<tr>
<th>Days fed</th>
<th>Ration ingredient</th>
<th>Amount fed/day</th>
<th>Price/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(If you do not feed the same ration the entire feeding period, denote how many days each ration ingredient is fed.)

D. Could you please provide the following information on your 1969 cattle feeding program?
1. How many cattle will you feed during the 1969 feeding year?
    Number of steers _______________________
    Number of heifers _______________________

2. What will be the average purchase weight of the feeder cattle?
    Average purchase weight of steers _______________________
    Average purchase weight of heifers _______________________
3. What will be the average price per hundredweight that you will pay for feeders?

Average price of steers
Average price of heifers

4. What will be the average sale weight of your fed cattle?

Average sale weight of steers
Average sale weight of heifers

5. Will the total hundredweight of gain produced be approximately the following?

\[
\begin{align*}
(a) \quad \text{(Number of steers)} & \quad \left( \frac{\text{(average sale weight)}}{\text{(average purchase weight)}} \right) = \text{lbs. of gain} \\
(b) \quad \text{(Number of heifers)} & \quad \left( \frac{\text{(average sale weight)}}{\text{(average purchase weight)}} \right) = \text{lbs. of gain}
\end{align*}
\]

Total feedlot gain \( a + b \)

6. What is the annual interest rate charged on your operating capital?

Operating capital interest rate

7. If your ration ingredients and/or length of feeding period differ considerably during other seasons from those of your most recent lot placed on feed, would you please outline how they differ?

E. Would you please provide the following information on the changes in your feeding operation since our discussion of your October 1966-October 1967 feeding period?

1. Have you changed your method of feeding since the 1966-1967 period?

2. Have you added any additional feedlot facilities, milling facilities, or equipment since the 1966-1967 feeding period?

<table>
<thead>
<tr>
<th>Description of item</th>
<th>New cost</th>
<th>When purchased</th>
</tr>
</thead>
</table>

3. What is the current interest rate that is charged on your capital improvement loans?

Capital improvement interest rate

F. Would you please provide the following information on feedlot utilization?

1. Could you be feeding more cattle at this time than you have in your lot?
   Yes
   No

2. If yes, would you give those reasons why you choose not to feed more?

3. If no, what factors in your current operation restrict the feeding of additional cattle?
G. Could you provide the following information on your buying and selling activities?

1. When you buy concentrates, does the volume purchased at any one time or yearly volume affect the price you pay?

2. Is the same true for your roughage purchases?

3. What factors are most important in selling your fed cattle (lot size, even flow, annual volume)?

APPENDIX B

Assumption of Competitive Output Price

Data obtained from the feedlot operators interviewed were used to determine if there were any selling economies associated with feedlot size. The question asked of each feedlot operator was “What factors are most important in selling your fed cattle (lot size, even flow, annual volume)?” Response to the three factors suggested in the question were ranked, assigning “1” to the most important, “2” to second most important, and “3” to the third most important. Where two or more factors were felt of identical importance, the same rank was assigned to each. For those feedlot operators suggesting that none of the factors suggested had a measurable effect on selling price, “0” was assigned for the rank of each factor. The responses of all operators are summarized in Appendix Table 1.

Lot size was viewed as the least important factor in determining the selling price for fed cattle. Most operators suggested that as long as truckload lots of cattle were available for sale, no greater price would be received by having more than one truck-

Appendix Table 1. Ranking of factors viewed by feedlot operators as important in determining the selling price of fed cattle

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Lot size</th>
<th>Even flow</th>
<th>Annual volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
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<td>1</td>
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</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>10</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
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<td>0</td>
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<tr>
<td>15</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>2</td>
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</tr>
<tr>
<td>18</td>
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</tr>
<tr>
<td>21</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
load ready for shipment to slaughter at any one time. Several feeders who have less than a truckload available for sale at one time (less than 40 head) suggested that this was an accommodation to some buyers—especially small local packing plants.

Annual volume and an even flow of cattle from a feedlot were about equally important factors in determining selling price and both were more important than lot size in determining selling price. Several smaller volume operators stated that their production schedule is well known by the primary buyers, and that these buyers do not offer them less than the market price for their cattle. Their small annual volume is marketed unevenly throughout the year, but in a pattern that their primary buyers know.

There are few if any apparent internal selling economies related to size of feedlot, given that a firm is capable of selling truckload lots of cattle and its annual volume of output and production schedule is known to primary buyers. Large volume producers may attract a larger group of effective buyers, but there is no indication that this increases the price paid to them for their fed cattle.

Assumption of Variable Input Prices

Questions were asked to determine if the feedlot operators could affect the price of two purchased inputs—concentrates and roughages.

If purchase price was decreased by the quantity purchased at one time or the total quantity purchased annually this was shown as (-) in Appendix Table 2. If the input referred to was produced by the feedlot firm, the entry was designated by (H). If there was no effect, this was designated by “0” entry.

Few if any internal pecuniary buying economies were evident in the purchase of concentrates. On certain supplements, up to 5 percent price discounts were received by those purchasing in truckload lots. These were only reported by the smaller

Appendix Table 2. Effect of size of single purchase and volume purchased yearly on input prices of concentrates and roughages

<table>
<thead>
<tr>
<th>Firm code</th>
<th>Concentrates</th>
<th>Roughages</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Single purchase</td>
<td>Yearly volume</td>
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<tr>
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<tr>
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</tbody>
</table>
volume operators. Evidently, price discounting is discontinued on supplements at greater than local delivery truckload quantities, i.e., loads of 4 to 8 tons. Larger volume operators are evidently receiving the "delivery truck" discount, but no additional discounts for larger single delivery purchases.

One operator reported receiving a 15 percent price discount on the purchase of his annual requirements of low quality hay at harvest. No other internal pecuniary buying economies were achieved in roughage purchases.