

AN ABSTRACT OF THE THESIS OF

Jake Spratt for the degree of Master of Science in Economics presented on  
August 7, 2007.

Title: Parking Policy with Heterogeneous Agents

Abstract approved:

---

Bruce M. McGough

This paper investigates the potential welfare gains resulting from variable parking fees in the presence of heterogeneous agents. We begin by defining a modified linear city model with two lots located at separate distances from a central business district, and two types of agents with heterogeneous values of time. Agents maximize utility by minimizing transit costs, which include parameters dependent and independent of value of time. Simulation is used to model how various specifications affect the average travel costs, focusing on contrast between uniform pricing and variable pricing. We conclude by discussing the conditions which create a net welfare gain in light of a pricing gradient.

Parking Policy with Heterogeneous Agents

by  
Jake Spratt

A THESIS

submitted to

Oregon State University

in partial fulfillment of  
the requirements for the  
degree of

Master of Science

Presented August 7, 2007

Commencement June 2008

Master of Science thesis of Jake Spratt presented on August 7, 2007.

APPROVED:

---

Major Professor, representing Economics

---

Chair of the Department of Economics

---

Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

---

Jake Spratt, Author

## ACKNOWLEDGEMENTS

The author thanks professors Starr McMullen and Patrick Emerson quite sincerely for their time dedicated to this project, and professor Bruce McGough, without whom there is no possible way this project would have been completed. Further gratitude is extended to the author's wife for her patience and support, and to the Beanery girls for their chemical encouragement.

## TABLE OF CONTENTS

	<u>Page</u>
1 Introduction . . . . .	2
2 Literature Review. . . . .	4
3 The Model . . . . .	8
3.1 A Spatial Definition of the CBD . . . . .	8
3.2 The Variables . . . . .	9
3.3 Trip Generation. . . . .	11
4 Equilibrium Notion. . . . .	14
4.1 The Optimal Solution . . . . .	15
5 Simulation . . . . .	17
5.1 Definition of Variables . . . . .	19
5.2 Results . . . . .	21
5.3 Equity Concerns. . . . .	25
5.4 Policy Applications. . . . .	29
6 Conclusion . . . . .	31

## LIST OF TABLES

<u>Table</u>	<u>Page</u>
1 Variable Specifications . . . . .	20
2 Value of Time Specifications . . . . .	21
3 Simulation Results . . . . .	22
4 Gains from Pricing (base vs. gradient) . . . . .	24
5 Distribution of Agent Types . . . . .	27
6 Changes in ATC by Agent Type . . . . .	28

# Parking Policy with Heterogeneous Agents

Jake Spratt

August 7th 2007

## **Abstract**

This paper investigates the potential welfare gains resulting from variable parking fees in the presence of heterogeneous agents. We begin by defining a modified linear city model with two lots located at separate distances from a central business district, and two types of agents with heterogeneous values of time. Agents maximize utility by minimizing transit costs, which include parameters dependent and independent of value of time. Simulation is used to model how various specifications affect the average travel costs, focusing on contrast between uniform pricing and variable pricing. We conclude by discussing the conditions which create a net welfare gain in light of a pricing gradient.

# 1 Introduction

The study of parking, or lack thereof, has been identified as a curiously neglected field in the transportation literature. Virtually every article on the subject has drawn attention to the fact that there is relatively little formal analysis of this seemingly trivial, yet ubiquitous residue of modern life. Much attention has been paid to the idea of pricing roads, and justly so; it is a commonly held belief that time lost in transit due to congested roads produces a significant level of social inefficiency, and the road pricing literature suggests a number of realistic solutions to this problem. The study of pricing parking, however, has been relatively limited, despite a number of authors claiming optimal parking fees can effectively decrease congestion in a similar manner to road pricing.

Anderson and de Palma best summarize the situation in their 2004 paper:

[T]here is little formal economic analysis of parking, although technology for pricing parking is very simple (a parking meter!) and there is little social opprobrium for paying for parking. Arguably, inefficient search for parking may be at least as distortionary as excessive road use... [and] in the absence of road pricing, efficient pricing of parking may be an effective policy tool for combating congestion on the road and in parking.

While reducing congestion via parking tariffs is certainly a promising goal, it is not our primary concern in this work. Several papers (most notably Anderson and de Palma (2004), Arnott and Rowse (1999) and Arnott and Inci (2005)) have made progress modeling the effect parking tariffs have on congestion, in greater detail than the model presented here. However, each of these papers makes the simplifying assumption that all drivers are homogeneous, an assumption we wish to violate.

It is almost certainly true that commuters differ in their value of time (formally, studies of the SR91 commuters in California by Sullivan (1998) and Parkany (1999) have verified this claim). People earning higher salaries, for example, generally have higher opportunity costs of time, while people with low paying jobs do not forgo as much when not working. Similarly, some drivers



utilize their commute to listen to the morning radio or conduct business conversations, while others might detest traveling during peak hour traffic. It is reasonable to conclude, therefore, that agents with differing VOT will incur different travel costs *even if they both spend the same amount of time in transit*.

The aim of this paper is modest. Our central goal is to introduce heterogeneous agents into the literature by allowing commuters to vary in their value of time (VOT). The reason for doing so extends beyond the simple need to capture the effect this consideration has on the existing models, which are focused on congestion externalities. Rather, we introduce heterogeneous agents with the hypothesis that an optimal pricing gradient will produce an additional surplus gain for drivers by allowing those who highly value their time to pay a fee in order to park in a more desirable location. Thus, we are concerned not in the surplus gained from internalizing the social marginal cost of congestion, but rather the potential gain in allowing agents with high values of time to pay a premium for a parking location closer to their destination, thereby minimizing their travel time.

We continue the trend of using a central business district (CBD) to model parking behavior, although a new spatial specification will be employed. The reason for this is simple; the most obvious example of the distortionary effect of inefficient parking policies comes from densely populated metropolitan areas. Shoup (2005) reports that an average of 30% of vehicles in a sample of 13 downtown areas are ‘cruising’ for an available parking spot, with an average search time of 7.8 minutes. Accordingly, we will consider a model where agents travel to a CBD and search for parking amongst unassigned spaces, first in the presence of uniform prices and later with a gradient of priced parking lots<sup>1</sup>

To emphasize the net gains from variable pricing, we consider only revenue-

---

<sup>1</sup>Anderson and de Palma (2004) note that most daily commuters acquire long-term parking permits which allow them to avoid the daily parking search; we acknowledge this fact but employ the terms ‘workers’, ‘downtown’ and ‘CBD’ because of their ease of use in capturing the intuition of this model. Alternatively, one might consider students traveling to a campus or travelers to an airport, or the commonly applied tourist visiting the CBD.

neutral alternatives to a uniform pricing scheme. In doing so, we hope to show that a net welfare gain can be obtained via a pricing gradient even if tariff revenues are held constant. This is not the only possibility. Alternatively, policymakers could set a tariff revenue target above or below the amount generated under uniform pricing, which is a particularly useful trait if switching from a uniform to variable fee structure imparts additional costs to the planner. We leave these considerations for further research and a more direct employment of this model.

Despite the illustrative simplicity of the applications in this paper, the model was designed with the intent of providing a realistic framework for a (more) thorough application to a real parking district. Relevant steps to be taken in this regard are discussed in a later section, however it is worth noting from the outset that many of the simplifications made in the paragraphs to follow serve only to simplify the analysis and emphasize the principle results. In most instances, these restrictions can be loosened without significantly affecting the general form—or results—of the model.

The paper will take the following course. Section 2 provides a brief review of the literature, focusing on the most relevant work. Section 3 will introduce the model and all relevant assumptions, and section 4 will discuss the equilibrium conditions in the model. We will simulate the model in section 5 and discuss the practical concerns of implementing the model in specific policy applications. Section 6 will provide concluding remarks and suggestions for future research.

## 2 Literature Review

Past work of relevance to this paper falls in one of two categories: the formal modeling of parking behavior and the value of time literature, specifically VOT within the transportation literature. Before now, the two areas have yet to overlap. We begin with the former.

The earliest attempt to analyze parking as more than just a cost incurred at the end of a trip is Vickrey’s informal paper published in 1959, which provided a non-technical survey of current and potential issues in parking. Since then, there have been limited attempts to model parking behavior. Voith (1997) considers the effect parking policies and transit costs have on employment and land allocation in a central business district. Anderson and de Palma (working paper) endogenize the optimal number of parking spaces in a CBD where land is divided between parking and residential zones according to land use rents (which are determined endogenously). Their results support the intuitive notion that land is both increasingly valuable and increasingly scarce in areas closer to the CBD, a result our spatial specification will capture nicely.

A more direct analysis of parking behavior is provided by Arnott, de Palma and Lindsey (1991) who model parking given a morning commute to a CBD in the presence of bottleneck congestion, and Arnott and Rowse (1999), who use a circular city framework and model the uncertainty in searching for a parking space. In the latter paper, agents live around a central business district occupying a thin annulus of a circle, and are randomly offered trip opportunities which are uniformly distributed both spatially and temporally around the city. While this treatment realistically captures the inherent uncertainty of finding a parking space, the model quickly becomes complex, this despite the relatively parsimonious assumptions and modest illustrative intentions of the authors.

A much simpler treatment is provided by Anderson and de Palma (2004), who use a linear city CBD model to capture the effects a pricing gradient on search costs and congestion. In their model, the optimally priced solution results in agents parking across a greater ‘span’ from the CBD, compared to a more dense parking allocation in the unpriced solution. The authors find that agents ignore the marginal impact they impose by cruising for a location closer to the CBD, which results in an inefficient congestion externality. In a monopolistically competitive market structure, the optimal pricing gradient can

be achieved via private ownership of parking spaces. While we do not consider the welfare effects of congestion, the model used here, most specifically the cost specification, strongly resembles this 2004 paper.

The most recent development in modeling parking behavior comes from Arnott and Inci (2005), who capture the interaction between drivers cruising for parking and through traffic in a ‘Manhattan geometry’ city. The authors conclude that optimal parking fees will deter enough commuters from the CBD to eliminate ‘cruising’ altogether, which they find to be pure dead weight loss, while allowing parking spaces to remain fully saturated. This paper employs perhaps the most realistic spatial representation of a central business district, but again focuses on the role of parking fees in reducing congestion. Again in this paper, along with all previously mentioned publications, agents are assumed to be homogeneous.

To date, the author is aware of only one paper in the literature which attempts to model heterogeneous drivers in a parking framework, that of Glazer and Niskanen (1992). In this paper, a continuum of agents are ordered according to their willingness to pay for an additional minute of parking. Agents in this model value a minute of parking differently from one another, and thus respond differently to a positive parking tariff. However, this assumption is made only insofar as to allow some (but not all) drivers to be deterred from the congested area given a positive parking fee, and does not capture the search costs-walking costs trade off present in the more spatially accurate models. In fact, in this model drivers are concerned only with finding an available spot; they are indifferent between locations. Furthermore, the analysis is again concerned with the welfare gains arising from a reduction of congestion, not with any potential gains in allowing people to pay according to their value of time preferences.

In modeling the VOT gains in pricing, we borrow heavily from the work of Small and Yan (2004), who test a hypothesis similar to ours in a road pricing framework. The authors consider two symmetric roads of equal length and

capacity, where travel on the first road is ‘free’ (no road use tariff) and the second subject to a usage fee<sup>2</sup>. The authors assume there are two types of agents who differ in their value of time, and who choose which road to travel in order to minimize total costs. The authors conclude when both roads are unpriced (or, alternatively, priced at the same level), congestion and travel time is symmetric between the two roads; when one road is subject to a positive usage fee, the non-priced road is more congested than the priced road and travel times vary accordingly. The model is simulated with varying tariffs and differences in VOT, and welfare gains are reported for each specification.

We extend the Small and Yan framework as far as possible, however our model deviates in several key areas. Most notably, two commuters in the Small and Yan model arriving entering the roadway exactly 1 hour apart will incur the same travel costs *so long as congestion remains constant*. Thus, an individual entering the roadway later in the day incurs the same travel costs as an individual who leaves earlier in the morning commute (this is explained by the absence of any early arrival costs in their model). The structure of our cost specification precludes this result, as will become clear later in the paper<sup>3</sup>. This complication does not allow us to directly apply the equilibrium results presented in Small and Yan, although intuitively we follow a very similar notion<sup>4</sup>.

---

<sup>2</sup>This is one of several pricing structures considered in the article; others include a no-toll regime, a profit-maximizing structure, and a variable toll regime subject to a ‘level of service’ constraint.

<sup>3</sup>Namely, in our model, those who leave later in the trip sequence will face weakly higher costs than those who left before them.

<sup>4</sup>Small and Yan apply the Wardrop conditions, which state, in equilibrium, 1) agents choose which road to travel in order to minimize costs and 2) any two agents with the same VOT traveling on different roads must incur the same total cost; the second condition is problematic, since our model will allow agents of the same type parking in the same lot to incur strictly unequal costs, resulting from variations in their arrival time.

### 3 The Model

#### 3.1 A Spatial Definition of the CBD

We begin by developing a suitable spatial model for our city. Arnott and Rowse (1999) use a circular city model with the CBD stretched uniformly around the circumference of a circle (the city occupies a thin annulus, with parking distributed homogeneously around the city). The circular city model provides an intuitive geometrical representation but does not lend itself readily to alternative city specifications. Anderson and de Palma (2004) employ a linear city model, placing the CBD at the end of a series of parallel access roads with perpendicular 'side roads' serving as parking lots. This model provides a much simpler framework but is less realistic spatially. We propose a combination of the two.

Consider a downtown district located at the center of a large circle representing city limits. All agents live in neighborhoods uniformly distributed outside of city limits. The CBD is served by multiple access roads spanning a fixed distance  $\bar{x}$  from the outer circle and proportionately spaced throughout the city. There are  $j$  parking lots represented as a series of concentric circles centered at the CBD and located distance  $x_j$  from the CBD, where  $j = A, B, \dots, J$ . Each lot contains  $k_j$  number of parking spaces which is increasing with  $x$ . That is,

$$k_j = f(x_j) \tag{1}$$

where  $f$  is increasing in  $x$ <sup>5</sup>. Thus, parking lot  $B$  is located at distance  $x_B > x_A$  from the CBD and provides  $k_B > k_A$  parking spaces<sup>6</sup>. The monocentric city takes the form of a dartboard, with multiple access roads, residential zones

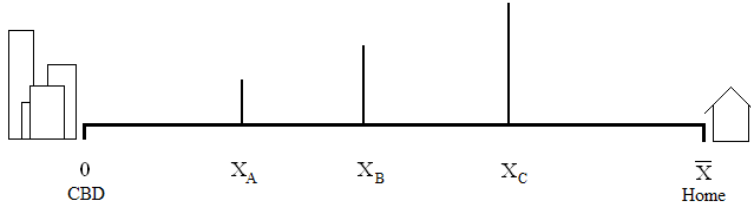
---

<sup>5</sup> $f(x)$  can be linear or non-linear, although in this analysis we will assume it to be linear for simplicity.

<sup>6</sup>this assumption is consistent with the results of Anderson and de Palma (2004), who endogenize land use and rents in a CBD and find less space devoted to parking in high value areas

outside of the city, and a decreasing number of parking spots closer to the CBD. However, it is difficult to manipulate analytically. To avoid these problems and capitalize on the intuitive properties of the specification, we aggregate the access roads and neighborhoods into a linear city format, where all agents live in a single neighborhood and travel a single access road to the downtown district. The city takes the following form:

Figure 1: Modified Linear City



### 3.2 The Variables

There are  $I$  types of agents in the economy with corresponding values of time  $\alpha_i$ , where  $i = 1, 2, \dots, I$  and  $\alpha_i > \alpha_{i+1}$  (measured in dollars per minute). Agents receive fixed utility  $v$  from each completed trip to and from the CBD, where  $v$  is constant for all  $i$ . Given a trip opportunity, a driver will travel distance  $\bar{x} - x_j$  at a rate  $V_D$ , at which point the agent turns into lot  $j$  and begins to search for an available parking space. Upon finding a space, the driver then walks the remaining distance  $x_j$  at rate  $V_W$ .

Let  $n_j$  denote the number of parked cars at lot  $j$ . Then  $k_j - n_j$  represents the number of available spots and  $\frac{k_j - n_j}{k_j}$  the proportion of available spaces at lot  $j$ . Thus an agent parking a distance  $x_j$  from the CBD can expect to search  $\frac{k_j}{k_j - n_j}$  number of spaces before finding an available spot<sup>7</sup>. Define  $\gamma$  as the time

<sup>7</sup>For a clever explanation of this result, see Anderson and de Palma (2004), from whom this result is borrowed.

necessary to check a single spot for availability; then  $\frac{k_j}{k_j - n_j}\gamma$  is the expected time spent searching for an available parking spot. Parking tariffs are fixed, and denoted  $\tau_j$ , where  $\tau_j \geq 0$  for all  $j$ <sup>8</sup>. Accordingly, a driver of type  $i$  parking at lot  $j$  faces the following costs:

$$C_{ij} = \alpha_i[(\bar{x} - x_j)V_D^{-1} + \frac{k_j}{k_j - n_j}\gamma + x_jV_W^{-1}] + \tau_j \quad (2)$$

It can easily be shown equation (2) is increasing in  $\alpha, \gamma, \tau_j, \bar{x}, x_j$  and  $n_j$ . If we impose the additional assumption that  $V_D > V_W$ , then (2) is decreasing in  $V_D, V_W$ , and  $k_j$  as well.

This cost structure displays a number of desirable properties. If agents drive faster than they walk (which is almost certainly true), then uniform tariffs imply lots closer to the CBD impart a lower cost than lots further away, since it is faster to drive as far as possible and walk the shortest distance possible. It is natural, therefore, for everyone to want to park at a closer lot. However, given the search costs specification, as a lot approaches capacity ( $n_j \rightarrow k_j$ ), search costs, therefore total costs, increase at an increasing rate (in fact, if we consider a continuous specification of  $n$  and  $k$ , search costs approach infinity as the lot approaches capacity). Thus, at some point it becomes faster to park at a lot further from the CBD and incur the extra walking costs to avoid the excessive search costs at a closer lot. As the lots on the peripheral of the CBD fill in turn, it becomes cheaper once more to park at a closer lot. As a result, the distribution of cars between lots ‘oscillates’ into equilibrium instead of simply filling up from the inner ‘rings’ outward.

Furthermore, only the driving, walking, and search costs are scaled by value of time  $\alpha$ , whereas  $\tau$  is independent of VOT. If an agent highly values their time, they will be willing to incur a higher parking tariff to park at a closer lot so long as the higher fee offsets the higher search and walking costs they would have

---

<sup>8</sup>As an alternative interpretation, assume drivers pay by the minute but visit the CBD for a fixed period of time each day



faced had they parked at a more saturated lot. An agent who does not value their time highly can park at a further lot and avoid a high fixed tariff. Here the foundation of our hypothesis emerges; by charging higher tariffs at closer lots and lower tariffs at further lots, the city planner will (presumably) scare away those with lower VOT who do not benefit as greatly from the reduced congestion and walking distance in the closer lot while allowing those with higher VOT to avoid high search and walking costs by paying a higher tariff.

We further assume, somewhat naturally, that parking is never over saturated. If  $\bar{K} = \sum_{j=A} k_j$  and  $\bar{N} = \sum_{i=1} n_i$ , this assumption becomes

$$\bar{N} \leq \bar{K} \quad (3)$$

Equation (3) further simplifies the model by allowing us to ignore instances of excess demand, where agents in queue form lines waiting for an available spot<sup>9</sup>. In our simulations, we will consider several specifications of lot saturation, each of which satisfies this condition.

### 3.3 Trip Generation

In order to simulate the model and obtain equilibrium results, we first must define a technology for generating trips in the economy. This problem has resulted in a number of solutions in the literature. Arnott and Rowse (1999) uniformly allocate trips in the economy using a Poisson process, where trip allocation is a stochastic process in a steady-state. Anderson and de Palma (2004) use a clever heuristic argument to skirt the issue altogether by claiming the trade-off between early arrival costs and higher search costs forces transit costs to be equal across all agents. While this seems a natural argument to make, it relies heavily on the assumption of homogeneous drivers which, of course, is

---

<sup>9</sup>Arnott and Inci (2006) consider the effect of over saturation of parking on both cruising costs and costs incurred by through drivers

the aim of this paper.

In the absence of any standard, we develop our own methodology for generating trip opportunities with the principle aim of modeling stochastic arrival times. The story is as follows.

Each day, agents sit at home waiting for a trip opportunity. An exogenous, stochastic process then assigns each agent in the economy a number corresponding with their order in a trip sequence. An agent assigned the number 17 would be the 17th person in the sequence. Agents then leave their homes and travel to the CBD *in the order of their number assignment* (the agent assigned number 1 would leave first, the agent assigned number 2 would leave second, and so on). Since all neighborhoods are equidistant to the CBD and all agents travel at rate  $V_D$ , the order of departure immediately becomes the order of arrival at a given lot. Drivers then park, walk the remaining distance to the CBD, receive a fixed amount of utility  $v$  and return home. The process is repeated the following day.

Each day, which we will call a single iteration, every agent is given a number and must take the trip to the CBD. Agents are aware of their place in the sequence and of the total number of parkers in the economy, thus are able to correctly predict their total travel costs *on any particular day*. We formalize the technology below.

Let  $G$  be the set of all agents in the economy. Let  $\omega$  be a bijective mapping of  $\bar{N}$  onto the interval  $1, \dots, \bar{N} \subset \text{Naturals}$ , where  $\bar{N} = \sum_i n_i$  is the number of agents in the economy. Now let  $S_G$  be the set of all possible orderings  $\omega$ :

$$S_G = \{\omega : G \rightarrow \text{ s.t. } \omega \text{ is bijective}\} \quad (4)$$

Thus,  $S_G$  is an ordering of all the agents in the economy from  $1, 2, \dots, \bar{N}$ . We define  $P(S_G)$  as the power set OF  $S_G$ ; that is, the set containing all possible subsets of  $S_G$ . Now, define  $\mu$  as a map of the elements of  $S_{\bar{N}}$  onto the interval

$[0, 1]$

$$\mu : P(S_{\bar{N}}) \rightarrow [0, 1] \quad (5)$$

according to the following, where  $E$  is an element of  $S_G$ :

$$\mu(E) = \frac{|E|}{\bar{N}} \quad (6)$$

Thus,  $\mu$  is a probability measure and, for a given  $\omega \in S_{\bar{N}}$ ,

$$\mu(\omega) = \frac{1}{\bar{N}} \quad (7)$$

The above implies that  $\mu$  identifies a uniform distribution on  $S_G$ . Thus, the distribution of orderings is uniform and the selection process independent and identically distributed. This implies each ordering has an equal probability of being selected at any one time, and the probability that a given ordering is chosen is independent of which orderings, if any, were chosen before it. We employ a uniform distribution for simplicity only, and by no means claim this specification is either unique or most realistic. In fact, there may be good reason to suspect that an alternative distribution might better capture actual departure times according to value of time <sup>10</sup>

The intuition behind this technology arises from the fact that, in reality, drivers experience unexpected ‘shocks’ in their commuting routine which result in different departure *and* arrival times. Despite our simplifying assumption of distinct time preferences, in reality even the most similar of people are not likely to depart a location at exactly the same moment even if they live right next to one another. Likewise, real-life commuters encounter unexpected delays almost every day, ranging from serious (road closure due to an accident) to

---

<sup>10</sup>Consider, for example, a distribution which places a heavier weight on agents with high VOT. Then those agents who value their time more would be more likely to be chosen earlier (i.e. leave earlier) in the sequence. Generally, the distribution can be chosen arbitrarily to capture whatever properties a practitioner might desire.

minor (a string of red lights) variations in arrival time, *even in the presence of homogeneous departure times*. In fact, the very nature of public roads almost requires that no two people arrive at the same location at the same time, thereby forcing some sort of an ordering even between vehicles immediately following one another. The technology proposed here captures these realistic complications, albeit imperfectly, while allowing for suitable variation in the assignment of parking spaces.

This technology is by no means invulnerable to criticism. Most notably, we do not endogenize the departure time decision, which clearly is affected by value of time in reality. Some might claim, quite naturally, that those with higher value of time would choose to leave earlier in the day to avoid facing the higher search costs associated with a later departure. This claim seems very natural if we consider a world where agents can utilize early arrival time to complete non-transit tasks, such as beginning work early. In this model, however, we assume early arrival costs are treated similarly to transit costs, and hence subject to the same per minute costs as time spent in actual transit. A more ambitious extension of this paper would incorporate endogenous departure times, but for now we limit the model to our exogenous stochastic process.

## 4 Equilibrium Notion

Recall drivers seek to minimize total travel costs on a given trip. Since each trip yields a fixed benefit  $v$ , agents maximize their utility by minimizing costs. Thus, an agent of type  $i$  seeks to solve the following equation

$$\text{Min}_j C_j \tag{8}$$

where  $C_{ij} = \alpha_i[(\bar{x} - x_j)V_D^{-1} + \frac{k_j}{k_j - n_j}\gamma + x_j V_W^{-1}] + \tau_j$  from equation (2) above. We assume all parking spaces are owned by a single city planner, who sets parking

tariffs with the goal of minimizing total costs incurred by all commuters. There are no alternative areas for parking, and the city planner has full pricing power accordingly.

In setting parking fees, the city planner seeks to minimize the average expected travel costs across all agents. The planner does not necessarily minimize average total costs each trip, due to the stochastic ordering process. Instead, the planner attempts to minimize *average* expected costs across numerous trip iterations. Thus, in maximizing total welfare the planner seeks to solve the following<sup>11</sup>:

$$\text{Min}_{\tau_j} E \sum_i C_{ij} \quad (9)$$

There are two aspects of equation (9) worth discussing. First, recall consumers receive fixed utility  $v$  for each trip. Since  $v$  is constant across all consumers, we can maximize consumer welfare strictly by minimizing consumer costs. Second, here total welfare is a function of consumer cost only; the city planner does not gain additional surplus from a tariff increase in the form of tax revenue. This seems unnatural. However, we justify this specification by concerning ourselves only with alternative pricing schemes that are revenue-neutral. In this way we can ignore any potential gain in producer surplus and greatly simplify our analysis without precluding our principle aim.

## 4.1 The Optimal Solution

We proceed by defining the optimal solution in terms of allocation of parking spaces to agents in the presence of uniform parking fees. We will then consider how and if a tariff gradient can achieve the optimal solution in light of the stochastic trip generation.

---

<sup>11</sup>Optimal tariffs would in fact minimize Eq. 9; however, the technical complications of computing unique, optimal tariffs prohibit us from doing so in this paper, as will be discussed in the Results section

**Proposition 1:** *Allocating the most desirable spots to the agents with the highest values of time produces the highest value of social welfare<sup>12</sup>.*

**Proof:** Assume parking spaces are in fact allocated according to value of time. Since  $V_D > V_W$  it can be shown algebraically that, in the absence of all other vehicles, the travel cost of parking in lot  $A$  is strictly less than the that of parking in lot  $B$ . Since search costs are defined functionally as  $\frac{k_j}{k_j - n_j}$ , search costs (therefore total costs) are strictly increasing as  $n_j \rightarrow k_j$ . Given equation (6) above, an agent who arrives earlier will spend (weakly) less time in transit than an agent chosen immediately afterward.

Let  $C^t$  denote the cost incurred by the  $t^{th}$  person to arrive at a parking lot, where  $C^t < C^{t+1}$ . Without loss of generality, let  $A^1, A^2$  denote the first and second drivers in the trip sequence corresponding with  $\alpha_1 > \alpha_2$  respectively. Now, assume  $A^1, A^2$  switch positions in the sequence, so that the agent corresponding with  $\alpha_2$  parks at a more desirable location than the agent with  $\alpha_1$ . Since  $\alpha_1 > \alpha_2$  and  $C^{t+1} > C^t$ , this implies total costs are strictly greater after the switch, which, with equation (7), implies total welfare decreases. •

**Proposition 2:** *Switching any two agents in the sequence will not change total welfare iff the two agents have the same value of time*

**Proof:**

i  $B$  implies  $A$ : Follows directly from distributive law

Without loss of generality, let the first and second agents in the sequence have the same value of time, call it  $\tilde{\alpha}$ , and again let  $C^1, C^2$  denote the cost incurred by the first and second agent to arrive at a parking lot. If agents are ordered according to the process described above, total costs

---

<sup>12</sup>We define ‘most desirable’ as the spaces which produce the lowest total travel costs.

are  $\tilde{\alpha}C^1 + \tilde{\alpha}C^2$ , or  $\tilde{\alpha}(C^1 + C^2)$ . If we switch the first two agents, total costs are  $\tilde{\alpha}C^2 + \tilde{\alpha}C^1$ , or  $\tilde{\alpha}(C^2 + C^1) = \tilde{\alpha}(C^1 + C^2)$ . •

ii *A implies B*: By contradiction

To prove by contradiction, we need only to show that switching agents with different VOT *will* change total welfare. Conveniently, we have already shown this in the proof for proposition 1. •

Propositions 1 and 2 above define the solution to equation (7) in terms of ordering agents in a trip sequence. This is our optimal benchmark solution; if the planner had complete control over all agents in the economy, she would order them according to VOT to maximize social welfare. Clearly this is unrealistic, but it does provide a useful benchmark solution to contrast against more realistic solutions in our stochastic framework.

## 5 Simulation

We proceed by fitting the model with values for each of the exogenous variables and simulating. We use MATLAB to randomly generate a trip ordering of all  $\bar{N}$  agents according to equations (4) and (7). Each agent chooses which lot to begin ‘cruising’ according to equation (8) and their position in the sequence. Total costs are tracked for each agent and averaged, allowing us to compute welfare changes for a given  $v$ . A new ordering is then selected and the process is repeated  $I$  times.

To illustrate the impact differences in VOT have on welfare gains we define a number of possible VOT specifications, with varying degrees of heterogeneity. It is reasonable to predict that a larger discrepancy between type 1 and type 2 agents will result in larger welfare gains. We test this hypothesis directly by allowing type 1 and type 2 VOT to diverge, and comparing the results. For simplicity of illustration, we will also assume there are only two available parking

lots (A and B) and two types of agents (with value of time  $\alpha_1$  and  $\alpha_2$ ), which we arbitrarily assign in equal proportions to the agents in the economy<sup>13</sup>. Thus, for  $\bar{N} = 1,000$ ,  $N_1 = 500$  and  $N_2 = 500$ . A more realistic extension allowing for more of either category may be conducted using the general form of the model.

**Base Scenario** We begin by defining a ‘base scenario’ pricing scheme where  $\tau_A = \tau_B > 0$ , which we assume to be the existing price structure for a given city. Using the values in the base scenario, we calculate total revenue under the existing, uniform prices, which will allow us to track changes in welfare under different pricing while maintaining revenue-neutrality.

To track the impact value of time has on welfare, we will allow for a variety of VOT specifications, beginning with homogeneous commuters ( $\alpha_1 = \alpha_2$ ) and continuing as  $\alpha_1 - \alpha_2$  increases. Thus for the base scenario, we will simulate parking with uniform tariffs under each of the VOT specifications and record the distribution of agents between lots, the average cost incurred by agents, and the total tariff revenue.

**Pricing Gradient Scenario** Next we consider a ‘pricing gradient’ scenario where  $\tau_A$  is allowed to grow relative to  $\tau_B$ . In order to emphasize gains from a pricing gradient resulting specifically from heterogeneous VOT, we will restrict all of our pricing schemes to be revenue-neutral relative to the base scenario<sup>14</sup>. For each VOT specification, we will simulate for a given  $\tau_A > \tau_B$  subject to the revenue-neutrality constraint, again recording the distribution of agents between lots, the average cost incurred by agents, and the total tariff revenue generated. A comparison can then be drawn between the two policies. An example is helpful.

---

<sup>13</sup>This specification, albeit poorly justified, simplifies the analysis and is consistent with the work of Small and Yan (2004)

<sup>14</sup>For example, if a sample city with population 2,000 charges a uniform \$2.00 to park anywhere in the city, all alternative parking fee schemes *must* generate the equivalent \$4,000 in parking fee revenue



Suppose, for purposes of illustration,  $\tau_A = \tau_B = \$2$  in our base example, and we have specified three possible VOT distributions:  $[\alpha_1, \alpha_2] = (0.4, 0.4)$ ,  $(0.3, 0.5)$ ,  $(0.2, 0.6)$ . Using our tariff structure we will simulate parking behavior and welfare results for each of the three VOT specifications. Now, suppose in our pricing gradient scenario,  $\tau_A = \$3$  and  $\tau_B = \$1$ . Using this tariff structure, we simulate with all three VOT possibilities. We can then compare total welfare between the base scenario when  $\text{VOT} = (0.3, 0.5)$  and the pricing gradient when  $\text{VOT} = (0.3, 0.5)$  to identify any gains resulting from the pricing gradient, and repeat for each value of time specification.

**Optimal Scenario** Using the fixed tariff structure and tariff revenue from the base scenario, we lastly simulate using the optimal ordering solution defined in section 4.1. Here, agents are ordered from highest value of time to lowest value of time, which we have shown to be the ordering which produces the lowest possible total costs. The resulting values serve as a useful benchmark with which to compare the relative effectiveness of a pricing gradient in maximizing social welfare.

Finally, to capture the impact the ‘fullness’ of the lots has on welfare, we will simulate our model under different levels of lot saturation. Recalling the notation in equation (3), we will denote the total lot saturation as

$$\delta = \frac{\bar{N}}{\bar{K}} \quad (10)$$

where equation (3) implies  $0 \leq \delta \leq 1$ .

## 5.1 Definition of Variables

A fictional city is constructed in order to fit the model with actual parameters. We restrict the number of parking lots to two, denoted  $A$  and  $B$ . Lot  $A$  is located distance  $X_A = 1,000$  ft from the CBD, while lot  $B$  is located distance

$X_B = 1,500$  from the CBD. There are two types of agents in the economy who differ according to value of time, denoted  $\alpha_1$  and  $\alpha_2$ . As discussed earlier, the distribution of VOT will be evenly divided between the total number of agents  $\bar{N}$ , where  $\bar{N} = 1,000$  in this simulation. Thus, there are 500 Type 1 agents and 500 Type 2 agents. Again, it should be mentioned that the numbers used in this simulation were picked arbitrarily for purposes of illustration and simplicity. In a real-life application of the model, less restrictive parameters may be employed<sup>15</sup>.

The remaining values, which are constant across VOT specifications and pricing scenarios, are recorded in Table ???. The value of time specifications that will be used are recorded in Table ???.

Table 1: Variable Specifications

Variable	Value
$V_D$	30 mi/hr
$V_W$	3 mi/hr
$\gamma$	3 seconds
$k_A$	1000
$k_B$	1500
$X_A$	1000 ft
$X_B$	1500 ft
$\bar{X}$	5 mi

We set lot saturation at  $\delta = 0.9, 0.96$  corresponding to 90% and 96% of lot capacity, simulating each scenario for both values<sup>16</sup>. In the base scenario, we set tariffs equal in both lots to \$0.75 per trip ( $\tau_A = \tau_B$ ). When  $\delta = 0.9$ , this corresponds to daily tariff revenue of \$1687.5. When  $\delta = 0.96$ , uniform tariffs  $\tau = \$0.75$  generate \$1,800 in daily tariff revenue, which will remain the constraint for all simulations under the high saturation specification. Comparisons are then made between the different levels of saturation for each VOT

<sup>15</sup>Further discussion is deferred to the conclusion.

<sup>16</sup>In the absence of actual empirical data, these values capture what we consider to be realistic levels of parking lot use

Table 2: Value of Time Specifications

Group	$\alpha_1$	$\alpha_2$
a)	0.1 (\$6/hr)	0.1 (\$6/hr)
b)	0.2 (\$12/hr)	0.1 (\$6/hr)
c)	0.3 (\$18/hr)	0.1 (\$6/hr)
d)	0.4 (\$24/hr)	0.1 (\$6/hr)

specification.

## 5.2 Results

We simulate each scenario 1,000 times, where a scenario is defined as a VOT specification, a pricing structure, and a saturation level. Our principle results are recorded in Table ??<sup>17</sup>.

Support for the hypothesis is found by comparing the average total cost (ATC) values in the base and gradient scenarios. In all three heterogeneous value of time specifications, there is a net decrease in the average total costs incurred by commuters when pricing varies across lot location, even when tariff revenue is held constant. As expected, we find the optimal ordering yields the lowest average travel costs among the three pricing schemes for each VOT specification, while uniform lot pricing in the base scenario produces the highest overall ATC. For all three VOT pairs, the pricing gradient solution yields an average total cost somewhere between the base and optimal solution.

The distribution of cars between lots is also affected by the pricing scheme. In the base and optimal solution, tariffs are fixed, leading agents to minimize costs according to the search cost-walking cost trade off alone. The result is

<sup>17</sup>ATR is average tariff revenue (per ‘day’), ATC is the average total cost (in dollars) incurred by all agents across all 1,000 simulations, and parking fees are in dollars per trip. Some values have been rounded to the nearest value recorded.

Table 3: Simulation Results

Variable	Base	Gradient	Optimal	Base	Gradient	Optimal
<b><math>\delta=0.9</math></b>						
<b><math>\alpha_1=\\$0.1, \alpha_2=\\$0.1</math></b>						
$\tau_A$	\$0.75	\$0.75	\$0.75	\$0.75	\$0.75	\$0.75
$\tau_B$	\$0.75	\$0.75	\$0.75	\$0.75	\$0.75	\$0.75
$N_A$	977	977	977	982	982	982
$N_B$	1273	1273	1273	1418	1418	1418
ATR	\$1,687.5	\$1,687.5	\$1,687.5	\$1,800	\$1,800	\$1,800
ATC	\$2.2265	\$2.2265	\$2.2265	\$2.2337	\$2.2337	\$2.2337
<b><math>\alpha_1=\\$0.2, \alpha_2=\\$0.1</math></b>						
$\tau_A$	\$0.75	\$0.9573	\$0.75	\$0.75	\$0.96	\$0.75
$\tau_B$	\$0.75	\$0.61	\$0.75	\$0.75	\$0.61	\$0.75
$N_A$	977	907	977	982	960	982
$N_B$	1273	1343	1273	1418	1440	1418
ATR	\$1,687.5	\$1,687.5	\$1,687.5	\$1,800	\$1,800	\$1,800
ATC	\$2.9640	\$2.9339	\$2.9263	\$2.9750	\$2.9409	\$2.9381
<b><math>\alpha_1=\\$0.3, \alpha_2=\\$0.1</math></b>						
$\tau_A$	\$0.75	\$1.1	\$0.75	\$0.75	\$1.0	\$0.75
$\tau_B$	\$0.75	\$0.5498	\$0.75	\$0.75	\$0.5813	\$0.75
$N_A$	977	814	977	982	967	982
$N_B$	1273	1436	1273	1418	1433	1418
ATR	\$1,687.5	\$1,687.5	\$1,687.5	\$1,800	\$1,800	\$1,800
ATC	\$3.7016	\$3.6555	\$3.6261	\$3.7162	\$3.6477	\$3.6425
<b><math>\alpha_1=\\$0.4, \alpha_2=\\$0.1</math></b>						
$\tau_A$	\$0.75	\$1.15	\$0.75	\$0.75	\$1.15	\$0.75
$\tau_B$	\$0.75	\$0.4769	\$0.75	\$0.75	\$0.4820	\$0.75
$N_A$	977	913	977	982	962	982
$N_B$	1273	1337	1273	1418	438	1418
ATR	\$1,687.5	\$1,687.5	\$1,687.5	\$1,800	\$1,800	\$1,800
ATC	\$4.4392	\$4.3402	\$4.3259	\$4.4573	\$4.3537	\$4.3469

a fixed number of cars in each lot for each VOT pair (e.g. 977 agents park in lot A in the base and optimal solution when  $\delta = 0.9$ , regardless of the VOT specification). This result is initially surprising, but is easily explained; since tariffs are fixed, agents decide where to park solely on the basis of the search costs-walking costs trade off, both of which are scaled by  $\alpha_i$ . Although agents differ in their value of time, all agents are cost minimizers. As lot A fills and

search costs become excessively high, agents begin to park at lot B. As B fills in turn, agents return to lot A, and so forth. In the presence of fixed tariffs, lot A and lot B will fill at the same rate, regardless of agents VOT.

We find overall parking in lot A to decrease in the pricing gradient scenario, while lot B becomes more saturated. This is a direct consequence of the relatively higher tariffs charged at lot A in the gradient solution and the structure of the cost equation. Since tariffs are independent of VOT, higher tariffs at lot A effectively ‘scare away’ agents with lower VOT (who are willing to incur higher walking and search costs) while enabling those with higher VOT to avoid higher walking costs (which they are sensitive) by paying a fixed fee. As lot A fills, the additional search costs, along with the higher tariffs, force agents back to lot B at a faster rate than when tariffs were equal, hence the more uneven parking distribution.

Finally, as  $\alpha_1 - \alpha_2$  increases, the difference in revenue-neutral tariffs  $\tau_A - \tau_B$  increases as well. This result is intuitive; as agents value their time more and more, they are willing to pay an even higher tariff to avoid increasingly costly time spent searching and walking.

Another useful metric can be applied to interpret the results: since the optimal ordering scenario produces the lowest possible ATC among all orderings, we can use average total costs from the base scenario to calculate the amount of unclaimed ‘surplus’ by subtracting  $ATC_{Base}$  from  $ATC_{Opt}$ . This captures any potential surplus a pricing gradient can hope to capture, and allows us to measure the effectiveness of variable pricing. Table ?? summarizes the performance of the pricing gradient.

The second and third column show average total costs decrease at an increasing rate as  $\alpha_1 - \alpha_2$  grows, which supports the intuitive argument that agents are increasingly time-sensitive as VOT increases, thus stand more to gain from a pricing gradient. The third column suggests that the variable pricing scheme is very effective in capturing the lost surplus resulting from uniform tariffs. In

Table 4: Gains from Pricing (base vs. gradient)

	$(\alpha_1, \alpha_2)$	$\Delta$ ATC	% $\Delta$ ATC	% of Surplus	\$ Saved
$\delta=0.9$	0.1, 0.1	-	-	-	-
	0.2, 0.1	-\$0.0301	-1.016	79.84	\$67.73
	0.3, 0.1	-\$0.0461	-1.245	61.06	\$103.73
	0.4, 0.1	-\$0.0990	-2.230	87.38	\$222.75
$\delta=0.96$	0.1, 0.1	-	-	-	-
	0.2, 0.1	-\$0.0341	-1.146	92.41	\$81.84
	0.3, 0.1	-\$0.0685	-1.843	92.94	\$164.4
	0.4, 0.1	-\$0.1036	-2.324	93.84	\$248.64

fact, in the high saturation scenario, a variable pricing policy captures over 90% of the inefficiency present in the base scenario, cutting average travel costs up to 2.324% when  $VOT=(0.4, 0.1)$ . This result holds despite the fact that tariffs have not been optimally determined to minimize costs in this study. A more sophisticated simulation able to identify cost-minimizing tariffs would strengthen these results even further.

The fourth column reports the average *daily* savings economy-wide under a variable tariff scheme, calculated using the difference in ATC and the number of agents  $\bar{N}$  in the economy<sup>18</sup>. At first glance, the gains seem trivial. However, when considering the accumulation of these saved costs over one or many years, the case for variable parking fees is made quite easily.

Regardless of the metric, there is one trend which persists throughout the results: as lots become more congested, gains from heterogeneous pricing increase. This is clear throughout Table ?? and ??, where gains under the  $\delta = 0.96$  scenario consistently out measure those of the less congested  $\delta = 0.90$  simulations. This is true for all value of time specifications, and supports what one might guess *a priori*: as lots become more saturated, search costs become even more of a factor in the parking decision, meaning late arrivals with high VOT must incur even higher costs. By pricing, those who are not as sensitive to costs

<sup>18</sup>  $\bar{N} = 2,250$  and  $2,400$  when  $\delta = 0.9$  and  $0.96$  respectively

associated with travel time are deterred to lots further from the CBD, while the high VOT agents gladly pay a fixed fee to minimize time spend in transit. In short, there is more to gain from heterogeneous pricing as parking lots become more and more saturated, and more *is* gained.

*A note on tariffs:* It is important to note that the tariffs used in the pricing gradient scenarios are not unique. There exists at least two—and likely many more—alternative specifications of  $\tau_A$  and  $\tau_B$  which satisfy the revenue-neutrality constraint<sup>19</sup>. The values recorded in Table ?? were found using a crude algorithm of ‘guess and check’; beginning with the base tariffs,  $\tau_A - \tau_B$  was allowed to increase until total revenue neared the required level. From there, one fee was held constant and the other ‘fine tuned’ to generate the required level of revenue.

We provide no proof that the tariffs presented here are the optimal, cost-minimizing values subject to the revenue neutrality constraint. Given the stochastic ordering process defined by equations (5) and (7) and the inherently dynamic relationship between tariffs, parking distribution, and total revenue, we find this to be an enormously complicated task, which we humbly defer in this paper. However, the reader is encouraged to keep the results in perspective; the goal of this paper is to show a net welfare gain resulting from a pricing gradient in light of heterogeneous values of time, which has been shown. Any further improvements via ‘optimized’ parking tariffs only serve to further our hypothesis and results.

### 5.3 Equity Concerns

Although this project focuses on the efficiency of parking policies, it is important to discuss the equity concerns the results might raise. To this aim, it is useful to identify how a change in parking fees affects the distribution of agents across

---

<sup>19</sup>Consider an example: set tariffs in one lot equal to zero and tariffs at the other lot such that once the ‘free’ lot fills, the remaining agents (who *must* now park at the priced lot) generate a sufficient level of revenue

lots. Table ?? summarizes, where  $N_{1A}$  is the average number of type 1 agents parked in lot A, and  $N_1$  is the average number of *total* cars parked in lot A<sup>20</sup>.

Table 5: Distribution of Agent Types

$\alpha_1=\$0.2, \alpha_2=\$0.1$			
		$\delta = 0.9$	$\delta = 0.96$
Uniform:	$N_{1A}$	49.45	99
	$N_1$	98	48.643
	$N_{1A}/N_1$	50.46%	49.13%
Gradient:	$N_{1A}$	90.529	95.364
	$N_1$	90.529	95.386
	$N_{1A}/N_1$	100%	99.98%
$\alpha_1=\$0.3, \alpha_2=\$0.1$			
		$\delta = 0.9$	$\delta = 0.96$
Uniform:	$N_{1A}$	48.47	49
	$N_1$	98	99
	$N_{1A}/N_1$	49.46%	49.49%
Gradient:	$N_{1A}$	80.293	96.628
	$N_1$	80.293	96.628
	$N_{1A}/N_1$	100%	100%
$\alpha_1=\$0.4, \alpha_2=\$0.1$			
		$\delta = 0.9$	$\delta = 0.96$
Uniform:	$N_{1A}$	49.27	48.70
	$N_1$	98	99
	$N_{1A}/N_1$	50.28%	49.19%
Gradient:	$N_{1A}$	91.350	95.639
	$N_1$	91.350	95.639
	$N_{1A}/N_1$	100%	100%

First, in the uniform pricing scheme the distribution of agent types in lot A is approximately fifty-fifty. This is expected. Since both types seek to minimize costs, uniform tariffs imply drivers base their decisions on the walking cost-search cost trade off alone. Drivers of different types will react identically to this trade off, although they may incur different time costs. Since there are equal

<sup>20</sup>In order to expedite the simulation process in this secondary analysis, the number of agents, parking spaces, and iterations were decreased by a magnitude of 10. This reduction, if made earlier, might have unduly affected the principle results, but we find it an acceptable convenience in the secondary analysis.



numbers of each agent type and each agent has an equal likelihood of being chosen at a given time in the trip sequence, the (almost) even distribution of agent types in lot A is expected.

The most surprising result of Table ?? is the fact that a fee gradient deters type 2 drivers from lot A *almost 100% of the time*. The only exception to this appears in the high saturation scenario when  $\alpha_1 = \$0.2$  and  $\alpha_2 = \$0.1$ . In this scenario, there is *one* iteration in which a type 2 agent parks in lot A. In addition to underlining the effectiveness of the pricing gradient in moving towards the optimal scenario described in section 4.1, this result begs the question: under what circumstances would a type 2 agent find it optimal to park in lot A, and why does this happen so infrequently?

The answer lies in the nature of the trip generation. Since type 1 agents are particularly averse to spending time walking or searching for a spot, they willingly incur the higher tariff in lot A to avoid more travel time. Type 2 agents, however, are more willing to spend time walking or searching for an available space. This means they will normally avoid the higher fee and incur the higher time costs by parking in lot B. However, in the event that a particular trip generation randomly selects a high proportion of type 2 agents early in the sequence, it may be the case that lot B becomes disproportionately full, since type 2 agents tend to park in lot B. If lot B is full enough to impart sufficiently high search costs, a type 2 agent may find it cheaper to park in lot A *even with the higher tariff*. As noted by the results in Table ??, this situation, although rare, is absolutely possible given the stochastic trip generation process.

At first glance, Table ?? seems to imply type 2 agents are worse off under the gradient fee structure than they were in the uniform fee structure. In the gradient scenario, type 2 agents are effectively barred from parking at the closer lot as they most often find lot A prohibitively expensive. This raises important equity concerns. If differences in value of time are attributed to differences in opportunity costs associated with forgone income, as they frequently are, then

this seems suggest that the poor suffer at the hands of a parking gradient while the rich benefit. Upon closer inspection, the opposite proves true.

While it is true type 2 agents are effectively barred from more preferable parking spaces under the gradient policy, it is also true that type 2 agents will pay less for parking at a further lot. This is an interesting trade off: Type 2 agents are barred from the preferable lot, which they formerly parked in 50% of the time, but now pay less to park at a less desirable lot. Similarly, type 1 agents now pay more if they choose to park at lot A, but do not face as high of search costs due to the removal of type 2 agents from lot A. To determine which effect dominates, we calculate the changes in average total costs by agent type<sup>21</sup>. Table ?? summarizes.

Table 6: Changes in ATC by Agent Type

Lot Saturation		$ATC_1$	$ATC_2$	$(\alpha_1, \alpha_2)$
$\delta = 0.9$	Base	3.9504	2.0768	
	Gradient	3.9547	2.0174	(\$0.2, \$0.1)
	% $\Delta$ ATC	0.109%	-2.860%	
	Base	5.5236	2.0774	
	Gradient	5.5542	1.9659	(\$0.3, \$0.1)
	% $\Delta$ ATC	0.554%	-5.367%	
	Base	7.0999	2.0770	
	Gradient	7.1013	1.18927	(\$0.4, \$0.1)
	% $\Delta$ ATC	0.020%	-8.873%	
$\delta = 0.96$	Base	3.9504	2.0768	
	Gradient	3.9547	2.0174	(\$0.2, \$0.1)
	% $\Delta$ ATC	-0.054%	-2.970%	
	Base	5.5236	2.0774	
	Gradient	5.5542	1.9659	(\$0.3, \$0.1)
	% $\Delta$ ATC	-0.737%	-4.402%	
	Base	7.0999	2.0770	
	Gradient	7.1013	1.18927	(\$0.4, \$0.1)
	% $\Delta$ ATC	-0.139%	-8.803%	

Somewhat surprisingly, agents with lower VOT gain the most from variable fees while agents with higher VOT gain relatively little. In fact, type 1 agents

<sup>21</sup>Again, to speed the simulation process, relevant variables are decreased by a magnitude of ten.

are worse off under variable pricing when lot density  $\delta = 0.9$ , albeit marginally. If we operate under the assumption that value of time is directly related to income, Table ?? suggests that a fee gradient favors the poor over the rich. Social philosophy aside, this begs the question of whether punishing the rich to benefit the poor is a desirable policy outcome. However, given the relatively marginal increase in the average total costs of type 1 agents—when in fact they do increase—and the significant reduction in type 2 average total costs, the sacrifice of the rich is in this case well justified.

## 5.4 Policy Applications

This model was constructed with realistic policy applications in mind. While the variables were chosen in order to illustrate the underlying results of the model without complicating the analysis with additional considerations, this does not have to be the case. The following steps describe how to best apply this framework to a real-world business district:

First, a structure must be imposed on the CBD identifying parking 'rings' and a location determined to be the center of the CBD. From here, parking lots of similar distance can be aggregated into the linear city form and relevant measures taken (i.e. distances, number of spaces). Second, average lot capacity should be calculated during peak driving time. While it may be true that lots near the center of the city will be full, it is likely that lots on the peripheral will not be filled. From the total number of parking spaces and of parked cars in the CBD, an average lot density can be calculated. Third, the remaining variables from Table ?? need to be approximated. This can be achieved a number of ways, and we defer the details here. Finally, it is necessary to capture the value(s) of time of the drivers. This might be the most potentially difficult task, likely requiring a lengthy and sufficiently broad survey of commuters. However, this complication is not unique to this model, but to any and all models requiring

survey data<sup>22</sup>.

Once the relevant variables are collected, a city planner can use the actual tariff revenue generated under the status quo fee structure to establish a neutral revenue target. Alternatively, a new tariff revenue target can be established to meet the needs of the city<sup>23</sup>. Either way, simulations can be executed and average total costs compared given the revenue target. The fee structure providing the lowest average total costs across commuters—while maintaining target revenues—would be the recommended fee structure for the city.

There are a number of useful and realistic settings in which this model, complete with the fundamental assumptions, might be applied. A college campus, for example, has complete authority over the pricing of parking spaces. A large sports arena or convention center are also potential candidates. In an obvious application, an airport providing long-term ‘economy’ and ‘premium’ parking spaces to travelers could use this model to set tariffs across lots.

However, the easiest and most cost-effective application of this model to public policy would be the incorporation of our principle results into policy decisions; namely, that planners should charge higher rates for more preferable spaces. On the surface, it seems unnecessary to provide proof that drivers gain when you allow those who are willing to pay for a closer parking space to do just that. It seems so, but a surprising number of cities and universities still charge a uniform fee across all lots within their control. If nothing else, this paper suggests to these planners and policymakers that parking policy should acknowledge the existence of heterogeneous agents and vary its parking fees accordingly, even if those fees are arrived at in a less technical manner than presented here.

---

<sup>22</sup>Small, Yan and Winston tackled the econometric details of such a task in a 2005 paper, which might prove useful in an empirical application of this model.

<sup>23</sup>For example, if the city believed changing parking fees would impart some fixed cost that needed recovering.

## 6 Conclusion

In summary, this paper advances two new ideas into the parking literature. The first is a realistic—yet simple—spatial specification of a central business district capturing the intuitive relationship between parking spaces and distance from the CBD. Second, we show a net decrease in average travel costs of heterogeneous agents when parking fees are variable rather than uniform. The results focus on the comparison of uniform parking fee policies with revenue-neutral fee gradients. Thus, the model shows planners can decrease the average travel costs of agents in the economy *even when tariff revenues are held constant*.

Two important relationships are revealed in the simulations. The first is that gains from variable pricing increase as agents VOT increases. This is an intuitive result; as people value their time more, they are increasingly averse to time spent in transit. Second, as overall lot saturation increases, both the potential and actual gains from variable pricing increase, due largely to increasingly influential search costs. This suggests that cities where ‘cruising’ is a particularly serious problem stand to gain the most from implementing a fee gradient policy.

Future research in this area might include alternative trip generation technologies, perhaps incorporating how value of time affects the departure time of heterogeneous agents. Another project might consider how unexpected daily ‘shocks’ affect an agent’s willingness to pay for reliable parking. A more ambitious study would propose a single model incorporating heterogeneous agents while considering how congestion—both in parking and multi-use access roads— affects overall transit time in the economy. However, the most obvious extension of this paper would tackle the issue of calculating optimal, cost-minimizing tariff levels in a revenue-neutral environment without sacrificing the realism brought by heterogeneous agents and stochastic arrival times.

We conclude by reaffirming the study of optimal parking policies as realistic alternatives to combating congestion. The results derived here do not aim to

complicate the literature any more than as to show the existence of a benefit arriving from variable pricing *in addition to* the previously established benefit of reducing congestion. Even if future models do not directly incorporate the results here, this model, if nothing else, augments the gains found in the existing literature. As urban development continues to swell the boundaries of what would be considered a central business district, city planners will be forced to consider policies that minimize increasingly costly time spend in transit. This paper serves as another voice in the call for more research in the understudied, yet potentially rewarding, field of urban parking policy as a solution to these problems.

## References

- [1] S. Anderson and A. de Palma, The Economics of Pricing Parking, *Journal of Urban Economics*, **55**, 1-24 (2004).
- [2] S. Anderson and A. de Palma, Parking in the City (working paper).
- [3] R. Arnott and E. Inci, An Integrated Model of Downtown Parking and Traffic Congestion, *Journal of Urban Economics*, **60**, 418-442, (2006)
- [4] R. Arnott and J. Rowse, Modeling Parking, *Journal of Urban Economics*, **45**, 97-124, (1999).
- [5] E. Calthrop, Evaluating On-street Parking Policy (working paper).
- [6] E. Calthrop and S. Proost, Regulating On-street Parking *Regional Science and Urban Economics*, **36**, 29-48, 2006.
- [7] A. Glazer and E. Niskanen, Parking Fees and Congestion, *Regional Science and Urban Economics*, **22**, 123-132, (1992).
- [8] E. Parkany, "Traveler Responses to New Choices: Toll vs. Free Alternatives in a Congested Corridor," Ph.D. Dissertation, University of California-Irvine (1999)
- [9] D. Shoup, *The High Cost of Free Parking*, American Planning Association, (2005)
- [10] K. Small and J. Yan, The Value of "Value Pricing" of Roads: Second-Best Pricing and Product Differentiation, *Journal of Urban Economics*, **49**, 310-336, (2001).
- [11] K. Small, C. Winston, and J. Yan, Uncovering the Distribution of Motorists' Preferences for Travel Time and Reliability, *Econometrica*, **73**, 1367-1382, (2005).

- [12] E. Sullivan, "Evaluating the Impacts of the SR 91 Variable-Toll Express Lane Facility: Final Report," report to California Department of Transportation, Dept. of Civil and Environmental Engineering, Cal Polytechnic State University, San Luis Obispo, CA (1998).
- [13] W. Vickery, Congestion Theory and Transport Investment, *American Economic Review Papers and Proceedings*, **59**, 251-260, (1969)
- [14] R. Voith, Parking, Transit, and Employment in a Central Business District, *Journal of Urban Economics*, **44**, 43-58, (1998).