THE EFFECTIVE STIFFNESS OF A STIFFENER ATTACHED TO A FLAT PLYWOOD PLATE

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THE EFFECTIVE STIFFNESS OF A STIFFENER ATTACHED TO A FLAT PLYWOOD PLATE[1]

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Summary

This report presents a mathematical analysis of the effective stiffness of a stiffener attached to a flat plywood plate. The analysis yields formulas for computing the stiffness added to a plywood plate by gluing a stiffener to one face of the plywood. The data obtained at the Forest Products Laboratory from 107 tests of 17 plates with stiffeners of differing depths supply reasonable confirmation of the analysis. Formulas for computing the effective stiffness of the stiffener are given in equations (68), (69), and (70). An explanation of the constants involved is given with these equations. Equations (71) and (72) and their accompanying explanation give a similar summary for the case in which both plate and stiffener are of an isotropic material.

Introduction

It is well known that the buckling load of a rectangular plate can be considerably increased by adding a centrally located stiffener attached to the

[1] This is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in aircraft. Results here reported are preliminary and may be revised as additional data become available. Original report published 1946.

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

plate and perpendicular to an edge. In a theoretical determination of this buckling load it is necessary to know the stiffness added to the plate by the attached stiffener. Since the stiffener is rigidly attached to the plate, it is at once apparent that the added stiffness will not be merely the flexural rigidity of the stiffener alone but will be considerably more. Several theoretical discussions[3] of the buckling of rectangular plates reinforced by stiffeners have been given elsewhere, but no methods for determining the added stiffness due to the reinforcing stiffeners have been included.

Separate theoretical discussions pertaining to the added stiffness due to the attached stiffener have been presented by several authors,[4] but unless the plate is very thin their discussions are inadequate. If a stiffened plate of this type is bent while it is supported on all four edges, the neutral surface in the neighborhood of the stiffener will lie between the middle plane of the plate and the middle plane of the stiffener. It will be convenient to think of this as a shift of the neutral surface from the middle plane of the plate. The added stiffness will then be due to the plate-stiffener combination being bent about the shifted neutral surface rather than due to the stiffener being bent about its own neutral surface. If the plate and stiffener have small cross-sectional dimensions, the neutral surface will not, in most cases, be shifted very far from the middle plane of the plate, and a rough estimate of its position will furnish a good approximation of the added stiffness. If the cross-sectional dimensions are relatively large, however, considerable error can result in the calculation of the added stiffness unless the position of the neutral surface can be accurately determined. For metallic plates and stiffeners the cross-sectional dimensions will frequently be small, and an approximate method of determining the position of the neutral surface may suffice; but for plywood plates and solid wood stiffeners the cross-sectional dimensions are large enough to demand a more accurate method for locating the position of the neutral surface.

It is the purpose of this report to present an approximate theoretical method of determining the stiffness added to a plywood plate by a reinforcing solid wood stiffener and the results of a series of experimental tests

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[3] Timoshenko, S., Theory of Elastic Stability, page 371.
Lindquist, E., Journal Aer. Sciences, Vol. 6, page 269, May 1939.
Ratzersdorfer, J., Rectangular Plates with Stiffeners, Aircraft Engineering, September 1942. This paper appears to be very similar to the following one in Russian: Lokshin, A. S., Journal of Applied Math. and Mech. Vol. 2 (1935) page 225.

[4] Timoshenko, S., Theory of Elasticity, page 156.
Karman, Th. Von. "Festschrift August Foppls," page 114, 1923.
Reissner, E. Stahlbau, 1934, S. 206.

confirming the main feature of the theoretical analysis. The theoretical method is also extended to the case in which the plate and stiffener are both of an isotropic material. In the analysis, the position of the neutral surface in a plywood plate-stiffener combination is obtained theoretically by making an approximate mathematical study of the stress distribution. The analysis makes use of the minimum energy principle. The general equation obtained for the neutral surface is complicated; at the stiffener, however, the equation reduces to a simple form. If the position of the neutral surface at the stiffener is known, it is possible to compute approximately the added stiffness due to the stiffener.

Mathematical Analysis

Choice of Axes and Orientation of Plywood and Stiffener

In the theoretical analysis plywood is assumed to be an orthotropic material, that is, a material possessing three mutually perpendicular planes of elastic symmetry.[5]

Consider a rectangular plywood plate with its area bisected by a stiffener perpendicular to an edge. The stiffener is of rectangular cross-section. The grain of the face plies of the plate is taken to make an angle of either 90° or 0° with the stiffener. The plate is assumed supported on all four edges and to have a length 2ℓ parallel to the stiffener and a rather large width in the other direction.[6] The plate has a thickness of h. The stiffener is of length 2ℓ. The plate is loaded in flexure with loads applied immediately over the stiffener and symmetrically situated with respect to the middle of the span of 2ℓ. The origin of coordinates is taken in the middle plane of the plate directly over the center of the stiffener as shown in figure 1. The x-axis is taken parallel to the stiffener. Due to symmetry only one-half of the plate need be considered, say that corresponding to positive y, where the middle plane of the plate is the xy-plane. The z-axis is directed upward.

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[5] Forest Products Laboratory Reports Nos. 1300, 1312, 1316.

[6] Experimental confirmation of the mathematical analysis indicates that the formulas are approximately correct for any width equal to or greater than 1.6ℓ. See appendix.

Stress-strain Components

It will be assumed that the stress component[7] Z_z vanishes throughout the plate and stiffener. It will also be assumed that throughout the plate and stiffener the strain components can be expressed in the following manner:

$$
\begin{aligned}
e_{xx} &= (e_{xx})_o + z\,(e_{xx})_1 \\
e_{yy} &= (e_{yy})_o + z\,(e_{yy})_1 \\
e_{xy} &= (e_{xy})_o + z\,(e_{xy})_1
\end{aligned}
\tag{1}
$$

where $(e_{xx})_o$, $(e_{xx})_1$, $(e_{yy})_o$, $(e_{yy})_1$, $(e_{xy})_o$, $(e_{xy})_1$ are functions of x and y and that do not vary from layer to layer in the plate or in the stiffener. These assumptions made regarding the strains can be interpreted as follows: In the stiffener and the part of the plate directly over it a strain distribution is assumed which varies linearly with z. These strains are then assumed to be transmitted continuously out into the plate.

At a point in a given ply

$$
\begin{aligned}
X_x &= \frac{E_x}{\lambda}\{e_{xx} + \sigma_{yx}\, e_{yy}\} \\
Y_y &= \frac{E_y}{\lambda}\{e_{yy} + \sigma_{xy}\, e_{xx}\} \\
X_y &= \mu_{xy}\, e_{xy}
\end{aligned}
\tag{2}
$$

where $\lambda = 1 - \sigma_{xy}\, \sigma_{yx}$ (3)

In these equations E_x and E_y are Young's moduli in the x- and y-directions, respectively. Poisson's ratio σ_{xy} is the ratio of the contraction parallel to the y-axis to the extension parallel to the x-axis associated with a tension parallel to the x-axis and similarly for σ_{yx}. The quantity μ_{xy} is the modulus of rigidity associated with the directions of x and y. If the plies in the plate are all rotary cut, the elastic constants in the various layers can be described as follows: The subscripts L and T are used to refer to the longitudinal and tangential directions in the wood. Then for the plies in which the grain of the wood is parallel to the x-axis,

[7] The notation is that of A. E. H. Love, see Love, A. E. H., Mathematical Theory of Elasticity.

$$E_x = E_L, \quad E_y = E_T$$

$$\sigma_{xy} = \sigma_{LT}, \quad \sigma_{yx} = \sigma_{TL} \tag{4}$$

$$\mu_{xy} = \mu_{LT}$$

while in plies in which the grain of the wood is parallel to the y-axis

$$E_x = E_T, \quad E_y = E_L$$

$$\sigma_{xy} = \sigma_{TL}, \quad \sigma_{yx} = \sigma_{LT} \tag{5}$$

$$\mu_{xy} = \mu_{TL} = \mu_{LT}$$

Substituting (1) in (2), it follows that in a given ply

$$X_x = \frac{E_x}{\lambda} \{ (e_{xx})_o + \sigma_{yx} (e_{yy})_o + z [(e_{xx})_1 + \sigma_{yx} (e_{yy})_1] \} =$$

$$(X_x)_o + z (X_x)_1$$

$$Y_y = \frac{E_y}{\lambda} \{ (e_{yy})_o + \sigma_{xy} (e_{xx})_o + z [(e_{yy})_1 + \sigma_{xy} (e_{xx})_1] \} = \tag{6}$$

$$(Y_y)_o + z (Y_y)_1$$

$$X_y = \mu_{LT} \{ (e_{xy})_o + z (e_{xy})_1 \} = (X_y)_o + z (X_y)_1$$

Membrane Stresses

It may be seen that each stress component in equations (6) consists of two parts: a part with a zero subscript and a part multiplied by z. The first part is independent of z, and it will be called a membrane stress as in the membrane theory of shells. The remaining part varies linearly with z, and it will be called a bending stress. The average values over the width of the plate of these membrane stress components P_x, P_y, and P_{xy} are given by the following equations:

$$P_x = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} X_x\, dz, \quad P_y = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} Y_y\, dz, \quad P_{xy} = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} X_y\, dz \tag{7}$$

Using equations (6), equations (7) for a plate whose construction is symmetrical with the xy-plane become

$$\begin{aligned} P_x &= \frac{E_a}{\lambda}\left\{(e_{xx})_o + \bar{\sigma}_{yx}(e_{yy})_o\right\} \\ P_y &= \frac{E_b}{\lambda}\left\{(e_{yy})_o + \bar{\sigma}_{xy}(e_{xx})_o\right\} \\ P_{xy} &= \mu_{LT}\left\{(e_{xy})_o\right\} \end{aligned} \tag{8}$$

where

$$E_a = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} E_x\, dz, \qquad E_b = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} E_y\, dz$$

$$\bar{\sigma}_{xy} = \frac{\sigma_{TL}E_L}{E_b}, \quad \bar{\sigma}_{yx} = \frac{\sigma_{TL}E_L}{E_a}, \quad \lambda = 1 - \sigma_{LT}\,\sigma_{TL} \tag{9}$$

From the conditions for the equilibrium of a rectangular element, it follows that[8]

$$\begin{aligned} \frac{\delta P_x}{\delta x} + \frac{\delta P_{xy}}{\delta y} &= 0 \\ \frac{\delta P_{xy}}{\delta x} + \frac{\delta P_y}{\delta y} &= 0 \end{aligned} \tag{10}$$

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[8] The vertical shearing forces on the sides of the rectangular element are neglected.

Equations (10) assure the existance of a stress function F such that

$$P_x = \frac{\delta^2 F}{\delta y^2}, \quad P_y = \frac{\delta^2 F}{\delta x^2}, \quad P_{xy} = -\frac{\delta^2 F}{\delta x \delta y} \tag{11}$$

Solving equations (8) for the quantities $(e_{xx})_o$, $(e_{yy})_o$, $(e_{xy})_o$, it follows that

$$(e_{xx})_o = \frac{1}{\overline{E}_x}\{P_x - \overline{\sigma}_{xy} P_y\}, \quad (e_{yy})_o = \frac{1}{\overline{E}_y}\{P_y - \overline{\sigma}_{yx} P_x\},$$

$$(e_{xy})_o = \frac{1}{\mu_{LT}} P_{xy} \tag{12}$$

where

$$\overline{E}_x = \frac{E_a (1 - \overline{\sigma}_{xy}\overline{\sigma}_{yx})}{\lambda}, \quad \overline{E}_y = \frac{E_b (1 - \overline{\sigma}_{xy}\overline{\sigma}_{yx})}{\lambda} \tag{13}$$

The equation of compatibility connecting e_{xx}, e_{yy}, and e_{xy} leads to the following relation containing $(e_{xx})_o$, $(e_{yy})_o$ and $(e_{xy})_o$:

$$\frac{\delta^2 (e_{xx})_o}{\delta y^2} + \frac{\delta^2 (e_{yy})_o}{\delta x^2} = \frac{\delta^2 (e_{xy})_o}{\delta x \delta y} \tag{14}$$

On substituting equations (11) in equations (12) and then substituting the resulting equations in equations (14), it results that

$$\frac{1}{\overline{E}_y}\frac{\delta^4 F}{\delta x^4} + \left(\frac{1}{\mu_{LT}} - \frac{2\overline{\sigma}_{xy}}{\overline{E}_x}\right)\frac{\delta^4 F}{\delta x^2 \delta y^2} + \frac{1}{\overline{E}_x}\frac{\delta^4 F}{\delta y^4} = 0 \tag{15}$$

where the relation

$$\frac{\overline{\sigma}_{xy}}{\overline{E}_x} = \frac{\overline{\sigma}_{yx}}{\overline{E}_y} \tag{16}$$

which follows from equations (9) and equations (13), has been used. It will be convenient to write equation (15) in the following form:

$$\frac{\delta^4 F}{\delta x^4} + 2\kappa_o \frac{\delta^4 F}{\delta x^2 \delta \eta^2} + \frac{\delta^4 F}{\delta \eta^4} = 0 \tag{17}$$

where

$$\kappa_o = \sqrt{\frac{\overline{E}_x \overline{E}_y}{2}} \left(\frac{1}{\mu_{LT}} - \frac{2\overline{\sigma}_{xy}}{\overline{E}_x} \right) \tag{18}$$

$$\eta = \epsilon_o y, \tag{19}$$

with

$$\epsilon_o = \sqrt[4]{\frac{\overline{E}_x}{\overline{E}_y}} \tag{20}$$

A general solution of equation (17) will be of the form[9]

$$F = R \left[F_1 \ (x + i\alpha\eta) + F_2 \ (x + i\beta\eta) \right] \tag{21}$$

where

$$i = \sqrt{-1}$$

$$\alpha = \sqrt{\kappa_o + \sqrt{\kappa_o^2 - 1}}, \qquad \beta = \sqrt{\kappa_o - \sqrt{\kappa_o^2 - 1}} \tag{22}$$

F_1 and F_2 are arbitrary analytic functions of their respective complex variables, and the letter R means that the real part of the expression in the bracket is to be taken. Substituting equation (21) in equation (11) it results that

$$P_x = \frac{\delta^2 F}{\delta y^2} = R \left[- \alpha^2 \epsilon_o^2 \frac{\delta^2 F_1}{\delta \zeta_1^2} - \beta^2 \epsilon_o^2 \frac{\delta^2 F_2}{\delta \zeta_2^2} \right]$$

$$P_y = \frac{\delta^2 F}{\delta x^2} = R \left[\frac{\delta^2 F_1}{\delta \zeta_1^2} + \frac{\delta^2 F_2}{\delta \zeta_2^2} \right] \tag{23}$$

$$P_{xy} = - \frac{\delta^2 F}{\delta x \delta y} = R \left[- i\alpha\epsilon_o \frac{\delta^2 F_1}{\delta \zeta_1^2} - i\beta\epsilon_o \frac{\delta^2 F_2}{\delta \zeta_2^2} \right]$$

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[9] Forest Products Laboratory Report No. 1510, pages 4-5.

where

$$\zeta_1 = x + i\alpha\eta, \quad \zeta_2 = x + i\beta\eta \tag{24}$$

On substituting the results of equations (23), equations (12) become

$$(e_{xx})_o = R\left\{\frac{1}{\overline{E}_x}\left[-f\,\frac{\delta^2 F_1}{\delta\zeta_1^2} - g\,\frac{\delta^2 F_2}{\delta\zeta_2^2}\right]\right\}$$

$$(e_{yy})_o = R\left\{\frac{1}{\overline{\overline{E}}}\left[f'\,\frac{\delta^2 F_1}{\delta\zeta_1^2} + g'\,\frac{\delta^2 F_2}{\delta\zeta_2^2}\right]\right\} \tag{25}$$

$$(e_{xy})_o = R\left\{\frac{1}{\mu_{LT}}\left[-i\alpha\epsilon_o\,\frac{\delta^2 F_1}{\delta\zeta_1^2} - i\beta\epsilon_o\,\frac{\delta^2 F_2}{\delta\zeta_2^2}\right]\right\}$$

where

$$f = \alpha^2\epsilon_o^2 + \overline{\sigma}_{xy}, \qquad g = \beta^2\epsilon_o^2 + \overline{\sigma}_{xy}$$

$$f' = 1 + \overline{\sigma}_{yx}\,\alpha^2\epsilon_o^2, \qquad g' = 1 + \overline{\sigma}_{yx}\beta^2\epsilon_o^2 \tag{26}$$

Equations (26) can be simplified if the following substitutions and rearrangements are made. From equations (16) and (20), it follows that

$$\overline{\sigma}_{yx} = \frac{\overline{\overline{E}}_y}{\overline{\overline{E}}_x}\,\overline{\sigma}_{xy} = \frac{\overline{\sigma}_{xy}}{\epsilon_o^4} \tag{27}$$

Using this relation, f' becomes

$$f' = 1 + \frac{\overline{\sigma}_{xy}{}^2\alpha^2}{\epsilon_o^2}$$

From (22) it follows that $\alpha = \frac{1}{\beta}$, and f' can finally be written as

$$f' = \frac{\beta^2 \epsilon_o^2 + \bar{\sigma}_{xy}}{\beta^2 \epsilon_o^2} = \frac{g\alpha^2}{\epsilon_o^2} \tag{28}$$

Similarly it is possible to write

$$g' = \frac{f\beta^2}{\epsilon_o^2} \tag{29}$$

It is now possible to write equations (25) as

$$\begin{aligned}
(e_{xx})_o &= R\left\{\frac{1}{\bar{\bar{E}}_x}\left[-f\frac{\delta^2 F_1}{\delta\zeta_1^2} - g\frac{\delta^2 F_2}{\delta\zeta_2^2}\right]\right\} \\
(e_{yy})_o &= R\left\{\frac{1}{\epsilon_o^2 \bar{E}_y}\left[f\alpha^2\frac{\delta^2 F_1}{\delta\zeta_1^2} + g\beta^2\frac{\delta^2 F_2}{\delta\zeta_2^2}\right]\right\} \\
(e_{xy})_o &= R\left\{\frac{1}{\mu_{LT}}\left[-i\alpha\epsilon_o\frac{\delta^2 F_1}{\delta\zeta_1^2} - i\beta\epsilon_o\frac{\delta^2 F_2}{\delta\zeta_2^2}\right]\right\}
\end{aligned} \tag{30}$$

Bending Stresses

So far a method for deriving the membrane strain components $(e_{xx})_o$, $(e_{yy})_o$, $(e_{xy})_o$ has been obtained by requiring that a small rectangular element of the plate is in equilibrium under the action of the membrane stresses. A method for deriving the strain components of bending $z\,(e_{xx})_1$, ... will be obtained by requiring the vanishing of the resultant couples acting on a similar rectangular element of the plate. It will be assumed that the bending strains are expressible in terms of the deflection of the middle plane of the plate as in the plywood plate theory,[10] and that the membrane stresses are small and do not affect the bending. Then from the requirement that the resulting couples acting on the rectangular element vanish, the following differential equation for the deflection w of the middle plane of the plate results:

[10] Forest Products Laboratory Report No. 1312, page 35.

$$\frac{\delta^4 w}{\delta x^4} + 2\kappa_1 \frac{\delta^4 w}{\delta x^2 \delta \eta_1^2} + \frac{\zeta^4 w}{\delta \eta_1^4} = 0 \qquad (31)$$

where

$$\kappa_1 = \frac{2\lambda\mu_{LT} + \sigma_{TL} E_L}{\sqrt{\overline{E}_1 \overline{E}_2}} \qquad (32)$$

$$\eta_1 = \epsilon_1 y \qquad (33)$$

$$\epsilon_1 = \sqrt[4]{\frac{E_1}{E_2}} \qquad (34)$$

$$E_1 = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} E_x z^2 dx}{\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 dz} = \frac{12}{h^3} \int_{-\frac{h}{2}}^{\frac{h}{2}} E_x z^2 dz \qquad (35)$$

$$E_2 = \frac{12}{h^3} \int_{-\frac{h}{2}}^{\frac{h}{2}} E_y z^2 dz \qquad (36)$$

The quantities[11] E_1 and E_2 are called the "mean moduli in bending."

The solution of equation (31) is somewhat different from that of equation (18) since κ_1 is apparently always less than 1 for plywood. If $\kappa_1 \geq 1$ appropriate changes in the following solution can be made.

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[11] For example Forest Products Laboratory Report No. 1312, page 39.

Substituting $w = W(x + \nu\eta_1)$ in equation (31) leads to the following equation for ν:

$$\nu^4 + 2\kappa_1\nu^2 + 1 = 0 \tag{37}$$

or

$$\nu^2 = -\kappa_1 \pm \sqrt{\kappa_1^2 - 1} \tag{38}$$

Let

$$\kappa_1 = \cos\psi \tag{39}$$

Then

$$\nu^2 = -\cos\psi \pm i\sin\psi = -e^{\mp i\psi} \tag{40}$$

Hence, the roots of equation (37) are

$$\nu_1 = +ie^{i\frac{\psi}{2}}, \quad \nu_2 = ie^{-i\frac{\psi}{2}}, \quad \nu_3 = -ie^{i\frac{\psi}{2}}, \quad \nu_4 = -ie^{-i\frac{\psi}{2}}$$

The general solution of equation (31) takes the form

$$\begin{aligned} w = R\{ & w_1\left[x + i\left(\cos\frac{\psi}{2} + i\sin\frac{\psi}{2}\right)\eta_1\right] \\ & + w_2\left[x + i\left(\cos\frac{\psi}{2} - i\sin\frac{\psi}{2}\right)\eta_1\right]\} \\ = {} & R\{w_1(x_1 + iy_1) + w_2(x_2 + iy_2)\} \end{aligned} \tag{41}$$

where

$$x_1 = x - \eta_1\sin\frac{\psi}{2}, \quad x_2 = x + \eta_1\sin\frac{\psi}{2}, \quad y_1 = y_2 = \eta_1\cos\frac{\psi}{2} \tag{42}$$

and w_1 and w_2 are any analytic functions of the complex variables $x_1 + iy_1$ and $x_2 + iy_2$ respectively.

Choice of the Functions F_1, F_2, w_1, and w_2

The selection of the functions F_1, F_2, w_1, w_2 is an important part of the analysis. The symmetrical form of the loading with respect to the origin suggests that these functions can be expressed in the form of trigonometric series involving cosine functions of x. As the plate is supported along the edges parallel to the stiffener, the deflection must vanish for large values of y. Consequently the accompanying exponential function of y in w_1 and w_2 is taken to have a negative exponent. The membrane stresses are assumed to be of a localized character in the neighborhood of the stiffener. Hence, the accompanying exponentials in F_1 and F_2 will also have negative exponents and will be of the form $e^{-\alpha j \eta_o}$ and $e^{-\beta j \eta_o}$ respectively where j is a constant. Since $\beta < \alpha$, it is evident that F_2 does not vanish very rapidly for large values of y as compared to F_1; hence, the function F_2 will be assumed identically zero. In the stiffener and in the portion of the plate directly over the stiffener the strain distribution will be taken to be independent of y. In the region $y \geq \frac{t}{2}$, where t is the width of the stiffener, the functions F_1, F_2, w_1, and w_2 will be taken to be

$$F_1 = R \sum_{m=1}^{\infty} A_m e^{imk\,(\zeta_1 - i\alpha_o \frac{t}{2})}, \quad F_2 = 0$$

$$w_1 = R\left(\frac{1}{2} \sum_{m=1}^{\infty} \{iB_m e^{imk\,(x'_1 + iy'_1)} + C_m e^{imk\,(x'_2 + iy'_1)}\}\right) \qquad (43)$$

$$w_2 = R\left(\frac{1}{2} \sum_{m=1}^{\infty} \{iB_m e^{imk\,(x'_2 + iy'_2)} + C_m e^{imk\,(x'_2 + iy'_2)}\}\right)$$

where

$$k = \frac{\pi}{\ell}$$

$$x'_1 = x - \epsilon_1 \left(y - \frac{t}{2}\right) \sin\frac{\psi}{2}, \quad x'_2 = x + \epsilon_1 \left(y - \frac{t}{2}\right) \sin\frac{\psi}{2}$$

$$y'_1 = y'_2 = \epsilon_1 \left(y - \frac{t}{2}\right) \cos\frac{\psi}{2}$$

Equations (43) hold only in the region $y \geq \frac{t}{2}$.

Strain Components and Determination of the Arbitary Constants

Since the quantities $z(e_{xx})_1$, $z(e_{yy})_1$, $z(e_{xy})_1$ are given by[12]

$$z(e_{xx})_1 = -z\frac{\delta^2 w}{\delta x^2}, \qquad z(e_{yy})_1 = -z\frac{\delta^2 w}{\delta y^2}, \qquad z(e_{xy})_1 = -2z\frac{\delta^2 w}{\delta x \delta y}$$

and $(e_{xx})_o$, $(e_{yy})_o$, $(e_{xy})_o$ are given by equations (30), it follows from equations (43) that

$$(e_{xx})_o = \frac{k^2}{E_x}\sum_{m=1}^{\infty} m^2 A_m f e^{-\alpha mk\epsilon_o y'} \cos m kx$$

$$(e_{yy})_o = \frac{-k^2}{\epsilon_o^2 E_y}\sum_{m=1}^{\infty} m^2 A_m g\alpha^2 e^{-\alpha mk\epsilon_o y'} \cos m kx$$

$$(e_{xy})_o = \frac{-k^2\epsilon_o}{\mu_{LT}}\sum_{m=1}^{\infty} m^2 A_m \alpha e^{-\alpha mk\epsilon_o y'} \sin m kx \qquad (44)$$

$$(e_{xx})_1 = k^2\sum_{m=1}^{\infty} m^2 \{B_m \sin mk\,\delta\epsilon_1 y'$$

$$+ C_m \cos mk\,\delta\epsilon_1 y'\}\ e^{-mk\rho\epsilon_1 y'} \cos m kx$$

$$(e_{yy})_1 = -k^2\epsilon_1^2\sum_{m=1}^{\infty} m^2 \{(B_m\kappa_1 + 2C_m\delta\rho) \sin mk\delta\epsilon_1 y'$$

$$+ (C_m\kappa_1 - 2B_m\delta\rho) \cos mk\delta\epsilon_1 y'\}\ e^{-mk\rho\epsilon_1 y'} \cos m kx$$

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[12] Forest Products Laboratory Report No. 1312, page 35.

$$(e_{xy})_1 \doteq 2k^2\epsilon_1 \sum_{m=1}^{\infty} m^2 \{ -(B_m\rho + C_m\delta) \sin mk\delta\epsilon_1 y'$$

$$+ (B_m\delta - C_m\rho) \cos mk\delta\epsilon_1 y \} e^{-mk\rho\epsilon_1 y'} \sin m\,kx$$

where

$$\delta = \sin\frac{\psi}{2} = \sqrt{\frac{1-\kappa_1}{2}}, \quad \rho = \cos\frac{\psi}{2} = \sqrt{\frac{1+\kappa_1}{2}}, \quad y' = y - \frac{t}{2} \tag{45}$$

These strain components hold in the region $y \geq \frac{t}{2}$. In the portion of the plate directly over the stiffener and in the stiffener, the strain components are taken to be independent of y and to be obtained by setting $y = \frac{t}{2}$ in each of equations (44).

The expressions for the strain energy of the plate and the strain energy of the stiffener will now be obtained.

Substituting equations (6) in the expression for the strain energy

$$V_P = \int_{\frac{t}{2}}^{\infty} \int_{-\ell}^{\ell} \int_{-\frac{h}{2}}^{\frac{h}{2}} (X_x e_{xx} + Y_y e_{yy} + X_y e_{xy})\, dzdxdy \tag{46}$$

the strain energy of the plywood plate is given by [13]

$$V_P = \frac{h}{\lambda} \int_{\frac{t}{2}}^{\infty} \int_{-\ell}^{\ell} \{E_a (e_{xx})_o^2 + E_b (e_{yy})_o^2 + 2\sigma_{TL} E_L (e_{xx})_o (e_{yy})_o$$

$$+ \lambda\mu_{LT} (e_{xy})_o^2 \}\, dxdy + \frac{h^3}{12\lambda} \int_{\frac{t}{2}}^{\infty} \int_{-\ell}^{\ell} \{E_1 (e_{xx})_1^2 + E_2 (e_{yy})_1^2$$

$$+ 2\sigma_{TL} E_L (e_{xx})_1 (e_{yy})_1 + \lambda\mu_{LT} (e_{xy})_1^2 \}\, dxdy \tag{47}$$

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[13] The strain energy of the portion of the plate directly over the stiffener will be included in the strain energy of the stiffener.

Substituting the expressions (44) for the strain components and performing the integrations, equation (47) becomes

$$V_P = \frac{\ell k^3}{2} \sum_{m=1}^{\infty} m^3 \left[t_1 A_m{}^2 + t_2 B_m{}^2 + t_3 C_m{}^2 + t_4 B_m C_m \right] \tag{48}$$

where

$$t_1 = \frac{h}{\lambda \alpha \epsilon_o} \left\{ \frac{E_a f^2}{\overline{E}_x{}^2} + \frac{E_b g^2 \alpha^4}{\epsilon_o^4 \overline{E}_y{}^2} - \frac{2\sigma_{TL} E_L f g \alpha^2}{\epsilon_o^2 \overline{E}_x \overline{E}_y} + \frac{\lambda \epsilon_o^2 \alpha^2}{\mu_{LT}} \right\}$$

$$t_2 = \frac{h^3}{24\lambda\epsilon_1} \frac{(1 - \kappa_1)}{\sqrt{2(1 + \kappa_1)}} \{E_1 + E_2 \epsilon_1^4 (3 + 2\kappa_1) + 2\sigma_{TL} E_L \epsilon_1^2 + 4\lambda\mu_{LT}\epsilon_1^2\}$$

$$\tag{49}$$

$$t_3 = \frac{h^3}{24\lambda\epsilon_1\sqrt{2(1 + \kappa_1)}} \{E_1 (3 + \kappa_1) + E_2 \epsilon_1^4 (1 + \kappa_1 + 2\kappa_1^2)$$

$$- 2\sigma_{TL} E_L \epsilon_1^2 (1 + 3\kappa_1) + 4\lambda\mu_{LT}\epsilon_1^2 (3 + \kappa_1)\}$$

$$t_4 = \frac{h^3}{24\lambda\epsilon_1} \sqrt{2(1 - \kappa_1)} \{E_1 - E_2 \epsilon_1^4 (1 + 2\kappa_1) + 2\sigma_{TL} E_L \epsilon_1^2$$

$$- 4\lambda\mu_{LT}\epsilon_1^2\}$$

The strain energy of the stiffener and the portion of the plate directly over it is given by [14]

$$V_S = \frac{t}{2} \int_{-(\frac{h}{2} + d)}^{\frac{h}{2}} \int_{-\ell}^{\ell} E_x \left[(e_{xx})_{y = \frac{t}{2}} \right]^2 dxdz \tag{50}$$

---

[14] It is assumed that in the stiffener and the portion of the plate directly over it only the strain component in the x-direction is important.

where t and d are the width and depth of the stiffener, respectively. Young's modulus, E_x, has three different values in the evaluation of the integral. In the stiffener $E_x = E_s$, where E_s is the Young's modulus of the stiffener in the longitudinal direction and $E_x = E_L$ or E_T in the plate depending on the layer considered. Obtain from equations (44) the expression for $e_{xx} = (e_{xx})_o + z(e_{xx})_1$. After setting $y = \frac{t}{2}$ and substituting the result in equation (50) it follows that

$$V_S = \frac{\ell k^3}{2} \sum_{m=1}^{\infty} m^4 \{ r_1 A_m^2 + r_2 C_m^2 + r_3 A_m C_m \} \tag{51}$$

where

$$r_1 = \frac{tE_s kf^2 d}{\overline{E}_x^{\,2}} + \frac{tE_a kf^2 h}{\overline{E}_x^{\,2}}$$

$$r_2 = \frac{tE_s k}{12} d\,(3h^2 + 6hd + 4d^2) + \frac{tkh^3 E_1}{12} \tag{52}$$

$$r_3 = -\frac{tE_s k}{\overline{E}_x} fd(h + d)$$

The total strain energy V is equal to $V_P + V_S$. Adding equations (48) and (51), it follows that

$$V = \frac{\ell k^3}{2} \sum_{m=1}^{\infty} m^3 \{A_m^2 (t_1 + mr_1) + B_m^2 t_2 + C_m^2 (t_3 + mr_2)$$

$$+ r_3 m A_m C_m + t_4 B_m C_m \} \tag{53}$$

The total bending moment transmitted at any cross section perpendicular to the stiffener can be represented for the symmetrical case considered by the series

$$M = \sum_{m=1}^{\infty} M_m \cos mkx \tag{54}$$

From statics the normal stresses over the cross section of the plate and stiffener must satisfy the following conditions:

$$2\int_{\frac{t}{2}}^{\infty}\int_{-\frac{h}{2}}^{\frac{h}{2}} X_x \; dzdy + t \int_{-(\frac{h}{2}+d)}^{\frac{h}{2}} E_x \, (e_{xx})_{y=\frac{t}{2}} \; dz = 0$$

$$2\int_{\frac{t}{2}}^{\infty}\int_{-\frac{h}{2}}^{\frac{h}{2}} X_x \, z \; dzdy + t \int_{-(\frac{h}{2}+d)}^{\frac{h}{2}} E_x \, (e_{xx})_{y=\frac{t}{2}} \; zdz = \sum_{m=1}^{\infty} M_m \cos mkx \qquad (55)$$

where E_x will again take on the three values E_s, E_L, or E_T in evaluating the integral. Substituting from equations (6) and using the notation introduced in equations (9), (35), (36), it follows that equations (55) can be written as

$$\frac{2hE_a}{\lambda}\int_{\frac{t}{2}}^{\infty} \{(e_{xx})_o + \overline{\sigma}_{yx}(e_{yy})_o\} \, dy + t \int_{-(\frac{h}{2}+d)}^{\frac{h}{2}} E_x \, (e_{xx})_{y=\frac{t}{2}} \; dz = 0$$

$$\frac{h^3}{6\lambda}\int_{\frac{t}{2}}^{\infty} \{E_1 \, (e_{xx})_1 + \sigma_{TL}E_L \, (e_{yy})_1\} \, dy + t \int_{-(\frac{h}{2}+d)}^{\frac{h}{2}} E_x \, (e_{xx})_{y=\frac{t}{2}} \; zdz \qquad (56)$$

$$= \sum_{m=1}^{\infty} M_m \cos mkx$$

Substituting from equations (44), equations (56) become after integrating

$$A_m \, s_{11} + C_m \, s_{13} = 0$$

$$A_m \, s_{21} + B_m \, s_{22} + C_m \, s_{23} = M_m \qquad (57)$$

where

$$s_{11} = \frac{2hE_a}{\alpha k\lambda\epsilon_o^3\overline{\overline{E}}_x}(f\epsilon_o^2 - \overline{\sigma}_{xy}\, g\alpha^2) + \frac{tE_s mfd}{\overline{\overline{E}}_x} + \frac{tE_a mfh}{\overline{\overline{E}}_x}$$

$$s_{13} = -\frac{tE_s md}{2}(h + d)$$

$$s_{21} = -\frac{tE_s k^2}{2\overline{\overline{E}}_x} m^2 fd\,(h + d) \tag{58}$$

$$s_{22} = \frac{kh^3 m}{6\lambda\epsilon_1}\sqrt{\frac{1 - \kappa_1}{2}}\,(E_1 + \sigma_{TL}E_L\epsilon_1^2)$$

$$s_{23} = \frac{kh^3 m}{6\lambda\epsilon_1}\sqrt{\frac{1 + \kappa_1}{2}}\,(E_1 - \sigma_{TL}E_L\epsilon_1^2)$$

$$+ \frac{tk^2m^2}{12}\{E_s d\,(3h^2 + 6hd + 4d^2) + h^3\}$$

Solving equations (57) for B_m and C_m, it results that

$$C_m = a_{11} A_m$$

$$\tag{59}$$

$$B_m = a_{21} A_m + a_{22} M_m$$

where

$$a_{11} = -\frac{s_{11}}{s_{13}}, \qquad a_{21} = \frac{s_{11}s_{23} - s_{21}s_{13}}{s_{13}s_{22}}, \qquad a_{22} = \frac{1}{s_{22}} \tag{60}$$

Substituting equations (59) in the expression for the total strain energy V given in equation (53), it follows that

$$V = \frac{\ell k^3}{2} \sum_{m=1}^{\infty} m^3 \{A_m^2 (t_1 + mr_1) + (a_{21} A_m + a_{22} M_m)^2 t_2$$

$$+ a_{11}^2 A_m^2 (t_3 + mr_2) + r_3 m a_{11} A_m^2$$

$$+ t_4 a_{11} A_m (a_{21} A_m + a_{22} M_m)\} \tag{61}$$

To find the value of A_m that makes the total energy a minimum, it is only necessary to set the derivative of V with respect to A_m in equation (61) equal to zero and solve for A_m, since the work done by the external moment is independent of A_m. Hence

$$A_m = -\frac{M_m (2a_{21} a_{22} t_2 + a_{11} a_{22} t_4)}{2\left[t_1 + a_{21}^2 t_2 + a_{11}^2 t_3 + a_{11} a_{21} t_4 + m(r_1 + a_{11}^2 r_2 + a_{11} r_3)\right]} \tag{62}$$

Stiffness Added by the Stiffener for a Bending Moment of M_1 cos kx

It is now possible to determine approximately the position of the neutral surface of a plywood plate with an attached stiffener when the plate-stiffener combination is slightly buckled into one half-wave in the direction parallel to the stiffener. For this important case the bending-moment diagram is a simple cosine curve,[15] say $M = M_1 \cos kx$. In this case in accordance with equations (59) and (62) all of the coefficients A_m, B_m, and C_m vanish with the exception of A_1, B_1, and C_1. It follows from equations (44) that

$$e_{xx} = k^2 \left\{ \frac{A_1}{\overline{E}_x} fe^{-\alpha k \epsilon_1 y'} \right.$$

$$\left. + z\left[B_1 \sin k\delta\epsilon_1 y' + C_1 \cos k\delta\epsilon_1 y'\right] e^{-k\rho\epsilon_1 y'} \right\} \cos kx \tag{63}$$

and the equation of the neutral surface in the region $y \geq \frac{t}{2}$ is obtained by setting $e_{xx} = 0$ in equation (63), or

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[15] In the experimental part of this report the bending-moment diagram in the form of a cosine curve is approximated by a bending-moment diagram resulting from a concentrated load at the center of the plate.

$$A_1 fe^{-\alpha k\epsilon_1 y'} + z\left[B_1 \sin k\delta\epsilon_1 y' + C_1 \cos k\delta\epsilon_1 y'\right] e^{-k\rho\epsilon_1 y'} = 0 \qquad (64)$$

The stress-strain distribution is assumed to be independent of y in the stiffener and in the portion of the plate directly over the stiffeners. Hence, the neutral surface will be represented by the plane $z = -z_n$ in the region $0 \leq y \leq \frac{t}{2}$, where $-z_n$ is obtained by setting $y = \frac{t}{2}$ in equation (64) and solving for z. Thus the shift of the neutral surface z_n at the stiffener is

$$z_n = \frac{A_1 f}{C_1 \overline{E}_x} \qquad (65)$$

A sketch of the trace of the neutral surface on a plane perpendicular to the stiffener is shown in figure 2 for a typical case. Substituting from equations (58) and (59), the expression for z_n becomes

$$z_n = \frac{tE_s fd\,(h + d)}{2\left\{\dfrac{2hE_a(f\epsilon_o^2 - \overline{\sigma}_{xy} g\alpha^2)}{\alpha\lambda k\epsilon_o^3} + tf\,(E_s d + E_a h)\right\}} \qquad (66)$$

Substituting from equations (26) and rearranging, equation (66) becomes

$$z_n = \frac{h + d}{2\left\{\dfrac{2hE_a \alpha\epsilon_o\,(1 - \overline{\sigma}_{xy}\overline{\sigma}_{yx})}{tk\lambda E_s fd} + 1 + \dfrac{E_a}{E_s}\dfrac{h}{d}\right\}} \qquad (67)$$

Since it is approximately correct to write $(1 - \overline{\sigma}_{xy}\overline{\sigma}_{yx}) = \lambda = 1$ and $f = \alpha^2\epsilon_o^2$, it is possible to write the following approximate equation for z_n (fig. 2):

$$z_n = \frac{(h + d)}{2\left\{\dfrac{2hE_a}{tkE_s \alpha\epsilon_o d} + 1 + \dfrac{E_a}{E_s}\dfrac{h}{d}\right\}} \qquad (68)$$

The constants that appear in equation (68) have the following meaning: The plies in the plywood plate will be considered to be numbered consecutively from one face to the other. The elastic constants in the i^{th} ply will be denoted by $(E_x)_i$, $(E_y)_i$, $(\mu_{xy})_i$, and $(\sigma_{xy})_i$, where the xy-plane is the middle plane of the plywood, and the x-axis is in the direction of the stiffener. The thickness of the i^{th} ply is h_i, that of the plate, h.

Then approximately[16]

$$\overline{E}_x = E_a = \frac{\sum (E_x)_i h_i}{h}$$

$$\overline{E}_y = E_b = \frac{\sum (E_y)_i h_i}{h}$$

$$\mu_{xy} = \frac{\sum (\mu_{xy})_i h_i}{h}$$

$$\overline{\sigma}_{xy} = \frac{\sum h_i (E_y)_i \, (\sigma_{xy})_i}{h E_b}$$

$$\epsilon_o = \sqrt[4]{\frac{\overline{E}_x}{\overline{E}_y}} = \sqrt[4]{\frac{E_a}{E_b}}$$

$$\alpha = \sqrt{\kappa + \sqrt{\kappa^2 - 1}}$$

$$\kappa = \frac{\sqrt{\overline{E}_x \overline{E}_y}}{2} \left(\frac{1}{\mu_{xy}} - \frac{2\overline{\sigma}_{xy}}{\overline{E}_x}\right)$$

h = thickness of the plywood plate
t = width of the stiffener
d = depth of the stiffener
E_s = Young's modulus of the stiffener in the longitudinal direction.

For a stiffened plywood plate slightly buckled the value of k will depend on whether the edges perpendicular to the stiffener are simply supported or clamped. If these edges are simply supported and a is the width of the plate in the direction of the stiffener, then $k = \frac{\pi}{a}$. If these edges are clamped, then $k = \frac{2\pi}{a}$. In either case the approximate added stiffness is given by

[16] For more accurate expressions see U. S. Forest Products Laboratory Report No. 1503.

$$\frac{tdE_s}{12}\{d^2 + 3(h + d - 2z_n)^2\} \tag{69}$$

where z_n is given by equations (68). A more accurate expression is

$$\frac{tdE_s}{12}\{d^2 + 3(h + d - 2z_n)^2\} + th\overline{E}_a\, z_n^{\,2} \tag{70}$$

where the extra term gives the stiffness added by the portion of the plate directly over the stiffener. In most cases equation (70) will give values only a few percent larger than equation (69).

Stiffness Added when Both the Plate and Stiffener are Isotropic

When both the plate and stiffener are isotropic formula (68) for the shift of the neutral surface at the stiffener reduces to

$$z_n = \frac{(h + d)}{2\left\{\dfrac{2h}{dtk\,(1 + \sigma)} + 1 + \dfrac{h}{d}\right\}} \tag{71}$$

where h is the thickness of the plate, t is the width of the stiffener, d is the depth of the stiffener, and σ is Poisson's ratio. The value of k will depend on whether the edges of the plate perpendicular to the stiffener are simply supported or clamped. If these edges are simply supported and a is the width of the plate in the direction of the stiffener, then $k = \frac{\pi}{a}$. If these edges are clamped then $k = \frac{2\pi}{a}$. In either case the approximate added stiffness is given by

$$\frac{tdE}{12}\{d^2 + 3(h + d - 2z_n)^2\} + thEz_n^{\,2} \tag{72}$$

where E is Young's modulus, and z_n is given by equation (71).

Description of Stiffened Plywood Plates

All specimens were fabricated of aircraft-grade plywood and Sitka spruce stiffeners. The plywood was made of yellow birch or yellow-poplar veneers rotary cut at the Forest Products Laboratory. The plate members, veneer species, number and thickness of plies and the plywood constructions are shown in table 1. Plates having numbers ending in 1 had the stiffener perpendicular to the direction of the grain of the face plies, and those having numbers ending in 2 had the stiffener parallel to the direction of the grain of the face plies. The dimensions of the specimens were 13-5/8 inches parallel to the direction of the stiffener and varied from 10-1/2 to 24-1/2 inches perpendicular to the direction of the stiffener.

The plates each had a stiffener glued to one face of the plywood and on the centerline. The width of the stiffeners, parallel to the face of the plywood, varied from 1/8 to 1/2 inch; the depth, perpendicular to the face, varied from 1 inch to 0; and the length of the stiffener was equal to the width of the plate.

Before assembling, both the plywood and the stiffeners were conditioned in an atmosphere of 65 percent relative humidity and 70° F. temperature. After assembly, they were stored under the same conditions until the time of test.

Methods of Test

The specimen was measured; the length and width to 0.01 inch and the thickness of the plywood to 0.001 inch; the width and depth of the stiffener to 0.001 inch.

The ends of the stiffener were restrained by means of metal clamps to avoid separation of the stiffener from the plywood.

The specimen was placed on the supporting frame, figure 3, which furnished simple support along all four edges of the plate. The end supports were so adjusted as to allow 1/4-inch overhang. The edge supports were so positioned that there was 9/16-inch overhang. The knife edges were notched to take the ends of the stiffener and the metal clamps when the specimen was placed with the stiffener down. The supporting frame is shown in figure 3.

Loads were applied to the center of the specimen by a 100,000-pound screw-type testing machine through a loading head of 3/4-inch radius and a wood block 3/8 inch wide, 1/2 inch deep, and 1 inch long. Loads were measured by a 100-pound springless scale.

Load increments were picked so as to give a clearly defined load-deflection curve. Deflections were measured by a 0.001-inch compression-dial in contact with the plate directly under the load. Each specimen was tested twice with the stiffener up and twice with the stiffener down. The plate was turned end for end between runs.

Each specimen was tested with a stiffener of 1-inch depth. The depth was then reduced and the specimen tested again. This procedure was repeated with the depth of the stiffener reduced after each test until finally the depth became zero and the plate was tested with no stiffeners.

From each test, a ratio of load to deflection was picked from the straight-line portion of the load-deflection curve. An average value was obtained for the stiffener-up position and for the stiffener-down position. Often at low loads the curve varied from a straight line due to lack of original flatness of the specimen. Also at high loads, the rate of deflection with respect to the load decreased because of the presence of membrane stresses.

Analysis of Results

Formula (70) of the mathematical analysis applies only if the bending moment of the stiffener is distributed according to a cosine curve. Such a curve is expected when the stiffener is attached to a plate supported at its edges and the plate buckled under edgewise compression. It is difficult under these conditions to determine accurately the added stiffness due to the stiffener by experimental means. For this reason a concentrated load applied to the center of the stiffener and normal to the surface of the plate was used instead of edgewise compression and the plate was simply supported at its edges. Under these conditions the bending moment of the stiffener is not distributed according to a cosine curve, but the distribution is not too different from such a curve. The mathematical analysis, therefore, should apply approximately to the conditions under which the tests were made.

The data obtained from these tests were expressed as load-deflection ratios (as previously described) for unstiffened plywood plates and for the same plywood plates stiffened with stiffeners of various sizes. The difference between the ratios for a stiffened plate and those for the plate with no stiffener is a measure of the effective stiffness of the stiffener. Formula (70) gives a theoretical effective stiffness value of the stiffener. Although the observed ratio and the theoretical value are both stiffness factors, they are not comparable terms. The theoretical EI_s is converted into a P/δ ratio by using the standard static bending formula, $E = \frac{PL^3}{48I\delta}$, which becomes, $P/\delta = \frac{EI_s}{40.7}$, when the value of the span is substituted for L. This makes it possible to compare observed and computed stiffness ratios.

Figures 4 and 5 were prepared to present comparisons between observed and computed stiffness ratios of the stiffeners. In figure 4 the ordinates are stiffness ratios and the abscissas are stiffener depths. The points indicated by the circles and squares are observed values from testing with the stiffener up and down, respectively. The solid line in each graph represents the values of stiffness ratios, computed from formula (70). In general, the observed values are slightly higher than the computed ones. This is probably due to the approximations made in the theoretical treatment and to the effect of original lack of flatness in the plywood. In figure 5 the ordinates are the observed stiffness ratios of the stiffened plates and the abscissas are sums of the stiffness ratios of the stiffeners according to formula (70) and those of the unstiffened plates; both are divided by the sums of the stiffness ratios of the stiffener alone and those of the unstiffened plates. These parameters make it possible to check all the results for accuracy in one graph. For the stiffnesses of the unstiffened plates the observed values were used. The 45° line is the locus of perfect checks between theory and experiment. The scatter of the points is probably due to the original lack of flatness of the plywood tested, but a definite correlation between theory and experiment is indicated.

Conclusions

The mathematical analysis presented in this report represents with reasonable accuracy the results of experimental tests to determine the stiffness of a plywood plate reinforced by an attached stiffener. The formulas developed make it possible to locate the position of the neutral surface and to compute the stiffness added to the plate by the attached stiffener.

Notation

The following symbols are the major ones used in this report. All quantities are in inch and pound units.

a = the length of the sides of the plywood plate, parallel to the length of the attached stiffener.

b = the length of the sides of the plywood plate, perpendicular to the length of the attached stiffener.

d = the depth of the stiffener, measured perpendicular to the face of the plywood.

h = thickness of the plywood.

k = a constant in equation (68) depending on the condition of support of the edges perpendicular to the stiffener.

t = the width of the stiffener, measured parallel to the face of the plywood.

z_n = distance from the center of the plywood to the neutral surface, which has been shifted by attaching a stiffener to the plywood.

E_a = effective modulus of elasticity of plywood in compression measured parallel to the side of length a of plywood plates.

E_b = effective modulus of elasticity of plywood in compression measured parallel to the side of length b of plywood plates.

E_s = modulus of elasticity of Sitka spruce stiffener in the direction parallel to the grain, as determined from a static bending test.

Computations

Observed stiffness factors (P/δ) of the plate-stiffener combinations were obtained from the straight-line portion of load-deflection curves by dividing a load by the corresponding deflection. This factor was determined for the stiffener-up and the stiffener-down positions and presented in table 3 in columns 2 and 3 respectively.

Observed stiffness factor (P/δ) of the plywood plates alone were obtained in the same way from data taken after the stiffener had been completely removed. These data are given in column 4 of table 3.

The observed additional stiffness due to the attached stiffener is the difference between the observed stiffness factors of the plate-stiffener combination and of the plywood plate alone. These values are presented in column 5 for the stiffener up and column 6 for the stiffener down.

The computed stiffness factors of the stiffeners about their own axes were computed from the formula:

$$\frac{P}{\delta} = \frac{4E_s td^3}{L^3} = \frac{E_s td^3}{488.3}$$

and presented in column 7 of table 3.

The computed values of the additional stiffness due to the attached stiffener were obtained from the formula:

$$\frac{P}{\delta} = \frac{48EI_s}{L^3} = \frac{EI_s}{40.7}$$

where values of EI_s were obtained from equation (70). These values are found in table 3, column 8. The ratios shown in columns 9 and 10, table 3 were obtained by dividing the values in columns 2 and 3, respectively, by the sums of the corresponding values in columns 4 and 7. Column 11 contains ratios obtained by dividing the sum of the corresponding values in columns 4 and 8 by the sum of the corresponding values in columns 4 and 7.

Tests of Minor Specimens

Coupons were cut from the specimen after test, from which the elastic properties of the specimen were obtained. They included: one for a static bending test, the face grain parallel to the direction of applied stress; two for compression tests, the face grain perpendicular to the direction of applied stress; and one for a plate shear test.

The static bending tests were made to determine the effective modulus of elasticity in bending when the face grain is parallel and when it is perpendicular to the span. The specimens were tested as simply supported beams, centrally loaded. The coupons were 2 inches wide and the spans used were 48 or 24 times the thickness of the coupon when the face grain was parallel or perpendicular to the span, respectively. Effective moduli of elasticity in bending were computed from the formula:

$$E_1 \text{ or } E_2 = \frac{PL^3}{4bd^3y}$$

where:

- P = load, pounds
- L = span, inches
- b = width, inches
- d = depth, inches
- y = deflection, inches

The coupons that were tested in compression were 4 inches long by 1 inch wide by the thickness of the plywood and were tested in a device which provided lateral support to keep the specimen in the plane of the load. The effective moduli of elasticity in compression were thus obtained in the two different grain directions, using the formula:

$$E_a \text{ or } E_b = \frac{PL}{Ay}$$

where:

P = load, pounds
L = gage length, inches
A = cross-sectional area, square inches
y = deformation, inches

Plate shear tests were made to determine the shear modulus of the plywood. The coupons were cut into squares of which the dimensions of the sides were 30 to 40 times the thickness. The test is described in Forest Products Laboratory Report 1301. Shear moduli were computed using the formula:

$$\mu_{LT} = \frac{3}{2}\frac{Pu^2}{wh^3}$$

where:

P = load, pounds
u = gage length, inches
w = deflection, inches
h = thickness of plywood, inches

The stiffeners, prior to attachment to the plywood, were tested as simply supported beams, centrally loaded, to determine the modulus of elasticity. The span used was 14 inches. Moduli of elasticity were computed by the formula:

$$E_s = \frac{PL^3}{4bd^3y}$$

where:

P = load, pounds
L = span, inches
b = width, inches
d = depth, inches
y = deflection, inches

APPENDIX

Some exploratory tests were run to determine the accuracy of the theory as applied to plates having various ratios of length to width. Two pairs of matched plates were cut from 1/4-inch, five-ply, yellow birch plywood. The direction of the face grain of one pair was parallel to the long sides of the plates and of the other pair, perpendicular to the long sides. One of each pair was stiffened with a rectangular stiffener, three-eighths inch wide and one-half inch deep, and attached at the short centerline of the plates. Each plate, with all four edges simply supported, was tested by the application of a normal load at a point in the center of the plate, in the manner described in the body of this report. The lengths of the plates were then reduced and the plates tested again. This procedure was repeated until a range of length-to-width ratios of 2.17 to 0.4 was obtained.

A load-to-deflection ratio (stiffness ratio) was obtained from each test. A similar ratio was obtained for the matched unstiffened plate having the same dimensions. The difference between the ratios in each case is a measure of the effective stiffness of the attached stiffener. These values are presented in table No. 4 with the corresponding dimensions of the plates and the length-to width ratios (b/a). Computed stiffness ratios of the stiffeners were obtained using equation (70) and are also presented in table No. 4. Due to lack of minor data on these particular plates the following values, which are averages of values obtained from plywood of the same construction and from the same source of supply, were assumed: the effective modulus of elasticity of the plywood in compression measured parallel to the grain direction of the face plies -- 1,360,000 pounds per square inch; the effective modulus of elasticity of the plywood in compression measured perpendicular to the grain direction of the face plies -- 940,000 pounds per square inch; and the modulus of rigidity associated with shear deformations in the xy-plane resulting from shear stresses in the xz- and zy-planes -- 160,000 pounds per square inch.

In figure No. 6, observed and computed effective stiffness ratios (P/δ) of the attached stiffeners are plotted against length-to-width ratios (b/a). These graphs show that the theory presented in this report applies to plates having length-to-width ratios of about 0.8 or over. Computed values are too low when length-to-width ratios are much below 0.8.

Table 1.--Construction of plywoods tested

Plate No.	Species	No. of plies	Thickness of plies	Construction
(1)	(2)	(3)	(4)	(5)
			Inch	
3x-1 3x-2 3bb-1	Yellow birch	3	1/16	The direction of the grain of the core perpendicular to that of the faces.
12bb-1 12bb-2	Yellow birch faces Yellow-poplar core	3	1/16	
25bb-1 25bb-2	Yellow birch	4	1/48	The direction of the grain of the two inner plies parallel to each other and perpendicular to that of the outer plies.
26bb-1 26bb-2	Yellow birch	4	1/20	
41x-1 41x-2 41bb-1 41bb-2	Yellow-poplar	4	1/20	
14x-1 14x-2	Yellow birch	5	1/20	The direction of the grain of each ply perpendicular to that of adjacent plies.
15xa-1 15xa-2	Yellow birch	5	1/16	

Table 2.--Dimensions and elastic properties of stiffened plates

Plate No.	Dimensions h	a	b	t	Information from minor tests E_1	E_2	E_a	E_b	μ_{LT}	E_s
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Inch	Inches	Inches	Inch	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.
3x-1	0.177	12.46	18.00	0.249	180	2,267	786	1,522	162	1,595
3x-2	.177	12.46	11.00	.243	2,267	180	1,686	810	162	1,671
3bb-1	.174	12.46	24.00	.252	170	2,240	786	1,435	172	1,627
12bb-1	.180	12.46	24.00	.186	156	2,395	518	1,575	151	1,659
12bb-2	.179	12.46	10.00	.178	2,347	162	1,434	584	170	1,631
25bb-1	.080	12.46	24.00	.124	445	2,388	1,198	1,216	253	1,488
25bb-2	.080	12.46	10.00	.124	2,028	460	1,181	1,136	230	1,642
26bb-1	.184	12.46	24.00	.374	415	1,826	1,275	1,172	137	1,590
26bb-2	.181	12.46	10.00	.376	1,826	415	1,172	1,275	137	1,713
41x-1	.175	12.46	18.00	.248	244	1,395	895	996	125	1,726
41x-2	.175	12.46	11.00	.252	1,395	244	967	955	125	1,750
41bb-1	.184	12.46	24.00	.245	292	1,530	872	892	107	1,617
41bb-2	.186	12.46	10.00	.247	1,491	283	902	960	105	1,650
14x-1	.239	12.46	18.00	.250	620	1,764	1,010	1,473	166	1,703
14x-2	.240	12.46	11.00	.250	1,764	620	1,380	976	166	1,751
15xa-1	.285	12.46	24.00	.506	673	2,173	1,182	1,712	162	1,764
15xa-2	.284	12.46	10.00	.503	2,142	662	1,642	1,158	161	1,537

Table 3.--Observed and computed stiffness ratios (P/δ) of plywood plates, stiffeners, and combinations

Depth of stiffeners	Stiffness factors (P/δ): Observed: Combination: Stiffener up	Stiffness factors (P/δ): Observed: Combination: Stiffener down	Stiffness factors (P/δ): Observed: Plywood plate alone	Stiffness factors (P/δ): Observed: Additional due to stiffener: Stiffener up	Stiffness factors (P/δ): Observed: Additional due to stiffener: Stiffener down	Stiffness factors (P/δ): Computed: Stiffener about its own axis	Stiffness factors (P/δ): Computed: Additional due to stiffener	Ratios: Observed: $\frac{(2)}{(4)+(7)}$	Ratios: Observed: $\frac{(3)}{(4)+(7)}$	Ratios: Computed: $\frac{(4)+(8)}{(4)+(7)}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Inches	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch
					Plate No. 41x-2					
0.486	540.6	550.5	158.6	382.0	391.9	104.7	409.2	2.05	2.09	2.16
.373	380.1	411.3	158.6	221.5	252.7	47.3	230.5	1.85	2.00	1.89
.255	249.0	279.0	158.6	90.4	120.4	15.1	104.5	1.43	1.61	1.51
.190	207.3	207.4	158.6	48.7	48.8	6.2	58.6	1.26	1.26	1.32
.122	214.2	174.2	158.6	55.6	15.6	1.7	26.1	1.34	1.09	1.15
					Plate No. 41bb-2					
.992	2,204.0	2,162.2	174.4	2,029.6	1,987.8	822.7	1,958.5	2.21	2.17	2.14
.887	1,742.9	1,777.8	174.4	1,568.5	1,603.4	568.4	1,471.4	2.35	2.39	2.22
.775	1,419.1	1,400.4	174.4	1,244.7	1,226.0	392.3	1,107.9	2.50	2.47	2.26
.626	1,019.1	1,079.3	174.4	844.7	904.9	206.7	683.9	2.67	2.83	2.25
.502	689.8	698.8	174.4	515.4	524.4	106.6	419.6	2.45	2.49	2.11
.249	298.5	312.9	174.4	124.1	138.5	13.0	96.7	1.59	1.67	1.45
					Plate No. 12bb-2					
.992	1,832.1	1,860.5	205.6	1,626.5	1,654.9	586.0	1,910.1	2.31	2.35	2.67
.876	1,476.1	1,562.5	205.6	1,270.5	1,356.9	403.6	1,413.6	2.42	2.56	2.66
.775	1,219.9	1,220.2	205.6	1,014.3	1,014.6	279.4	1,052.1	2.51	2.52	2.59
.626	857.2	900.9	205.6	651.6	695.3	147.3	631.5	2.43	2.55	2.37
.499	606.1	670.4	205.6	400.5	464.8	74.6	370.2	2.16	2.39	2.05
.251	298.8	315.0	205.6	93.2	109.4	9.5	79.6	1.39	1.46	1.33
					Plate No. 3x-2					
.486	657.6	660.9	179.3	478.3	481.6	96.4	453.1	2.38	2.40	2.29
.361	452.9	449.6	179.3	273.6	270.3	39.5	231.8	2.07	2.05	1.88
.312	384.6	368.5	179.3	205.3	189.2	25.5	168.3	1.88	1.80	1.70
.247	316.1	275.7	179.3	136.8	96.4	12.6	102.4	1.65	1.44	1.47
.130	231.2	173.2	179.3	51.9	-6.1	1.8	29.2	1.28	.96	1.15
.069	198.9	169.5	179.3	19.6	-9.8	.3	10.1	1.11	.94	1.05
					Plate No. 25bb-2					
.992	805.8	817.4	116.8	689.0	700.6	411.0	1,004.0	1.53	1.55	2.12
.876	686.1	718.2	116.8	569.3	601.4	283.0	751.0	1.72	1.80	2.17
.751	499.3	554.2	116.8	382.5	437.4	178.3	501.5	1.69	1.88	2.10
.625	347.7	399.6	116.8	230.9	282.8	102.8	317.1	1.58	1.82	1.98
.501	229.4	278.4	116.8	112.6	161.6	52.9	182.7	1.35	1.64	1.76
.252	71.0	122.2	116.8	-45.8	5.4	6.7	33.6	.57	.99	1.22
					Plate No. 26bb-2					
.999	3,158.0	2,904.4	259.5	2,898.5	2,644.9	1,327.9	2,923.0	1.99	1.83	2.00
.875	2,540.8	2,410.4	259.5	2,281.3	2,150.9	892.2	2,145.1	2.21	2.09	2.09
.770	2,072.1	1,971.6	259.5	1,812.6	1,712.1	608.0	1,598.5	2.39	2.27	2.14
.629	1,444.1	1,449.4	259.5	1,184.6	1,189.9	331.4	1,012.1	2.44	2.45	2.15
.508	999.4	1,049.9	259.5	739.9	790.4	174.6	629.0	2.30	2.42	2.05
.269	446.9	480.5	259.5	187.4	221.0	25.9	166.9	1.57	1.69	1.49
					Plate No. 14x-2					
.487	1,105.0	1,014.6	528.6	576.4	486.0	104.5	589.3	1.75	1.60	1.77
.375	872.1	895.5	528.6	343.5	366.9	47.7	336.5	1.51	1.55	1.50
.246	692.9	694.5	528.6	164.3	165.9	13.5	143.4	1.28	1.28	1.24
.127	587.1	575.8	528.6	58.5	47.2	1.8	43.8	1.11	1.09	1.08
.069	543.5	561.8	528.6	14.9	33.2	.3	17.2	1.03	1.06	1.03
					Plate No. 15xa-2					
1.004	5,431.4	5,463.1	1,018.6	4,412.8	4,444.5	1,617.8	5,240.6	2.06	2.07	2.37
.862	4,507.4	4,305.6	1,018.6	3,488.8	3,287.0	1,023.9	3,722.0	2.21	2.11	2.32
.745	3,671.9	3,872.9	1,018.6	2,653.3	2,854.3	661.0	2,694.7	2.19	2.31	2.21
.625	2,998.2	2,136.5	1,018.6	1,979.6	1,117.9	390.3	1,840.9	2.13	1.52	2.03
.498	2,303.5	2,335.2	1,018.6	1,284.9	1,316.6	197.4	1,138.7	1.89	1.92	1.77
.250	1,412.1	1,375.0	1,018.6	393.5	356.4	25.0	295.3	1.35	1.32	1.26

(Sheet 1 of 2)

Table 3.--Observed and computed stiffness ratios (P/δ) of plywood plates, stiffeners, and combinations (continued)

Depth of stiffeners	Stiffness factors (P/δ): Observed: Combination: Stiffener up	Combination: Stiffener down	Plywood plate alone	Additional due to stiffener: Stiffener up	Additional due to stiffener: Stiffener down	Computed: Stiffener about its own axis	Computed: Additional due to stiffener	Ratios: Observed: $\frac{(2)}{(4)+(7)}$	Ratios: Observed: $\frac{(3)}{(4)+(7)}$	Ratios: Computed: $\frac{(4)+(8)}{(4)+(7)}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Inches	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch
				Plate No. 41x-1						
0.486	523.6	510.2	94.9	428.7	415.3	101.6	394.8	2.66	2.60	2.49
.372	347.2	351.7	94.9	252.3	256.8	45.6	221.2	2.47	2.50	2.25
.252	221.4	228.1	94.9	126.5	133.2	14.2	98.6	2.03	2.09	1.77
.127	125.8	131.4	94.9	30.9	36.5	1.8	27.1	1.30	1.36	1.26
.066	102.2	108.9	94.9	7.3	14.0	.2	9.4	1.07	1.15	1.10
				Plate No. 41bb-1						
1.032	1,933.8	1,787.2	72.2	1,861.6	1,715.0	900.3	2,090.8	1.99	1.84	2.22
.857	1,513.9	1,493.1	72.2	1,441.7	1,420.9	515.6	1,357.3	2.58	2.54	2.43
.745	1,165.6	1,139.4	72.2	1,093.4	1,067.2	338.7	984.6	2.84	2.77	2.57
.637	860.7	841.6	72.2	788.5	769.4	211.7	690.8	3.03	2.96	2.69
.383	339.2	343.8	72.2	267.0	271.6	46.0	226.9	2.87	2.91	2.53
				Plate No. 12bb-1						
1.002	1,472.9	1,386.9	65.8	1,407.1	1,321.1	641.9	1,569.0	2.08	1.96	2.31
.872	1,212.1	1,243.5	65.8	1,146.3	1,177.7	423.1	1,135.5	2.48	2.54	2.46
.746	976.6	917.8	65.8	910.8	852.0	264.9	792.8	2.95	2.78	2.60
.630	683.8	674.2	65.8	618.0	608.4	159.6	539.7	3.03	2.99	2.69
.502	436.5	445.3	65.8	370.7	379.5	80.7	324.8	2.98	3.04	2.67
.246	146.1	146.1	65.8	80.3	80.3	9.5	71.5	1.94	1.94	1.82
				Plate No. 3x-1						
.489	526.9	532.3	116.5	410.4	415.8	96.0	393.0	2.48	2.50	2.40
.243	212.6	209.5	116.5	96.1	93.0	11.8	88.9	1.66	1.63	1.60
.092	215.1	171.0	116.5	98.6	54.5	.6	15.1	1.84	1.46	1.12
.066	136.6	156.3	116.5	20.1	39.8	.2	9.0	1.17	1.34	1.08
				Plate No. 3bb-1						
.492	498.3	497.5	95.7	402.6	401.8	101.0	405.7	2.53	2.53	2.55
.369	311.8	316.2	95.7	216.1	220.5	42.6	216.1	2.25	2.28	2.25
.132	121.4	105.3	95.7	25.7	9.6	2.0	28.2	1.24	1.08	1.27
				Plate No. 25bb-1						
1.000	874.0	765.8	29.6	844.4	736.2	381.5	988.8	2.12	1.86	2.48
.840	621.1	601.5	29.6	591.5	571.9	226.1	638.0	2.43	2.35	2.61
.755	518.1	500.0	29.6	488.5	470.4	164.2	487.6	2.67	2.58	2.67
.627	347.9	356.1	29.6	318.3	326.5	94.0	305.2	2.81	2.88	2.71
.505	207.9	212.6	29.6	178.3	183.0	49.1	176.8	2.64	2.70	2.62
.251	49.0	46.4	29.6	19.4	16.8	6.0	31.0	1.37	1.30	1.70
				Plate No. 26bb-1						
.999	2,820.5	2,762.4	107.6	2,712.9	2,654.8	1,225.9	2,903.2	2.12	2.07	2.26
.875	2,338.1	2,285.7	107.6	2,230.5	2,178.1	823.7	2,131.0	2.51	2.45	2.40
.758	1,894.2	1,725.9	107.6	1,786.6	1,618.3	535.5	1,531.5	2.94	2.60	2.55
.500	833.3	864.2	107.6	725.7	756.6	153.7	602.7	3.19	3.31	2.72
.242	272.0	267.0	107.6	164.4	159.4	17.4	132.0	2.18	2.14	1.92
				Plate No. 14x-1						
.488	869.6	890.2	308.3	561.3	581.9	102.3	549.4	2.12	2.17	2.09
.385	802.3	654.4	308.3	494.0	346.1	50.2	332.3	2.24	1.82	1.79
.251	484.7	473.9	308.3	176.4	165.6	13.9	140.6	1.50	1.47	1.39
.106	386.0	332.5	308.3	77.7	24.2	1.0	31.2	1.25	1.07	1.10
				Plate No. 15xa-1						
.996	5,166.0	2,745.1	496.9	4,669.1	2,248.2	1,824.0	5,069.4	2.22	1.18	2.40
.860	4,000.0	2,958.6	496.9	3,503.1	2,461.7	1,174.0	3,669.9	2.39	1.77	2.49
.744	3,125.0	2,525.2	496.9	2,628.1	2,028.3	760.0	2,682.3	2.49	2.01	2.53
.635	2,478.1	2,141.0	496.9	1,981.2	1,644.1	472.5	1,918.8	2.56	2.21	2.49
.518	1,886.8	1,682.2	496.9	1,389.9	1,185.3	256.6	1,261.8	2.50	2.23	2.33
.266	892.8	876.8	496.9	395.9	379.9	34.7	352.8	1.68	1.65	1.60

Table 4.--Dimensions and stiffness ratios (P/δ) of plywood plates

Plate No.	h	b	a	$\frac{b}{a}$	Stiffness ratios: Plywood plates: Stiffened	Stiffness ratios: Plywood plates: Unstiffened	Stiffness ratios: Attached stiffeners: Observed	Stiffness ratios: Attached stiffeners: Computed	t	d	E_s
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Inch	Inches	Inches		Pounds per inch	Pounds per inch	Pounds per inch	Pounds per inch	Inch	Inch	1,000 p.s.i.
14A	0.242	23.0	12.46	1.845	1,384	560	824	815	0.378	0.497	1,720
	.242	20.0	12.46	1.605	1,394	563	831	815	.378	.497	1,720
	.242	16.0	12.46	1.284	1,377	558	819	815	.378	.497	1,720
	.242	12.0	12.46	.963	1,415	618	797	815	.378	.497	1,720
	.242	8.0	12.46	.642	1,663	789	874	815	.378	.497	1,720
	.242	6.0	12.46	.482	2,133	1,190	943	815	.378	.497	1,720
	.242	5.0	12.46	.401	2,580	1,556	1,024	815	.378	.497	1,720
14B	.245	27.0	12.46	2.167	1,136	330	806	817	.376	.506	1,836
	.245	24.0	12.46	1.925	1,130	333	797	817	.376	.506	1,836
	.245	20.0	12.46	1.605	1,130	347	783	817	.376	.506	1,836
	.245	16.0	12.46	1.284	1,198	413	785	817	.376	.506	1,836
	.245	12.0	12.46	.963	1,409	583	826	817	.376	.506	1,836
	.245	8.0	12.46	.642	2,062	1,250	812	817	.376	.506	1,836
	.245	6.0	12.46	.482	3,307	2,062	1,245	817	.376	.506	1,836
	.245	5.0	12.46	.401	4,124	2,703	1,421	817	.376	.506	1,836

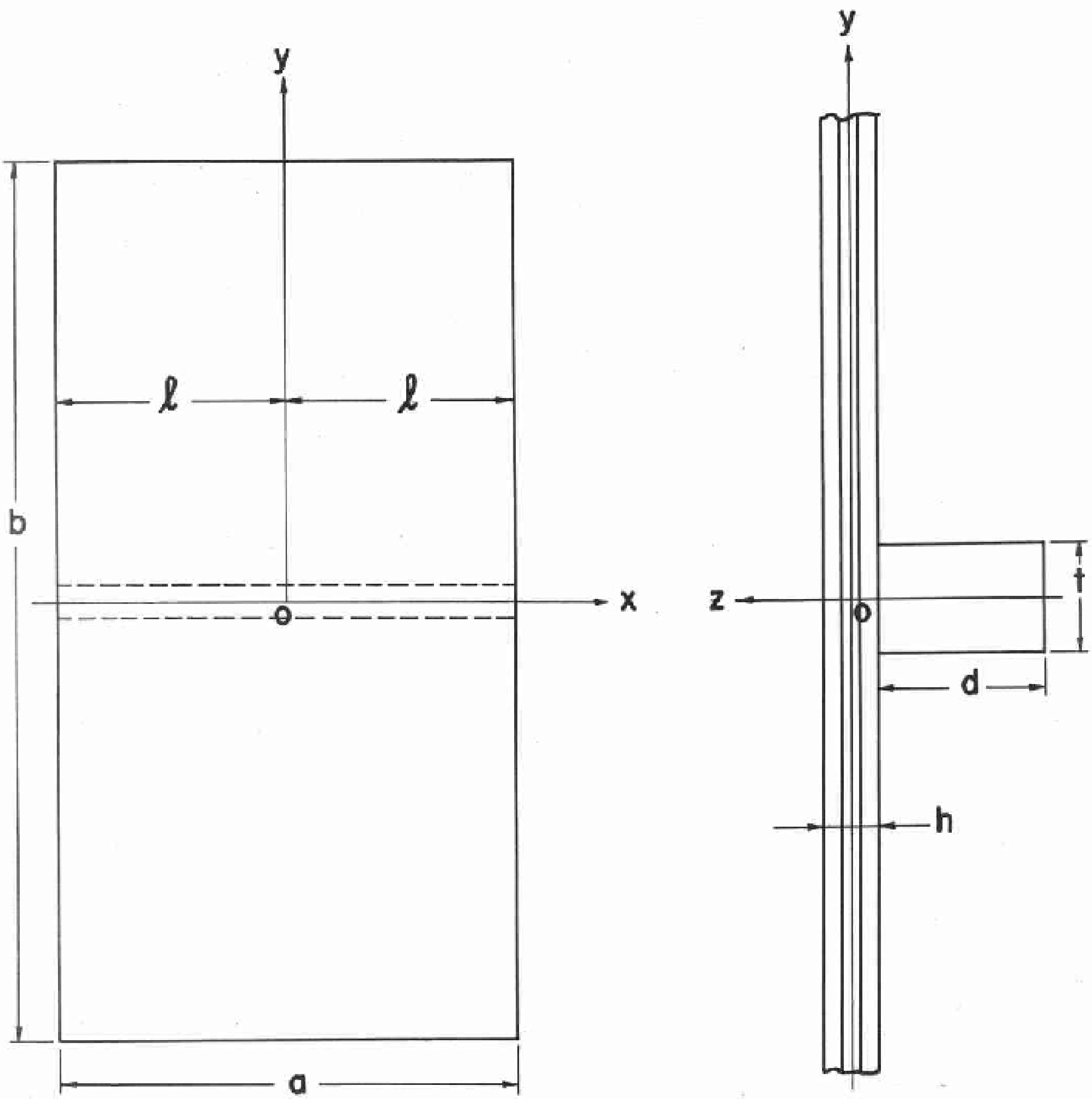
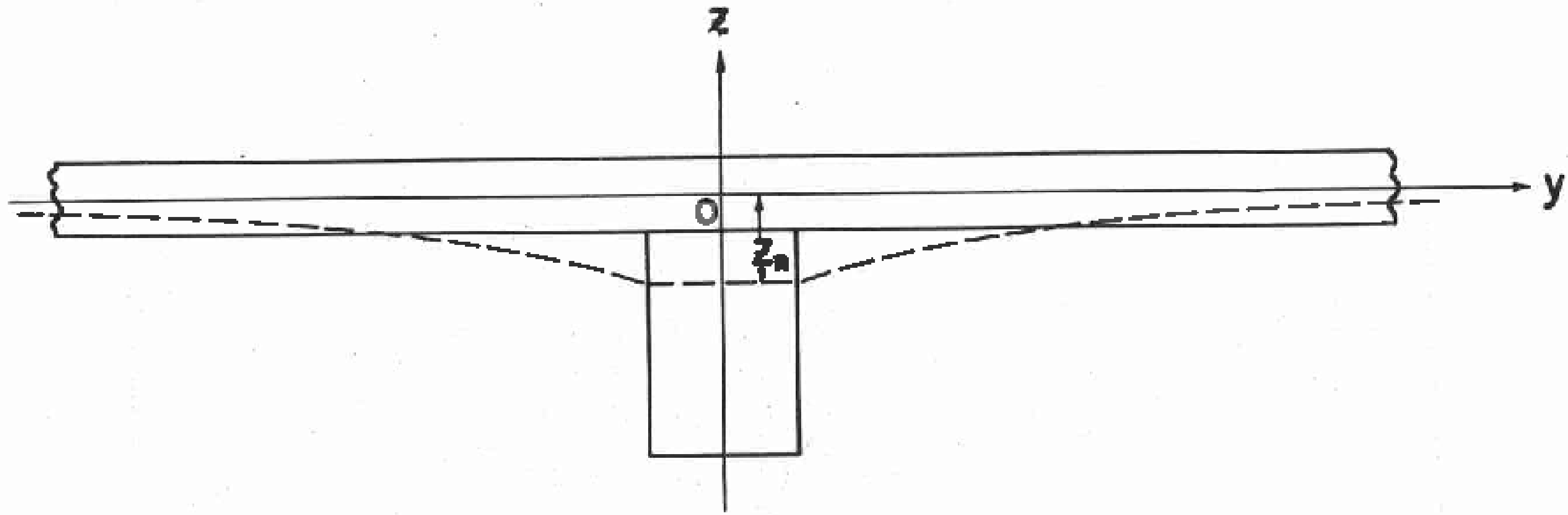


Figure 1.--Choice of axes and dimensions of plywood plate and stiffener.

Z M 68615 F



Z M 68612 F

Figure 2.--Approximate location and shape of neutral surface.

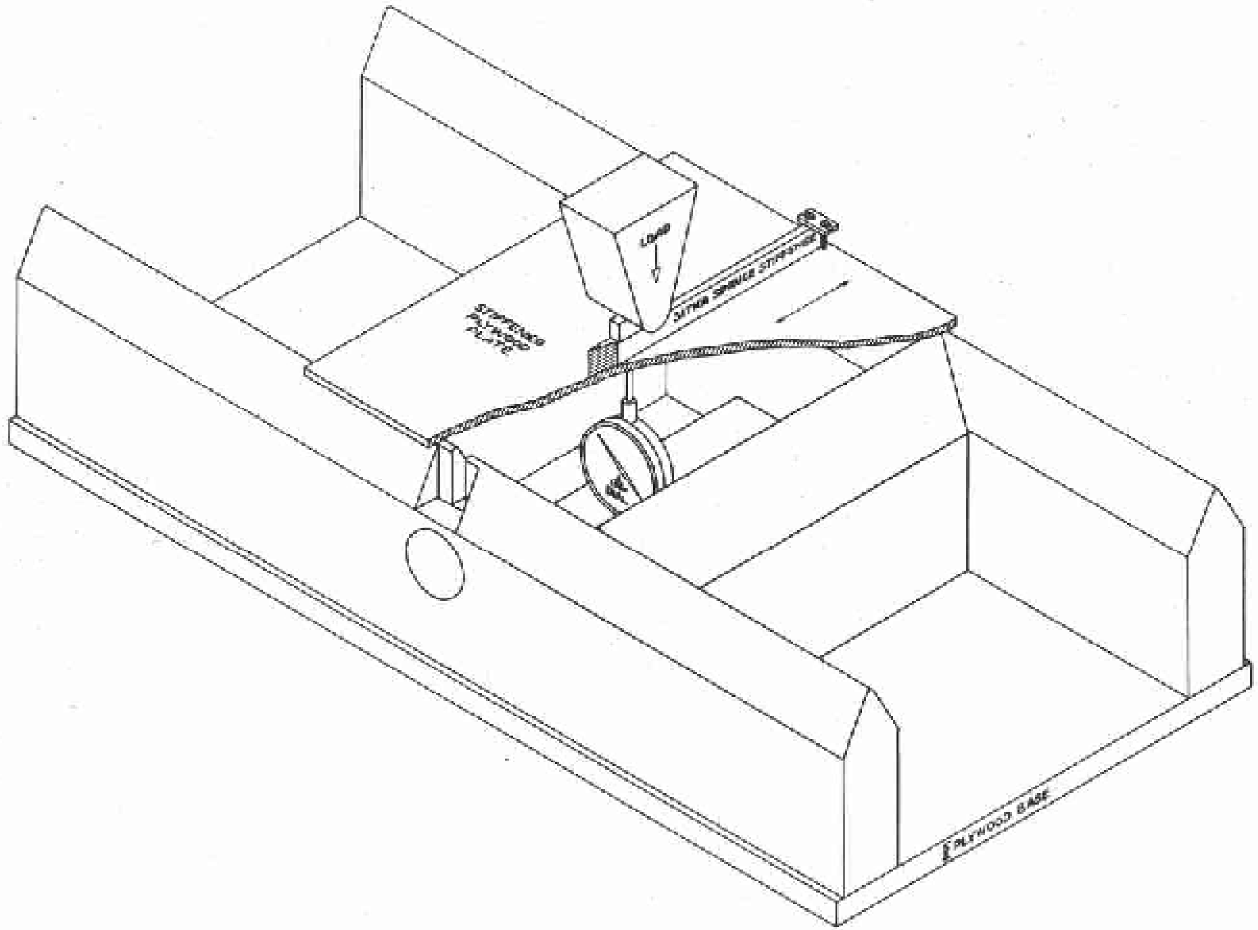


Figure 3.--Supporting frame for stiffened plywood plate bending test.

Z M 64155 F

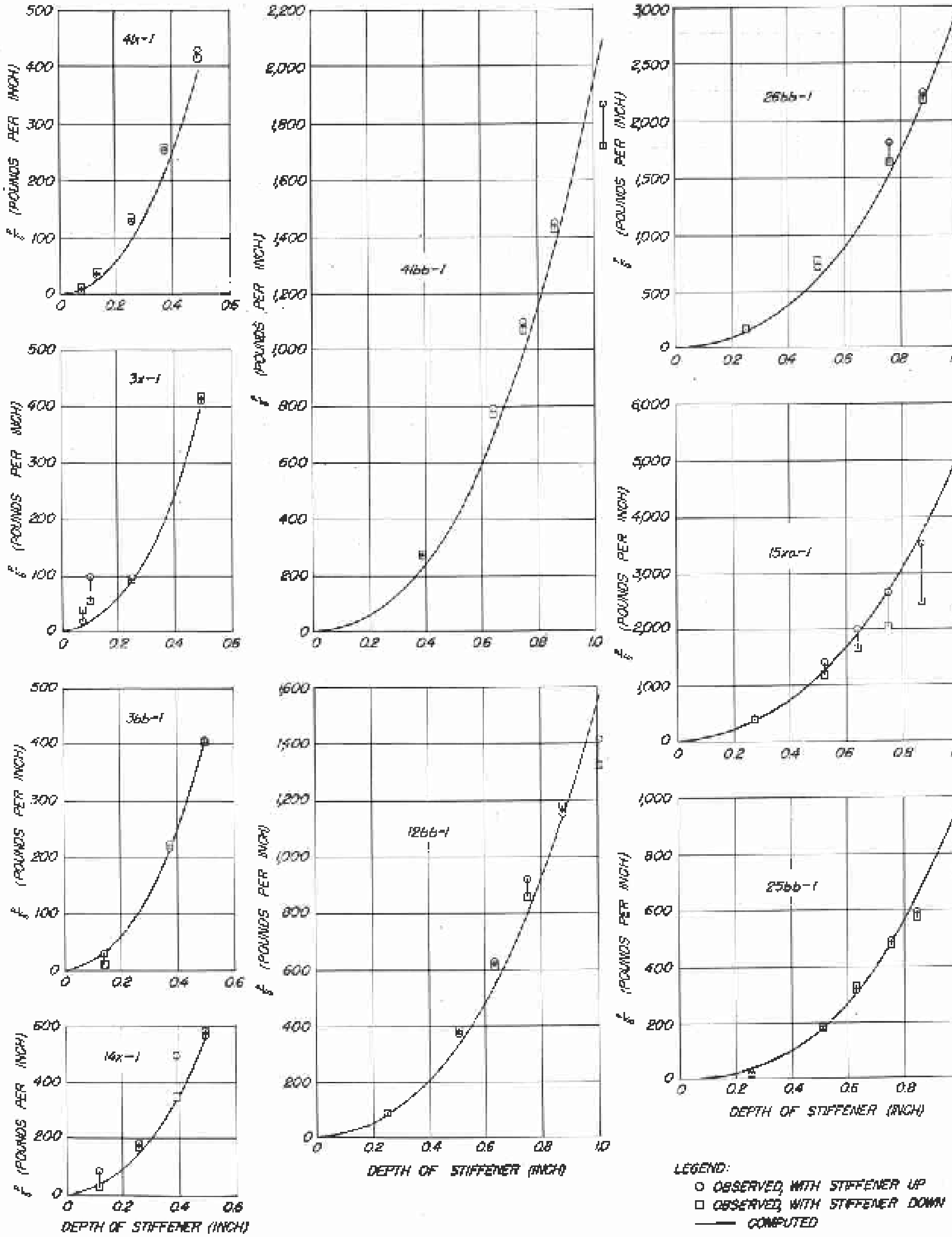


Figure 4A.--Comparison of observed and computed stiffness ratios of attached stiffeners. (Stiffeners perpendicular to the direction of the grain of the face plies.)

Z M 68213 F

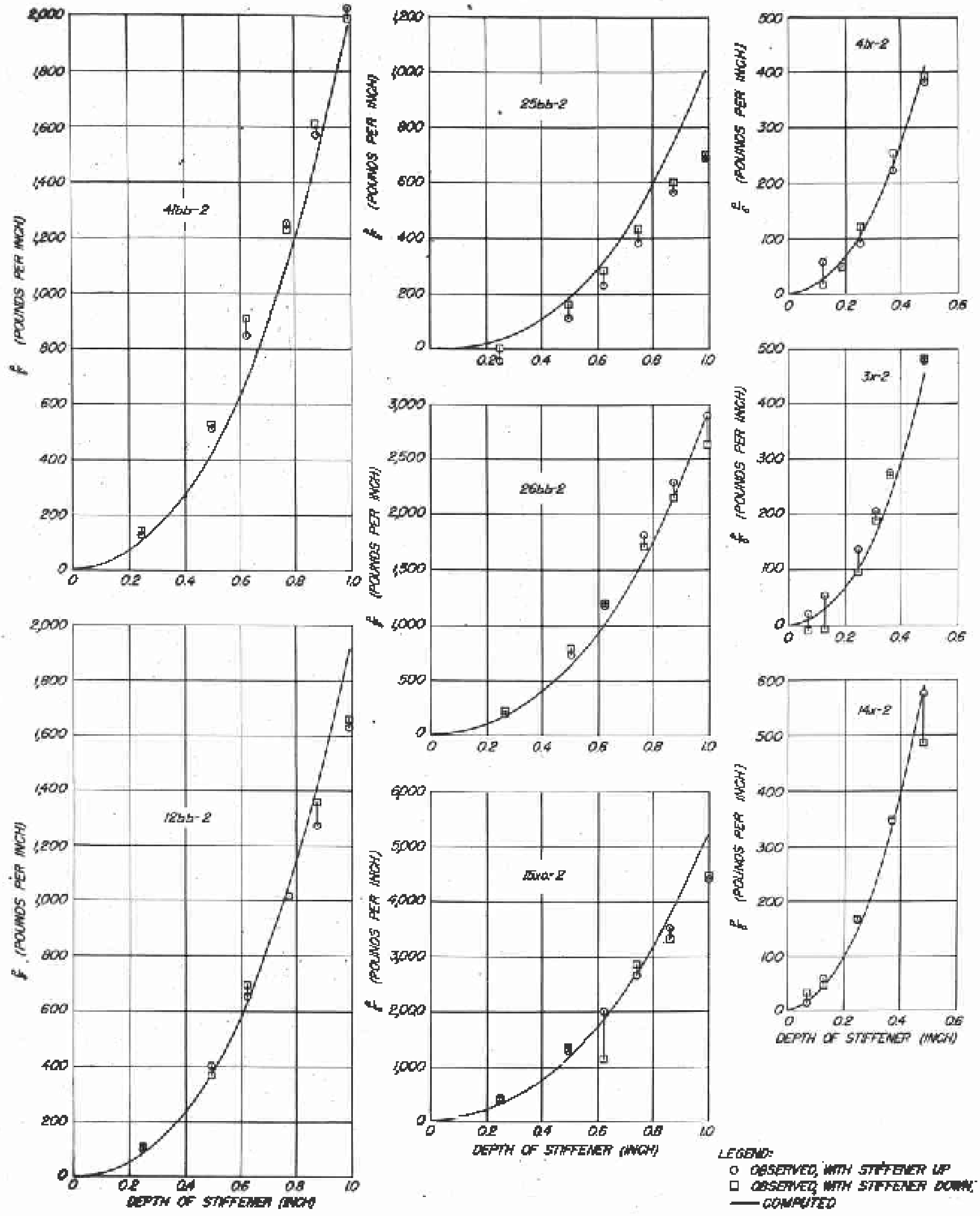


Figure 4B.--Comparison of observed and computed stiffness ratios of attached stiffeners. (Stiffeners parallel to the direction of the grain of the face plies.)

Z M 68616 F

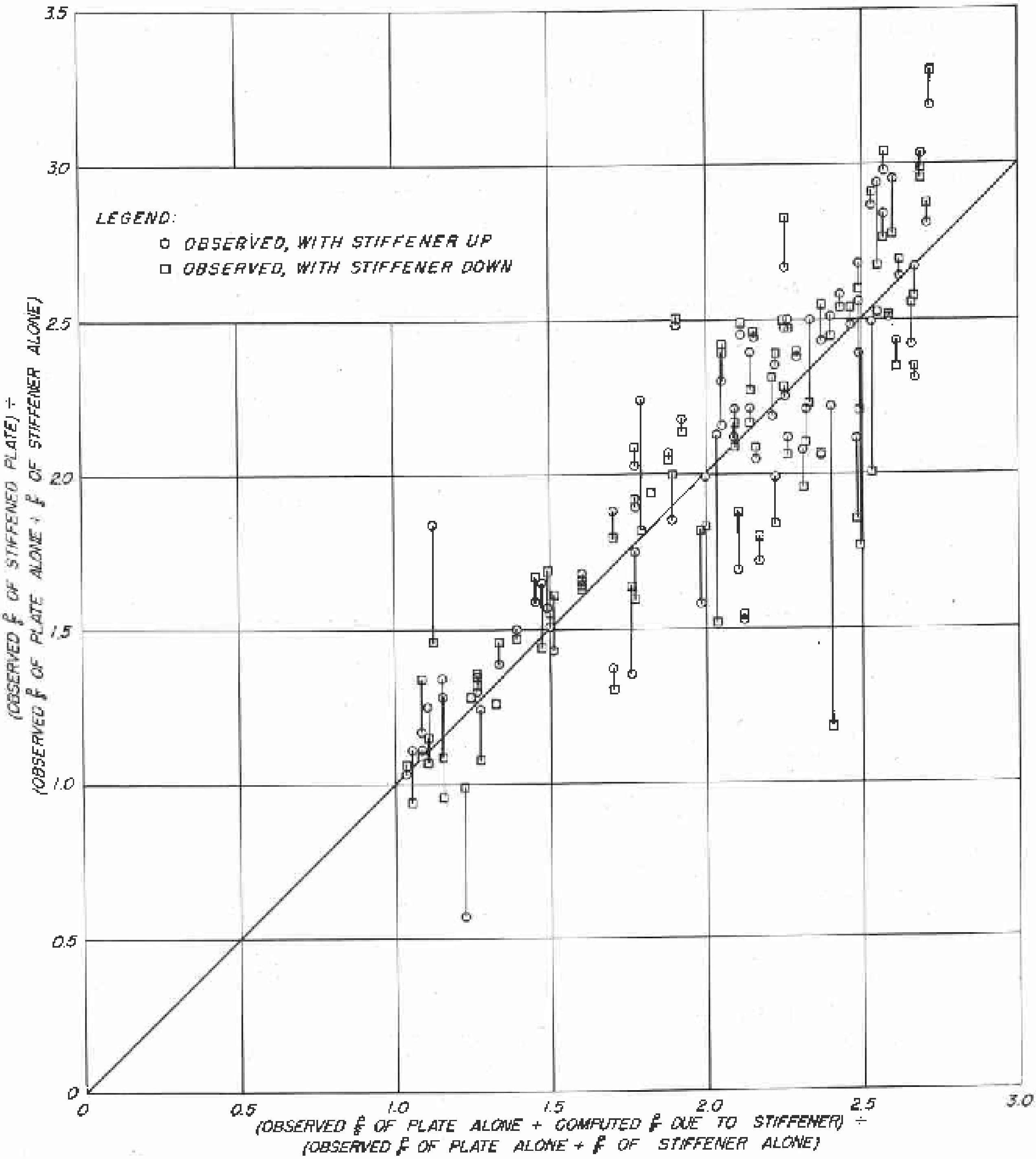


Figure 5.--A comparison of observed and computed stiffness ratios (P/δ) of stiffened plywood plates. (Both expressed as ratios to the sums of the stiffness factors (P/δ) of the plates alone and the stiffeners alone.)

Z M 58617 F

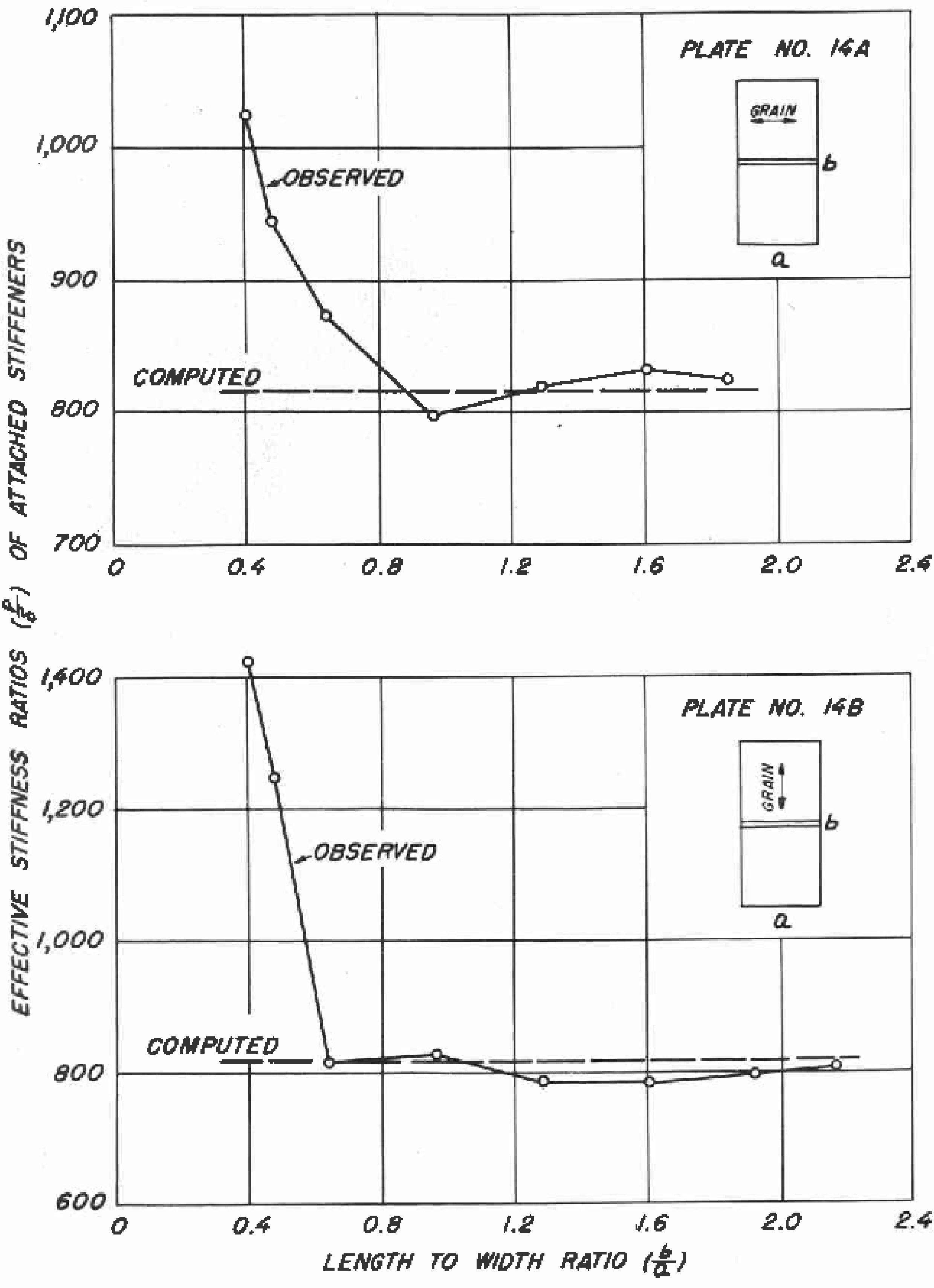


Figure 6.--Comparison of observed and computed stiffness ratios of stiffeners attached to plywood plates of various length to width ratios.

Z M 60614 F