Data Driven Study of Neutron Response in Minerva Using Quasielastic Neutrino Scattering

By
Evan Peters

A THESIS

submitted to
Oregon State University
University Honors College

in partial fulfillment of
the requirements for the
degree of

Honors Baccalaureate of Science in Physics and Nuclear Engineering
(Honors Associate)

Presented May 25, 2017
Commencement June 2017
AN ABSTRACT OF THE THESIS OF

Evan Peters for the degree of Honors Baccalaureate of Science in Physics and Nuclear Engineering presented on May 25, 2017. Title:
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Understanding how particles behave in detectors is a critical part of analyzing data from neutrino experiments. There are methods in place to track charged particles in scintillator material, but neutral particles such as neutrons are more difficult to characterize. The purpose of this project was to assess and calibrate methods for predicting neutron behavior in quasielastic antineutrino scattering (QE) events in the Minerva detector. Here, relativistic kinematics for fully elastic scattering were used to develop a kinematic model that could predict the properties of outgoing neutrons using known muon behavior. These kinematic neutrons compared first to neutrons in Monte Carlo simulations, and then disassociated energy “blobs” reconstructed from simulated data that were hypothesized represent neutron energy deposition. In selected events (QE selection efficiency = 49%, neutron detection efficiency = 3.1%) kinematic neutrons predict relative blob position with an average angular error of 20 degrees but only poorly track blob energy trends. Therefore the kinematic neutron model cannot be applied to neutron detection in real data until further calibrations are developed for the kinematics equations.

Key Words: Neutrino, Minerva, Computational, Physics

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# Contents

1 Background  
1.1 Motivation ........................................... 4  
1.2 NUMI and Minerva: Detecting Neutrino Interactions ................................................... 4  
1.3 Neutrons in Quasielastic Neutrino Scattering ......................................................... 5  
1.3.1 QE Event Description ........................................... 6  
1.3.2 QE Kinematics ........................................... 7  
1.3.3 Limitations to QE Kinematics ........................................... 8  
1.3.4 Reaction Deception ........................................... 9  
1.3.5 Neutrons Traveling Through Matter ........................................... 11  
1.3.6 Neutrons Scattering in Matter ........................................... 11  
1.4 Neutrino Algorithms and Simulations ......................................................... 12  
1.4.1 Minerva Reconstructed Events ........................................... 12  
1.4.2 Monte Carlo Simulations ........................................... 14  
1.4.3 GEANT4 and GENIE Simulations ........................................... 14  

2 Methods ........................................... 15  
2.1 Neutron Variations ........................................... 15  
2.2 Event Selection Cuts ........................................... 17  
2.2.1 QE Selection in truth Data ........................................... 17  
2.2.2 QE Selection in Reconstructed Data ........................................... 17  
2.3 Data Quality Filters ........................................... 19  
2.3.1 Filter optimization ........................................... 20  
2.4 Analysis Tools ........................................... 20  

3 Analysis ........................................... 22  
3.1 Implementing selection cuts in mc simulation ......................................................... 22  
3.1.1 Blob detection efficiency ........................................... 23  
3.2 Optimizing data quality filters ......................................................... 24  
3.2.1 Statistical techniques ........................................... 24  
3.2.2 Filter parameterization ........................................... 25  
3.3 Simulated blob behavior ......................................................... 27  
3.3.1 Distribution of $|r_{vb}|$ ........................................... 27  
3.3.2 Blob Energy Deposition Patterns ........................................... 27  
3.3.3 Validation by conservation of transverse momentum ......................................................... 28  
3.4 Performance of truth neutron at predicting blob behavior ......................................................... 29  
3.4.1 Distribution of $\Delta \theta_{n,mc}$ ........................................... 29  
3.5 Performance of kinematic neutrons in simulated data ......................................................... 30  
3.5.1 Kinematic neutron, recon vs. truth neutron ........................................... 30  
3.5.2 Kinematic neutron, recon vs. energy blob ........................................... 32
<table>
<thead>
<tr>
<th>4 Conclusion</th>
<th>34</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Future Work</td>
<td>34</td>
</tr>
<tr>
<td>4.2 Limitations</td>
<td>34</td>
</tr>
<tr>
<td>5 Acknowledgments</td>
<td>35</td>
</tr>
<tr>
<td>6 Appendix A: Kinematic neutron derivation</td>
<td>38</td>
</tr>
<tr>
<td>7 Appendix B: Optimization plots</td>
<td>40</td>
</tr>
<tr>
<td>8 Appendix C: Source code</td>
<td>42</td>
</tr>
</tbody>
</table>
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Minerva Detector Schematic</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>QE scattering Diagram</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Minerva QE sample event</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>Kinematic representation of QE Scattering</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>QE False positives and negatives</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Alternatives to QE reactions</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>Reconstructed particle paths</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>Cartoon representation of simulated neutrons</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>$\Delta \theta_{n,mc}$ vs. $E_n$ $d\phi_T$ distribution</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>Sample filter correlation matrix</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>Efficiency and scores vs. $E_{blob}$</td>
<td>26</td>
</tr>
<tr>
<td>12</td>
<td>$</td>
<td>\vec{r}_{vb}</td>
</tr>
<tr>
<td>13</td>
<td>$E_{blob}$ distribution</td>
<td>28</td>
</tr>
<tr>
<td>14</td>
<td>$\Delta \phi_{n,\text{recon}}$ distribution</td>
<td>28</td>
</tr>
<tr>
<td>15</td>
<td>$\Delta \theta_{n,mc}$ distribution</td>
<td>29</td>
</tr>
<tr>
<td>16</td>
<td>$\Delta E_{\text{recon}}$ distribution</td>
<td>30</td>
</tr>
<tr>
<td>17</td>
<td>$\Delta \theta_{mc,\text{recon}}$ distribution</td>
<td>31</td>
</tr>
<tr>
<td>18</td>
<td>$\Delta E_{mc,\text{recon}}$</td>
<td>31</td>
</tr>
<tr>
<td>19</td>
<td>Blob energy vs. $E_{\text{recon}}$</td>
<td>32</td>
</tr>
<tr>
<td>20</td>
<td>$\Delta \theta_{n,\text{recon}}$</td>
<td>33</td>
</tr>
<tr>
<td>21</td>
<td>$E_{blob}$ vs. $E_{\text{recon}}$</td>
<td>33</td>
</tr>
<tr>
<td>22</td>
<td>Optimization on $E_{mc}$</td>
<td>40</td>
</tr>
<tr>
<td>23</td>
<td>Optimization on $\phi_T$</td>
<td>40</td>
</tr>
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<td>24</td>
<td>Optimization on $E_{\text{recon}}$</td>
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<td>Optimization on $</td>
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</table>
1 Background

This section provides an overview of the background information and previous work used as a basis for this research project. Section 1.1 describes the status of the field and provides an overview of the neutron tracking problem. Section 1.2 details the Minerva neutrino experiment and the relevant geometrical and signal processing concerns. Section 1.3 describes the kinematics of neutrons produced in neutrino interactions and gives context for the main focus of this research. Then, Section 1.4 overviews algorithms and simulations that contributed to the data analyzed in this project.

1.1 Motivation

Neutrino science is a cornerstone of current models for fundamental interactions and particles in nature. The field recently received recognition with the 2015 Nobel Prize in physics for the confirmation of neutrino oscillations and mass. These discoveries were made possible through extensive use of neutrino detectors. Minerva is one such experiment which detects interactions of high energy neutrinos in a large volume of scintillator material. For each reaction in the detector volume, scintillators track energy deposition and then reconstruction algorithms combine detector geometry with knowledge of particle behavior to identify the particles involved and reconstruct a set of particle paths. However, this method is unreliable for tracking neutral particles, which do not interact with the scintillator material in the same way as charged particles. Neutrons specifically have a low interaction probability moving through material, and therefore appear as a series of disconnected “blobs” of energy deposition that tell very little about the actual particle trajectory.

The motivation for this research project is to develop and calibrate kinematic methods for tracking neutrons in quasielastic (QE) neutrino scattering so that the detector response to these neutrons can be better understood. Findings may be useful for identifying the source of blob energy depositions and characterizing the energies of particles in QE events.

1.2 NUMI and Minerva: Detecting Neutrino Interactions

The first step towards detecting neutrinos is to produce them: The Neutrinos at Main Injector (NUMI) neutrino beamline produces a steady flow of high-energy neutrinos by bombarding a carbon target with 120 GeV protons. After focusing and filtering, a neutrino beam emerges at an angle of roughly 3.1 degrees downwards (herein used to describe the “neutrino coordinate system”). These weakly-interacting particles are then ready to be measured by the specialized Minerva neutrino detector.
The Minerva detector consists of roughly 9 tons of plastic scintillator (PPO/POPOP-doped polystyrene [1]) that perform active tracking, surrounded by calorimeters that provide information on total energy deposition. The front of the detector volume houses targets for experiments on neutrino interaction cross section, and the rear has interspersed steel and lead plates to capture escaping particles and increase the chance of energy detection. Figure 1 shows a side-view of the detector.

Scintillator detectors consist of specialized material that absorb energy from passing radiation and emits light proportional to the measured energy. When a scintillator strip is excited, the emitted light is carried down an optical fiber located in a central channel to be collected and amplified into a signal. By clever use of timing and data acquisition, these signals are used to reconstruct information on the time and place of an interaction [1]. In Minerva, scintillators planes are divided into three categories (X, U, V) based on the orientation of the view they provide of the detector (X planes provide a top-down view, and U and V planes are offset by 60 degrees clockwise and counterclockwise, respectively. Combined, these three orientations can reconstruct events in 3D and verify one another.

![Figure 1: A cross-section of the Minerva particle detector](image)

1.3 Neutrons in Quasielastic Neutrino Scattering

The QE scattering studied in this research involves an incident antineutrino scattering from a nucleus, resulting in an outgoing muon and neutron. This is an ideal candidate for modeling neutron behavior because QE events can be approximated as deterministic, two-body collisions. In the detector, neutrons are not tracked and will instead appear as a
“blob” of unassociated energy deposited near the interaction vertex. Therefore, choosing QE events for analysis allows us to compare a single kinematic neutron to a single energy blob in experimental data.

The goal of this research is to develop a model for the behavior of neutrons produced in QE events using relativistic kinematics. Given the energy and momentum of a muon in Minerva, this model will provide a complete description of the outgoing neutron. The direction of this outgoing neutron can then be compared to the location of the energy blob in experimental data in order to develop scaling relations between the kinematic model and actual data.

1.3.1 QE Event Description

This research will focus on QE antineutrino scattering involving the reaction \( \bar{\nu} + p \rightarrow \mu^+ + n \). This means a QE event will be considered for analysis if it has exactly one outgoing muon and neutron and no other outgoing particles (some exceptions to this rule are discussed in Section 1.3.4). Figure 2 shows a schematic of QE scattering. The W-boson shown carries a force that is analogous to electromagnetism in a classical, “billiard-ball” collision of charged particles.

![Feynman Diagram](image)

Figure 2: QE scattering can be represented with a Feynman diagram. The incoming and outgoing nucleons can be switched (and the leptons exchanged for their antiparticles) and the event will still be QE.

Figure 3 shows a reconstructed QE event in Minerva detector space (image generated by the Arachne web application that shows real-time events in Minerva [2]). The event depicted involves an antineutrino entering Minerva from the left, then undergoing QE scattering at X=50 to produce an outgoing muon track that exits on the right. A neutron track is not created, but the neutron appears as an energy blob at X=80 near the bottom of the figure.
1.3 Neutrons in Quasielastic Neutrino Scattering

Figure 3: A sample reconstructed QE event, as seen in the Arachne app. The horizontal axis corresponds to length in the detector (Z), which moves through the active tracking region to a series of calorimeters from left to right. The vertical axis roughly maps to horizontal displacement (the U-plane of scintillators).

Because of the specific set of conditions that must be met for an event to be considered QE, filters must be applied to data to determine whether an event was actually QE. Section 2.2 gives detailed methods for determining QE events.

1.3.2 QE Kinematics

The two-body nature of QE scattering means that relativistic conservation of energy can be used to determine some of the expected behavior of particles produced. Figure 2 shows a schematic representation of a QE reaction, and the balanced reaction is:

\[ \bar{\nu} + p \rightarrow \mu^+ + n \]  

(1)

Table 1 shows the known and unknown (denoted by "?" ) quantities at the beginning of the kinematic analysis. Capital \( \vec{P} \) denotes the 4-vector describing a particle, while \( \vec{p} \) describes the particle momentum with components \( p_i \). For simplicity of unit conversion, we set all physical constants (such as \( c \)) to 1.

We make the following assumptions to simplify the calculation:

1. \( P_\mu \) is fully determined - CCQEAntiNuTool reconstructs all information for \( P_\mu \), and
Table 1: Relevant quantities for a QE event, in the neutrino beam coordinate system.

<table>
<thead>
<tr>
<th>4-vector</th>
<th>E</th>
<th>$p_x$</th>
<th>$p_y$</th>
<th>$p_z$</th>
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<tbody>
<tr>
<td>$P_\nu$</td>
<td>?</td>
<td>0</td>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>$P_\mu$</td>
<td>$E_\mu$</td>
<td>$p_{\mu,x}$</td>
<td>$p_{\mu,y}$</td>
<td>$p_{\mu,z}$</td>
</tr>
<tr>
<td>$P_p$</td>
<td>$m_p$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P_n$</td>
<td>?</td>
<td>$-p_{\mu,x}$</td>
<td>$-p_{\mu,y}$</td>
<td>?</td>
</tr>
</tbody>
</table>

we take these values to be accurate.

2. $E_p = m_p$ - Assume the target proton is at rest. A nuclear binding energy of $BE_p$ is required to liberate the proton from the nucleus.

3. $p_{n,x} = -p_{\mu,x}$, $p_{n,y} = -p_{\mu,y}$ - By inspection, conservation of momentum in the $xy$-plane immediately yields information on neutron $x$- and $y$-momenta.

The neutrino energy is unknown because the Minerva neutrino beam has a wide range of energies between 1 and 10 GeV (depending on the NUMI operation mode) and a large uncertainty in these energy distributions (up to 16%) [3].

As it turns out, this system is well-determined with four equations and four unknowns. We arrive at the following expressions for neutron energy and momentum (derived in Appendix A):

$$E_n = \frac{(p_{\mu,x}^2 + p_{\mu,y}^2 + m_n^2) + (p_{\mu,z} + m_p - BE_p - E_\mu)^2}{2(p_{\mu,z} + m_p - BE_p - E_\mu)}$$  \hspace{1cm} (2)

$$p_{n,z} = \frac{(p_{\mu,x}^2 + p_{\mu,y}^2 + m_n^2) - (p_{\mu,z} + m_p - BE_p - E_\mu)^2}{2(p_{\mu,z} + m_p - BE_p - E_\mu)}$$  \hspace{1cm} (3)

We have now solved for all unknown quantities in Table 1 and shown the QE reaction to have well-defined kinematics.

### 1.3.3 Limitations to QE Kinematics

The outgoing kinematics of interactions involving an incoming neutrino can be confounded due to effects within the nucleus. Some of the confounding factors are the following [2]:

1. **Fermi motion** represents the thermal energy of nucleons bound in a nucleus - particles may exist within the nucleus with momenta up to 200 MeV/c [4]
1.3 Neutrons in Quasielastic Neutrino Scattering

Figure 4: $\nu + N \rightarrow l' + N'$. The transverse projections of the momenta of outgoing particles indicate loss of momentum due to FSIs [4]

2. **Binding energy** is the excess energy needed to liberate a nucleon stably bound in the nucleus. Any reaction that involves the liberation of a nucleon also involves the absorption of binding energy by the nucleus

3. **Final State Interactions** (FSIs) - A neutrino impacting a nucleus can produce a hadron that interacts with other nucleons on its way out of the nucleus. This process may lead to a shower of outgoing hadrons, or may be undetectable as the particles interact and recombine within the nucleus.

Because the Fermi motion of the target proton is not included in the kinematics derivation of Section 1.3.2, the simplified model for kinematic neutrons will introduce some error.

1.3.4 Reaction Deception

Since neutrons are not easily tracked and sets of product particles from reactions are not unique, determining whether a reaction is QE is prone to false negatives and positives. A QE event that also appears to be QE ("QE-like") consists of an incoming neutrino scattering to produce an outgoing neutron and muon (Figure 5, top left). A QE event may be misidentified as not QE if the initial outgoing neutron interacts to produce more particles than would be expected in a true QE event (Figure 5, top right). Conversely, a
1.3 Neutrons in Quasielastic Neutrino Scattering

A non-QE event (e.g., an incoming neutrino impacts a nucleon to form an excited hadron) will appear to be QE if its products interact undergo FSIs to produce a single outgoing neutron (Figure 5, bottom left), making the final state indistinguishable from a true QE event. The possibility of misidentifying QE events based on final state particles introduces error to process for selecting QE events.

Figure 5: The identity of outgoing particles does not fully determine the type of nuclear reaction that occurred [5]

A number of other nuclear reactions occur in the presence of neutrino flux, including deep inelastic scattering and resonant pion production, for example. These interactions are important to this analysis only in that they must be filtered out of the experimental data, and therefore some knowledge is required of the incoming and outgoing particles when such interactions occur. Figure 6 shows a few examples of reactions that must be accounted for.

Figure 6: The input particles that result in QE scattering can undergo alternative reactions [5]
1.3 Neutrons in Quasielastic Neutrino Scattering

1.3.5 Neutrons Traveling Through Matter

Once neutrons are produced, some statistical quantities can be used to determine their path length and interaction probability in a given material. The most basic description of how particles travel through matter involves the use of an interaction cross section $\Sigma_t$ which is the fractional change in particle beam intensity per unit distance traveled [6]:

$$\Sigma_t = -\frac{1}{I(x)} \frac{dI}{dx}$$  \hspace{1cm} (4)

$\Sigma_t$ represents the total cross section for interaction, which is a combination of cross sections for absorption, scattering, and other specific interactions. This relationship can be solved to give an exponential decay in particle intensity:

$$\frac{I(x)}{I_0} = e^{-\Sigma_t x}$$

$$= e^{-x/\lambda}$$  \hspace{1cm} (6)

Where $I_0$ is the initial intensity and $I(x)$ is the intensity at some distance traveled, and $\lambda$ is the mean free path (mfp) in the second equation. $I(x)$ also represents the probability that a particle travels a distance $dx$ without interacting. Using statistics to solve for the average distance traveled before interacting, we have

$$\bar{x} = \int_0^\infty xe^{-\Sigma_t x} dx = \frac{1}{\Sigma_t}$$  \hspace{1cm} (7)

This relation ties together the distance-decay rate of a beam of neutrons with the mean free path that predicts the distance traveled by any single neutron. For example, a 100 MeV neutron has an interaction cross section in hydrogen on the order of 10 millibarn [12]. With a density on the order of 1 g/cm, the above calculations yield a mfp on the order of 100 cm.

1.3.6 Neutrons Scattering in Matter

In the classical approximation, non-relativistic neutrons undergo energy loss from $E_1 \rightarrow E_2$ in scattering events according to the relationship [6]:

$$\ln \left( \frac{E_1}{E_2} \right) = 1 + \frac{\alpha}{1 - \alpha} \ln \alpha$$  \hspace{1cm} (8)

where $\alpha = \frac{A+1}{A-1}$ and $A$ is the atomic mass of the target nucleus. Scintillator material consists of mostly hydrogen and carbon, and therefore the fraction of initial kinetic energy remaining in a neutron after a scattering event should range between 85% (C) and 37%.
(H). While this is a rough approximation, these results show that the energy that a scattering neutron deposits will only represent some 15%-60% of the neutrons kinetic energy. This information can be used to convert the deposited blob energy to the total energy of the original neutron created in the QE scattering event.

1.4 Neutrino Algorithms and Simulations

Minerva is an advanced experiment with many tools available to conduct simulations and analysis of results. This section gives an overview of the algorithms and simulations that are used by others to produce the data that is analyzed in this project.

1.4.1 Minerva Reconstructed Events

The Minerva experiment relies on complex algorithms written by the Minerva scientific team to convert raw scintillator signals into particle paths. The details of this process are important for making corrections or finding discrepancies in reconstructed data during secondary analysis of events. An overview of the process is as follows [1]:

1. **Cluster Formation** - When measurements indicate that energy was deposited in two or more (physically) adjacent scintillators in the same plane, the corresponding signals are grouped together as a “cluster”.

2. **Seed Formation** - Groups of three clusters in consecutive planes of a given type (X, U, V) are grouped as a “seed”.

3. **Candidate Formation** - For every pair of seeds which shares a cluster, formed in the same plane type (X, U, V), and conformed to linear track requirements, a candidate is formed. Candidates of (X, U, V) type are also merged using a similar set of requirements.

4. **Track Formation** - If a set of three candidates (one for each plane type) represents a consistent 3-dimensional path in the detector, the candidates are combined into a “track”.

5. **Vertex** - Tracks are cleaned, filtered, combined, and analyzed to find a common point of origin - this is labeled as the Vertex, and represents the location of the initial neutrino interaction.

6. **Blob Formation** - After energy hits have been grouped into the vertex and particle tracks, the remaining unassociated energy signatures may be formed into “blobs” - these represent energy deposited in a location where no charged particles were
detected. The algorithms for this process are complex and statistical, but the presence of an energy blob can indicate an uncharged particle scattering event.

The reconstruction process results in a diagram of the particles approximate trajectories in physical space as in Figure 7.

![Figure 7: Particle paths in Minerva are constructed using a hierarchal reconstruction algorithm](image)

Once particles paths are determined, the particles are assigned physical attributes and identities based on the detector measurements taken at corresponding event times:

- **Identity** - The identity of a particle is determined by analyzing its energy deposition as a function of distance traveled. This curve has a unique shape for a given type of particle. Heavy, highly charged particles tend to deposit most of their energy in the region where they begin to slow down, resulting in concentrated energy deposition known as a “Bragg Peak”. Lighter, lesser-charged particles deposit energy more linearly with distance.

- **Muon Charge Sign** - The charge of a muon is determined simply by its curvature in the MINOS B-field.

- **Momentum** - The momentum of a charged particle can be determined using either its path curvature in the MINOS B-field or its total energy deposition and track length in the MINOS detector volume.

- **Energy** - Particle energy is determined by the space-integrated energy deposited in the scintillators combined with the total calorimeter deposition.
1.4.2 Monte Carlo Simulations

The next step in the process is to simulate quasielastic events using a separate tool designed specifically to trace outgoing neutrons. To fully model the stochastic nature of FSIs and the displacement of nucleons, the code randomly assigns a location of reaction within the nucleus and then enforces the appropriate inter-nucleon interactions. This simulation models an isolated nuclear interaction on the microscopic scale using theory for neutrino interactions. The simulation can run for QE, DIS, RES, (Figure 6) and other scattering processes and produces data for an interaction vertex, incoming and outgoing particle 4-vectors, and the interaction type.

For an incoming antineutrino the code uses MC methods to simulate a point of origin for the outgoing neutron (most likely nearby the proton or correlated nucleons impacted by the antineutrino). This allows for FSIs which may disturb the outgoing particle from its original course and result in a set of 4-vectors that do not obey kinematics based on observable incoming and outgoing particles. This results in possible confusion as to the actual of identity of the interaction (detailed in Figure 5).

1.4.3 GEANT4 and GENIE Simulations

To develop analysis tools for live data, simulations can be used prepare a batch of simulated events with controlled behavior. The GENIE code simulates neutrino interactions based on the geometry, composition, and cross sections of the Minerva detector [8]. GE-NIE then employs a host of other models and packages to account for Fermi motion of the target nucleus, results of QE scattering, and final state interactions - this produces an interaction vertex and incoming and outgoing particles with known momenta and identity [7]. These simulated particles can be analyzed in data and are referred to as “truth” (for what really happened) or “MC” interchangeably.

Once GENIE has simulated an event, the particles’ information are fed into the GEANT4 code [9] which simulates particle propagation and energy deposition on a macroscopic scale and provides geometry and cross section data for GENIE. The simulated particle event combined with Minerva’s geometry and composition returns a full account of a neutrino interaction and the energy deposited by the particles in the detector.

Reconstruction algorithms of Section 1.4.1 are then used to interpret the events as they would be seen in the real experiment. The output of this process is labeled “CCQEAntiNuTool” (for “Charged Current, Quasielastic”) and provides a macroscopic description of particle interactions in Minerva. The values corresponding to CCQEAntiNuTool will also be referred to as “reconstructed”, as they represent what is essentially reconstruction algorithms being applied to simulated events.
With the correct settings, GENIE can effectively simulate reconstructed data for outgoing particles - the effectiveness of these programs is demonstrated in [10].

2 Methods

This section presents some of the tools and processes developed specifically for this project, and the manner in which the interface with the previously described algorithms. Section 2.1 introduces methods for modeling kinematic neutrons and diagrams, notations, and particle descriptions to familiarize the reader with notation used in the analysis. Section 2.2 then describes how cuts and filters are used to improve data quality and QE event selection efficiency. Section 2.4 introduces some of the code and computational tools used to conduct the research.

2.1 Neutron Variations

The first step of this project was to develop code to implement the QE kinematics developed in Section 1.3.2. Equation 2 solves for neutron characteristics given a muon energy and momentum, so every muon representation has an associated kinematic neutron. The Monte Carlo and reconstruction algorithms of Section 1.4 can produce the following types of muons:

- Muons generated in MC data
- Muon tracks reconstructed from simulated data
- Muon tracks reconstructed from real data

Each of these muons has a corresponding kinematic neutron. Since the goal is to develop models for the detector’s response to neutrons, the energy and location of energy blobs in an event will be used to represent neutrons in real data. Specifically, the path from the position of the interaction vertex ($\vec{r}_v$) to the energy blob ($\vec{r}_b$) represents the simplest, straight line path a neutron could have taken.

In summary, the analysis will involve four distinct representations of neutrons:

1. **MC neutron** - This is the neutron generated by the MC code. In actual data, a complete description of the QE neutron is unavailable.

2. **Kinematic neutron, MC** - This is the neutron 4-momentum reconstructed using Equation 2 with muon traits corresponding to the MC muon
2.1 Neutron Variations

(a) Raw energy deposition hits in the scintillator material.

(b) Reconstructed vertex, blob, and muon track. \( \vec{k}_{\text{recon}} \) constructed using \( \mu_{\text{recon}} \)

(c) MC code introduces simulated \( \mu_{mc} \) and \( \vec{n}_{mc} \). \( \vec{k}_{mc} \) produced using \( \mu_{mc} \)

(d) Probable path for “real” neutron represented by \( \vec{r}_{vb} \)

Figure 8: Cartoon representation of the neutrons simulated in this research. Bold lines represent particles taken directly from data or MC results, dashed lines represent particle momenta constructed using Equation 2.

3. **Kinematic neutron, reconstructed** - This is the neutron constructed as above, but with muon traits corresponding to the reconstructed muon (available for MC and real data)

4. **Blob-vertex neutron** - This is comprised of the total energy of a blob and the vector connecting the vertex to the center of the blob. This represents data that will be available in real data and simulation.

Figure 8 is a graphic showing how these variations on simulated neutrons are created using a real data input.
2.2 Event Selection Cuts

This project involves analysis of two distinct kinds of data: Monte Carlo simulated data and “real” data reconstructed from events in the Minerva experiment. The following is an overview of the process by which QE events are identified in these data.

2.2.1 QE Selection in truth Data

Selecting for truth CCQE events is straightforward since the simulation provides a full description of truth particles’ behavior each event. The following is a list of filters that were applied to truth events create a pure dataset of only CCQE events:

- **Incoming particle** - Events must be caused by an incoming antineutrino
- **Is QE-like** - Sort reactions by event type, selecting for QE-like (but not necessarily QE) events
- **MC current** - Select reactions with charged current
- **Charm quark** - Remove any events that had charm quark interactions

2.2.2 QE Selection in Reconstructed Data

The following list of cuts identifies QE events in data samples using information available from the detector readout (due to either simulated or real signals).

- **Fiducial Vertex** - allow only events where the reaction occurred in the main detector volume
- **Muon Charge** - allow only events with the properly-charged muon
- **Extra outgoing tracks** - allow only events with no extra outgoing charged particle tracks
- **Veto pass** - allow only events for which the rock muon was “vetoed”
- **Dead Discriminators** - Check that the event occurred in a fully readable region of the detector

Note that these cuts do not account for the presence of the QE-like, false positive events discussed in Section 1.3.4. This represents error intrinsic to real data that must be
accounted for in judging the accuracy of kinematic neutron models.

To characterize the effectiveness of these cuts at creating a sample of CCQE events from a larger population of events, the following quantities are defined:

\[
\text{selection purity} = \frac{(\# \text{ events surviving truth CCQE cuts and recon CCQE cuts})}{(\text{Total } \# \text{ events surviving recon CCQE cuts})} \tag{9}
\]

\[
\text{selection efficiency} = \frac{(\# \text{ events surviving recon CCQE cuts and truth CCQE cuts})}{(\text{Total } \# \text{ events surviving truth CCQE cuts})} \tag{10}
\]

Therefore, the purity of the sample measures the chance that reconstructed CCQE selection cuts correspond to an initial signal that originated as CCQE. As the purity approaches 100%, all of the events that passed reconstructed CCQE selection also pass truth CCQE selection, demonstrating that the entirety of the reconstructed-selected sample originated from a CCQE signal. By another definition, the impurity (1 - purity) is a measure of false positive rate in reconstructed CCQE selection.

The efficiency of the sample measures how much of the population of truly CCQE events (by truth selection standards) is captured by reconstructed selection cuts. As efficiency approaches 100%, all of the events originating as CCQE pass reconstructed CCQE selection cuts.

Efficiency and purity can be calculated using a table such as Table 2. This is an example of a “signal table”, which shows comparative numbers of events passing different selection criteria. An example calculation is provided:

\[
\text{selection efficiency} = \frac{142,029}{142,029 + 164,646} = 46.3\% \tag{11}
\]

Table 2: Signal table comparison of events surviving reconstructed CCQE selection (“recon CCQE”) and/or truth selection cuts (“truth CCQE”)

<table>
<thead>
<tr>
<th>not truth CCQE</th>
<th>recon CCQE</th>
<th>recon CCQE</th>
</tr>
</thead>
<tbody>
<tr>
<td>not truth CCQE</td>
<td>1352456</td>
<td>164646</td>
</tr>
<tr>
<td>truth CCQE</td>
<td>132067</td>
<td>142029</td>
</tr>
</tbody>
</table>
2.3 Data Quality Filters

An important question to address is the effectiveness of the above cuts at selecting a sample that is completely CCQE-like. While the truth CCQE-like event selection is 100% accurate (events are generated to meet these standards), QE selection in reconstructed data in on the order of 80% efficient [7].

The sample purity and efficiency can be improved by further constraining certain physical parameters that are associated with lower data quality. These constraints will be referred to as “filters” and make up a stage of sample improvement distinct from CCQE selection. The following is a list of data quality filters:

- **Blob Location** $\vec{r}_{vb}$ - The algorithm for producing blobs induces spurious blobs that appear nearby the interaction vertex. Furthermore, there is poor angular resolution for neutron direction for blobs that are close to the vertex, which makes these cases difficult to analyze.

- **Neutron Energy** $E_{inc}, E_{recon}$ - Low energy neutrons’ energy depositions are indistinguishable from hadron showers near the nucleus. Reconstruction algorithms avoid trimming and blobbing energy that is nearby the nucleus [3].

- **Transverse Angle** $\phi_T$ - Conservation of momentum in the plane transverse to the neutrino beam line requires that the CCQE muon and neutron are emitted opposite to each other.

- **Blob-Muon Separation** - Diffuse energy from the muon track can interfere with formation of energy clusters. This effect can be avoided by filtering on the angle between the blob-vertex neutron and the muon track, or by filtering on the blob’s distance of closest approach (DCA) to the muon track.

With these filters introduced, the definitions of **purity** and **efficiency** are modified to account for the results of applying data quality filters:

\[
\text{filter purity} = \frac{\left( \text{# events surviving truth CCQE cuts} \right)}{\left( \text{Total # events surviving (recon CCQE cut and quality filter)} \right)}
\]

\[
\text{filter efficiency} = \frac{\left( \text{# events surviving (recon CCQE cut and quality filter)} \right)}{\left( \text{Total # events surviving truth CCQE cuts} \right)}
\]
2.3.1 Filter optimization

Since filters are imposed on parameters of continuous, ratio datatypes, the acceptable ranges of parameters can be adjusted until a desired purity or efficiency is reached. Thus an optimization process can be used to maximize efficiency and data quality simultaneously for a given set of filters.

Filter efficiency is a byproduct of imposing filters that improve the likelihood that an energy blob represents a neutron interaction in a given event. The filters cannot be optimized on filter efficiency since it is highest when no filters are applied at all! Instead, filters must be optimized over quantities that improve the likelihood that an energy blob in a given event represents a neutron interaction.

One useful parameter for gauging filter performance is blob detection efficiency, an extension of event selection efficiency that correlates events surviving selection and filters with the presence of a blob in reconstructed data. The efficiency for blob detection in truth or recon selected CCQE events is then defined as:

\[
\eta_{\text{mc}} = \frac{\# \text{ events surviving (truth CCQE cut and quality filter)} \text{ with a blob present}}{\# \text{ events surviving (truth CCQE cut and quality filter)}} \tag{14}
\]

\[
\eta_{\text{recon}} = \frac{\# \text{ events surviving (recon CCQE cut and quality filter)} \text{ with a blob present}}{\# \text{ events surviving (recon CCQE cut and quality filter)}} \tag{15}
\]

Evidently, \( \eta \) is a function of each of the filter parameters on which cuts are made, since the allowable range for each filter parameter will affect the number events that pass data quality filters and the number of events in which a blob is detected in different ways.

Table 3 describes event parameters that can be modified directly to improve the performance of the neutron models. This performance is characterized by the quantities in Table 4. In real data, filters are things we see and expect physically, while scored parameters are things we want to refine without constraining directly.

2.4 Analysis Tools

Given the highly statistical nature of the experiment goals, Minerva generates many gigabytes of event data per day. Proper code infrastructure is necessary analyze these data, and a summary of some of the common tools is as follows:

- **ROOT** - ROOT is a C++ framework for data analysis. It handles large datasets
Table 3: Filter quantities may influence the quality of data, and can be constrained directly in order to improve data quality.

<table>
<thead>
<tr>
<th>Filter Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{blob}}$</td>
<td>Highest blob energy</td>
</tr>
<tr>
<td>$E_{\text{kine}}$</td>
<td>Expected neutron energy</td>
</tr>
<tr>
<td>$E_{\text{n,mc}}$</td>
<td>Truth neutron energy</td>
</tr>
<tr>
<td>$r_{\text{vb}}$</td>
<td>Blob-vertex distance</td>
</tr>
<tr>
<td>$\phi_{T}$</td>
<td>Blob vs. reconstructed muon transverse angle</td>
</tr>
</tbody>
</table>

Table 4: Scored quantities are a measure of how well the behavior of an energy blob matches that of a truth neutron. The general goal of the analysis is to minimize these quantities.

<table>
<thead>
<tr>
<th>Scored Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \theta_{\text{n,mc}}$</td>
<td>Truth neutron vs. blob angle</td>
</tr>
<tr>
<td>$\Delta E_{\text{mc}}$</td>
<td>Truth neutron vs. blob energy</td>
</tr>
<tr>
<td>$\Delta \phi_{\text{n,mc}}$</td>
<td>blob vs. Truth neutron transverse angle</td>
</tr>
<tr>
<td>$\Delta \theta_{\text{n,recon}}$</td>
<td>Recon kinematic neutron vs. blob angle</td>
</tr>
<tr>
<td>$\Delta \phi_{\text{n,recon}}$</td>
<td>blob vs. Recon kinematic neutron transverse angle</td>
</tr>
<tr>
<td>$\Delta E_{\text{recon}}$</td>
<td>Recon kinematic neutron vs. blob energy</td>
</tr>
</tbody>
</table>

and produces useful visualizations. This project relies heavily on histograms and data structures developed in ROOT.

- **PyROOT** - This is a module developed for interfacing ROOT data structures with Python scripts. This allowed the bulk of the analysis to be written in the Python scripting language, for which the author is very thankful.

- **rootpy** - Another module for ROOT interfacing, with more pythonic objects and functionality.

- **numpy** - This module contains a host of mathematical functions for efficient arrays storage, vector and matrix calculations.

- **pandas** DataFrame objects support fast statistical functions for filtering, correlating, and plotting data.

- **multiprocessing** - The analysis and histogram construction was parallelized across many cores. The routine was I/O-heavy and relied on file splitting and recombination to run across many threads.
3 Analysis

Analysis of reconstruction signature due to simulated truth events was conducted according to the following procedure (Section 3):

1. Implement selection (Section 3.1) cuts and data quality filters (Section 3.2) for reconstructed data due to truth simulation events. Assess purity and efficiency for CCQE event selection and blob detection.

2. Compare truth neutron against simulated blobs to assess the performance of blob/-clustering algorithms (Section 3.4)

3. Analyze simulated blob behavior in reconstructed data to gather information on neutron detection behavior and blob detection efficiency (Section 3.3)

4. Compare kinematic neutron, recon against to truth neutron to validate kinematic model for reconstructed data, and then repeat comparison against blob behavior (Section 3.5).

3.1 Implementing selection cuts in mc simulation

Table 5 categorizes the events in the simulated dataset according to which cuts they passed. The selection efficiency and purity (Section 2.2.2) calculated from these data are:

<table>
<thead>
<tr>
<th></th>
<th>not recon CCQE</th>
<th>recon CCQE</th>
</tr>
</thead>
<tbody>
<tr>
<td>not truth CCQE</td>
<td>1352456</td>
<td>164646</td>
</tr>
<tr>
<td>truth CCQE</td>
<td>132067</td>
<td>142029</td>
</tr>
</tbody>
</table>

Table 5: Event signal table for selection by truth CCQE vs. reconstructed CCQE cuts.

The prominent off-diagonal terms in Table 5 represent false positive and false negatives for selection by reconstructed cuts. An event that was truth CCQE has a nearly 50% chance of being selected as reconstructed CCQE, as does an event that was not truth CCQE. To reduce the influence of selection inefficiency, the following analysis includes events that passed both truth- and recon- CCQE selection cuts (effectively assuming 100% efficient selection for the data sample).
3.1 Implementing selection cuts in mc simulation

3.1.1 Blob detection efficiency

The blob detection efficiency (Section 2.3.1) was found by comparing statistics of surviving events before and after selection cuts were applied. Since there is a distinct selection cut for each datatype, this efficiency was assessed for both truth CCQE event selection (Table 6) and recon CCQE event selection (Table 7) in simulated data.

Table 6: Event selection table for truth CCQE cuts vs. blob detection compares the number of events that did/did not pass truth CCQE selection vs. the number of events with at least 1 blob

<table>
<thead>
<tr>
<th></th>
<th>no blob</th>
<th>blob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not CCQE</td>
<td>248406</td>
<td>1278244</td>
</tr>
<tr>
<td>CCQE</td>
<td>241466</td>
<td>53167</td>
</tr>
</tbody>
</table>

Table 7: Event selection table for recon cuts vs. blob detection compares the number of events that did/did not pass reconstructed CCQE selection vs. the number of events with at least 1 energy blob

<table>
<thead>
<tr>
<th></th>
<th>no blob</th>
<th>blob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not CCQE</td>
<td>318563</td>
<td>1165960</td>
</tr>
<tr>
<td>CCQE</td>
<td>146285</td>
<td>160390</td>
</tr>
</tbody>
</table>

Table 8 compares the presence of truth neutrons to the detection of energy blobs for events passing both truth-CCQE and recon-CCQE selection. A majority of selected events have no energy blob, resulting in an approximate upper limit of $\eta = 0.168$ for filtered blob detection efficiency (which is a rough approximation, not accounting for the total number of truth neutrons in one-blob-many-neutrons events).

Table 8: Event counts with comparison between the number of truth neutrons and the detection of energy blobs.

<table>
<thead>
<tr>
<th>Truth neutrons</th>
<th>Reconstructed blobs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
</tr>
<tr>
<td>None</td>
<td>175</td>
</tr>
<tr>
<td>1</td>
<td>87534</td>
</tr>
<tr>
<td>Many</td>
<td>38710</td>
</tr>
</tbody>
</table>
3.2 Optimizing data quality filters

As described in Section 2.3.1, data quality filters necessarily reduce blob detection efficiency from the values found using event selection alone. The ideal choice of filters maximizes detection efficiency while minimizing energy and angle differences between the energy blob and the truth neutron.

An intuitive method for determining ideal filters was to compare the scored parameters to the filters graphically. The significant population in the bottom right corner in Figure 9a captures events with very low neutron energies that resulted in energy blobs forming roughly 90 deg. offset from the neutron direction. This suggests a cut on the minimum energy of truth neutrons.

Figure 9: Comparison of $\Delta \theta_{n,mc}$ (vertical axis) vs. truth neutron energy (horizontal axis)
(a) Blobs in unfiltered events with low neutron energy appear in unrealistic locations (b) Filtered on ranges from Table 9, anomalous blob behavior is removed

3.2.1 Statistical techniques

Figure 10 shows linear correlation values (Pearson r-value) between various filter and scoring parameters. However, since the scored parameters for blob performance listed in Table 4 have nonlinear behavior they are poor candidates for correlation with the filter parameters.
3.2 Optimizing data quality filters

Figure 10: Sample correlation matrix for scored vs. filter parameters, using Pearson correlation coefficient scoring. Lack of any significant correlation implies nonlinear relationships between the various event parameters.

One method for refining these correlations involved applying Singular Value Decomposition and Principal Component Analysis to the correlation matrices and then optimizing the filters to maximize correlation r-values. This method was ineffective without some means of linearizing the scored parameters, and was abandoned in favor of a filter parameterization study.

3.2.2 Filter parameterization

Because of the drawbacks of filtering on correlations this strategy was not pursued any further. Instead, statistics were collected on the effect of filters on sample population and scored parameter performance, over a range of filters. For instance, Figure 11 compares $\eta_{blob}$ and scored parameter performance against different cuts on $E_{blob}$. The roughly linear decrease of $\eta_{blob}$ and $\Delta E_{mc}$ in Figure 11b compared to a flattening, decreasing trend in $\Delta \theta_{n,mc}$ suggests that an energy filter should be imposed somewhere before the minimum in $\Delta \theta_{n,mc}$.
3.2 Optimizing data quality filters

Figure 11: $\eta_{\text{blob}}$ trends downwards with increasing constraints on minimum $E_{\text{blob}}$ (a) Mean $\Delta \theta_{n,mc}$ decreases towards a minimum when $E_{\text{blob}} > 160$ MeV while (b) $\Delta E_{mc}$ steadily increases with energy constraint.

Repeating this comparison between scored parameters, $\eta_{\text{blob}}$, and minimum and maximum constraints on filter parameters allows for a “fuzzy” determination of ideal filters. Table 9 lists the final filters selected for data quality improvement, chosen using trends in efficiency and blob vs. truth neutron behavior in Figure 11 and others in Appendix C (Section 7).

Table 9: Final filter settings were chosen by “fuzzy” maximization of efficiency vs. values of scored parameters. Descriptions of these filter parameters are given in Table 3

<table>
<thead>
<tr>
<th>Filter Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{blob}}$</td>
<td>90 MeV</td>
<td>5000 MeV</td>
</tr>
<tr>
<td>$E_{\text{kin}}$</td>
<td>140 MeV</td>
<td>5000 MeV</td>
</tr>
<tr>
<td>$E_{n,mc}$</td>
<td>90 MeV</td>
<td>5000 MeV</td>
</tr>
<tr>
<td>$</td>
<td>\vec{r}_{\text{vb}}</td>
<td>$</td>
</tr>
<tr>
<td>$\phi_T$</td>
<td>160 deg</td>
<td>200 deg</td>
</tr>
</tbody>
</table>

4639 events survived the filters, from initial number of 142029 events selected for truth and recon CCQE. This results in a final blob detection efficiency $\eta = .0327$. 
3.3 Simulated blob behavior

3.3.1 Distribution of $|r_{vb}|$

The distribution of blob-vertex separation in MC simulated data with no filter restrictions Figure 12a. Also shown is an exponential fit with a slope of -0.00171. Assuming this distribution describes all neutrons interacting in the detector, this corresponds to a mean free path of 58.5 cm (described in Section 1.3.5) for neutrons in Minerva scintillator material, which is on the order of the expected mean free path. However, a majority of neutrons escape the detector and this MFP therefore represents a characteristic blob detection distance rather than a physical property of neutrons in QE events.

![Figure 12](image)

Figure 12: (a) The exponential falloff in $|r_{vb}|$ relates to the MFP of simulated particles that eventually form blobs in the detector. (b) With filters applied, the distribution is similarly exponential, but with an artificially lowered MFP

3.3.2 Blob Energy Deposition Patterns

The distribution of energies for blobs in the filtered sample is plotted in Figures 13a-b. Note that this distribution only tracks the energy of the best candidate for blob selection (the blob closest to the truth neutron direction).
3.3 Simulated blob behavior

Figure 13: $E_{\text{blob}}$ distribution. (a) Unfiltered events consist largely of low-energy blobs. (b) The blob energy filter is applied to cut events with blob energies less than 100 MeV.

### 3.3.3 Validation by conservation of transverse momentum

Conservation of momentum in the transverse (XY) plane requires that the outgoing muon and neutron are emitted 180 degrees apart from each other. Figure 14a plots $\phi_T$ and demonstrates that the energy blobs in selected CCQE events generally obey this kinematic requirement. There is some bad behavior in which blob energy appears aligned with the muon track (far left and right fringes of Figure 14a); this behavior is suppressed by imposing a requirement that $\phi_T$ be within $20^\circ$ of $180^\circ$.

Figure 14: Angular error, truth vs. recon kine ($\Delta \phi_{n,\text{recon}}$) (a) In unfiltered events, the angular distribution of blobs mostly agrees with conservation of transverse momentum in the CCQE event (b) Applied filters requires blobs appear at $180^\circ \pm 20^\circ$ from the muon.
3.4 Performance of truth neutron at predicting blob behavior

3.4.1 Distribution of $\Delta \theta_{n,mc}$

The most important assessment of blob behavior is based on comparison to the truth neutron. Simulated “truth” events are the original source of neutrons that deposit (or don’t deposit) energy in the detector, so comparison of the simulated blob to the truth neutron should show the strongest correlation out of all the neutron models.

Figures 15a shows the distribution of difference in 3D angle between $\vec{r}_{vb}$ and $\vec{n}_{mc}$. The peak near 0° shows that there is a strong correlation between the directions of the MC neutron and the blob (a cute feature of coordinates: the peak cannot occur at exactly 0° since the solid angle subtended by $\vec{r}_{vb}$ has magnitude 0 at a relative angle of $\theta = 0$ to $\vec{n}_{mc}$). In Figures 15b, applied filters successfully remove the anomalous peak at roughly 90°.

![Figure 15](image)

Figure 15: (a) The truth neutron direction often aligns with the blob direction, with some anomalous misalignment. (b) Only blobs with reasonable $\vec{r}_{vb}$ direction remain in the filtered sample.

Figure 16 compares the kinetic energy of the truth neutron to the energy deposited by the simulated blob. As explained in Section 1.3.6, an energy blob resulting from a neutron collision would receive only a fraction of the neutron’s total kinetic energy.
3.5 Performance of kinematic neutrons in simulated data

3.5.1 Kinematic neutron, recon vs. truth neutron

This section presents analysis of the neutrons modeled using the kinematics equations from Section 1.3.2, compared to the truth neutron that represents how the outgoing neutron really behaved.

In filtered data, the 3D angle between the truth and kinematic neutrons agrees as demonstrated in Figure 17.

Figure 16: Energy comparison ($\Delta E_{\text{recon}}$) of the blob energy (y-axis) vs. truth neutron energy (x-axis) has a disproportionately large number of low-energy truth neutrons in (a) filtered data. (b) In filtered data, there is no obvious linear trend in relative energies
3.5 Performance of kinematic neutrons in simulated data

Figure 17: Angle difference between truth and kinematic neutrons ($|\Delta\theta_{\text{mc,recon}}|$). (a) The neutrons generally align even in unfiltered events (b) Applying filters further approves this alignment

The energies of the kinematic neutron and truth neutron are related in Figure 18a-b, where a 45° line represents matched energies between the models. There appears to be anomalous energy disagreement between truth and kinematic neutrons for low kinematic neutron energies.

Figure 18: Energy difference ($\Delta E_{\text{mc,recon}}$) between truth neutron (y-axis) and kinematic neutron (x-axis). (a) In unfiltered events, the neutron energies fall into a roughly linear trend (b) Applied filters reduce any obvious agreement between the energies.

In addition to energy distribution, Figure 19 shows the ratio of blob energy to kinematic neutron, reconstructed energy for the filtered data sample. These results show that the
energy blobs (corresponding to neutrons) in experimental data contained an average of 46% as much energy as the corresponding kinematic neutron model.

Figure 19: The normalized ratio of blob energy / kinematic neutron $\frac{E_{\text{recon}} - E_{\text{blob}}}{E_{\text{recon}} + E_{\text{blob}}}$ is less than unity, which indicates that experimental neutrons deposit a fraction of their energy into the scintillator material when they eventually scatter.

These results indicate that only a small fraction of the energy of a neutron traveling through the detector is deposited in the scintillator material - this serves as validation for the theory discussed in Section 1.3.6.

### 3.5.2 Kinematic neutron, recon vs. energy blob

Repeating the comparison between angles for neutrons in Section 3.4, Figure 20a-b compares the kinematic neutron direction to the energy blob in raw and filtered data.
3.5 Performance of kinematic neutrons in simulated data

Figure 20: Angular error between kinematic neutron and blob ($\Delta \theta_{n,\text{recon}}$). (a) The energy blobs in unfiltered data exhibit similar anomalous angle behavior compared to kinematic neutrons (b) Filters similarly remove these angle anomalies

The energies of $\vec{k}_{\text{recon}}$ and $r_{vb}$ are compared in Figure 21a-b. The reconstructed kinematic neutron is unsuccessful at predicting the energy deposited by blobs in filtered events, which poses a challenge for determining whether blobs in filtered events are truly the result of CCQE neutrons. This does not account for fractional energy deposition by neutrons in scintillator material (discussed in Section 1.3.6), but it is unlikely that the discrepancy in Figure 21 could be corrected by adding a scaling factor alone.

Figure 21: Energy difference between the blob and kinematic neutron ($E_{\text{blob}}$ vs. $E_{\text{recon}}$) shows poor agreement between the models for both raw and filtered CCQE event samples.
4 Conclusion

The kinematic model for neutrons in quasielastic neutrino scattering events compared favorably against QE neutrons generated from Monte Carlo signal, but failed to predict characteristics of disassociated energy blobs hypothesized to represent neutron scattering in QE. Events were filtered for data quality resulting in an analyzed sample with 3.27% blob detection efficiency. Kinematic neutrons created from simulated reconstructed data agreed with truth neutrons with an average angular error of 16.5° and energy error within 25%. Kinematic neutron behavior agreed with the blob direction with an average of 20° angular error and failed to provide a reasonable prediction of blob energy.

4.1 Future Work

The primary goal of this work was to analyze performance of models for predicting neutron behavior in real Minerva data. Such analysis of reconstructed data due to real events might look like:

1. Analyze blob behavior in reconstructed data from real events. Gather information on neutron detection behavior and blob detection efficiency.

2. Compare kinematic neutron, recon against real blobs to assess the performance of the kinematic model in real data.


In order to apply the kinematic neutron to real data, calibrations must be developed to improve the ability of the kinematic model to predict blob position and energy. These results would also benefit from a more rigorous method of choosing filters.

4.2 Limitations

These results are limited by the size of the data sample: The experimental data set of QE events is too small to finalize these calibrations; future work will require a larger sample of QE events for analysis. The accuracy of the kinematic model could be improved on the front end with better knowledge of FSI probability, neutrino energy, and the magnitude of kinematic imbalances [4].
5 Acknowledgments

Huge thanks to Dr. Heidi Schellman for ongoing mentoring, academic advice, funding assistance, and time spent debugging my research. She has also helped me dive head-first into the world of particle/nuclear physics by having me run shifts on Minerva, sponsoring my trip to the 2016 DNP conference as well as a trip to FNAL during spring break 2017, and just knowing a ton about the world of HEP/neutrino physics. Many thanks to Professor Tate for continuous guidance and hands-on involvement in getting me to finish my thesis. Thank you to my group members Gabe and Cheryl for contributions to code base and shared ROOT frustrations. Finally, I appreciate the attention of my fellow physics students of PH 403 and nuclear engineers of the ANS for their feedback on my posters, presentations, and dress code (!) related to this research.
References


6 Appendix A: Kinematic neutron derivation

The following is a derivation of the energy and momentum for a QE neutron. For this analysis, we use the reference frame of the incoming neutrino \( (p_\nu \rightarrow p_{\nu,z}) \) and make the following assumptions with corresponding mathematical consequences:

1. Target proton is at rest \( \rightarrow E_p = m_p \)
2. Target proton requires \( BE_p \) input energy to be freed from the nucleus
3. Neutrino mass is negligible compared to its energy \( \rightarrow E_\nu = p_\nu \)
4. The outgoing muon is fully described by measurements

The following equation describes relativistic conservation of energy \([11]\)

\[
E^2 = \vec{p}^2 + m^2
\]  

(16)

Since the mass of a neutrino is very small compared to the energy of neutrino produced in the NUMI beamline (roughly 5 GeV neutrinos), we neglect \( m_\nu \) and therefore state

\[
E_\nu = p_{\nu,z} \]  

(17)

Both of these values are unknown. Conservation of momentum in the z-direction gives

\[
p_{\nu,z} = p_{\mu,z} + p_{n,z} \]  

(18)

which brings the unknown \( p_{n,z} \) into the equations. Conservation of energy for incoming and outgoing particles yields

\[
E_\nu + (m_p - BE_p) = E_\mu + E_n
\]  

(19)

Which further involves the unknown \( E_n \). Finally, relativistic conservation of energy yields the following:

\[
E^2_n = \vec{p}_n^2 + m^2_n
\]  

(20)

Which introduces no new variables, since the mass \( m_n \) of the neutron is known. With Equations 17-20 we have a system of four equations and four unknowns.

Substituting Equation 18 into Equation 19 and reorganizing results in

\[
E_n = p_{n,z} + p_{\mu,z} + (m_p - BE_p) - E_\mu
\]  

(21)
which is still a statement of conservation of total energy. Expanding the momentum term in Equation 20 and rearranging gives

\[ E_n^2 = (p_{n,x}^2 + p_{n,y}^2 + p_{n,z}^2) + m_n^2 \]

Or, with the proper substitutions,

\[ p_{n,z}^2 = E_n^2 - (p_{\mu,x}^2 + p_{\mu,y}^2 + m_n^2) \]  
(22)

To simplify the algebra, we make the following definitions using only quantities that are known:

\[ A \equiv (p_{\mu,z} + (m_p - BE_p) - E_\mu) \]  
(23)
\[ B \equiv (p_{\mu,x}^2 + p_{\mu,y}^2 + m_n^2) \]  
(24)

Now Equations 21 and 22 are rather inviting:

\[ E_n = p_{n,z} + A \]  
(25)
\[ p_{n,z}^2 = E_n^2 - B \]  
(26)

This leads immediately to the solutions for neutron energy and momentum

\[ E_n = \frac{B + A^2}{2A} \]  
(27)
\[ p_{n,z} = \frac{B - A^2}{2A} \]  
(28)
7 Appendix B: Optimization plots

This appendix includes overflow comparisons between efficiency and scored parameters during the filter optimization process. Figures 22-25 provide additional context for the choice of filters presented in Table 9.

![Figure 22: (a)](image1)

Filters on $\phi_T$ have a drastic impact on the population of events. Selecting for events with $\phi_T \approx 180$ results in blob detection efficiencies of $> 1$ percent, requiring a generous allowance for conservation of transverse momentum in blob selection.

![Figure 23: (b)](image2)

Erratic behavior in $\Delta E_{n,mc}$ is due to averaging quantities over a statistically insignificant subset of the original selected events.
Figure 24: Efficiency vs. (a) $\Delta \theta_{n,mc}$ and (b) $\Delta E_{n,mc}$ follow similar trends as for parameterization of the filter on $E_{blob}$.

In Figure 25, $\Delta \theta_{n,mc}$ reaches a local minimum as $|\vec{r}_{vb}|$ is constrained, before rising with further constraint. Judging by trends in $\Delta E_{n,mc}$ and $E_{blob}$, this could be due to the greater population of lower-energy blobs that occur further from the CCQE vertex.

Figure 25: Trends in (a) $\Delta \theta_{n,mc}$ and (b) $\Delta E_{n,mc}$ suggest that filters on blob-vertex distance select for a poorly-behaved subset of blobs occurring far away from the vertex.
8 Appendix C: Source code

This project’s analysis tools are completely open source and available at:

HTTPS://GITHUB.COM/PETERSE/NEUTRONPARSER