DEVELOPMENT OF BARK EQUATIONS FOR SITKA SPRUCE IN SOUTHEAST ALASKA

by

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At present, bark has little economic value. In 1966 alone, six to seven million tons of bark were produced in the Pacific Northwest and British Columbia. Of this amount most was disposed of by burning, either for fuel or in wigwam burners (Bollen 1969). Because bark has been considered a waste product there has been little reason to measure it in the past. However, with pollution legislation and increasing stumpage and wood processing costs, there is an increasing need to develop ways to profitably utilize the bark resource. Procedures are needed to estimate the amount of bark that is being produced. The development of optical dendrometers, optical calipers, and other instruments (Bruce 1967) for measuring upper stem diameters of standing trees is also directing attention to the need for bark information, as upper stem diameter measurements must first be converted to diameter inside bark before a tree's solid wood content can be estimated.

This paper reviews techniques that have been used to estimate bark growth, upper stem bark thickness, and bark volume. Two new equation forms have been developed and tested for estimating upper stem bark thickness using, as an example, data from Sitka spruce (Picea Sitchensis (Bong.) Carr.) in southeast Alaska.

REVIEW OF THE LITERATURE

Accuracy of Bark Thickness Measurements

Bark thickness is usually measured with some type of bark punch; the most common type being the Swedish bark gauge. Other models have been described by Lewis (1953), Furniss (1962), Rideout (1966), and Carron (1968).

Conceptually, the bark gauge is a good idea, but in practice its use may lead to biased measurements. When using the bark gauge the assumption is made that the bark will be penetrated but the wood will not be penetrated. Unfortunately, no sure way has been found to judge when the punch has passed through the bark and is about to penetrate the wood. Althen (1964) noted that the most serious mistake was the practice of "whacking" the gauge with the palm of the hand to drive the punch through the bark. This technique nearly always resulted in penetration of the wood. Althen's study of the accuracy of bark thickness measurements on 158 plantation-grown red pine indicated a positive bias when the bark gauge was used. Penetration of the wood was found to occur most frequently during the growing season at points on the stem lying near the start of the live crown. Average percentage deviation in bark thickness disregarding sign ranged from 9.6 to 18.0 percent, being least at the base of the tree and greatest at half the height of the tree.

In a Norwegian study of measurement accuracy, Dahl (1965) calipered 4,530 spruce and 741 pine logs both outside and inside bark.

He then compared the true bark thickness with estimated thickness based on sampling. Estimated thickness averaged 5.8 percent too high for spruce and 33.3 percent too high for pine.

In a recent study of shortleaf pine in Arkansas, Mesavage (1969) found that estimated bark thickness using the Swedish bark gauge was rarely within 0.1 inch and that errors as great as 0.6 inch were not uncommon. Mesavage attributed the errors to 1) ambiguous gauge readings caused by unevenness of the bark surface at the point of measurement, 2) inadequate sampling, and 3) incorrect seating of the chisel of the bark measuring gauge. Penetration was usually short of the wood, but occasionally too deep. For rough-barked trees Mesavage suggested that, at the point of measurement, a diameter tape be wrapped around the tree. Bark thickness is measured as the distance from tape to wood. With this procedure the errors due to unevenness of bark thickness are mostly avoided.

Another problem is that bark thickness may vary greatly around the stem. It is generally suggested, therefore, that at least two measurements be taken on opposite sides of the tree when using a bark gauge. Mesavage (1969) suggested that one pair of measurements be taken. Then, if one measurement of the pair is 30 percent or more of the other, an additional pair be taken. If the total of one pair is 30 percent or more greater than the total of the other, measure two more pair, etc.

Unless destructive sampling methods can be used there appears no alternative but to measure bark thickness with some type of bark gauge, being careful to avoid the sources of error that have been discussed. For felled trees or logs, at least two other methods have been used. Chips of bark are sometimes cut out and measured, or diameter inside bark is measured directly by peeling off the bark before measurement. This latter technique is by far the most accurate and should be used for detailed mensurational studies of tree bark.

Bark Growth at Breast Height

Knowledge of bark growth at breast height is often needed in growth studies to estimate past tree diameters. Wood growth is readily obtained from increment cores but some indirect approach is needed to estimate bark growth. If bark growth is ignored and past diameter is obtained by subtraction of wood growth alone, diameter growth will be underestimated.

Two techniques have been used to estimate past diameters. At first glance they may appear to be different but both give the same results. The first uses as a starting point the regression of diameter inside bark over diameter outside bark. The second uses a regression of double bark thickness over diameter outside bark. Substitution of bark thickness for diameter inside bark is just a restatement of the same relation.

The first procedure uses a regression of the form

$$d_{pr} = b_o + b_1 D_{pr} \qquad \text{(Eq. 1)}$$

where:

d_{pr} = present diameter inside bark

D_{pr} = present diameter outside bark

Past diameter inside bark is found by subtracting diameter wood growth measured on increment cores from present diameter inside bark.

Or, $$d_{pa} = d_{pr} - Wg \qquad \text{(Eq. 2)}$$

where:

d_{pa} = past diameter inside bark

W_g = diameter wood growth

Then past diameter outside bark is calculated using the slope coefficient (b_1). Substitution of equation 1 into equation 2 gives

$$d_{pa} = b_o + b_1 D_{pr} - W_g \qquad \text{(Eq. 3)}$$

And equation 1 rewritten for past diameters is given by

$$d_{pa} = b_o + b_1 D_{pa}$$

Or,

$$D_{pa} = (d_{pa} - b_o) / b_1$$

Substituting for d_{pa} gives

$$D_{pa} = (\cancel{b_o} + b_1 D_{pr} - W_g - \cancel{b_o}) / b_1$$

Or,

$$D_{pa} = D_{pr} - W_g/b_1 \qquad \text{(Eq. 4)}$$

An estimate of bark thickness itself does not enter into the calculations but can easily be obtained from

$$BT = D_{pr} - d_{pr} \qquad \text{(Eq. 5)}$$

where:

BT = double bark thickness at breast height

A second procedure is to fit a regression of double bark thickness over diameter outside bark at breast height of the form

$$BT = b'_o + b'_1 D_{pr} \qquad \text{(eq. 6)}$$

The regression coefficient b'_1 is an estimate of the rate of bark growth or change in bark thickness per unit change in diameter outside bark. Thus,

$$B_g = b'_1 D_g$$

where:

B_g = bark growth

D_g = diameter growth outside bark

And,

$$D_g = W_g + b'_1 D_g$$

Therefore,

$$D_g = W_g / (1-b'_1) \qquad \text{(Eq. 7)}$$

And,

$$D_{pa} = D_{pr} - W_g / (1 - b'_1) \qquad \text{(Eq. 8)}$$

The two methods give the same results. Comparison of the two estimates of past diameter (equations 4 and 8) show that, for the coefficients:

$$b_1 = 1 - b'_1$$

Or,

$$b'_1 = 1 - b_1$$

Equations to estimate bark growth and past diameters at breast height have been prepared by many investigators. Some have used a regression of diameter inside bark over diameter outside bark (Finch 1948, McCormack 1955, Loetsch 1957, Myers 1958 and 1963). Others have preferred to use a regression of double bark thickness over diameter outside bark (Johnson 1955 and 1956, Maezawa 1956, Spada 1960, Kalinin 1966, and Powers 1969).

The importance of the bark growth correction varies with species. For the northern Rocky Mountains Finch (1948) found bark growth to vary from 17.5 percent of wood growth for western larch and 15.4 percent for Douglas fir to 3.1 percent for alpine fir and 1.4 percent for Engelmann spruce. A correction for the thin-barked fir and spruce may be relatively unimportant. If, however, the correction for the thick-barked species is ignored, diameter growth will be greatly underestimated. Spurr (1950) suggests that, in the absence of better information, a good rule of thumb is to consider bark growth to be 5 percent of wood growth for thin-barked species, 10 percent for trees with average bark thickness, and 15 percent for trees with exceptionally thick bark.

Effects of Site, Age, Latitude, and Genetic Variation on Bark Thickness

Few studies have dealt with the effects that differences in site, age, latitude, and genetic makeup of the species have on bark thickness. Schoenike (1963) gave an account of a German study in which Vanselow (1934) reported that bark thickness among 9 seed sources of Scotch pine varied from 9 to 16 percent of the outside bark diameter at 25 years of age. In terms of volume, 1 source had 15 percent more wood than the other for the same diameter and height. Schoenike examined the bark thickness and bark thickness--diameter outside bark at breast height relationship among 25 seed sources of 18-21-year-old jack pine in a plantation at Cloquet, Minnesota. He found that, although there was considerable tree-to-tree variation, bark thickness declined from south to north for a given tree diameter. In a Swedish study, Östlin (1963) found the same relationship for Scotch pine. From a study of 20,000 measurements from throughout Germany, Wiebermann (1934) found thin bark in all coastal regions and thick bark in all dry regions of eastern Germany.

The effect of site on bark thickness has been studied by Spurr (1950), Miller (1961), and Powers (1969). Spurr found from his study of 192 black spruce from the Superior National Forest in Minnesota that bark was thickest on the trees in the swamp sites, nearly as thick on trees from upland sites, and thinnest on trees from the lowland site.

Power's study of ponderosa pina in northern California and Miller's investigations of bark thickness in slash pine in Georgia also lend support to the proposition that bark tends to be thicker on poorer wet sites than on drier sites.

Bark thickness is also generally dependent upon tree age, being thickest on old trees. Hale (1955) found this to be true for five of the six species of trees he studied north of Lake Superior. The one exception was with 41-60-year-old white spruce which, because of rapid growth, had thicker bark than much older trees of the same diameter. In California, Pemberton (1924) also found that, for the same diameter, the older redwood had thinner bark than did young-growth redwood. For young-growth from 6 locations bark percent averaged 15.6 percent of the diameter outside bark at breast height while, for older trees, bark percent averaged 12.7 percent. Pemberton attributed the relative decline with age to abrasion over the years. Bark thickness of lodgepole pine is also known to vary with age. For a given site, the change in bark thickness with diameter is slightly greater in young than in old stands (Parker 1950).

It seems clear that many factors affect bark thickness within and between species. Because of this it is difficult and unwise to generalize on the effect that various factors have on bark development.

Upper Stem Bark Thickness

Few attempts have been made to develop equations for predicting upper stem bark thickness, bark factor,1/ or bark percent.2/ Most studies of bark thickness, mainly in European literature, have used graphic techniques to show trends in bark thickness from base to tip of the tree.

One of the first detailed studies of bark thickness in this country was conducted by Bruce and Rieneke (1931). The objective of their study was to find for shortleaf pine the factors giving the best indication of bark thickness and to incorporate them into an equation for predicting bark thickness at any point on the stem. Their first equation including eight significant variables was of the form

$$BT_{us} = b_o + b_1 S + b_2 D + b_3 H + b_4 A_D + b_5 d_I + b_6 h + b_7 h/H + b_8 A_s \quad \text{(Eq. 10)}$$

where:

BT_{us} = double bark thickness at point on the upper stem where bark thickness is to be determined

S = site index

D = diameter outside bark at breast height

H = total height

1/ Bark factor is the ratio of diameter inside bark to diameter outside bark.

2/ Bark percent is the ratio of double bark thickness to diameter outside bark. Bark percent = 1 - bark factor.

A_D = age at breast height

d_I = diameter inside bark at point on the upper stem where bark thickness is to be determined

h = distance up stem from ground

A_s = section age

The authors then removed the variables site, age at breast height, section age, and section height as these variables were considered to be difficult to measure. This left an equation of the form

$$BT_{us} = b_o + b_1D + b_2H + b_3d_I + b_4H/H \qquad \text{(Eq. 11)}$$

Although significant, the four eliminated variables actually contributed little toward increasing precision. Bruce and Rieneke also tried two other combinations of variables. They were:

$$BT_{us} = b_o + b_1D + b_2H + b_3h/H \qquad \text{(Eq. 12)}$$

and,

$$BT_{us} = b_o + b_1D + b_2J + b_3d_I \qquad \text{(eq. 13)}$$

Percentages of bark thickness variation accounted for were 42.5 for equation 10, 42.3 for equation 11, 41.0 for equation 12, and 27.5 for equation 13. Bruce and Rieneke concluded that equation 12 was adequate as it only required the measurement of the three easily measured variables, diameter at breast height, total height, and section height.

In recent years, investigators have applied graphic and regression techniques to develop procedures to estimate upper stem bark thickness, bark factors, or bark percentages. European investigators such as

Krastanov (1964), Vasilev, Andonov, and Taskov (1964), Bojanin (1966), and Kalinin (1966) constructed tables and graphs of double bark thickness for absolute or relative heights in the stem.

In this country Miller (1961) used regression to develop equations for estimating upper stem bark thickness of slash pine growing on different topographic sites in Georgia. Miller's equations were of the form:

$$BT_{us} = b_o + b_1 D + b_2 \log_{10} h \qquad \text{(Eq. 14)}$$

Johnson's (1966) study of Douglas-fir bark led him to the use of bark factor instead of bark thickness as the dependent variable. Many single variables and combinations of variables were considered. One of his final equations was of the form:

$$BF_{us} = b_o + b_1 A_T + b_2 D + b_3 (h/H)^2 + b_4 (h/H)(d_o/D) + b_5 BF_{LS} \qquad \text{(Eq. 15)}$$

where:

BF_{us} = upper stem bark factor

A_T = tree age

d_o = diameter outside bark at point on the upper stem where bark factor is to be determined

BF_{LS} = bark factor at stump

Johnson also developed two other forms of the equation. They were:

$$BF_{us} = b_o + b_1 A_T + b_2 h + b_3 (d_o/D) + b_4 (d_o/D)^2 + b_5 BF_{LS} \qquad \text{(Eq. 16)}$$

and,

$$BF_{us} = b_o + b_1h + b_2(d_o/D) + b_3(d_o/D)^2 + b_4BF_{LS} \quad \text{(eq. 17)}$$

Equation 16 lacks the variable total height and equation 17 the variables total height and age. Percentages of bark factor variation accounted for were 40 by equation 15, 37 by equation 16, and 27 by equation 17.

In Australia, Lawrence (1965) developed a multiple regression program for the I.C.T. 1301 computer. For an example to show how the program worked he chose to develop an equation to predict upper stem bark thickness of radiata pine growing in N.W. Tasmainia. For independent variables he chose only those variables capable of being measured by a dendrometer or by a climber not using a bark gauge. Lawrence developed his equation by a stepwise procedure by adding one variable or a function of it at a time. After each step he studied plots of the residuals over the independent variables before choosing a variable for the next step. Lawrence did not give the final solution in his paper, but from his discussion it appeared that his final equation was of the form:

$$BT_{us} = b_o + b_1D^2 + b_2(h/H) + b_3(H/h) + b_4H + b_5T_B \quad \text{(Eq. 18)}$$

where:

T_B = bark percent at breast height

Smith and Kozak (1967) studied bark thickness and bark percentage and their relation with various section characteristics for 19 species or species groups in British Columbia. They found diameter and bark thickness at breast height and tree age to be the best indicators of bark thickness. The best section variable was diameter outside bark. Age was not used, however, because of the difficulty of measuring it in mature trees.

Bark thickness is obviously related to many variables and combinations of variables as has been demonstrated by many investigators. What has not been considered, however, is that the equations developed so far do not meet certain basic conditions, the most important of which is that $BT_{us} = BT_D$ (double bark thickness at breast height), when $h = 4.5$. Johnson[3/] pointed this out early in this study and suggested, as a possible solution, the use of an equation of the form:

$$BT_{us} - BT_D = f(D, H, h, \text{ etc.})Z$$

where:

$$Z = h - 4.5$$

Thus, when $Z = O$, $BT_{us} - BT_D = O$, and $BT_{us} = BT_D$ as it should.

Another, but perhaps less important condition is that $BT_{us} = O$, when $h = H$. That is, at the tip of the tree. While it is true that normally bark thickness will not be estimated near the base or tip of the tree, it is somewhat disconcerting that these conditions are not generally met. In this paper equations will be developed which meet these constraints.

3/ Personal correspondence with Mr. Floyd Johnson of the Pacific Northwest Forest and Range Experiment Station, Forest Service, Portland, Oregon, on file at the Institute of Northern Forestry, P.O.Box 909, Juneau, Alaska.

Bark Volume

Studies of bark volume have been carried out by a number of investigators including Pemberton (1924), Ostlin (1930), Wiedermann (1932), Nilsen (1934), Parker (1950), Warner (1963), Kalinin (1966), Krier and River (1968), and Rymer (1968). Techniques have varied. Investigators have worked with bark volume percent as related to 1) diameter at breast height (Kalinin 1966 and Osanai 1968), 2) bark thickness percent at breast height (Rymer 1968), or 3) total height (Osanai 1968). Still others have preferred to develop bark volume equations (Warner and Goebel 1963).

Whenever a wide range of tree diameters is studied it is usually found that bark volume percent decreases with an increase in diameter (for example, Hale 1955 and Kalinin 1966). For some species and conditions, however, a single bark percentage correction has proved satisfactory. In one area of Australia, for radiata pine, bark percent for most purposes can be considered as being equal to 15 percent of the outside bark volume (Carron 1968). For black spruce on the Superior National Forest in Minnesota Spurr (1950) found no relationship between bark volume percent and diameter. Bark volume averaged 16 percent of total volume. And for young-growth redwood in California, Pemberton (1924) found bark volume to be 27 percent of total volume. Meyer (1946) also presented estimates of average bark volume percent for seven species in Pennsylvania and two in Venezuela, growing on selected average sites. Meyer recognized that the values would be different for different sites.

Average correction factors may be useful where there are small differences over a wide range of diameters, sites, and localities. For more precise estimates, corrections should be made for those factors which are strongly correlated with bark volume percent.

THE STUDY

Objectives

The study objectives were to:

1. Investigate the merits of using bark thickness versus bark factor or bark percent as a dependent variable.

2. Develop an equation to estimate past tree diameters.

3. Determine which variables best predict upper stem bark thickness.

4. Develop improved techniques for estimating upper stem bark thickness subject to conditions that $BT_{us} = BT_D$, when $h = 4.5$ and $BT_{us} = 0$, when $h = H$.

5. Develop equations and tables for predicting cubic-foot volumes inside and outside bark and bark volume.

Basic Data

The basic data consisted of measurements taken on 267 felled Sitka spruce from even-aged stands of Sitka spruce-western hemlock (Tsuga heterophylla (Raf.)Sarg.) located throughout southeast Alaska. Sample trees were generally measured at heights of 1 foot, d.b.h., and then at 8.15-foot intervals up the stem to a 4-inch top inside bark. Diameters at breast height ranged from 3.7 to 33.8 inches outside bark, and total height from 24 to 170 feet. Bark thickness at breast height averaged 0.61 inch. Breast height age ranged from 40 to 150 years.

ANALYSIS

Bark Thickness Versus Bark Percent

The observation that standard deviation of bark thickness is generally proportional to diameter has led some investigators (Vasilev et al 1964, Johnson 1966) to the use of bark percent (BP) as the dependent variable in place of bark thickness (BT). For Sitka spruce, neither bark thickness nor bark percent is truly adequate to use as a dependent variable. Variation in bark thickness increases greatly with increasing diameter and variation in bark percent decreases with increasing diameter (tables 1 and 2).

This can also be demonstrated in another way by comparing the relative efficiency of weighted versus unweighted regression for the breast height relationship of bark thickness to diameter outside bark. The two forms of the equation fitted were:

Unweighted: $BT_D = b_o + b_1 D$ (Eq. 18)

Weighted: $BP_D = (BT_D/D) = (b_o/D) + b_1$ (Eq. 19)

Use of equations 18 and 19 led to:

Unweighted: $BT_D = 0.356 + 0.01695D$ (Eq. 20)

with $SE_{y.x} = 0.11055$ and $r^2 = 0.42$

Weighted: $BP_D = (BT_D/D) = (0.320/D) + 0.01947$

with $SE_{y.x} = 0.007295$

The weighted form of the regression is rearranged in terms of BT_D by multiplying each term by D.

Table 1. Mean and standard deviation of double bark thickness and bark percent at breast height for young-growth Sitka spruce in southeast Alaska.

Range in Breast ht. Diameter	Sample Size	Double Bark Thickness		Bark Percent	
		Mean	Standard Deviation	Mean	Standard Deviation
Inches	Number	Inches	Inches		
5.0 - 6.9	3	.33	.057	5.0	.92
7.0 - 8.9	22	.48	.081	6.0	1.02
9.0 - 10.9	44	.53	.069	5.2	.71
11.0 - 12.9	34	.55	.074	4.7	.60
13.0 - 14.9	36	.61	.092	4.4	.69
15.0 - 16.9	32	.63	.076	4.0	.49
17.0 - 18.9	31	.69	.144	3.8	.80
19.0 - 20.9	14	.69	.151	3.4	.82
21.0 - 22.9	13	.72	.101	3.3	.46
23.0 - 24.9	9	.88	.211	3.7	.91
25.0 - 26.9	9	.71	.093	2.8	.35
27.0 - 28.9	5	.72	.164	2.6	.59
29.0 - 30.9	4	.85	.252	2.8	.76
31.0 - 32.9	1	.90	—	2.7	—

Table 2. Mean and standard deviation of upper stem double bark thickness and bark percent for young-growth Sitka spruce in southeast Alaska.

Range in Upper Stem Diameter	Sample Size	Double Bark Thickness		Bark Percent	
		Mean	Standard Deviation	Mean	Standard Deviation
Inches	Number	Inches	Inches	Inches	Inches
3.0 - 4.9	268	.32	.075	7.46	1.62
5.0 - 6.9	396	.38	.073	6.02	1.10
7.0 - 8.9	648	.44	.083	5.35	1.00
9.0 - 10.9	471	.48	.084	4.85	0.82
11.0 - 12.9	438	.52	.087	4.36	0.73
13.0 - 14.9	334	.56	.093	4.07	0.72
15.0 - 16.9	248	.59	.112	3.73	0.69
17.0 - 18.9	192	.63	.132	3.50	0.72
19.0 - 20.9	124	.67	.134	3.34	0.65
21.0 - 22.9	86	.69	.145	3.14	0.65
23.0 - 24.9	66	.72	.194	3.01	0.81
25.0 - 26.9	41	.75	.216	2.90	0.79
27.0 - 28.9	24	.80	.236	2.86	0.82
29.0 - 30.9	15	.83	.159	2.77	0.50
31.0 - 32.9	10	.76	.178	2.39	0.58
33.0 - 34.9	8	.91	.304	2.69	0.88
35.0 - 36.9	9	.76	.159	2.03	0.45

Or, $BT_D = 0.320 + 0.01947D$ (Eq. 21)

An estimate of the standard error of estimate for the rearranged weighted regression can be determined in the manner suggested by Furnival (1961) where the standard error of estimate ($wt.SE_{y \cdot x}$) of the rearranged weighted regression is:

$$wt.SE_{y \cdot x} = SE_{y \cdot x} \cdot \text{antilog}_{10} \overline{(\log_{10} D)} \qquad \text{(Eq. 22)}$$

Or, $wt.SE_{y \cdot x} = 0.007295 \cdot 14.2799$

$= 0.10418$

In terms of the variances, the relative efficiency of weighted versus unweighted regression is expressed as ? :

$$\text{Relative efficiency } E = \frac{SE^2_{y \cdot x}}{wt.SE^2_{y.x}} \qquad \text{(Eq. 23)}$$

Or, $E = \frac{(0.11055)^2}{(0.10416)^2} = 1.126$

From a statistical point of view there is in this case about a 13 percent gain in using weighted regression. Practically, however, weighting by $1/D^2$ has done little to equalize the variance throughout the range of diameters, and there is little difference between the two equations. The relationship of bark thickness to diameter outside bark thickness is weak. Only 42 percent of the variation in bark thickness is accounted for by the unweighted regression. The coefficient of determination (r^2) could not be retrieved when weighting was used as much of the variation associated with diameter was removed by weighting.

Additional tests were made to investigate the possibility of deriving a better weighting equation to equalize the variance. To do this the data were divided into equal diameter groups and the variance of each group calculated (table 1). A plot of variance over D suggested that some form of a polynomial in D would best estimate the relation between variance and D. The derived equation was:

$$\frac{1}{\text{wt.}} = \text{variance in BT} = -0.0609 + \frac{0.42295}{D} + 0.000297D^2 - 0.000007304D^3$$

Then weighted regression of BT_D over D, with wt. = (1 / variance in BT) led to: $BT_D = 0.341 + 0.01805D$ (Eq. 24)

with wt. $SE_{y \cdot x} = 0.098915$

and relative efficiency, $E = \frac{(1.110552)^2}{(0.098915)^2} = 1.249$

By using as weights a function that adequately describes the variance, it was possible to increase the efficiency. It seems doubtful, however, that there is much to be gained by doing this as variation in the data and sample size distribution could greatly influence the form of the weighting equation. Just because there has been a gain in efficiency does not imply that the newly developed equation for bark thickness is much or any better than the equation weighted by $1/D^2$, or the unweighted regression. Within the range of bark data, the maximum difference in estimated bark thickness is very small.

For a 30-inch tree the three equations (Eq. 20, 21, and 24) estimate bark thickness as 0.86-, 0.90-, and 0.88-inch, respectively. The difference is hardly worth further consideration. For this study bark thickness was chosen as the dependent variable except for the section which deals with prediction of bark volume.

Attempts to reduce bark thickness variation further by the addition of more independent variables such as breast height age, total height, elevation, and latitude, were unsuccessful. All additional variables were nonsignificant given the relationship:

$$BT_D = b_o + b_1 D.$$

Bark Growth at Breast Height

In the previous section on the choice of the dependent variable, three possible choices for the relationship between BT_D and D were presented. If equation 20 is selected as a reasonable equation to show this relationship, then the slope coefficient ($b_1 = 0.01695$) could be considered an indirect measure of the rate of bark growth or the change in bark per unit change in diameter outside bark at breast height. In equation form:

$$B_g = 0.01695 D_g \qquad \text{(Eq. 25)}$$

where:

B_g = bark growth at breast height

D_g = diameter growth at breast height

Then, using equations 7 and 8 (page 7) diameter growth and past tree diameter can be estimated as:

$$D_g = \frac{W_g}{1 - b_1}$$

or, $$D_g = \frac{W_g}{0.983}$$

and, $$D_{pa} = D_{pr} - \frac{W_g}{0.983} \qquad \text{(Eq. 26)}$$

Development of Upper Stem Bark Thickness Equations

1. Bark thickness with no constraints.

Many possibilities exist for the choice of independent variables to be used in an equation to predict upper stem bark thickness. There are the easily measured tree variables of breast height diameter (D), total height (H), and bark thickness at breast height (BT_D), plus those variables which characterize the position within the tree where bark thickness is to be estimated, including height above ground (h), height expressed as a percent of total height ($\frac{h-4.5}{H}$), and diameter outside bark at the upper stem point (d_o). In equation form:

$$BT_{us} = f(D, H, BT_D, h, \frac{h-4.5}{H}, d_o)$$

Analysis of the Sitka spruce data, using stepwise regression (program BMDO2R, Dixon 1968), indicated that the use of two or three independent variables was about as effective as many independent variables (table 3). The most important variables were BT_D and $\frac{h-4.5}{H}$. With a large number of observations such as the 3,383 we have for Sitka spruce, many significant variables will enter the regression. However, only a few will account for most of the variation.

Seventy-four percent of the bark thickness variation was accounted for by the equation:

$$BT_{us} = 0.245 + 0.619BT_D - 0.343\frac{h-4.5}{H} \qquad \text{(Eq. 27)}$$

with $SE_{y.x}$ = 0.0 781 or 15.45 percent of the mean bark thickness.

Table 3. Regression equation for estimation of upper stem bark thickness of Sitka spruce in southeast Alaska.

Constant	d_o 1/	BT_D 2/	D 3/	h 4/	H 5/	$\frac{h-4.5}{H}$	$SE_{y \cdot x}$	r^2 or R^2
Inches	Inches	Inches	Inches	Feet	Feet	Feet	Inches	
0.276	0.0194						0.1041	.54
0.088	0.0156	0.361					0.0883	.67
0.198	0.0062	0.524				-0.254	0.0748	.76
0.182	0.0087	0.540	-0.0025				0.0745	.76
0.141		0.565					0.1218	.37
0.246		0.619				-0.343	0.0781	.74
0.207		0.564			0.00072	-0.349	0.0763	.75
0.167		0.563		-0.00097	0.00110	-0.243	0.0760	.75

1/ d_o = diameter outside bark at point on the upper stem where bark thickness is to be determined.

2/ BT_D = double bark thickness at breast height.

3/ D = diameter outside bark at breast height.

4/ h = height above ground to point on upper stem where bark thickness is to be determined.

5/ H = total tree height.

Only an additional two percent of the variation was accounted for by adding the variable d_o. Addition of this variable gave the equation:

$$BT_{us} = 0.198 + 0.0062D_o + 0.524BT_D - 0.254\,\frac{h-4.5}{H} \qquad \text{(Eq. 28)}$$

with $SE_{y.x} = 0.0748$ or 14.79 percent of the mean bark thickness.

The addition of D, h, and H added little to the regression even though, because of the large amount of data, they were significant.

2. Bark thickness with one constraint

One objection to the use of equations 27 and 28 is that when solving the equation at breast height ($h = 4.5$), $BT_{us} \neq BT_D$. A solution to this problem is to develop an equation of the form:

$$BT_{us} - BT_D = f\left[(D,\ H,\ h,\ BT_D)Z + X\right] \qquad \text{(Eq. 29)}$$

where:

$$Z = h - 4.5$$

$$X = \frac{h-4.5}{H}$$

Rearrangement gives:

$$BT_{us} = f\left[(D,\ H,\ h,\ BT_D)Z + X\right] + BT_D$$

As with previous equations additive stepwise regression was used to fit equation 29. To meet the condition that $BT_{us} = BT_D$, when $h = 4.5$, the intercept was left out. This led to the following three regression steps:

step 1. $BT_{us} = BT_D - 47.0902(10^{-4})\ (BT_D)(Z)$

with $SE_{y.x} = 0.0877$

step 2. $BT_{us} = BT_D - 89.7669(10^{-4})(BT_D)(Z) + 25.4339(10^{-6})(H)(Z)$

with $SE_{y.x} = 0.0768$

step 3. $BT_{us} = BT_D - 77.5936(10^{-4})(BT_D)(Z) + 30.1277(10^{-6})(H)(Z)$

$$- 18.1283(10^{-2})(X) \qquad \text{(Eq. 30)}$$

with $SE_{y.x} = 0.0700$.

Addition of more significant variables added little to the reduction of variance.

3. Bark thickness with two constraints.

In the previous section an estimation equation was developed with the single constraint that $BT_{us} = BT_D$, when $h = 4.5$. If an additional constraint is added that $BT_{us} = 0$, when $h = H$, an equation of still greater complexity must be developed. The equation form used here is that of fitting polynomials similar to the procedure used by Bruce, Curtis, and Vancoevering (1968).

Initially the equation was written in the form:

$$\frac{BT_{us}}{BT_D} - 1 = b_1(x^{n_1} - 1) + b_2(x^{n_2} - 1) + \cdots + b_k(x^{n_k} - 1) \qquad \text{(Eq. 31)}$$

where:

$$x = \frac{H - h}{H - 4.5}$$

This form of the equation provided the condition that $\frac{BT_{us}}{BT_D} = 0$, when $x = 1$. The condition that $\frac{BT_{us}}{BT_D} = 0$, when $x - 0$, was met by adding weighted "observations" representing the tree tip.4/

Numerous trials were run. Data were grouped by tree size and a range of values of n_i were tested. Stepwise fitting of equation 31 led to the selection of the 1/20th, 2/5ths, and 15th powers of x to approximate the double bark thickness profile.

A fit of all data led to the equation:

$$\frac{BT_{us}}{BT_D} - 1 = 0.19021\ x^{1/20} + 0.75262\ x^{2/5} + 0.05699\ x^{15} \quad \text{(Eq. 32)}$$

Rearranged, this equation becomes:

$$BT_{us} = BT_D \left[0.19021\ x^{1/20} + 0.75262\ x^{2/5} + 0.05699\ x^{15} + 1\right]$$

with $SE_{y \cdot x} = 0.0700$.

Fit of this equation, as measured by Furnival's (1961) index of fit, was

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4/ From page 314, Bruce, Curtis, and Vancoevering 1968. "Note that once appropriate powers of x are determined, $\sum_1^k b_i = 1$, $b_1 = 1 - \sum_2^k b_i$, and these conditions are met by

$$(BT_{us}/BT_D) = 1{:}x^{n_1} + b_2(x^{n_2} - x^{n_1}) + b_3(x^{n_3} - x^{n_1}) + \cdots + b_i(x^{n_i} - x^{n_1}),$$

without further need for this procedure.

about the same as that of equation 30.

When interaction terms of D, H, and BT_D, and the differences of powers of x were added, all x^{n_i} were replaced except $x^{2/5}$. The resulting equation was of the form:

$$\frac{BT_{us}}{BT_D} - x^{2/5} = b_i(x^{n_i} - x^{n_j})(f(D,\ H,\ BT_D))$$

A fit of all data gave in rearranged form:

$$\begin{aligned} BT_{us} = BT_D\Big[x^{2/5} &- 7.249(10^{-1})(BT_D)(x^{1/20} - x^{2/5}) \\ &+2.221(10^{-1})(BT_D)(x^{15} - x^{2/5}) \\ &+4.929(10^{-2})(H^{1/2})(x^{1/20} - x^{2/5}) \\ &-7.705(10^{-3})(H^{1/2})(x^{15} - x^{2/5}) \\ &+1.322(10^{-5})(H^2)(x^{1/20} - x^{2/5}\Big] \end{aligned} \qquad \text{(Eq. 33)}$$

The estimated standard error using Furnival's index of fit was 0.0654. A number of alternative equations of slightly differing form gave nearly identical results.

For equations 30, 32, and 33 the significant independent variables were BT_D, h, H, and combinations of these variables. Diameter at breast height (D) did not appear to be important, probably because it is highly correlated with BT_D.

In terms of relative efficiency, equation 33 was superior to equations 27, 28, 30, and 32. A comparison of the equations is given below:

		Relative Efficiency
Equation (i)	$SE_{y.x}$	Eq. 33 vrs. Eq. (i)
27	0.0781	1.19
28	0.0748	1.14
30	0.0700	1.07
32	0.0700	1.07
33	0.0654	--

Bark Volume

Smalian's formula was used to calculate cubic-foot volume of sample trees both outside (V_O) and inside (V_I) bark. Volume was found between a 1-foot stump and a 4-inch top inside bark.

Bark volume (V_B) was calculated as the difference between volume outside and inside bark. To avoid differences in equation form and subsequent inconsistencies between equations for predicting wood and bark volumes, three equations of the same form were developed. One for volume outside bark, another for volume inside bark, and a third for bark volume.

Because standard error of volume is proportional to D^2H, weighted regression was used of the form:

$$\frac{V_i}{D^2H} = \frac{b'_o}{D^2H} + \frac{b'_1}{DH} + \frac{b'_2}{H} + \frac{b'_3}{D^2} + b'_4 + \frac{b'_5}{D^4H}$$

Stepwise fitting of the above equation led to the following rearranged equations:

$$V_O = 1.1785D - 0.106776D^2 - 0.3744H + 0.0028791D^2H - \frac{114.75}{D^2} \qquad \text{(Eq. 34)}$$

with $SE_{y.x} = 4.582$ or 6.46 percent of the mean volume outside bark.

$$V_I = 1.0041D - 0.094337D^2 - 0.4088H + 0.00267757D^2H - \frac{85.00}{D^2} \qquad \text{(Eq. 35)}$$

with $SE_{y.x}$ = 4.345 or 6.13 percent of the mean volume inside bark.

$$V_B = 0.1744D - 0.012439D^2 + 0.00344H + 0.0002016D^2H - \frac{29.75}{D^2} \qquad \text{(Eq. 36)}$$

with $SE_{y.x}$ = 0.685 or 12.69 percent of the mean bark volume.

Volume tables prepared from these equations are given on pages 36 to 38.

Bark volume percent.

Bark volume percent was found to be related to diameter and bark thickness at breast height. Bark volume percent ($\%V_B$) decreased with an increase in diameter as given by:

$$\%V_B = 12.982 - 0.2673D \qquad \text{(Eq. 37)}$$

with $SE_{y.x}$ = 1.356 or 15.21 percent of the mean bark volume percent, and $r^2 = 0.55$.

The addition of bark thickness at breast height led to the equation:

$$\%V_B = 10.122 - 0.4034D + 0.0883BT_D \qquad \text{(Eq. 38)}$$

with $SE_{y.x}$ = 1.027 or 11.52 percent of the mean bark volume percent, and $R^2 = 0.74$.

Table 4.--Cubic-foot tree volumes including bark (1-foot stump to 4-inch top d.i.b.) Smalian's formula, for Sitka spruce, southeast Alaska[1]

D.b.h. (D)[2]	Total height in feet (H)[3]																Basis: trees measured[4]
	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	
Inches																	Number
6	2.0	2.7	3.3	4.0	4.7												--
7	3.8	4.8	5.9	6.9	7.9	9.0											7
8	5.2	6.7	8.1	9.6	11.1	12.5	14.0										14
9	6.4	8.4	10.3	12.3	14.2	16.2	18.2	20.1									16
10	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5								20
11	8.4	11.5	14.6	17.8	20.9	24.0	27.1	30.2	33.3	36.4							24
12		13.1	16.8	20.6	24.4	28.1	31.9	35.7	39.5	43.2	47.0						15
13		14.6	19.1	23.5	28.0	32.5	37.0	41.5	46.0	50.5	55.0						21
14		16.1	21.3	26.6	31.9	37.1	42.4	47.7	52.9	58.2	63.5	68.7					15
15		17.6	23.7	29.8	35.9	42.0	48.1	54.2	60.3	66.4	72.5	78.6					18
16			26.1	33.0	40.0	47.0	54.0	61.0	68.0	75.0	82.0	89.0					16
17			28.5	36.5	44.4	52.3	60.3	68.2	76.2	84.1	92.1	100	108				13
18				40.0	48.9	57.9	66.8	75.8	84.8	93.7	103	112	121				17
19				43.6	53.7	63.7	73.7	83.7	93.7	104	114	124	134				10
20					58.6	69.7	80.9	92.0	103	114	125	137	148	159			3
21					63.7	76.0	88.3	101	113	125	138	150	162	175			12
22					68.9	82.5	96.1	110	123	137	150	164	177	191	205		6
23					74.4	89.3	104	119	134	149	164	178	193	208	223		5
24					80.0	96.3	112	129	145	161	177	194	210	226	242		6
25					85.9	104	121	139	156	174	192	209	227	244	262		5
26					91.9	111	130	149	168	187	206	226	245	264	283		2
27							139	160	181	201	222	242	263	284	304		4
28								171	193	216	238	260	282	304	326		3
29									206	230	254	278	302	326	350		--
30										246	271	297	322	348	373	399	3
31											289	316	343	370	398	425	1
32											307	336	365	394	423	452	--
33											325	356	387	418	449	480	--
34											344	377	410	443	476	509	1
35											364	399	434	469	504	538	--

1/ Based on weighted regression: $V = 1.1785D - 0.106776D^2 - 0.03744H + 0.0028791D^2H - \frac{114.48}{D^2}$. Standard error of estimate = 4.582 or 6.46% of the mean volume.

2/ 10-inch class includes trees 9.6 through 10.5 inches d.b.h.

3/ 90-foot class includes trees 86 through 95 feet in height.

4/ Number of trees; range of data for 257 trees enclosed by solid lines.

Table 5.--Cubic-foot volume of wood inside bark (1-foot stump to 4-inch top d.i.b.) Smalian's formula, for Sitka spruce, southeast Alaska[1/]

D.b.h. (D)[2/]	Total height in feet (H)[3/]																Basis: trees measured[4/]
	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	
Inches																	Number
6	1.9	2.5	3.0	3.6	4.2												--
7	3.4	4.3	5.2	6.1	7.0	7.9											7
8	4.6	5.9	7.2	8.5	9.8	11.1	12.4										14
9	5.6	7.4	9.1	10.9	12.7	14.4	16.2	17.9									16
10	6.6	8.8	11.1	13.4	15.6	17.9	20.2	22.4	24.7								20
11	7.4	10.3	13.1	15.9	18.7	21.6	24.4	27.2	30.1	32.9							24
12		11.7	15.1	18.6	22.0	25.4	28.9	32.3	35.8	39.2	42.7						15
13		13.1	17.2	21.3	25.4	29.5	33.7	37.8	41.9	46.0	50.1						21
14		14.5	19.3	24.2	29.0	33.8	38.7	43.5	48.4	53.2	58.0	62.9					15
15		15.9	21.5	27.2	32.8	38.4	44.0	49.6	55.2	60.8	66.5	72.1					18
16			23.8	30.3	36.7	43.1	49.6	56.0	62.5	68.9	75.4	81.8					16
17			26.2	33.5	40.8	48.1	55.5	62.8	70.1	77.5	84.8	92.1	99.5				13
18				36.8	45.1	53.4	61.6	69.9	78.2	86.4	94.7	103	111				17
19					49.6	58.8	68.1	77.4	86.6	95.9	105	114	124				10
20					54.2	64.5	74.8	85.1	95.5	106	116	126	137	147			3
21					59.1	70.5	81.9	93.3	105	116	128	139	150	162			12
22					64.1	76.7	89.2	102	114	127	139	152	164	177	190		6
23					69.3	83.1	96.8	111	124	138	152	166	179	193	207		5
24					74.7	89.7	105	120	135	150	165	180	195	210	225		6
25					80.3	96.6	113	129	146	162	178	195	211	227	244		5
26					86.1	104	121	139	157	174	192	210	228	245	263		2
27							130	149	168	188	207	226	245	264	283		4
28								160	180	201	222	242	263	283	304		3
29									193	215	237	259	281	303	326		--
30										229	253	277	300	324	348	372	3
31											270	295	320	346	371	396	1
32											287	314	341	368	395	422	--
33											304	333	362	390	419	448	--
34											322	353	383	414	444	475	1
35											341	373	405	438	470	503	--

1/ Based on weighted regression: $V = 1.0041D - 0.094337D^2 - 0.04088H + 0.0026776D^2H - \frac{85.00}{D^2}$. Standard error of estimate = 4.34 or 6.13% of the mean volume.

2/ 10-inch class includes trees 9.6 through 10.5 inches d.b.h.

3/ 90-foot class includes trees 86 through 95 feet in height.

4/ Number of trees; range of data for 257 trees enclosed by solid lines.

Table 6.--Cubic-foot volume of bark (1-foot stump to 4-inch top d.i.b.) Smalian's formula, for Sitka spruce, southeast Alaska[1]

D.b.h. (D)[2]	Total height in feet (H)[3]: 40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	Basis: trees measured[4]
Inches																	Number
6	.1	.2	.3	.4	.5												--
7	.4	.5	.7	.8	.9	1.1											7
8	.6	.8	1.0	1.1	1.3	1.4	1.6										14
9	.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2									16
10	.9	1.1	1.4	1.6	1.9	2.1	2.3	2.6	2.8								20
11	1.0	1.3	1.6	1.8	2.1	2.4	2.7	2.9	3.2	3.5							24
12		1.4	1.7	2.0	2.4	2.7	3.0	3.3	3.7	4.0	4.3						15
13		1.5	1.9	2.2	2.6	3.0	3.4	3.7	4.1	4.5	4.9						21
14		1.6	2.0	2.4	2.9	3.3	3.7	4.1	4.6	5.0	5.4	5.9					15
15		1.6	2.1	2.6	3.1	3.6	4.1	4.6	5.1	5.5	6.0	6.5					18
16			2.2	2.8	3.3	3.9	4.4	5.0	5.5	6.1	6.6	7.2					16
17			2.4	3.0	3.6	4.2	4.8	5.4	6.1	6.7	7.3	7.9	8.5				13
18				3.1	3.8	4.5	5.2	5.9	6.6	7.3	8.0	8.6	9.3				17
19					4.1	4.8	5.6	6.4	7.1	7.9	8.6	9.4	10.2				10
20					4.3	5.2	6.0	6.8	7.7	8.5	9.4	10.2	11.0	11.9			3
21					4.6	5.5	6.4	7.3	8.3	9.2	10.1	11.0	12.0	12.9			12
22					4.8	5.8	6.8	7.9	8.9	9.9	10.9	11.9	12.9	13.9	14.9		6
23					5.1	6.2	7.3	8.4	9.5	10.6	11.7	12.8	13.9	15.0	16.1		5
24					5.3	6.5	7.7	8.9	10.1	11.3	12.5	13.7	14.9	16.1	17.3		6
25					5.6	6.9	8.2	9.5	10.8	12.1	13.4	14.7	15.9	17.2	18.5		5
26					5.9	7.3	8.7	10.1	11.4	12.8	14.2	15.6	17.0	18.4	19.8		2
27							9.1	10.6	12.1	13.6	15.1	16.7	18.2	19.7	21.2		4
28								11.2	12.9	14.5	16.1	17.7	19.3	20.9	22.5		3
29									13.6	15.3	17.0	18.8	20.5	22.2	24.0		--
30										16.2	18.0	19.9	21.7	23.6	25.4	27.3	3
31											19.0	21.0	23.0	25.0	26.9	28.9	1
32											20.1	22.2	24.3	26.4	28.5	30.6	--
33											21.2	23.4	25.6	27.9	30.1	32.3	--
34											22.3	24.6	27.0	29.4	31.7	34.1	1
35											23.4	25.9	28.4	30.9	33.4	35.9	--

1/ Based on weighted regression: $V = 0.1744D - 0.012439D^2 + 0.00344H + 0.0002016D^2H - \frac{29.75}{D^2}$. Standard error of estimate = 0.68 or 12.69% of the mean volume.

2/ 10-inch class includes trees 9.6 through 10.5 inches d.b.h.

3/ 90-inch class includes trees 86 through 95 feet in height.

4/ Number of trees; range of data for 257 trees enclosed by solid lines.

DISCUSSION

Sitka spruce has thin bark. Single-bark thickness for this species within Alaska seldom exceeds 0.6 inch in young-growth stands, even on trees 30 or more inches in diameter. Bark thickness is fairly uniform around the stem and easily measured as its surface is scaly but not furrowed.

1. Bark thickness at breast height.

Bark thickness varies considerably from tree to tree, even within the same diameter class. Only 42 percent of the bark thickness variation at breast height was accounted for by the single variable -- diameter outside bark (equation 20, page 19). Additional variables such as breast height age, total height, elevation, and latitude were not significant given the relation:

$$BT_D = b_o + b_1 D$$

Because spruce has thin bark, no correction for bark growth is necessary for most growth studies. Even with no correction, the maximum error should be less than two percent of wood growth. This error is well within the limits of precision normally achieved where diameters are measured to the nearest 0.1 inch. Most tree species have bark much thicker than Sitka spruce. With thicker-barked species, failure to correct for bark growth could result in considerable bias.

2. Effects of age, elevation, latitude, and site.

No meaningful relationships were found between bark thickness and breast height age, elevation, or latitude. The influence of site was not evaluated in this study. If these factors do affect bark thickness of Sitka spruce it is possible that the normal variation in bark thickness could mask their effect unless a large number of very accurate measurements were taken and analyzed.

3. Prediction of upper stem bark thickness.

The major effort in this study was directed toward development of suitable equations to predict upper stem bark thickness. At first, many variables and combinations of variables were screened to evaluate their importance. Similar analyses have been made by Bruce and Rieneke (1931), Miller (1961), Lawrence (1965), Johnson (1966), and Smith and Kozak (1967). Results from these trials indicated that the most important variables for predicting upper stem bark thickness were bark thickness at breast height (BT_D), diameter outside bark at the point on the upper stem where bark thickness is to be determined (d_o), and section height (h) expressed as a percent of total height (H) expressed in the form $(\frac{h-4.5}{H})$. Upper stem diameter was later removed as it contributed little to the reduction in variance after the variables BT_D and $\frac{h-4.5}{H}$ were included in the prediction equation, and also because upper stem diameter is difficult to measure.

Two extentions of multiple regression were developed to meet the constraints that $BT_{us} = BT_D$, when $h = 4.5$, and $BT_{us} = 0$, when $h = H$. If BT_D is used as an independent variable, it is desirable that these two constraints be met even though we seldom would want to estimate bark thickness near breast height or near the tree's tip. Equation 30 (page 29) satisfies the first constraint and equation 33 (page 31) satisfies both constraints.

For most purposes an equation as complex as equation 33 is unnecessary. This equation best conforms to certain desired constraints and has the lowest standard error. However, because of the number and complexity of terms, the equation may fit this particular set of data better than the others, but for new observations may provide no better estimates than an equation of simpler form.

Equation 30, whose general form (equation 29) was first suggested by Johnson[5/] seems in many ways to be superior in that 1) it meets the basic criticisms of equations developed previously in that the constraint that $BT_{us} = BT_D$, when $h = H$ is satisfied, 2) its index of fit (Furnival 1961) is similar to that of the more complex equation 33 (table 3, page 27), and 3) it is much easier to develop.

5/ See footnote 3, page 15.

4. Use of a constant for bark percent and bark volume percent.

In the absence of better information, constants are generally used for bark percent and bark volume percent. For some species and conditions this may be adequate. Pemberton (1924), Spurr (1950), and Carron (1968) reported that a single bark volume percent would be adequate for the species and conditions they studied. For most species and conditions, however, additional corrections should be made for diameter and any other factors that prove to be important. For Sitka spruce, bark percent (table 2, page 21) and bark volume percent (equation 37, page 35) decreased with an increase in diameter. Bark volume percent in 5-, 15-, and 30-inch trees is about 11.6, 9.0, and 5.0 percent, respectively. So, even with a thin-barked species, there can be a considerable range in the correction for bark.

When measuring the upper stem diameter of standing trees, there is little need for outside bark measurements to an accuracy of 0.1 inch unless there is some assurance that a correction for bark thickness is of similar accuracy or unless repeated measurements are to be made over time on the sample trees. This is less of a problem with thin- than with thick-barked species but it should be considered when there is a wide choice and cost of instruments for doing the job. For the spruce in this study, double bark thickness ranged from an average of about 0.3 inch for a diameter of 4 inches to about 0.8 inch for a diameter of 30 inches. Bark percent for the same diameters averaged 7.5 and 2.8, respectively (table 2, page 21).

CONCLUSION

This paper summarizes existing literature on estimation techniques for bark growth, bark thickness, and bark volume. In addition two new equation forms are developed for predicting upper stem bark thickness subject to constraints that, at breast height, predicted bark thickness equals actual bark thickness and that, at the tip of the tree, predicted bark thickness equals zero. The methods developed here should be applicable to other species. Bark weight, another important variable, particularly when estimates of bark production are desired, was not discussed. Little is currently known about this topic.

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