research bulletin 31b october 1980

TREES: timber resource economic estimation system

volume II mathematical analysis and policy guide

James S. Schmidt
Philip L. Tedder

FOREST RESEARCH LAB

Oregon State University School Of Forestry Corvallis, Oregon 97331

Since 1941, the Forest Research Laboratory—part of the School of Forestry at Oregon State University in Corvallis—has been studying forests and why they are like they are. A staff of more than 50 scientists conducts research to provide information for wise public and private decisions on managing and using Oregon's forest resources and operating its wood-using industries. Because of this research, Oregon's forests now yield more in the way of wood products, water, forage, wildlife, and recreation. Wood products are harvested, processed, and used more efficiently. Employment, productivity, and profitability in industries dependent on forests also have been strengthened. And this research has helped Oregon to maintain a quality environment for its people.

Much research is done right in the Laboratory's facilities on the campus. But field experiments in forest genetics, young-growth management, forest hydrology, harvesting methods, and reforestation are conducted on 12,000 acres of School forests adjacent to the campus and on lands of public and private cooperating agencies throughout the Pacific Northwest.

With these publications, the Forest Research Laboratory supplies the results of its research to forest land owners and managers, to manufacturers and users of forest products, to leaders of government and industry, and to the general public.

As a **research bulletin**, this publication is one of a series that comprehensively and in detail discusses a long, complex study or summarizes available information on a topic.

the authors

James S. Schmidt, formerly a forest economics consultant, is now a forest economist for the U.S. Forest Service, Mt. Baker-Snoqualmie National Forest, Seattle, Washington. Philip L. Tedder is Assistant Professor, Department of Forest Management, School of Forestry, Oregon State University, Corvallis.

disclaimer

This project was accomplished through partial funding by the Pacific Northwest Regional Commission. The statements findings, conclusions, recommendations and other data are solely those of the Grantee, and do not necessarily reflect the view of the Pacific Northwest Regional Commission.

acknowledgments

We extend our thanks to the Oregon State University School of Forestry for its multifaceted support of this project—in particular, to Carol Small who singlehandedly edited the four TREES manuals.

to order copies

The TREES manuals may be ordered from

Forestry Accounting
School of Forestry
Oregon State University
Corvallis, Oreg. 97331

at $6 per manual prepaid (including postage and handling). Contact P. L. Tedder directly for information on the TREES tape and a source listing. Copies of *Timber for Oregon's Tomorrow* (Beuter et al. 1976), Res. Bull. 19, as well as other Forest Research Laboratory publications, are available from

Forest Research Laboratory
School of Forestry
Oregon State University
Corvallis, Oreg. 97331

Please include author(s), title, and publication number if known.

contents

3 **Preface**
3 **Introduction**
4 **Harvest Scheduling Methods**
4 Absolute Amount
- 4 applying the method
- 4 policy considerations

5 Percent of Inventory
- 5 applying the method
- 5 policy considerations

6 Area Control
- 6 applying the method
- 6 policy considerations

7 Even-Flow of Volume (EFV)
- 8 applying simple EFV (AD)
- 11 applying simple EFV (DD)
- 14 policy considerations: the simple case
- 15 applying sequential EFV (AD and DD)
- 15 policy considerations: the sequential case

16 Even-Flow of a Function of Volume (EFFV)
- 16 applying simple EFFV (AD)
- 17 applying simple EFFV (DD)
- 17 applying sequential EFFV (AD and DD)
- 17 policy considerations: simple and sequential cases

18 Present Net Worth (PNW)
- 21 solution methods: optimality and feasibility
- 24 stability characteristics
- 28 optimality re-examined
- 30 linkage equation with a changing demand intercept
- 31 linkage equation with age-dependent harvest price and costs
- 33 cultural treatment costs
- 33 exogenous harvests
- 34 incorporating productivity classes
- 35 ending-condition checks
- 35 policy considerations

35 Present Net Benefit (PNB)
- 35 applying the method
- 36 policy considerations

38 **Growth Algorithms**
38 AD Options
- 38 standard yield
- 38 approach to normality
- 40 growth after thinning

41 DD Options
- 41 mortality
- 42 diameter growth
- 43 ingrowth and upgrowth

51 **Literature Cited**
52 **Appendix A—Detailed Algorithms for the Variable Harvest Scheduling Methods**
52 Variable Definitions
53 EFV and EFFV
54 PNW and PNB
56 **Appendix B—Optimization Detail Report**
56 Fixed Method Report
56 Even-Flow Report
57 PNW and PNB Report
59 **Appendix C—Quadratic Interpolation Routine for PNW and PNB Algorithms**
63 **Appendix D—Equation Limits**

preface

TREES (Timber Resource Economic Estimation System) is a forest management and harvest scheduling simulation model designed to predict future timber harvest volumes and the ensuing effects on forest inventory. The system can respond to a wide variety of management problems at a reasonable cost from the national to local woodlot level.

The model was developed by K. Norman Johnson, H. Lynn Scheurman, and John H. Beuter to provide a means of answering questions about future timber harvests in Oregon and resulting impacts. Their findings are reported in *Timber for Oregon's Tomorrow, An Analysis of Reasonably Possible Occurrences* (1976), familiarly known as the Oregon Timber Study.

The TREES package—a set of four manuals—contains explicit information on all aspects of the system:

TREES—Vol. 1: A User's Manual for Forest Management and Harvest Scheduling

TREES—Vol. II: Mathematical Analysis and Policy Guide

TREES—Vol. III: Example Problem Guide

TREES—Vol. IV: Computer Analyst's Guide

The **User's Manual** and **Math/Policy Guide** not only supply complete operating instructions but also, for the first time, present detailed analyses of modern forest management techniques, harvest scheduling methods, and interaction of the two.

introduction

This **Mathematical Analysis and Policy Guide** reveals the mathematical relationships underlying the harvest scheduling methods, growth algorithms, and special functions in the TREES model.

Algorithm steps are presented for each of the fixed harvest scheduling methods (absolute amount, percent of inventory, and area control) and for the more complex variable methods (even-flow of volume, even-flow of a function of volume, present net worth, and present net benefit), which require multiple iterations to set the harvest level. Consideration of the effects of each method on key harvest policy variables, including ending inventory, harvest volume flow, acres cut, and cash flow, make this manual an invaluable guide for forest managers and resource analysts both to more accurately determine which method(s) meet policy goals and to avoid unnecessary and expensive computer runs.

Growth algorithms are explained to show how the user-specified yield values (for even-aged inventories) and growth rates (for uneven-aged inventories) are handled in the model. For both approaches, explicit detail is provided in order that the user may fully understand the complex relationships that mortality, ingrowth, upgrowth, and basal area have within the growth routines. Growth options for the period after thinning in the even-aged approach are detailed, and the uneven-aged approach is contrasted with the nonlinear interpolation and big Q methods of TRAS (Larson and Goforth 1974).

The appendices will be especially useful to the computer specialist. Algorithm steps for the variable harvest scheduling methods are expressed in terms of computer program variables. Program variables also are defined for the optimization detail, available in three distinct reports. Also detailed are routines for quadratic interpolation, which is used by the PNW and PNB methods to smooth the transition between discontinuous yield-table values and equilibrium conditions, and for setting the equation limits that produce the internal program table values used in the model.

The **Math/Policy Guide**, in conjunction with the other three TREES volumes, will enhance users' understanding of the structure of TREES and increase their facility in running the model.

harvest scheduling methods

TREES incorporates both fixed and variable harvest scheduling methods (see the harvest scheduling methods sections, **User's Manual**).

The three fixed methods—**absolute amount, percent** of **inventory,** and **area control**—are those for which entries in the allowable cut (ACC) file predetermine harvest levels. Absolute amount and percent of inventory may be applied to either even-aged (AD) or uneven-aged (DD) stands; area control applies to AD only. These methods do not use an iterative search to find the harvest level. No optimization occurs.

The four variable methods—**even-flow of volume, even-flow of a function of volume, present net benefit (PNB)**, and **present net worth (PNW)**—use a multiple iteration search to set the harvest level. The even-flow methods may be used for either AD or DD stands; however, PNB and PNW apply to AD only.

Absolute Amount

applying the method

The user selects a harvest priority (oldest age first, maximum value, or minimum growth) and requests N volume to be harvested for each period the method is in effect. (Harvest priorities are described in Appendix C, **User's Manual**, and in later sections of this volume.) The algorithm proceeds:

Step 1. Check the requested volume for feasibility. If the requested volume is less than the exogenous harvest, take the exogenous harvest; if it is greater than the volume available, take the volume available according to the harvest priority chosen. If neither, take the volume consistent with the selected harvest priority.

Step 2. Check the period. If the last period, stop. If not, grow the inventory and go to Step 1.

Exogenous harvests are thinnings, mortality salvage, and species conversion and will always be taken if user specified.

policy considerations

Absolute amount offers extreme flexibility for achieving a given harvest policy. Its application is simple but predicting its effects difficult.

For a nondeclining, even-flow harvest, absolute amount will be successful for the planning horizon if current and future inventories can fill the volume requested. If the forest is not initially regulated[1], acres harvested each period may fluctuate dramatically. Additionally, unless the volume requested can be sustained by the forest when regulated at a desired rotation age or diameter-class distribution, inventory will either be depleted or continue to become regulated at a rotation age or diameter distribution other than that desired. If the volume requested can be sustained for the planning horizon to just equal growth, regulation will occur, with the volume harvested each period equal to growth. The time required to achieve regulation will depend on how close the forest initially is to this goal. If the forest is irregular in stocking or

[1] For AD forests, regulation occurs when equal numbers of acres are in every age class in a given productivity class, with the oldest age class at the desired harvest age. For DD forests, regulation theoretically occurs when the ideal diameter-class distribution has been reached.

age, the conversion period may be several rotations long.

When applied to a nondeclining even-flow harvest, absolute amount is insensitive to economic conditions. For example, if mill demand or capacity is balanced with harvest, economic stability in an area could be reasonably assured. But if the real price of timber were rising and the absolute volume requested caused excess inventory buildup, resulting low stumpage prices would probably attract new mills. Forest inventories would be pressed to produce even greater harvests. In this example, absolute amount is actually the same as even-flow of volume (EFV), except that no ending condition can be specified.

Where volume requests are calculated by an exogenous method not directly associated with the forest, such as econometric analysis, future supply-and-demand equilibrium points could be calculated and those volume levels entered as absolute requests.

Absolute Amount Highlights

- Simple to execute, but its effects on the forest are difficult to project.
- Would not be affected by adding or deleting inventory unless the amount requested were not achievable.
- Regulation may occur but at an unknown time.
- Ending conditions cannot be specified.
- Volume harvested is insensitive to economic conditions.

Percent of Inventory

applying the method

The user requests a proportion of the available inventory to be harvested each period. This proportion may be constant or a linear function of the period, with minimum and maximum limits. The algorithm proceeds:

Step 1. Calculate the proportion of available inventory to be harvested in the period.

Step 2. Multiply that proportion by the inventory above minimum age class[2] or diameter class for harvest to arrive at a volume request.

Step 3. Check for feasibility. If the volume requested is less than the exogenous harvest, take the exogenous harvest; if it is greater than the harvest volume available, take the volume available according to the harvest priority chosen. If neither, take the volume requested consistent with the selected harvest priority.

Step 4. Check the period. If the last period, stop. If not, grow the inventory and go to Step 1.

policy considerations

Percent of inventory ties harvest volume to the physical inventory on hand for a specific time period. Although applying percent of inventory is straightforward, evaluating its long-term impact is difficult.

The percent of inventory to be harvested is critical for determining at which rotation age or diameter-class distribution the forest will become regulated. The lower the percentage specified, the larger the resulting rotation age (assuming, of course, that the proportion harvested remains constant). The proportion may be held constant or varied, but when the proportion is varied, ending inventory is uncertain. Therefore, final inventory must be examined thoroughly to determine its desirability. Adding or deleting inventory or intensifying management practices will certainly affect future volumes harvested.

Percent of inventory will probably result in either even-flow of volume or gently rising (or falling) harvest flows with an even-flow ending. No ending conditions apply. Like absolute amount, percent of inventory is insensitive to economic conditions.

Percent of Inventory Highlights

- Simple to specify, but long-term effects are difficult to assess.
- Percentage may be constant or variable.
- Adding or deleting inventory will immediately change harvest volume.
- Acres harvested may fluctuate.
- Ending inventories are not considered.
- Regulation will occur eventually but at an unknown time if a constant percentage is used.
- Volume harvested is insensitive to economic conditions.

[2] For shelterwood, available volume includes residual volumes from various shelterwood cuts plus the volume harvested at first entry of uncut stands.

Area Control

applying the method

The user first selects a desired rotation length (in periods). The algorithm proceeds:

Step 1. Divide total number of acres available for harvest by rotation length to determine the cut.

Step 2. Calculate harvest volume by determining the volume available if acres are harvested on an oldest-first priority; add the exogenous harvest volume.

Step 3. Allocate the harvest according to the priority chosen.

Step 4. Check the period. If the last period, stop. If not, grow the inventory and go to Step 1.

Area control is designed to achieve forest regulation as rapidly as possible. A forest so structured will yield equal harvests every period. However, this method may *not* produce perfect regulation because:

- Acres may be shifted into or out of the inventory over time, causing each period's harvest to vary.

- Many productivity classes may be combined into one harvest (allowable cut) unit. Even if the same number of acres exists in every age class, age classes may be distributed among different sites. Harvest volumes will fluctuate as acres of differing productivity are cut.

policy considerations

Area control, one of the oldest, simplest types of harvest scheduling methods, is designed to regulate the AD forest in a specific time period (see footnote 1; also, Davis 1966). Stated differently: Area control creates areas of equal physical productivity in each age class after a conversion period.

The number of acres to be harvested each period is determined by dividing total available acres by prespecified rotation age. If inventory remains unchanged, the forest will convert to a fully regulated state in the time required for newly regenerated acres to reach rotation age. A harvest priority is then determined and acres are cut. To ensure constant harvest levels in the post-conversion period, the average productivity of acres harvested should be the same each period.

Area control is simple as long as inventory remains constant; however, adding or subtracting acres means that regulation must start all over again. For example, if a large section of forest (under area control) were sold, achieving regulation would require a new conversion period the length of one rotation age. Similarly, changing management intensity (MI) during the conversion period will cause productivity differences among age classes and changes in volume flows from future harvests.

It is vitally important to consider the volume flow of timber needed during conversion and after regulation. If the stocking levels of unconverted inventory vary greatly, harvesting the same number of acres each year will cause the conversion-period harvest to fluctuate wildly. Rotation age determines volume production in the post-conversion period. Rotation ages less than or greater than the age at which volume per acre divided by age culminates result in reduced post-conversion harvest.[3]

Fluctuating harvest-volume flow during conversion goes hand-in-hand with fluctuating net revenue. Because area control is insensitive to economic conditions, using this harvest scheduling method on large parcels could aggravate the local economic situation depending on the prevailing economic climate.

Area control may be most useful when a forest is almost regulated at the predetermined rotation age. Harvest-volume fluctuations will be minimized, although the problem of economic insensitivity will remain.

Area Control Highlights

- Simple to execute.
- Equal areas harvested each period.
- Achieves regulation in a specified time period.
- Needs a specified conversion period (rotation age).
- Regulation must start over after adding or deleting inventory.
- Fluctuating harvest volumes will probably yield fluctuating revenues during conversion.
- Volume harvested is insensitive to economic conditions.

[3]Rotation ages with growth rates less than or greater than the discount rate may reduce post-conversion economic returns.

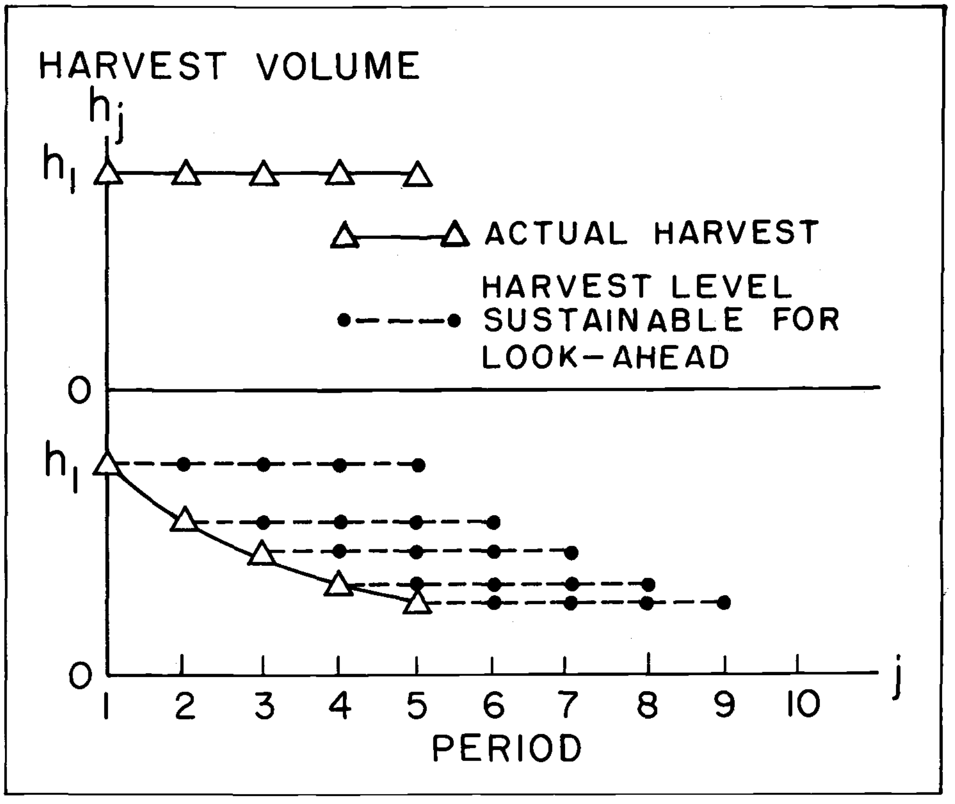


Figure 1.

Simple and sequential EFV harvest schedules.

Even-Flow of Volume (EFV)

EFV may be either simple or sequential.

Simple EFV is patterned after the SIMAC technique used by the Bureau of Land Management (Sassaman et al. 1972). The algorithm searches for the highest EFV harvest sustainable over a specified look-ahead period (the inner or optimization cycle) compliant with the ending condition selected. The harvest level found is the harvest taken in *each* period of the simulation.

Sequential EFV, patterned after the SORAC technique (Chappelle and Sassaman 1968), solves the simple EFV problem starting anew each simulation period. The algorithm searches for the highest EFV sustainable over a specified look-ahead compliant with the ending condition imposed. But the harvest level found is the harvest taken in the *first period of the look-ahead only.* After first-period harvest, the remaining inventory is adjusted for growth and the highest EFV harvest again calculated. The new (adjusted) inventory is the starting point for the next look-ahead, which extends one period further into the future. Sequential EFV continues for the number of periods specified (the outer or number-of-optimizations cycle).

Simple and sequential EFV schedules are compared in Figure 1. The declining sequential EFV schedule indicates a forest with excess inventory; conversely, a rising sequential EFV would indicate an understocked forest.

applying simple EFV (AD)

The simple EFV problem may be formulated as a linear program (LP) for AD stands (Johnson and Scheurman 1977). To simplify notation, assume a single site class, species type, stocking level, and MI; instantaneous regeneration; and no thinning.

Let

x_{ij} = acres harvested from age class i in period j, where i is the period of stand establishment. For a given acre, i changes only at harvest.

V_{ij} = volume per acre, age class i, period j (changes with the period).

h_j = total harvest, period j

$$= \sum_{i=-M}^{j-Z} V_{ij} x_{ij}.$$

N = number of periods in the look-ahead.

Z = number of periods before regenerated stands are eligible for harvest.

-M = period of stand establishment for the oldest age class relative to the start of the simulation. The minus indicates that the stand was established M periods *before* the simulation began.

A_i = number of acres in age class i at the start of the simulation.

The objective function is:

$$\text{Maximize} \sum_{i=-M}^{j-Z} V_{ij} x_{ij} \qquad [1]$$

or, equivalently, maximize h_1 (first-period harvest).

Constraints on the objective function are:

$$\sum_{j=1}^{N} x_{ij} \leq A_i \qquad [2]$$

$i = -M, \ldots, 0.$

Acres harvested from an age class existing at the start of the planning period cannot exceed the acres originally in that age class.

$$\sum_{k=j+Z}^{N} x_{jk} \leq \sum_{i=-M}^{j-Z} x_{ij} \qquad [3]$$

$j = 1, 2, \ldots, N.$

Acres harvested from the age class (x_{jk}) created by harvest in year j cannot exceed the acres harvested in year j (x_{ij}).

$$h_j = h_1 \qquad [4]$$

$j = 2, \ldots, N.$

Harvest in all periods must equal the first-period harvest.

$$x_{ij} \geq 0 \qquad [5]$$

$i = -M, \ldots, j-Z$
$j = 1, \ldots, N.$

Acres harvested cannot be negative.

In addition to the preceding constraints, the user may impose either of two ending conditions:[4]

$$h_N = \sum_{i=-M}^{0} (A_i - \sum_{j=1}^{N-1} x_{ij})(V_{iN} - V_{i,N-1}) + \sum_{j=1}^{N-1} (\sum_{i=-M}^{j-Z} x_{ij} - \sum_{k=j+Z}^{N-1} x_{jk})(V_{jN} - V_{j,N-1}). \qquad [6]$$

Harvest in the last period of the look-ahead (h_N) equals growth since the previous harvest, both on the original inventory remaining uncut and on the age classes created through harvesting.

$$h_N = \sum_{i=-M}^{0} (A_i - \sum_{j=1}^{N-1} x_{ij})V_{iN} + \sum_{j=1}^{N-R} (\sum_{i=-M}^{j-Z} x_{ij} - \sum_{k=j+Z}^{N-1} x_{jk})V_{jN}. \qquad [7]$$

Harvest in the last period of the look-ahead (h_N) equals the volume in all age classes above age class R in period N. If R is less than N, all acres will be cut over at least once during the look-ahead, and the first term of Eq. [7] will drop out.

Solution methods: optimality and feasibility. As formulated, the simple EFV problem could be solved with an LP algorithm, which simultaneously determines harvest level and harvest priority. The EFV algorithm in TREES, on the other hand, requires selecting a harvest priority *before* solving for the harvest level. One of three harvest priorities (oldest age first, maximum value, or minimum growth) may be chosen. Preselecting the harvest priority greatly simplifies finding the highest EFV because harvest levels are varied only until the highest level meeting all constraints is found. The algorithm used in subroutine ACSTHV (see the **Analyst's Guide**) proceeds:

Step 1. Set the first-period harvest level (h_1).

Step 2. Check the harvest level to ensure it is greater than or equal to 0 and less than or equal to the total harvest volume available. If so, go to Step 3. If not, go to Step 1 and adjust the first-period harvest level up or down.

Step 3. Allocate the harvest according to the chosen priority.

Step 4. Check the period. If the last period, go to Step 5. If not, grow the inventory, set next-period harvest equal to first-period harvest, and go to Step 2.

Step 5. Check the ending condition. If satisfied, stop. If not, go to Step 6.

Step 6. Check the range of possible first-period harvests. If the difference between the last trial harvest level that proved too high and the last trial harvest level that proved too low has become too small, stop—assume the problem has no feasible solution. If not, go to Step 1 and adjust the first-period harvest level.

(See Appendix A for more detailed descriptions of simple and sequential EFV algorithms; see Appendix B for a discussion of the optimization detail report which aids interpretation of the iterative search.)

Imposing a harvest priority adds a further constraint. At best, the EFV algorithm in TREES can produce harvest levels as high as those produced by the LP solution, which has no such constraint. But EFV differs from LP to the degree that the harvest priority chosen is not optimal (i.e., is not the priority that would have resulted from solving the LP). For the simple EFV problem (formu-

[4]If no ending condition is imposed, the EFV algorithm would not try to maximize harvest level but would choose the first feasible even-flow harvest level sustainable for the N-period look-ahead.

lated in Eqs. [1] to [5]), an oldest-first priority should prove optimal if increases in age yield decreasing growth rates. For more complex problems, where parameters such as multiple site classes and stocking levels introduce variations in volume-to-age relationships, simple harvest priorities such as those available in TREES may prove suboptimal. Although growth rates of current stands and stands replacing current stands affect the optimal harvest priority, strong practical arguments can still be made for choosing EFV over LP. LP algorithms are limited in the complexity of the problem they can solve. Adding multiple site classes, species types, stocking levels, and MIs introduces considerable complexity and may rapidly outstrip the LP's capacity or cause unacceptable run costs. The EFV algorithm can handle this with relative ease and at a substantially lower cost.

TREES can simultaneously consider up to 500 grouped resource units (GRUs). Each GRU may include up to 33 age classes, 3 stocking levels, and 7 MIs. Theoretically, nearly 350,000 separate classes of timber could be eligible for harvest in a period.

To eliminate individually assigning harvest priorities to all those timber classes, the subroutine ACALAD averages age-class volumes, net values, and growth for all GRUs into 99 composite classes (33 age classes x 3 species report-groups). The subroutine ACALAG assigns the oldest-first priority; the subroutine ACALGD assigns maximum-growth or minimum-value priorities. Given a harvest level and harvest priority, these subroutines determine the proportion cut from each AD composite class. The subroutine RUHRVA applies these proportions to the corresponding age classes in individual GRUs irrespective of stocking level and MI. (See the **Analyst's Guide** for subroutine details.)

Of course, amalgamating age classes in this fashion limits the ability to order harvests among or within individual GRUs. Consequently, the harvest priority used in the EFV algorithm will probably not be strictly optimal for complex inventories. The potential EFV will be understated, although underestimation may not be significant because, in practice, harvests rarely conform strictly to any priority specified or solved for in a harvest scheduling model.

In general, the EFV algorithm will iterate until a feasible solution is found. However, certain combinations of inventory and ending conditions can make solution impossible. Large gaps in age-class distribution commonly cause such problems. For example, if the starting inventory contains a preponderance of age classes too young for harvest, the even-flow harvest will be constrained to a low level. Low harvest levels allowing inventories to increase over time could make it impossible for harvest to equal growth by the end of the look-ahead.

Exogenous harvests. Thinning, species conversion, and mortality salvage may be incorporated into the EFV algorithm but only as exogenous harvests. How those harvests will take place must be specified before the solution process begins. Step 2 of the algorithm must be revised to ensure that trial harvest levels are at least as high as the exogenous harvest for the period—this is equivalent to adding a new constraint.

Let

EX_j = exogenous harvest volume for period j.

The added constraint would be:

$$h_j \geq EX_j \qquad [8]$$

$$j = 1, \ldots, N.$$

Ending conditions must also be appropriately modified.

To treat thinning as an exogenous harvest, we assume that the specified thinning regime is optimal for volume production and that volume from thinning is as desirable as volume from final harvest. But solving for the optimal thinning regime and harvest level requires mathematical programming techniques or iterating with the EFV method using different thinning regimes to approximate the optimal regime.

Difference tolerances. Even-flow and ending-condition constraints may be modified to allow harvest levels that almost meet these constraints. This *difference tolerance* is defined by multiplying a specified proportion (PVLDTP in the ACC file) by the first-period harvest level (h_1). The algorithm will consider harvest levels within $h_1 \pm$ (PVLDTP x h_1) sufficiently close to the even-flow level. Likewise, it will consider harvest levels within $\pm$ (PVLDTP x h_1) of the volume required to meet the ending condition to have met that constraint. Steps 2 and 5 of the algorithm must be modified accordingly.

Choosing the look-ahead period and ending condition. Long-run sustained yield (LRSY) of volume production occurs when each distinct productivity class has an equal number of acres in each age class and when the harvest age of each age class is at maximum mean annual increment (MAI). MAI is volume per acre divided by stand age. If LRSY does not equal maximum obtainable MAI, the forest manager must decide how best to convert to the ideal structure. On public lands in the western United States, where the EFV algorithm technique has most frequently been used, the problem is usually one of converting from old-growth inventory with high volumes per acre to younger inventory with lower volumes per acre. Length of the conversion period, harvest volume in the conversion period, and post-conversion harvests are variables that must be considered.

If volumes per acre and productivity are relatively uniform, then simply taking the maximum even-flow harvest over the length of one maximum MAI rotation and using the ending condition *harvest all age classes above the age class indicated by maximum MAI* (see Eq. [7]) will convert the forest to the desired structure. Because volumes per acre in old-growth stands are high, harvest levels will usually drop to the LRSY level after conversion. If such a falldown is undesirable, the user can lengthen the conversion period, impose the ending condition *harvest equals growth* (see Eq. [6]), or both. Lengthening the conversion period will ration excess inventory over a longer period. Imposing *harvest equals growth* will also meter out existing inventory for a smoother transition to LRSY. Combining the two strategies will produce similar results.

applying simple EFV (DD)

We may formulate the simple EFV problem for DD stands as a nonlinear program (NLP) (Adams and Ek 1974). To simplify notation, assume one site class, species type, MI, stocking level, and stand size-class.

Let

t_{dj} = number of trees in diameter class d, period j, after harvest in period j.

T_{Bj} = vector of trees in each diameter class, period j, before harvest in period j (Fig. 2).

T_{Aj} = vector of trees by diameter class, period j, after harvest in period j (Fig. 2).

$G(T_{Aj})$ = growth function yielding trees by diameter class, period j+1, given trees by diameter class after harvest in period j $= T_{B,j+1}$

$V(T_{\cdot j})$ = volume function giving total volume in period j based on the distribution of trees by diameter class $T_{\cdot j}$.

The dot notation indicates that either A or B may be used.

H_j = vector of trees harvested by diameter class, period j.

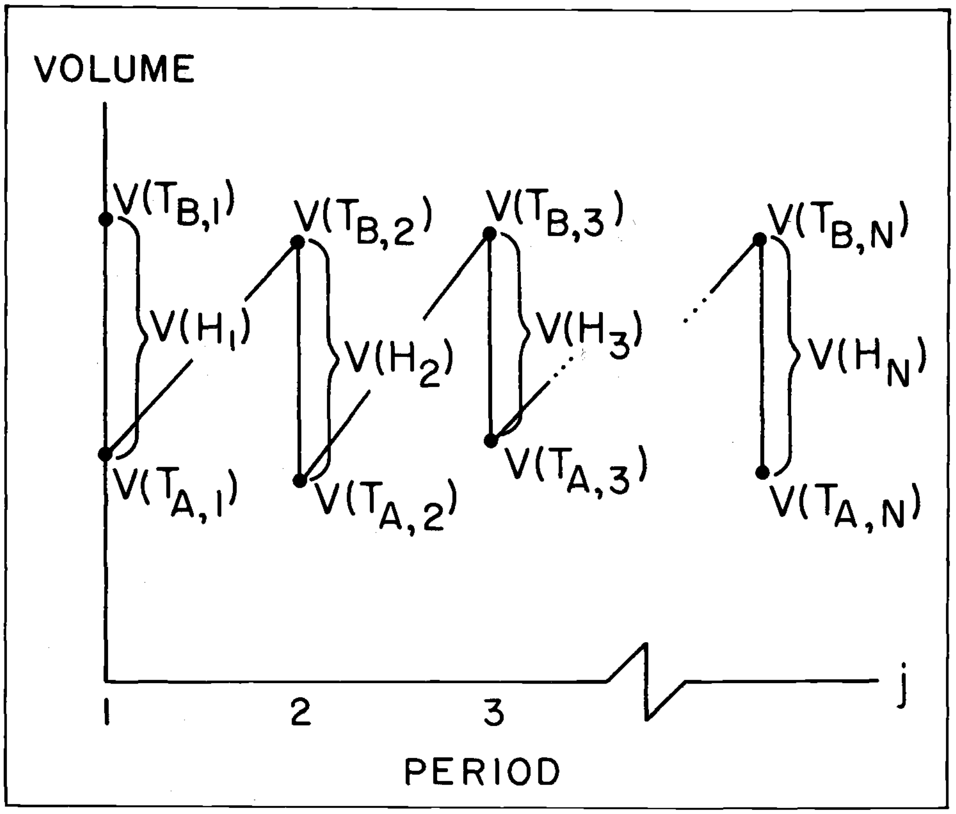


Figure 2.

Harvest and growth pattern in an uneven-aged forest.

N = number of periods in the look-ahead.

D = number of diameter classes into which the inventory will be divided.

$\hat{d}_H$ = largest diameter class left after last-period harvest.

The objective function is:

$$\text{Maximize } V(T_{B1} - T_{A1}) \qquad [9]$$

or, equivalently, maximize $V(H_1)$.

The constraints are:

$$V(H_j) = V(H_1) \qquad [10]$$

$$j = 2, \ldots, N.$$

Harvests in all periods must equal first-period harvest. Equation [10] may be rewritten as either

$$V(T_{Bj} - T_{Aj}) = V(H_1) \qquad [11]$$

$$j = 2, \ldots, N$$

or

$$V[G(T_{A,j-1}) - T_{Aj}] = V(H_1)$$

$$j = 2, \ldots, N. \qquad [12]$$

$$T_{Aj} \geq 0 \qquad [13]$$

$$j = 1, \ldots, N.$$

Harvest can never take more than available inventory.

Either of two ending-condition constraints may be imposed:

$$V(H_N) = V(T_{BN} - T_{A,N-1}). \qquad [14]$$

Harvest equals growth. Equation [14] may be rewritten:

$$V[G(T_{A,N-1}) - T_{AN}] = V[G(T_{A,N-1})$$

$$t_{dN} = 0 \qquad [16]$$

$$d = \hat{d}_H + 1, \ldots, D.$$

The last-period harvest must leave no trees in diameter classes above the designated diameter class d.

Solution methods: optimality and feasibility. The growth function in a DD forest tends to introduce nonlinearities into the objective function. Trees cut from one diameter class affect growth rates in other diameter classes. Therefore, we cannot consider the growth and yield of single diameter classes when selecting trees for harvest, as is possible in AD stands. Provided the growth function is relatively simple, we can use an NLP algorithm (Adams and Ek 1974) to find trees to harvest from each diameter class (cutting rules) in each period and harvest level.

In the EFV algorithm in TREES, the user specifies a cutting rule including the requested harvest-volume proportion for each diameter class and size classes 2, 3, and 4 [PVLRDC(di,sz) in the ACC file] *before* the simulation begins. Each period, the subroutine ACALDD multiplies PVLRDC(di,sz) by the volume in each corresponding diameter class and sums the products. ACALDD then divides the harvest requested for the period by that sum to form an adjustment ratio. This ratio, multiplied by the original

$- T_{A,N-1}]$. [15]

proportion [PVLRDC(di,sz)], gives the adjusted proportion. If the adjusted proportion is greater than 1, all volume in that diameter class will be harvested; if not, the adjusted proportion will differ from the user-designated proportion by a constant factor. (See Appendix C, **User's Manual.**)

As in the AD case, preselecting the harvest priority greatly simplifies searching for the maximum EFV harvest. The subroutine ACSTHV tries ending-condition checks to see if harvest levels meet all constraints and adjusts levels until the highest feasible harvest is found. [See the simple EFV (AD) section for an outline of the algorithm.]

Imposing a harvest priority adds a further constraint. At best, the EFV algorithm in TREES will match harvest levels found through an NLP solution. Because the optimal cutting strategy is a multidimensional problem that can change over time, a cutting rule as specified in TREES will probably not be strictly optimal. For complex problems, however, NLPs are even more limited than LPs; thus, the EFV approach may provide the only satisfactory solution now available. By varying harvest priorities in successive runs of the EFV algorithm, we can arrive at cutting rules close enough to the optimal for practical purposes.

Each GRU may contain as many as four stand size-classes[5], three stocking levels, and seven MIs. Each productivity class may comprise up to 13 diameter classes and two fiber types. Thus, for each GRU, up to 84 x 26 separate classes of timber are possible. Because up to 500 GRUs may be defined, a single harvest calculation could require a cutting rule to distinguish among more than one million separate, possible classes of timber.

To reduce harvest-priority specifications to reasonable dimensions and reduce computer storage costs, before each harvest the subroutine RUAGVD sums acres and volumes in each diameter class across all GRUs by stand size-class and diameter class. The user may enter cutting rules for size classes 2, 3, and 4 eligible for harvest. Subroutine ACALDD uses the 52 DD composite classes to determine volume proportions to harvest from each diameter class in each size class. The subroutine RUHRVD then applies the proportion to individual GRUs by diameter and size classes regardless of species and fiber types and stocking level. Although this harvest-allocation method is undeniably efficient, it sacrifices flexibility in tailoring cutting rules to individual stands.

Generally, the EFV algorithm will iterate to a feasible solution. Empirical stocking guides or NLP solutions to sample problems may provide clues for starting points in the iterative search. But when initial inventories are low, even-flow harvest levels may be so low that the ending condition imposed may prove impossible to meet.

Exogenous harvests and difference tolerances. Mortality salvage is the only exogenous harvest that may be specified for DD. The even-flow harvest level must be at least as great as the mortality salvage proposed.

Difference tolerances identical to those described in the AD case may also be specified to modify even-flow and ending-condition constraints.

Choosing the look-ahead period and ending condition. If long-run sustained yield (LRSY) is the goal of the forest manager[6], the forest structure for producing such a harvest must be determined. Adams and Ek (1974) have demonstrated that an NLP algorithm can find the best sustainable

[5]A stand size-class is determined by assigning individual diameter classes a size class (e.g., pole, sawlog). The size class with the largest basal area in the stand determines the stand size-class.

[6]Not necessarily the goal of *every* forest manager, LRSY has been the traditional goal of public forest managers.

diameter distribution as well as the numbers of trees to remove from each diameter class each period. Lacking such a solution, we could use empirically based stocking guides; or a series of one-period simulations could be run using different initial diameter distributions and no harvest to determine the distribution producing the highest volume growth. The latter should reasonably approximate the best possible LRSY distribution.

For stands lacking the desired structure, the question is how best to convert to that structure. If an even-flow harvest is desired during the conversion period, the simple EFV algorithm could be applied [or a second NLP problem solved, as in Adams and Ek (1974)]. Set the look-ahead period equal to the desired conversion period; use the ending condition *harvest all volume above the highest diameter class* (see Eq. [16]); and set the harvest proportions to reflect the initial distribution, desired final distribution, and length of the conversion period. Some experimenting with harvest proportions probably will be necessary because no hard-and-fast rule exists for setting the conversion period. For overstocked stands, the longer the conversion period, the longer the delay before achieving maximum volume growth. Overall volume production will suffer. For understocked stands, the shorter the conversion period, the more difficulty in reaching the desired distribution.

When stands are initially overstocked, converting to a specified final distribution will cause a production drop in the post-conversion period (paralleling the AD case). To avoid this falldown, specify the ending condition *harvest equals growth* (see Eq. [14]) or lengthen the conversion period. Such an approach may make achieving the desired final distribution difficult or may delay conversion to LRSY.

policy considerations: the simple case

Simple EFV seeks the highest level of even-flow harvest that can be maintained throughout a single, specified planning horizon and still meet the prescribed ending condition. The highest harvest level is determined through an iterative search; but the smallest amount of inventory available for harvest *in any one period* dictates the highest possible harvest level for *all* periods. If management practices do not increase volume available in the critical period, no increase can result. In some circumstances, no reasonable harvest schedule can be found due to the ending condition specified and inventory structure.

Adding or deleting inventory or intensifying management practices will affect the immediate harvest level (allowable cut effect). Although an even-flow of volume is ensured over the specified planning horizon, the number of acres to be harvested each period may be erratic. Forest regulation would be purely coincidental—the only certainty is that the ending condition would be met.

Simple EFV is generally insensitive to economic conditions. However, if a particular area were economically stable, this method might be a sound choice because harvesting an even-flow of volume would continue to promote employment and income stability.

Simple EFV Highlights

- Complex to specify.
- Maximum harvest level limited to the smallest inventory available for harvest in any period.
- Ending conditions may be specified.
- Acres harvested each period probably will be erratic.
- Adding or deleting inventory may or may not immediately change volume harvested.
- Regulation would be coincidental.
- Volume harvested generally is insensitive to economic conditions.

applying sequential EFV (AD and DD)

Sequential EFV can be viewed as a series of simple EFV problems (except for harvest-flow constraints, discussed next; see the simple EFV sections for amplification).

Harvest-flow constraints. In sequential EFV, the user may specify harvest-flow constraints to limit the period-to-period change in harvest levels. Proportional harvest increase (PVLICP) and decrease (PVLDCP) are specified separately in the ACC file and take priority over the even-flow constraint.

Figure 3.

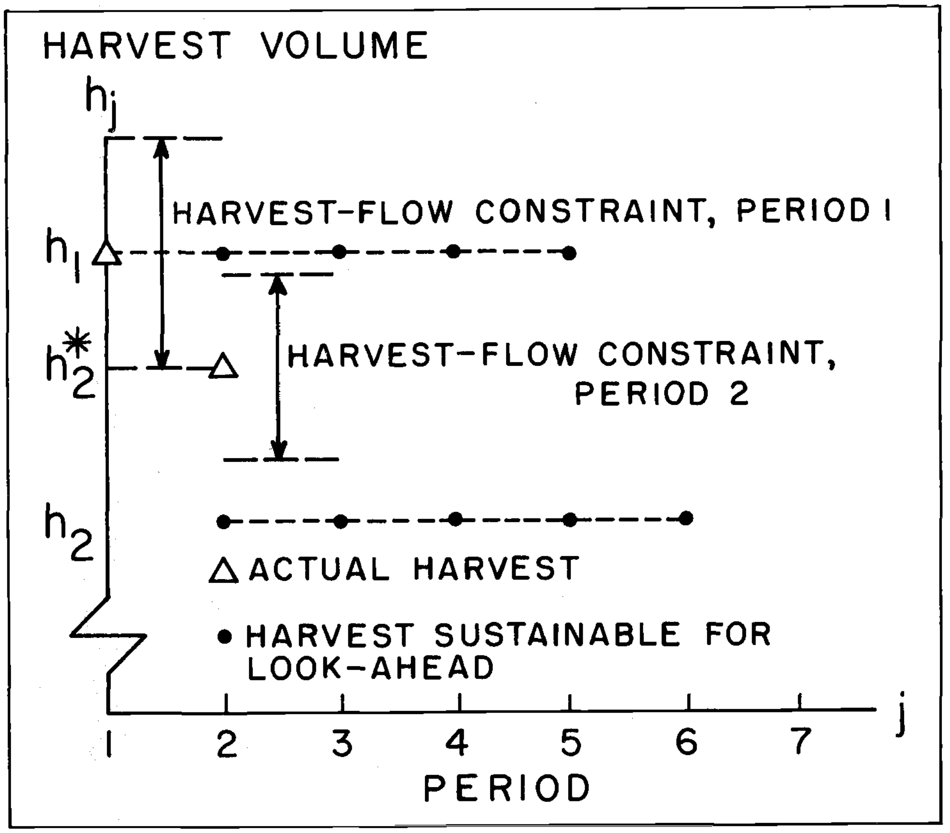
Second-period harvest h^*_2 constrained above maximum EFV level h_2.

Where the proportional increase and decrease are set to 0.10 (PVLICP = PVLDCP = 0.10) and h_1 is the maximum EFV harvest level (five-period look-ahead) for the first period, the harvest taken in the second period must be written $\pm$ $(0.10 \times h_1) + h_1$ of h_1 (Fig. 3). If the maximum EFV starting in period 2 falls outside that range, the even-flow requirement is ignored and the harvest-flow constraint (h^*_2) met, if feasible. If h^*_2 is taken in period 2, harvest in period 3 must be within $\pm$ $(0.10 \times h_2) + h^*_2$ of h^*_2. Where a difference tolerance has been specified (PVLDTP $\neq$ 0), harvests within $\pm$ (harvest-flow level) are considered satisfactory.

Choosing the look-ahead period and ending condition. Sequential EFV allows harvests to be gradually phased into an LRSY level. For understocked stands, harvest levels will rise to the LRSY level. To achieve such a phase-in, set the look-ahead (NPDVPP) to the rotation age where MAI is maximum and require all volume in age or diameter classes above maximum MAI to be harvested by the end of each look-ahead. Of course, no single maximum MAI rotation length can be applied to *all* timber classes with multiple site classes and MIs—an average figure must be chosen for the entire inventory. The time required to reach LRSY will depend on the inventory structure; in complex situations, a stable LRSY level may never be reached. Set the number of outer cycle periods (NPDOCP) equal to the number of periods for which harvest reports are desired.

policy considerations: the sequential case

Sequential EFV attempts to determine the maximum volume that can be harvested in a specific



period and sustained for a predetermined number of periods yet still meet the prespecified ending condition. This period of sustainability (look-ahead) and the ending condition will dictate whether the forest will be regulated at the end of the planning horizon; however, unless the forest is approaching regulation, volume flow and acres cut per period will probably be uneven due to composition of current inventory, number of acres initially cut, and future production. If we specify a sustainability check and an ending condition, the forest should gradually approach regulation, but composition at regulation will depend primarily on the ending condition specified. Adding or deleting inventory or intensifying management practices in future periods could affect current harvest levels due to sustainability requirements.

Economic conditions will not alter the calculated harvest schedule because harvest flow is based solely on the productive capacity of the timber land.

Sequential EFV Highlights

- Complex to specify.
- Ensures a period of sustainability.
- Ending conditions may be specified.
- Regulation will eventually occur.
- Based solely on the productive capacity of the forest.
- Volume harvested is insensitive to economic conditions.

Even-Flow of a Function of Volume (EFFV)

EFFV is identical to EFV *except* that the variable for which a maximum sustainable level is sought is a *linear function of volume* rather than volume itself. Volume-dependent variables might be employment or gross revenue. The linear function, which predicts the dependent variable, is user specified. Slope and intercept of the linear function may be shifted over time to reflect changes in the relationship between the dependent variable and harvest volume—for example, slope and intercept might be shifted over time to simulate reduced employment per unit of volume harvested due to technological change. Like EFV, EFFV may operate in the simple or sequential mode and, in either case, may be applied to AD or DD stands.

applying simple EFFV (AD)

The simple even-aged EFFV problem requires only that the even-flow-of-volume constraint (Eq. [4]) be changed to an even-flow of the *volume-dependent* variable.

Let

$\mathbf{Q_j}$ = quantity dependent on volume harvested in period j $= C_{1j} + C_{2j}h_j$.

$\mathbf{C_{1j}}$ = intercept of the function relating Q_j to h_j.

$\mathbf{C_{2j}}$ = slope of the function relating Q_j to h_j.

The new constraint may be written:

$$Q_j = Q_1 \quad [17]$$
$$j = 2, \ldots, N.$$

The dependent variable in all periods must equal the first-period value. Equation [17] may be restated:

$$C_{1j} + C_{2j}h_j = Q_1 \quad [18]$$
$$j = 2, \ldots, N$$

or, rearranging terms,

$$h_j = (Q_1 - C_{1j})/C_{2j}. \quad [19]$$

Solving for EFFV and EFV is the same, except that Step 4 of the EFV algorithm must be modified to:

Step 4. Check the period. If last period, go to Step 5. If not, grow the inventory and set next-period harvest (h_j) equal to $(Q_1 - C_{1j})/C_{2j}$.

(See Appendix A for more detailed explanation of the EFFV algorithm; see Appendix B for the optimization detail report, which aids interpretation of the iterative search.)

applying simple EFFV (DD)

The simple uneven-aged EFFV option requires the following change in the even-flow constraint (Eq. [10]):

Let

Q_j = quantity dependent on volume harvested in period j $= C_{1j} + C_{2j}[V(H_j)]$.

C_{1j} = intercept of the function relating Q_j to harvest volume $V(H_j)$.

C_{2j} = slope of the function relating Q_j to harvest volume $V(H_j)$.

The new constraint may be written:

$$Q_j = Q_1 \quad [20]$$
$$j = 2, \ldots, N.$$

Equation [20] may be rewritten:

$$V(H_j) = (Q_1 - C_{1j})/C_{2j} \quad [21]$$
$$j = 2, \ldots, N.$$

Solving for EFFV and EFV is the same, except that Step 4 of the EFV algorithm must be modified to:

Step 4. Check the period. If last period, go to Step 5. If not, grow the inventory and set next-period harvest $V(H_j)$ equal to $(Q_1 - C_{2j})/C_{2j}$.

applying sequential EFFV (AD and DD)

Sequential EFFV and EFV are identical, except for changes in the constraints noted in Eqs. [17] and [20]. Note that the harvest-flow constraints continue to operate on harvest volumes, not on the quantity dependent on harvest. If the even-flow of the dependent variable Q_j requires a harvest level that violates a harvest-flow constraint, the even-flow constraint will be ignored and the harvest-flow constraint satisfied.

policy considerations: simple and sequential cases

Like EFV, EFFV may be simple (one look-ahead) or sequential (multiple look-aheads). In either case, remember that the objective value—the variable for which an even-flow is specified—is *not* volume. Volume may vary over time to achieve even-flow of the objective value—which might be employment, dollars, or anything else that can be specified as a *linear function of volume*. Thus, depending on the nature of the objective, this policy could become sensitized to economic conditions.

Although key factors cannot be described without knowing the objective value, we can safely assume that volume flow and acres cut each period will probably be uneven. The forest may or may not become regulated even though an ending condition is specified. However, it is difficult to predict how adding or deleting inventory or intensifying management practices will affect the objective value.

Simple and Sequential EFFV Highlights

- Complex to specify.
- Objective value is the key variable.
- Ending conditions may be specified.
- Regulation may or may not occur.
- Volume and acres harvested (by period) would probably be uneven.
- Effects of adding or deleting inventory or intensifying management practices would be difficult to assess.
- Economic sensitivity may be increased, depending on the objective value.

Present Net Worth (PNW)

The PNW algorithm is an advanced version of the original ECHO algorithm developed by Walker (1971) to "provide a means of optimizing harvest rates for a timberland owner facing a negatively sloped demand curve. This, in turn, defines the optimal time path to a long-run equilibrium forest, starting from a non-equilibrium condition." Available for AD stands only, the PNW algorithm uses the same basic linkage equation as ECHO but may be extended to more complex situations.

The PNW problem may be formulated as a quadratic program (QP) (Johnson and Scheurman 1977). In this simple formulation, we consider only one site class, species type, and stocking level; a linear demand curve that is constant over time; and harvest costs that are constant per acre harvested. All harvests are taken at the midpoint of each period.

Let

x_{ij} = acres cut in period j from age class i, where i is the period of stand establishment, which does not change as the stand ages.

V_{ij} = volume per acre, age class i, period j.

h_j = total harvest, period j

$$= \sum_{i=-M}^{j-Z} V_{ij}x_{ij}.$$

N = number of periods in the planning horizon.

-M = period of stand establishment for the oldest age class in the starting inventory relative to the start of the planning horizon. The minus indicates that age class -M was established M periods *before* the start of the planning period.

Z = minimum number of periods before regenerated stands are eligible for harvest.

$\mathbf{C_1}$ = demand-curve intercept (assumed constant over time).

$\mathbf{C_2}$ = demand-curve slope (assumed constant over time), where $C_1 - C_2h_j$ is price per unit volume.

$\mathbf{C_3}$ = harvest cost per unit (assumed constant over time and unaffected by age class harvested).

r = annual discount rate.

p = period length (in years).

$\mathbf{D^j}$ = discount factor, period j[7].

$$= [(1 + r)^{p(j-1/2)}].$$

$\mathbf{TC_j}$ = total harvest cost, period j, undiscounted
$= C_3h_j$.

$\mathbf{TR_j}$ = total revenue, period j, undiscounted
= price multiplied by harvest level
$= [(C_1 - C_2h_j)h_j]$.

[7]Like harvesting, discounting also is calculated midperiod.

A_i = acres in age class i in initial inventory.

The objective function may be written:

$$\text{Maximize} \sum_{j=1}^{N} \frac{TR_j - TC_j}{D^j}. \quad [22]$$

If we substitute for TR_j and TC_j, the equation may be rewritten:

$$\text{Maximize} \sum_{j=1}^{N} \frac{C_1 h_j - C_2 h_j^2 - C_3 h_j}{D^j}. \quad [23]$$

This formulation clearly shows the nonlinearity of the objective function, given a linear demand function. The use of a nonlinear demand function would lead to third-degree or greater polynomials in the objective function, which causes nonconvexity problems—the Kuhn-Tucker (K-T) conditions[8] are no longer sufficient for maximization.

Additional substitution for h_j and D^j gives the form:[9]

$$\text{Maximize} \sum_{j=1}^{N} \frac{\left(C_1 - C_2 \sum_{i=-M}^{j-Z} V_{ij} x_{ij}\right) \sum_{i=-M}^{j-Z} V_{ij} x_{ij} - C_3 \sum_{i=-M}^{j-Z} V_{ij} x_{ij}}{[(1 + r)^p]^{(j-1/2)}}. \quad [24]$$

[8] For discussion of Kuhn-Tucker conditions, see Zangwill (1966); Wolfe (1962), p. 365; Kunzi et al. (1966), p. 75; Wagner (1969), p. 600; or other nonlinear programming textbooks.

[9] The QP solution is biased when N is finite because harvests in periods beyond N have no value in the objective function. Consequently, merchantable inventory will usually be exhausted by the N^{th} period. However, the larger N becomes and the higher the discount rate r, the closer the QP solution comes to the true, optimal solution. In the ensuing discussion, we assume that N and r are sufficiently large to make the difference negligible.

For this formulation, we recognize the following constraints:

$$\sum_{j=1}^{N} x_{ij} \leq A_i \qquad [25]$$

$i = -M, \ldots, 0.$

Acres harvested (x_{ij}) from an age class i existing in the starting inventory cannot exceed acres in the inventory for that age class (A_i).

$$\sum_{k=j+Z}^{N} x_{jk} \leq \sum_{i=-M}^{j-Z} x_{ij} \quad . \qquad [26]$$

$j = 1, \ldots, N.$

Acres harvested from an age class (x_{jk}) established in period j by harvesting previous age classes (x_{ij}) must not exceed the sum of those age classes (Σx_{ij}). To simplify notation, we assume no regeneration lag.

$$x_{ij} \geq 0 \qquad [27]$$

$i = -M, \ldots, j - Z$
$j = 1, \ldots, N.$

Acres harvested from any age class must not be negative.

Because the constraints are linear and the quadratic objective function is concave, the K-T conditions are both necessary and sufficient for optimal harvest schedules. K-T conditions for an optimal harvest schedule are:

K-T1

$$A_i - \sum_{j=1}^{N} x_{ij} \geq 0$$

$i = -M, \ldots, 0$

$$\sum_{i=-M}^{j-Z} x_{ij} - \sum_{k=j+Z}^{N} x_{jk} \geq 0$$

$j = 1, \ldots, N$

$$x_{ij} \geq 0$$

$i = -M, \ldots, j - Z$
$j = 1, \ldots, N.$

An optimal solution must meet all the constraints (i.e., must be feasible).

K-T2

There must exist multipliers λ_i, ℓ_j, and μ_{ij} such that $\lambda_i \geqslant 0$, $\ell_j \geqslant 0$, and $\mu_{ij} \geqslant 0$, and

$$\lambda_i(A_i - \sum_{j=1}^{N} x_{ij}) = 0$$

$i = -M, \ldots, 0$

$$\ell_j(\sum_{i=-M}^{j-Z} x_{ij} - \sum_{k=j+Z}^{N} x_{jk}) = 0$$

$j = 1, \ldots, N$

$$\mu_{ij}(x_{ij}) = 0$$

$i = -M, \ldots, j - Z$
$j = 1, \ldots, N.$

These are "complementary slackness" conditions.

K-T3

$$\frac{(C_1 - 2C_2h_j)V_{ij} - C_3V_{ij}}{D^j} - \lambda_i + \ell_j + \mu_{ij} = 0$$

$i = -M, \ldots, j - Z$
$j = 1, \ldots, N.$

Discounted marginal net revenue must offset the opportunity costs of harvests.

Noting that $(C_1 - 2C_2h_j)V_{ij}$ is $\delta TR/\delta x_{ij}$, we can define MR_{ij}, the marginal change in revenue due to a one-acre change in the acres harvested from age class *i* in period j.

Similarly, C_3V_{ij} is $\delta TC/\delta x_{ij}$, defined as MC_{ij}, the marginal change in total cost due to a one-acre change in acres harvested from age class i in period j.

Therefore, we can rewrite K-T3:

$$\frac{MR_{ij} - MC_{ij}}{D^j} = \lambda_i - \ell_j - \mu_{ij} \qquad [28]$$

$i = -M, \ldots, j - Z$
$j = 1, \ldots, N.$

The marginal discounted net revenue from harvesting an additional acre of age class i in period j must equal the quantity on the right-hand side of the equation.

The multipliers λ_i, ℓ_j, and μ_{ij} may be interpreted as the "shadow prices" of the constraints, representing the increase in the objective function that could be achieved by relaxing the constraints by one unit.

- The multiplier λ_i is the increase in the objective function from adding one unit to the acres in age class i. Alternately, λ_i could be viewed as the increase in net discounted returns from future harvests due to a one-acre decrease in the cut from age class i in period j—or the opportunity cost of *cutting* an acre from age class i in period j. (The less cut from age class i in period j, the more can be cut in period j+1.)
- The multiplier ℓ_j is the increase in the objective function from harvests in stands replacing age class i due to a one-acre increase in x_{ij}. (The more cut now, the sooner regenerated stands will be available for cutting.) Thus, ℓ_j is the opportunity cost of *not cutting* an acre from age class i in period j. This term has also been called the land-holding cost because it represents the cost of maintaining land in its present state.
- The multiplier μ_{ij} is the total net discounted return from cutting an additional acre of age class i in period j, including current return and opportunity costs.

solution methods: optimality and feasibility

The QP solution to the constrained maximization problem described in Eqs. [24] through [27] could be finding the one harvest schedule that satisfies all K-T conditions. In contrast, the PNW algorithm uses K-T1, parts of 2 and 3, and several additional assumptions about harvest priority to search for the optimal harvest schedule for period 1, which in turn determines all subsequent harvest levels.

Johnson and Scheurman (1977) assume the following three conditions must be met for the PNW algorithm to yield a harvest schedule equivalent to QP:

1—Harvests can be optimally pre-ordered by age class. The PNW algorithm in TREES can use any of the three harvest priorities available in the model.[10]

2—In every period j, one age class exists for which $x_{ij} > 0$ and $x_{i,j+1} > 0$—this is the "linkage" age-class, which is harvested in both the current and succeeding periods.

3—For the linkage age-class harvested in periods j and j+1, $\ell_j - \ell_{j+1} = 0$. That is, the opportunity cost of delaying regeneration and future harvests on an acre of the linkage age-class is constant between periods j and j+1. Because constant costs do not affect optimality, assuming $\ell_j = \ell_{j+1}$ is equivalent to ignoring the opportunity cost of delaying future harvests. This assumption is equivalent to the "simple financial maturity" approach (the Type A analysis) in Duerr (1960).

[10]The original ECHO algorithm (Walker 1971) was based on an oldest-first harvest priority. A newer version, developed by Walker and Hamilton (personal communication, 1978) for the Simpson Timber Company, Shelton, Washington, allows a variety of harvest priorities to be specified.

In addition to the three preceding Johnson-Scheurman (J-S) assumptions, we will add:

4—The first harvest schedule feasible over N periods and meeting the requirements of the PNW linkage equation is also optimal.

Suppose we know the harvest level in period j. From J-S1, we can determine the last age class harvested in period j (x_{ij}), where age class i is the linkage age-class. K-T3 requires that, for x_{ij}:

$$[(MR_{ij} - MC_{ij})/D^j] - \lambda_i + \ell_j + \mu_{ij} = 0. \quad [29]$$

We can derive a similar condition for $x_{i,j+1}$:

$$[(MR_{i,j+1} - MC_{i,j+1})/D^{j+1}] - \lambda_i + \ell_{j+1} + \mu_{i,j+1} = 0. \quad [30]$$

From K-T2:

$\mu_{ij}x_{ij} = 0$ and $\mu_{i,j+1}x_{i,j+1} = 0.$

From J-S2:

$x_{ij} > 0$ and $x_{i,j+1} > 0.$

Therefore, μ_{ij} and $\mu_{i,j+1} = 0$.

Equating Eqs. [29] and [30] and combining terms:

$$(MR_{ij} - MC_{ij})D^j = [(MR_{i,j+1} - MC_{i,j+1})/D^{j+1}] - (\ell_j - \ell_{j+1}). \quad [31]$$

From J-S3:

$\ell_j - \ell_{j+1} = 0.$

We can rewrite Eq. [31]:

$$(MR_{ij} - MC_{ij})/D^j = (MR_{i,j+1} - MC_{i,j+1})/D^{j+1}. \quad [32]$$

Substituting for MR and MC, we can rewrite Eq. [32]:

$$[(C_1 - 2C_2h_j)V_{ij} - C_3V_{ij}]/D^j = [(C_1 - 2C_2h_{j+1})V_{i,j+1} - C_3V_{i,j+1}]/D^{j+1}. \qquad [33]$$

An acre cut from age class i must return the same discounted marginal net revenue in each pair of periods harvested.

Because we have already chosen h_j and because C_1, C_2, C_3, V_{ij}, $V_{i,j+1}$, D^j, and D^{j+1} are known, we can solve for the harvest level required in j+1 to meet the linkage equation [33].

Let

$$C_4 = C_1 - C_3.$$

$$h_{j+1} = C_4/2C_2[1 - D(V_{ij}/V_{i,j+1})] + D(V_{ij}/V_{i,j+1})h_j. \qquad [34]$$

The PNW algorithm, derived from Eq. [34], is outlined as:

Step 1. Select a harvest level (h_j) for the first period (j = 1).

Step 2. Check the feasibility of the harvest level:

- If $h_j >$ volume available for harvest, go to Step 1 and reduce the first-period harvest level.
- If $h_j < 0$, go to Step 1 and increase the first-period harvest level. If exogenous harvests are included, exogenous harvest volume replaces 0.
- If $0 \leqslant h_j \leqslant$ available volume, allocate the harvest according to the priority chosen and go to Step 3. If exogenous harvests are included, exogenous harvest volume replaces 0.

Step 3. Check the period. If the last period, stop. If not, grow the inventory and go to Step 4.

Step 4. Using J-S1, find the last age class harvested in period j. Using J-S2 and Eq. [34], determine h_{j+1} (the next-period harvest level). Set j = j+1 (increment the period), and go to Step 2.

The PNW algorithm stops: (1) when a feasible harvest schedule for all N periods is found using Eq. [34], based on our implicit assumption that the first feasible schedule is also optimal, or (2) if no feasible harvest schedule can be found. The search terminates when the difference between successive trial levels for the first-period harvest becomes smaller than the user-specified tolerance or when h_1 becomes less than 1. (See Appendix A for details on the PNW algorithm and Appendix B for details on the iterative search process.)

The PNW algorithm may be viewed as attempting to find a "window" through which the harvest path must travel to assure feasibility. Initial harvest levels are adjusted until one "makes it through the window" or until it is apparent that no window exists.

The ECHO algorithm (Walker 1971) differs from PNW in that run length is not specified. The simulation proceeds until harvest levels in successive periods are close enough to meet certain stability criteria. If no feasible harvest schedule meeting these criteria can be found and if the last two trial levels are arbitrarily close, the lower schedule is followed until it begins to diverge from the higher schedule. At that point, the linkage equation is ignored, and the algorithm begins again using the inventory at the point of divergence. ECHO allows a harvest schedule to be specified in any situation but does so by violating the optimality criteria embodied in the linkage equation.

For example, in a three-iteration PNW search (Fig. 4):

- The first-period harvest level ($h_1{}^1$) causes negative harvest volume in a later period.

- The first-period harvest volume is raised to $h_1{}^2$, resulting in an h_j greater than the volume available for some j less than N.

- The first-period harvest volume ($h_1{}^3$) is set midway between the previous harvest levels, producing a feasible harvest sequence for all periods.

For a given demand function, the starting point of each iteration determines all subsequent harvests. Increasing h_1 raises all subsequent harvests, though not necessarily uniformly.

stability characteristics

We will now examine the ability of the PNW and ECHO algorithms to converge on an equilibrium—and then return to the question of optimality.

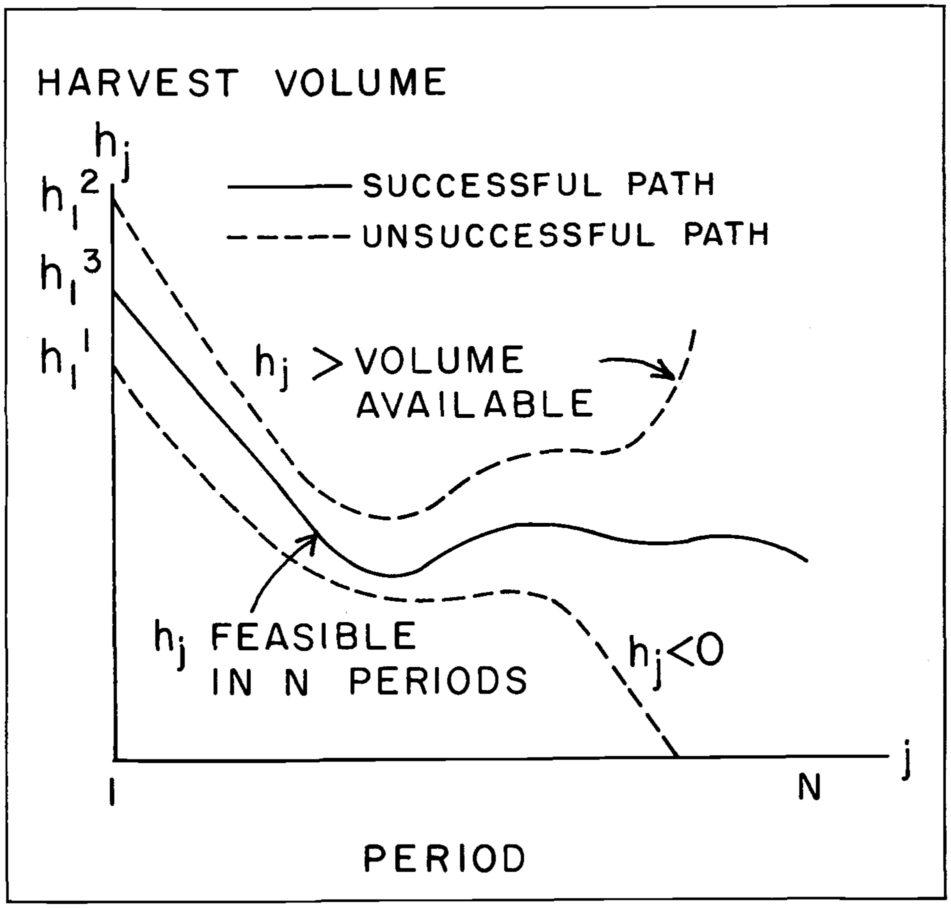


Figure 4.

Three-iteration PNW search.

Sessions (1978) outlined the conditions required for a stable harvest level for the simple linkage equation (Eq. [34]):

- No change in demand curve over time
- Constant unit costs
- No thinning
- No quality premium[11]

Let

$$A = \frac{D}{(V_{i,j+1}/V_{ij})} = \frac{\text{one-period discount factor}}{\text{one-period volume growth ratio}}.$$

Substituting in Eq. [34]

$$h_{j+1} = (C_4/2C_2)(1-A) + Ah_j. \quad [35]$$

Noting that

$$Ah_j = (1 - A)(-h_j) + h_j,$$

$$h_{j+1} = h_j + (1 - A)[(C_4/2C_2) - h_j]. \quad [36]$$

If we assume negative net marginal revenues cannot be optimal in any period, then $C_4 - 2C_2h_j \geqslant 0$ or $C_4/2C_2 \geqslant h_j$. Therefore, the term $[(C_4/2C_2) - h_j]$ in Eq. [36] must be greater than or equal to 0. This allows us to state the following stability conditions for harvest levels:

1. **$A = 1$.** If the volume growth ratio exactly equals the discount factor, then $h_{j+1} = h_j$.

2. **$(C_4/2C_2) - h_j = 0$.** If the marginal net revenue in period h_j is 0, then $h_{j+1} = h_j$.

[11]Quality premium is the increase in value per unit of wood due to improved wood quality as trees grow larger.

When neither of these conditions is met, harvest levels will change from period to period. For positive marginal net revenues, the direction of change is determined by A:

3. **$A > 1$.** If the volume growth ratio is less than the discount factor, $h_{j+1} < h_j$, and harvest falls.

4. **$A < 1$.** If the volume growth ratio is greater than the discount factor, $h_{j+1} > h_j$, and harvest rises.

If marginal net revenues are negative, conditions 3 and 4 are reversed: $A > 1$ increases harvest, and $A < 1$ decreases it.

For an overstocked forest in a typical PNW run (Fig. 5), the volume growth ratio of the linkage age-class is less than the discount factor in the early periods ($A > 1$), and harvest levels fall. As the last age class cut each period declines in age, the growth ratio increases and A becomes smaller. If the linkage age-class in a period has a volume growth ratio equal to the discount factor (i.e., is the "equilibrium" age-class), then $A = 1$; the next period's harvest will remain at the same level. Continued stability requires that the last age class cut in any subsequent period be the equilibrium age-class. Such a requirement implies that the steady-state harvest in all periods must be greater than growth into age classes above the equilibrium age-class but less than that growth plus the volume in the equilibrium age-class. That, in turn, implies that the forest is nearly regulated such that harvest equals growth in each period.

We can restate the stability conditions when marginal net revenue is positive:

1—An age class exists for which $A = 1$ (equilibrium age-class).

2—The equilibrium age-class must be the last age class harvested in some period j (i.e., $h_{j+1} = h_j$).

3—The term h_j must be greater than the volume growing into age classes above the equilibrium age-class in j+1 but less than that volume plus the volume in the equilibrium age-class. Otherwise, the equilibrium age-class will not be the last class harvested in period j+1 and $h_{j+2} \neq h_{j+1}$.

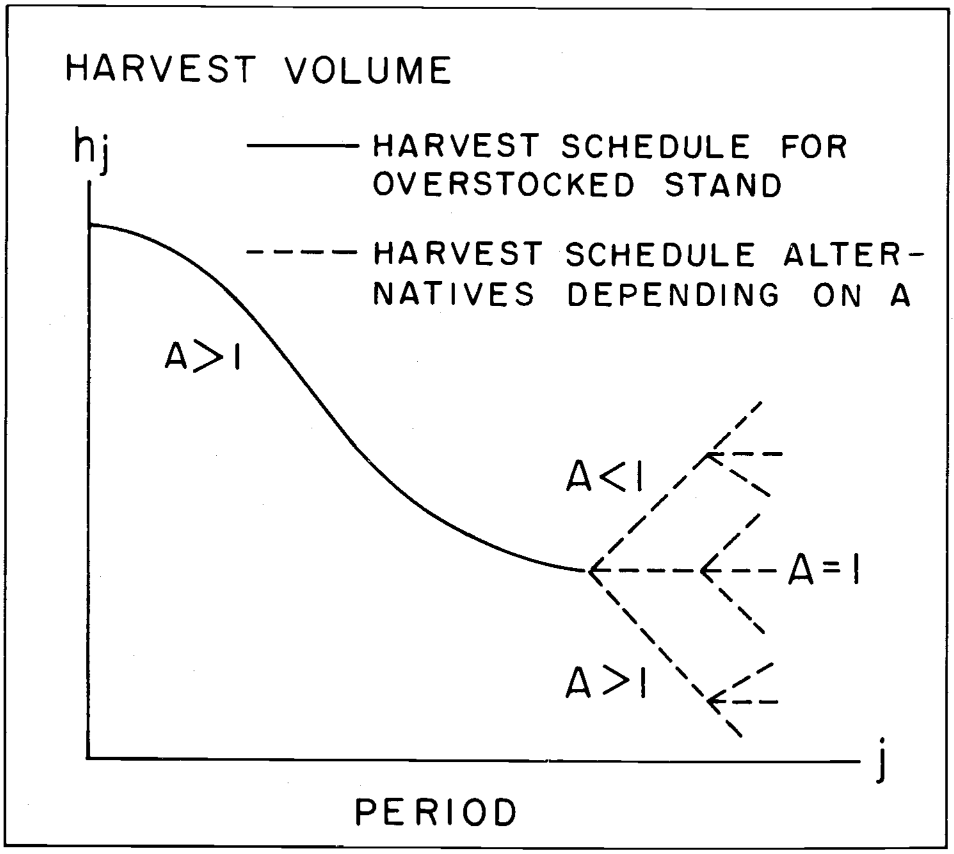


Figure 5.

Possible PNW harvest schedules.

4—The forest must be nearly regulated in period j so growth and harvest are balanced. Otherwise, stability (which implies forest regulation) is not guaranteed beyond j+2.

However, simultaneously fulfilling these stability conditions is *not* guaranteed by the PNW solution procedure.

First, condition 1 may not hold. If the discount rate is high, no age classes may be growing fast enough for A = 1. Or, because we are using discrete age classes, age classes may exist for which A > 1 and A < 1 but not for which A = 1.

The original ECHO program (Walker 1971) used annual age classes and harvest periods to minimize the discontinuity in growth rates between age classes. The PNW option in TREES uses a unique quadratic interpolation routine to make age-class growth rates continuous even with 5- or 10-year age classes. Volume per acre is assumed a quadratic function of age, and age of the last acre harvested is assumed to vary in proportion to the amount of the last age class harvested (see Appendix C).

Even if we use the quadratic interpolation routine, condition 2 may not be met. Harvesting exactly the right proportion of the last age class so that A = 1 may be difficult. Very small changes in initial harvest level can cause substantial differences in the proportion of an age class harvested 10 or 20 decades in the future.

Even if we assume that conditions 1 and 2 hold, conditions 3 and 4 may not. If 4 is true but not 3, then the harvest level in period j does not equal volume growth. A harvest level in excess of growth will eventually cause the age of the last age class harvested to fall below the equilibrium age, increasing A above 1. Harvest levels will rise, further depleting inventory and increasing A. At some point, inventory will be exhausted. Conversely, a harvest level less than growth will cause the age of the last age class harvested to rise above the equilibrium age (A < 1). Harvest levels will fall, further increasing inventory and reducing A. Eventually, harvest levels will become negative.

If 3 is true but not 4, then harvests may oscillate for a time as the linkage age-class rises and falls with growth. Because growth in excess of harvest levels will cause harvests to decline and growth below harvest levels will cause harvests to increase, the algorithm is inherently unstable.[12] If N is large enough, the harvest level will eventually either exceed available inventory or become negative.

In searching for harvest schedules that remain feasible over N periods, the PNW algorithm seeks the most stable harvest schedules. (Harvest levels that become stable are more likely to "make it through the window.") However, the search process does not guarantee ultimate stability. The smaller N is, the larger the "window" through which harvest schedules must pass, in which case feasible harvest levels need exhibit little or no stability. On the other hand, large N values, small inventories, high interest rates relative to growth rates, or nearly horizontal demand curves can cause instances where no harvest schedule is sufficiently stable to remain feasible over the projection period.

How different demand curves affect the harvest schedule is illustrated in Figure 6. As intercept increases and slope decreases (i.e., as the demand curve becomes more horizontal), the term $C_4/2C_2$ in Eq. [36] becomes larger. Period-to-period changes in harvest levels increase, affecting forest structure as the equilibrium age-class is approached and increasing subsequent stability problems.

[12] Walker (1971) refers to this instability as an oscillating equilibrium, but it is not clearly an equilibrium.

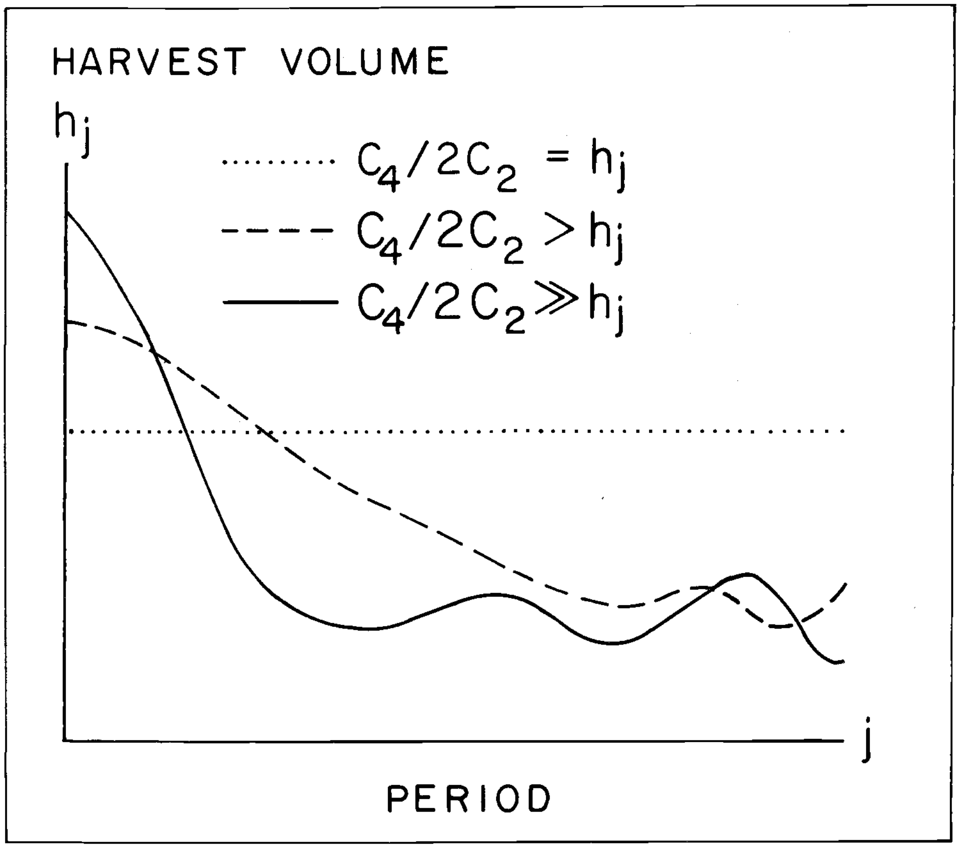


Figure 6.

Effect of changing demand intercept and slope on PNW schedule.

To summarize: The PNW algorithm with stable demand curve will often find harvest paths leading to approximately regulated forest structures simply because such structures lead to greater harvest stability and the likelihood of a feasible harvest path in all periods. How close the algorithm comes to equilibrium depends on N, inventory size, demand-curve slope and intercept, and growth and discount rates. Where demand curves change over time, stable harvest levels and harvests equaling growth are not compatible.

Starting the algorithm again at the point of divergence (as ECHO does) may postpone infeasible harvests and improve the algorithm's ability to achieve regulation. In some situations, however, the original ECHO algorithm (Walker 1971) is so unstable that it must start over every period. Incorporating the quadratic interpolation routine might improve performance.

optimality re-examined

Under what conditions do J-S assumptions 1 through 3 and our assumption 4 hold? If these are violated, how close does the PNW algorithm come to finding an optimal harvest sequence?

Definitive answers to these questions await further development of the ECHO theory. However, we can offer some tentative conclusions.

J-S1—harvest priority is optimal. If volume per acre increases with age but at a decreasing rate, then an oldest-first harvest priority will be optimal (Walker 1971). For the simple case with which we have been concerned, such a rule is equivalent to harvesting age classes in order of decreasing marginal value growth percent (MVGP). Such an ordering is indeed optimal if we accept the "simple financial maturity" criterion. But when variable harvest costs and quality premiums are introduced or multiple productivity classes included, the relationship between MVGP and age may not be straightforward. MVGP may not decrease uniformly with age in a given site class and, in different site classes, may vary widely for a given age class.

Because the PNW option currently cannot order harvests in individual stands on the basis of MVGP, all stands are combined into 33 age classes and three species report-groups to allocate harvests. Thus, per-acre volumes, costs, and growth assigned to each age class or species report-group are weighted averages of the values for individual stands.

Harvest priorities are assigned to the combined harvest unit (ACU). The oldest-first priority can be modified by taking harvest proportions from various age classes; once those have been harvested, the algorithm reverts to oldest first. When the harvest level has been found, the proportion to be taken from each age class or species report-group is determined according to the harvest priority chosen and is then applied uniformly to the corresponding age class in each site class.

Although the PNW algorithm efficiently handles large numbers of stands, it does not specifically order stands according to MVGP and sacrifices gains in optimality by treating age classes on each site class individually. In contrast, although the most recent version of ECHO [13] apparently allows ordering according to MVGP among age or site classes, the number of productivity classes that may be considered simultaneously is limited.

J-S2—$x_{ij} > 0$ and $x_{i,j+1} > 0$ for the linkage age class, all j. This assumption is probably the most critical to PNW optimality. If harvesting acres in every period or harvesting part of some linkage age-class in every pair of successive periods is not optimal, then the PNW harvest schedule will differ from that of the QP solution. At

[13] A version of ECHO used by the Washington State Department of Natural Resources (personal communication, 1978) can consider up to 160 stands. In contrast, TREES can simultaneously consider 500 GRUs, 33 age classes, 3 stocking levels, and 7 MIs—or nearly 350,000 classes of timber—to specify growth and management.

least two cases exist where this assumption apparently holds.

- **Case 1: No feasible PNW solution can be found over N periods.** We know that QP has at least one feasible solution ($h_j = 0$, $j = 1, ..., N$); therefore, it has at least one optimal solution. But the PNW procedure cannot find an optimal schedule that involves no harvest in some period or a harvest that just exhausts the last age class cut in some period. Assumption (2) has been violated. When no feasible harvest level can be found, the PNW algorithm stops.

  In ECHO, a nonoptimal path may be followed until divergence occurs, at which point the search procedure starts again. Implicit is the assumption that, at the point of divergence, exhausting the last age class harvested (thus destroying the linkage) would have been optimal. Although this procedure allows a harvest to be determined for every period, the computational requirements may be significant.

- **Case 2: Marginal net revenue for the last age class cut in some period is negative.** Recall Eq. [29]:

$$(MR_{ij} - MC_{ij})/D^j = \lambda_i - \ell_j. \quad [37]$$

($\mu_{ij} = 0$ because $\mu_{ij}x_{ij} = 0$ and $x_{ij} > 0$.)

If the marginal net revenue is less than 0, then $\lambda_i < \ell_j$. In other words, the discounted return from cutting an acre of age class i in period j+1 is exceeded by the discounted return from future harvests by cutting that acre in period j. This could occur if age class i were poorly stocked and would be replaced by a much more vigorous stand or if the demand curve were shifting over time. However, under a stable demand curve and single stocking level, a negative net marginal revenue will probably not be optimal. Thus, J-S2 may lead to a nonoptimal, though feasible, harvest schedule.

Using a "simple financial maturity" criterion, we may assume that $\ell_j = 0$ and add a check to the algorithm that treats negative marginal net revenue as infeasible. ECHO incorporates such a check but PNW does not. Refusal to consider negative marginal net revenue does not ensure that the chosen level of marginal net revenue is optimal (see the assumption 4 discussion).

Case 1 conditions arise more frequently where the demand-curve slope is small relative to the intercept ($C_4/2C_2$ is large) or where demand is large relative to inventory size. As previously noted, extremely elastic demand curves require higher initial levels. If inventories are not large enough to support those levels, infeasibilities result. On the other hand, case 2 conditions may occur with large inventories and steep demand curves. Although case 2 causes rising harvests over time (Fig. 5), large inventories may avoid feasibility problems over the N periods of the planning horizon.

We may then hypothesize that extremes in demand-curve slope and in the relationship between demand and inventory will cause PNW solutions to diverge from the optimal solution to the problem stated in Eqs. [24] through [27]. If growth rates were continuous, rather than changing discretely between age classes, then harvesting only part of an age class every period (leaving the rest for the next period) might be optimal. However, J-S2 continues to be violated, though less frequently, even using the PNW quadratic interpolation routine (Scheurman and Johnson 1975).

J-S3—ℓ_j - ℓ_{j+1} = 0. Ignoring the opportunity costs of delaying future harvests will generally lead to smaller immediate and larger future harvests than are optimal. The lower the discount rate and the higher the growth rate, the greater the difference between the optimal and PNW schedules.

Assumption 4—the first feasible PNW solution is optimal. Even if the three J-S assumptions hold, assumption 4 may not. Often, several feasible harvest schedules with different net returns can be found. The range of possible solutions widens as N decreases and inventory increases.

Given J-S 1 through 3, the existence of multiple solutions implies that the linkage equation provides necessary but not sufficient conditions for an optimal solution. Insufficiency follows because *only part* of Eq. [28] (rewritten K-T3) was used in developing the algorithm. Though the algorithm ensures that marginal net revenues are equated over time, it cannot determine that the *level* of marginal net revenue is optimal.

To summarize: It is not clear when the PNW or ECHO solution procedures will yield harvest schedules that closely parallel the QP solution. How close the solutions come depends on the demand-curve slope, inventory size, number of simulation periods, harvest-priority specification, and cost of delaying future harvests.

linkage equation with a changing demand intercept

In the PNW algorithm, the demand-price intercept may be varied at a constant or changing rate between the first and any future period. Let α = rate of change in the demand-equation intercept such that

$$(1 + \alpha)C_{1j} = C_{1,j+1}. \qquad [38]$$

Substituting into Eq. [33], we can derive the new linkage equation:

$$\frac{(C_{1j} - 2C_2h_j)V_{ij} - C_3V_{ij}}{D^j} = \frac{[(1 + \alpha)C_{1j} - 2C_2h_{j+1}]V_{i,j+1} - C_3V_{i,j+1}}{D^{j+1}}. \qquad [39]$$

Letting

$$C_4 = (C_{1j} - C_3),$$

$$h_{j+1} = \frac{C_4}{2C_2}\left[1 - D\left(\frac{V_{ij}}{V_{i,j+1}}\right)\right] + D\left(\frac{V_{ij}}{V_{i,j+1}}\right)h_j + \alpha\left(\frac{C_{1j}}{2C_2}\right). \qquad [40]$$

Thus, we see that h_{j+1} will be increased by $\alpha(C_{ij}/2C_2)$ when α is positive and the demand intercept shifts by (α x 100) percent (Fig. 7). The starting harvest level may need to be reduced to maintain feasibility. Increasing the demand-curve intercept also raises the age of the equilibrium age-class.

From Eq. [40], letting

$$A = \frac{D}{\frac{V_{i,j+1}}{V_{ij}}} = D\left[\frac{V_{ij}}{V_{i,j+1}}\right],$$

$$h_{j+1} = \frac{C_4}{2C_2}\left(1 - A\right) + Ah_j + \alpha\left(\frac{C_{1j}}{2C_2}\right). \qquad [41]$$

Rearranging terms in Eq. [41] and noting that $Ah_j = (1\text{-}A)(\text{-}h_j) + h_j$,

$$h_{j+1} = h_j + (1 - A)(C_4/2C_2 - h_j) + \alpha\left(\frac{C_{1j}}{2C_2}\right). \qquad [42]$$

Therefore, h_{j+1} will equal h_j when

$$(1 - A)(C_4/2C_2 - h_j) + \alpha\left(\frac{C_{1j}}{2C_2}\right) = 0 \qquad [43]$$

or, equivalently, when

$$A = 1 + \alpha\left(\frac{C_{1j}}{C_4 - 2C_2h_j}\right). \qquad [44]$$

Assuming the marginal net revenue is positive ($C_4 - 2C_2h_j > 0$) and $\alpha > 0$, then $A > 0$ when $h_{j+1} = h_j$. The growth rate must be less than the discount rate.

A decreasing growth rate in the last age class cut implies that the age of that class is increasing and that harvest is less than growth. Forest regulation and stable harvests seem impossible to achieve simultaneously when the demand curve is shifting each period.

linkage equation with age-dependent harvest price and costs

Price per unit harvested may depend on stand age as well as overall demand because wood quality changes with age. Per-acre and per-unit harvest costs also are often age related. The PNW algorithm incorporates age-dependent quality premiums and harvest costs as follows:

Let

BP_{ij} = base price per unit of volume harvested, age class i, period j. Base price is the price received in the base year for timber that was the same age as age class i in year j.

BP_{av} = price per unit of volume received for an average-aged tree in the base year.

CA_{ij} = cost per acre harvested from age class i, period j. Cost per acre as a function of age remains constant over time in the PNW option but may change as stand age changes.

CU_{ij} = cost per unit of volume harvested, age class i, period j. Cost per unit of volume as a function of age remains constant over time in the PNW option but may change as stand age changes.

MC_{ij} = change in total cost in period j due to a one-acre change in the harvest from age class i.

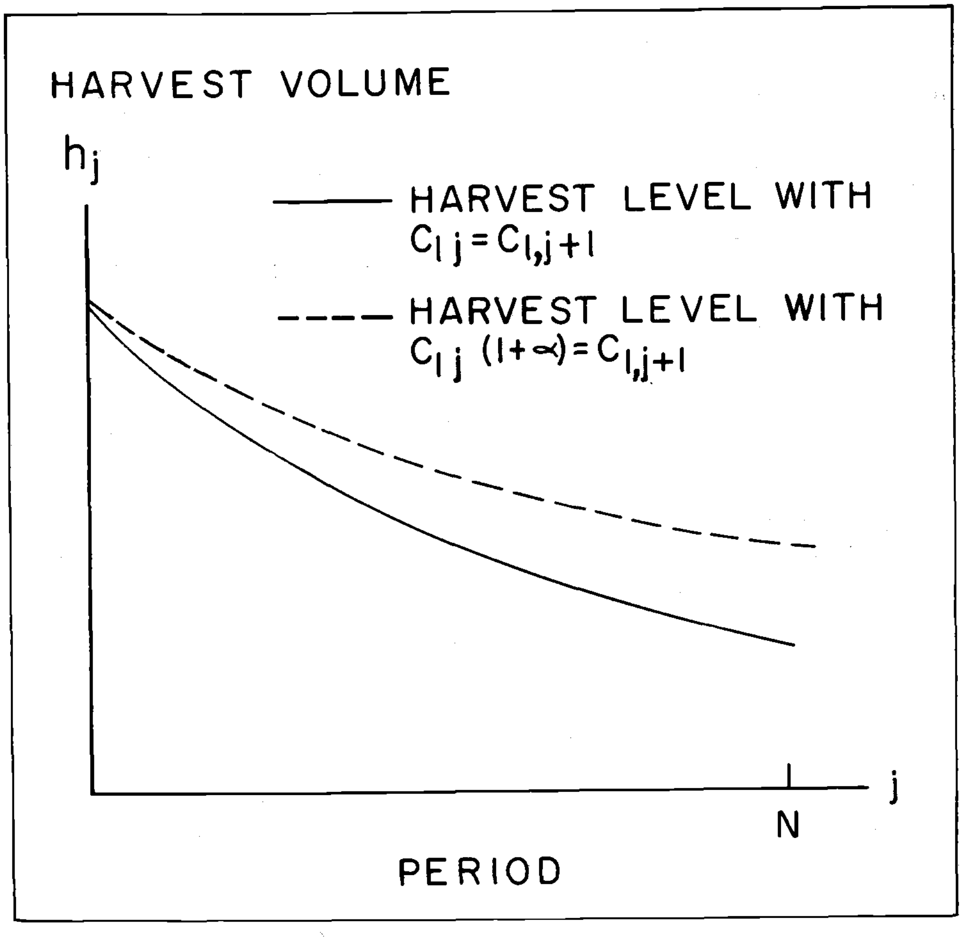


Figure 7.

Effect of demand intercept increase when the starting harvest level is held constant.

These are incorporated into the linkage equation [39] :

$$\left[\frac{BP_{ij}}{BP_{av}}\right]\left[\frac{(C_{1j} - 2C_2h_j)V_{ij}}{D^j}\right] - \left[\frac{CA_{ij} + CU_{ij}V_{ij}}{D^j}\right] = \left[\frac{BP_{i,j+1}}{BP_{av}}\right]\left[\frac{[(1 + \alpha)C_{1j} - 2C_2h_{j+1}]V_{i,j+1}}{D^{j+1}}\right] -$$

Let

$$MC_{ij} = CA_{ij} + CU_{ij}V_{ij}.$$

$$MC_{i,j+1} = CA_{i,j+1} + CU_{i,j+1}V_{i,j+1}.$$

Rearranging terms:

$$h_{j+1} = \frac{C_{1j}}{2C_2}\left[(1 + \alpha) - D\left(\frac{V_{ij}}{V_{i,j+1}}\right)\left(\frac{BP_{ij}}{BP_{i,j+1}}\right)\right] + D\left(\frac{V_{ij}}{V_{i,j+1}}\right)\left(\frac{BP_{ij}}{BP_{i,j+1}}\right)h_j$$

$$+ \left(\frac{BP_{av}}{BP_{i,j+1}}\right)\left(\frac{1}{2C_2}\right)\left(\frac{1}{V_{i,j+1}}\right)[D(MC_{ij}) - MC_{i,j+1}]. \qquad [46]$$

Let

$$R = D/\left[\left(\frac{V_{i,j+1}}{V_{ij}}\right)\left(\frac{BP_{i,j+1}}{BP_{ij}}\right)\right] = \frac{\text{discount factor}}{\text{value growth ratio}}.$$

$$h_{j+1} = \frac{C_{1j}}{2C_2}(1 + \alpha) - R + Rh_j + \left[\frac{BP_{av}}{BP_{i,j+1}}\right]\left[\frac{D(MC_{ij}) - MC_{i,j+1}}{2C_2V_{i,j+1}}\right]. \qquad [47]$$

$$\left[\frac{CA_{i,j+1} + CU_{i,j+1}V_{i,j+1}}{D^{j+1}}\right]. \quad [45]$$

The quality-premium feature of the PNW algorithm presumes that price calculated from the demand curve is based on an average-sized tree, which remains constant over time. Trees differing from average size would command different prices. The average or base price determined from the demand curve is adjusted to reflect those differences according to the relationships existing between tree age and price in the base year. However, even though demand curves in all periods are based on the average-sized tree in the base period, actual tree size is *not* assumed constant over time. In each period, the algorithm adjusts the base demand-curve to reflect the age of trees actually harvested, in effect creating a new demand curve for each age class (Fig. 8). Price per unit harvested adjusts to reflect quality premiums existing in the base year.

cultural treatment costs

Harvest costs may include future expenditures necessitated by harvesting (for example, planting costs). To incorporate these into the linkage equation:

Let

CTC = future costs per acre attributable to harvest, discounted to the time of harvest.

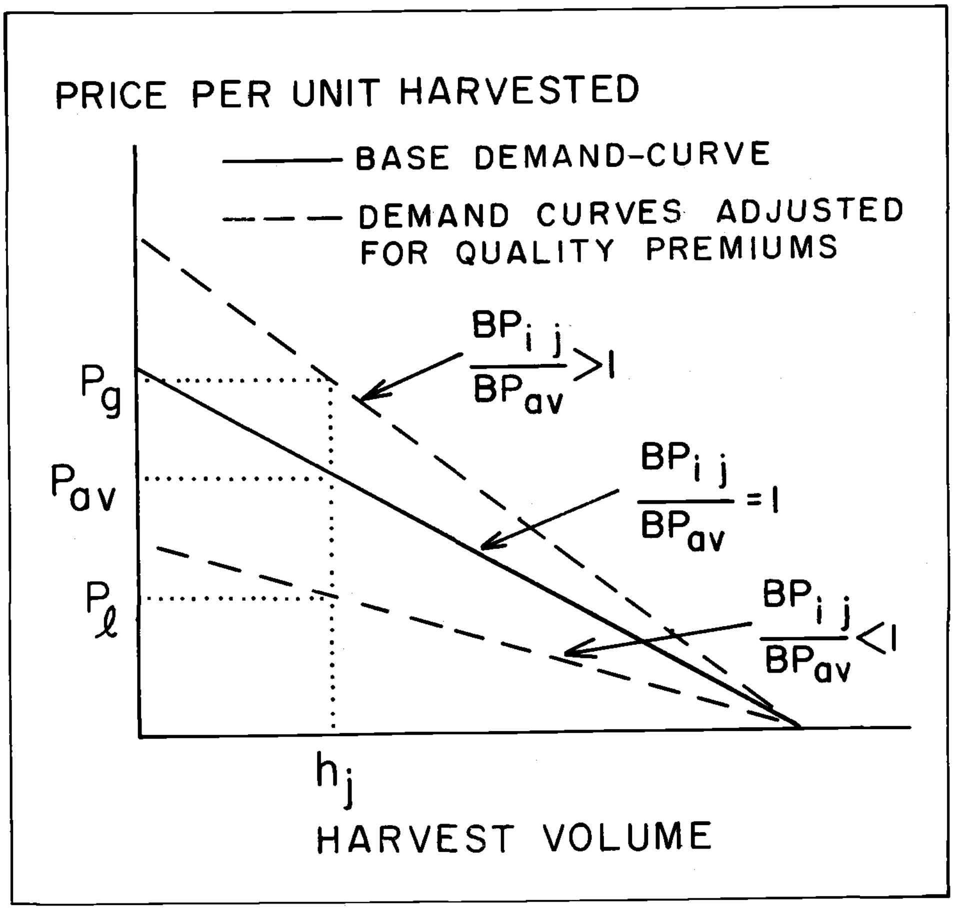


Figure 8.

Demand curve adjustments for quality premiums. For a given harvest level (h_j), P_g is the price for age classes with $BP_{ij} > BP_{av}$; P_{av} for age classes with $BP_{ij} = BP_{av}$; and P_ℓ for age classes with $BP_{ij} < BP_{av}$.

$$MC_{ij} = CA_{ij} + CU_{ij}V_{ij} + CTC.$$

$$MC_{i,j+1} = CA_{i,j+1} + CU_{i,j+1}V_{i,j+1} + CTC.$$

Then the linkage equation [45] still holds.

exogenous harvests

Harvests for commercial thinning, mortality salvage, and species conversion require two changes in the PNW algorithm. First, because these harvests are predetermined, they must be taken as specified—that is, thinning cannot be reduced when harvest calculated from the linkage equation is less than the amount to be taken in thinning.

Second, it is assumed in the linkage age-class that acres cannot be clearcut and thinned in the same period. This assumption is made *only* for the linkage age-class and *only* to compute the net marginal return of the last acre cut. Reported volumes and costs of intermediate (thinning) and final harvests are unaffected by this assumption. If the last acre clearcut in a period were thinned the same period, the net return from clearcutting would be the increase in marginal net revenue from clearcutting over and above what would have been gained from thinning.

The following linkage equation is used in TREES:

Let

MR_{ij} = marginal revenue from a clearcut on age class i, period j
$= (C_{1j} - C_2 h_j) V_{ij}$.

$MR_{i,j+1}$ = marginal revenue from a clearcut on age class i, period j+1
$= [(1+\alpha)C_{1j} - 2C_2 h_{j+1}] (V_{i,j+1} - T_{ij})$.

MC_{ij} = marginal cost from a clearcut on age class i, period j
$= CA_{ij} + CU_{ij}V_{ij} + CTC$.

$MC_{i,j+1}$ = marginal cost from a clearcut on age class i, period j+1
$= CA_{i,j+1} + CU_{i,j+1} (V_{i,j+1} - T_{ij}) + CTC$.

MRT_{ij} = marginal revenue from thinning age class i, period j
$= (C_{1j} - C_2 h_j) T_{ij}$.

MCT_{ij} = marginal cost from thinning age class i, period j
$= CAT_{ij} + CUT_{ij}T_{ij}$.

T_{ij} = thinning taken in age class i, period j.

CAT_{ij} = cost per acre of thinning age class i, period j.

CUT_{ij} = cost per unit of volume taken in thinning age class i, period j.

Thus the linkage equation [45] becomes

$$\left[\left(\frac{BP_{ij}}{BP_{av}}\right)\left(\frac{MR_{ij}}{D^j}\right) - \frac{MC_{ij}}{D^j}\right] - \left[\left(\frac{BP_{ij}}{BP_{av}}\right)\right.$$

Next-period harvest could be found by rearranging terms and breaking h_j and h_{j+1} out of the marginal revenue terms:

Let

$$R' = D/\left[\left(\frac{BP_{ij}}{BP_{i,j+1}}\right)\left(\frac{V_{ij} - T_{ij}}{V_{i,j+1} - T_{i,j+1}}\right)\right.$$

$$h_{j+1} = \frac{C_{1j}}{2C_2}(1 + \alpha) - R' + R'h_j +$$

incorporating productivity classes

To calculate the proportions of each age class to be harvested on all management units, unit volumes are summed together by age class and species report-group (see **User's Manual**). Cutting proceeds according to the harvest priority on the summed volumes until the harvest is satisfied. The last age class and species report-group to be cut become the linkage age-class and species report-group. For each GRU, the proportion harvested from the linkage age-class is applied to the proper age class and species report-group for all MIs and stocking classes. Resulting harvest volumes, harvest costs, and base revenues are adjusted by quadratic interpolation of the standard values for the appropriate

$$\left(\frac{MRT_{ij}}{D^{j}}\right) - \frac{MCT_{ij}}{D^{j}}\Bigg] = \left[\left(\frac{BP_{i,j+1}}{BP_{av}}\right)\left(\frac{MR_{i,j+1}}{D^{j+1}}\right) - \frac{MC_{i,j+1}}{D^{j+1}}\right]. \quad [48]$$

$$\Bigg] = \frac{\text{discount factor}}{\text{value growth ratio, net of thinning}}.$$

$$\left(\frac{BP_{av}}{BP_{i,j+1}}\right)\left(\frac{1}{2C_2}\right)\left[\frac{D(MC_{ij} - MCT_{ij}) - MC_{i,j+1}}{V_{i,j+1} - T_{i,j+1}}\right]. \quad [49]$$

MIs. Adjusted quantities are summed over all GRUs. The sums of harvest volumes in the last age class, harvest costs, and base-price total revenues are then divided by total acres harvested in the final age class to determine per-acre values. The final values represent weighted averages of the values for individual management units.

ending-condition checks

One of two ending conditions may be specified in the PNW algorithm in TREES: (1) harvest equals growth (see Eq. [6]), or (2) harvest all volume above a specified age class (see Eq. [7]). Using an ending condition requires an additional feasibility check on last-period harvest—if last-period harvest is too high, lower the harvest and begin again; if too low, return to the first period and raise the harvest. Ending conditions may limit feasible PNW harvest schedules to a narrower range than was previously the case. In fact, including ending conditions often will cause infeasibilities.

policy considerations

The PNW method, highly complex, produces a harvest schedule that would maximize profits to the producer. Producer profits are at a maximum *when marginal cost (including opportunity costs) of the last unit harvested equals marginal revenue.* PNW finds a harvest schedule for which the last (marginal) acre in any period would not contribute more to net discounted profits if held until the next period's harvest. A demand curve must be specified to determine marginal revenues. Compared with PNB, PNW reduces harvest volume and raises stumpage prices. (See the policy considerations for the PNB approach for more detail.)

Present Net Benefit (PNB)

applying the method

The PNB harvest scheduling method is analogous to PNW *except* that, instead of attempting to maximize discounted net revenue, PNB seeks to maximize consumer-plus-producer surplus, the net discounted difference between the area under the demand curve (a measure of the consumer's "willingness to pay") and total cost. Consumers, in this case, would be purchasers of stumpage.

Assuming a static linear demand curve and constant unit costs, we may formulate the maximization problem as:

$$\text{Maximize} \sum_{j=1}^{N} \frac{1}{D^j} \int_0^{h_j} \left[C_1 - C_2 h_j - C_3\right] dh_j. \qquad [50]$$

Once integrated, Eq. [50] becomes

$$\text{Maximize} \sum_{j=1}^{N} \frac{C_1 h_j - \frac{C_2 h_j^2}{2} - C_3 h_j}{D^j}. \qquad [51]$$

For this objective function and the constraints previously mentioned, we can derive K-T3 by differentiation with respect to x_{ij}. Equation [51] (the profit function π) becomes

$$\frac{\delta \pi}{\delta x_{ij}} = \sum_{j=1}^{N} (C_1 - C_2 h_j) V_{ij} - C_3 V_{ij} = \lambda_i - \ell_j - \mu_{ij} \qquad [52]$$

$i = 1, \ldots, j - Z$
$j = 1, \ldots, N.$

This differs from K-T3 for PNW only in that the slope term C_2 has a coefficient of 1 rather than 2. Multiplying the slope term by 1 will generally produce higher harvest levels in early periods and lower levels later on (Fig. 9). Stability conditions remain unchanged.

policy considerations

The PNB method is extremely complex, producing a harvest schedule maximizing the sum of the benefits accruing to consumers and producers of stumpage. Net discounted benefits are at a maximum when *marginal costs of the*

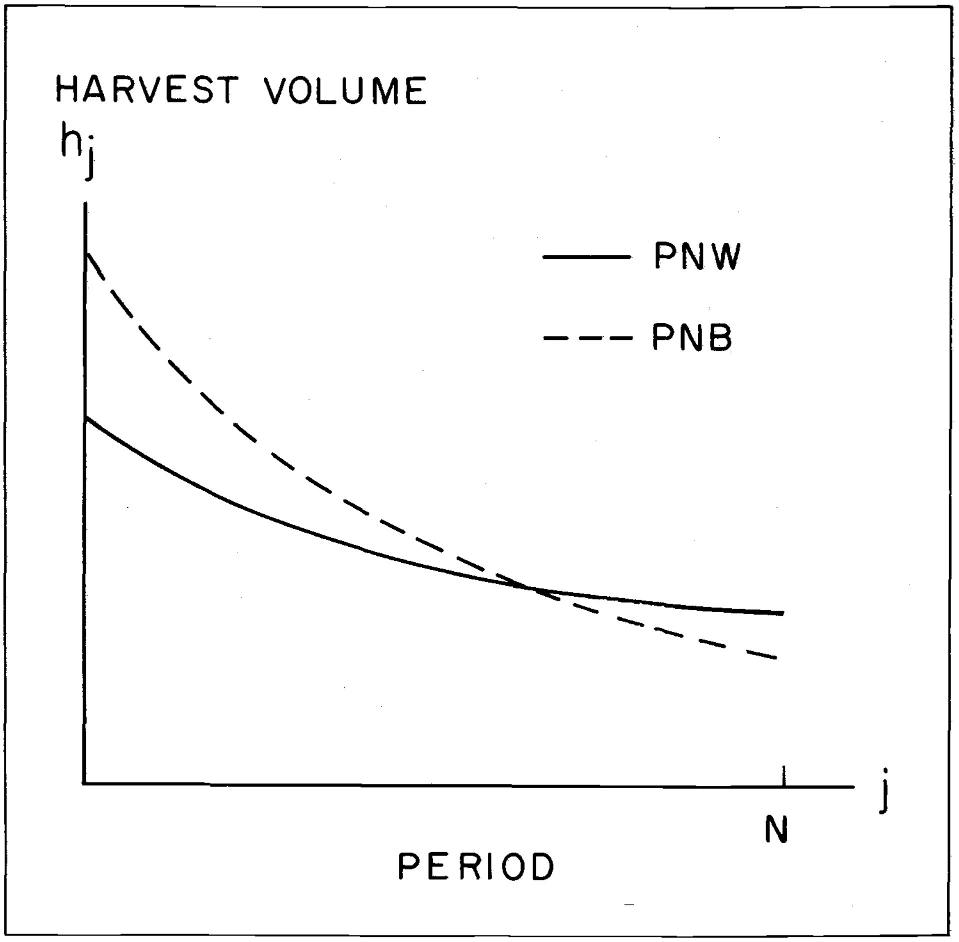


Figure 9.

PNW and PNB harvest schedules.

last unit harvested equal price. (Marginal costs must include opportunity costs as well as direct harvest costs.) Harvest volumes are allocated over time so that the last (marginal) acre harvested in any period would not contribute more to net discounted benefits if held for harvest until the next period. A demand curve for stumpage from the stumpage supplier(s) must be specified to calculate marginal benefit.

Three vital elements must be weighed in examining such a harvest schedule: demand curve, value growth rate, and discount rate. Rotation age will equal the age where the value growth rate just equals the discount rate. Although the growth rate (and, thus, value growth rate) is largely uncontrollable, we know that the specified demand curve and discount rate will significantly affect the resulting harvest schedule, assuming a feasible harvest schedule exists. Therefore, determining a demand curve and discount rate is a major policy step.

Volume flows and acres cut will initially be erratic, depending on starting inventory, but regulation may occur if a feasible solution is found. Yet the effects of adding or deleting inventory or intensifying management practices are difficult to identify. An overabundance of species or sites may increase stability problems and, thus, the chances of an infeasible harvest.

PNB and PNW Highlights

- Very complex to specify.
- Volume flows are probably erratic, depending on initial inventories.
- Acres initially cut will be erratic.
- Adding or deleting inventory produces uncertain results.
- Regulation (or close to it) may occur during the planning horizon if a feasible harvest is found.
- Only economic criteria are considered in setting harvests.

growth algorithms

AD Options

Growth and yield in AD stands are based on volume tables. Seven different tables, one for each MI, may be entered for each GRU; each one gives "standard" or "normal" estimates of live volume per acre for every age class for the MI selected. These volume tables serve as base estimates of yield that may be used in several different ways, depending on the stocking level of the stand and the growth option chosen.

Volume tables may be entered as tables, as equations predicting net yield as a function of age, or as a combination of the two. Constant, linear, quadratic, or cubic functions of age may be used. Equation limits may be specified to narrow the range of estimates predicted by the equation (see Appendix D).

standard yield

We will assume that standard yields are defined by the cubic equation

$$SV_i = b_0 + b_1A_i + b_2(A_i)^2 + b_3(A_i)^3 \qquad [53]$$

where:

SV_i = standard volume per acre, net of mortality, age class i.

A_i = midpoint age of age class i.

b_0, b_1, b_2, b_3 = user-defined coefficients.

In the standard yield option, the volume per acre in each age class in the first simulation period is the volume entered in the BRU inventory. After the first harvest, all acres in a GRU are assumed to have the standard volume for their respective age classes and MIs. No adjustments are made for differences in stocking levels defined by the starting inventory. For second-entry shelterwood stands, volume is assumed to be the standard volume multiplied by the proportion of volume left after first entry. If thinning is to occur, both the volume decrease and the growth pattern after thinning must be reflected in the standard volume equation itself.

approach to normality

The standard yield option generally is applied to cases in which the user wants simply to estimate inventory and growth. But for more pragmatic problems, an approach-to-normal option[14] may prove more satisfactory. Approach to normality is based on the presumption that stands not normally stocked will tend to become so over time (see Bruce's treatment in McArdle et al. 1949).

In TREES, the approach-to-normal option can be applied either to normal net growth (the difference between normal yields in two periods) (U.S. Forest Service 1963) or to normal volume (McArdle et al. 1949). In either case, the user estimates the normal or standard yield per acre by age class for each MI and specifies a linear approach-to-normal function to predict the proportion of normality *after* growth, given the

[14] Normality is defined here as full or optimal stocking with respect to volume.

proportion *before*. Yield prediction proceeds as described in Johnson et al. (1976).

Approach-to-normal volume:

$$V_{i+1,j+1} = SV_{i+1} \left[\alpha + \beta\left(\frac{V_{ij}}{SV_i}\right)\right]. \quad [54]$$

Equation [54] may be rewritten:

$$V_{i+1,j+1} = \left[b_0 + b_1A_{i+1} + b_2\left(A_{i+1}\right)^2 + b_3\left(A_{i+1}\right)^3\right] \times \left[\alpha + \beta\left(\frac{V_{ij}}{b_0 + b_1A_i + b_2(A_i)^2 + b_3(A_i)^3}\right)\right] \quad [55]$$

where:

V_{ij} = actual net volume per acre, age class i, after harvest in period j.

$V_{i+1,j+1}$ = volume per acre, age class i+1, before harvest in period j+1.

A_i = midpoint age (in years) of age class i.

A_{i+1} = midpoint age (in years) of age class i+1.

SV_i = standard or normal volume per acre, age class i.

SV_{i+1} = standard or normal volume per acre, age class i+1.

b_0, b_1, b_2, b_3 = user-supplied constants defining standard values as functions of age.

α, β = user-supplied constants defining the approach-to-normal function.

If A_i is the maximum age class, then $SV_{i+1} = SV_i$ (i.e., no change in standard volume, but actual volume may change through the approach-to-normal function).

Approach-to-normal growth:

$$V_{i+1,j+1} = V_{ij} + \left(SV_{i+1} - SV_i\right)\left[\alpha + \beta\left(\frac{V_{ij}}{SV_i}\right)\right] \quad [56]$$

where:

$SV_{i+1} - SV_i$ = normal growth

or, equivalently,

$$V_{i+1,j+1} = V_{ij} + \left[b_0 + b_1 A_{i+1} + b_2 (A_{i+1})^2 + b_3 (A_{i+1})^3 - b_0 - b_1 A_i - b_2 (A_i)^2 - b_3 (A_i)^3 \right]$$

$$\times \left[\alpha + \beta \frac{V_{ij}}{b_0 + b_1 A_i + b_2 (A_i)^2 + b_3 (A_i)^3} \right] . \qquad [57]$$

When A_i is the maximum age class, SV_{i+1} equals the standard volume for the oldest age class and SV_i, the standard volume for the next oldest age class (i.e., standard normal growth of the last age class is the same as standard growth of the next-to-last age class).

growth after thinning

If the user chooses approach to normality, three options are available for growth after thinning. [15] Growth may (1) continue according to approach to normality, (2) be specified as a percentage of normal net growth, or (3) be specified as a percentage of normal gross growth.

For the first option, yield after growth is defined by either Eq. [54] or [56], where V_{ij} is the volume per acre left in age class i after thinning.

For the second option,

$$V_{i+1,j+1} = V_{ij} + p(SV_{i+1} - SV_i) \qquad [58]$$

[15] The only option available to the oldest and next oldest age classes for growth after thinning is approach to normality. Thinning would not normally occur in these age classes; however, in areas where the rotation age is low, this restriction for the two oldest age classes will hold.

where:

V_{ij} = volume per acre remaining in age class i after thinning in period j.

p = user-specified proportion.

For the third option,

$$V_{i+1,j+1} = V_{ij} + p(SV_{i+1} - SV_i + SM_{i+1}) \quad [59]$$

where:

SM_{i+1} = standard mortality per acre for age class i+1, specified as a function of age in the same manner as standard net yield.

DD Options

Growth in DD stands is assumed to take place between harvests (i.e., between the midpoints of each simulation period). Estimating growth includes estimating (1) mortality, (2) diameter growth, and (3) ingrowth and upgrowth, all of which are calculated separately for softwoods and hardwoods in each stand size-class in a GRU.

mortality

The proportion of trees in a diameter class that die each period is considered a function of the midpoint diameter of that class and of the total stand basal area.[16] After mortality rate for each diameter class has been determined, the appropriate number of trees is removed from each class and diameter growth estimated on the remaining live trees.

The assumed form of the relationship is:

$$MRP_{ij} = b_0 + b_1[BA_j/(D_i)^3] \quad [60]$$

where:

MRP_{ij} = predicted mortality rate for diameter class i in period j.

BA_j = total stand basal area per acre (in square feet) after harvest in period j

$$= \sum_i [(D_i)^2(T_{ij} - C_{ij})(0.00545415)].$$

D_i = midpoint diameter (in inches) of the i^{th} diameter class.

T_{ij} = trees per acre in diameter class i before harvest in period j.

C_{ij} = trees per acre harvested from diameter class i in period j.

b_0, b_1 = user-supplied constants entered for softwoods and hardwoods.

Estimates of the coefficients b_0 and b_1 may be based on data from a wide variety of stand types and site classes. The program calibrates estimates to the stand conditions for an individual GRU by the following adjustment:

Let

MRS_i = standard mortality rate (mortality rate in diameter class i) measured for the stand in a chosen base period.

BAS = standard basal area (basal area of the stand in the base period for which stand mortality rate is measured).

MR_{ij} = mortality rate calibrated to stand conditions, diameter class i, period j.

$MRSP_i$ = mortality rate predicted for diameter class i using Eq. [60] with standard basal area substituted for total stand basal area.

The adjusting equation is:

$$MR_{ij} = MRS_i \left[\frac{b_0 + b_1(BA_j/D_i^3)}{b_0 + b_1(BAS/D_i^3)}\right] = MRS_i \left[\frac{MRP_{ij}}{MRSP_{ij}}\right]. \quad [61]$$

When the basal area diverges from the standard, the change in actual mortality rate (MR_{ij}) from standard mortality rate (MRS_i) is proportional to the change in mortality rate predicted using Eq. [60].

[16] Stand refers to a given size class in a GRU.

Equation [60] and its adjustment in [61] are derived from the U.S. Forest Service TRAS growth-estimation procedure used in the Pacific Northwest (Larson and Goforth 1974). Equation [60] predicted annual mortality rates on U.S. Forest Service growth plots with R^2 = 0.80 and SE = ±0.0017 (personal communication from Thomas D. Farrenkopf, U.S. Forest Service, Portland, Oregon, 1971).

Over the long term, basal areas may build up beyond the range used in estimating Eq. [60]. At such high basal areas, mortality rates predicted by Eq. [61] may prove too low. A maximum limit may be placed on basal area to prevent unreasonable buildup so when that limit is reached, the stand is frozen; no further changes occur in stand distribution through mortality or growth. Should harvest reduce the stand basal area below the maximum limit, mortality again will be estimated with Eq. [61].

diameter growth

Estimating diameter growth is similar to estimating mortality rates. After diameter growth for each diameter class has been determined, trees are moved from one diameter class to another to reflect this growth. Equations are used to predict standard diameter growth rates and to adjust those rates for changes in basal area. Diameter growth is assumed to be a function of the diameter-class midpoint, total net basal area per acre, and the cross product of the two.

$$DGP_{ij} = b_0 + b_1D_i + b_2BAN_j + b_3D_i(BAN_j)^2 \qquad [62]$$

where:

DGP_{ij} = diameter growth (in inches) predicted for diameter class i, period j.

D_i = midpoint diameter of diameter class i.

BAN_j = net basal area per acre, period j, after harvest and mortality have been subtracted.

b_0,b_1,b_2,b_3 = user-estimated coefficients.

Equation [62] is again taken from TRAS estimates of growth rates in the Pacific Northwest (Larson and Goforth 1974). When applied to U.S. Forest Service plot data, it predicted annual radial growth with R^2 = 0.95 and SE = ±0.008 inches (personal communication from Thomas D. Farrenkopf, U.S. Forest Service, Portland, Oregon, 1971). To calibrate Eq. [62] to the growth of an individual stand, the user estimates stand diameter growth at the same standard basal area used to estimate mortality. The adjustment equation is:

$$DG_{ij} = DGS_i \left[\frac{b_0 + b_1D_i + b_2BAN_j + b_3D_i(BAN_j)^2}{b_0 + b_1D_i + b_2BAS + b_3D_i(BAS)^2} \right]$$

$$= DGS_i \left[\frac{DGP_{ij}}{DGSP_i} \right] \qquad [63]$$

where:

DG_{ij} = diameter growth (in inches) for diameter class i, period j.

DGS_i = standard diameter growth, diameter class i, at the standard basal area per acre.

BAS = standard basal area.

$DGSP_i$ = predicted diameter growth using Eq. [62] with standard basal area substituted for net basal area.

In Eq. [63], actual diameter growth is adjusted for changes in basal area from the standard basal area in the same proportion as the predicted diameter growth rates using Eq. [62]. This adjustment procedure, as well as that estimating mortality, allows Eqs. [60] and [62] to be based on different period lengths than the standard diameter growth and mortality rates. In *Timber for Oregon's Tomorrow* (Beuter et al. 1976), for example, Eqs. [60] and [62] predicted annual mortality rates and radial growth, whereas Eq. [61] predicted mortality rates for a 10-year period.

ingrowth and upgrowth

Ingrowth—the number of trees growing into the smallest diameter class—is assumed to be at least enough to maintain a constant number of trees in that diameter class at the level of the starting inventory. The user may increase ingrowth into that diameter class but may not decrease it below the minimum. Thus, in each period j:

$$I_{1j} = C_{1j} + M_{1j} + U_{1j} + IN \quad [64]$$

where:

I_{1j} = ingrowth into the smallest diameter class, period j.[17]

C_{1j} = trees cut from the smallest diameter class, period j.

M_{1j} = trees that die in the smallest diameter class, period j.

U_{1j} = trees that grow out of the smallest diameter class, period j.

IN = additional ingrowth specified by the user (entries may be made for both softwoods and hardwoods).

Upgrowth is the number of trees growing into higher diameter classes. The number of trees available for harvest in period j+1 in diameter class i is

$$T_{i,j+1} = T_{ij} - C_{ij} - M_{ij} + U_{i-1,j} - U_{ij} \quad [65]$$

where:

T_{ij} = trees in diameter class i, period j, before harvest in period j.

$T_{i,j+1}$ = trees in diameter class i, period j+1, before harvest in period j+1.

C_{ij} = trees harvested from diameter class i, period j.

M_{ij} = trees that die in diameter class i, period j, after harvest in period j.

$U_{i-1,j}$ = upgrowth into diameter class i from smaller diameter classes in period j.

U_{ij} = upgrowth from diameter class i, period j.

Upgrowth for each class is estimated with a unique combination of curvefitting to the distribution of trees by diameter and calculation of areas beneath the estimated curve (Fig. 10). This distribution of trees by diameter is

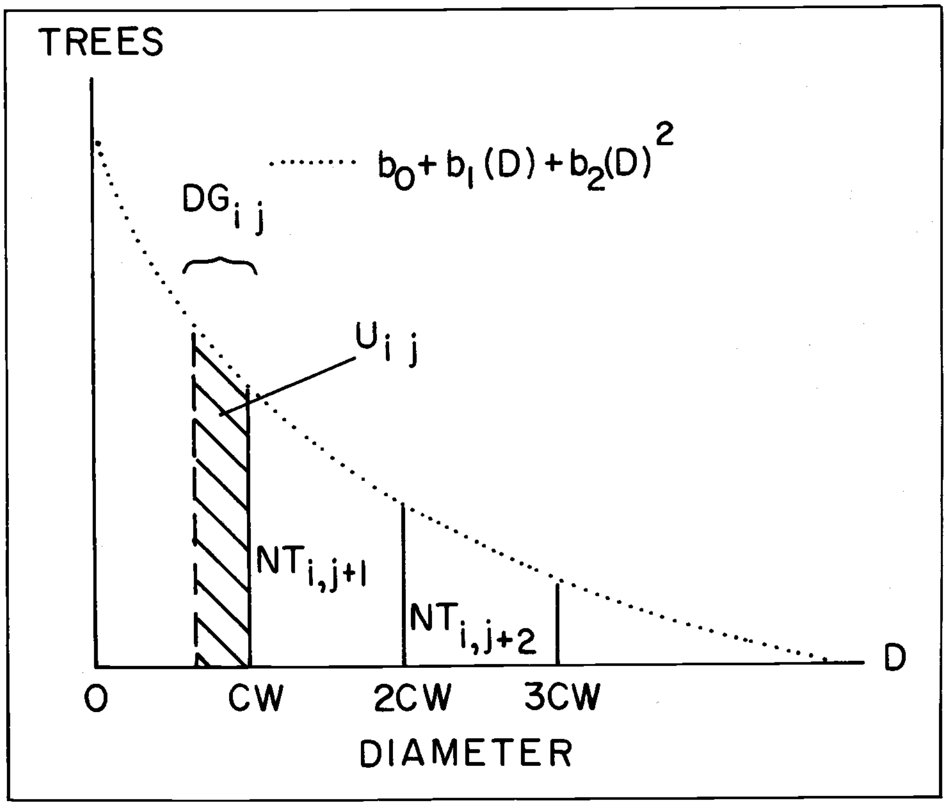


Figure 10.

Estimating upgrowth for DD stands. The area under the curve from 0 to CW is NT_{ij}.

[17] To avoid confusing and complex subscripts, the theory presented is based on one growth cycle per period. The TREES model is capable of growing the stand a variable number of times during a period interval.

assumed to be a quadratic function of diameter whose parameters (b_0, b_1, and b_2) are estimated within the program using the trees in diameter class i and the next two larger diameter classes. If trees are quadratically distributed, the area under the estimated curve in each diameter class must equal the number of trees in that class. Therefore, we can form three equations in the three unknowns (b_0, b_1, and b_2) and solve for the coefficients:

$$\int_0^{CW} b_0 + b_1(D) + B_2(D)^2 = NT_{ij} \qquad [66]$$

$$\int_{CW}^{2CW} b_0 + b_1(D) + b_2(D)^2 = NT_{i+j,j} \qquad [67]$$

$$\int_{2CW}^{3CW} b_0 + b_1(D) + b_2(D)^2 = NT_{i+2,j} \qquad [68]$$

where:

CW = width (in inches) of one diameter class.

D = tree diameter.

NT_{ij} = net trees per acre in period j, diameter class i, after subtracting harvest and mortality.

Performing the integrations, we may rewrite Eqs. [66] through [68] as:

$$b_0(CW) + b_1\left(\frac{CW^2}{2}\right) + b_2\left(\frac{CW^3}{3}\right) = NT_{ij}. \qquad [69]$$

$$b_0(CW) + b_1\left(\frac{3CW^2}{2}\right) + b_2\left(\frac{7CW^3}{3}\right) = NT_{i+1,j}. \qquad [70]$$

$$b_0(CW) + b_1\left(\frac{5CW^2}{2}\right) + b_2\left(\frac{19CW^3}{3}\right) = NT_{i+2,j}. \qquad [71]$$

Solving this system, we find:

$$b_0 = \frac{[11(NT_{ij}) - 7(NT_{i+1,j}) + 2(NT_{i+2,j})]}{6CW}. \qquad [72]$$

$$b_1 = \frac{[-2(NT_{ij}) + 3(NT_{i+1,j}) - NT_{i+2,j}]}{6CW^2}. \qquad [73]$$

$$b_2 = \frac{[NT_{ij} - 2(NT_{i+1,j}) + NT_{i+2,j}]}{2CW^3}. \qquad [74]$$

Diameter growth (DG_{ij}) is calculated for diameter class i as shown in Eq. [63]. If we assume all trees in a diameter class grow at the same rate as trees having the midpoint diameter[18], all trees in class i within DG_{ij} of the upper class boundary will grow out of the class. The number of trees growing out of the class can be estimated by finding the area under the curve between $CW-DG_{ij}$

[18] As diameter class width increases or as growth periods decrease, the average diameter of trees moving out of a class is probably above midpoint. If rate of diameter growth is increasing with diameter, then upgrowth will be underestimated; if the converse, upgrowth will be overestimated.

and CW (Fig. 10). This can be written:

$$\int_{CW-DG_{ij}}^{CW} b_0 + b_1(D) + b_2(D^2) = U_{ij}. \quad [75]$$

If we use the values b_0, b_1, and b_2 from Eqs. [72] through [74], integrating and evaluating Eq. [75] yields the upgrowth estimate:

$$U_{ij} = \frac{DG_{ij}}{6CW^3}\Big[NT_{ij}(CW + DG_{ij})(2CW + DG_{ij})$$

$$+ NT_{i+1,j}(5CW + 2DG_{ij})(CW - DG_{ij})$$

$$- NT_{i+2,j}(CW + DG_{ij})(CW - DG_{ij})\Big]. \quad [76]$$

After upgrowth from diameter class i has been estimated, U_{ij} is subtracted and $U_{i-1,j}$ added to the number of trees in diameter class i; then i is incremented (i=i+1) and upgrowth calculated for the succeeding diameter classes using the same procedure until only two classes remain. When i is the next-to-last class, upgrowth is calculated using a curve fitted to the trees in classes i-1, i, and i+1. Because the trees in class i+1 have already been adjusted to reflect growth, this procedure may cause some inconsistency in upgrowth estimates for the next-to-last diameter class. No upgrowth is calculated for the last diameter class, which is assumed to include all trees above the lower class boundary.

As upgrowth is calculated, the decision must be made into what class or classes trees will be moved. In TREES, the following rules apply:

- If $DG_{ij} \leqslant CW$, move U_{ij} to one class above.
- If $DG_{ij} \geqslant 2CW$, move U_{ij} to two classes above.
- If $CW < DG_{ij} < 2\,CW$, move $U_{ij}[2 - (DG_{ij}/CW)]$ trees one class above and move U_{ij} $[(DG_{ij}/CW) - 1]$ trees two classes above.

The third rule assumes that trees to be moved are evenly distributed by diameter (in contrast to the assumption used to calculate upgrowth). For most downward-sloping distribution curves, trees moving into the next diameter class will be underestimated; those moving to the diameter class two classes above will be overestimated. Trees can, at most, skip one diameter class during the growth period; if selected growth periods are long or diameter class widths narrow, this limit suppresses the specified diameter growth.

Calculating upgrowth in TREES may be contrasted with the procedures used in TRAS (Larson and Goforth 1974), which uses either the Q method or the nonlinear interpolation (NLI) method. In both instances, curves are fitted successively to *cumulative* distribution curves, and upgrowth is calculated by shifting the cumulative curves by the amount of diameter growth.

The Q method assumes that the cumulative distribution curve is exponentially distributed (Fig. 11).

$$AT = Ke^{-a(D)} \qquad [77]$$

where:

AT = cumulative number of trees larger than diameter D.

D = diameter.

K, a = constants to be estimated.

For a given diameter class i, upgrowth is calculated by fitting an exponential curve at AT_{ij} and $AT_{i+1,j}$ where AT_{ij} is the number of trees greater than the lower limit of diameter class i (DL_i) in growth period j, *including the trees that will die in growth period j.*

The estimated curve is shifted horizontally by the diameter growth of a tree with the diameter DL_{i+1} (i.e., trees growing out of diameter class i are assumed to be growing at the same rate as trees at the lower limit of the next class). Upgrowth can then be directly calculated as the difference in number of trees larger than DL_{i+1} before and after growth ($AT_{i+1,j+1}$ - $AT_{i+1,j}$). A new curve is estimated and upgrowth similarly calculated for each diameter class.

Assuming that the cumulative distribution curve is exponentially distributed between DL_i and DL_{i+1} implies that the underlying distribution of trees in diameter class i is also exponential. If f(D) is the distribution of trees per acre by diameter, then according to the Q method

$$AT_{ij} - AT_{i+1,j} = T_{ij} - C_{ij} \qquad [78]$$

or, equivalently,

$$Ke^{-a(DL_i)} - Ke^{-a(DL_{i+1})} = \int_{DL_i}^{DL_{i+1}} f(D). \qquad [79]$$

Equation [79] can be true only if $f(D) = aKe^{-a(D)}$ [i.e., if f(D) is exponentially distributed over diameter class i]. Because the cumulative distribution curve must

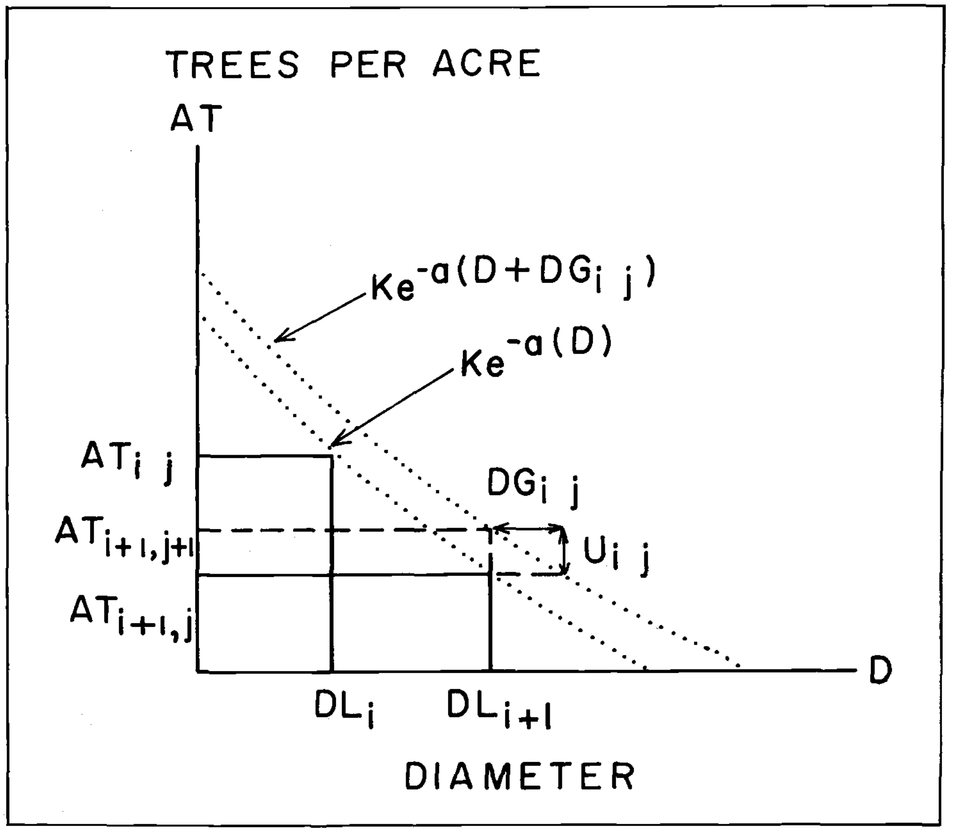


Figure 11.

The Q method.

decline with diameter, "a" must be positive, implying that *trees per acre must decline exponentially within a diameter class.*

The NLI method assumes that the cumulative distribution curve over any three lower-class boundaries can be described by a second-degree function of diameter (a parabola, as in Fig. 12). In contrast to the Q method, the curve is fitted *after* diameter growth has taken place; but like the Q method, a new curve is estimated to calculate upgrowth for each diameter class except the last two.

A cumulative distribution curve that is quadratic implies that the actual distribution of trees per acre by diameter is linear over classes i and i+1. Let f(D) be the distribution of trees by diameter. Then the NLI method implies:[19]

$$AT_{i,j+1} - AT_{i+1,j+1} = T_{i,j+1} \qquad [80]$$

and $$AT_{i+1,j+1} - AT_{i+2,j+1} = T_{i+1,j+1} \qquad [81]$$

or, equivalently,

$$b_0 + b_1(DL_i) + b_2(DL_i)^2 - [b_0 + b_1(DL_{i+1}) + b_2(DL_{i+1})^2]$$

$$= \int_{DL_{i+1}}^{DL_{i+2}} f(D). \qquad [82]$$

But Eq. [82] implies that f(D) = $b_1 + 2b_2(D)$ (i.e., the distribution of trees by diameter over diameter classes i and i+1 is linear).

[19] $T_{i,j+1}$ includes mortality for growth period j.

Several differences between the TREES and TRAS methods for calculating upgrowth are significant:

- **Mortality**—TREES subtracts mortality *before* fitting a curve to calculate upgrowth, but TRAS carries mortality until *after* upgrowth has been calculated. Where mortality is substantial, the differences in treatments could cause differences in upgrowth estimates.
- **Trees moving to higher classes**-TREES allows trees to be moved up two diameter classes in one growth period; in TRAS, trees may move up one class only.
- **Diameter growth**—In TREES, diameter growth is specified at the diameter-class midpoint; in TRAS, at the upper class boundary. If diameter growth is increasing with diameter, TRAS will calculate greater upgrowth estimates.
- **Last class**—Using a cumulative distribution curve in TRAS avoids problems in estimating

distributions when the last diameter class is open ended. In TREES, trees per acre in the last class are treated as if distributed over the width of one class when estimating curves for the next two lower classes. If trees accumulate in the last class, upgrowth from lower classes may be overestimated.

- **Type of distribution assumed**—As indicated previously, the Q method assumes that the distribution of trees within each diameter class is exponential [$aKe^{-a(D)}$]. Because the cumulative distribution curve is always downward sloping, "a" is necessarily positive. This implies that the number of trees must decline exponentially within each class.

The NLI method was designed for cases in which the exponential decline does not accurately portray the distribution. However, NLI presumes a linear distribution of trees within each diameter class, in effect describing a distribution with a series of linear segments. Inevitably, applying NLI to curvilinear distributions causes some inaccuracy. For example, NLI overestimates the number of trees at the lower boundary of diameter class i (Fig. 13); thus, upgrowth estimates will be biased downward.

TREES assumes that the distribution of trees by diameter

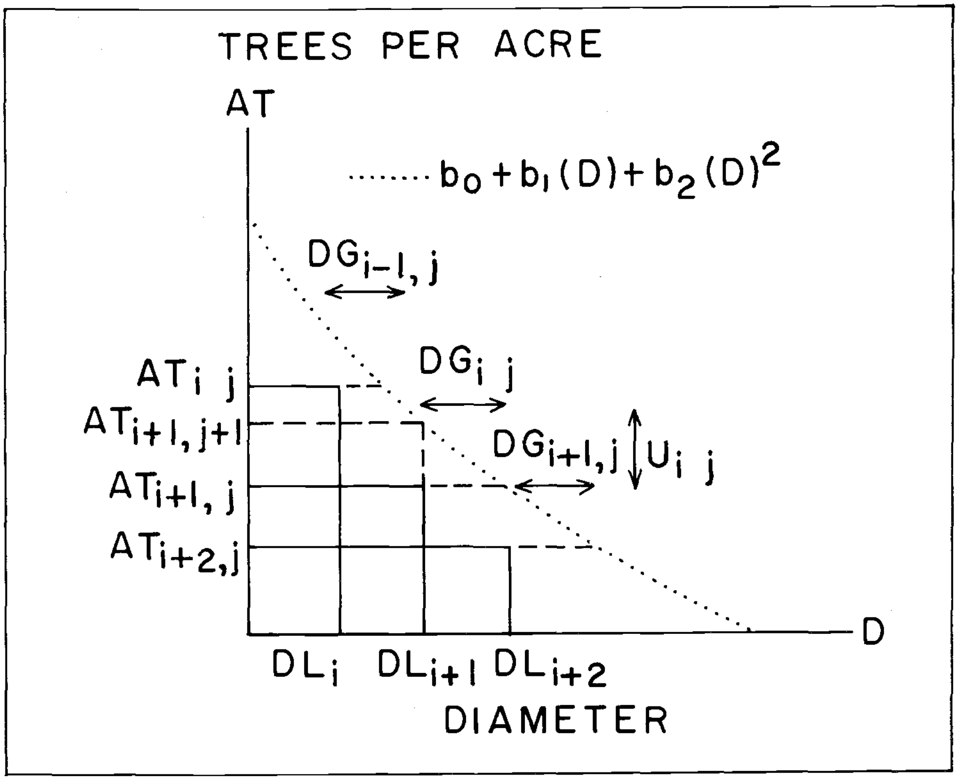


Figure 12.

The NLI method.

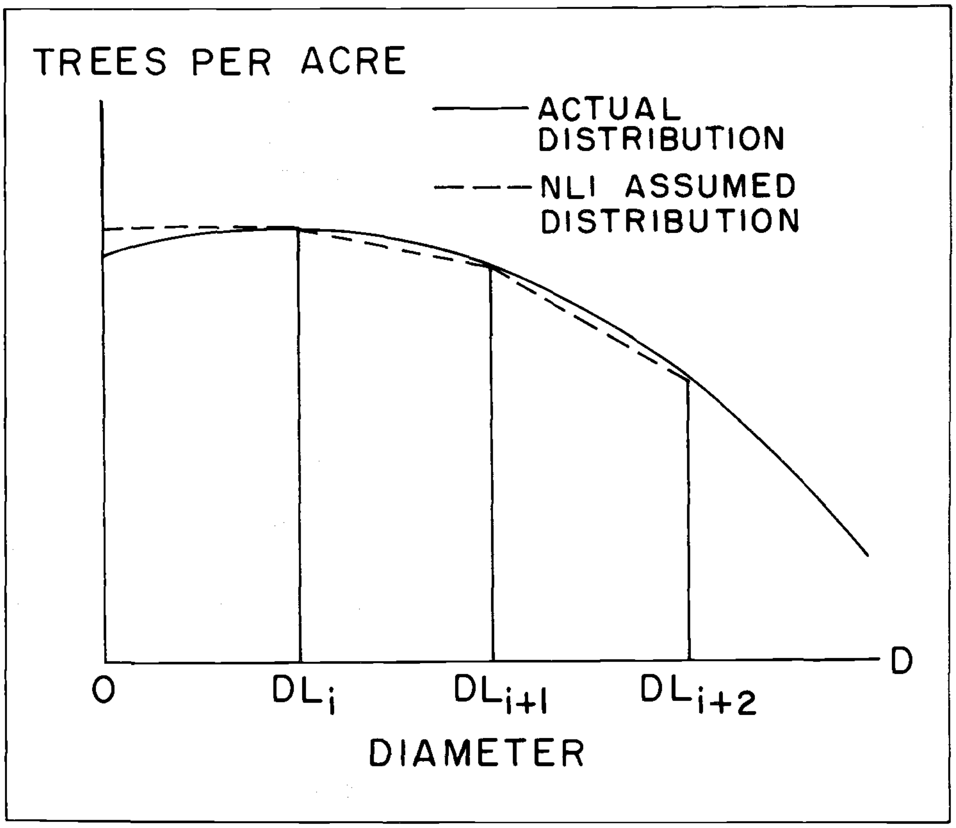


Figure 13.

Overestimating trees at lower boundary of diameter class i when using NLI method.

within each diameter class may be described by a quadratic function of diameter. Because quadratic curves can closely approximate exponential and linear forms as well as other curvilinear distributions, TREES offers greater flexibility than TRAS.

- **Inconsistent upgrowth estimates**— Unlike the Q method, NLI and TREES may give rise to upgrowth estimates exceeding the trees available in the class. When large differences exist between trees per acre in adjacent classes, the curves fit using NLI and TREES may become negative within the class for which upgrowth is to be calculated. (Exponential curves cannot become negative.) For the distribution shown in Figure 14 (no trees in diameter class i), the curve fit using TREES has a net area of 0 over class i, as required. However, the zero net area is achieved by a large negative area offsetting an equally large positive area. The positive area occurs at the upper class boundary, causing a positive estimate of upgrowth.

Such occurrences can be minimized by aggregating stands over sufficient area to eliminate gaps in diameter distribution. In addition, TREES requires that harvest increase with increasing diameter, which may prevent such gaps from appearing during a simulation.

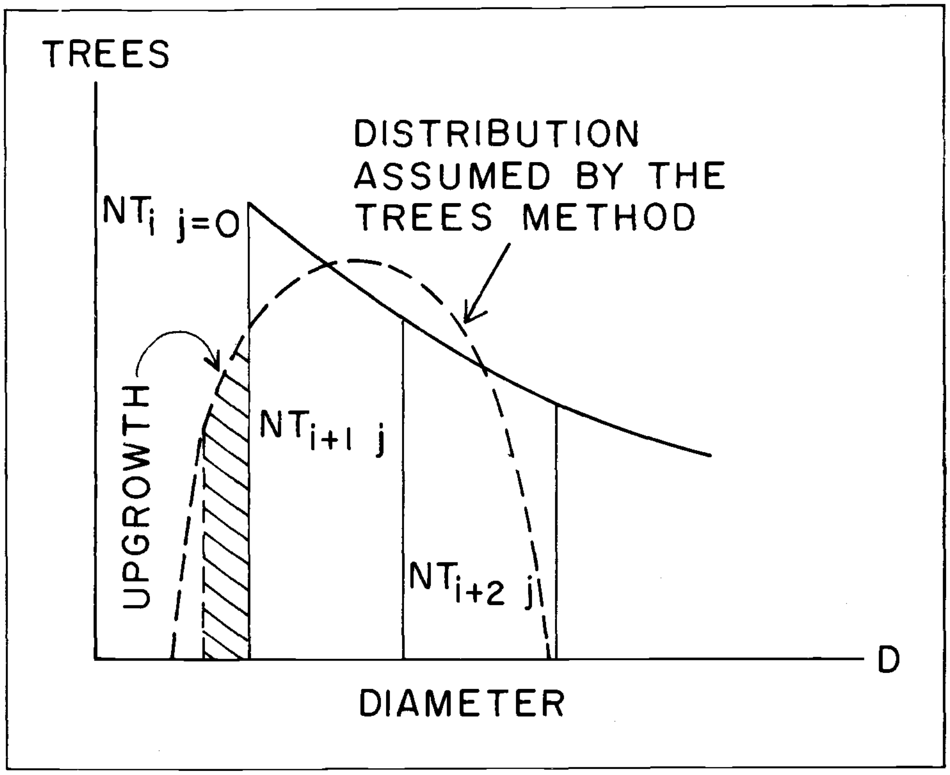


Figure 14.

Positive upgrowth predicted for diameter class i in period j when the number of trees in that class is 0.

literature cited

ADAMS, D. M. and A. R. EK. 1974. Optimizing the management of uneven-aged forest stands. Can. J. For. Res. 4(3):274-297.

BEUTER, J. H., K. N. JOHNSON, and H. L. SCHEURMAN. 1976. Timber for Oregon's tomorrow, an analysis of reasonably possible occurrences. Oreg. State Univ., For. Res. Lab., Corvallis. Res. Bull. 19. 111 p.

CHAPPELLE, D. E. and R. W. SASSAMAN. 1968. A computer program for scheduling allowable cut using either area or volume regulation during sequential planning periods. U.S. For. Serv., Pac. Northwest For. Range Exp. Stn., Portland, Oreg. Res. Note PNW-93. 9 p.

DAVIS, K. P. 1966. Forest Management. McGraw-Hill, New York, N.Y. 519 p.

DUERR, W. A. 1960. Fundamentals of Forestry Economics. McGraw-Hill, New York, N.Y. 579 p.

JOHNSON, K. N. and H. L. SCHEURMAN. 1977. Techniques for prescribing optimal timber harvest and investment under different objectives—discussion and synthesis. For. Sci. Monogr. 18. 31 p.

JOHNSON, K. N., H. L. SCHEURMAN, and J. H. BEUTER. 1976. The Oregon timber resources model—a brief introduction, p. 148-163 *in* Systems Analysis and Forest Resource Management. J. Meadows et al., eds. Society of American Foresters, Bethesda, Md.

KUNZI, H. P., W. KRELLE, and W. OUTTLI. 1966. Nonlinear Programming. Blaisdell Publishing Co., Waltham, Mass. 240 p.

LARSON, R. W. and M. H. GOFORTH. 1974. TRAS, a timber volume projection model. U.S. For. Serv., Washington, D.C. Tech. Bull. No. 1508. 15 p.

McARDLE, R. E., W. H. MEYER, and D. BRUCE. 1949. The yield of Douglas-fir in the Pacific Northwest. U.S. For. Serv., Washington, D.C. Bull. No. 201. 74 p.

SASSAMAN, R. W., E. HOLT, and K. BERGSVIK. 1972. User's manual for a computer program for simulating intensively managed allowable cut. U.S. For. Serv., Pac. Northwest For. Range Exp. Stn., Portland, Oreg. Tech. Rep. PNW-1. 50 p.

SCHEURMAN, H. L. and K. N. JOHNSON. 1975. Computational experience with a renewable resource (timber) harvest model. Oreg. State Univ., Dep. Stat., Corvallis. Tech. Rep. No. 47. 35 p.

SESSIONS, J. 1978. Stability conditions in ECHO modelling. For. Sci. 33(4):446-449.

U.S. FOREST SERVICE. 1963. Timber trends in western Oregon and western Washington. Pac. Northwest For. Range Exp. Stn., Portland, Oreg. Res. Pap. No. PNW-5. 154 p.

WAGNER, H. M. 1969. Principles of Operations Research with Applications to Management Decisions. Prentice-Hall, Inc., Englewood Cliffs, N.J. 937 p.

WALKER, J. L. 1971. An economic model for optimizing the rate of timber harvesting. Ph.D. Thesis, Univ. Wash. (Lib. Congr. Card Microfiche No. 71-28489). 117 p. Univ. Microfilms, Ann Arbor, Mich. [Diss. Abstr. 32(5):2276-A.]

WOLFE, P. 1962. Recent developments in nonlinear programming. Rand Corp., Santa Monica, Calif. 40 p.

ZANGWILL, W. I. 1966. Nonlinear Programming—A Unified Approach. Prentice-Hall, Inc., Englewood Cliffs, N.J. 356 p.

appendix A — detailed algorithms for the

Variable Definitions

All capitalized names are computer program variables. When program variables are used in expressions, FORTRAN notation, including computational hierarchies, is used.

ITRMCZ = harvest flag: 0 = unsuccessful—try again; 1 = infeasible—stop; or 2 = successful—go to the next period.

NPDECG = period to check ending condition. Last period of the look-ahead for EFV and EFFV; last period of the inner cycle for PNW and PNB.

NPDIG = current inner-cycle period.

NPDIX = NPDIG+1, the FORTRAN subscript corresponding to NPDIG.

NPDOG = current outer-cycle period.

NVPOPC = number of outer cycles. Because TREES starts with cycle 0, NVPOPC-1 is used for comparison.

MXPDOG = last period of the outer cycle (planning horizon).

OBJVL = objective function for EFV and EFFV.

PVLBIP = proportion used to set the initial binary search step-size.

PVLDCP = proportion used to set the maximum decrease in outer-cycle[1] harvest volumes from one period to the next for EFV and EFFV.

PVLDTP = proportion used to set the difference tolerance (i.e., how much the harvest can differ from the volume meeting the constraints and still be considered satisfactory).

PVLICP = proportion used to set the maximum increase in outer-cycle harvest volumes from one period to the next for EFV and EFFV.

PVLITP = proportion used to set the minimum binary search step-size.

VLBSIL = binary search step-size (amount by which VLBSPL is changed in the iterative search).

VLBSPL = trial harvest level.

VLEXC = exogenous harvest volume (including thinnings, mortality salvage, and species conversion).

VLDIFL = difference between proposed harvest volume and a harvest meeting one of the constraints.

VLHPPP = first trial harvest level, when no fixed harvest

[1]Outer cycle harvests are those *actually taken* in the sequential EFV option, as opposed to inner cycle harvests, which are *potentially taken*. The current outer cycle is the first period of the current inner cycle.

scheduling method precedes the variable method.

VLPRQL = harvest volume exactly satisfying the linkage equation.

VLRQCL = harvest volume exactly satisfying the ending condition.

VLTAVL = volume available for harvest (including exogenous harvest).

VLTRC(NPDIX) = harvest level for the current inner cycle.

VLTRC(NPDIX-1) = harvest level for the last period.

EFV and EFFV

Step 1. Initiate the outer and inner cycles.

(a) Set NPDOG = 0 plus the number of fixed method periods if a fixed harvest scheduling method precedes EFV or EFFV.

(b) Set NPDIG = NPDOG.

Step 2. Shift utilization standards and MIs; adjust stocking levels; calculate the exogenous harvest; and check the period.

(a) If the first period of the look-ahead (NPDIG = NPDOG), go to Step 3.

(b) If not, go to Step 5.

Step 3. Set the trial harvest level for the first period of the look-ahead and the initial search step-size.

(a) Set VLBSPL = VLTRC (NPDIX-1). If (NPDOG = 0), set VLBSPL = VLHPPP.

(b) Set VLBSIL = VLBSBL * PVLBIP.

(c) Set VLTRC(NPDIX) = VLBSPL.

(d) If EFV, OBJVL = VLTRC(NPDIX); if EFFV, set OBJVL = C_{1j} + C_{2j} [VLTRC(NPDIX)] where C_{1j} and C_{2j} are the EFFV coefficients (see ACC, Card 08, **User's Manual**).

(e) If no harvest-flow constraints are imposed (PVLDCP = PVLICP = 0), go to Step 7. Otherwise, continue on to Step 4.

Step 4. Check the trial harvest level to see that it meets the harvest-flow constraints. If it does not, reset the harvest so the constraints are met.

(a) If VLTRC(NPDIX) > VLTRC(NPDIX-1) * (1+PVLICP), reset VLTRC(NPDIX) = VLTRC(NPDIX-1) * (1+PVLICP).

(b) If VLTRC(NPDIX) < VLTRC(NPDIX-1) * (1-PVLDCP), reset VLTRC(NPDIX) = VLTRC(NPDIX-1) * (1-PVLDCP).

(c) Go to Step 7.

Step 5. In the period immediately preceding the ending-condition check, store the target harvest volume. If NPDIG = NPDECG-1 *and* an ending condition is imposed, set VLRQCL equal to the harvest volume required to meet the ending condition.

Step 6. Set the target harvest for the next period.

(a) If EFV, VLTRC(NPDIX) = OBJVL = VLBSPL.

(b) IF EFFV, VLTRC(NPDIX) = OBJVL - C_{1j}/C_{2j}.

Step 7. Check the trial harvest level to see that it is greater than the exogenous harvest and less than the volume available for harvest.

(a) If VLTRC(NPDIX) < VLEXC, set VLDIFL = VLEXC - VLTRC(NPDIX), reset VLTRC(NPDIX) = VLEXC, and go to Step 8.

(b) If VLTRC(NPDIX) > VLTAVL, set VLDIFL = VLTAVL - VLTRC(NPDIX), reset VLTRC(NPDIX) = VLTAVL, and go to Step 8.

(c) If neither, go to Step 9.

Step 8. Check to see that the revised harvest level is sufficiently close to the target level to satisfy the difference tolerance.

(a) If the absolute value of VLDIFL ⩽ VLBSPL * PVLDTP, go to Step 9.

(b) If the requested harvest has been reset to meet the harvest-flow constraint (Step 4), go to Step 9.

(c) If neither, go to Step 14.

Step 9. Set the successful harvest flag and write the optimization detail.

(a) Set ITRMCZ = 2.

(b) Write the optimization detail on TAPE33 (see Appendix B).

Step 10. Determine the proportion of acres or trees to be harvested from each composite class according to the harvest priority chosen and increment the period (set NPDIG = NPDIG+1).

Step 11. Apply the calculated proportion uniformly to the individual components of each composite class.

Step 12. Adjust the inventory for changes in utilization standards, regeneration, and growth; check the period.

(a) If not the period for checking the ending condition (NPDIG < NPDECG), go to Step 2.

(b) If the period for checking the ending condition, continue on to Step 13.

Step 13. Check the ending condition.

(a) Set VLDIFL = VLRQCL - VLTRC(NPDIX).

(b) If absolute value of VLDIFL ⩽ VLBSPL * PVLDTP, or if the harvest has been forced to a harvest-flow constraint in Step 4:

(1) Set ITRMCZ = 2.

(2) Write the optimization detail on TAPE33 (see Appendix B).

(3) Go to Step 15.

(c) If not, go to Step 14.

Step 14. Adjust the first-period trial harvest through the binary-search routine.

(a) If the current step VLBSIL is less than the step-size tolerance VLBSPL * PVLITP, set ITRMCZ = 1, write the optimization detail, and stop.

(b) If not, calculate a new step size using the binary-search technique and reset VLBSPL = VLBSPL ± VLBSIL.

(c) If VLBSPL < 1, set ITRMCZ = 1, write the optimization detail, and stop. If not, set ITRMCZ = 0, write the optimization detail, and go to Step 3(c).

Step 15. Check if this is the last period of the planning horizon.

(a) If NPDOG < MXPDOG, set NPDOG = NPDOG+1, and go to Step 1(b).

(b) If NPDOG = MXPDOG, prepare reports and stop.

PNW and PNB

Step 1. Initiate the outer cycle.

(a) Set NPDOG = 0 plus the number of fixed method periods if a fixed harvest scheduling method precedes PNW or PNB.

(b) Set NPDIG = NPDOG.

Step 2. Shift utilization standards and MIs; adjust stocking levels; calculate the exogenous harvest; and check the period.

(a) If the first period of the look-ahead (NPDIG = NPDOG), go to Step 3.

(b) If not, go to Step 4.

Step 3. Set the trial harvest level for the first period of the look-ahead and the initial search step-size.

(a) Set VLBSPL = VLTRC (NPDIX-1). If NPDOG = 0, set VLBSPL = VLHPPP.

(b) Set VLBSIL = VLBSPL * PVLBIP.

(c) Set VLTRC(NPDIX) = VLBSPL.

(d) Set VLPRQL = VLTRC (NPDIX).

(e) Go to Step 6.

Step 4. In the period immediately preceding the ending-condition check, store the target harvest volume. If NPDIG = NPDECG-1 *and* an ending condition is imposed, set VLRQCL equal to the harvest volume necessary to meet the ending condition.

Step 5. Calculate the harvest volume required in this period based on VLPRQL and on values calculated in Step 11 in the previous period and the linkage equation (see Eq. [49] in PNW harvest scheduling method section).

(a) Set VLTRC(NPDIX) = linkage equation value where h_j = VLPRQL.

(b) Set VLPRQL = VLTRC (NPDIX).

Step 6. Check the trial harvest level to see if it is greater than the exogenous harvest for the period and less than the volume available.

(a) If VLTRC(NPDIX) < VLEXC, set VLDIFL = VLEXC - VLTRC(NPDIX); reset VLTRC(NPDIX) = VLEXC; and go to Step 7.

(b) If VLTRC(NPDIX) > VLTAVL, set VLDIFL = VLEXC - VLTRC(NPDIX); reset VLTRC(NPDIX) = VLTAVL; and go to Step 7.

(c) If neither, go to Step 8.

Step 7. Check to see if the revised harvest level is sufficiently close to the target harvest level to satisfy the difference tolerance.

(a) If the absolute value of VLDIFL ⩽ VLBSPL * PVLDTP, go to Step 8.

(b) If not, go to Step 14.

[Note that the difference tolerance is defined using *first-period* rather than *current-period* harvest level (VLPRQL)].

Step 8. Set the successful harvest flag and write the optimization detail.

(a) Set ITRMCZ = 2.

(b) Write the optimization detail on TAPE25 (see Appendix B).

Step 9. Determine the proportion of acres to be harvested from each composite class according to the harvest priority chosen. Determine the last composite class harvested (i.e., linkage class) and increment the period (set NPDIG = NPDIG+1).

Step 10. Apply the calculated proportion uniformly to the individual components of each composite class.

Step 11. Compute values needed for the linkage equation (Step 5).

(a) For each GRU containing the linkage class, interpolate the volume and cost per acre and the cost and revenue per unit volume for the last acre harvested this period and the first acre harvested next period. Calculate volume, cost, and revenue by multiplying per-acre values by number of acres and per-unit-volume values by volume. The number of acres for the last class harvested this period is the number cut this period; the number for the first class harvested next period is the number remaining after this period's harvest. Sum number of acres, volume, cost, and revenue for the last and first classes over GRUs.

(b) Divide sums by total number of acres to compute volume, cost, and revenue, all per acre, for the last and first classes.

Step 12. Adjust the inventory for changes in utilization standards, regeneration, and growth; check the period.

(a) If not the period for checking the ending condition (NPDIG < NPDECG), go to Step 2.

(b) If the period for checking the ending condition (NPDIG = NPDECG), continue on to Step 13.

Step 13. Check the ending condition.

(a) Set VLDIFL = VLRQCL - VLTRC(NPDIX).

(b) If the absolute value of VLDIFL ⩽ VLBSPL * PVLDTP:

(1) Set ITRMCZ = 2.

(2) Write the optimization detail on TAPE25 (see Appendix B).

(3) Prepare reports and stop.

(c) If not, go to Step 14.

Step 14. Check the binary search step-size. Reset the first-period search level.

(a) If VLBSIL ⩽ VLBSPL * PVLITP, set ITRMCZ = 1; write the optimization detail; and stop.

(b) If not, set VLBSIL equal to the new binary search step-size.

(c) Set VLBSPL = VLBSPL ± VLBSIL.

(d) If VLBSPL < 1, set ITRMCZ = 1; and stop.

(e) If not, set ITRMCZ = 0; write the optimization detail; and go to Step 3(c).

appendix B — optimization detail report

The optimization detail lists significant variables primarily to allow interpretation of the success or failure of the variable (multiple iteration) harvest scheduling methods. This report is particularly valuable for determining why a variable method stops before finding a solution for all periods. The optimization detail, written on TAPE25 (see the **Example Guide**), appears in three versions: (1) fixed method report (for absolute amount, percent of inventory, and area control); (2) even-flow report (for EFV and EFFV); and (3) PNW and PNB report. When switching from a fixed harvest scheduling method to a variable one, the fixed method report precedes the variable report.

Fixed Method Report

Five variables are printed for each outer cycle. In order of appearance across the page, variables are:

NPDOG = outer cycle (planning period), with 0 as the first period.

NPDIG = inner cycle (look-ahead); for fixed methods, NPDIG = NPDOG.

VLTAVL = total volume available for harvest in the period.

VLTRC(NPDIX) = harvest volume for the period (NPDIX = NPDOG+1, for subscripting). VLTRC(NPDIX) may differ from the requested volume in the ACC file if the requested volume is less than exogenous harvests or greater than the volume available.

ITRMCZ = harvest flag (2 for all fixed method periods).

Even-Flow Report

Thirteen variables are printed on a line for each period of an inner cycle iteration or trial; two lines are printed for the last period, one before the ending-condition check and one after.

In order of appearance across the page, the variables are:

NPDOG = outer cycle period. The first outer-cycle period is 0; the last outer-cycle period is NVPOC-1, where NVPOCP is the number of optimization cycles entered in the ACC file. If any fixed method precedes an even-flow method, then NPDOG includes the fixed method periods.

NPDIG = inner cycle period. NPDIG = NPDOG in the first period of each inner cycle (look-ahead). NPDIG = NPDOG + NPDVPP in the last period of each successful inner-cycle trial. NPDVPP is entered in the ACC file as the inner cycle length.

VLTAVL = total volume available for harvest in the period.

VLTRC(NPDIX) = total harvest for the period (NPDIX = NPDIG+1, for subscripting). VLTRC(NPDIX) is the EFV or EFFV harvest level unless that level is greater than the amount available or less than the exogenous harvest. In those cases, VLTRC(NPDIX) is reset to the volume available, exogenous harvest level, or level required by the harvest-flow constraint. VLDIFL will represent the difference between the EFV or EFFV harvest level and VLTRC(NPDIX).

ITRMCZ = harvest flag.

0 if VLDIFL > VLBSPL * PVLDTP, VLBSIL > VLBSPL * PVLITP, and VLBSPL > 1. The difference between VLTRC(NPDIX) and some desired harvest level exceeds the difference tolerance. Find a new level for harvest in the first period of the inner cycle and try again.

1 if VLDIFL > VLBSPL * PVLDTP and (a) VLBSIL ⩽ VLBSPL * PVLITP, or (b) VLBSPL < 1. In (a), the binary search step-size has become smaller than the step-size tolerance; in (b), the first-period harvest level for the next iteration is < 1. In either case, the search for an available harvest in the outer cycle period is abandoned. If only one look-ahead period is used (NVPOCP = 1), the run will terminate. If NVPOCP > 1, proceed to the next outer-cycle period *without taking a harvest in the current outer-cycle period and without growing the stand.*

2 if (a) harvest level successfully meets the even-flow or ending conditions or (b) trial harvest level has been forced to the harvest-flow constraint boundaries. When ITRMCZ = 2, the algorithm proceeds to the next inner-cycle period or, if the ending condition has been met, to the next outer-cycle period.

IBGOPP = optimization phase.

0 if in the second (or greater) iteration in searching for an inner cycle harvest schedule.

1 if in the inner cycle trial of the first variable harvest period.

2 if in the first inner-cycle trial after incrementing the outer cycle period.

IBSSTL = binary-search stage.

1 if (a) the first inner-cycle trial of the first period of a variable harvest scheduling method or (b) the last period of the inner cycle has been reached in two successive iterations, but the harvest level has not met the ending condition and upper and lower harvest bounds have not been found. In this case, the step size will be increased by multiplying VLBSIL by NSTPL (number of steps). NSTPL depends on the magnitude of VLDIFL or on the distance to an upper or lower periodic harvest-flow constraint, if in effect.

2 if upper and lower bounds have not yet been found.

3 if the trial harvest level has been forced to one of the harvest-flow constraint boundaries. If IBSSTL = 3, the target harvest level is the constraint level in subsequent trials. If that level is not feasible, the closest feasible harvest is taken.

4 if upper and lower bounds on the optimal harvest level have been found.

VLBSIL = binary search step-size, or change in the trial harvests for successive inner-cycle iterations, except when IBSSTL = 1, case (b). When VLBSIL < VLBSPL * PVLITP and the difference tolerance is violated, the search for a harvest level for the current outer-cycle period is abandoned (see ITRMCZ = 1).

VLBSPL = trial or target harvest level for the first period of the inner cycle, when NPDIG = NPDOG. VLBSPL remains the same as long as ITRMCZ = 2. When ITRMCZ = 0, VLBSPL is adjusted by the amount of VLBSIL. If VLBSPL < 1, then ITRMCZ = 1, and the search for a current outer-cycle harvest level ceases.

OBJVL = objective value. OBJVL = VLBSPL for the EFV method, or the target level for the volume-dependent variable for the EFFV method.

VLHPOL = volume harvested in the previous outer-cycle period.

VLHPIL = volume harvested in the previous inner-cycle period, defined only if the periodic harvest-flow constraints are in effect.

VLDIFL = volume difference; can be (a) the exogenous harvest, when the target harvest is less than the exogenous harvest; (b) the volume available, when the target harvest is greater than the volume available; or (c) the ending-condition volume, when checking the ending condition at the last period of each inner cycle. If the absolute value of VLDIFL, as defined in all three cases, exceeds the different tolerance (i.e., VLDIFL > VLBSPL * PVLDTP), then a new target harvest level is computed, beginning a new inner-cycle iteration.

PNW and PNB Report

The 14 variables, printed in order of appearance across the page, are:

NPDOG = outer cycle period (constant in PNW and PNB runs).

NPDIG = inner cycle period. In the first inner-cycle period, NPDIG = NPDOG.

VLTAVL = total volume available for harvest in the period.

VLTRC(NPDIX) = harvest for the current period.

ITRMCZ = harvest flag.

0 if (a) harvest determined from the linakge equation is greater than the volume available (VLTAVL), and the absolute value of VLDIFL (the difference between the two) is greater than the difference tolerance [ABS(VLDIFL) > VLBSPL * PVLDTP], or (b) harvest determined from the linkage equation is less than the exogenous harvest (VLEXC), and the absolute value of VLDIFL (the difference between the two) is greater than the difference tolerance [ABS(VLDIFL) > VLBSPL * PVLDTP]. In either case, a new value will be chosen for VLBSPL, the harvest in the first period of the inner cycle.

1 if (a) the next trial harvest level for the first period of the inner cycle is < 1, or (b) the difference tolerance has been violated (see ITRMCZ = 0) and the current binary search step-size (VLBSIL) is not greater than the step-size tolerance (VLBSIL ⩽ VLBSPL * PVLITP). In either case, the search for a feasible PNW or PNB schedule will be abandoned.

2 if the harvest taken [VLTRC(NPDIX)] is within the difference tolerance of the linkage-equation harvest or the ending condition, if specified. If not in the last period, the next-period harvest is calculated from the linkage equation and from the linkage-equation harvest level in the current period (VLPRQL), which may differ from VLTRC(NPDIX) by VLDIFL.

IBGOPP = optimization-phase indicator.

0 if in the second (or greater) iteration in searching for a PNW or PNB solution.

1 if in the first iteration in searching for a PNW or PNB solution.

IBSSTL = binary-search stage.

1 if (a) in the first iteration of the search for a PNW or PNB solution, or (b) ending conditions are specified; two successive iterations fail as a result of not meeting the ending conditions; upper and lower harvest bounds have not yet been found; and the change in VLDIFL in the two previous iterations is less than one-fifth of VLDIFL. If all these conditions prevail, then the binary search step-size (VLBSIL) will be multiplied by VLDIFL divided by the change in VLDIFL.

2 if upper and lower harvest bounds have not yet been found.

4 if upper and lower harvest bounds have been found.

VLBSIL = binary search step-size (the change in first-period harvest levels in successive iterations). When IBSSTL = 1 or 2, VLBSIL = VLBSPL * PVLBIP. When IBSSTL = 4, VLBSIL = VLBSIL/2.

VLBSPL = trial harvest level for the first period of the current iteration. If ITRMCZ = 0, VLBSPL is the trial harvest level for the next iteration.

PDLDRC(NPDIX) = current-period discount factor.

-DAFHLL = marginal net return, undiscounted, from clearcutting the last acre in the previous period. A negative value in the printout indicates a negative net return for the last acre cut.

-DATHLL = marginal net return, undiscounted, from thinning on the last acre clearcut in the previous period if that acre was thinned in the period. A negative value in the printout indicates a negative marginal net return for the last acre thinned.

DVDRVL = gross marginal revenue per unit volume, undiscounted, in PNW runs or gross marginal benefit per unit volume, undiscounted, in PNB runs.

VLDIFL = volume difference; can be: (a) VLEXC minus the linkage-equation harvest level, where VLEXC is exogenous harvest volume; (b) VLTAVL minus the linkage-equation harvest level, where VLTAVL is total volume available for harvest; or (c) VLPRQL minus the linkage-equation harvest level, where VLPRQL is the harvest needed to meet the ending condition. VLDIFL is redefined only when the harvest does not exactly meet all requirements.

appendix C — quadratic interpolation routine for PNW and PNB algorithms

If we assume ages are evenly distributed within each age class, the proportion of the last age class harvested will determine the exact age of the last acre harvested. The volume growth ratio for that acre is found by XXQINF using a quadratic interpolation. A quadratic function of age is fit to the standard volume per acre for the last age class cut and for the next oldest and next youngest age classes (Fig. C-1). From the resulting curve, we can estimate the standard growth rate for the last acre cut. In addition to adjusting volume growth, the quadratic interpolation can adjust harvest costs, base price per unit volume, and thinning volumes on the last acre harvested.

Suppose 80 percent of the 45-year-old age class, the last to be harvested in period j, is to be taken. We estimate the age of the last acre to be cut at 42 years. From our first estimated volume curve, we find the volume per acre at age 42 to be 500 cubic feet. At age 52 (after one growth period), the volume of the last acre, estimated from the second volume curve, would be 600 cubic feet for a 10-year growth rate of 600/500 or 1.2 (slightly less than the growth rate estimated from the age-class midpoints). Consequently, h_{j+1} is lower than if no interpolation is done.

Using this routine, we can focus on virtually any growth rate between that of the fastest and slowest growing age classes. The chances of finding a growth rate close to the discount rate are much improved, and harvests tend to remain stable longer (Scheurman and Johnson 1975). However, the interpolation relies entirely on the standard volume estimates for the MI of the last class harvested. No adjustments are made for the effects of stocking level on growth rates (i.e., approach to normality) nor are changes in MI reflected in growth rates.

Mathematically, the adjustment process is:

Let

V_{ij} = volume per acre, age class i, period j, where age class i is the last age class harvested.

SV_i = standard volume per acre, age class i.

SV'_i = interpolated standard volume for last acre harvested from age class i.

SV'_{i+1} = interpolated standard volume for the first acre harvested from age class i+1.

V'_{ij} = interpolated actual volume from the last acre harvested, age class i, period j.

$V'_{i,j+1}$ = interpolated actual volume from the first acre harvested, age class i, period j+1.

$$V'_{ij} = V_{ij}\left(\frac{SV'_i}{SV_i}\right).$$

$$V'_{i,j+1} = V_{ij}\left(\frac{SV'_{i+1}}{SV_i}\right).$$

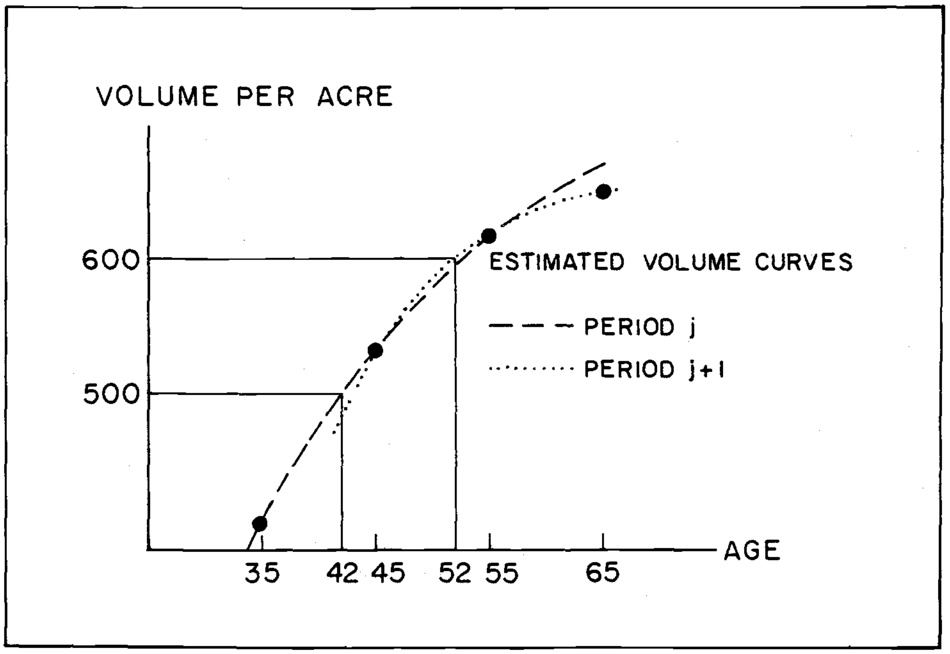


Figure C-1.

Interpolated volume per acre for last acre cut in period j and the same acre if left to grow to period j+1.

$$\frac{V'_{i,j+1}}{V'_{ij}} = \frac{SV'_{i+1}}{SV'_{i}} = \text{interpolated volume growth ratio .}$$

Equivalent terms used in TREES are:

V_{ij} = VAMA (IAGX, IMIX, I2SLX)

SV_i = VATTA (IAGX, IMIX)

SV'_i = XXQINF (PACHVC (IAGX, ISPCX), IAGX, VATTA (1, IMIX), 1)

SV'_{i+1} = XXQINF (PACHVC (IAGX, ISPCX), IAGX+1, VATTA (1, IMIX), 1)

V'_{ij} = VLFHLC/ACFHLC

$V'_{i,j+1}$ = VLFHFC/ACFHFC

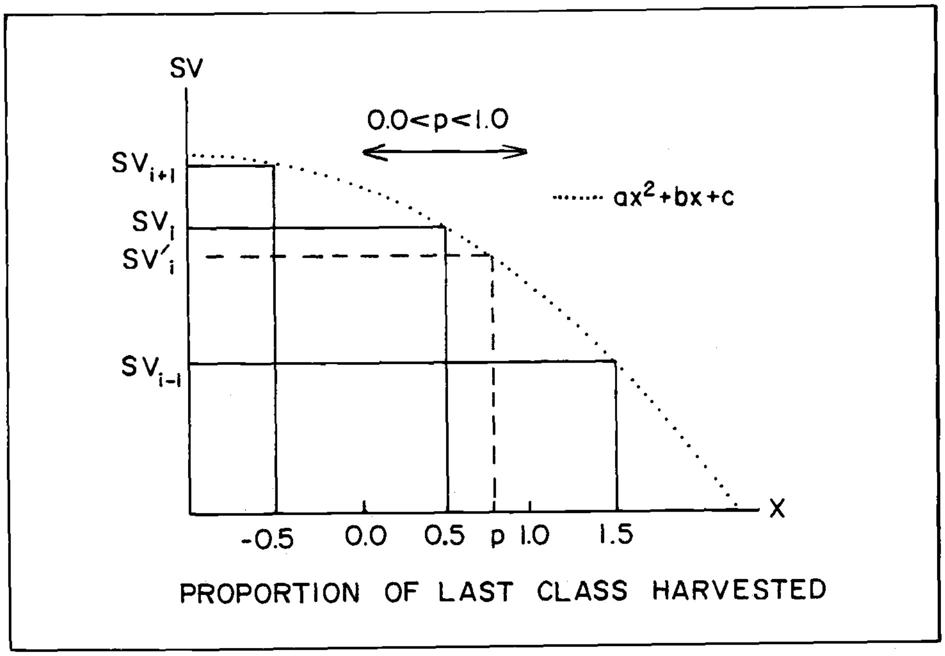


Figure C-2.

Relationship between standard volume (SV) and proportion of last class harvested.

To adjust volume per acre on the last acre harvested, the interpolation process proceeds:

Let

X_1, Y_1 = -0.5, SV_{i+1}

X_2, Y_2 = +0.5, SV_i

X_3, Y_3 = 1.5, SV_{i-1}

(See Fig. C-2.)

From three equations with three unknowns a, b, and c:

$$a + b(-0.5) + c(-0.5)^2 = SV_{i+1}.$$

$$a + b(+0.5) + c(+0.5)^2 = SV_i.$$

$$a + b(1.5) + c(1.5)^2 = SV_{i-1}.$$

Solving for a, b, and c:

$$a = \frac{SV_{i-1}(-0.25) + SV_i(1.5) + SV_{i+1}(0.75)}{2}.$$

$$b = \frac{SV_i(2.0) + SV_{i+1}(2.0)}{2}.$$

$$c = \frac{SV_{i-1} + SV_i(-2.0) + SV_{i+1}}{2}.$$

Substituting p (proportion of the last class harvested) into the estimated equation, we obtain SV_i', the adjusted standard volume for the last acre harvested.

The equivalent equation in coded form is:

```
XXQINF = [XXTBLZ (IAGX-1)
          * PM1HL * PP1HL - 2.
          * XXTBLZ   (IAGX)
          * PP1HL  *  PM3HL
          + XXTBLZ   (IAGX+1)
          * PM1HL * PM3HL]/2.
```

where:

XXQINF = SV_i'

XXTBLZ(IAGX-1) = SV_{i-1}

PM1HL = (p - 0.5)

PP1HL = (p + 0.5)

XXTBLZ(IAGX) = SV_i

PM3HL = (p - 1.5)

XXTBLZ(IAGX+1) = SV_{i+1}

When the last age class cut is age class 33 (oldest age class), the quadratic curve is estimated using the standard values for age class 33 and the two preceding classes. When the next-to-last age class harvested has a zero standard value, the quadratic curve is estimated by assuming that, at the lower boundary of the last class harvested, the curve crosses the x-axis (Fig. C-3).

$$SV_i = a + bp + cp^2$$

$$= \frac{SV_{i-1}(-0.25 + p^2) - 2SV_i(-0.75 - p + p^2) + SV_{i+1}(0.75 - 2p + p^2)}{2}$$

$$= \frac{SV_{i-1}(p - 0.5)(p + 0.5) - 2SV_i(p + 0.5)(p - 1.5) + SV_{i+1}(p - 0.5)(p - 1.5)}{2}.$$

In this instance,

$$SV_i' = SV_i(-2)(p - 1)(p + 0.5)$$

$$+ SV_{i+1}(2/3)(p - 0.5)(p - 1).$$

When adjusting thinning volume per acre for the last class harvested, the estimated curve used when the class above the last class harvested has no thinning volume is:

$$STV_i' = STV_{i-1}(2/3p)(p-0.5)$$

$$+ STV_i(-2p)(p - 1.5)$$

where:

STV = standard thinning volume.

In that case, we assume that thinning ceases at the upper class boundary (Fig. C-4).

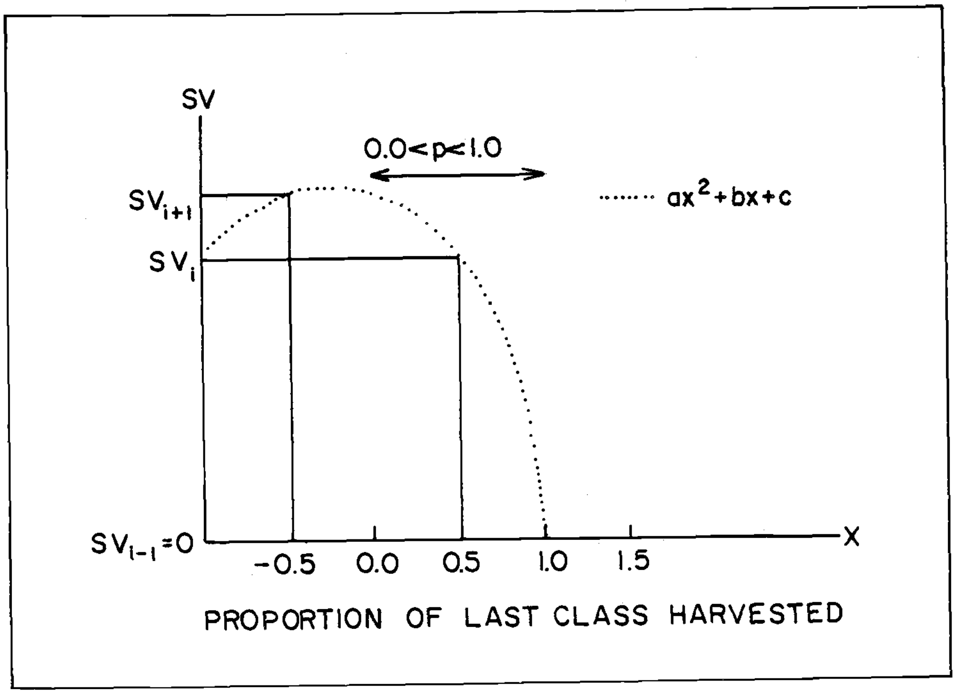


Figure C-3.

Estimating the SV curve when the class below has no volume.

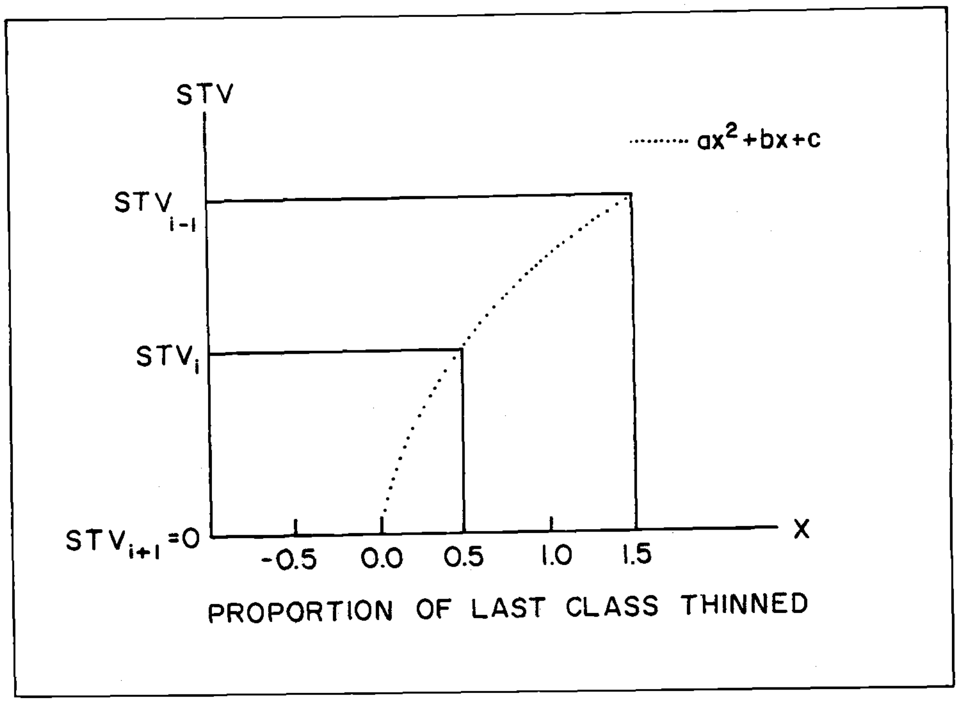


Figure C-4.

Estimating the STV curve when the class above has no thinning volume.

appendix D — *equation limits*

If an equation is used to define values for a variable, Generator subroutines STCEAD and EQTBAD evaluate the equation and construct a table containing one entry for each age- or diameter-class midpoint. The values predicted by the equation can be modified using an indicator for the type of equation limit (ITEQ). Table values constructed by combining an equation and ITEQ may be overridden by direct tabular entries for chosen age or diameter classes. This appendix describes the table values resulting from all possible combinations of equation type and limit. (For flowcharts and further discussion of STCEAD, see the **Analyst's Guide.**)

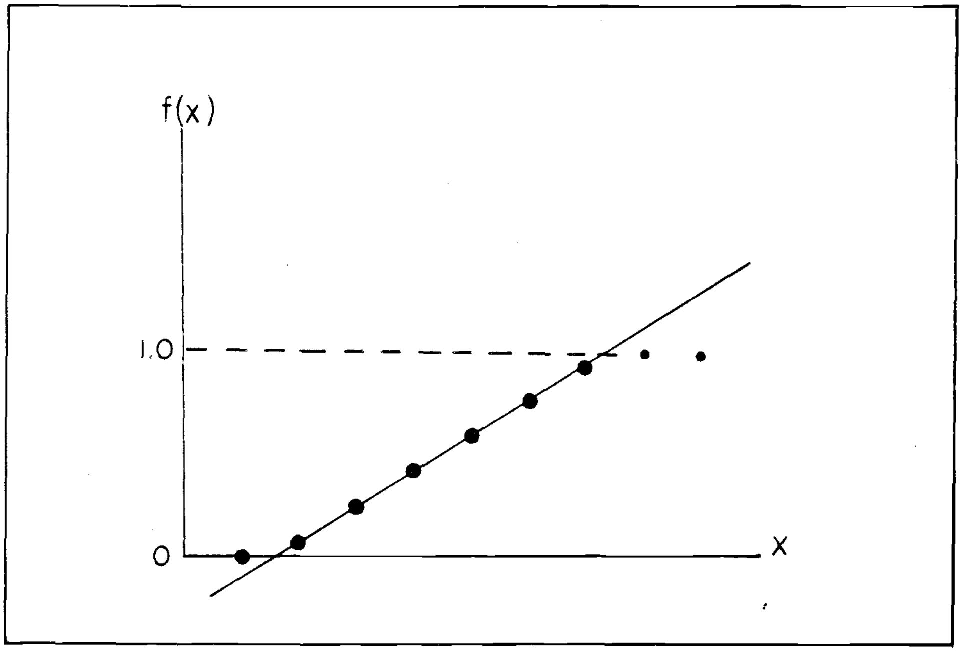


Figure D-1.

Linear equation $(ax + b)$ with $a > 0$ and ITEQ = 5. The · indicates values inserted in tables for Figures D-1 through D-15.

Possible entries for ITEQ and corresponding restrictions on equation values are:

ITEQ	Constraint
0	$f(x) = 1$
1	$f(x) \geqslant 1$ and nonincreasing $[f'(x) \leqslant 0]$
2	$f(x) \geqslant 1$ and nonincreasing $[f'(x) \leqslant 0]$
3	$f(x) \geqslant 0$
4	$f(x) \geqslant 0$ and nondeclining $[f'(x) \geqslant 0]$
5	$0 \geqslant f(x) \geqslant 1$ and nondeclining $[f'(x) \geqslant 0]$

Generally, these constraints operate as described. An asterisk (*) denotes the constraints actually imposed that differ from those previously described; the deviation, in *italics*, follows the description. If the constraints described do not meet the user's needs, we suggest that ITEQ be set to 3 and table values substituted where equation values are unsatisfactory.

(1) When no equation values meet the constraint imposed, the default value for all table values is set to 1.0. The default value 1.0 is also selected for all table values when all coefficients entered for an equation are 0 or when ITEQ = 0.

***(2)** Only a nonzero constant is entered for the equation $[f(x) = a]$ and ITEQ > 0. The table is entirely filled with the value entered for the nonzero constant. *Does not impose stated limits.*

(3) The equation entered is linear $[f(x) = ax + b]$.

(a) The function is increasing $[f'(x) = a > 0]$.

(a-1) ITEQ = 1. Default values of 1.0 are used for all table values.

(a-2) ITEQ = 2. Default values of 1.0 are used for all table values.

(a-3) ITEQ = 3. Equation values are used when positive; 0s otherwise.

(a-4) ITEQ = 4. Equation values are used when positive; 0s otherwise.

(a-5) ITEQ = 5. Zeros or equation values, when positive, are used until equation values exceed 1.0. The value 1.0 is used for all other table values (Fig. D-1).

(b) The function is decreasing $[f'(x) = a < 0]$.

(b-1) ITEQ = 1. Equation values are used when ≥ 1.0; the value 1.0 is used for the remaining table values.

(b-2) ITEQ = 2. Equation values are used when ≥ 1.0; the value 1.0 is used for the remaining table values.

(b-3) ITEQ = 3. Equation values are used when positive; 0s otherwise.

(b-4) ITEQ = 4. Default values of 1.0 are used for all table values.

(b-5) ITEQ = 5. Default values of 1.0 are used for all table values.

(4) The equation entered is quadratic $[f(x) = ax^2 + bx + c]$.

(a) The function has a minimum, i.e., the second derivative is positive $[f''(x) = 2a > 0]$ (see Fig. D-2).

(a-1) ITEQ = 1.

*$c - b^2/4a \geq 1$. Equation values are used for all table values. *Increasing values are allowed.*

$c - b^2/4a < 1$. Equation values are used until 1.0 is reached; the value 1.0 is used for all subsequent table values (Fig. D-3).

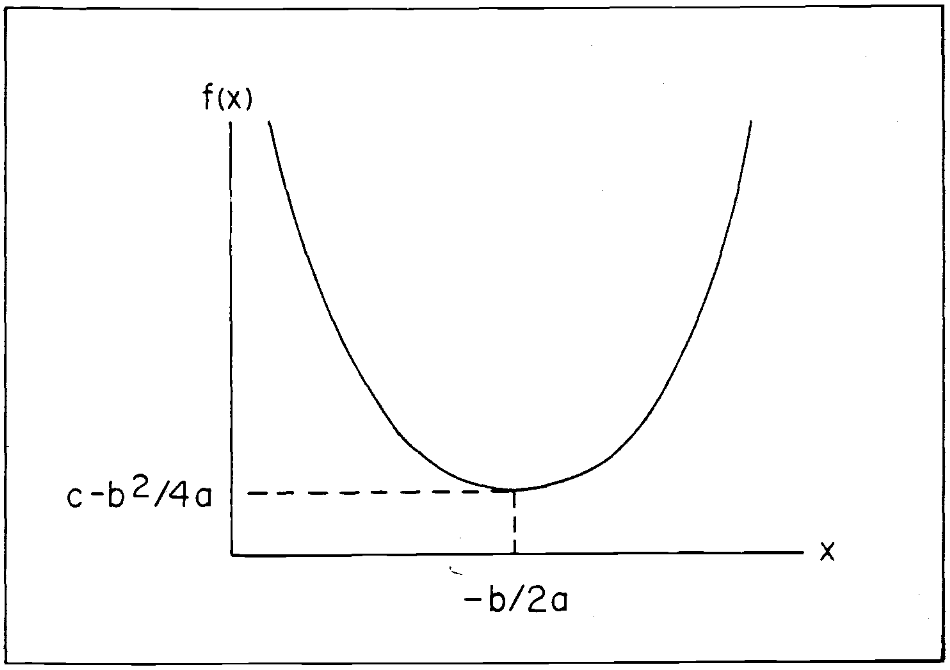


Figure D-2.

Quadratic equation $(ax^2 + bx + c)$ with $a > 0$.

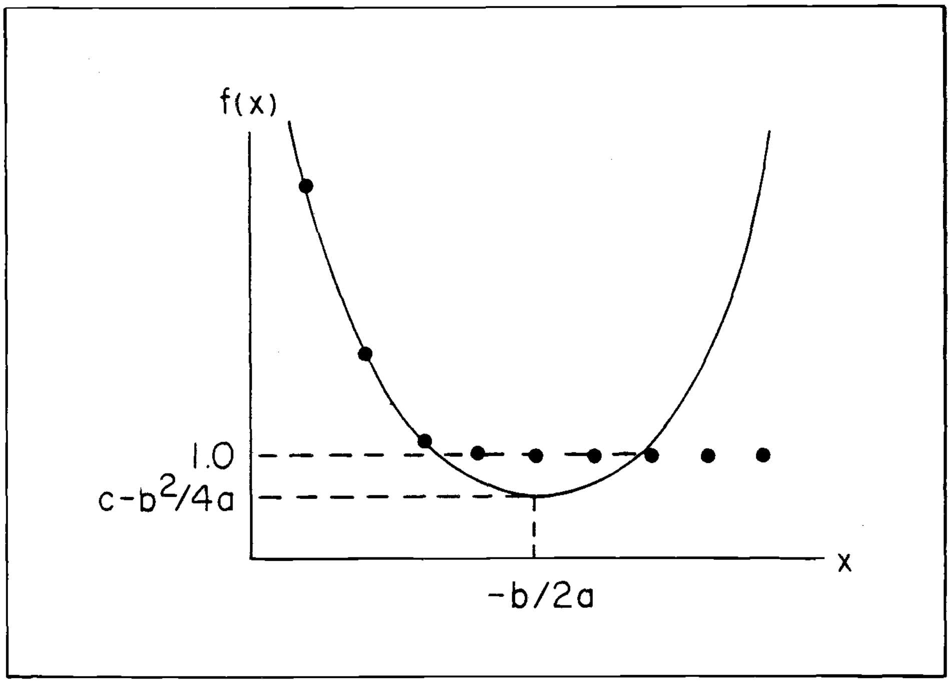


Figure D-3.

Quadratic equation $(ax^2 + bx + c)$ with $a > 0$, ITEQ = 1 or 2, and $(c - b^2/4a) < 1$.

(a-2) ITEQ = 2.

$c - b^2/4a \geqslant 1$. Equation values are used as long as they are decreasing. When the minimum is reached, all subsequent values are set equal to $c - b^2/4a$ (the minimum) (Fig. D-4).

$c - b^2/4a < 1$. Equation values are used until 1.0 is reached. The value 1.0 is used for all subsequent table values.

(a-3) ITEQ = 3. Equation values are used if positive; 0s otherwise.

*(a-4) ITEQ = 4. Default values of 1.0 are used for all table values. *Does not use the increasing portion of the curve.*

*(a-5) ITEQ = 5. Default values of 1.0 are used for all table values. *Does not use the increasing portion of the curve.*

(b) The function has a maximum, i.e., the second derivative is negative $[f''(x) = 2a < 0]$ (Fig. D-5).

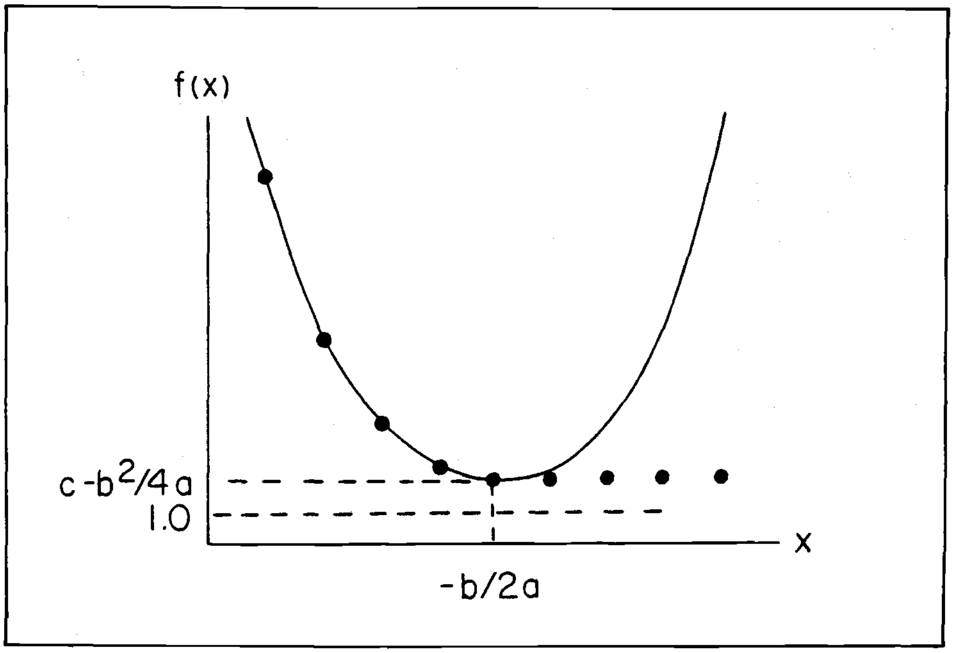


Figure D-4.

Quadratic equation ($ax^2 + bx + c$) with $a > 0$, ITEQ = 2, and ($c - b^2/4a$) > 1.

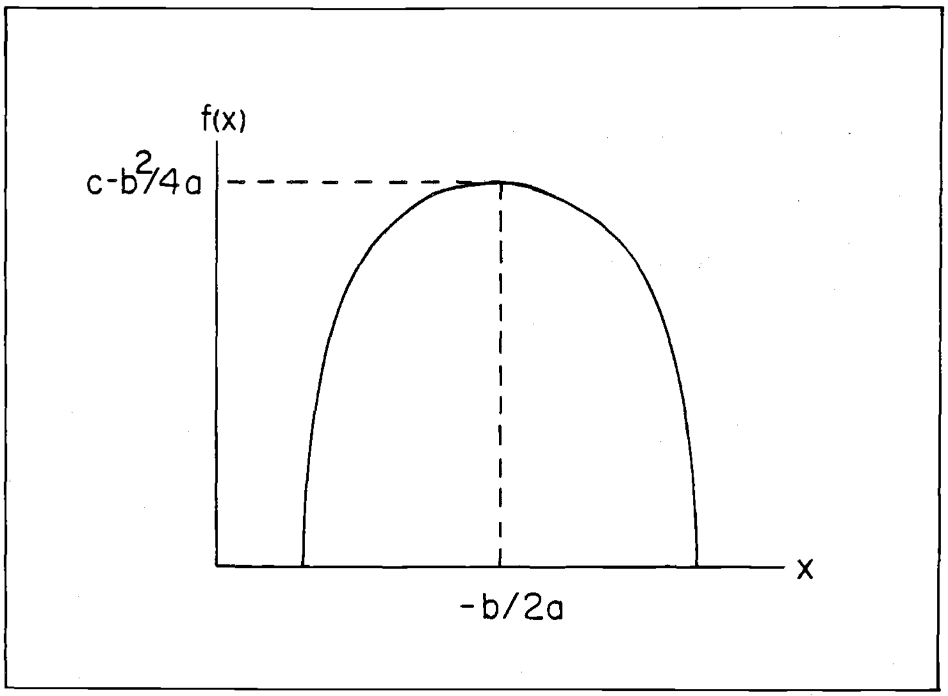


Figure D-5.

Quadratic equation ($ax^2 + bx + c$) with $a < 0$.

*(b-1) ITEQ = 1. Default values of 1.0 are used for all table values. *Does not use the decreasing portion of the curve.*

*(b-2) ITEQ = 2. Default values of 1.0 are used for all table values. *Does not use the decreasing portion of the curve.*

(b-3) ITEQ = 3.

$c - b^2/4a \leqslant 0$. Default values of 1.0 are used for all table values.

$c - b^2/4a > 0$. Equation values are used where positive; 0s otherwise.

(b-4) ITEQ = 4.

$c - b^2/4a \leqslant 0$. Default values of 1.0 are used for all table values.

$c - b^2/4a > 0$. Equation values are used when increasing and positive; the value at the maximum $(c - b^2/4a)$ is used thereafter (Fig. D-6).

(b-5) ITEQ = 5.

$c - b^2/4a \leqslant 0$. Default values of 1.0 are used for all table values.

$0 < c - b^2/4a \leqslant 1.0$. Positive equation values are used up to the maximum; the maximum value is used thereafter (similar to Fig. D-6).

$c - b^2/4a > 1.0$. Positive equation values are used up to 1.0; the value 1.0 is used thereafter (Fig. D-7).

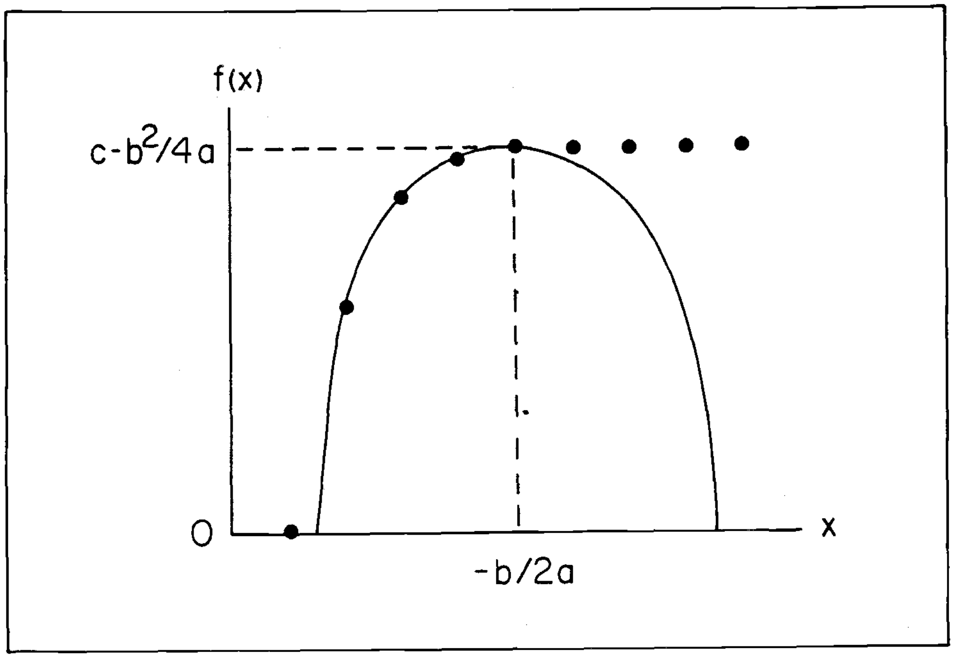


Figure D-6.

Quadratic equation $(ax^2 + bx + c)$ with $a < 0$, ITEQ = 4, and $(c - b^2/4a) > 0$.

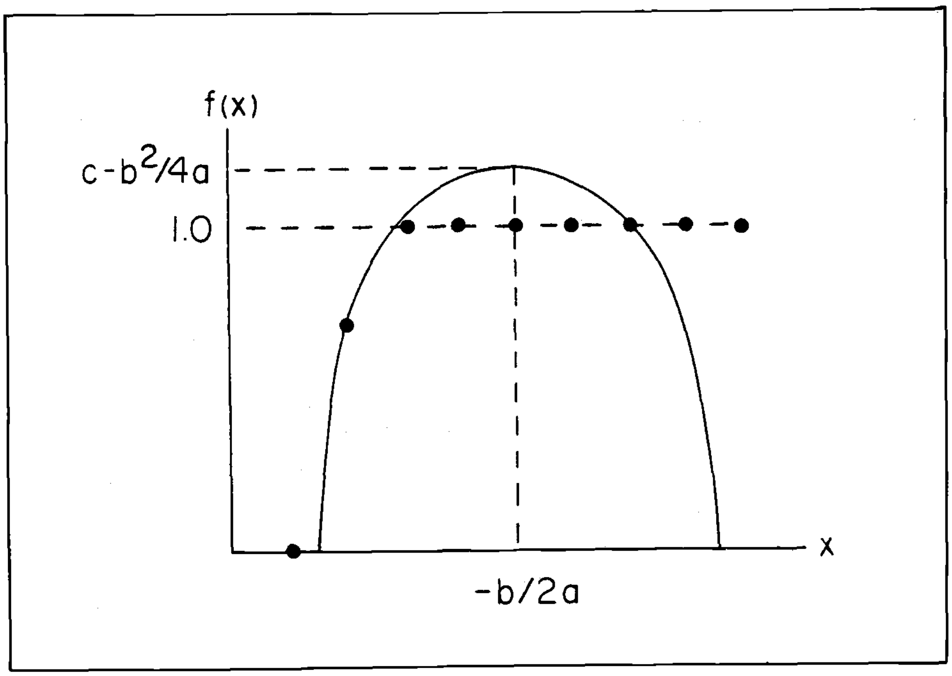


Figure D-7.

Quadratic equation $(ax^2 + bx + c)$ with $a < 0$, ITEQ = 5, and $(c - b^2/4a) > 1$.

(5) The equation entered is cubic $[f(x) = ax^3 + bx^2 + cx + d]$. Local extrema can be found by setting the first derivative equal to 0 and solving for x by the quadratic formula:

$$f'(x) = 3ax^2 + 2bx + c = 0.$$

$$x = \frac{-b + \sqrt{b^2 - 3ac}}{3a}.$$

When $b^2 - 3ac < 0$, x is not a real number (i.e., no real extrema exist). If $b^2 - 3ac = 0$, then $x = -b/3a$. But $x = -b/3a$ implies that f(x) is ,an inflection point because the second derivative $[f''(x) = 6ax + 2b]$ is 0 only if $x = -b/3a$.

(a) No local extrema exist ($b^2 - 3ac \leqslant 0$), and the function is decreasing ($a < 0$) (Fig. D-8).

(a-1) ITEQ = 1.

$b^2 - 3ac < 0$. Equation values are inserted unless $f(x) < 1.0$, in which case table values are set to 1.0.

*$b^2 - 3ac = 0$. When $f(-b/3a) \leqslant 1$, equation values are inserted unless $f(x) < 1.0$, in which case table values are set to 1.0. When $f(-b/3a) > 1$, an error exists in the code and resulting values are uncertain. (Statement 250+1 should read GO TO 280 rather than GO TO 480.) *The code error is the deviation from the stated restrictions.*

(a-2) ITEQ = 2

*$f(-b/3a) > 1.0$. Uses equation values to the left of the inflection point and the value at the inflection point $[f(-b/3a)]$ thereafter. *Does not use the portion of the curve between the inflection point and 1.0 (Fig. D-9).*

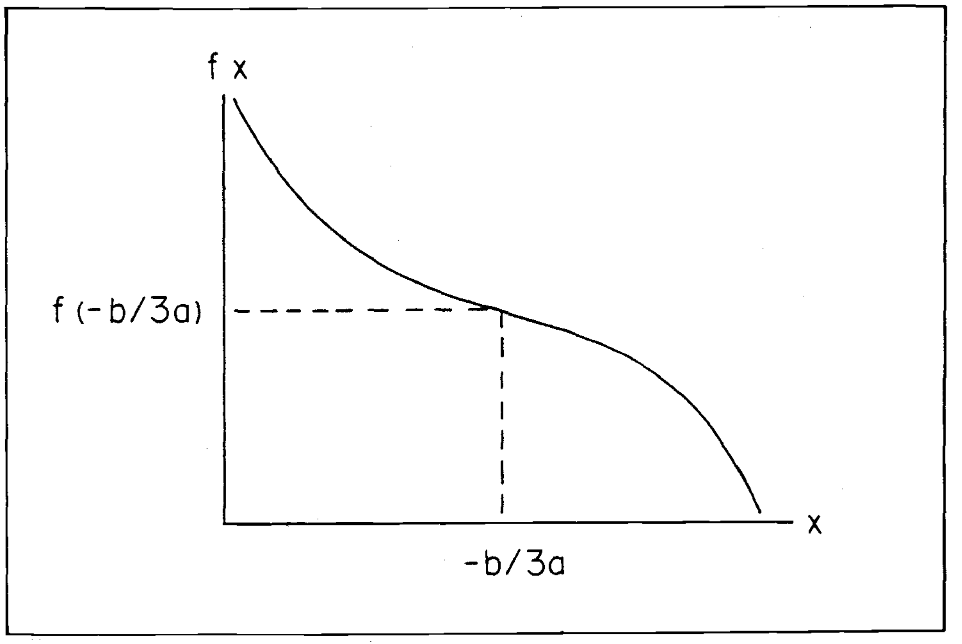


Figure D-8.

Cubic equation ($ax^3 + bx^2 + cx + d$) with no local extrema ($b^2 - 3ac \leqslant 0$) and $a < 0$.

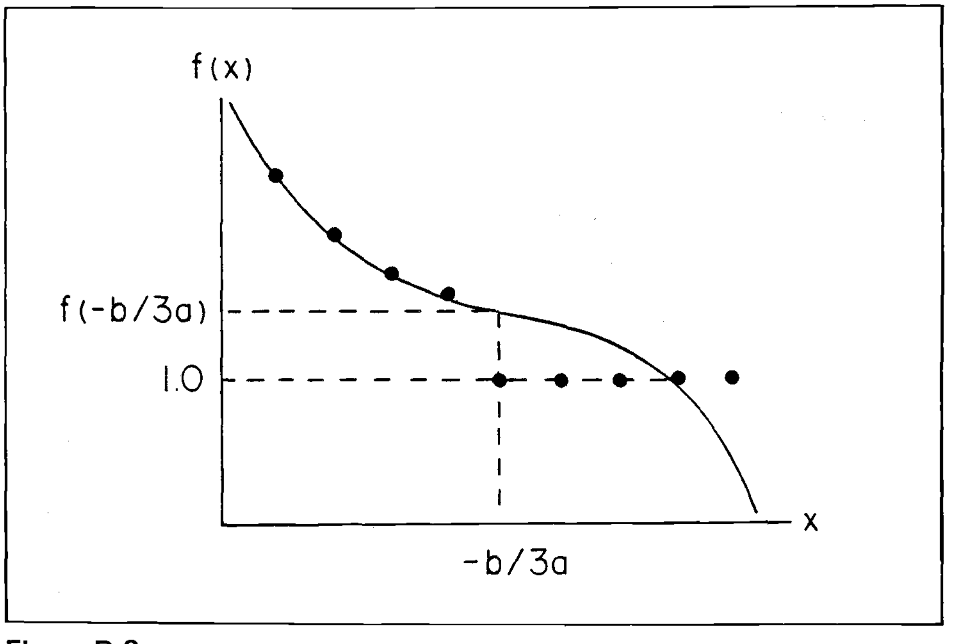


Figure D-9.

Cubic equation ($ax^3 + bx^2 + cx + d$) with no local extrema ($b^2 - 3ac \leqslant 0$), $a < 0$, ITEQ = 2, and $f(-b/3a) > 1$.

$f(-b/3a) \leqslant 1.0$. Equation values are inserted unless $f(x) < 1.0$, in which case table values are set to 1.0.

(a-3) ITEQ = 3.

Equation values are inserted unless $f(x) < 0$, in which case table values are set to 0.

(a-4) ITEQ = 4.

$b^2 - 3ac < 0$. Default values of 1.0 are used for all table values.

*$b^2 - 3ac = 0$. If $f(-b/3a) \leqslant 0$, default values of 1.0 are used for all table values. If $f(-b/3a) > 0$, equation values are used to the left of the inflection point and the value at the inflection point is used thereafter. *Uses a decreasing function.*

(a-5) ITEQ = 5.

$b^2 - 3ac < 0$. Default values of 1.0 are inserted for all table values.

*$b^2 - 3ac = 0$. If $f(-b/3a) \leqslant 0$ or > 1, default values of 1.0 are inserted for all table values. If $0 < f(-b/3a) \leqslant 1$, equation values are used to the left of the inflection point and the value at the inflection point is used thereafter. *Uses a decreasing function.*

(b) No local extrema exist ($b^2 - 3ac \leqslant 0$), and the function is increasing ($a > 0$) (Fig. D-10).

(b-1) ITEQ = 1.

$b^2 - 3ac < 0$. Default values of 1.0 are used for all table values.

*$b^2 - 3ac = 0$. If $f(-b/3a) \leqslant 1$, equation values are used unless $f(x) < 1.0$, in which case table values are set to 1.0. If $f(-b/3a) > 1$, an error in the code is indicated and resulting values are uncertain. *Increasing values are used in the first case; the code error is also a deviation from the stated restriction.*

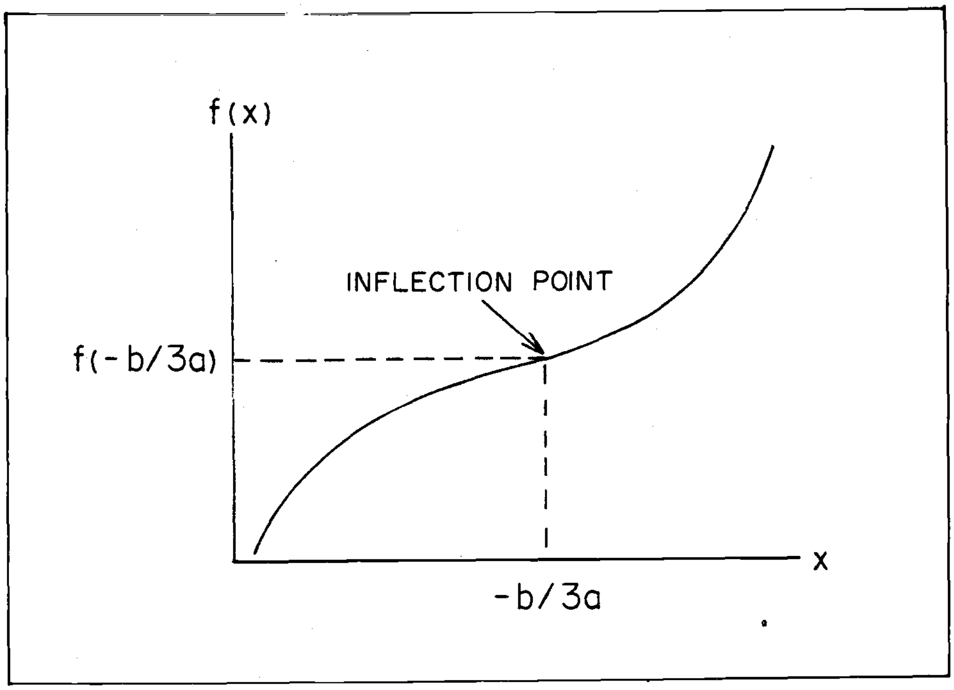


Figure D-10.

Cubic equation ($ax^3 + bx^2 + cx + d$) with no local extrema ($b^2 - 3ac \leqslant 0$) and $a > 0$.

(b-2) ITEQ = 2.

$b^2 - 3ac < 0$. Default values of 1.0 are used for all table values.

*$b^2 - 3ac = 0$. If $f(-b/3a) \geqslant 1$, use positive equation values up to the inflection point and $f(-b/3a)$ thereafter. If $f(-b/3a) < 1$, equation values are used unless $f(x) < 1.0$, in which case table values are set to 1.0. *Uses an increasing function in either case.*

(b-3) ITEQ = 3. Equation values are used if positive; 0s otherwise.

(b-4) ITEQ = 4.

*$f(-b/3a) \leqslant 0$. Default values of 1.0 are used for all table values. *Does not use the increasing portion of the curve to the right of the inflection point.*

*$f(-b/3a) > 0$. If positive, equation values are used up to the inflection point; thereafter, table values are set equal to $f(-b/3a)$. *Does not use the increasing portion of the curve to the right of the inflection point.*

(b-5) ITEQ = 5.

*$f(-b/3a) \leqslant 0$. Default values of 1.0 are used for all table values. *Does not use the increasing portion of the curve to the right of the inflection point.*

*$0 < f(-b/3a) \leqslant 1$. Equation values, if positive, are used up to the inflection point; thereafter, the value at the inflection point [$f(-b/3a)$] is inserted. *Does not use the increasing portion of the curve between the inflection point and 1.0.*

$f(-b/3a) > 1$. Equation values, if positive, are used up to 1.0; thereafter, 1.0 is inserted as the table value.

(c) Local extrema exist ($b^2 - 3ac > 0$), and the maximum is first ($a > 0$) (Fig. D-11). The equations for local

maximum (XMAX) and minimum (XMIN) are

$$XMAX = \frac{-b - \sqrt{b^2 - 3ac}}{3a}.$$

$$XMIN = \frac{-b + \sqrt{b^2 - 3ac}}{3a}.$$

(c-1) ITEQ = 1.

*f(XMIN) $\leqslant$ 1.0. Find the highest class midpoint (MP) less than XMIN for which f(x) > 1.0. Equation values, if positive, are used for all x $\leqslant$ MP; for all x > MP, the value 1.0 is inserted (Fig. D-12). *May use an increasing portion of the curve where x < XMAX.*

*f(XMIN) > 1.0. An error in the code makes values uncertain. *The error is the deviation from stated restrictions.*

(c-2) ITEQ = 2.

*f(XMIN) $\leqslant$ 1. Equation values are used for x < MP where MP < XMIN and f(MP) > 1.0; for x > MP, the value 1.0 is inserted. *May use an increasing portion of the curve where x < XMAX.*

*f(XMIN) > 1. Equation values, if positive, are used until XMIN is reached; for x $\geqslant$ XMIN, table values are set equal to f(XMIN). *May use an increasing portion of the curve where x < XMAX.*

(c-3) ITEQ = 3.

Equation values are used, if positive; 0s otherwise.

(c-4) ITEQ = 4.

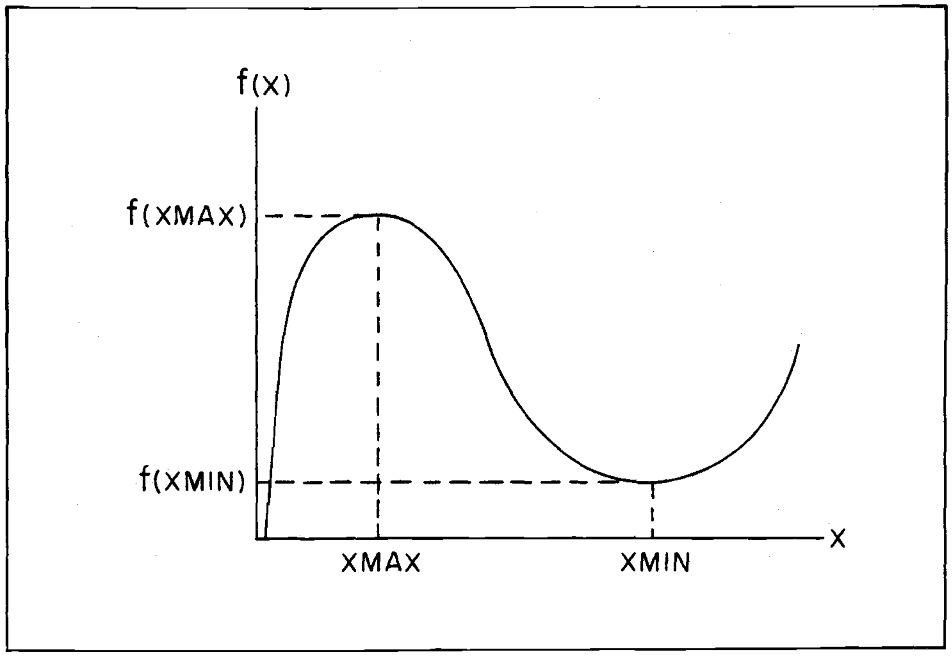


Figure D-11.

Cubic equation ($ax^3 + bx^2 + cx + d$) with local extrema ($b^2 - 3ac > 0$) and $a > 0$.

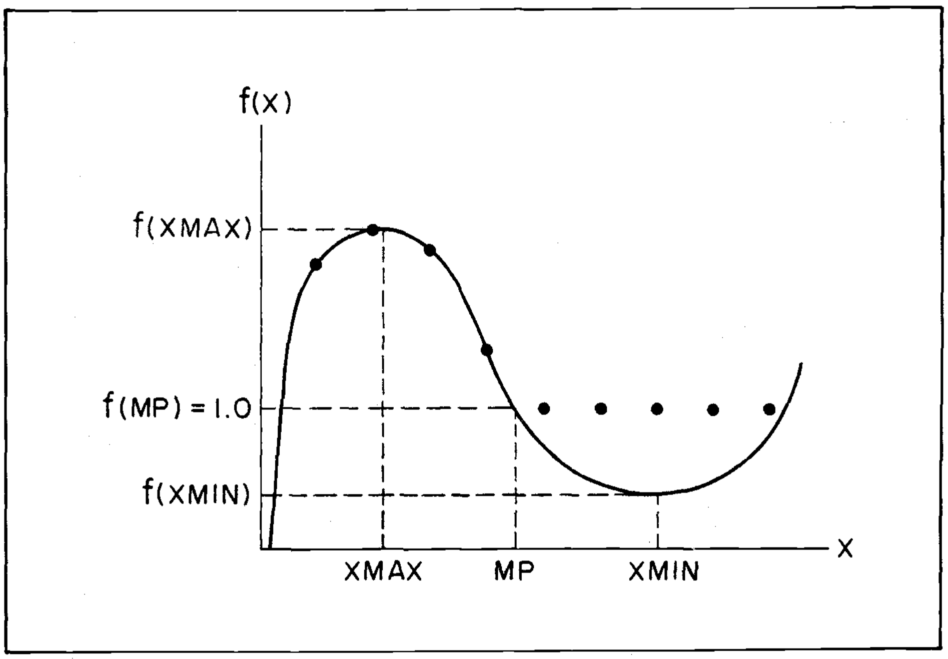


Figure D-12.

Cubic equation ($ax^3 + bx^2 + cx + d$) with local extrema ($b^2 - 3ac > 0$), $a > 0$, ITEQ = 1, and f(XMIN) < 1.

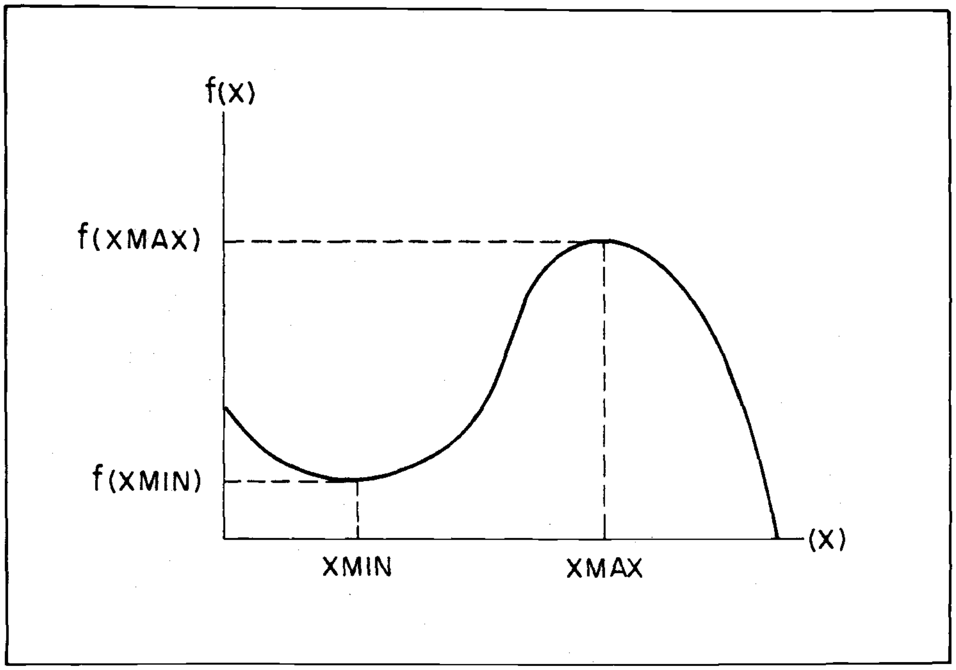


Figure D-13.

Cubic equation ($ax^3 + bx^2 + cx + d$) with local extrema ($b^2 - 3ac > 0$) and $a < 0$.

*f(XMAX) ⩽ 0. Default values of 1.0 are used for all table values. *Does not use the increasing portion of the curve for x > XMIN.*

*f(XMAX) > 0. Equation values, if positive, are used for all x ⩽ XMAX. For x > XMAX, f(XMAX) is inserted. *Does not use the increasing portion of the curve for which x > XMIN.*

(c-5) ITEQ = 5.

f(XMAX) ⩽ 0. Default values of 1.0 are used for all table values. *Does not use the increasing portion of the curve for which x > XMIN.*

*0 < f(XMAX) ⩽ 1. Equation values, if positive, are used for x ⩽ XMAX; f(XMAX) is inserted for table values if x > XMAX. *Does not use the increasing portion of the curve where x > XMIN.*

*f(XMAX) > 1. Equation values, if positive, are used for x < MP where MP < XMAX and f(MP) < 1.0; for x > MP, the value 1.0 is inserted. *Does not use the increasing portion of the curve for x > XMIN.*

(d) Local extrema exist ($b^2 - 3ac > 0$), and the minimum is first ($a < 0$) (Fig. D-13). The equations for local minimum (XMIN) and maximum (XMAX) are

$$XMIN = \frac{-b - \sqrt{b^2 - 3ac.}}{3a}$$

$$XMAX = \frac{-b + \sqrt{b^2 - 3ac.}}{3a}$$

(d-1) ITEQ = 1.

*f(XMIN) ⩽ 1.0. Equation values are used for x < MP where MP < XMIN and f(MP) > 1.0; for x > MP, table values are set to 1.0. *Does not use the decreasing portion of the curve for x > XMAX.*

*f(XMIN) > 1.0. *An error in the code results in uncertain values.*

(d-2) ITEQ = 2.

*f(XMIN) ⩽ 1.0. Equation values are used for x < MP where MP < XMIN and f(MP) = 1.0. For x > MP, table values are set to 1.0. *Does not use the decreasing portion of the curve for x > XMAX.*

*f(XMIN) > 1.0. Equation values are used for x < XMIN. Values are set to f(XMIN) for x > XMIN. *Does not use the decreasing portion of the curve for x > XMAX.*

(d-3) ITEQ = 3.

*Where f(XMIN) ⩽ 0 and f(XMAX) > 0, equation values are used for x < XMIN; 0s are inserted thereafter. *Does not use the positive equation values for x ⩾ XMIN.*

For all other cases, equation values, if positive, are used; 0s otherwise.

(d-4) ITEQ = 4.

$f(XMAX) \leq 0$. Default values of 1.0 are used for all table values.

*$f(XMAX) > 0$. Equation values, if positive, are used for $x \leq XMAX$. For $x > XMAX$, values are set to f(XMAX). *May use decreasing portions of the curve when $x < XMIN$ (Fig. D-14).*

(d-5) ITEQ = 5.

$f(XMAX) \leq 0$. Default values of 1.0 are used for all table values.

*$0 < f(XMAX) \leq 1$. Equation values, if positive, are used for all $x \leq XMAX$; for $x > XMAX$, f(MAX) is inserted. *May use declining values for $x < XMIN$.*

*$f(XMAX) > 1$. For $x \leq MP$, where $MP < XMAX$ and $f(MP) < 1.0$, use equation values, if positive. For $x > MP$, set values to 1.0 (Fig. D-15). If no such MP exists, then set all values equal to 1.0. *May use declining values when $x < XMIN$.*

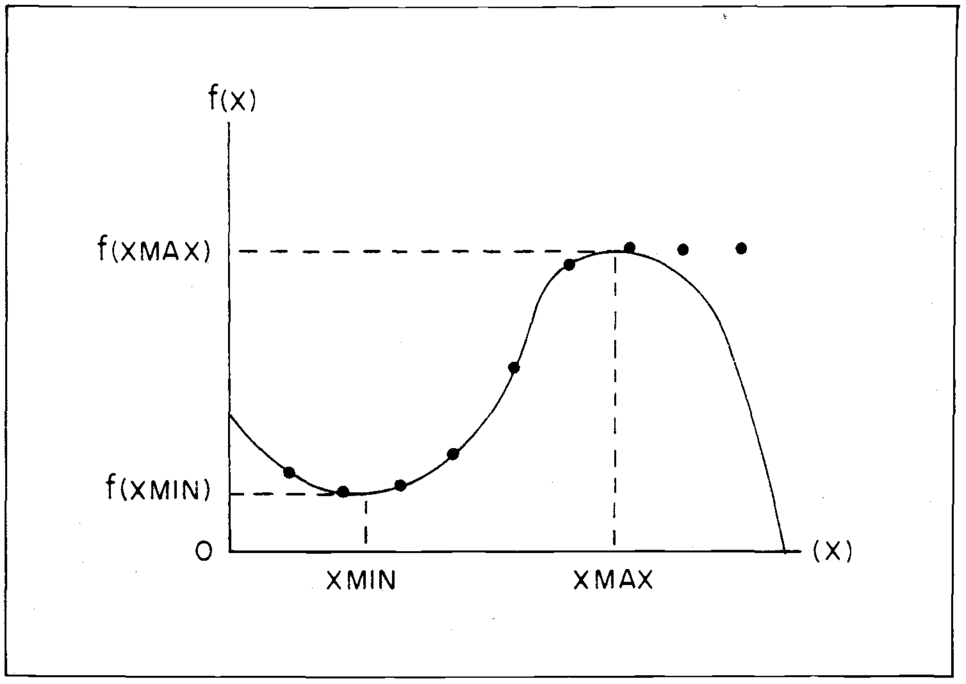


Figure D-14.

Cubic equation ($ax^3 + bx^2$ $cx + d$) with local extrema ($b^2 - 3ac > 0$), $a < 0$, ITEQ = 4, and $f(XMAX) > 0$.

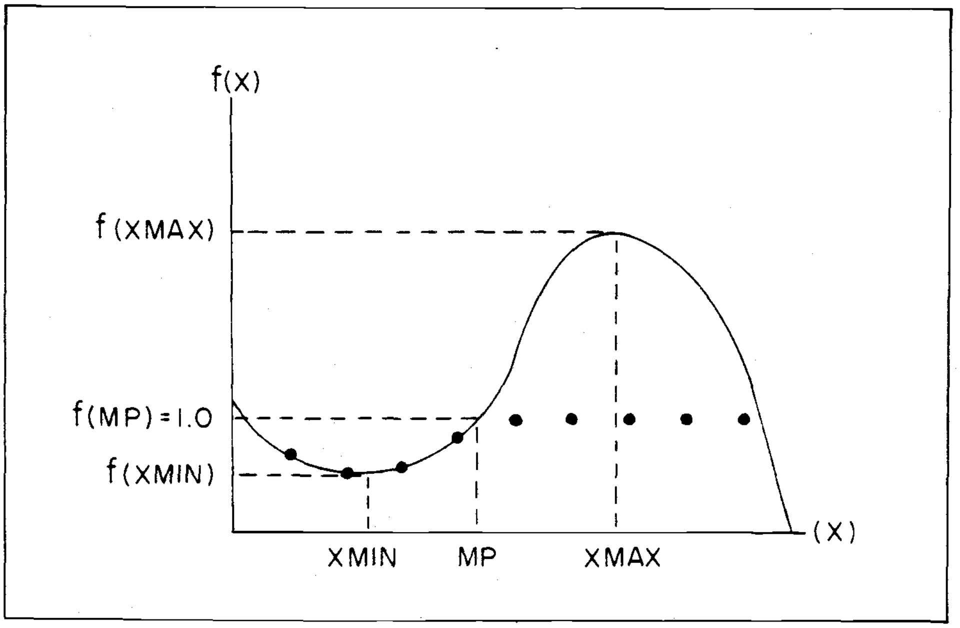


Figure D-15.

Cubic equation ($ax^3 + bx^2 + cx + d$) with local extrema ($b^2 - 3ac > 0$), $a < 0$, ITEQ = 5, and $f(XMAX) > 1$.

Schmidt, James S., and Philip L. Tedder. **TREES, Timber Resource Economic Estimation System. Volume II: mathematical analysis and policy guide.** Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 31b. 71 p.

The mathematical relations implicit in TREES (Timber Resource Economic Estimation System), a forest management and harvest scheduling model, are explained in this second of a four-volume series. Algorithm steps for applying fixed harvest scheduling methods (absolute amount, percent of inventory, and area control) are outlined and effects on harvest policy considered. Algorithms for the more complex variable methods (even-flow of volume, even-flow of a function of volume, present net benefit, and present net worth) are detailed; solution feasibility, optimality, and stability evaluated; and effects on harvest policy again considered. Growth options for even-aged stands (standard yield and approach-to-normal growth or volume) and uneven-aged stands (mortality, diameter growth, and ingrowth and upgrowth) are described. Appendices connect algorithms for the variable methods to their computer implementation and present supporting routines for performing quadratic interpolation and setting equation limits.

Key words: forest management, forest economics, harvest scheduling, simulation.

Schmidt, James S., and Philip L. Tedder. **TREES, Timber Resource Economic Estimation System. Volume II: mathematical analysis and policy guide.** Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 31b. 71 p.

The mathematical relations implicit in TREES (Timber Resource Economic Estimation System), a forest management and harvest scheduling model, are explained in this second of a four-volume series. Algorithm steps for applying fixed harvest scheduling methods (absolute amount, percent of inventory, and area control) are outlined and effects on harvest policy considered. Algorithms for the more complex variable methods (even-flow of volume, even-flow of a function of volume, present net benefit, and present net worth) are detailed; solution feasibility, optimality, and stability evaluated; and effects on harvest policy again considered. Growth options for even-aged stands (standard yield and approach-to-normal growth or volume) and uneven-aged stands (mortality, diameter growth, and ingrowth and upgrowth) are described. Appendices connect algorithms for the variable methods to their computer implementation and present supporting routines for performing quadratic interpolation and setting equation limits.

Key words: forest management, forest economics, harvest scheduling, simulation.

Schmidt, James S., and Philip L. Tedder. **TREES, Timber Resource Economic Estimation System. Volume II: mathematical analysis and policy guide.** Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 31b. 71 p.

The mathematical relations implicit in TREES (Timber Resource Economic Estimation System), a forest management and harvest scheduling model, are explained in this second of a four-volume series. Algorithm steps for applying fixed harvest scheduling methods (absolute amount, percent of inventory, and area control) are outlined and effects on harvest policy considered. Algorithms for the more complex variable methods (even-flow of volume, even-flow of a function of volume, present net benefit, and present net worth) are detailed; solution feasibility, optimality, and stability evaluated; and effects on harvest policy again considered. Growth options for even-aged stands (standard yield and approach-to-normal growth or volume) and uneven-aged stands (mortality, diameter growth, and ingrowth and upgrowth) are described. Appendices connect algorithms for the variable methods to their computer implementation and present supporting routines for performing quadratic interpolation and setting equation limits.

Key words: forest management, forest economics, harvest scheduling, simulation.