REFLECTION AND TRANSMISSION OF MICROWAVES IN PRISMS OF WOOD, SULFUR, AND SODIUM CHLORIDE

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TABLE OF CONTENTS

REFLECTION AND TRANSMISSION OF
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INTRODUCTION

When an electromagnetic plane wave crosses the boundary between two dielectric media of different permittivities, part of the wave will be reflected and part will be refracted. If the boundary between the two media is a plane surface, the incident, reflected and refracted waves will have wave normals which lie in a plane. The plane passes through a normal to the boundary and is called the plane of incidence (2, p.4). The angles that the incident, reflected and refracted wave normals make with the boundary normal are known as the angle of incidence, φ, the angle of reflection, φ', and the angle of refraction, ψ. The angle of reflection will be equal to the angle of incidence. The angle of refraction is related to the angle of incidence and the indices of refraction of the two media by Snell's law which states that

$$n_1 \sin\varphi = n_2 \sin\psi$$

where the radiation passes from medium one to medium two with indices of refraction n_1 and n_2 respectively. For the radiation passing from a dense medium to a rarer medium, so that $n_1 > n_2$, total reflection will occur when φ exceeds a critical angle, φ_c, which is obtained from

Snell's law by setting $\psi = 90^\circ$. φ_c will be given by,

$$\varphi_c = \sin^{-1} \frac{n_2}{n_1} = \sin^{-1} n$$

where n_2/n_1 is the relative index of refraction, n (2, p.5). There will be no refracted wave when φ_c is exceeded under these conditions and no energy is lost upon reflection. It is known that the totally reflected wave penetrates into the rarer medium approximately one wave length (1, p.73). If two dense media are separated by a rarer medium with a thickness less than a wave length it is possible for part of the incident energy to pass completely through the rarer medium for $\varphi > \varphi_c$ (10, p.374).

The electromagnetic theory affords a method of deriving equations which will give the fraction of the incident energy which is reflected or refracted for two samples of a medium separated by a less dense medium. The equations give reflection and transmission power coefficients which are functions of φ, n, λ_2 and d, where λ_2 is the wave length of the electromagnetic wave in the rarer medium and d is the thickness of the rarer medium. Four equations are derived, two for the electric vector plane polarized perpendicular to the plane of incidence, and two for the electric vector plane polarized parallel to the plane of incidence. The theory was worked out essentially by E. E. Hall in 1902 (1, pp.93-95). He performed various optical experiments with glass prisms to check his theory but due to the extremely small wave length of light, he could only get qualitative results. One of his qualitative experiments was to use 45°-45°-90° prisms (1, p.74). The critical angle for glass to air is about 42°. When light is incident perpendicular to one of the right angle faces of the prisms, total reflection will occur at the hypotenuse surface since the angle of incidence at that surface is 45° which exceeds the critical angle. Hall placed two identical prisms together to form a cube. Light incident perpendicular to one of the right angle faces was transmitted through the cube when the prisms were pressed together to exclude the air between the hypotenuse surfaces. The two prisms were then effectively a solid cube of glass and no reflection would take place at the hypotenuse surfaces. If prism 1 is designated as a stationary prism upon which the light is incident and prism 2 is a movable prism, so that the air gap between the hypotenuse surfaces can be varied, then as prism 2 is moved away from prism 1 more and more of the incident energy will be reflected from the hypotenuse surface. As the air gap is increased to approximately one wave length, essentially all of the incident energy will be totally reflected by prism 1.

Hall mentioned in his analysis that his theory should be checked with centimeter waves and, in 1910, Schaefer and Cross performed a similar experiment. They used 45°-45°-90° prisms made of paraffin which were 55 cm. square on the right angle faces and an electromagnetic radiation with a wave length of 15 cm. (6, pp.648-672). The critical angle for paraffin is about the same as for glass, thus 45°-45°-90° prism could be used. Again the experiments were not conclusive since the transmitter was not strictly monochromatic and the prisms were not very large as compared to the wave length of the radiation, a condition which may have produced edge effects. In 1950, precision measurements were made with modern microwave equipment by M. D. Pearson (5, pp.20-31). He also used 45°-45°-90° paraffin prisms. The right angle faces were about 25 cm. square and the wave length of the radiation was 3.2 cm. He found that the transmitted radiation fell to about 1% of the incident power when the prisms were separated 1.2 wave lengths for $E_{\perp}$ and 1.4 wave lengths for $E_{\parallel}$ where $E_{\perp}$ is for a plane polarized wave which has the electric vector, E, perpendicular to the plane of incidence and $E_{\parallel}$ is for a plane polarized wave with the electric vector parallel to the plane of incidence. The agreement between experimental and theoretical values was excellent since the average deviations were of the order of 5% for power coefficients greater than 0.2.

Even though Pearson's work agreed very well with the theory, it became desirable for the following reasons to make a further check using other prism materials. The reflection and transmission coefficients vary greatly over a small range of index of refraction. For example, the theory predicts that as the index of refraction is increased beyond a value $n_1 = 1.732$ and for $\varphi = 45^\circ$, the penetration is at first greater for $E_{\parallel}$ and then greater for $E_{\perp}$. The theory also predicts that the penetration is considerably less as n_1 is increased. Therefore, the purpose of this research is to give a further check of the theory and to show the marked variation of the transmission coefficients as n_1 is varied.

TRANSMISSION AND REFLECTION COEFFICIENTS

The transmission and reflection coefficients are derived from the electromagnetic theory by assuming a plane wave solution which satisfies Maxwell's equations. For a uniform nonconducting medium having a dielectric constant $k_e = \frac{\epsilon}{\epsilon_0}$ and magnetic permeability $k_m = \frac{\mu}{\mu_0} \cong 1$, Maxwell's equations and the constitutive equations can be resolved into two vector equations in E and H (8, p.91). These equations are in the M.K.S. system of units:

$$(1) \quad \nabla^2 E - \epsilon\mu \frac{\partial^2 E}{\partial t^2} = 0$$

$$(2) \quad \nabla^2 H - \epsilon\mu \frac{\partial^2 H}{\partial t^2} = 0$$

Referring to Figure 1, consider two slabs of a medium, 1, with constants μ_1, ϵ_1 separated by a rarer medium, 2, with constants μ_2, ϵ_2. Let the rarer medium be of thickness d and the planes of separation be infinite planes at $z = 0$ and $z = d$. Assuming a plane wave solution of equations (1) and (2) where the wave normal is in the x-z plane, the incident wave in the negative z region will be of the form

$$U = U_o \, e^{i\omega\left(t - \frac{x \sin\varphi + z \cos\varphi}{v_1}\right)}$$

where v_1 is the velocity of propagation of the electromagnetic wave and ω is the angular frequency of the wave. U is a scalar which can stand for any one of the components of E or H and U_o is a constant. For the plane wave the E and H vectors will be at right angles to the direction of propagation and also at right angles to each other (8, p.94). The plane wave can be considered for two special cases (8, p.121).

Case I. The electric vector is perpendicular to the plane of incidence, $E_\perp$.

Case II. The electric vector is parallel to the plane of incidence, $E_\parallel$.

For Case I where the E vector is polarized in the y direction the incident wave will be

$$(3)\qquad \begin{aligned} E_\perp &= E_o\, e^{i\omega\left(t - \frac{x\sin\varphi + z\cos\varphi}{v_1}\right)} \\ H_x &= -\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\varphi\, E_\perp \end{aligned}$$

where the amplitude E_o is assumed to be unity and where H_x is the component of H in the x direction.

For a wave reflected in the first medium which will be denoted by double primes,

$$(4)\qquad \begin{aligned} E''_\perp &= R\, e^{i\omega\left(t - \frac{x\sin\varphi - z\cos\varphi}{v_1}\right)} \\ H''_x &= \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\varphi\, E''_\perp \end{aligned}$$

where R is the amplitude of tangential E in the reflected wave.

For medium 2 the tangential components of E and H are given in two sets of equations corresponding to waves traveling to the left and to the right. For increasing Z, denoted by the superscript +,

$$(5)\qquad \begin{aligned} E^+_\perp &= T^+\, e^{i\omega\left(t - \frac{x\sin\psi \;\; z\cos\psi}{v_2}\right)} \\ H^+_x &= -\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\psi\, E^+_\perp \end{aligned}$$

For decreasing Z, denoted by the superscript -.

$$(6)\qquad \begin{aligned} E^-_\perp &= T^-\, e^{i\omega\left(t - \frac{x\sin\psi - z\cos\psi}{v_2}\right)} \\ H^-_x &= \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\psi\, E^-_\perp \end{aligned}$$

Equations (5) and (6) are summations which take into account multiple reflections. The phase differences which are introduced are included in the amplitude T^+ and T^- which are to be regarded as complex (1, p.94).

For the transmitted wave in the first medium where $z = d$, denoted by a single prime,

$$(7) \quad \begin{aligned} E'_\perp &= T\, e^{i\omega\left(t - \frac{x \sin\varphi + z \cos\varphi}{v_1}\right)} \\ H'_x &= -\sqrt{\frac{\epsilon_1}{\mu_1}} \cos\varphi \; E'_\perp \end{aligned}$$

Equations (3), (4), (5), (6) and (7) must satisfy the following boundary conditions. At a surface of discontinuity between two media, the tangential components of E and H are continuous at a surface that contains no charge or current (8, p.117). This is stated by the equations

$$(8) \quad \begin{aligned} E_\perp + E''_\perp &= E^-_\perp + E^+_\perp \\ H_x + H''_x &= H^-_x + H^+_x \quad \text{at } Z = 0, \end{aligned}$$

and

$$\begin{aligned} E^-_\perp + E^+_\perp &= E' \\ H^-_x + H^+_x &= H'_x \quad \text{at } Z = d. \end{aligned}$$

Applying the boundary conditions to equations (2) through (6) and rewriting $\cos\psi$ in terms of $\cos\varphi$, for the case where $\varphi > \varphi_c$, the transmission and reflection coefficients for $E_\perp$ can be found. Similarly, the transmission and reflection coefficients can be found for $E_{\parallel}$. This is worked out in detail by Pearson for both cases (5, pp.2-20). The transmission coefficient, $T_{\perp}$, for the electric vector perpendicular to the plane of incidence is

$$(10)\quad T_{\perp} = \left[\frac{(\sinh^2 u)(1-n^2)^2}{(4\cos^2\varphi)(\sin^2\varphi - n^2)} + 1\right]^{-1}$$

The reflection coefficient, $R_{\perp}$, for the electric vector perpendicular to the plane of incidence is

$$(11)\quad R_{\perp} = \left[\frac{(4\cos^2\varphi)(\sin^2\varphi - n^2)}{(\sinh^2 u)(1-n^2)^2} + 1\right]^{-1}$$

The transmission coefficient, $T_{\parallel}$, for the electric vector parallel to the plane of incidence is

$$(12)\quad T_{\parallel} = \left[\frac{(\sinh^2 u)(1-n^2)^2\,(\sin^2\varphi - n^2\cos^2\varphi)^2}{(4n^4\cos^2\varphi)(\sin^2\varphi - n^2)} + 1\right]^{-1}$$

The reflection coefficient, $R_{\parallel}$, for the electric vector parallel to the plane of incidence is

$$(13)\quad R_{\parallel} = \left[\frac{(4n^4\cos^2\varphi)(\sin^2\varphi - n^2)}{(\sinh^2 u)(1-n^2)^2\,(\sin^2\varphi - n^2\cos^2\varphi)^2} + 1\right]^{-1}$$

When φ is less than the critical angle, φ_c, the value of u that appears in the equations will be

$$u = \frac{i\,2\pi}{\lambda_2} d \cos$$

where λ_2 is the wave length of the electromagnetic radiation in medium 2. When φ is greater than φ_c, which is the case for total reflection, $\sin\psi$ will be greater than one. This means that ψ must be imaginary and will be of the form

$$\cos\psi = -i\sqrt{\frac{\sin^2\varphi}{n^2} - 1}$$

where the negative sign indicates a disturbance in the rarer medium decreasing in amplitude with the distance of penetration (5, p.10). The expression for u now becomes

$$(14) \qquad u = \frac{2\pi}{n}\frac{d}{\lambda_2}\sqrt{\sin^2\varphi - n^2}$$

The indices of refraction of medium 1 and 2 are related to the dielectric constants and the magnetic permeabilities by the equations

$$(15) \qquad n_1 = \sqrt{k_{e_1} k_{m_1}}$$

$$(16) \qquad n_2 = \sqrt{k_{e_2} k_{m_2}}$$

where k_e and k_m are the dielectric constant and the magnetic permeability respectively (8, p.93). For a uniform nonconducting medium, k_m is approximately one and the indices of refraction will be the square roots of their respective dielectric constants. The velocity of propagation of the undamped wave can be expressed in terms of the index of refraction of the medium and the velocity of light by the equations

$$v_1 = \frac{c}{n_1}$$

$$v_2 = \frac{c}{n_2}$$

where v_1 and v_2 are the velocities of the wave in medium 1 and 2 respectively and c is the velocity of light in empty space (8, pp.92, 93).

DESCRIPTION OF EQUIPMENT

The equipment consisted of a microwave transmitter and receiver, an absorbing screen, wooden, sulfur and salt prisms and a movable platform as shown in Figure 2.

The transmitter consisted of a Varian x-13 reflex klystron(with cooling fan)as a source of microwave power, an attenuator, a cavity type wavemeter and an electromagnetic horn. The reflector supply voltage of the klystron tube was modulated with a 1000 cycle per second square wave. A square wave was necessary to insure monochromatic radiation since the generated wave length will vary with a slight change in the reflector supply voltage. The cavity of the tube was tunable over a range of 8.2 kilomegacycles to 12.4 kilomegacycles. The power output of the tube was approximately 150 milliwatts. The tube was fastened directly to a 1/2 inch by 1 inch rectangular wave guide which propagated the $TE_{o,1}$ mode (7, p.228). The wave was generated into the wave guide through a thin transparent window in the tube as shown in Figure 5. The wave guide had a variable knife edge type attenuator in it to regulate the intensity of the output power.

Connected to the attenuator was a General Electronics model 1521 cavity wave meter which was capable of wave length measurements to three significant figures. The wave meter was terminated with an electromagnetic horn. The aperture of the horn was 15 centimeters by 12 centimeters in the H and E planes respectively. The flare angle of the horn was about 18.5°. The transmitter was very stable over a long period of time and it was not necessary to have a reference signal for monitoring purposes. The polarization of the transmitted wave was changed from $E_{\perp}$ to $E_{\parallel}$ by inserting a piece of wave guide, which was twisted 90° along its axis of symmetry, between the horn and the wave meter as shown in Figure 4. The transmitter is shown set up to transmit $E_{\perp}$ in Figure 2 and $E_{\parallel}$ in Figure 4.

The receiver consisted of a receiving horn with a bolometer type detector mounted in it, an audio amplifier, and a Ballantine Model 300 voltmeter. The receiving horn had an aperture of 6 centimeters by 4.8 centimeters in the H and E planes respectively. The flare angle was about 10.5°. The horn was terminated with an 82x Sperry Barretter bolometer. The bolometer was biased with about 8 milliampers of D. C. current to give a linear response to the incident energy. When the electromagnetic energy is incident upon the bolometer wire the wire will heat up and its resistance will change. The resistance change is periodic due to the 1000 cycle per second modulation of the transmitted wave and a periodic current change proportional to the power flows through the primary of a transformer located in the amplifier (7, p.602). The amplifier amplifies the 1000 cycle variation in current in order to obtain a measurable reading on the Ballantine voltmeter. The amplifier and voltmeter were designed to operate as a unit which was linear over a range of 100 decibels. A schematic diagram of the amplifier is given (7, p.605). The output of the amplifier was coupled to the Ballantine voltmeter by a coaxial cable.

The absorbing screen was made of a sheet of 1/4 inch fir plywood 100 centimeters by 122 centimeters. One side of the plywood was covered with aluminum foil and the other side with aquadag impregnated cloth. Rubber cement was used to apply both coverings. A 12 centimeter square hole was cut in the center of the screen to allow the microwave beam to pass through to the prisms. The screen was placed with the absorbing cloth towards the transmitter. An absorbing screen of this type operates on the following principle. Microwave radiation incident upon the screen is reflected by the aluminum foil and standing waves are set up between the aluminum foil and the transmitter. If a sheet of absorbing material is placed a quarter wave length from the foil, where the first antinode is located, the maximum amount of energy will be absorbed (2, p.209). The wave length of the microwaves in fir is about one inch; therefore, the 1/4 inch plywood served to separate the cloth and the foil by a quarter wave length. Not all of the reflected energy is absorbed by a screen of this type and further precautions must be taken. A considerable error will result in the measurements if waves which are reflected about the room arrive at the receiver in phase. This type of error can be greatly reduced by disturbing the phase of the reflected waves. This was done by placing the equipment at an angle such that the transmitted beam made an acute angle with the walls of the room, and by using a curved absorbing screen. The concave side of the screen was placed next to the prisms so the energy reflected from the other side would be scattered about the room. The aluminum side of the screen is shown in Figure 2, and the absorbing cloth side and the hole are shown in Figure 6. The description of the prisms is given in the following section. Table I gives the essential information for the materials used. The dielectric constant is given for a frequency of 1×10^{10} cycles per second (4, pp.4, 41, 42). The index of refraction was computed from the dielectric constant by equation

(15) where k_m is one. The critical angle is computed from Snell's law where $\psi = 90^{\circ}$.

CONSTRUCTION OF PRISMS

The wooden prisms were made of kiln dried fir. A six-foot board of select fir was obtained which had a good even grain. The board was run through a planer and then cut into lengths with the proper angles to make a prism when stacked. For fir the critical angle is about 48° and right angle prisms could not be used. The prisms had angles of 52°-52°-76° with sides 26 cm. by 26 cm. by 32 cm. and were about 30 cm. high. Two pairs of prisms were made with the grain going in a different direction for one pair than in the other pair. The first pair of prisms were glued by spreading glue on the ends of the prism and then clamping a triangular shaped piece of plywood on the ends. One of the prisms is shown on the far right in Figure 6. The prisms looked very good at first but several days later the wood warped and cracks about 1/16 inch wide appeared where the microwave beam was to pass through the prisms. A different method was used in gluing the second pair to try to eliminate the warping of the wood. Grooves were cut near the ends of each piece of wood as shown in the prism second from the right, Figure 6. The pieces of wood were glued on the surfaces to the outside of the grooves and then clamped. The grooves were cut into the wood to prevent glue from running between the layers of wood where the microwave beam would pass. This set of prisms also warped slightly after several days and cracks about 1/32 inch wide appeared between the layers of wood.

The sulfur prisms were cut from a square block of sulfur which was cast from molten sulfur. The sulfur was poured into a rectangular wooden box with a square base. The box was 30 cm. by 30 cm. by 40 cm. and was coated with shellac to keep the sulfur from sticking to the wood. The main problem in making a homogeneous cube was in cooling the molten sulfur so that no large crystals or air spaces would be formed. In cooling small amounts of sulfur, it was noted that if a crust was allowed to form over the top, large crystals and air spaces would be formed underneath the crust as the remaining sulfur solidified. It was decided that the sulfur should be cooled from the bottom, if possible, and stirred constantly to keep the crust from forming over the top. The sulfur was poured from a large vat of molten sulfur which was kept at a constant temperature of about 120° C. The box was filled to a height of 34 cm. and then placed in a cold water bath which circulated about the bottom and about 10 cm. up the sides of the box. The top of the box was covered with paper to insulate the surface of the sulfur and the sulfur was stirred constantly through a hole in the paper to prevent the surface from crusting over. Due to the large amount of sulfur, 1.08 cubic feet, it was impossible to solidify the sulfur as planned. As soon as a layer about 5 cm. thick solidified about the bottom and sides of the box, it insulated the remaining molten sulfur from the cold water and the sulfur would not solidify near the bottom. The remaining sulfur cooled very slowly and did not begin to solidify until about 3 hours after it was poured. Since the sulfur was stirred constantly, the liquid gradually became thick and granular and after about another hour it began to set up enough so that it was very difficult to stir. During the final stages of cooling, the stirring rod was raised slowly so that the bottom would have a chance to solidify first. Since the sulfur was getting very thick, the rod was moved back and forth along a diagonal as it was raised so that if any spaces were left due to the rod there would be a chance of cutting them out by cutting a slice from the hypotenuse surfaces of the prisms after the first diagonal cut was made on the block of sulfur. When the sulfur could no longer be stirred, the stirring rod was removed and the sulfur was allowed to solidify over the top. The sulfur was cooled over night. The sides of the box were then removed from the block and a test slice 3 cm. thick was cut from the top of the block with a hand saw. The slice was very porous so another slice 3 cm. thick was cut. This slice looked very good on the side where the last cut was made. There were no air spaces and the sulfur had a hard granular texture. The block was then cut along the diagonal. This was done by making a wooden miter box type saw guide so that a straight cut could be made. Care was taken to keep the sulfur dust out of the adjoining laboratories as much as possible. A thin slice was then cut from each hypotenuse surface to remove some small imperfections due to the stirring rod during the final stages of cooling. The hypotenuse surfaces were then scraped and cleaned with a damp cloth. Several cracks were present in these surfaces due to uneven cooling so molten sulfur was poured into the cracks to fill them up as much as possible. The faces of the prism were scraped by hand with a straight edge to form two identical 45°-45°-90° prisms. The scraping was done by hand since it was undesirable to have the sulfur in contact with any machinery as it may have corroded them. The prisms were then tried out and several runs were made of the transmission and reflection coefficients versus the distance of separation of the hypotenuse surfaces. The curves were displaced considerably along the distance axis and it was believed to be due to the hypotenuse surfaces not being plane enough. New hypotenuse surfaces about 1/8 inch thick were poured onto the prisms and the faces were scraped to a plane surface before the sulfur had a chance to get extremely hard. These surfaces turned out to be much better and were used for the final experimental curves. The final size of the prisms was 28 cm. square on the right angle faces. The prisms are shown in Figure 2 and Figure 6.

The salt prisms were cut with a hand saw from blocks of pasture salt obtained from a farm feed store. One prism was cut from each block. The blocks which were finally used had absorbed very little moisture since they were stored in a window which was exposed to direct sunlight. Previously, a block which had been stored in a damp place was tried out for transmission and the microwave beam would not pass through the block. The block was then baked for 24 hours and when the microwaves were directed at the block, the transmission was almost 100%. Thus care was taken to keep the blocks as dry as possible. The same method was used in cutting and scraping the salt blocks as that used for the sulfur block. The blocks were cleaned with a dry cloth instead of a damp cloth and to obtain a better plane hypotenuse surface, machinist layout dye was spread over a flat piece of plate glass and allowed to dry. The hypotenuse surface was then placed face down on the glass and moved about slightly. The prism was turned over and scraped where the dye had rubbed off onto the high spots on the salt. This method worked very well in obtaining a plane surface. Only the hypotenuse surfaces were treated in this manner since the other surfaces were not so critical. Since the critical angle for salt is less than 45°, 45°-45°-90° prisms could be used. The size of the prisms was 21.6 cm. square on the right angle surfaces with angles of 45°-45°-90°.

MEASUREMENT OF THE REFLECTION AND TRANSMISSION COEFFICIENTS

The microwave equipment was warmed up for about one hour to allow it to become stabilized for constant power output. A photograph of the equipment is shown in Figure 2 and a schematic sketch is shown in Figure 3. The intensity of the transmitted wave was measured in the following manner. Referring to Figure 3, prism 2 was moved towards prism 1 until the separation of the hypotenuse surfaces, d, was zero. A homogeneous cube was formed and total transmission occurred. The receiver was placed 20 cm. away from the face of prism 2 to receive the transmitted power. The attenuator in the transmitter was adjusted to give a full scale reading on the voltmeter. Prism 2 was then moved away from prism 1, parallel to the incident beam, such that the separation, d, was 1 mm. The receiver horn was moved parallel to the prism face, along AB, to obtain a maximum reading on the voltmeter. This reading was recorded and then d was increased in 1 mm. steps out to 1 cm. The maximum reading was obtained and recorded for each step. Beyond 1 cm. the readings were recorded in 2 mm. steps until the transmitted power dropped to the noise level of the voltmeter which was approximately 0.005 volt. To measure the intensity of the reflected wave, the prisms were again moved together to form a cube and the transmitted wave was checked for a full scale reading. The receiver was then placed 20 cm. away from the face of prism 1 to receive the reflected energy as shown by the dashed line sketch of the receiver in Figure 3. Prism 2 was then moved away from prism 1 in the same manner as for the transmitted energy run and the receiving horn was moved parallel to the prism face, along CD, to obtain the maximum reading for each step. The reading was recorded each time until the reflected energy increased no more. Several runs were made alternately of the transmitted and reflected energy.

Measurements were made for $E_{\perp}$ and then for $E_{\parallel}$ by changing the polarization of the transmitted microwaves. The receiver had a similar piece of twisted wave guide as shown for the transmitter in Figure 4 to receive the change of polarization. Figure 2 shows the equipment set up to receive the energy where the electric vector is polarized perpendicular to the plane of incidence. The transmitter horn was placed one meter from the screen to approximate the condition that the incident wave be a plane wave. The boundary condition that the separation of medium 1 and 2 be infinite planes was approximated by keeping the prisms as large as possible compared to the wave length of the microwave beam. The smallest prisms were over 6 times λ_2 to approximate the last condition. The hole in the screen was made large enough to enable the receiver to be placed in the Frenel region where serious diffraction scattering would not occur (7, p.574). The hole was about 3.4 times λ_2 which kept the beam from coming too close to the edges of the prisms. The screen was placed next to prism 1 rather than some distance away because it was observed that the voltmeter reading would vary considerably upon any slight movement of the screen. This was probably due to standing waves set up between the screen and the face of the prism. Although it may seem objectionable to place the receiver in medium 2 (air), it is proper to do this provided that the distance the transmitted and reflected waves travel in air are the same for all readings. This requirement is observed by keeping the receiving horn opening along the line AB or CD as shown in Figure 3, where the lines are stationary with respect to prism 1.

The materials used to make the prisms must be transparent to the microwaves. The transparency is determined from the loss tangent of the material. The loss tangent is given as

$$\text{loss } \tan \delta = \frac{\epsilon_2}{\epsilon_1}$$

where the permittivity ϵ is considered to be complex and of the form

$$\epsilon = \epsilon_1 - i\epsilon_2$$

The power dissipated in a dielectric in terms of the loss tangent is

$$P = \frac{\epsilon_1 E^2}{4\pi} \omega \tan \delta$$

where ω is the angular frequency of the radiation and ϵ_1 is the real dielectric constant (3, pp.108-110). For a good insulator the loss tangent is usually below 0.001.

The reflection and transmission coefficients were computed from the data by the following method. An average of the meter readings was computed for the reflected and the transmitted energy for each distance of separation, d. The sum of the average reflected and the average transmitted energy was then computed for each value of d and averaged to give a measure of the intensity of the incident microwave beam. The absorption was considered to be negligible for the sulfur and salt because of the very low loss tangents of the materials. By taking the ratio of the average reflected, or transmitted, energy value to the intensity of the incident microwave beam, the reflection or transmission was found for each distance of separation of the hypotenuse surfaces of the prisms. The power coefficients were then plotted against d/λ_2 to give a direct comparison with the wave length of the microwave beam.

RESULTS AND CONCLUSIONS

Only qualitative results were obtained for the wood prisms. The fir would not exhibit total reflection and absorbed a considerable amount of the incident energy. This was believed to be due partly to the cracks between the layers of wood and later it was found that fir has a comparatively high loss tangent for microwave radiation (4, p.42). When prism 2 was moved entirely away from the microwave beam, the transmitted energy increased about 20 times over the reading when d was zero. The reflected energy increased to only about 0.8 of the transmitted energy when d was zero. For these reasons, wood was abandoned as a good material for prisms.

Much better results were obtained for sulfur and salt. The experimental and theoretical values of the transmission and reflection coefficients for sulfur are shown graphically in Figure 7 and Figure 8. The corresponding curves for salt are shown in Figure 9 and Figure 10. The theoretical curves were plotted for $\varphi = 45^\circ$ and a relative index of refraction of 0.329 and 0.412 for sulfur and salt respectively.

The coefficients are also shown in tabular form for sulfur and salt in Tables II and III respectively. No curves or tables are shown for wood since the measurements were of no value for these studies. The curves for salt and sulfur show the very pronounced decrease in depth of penetration into the rarer medium as n_1 is increased. The reversal of the penetration of $E_{\parallel}$ and $E_{\perp}$ as n_1 increases beyond 1.732 is also shown by noting that for salt, the transmission coefficients show a greater penetration for $E_{\perp}$ whereas for paraffin the penetration was greater for $E_{\parallel}$. The error for the power coefficients above 0.15 is given in Tables II and III for each power coefficient. The error in the angles for the sulfur and salt prisms was less than 0.5° which was believed to cause a negligible error in the curves. The large error for $E_{\perp}$ in the transmission coefficients for sulfur is believed to be due to a systematic distance measurement error. It is possible that the planes were not kept exactly parallel as d was increased, or it may have been due to d not being exactly zero when the prisms were moved together. This error could be corrected by shifting the experimental points to the right corresponding to a distance of about 0.5 mm. which would bring them into much better agreement with the theoretical curve. Some of the curves for salt also show a considerable error but it is believed to be due to a distance measurement error of less than 0.5 mm. which would bring the experimental points into excellent agreement with the theoretical curves.

Since the theoretical curves for the theory of penetration into the rarer medium have been verified for two more substances and a predicted reversal of the penetration of $E_{\perp}$ and $E_{\parallel}$ as n_1 is varied beyond 1.732 has also been verified the purpose of this research has been accomplished.

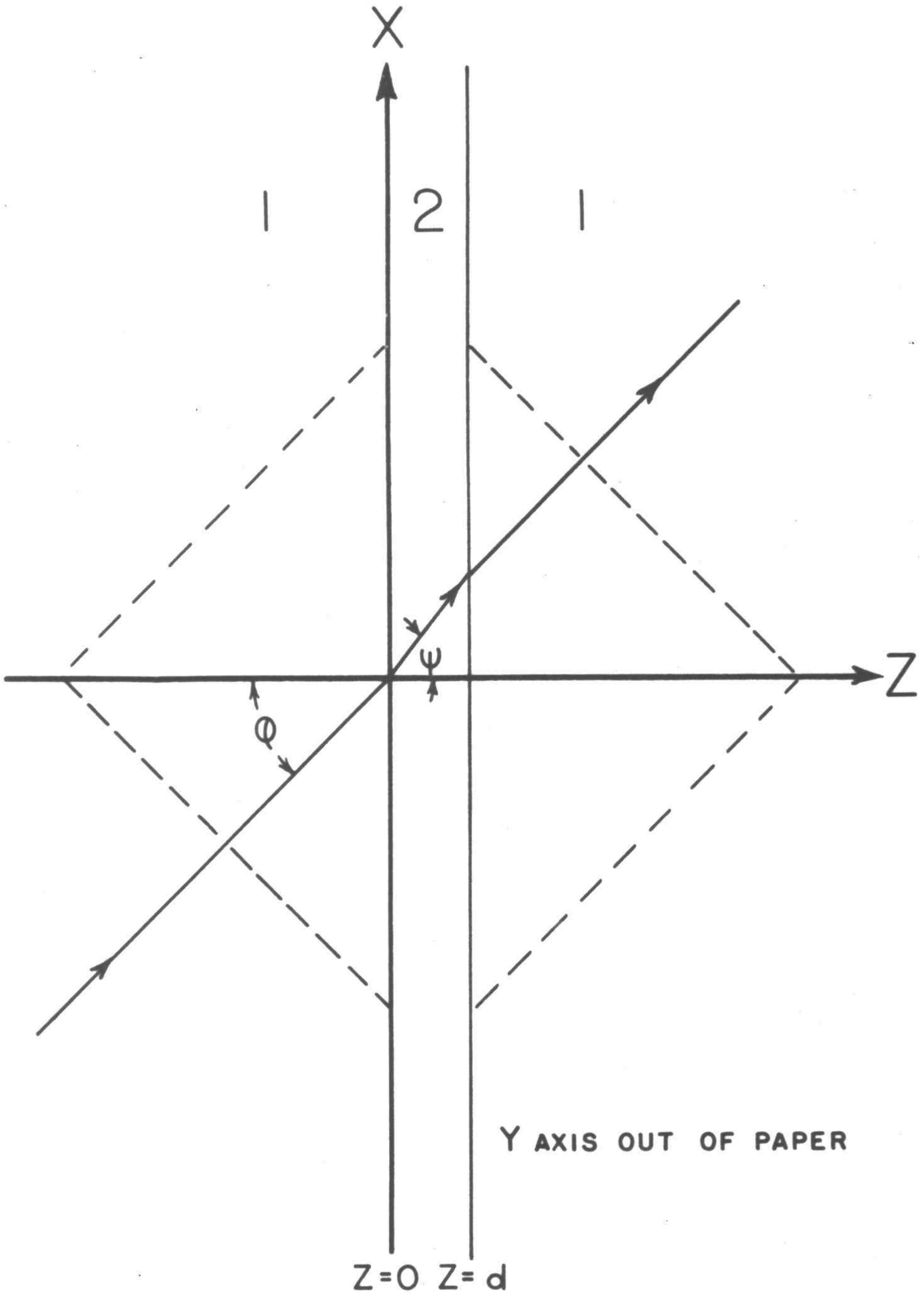

FIG. 1. COORDINATE SYSTEM SHOWING OUTLINE OF PRISMS.

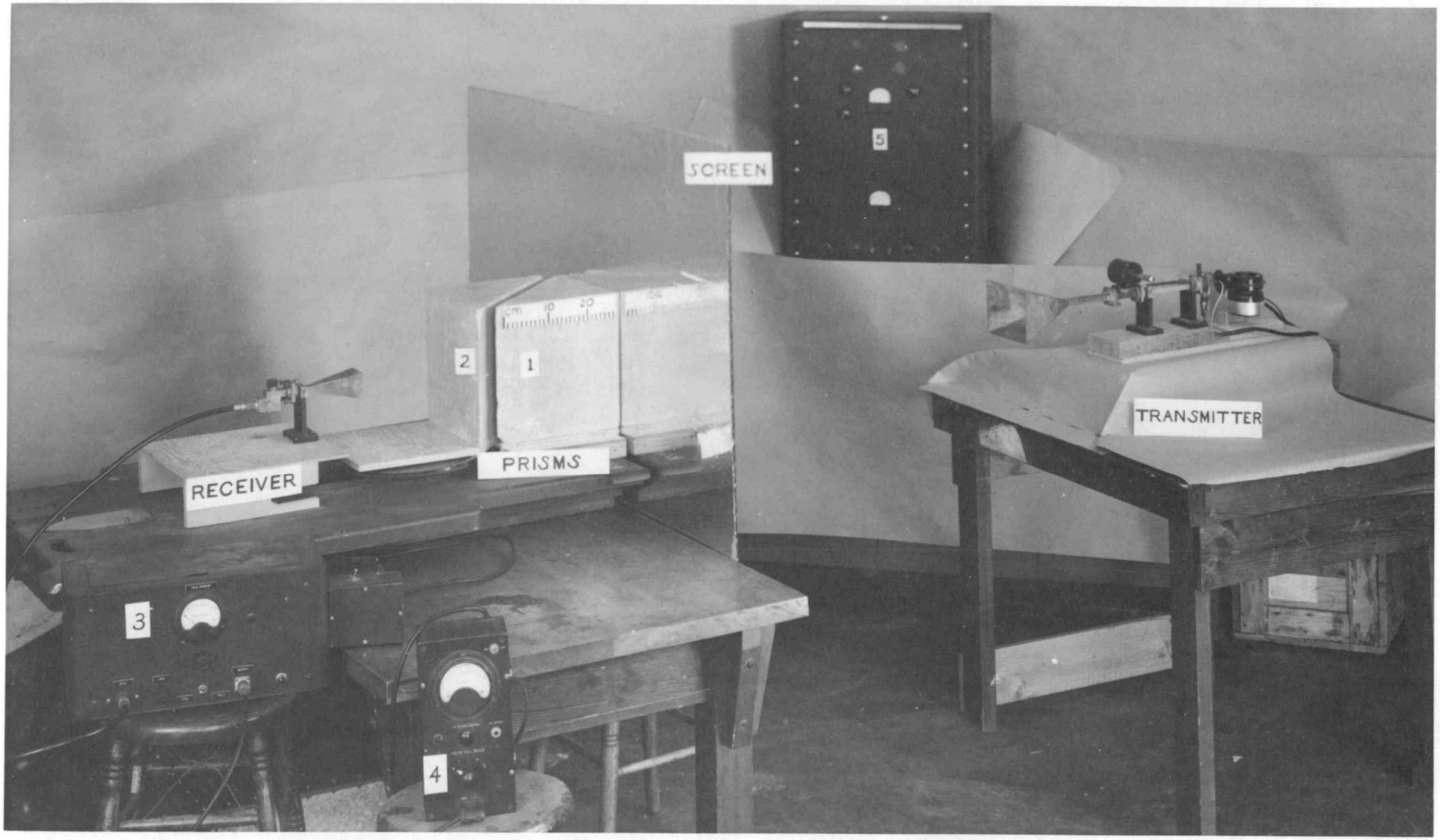

Fig. 2. The equipment set up to measure the transmitted wave for sulfur. 1. Stationary prism; 2. Movable prism; 3. Bolometer amplifier; 4. Ballantine voltmeter; 5. Transmitter power supply.

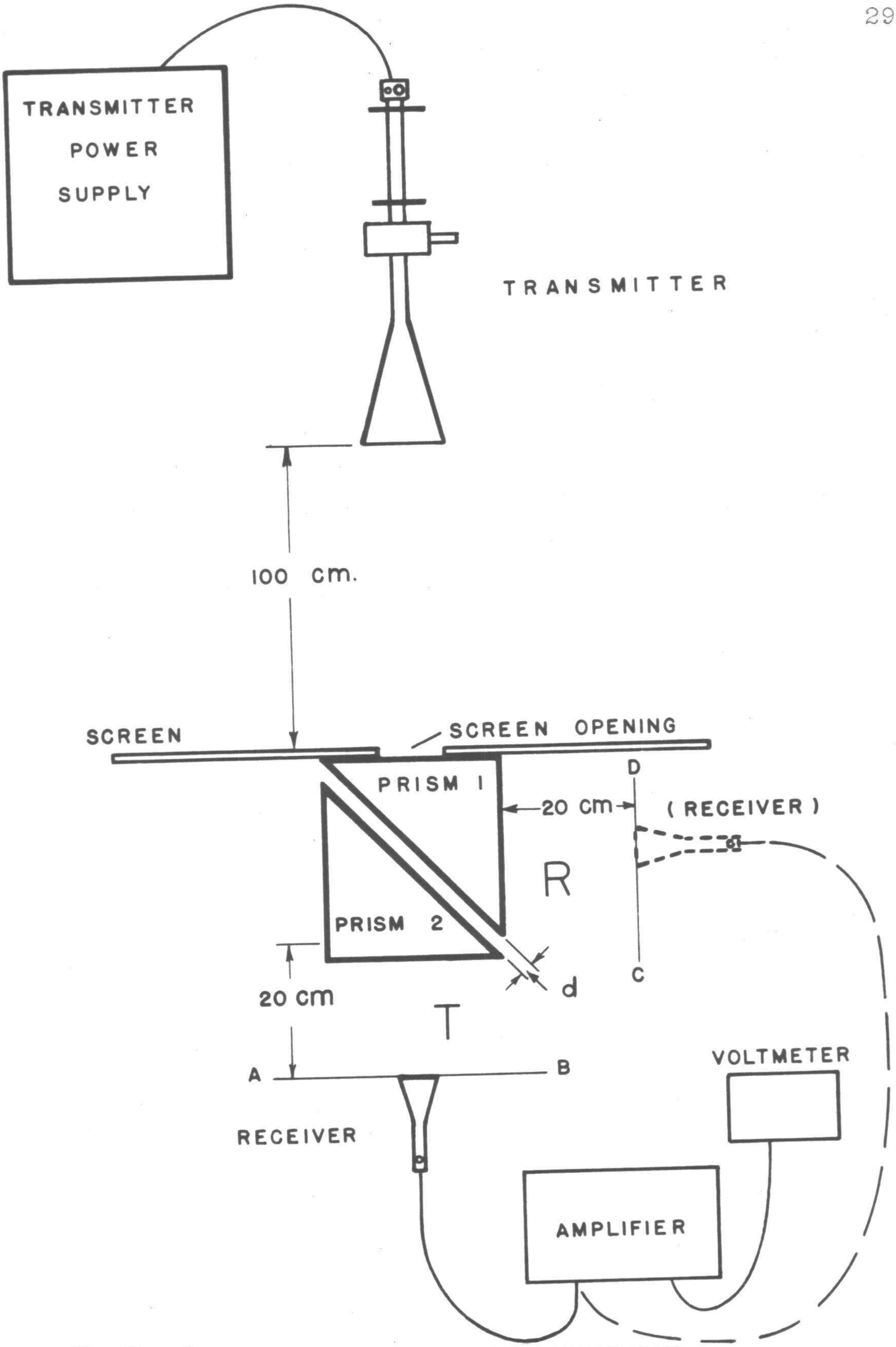

FIG. 3. SCHEMATIC SKETCH OF APPARATUS.

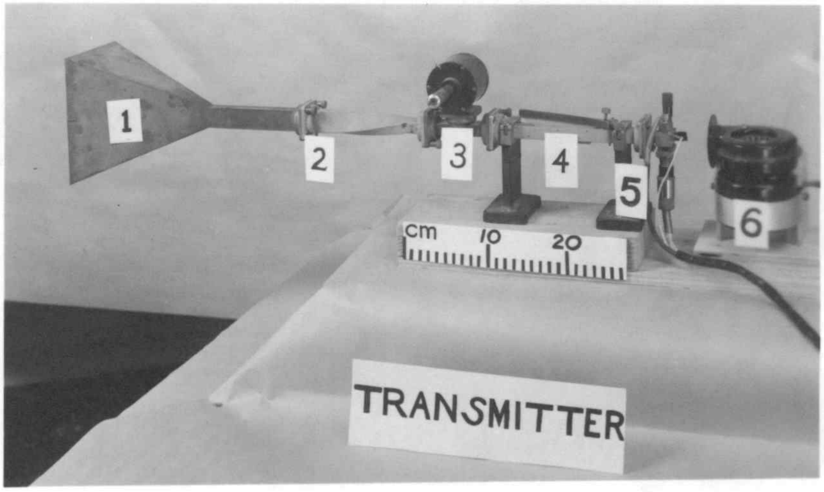

Fig. 4. 1. Horn; 2. Twisted wave guide;
3. Wave meter; 4. Attenuator
5. Klystron; 6. Cooling fan.

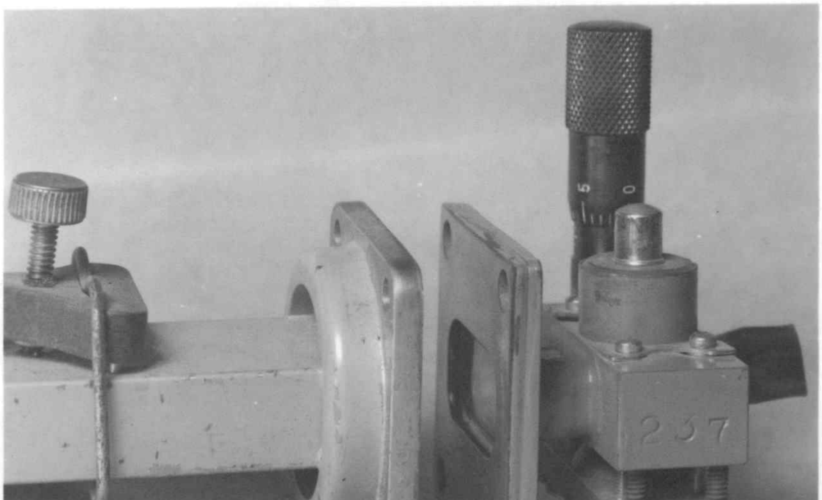

Fig. 5. Klystron showing the transparent window.

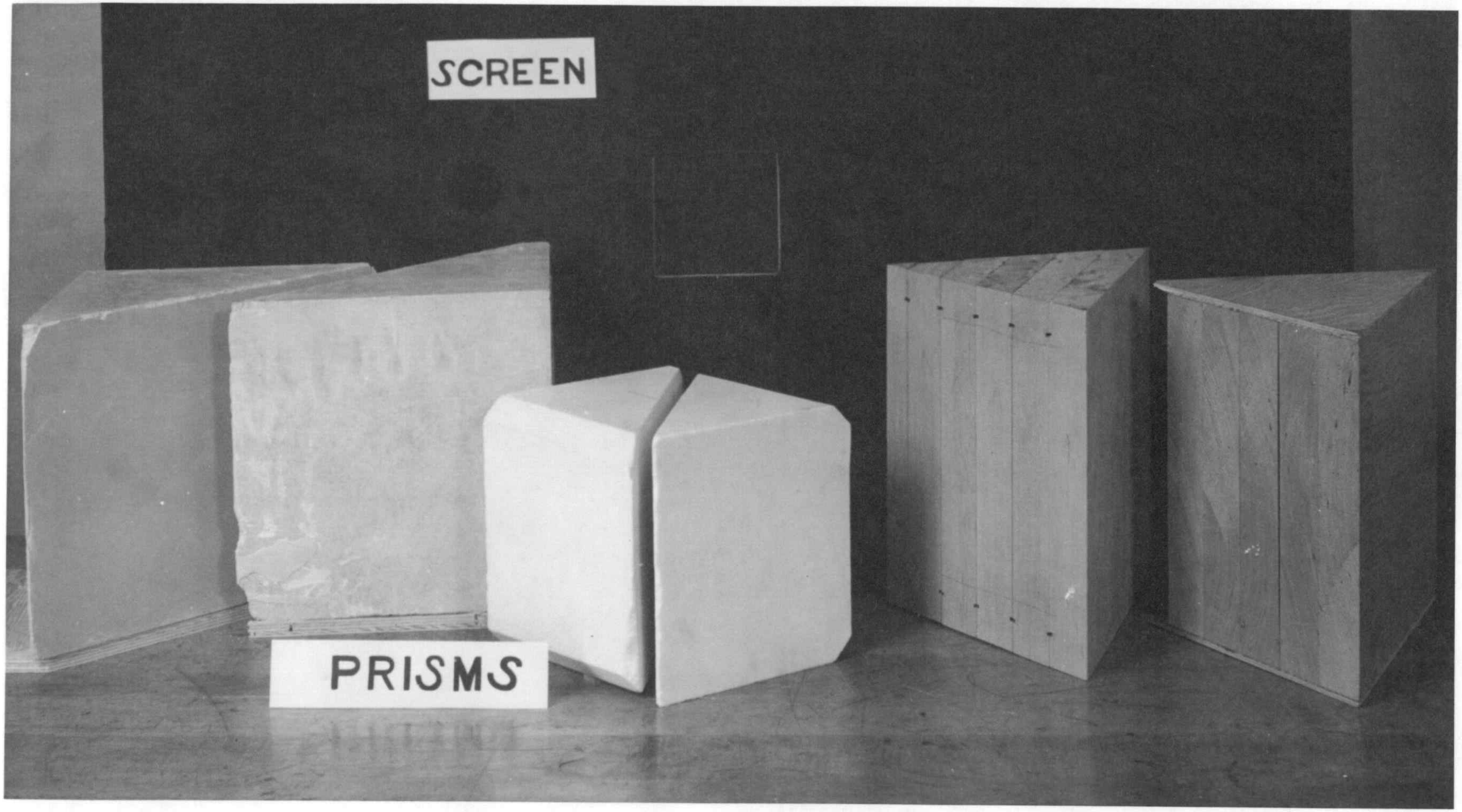

Fig. 6. Prisms - Left to right. Sulfur, Salt and Wood.
Screen showing the side covered with
impregnated cloth and the 12 cm. square opening.

TABLE I

PRISM MATERIAL CONSTANTS AND PRISM SIZES

PRISM MATERIAL	CONSTANTS					PRISM SIZE	
	k_e	n_2	** φ_c	φ	Loss tangent	Size (cm.)	Angles
Wood (Fir)	1.82	1.35	48°	52°	0.0290	26 x 26 x 32	52°-52°-76°
Sulfur	3.58	1.90	32°	45°	.00015	28 x 28 x 39.6	45°-45°-90°
Salt (NaCl)	5.90	2.43	24°	45°	.00050	21.6 x 21.6 x 30.6	45°-45°-90°
*Paraffin	2.25	1.5	42°	45°	.00021	25.6 x 25.6 x 36	45°-45°-90°

*Prism material used by Pearson (5, p.23)

** φ_c computed for n_1 = 1. (air)

POWER COEFFICIENTS VS. PRISM SPACING FOR SULFUR

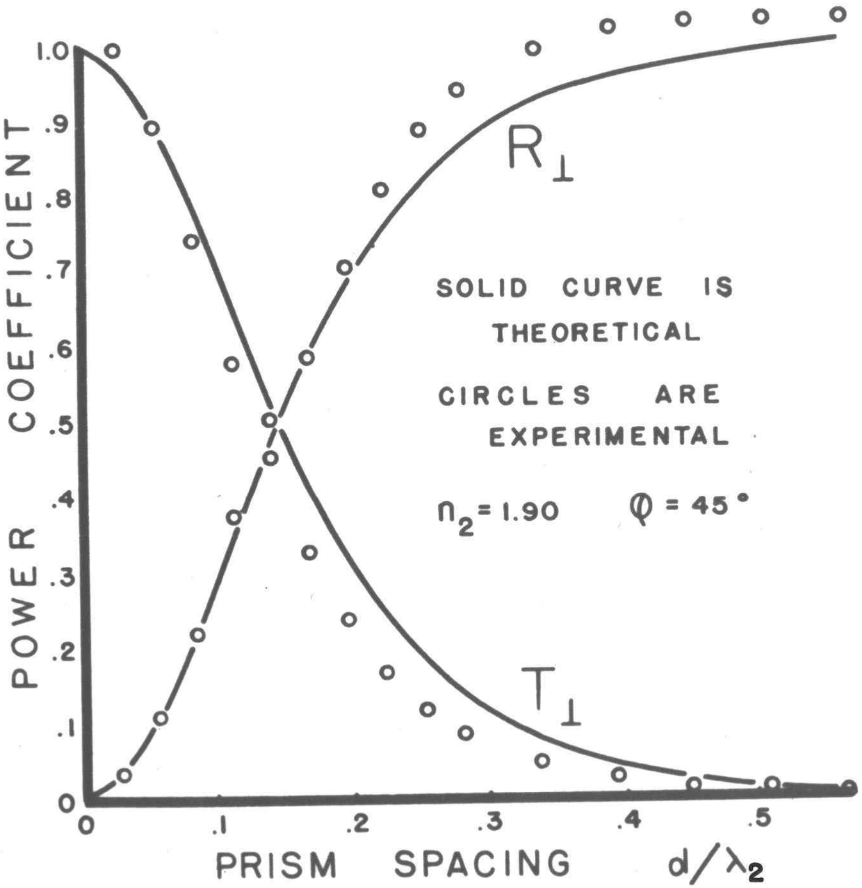

FIG. 7. E PERPENDICULAR TO PLANE OF INCIDENCE.

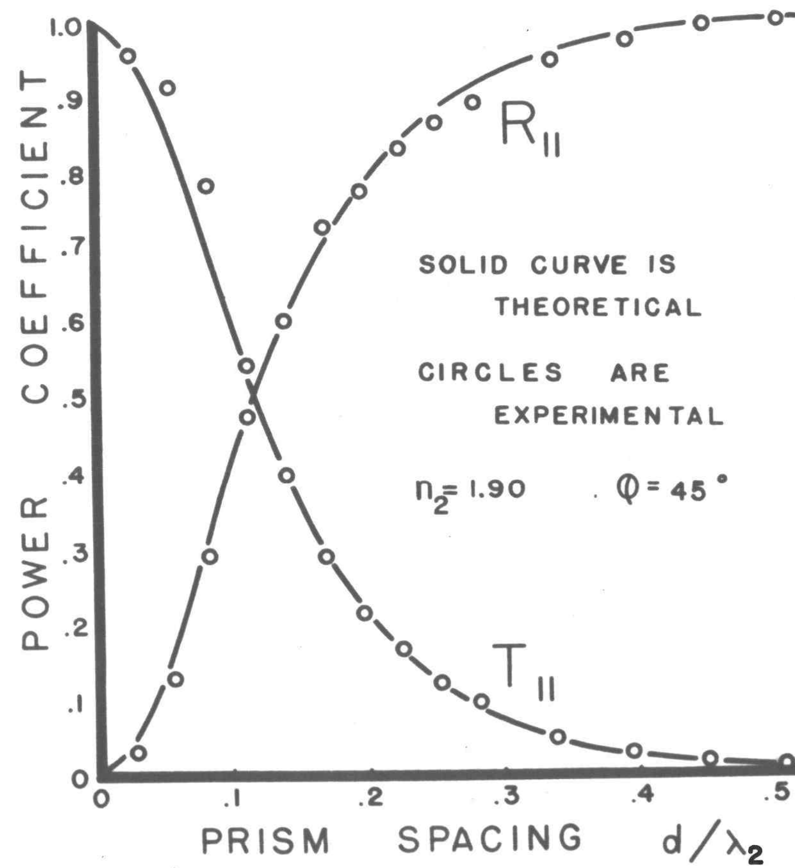

FIG. 8. E PARALLEL TO PLANE OF INCIDENCE.

TABLE II

NUMERICAL VALUES OF COEFFICIENTS FOR SULFUR

(a) Power Coefficients for $E_\perp$

PRISM SPACING		EXPERIMENTAL		THEORETICAL		% ERROR FOR COEFFICIENTS > 0.15	
d(cm.)	d/λ_2	$T_\perp$	$R_\perp$	$T_\perp$	$R_\perp$	$T_\perp$	$R_\perp$
0.0	0.000	1.028	0.005	1.000	0.000	3.6	
.1	.028	0.992	.035	0.972	.028	2.1	
.2	.056	.890	.103	.892	.108	0.2	
.3	.085	.740	.218	.780	.220	5.1	0.9
.4	.113	.578	.374	.652	.348	11.3	7.5
.5	.141	.452	.503	.529	.471	14.5	6.8
.6	.169	.327	.586	.416	.584	21.4	0.3
.7	.197	.239	.704	.321	.679	31.5	3.7
.8	.225	.167	.806	.244	.756	36.4	6.6
.9	.254	.117	.886	.184	.816		8.6
1.0	.282	.083	.935	.137	.863		8.4
1.2	.338	.049	.992	.075	.925		7.2
1.4	.394	.027	1.018	.045	.955		6.6
1.6	.451	.013	1.028	.027	.973		5.7
1.8	.507	.007	1.028	.012	.988		4.1
2.0	.563	.005	1.028	.006	.994		3.4
					Average	14.0	5.4

(b) Power Coefficients for $E_\parallel$

d(cm.)	d/λ_2	$T_\parallel$	$R_\parallel$	$T_\parallel$	$R_\parallel$	$T_\parallel$	$R_\parallel$
0.0	0.000	1.010	0.011	1.000	0.000	1.0	
.1	.028	0.950	.027	0.953	.047	0.3	
.2	.056	.910	.125	.833	.167	9.2	25.1
.3	.085	.778	.288	.680	.320	14.4	10.0
.4	.113	.542	.475	.530	.470	2.3	1.1
.5	.141	.394	.602	.403	.597	2.2	0.8
.6	.169	.284	.723	.300	.700	5.3	3.3
.7	.197	.208	.768	.221	.779	5.9	1.4
.8	.225	.167	.824	.163	.837	2.5	1.6
.9	.254	.124	.859	.119	.881		2.5
1.0	.282	.093	.884	.087	.913		3.2
1.2	.338	.044	.940	.047	.953		1.4
1.4	.394	.021	.965	.025	.975		1.0
1.6	.451	.014	.985	.013	.987		0.3
1.8	.507	.011	.990	.007	.993		0.3
2.0	.563	.006	.970	.004	.996		2.6
					Average	4.8	3.9

POWER COEFFICIENTS VS. PRISM SPACING FOR SALT

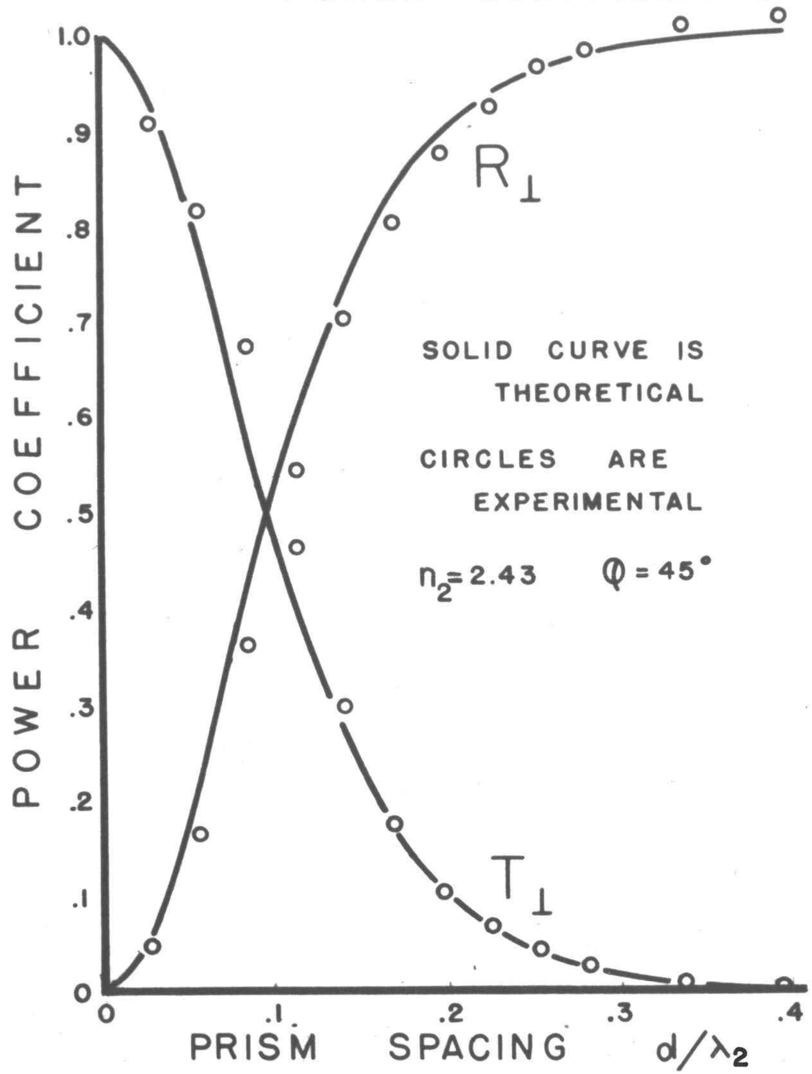

FIG. 9. E PERPENDICULAR TO PLANE OF INCIDENCE.

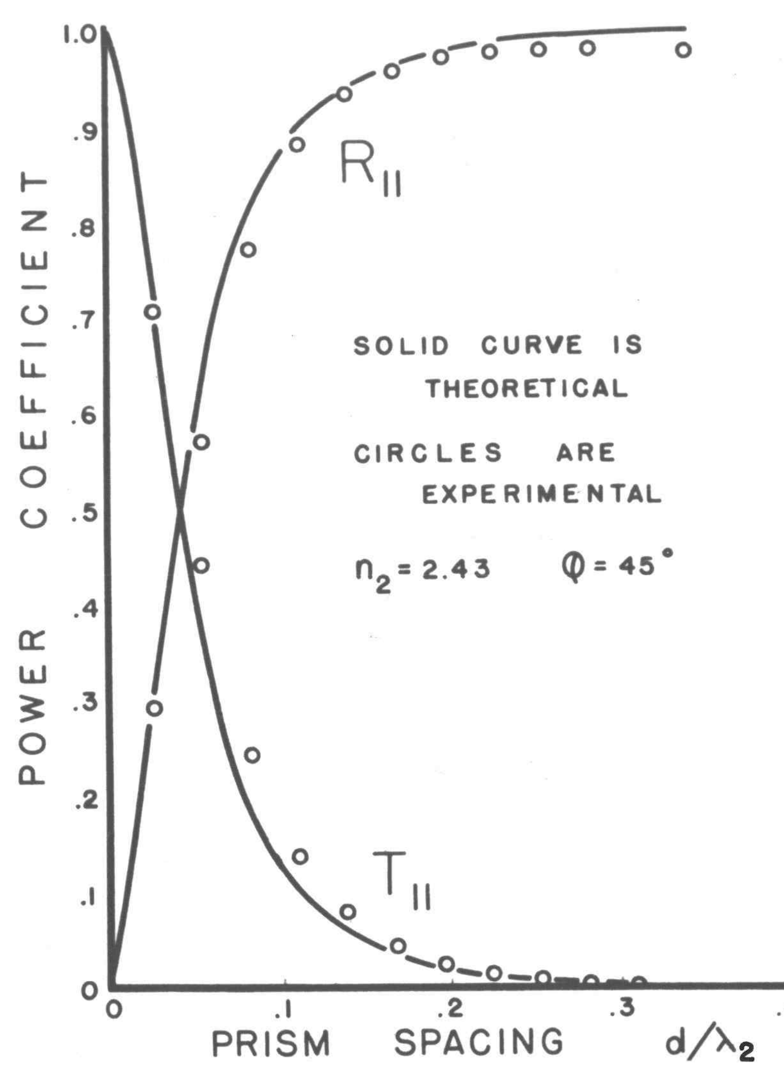

FIG. 10. E PARALLEL TO PLANE OF INCIDENCE.

TABLE III

NUMERICAL VALUES OF COEFFICIENTS FOR SALT

(a) Power Coefficients for $E_\perp$

PRISM SPACING		EXPERIMENTAL		THEORETICAL		% ERROR FOR COEFFICIENTS > 0.15	
d(cm.)	d/λ_2	$T_\perp$	$R_\perp$	$T_\perp$	$R_\perp$	$T_\perp$	$R_\perp$
0.0	0.000	-	-	1.000	0.000	-	
.1	.028	0.909	0.050	0.936	.064	2.9	
.2	.056	.815	.164	.774	.226	5.3	27.4
.3	.085	.673	.362	.579	.421	16.2	14.0
.4	.113	.463	.546	.402	.598	15.2	8.7
.5	.141	.297	.701	.265	.735	12.1	4.6
.6	.169	.176	.806	.169	.831	4.1	3.0
.7	.197	.104	.876	.105	.895		2.1
.8	.225	.070	.923	.065	.935		1.3
.9	.254	.044	.963	.040	.960		0.3
1.0	.282	.030	.983	.024	.976		0.7
1.2	.338	.013	1.001	.009	.991		1.0
1.4	.394	.005	1.020	.003	.997		2.3
				Average		9.3	5.9

(b) Power Coefficients for $E_\parallel$

d(cm.)	d/λ_2	$T_\parallel$	$R_\parallel$	$T_\parallel$	$R_\parallel$	$T_\parallel$	$R_\parallel$
0.0	0.000	-	-	1.000	0.000	-	
.1	.028	0.708	0.294	0.707	.293	0.1	
.2	.056	.443	.557	.362	.638	22.4	12.7
.3	.085	.244	.772	.185	.815	31.9	5.3
.4	.113	.139	.881	.100	.900		2.1
.5	.141	.080	.933	.056	.944		1.2
.6	.169	.046	.957	.032	.968		1.1
.7	.197	.025	.970	.019	.981		1.1
.8	.225	.015	.974	.011	.989		1.5
.9	.254	.009	.974	.007	.993		1.9
1.0	.282	.006	.977	.004	.996		1.9
1.2	.338	.005	.977	.001	1.000		2.3
				Average		18.1	3.1

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