Estimation of Surface Heat Flux

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ABSTRACT

The authors reconsider the problem of estimating the sensible heat transfer at the earth's surface from direct measurements of turbulent fluxes in the atmospheric boundary layer. For simplicity, only horizontally homogeneous conditions are considered for a thin atmospheric layer containing no liquid water, adjacent to the earth's ground surface. Applying the first law of thermodynamics to the thin interfacial layer, an expression is obtained for thermal conduction at the surface in terms of the traditionally defined sensible heat flux by turbulence and a set of correction terms including the so-called moisture correction term. A scale analysis is presented to suggest that the magnitudes of the miscellaneous correction terms are usually negligible. Previous literature on estimation of the sensible heat flux is critically reviewed in light of the new result.

1. Introduction

The interpretation of observational estimates of sensible heat flux across the earth's surface suffers from ambiguous discussions in the literature, especially regarding the influence of moisture flux on the calculation of the sensible heat transfer. Although molecular processes are primarily responsible for the transfer of sensible heat and water vapor from a land or water surface to the air, it is a practical impossibility to measure molecular conduction and diffusion over meteorologically relevant time and space scales. Real surfaces are thermodynamically and geometrically complex. A method is needed to integrate the effects of the surface molecular processes over time and space to estimate the thermal conduction at the surface.

In principle, fast response measurements of wind, temperature, humidity, and pressure from boundary layer platforms such as airplanes, towers, and ships may be used to infer the heat transfer from the earth's surface to the air. The strategy for estimating the surface energy balance over homogeneous terrain conditions is illustrated in Fig. 1 for the thin layer of air adjacent to the surface containing no liquid water.

The first step is to write conservation relations for heat and mass for a layer of moist air adjacent to the earth's surface that is thin enough to ignore local storage and horizontal flux convergence terms within the layer, and yet thick enough to ignore the net vertical heat flux by molecular diffusion and thermal conduc-

\[ R_{\text{net}} = Q_H + Q_E + Q_G + \Delta Q. \]  

where \( R_{\text{net}} \) is the net radiation absorbed by the earth, \( Q_H \) is the heat lost to the air by molecular conduction, and \( Q_E \) is the energy used to evaporate water as it moves from the earth's surface to the air. The quantity \( Q_G \) is the energy used to heat subsurface soil or water. Over land \( \Delta Q = \Delta Q_e + \Delta Q_p \), where \( \Delta Q_e \) represents the canopy energy storage and \( \Delta Q_p \) represents the energy used in photosynthesis. Over a water body, \( \Delta Q \) might be associated with the canopy formed by water plants and surface debris, if any, and \( \Delta Q_p \) with photosynthesis by aquatic biota in the canopy.

The term \( R_{\text{net}} \) may be partitioned into components by the expression

\[ R_{\text{net}} = S_1 - S_1 + L_4 - L_4, \]

where \( S_1, S_1, L_4, \) and \( L_4 \) are the downward and upward shortwave and longwave irradiance.

Implementing the budget calculations described above appears to be a straightforward exercise, and yet there has been considerable confusion and debate in the literature about how to connect the directly measured turbulent fluxes of sensible heat and water vapor to the related quantities at the surface, \( Q_H \) and \( Q_E \). The difficulties appear to arise from a lack of agreement on the nature of the moisture flux contribution to the sensible heat flux (Montgomery 1948), and to an unresolvable
ambiguity in the magnitude of the sensible heat flux due to the arbitrary nature of the thermodynamic reference state for moist air processes.

Much of the discussion in the literature has centered on whether the enthalpy flux by water vapor can be neglected in calculations of the sensible heat flux (Brook 1978, 1982; Reinking 1980; Webb et al. 1980; Frank and Emmitt 1981; Leunung and Legg 1982; Webb 1982; Businger 1982). The confusion in the literature is partly due to discrepancies in the usage of the term "sensible heat flux." The Glossary of Meteorology (Huschke 1959) uses sensible heat flux and enthalpy flux as interchangeable terms. It is therefore traditional in meteorology to assume that the thermal conduction, $Q_H$, is directly proportional to the turbulent enthalpy flux just above the surface. It is also traditional to define the "turbulent sensible heat flux" $H$ as the covariance between vertical velocity and temperature perturbations; that is, $H = \overline{\rho d c_{pd} + \rho e c_{pe}} \overline{w' T'}$, where $(\overline{\rho d}, c_{pd})$ and $(\overline{\rho e}, c_{pe})$ are the mean density and the specific heat at constant pressure for dry air and water vapor, respectively (Stull 1988).

The question of whether the enthalpy flux properly accounts for the contribution of the water vapor to the sensible heat flux depends therefore on the definition of the sensible heat flux. In this paper, we focus on the estimation of the thermal conduction $Q_H$ at the earth’s surface [see Eq. (1)] using estimates of turbulent fluxes in the atmospheric boundary layer. Without relying on a particular definition of the sensible heat flux, we carefully derive the relationship between $Q_H$ and the directly measured turbulent fluxes. In the development of this relationship, the physical meaning of the traditionally defined sensible heat flux and the distinction between the turbulent sensible heat flux and enthalpy flux become clear.

A number of authors have addressed the issue of whether the mean vertical motion due to water vapor flux can be neglected in calculations of the sensible heat flux (Brook 1978; Webb et al. 1980; Webb 1982; Businger 1982). Webb et al. (1980) show that the mean upward movement of the water vapor is, in fact, an important contributor to the total sensible heat flux in the surface boundary layer. When the concentration of a quantity (e.g., water vapor, CO$_2$, or other trace gases) is expressed as a mass density, Webb et al. (1980) show that a mean mass flux "correction" is required for computation of the total vertical flux. The work of Webb et al. (1980) is widely cited, and the effect of the mean motion is commonly called Webb’s correction.

Brook (1978, 1982) argues that the sensible heat transport by the mean vertical motion due to water vapor flux should not be considered as part of the turbulent sensible heat flux. Brook (1978) concludes that turbulent water vapor flux makes a significant contribution to the turbulent sensible heat flux when evaporation is large. According to Brook’s analysis, the contribution of the turbulent water vapor flux can be as large as the traditionally defined sensible heat flux $H$ over evaporating land surfaces and most of the global ocean. Brook’s magnitude estimate, however, is closely linked to the implicit choice of a thermodynamic reference state. After properly accounting for the relationship between the turbulent mass flux and the mean mass flux of water vapor, Frank and Emmitt (1981) and Businger (1982) show that Brook’s analysis implicitly assumes that the temperature of the thermodynamic reference state is absolute zero.
Webb (1982) and Leuning and Legg (1982) criticize Brook's implicit choice of an absolute zero reference state and define the sensible heat in terms of the enthalpy of an air with reference to a "base" temperature at which the air parcel is warmed or cooled. These arguments are physically reasonable, but the base temperature is not clearly defined.

A different perspective on the choice of reference state is provided by the work of Businger (1982) and Nicholls and Smith (1982). They suggest that the proper context for the discussion of the relationship between $Q_H$ and $H$ is within the boundary layer heat and moisture budgets. From this perspective, the water vapor correction term represents the rate of change of water vapor enthalpy as it passes through the layer, due to the water vapor entering and leaving the surface boundary layer at different temperatures.

The practical objective of the calculations outlined in this paper is to provide an explicit estimate of $Q_H$ from turbulence data taken at tower, ship, or aircraft levels for use in closing energy budgets of the form given by Eq. (1). The formulation for $Q_H$ is developed in section 2 using the heat and mass balance of thin horizontally homogeneous interfacial layers. This development can be regarded as an extension of the work by Businger (1982) and Nicholls and Smith (1982).

The results show that the formula for $Q_H$ can be expressed in terms of the traditionally defined sensible heat flux $H$ and correction terms, which includes the so-called moisture correction term. Quantitative estimates of the magnitude of the correction terms are presented in section 3. In section 4, we return to our discussion of the previous literature in light of this new result. We find that the developments of Webb et al. (1980) and Businger (1982) provide good approximations to the total enthalpy flux, which in turn leads to a good approximation of the surface conduction term, subject to precise definition of the thermodynamic reference temperature, certain corrections identified in appendix B, and elimination of certain extreme atmospheric conditions (section 4).

2. Flux formulation

Using the first law of thermodynamics and the ideal gas law, the energy conservation of a moist air parcel can be expressed as

$$\rho \dot{Q} - \nabla \cdot \mathbf{J} = (\rho_d c_{pd} + \rho_v c_{pv}) \frac{dT}{dt} - \frac{dp}{dt}, \quad (3)$$

where the variables $T$, $t$, and $p$ in Eq. (3), are temperature, time, and pressure, respectively; and

$$\frac{d}{dt} \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

$$\rho = \rho_d + \rho_v \quad (5)$$

The reader is referred to deGroot and Mazur (1984) for a more complete discussion. The terms on the left side of Eq. (3) are the nongradient heat source term, $\rho \dot{Q}$, which may include heat generation due to viscosity or chemical reactions; and the gradient heat source term, $-\nabla \cdot \mathbf{J}$. The energy flux vector $\mathbf{J}$ includes the contributions by radiation, $\mathbf{J}_R$, and thermal conduction, $\mathbf{J}_H$, respectively. The terms $\mathbf{M}_d$ and $\mathbf{M}_v$ for dry air and water vapor, respectively, are defined as

$$\mathbf{M}_d = \rho_d (\mathbf{v}_d - \mathbf{v}) \quad (8)$$

$$\mathbf{M}_v = \rho_v (\mathbf{v}_v - \mathbf{v}) \quad (9)$$

Here $\rho_d$, $\rho_v$, $\mathbf{v}_d$, and $\mathbf{v}_v$ are the dry air density, the water vapor density, and the velocity vectors of the mass centers for dry air and water vapor, respectively. The variables $\mathbf{M}_d$ and $\mathbf{M}_v$ are nonzero if the mass centers of dry air and water vapor move at different velocities as is the case over an evaporating land or ocean surface. The $\mathbf{M}_d$ and $\mathbf{M}_v$ terms arise from expressing the advective velocity in Eq. (4) in terms of the mass-weighted velocity, $\mathbf{v}$, rather than the dry air and water vapor velocities, $\mathbf{v}_d$ and $\mathbf{v}_v$, separately.

The mass conservation principle can be expressed as

$$\frac{dp}{dt} = -\rho \nabla \cdot \mathbf{v}. \quad (10)$$

Combining Eqs. (3) and (10), the first law of thermodynamics can be expressed in flux form as

$$\frac{\partial (\rho_d c_{pd} + \rho_v c_{pv})}{\partial t} + \nabla \cdot (\mathbf{J}_R + \mathbf{J}_H + \rho_d c_{pd} \mathbf{T} \mathbf{v}_d - \frac{dp}{dt}) = \rho \dot{Q}, \quad (11)$$

where $\rho_d c_{pd} \mathbf{T} \mathbf{v}_d$ and $\rho_v c_{pv} \mathbf{T} \mathbf{v}_v$ are the enthalpy fluxes for dry air and water vapor, respectively.

For simplicity, we apply Eq. (11) to a thin layer of moist air adjacent to a flat surface as shown in Fig. 1 and make the following assumptions:

1. The vertical thickness of the layer, $\delta z$, is sufficiently small that time-averaged heat and mass storage and the advection of time-averaged heat and mass can be neglected within the layer.

2. The top of the layer is above the viscous surface sublayer where thermal conduction and molecular diffusion are important. In other words, at the top of the layer, the net energy transfer by thermal conduction and molecular diffusion is much smaller than the energy transfer by turbulent fluxes. Symbolically, the assumption is written as $[\mathbf{J}_H]_{\delta z} \approx 0$, and $[\mathbf{M}_d]_{\delta z} = -[\mathbf{M}_d]_{\delta z} \approx 0$. 

\[ \begin{align*} v &= \frac{\rho_d N_d + \rho_v N_v}{\rho} \quad (6) \\
J &= J_r + J_h + c_{pd} T \mathbf{M}_d + c_{pv} T \mathbf{M}_v. \quad (7) \end{align*} \]
3) The time-averaged air properties within the layer are in equilibrium with a flat homogeneous ground surface.

4) The nongradient heat source term, $\rho Q$, is negligible in the layer compared with other terms in Eq. (11).

5) At the surface, the mean diffusive flux of water vapor away from the surface is much larger than the net exchange of dry air constituents. For the purposes of this paper we will adopt the common assumption that the exchange of dry air constituents at the surface is zero (e.g., Webb et al. 1980).

With the assumptions listed above, Eq. (11) can be integrated from the ground, $z = 0$, to the top of the thin layer, $z = \delta z$, and averaged in time or in space, 

$$
(\rho_d c_{pd} T_{wd} + \rho_v c_{pv} T_{wc})_{\delta z} = (J_{Hc} + \rho c_{pd} T_{wd})_{\delta z} + \rho c_{pv} T_{wc} \delta z + \Delta P + Q_R, \tag{12}
$$

where

$$
\Delta P = \int_0^{\delta z} \frac{dp}{dt} \ dz, \tag{13}
$$

$$
Q_R = [J_{Hc}]_0, \tag{14}
$$

and $w_d$, $w_v$, $J_{Hc}$, and $J_{Hv}$ are the vertical components of $v_d$, $v_v$, $\overline{J}_{Hc}$, and $\overline{J}_{Hv}$, respectively. Applying Reynolds averaging to Eq. (12), we obtain

$$
[(c_{pd}\overline{\rho w_d}^\prime + c_{pv}\overline{\rho w_v}^\prime) \overline{T} + c_{pd}(\overline{\rho w_d})^\prime]_{\delta z} = [J_{Hc}]_0 + [(c_{pd}\overline{\rho w_d} + c_{pv}(\overline{\rho w_v}))^\prime]_{\delta z} + \Delta P + Q_R. \tag{15}
$$

Applying assumptions 1 and 5 listed above, the conservation of dry air and water vapor mass in the thin layer implies that

$$
(\overline{\rho w_v})_{\delta z} = (\overline{\rho w_v})_0, \tag{16}
$$

$$
(\overline{\rho w_d})_{\delta z} = (\overline{\rho w_d})_0 = 0, \tag{17}
$$

as well as

$$
(\overline{\rho w_d})_0 = 0. \tag{18}
$$

Notice that Eq. (17) is expressed in terms of the dry airmass velocity, whereas the comparable expression in Webb et al. (1980) is expressed in terms of the mass-weighted vertical velocity $(\overline{\rho w}) = 0$. With assumption 2 listed above, it can be seen that the comparable expression in Webb et al. (1980) applies to the top of the thin atmospheric layer, or at the observational height where $w_d \approx w_v \approx \overline{w}$.

Substituting Eqs. (16)–(18) into Eq. (15), we have

$$
[c_{pd}(\overline{\rho w_d})^\prime]_{\delta z} + [c_{pv}(\overline{\rho w_v})^\prime]_{\delta z} + \overline{\rho w_v} \overline{J}_{Hc} - [c_{pv}(\overline{\rho w_v})^\prime]_{\delta z} = [J_{Hc}]_0 + \Delta P + Q_R. \tag{19}
$$

Here $T_0$ is the surface temperature.

The energy budget for the atmospheric surface layer given by Eq. (19) can now be matched with the energy budget at the earth’s surface, Eq. (1), by noting that

$$
Q_R = [J_{Hc}]_0. \tag{20}
$$

Both $Q_R$ and $[J_{Hc}]_0$ represent the mean molecular thermal conduction of heat across the interface between the earth’s surface and the air. This is the portion of the energy transfer that may occur in the absence of mass transfer between the earth’s surface and the air. A more detailed discussion of heat transfer in a multicomponent diffusive system can be found in deGroot and Mazur (1984).

Applying assumption 2 listed above—that is, $[w_v]_{\delta z} \approx [w_d]_{\delta z} \approx [\overline{w}]_{\delta z}$—and substituting Eq. (20) into Eq. (19), the thermal conduction, $Q_R$, at the earth’s surface can be expressed in terms of the turbulent fluxes measured at the observational height, $\delta z$,

$$
Q_R = H + R - \Delta P - Q_R, \tag{21}
$$

where

$$
H = (\overline{\rho c_{pd}} + \overline{\rho c_{pv}})\overline{T}^\prime \overline{T}^\prime, \tag{22}
$$

$$
R \equiv \overline{\rho w c_{pv}}(\overline{T} - \overline{T}_0) + \overline{\rho c_{pd} \overline{w}_d - \overline{w}_d^\prime \overline{T}^\prime + \overline{c_{pv} \rho c_{pv} \overline{T}^\prime}} - \overline{\rho}(\overline{\rho w_v})^\prime \overline{T}^\prime \overline{T}^\prime \overline{T}_0. \tag{23}
$$

Expressing $\rho_d$ and $\rho_v$ in terms of $\rho$ and $q$ where $q$ is the specific humidity; that is,

$$
\rho_d = \rho(1 - q), \tag{24}
$$

$$
\rho_v = pq, \tag{25}
$$

and substituting them into Eq. (23), $R$ becomes

$$
R \equiv \overline{\rho c_{pd}}\overline{w q}^\prime (\overline{T} - \overline{T}_0) + \overline{\rho c_{pv}[(c_{pd}(1 - q) + \overline{c_{pv}} \overline{q}^\prime \overline{T}^\prime + \overline{c_{pv} c_{pd} \overline{q}^\prime \overline{T}^\prime}] + \overline{\rho c_{pd} \overline{w}^\prime \overline{q}^\prime (\overline{T} - \overline{T}_0)} + \overline{\rho c_{pd} \overline{w}^\prime \overline{q}^\prime (\overline{T} - \overline{T}_0)} - \overline{c_{pv}}[(\overline{\rho w_v})^\prime \overline{T}^\prime \overline{T}^\prime]_0 = R_w + R_p + R_0 - \overline{c_{pv}}[(\overline{\rho w_v})^\prime \overline{T}^\prime \overline{T}^\prime]_0, \tag{26}
$$

where

$$
R_w \equiv \overline{\rho c_{pd} \overline{w}^\prime q^\prime (\overline{T} - \overline{T}_0)} \tag{27}
$$

$$
R_p \equiv \overline{\rho c_{pd} \overline{w}^\prime (1 - q)} + \overline{c_{pv} \overline{q}^\prime \overline{T}^\prime} + \overline{c_{pv} \overline{c_{pd} \overline{q}^\prime \overline{T}^\prime}} - \overline{c_{pv}}[(\overline{\rho w_v})^\prime \overline{T}^\prime \overline{T}^\prime \overline{T}_0)] \tag{28}
$$

$$
R_0 = \overline{\rho c_{pd} \overline{w}^\prime \overline{q}^\prime (\overline{T} - \overline{T}_0)}. \tag{29}
$$

All the triple correlation terms in Eq. (26) have been neglected. Eliminating density fluctuations in $R_s$, $R_0$, and the mean vertical velocity in terms of $p'$ and $T'$ by using the equation of the state, one obtains (appendix A):
\[ R_p \approx \frac{\bar{w}}{R_d} \left[ \frac{\bar{p}^\prime T^\prime}{\bar{T}} + \frac{\bar{p}^\prime T^\prime}{\bar{T^2}} \right] + \bar{\rho} q^\prime T^\prime \left[ c_{pv} - c_{pd} \frac{1}{\epsilon} \right] + c_{pm} \bar{\rho} \bar{q} (\bar{T} - \bar{T}_0) \]  

(30)

\[ R_0 \approx \bar{q} c_{pm} (\bar{T} - \bar{T}_0) \times \left[ \frac{w^\prime p^\prime}{R_d \bar{T}} - \frac{\bar{\rho}}{\bar{T}} w^\prime T^\prime - \frac{1 - \epsilon}{\epsilon} \bar{\rho} q^\prime q^\prime \right] \]  

(31)

\[ \bar{w} \approx - \frac{1}{\bar{p}} \frac{w^\prime p^\prime}{\bar{T}^2} + \frac{1}{\bar{T}} \frac{w^\prime w^\prime}{\bar{T^2}} + \frac{1}{\epsilon} \frac{w^\prime q^\prime}{\bar{T}} \]  

(32)

The term, \( Q_{E} \), in Eq. (21) measures a divergence of the radiant energy flux across the layer, while \( \Delta P \) measures integrated pressure changes within the layer. For convenience, only variables at \( z = 0 \) are labeled and \( \delta z \) is omitted.

From the above derivation of the energy balance for the atmospheric surface layer, it is clear that the traditionally defined sensible heat flux \( H \) represents only part of the thermal conduction at the surface, \( Q_{H} \), and only part of the enthalpy flux as well. To estimate \( Q_{H} \), it is necessary to “correct” \( H \) for effects related to the presence of water vapor, pressure change, and radiation. In section 3, we will see that the most important physical effect among all the water vapor correction terms is the enthalpy divergence due to the fact that water vapor crosses the earth’s surface at a temperature near \( \bar{T}_0 \) and crosses the top of the layer near \( \bar{T} \).

3. Estimation of the correction terms

The correction terms to \( H \) in Eq. (21) will be evaluated in this section based on a scale analysis. In general, the correction terms become comparable with \( H \) when the value of \( H \) becomes small and the values of water vapor content and the moisture flux are large.

a. Moisture flux term, \( R_w \)

The term, \( R_w \), defined in Eq. (27) corresponds to the controversial moisture correction term in the literature (Brook 1978; Reinking 1980; Webb et al. 1980; Frank and Emmitt 1981; Webb 1982; Leuning and Legg 1982; Nicholls and Smith 1982; Businger 1982). Its contribution to \( H \) can be measured by the ratio to the traditional sensible heat flux term such that

\[ \frac{\bar{p} c_{pv} w^\prime q^\prime (\bar{T} - \bar{T}_0)}{H} \approx 7.8 \times 10^{-4} \left( \frac{\bar{T} - \bar{T}_0}{B} \right). \]  

(33)

This ratio does not vary rapidly over land because increasing moisture flux is normally associated with decreasing air–surface temperature difference. For a value of the Bowen ratio \( B \) of \( O(1) \), the air–ground temperature difference is typically \( O(10 \text{ K}) \) in which case the above ratio is of order \( 10^{-2} \). However, in near-neutral cases such as windy flow over a wet surface, the Bowen ratio may approach very small values with \( (\bar{T} - \bar{T}_0) \) becoming positive and nonzero. In this case, the ratio Eq. (33) can become \( O(1) \) or greater. However, the moisture flux correction term, \( R_w \), appears to be, percentagewise, important only when the total heat conduction is small. The absolute value of the moisture flux term is expected to be at most \( \sim 5 \text{ W m}^{-2} \), as might occur under conditions of maximum solar forcing of a saturated land surface.

b. Pressure covariance

Evaluation of \( R_p \) from Eq. (30) and \( R_0 \) from Eq. (31) requires evaluation of the pressure covariance terms. Recent measurements of Wilczak (unpublished) suggest that the pressure–vertical velocity covariance term \( w^\prime p^\prime \) can be formulated as \( C_{p} u_{*}^3 \), where \( C \) is between 1 and 5. McBean and Elliot (1975) find \( C \) to be a function of \( z/L \) with a typical value between 0.5 and 1.0, although they suggest that instrumentally induced dynamic pressure effects could lead to underestimation of the true value of \( w^\prime p^\prime \); here \( z \) is the observational height and \( L \) is the Obukhov length. Assuming that \( p^\prime \) scales as \( 0.2 \bar{\rho} u_{*}^2 \), Wyngaard and Coté (1971) estimate \( C \) to be about 5 for \( z/L = -1.0 \). The ratio of the pressure covariance term in Eqs. (31) and (32) to the heat flux term scales as

\[ S = \frac{w^\prime p^\prime}{w^\prime T^\prime} \frac{\bar{T}}{\bar{p}} \approx \frac{C u_{*}^3}{R_d w^\prime T^\prime}. \]  

(34)

Clearly, with large surface drag and weak heat flux, the pressure covariance term is significantly influenced by pressure fluctuations. This situation could arise in moderate to strong winds over a rough surface with weak downward shortwave radiation and/or large surface evaporation. As a plausible numerical example, choosing \( w^\prime T^\prime = 0.01 \text{ K m s}^{-1} \), \( u_{*} = 1 \text{ m s}^{-1} \), \( C = 3 \), the ratio \( S \) in Eq. (34) is of order unity; then the pressure–velocity covariance term \( w^\prime p^\prime \) in Eqs. (31) and (32) must be included. However, in general, the pressure covariance term is unimportant in which case Eq. (32) becomes

\[ \bar{w} \approx - \frac{1}{\bar{T}} \frac{w^\prime p^\prime}{\bar{T}^2} + \frac{1}{\epsilon} \frac{w^\prime q^\prime}{\bar{T}}. \]  

(35)

The behavior of the pressure and temperature covariance term in Eq. (30) is less known. The relative importance of the pressure–temperature covariance term can be estimated by first evaluating the ratio

\[ G = \frac{p^* T}{T^* p^*}. \]  

(36)

where

\[ \bar{p} = R_d \bar{p} \bar{T}. \]

Here \( T^* \) and \( p^* \) represent the scales of temperature and
pressure perturbations, respectively. Normally $G$ is assumed to be small compared to one although such an assumption is not categorically valid (Mahrt 1986). Pressure fluctuations are historically assumed to scale with horizontal velocity fluctuations (e.g., Batchelor 1951)

$$ p^* = C_G \bar{\rho} \sigma_u^2. $$

(37)

Here $\sigma_u$ is the standard deviation of the horizontal velocity fluctuations and $C_G$ is a nondimensional coefficient.

Using Eq. (37), the term $G$ in Eq. (36) becomes

$$ G \approx C_G \frac{\sigma_u^2}{R_d T^*}. $$

(38)

For strong turbulence, $\sigma_u$ can be on the order of 1 m s$^{-1}$ in which case $p^*/\bar{p}$ is predicted to be on the order of $10^{-5}$. The ratio $T^*/\bar{T}$ is typically on the order of $10^{-3}$ in which case $G$ is on the order of 10$^{-2}$, provided that the coefficient, $C_G$, in Eq. (37) is of order unity.

Recent improvements in measurements of pressure fluctuations allow direct estimation of the magnitude of pressure fluctuations. Conklin and Knoerr (1994) and Zhuang and Amiro (1994) found $p^*/\bar{p}$ on the order of $10^{-5}$ for light wind conditions. Schmidt et al. (1995) found $p^*/\bar{p}$ to exceed $10^{-4}$ in moderate winds over shelterbelts, presumably representative of scattered isolated obstacles. Massman (personal communication) has found similar values in very windy conditions over a rough surface. Therefore, in conditions of strong wind and weak temperature fluctuations, the pressure fluctuation ratio $p^*/\bar{p}$ cannot necessarily be ignored in comparison with $T^*/\bar{T}$. In the subsequent analysis, we will assume that $G \ll 1$ and $S \ll 1$ with the understanding that this assumption appears to break down with strong winds and weak temperature fluctuation.

c. **Simplified correction term $R$**

Neglecting pressure covariance terms in Eq. (30) based on the analysis of section 3b, using Eq. (35), and again assuming $\bar{q} \ll 1$, the term $R_\rho$ becomes

$$ R_\rho \approx \left( \frac{1}{\epsilon} w^* \bar{w} T^* \right) \left[ - \frac{c_{pd}^2 T^*}{R_d \bar{T}} \left( \frac{T^*}{\bar{T}} \right)^2 \right] \left[ - \frac{c_{pd}^2 T^*}{R_d \bar{T}} \left( \frac{T^*}{\bar{T}} \right)^2 \right] \left[ - \frac{c_{pd}^2 T^*}{R_d \bar{T}} \left( \frac{T^*}{\bar{T}} \right)^2 \right]. $$

(39)

The following analysis will be simplified by assuming that $w^* \bar{w} T^*/\epsilon$ does not exceed $w^* \bar{T}^*/\bar{T}$ in order of magnitude and, therefore, the order of magnitude of $\bar{w}$ can be represented by $w^* \bar{T}^*/\bar{T}$ alone. With a more lengthy development, it can be shown that this simplification does not affect the conclusions below.

Assuming $|T^*| / \bar{T} \ll 1$, then the term $T^* / \bar{T}^2$ can be neglected in Eq. (39). Then with the restriction $|q^* T^*| \ll O(q^* T^*)$, where $q^*$ is the scale value for moisture fluctuations, the ratio of the term $R_\rho$ to the traditional heat flux term $\bar{\rho} c_{pm} w^* \bar{T}$ scales as

$$ \frac{R_\rho}{\bar{\rho} c_{pm} w^* \bar{T}} \ll \frac{q^* T^*}{\bar{T}} \left( \frac{c_{pm}}{c_{pd}} - \frac{1}{\epsilon} \right) + \frac{\bar{q} c_{pm} \Delta T}{c_{pd} \bar{T}}. $$

(40)

where $\Delta T = \bar{T} - \bar{T}_0$.

If we choose $T^* = 3 \times 10^{-1}$ K, $\bar{T} \approx 300$ K, $\Delta T = O(10 \text{ K})$, $q^* = 3 \times 10^{-1}$ g kg$^{-1}$, and $\bar{q} = 10$ g kg$^{-1}$, then the scaled first and second terms on the right-hand side of Eq. (40) are $O(10^{-2})$ and $O(10^{-4})$, respectively.

The magnitude of the term $R_\rho$ in Eq. (29) can be evaluated by neglecting the pressure covariance term in Eq. (31) based on section 3b in which case

$$ R_\rho \approx \frac{\bar{q} c_{pm} (\bar{T} - \bar{T}_0)}{\bar{T}} \left[ - \frac{\bar{p}}{\bar{T}} w^* \bar{T}^* - \frac{1 - \epsilon}{\epsilon} \bar{\rho} w^* q^* \right]. $$

(41)

The ratio of the maximum value of the heat flux term on the right-hand side of Eq. (41) to the traditional term $\bar{\rho} c_{pm} w^* \bar{T}$ scales as

$$ \left| \frac{\bar{q} c_{pm} (\bar{T} - \bar{T}_0)}{\bar{T}} \left[ - \frac{\bar{p}}{\bar{T}} w^* \bar{T}^* - \frac{1 - \epsilon}{\epsilon} \bar{\rho} w^* q^* \right] \right| $$

$$ \leq \left| \bar{q} c_{pm} \Delta T \right| \left| \frac{c_{pm}}{c_{pd}} \right|. $$

(42)

The ratio of the moisture flux term on the right-hand side of Eq. (41) to the term $R_\rho$ in Eq. (27) is

$$ \left| \frac{\bar{q} c_{pm} (\bar{T} - \bar{T}_0)}{\bar{T}} \left[ - \frac{1 - \epsilon}{\epsilon} \bar{\rho} w^* q^* \right] \right| = \frac{1 - \epsilon}{\epsilon} \bar{q}. $$

(43)

Using the same parameter values as above, the scaled terms in (42)–(43) are $O(10^{-2})$ and $O(10^{-4})$, respectively. The small value of Eq. (43) implies that this moisture flux term is unimportant compared to $R_\rho$, which in turn is small compared to the traditional heat flux term.

The assumption $T_0 \approx 0$, which is implicitly assumed in Businger (1982), is used here to neglect the last term in $R$ [Eq. (26)]. Surface temperature varies significantly in time and space over land surfaces, but the covariance of surface temperature with water vapor flux cannot be evaluated with current available observations.

Based on the above order of magnitude estimates, the largest term in $R$ is $R_\rho$ defined by Eq. (27). The correction term $R$ can therefore be approximated as

$$ R \approx \bar{\rho} c_{pm} w^* q^* (\bar{T} - \bar{T}_0), $$

(44)

the so-called moisture correction term.
d. The Lagrangian pressure change, $\Delta P$

The term $\Delta P$ defined in Eq. (13) can be formally analyzed by expanding $dp/dt$ into local change and advective terms. Making use of the budgets of kinetic energy $K$ and potential energy $\Phi$, the magnitude of the term $\Delta P$ can be estimated as

$$\Delta P = \int_0^{z_c} \left( \frac{\partial p}{\partial t} - \rho \frac{dK}{dt} - \rho \frac{d\Phi}{dt} - \rho \nu \cdot \mathbf{F} \right) dz,$$

(45)

where $\nu$ is the three-dimensional velocity vector, and $\mathbf{F}$ is the frictional force per unit mass. An order of magnitude argument for the thin surface layer suggests that

$$\Delta P \approx \int_0^{z_c} \left( -\rho \frac{d\Phi}{dt} \right) dz,$$

(46)

where the right-hand side represents the vertically integrated change of potential energy over the thin atmospheric surface layer. Using assumptions 1 and 3 from section 2, and the resulting Eqs. (16)–(18), this term may be integrated to give

$$\int_0^{z_c} \left( -\rho \frac{d\Phi}{dt} \right) dz = \frac{g \delta z}{0} g \delta z,$$

(47)

where $g$ is the gravity constant. For $\delta z \sim 10$ m and $Q_e \sim 400$ W m$^{-2}$, the right-hand side of Eq. (47) is on the order of $O(10^{-2})$ W m$^{-2}$. In other words, the amount of work that it takes to move water vapor from the surface to $\delta z$ is negligible compared to the other terms in the thermodynamic energy budget.

A more rigorous treatment of this term requires consideration of the maintenance of the mean kinetic energy, and is therefore beyond the scope of this analysis where we have not explicitly considered the role of the mean horizontal pressure gradient force.

e. Simplified surface heat transfer, $Q_H$

Accumulating the assumptions in the above subsections, the thermal conduction in the surface energy balance (21) can be approximated as

$$Q_H \approx (\bar{\rho} c_{pm} + \bar{\rho} c_p) w' T' + \bar{\rho} c_p w' q' (\bar{T} - \bar{T}_0) - Q_h.$$  

(48)

The comparison between this derivation of $Q_H$ and key papers in the literature is given in section 4.

Due to the sharp change of the humidity and temperature with height, the radiative cooling rate can be 1–3 K h$^{-1}$ within a few meters of the ground (Stull 1988). If we choose the radiative cooling rate to be 3 K h$^{-1}$ for the lower 3 m of the atmosphere, then the radiation term $Q_R$ in Eq. (21) close to the earth’s surface is approximately 3 W m$^{-2}$. In this case, the term $Q_R$ may be comparable with $H$ and should be included in the estimate of $Q_H$ in Eq. (48).

In general, the thermal conduction can be approximated by the net enthalpy flux between the ground surface and the observational level, which in turn can be normally approximated by the traditional sensible heat flux at the observational level. With large solar radiation and moist surface conditions, the resulting small traditional sensible heat flux may require correction for the net enthalpy flux of water vapor between the two levels.

4. Discussions on the surface energy balance

We are now in a position to evaluate previous suggestions in the literature for the computation of $Q_H$, the molecular thermal conduction of heat at the earth’s surface. It is clear from our analysis and the numerous comments in the literature (Frank and Emmitt 1981; Businger 1982; Webb 1982; Nicholls and Smith 1982) that the analyses by Brook (1978) and Reinking (1980) are not directly relevant to the problem at hand. Brook’s (1978) analysis was intended to provide an estimate of the turbulent sensible heat flux in the atmospheric surface boundary layer, neglecting the vertical enthalpy flux by the mean motion.

Webb et al. (1980) suggest that the heat flux (i.e., the enthalpy flux) carried by a mixture of moist and dry air may be expressed as

$$Q_H |_{z=0} = c_{pm} \bar{w} \rho_a (T - T_b) + c_{pm} \bar{w} \rho_a (T - T_b),$$

(49)

where $T_b$ is an assumed initial “base” temperature. The base temperature chosen by Webb et al. (1980) implicitly adds the physics of heat sources and sinks at levels in the atmosphere above or below $z = \delta z$. Comparing Eq. (49) with Eq. (48), it is clear that the heat flux in Webb et al. (1980) is actually the approximate form of the thermal conduction if $Q_H$ in Eq. (49) is equal to $T_0$. From this perspective, heat might be added to an air parcel through molecular diffusion at the earth’s surface having temperature $T_0$.

The idea that vertically integrated surface layer budgets of total thermodynamic energy should be used to define the moisture correction can be traced to the work of Businger (1982) and Nicholls and Smith (1982). Businger (1982) derives an expression for the total specific enthalpy flux in the form

$$\rho w h_m |_{z=0} = \bar{\rho} c_{pm} \bar{w} T' + \bar{\rho} w' q' [c_{pm} (T - T_0) + L_0],$$

(50)

where $\bar{T}_0$ is the averaged surface temperature; $h_m$ and $c_{pm}$ represent the moist enthalpy and the specific heat capacity under constant pressure, respectively; and $L_0$ is the latent heat of vaporization at the surface. Here Eq. (50) is the corrected Businger (1982) formula (appendix B). Note that Eq. (50) contains an additional latent heat flux term since the expression was derived from consideration of the total enthalpy budget.

Businger (1982) notes that if $T_0$ is formally set to 0 K in Eq. (50), and the flux of enthalpy due to rain is
neglected, the Frank and Emmitt (1981) expression for their total turbulent flux is obtained in the form
\[ \rho w_{h_m} | z = 0 = \bar{\rho} c_p m \bar{w} \bar{T}^T + \bar{\rho} \bar{w} \bar{q} \left[ \left( c_l - c_{pd} \right) \bar{T} + \bar{L} \right], \] (51)

where \( c_l \) is the specific heat of liquid water. There is a subtle but extremely important difference between Eqs. (50) and (51). Equation (50) is the result of applying boundary conditions for sensible and latent heat flux at the earth’s surface. The left-hand side of Eq. (50) represents the flux at the surface, \( z = 0 \), while the left-hand side of Eq. (51) represents the flux at some level above the surface, \( z = \delta z \), within the turbulent boundary layer. Frank and Emmitt’s (1981) expression, as well as the moisture correction terms of Brook (1978), Webb et al. (1980), Webb (1982), and Leuning and Legg (1982), are not directly connected to the flux at the surface through budget equations and appropriate boundary conditions.

5. Conclusions

We have derived an approximate expression, Eq. (48), for the spatially or temporally averaged molecular thermal conduction at the earth’s surface, \( Q_h \). The expression permits the evaluation of \( Q_h \) with the use of conventional fast-response aircraft, tower, or shipboard measurements of wind, temperature, and humidity in the atmospheric boundary layer. A scale analysis indicates that errors introduced by using the traditionally defined turbulent sensible heat flux to estimate \( Q_h \) at the surface are normally a few percent of \( Q_h \) under homogeneous, stationary conditions in the thin layer of air containing no liquid water, adjacent to the earth’s surface. The expression for \( Q_h \) is consistent with the total enthalpy flux formula at the ground surface presented by Businger (1982) with corrections in appendix B. To obtain the total enthalpy flux at the observational level from \( Q_h \), it is necessary to add the latent heat flux of the water vapor evaporated from the surface.

The derivation of the expression for \( Q_h \) involves detailed consideration of the vertically integrated budgets of mass, heat, and moisture, adjacent to the earth’s surface. Reliance on first principles creates complexity, but results in the possibility of precise and quantitative representations of the physical mechanisms for earth–air exchanges of mass, heat, and water vapor.

We have examined the question regarding the so-called moisture “correction” for the sensible heat flux near the earth’s surface. It is confirmed that the moisture flux correction is generally less than a few percent of the traditionally defined sensible heat flux \( H \). With weak temperature flux and strong surface evaporation, the moisture correction term may become important (section 3a). In windy conditions over a rough surface with weak temperature flux, additional pressure covariance terms become important (section 3b). Radiative flux divergence may also play an important role when the water vapor content of the air is large and the traditional sensible heat flux is small.

We note that our derivation of the vertically integrated thermodynamic energy equation can be generalized using the entropy of moist air (e.g., Ooyama 1990). The advantage of a generalized approach is that entropy is a conserved variable for both dry and moist adiabatic processes, whereas enthalpy is approximately conserved. Thus, the right-hand side of Eq. (3) may be expressed in flux form,
\[ \rho \frac{\bar{Q}}{T} - \frac{1}{T} \nabla \cdot \bar{J} = \frac{\partial \bar{p}_s}{\partial t} + \nabla \cdot (\rho \bar{v} \bar{s}), \] (52)

where \( s \) is the specific entropy of moist air. The disadvantage of this approach is that the vertical integral of the nonadiabatic terms on the left-hand side become more complex, and the logarithmic terms in \( s \) must be linearized to express the heat flux in terms of \( \bar{w} \bar{T}^T \) and \( \bar{w} \bar{q}^T \). We have verified, but will not show here, that the two approaches yield the same results at the same level of approximation in the absence of cloud water and ice.

When the observing platform is more than a few tens of meters above the surface, or when the underlying surface or atmospheric conditions have significant mesoscale horizontal variations, estimation of \( Q_h \) may require advection terms (not considered here) in the vertically integrated mass, heat, and moisture budgets (Sun and Mahrt 1994).

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APPENDIX A

Density Covariances in \( R_p \) and \( R_0 \)

To evaluate the terms in \( R_p \) and \( R_0 \), information on density fluctuations for dry air, water vapor, and moist air is required. Density fluctuations are not directly measured and are traditionally eliminated by applying the linearized equation of state.

The equation of state for moist air is
\[ p = \rho R_t T_v, \] (A1)

where the virtual temperature can be expressed as
\[ T_v = T \left( 1 + \frac{1 - \varepsilon}{\varepsilon} q \right) \] (A2)

and \( \varepsilon = R_v/R_t = 0.622 \). Substituting the expression for virtual potential temperature into the equation of state (A1), decomposing variables into mean and pertur-
bation parts and neglecting triple correlation terms, the mean pressure becomes
\[
\bar{p} = R_g \left[ \bar{T}_p + \bar{\rho} \bar{T}' + \frac{1 - \epsilon}{\epsilon} (\bar{\rho} \bar{q} \bar{T}' + \bar{q} \bar{\rho} T' + \bar{T} \bar{\rho} q') + \frac{1 - \epsilon}{\epsilon} (\bar{\rho} \bar{p} q' + \bar{q} \bar{\rho} q' + \bar{T} \bar{\rho} q') \right]. \tag{A3}
\]
and the pressure perturbation with \( \bar{q} \ll 1 \) becomes
\[
p' = R_g \left\{ \rho' T' - \bar{\rho} \bar{T}' + \bar{\rho} \bar{T}' + \frac{1 - \epsilon}{\epsilon} \left[ \bar{T} \bar{p} q' + \bar{q} \bar{\rho} q' + \bar{T} \bar{\rho} q' \right] + \frac{1 - \epsilon}{\epsilon} \left[ \bar{T} \bar{p} q' + \bar{q} \bar{\rho} q' + \bar{T} \bar{\rho} q' \right] \right\}. \tag{A4}
\]

Multiplying Eq. (A4) by \( T' \), averaging and neglecting all triple perturbation terms, \( \bar{\rho} \bar{T}' \) becomes
\[
\bar{p}' T' \approx R_g \left[ \frac{\rho' T'}{\bar{T}} + \frac{1 - \epsilon}{\epsilon} \bar{T} \bar{p} q' \right]. \tag{A5}
\]
Similarly, multiplying Eq. (A4) by \( w' \), averaging and neglecting all triple perturbation terms, \( \bar{w}' \bar{p}' \) becomes
\[
\bar{w}' p' \approx R_g \left[ \frac{\rho' T'}{\bar{T}} + \frac{1 - \epsilon}{\epsilon} \bar{T} \bar{p} q' \right]. \tag{A6}
\]

Assuming \( |q'|/\bar{q} | \ll 1 \) and \( |\rho'/\bar{\rho}| \ll 1 \), then the term \( \bar{\rho} \bar{q} q' / \bar{T} \) can be ignored in comparison with \( \bar{\rho} \bar{q} T' / \bar{T} \) in \( \bar{T} \bar{p} q' \) in \( \bar{T} \bar{p} \bar{q} \) in \( \bar{T} \). Then, solving for \( \bar{\rho} \bar{T}' \) from Eq. (A5), substituting it into the expression for \( \bar{R}_g \), and noting \( \bar{q} \ll 1 \), \( R_g \), becomes
\[
R_g \approx w \left[ \frac{c_{pd}}{R_g} \left[ \frac{\rho' T'}{T} - \frac{\bar{\rho} \bar{T}'}{T} \right] + \frac{1}{\epsilon} \bar{T} \bar{p} q' \right] \times \left[ \frac{c_{pr} - c_{pd}}{\epsilon} + c_{pd} \frac{\bar{q}}{\bar{T} - \bar{T}_0} \right], \tag{A7}
\]
where
\[
\bar{T} \equiv R_g \bar{\rho} \bar{T}.
\]

Decomposing vertical motion and dry air density in Eq. (17) and applying Eqs. (18), (24), and assumption 2 in section 2, the Reynolds averaged mean vertical motion is
\[
\bar{w} = \frac{w' \bar{\rho}_d}{\bar{\rho}_d} = -\frac{w' \bar{\rho} (1 - \bar{q}) - \bar{\rho} \bar{w}' q'}{\bar{\rho}_d}. \tag{A8}
\]

Solving for \( \bar{w}' \bar{q}' \) from Eq. (A6) and assuming \( \bar{q} \ll 1 \) and \( \bar{\rho} \approx \bar{\rho}_d \), then \( \bar{w} \) [Eq. (A8)], and \( R_0 \) [Eq. (29)] become approximately
\[
\bar{w} \approx -\frac{1}{\bar{\rho}} \bar{w}' \bar{p}' + \frac{1}{\bar{\rho}} \bar{w}' \bar{T}' + \frac{1}{\bar{\rho} \bar{w}' q'} \tag{A9}
\]
and
\[
R_0 \approx \bar{q} c_{pd} (\bar{T} - \bar{T}_0) \times \left[ \frac{w' \bar{p}'}{R_g T} - \frac{\bar{T}}{\bar{\rho} \bar{w}' q'} - \frac{1}{\epsilon} \frac{\bar{T}}{\bar{\rho} \bar{w}' q'} \right]. \tag{A10}
\]

APPENDIX B

Heuristic Derivation of Eq. (48)

Following the outline of Businger (1982), the enthalpy may be written as
\[
\rho w h_m = \rho w T' \left[ c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right] + \bar{\rho} w' q' (c_{pd} + \bar{q} (c_{pm} - c_{pd}) + \bar{\rho} w' q' \left( c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right) + \frac{1}{\bar{\rho} \bar{w}' q'} \right]. \tag{B1}
\]

Here Businger’s (1982) notation for the specific heat of dry air is changed to \( c_{pd} \). The quantity \( b_3 \) is the reference state value for specific enthalpy of liquid water, which, following Businger (1982), can be expressed as
\[
b_3 = b_2 + (c_{pm} - c_1) T - L. \tag{B2}
\]
The term \( b_2 \) is the reference state for specific enthalpy of water vapor. At the surface, the total enthalpy transfer is the sum of the thermal conduction and the enthalpy flux associated with the evaporation, \( E \), written symbolically
\[
Q_H + (c_{pd} T_0 + b_2) E. \tag{B3}
\]
If we assume that the flux given by expression (B3) is equal to \( \rho w h_m \), at \( z = \bar{z} \), and that \( E \) is approximately \( \bar{\rho} w' q' \), then
\[
Q_H = -\left( c_{pd} T_0 + b_2 \right) \bar{\rho} w' q' + \bar{\rho} w T' \left[ c_{pd}
\right.

+ \bar{q} (c_{pm} - c_{pd}) \left. \right] + \bar{\rho} w' q' \left( c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right) + \frac{1}{\bar{\rho} \bar{w}' q'} \left( c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right) + \frac{1}{\bar{\rho} \bar{w}' q'} \left( c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right). \tag{B4}
\]
Substituting Eq. (B2) into Eq. (B4), we obtain
\[
Q_H = \bar{\rho} w T' \left[ c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right] + \bar{\rho} w' q' \left( c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right) + \frac{1}{\bar{\rho} \bar{w}' q'} \left( c_{pd} + \bar{q} (c_{pm} - c_{pd}) \right). \tag{B5}
\]
Note that Eq. (B5) is equivalent to Eq. (48) with the exception of the radiative flux term. The benefit of (B1)–(B5) is independent from the thermodynamic reference state. Businger’s (1982) Eq. (16), which requires that the constant \( b_3 \) be set to \( c_1 T_0 \), is not needed. Even for an open system, it is not necessary to choose the surface temperature as the thermodynamic reference state. The elimination of this requirement is advantageous since the ground surface temperature can vary dramatically in time and space.
Finally, note that substituting Businger's (1982) Eq. (16) into his Eq. (13) does not give the relationship immediately following his Eq. (16). The correct derivation is obtained by substituting
\[ \overline{L_0} = \overline{L} - (c_{pv} - c_l)(\overline{T} - \overline{T}_0) \] (B6)
into
\[ \frac{\rho w h_m}{\overline{w'}} = \frac{\overline{\rho w'T'}}{[c_p + \overline{q}(c_{pv} - c_p)]} \]
\[ + \frac{\overline{\rho w'q'}}{[\overline{L} + c_l(\overline{T} - \overline{T}_0)]]} \] (B7)
to obtain
\[ \frac{\rho w h_m}{\overline{w'}} = \frac{\overline{\rho w'T'}}{[c_p + \overline{q}(c_{pv} - c_p)]} \]
\[ + \frac{\overline{\rho w'q'}}{[\overline{L_0} + c_{pv}(\overline{T} - \overline{T}_0)]}. \] (B8)

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