**Integrated modeling framework to quantify the coastal protection services supplied by vegetation**


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Integrated modeling framework to quantify the coastal protection services supplied by vegetation

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Abstract Vegetation can protect communities by reducing nearshore wave height and altering sediment transport processes. However, quantitative approaches for evaluating the coastal protection services, or benefits, supplied by vegetation to people in a wide range of coastal environments are lacking. To begin to fill this knowledge gap, we propose an integrated modeling approach for quantifying how vegetation modifies near-shore processes—including the attenuation of wave height, mean and total water level—and reduces shore-line erosion during storms. We apply the model to idealized seagrass-sand and mangrove-mud cases, and illustrate its potential by quantifying how those habitats reduce water levels and sediment loss beyond what would be observed in the absence of vegetation. The integrated modeling approach provides an efficient way to quantify the coastal protection services supplied by vegetation and highlights specific research needs for improved representations of the ways in which vegetation modifies wave-induced processes.

1. Introduction

Coastal vegetation (e.g., seagrass beds, kelp forests, marshes, mangroves, and coastal forests) protects shorelines, human development, and economic activity by reducing the impacts of coastal hazards. Specifically, vegetation has been shown to attenuate wave height, moderate the strength of wave-induced currents, and decrease the extent of wave runup on beaches [Lovas and Torum, 2001; Bridges, 2008; Luhar et al., 2010]. These effects result in lower water levels and reduced shoreline erosion, functional benefits that can save lives, and prevent millions of dollars in property damage [Mazu, 1997; Das and Vincent, 2009].

The most commonly reported measure of the coastal protection services, or benefits, provided by vegetation to people is the attenuation of wave height. The use of that metric has been facilitated by the wide availability of engineering models for wave evolution in the presence of vegetation (see e.g., Suzuki et al. [2012], or reviews from Anderson et al. [2011] and McVor et al. [2012]). However, some of the most important coastal protection metrics needed by engineers and scientists are the amount of avoided coastal erosion and inundation during storms due to the presence of vegetation. To compute these metrics, it is necessary to quantify how different plants, under storm forcing conditions, alter total water levels and the erosion processes of sandy or muddy shorelines.

Most of the numerical models that examine the detailed response and impacts of particular types of vegetation on nearshore processes and on the shoreline are often developed for specific habitats under specific settings [see e.g., Augustin et al., 2009; Li and Zhang, 2010; Maza et al., 2013]. Even when general approaches are proposed to quantify how vegetation alters sediment transport and shoreline erosion, they consider only a limited range of vegetation types, rely heavily on models suited for sandy beaches devoid of vegetation, or are too complex for general use [Chen et al., 2007; Mariotti and Fagherazzi, 2010; Karambas et al., 2012]. Some of these deficiencies are attributable to an incomplete understanding of the way in which vegetation modifies sediment transport processes [Le Hir et al., 2007]. However, these deficiencies are also due to the absence of a modeling framework that links all the relevant processes governing the interaction of waves and vegetation.
In a first step toward filling this gap, we introduce an integrated modeling framework to quantify how vegetation attenuates wave energy, moderates water levels, and avoids sand or mud loss during storms. This modeling approach relies on well-established models and modifications thereof, links the most relevant processes responsible for inundation and shoreline erosion, and provides an initial practical framework for deepening our understanding of how vegetation alters key nearshore processes and thus supplies coastal protection services to people.

2. Nearshore Wave and Erosion Model

Vegetation supplies coastal protection services by modifying nearshore waves, water levels, and sediment transport. In this section, we present the framework of a process-based model to quantify these services. After defining the study domain and the model’s main variables, we briefly review the computational method for the evolution of a wavefield in the presence of vegetation. Then we describe methods to estimate mean and total water levels at the shoreline as well as the amount of beach retreat or mud bed scour in the presence of vegetation. Throughout, we highlight our assumptions, the model’s limitations, and, importantly, key areas for future improvements.

2.1. Model Setup and Definition of Variables

The study domain is defined in a longshore uniform, two-dimensional vertical (2DV) coordinate system $(x, z)$, with $x$ pointing shoreward, $z$ pointing upward and the origin at the still water level (Figure 1; see also notation section for a list of commonly used symbols). The coast may either be sandy or muddy. Wave properties, such as the free surface elevation $\eta$, are expressed using linear wave theory (LWT). Wave crests are propagating parallel to the coast and are modeled along a shore-perpendicular one-dimensional transect. Two-dimensional processes, including longshore currents, are neglected.

A storm approaching the region of interest generates a wavefield with a deep water significant wave height $H_o$, peak period $T_p$, and horizontal and vertical water particles velocities $u$ and $w$, respectively. Velocity component are decomposed into short-wave and mean values [Svendsen, 2006, chap. 10], ignoring turbulent motions, e.g., $u = u_{ow} \cos \psi + U$, where $u_{ow}$ is the short-wave amplitude, $\psi(x, t)$ the wave phase, and $t$ represents time. $U$ is the time averaged velocity, $U = \overline{U}$, and it is weak compared to the short-wave velocity: $|U|/u_{ow} < 1$ and $(U/u_{ow})^2 \ll 1$ [Svendsen, 2006, chap. 10]. The near bed mean velocity is expressed as $U_b$.

As waves progress shoreward, they encounter a vegetation field (e.g., a seagrass meadow, marsh, or a mangrove forest). Marshes and seagrass meadows are represented as single stem elements, with diameter $d_v$, height $h_v$, and a density of $N_v$ stems per m² (Figure 1). Mangroves and forests may have supra-aerial roots, trunk, and...
canopies. Each of these components has a diameter \(d_i\), height \(h_i\), and density \(N_i\), where the subscript \(i\) represents roots, trunk, or canopy.

We assume that vegetation elements do not sway with the flow and are represented by vertical cylinders of uniform diameter [Dalymple et al., 1984]. Thus waves exert a horizontal drag force \(F_d\) on the stems in the cross-shore direction [Dalymple et al., 1984]:

\[
F_d = \frac{1}{2} \rho C_d d_i N_i \bar{u}^2 |\bar{u}|,
\]

where \(C_d\) is a drag coefficient associated with the vegetation field. Note that estimates of the drag force \(F_d\) using LWT in equation (1) can differ substantially from more accurate computational approaches (Appendix A).

Storms often generate surges at the shoreline, which can be altered by vegetation. However, since the primary focus of this paper is the modification of wave processes by vegetation, we neglect surge attenuation by plants and refer interested readers to, e.g., Wamsley et al. [2010] or Zhang et al. [2012].

### 2.2. Wave Model

The evolution of a wavefield of root-mean square (rms) wave height \(H\) in a 2DV coordinate system with weak mean currents is computed by solving the wave energy balance equation:

\[
\frac{\partial E_w}{\partial x} = -D_b - D_f - D_v,
\]

where \(E_w\) is the wave energy density and \(C_g\) the group velocity. The dissipation of wave energy flux is caused by wave breaking \(D_b\), bottom friction \(D_f\), and the presence of vegetation in the water column \(D_v\). \(D_b\) and \(D_f\) are expressed following Thornton and Guza [1983]:

\[
D_b = \frac{3}{32 \sqrt{\pi}} \rho g \frac{a^3}{H^3} H^3,
\]

\[
D_f = \frac{C_f}{16 \sqrt{\pi}} \left( \frac{\sigma}{\sinh kh} \right)^3 H^3,
\]

where \(\rho\) is the water density, \(g\) the constant of gravity, \(k\) the wave number, and \(\sigma\) the wave frequency. The breaking coefficient \(b\) and breaker index \(\gamma\) have default values of 1.0 [Thornton and Guza, 1983] and 0.78 [Dean and Dalymple, 1984, chap. 4.8.4], respectively.

\(D_f\) is the transfer of wave energy flux to the turbulent bottom boundary layer, in the absence of vegetation, and \(C_f\) is a bottom friction coefficient, typically a function of bed roughness. Because \(D_f\) is usually small, we ignore the presence of vegetation in equation (4) (see e.g., Nepf, 1999), and use a default value of \(C_f = 0.01\) for sand beds [Thornton and Guza, 1983]. For mud beds, \(C_f = 0.0521 \left( \frac{u_*}{\alpha_1} \right)^{0.187}\) [Whitehouse et al., 2001], where \(\nu\) is the water eddy viscosity and \(u_0\) is the near bed velocity computed using LWT. This measure of energy dissipation does not explicitly account for the presence of vegetation other than in the reduction of near-bed velocity caused by the attenuation of wave height due to the presence of vegetation. (We performed a sensitivity analysis (not shown) to the expression of \(C_f\), following recommendations in, e.g., Nielsen [1992] and Whitehouse et al. [2001], and found that the expression of that coefficient had little impact on the final results since dissipation of wave energy via bottom friction is small in comparison to the other dissipative mechanisms.)

Finally, dissipation due to vegetation \(D_v\) is expressed as [Suzuki et al., 2012]:

\[
D_v = \frac{1}{2 \sqrt{\pi}} \rho \left( \frac{kg}{2\pi} \right)^{3/2} \sum_{i=1}^{3} \frac{D_{vi}}{3k \cosh kh} H^3
\]

where for mangroves and trees, \(D_{vi} = 1, 2\) represent the contributions of roots \(i = 1\), trunk \(i = 2\), and canopy \(i = 3\). Detailed expressions for \(D_{vi} = 1, 2\) are provided in Appendix B. For seagrasses and marshes, \(i \equiv 1\) to represent stems, and the expression of \(D_v\) is identical to the expression of Mendez and Losada [2004].

Equation (5) has been extensively validated [see e.g., Pinsky et al., 2013]. It assumes that dissipation is dominated by short-wave forces and neglects the role of mean currents, wave reflection, and other nonlinear
processes that occur near and within vegetated fields. Outputs from equation (2) are used to estimate the total water level at the shoreline.

2.3. Total Water Level Model

The total water level at the shoreline is a critical driver of coastal flooding and shoreline erosion. It is, in part, a function of the astronomical tide, the storm surge, and the runup level of waves [Ruggiero et al., 2001]. The runup is the sum of the wave setup at the shoreline and the height of the incident and infragravity band swash [Stockdon et al., 2006]. In this section, we first derive an equation for the mean water level in the presence of vegetation. Then we propose a method to compute wave runup on beaches in the presence of vegetation.

2.3.1. Computation of Wave Setup

Dean and Bender [2006] heuristically developed an expression for the mean water level $\eta$ in the presence of vegetation by first arguing that stems exert a stress on the water column, then by inserting that stress directly into the mean momentum equation. Here, we confirm that vegetation has the potential to modify $\eta$ by deriving an expression for the mean momentum equation in the presence of vegetation, starting from the governing equations of fluid motion. We present this derivation for single stem vegetation elements in a monochromatic wavefield, then expand it to all vegetation types in a random wavefield. Throughout, we highlight both theoretical and observational knowledge limitations and gaps that are necessary to fully validate and eventually improve that expression.

To express $\eta$ in the presence of vegetation, it is necessary to first assume that the drag coefficients $C_d$ in the momentum equation (7) and in the wave evolution equation (5) are identical. Similarly, $C_d$ associated with short-wave and mean velocities, $u_{caw}$ and $U$, are identical [Zhou and Graham, 2000; Luhar et al., 2010].

The 2DV conservation of mass and momentum equations in the presence of vegetation are [Augustin et al., 2009; Li and Zhang, 2010]:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{6}$$

$$\begin{cases}
\rho \frac{\partial u}{\partial t} + \rho \frac{\partial u^2}{\partial x} + \rho \frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right) - F_d \\
\rho \frac{\partial w}{\partial t} + \rho \frac{\partial uw}{\partial x} + \rho \frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z} - \rho g + \left( \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right)
\end{cases} \tag{7}$$

where $\rho$ represents the hydrostatic pressure, and $\tau_{ij}$ represent the shear stress components, in tensor notation. The drag force $F_d$ (equation (1)) is only included in the horizontal component of the momentum equation. That force is null in the vertical direction since stems are treated as rigid vertical cylinders [Dalrymple et al., 1984].

We integrate Equations (6) and (7) through the water column and time average them, we decompose the velocity terms, into oscillatory and mean components, and we apply dynamic and kinematic boundary conditions. For brevity, only calculations involving vegetation variables are shown below—interested readers are referred to Longuet-Higgins and Stewart [1963, 1964] or Svendsen [2006, chap. 11] for more details. These equations become, for steady state conditions,

$$\frac{\partial Q}{\partial x} = 0, \tag{8}$$

$$\rho g (h + \eta) \frac{\partial \eta}{\partial x} + \frac{\partial S_{ax}}{\partial x} + \int_{-h}^{0} F_d dz + \tau_b = 0, \tag{9}$$

where $Q$ is the shoreward mass flux of the waves, $\tau_b$ is the time averaged bottom shear stress, and $S_{ax}$ is the radiation stress generated by both waves and rollers. The energy of the roller $E_r$ is modeled following Reniers and Battjes [1997] and Apotsos et al. [2007].

Equation (8) indicates that, since $Q \equiv 0$ at the coast, waves generate a steady current $U(x, z)$, the undertow, with a depth averaged value $U_{off}(z)$ that balances the shoreward mass flux of waves [Guannel and Özkan-Haller, 2014]:
\[
\rho h \frac{\partial U_{\text{off}}}{\partial t} = -\frac{E_w + 2E_i}{C}, \tag{10}
\]

where \( C \) is the wave celerity. This equation indicates that because vegetation reduces the energy density of waves (equation (2)), vegetation also moderates the depth-average undertow \( U_{\text{off}} \). As the undertow is one of the principal mechanisms for offshore sediment transport during storms [Guannel, 2009], equation (10) suggests that submerged vegetation is likely to reduce coastal erosion.

The mean momentum equation (9) is identical to the classic expression derived by Longuet-Higgins and Stewart [1963, 1964], with the addition of a stress term due to the presence of vegetation elements. Assuming that the short-wave velocity is constant between the still-water level and the wave crest [Dean, 1965], this equation becomes, for monochromatic waves:

\[
\tilde{F}_d \equiv \Phi \left[ \alpha_v u_{\text{ow}}^2 \cos \psi \cos \psi \right]_0^h + \int_{-h}^0 2U_{\text{ow}} \cos \psi |\psi| \, dz, \tag{12}
\]

where \( \alpha_v = \min \left\{ h_v / h, 1 \right\} \). This approximation is reasonable because \( \alpha_v \), \( F_d \) values computed using LWT versus \( F_d \) values computed at various water depths using the more exact nonlinear stream function theory [Dean, 1965] can be linearly related throughout the water column and at the water surface \( (R^2 > 0.8 \text{ and } R^2 > 0.9, \text{ respectively; see Appendix A}) \).

Assuming that the short-wave velocity is constant between the still-water level and the wave crest [Dean and Bender, 2006; Svendsen, 2006, chap. 11], equation (12) becomes, for monochromatic waves:

\[
\tilde{F}_d \equiv \Phi \left[ \alpha_v u_{\text{ow}}^2 \cos \psi \cos \psi \right]_0^h + \int_{-h}^0 2U_{\text{ow}} \cos \psi |\psi| \, dz,
\]

\[
\equiv \Phi \left( \frac{\alpha_v g k}{6 \pi \sinh kh} H_{\text{mon}}^2 + \frac{2 \nu_{\text{ow}} c \sinh \left( k \alpha_v h \right)}{\nu_{\text{ow}} k h} \right), \tag{14}
\]

where \( H_{\text{mon}} \) is a monochromatic wave height representative of the wavefield. In equation (14), the mean horizontal current within the vegetated field \( U_{\text{veg}} \) is assumed, as a first approximation, to be constant in the

\[\text{Equation (10)}\]

\[\text{Equation (12)}\]

\[\text{Equation (14)}\]
vertical [Luhar et al., 2010]. This assumption is reasonable, based on results of numerical simulations and observations [Lovas and Torum, 2001; Maza et al., 2013]. It also avoids including more complexity in the model, an approach justified by our limited understanding of how vegetation modifies the undertow, as discussed in section 3.

Substituting equation (14) into equation (9) yields:

$$\frac{\rho g (h+\eta)}{\gamma} \frac{\partial \eta}{\partial t} + \frac{\partial S_{xx}}{\partial x} + \frac{1}{2} \gamma C_d d_s N_v \frac{\eta x_v g_k}{\tanh kh} \left( \frac{\eta^3}{\delta} \right) + \frac{\rho C_d d_s N_v}{\pi \sinh \chi} H_m^2 + \frac{\rho C_d d_s N_v}{\pi \sinh \chi} U_{\text{vex}C \sinh (k \alpha w)} H_m + \tau_b = 0. \quad (15)$$

The first term on the LHS (term I) represents the classic mean momentum balance in nearshore regions without vegetation [Dean and Dalrymple, 1984, chap. 10.5]. Vegetation alters the mean water level via the addition of a short-wave-induced stress on the water column (term II), the advection of mean currents (term III) and by changing the strength of the bed shear stress (term IV). Indeed, the bed shear stress can be expressed as $$\tau_b \equiv \frac{2}{\pi} \gamma p C_d u_{ow} U_b$$ [Svendsen, 2006, chap. 10], which is likely affected by the presence of vegetation (equation (10), see also observations by, e.g., Lovas and Torum [2001] or Luhar et al. [2010]).

To generalize the expression for $$\eta$$ to all vegetation types, the stresses associated with each layer that makes up the vegetated field (e.g., roots, trunk, and canopy) are linearly added following Suzuki et al. [2012], and the resulting equation is converted to random wavefields following Mendez and Losada [2004]. We obtain:

$$\frac{\rho g (h+\eta)}{\gamma} \frac{\partial \eta}{\partial t} + \frac{\partial S_{xx}}{\partial x} + \left( \frac{1}{16\sqrt{\pi}} \sum_{n=1}^{3} \rho C_d d_s N_v \eta x_v g_k \frac{\eta^3}{\tanh kh} + \frac{1}{2\sqrt{\pi}} \sum_{n=1}^{3} \rho C_d d_s N_v U_{\text{vex}C} \sinh \chi H_m \right) + \tau_b = 0. \quad (16)$$

where $$H$$ is the RMS wave height, and the subscript “v” represents roots, trunk, or canopy. Expressions for the coefficients $$x_v$$ and $$S_v$$ are provided in Appendix B.

Finally, to remain consistent with the approximations used in the derivation of equation (5), the mean current term is neglected (see e.g., Dalrymple et al. [1984] and Mendez and Losada [2004]; see also section 3 where the relative importance of the neglected mean current term is examined) to obtain:

$$\frac{\rho g (h+\eta)}{\gamma} \frac{\partial \eta}{\partial t} + \frac{\partial S_{xx}}{\partial x} + \frac{1}{16\sqrt{\pi}} \sum_{n=1}^{3} \rho C_d d_s N_v \eta x_v g_k \frac{\eta^3}{\tanh kh} H^3 + \tau_b = 0. \quad (17)$$

For vegetation without supra-aerial roots or canopy (e.g., seagrasses or marshes), this equation becomes

$$\frac{\rho g (h+\eta)}{\gamma} \frac{\partial \eta}{\partial t} + \frac{\partial S_{xx}}{\partial x} + \frac{1}{16\sqrt{\pi}} \rho C_d \frac{\eta x_v g_k}{\tanh kh} H^3 + \tau_b = 0. \quad (18)$$

Ignoring the bed shear stress and the presence of a roller, equation (18) reduces to equation (1) of Dean and Bender [2006], for monochromatic wavefields.

Equations (17) and (18) provide an approach for estimating the setup nearshore $$\eta_{\text{shore}}$$ ($$\eta_{\text{shore}} = \eta(x = \text{shore})$$), a significant component of the runup and total water level, in the presence of most vegetation types. While commonly used approximations were used to derive equations (17), we are not aware of a data set presently available to validate it. Outputs of this equation are used to estimate runup and total water level at the shoreline.

### 2.3.2. Wave Runup on Beaches

The 2% exceedence level of wave runup maxima generated by random wavefields on open coast sandy beaches in the absence of vegetation can be estimated by the expression proposed by Stockdon et al. [2006]:

$$R_u = 1.1 \left( 0.35 m \sqrt{H_{L_0} \left( 0.563 m^3 H_{L_0} + 0.004 H_{L_0}^2 \right)} \right) = 1.1 \left( \frac{H_{L_0} + \sqrt{H_{L_0}^2 + S_w^2}}{2} \right). \quad (19)$$

where $$L_0$$ is the offshore wavelength, and $$m$$ the foreshore slope of the beach. This equation expresses runup as a function of empirical estimates of incident wave setup at the shoreline $$\eta_{5\theta}$$, and incident and
infragravity band swash $s_{inc}$ and $z_g$. As shown by Raubenheimer et al. [2001], this empirical estimate of setup is a priori different from the nearshore setup, $\eta_{shore}$, computed using the process-based model described by equation (17).

We modify the empirical expression of runup to take into account the presence of vegetation by first assuming that plants only affect setup and incident band swash; vegetation has a limited impact on the propagation of infragravity waves (see observations in, e.g., Møller et al. [1999], Manca et al. [2012], or JadHAV et al. [2013]), and thus on low frequency swash. Next, we relate mean water level values nearshore computed in the presence and absence of vegetation from equation (17) by a proportionality coefficient $\varepsilon$:

$$\eta^ {veg}_{shore} = \varepsilon \eta_{shore}$$

Where the superscript $veg$ indicates that vegetation is present. Then, we modify the empirical estimate of setup $\eta_{shore}$ at the shoreline in equation (19) to take into account the presence of vegetation by using the same proportionality coefficient $\varepsilon$.

$$\eta^ {veg}_{St} = \varepsilon \eta_{St}$$

This approximation is justified by the fact that, in the absence of vegetation, the nearshore setup $\eta_{shore}$ can be linearly related to the empirical estimate of setup $\eta_{St}$ at the shoreline ($R^2 > 0.7$; Appendix C). Applying the same proportionality coefficient $\varepsilon$ to the incident swash variable, equation (19) becomes, in the presence of vegetation:

$$R_u = 1.1 \left( 0.35mc \varepsilon \sqrt{H_0 L_0} + \frac{\sqrt{0.563m^2c^2H_0 L_0 + 0.004H_0 L_0}}{2} \right) = 1.1 \left( \varepsilon \eta_{St} + \frac{\sqrt{z^2 + s_{inc}^2}}{2} \right). \quad (20)$$

In cases where vegetation produces a setdown near the shoreline, we set $\varepsilon \equiv 0$, which is likely to underestimate the potential for vegetation to reduce runup. While we justify the assumptions used herein in Appendix C, we are not aware of a data set to validate equation (20).

For sandy beaches, the runup is added to the storm surge to yield the total water level in the presence of vegetation. For mud beds, or in cases when dunes fail, swash processes are ignored and equation (17) is used instead. Wave characteristics, mean and total water level values are used to compute shoreline change.

### 2.4. Coastal Change Model

The supply of coastal protection services by plants can be quantified by taking the difference in shoreline erosion observed in their presence and absence. In this section, we modify existing models developed in the absence of vegetation to estimate the amount of sandy beach retreat and mud bed scour in the presence of vegetation.

#### 2.4.1. Erosion of Sandy Beaches

We compute the amount of beach erosion $E_b$ (m) during a storm in the presence of vegetation by adapting the erosion model of Kriebel and Dean [1993]:

$$E_b = \Theta S \left( x_b h_b - s_{inc} / m \right) - W \left( B + h_b - S/2 \right) \frac{B + D + h_b - S/2}{B + D + h_b - S/2}, \quad (21)$$

where $B$ and $W$ are the beach berm height and width and $D$ is the dune height (Figure 1). $S = S + R_s$ is the total water level, the sum of the surge $S$ and runup $R_s$ [Mull and Ruggiero, 2014], and $h_b$ and $x_b$ represent the depth and distance from the shoreline of the breakpoint, respectively. $\Theta$ is a convolution integral that limits the beach erosional response as a function of wave breaking characteristics, storm duration, and a characteristic erosion time scale [Kriebel and Dean, 1993]. The location of wave breaking $x_b$ is identified from the profile of wave height (equation (2)) as $H(x_b) = \gamma h$. Consequently, changes in total water level and in breaking characteristics of incoming waves caused by vegetation dictate the amount of erosion modeled.

#### 2.4.2. Erosion of Muddy Shores

Muddy shoreline erosion is a function of mud composition, plant biomass and physical characteristics, and of incident wave characteristics [Whitehouse et al., 2001; Feagin et al., 2009; Mariotti and Fagherazzi, 2010]. Here, we quantify the rate of mud bed scour $E_m$ (cm/h) in the presence of vegetation by adapting the method presented in Whitehouse et al. [2001]:

$$E_m = \Theta s_{inc} m c \varepsilon \eta_{St}$$

Where $\eta_{St}$ is estimated using equation (19) with $\varepsilon \equiv 1$. The mud bed scour rate is a function of the mud composition and the characteristic erosional response as a function of wave breaking characteristics, storm duration, and a characteristic erosion time scale [Whitehouse et al., 2001].
3. Model Application

In this section, we first apply and test the robustness of the modeling framework by quantifying the protective service of a seagrass meadow offshore of a sheltered sandy beach, and of a mangrove forest on a mud bed, for three hypothetical storms (Table 1). For both vegetation cases, wave properties (\(H, T, \eta_{w} \), etc.) are computed from the offshore extent of the profile to the nearshore, where the minimum water depth is 10 cm. Throughout, plant characteristics and drag coefficient values are based on U.S. Federal Emergency Management Agency guidelines (FEMA, 2007; see Pinsky et al. [2013] and Appendix D for other examples of formulations) and are kept constant. We also assess the sensitivity of the model to the uncertainty associated with the choice of plant parameters and discuss the practical ramifications of the results.

3.1. Seagrass Meadow Fronting a Dune-Backed Sandy Beach

In this example, a seagrass meadow grows from \(-5.0 \) to \(-0.5 \) m on a beach exhibiting an equilibrium profile. The beach has a 3 m high dune, a profile scale factor \(A=0.0115\), corresponding to a sand diameter \(d_{50}=0.25\) mm [Dean and Dalrymple, 2002, chapter 7], and a 1:15H foreshore slope (Figure 2d). The physical parameters of the seagrass are similar to the ones observed by Huber [2003], with \(h_{v}=40\) cm, \(d_{v}=0.6\) cm, and \(N_{v}=1,200\) blades/m\(^2\) (Figure 1). In accordance with FEMA [2007], \(C_{d}=0.1\).

### Table 1. Forcing Conditions for the Seagrass and Mangrove Case

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<th>Storm</th>
<th>Wave Height (m)</th>
<th>Wave Period (s)</th>
<th>Surge (m)</th>
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<tbody>
<tr>
<td>A</td>
<td>1.5</td>
<td>4.0</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>2.5</td>
<td>6.0</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>7</td>
<td>2</td>
</tr>
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</table>

In addition to dissipating wave energy flux, the stems also reduce \(\eta_{w}\) throughout the surf zone, yielding a 41 cm, or 83%, reduction in setup nearshore (\(\eta_{w,\text{near}}\) Figure 2b). If vegetation-induced stresses in the cross-shore momentum balance are ignored, \(\eta_{w,\text{near}}\) still decreases by 12 cm, relative to the no-vegetation case, due
to the influence of vegetation on the radiation stress gradient alone. Hence, regardless of the limitations of equation (17), vegetation moderates the wave-induced mean water level, and hence the total water level. Note that we report values of $\eta_{\text{shore}}$, the setup near the shoreline computed from equation (18). This setup is modified to compute values of runup, and setup $\pi_{\text{St}}$, from equation (20) (see section 2.3.2). In this example, values of $\pi_{\text{St}}$ are lower than values of $\eta_{\text{shore}}$ (not shown; see also Appendix C).

In addition to lowering the mean water level, the stems lower wave runup by nearly 32 cm, or 39%, at the shoreline (Figure 3a, Storm B). The reduction in runup and breaking wave height by seagrasses yields 2.2 m of avoided erosion (4.2 m of erosion in their absence compared to 2.0 m in their presence, Figure 3a). This estimate of avoided erosion does not take into account the fact that the depth-averaged undertow, the main mechanism for offshore sediment transport during storms, is weakened by the presence of vegetation. Indeed, as shown in Figure 2c, since wave height and, by extension, the mass flux of the waves are reduced (equation (10)), the strength of the depth-averaged undertow $U_{\text{off}}$ decreases by more than 25% over the vegetated bed. Hence, it is likely that vegetation also reduces offshore sediment flux during storms and that mobilized sediments resettle near the shoreline. The combination of those factors may facilitate a more efficient poststorm recovery.

We estimate the services of submerged vegetation under two additional simulated storm conditions (Table 1 and Figure 3). For storm A, vegetation reduces $\eta_{\text{shore}}$ and total water level by 17 and 15 cm, respectively.
For storm B, $\eta_{\text{shore}}$ and total water level are reduced by 50 and 47 cm, respectively. Finally, through a combination of lower runup on the beach and reduced wave heights, the amount of shoreline erosion is reduced by roughly 50% (60 cm for storm A, and 5 m for storm C).

3.1.2. Role of Mean Current Advection and Bed Shear Stresses on the Mean Water Level

The influence of mean currents is ignored in the derivation of equation (5) [Dalrymple et al., 1984]. Accordingly, the undertow was also neglected in the expression of $\eta_j$ and equation (16) simplified to equation (17). In this section, we examine the role of mean current advection, and bed shear stresses to a lesser extent, in the computation of $\eta_j$ using equation (16).

In the absence of a reliable model for the expression of the undertow in the presence of vegetation, the undertow is assumed to be, as a first approximation, depth uniform in the vegetated field (i.e., $U_{\text{veg}}(x, z) \equiv U_{\text{veg}}(x)$), and approximately an order of magnitude weaker than the orbital velocity, i.e., $|U_{\text{veg}}| \approx u_{\text{ave}}/10$. These assumptions are reasonable based on experimental and numerical observations [Lovas and Torum, 2001; Svendsen, 2006, chap. 12; Guannel, 2009; Maza et al., 2013]. At the bed, $U_b$ is approximated by $U_{\text{off}}$ [Guannel, 2009].

The direction of the mean velocity in the presence of vegetation is still an active subject of research. Mean velocities in meadows have been observed to be directed both shoreward and offshore [Lovas and Torum, 2001; Luhar et al., 2010, 2013]. This uncertainty is taken into account by considering both an offshore and a shoreward oriented mean velocity in the water column and at the bed. Thus, the effect of the formulation of mean stress on the water column is modeled using four combinations of velocity direction in the water column and at the bed.

For all combinations of stress formulation considered, vegetation lowers the mean water level at the shoreline, $\eta_{\text{shore}}$ (Figure 3b). First, when vegetation-induced stresses are not taken into account ($F_d = 0$), $\eta_{\text{shore}}$ decreases by 11 cm on average (Figures 2b and 3b). This reduction is solely due to the influence of vegetation on wave energy flux and radiation stress gradients (Figures 4a and 4b). When mean currents in the water column are oriented offshore ($U_{\text{veg}} < 0$), the setup is reduced by 21 cm, on average. Changing the orientation of the bed shear stress ($U_b > 0$ or $U_b < 0$) only yields differences on the order of 6 cm. Finally, when the mean current in the meadow is oriented shoreward, $\eta_{\text{shore}}$ is further lowered, resulting in an average reduction of 44 cm.

We also evaluated the influence of mean current and bed shear stresses on runup and erosion estimates (Figure 3b). In all cases, even when vegetation-induced stresses are ignored in the mean momentum equation, vegetation reduces total water level and shoreline retreat. On average, runup decreases by 12 cm when $F_d = 0$, and by 38 cm when mean currents and bed shear stresses are included in the computation of $\eta$. The impacts of a reduced mean water level on erosion estimates are more subtle, with erosion values
varying by less than 1 m. This is likely due to the fact that the erosion model used herein is primarily controlled by wave breaking characteristics (equation (21)).

3.1.3. Balance of Stress Terms in the Mean Momentum Equation

To gain more insight into the potential role of the vegetation-induced stresses on $\Gamma_{22}$, we examine the cross-shore structure of the various components of the mean momentum equation, for storm B. The balance of stress terms shows that, in the absence of vegetation, gradients in radiation stress ($\partial S_{xx}$, Figure 4a) oppose the pressure gradients caused by changes in $\Gamma_{22}$ ($\partial \Gamma_{22}$), yielding a 49 cm setup at the shoreline. This balance is altered by the presence of vegetation.

The decrease in wave height in the shoaling region caused by vegetation (Figure 2a) results in a decrease in radiation stress $S_{xx}$, which causes the gradient of $S_{xx}$ to change sign earlier along the profile ($x \approx 300$ m in Figure 4b compared to $x \approx 200$ m in Figure 4a). As wave height decreases in the meadow, the radiation stress gradient becomes approximately 40% weaker in the surf zone than when seagrasses are absent. This weakening of the radiation stress gradient alone decreases wave setup to 37 cm, as observed in Figure 2b. The vegetation-induced stress $\sim F_d$ reduces the setup further to 8 cm. In the shoaling and surf zone regions ($x < 200$ m, Figure 4b), $\sim F_d$ is approximately twice as strong as the radiation stress gradient. This causes gradients in $\partial \Gamma_{22}$ and $S_{xx}$ to act together to oppose this stress, pushing shoreward the location of the setdown, where the pressure gradient changes sign, and lowering the setup at the shoreline.

Finally, when offshore oriented mean currents are included in $\sim F_d$ (Figure 4c), they generate a stress almost equal and opposite to the short-wave stress, cancelling the effect of $\sim F_d$ on the water column. As a consequence, the location of the maximum setdown moves further offshore than when we only considered short-wave stresses, and the mean water level at the shoreline increases to 32 cm. However, this setup is lower than when vegetation is absent because the vegetation still modifies the profiles of the radiation stress gradient, as noted earlier. The influence of bed shear stress is negligible in all cases.

3.2. Mangrove Forest on a Mud Bed

The mangrove forest grows on a uniform 1V:60H mud bed. Seaward of the mangroves, the seabed has a uniform slope of 1V:60H. The forest is composed of black mangroves (Avicennia germinans species, see also Figure 1), and it extends landward from the shoreline for 250 m (Figure 5d). The physical
characteristics of the aerial mangrove roots (pneumatophores) and the trunks are, respectively: $h_v^1 = 0.5$ and $h_v^2 = 5$ m, $d_v^1 = 0.2$ and $d_v^2 = 30$ cm, and $N_v^1 = 50$ and $N_v^2 = 0.7$ units/m$^2$ (Figure 1), where the subscripts 1 and 2 correspond to roots and trunks, respectively. Following FEMA [2007], $C_{d1} = 1$.

### 3.2.1. Quantification of Coastal Protection Services

The mangrove forest is subjected to the same storm conditions as the seagrass meadow (Table 1). During storm B, waves break offshore of the mangroves (Figure 5a). In the surf zone, wave height is reduced by an average of 73% across the forest, relative to the case without vegetation (Figure 5a). The far-field effect of the mangroves is more pronounced than in the seagrass example, as waves exiting the forest remain lower than waves propagating over a non-vegetated bed for more than 250 m. This pronounced far-field effect is due to the fact that mangroves are on a flatter bed than seagrasses and have a higher drag coefficient. Analysis of profiles of wave energy dissipation (not shown) reveal that, because of the relatively shallow water depth, the roots dissipate, on average, 80% more energy than the trunks.

In addition to attenuating waves, the mangrove forest also lowers $\eta_{\text{shore}}$ by 13 cm, or 29%, compared to values obtained in the absence of vegetation (Figure 5b). Even if $F_d$ is neglected in the mean momentum equation, $\eta_{\text{shore}}$ still decreases by 4 cm.

Finally, the attenuation of wave height due to the presence of vegetation nearly prevents any sediment loss in the forest. When mangroves are present, the bed is only scoured over the first 15 m (Figure 5c, $K_{\text{veg}} = 4$). By contrast, in the absence of mangroves, the bed is scoured over its entire length. Even if the effect of the vegetation biomass is not taken into account (i.e., $K_{\text{veg}} = 1$), the reduced wave height and orbital velocity—due to the presence of the mangroves—limit any scour to the first 60 m of the forest.
3.2.2. Role of Mean Current Advection and Bed Shear Stresses on the Mean Water Level

To evaluate the relative importance of mangrove-induced stresses, and the potential role of mean currents on the mean water level, we assume that \( U_{\text{off}} \) is depth uniform, and that \( U_b \equiv U_{\text{off}} \). However, since the trees are emerged, the shoreward mass flux of the waves has to be exactly balanced by the undertow (equation (10)). Thus, we only consider the case when \( U_{\text{off}} \) is oriented offshore: \( U_{\text{off}} = -|U_{\text{off}}| \). The bed shear stress can act in either direction.

The role of the mangrove forest-induced stresses on \( \eta_{\text{shore}} \) for the three storm cases (Figure 6b), differs from what we observed in the seagrass example. First, when vegetation is present but vegetation-induced stresses are ignored \( F_{\text{d}} = 0 \), \( \eta_{\text{shore}} \) is reduced by 8 cm, on average. However, the inclusion of offshore oriented mean currents and bed shear stresses increases \( \eta_{\text{shore}} \) to the point that it is nearly equal to the no-vegetation case. When the mean current is still oriented offshore, but the bed shear stress is oriented shoreward, \( \eta_{\text{shore}} \) decreases again by 9 cm, on average. However, it has nearly the same value of \( \eta_{\text{shore}} \) obtained when we took \( F_{\text{d}} = 0 \).

These results indicate that the vegetation-induced mean current stress is likely to play an important role when vegetation is shoreward of the breaking region and waves are relatively small. However, our findings also point to the greater role played by bed shear stress in this example, as compared to the seagrass example. This is likely due to the fact that the depth average mean current is greater in that vegetation field by 30% on average.

3.2.3. Balance of Stress Terms in the Mean Momentum Equation

We again examine the balance of terms in the mean momentum equation (Figure 4), for storm B, to better understand the potential importance of mangrove-induced stresses on \( \eta \). In the absence of vegetation, gradients of \( S_{xx} \) and \( \eta \) oppose each other (Figure 4d), yielding a setup of 45 cm at the shoreline. The rapid reduction of wave height caused by mangroves in the surf zone yields an initial 18% increase in the strength of the radiation stress gradient (Figure 4d and 4e, \( x \approx 0 \) m). However, as waves become smaller, the radiation stress gradient decreases by half near the landward edge of the mangrove \( x \approx 50 \) m). This reduction alone would yield a lower setup of 41 cm at the shoreline (Figure 5b). But, because we take into account the vegetation-induced stress, which is, at its maximum, almost 40% as strong as the radiation stress gradient, the pressure gradient rapidly reduces to zero, further decreasing the setup at the shoreline to 33 cm. In contrast from the seagrass example, where seagrasses altered the shoaling and breaking process, mangroves are in the surf zone and the mangrove-induced stress works together with the pressure gradient to counteract the stress generated by the radiation stress gradient (Figures 4d and 4e). Hence, the location of setdown remains the same (Figures 4e and 5b).
Lastly, when we include the role of the offshore oriented undertow (equation (16)), the stress that it generates opposes the short-wave stress (Figure 4f), resulting in a higher pressure gradient in the first few meters of forest (increase of approximately 30% at $x \approx 0$). However, since the radiation stress gradient decreases as waves exit the forest, the pressure gradient also decreases. As a result, the setup at the shoreline increases to 40 cm. The bed shear stress plays a minor role.

Results presented in sections 3.1 and 3.2 show that vegetation has the potential to modify mean water levels by altering the strength of radiation stress gradients, but also by generating relatively strong stresses in the water column. Onshore currents seem to reinforce the importance of the vegetation-induced stress while offshore currents seem to diminish it. We confirmed this finding (not shown) by rerunning the two cases presented above for storms A and C, assuming that vegetation could be represented using emergent stems (i.e., $F_d$ is not approximated with equation (13)), or by estimating $U_{huv}$ and $U_b$ as 1/20 and 1/5 of the value of the orbital velocity, respectively, and by estimating $U_b$ with the formulation proposed by Luhar et al. [2010].

In summary, the application of our integrated modeling framework to two idealized examples demonstrates that vegetation has the potential to reduce the impacts of storms and thus provide important coastal protection services to people. These results also show that reliable models for the undertow in the presence of vegetation are imperative to better quantify the protective services provided by vegetation. Note that we only evaluated the potential role of the undertow in estimates of mean water level. The relative importance of this current in the dissipation of wave energy in the presence of vegetation (equation (5)) was not quantified. In the next section, we examine the model’s sensitivity to changes in vegetation characteristics, and suggest steps to improve the framework presented herein.

3.3. Relative Importance of Plant Parameterization and Choice of Drag Coefficient

The application of the proposed framework, or any model, to a particular site requires estimates of the physical characteristics of the vegetation at that site, along with an appropriate value of drag coefficient $C_d$. Plants’ physical parameters are often challenging to measure [de Vos, 2004], and vary with regions, seasons, and ecosystem’s health [Feagin et al., 2011; Paul and Amos, 2011; Pinsky et al., 2013]. Still, choosing an appropriate value of $C_d$ is even trickier, as it requires wave measurements across the vegetation. Thus, it is common to use values of plant parameters found in the literature, which are measured under specific settings, forcing and plant conditions. In this section, we investigate the sensitivity of the model to plant parameters.

We assess the model’s sensitivity to the choice of vegetation parameters using an uncertainty analysis method called tornado analysis [Howard, 1988; Celona and McNamee, 2001]. For simplicity, we use the seagrass meadow example (section 3.1)—we obtained similar results for mangroves using values from Narayan [2009], or for marshes, using values from Feagin et al. [2011].

The first step in the tornado analysis is to define a range of possible values (minimum, typical, maximum) for $N_v$, $h_v$, $d_v$, and $C_d$ (Table 2). Values for $N_v$ and $h_v$ were obtained from Huber [2003], and for $d_v$ by varying the diameter observed in Huber [2003] by a few millimeters, based on ranges observed elsewhere [Pinsky et al., 2013]. Finally, a range of $C_d$ values was generated from typical estimates obtained for seagrass meadows (see e.g., Pinsky et al. [2013] or Appendix D). Next, the model is run multiple times for each value of a particular input variable (e.g., $N_v$), while keeping the other input variables (e.g., $h_v$, $d_v$ and $C_d$) constant at their typical values. This step produces three model outputs for each particular variable. Differences between the output from the typical value and maximum and minimum outputs values (the “swing”) are plotted as horizontal bars. The longer the bar (or “swing”) associated with a particular variable, the more sensitive the model is to changes in that variable.

Results indicate that variations in vegetation parameters can change the amount of protection services supplied (Figure 7). More importantly, results demonstrate that the choice of $C_d$ dictates most of the variance in modeled wave height, setup, and shoreline erosion. Changing the value of $C_d$ has the same effect on mean water level as the inclusion of mean current in the mean momentum equation (sections 3.1.2 and 3.2.2). Thus, the high uncertainty

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum Value</th>
<th>Typical Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $N_v$ (#/m²)</td>
<td>240</td>
<td>1200</td>
<td>1500</td>
</tr>
<tr>
<td>Diameter $d_v$ (mm)</td>
<td>0.4</td>
<td>6.0</td>
<td>7.5</td>
</tr>
<tr>
<td>Height $h_v$ (cm)</td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Drag coefficient $C_d$</td>
<td>0.05</td>
<td>0.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>
in the exact value of drag coefficient yield model results that mask the uncertainty in the theoretical framework.

This finding is somewhat troublesome, because it indicates the need for precise values of $C_d$ in the modeling framework. $C_d$ values are difficult to obtain and measure, and are highly variable in both time and space. Indeed, predictive formulas for $C_d$ found in the literature are highly dependent not only on the site where the study is conducted, but also on the wave model used in the study (Appendix D). The sensitivity of the drag coefficient to setting and to the choice of wave model is partially due to the fact that many detailed processes are typically ignored in equation (5) (e.g., mean currents and other nonlinear processes are neglected). In addition, little is known about the specifics of how vegetation modifies wave breaking, which is typically modeled based on tunable empirical formulations developed for beaches without vegetation [Mendez and Losada, 2004; Suzuki et al., 2012]. Finally, the model’s sensitivity to $C_d$ is also due to the fact that the use of LWT yields an approximate estimate of the drag force generated by vegetation—estimates of $F_d$ using LWT differ by a factor of at least two from estimates made using the more accurate stream function theory (Appendix A).

These results suggest that formulations of drag coefficient might only be reliable when used at the site where they were derived. If used at other sites, an uncertainty analysis might be required to capture the variability in protection metric estimates. However, some of the deficiencies of current estimates of $C_d$ are currently being addressed by, e.g., decoupling $C_d$ calibration from measurements using LWT [Zeller et al., 2014]. More observations of this type should help reduce model uncertainty.

4. Conclusion

This paper presents an integrated modeling framework for quantifying how vegetation can reduce the impacts of coastal hazards on coastal communities. Using two illustrative examples (a seagrass meadow offshore of a sandy beach backed by a dune and a mangrove forest on a mud bed), we first estimate changes in wave height during storms due to the presence of vegetation. We show that aquatic vegetation has the potential to lower mean water levels, using a newly derived formulation of the mean momentum equation. Then, relying on adapted formulations of runup and shoreline erosion models, we demonstrate that vegetation can reduce total water level, beach erosion and mud bed scour during storms.

By linking together some of the key wave-induced processes that lead to shoreline retreat, sediment loss, and inundation, the framework allows for the quantification of coastal protection services supplied by coastal vegetation. Outputs from this model are in the units and of the type most relevant to practitioners, policymakers, and stakeholders. If necessary, they can be converted to monetary units as well [Sharp et al., 2014].

In addition to developing one of the first production functions to quantify the coastal protection services supplied by vegetation, this paper also highlights research needs that would allow for more accurate estimates of the protective role of vegetation. Specifically, there is a need for a better understanding, and models, of the modification of wave breaking processes in the presence of vegetation. More importantly, additional field and laboratory observations of wave-induced velocity, mean water level, runup, beach erosion, and mud bed...
scour in the presence of vegetation are critical to validate, improve, or modify the integrated modeling framework proposed herein. Those observations are also necessary to better understand the role of mean currents on wave dissipation, mean water level and bed erosion in the presence of vegetation.

Lastly, this paper demonstrates that the variability of outputs from the model is most strongly dictated by the choice of drag coefficient. This can be interpreted positively since drag coefficient formulas, as currently computed and employed, are essentially calibration coefficients that capture processes that are ignored or approximated [Mendez and Losada, 2004; Pinsky et al., 2013]. Thus, the filling of the theoretical gaps highlighted in this paper are likely to improve our ability to predict and choose a correct drag coefficient, which will allow for a more complete understanding and more accurate estimates of shoreline response in the presence of vegetation.

In conclusion, the integrated modeling framework presented in this paper (and used in the InVEST model [Sharp et al., 2014]) is a critical first step toward the quantification of coastal protection services supplied by vegetation. It shows that vegetation can protect against storms, but that any quantification of that protection will contain significant uncertainty. This uncertainty will likely remain until there is a better understanding of the functional relationship between drag coefficients and wave parameters, as well as an understanding of the way in which vegetation modifies wave-induced nearshore processes. Thus, the framework, in its present form, is most appropriate for initial estimates of the effectiveness of nature-based solutions to coastal hazards and for developing initial alternative strategies for coastal disaster management.

**Appendix A: Comparison of Drag Force Computed Using Linear Wave Theory and Stream Function Theory**

To estimate the error associated with using LWT to compute $F_d$ (equation (1)) and the stress exerted by vegetation stems on the water columns (equation (13)), estimates of the nondimensional wave-averaged drag force $F_{nd}$ computed using LWT are compared to values obtained using stream function theory (SFT) [Dean, 1965]. $F_{nd}$ is expressed as [Dean, 1974]:

$$F_{nd}(z_0) = \frac{\int_{\frac{1}{h}}^{\infty} \frac{F_d}{\mu C_d B_v h H^2 / T^2} \, dz}{\frac{h}{\mu C_d B_v h H^2 / T^2}}$$  \hspace{1cm} (A1)

$F_{nd}$ is computed with SFT following Dean [1974], and with LWT as $F_{nd}^{LWT}(z_0) = F_{nd}^{LWT}(h) \frac{z_0}{h}$ for $z_0 < h$.

We computed $F_{nd}$ at 10 discrete elevations ($z_0 = 0.1h, 0.2h, \ldots, h$) through the water column, using various combinations of relative depths ($h/L_o$) and ratios of wave height over breaking wave height ($H/H_b$),

![Figure A1](image-url)
assuming that \( H_0 = 0.78h \) (we used Cases 3, 4, and 5, A through D in the lookup tables of Dean [1974]). These combinations are reasonable representations of nearshore environments where vegetation exists.

The computed drag force using LWT varies almost linearly with the computed drag force using SFT (\( R^2 > 0.8 \), Figure A1), and the drag force computed from SFT also varies linearly with depth (\( R^2 > 0.9 \) not shown). Thus, the assumption of linearity between submerged and emerged stress in equation (13) appears reasonable.

It also appears that estimates of the drag force using LWT can both over and underestimate the real drag force (Figure A1). For small relative depths (\( h/L_o = 0.01 \)) and small waves (small ratios of \( H/H_0 \)), LWT under-predicts the drag force compared to SFT (Figure A1). As the wave height increases toward its breaking height (\( H/H_b \approx 1 \)), LWT over-predicts the force by more than factor of two. Further, as the relative depth increases, the overprediction of the force by LWT increases. For \( h/L_o = 0.05 \), LWT overestimates the drag force by a factor of 2–4. When waves are in intermediate and deep water (\( h/L_o > 0.05 \)), that factor is over 25 (Case 10, not shown).

**Appendix B: Wave Dissipation and Mean Stress Terms Due to the Presence of Vegetation**

The dissipation terms due to the presence of vegetation in equation (5) are expressed following Suzuki et al. [2012] as:

\[
\begin{align*}
D_{v1} &= C_{G1}N_2d_1(\sinh^3 k_{x_1} + 3\sinh k_{x_1}) \\
D_{v2} &= C_{G2}N_2d_2(\sinh^3 k_{x_1} + 3\sinh k_{x_1} - \sinh^3 k_{x_2} + 3\sinh k_{x_2}) \\
D_{v3} &= C_{G3}N_2d_3(\sinh^3 k_{x_1} + 3\sinh k_{x_1} - \sinh^3 k_{x_2} + 3\sinh k_{x_2}) \\
&- 3\sinh k_{x_2}x_{x_2} - 3\sinh k_{x_1}x_{x_1} \quad (\text{B1})
\end{align*}
\]

where \( x_{i} = a\left[1 - \sum_{k=1}^{i-1}h_{i}k\right]/h - 1 - \left(\sum_{k=1}^{i}h_{i}k\right)/h \) \( \Sigma \), with \( \Sigma \) the Heaviside step function. For mangroves and trees, \( i = 1 \) corresponds to roots, \( i = 2 \) to trunks and \( i = 3 \) to canopies. For seagrasses and marshes, \( i = 1 \) to represent stems and the subscript is omitted.

In the mean momentum equation, the mean current advection factors \( S_{i} \) (section 2.3.1) are obtained by integrating \( \cosh[k(h+z)] \) between the different layers that compose the vegetation field:

\[
\begin{align*}
S_{i1} &= \sinh k_{x_1}h \\
S_{i2} &= \sinh k_{x_2}x_{x_2} + \sinh k_{x_1}h \\
S_{i3} &= \sinh k_{x_2}x_{x_2} + \sinh k_{x_2}x_{x_3} + \sinh k_{x_1}x_{x_2} + \sinh k_{x_1}x_{x_3} \quad (\text{B2})
\end{align*}
\]

where expressions for the coefficients \( S_{i} \) are presented above.

**Appendix C: Runup**

In order to test the assumption of linearity between estimates of \( \eta_{\text{shore}} \), the mean water level at the shoreline computed from equation (17), to \( \eta_{a} \), the empirical estimate of setup computed from equation (19), \( \eta_{\text{shore}} \) is compared with \( \eta_{a} \) for a wide range of equilibrium beach profiles, in the absence of vegetation. The equilibrium beach profiles were generated from sediment sizes (\( d_{50} \)) ranging from 0.1 to 1.09 mm [Dean and Dalrymple, 2002]. For each \( d_{50} \), two types of equilibrium profiles were created. The first type had a foreshore slope starting at 10 cm of water depth, and the second had a foreshore slope that started at the intersection of the equilibrium profile and a planar beach profile [Kriebel and Dean, 1993]. Foreshore slopes \( m \) varied from 1V:100H to 1V:10H and each \( d_{50} \) was related to four \( d_{50} \) values. Three \( m \) values were obtained from Wiegel [1964], who proposed relationships for sheltered, moderately exposed and exposed beaches. The fourth \( m \) value was obtained from McLachlan and Darvall [2005], who reported observations of \( m \) as a function of \( d_{50} \) from beaches around the world. In total, eight different profiles types were generated per \( d_{50} \), for a total of 112 profiles.

To compute the maximum setup at the shoreline for a range of hydrodynamic conditions, we generated a distribution of wave heights between 0.5 and 12 m. For a given wave height value prescribed at the offshore boundary, a suite of associated wave periods was computed by letting offshore wave steepness range
between $2 \times 10^{-3}$ and 0.1 [Holthuijsen and Herbers, 1986], and limiting wave period to values lower than 20 s. This yielded a total of 402 combinations of wave height and period.

Results (Figure C1) indicate that, for a given foreshore slope value, there is a relatively strong linear relationship ($R^2 > 0.7$) between modeled maximum setup $\eta_{\text{shore}}$ and $\eta_{\text{St}}$.

**Appendix D: Influence of Wave Model on Drag Coefficient Estimates**

Here, we examine whether calibrated values of $C_d$ are sensitive to the choice of wave model and formulations of the Reynolds number ($Re$). The drag coefficient $C_d$ is generally expressed as a function of $Re$ or Keulegan-Carpenter ($KC$) numbers (see Anderson et al. [2011] or McIvor et al. [2012] for a review). Table D1

<table>
<thead>
<tr>
<th>Study</th>
<th>Vegetation</th>
<th>Empirical Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kobayashi et al. [1993]</td>
<td>Artificial kelp</td>
<td>$C_d = 0.08 + \left(\frac{500}{Re}\right)^{0.4}$</td>
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<td></td>
<td></td>
<td>$2,000 &lt; Re &lt; 18,000$</td>
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<tr>
<td>Méndez et al. [1999]</td>
<td>Artificial kelp-rigid</td>
<td>$C_d = 0.08 + \left(\frac{400}{Re}\right)^{0.2}$</td>
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<td></td>
<td></td>
<td>$200 &lt; Re &lt; 15,000$</td>
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<tr>
<td>Méndez et al. [1999]</td>
<td>Artificial kelp-swaying</td>
<td>$C_d = 0.40 + \left(\frac{300}{Re}\right)^{0.9}$</td>
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<td>$2,300 &lt; Re &lt; 20,000$</td>
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<tr>
<td>Bradley and Houser [2009]</td>
<td>Seagrass</td>
<td>$C_d = 0.1 + \left(\frac{90}{Re}\right)^{1.16}$</td>
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<tr>
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<td></td>
<td>$200 &lt; Re &lt; 800$</td>
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<tr>
<td>Manca et al. [2012]</td>
<td>Seagrass</td>
<td>No relationship; data points only</td>
</tr>
<tr>
<td>Koftis et al. [2013]</td>
<td>Seagrass</td>
<td>$C_d = \left(\frac{300}{Re}\right)^{0.77}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Re &lt; 2,500$</td>
</tr>
<tr>
<td>Paul and Amos [2011]</td>
<td>Marsh</td>
<td>$C_d = 0.06 + \left(\frac{110}{Re}\right)^{1.45}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$100 &lt; Re &lt; 1,000$</td>
</tr>
<tr>
<td>Jadhav et al. [2013]</td>
<td>Marsh</td>
<td>$C_d = 0.02 + \left(\frac{400}{Re}\right)^{0.78}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$200 &lt; Re &lt; 3,500$</td>
</tr>
<tr>
<td>Augustin et al. [2009]</td>
<td>Cylinders</td>
<td>No relationship; data points only</td>
</tr>
<tr>
<td>Luhar et al. [2013]</td>
<td>Seagrass</td>
<td>$C_d = \left(\frac{80}{Re}\right)^{1.6}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$500 &lt; Re &lt; 2,500$</td>
</tr>
<tr>
<td>Jadhav and Chen [2012]</td>
<td>Marsh</td>
<td>$C_d = 2\left(\frac{150}{Re} + 0.18\right)^{0.78}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$600 &lt; Re &lt; 3,200$</td>
</tr>
<tr>
<td>U.S. Army Corps of Engineers (USACE) [2002]</td>
<td>Cylinders</td>
<td>$C_d = 1.2 - 0.5\left(3.3 \times 10^{-6}Re - 0.667\right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2 \times 10^5 &lt; Re &lt; 8 \times 10^7$</td>
</tr>
</tbody>
</table>
lists several of the formulations of $C_d$ as a function of $Re$ found in the literature; studies that did not use equations (2) and (5) to compute $C_d$ were neglected.

To test whether a general expression of $C_d$ exists, we plotted together formulations of $C_d$ versus $Re$, regardless of the vegetation type (Figure D1). The scatter in the figure makes it clear that a strong universal relationship between these two dimensionless quantities is unlikely. Similar results were obtained when $C_d$ was plotted as a function of $KC$, or when relationships were discriminated by taxa (results not shown). Consequently, many of the formulations for $C_d$ proposed in the literature are specific to the site and hydrodynamic conditions during which measurements were made. Next, we explore the sensitivity of $C_d$ to the choice of wave model.

Pinsky et al. [2013] estimated the drag coefficient of a very large number of plants, under a wide range of hydrodynamic conditions, by fitting equations (2) through (5) to observations. They also used the wave breaking model of Baldock et al. [1998] in equation (2). However, formulations of $C_d$ presented in the literature were obtained using other expressions of breaking dissipation as well as various definitions of the velocity $u$ used to compute $Re$. Indeed, $u$ was defined as either the velocity at the beginning or at the middle of the vegetated field, and was computed either at the top of the submerged stems or near the bed [Kobayashi et al., 1993; Mendez and Losada, 2004; Le Hir et al., 2007; Paul and Amos, 2011; Manca et al., 2012; Jadhav et al., 2013; Maza et al., 2013; Wu et al., 2013].

To explore the sensitivity of the drag coefficient to the choice of a particular wave model and method to compute $Re$, we reanalyzed the data in Pinsky et al. [2013]. In addition to the model of Baldock et al. [1998], we used the popular breaking dissipation models proposed by Ruessink et al. [2003], Battjes and Stive [1985], and by Thornton and Guza [1983] with breaking index values of $\gamma=0.78$ and $\gamma=0.42$. For each model, we set the breaking coefficient $b$ equal to 0.6, 0.8, and 1.4, typical limits found in the literature. Finally, for each value of $C_d$ obtained by fitting observations to the wave model, we computed four values of $Re$ by evaluating $u$ at the different locations described above.

Results are presented in Figure D1, where the gray area represents the range and envelope of the $C_d$ and $Re$ values computed. They indicate that the choice of breaking dissipation model, as well as the way in which $Re$ is computed, significantly influence the value of the drag coefficient.
Notation

$\alpha_{vi}$ ratio of the height $h_{vi}$ of vegetation element $i$ over the water depth, $h$.

$\eta$ free surface elevation, m.

$\bar{\eta}$ mean water level (e.g., wave setup or setdown), m.

$\eta_{shore}$ mean water level at the shoreline computed from the mean momentum equation (17), m.

$\eta_{St}$ mean water level at the shoreline as computed in Stockdon et al. [2006] (see equation (20)), m.

$\sigma$ wave radial frequency, s$^{-1}$.

$\tau_b$ bed shear stress, N/m$^2$.

$\psi$ wave phase ($\phi = kx - \sigma t$).

$d_{vi}$ frontal distance (diameter) of vegetation elements, m.

$h$ still water level, m.

$h_{vi}$ height of vegetation elements, m.

$i$ vegetation element: $i = 1 \ldots 3$, represents roots, trunk, and canopy, respectively. For seagrasses, $i \equiv 1$ and that subscript is omitted.

$k$ wave number, m$^{-1}$.

$m$ beach foreshore slope.

$u$ horizontal velocity of water particles, m/s.

$u_b$ short-wave orbital velocity at the bed, m/s.

$u_{ow}$ magnitude of short-wave (orbital) velocity of water particles, m/s.

$x$ cross-shore position, m.

$z$ vertical position, m.

$B$ beach berm height, m.

$C$ wave celerity, m/s.

$C_{di}$ drag coefficient of vegetation elements.

$C_f$ friction coefficient.

$D$ beach dune height, m.

$D_b$ time averaged energy dissipation rate due to wave breaking, kW/m$^2$.

$D_t$ time averaged energy dissipation rate due to bottom friction, kW/m$^2$.

$D_v$ time averaged energy dissipation rate due to the presence of vegetation, kW/m$^2$.

$E_r$ roller energy, J/m$^2$.

$E_w$ wave energy density, J/m$^2$.

$F_d$ drag force exerted by waves on vegetation stem or element, N/m$^2$.

$F_{Wave}^{\alpha}$ vegetation-induced stress in the water column, N/m$^2$.

$F_{uv}^{\alpha}$ vegetation-induced stress in the water column generated by short-waves only, N/m$^2$.

$F_{v}^{\alpha}$ vegetation-induced stress in the water column generated by mean current in the vegetated field, N/m$^2$.

$H$ root-mean square wave height, m.

$H_o$ deep water significant wave height, m.

$N_{vi}$ density of vegetation field for vertical vegetation elements, units/m$^2$.

$R_{run}$ wave runup, m.

$R_e$ Reynolds number.

$S_{SW}$ radiation stress generated by waves and rollers, J/m$^2$.

$T_p$ wave group peak period, s.

$U$ time average (mean) velocity of water particles, m/s.

$U_b$ near bed mean velocity, m/s.

$U_{off}$ mean velocity inside a vegetated field, m/s.

$U_{off}$ depth averaged mean current (undertow) in the water column, m/s.

$W$ beach berm width, m.

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