In order to analyze the accuracy of the navigation system LORAN-C, we have collected time-difference (TD) data from the U.S. west coast chain (9940) at a fixed receiver location (Corvallis, Oregon). The analysis of the TD data shows an unexpected high correlation between the TDs as well as a repetitive error pattern with a period of 119.1 seconds which is traced to cross-rate interference (CRI) from the Canadian west-coast chain (5990). For each TD, we extract the periodic error pattern, and then subtract it from the corresponding TD. For a receiver in Corvallis, the observed errors due to CRI ranged over 40 nanoseconds (TDW), 60 nanoseconds (TDX), and 120 nanoseconds (TDY). The constant and time-varying TD errors and the resulting constant and time-varying position errors for the location Corvallis are computed both before and after subtraction of the CRI component. As a result, we find that in addition to CRI another non-white TD error (yet unknown) must be considered.
LORAN-C Receiver Position Estimation
From Noisy Time-Difference Data

by

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A THESIS
submitted to
Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Completed June 12, 1986
Commencement June 1987
ACKNOWLEDGEMENT

I wish to express my thanks to my major professor and thesis advisor Dr. Rudolph S. Engelbrecht for his help and encouragement throughout my studies at O.S.U.
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LIST OF SYMBOLS

B[.] Bias
C Receiver in Corvallis
CD Coding delay (time-difference in microseconds of S receiving master pulse and transmitting its own pulse)
COV[i,j] Covariance between i and j
CRI Cross-rate interference
GRI Group repetition interval
IAT Latitude
LON Longitude
M Master station of a chain
R Receiver
R[i,j] Correlation coefficient between i and j
S Secondary station
S[.] Standard deviation
SF Secondary phase correction
T Propagation time
TDi Time-difference between master pulse and pulse of secondary i at the receiver
V[.] Variance
W Secondary station W
X Secondary station X
Y Secondary station Y
Z Secondary station Z
I. INTRODUCTION

I.1 LORAN principles

LORAN-C (Long Range Navigation) is a passive electromagnetic navigation system operating at 100 KHz [1], [2], [3]. A LORAN-C configuration for a particular region is called a chain. Each chain consists of at least three fixed transmitters which emit pulses at regular time intervals. One of these is the "master" station (M) and the others are "secondary" stations (W, X, Y, or Z). A receiver determines its position from the time-differences of the corresponding received pulses.

If we assume the earth's surface to be flat within the region of the chain, the so called "lines of position" (LOP) for which the time-differences between received pulses from any two transmitters are constant form a family of hyperbolas with the transmitters as foci. Thus, with three (non-collinear) transmitters we obtain two LOPs intersecting at the receiver's location. To determine one's position, the measured time differences are converted into distance differences and ultimately into latitude/longitude coordinates.

The master station transmits a pulse at relative time t=0. A secondary station receives this pulse at time t=\(\beta\), and after a coding delay (CD) transmits its own pulse at time t=\(\beta+CD\). The coding delay is chosen so that the pulse from the secondary station is always received after the master pulse, regardless of the receiver position.

A receiver at point R (Fig. 1.1) measures the time-difference (TD) between the arrival of pulses from the master (M) and a secondary (say, X):

\[
T_{DX} = (\beta_X + CD_X) + T_{XR} - T_{MR} \quad \mu s
\]  

(1.1)
This time difference has upper and lower bounds:

\[ 2\beta_x + \text{CDX} \geq \text{TDX} \geq \text{CDX} \quad (1.2) \]

The extreme values occur for a receiver which is located on the "baseline" extension.

In order to calculate the receiver's position we also need to know the time-difference between the master (M) and another secondary station (say, Y). These two secondaries (X and Y) together with the master (M) form a "triad". The selection of triad MXY, for example, gives us the time-differences TDX and TDY (Fig. 1.2).

The conversion of TDX, TDY to receiver position is not unique, but leads to two solutions. However, one of these is, in general, outside the primary region served by the triad and can therefore be discarded.
Figure 1.2 LORAN-C chain with hyperbolic lines of position
I.2 Propagation principles and phase corrections

The measurements of time-differences (e.g., TDX and TDY) as outlined above involve four major potential sources of error:

a) The ground-wave pulse (Fig. 1.3) which is used to determine the time-differences is contaminated by sky wave pulses. However, the sky wave arrives at the receiver at least 30 μsec after the ground-wave. To avoid sky wave interference, the third cycle crossing (30 μsec at 100 KHz) of the received ground-wave pulse is used for determining the arrival time [1].

![Transmitted LORAN-C pulse](image)

Figure 1.3 Transmitted LORAN-C pulse

b) The flat-earth assumption is not valid within the entire region covered by a chain. Even the assumption of a spherical earth can lead to inaccuracies of up to 1000 meters [4]. A spheroidal earth model developed by Razin [5] gives about 1 meter accuracy.

c) The ground-wave velocity decreases with decreasing surface conductivity (by as much as 0.1% compared to infinite conductivity). Razin includes a nonlinear correction factor SF = f(t) (t is propagation time) for sea water which is called "secondary phase
correction" (SF), and which must be added to the propagation time of a wave in standard atmosphere. This correction, however, can only be determined by iteration after the receiver position is estimated since $T_{xa}$ and $T_{ma}$ must be known.

In order to eliminate this iteration, Newman linearizes the SF [4], [6]. The SF can then be directly calculated from the known TDs (after subtracting CDs) to within about $0.1 \mu$sec. A flow chart of Newman's algorithm and additional details are given in appendix A.

For over-land paths, an additional phase correction (ASF) can be added to SF to account for the lower ground conductivity of land compared to sea water.

d) The ground-wave path is increased by major topographical features such as mountains. Also, all calculations are usually made by assuming that the transmitters and the receiver are located at sea-level. So far, little work has been done in this area [7].
I.3 Temporal and spatial spectrum of TD errors

Since TD errors are converted directly into position errors, it is important that they are minimized.

Temporal TD errors at a fixed receiver location consist of:

* a constant term (or "bias") due to an inaccurate model of path topography, ground conductivity, etc. [7]

* a seasonal term due to seasonal weather changes (which also cause some change in ground conductivity) [8], [9]

* a diurnal term due to an increase in white noise, sky wave contamination, fading, etc. during night time [10]

* a periodic term with a period of about one to two minutes due to cross-rate interference (CRI) from another LORAN-C chain. This term is stronger at night time due to an increase in fading [11].

* a random noise term

The spatial TD errors between two receiver locations at a fixed time are due to:

* different path topographies and ground conductivities [7]

* different weather conditions [8], [9]

* different cross-rate interference

* different random noise
1.4 Transformation of TD errors to position errors

Small TD errors can lead to large position errors. In appendix B, the position errors are derived for:

a) Constant TD errors (bias)
Here, the latitude/longitude errors are determined by the partial derivatives $g_i = \partial \text{LAT}/\partial \text{TD}^k$ and $h_i = \partial \text{LON}/\partial \text{TD}^k$:

$$
\Delta \text{LAT} = g_i \Delta \text{TD}^i + g_j \Delta \text{TD}^j
$$
$$
\Delta \text{LON} = h_i \Delta \text{TD}^i + h_j \Delta \text{TD}^j
$$

b) Gaussian distributed $\text{TD}^i$ and $\text{TD}^j$ errors with zero means, variances $V[\text{TD}^i]$ and $V[\text{TD}^j]$, and correlation coefficient $R[i,j]$. Here, the latitude/longitude errors are described by average (bias) values $B[.]$, variances $V[.]$, and covariance $\text{COV}[.]$ in terms of the first and second partial derivatives ($g_i = \partial \text{LAT}/\partial \text{TD}^2$, $h_{ij} = \partial \text{LON}/(\partial \text{TD}^i \partial \text{TD}^j)$, etc.):

$$
B[\text{LAT}] = \frac{1}{2} V[\text{TD}^i]*g_i^2 + \frac{1}{2} V[\text{TD}^j]*g_j^2
+ R[i,j]*S[\text{TD}^i]*S[\text{TD}^j]*g_i*g_j
$$

$$
B[\text{LON}] = \frac{1}{2} V[\text{TD}^i]*h_i^2 + \frac{1}{2} V[\text{TD}^j]*h_j^2
+ R[i,j]*S[\text{TD}^i]*S[\text{TD}^j]*h_i*h_j
$$

$$
V[\text{LAT}] = V[\text{TD}^i]*g_i^2 + V[\text{TD}^j]*g_j^2
+ 2*R[i,j]*S[\text{TD}^i]*S[\text{TD}^j]*g_i*g_j
$$

$$
V[\text{LAT}] = V[\text{TD}^i]*h_i^2 + V[\text{TD}^j]*h_j^2
+ 2*R[i,j]*S[\text{TD}^i]*S[\text{TD}^j]*h_i*h_j
$$

$$
\text{COV}[\text{LAT,LON}] = V[\text{TD}^i]*g_i*h_i + V[\text{TD}^j]*g_j*h_j
+ R[i,j]*S[\text{TD}^i]*S[\text{TD}^j]*(g_i*h_j + g_j*h_i)
$$
II. TD ERROR MEASUREMENTS AND COMPUTATIONS

II.1 Data collecting and processing system

The following gives a brief explanation of the data collection and processing system (DCPS) shown in Fig. 2.1.

* The LORAN-C receiver (Apollo II Model 611, [12]) has a modified built-in interface which serially outputs measured and calculated parameters such as: TDs of the selected chain, signal-to-noise ratios (SNR) of the master signal and of the two selected secondaries, receiver latitude/longitude, etc. Here, only the TDs and SNRs are being used.

* The IBM PC selects and stores the TDs and SNRs from the LORAN-C receiver output. One set of TDs, etc. is called one sample. For the US-chain 9940, the sample rate is about 1.1 samples/sec. The IBM PC is also used to compute statistical averages such as means, standard deviations, etc. and to create graphs.

* The HP 1000 computer system is used to post-process the samples collected with the IBM PC. Data files are transferred between the two computers via "Kermit" (a terminal emulator and file transfer program).
II.2 Statistical averages over 1000 samples

The measured TDs were found to have nearly Gaussian distributions (Fig. 2.2, appendix C).

Figure 2.2 TDX-distribution over a 1,000-sample set

In order to estimate the TD distributions, means, variances, and correlations (which are all functions of time) we must perform averaging (i.e., "smoothing") over a set of successive samples. The number of samples used in each set is a compromise between low estimator variance (i.e., many samples) and high temporal resolution (i.e., few samples). For the TD estimates, 1000 samples per set (approx. 15 minutes) were found to be nearly optimum. Then, the standard deviation of the mean estimate is about 3.2% of the actual standard deviation and the standard deviation of the variance estimate is about 4.5% of the actual variance (assuming statistically independent Gaussian distributed samples).
II.2.1 Statistical averages for US West-Coast chain 9940

The 1000-sample estimates were taken over a period of 11 days. Appendix D contains all graphs of means, standard deviations, and correlation coefficients for TDW, TDX, and TDY as well as for the SNRs of the M-, W-, and X-signals. This section only contains a representative selection of the graphs.

A qualitative evaluation of the graphs shows that:

* The TD standard deviation is smallest for TDW and largest for TDY, and increases during the night (diurnal variations) by more than 50% (Fig. 2.3). The same diurnal variations have been reported in [10].

![Figure 2.3 TD standard deviations for 1000-sample sets](image)

* No systematic diurnal TD mean variations can be seen (Fig. 2.4).
Figure 2.4  TDX means for 1000-sample sets

* All three TD correlation coefficients are positive and show small diurnal variations. They increase in the order $R[X,Y]$, $R[W,Y]$, and $R[W,X]$ (Fig. 2.5).

Figure 2.5  TD correlation coefficients for 1000-sample sets
(a) $R[W,X]$  (b) $R[W,Y]$  (c) $R[X,Y]$
Figure 2.5 continued
* The SNR means and standard deviations also contain diurnal variations which are negatively correlated with the TD standard deviations with the exception of the W-signal (this might be due to the TDW path-topography) (appendix D).

* The SNR correlations for M, X, and Y seem to fluctuate randomly about zero (appendix D).

Our quantitative evaluation consists of three parts:

(a) the ratio of the three TD standard deviations

(b) the ratio of the three TD correlation coefficients

(c) the TD bias

We first develop a 1st order model for the ground-wave propagation:

The ground-wave from a transmitter to the receiver is influenced mainly by particles, clouds, fog, etc. ("random" scatterers) within the first Fresnel zone. For the wave length \( \lambda = 3 \text{km} \), distance \( L > 300 \text{km} \) between transmitter and receiver, and scatterer correlation distances \( < V/\lambda \text{km} = 30 \text{km} \), it can be shown [13] that the propagation time \( T \) is Gaussian distributed, with variance proportional to the mean \( E[T] \):

\[
V[T] = c \ E[T] \quad (c = \text{const.}) 
\]

(2.1)

The variance of the time-difference \( TD = T_1 - T_2 \) then in general is:

\[
V[TD] = V[T_1 - T_2] = V[T_1] + V[T_2] - 2 \ \text{COV}[T_1,T_2] 
\]

(2.2)

Table 2.1 shows the propagation times between the master, the three secondaries (W, X, and Y), and the receiver (Corvallis, lat/lon = 44.5675°/123.2745°). (The derivation is given in appendix E.)
In order to determine the TD variances we also need to know the path geometry (Fig. 2.6) and the correlations between different paths.

We can see that the paths for TDW and TDX (except the common master-receiver path) are nearly uncorrelated. Let the mean of the propagation time $T$ between M and C be denoted by $T_{MC}$, etc. The variance of TDW then yields:

$$V[TDW] = c(T_{MW} + T_{WC} - T_{MC}) = cV[T_{MW}] + cV[T_{WC}] + cV[T_{MC}] = c 6327 \quad (2.3)$$

$$V[TDX] = c(T_{MX} + T_{XC} - T_{MC}) = cV[T_{MX}] + cV[T_{XC}] + cV[T_{MC}] = c 5478 \quad (2.4)$$

For TDY, we see that $T_{YC} \approx T_{MC} + T_{MV}$. Hence, the M-Y and M-C paths can be assumed to be highly correlated with the corresponding segments of the Y-C path. Then, from (2.2):

<table>
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<th>M-W</th>
<th>M-X</th>
<th>M-Y</th>
<th>W-C</th>
<th>X-C</th>
<th>Y-C</th>
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<tr>
<td>2227.62</td>
<td>2796.91</td>
<td>1094.53</td>
<td>1967.32</td>
<td>1302.19</td>
<td>2155.87</td>
<td>4188.49</td>
</tr>
</tbody>
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Table 2.1 Propagation times (in $\mu$sec)
\[ V[TDY] = V[T_{MY} + T_{YC}] + V[T_{MC}] - 2COV[T_{MY}, T_{YC}] \]
\[ = V[T_{MY} + T_{YC}] + V[T_{MC}] - 2V[T_{MC}] \]  
(2.5a)

\[ V[T_{MY} + T_{YC}] = V[T_{MY}] + V[T_{YC}] + 2COV[T_{MY}, T_{YC}] \]
\[ = V[T_{MY}] + V[T_{YC}] + 2V[T_{MY}] \]  
(2.5b)

\[ V[T_{YC}] = V[T_{MY}] + V[T_{MC}] \]  
(2.5c)

The TDY variance then finally becomes:

\[ V[TDY] = 4 V[T_{MY}] = 4 \times 7869 \]  
(2.6)

Comparison of the measured TD variances with this 1st order model shows that the model is not sufficient to describe the TD variances since their ratios do not agree.

The theoretical correlation coefficient between the time-differences TDi and TDj is given by:

\[ R[i,j] = \frac{COV[TDi, TDj]}{S[TDi] S[TDj]} \]  
(2.7)

According to our model,

\[ COV[TDW, TDX] = V[T_{MC}] \]

and

\[ COV[TDW, TDY] = COV[TDX, TDY] = 0 \]

Thus,

\[ R[W,X] = \frac{c \frac{T_{MC}}{V[TDW] V[TDX]}}{V[TDW] V[TDX]} \approx 0.38 \]  
(2.8)

\[ R[W,Y] = R[X,Y] = 0 \]  
(2.9)

Again, the theoretical correlation coefficients differ from the measured values.
Even if we assume the variance of $T$ to be $V[T] = E^*[T]$ with $0.5 < x < 3$, the correlation coefficient will always yield $R[W,X] < 0.4$. Furthermore, the correlation during a 15 minute interval (1000-sample group) can only be due to the common master-receiver path, since all known propagation effects which extend over large regions (weather systems, ionosphere, etc.) fluctuate much more slowly.

In conclusion, the differences between the propagation model and measurements indicate that we have to consider an additional noise component. In sections II.3 and II.4 we shall see that a systematic component and random noise also must be taken into consideration.

From the propagation times and coding delays shown in Fig. 2.6, we obtain the theoretical TDs for US-chain 9940 transmitters and a receiver at lat/lon=44.5675°/123.2745° (O.S.U., Dearborn Hall):

\[
\begin{align*}
    TD_W &= 12871.48 \mu \text{sec} \\
    TD_X &= 28022.78 \mu \text{sec} \\
    TD_Y &= 43928.19 \mu \text{sec}
\end{align*}
\] 

(2.10a)  
(2.10b)  
(2.10c)

The measured (actual) and theoretical TDs show a difference of about -1.6 to -1.8 \(\mu\text{sec}\) for $TD_W$, 0 to -.10 \(\mu\text{sec}\) for $TD_X$, and 1.7 to 2.0 \(\mu\text{sec}\) for $TD_Y$. It is interesting to note that for $TD_W$ and $TD_X$ the measured (actual) TDs are smaller than the theoretical values. This could be due to (a) an additional propagation delay (ASF) in the master signal, or (b) the coding delay adjustments of secondaries $W$ and $X$ which are under the control of the 9940 monitor (at North Bend, OR).
II.2.2 Statistical averages for Canadian West-Coast chain 5990

The 1000-sample averages were taken over a period of about 60 hours. The graphs for estimates of mean and standard deviation for TDY and TDZ and of the correlation coefficient $R[Y,Z]$ are given in appendix F.

Here, the very high value of $R[Y,Z]$ ($\sim 0.8$) is of special interest. The high correlation cannot be explained by the path geometry, and therefore also suggests an additional systematic noise component.
II.3 Post-processing of TD-samples

In section II.2 we suggested that in addition to the path geometry a systematic noise component be included into our 1st order model which accounts for the high TD correlations.

In this section we compute and then extract a systematic noise component by post-processing raw data. Here, raw data refers to a set of some 13,500 successive TD-samples (approx. 3.4 hours). Altogether, three sets of raw data (data set no. 1, 2, and 3) are analyzed. Table 2.2 shows the date and time at which the collection of raw data was started.

<table>
<thead>
<tr>
<th>Data set no.</th>
<th>Date</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>no. 1</td>
<td>5/05/86</td>
<td>0:00 AM</td>
</tr>
<tr>
<td>no. 2</td>
<td>5/07/86</td>
<td>6:00 AM</td>
</tr>
<tr>
<td>no. 3</td>
<td>5/08/86</td>
<td>11:00 AM</td>
</tr>
</tbody>
</table>

Table 2.2 Date and starting time of data collection

In the remaining part of this chapter, all statistical evaluations and graphs are represented for TDX of data set number 2. The complete set of graphs for all three data sets can be found in the appendix.
II.3.1 TD-autocovariance of raw data

Fig. 2.7 shows the first 1000 successive TDX-samples of data set no. 2. The TDX-samples fluctuate about the mean value, mainly in the range of ± 60 nsec. However, it is not possible to trace a systematic noise component from this graph.

Figure 2.7 1000 successive TDX-samples of data set no. 2

The unbiased autocovariance estimate of TDX (data set no. 2) which is computed for 13,500 successive samples and up to 450 samples lag is shown in Fig. 2.8.
If the fluctuations were only due to white noise the autocovariance of TDX would show a high value about lag zero and values close to zero for all other lags. The graph in Fig. 2.8, however, shows that the TDX fluctuations are not only due to white noise (although it is the strongest noise component) since we can see significant correlation for lags of 133, 266, and 399 samples. Furthermore, the autocovariances of all TDs of the three data sets also show significant correlations for the same lags (appendix G). This result indicates that TDX and all other TDs contain a periodic noise component with period $T \approx 133$ samples.

In the following section we shall see that the periodic noise component is due to cross-rate interference (CRI) from the Canadian chain 5990.
In section II.3.1 we have seen that TDX of data set number 2 (and all remaining TDs) contains, besides white noise, a periodic noise component. In order to prove that this periodic noise component is due to CRI, we first present the timing of LORAN-C chains in detail:

The master and the secondaries of a chain emit LORAN-C pulses (Fig. 1.3) in a predefined LORAN-C pulse pattern. This pattern is partly a function of receiver position and therefore is presented in a general form (Fig. 2.9).

A receiver first receives a pulse train of 9 pulses from the master followed by the pulse trains of 8 pulses from each secondary (X, Y, and Z). The spacing between two pulses within a pulse group is 1000 μsec (except for the 9th master pulse). The duration of the pulse pattern is called Group Repetition Interval (GRI) and is fixed for each chain. In order to distinguish chains and to minimize cross-rate interference (mutual interference) each chain utilizes a different GRI. Furthermore, the individual LORAN-C pulses within a pulse train are phase-coded over 2 GRIIs so that successive ("even" and "odd") pulse patterns are distinct. The phase-coding for master pulse trains is different from secondaries but not distinct for different chains.

For US-chain 9940 and Can-chain 5990 the GRIIs are 99400 μsec and 59900 μsec, respectively (note that the GRI is given by the chain number). At relative time t=0 both masters of the two chains transmit a pulse train with the same phase-coding (say, both "odd"). These mark the
beginning of two pulse patterns (one for each chain) which repeat after
99400 µsec and 59900 µsec, respectively. After \( t = T = 99400 \times 599 \mu\text{sec} \) (smallest common multiple) the two master pulse trains again coincide in
time, but with different ("even", "odd") phase-coding. Finally, after
\( t = 2T = 119.0812 \text{ sec} \), the two master pulse trains coincide with the
same pulse-coding (both "odd"). We denote \( T_{\text{cri}} = 2T = 119.0812 \text{ sec} \) as
CRI-period. The same CRI-period applies to all secondaries of both
chains.

The two master pulse trains transmitted at relative time \( t=0 \) arrive
at O.S.U. after the propagation delays of \( T_{\text{MC}} = 2227.62 \mu\text{sec} \) for US-chain
9940 and \( T_{\text{MC}} = 2755.76 \mu\text{sec} \) for Can-chain 5990. Hence, the first pulse
pattern for Can-chain 5990 is delayed by approx. 528 µsec with respect
to the first US-chain 9940 pulse pattern. Each pulse pattern is then
repeated with the chain-GRI. After the CRI-period \( T_{\text{cri}} = 119.0812 \text{ sec} \),
the sequence of all received pulses repeats with the same relative tim-
ing and coding.

During one CRI-period, pulse trains from the two chains may collide
at the receiver, depending on its location. However, only colliding
master pulse trains or secondary pulse trains of the same coding (both
"even" or both "odd") can cause a TD error. Collisions of mixed pulse
trains (master and secondary or "even" and "odd"), can be detected and
hence eliminated.

Whether or not two colliding pulse trains of the same code cause a
TD error in the receiver depends on the time-difference between the two
pulse trains and on the error-detection circuit of the receiver. The
error modeling becomes complex since pulse trains of both the ground-
wave and sky waves from the interfering chain can collide with pulse
trains of the desired chain's ground-wave. Furthermore, fading cannot
be predicted satisfactorily. The receiver causes additional complexity
in error modeling since, for example, each sample is already an average
over a number of GRI-periods, and internal processing is nonlinear
(e.g., hard-limiting of the time interval for received LORAN-C pulses).
Since the collision of pulse trains (i.e., CRI) repeats periodically after $T_{\text{CRI}} = 119.0812$ sec, the TD error due to CRI also repeats periodically with period $T_{\text{CRI}}$. If we express $T_{\text{CRI}}$ in units of samples rather than seconds, the CRI-period becomes $T_{\text{CRI}} = 132.9$ samples (assuming a constant sample rate of 1.116 samples/sec) which is in accordance with our periodic noise component with period $T \approx 133$ samples.

For a simple TD error model, we assume that only pulse trains which arrive between 250 $\mu$sec earlier and 30 $\mu$sec later than the desired pulse train (i.e., window = $[250 \mu\text{sec}, -30 \mu\text{sec}]$) can cause a TD error. Also, we only account for the error direction (i.e., positive or negative) since any amplitude calculations would be highly speculative (see above).

An interfering pulse can cause a positive or negative TD error, depending on whether it (a) is a master or secondary pulse, and (b) has positive or negative phase ($\pm \Upsilon$ radians) relative to the desired pulse (Fig 2.10).

![Figure 2.10 TD error direction depending on phase](image)

Applying these conditions to the US-chain 9940 (desired) and Can-chain 5990 (interfering), we obtain the following results for TDx errors (Table 2.3). Here, the entries are the sample positions in each CRI-period starting at $t=0$. (Note that the entries in parenthesis have a very small phase shift (<100nsec) and therefore may have the opposite sign, depending on the exact TDs used in the model.)
Table 2.3 Modeled TDX error due to CRI from Can-chain 5990.  
(Entries are sample positions in each CRI-period)

<table>
<thead>
<tr>
<th>pos.</th>
<th>Can master</th>
<th>Can secondaries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 41 92</td>
<td>(27) 44 (58) 95 (109) 126</td>
</tr>
<tr>
<td>neg.</td>
<td>------</td>
<td>0 31 82</td>
</tr>
</tbody>
</table>

Since the TD error due to CRI is periodic, we can compute this error by averaging all samples of our raw data which are spaced one CRI-period and thus reduce the effect of random noise. Altogether, we average over 100 CRI-periods. The limit of 100 periods is forced by the available memory space (RAM). However, this averaging method involves two problems:

(a) We must choose a period of an integral number of samples (i.e., 133 samples). When averaging over 100 CRI-periods, with 132.9 samples each, the last samples (in the 100th period) would be up to 10 samples off the desired position.

(b) The sample rate varies between 132 and 134 samples/CRI-period. If it were constant, we could minimize problem (a) by just shifting every 10th period one sample towards the first period.

We can minimize both problems by recording time in the IBM PC while collecting samples and establishing the beginning of a new CRI-period after 119.0812 sec have elapsed. In each CRI-period, we use the first 132 samples, thus discarding sample number 133 and 134 when they occur. The reduced (compressed) data set \( c[n] \) consists of 13,200 samples. The TD error \( e[i] \) then becomes:

\[
e[i] = \frac{1}{100} \sum_{j=0}^{99} c[i + j \times 132] \quad (i=1, \ldots, 132) \tag{2.11}
\]

Figure 2.11 shows the TDX error due to CRI for all three data sets (appendix H contains graphs of TD errors due to CRI for all TDs). The observed TD errors range over approximately 30 nsec for TDW, 60 nsec for
TDX, and 140 nsec for TDY and increase during the night. The fact that the TD errors for all three data sets show nearly the same pattern indicates that TD errors due to CRI are highly repetitive and thus systematic. With the raw data variance of approximately $2 \times 10^{-3} \, \mu\text{sec}^2$ for data set number 1 and $10^{-3} \, \mu\text{sec}^2$ for data set number 2 and 3 the standard deviation for the TD error estimates due to CRI (assuming statistically independent Gaussian distributed samples) becomes:

$$S[\hat{e}[i]] = \sqrt{2 \times 10^{-3}/100} \, \mu\text{sec} = 4.5 \, \text{nsec} \quad (\text{no.1})$$

$$S[\hat{e}[i]] = \sqrt{10^{-3}/100} \, \mu\text{sec} = 3.2 \, \text{nsec} \quad (\text{no.2 & 3})$$

This is significantly less than the TD error range of 60 nsec.

Also included in Fig. 2.11 are arrows that show the calculated positions of the interfering pulses from Table 2.3. (Note that in Figure 2.11, 2.12, and all corresponding figures in the appendix, $i=128$ corresponds to $t=0$ as defined for Table 2.3.) An arrow in the lower half represents a negative TDX error, an arrow in the upper half a positive TDX error. For sky-wave interference, we cannot determine the error direction.
(relative phase), but only the possible positions of interfering pulses. Therefore, the dashed lines represent additional possible positions of interfering sky-wave pulses if we increase the window size to [500 μsec, -30 μsec]. The positions of interfering master pulses are also shown in the TDW and TDY graphs (appendix H).

The standard deviation of all 100 samples at the same sample position \( i \) in the CRI-period from the CRI-error component be calculated, using the notation \( c_j[i] = c[i+j*132] \), by:

\[
S[e[i]] = \left( \frac{1}{100} \sum_{j=0}^{99} (c_j[i])^2 - \left( \frac{1}{100} \sum_{j=0}^{99} c_j[i] \right)^2 \right)^{0.5}
\]

(2.13)

The graphs for TDX are shown in Fig. 2.12 (also see appendix I).

![Figure 2.12 Standard deviation of each sample \( i \) of TDX error due to CRI](image)

The following can be observed:

* The standard deviations are not constant over the 132 samples. This indicates that, besides white noise, fading and maybe an additional non-white noise component must be considered.
* The standard deviation for data set number 1 (night data!) is approx. 50% higher than the standard deviations for the two daytime data sets. This could be due to (a) an increase in white noise during the night, or (b) an increase in sky wave interference (and thus fading) during the night. The 50% increase, however, is in accordance with the observed diurnal variations (II.2.1).

The autocovariance of the TDX error is shown in Fig. 2.13. It can be observed that not only the peak-values at lag 133, 266, and 399 but also some of the details between the peaks match fairly well with the autocovariance for the raw data (Fig. 2.8) (appendix J).

Figure 2.13  TDX-autocovariance of CRI-data (set no.2)
II.3.3 Residual TD error

To improve the accuracy of our TD measurements we subtract the TD error due to CRI from the compressed raw data. Here, we assume that the TD-drift within one period is negligible. After centering each CRI-period of our raw data and the TD error by subtracting out the means (2.14), (2.15) we can subtract corresponding samples (2.16). Fig. 2.14 shows the principal idea:

![Diagram of jth period of compressed raw data and TD error due to CRI](image)

Figure 2.14: jth period of compressed raw data and TD error due to CRI

\[ \bar{c}_j = \frac{1}{132} \sum_{i=1}^{132} c_j[i] \]  
(2.14)

\[ \bar{e} = \frac{1}{132} \sum_{i=1}^{132} e[i] \]  
(2.15)

\[ n[i+j*132] = n_j[i] = (c_j[i] - \bar{c}_j) - (e[i] - \bar{e}) \]  
(2.16)

(i=1,...,132  j=0,...,99)

We call the remaining data after subtracting the TD error "residual" data. According to our model, the residual data \( n_j[i] \) should consist only of white noise. However, we have seen that fading may be an additional error source.

The TDX-autocovariance of the residual data (data set no.2) is shown in Fig. 2.15. Here, no recognizable periodic noise component with period \( T = 133 \) samples remains. The same can be observed for the majority of the graphs of the TD-autocovariances of residual data which are
In order to verify the assumption that the TD-drift within one period is negligible we computed the TD mean of each CRI-period from the compressed raw data (Fig. 2.16 and appendix L).

It can be observed that the means of successive CRI-periods differ by up 30 nsec. The fluctuations cannot only be due to white noise, for averaging over 132 samples (assuming statistically independent samples) and
raw data variance (set no. 2) of $0.9 \times 10^3 \mu\text{sec}^2$ the standard deviation for each mean estimate is:

$$S_j = \sqrt{0.9 \times 10^3 / 132} \mu\text{sec} = 2.6 \text{ nsec}$$  \hspace{1cm} (2.17)

If fading were the major cause of the observed TD-mean fluctuations our TD error due to CRI would not have a highly repetitive nature. Thus, we conclude once more that the TD errors are not only due to white noise and CRI but also to an additional non-white noise component which is as yet unidentified.

II.3.4 Statistical TD-error evaluation

In Table 2.3, all important statistical estimates for raw data, CRI-data, and residual data are listed:

<table>
<thead>
<tr>
<th>data set no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>variance of</td>
<td>W X Y</td>
<td>W X Y</td>
<td>W X Y</td>
</tr>
<tr>
<td>raw data ($*10^3 \mu\text{sec}^2$)</td>
<td>.90 2.1 20.</td>
<td>.39 .91 7.2</td>
<td>.38 1.0 6.4</td>
</tr>
<tr>
<td>CRI-data ($*10^3 \mu\text{sec}^2$)</td>
<td>.11 .30 1.0</td>
<td>.04 .14 .90</td>
<td>.03 .14 .55</td>
</tr>
<tr>
<td>residual data ($*10^3 \mu\text{sec}^2$)</td>
<td>.67 1.6 17.</td>
<td>.31 .72 5.6</td>
<td>.33 .80 5.2</td>
</tr>
<tr>
<td>correlation coefficient for R_{wx} R_{wy} R_{xy} R_{wx} R_{wy} R_{xy} R_{wx} R_{wy} R_{xy}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>raw data</td>
<td>.51 .15 .16</td>
<td>.56 .20 .24</td>
<td>.56 .23 .23</td>
</tr>
<tr>
<td>CRI-data</td>
<td>.46 .20 .14</td>
<td>.34 .26 .33</td>
<td>.36 .29 .21</td>
</tr>
<tr>
<td>residual data</td>
<td>.51 .15 .15</td>
<td>.60 .21 .24</td>
<td>.62 .23 .25</td>
</tr>
</tbody>
</table>

Table 2.3 Statistical estimates for raw data, CRI-data and residual data

It can be seen that the variance of the CRI-data is between 8% and 15% of the raw data variance. This is also observed from the TD-autocovariances. With the assumption that white noise and TD error due
to CRI are statistically independent and that these are the only noise components in the TDs, the variances of residual data and CRI-data should add up to the raw data variance \( V_{\text{RES}} + V_{\text{CRI}} = V_{\text{RAW}} \). However, the difference is always more than the variance of the CRI-data. We can count this remaining variance as variance \( V_{\text{ADD}} \) of an additional noise component which includes fading, drift, additional systematic noise, etc. (2.18).

\[
V_{\text{RAW}} = V_{\text{RES}} + V_{\text{CRI}} + V_{\text{ADD}}
\]  

(2.18)

The high TD-correlation led us to the TD error due to CRI. If our model for TD errors were complete the TD-correlation coefficients for the residual data should be close to the modeled values in eq. (2.8) and (2.9). However, this is not the case. Rather, the correlation coefficients are too high, which could be due to:

* introducing a systematic error when subtracting the computed TD error due to CRI from the compressed raw data.

* an additional non-white noise component which is also responsible for the high TD-correlations of the raw data.

II.3.5 Spectral TD error evaluation

For each TD, the power spectrum is computed by windowing the TD-autocovariance over two CRI-periods from sample number 66 to sample number 329 (rectangular window over 263 samples) and then computing the Discrete Fourier Transform (DFT). We thus reduce the white noise component and can better estimate periodic components. The period of \( T_{\text{CRI}} = 132 \) samples is chosen so that we can compare the power spectra of raw data, CRI-data, and residual data with each other.

Fig. 2.17 shows the TDX power spectrum of raw data set number 2.
Figure 2.17 TDX power spectrum of raw data (set no.2)

We can see that, besides the frequency $1/\text{CRI}$-period, the power spectrum contains strong harmonics and other frequency components which can also be observed in the autocovariance (Fig. 2.8). The frequency of the strongest component corresponds to a period of $132/5 \approx 26$ samples. The small band with very high frequencies ($62/\text{CRI}$-period) could be due to the receiver (appendix M).

The TDX power spectrum of the CRI-data (Fig. 2.18) shows an increase in the strongest frequency component ($10/2\text{CRI}$-periods), and, a decrease in the fundamental frequency ($1/\text{CRI}$-period). The strongest frequency component is a harmonic of the fundamental frequency and thus also due to CRI (appendix N).
The TDX power spectrum of the residual data (Fig. 2.19) shows a reduction in all spectral frequency lines. The remaining spectral lines could be due to an additional systematic noise component and due an inaccurate TD error computation (i.e., due to fading, drift, etc.) (appendix 0).

In conclusion, the spectral evaluation of extracting the TD error due to CRI is in accordance with the earlier analysis and also suggests an additional non-white noise component present in the TDs.
III. POSITION AND SPEED ERROR COMPUTATIONS

In this chapter, we evaluate the position and speed errors which are caused by the different types of TD errors that are discussed in chapter II. Here, we distinguish between

* constant position errors
* time-varying position errors
* time-varying speed errors due to CRI

for the three triads of US-chain 9940 (MWX, MWY, and MXY) and a receiver at O.S.U., Corvallis. In addition, we present position error maps for the US west-coast area.

### III.1 Constant position errors

Constant TD errors transform directly into constant position errors. This transformation can be described by partial derivatives \( g_k = \partial \text{LAT}/\partial \text{TD}_k \) and \( h_k = \partial \text{LON}/\partial \text{TD}_k \) and is presented in section 1.4 (eq. 1.3a,b).

Furthermore, the time-varying TD errors cause position errors which also have a constant component (bias). An approximation for this bias is given in section 1.4, eq. (1.4a,b).

With Razin's algorithm for calculating the receiver position from known TDs (appendix A) and using numerical differentiation (appendix P), the first and second partial derivatives \( (g_{ij} = \partial^2 \text{LAT}/\partial \text{TD}_i \partial \text{TD}_j, h_{ij} = \partial^2 \text{LON}/\partial \text{TD}_i \partial \text{TD}_j, \text{ etc.}) \) were computed for a receiver at O.S.U. \((\text{LAT}/\text{LON} = 44.5675°/123.2745°)\) and each of the three triads MWX, MWY, and MXY of US-chain 9940. They are listed in Table 3.1.
Table 3.1 First and second derivatives for TD-to-position conversion for US-chain 9940 and receiver at O.S.U.

<table>
<thead>
<tr>
<th>TRIAD (Mi)</th>
<th>MWX</th>
<th>MMY</th>
<th>MXY</th>
</tr>
</thead>
<tbody>
<tr>
<td>g_i</td>
<td>-1.80x10^3</td>
<td>-1.89x10^3</td>
<td>-1.41x10^-2</td>
</tr>
<tr>
<td>g_j</td>
<td>-4.40x10^4</td>
<td>6.77x10^2</td>
<td>-1.90x10^-1</td>
</tr>
<tr>
<td>h_i</td>
<td>-9.4x10^4</td>
<td>-2.01x10^2</td>
<td>-1.48x10^-2</td>
</tr>
<tr>
<td>h_j</td>
<td>-8.29x10^3</td>
<td>1.19x10^1</td>
<td>-9.77x10^-2</td>
</tr>
</tbody>
</table>

| g_ii       | 0.53x10^4   | 0.11x10^-5  | 0.81x10^4   |
| g_jj       | 0.15x10^5   | 0.92x10^3   | -0.98x10^-2 |
| h_ii       | 0.34x10^4   | 0.13x10^-5  | 0.95x10^4   |
| h_jj       | 0.82x10^5   | 0.13x10^-1  | -0.44x10^-2 |
| g_j        | 0.18x10^5   | -0.30x10^4  | 0.91x10^-3  |
| h_j        | 0.38x10^-5  | -0.72x10^-4 | 0.81x10^-2  |

The set of measured TD bias, standard deviations, and correlations (US-chain 9940) that are used in the following evaluations are listed in Table 3.2.

Table 3.2 TD bias, standard deviations, and correlations for US-Chain 9940 and receiver at O.S.U.

<table>
<thead>
<tr>
<th>bias (μsec)</th>
<th>TDW</th>
<th>TDX</th>
<th>TDY</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.8</td>
<td>-0.15</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>standard deviation (nsec)</td>
<td>15</td>
<td>20</td>
<td>70</td>
</tr>
<tr>
<td>correlation coefficient</td>
<td>R_{wx} = 0.5</td>
<td>R_{wy} = 0.2</td>
<td>R_{xy} = 0.15</td>
</tr>
</tbody>
</table>

Then, with (1.3a,b) and the values from Tables 3.1 and 3.2, the constant position errors were computed for US-chain 9940 and are listed in Table 3.3:
Table 3.3  Constant position error due TD bias for
US-chain 9940 and receiver at O.S.U.

Since 1 arc-min of latitude corresponds to 1852 meters and 1 arc-min of longitude to about 1852 meters * COS(LAT) (depends on latitude), we can convert the position error ΔLAT/ΔLON (in degrees) into ΔX/ΔY (in meters) and finally into the magnitude of the position error MPE = (∆X² + ∆Y²)¹/². The position errors (in meters) are also listed in Table 3.3.

The constant position errors due to the Gaussian distributed TD error components (time-varying components) are comparably small. They will be evaluated in the following section.

III.2  Time-varying position errors

Here, the time-varying position errors are described by variances and covariances only (section I.4).

If we treat the latitude and longitude errors as two random variables with joint Gaussian distribution, the region within the ellipse which is defined by the two standard deviations S[LAT], S[LON] and the correlation coefficient R[LAT,LON] then corresponds to approximately 68% of the joint distribution. This ellipse is called "error ellipse" (Fig. 3.1) [14], [15].
The standard deviations for latitude and longitude \((S[\text{LAT}], S[\text{LON}])\) as well as their correlation coefficient \((R[\text{LAT}, \text{LON}])\) were computed for a receiver at O.S.U. (Corvallis) and each of the three possible triads of US-chain \(9940\) (eqs. (1.5a,b) and (1.6)). They are listed in Table 3.4. (Note that the standard deviations are expressed in meters, see III.1). Also included is the constant position error \((B[\text{LAT}], B[\text{LON}])\) which is caused by time-varying TD errors.

\[
\begin{align*}
A &= S[\text{LAT}] \sqrt{1-R^2} = 1.51 \\
B &= S[\text{LON}] \sqrt{1-R^2} = 1.51 \\
R &= R[\text{LAT}, \text{LON}] \\
\tan 2\alpha &= \frac{2 \cdot R \cdot S[\text{LAT}] \cdot S[\text{LON}]}{V[\text{LAT}] - V[\text{LON}]} \\
\end{align*}
\]

**Figure 3.1** Error ellipse for jointly Gaussian distributed LAT/LON-errors

From the entries in Table 3.4, we generated three error ellipses (one for each triad). The error ellipses are shown in Fig 3.2.
Figure 3.2 Position error ellipses for data from Table 3.4:
(a) triad MWX  (b) triad MWY  (c) triad MXY
The following can be observed from Fig. 3.2:

* The constant error $B_{[LAT]}$, $B_{[LON]}$ is negligible.

* The receiver position can be estimated most accurately with triad MWX. The accuracy is less with triads MWY and MXY.

* For triads MWX and MWY, the latitude estimate is more accurate than the longitude estimate. For triad MXY the longitude estimate is more accurate.

III.3 Position-error maps for west coast area

For a given receiver position and corresponding TD variances and correlations, we could easily generate error ellipses to describe the position error. Therefore, knowing the TD error estimates for any location, would make it possible to generate ellipses for the entire region served by US-chain 9940. However, a model for predicting the TD variances and correlations for any given receiver position was not derived here, since we have to consider an additional (so far unknown) non-white TD-error component in our model. Therefore, we generated position-error maps for a few sets of TD variances and correlations (for triads MWX, MWY, and MXY) to illustrate the position error within the region served by US-chain 9940 (appendix Q). Here, we show a map for triad MWX using the TD variances and correlation that were observed with a receiver at O.S.U. (Fig. 3.3). To be able to compare error ellipses we magnified each ellipse by the indicated number (say, 512). The actual size of the ellipse can then be obtained by dividing the size of the plotted ellipse by the indicated number (say, 512). This means that positions with small errors are indicated by large numbers (e.g., 4096). For positions with missing ellipses, Newman's algorithm (converting TDs into LAT/LON) does not yield a solution.

We can see that the position error is smallest on the baselines (MX) and (MY) and largest on the baseline extensions as well as on the extension of the line between the secondaries W and X.
Figure 3.3
Position error map using Air TRM, with RLM
W, C indicates XW, D, E, F.
III.4 Position and speed error due to CRI

The TD errors due to CRI are time-varying and therefore cause a time-varying position error. This position error (due to CRI) was computed for TDW and TDX (triad MWX) of US-chain 9940 and is shown in Fig. 3.4.

![Figure 3.4 Position error due to CRI for data set no. 2 and triad MWX of US-chain 9940](image)

The position error shows positive correlation between latitude and longitude. We can also observe an accumulation of position errors in a region similar to an ellipse. This result shows that error ellipses illustrate well how position errors accumulate.

A time-varying position error results in an apparent receiver speed. It expresses how fast the apparent receiver position due to TD errors changes. Even for large position errors, the apparent receiver speed should be small (because of filtering in the receiver), i.e., the position error should change slowly with time.

The apparent receiver speed can be computed by calculating the distance difference (in meters) between two successive position samples. To express the speed in meters per second, we multiply the speed in meters/sample with the sample rate (approx. 1.1 samples/sec).
The smallest apparent receiver speed (receiver at O.S.U.) is expected to be obtained for triad MWX (US-chain 9940) since the TD error ellipse is smallest for this triad. Therefore, we computed the apparent receiver speed for triad MWX for the raw data, the CRI-data, and the residual data. The graphs are shown in Fig. 3.5.

We can observe that the apparent receiver speed due to CRI has a maximum of about 15 m/sec (34 mph) and is significantly less than the apparent receiver speed for the raw data and the residual data which is about the same.

Figure 3.5 Apparent receiver speed for (a) raw data (b) CRI-data (c) residual data
Figure 3.5 continued
IV. POSITION AND SPEED ERROR REDUCTION POSSIBILITIES

In this chapter, we discuss:
* Constant error reduction
* TD data smoothing
* Cross-rate interference removal
* Triad selection or multiple triad operation
* Predicted improvement for Corvallis location

IV.1 Constant error reduction

A constant position error is caused mainly by constant TD errors (see III.1). Thus, to reduce a constant position error, we must reduce the constant TD errors.

In section 1.3, we listed possible error sources which cause constant TD errors: an inaccurate model of path topography, ground conductivity, etc. An additional error source are the monitor stations which control the secondaries of a chain such that their own positions are accurately given by the TDs. This improvement, however, applies only to the area near the monitors (typically off shore) whereas for the remaining area served by the LORAN-C chain, the constant TD errors may actually increase.

All of these constant TD errors can be reduced by operating the LORAN-C receiver in a "differential mode" [16]. In a differential mode, reference monitors compute the constant TD errors (or slowly time-varying TD errors) for their position. If a receiver is near a reference monitor, the TD errors computed for the reference monitor position can be obtained by radio and used to correct the receiver TD measurements. Unfortunately, this method is not always practical, since the area served by a LORAN-C chain would have to be covered with a large number of reference monitors. However, it can be applied to a moving receiver when returning to a starting position with known constant (or nearly constant) TD errors.
Another method of reducing constant TD errors is to calibrate a chain by (a) systematically measuring topography, ground conductivity, etc. throughout the entire chain, or (b) predicting these parameters [7] and thus generating corresponding maps. Of these two methods, only calibration by prediction seems to be practical since measuring ground conductivity, etc. with sufficient spatial resolution is expensive and not feasible in regions such as mountains, etc.

IV.2 TD data smoothing

TD data smoothing can be used to filter out high frequency components of the TD errors. If the receiver is stationary (fixed), a low-pass filter with very low cut-off frequency (averaging over a long period of time, i.e., from several minutes up to several days) can be used to improve the TD accuracy and thus the position-fix accuracy. For moving receivers (especially airborne receivers), averaging can only be done over at most a few seconds. To increase the averaging period, many receivers utilize a linear-prediction filter to account for the distance the receiver travels during an averaging period.

IV.3 Cross-rate interference removal

A linear-prediction filter is optimum only if the noise, accompanying the signal is white (and Gaussian). The TD errors, however, have at least one systematic component which is due to CRI. Thus, to increase the accuracy of TD estimates, the TD error due to CRI must be subtracted prior to linear prediction. This can be achieved by eliminating all pulses that are distorted by an interfering pulse. The positions of possible interfering pulses within one CRI-period can be precomputed as done in section II.3.2, and the beginning of the CRI-period (arbitrarily chosen) can be obtained by measuring the time-difference between the pulses from the desired master and the interfering master at relative time t=0.
In our case, however, the TD errors due to CRI could only be computed after the TDs were processed by the receiver (including its non-linear operations such as hard-limiting).

IV.4 Triad selection or multiple triad operation

From Fig. 32, Fig. 3.3, and appendix Q we observe that for the majority of locations within the served area of US-chain 9940 (and in general), the position error is favored either in the latitude or longitude direction.

Since the position estimate becomes more accurate for smaller error ellipses, we generally select the triad with the smallest error ellipse for a given receiver position.

For some receiver positions, however, the accuracy of the position estimate can be further improved by using two triads simultaneously, where the latitude coordinate is obtained from one triad and the longitude coordinate from the other triad.

IV.5 Predicted improvement for Corvallis

The best position estimate can be obtained when selecting triad MWX. Multiple triad selection (using triads MWY and MXY), however, would give us a position estimate that is still more accurate than the position estimate with either triad MWY or MXY only (not considering the constant position errors).

The constant position errors can be obtained by correcting the measured TDs for the adjustments made by the monitor stations, and averaging over a long period of time. With the corrections, we can then establish additional phase factors (ASF) for Corvallis.

The subtraction of the computed TD error due to CRI yielded a reduction of about 10% of the total TD standard deviation. This reduc-
tion can be further improved by directly eliminating distorted pulses due to CRI since there will be no errors caused by fading, subtracting TD error due to CRI, drift, etc.
V. CONCLUSIONS AND RECOMMENDATIONS

We collected time-difference data (TDW, TDX, and TDY) from US-chain 9940 with a fixed receiver located at O.S.U. in Corvallis (Oregon). The TD data showed a constant TD error (which is partly due to timing adjustments done on secondaries by the monitor station at North Bend). The smallest resulting constant position error for our receiver at O.S.U. was 435 meters, which reduces the accuracy of LORAN-C considerably.

The correlation coefficients between the TDs were surprisingly high and could not be explained with a first order model for time-varying TD errors (path geometry, etc.). Likewise, all TD-autocovariances showed a periodic component with a period of about 119.1 sec. This periodic error (or noise) component was found to be caused by cross-rate interference from the Canadian west-coast chain (5990).

We established a CRI-period (132 samples) and extracted the TD error due to CRI by averaging over those samples that have the same position within a CRI-period (from 100 CRI-periods). The TD errors due to CRI were found to be highly repetitive and to range over approximately 30 nsec for TDW, 60 nsec for TDX, and 140 nsec for TDY. The computed correlation coefficients for the TD errors due to CRI showed a positive correlation around 0.3 (mainly because of common CRI of the master pulses).

After subtracting the TD errors due to CRI from the raw data, the correlations between the TDs of the resulting residual data did not change appreciably. The variance, however, decreased by approximately 20%.

This and the fact that the mean estimates over each CRI-period fluctuate significantly more (over 30 nsec) than the standard deviation of the mean estimates (less than 3 nsec, assuming independent successive samples) suggest that an additional error component be included into a TD-error model.

The apparent speed of our fixed receiver was computed from the TD
triad MWX). The maximum computed speed resulting from the TDs of the corresponding raw data and the residual data, however, was approx. 42 meters/sec or 94 mph.

To significantly reduce the TD errors and thus the position and speed errors, further research has to be done:

* Cross-rate interference must be minimized by eliminating pulses that are distorted by interfering pulses from other chains.

* After the errors due to CRI are minimized, the remaining error components must be traced. The receiver could also be a source of a systematic error (e.g., the PLL).

* A model to predict the time-varying TD errors (described by variances and covariances) as a function of the receiver position must be established. This model must also explain the high TD correlations.

* After a model is established, a position-error map can be generated for each triad of US-chain 9940. With these maps, it will be easy to select the triad which will give the smallest (time-varying) position error for a given receiver position. Furthermore, it can be determined whether multiple triad operation can improve the accuracy of the position estimate.

* The high constant TD errors must be reduced (here, constant TD errors include seasonal TD fluctuations, etc.), since they are the major position error components (e.g., the minimum position error for Corvallis was 435 meters). A reduction can be achieved by accounting for the timing adjustments made at the secondaries and controlled by monitor stations, and by modeling the topography and ground conductivity of the entire service area of the US-chain 9940.

* The improvements for US-chain 9940 can then be expanded to the Canadian west coast chain (5990) and other chains.
BIBLIOGRAPHY


[12] II MORROW INC. Salem, Oregon U.S.A.


APPENDICES
APPENDIX A

INPUT

\( P_i = \frac{(TD_i - CD_i + 0.2) \times RV}{G_{mi}} \)

\( b_i = \frac{\sin P_i}{\sin \Theta_{mi}} \)

\( a_i = \frac{\cos P_i}{\sin \Theta_{mi}} - \cot \Theta_{mi} \)

\( u = a_4 b_1 - a_1 b_4 \)

\( v = a_1 \sin k \)

\( w = a_1 \cos k - a_3 \)

\( c = \frac{uw + v(v^2 + w^2 - u^2)^{0.5}}{v^2 + w^2} \)

\( \Theta_{MS} = \tan^{-1}\left(\frac{a_1}{(b_1 + c)}\right) \quad (0 < \Theta_{MS} < 180^\circ) \)

\( \Theta_{S_1} = \Theta_{MS} + P_i \)

\( X_j = C_{Mj} \cos \Theta_{MS} + C_{ij} \cos \Theta_{S_1} + C_{2j} \cos \Theta_{S_2} + C_{oj} \)

\( (j = 1, 2, 3) \)

\( LON = \tan^{-1}\left(\frac{X_1}{X_2}\right) \)

\( LAT = \tan^{-1}\left[\frac{X_9}{(X_1^2 + X_2^2)^{0.5}}\right] \)

STOP

Figure A.1 Flow chart for simplified Razin algorithm. The plus sign is used in the expression for \( c \) unless the receiver is in a sector bounded by extensions of the arcs connecting the master and secondary stations [4,6].
The constants for the simplified Razin algorithm in Fig. A.1 are listed in Table A.1. Here, the latitude of the master station is denoted IAM, the longitude of the secondaries (labeled 1, 2, and 3) are L01, L02, and L03, etc.

<table>
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<th>IAW</th>
<th>IAX</th>
<th>IAY</th>
<th>LOA</th>
<th>LOX</th>
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<td>-0.0030171</td>
<td>11000</td>
<td>27000</td>
<td>40000</td>
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</table>

for triad:  
MWX (i=1,2)  

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<th>Cm1</th>
<th>Cm2</th>
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<td>40000</td>
</tr>
</tbody>
</table>

Table A.1 Constants for simplified Razin algorithm for US-chain 9940

The equations for the secondary phase correction (SF) in theory and in linearized form are:

\[ SF = T/1544 - (1.5 + 479/T)^2 \mu\text{sec} \]
\[ SF = T(1/0.9994 -1.0) - 0.2 \mu\text{sec} \]

Here, \( T \) is the propagation time in \( \mu\text{sec} \). SF must be added to \( T \) in order to obtain a secondary phase correction.
APPENDIX B

Derivation of position errors from TD errors

The position of a fixed receiver is expressed by latitude and longitude. Both, latitude and longitude, are functions of two different TDs (say, TDW and TDX). Here, all derivations are made in a general form, i.e., for two functions $U = g(X,Y)$ and $V = h(X,Y)$ of two random variables $X$ and $Y$ with zero mean, variances $V[X], V[Y]$, and covariance $\text{COV}[X,Y]$. ($U$ and $V$ can be replaced by LAT and LON, respectively, and $X$ and $Y$ by TDi and TDj, respectively.) Partial derivatives are then expressed as $g_x = \partial g / \partial X$, $g_y = \partial g / \partial Y$, $h_x = \partial h / \partial X$, $h_y = \partial h / \partial Y$, etc.

We expand $g(X,Y)$ in a Taylor series about $X_0, Y_0$ with $g(X_0, Y_0) = U_0$ (and similar for $h(X,Y)$ with $h(X_0, Y_0) = V_0$), where $g_x$ represents $g_x(X_0, Y_0)$, etc.:

$$
g(X,Y) \approx U_0 + g_x(X-X_0) + g_y(Y-Y_0) + \frac{1}{2} [g_{xx}(X-X_0)^2 + 2g_{xy}(X-X_0)(Y-Y_0) + g_{yy}(Y-Y_0)^2]
$$

Neglecting all derivatives higher than first order, we get for small changes $\Delta U = U - U_0$:

$$
\Delta U = g_x \Delta X + g_y \Delta Y
$$

and similarly for $V$:

$$
\Delta V = h_x \Delta X + h_y \Delta Y
$$

The mean $E[U] = E[g(X,Y)]$ becomes:

$$
E[g(X,Y)] = E[U_0] + g_x E[X-X_0] + g_y E[Y-Y_0] + 0.5 g_{xx} E[(X-X_0)^2] + g_{xy} E[(X-X_0)(Y-Y_0)] + 0.5 g_{yy} E[(Y-Y_0)^2]
$$
since \( E[X-X_\ast] = E[Y-Y_\ast] = 0 \), the mean of \( g(X,Y) \) becomes:

\[
E[g(X,Y)] \approx U_\ast + 0.5 \, V[X] \, g_{xx} + 0.5 \, V[Y] \, g_{yy} + \text{COV}[X,Y] \, g_{xy}
\]

(and similar for \( h(X,Y) \))

The second order moment \( E[g^2(X,Y)] \) is computed by neglecting all expectations for \( X \) and \( Y \) with terms higher than second order and yields:

\[
E[g^2(X,Y)] \approx U_\ast^2 + V[X] \, (g \, g_{xx} + g_x^2) + V[Y] \, (g \, g_{yy} + g_y^2) + + 2 \, \text{COV}[X,Y] \, (g \, g_{xy} + g_x \, g_y)
\]

(and similar for \( h(X,Y) \))

The square of the \( E[g(X,Y)] \) becomes when neglecting all products with \( V[X] \) and \( V[Y] \):

\[
E^2[g(X,Y)] \approx U_\ast^2 + V[X] \, g \, g_{xx} + V[Y] \, g \, g_{yy} + 2 \, \text{COV}[X,Y] \, g \, g_{xy}
\]

Finally, the variance \( V[g(X,Y)] \) yields:

\[
V[g(X,Y)] = E[g^2(X,Y)] - E^2[g(X,Y)] = = V[X] \, g^2_x + V[Y] \, g^2_y + 2 \, \text{COV}[X,Y] \, g_x \, g_y
\]

(and similar for \( h(X,Y) \))

The second order moment \( E[g(X,Y)h(X,Y)] \) becomes when neglecting all products with \( V[X] \) and \( V[Y] \):

\[
E[g(X,Y)h(X,Y)] = U_\ast \, V_\ast + 0.5 \, V[X] \, g \, h_{xx} + 0.5 \, V[Y] \, g \, h_{yy} + + \text{COV}[X,Y] \, g \, h_{xy} + 0.5 \, V[X] \, h \, g_{xx} + 0.5 \, V[Y] \, h \, g_{yy} + + \text{COV}[X,Y] \, h \, g_{xy}
\]

The covariance \( \text{COV}[g(X,Y),h(X,Y)] \) finally yields:

\[
\text{COV}[g(X,Y),h(X,Y)] = E[g(X,Y)h(X,Y)] - E[g(X,Y)] \, E[h(X,Y)] = = V[X] \, g_x \, h_x + V[Y] \, g_y \, h_y + \text{COV}[X,Y] \,(g_x \, h_y + g_y \, h_x)
\]
APPENDIX C

TD-distributions over 1,000 successive samples

TDW DISTRIBUTION

using 1,000 samples

TDY DISTRIBUTION

using 1,000 samples
APPENDIX D

Graphs of the statistical averages for US-chain 9940

TDW MEANS FOR 1000-SAMPLE SETS

21 Mar - 1 Apr 1988

TDY MEANS FOR 1000-SAMPLE SETS

21 Mar - 1 Apr 1988
MASTER-SNRs FOR 1000-SAMPLE SETS
21 Mar - 1 Apr 1986

W-SNRs FOR 1000-SAMPLE SETS
21 Mar - 1 Apr 1986
X-SNRs FOR 1000-SAMPLE SETS

M-SNR STD. DEVs. FOR 1000-SAMPLE SETS
W–SNR STD. DEVs. FOR 1000–SAMPLE SETS

21 Mar – 1 Apr 1986

X–SNR STD. DEVs. FOR 1000–SAMPLE SETS

21 Mar – 1 Apr 1986
SNR-CORR. COEFF. FOR 1000-SAMPLE SETS
21 Mar - 1 Apr 1986

SNR-CORR. COEFF. FOR 1000-SAMPLE SETS
21 Mar - 1 Apr 1988
APPENDIX E

Theoretical propagation times for spheroidal, sea water, standard atmosphere model [4]

The distance between two points (point 1 and point 2) with LAT1/LON1 and LAT2/LON2 can be obtained with the following equations:

1. Convert geodetic latitude LAT into parametric latitude P:
   \[ \tan (P) = \frac{b}{a} \tan (LAT) \quad \text{with } b = 6356.751, \quad a = 6378.135 \text{ (km)} \]

2. The great-circle distance between points 1 and 2 on a sphere is defined by the arc length:
   \[ \theta = \cos^{-1} [\sin(P1) \sin(P2) + \cos(P1) \cos(P2) \cos(LON1 - LON2)] \]

3. On a spheroid with small eccentricity, this arc length converts into a distance (\( \theta \) in rad):
   \[ d \approx a \theta \left( \frac{1}{4} (a-b) \left[ (\sin(P1) + \sin(P2))^2 \frac{\theta - \sin \theta}{1 + \cos \theta} \right. \right. \\
   + [\sin(P1) - \sin(P2))^2 \left( \frac{\theta + \sin \theta}{1 - \cos \theta} \right) \left. \right] \quad \text{km} \]

4. To obtain the propagation time, divide d by the propagation velocity (0.299691 km/\( \mu \)sec for standard atmosphere).

5. Correct for the "secondary phase factor" (SF) (sea water):
   \[ SF = \frac{T}{1544} - (1.5 + \frac{479}{T})^2 \quad \mu \text{sec} \]

6. Add and subtract corresponding propagation times and coding delays, to obtain the desired time-difference (TD).
APPENDIX F

Graphs of statistical averages for Can-chain 5990

TDY-MEANS FOR 1000-SAMPLE SETS
7-9 Mar (Canadian chain)

TDZ-MEANS FOR 1000-SAMPLE SETS
7-9 Mar (Canadian chain)
APPENDIX G

TD-autocovariances of raw data (13,500 successive samples)
(US-chain 9940)

TDW-AUTOCOVARIANCE OF RAW DATA

TDW-AUTOCOVARIANCE OF RAW DATA
TDX-AUTOCOVARIANCE OF RAW DATA

data set no.3

0.0011
0.001
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0
0.0002
0.0001

log n (samples)

TDY-AUTOCOVARIANCE OF RAW DATA

data set no.1

0.022
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.008
0.004
0.002
0
-0.002
-0.002

log n (samples)
APPENDIX H

TD error due to CRI
(US-chain 9940)

TDW-ERROR DUE TO CRI

to get TDW add 12870

Sample number I

TDY-ERROR DUE TO CRI

to get TDY add 43930

Sample number I
APPENDIX I

Standard deviation of each sample of TD error due to CRI (US-chain 9940)

**TDW-STD.DEV. OF SAMPLE i IN CRI-PERIOD**

![Graph showing TDW-STD.DEV. of sample i in CRI-period](image)

**TDY-STD.DEV. OF SAMPLE i IN CRI-PERIOD**

![Graph showing TDY-STD.DEV. of sample i in CRI-period](image)
APPENDIX J

TD-autocovariance of CRI-data
(US-chain 9940)

TDW-AUTOCOVARIANCE OF CRI-DATA

data set no.1

log n (samples)

TDW-AUTOCOVARIANCE OF CRI-DATA

data set no.2

log n (samples)
TDY-AUTOCOVARIANCE OF CRI-DATA

data set no.2

log n (samples)

TDY-AUTOCOVARIANCE OF CRI-DATA

data set no.3

log n (samples)
APPENDIX K

TD-autocovariance of residual data
(US-chain 9940)

TDW-AUTOCOVARIANCE OF RESIDUAL DATA

data set no.1

TDW-AUTOCOVARIANCE OF RESIDUAL DATA

data set no.2
TDX—AUTOCOVARIANCE OF RESIDUAL DATA

TDX—AUTOCOVARIANCE OF RESIDUAL DATA

log n (samples)

log n (samples)
APPENDIX L

TD mean of each CRI-period
(US-chain 9940)

TDW-MEAN OF EACH CRI-PERIOD

TDW-MEAN OF EACH CRI-PERIOD

TDW-MEAN OF EACH CRI-PERIOD

TDW-MEAN OF EACH CRI-PERIOD

data set no.3

TDX-MEAN OF EACH CRI-PERIOD

data set no.1
TDY-MEAN OF EACH CRI-PERIOD

data set no.2

TDY-MEAN OF EACH CRI-PERIOD

data set no.3
APPENDIX M

Power spectrum of raw data
(US-chain 9940)

TDW POWER SPECTRUM OF RAW DATA
data set no.1

TDW POWER SPECTRUM OF RAW DATA
data set no.2
APPENDIX N

Power spectrum of CRI-data
(US-chain 9940)

TDW POWER SPECTRUM OF CRI-DATA

data set no.1

TDW POWER SPECTRUM OF CRI-DATA

data set no.2
TDX POWER SPECTRUM OF CRI-DATA

data set no.3

TDY POWER SPECTRUM OF CRI-DATA

data set no.1
TDY POWER SPECTRUM OF CRI-DATA
data set no.2

TDY POWER SPECTRUM OF CRI-DATA
data set no.3
APPENDIX O

Power spectrum of residual data
(US-chain 9940)

TDW POWER SPECTRUM OF RESIDUAL DATA

TDW POWER SPECTRUM OF RESIDUAL DATA
TDW POWER SPECTRUM OF RESIDUAL DATA

data set no.3

TDX POWER SPECTRUM OF RESIDUAL DATA

data set no.1
TDY POWER SPECTRUM OF RESIDUAL DATA

data set no.3

TDY POWER SPECTRUM OF RESIDUAL DATA

data set no.1
TDY POWER SPECTRUM OF RESIDUAL DATA
data set no.2

td

TDY POWER SPECTRUM OF RESIDUAL DATA
data set no.3

APPENDIX P

Numerical differentiation

1. derivatives of a function $f(x, y)$ about $x_0, y_0$:

$$
\frac{\partial f}{\partial x} \approx m_1 = \frac{f(x_0, y_0) - f(x_0-h, y_0)}{h} \\
\frac{\partial f}{\partial x} \approx m_2 = \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h}
$$

(and similar for $\frac{\partial f}{\partial y}$)

2. derivatives of a function $f(x, y)$ about $x_0, y_0$:

$$
\frac{\partial^2 f}{\partial x^2} \approx \frac{m_2 - m_1}{h^2} = \frac{f(x_0+h, y_0) - 2f(x_0, y_0) + f(x_0-h, y_0)}{h^2}
$$

(and similar for $\frac{\partial^2 f}{\partial y^2}$)

$$
\frac{\Delta f}{\Delta x} = \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h} \\
\frac{\Delta f}{\Delta y} = \frac{f(x_0, y_0+h) - f(x_0, y_0)}{h}
$$

$$
\frac{\partial^2 f}{\partial x \partial y} \approx \frac{f(x_0+h, y_0+h) - f(x_0, y_0+h) - f(x_0+h, y_0) + f(x_0, y_0)}{h^2}
$$

where $h$ is the step size
TRIAD MXY: Position error-ellipses for $S(TDX) = .020$, $S(TDY) = .070$, and $R(X, Y) = .15$
TRIAD MWY: Position error-ellipses for S(TDW)=.015, S(TDY)=.070, and R(W,Y)=.20
TRIAD MWX: Position error-ellipses for $S_{(TM)} = .020$, $S_{(TDX)} = .020$, and $R_{(W,X)} = .40$
TRIAD MWY: Position error-ellipses for $S(TDW)=.020$, $S(TDY)=.020$, and $R(W,Y)=.40$
TRIAD MXY: Position error-ellipses for $S(TDX)=.020$, $S(TDY)=.020$, and $R(X,Y)=.40$