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Movement of water in unsaturated soil has recently been recognized as important in the design of drainage systems. Measurement has been difficult. Because the diffusion equation is gaining favor as a means of describing moisture flow, a study of the methods used to measure diffusivity and unsaturated hydraulic conductivity has become desirable.

Outflow data measured with time were obtained by use of a tension plate and horizontal burette apparatus. Small increments of tension were applied to a previously saturated soil sample and outflow was recorded with time.

Samples of natural Amity silt loam, Amity with the clay content increased, and Amity with the 2 to 20μ silt fraction decreased, were used.

Comparisons of methods of measuring diffusivity and hydraulic

conductivity, of the difference in diffusivity and hydraulic conductivity caused by change in clay content, and of differences in duplicate measurements of hydraulic conductivity were made.

The conclusions drawn from this thesis are:

1. A combination of methods of measuring diffusivity by the transient state methods gives best results.

2. Some methods suggested in the literature did not apply to the data obtained.

3. Plate impedance is not negligible in the moisture range near saturation.

4. The assumption of constant diffusivity is not valid in the drier range above about 50 cm. water tension.

5. Soil texture has an effect on diffusivity and hydraulic conductivity. The finer textures have lower values of D and K.

6. Many more trials are needed in order to draw conclusions about the ability of the methods to replicate the results obtained.

COMPARISON OF SELECTED METHODS OF MEASURING HYDRAULIC CONDUCTIVITY AND DIFFUSIVITY OF SOIL WITH VARIED CLAY CONTENT

by

GEORGE FRANK HAMLIN

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In Charge of Major

Redacted for Privacy

Head of Department of Agricultural Engineering

Redacted for Privacy

Dean of Graduate School

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TABLE OF SYMBOLS

D	=	diffusivity (cm^2/min)
H	=	tension (cm of water)
K	=	unsaturated hydraulic conductivity (cm/min)
L	=	length of soil sample or flow path (cm)
Q	=	outflow at time t (cm^3)
Q_o	=	total outflow for a particular pressure increment (cm^3)
T_{RP}	=	time reference point (min)
V	=	volume of soil sample (cm^3)
Z_m, Z_t	=	plate and sample impedance respectively, L/K, (min)
a	=	ratio of plate impedance to soil impedance, (dimensionless)
h	=	applied head causing outflow in steady state (cm of water)
q	=	volume rate of flow (cm/min)
t	=	time (min)
θ	=	volume of moisture per unit volume (dimensionless)
$\Delta\theta$	=	Q_o/V (dimensionless)
α_n	=	nth solution of the equation $a\,\alpha_n = \cot \alpha_n$, $n = 1, 2, 3, \ldots$
μ	=	(microns)

COMPARISON OF SELECTED METHODS OF MEASURING HYDRAULIC CONDUCTIVITY AND DIFFUSIVITY OF SOIL WITH VARIED CLAY CONTENT

INTRODUCTION

The study of unsaturated moisture flow in soils is of great importance in agriculture. At the present time most of the equations used for drainage system design are based on flow through the saturated zone only. It is now believed that design values for tile size, and depth of installation should also be dependent on how fast the water will travel in the unsaturated zone above the water table. Applications of irrigation water to soil are also dependent on the principles of unsaturated flow. It is important to know how far, in what direction, how fast, and how much moisture movement will take place beneath an irrigation furrow for instance. A significant portion of the flow through earth dams, roadways, and embankments is in the unsaturated zone.

For unsaturated soils, rate of movement is dependent on the moisture content of the soil itself. It is this property that makes unsaturated hydraulic conductivity and diffusivity difficult to calculate or measure experimentally. Much of the recent work done in measuring unsaturated hydraulic conductivity has been concerned with the drier range from one-tenth to 15 atmospheres tension.

Hydraulic conductivity is generally plotted as a function of moisture content.

The water holding capacity of a soil may be expressed in terms of the moisture-tension curve. Soils are tested to determine what percent moisture they will hold under certain tensions or pressures.

The diffusion equation has been used to express unsaturated moisture movement in soils. The relationship of diffusivity, D, to hydraulic conductivity, K, is given by $D = K\ dp/d\theta$ where dp is the change in pressure per unit change in moisture content, $d\theta$.

Ashcroft (1) gives a rather complete history of the application of the diffusion equation to flow of moisture in soil.

SCOPE OF RESEARCH

It was the purpose of this research to study moisture movement in the very wet range of soil moisture, starting with saturation and drying down to approximately one hundred centimeters of water tension. This range is of prime importance to design of drainage systems.

Field capacity of a soil can be described as the moisture content where the soil pores because of their smallness of diameter are able to overcome the force of gravity and to hold water by surface tension. Field capacity is generally considered to occur when the moisture tension approaches one-third atmosphere. Between saturation and one-third atmosphere, approximately 300 cm. of water tension, the water will move by gravity, with the greatest movement at the lowest tension. Hence the interest of one working with soil drainage problems is centered in the range near saturation.

It was the intent of this study to discover a reliable method of measuring diffusivity and unsaturated hydraulic conductivity in the moisture tension range from zero to 100 cm. of water. The study involves a soil that has been altered to give three textural classes. Amity silt loam was chosen because it is one of the drainage problem soils of the Willamette Valley. The soil texture was changed by removing the clay particles to make a sandy and silty textured soil and by adding clay particles to make a clayey textured soil. These two textures are compared to the unaltered soil to determine what effect soil texture has on hydraulic conductivity or diffusivity. This study is limited to the drying down case with no consideration for hysteresis on the wetting cycle.

In order to investigate the reliability of different methods of measuring hydraulic conductivity, a comparison was made among six methods reported in the literature. One method is to apply a steady state condition to a soil sample and measure the rate of outflow per unit time at different moisture tensions. Another way and at this time the more popular method is to apply a pressure or tension and measure the outflow per unit time during the transient state. This arrangement gives a value for diffusivity from which hydraulic conductivity may be calculated. The slope of the moisture-tension curve, another value needed for this calculation, can be obtained by measuring the change in pressure that caused the outflow and the corresponding change in moisture content.

The techniques of measuring diffusivity and calculating conductivity introduced by Gardner (6), Crank (11), Rijtema (15), Elrick (5) and Kunze and Kirkham (12) have been studied and are reported for comparison in this research. One steady state technique used by Elrick is also included in the report. In order to simplify the comparisons, only one soil texture is used for comparing methods of these authors. A similar comparison might have been made for the other textural classes or experimental runs of the same soil texture. The results of these comparative methods of measuring hydraulic conductivity and diffusivity and the comparison of effects of soil textural changes on conductivity and diffusivity are given in another section of this report.

REVIEW OF LITERATURE

Since 1956 several authors have published reports on laboratory methods of measuring hydraulic conductivity and soil diffusivity by pressure plate outflow data. Gardner (6) is the apparent initiator of these methods. In 1956 he determined capillary conductivity and diffusivity from outflow data by using the Darcy law and the equation of continuity in combination and solving by a separation of variables technique. He made three assumptions: one, that a small step change in pressure would give a constant K value; two, that the plate impedance would be negligible as compared to the soil impedance; three, that gravity can be neglected. Gardner's equation is given as

$$\ln(Q_0 - Q) = \ln(8Q_0/\pi^2) - \alpha^2 Dt \quad \text{.............. (1)}$$

where Q_0 is the total outflow, Q is the outflow at time t, and $\alpha = \pi/2L$ where L is the length of the soil sample. This equation is only the first term of a series solution but the other terms are negligible if time is taken large enough. It may be solved for D and then by the relationship $D = K\, dp/d\theta$, K may be calculated.

Miller and Elrick (13) in 1958 advanced this method by making allowance for plate or membrane impedance. They expanded a Fourier series expressing liquid content as a function of position and time, then integrated over a soil sample height, thus arriving at a general equation for transient flow. This was then rewritten for a particular set of variables, yielding a single-parameter family of almost-identical curves. At zero time the curves have their greatest differences, but they coalesce into a single curve for large values of time. Then by matching experimental plots of outflow vs. time to the theoretical curves by an overlay method the diffusivity can be calculated.

Rijtema (15) in 1959, working on the Gardner method, developed a way to take membrane impedance into account without having to measure it as did Miller and Elrick. Rijtema pointed out the difficulty of not knowing the contact impedance when measuring the membrane impedance separately. He plotted $\log(Q_0 - Q/Q_0)$ against Dt/L^2 and obtained a set of straight lines for various values of membrane impedances. Then by measuring the slope he could calculate D and get K from the same relationship as Gardner stated, $D = K d\phi / d\theta$.

In 1961 Kunze and Kirkham (12) started with Miller and Elrick's method and simplified it by using only the initial outflow and by providing graphical solutions to the equations. They plotted a series of theoretical curves on log log paper, then used an overlay curve matching process to fit the experimental curve to the theoretical one. Their method takes membrane impedance into account without having to measure it.

In 1962 Elrick (5) published a report on a simplified procedure of measuring capillary conductivity and diffusivity of unsaturated soil by use of a tension plate apparatus. He made both a steady state and a transient state measurement on a horizontal soil column with horizontal burettes connected to a sintered glass bead plate at either end. He could choose between a constant head steady state or a transient state by regulating flow with the burette stopcocks. He still relied on separate measurements of the plate impedance and appeared to have an air entry problem with his sandy soil. One of his equations is $Z_t = hA/q$ where $\underline{Z_t}$ is total impedance, $\underline{h}$ is applied head, $\underline{A}$ is cross sectional area of sample and $\underline{q}$ is volume flow rate. A second is $K = L/(Z_t - Z_m)$, where $\underline{K}$ is hydraulic conductivity, $\underline{L}$ is length of sample, and $\underline{Z_m}$ is plate impedance. These two equations enabled him to calculate K directly from his steady state data. He then started the transient state by closing the inflow stop cock and obtained outflow data for various steps of increasing tension. D could be calculated from the slope of his semi-log plots of $(Q_0 - Q)/Q_0$ vs. time.

Jackson, van Bavel and Reginato (11) in 1963 gave an overall report on the outflow method of measuring diffusivity. They compared Crank's square root of time method with Gardner's, Rijtema's, Miller and Elrick's, and Kunze and Kirkham's methods for plotting outflow vs. time and calculating diffusivity. Crank's method neglects plate impedance and can use only the initial portion of the outflow curve because of assuming a semi-infinite column. They point out that Gardner, Rijtema, and Miller and Elrick all assume, perhaps erroneously, that diffusivity is constant. Gardner and Rijtema also must have the intercept of their plots be at least ln $8/\pi^2$ for their methods. Kunze and Kirkham's method has the difficulty of matching experimental points to theoretical curves. They report a wide variation in measurements and state it is very difficult to obtain replicated results of diffusivity measurements. They conclude that data obtained by these methods may be uncertain by a factor of ten.

THEORY

Plate impedance and constant diffusivity are two of the biggest problems to overcome in measuring diffusivity by the pressure-outflow method. In order for Gardner's (6) method to work he had to assume constant diffusivity over a pressure increment and negligible plate impedance. In plotting his equation $\ln(Q_0 - Q) = \ln 8Q_0/\pi^2 - \alpha^2 Dt$, the intercept has to be $\ln 8Q_0/\pi^2$. If diffusivity is not constant or impedance is not negligible a straight line will not be obtained and the method will not work. Rijtema's method (15) must also have D constant and the intercept must be $\ln 8/\pi^2$ or larger. If it is larger, then impedance is not negligible and can be calculated. Miller and Elrick (13) assumed constant D and measured plate impedance separately. They then used an overlay and matched experimental plots of data to their theoretical curves. Kunze and Kirkham (12) simplified Miller and Elrick's method so that impedance could be accounted for without measuring it separately. They did not consider the assumption of constant diffusivity to be valid but hoped to minimize the error by taking small increments of pressure. They utilized only the first 10 to 15% of outflow to match with their theoretical curves.

It is the thought of the author that these limitations may be at least partially overcome by a combination of methods of measuring diffusivity. First the use of even smaller increments of pressure or tension than those reported by others (5;6;11;12;15) should have better results. Perhaps a check of the constancy of diffusivity can be made by plotting the outflow data vs. time according to Kunze and Kirkham and matching the resulting curves to the theoretical ones. Jackson, van Bavel and Reginato's plot (11) of the theoretical curves of Kunze and Kirkham suggest that if the experimental curves are flatter than the theoretical one for negligible plate impedance ($a = 0$) and constant diffusivity, then the diffusivity is not constant. This plot further suggests that any experimental data that fall below this line indicate negligible plate impedance. Also any experimental curves that rise above this line have constant diffusivity and non-negligible plate impedance.

The chance for non-constant diffusivity increases with the size of the pressure increment. A small increment used near saturation will provide nearly constant diffusivity for its initial outflow and still will be great enough to cause a measurable outflow. As the soil dries down, larger increments of pressure are needed to get a measurable outflow. This increases the range of diffusivity within the increment. Also, as the soil dries down only the smaller pores remain filled and the impedance of the soil increases rapidly, thus the plate impedance becomes insignificant.

The square root of time method depends on negligible plate impedance but is not affected by non constant diffusivity. Thus by using small increments of pressure or tension an average diffusivity can be calculated. Since the initial portion of the outflow curve is used in this calculation, it should follow that the diffusivity value should correspond to a pressure value between the two pressure steps but perhaps closer to the initial one.

For those outflow curves that would fit on Kunze and Kirkham's theoretical plot, plate impedance is accounted for and diffusivity can be calculated. Again it can be assumed to be nearer to the initial pressure because only the initial 10 to 15% of outflow is used.

Near saturation, the accuracy of the measured diffusivity will be affected by plate impedances more than it will be after the soil has dried down. As the soil dries, the water leaves the larger pores first and the impedance of the soil increases rapidly as the average size of the remaining saturated pores becomes smaller. At the same time, the plate presumably remains saturated and its impedance constant. Thus the ratio of plate impedance to soil impedance, Z_m/Z_t, becomes negligible at some moisture content. From this point on, a calculation of diffusivity by the square root of time method (11) should give good results. Kunze and Kirkham's method should also work if the curves for non-constant diffusivity were calculated and plotted on their theoretical plot.

Soil texture may have a bearing on use of the outflow method of measuring diffusivity. A coarse textured soil would tend to have larger pores that would empty faster than a fine textured soil. Plate impedance would be significant over a larger moisture content range in the coarser soil. In order to overcome this, a matching of plate to soil might be in order. Perhaps a coarse textured soil should be dried by two overlapping experiments, one with a coarse graded plate would work up to its ability to withstand tension and the finer plate would have a significant impedance in the wet range but would work after the soil became drier.

The pressure increments should be reduced for a coarse textured soil because it would take less pressure to cause outflow. Perhaps the reason some experiments fail is that researchers use too large a pressure increment and or too coarse a soil or other porous material to permit the assumption of constant diffusivity over the increment.

The objectives of this research are:

1. To compare different methods of measuring diffusivity and hydraulic conductivity for Amity silt loam.

2. To determine what effect altering the clay content has on measured values of diffusivity and hydraulic conductivity.

3. To determine the difference between corresponding values of hydraulic conductivity measured in two replications.

METHODS AND MATERIALS

Laboratory Techniques

Elrick's method of horizontal soil column and burettes was adapted because of its simplicity.

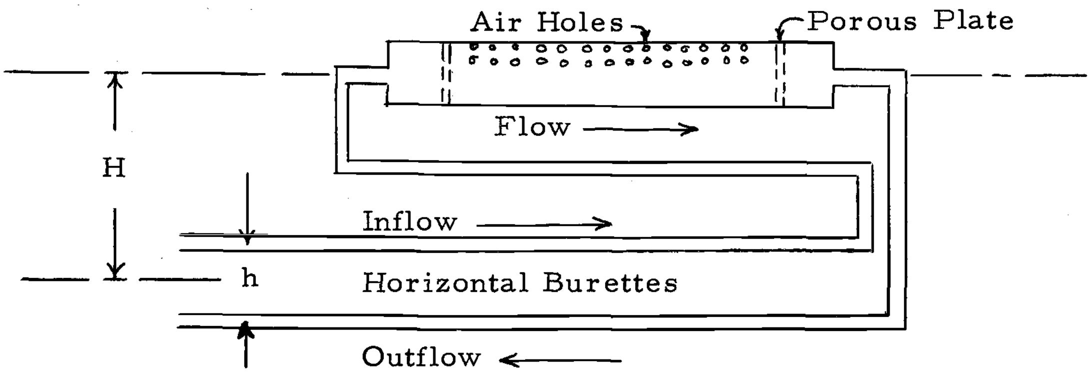

Figure 1. Diagram horizontal drydown apparatus.

A diagram of the apparatus is shown in figure 1. The soil column is connected to the horizontal burettes by tygon tubing. Branch lines for filling, draining and removing trapped air in the tubing were added.

The general technique used to measure steady state hydraulic conductivity was as follows: A constant head h, was maintained between the inflow and outflow burettes. When the rates of water movement in the inflow and outflow burettes were the same the total impedance was calculated from the equation

$$Z_t = hA/q \dots\dots\dots\dots (2)$$

where h is the applied head, A is the cross sectional area of the sample, and q is the volume rate of flow. The conductivity could then be calculated from the equation

$$K = L/(Z_t - Z_m) \dots\dots\dots\dots\dots\dots\dots (3)$$

where L is the length of the soil sample and Z_m is the previously measured plate impedance.

The transient case was then started by closing the stopcocks and increasing the tension H by lowering the outflow burette to the desired tension. H is measured for the transient case from the center line of the soil column to the center line of the outflow burette. The outflow burette stopcock was then opened and the outflow volume was measured with respect to time.

In early trials a single soil column with a double set of burettes was adapted so that only slight delays would be encountered while emptying and refilling the burettes. It was found that after the initial outflow for any given increment of suction that the time loss due to emptying the outflow burette is negligible. In later experiments two soil columns were run simultaneously so that there would be duplication of results. A typical procedure followed in the experimental runs is given in Table 1. The porous plates were measured for plate impedance prior to use on the soil columns. The soil used was an Amity silt loam taken from an experimental drainage site south of the high school at Philomath, Oregon. The soil was collected between four and 14 inches depth. It was rolled, sieved, air dried and packed on

Table 1. Procedure followed in experimental runs.

Step	State	inflow elevation	outflow elevation	soil col. elevation	h cm	H cm
1	steady	102.5	97.5	100	5	0
2	drawdown	off	95	100		5
3	steady	100	90	100	10	5
4	drawdown	off	90	100		10
5	steady	95	85	100	10	10
6	drawdown	off	85	100		15
7	steady	90	80	100	10	15
8	drawdown	off	80	100		20
9	drawdown	off	75	100		25
10	steady	80	70	100	10	25
11	drawdown	off	70	100		30
12	drawdown	off	60	100		40
13	drawdown	off	50	100		50
14	drawdown	off	40	100		60
15	drawdown	off	25	100		75
16	steady	30	20	100	10	75
17	drawdown	off	10	100		90
18	drawdown	off	0	100		100

a mechanical soil packer designed and built especially for this project. The uniformity of packing has been tested by experimentally sectioning packed soil columns and by use of a gamma scope.[1] The results indicate an accuracy of uniform bulk density of ± 0.01. Two sizes of lucite soil columns were used 2.5 and 3.2 centimeters in diameter. The porous plates were purchased from Corning Glass Co., Corning, New York. A medium grade of sintered glass beads seemed best for the desired range of 0-100 cm. water tension. The lucite columns were ventilated along the upper side with four rows of no. 12 size drill holes, spaced about 1/2 inch apart. The soil was packed directly on one porous plate and the other plate placed on top, both were rubber cemented and taped in place before saturating the soil column.

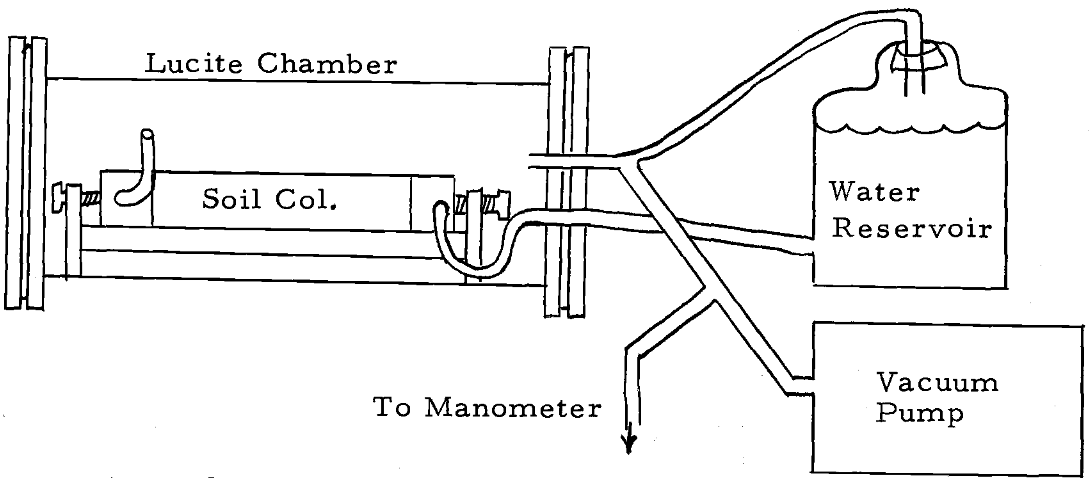

Figure 2. Saturation Chamber.

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[1] Gammascope equipment was used to measure density with a collimated beam of gamma rays 1 mm wide and 25.4 mm high; source was 250 milli-curies; readout equipment was a spectrometer and a rate meter made by Packard.

The soil columns were saturated in a vacuum chamber, see Figure 2, drawn down to 72 cm of Hg tension. After the soil air was evacuated, water under a low head was allowed to enter until the entire column was saturated. The measured moisture content at saturation was 57% by volume. The columns were then wrapped in Saran wrap and placed in position for the horizontal outflow experiment. Small (5 cm) increments of water tension were applied and constant attention given to the columns during the first hour of each increment. Tension instead of pressure was used in this experiment. Tension equipment is simpler to construct and operate than the pressure chambers used by others and still gives a high degree of accuracy. It was found that the columns could be left overnight on a transient state but must be watched closely when on a steady state phase. The soil columns and ends of the burettes were wrapped loosely with Saran wrap to reduce evaporation losses. The outflow burettes were fitted with stoppers and vented through a flask to catch any overflow if it should occur. The water used throughout the experiment was distilled water, boiled to remove air and treated with $CuSO4$ (one gram per 1000 ml) to prevent bacterial growth in the soil and porous plates. The burettes were 10 ml micro burettes chosen for their diameter. The meniscus would remain stable even though the burette was horizontal. This furnished a constant head with a high degree of accuracy as they were calibrated to the 0.02 ml mark. The small diameter of the burettes made it necessary to take them into account when measuring plate impedance. The plate impedances were measured using the same tubing used in the experimental outflow measurements. A falling head permeameter test as described in (17, p. 101-104) was used to determine the plate impedance. Initial and final moisture contents were calculated by weighing the entire soil column. Also, a soil sample was extracted and dried at the end of the experiment to determine the final moisture content.

The soil used in the three comparisons was a natural Amity silt loam, typical of the soils found in drainage problem areas of the Willamette Valley. One soil treatment was left in its natural state. The second soil was the same basic Amity, but an attempt was made to change it to a coarser textured soil by removing part of the clay fraction. Four percent of the soil sample was removed. The remainder was used as the sand and silt treatment. The third soil treatment had an addition of about 10% clay to its natural clay content.

The alteration was accomplished by weighing and placing fifty grams of soil in each of twelve quart jars and filling the jars up to the shoulder with water, about 10 cm above the soil. These were stirred by air jetting for a period of five minutes then allowed to stand for a period of seven hours to allow the sand and silt to settle to the bottom. The remaining liquid and soil particles were then siphoned off into a large stainless steel pan. The jars were refilled, air jetted and allowed to settle four more times with fresh water being added each time. The liquid removed was evaporated on electric hot plates till the remainder would fit into a 500 ml glass dish, this was placed in an electric oven and dried at 50° C until equilibrium was reached. After the five siphonings of the liquid, the remainder of soil and water in the jars was placed in a shallow pan in the oven and dried. The contents of both the glass dish containing the clay and the pan with the sand and silt were rolled and sieved to 2 mm again for use in the soil columns. The mechanical analysis of the three soil textures is shown in Table 2. The analysis shows that the second soil treatment had a structural breakdown. Of the 600 grams of soil, 25 grams of clay were removed, yet the soil treatment had about the same amount of clay as the natural soil, a reduction of 10 percentage points in the 2 to 20μ silt content, and a buildup of nine percentage points in the 20 to 50μ silt content.

Table 2. Mechanical composition of three soils as determined by pipette analysis.

Soil	Clay	Silt		Sand
	< 2μ	2 to 20μ	20 to 50μ	> 50μ
	%	%	%	%
Natural Amity	23.41	63.77	6.11	6.71
Amity with clay removed	24.24	53.93	14.92	6.91
Amity with clay added	33.21	47.63	13.22	5.94

Simplicity might be the key word to the methods used in this research. Data obtained by the simplest and easiest to follow methods should have less chance for experimental error than an elaborate method with more complicated steps. Selection of the horizontal burette seemed simpler than injecting an air bubble and timing it along an inclined and calibrated tube. Tension plates seemed simpler than the pressure plate membrane apparatus reported by Gardner (6), Miller and Elrick (13), Rijtema (15) and others. Obtaining data from both the steady state and transient state on the same soil column appeared to be worthwhile. There is no chance for differences of soil or packing in a comparison of the two methods when all data come from the same soil column. The air entry problem referred to in Elrick (5) was easily overcome by boring air holes of sufficient size along one side of the columns. These holes were taped closed while packing and saturating the columns. Then the tape was removed and the air having only short distances to travel to enter the pores, could easily fill in as the water drained away. Evaporation loss was checked by weight measurements on a similar column while the one was being dried down. The evaporation after the column is loosely wrapped in Saran wrap is small and can be accounted for.

Several difficulties were experienced, some of which were overcome. Plate impedance is the hardest item to measure in the Elrick (5) method. Before and after measurements of plates did not agree and an average impedance would only be a poor assumption. Contact impedance cannot be accounted for in this procedure. Kunze and Kirkham's (12) method of plotting the theoretical curves and matching the experimental outflow by an overlay could be done using the same data as Elrick. A move to a constant temperature room became desirable after observing a fluctuation in outflow curves that matched an increase in room temperature. Also another change of using several layers of absorbent paper towels folded over the soil columns and kept moist by a damp sponge was adapted to prevent evaporation from the air holes in the sides of the columns.

RESULTS AND DISCUSSION

Diffusivity and hydraulic conductivity calculations were made on the experimental outflow data of the various soil textures. In comparing methods of calculating diffusivity and conductivity one soil texture and two soil samples were used. Alternate steady state and transient outflow measurements were made. Five-centimeter increments of tension were used on the transient method. For the steady state, a ten-centimeter pressure potential between the horizontal burettes was used. See Figure 1.

Kunze and Kirkham Method

The first method tried was that of Kunze and Kirkham (12). The outflow Q/Q_0 versus time $\underline{t}$ was plotted on log log paper. See Figures 21 and 22. The plotted points are then fitted to the theoretical curves shown in Figure 23 by moving the experimental curve along in the horizontal direction until the best fit is obtained. The reference point T_{RP} is then read from the experimental plot. The α_1^2 value is read from the auxiliary curves found on the theoretical plot. Diffusivity, D, is then calculated by the equation $D = L^2/\alpha_1^2 T_{RP}$ where L is the length of the soil sample. Hydraulic conductivity, K, may then be calculated by the relationship $K = D\Delta\theta/(H_2-H_1)$ where $\Delta\theta$ is the total outflow Q_0 divided by volume, V, of the soil sample. For an illustration see the following calculations and Figure 3 where the experimental plot is fitted to the theoretical curve at a = 0.

$H_1 = 40$ cm. H_2O	$V = 113.73$ cm.^{3}
$H_2 = 50$ cm. H_2O	$D = L^2/\alpha_1^2 T_{RP}$
$\alpha_1^2 = 2.47$	$D = 1.25071$ cm^2/min.
$T_{RP} = 69$ min.	$K = D\Delta\theta/(H_2 - H_1)$
$Q_0 = 2.69$ cm.	$K = 0.00296$ cm./min.
$L = 14.6$ cm.	

Plots of D and K vs. θ by the Kunze and Kirkham method (12) are found in Figures 4, 5, 6, and 7.

Methods of Gardner, Elrick, Rijtema, and Crank

The next methods tried were those of Gardner (6), Elrick (5), Rijtema (15) and Crank's square root of time method as reported by Jackson, van Bavel and Reginato (11). The first three of these plotted the outflow vs. time on semi-log paper and measured the slope. Gardner and Elrick assumed negligible plate impedance. By using the first term of the equation

$$Q/Q_0 = (1 - 8/\pi^2) \sum_{n=0}^{\infty} \frac{\exp[-(2n+1)^2 \pi^2 Dt/4L^2]}{(2n+1)^2} \ldots\ldots\ldots\ldots (4)$$

for _t_ greater than $1.2L^2/\pi^2 D$, a plot of ln $(Q_0 - Q/Q_0)$ vs. _t_ should yield a straight line with an intercept of ln $8/\pi^2$. In order to find

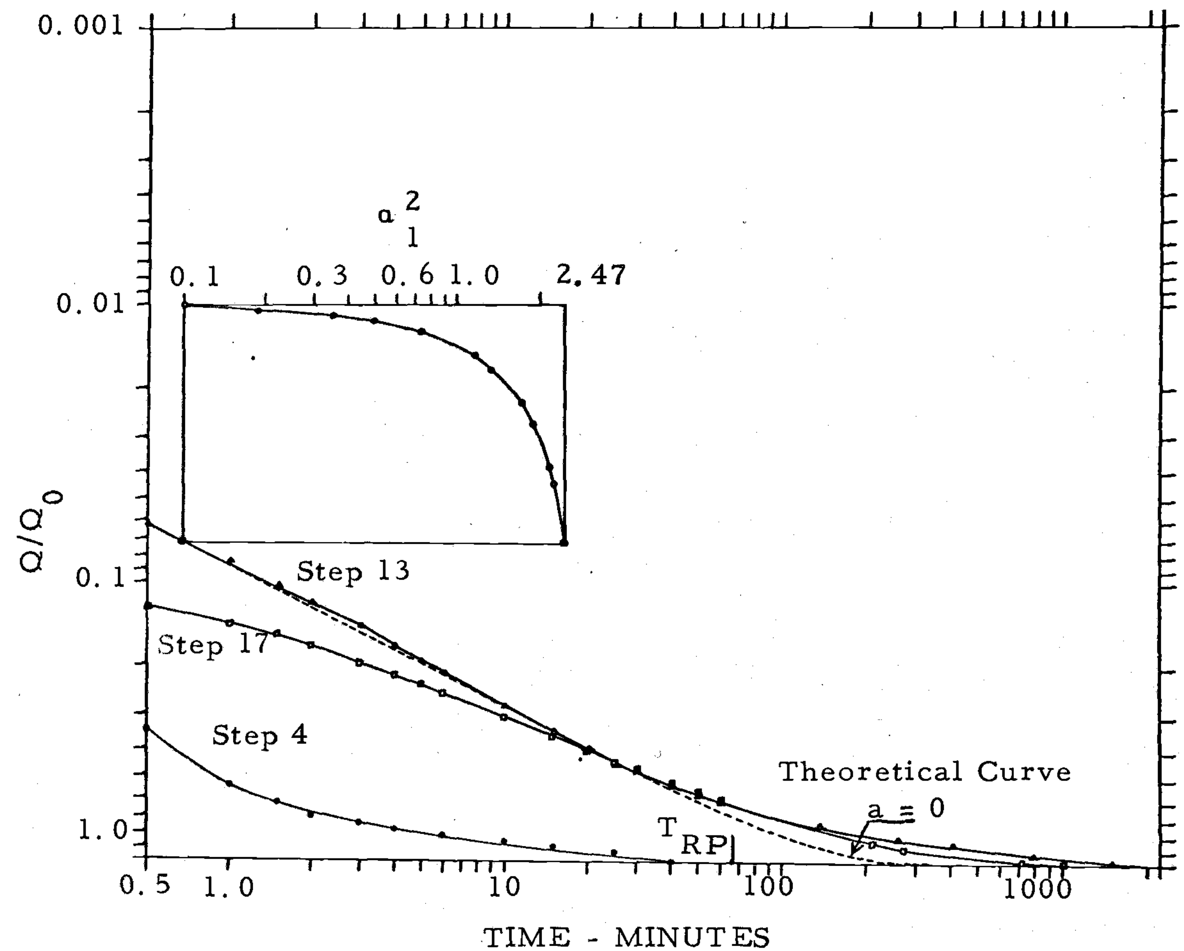

Figure 3. Outflow curves, soil column two, showing match of Step 13 experimental data to the a = 0 theoretical curve.

the slopes needed for these methods and also those of Rijtema and Crank a linear regression was run on the IBM 1620 computer using a ready-made program for linear regressions of semi-log plots.[2] The data selected for these regressions was determined by using the D values calculated by the Kunze and Kirkham method. Time _t_ for Gardner was required to be greater than $1.2\ L^2/\pi^2 D$ and for both Elrick and Rijtema, $t > 0.4L^2/\alpha_1^2 D$ was required. Rijtema used the intercept greater than $\ln 8/\pi^2$ to find plate impedance. It was found that in only one pressure increment out of 24 was the intercept greater than $\ln 8/\pi^2$ for the data used. This means that these three methods do not fit the data obtained and that more terms of equation 4 would have to be used in order to use these methods with this data. No attempt was made to do this. A computer program would have to be written to solve equation 4 for more than one term. The outflow plots of Gardner, Elrick, and Rijtema are included to show the variation in methods of plotting. See Figures 24, 25, 26.

Crank's square root of time method also used a slope measurement which was calculated at the same time as the others on the computer. The data was selected by first plotting $(Q/Q_0)^2$ vs. time on rectangular coordinate paper; then drawing a straight line through the initial outflow points. The point where the experimental curve

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[2] IBM 1620 computer at OSU Department of Statistics.

broke from the straight line was selected as the upper limit of time for data selection. See Figure 8. This was done because Crank assumed a finite column would act like one of semi-infinite length during the initial portion of outflow. His only other assumption was that plate impedance was negligible. Diffusivity was then calculated from the equation $D = L^2 \pi m/4$ where L is the length of soil sample and m is the slope of the plot (Q/Q_0) vs. time$^{\frac{1}{2}}$. A slight modification of plotting $(Q/Q_0)^2$ vs. time was made for ease of calculation. Hydraulic conductivity was then figured by the same method as in Kunze and Kirkham of $K = D\Delta\theta / (H_2 - H_1)$. See Figures 9 and 10 for plots of D and K vs. θ by the Crank method. For a comparison of methods see Figures 11 and 12 where the curves of the least squares fit, computed on the Alwac III e computer,[3] are plotted.

Comparison of Computer Solutions of Moisture Content to Experimental Measurements

One other comparison of methods of measuring diffusivity and hydraulic conductivity was made. The K and D values from the Crank and the Kunze and Kirkham methods were inserted into a computer program which solves the diffusion equation for the one-dimensional, vertical, drydown case. A comparison of the calculated moisture

[3] Operated by the Department of Mathematics, Oregon State University.

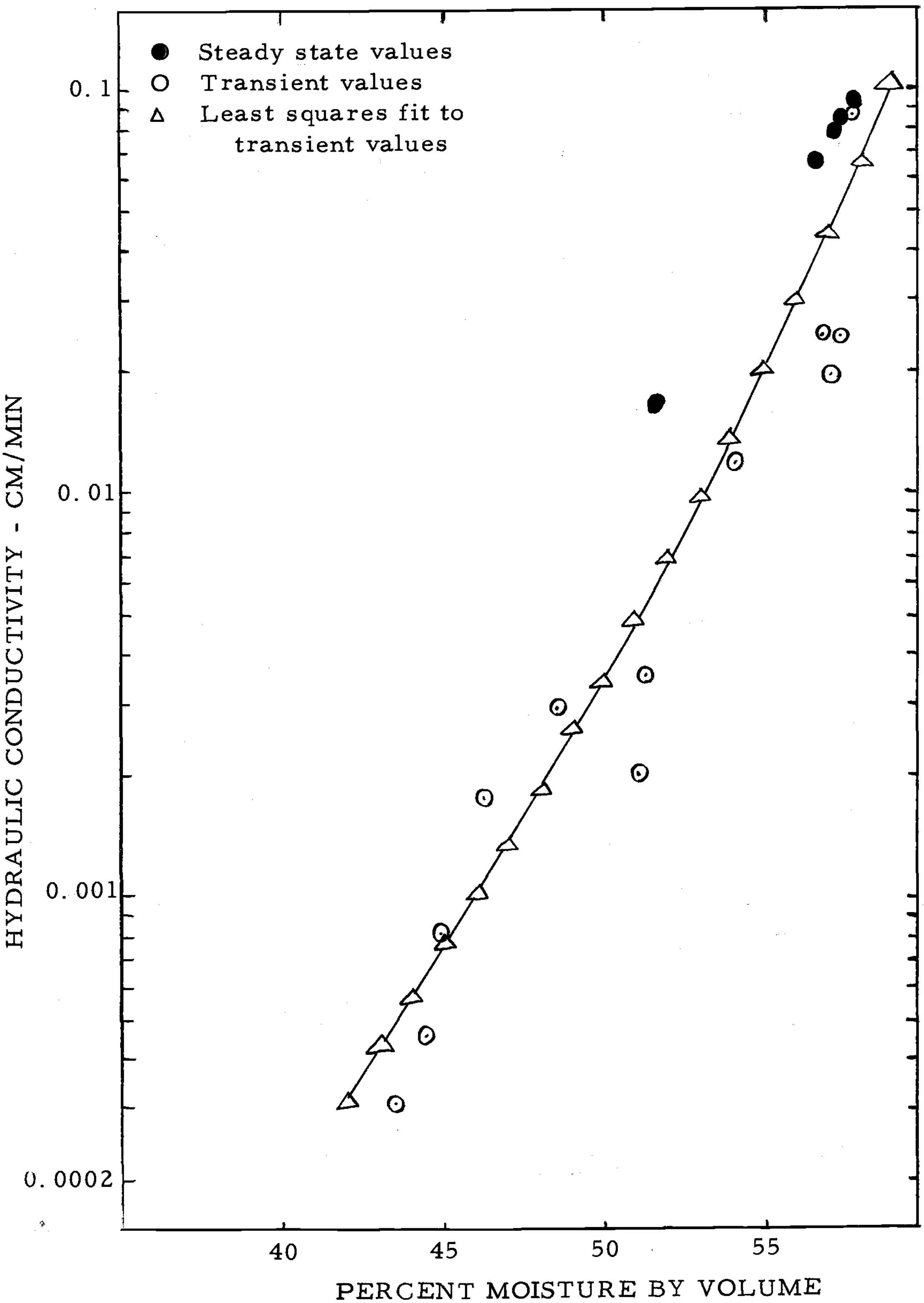

Figure 4. Hydraulic conductivity versus θ by Kunze and Kirkham method. Soil column two, Amity silt loam.

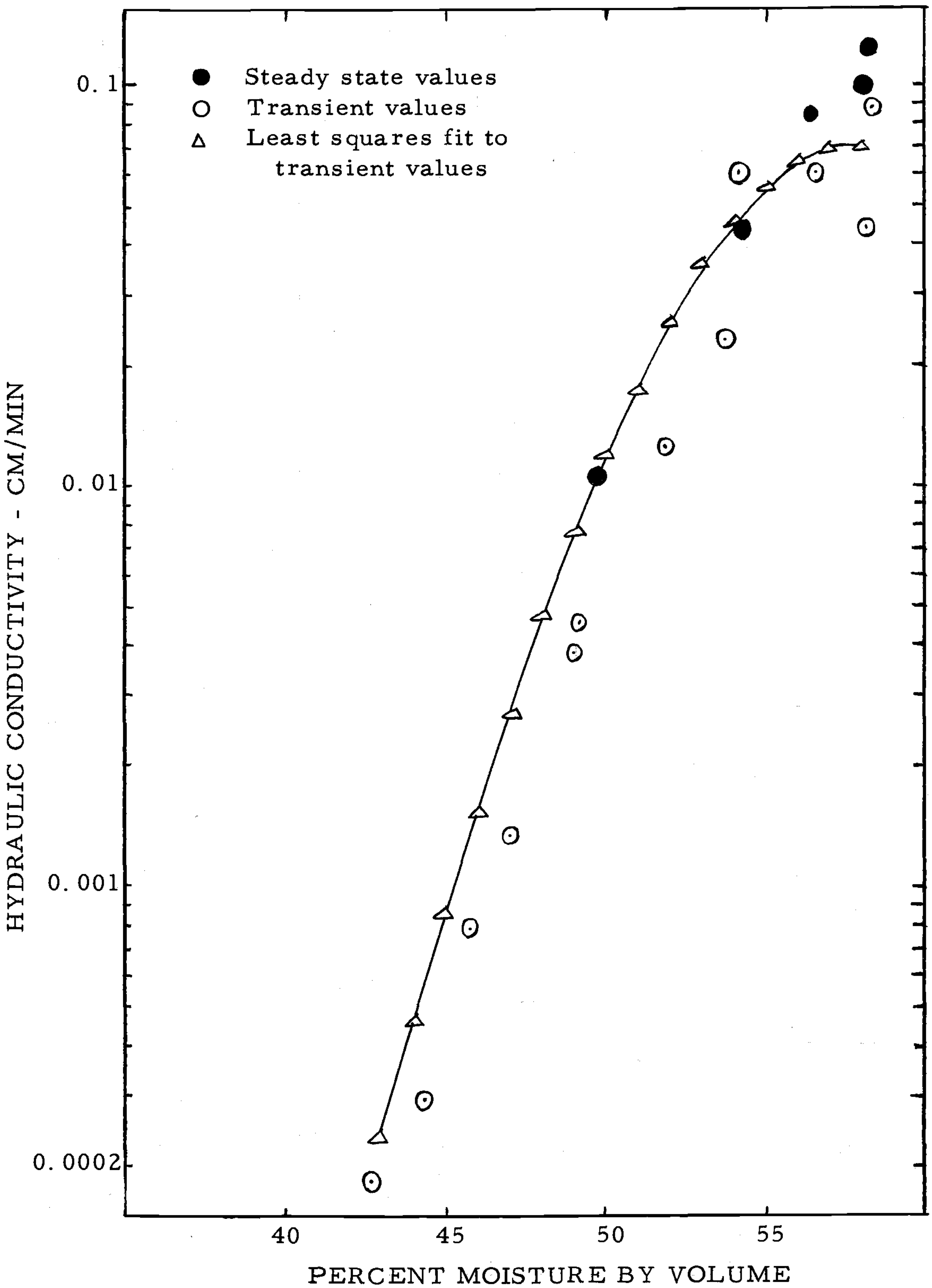

Figure 5. Hydraulic conductivity versus θ by Kunze and Kirkham method, soil column one, Amity silt loam.

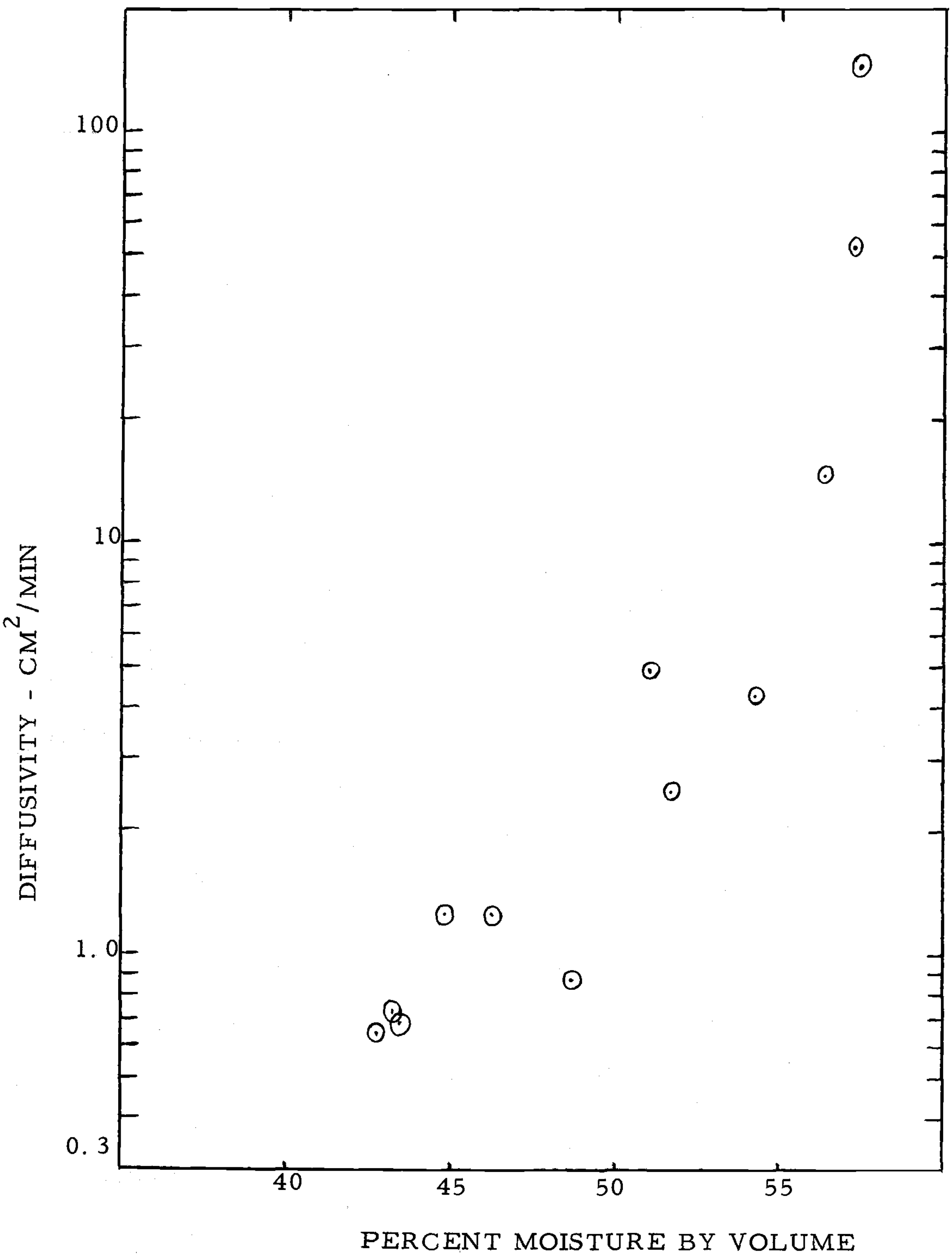

Figure 6. Diffusivity versus θ. By Kunze and Kirkham method, Amity silt loam, soil column two.

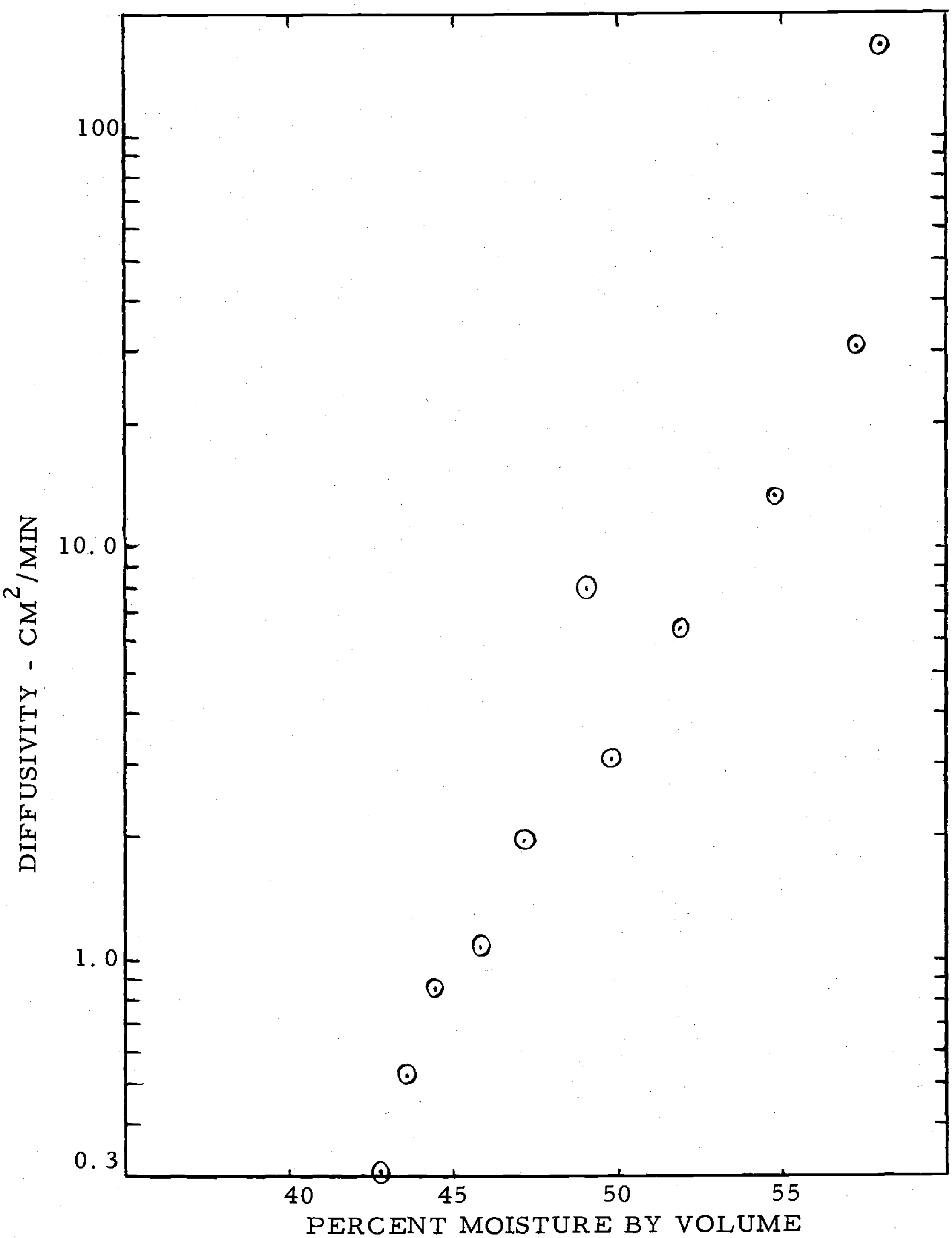

Figure 7. Diffusivity versus θ. By Kunze and Kirkham method, Amity silt loam, soil column one.

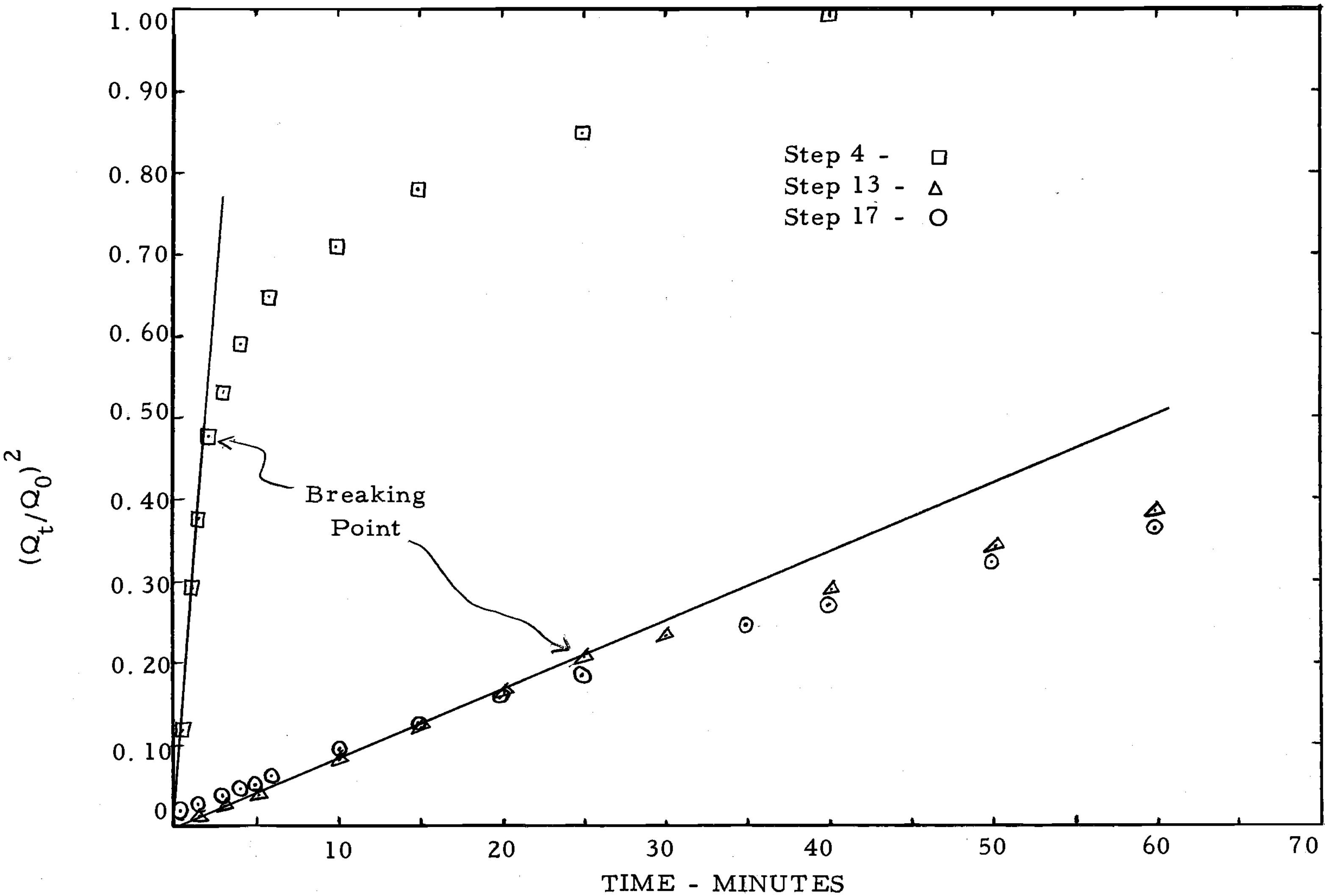

Figure 8. Outflow quantity squared versus time. Used with the Crank method. Amity silt loam, soil column two.

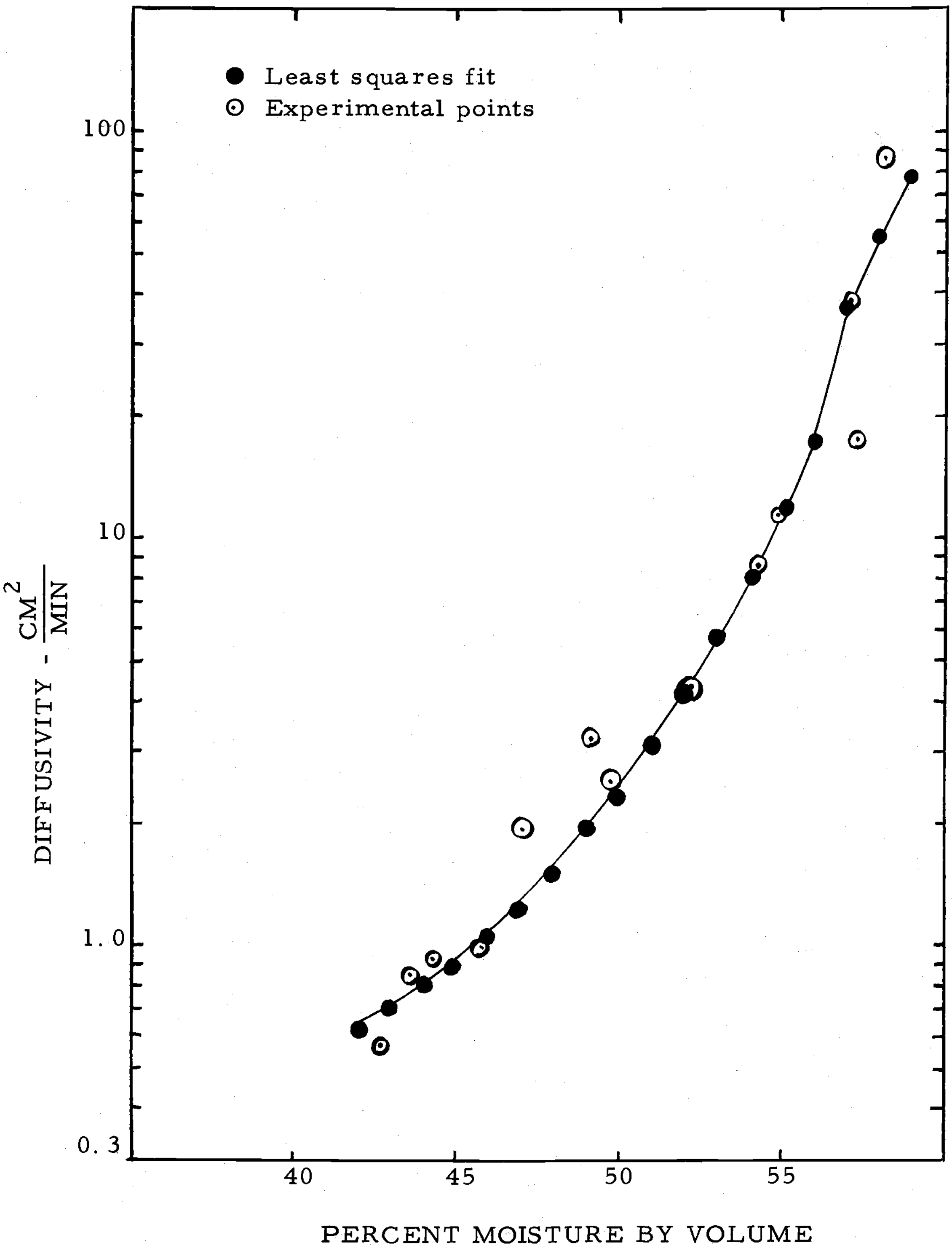

Figure 9. Diffusivity vs. θ by Crank method for soil column one, Amity silt loam.

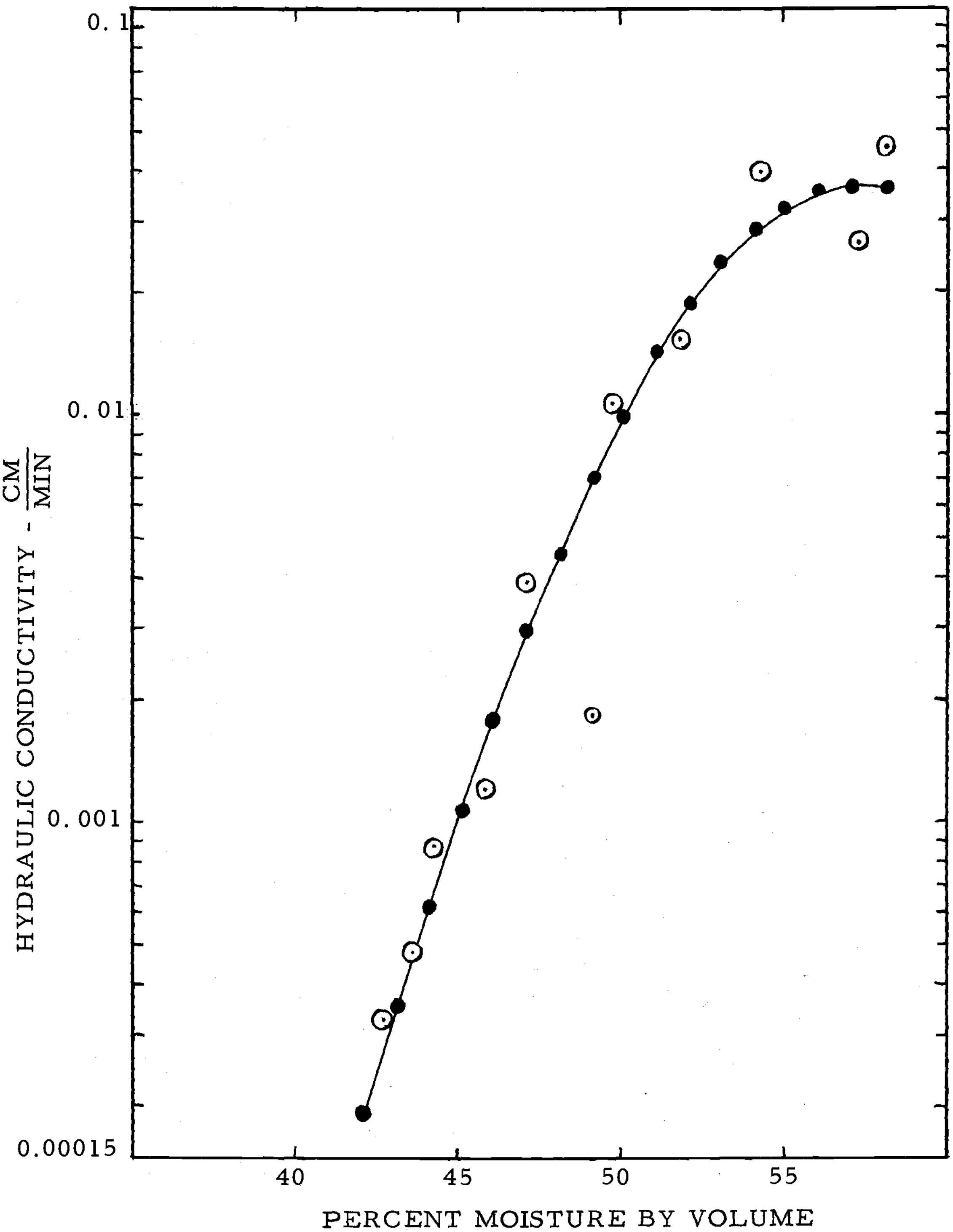

Figure 10. Hydraulic conductivity vs. θ by Crank method for soil column one, Amity silt loam.

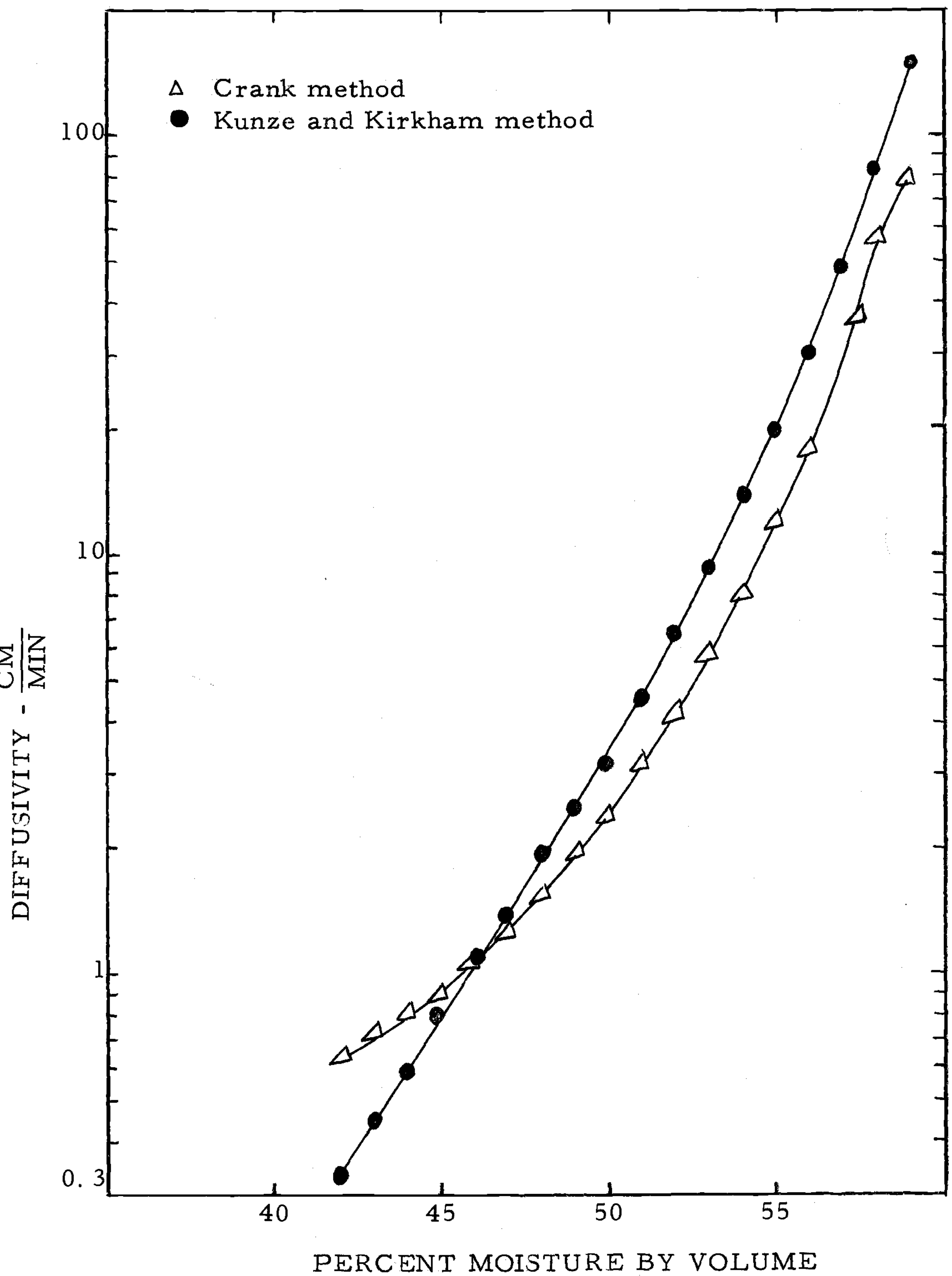

Figure 11. Diffusivity vs. θ for two methods, least squares fit for soil column one, Amity silt loam.

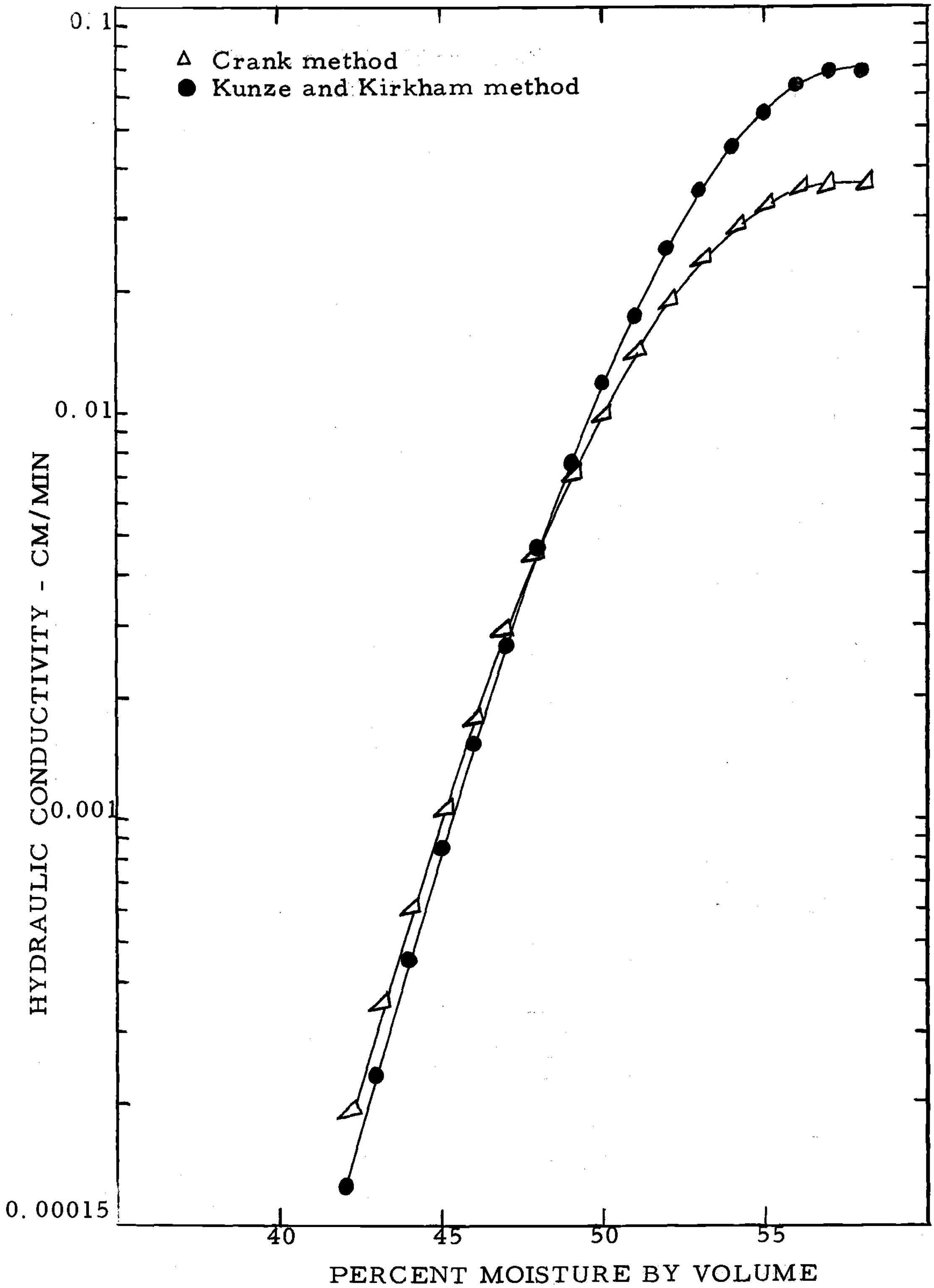

Figure 12. Hydraulic conductivity vs. θ for two methods, least squares for soil column one, Amity silt loam.

distribution in a 100 cm column after draining for 70 hours was made with actual measurements made after 70 hours in the laboratory. This computer program has been reported in the annual progress report of Oregon contributing project number 467 to Western Regional Research Project W-51, Drainage Design for Irrigation Agriculture, Oregon Agricultural Experiment Station, Oregon State University, October 24, 1963.

The following table shows calculated and measured moisture contents in each of six sections of a 100 cm column after they had nearly reached equilibrium (70 hours). The measured bulk densities are also shown. The average bulk density of the column used for the Kunze and Kirkham and the Crank methods was 1.1442 grams per cm^3. The values are listed in order from the top of the column down.

Table 3. Experimental moisture content compared to computer solutions using D and K values calculated by Crank and Kunze and Kirkham methods.

Experimental 100 cm col.		Calculated	
		Kunze	Crank
BD	θ	θ	θ
1.14906	0.4381	0.4247	0.4214
1.15613	0.4625	0.4326	0.4276
1.14818	0.4835	0.4431	0.4364
1.15293	0.4966	0.4600	0.4511
1.14817	0.5525	0.4953	0.4870
1.13878	0.5732	0.5544	0.5546

The soil used in the 100 cm column was the same Amity silt loam. It was packed on the mechanical packer and saturated in the vacuum chamber in approximately the same manner as the other soil columns used in this study. The column was packed in six sections and then put together. Figure 30 of the appendix is a plot of the values in Table 3. The experimental data apparently had a higher moisture content at saturation ($\theta = 0.58$) than was used on the computer program ($\theta = 0.57$). This could explain part of the difference between curves and in fact could account for most of the difference at the two end points. The two computed solutions are from the same experimental data and should start out from saturation together as they do in the lowest part of the curves in Figure 30. They draw apart because the ratio of the D values to K values of the Crank method are lower near saturation and therefore do not retain as much water as the computer solution of the Kunze and Kirkham method indicates. This is because diffusivity tends to hold water up in the soil and hydraulic conductivity tends to let water flow down in response to gravity. The variation in packing as shown by the bulk densities probably accounts for the differences between calculated and experimental values for water retained. The bulk densities being higher would tend to increase diffusivity and decrease hydraulic conductivity.

Comparison of Replications

A comparison of the attempt at duplication is shown in the following table 4. It is observed that the ratios of column two/column one ranged from 1.89 to 0.23 for the Crank method and from 1.76 to 0.24 by the Kunze and Kirkham method. Perhaps this difference could be attributed to the difference in packing the two soil columns. Column one had a bulk density of 1.1442 and column two had 1.1457 grams/cm^3.

Table 4. Comparison of K values from soil column one and column two.

	Crank Method			Kunze and Kirkham Method		
	Col. One	Col. Two		Col. One	Col. Two	
θ	K $\frac{cm}{min}$	K $\frac{cm}{min}$	Ratio	K $\frac{cm}{min}$	K $\frac{cm}{min}$	Ratio
.43	.00036	.00068	1.89	.00025	.00044	1.76
.44	.00064	.00078	1.22	.00047	.00058	1.23
.45	.00110	.00093	.85	.00088	.00078	.89
.46	.00185	.00112	.61	.00161	.00104	.65
.47	.00300	.00139	.46	.00287	.00141	.49
.48	.00471	.00174	.37	.00495	.00191	.39
.49	.00711	.00221	.31	.00823	.00260	.32
.50	.01033	.00284	.27	.01316	.00358	.27
.51	.01438	.00366	.25	.02016	.00497	.25
.52	.01915	.00484	.25	.02947	.00696	.24
.53	.02434	.00613	.25	.04096	.00986	.24
.54	.02944	.00789	.23	.05393	.01411	.26
.55	.03381	.01008	.30	.06702	.02045	.31
.56	.03677	.01275	.35	.07830	.03003	.39
.57	.03778	.01591	.42	.08568	.04472	.52
.58	.03659	.01951	.53	.08750	.06760	.77

Comparison of D and K Values on Different Soil Textures

The attempt to compare D and K values on different soil textures was hampered by an apparent breakdown of soil structure on the sand and silt phase. See the mechanical analysis, Table 2. The calculations of D and K for these different textures were made by the steady state and Kunze and Kirkham methods. Plots of D and K vs. θ are shown in Figures 13 to 20. Two soil samples were run of each soil texture. Tables 5 and 6 show the difference in D and K values for the different soil textures. The spaces shown between the D and K values in Tables 5 and 6 represent the total pressure increment over which the measurement was made. For example in soil column one of the clay soil, Table 5, the value found on the 65 cm. tension line, $D = 1.161$, is the measurement made for the 15 cm. increment of tension from 50 cm to 65 cm. This value presumably represents the diffusivity near the initial 50 cm pressure because only the first 10 to 15% of outflow is used to calculate D by the Kunze and Kirkham method. The larger increments were needed to get a sufficient total outflow to make an accurate measurement in the drier range. That portion of the columns having a number on each line, as in Table 6, lines 5 to 30 cm for soil column one of the natural soil, each represent a five centimeter increment. If the erratic values are discounted as being experimental error, then the natural Amity silt loam shows a higher diffusivity and conductivity than the other two columns. The column with clay added appears to have a set of D and K values that compare logically with those of the natural soil. Being a soil with a finer texture, it follows that its diffusivity and unsaturated hydraulic conductivity will be lower than the natural soil. The column that was supposed to have a coarser texture than the natural Amity silt loam, also had a lower diffusivity and hydraulic conductivity. It is believed by the author that this soil sample suffered a structural breakdown. The soils treatments were not dispersed before making the mechanical analysis, but the soil aggregates of the sand and silt treatment may have been dispersed by the air jetting and handling or perhaps by oven drying. The single-grained particles then had a smaller pore size and consequently a lower than normal diffusivity and hydraulic conductivity. Plots of the three soil textures at the same tension were made. See figures 27, 28, 29. These show the relationship of outflow vs. time for the different textures.

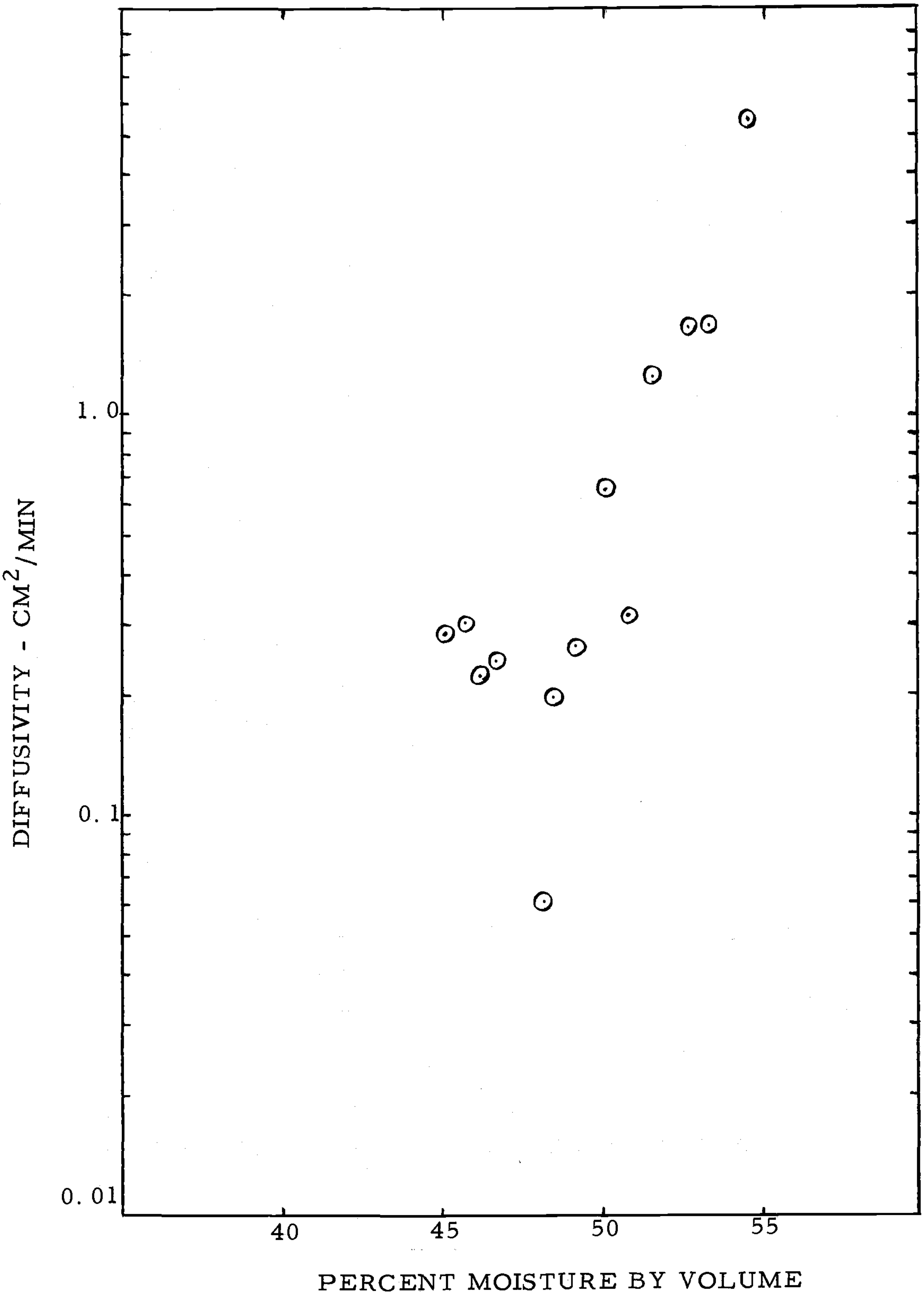

Figure 13. Diffusivity vs. θ on Amity silt loam with clay removed, soil column one. Calculations by Kunze and Kirkham method.

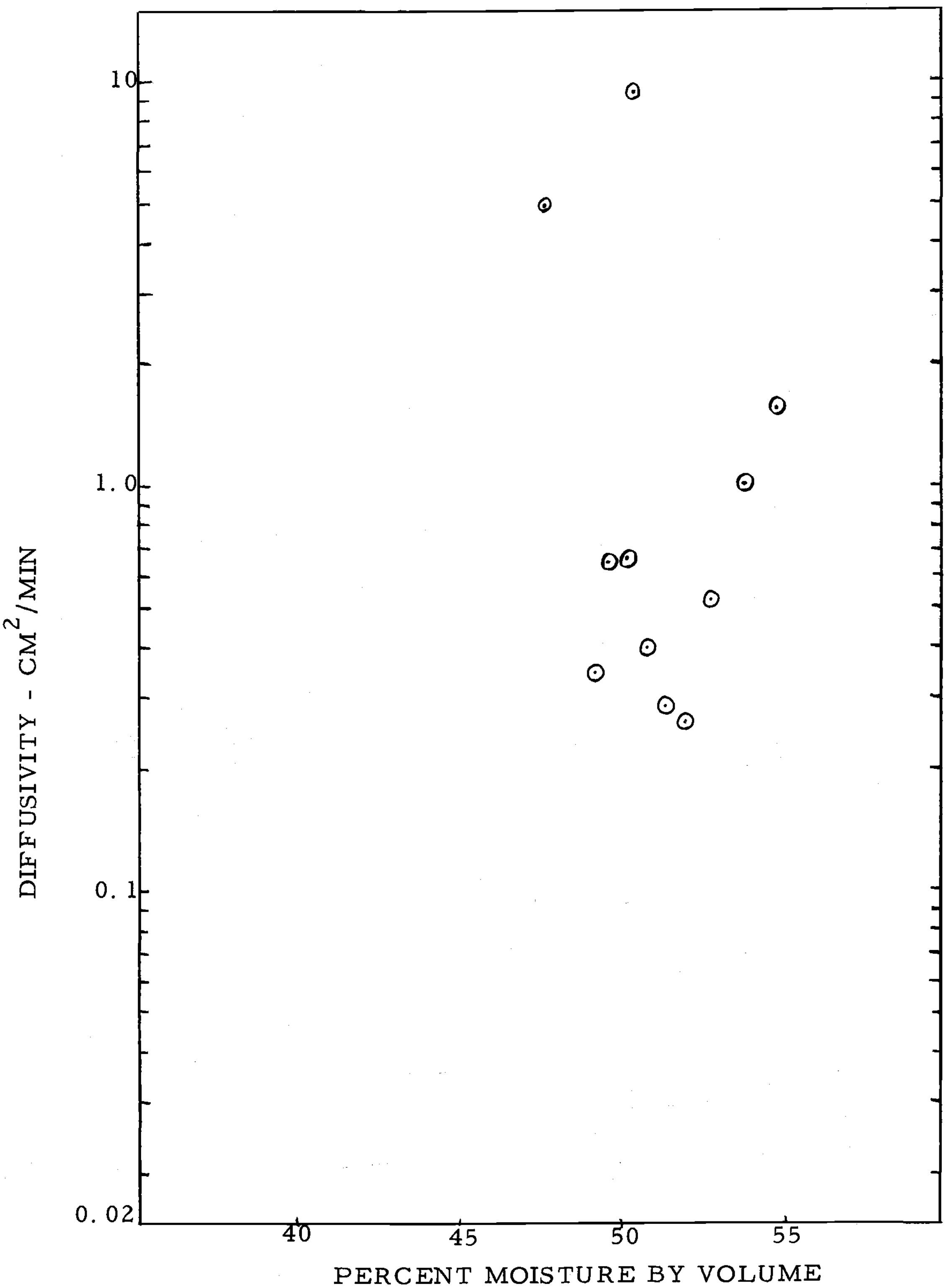

Figure 14. Diffusivity vs. θ on Amity silt loam with clay removed, column two. Calculations by Kunze and Kirkham method.

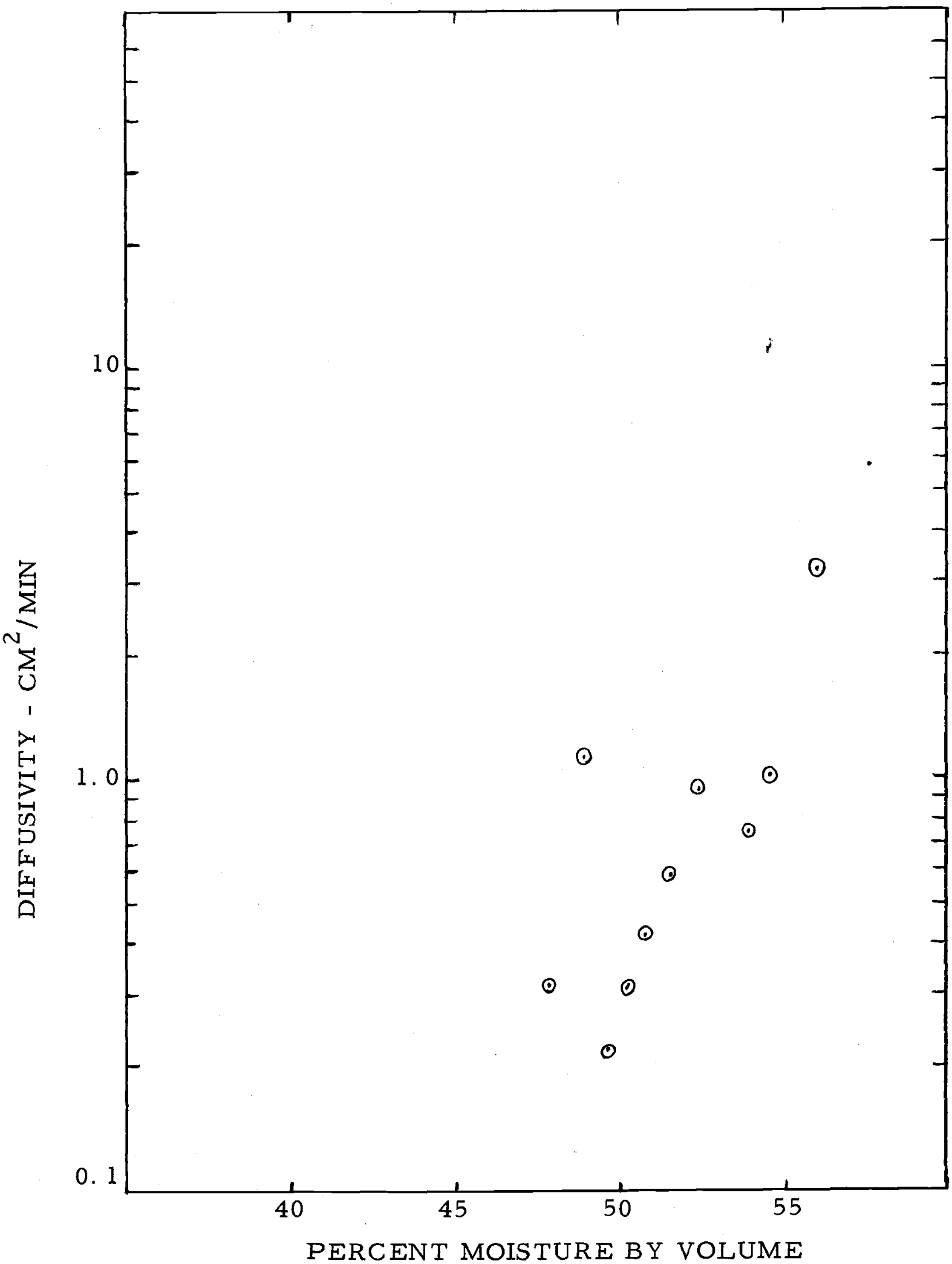

Figure 15. Diffusivity vs. θ on Amity silt loam with clay added, soil column one. Calculations by Kunze and Kirkham method.

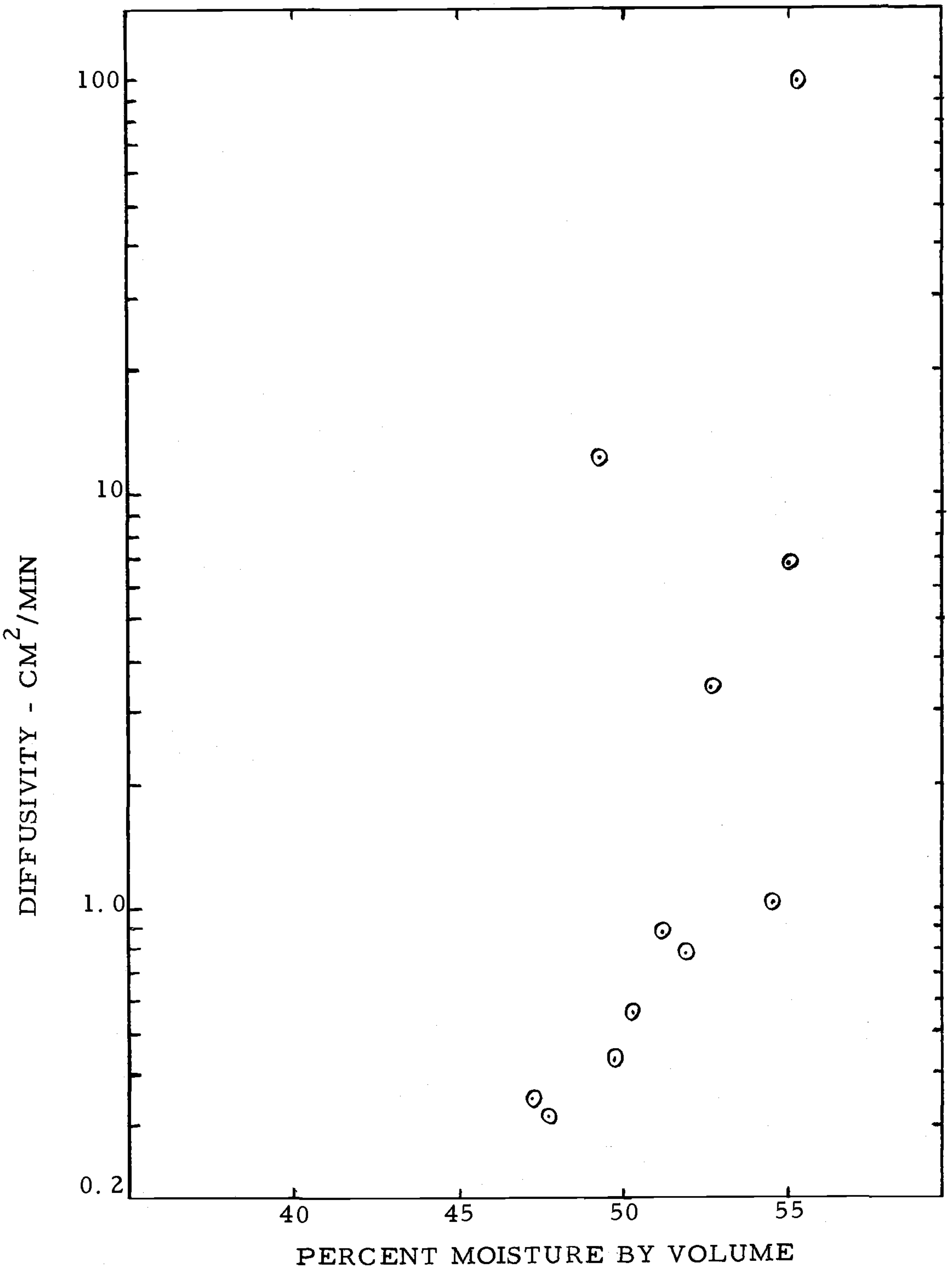

Figure 16. Diffusivity vs. θ on Amity silt loam with clay added, soil column two. Calculations by Kunze and Kirkham method.

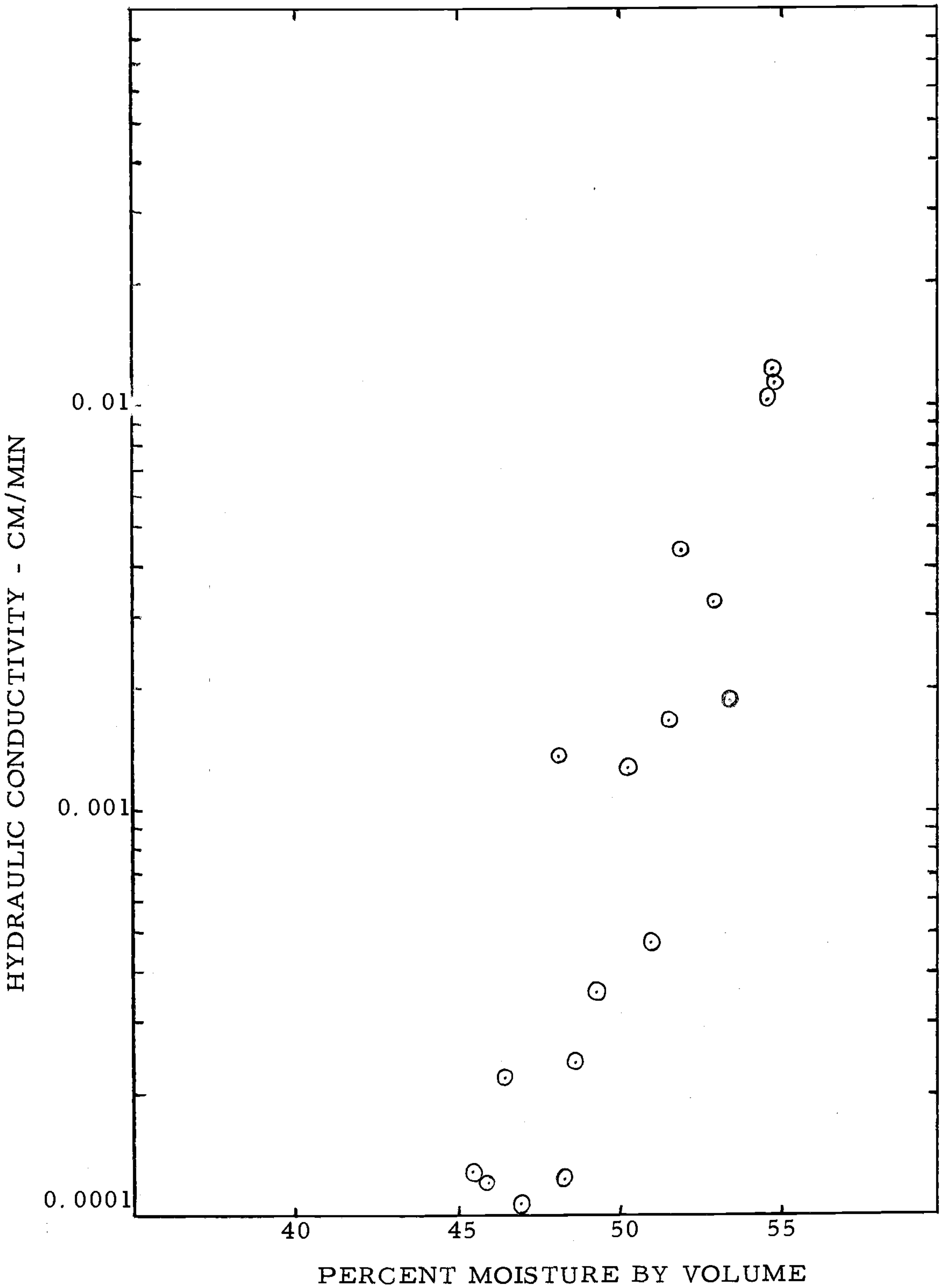

Figure 17. Hydraulic conductivity vs. θ for Amity silt loam with clay removed, column one. Calculations by Kunze and Kirkham method.

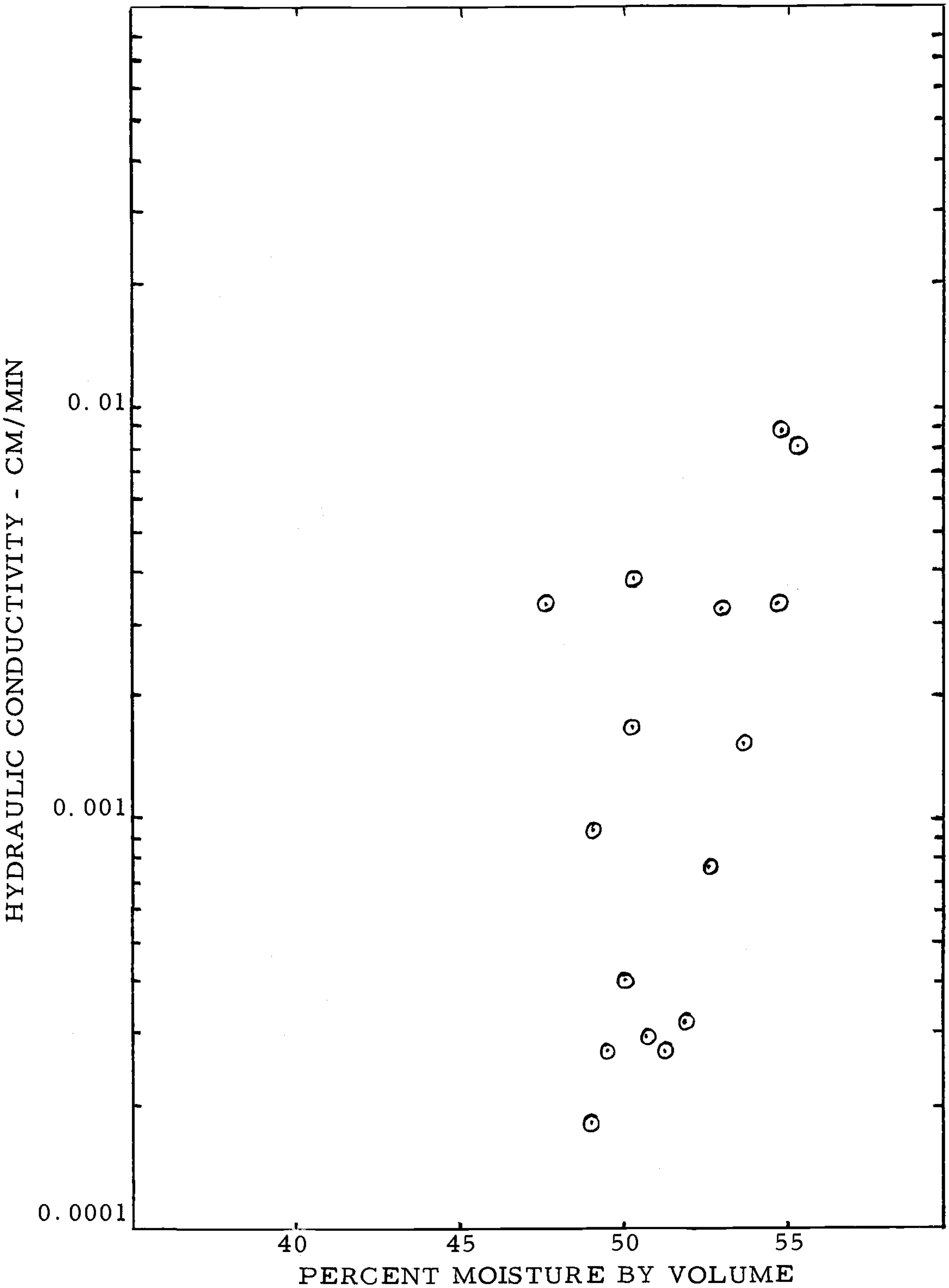

Figure 18. Hydraulic conductivity vs. θ for Amity silt loam with clay removed, column two. Calculations by Kunze and Kirkham method.

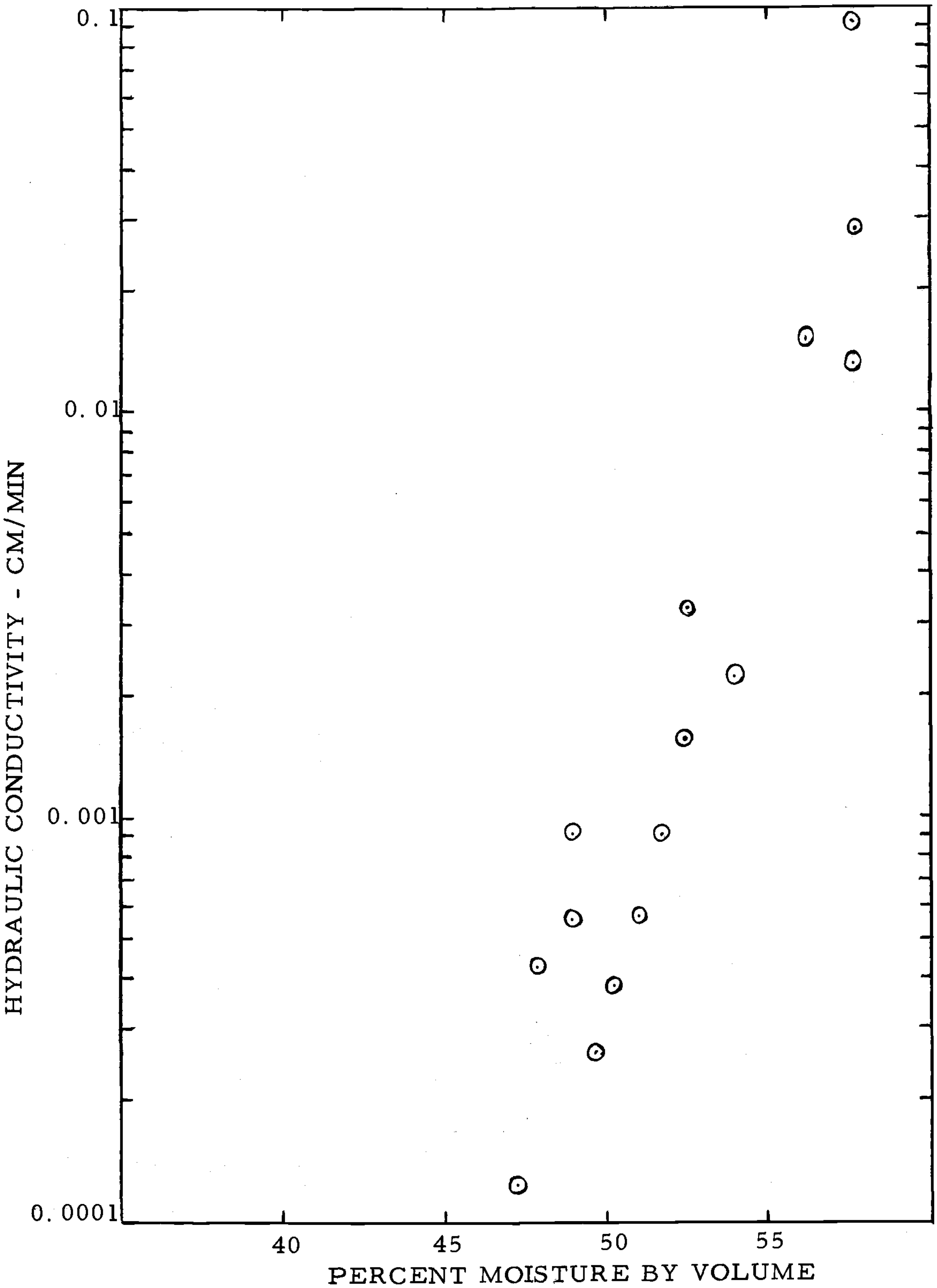

Figure 19. Hydraulic conductivity vs. θ for Amity silt loam with clay added, column one. Calculations by Kunze and Kirkham method.

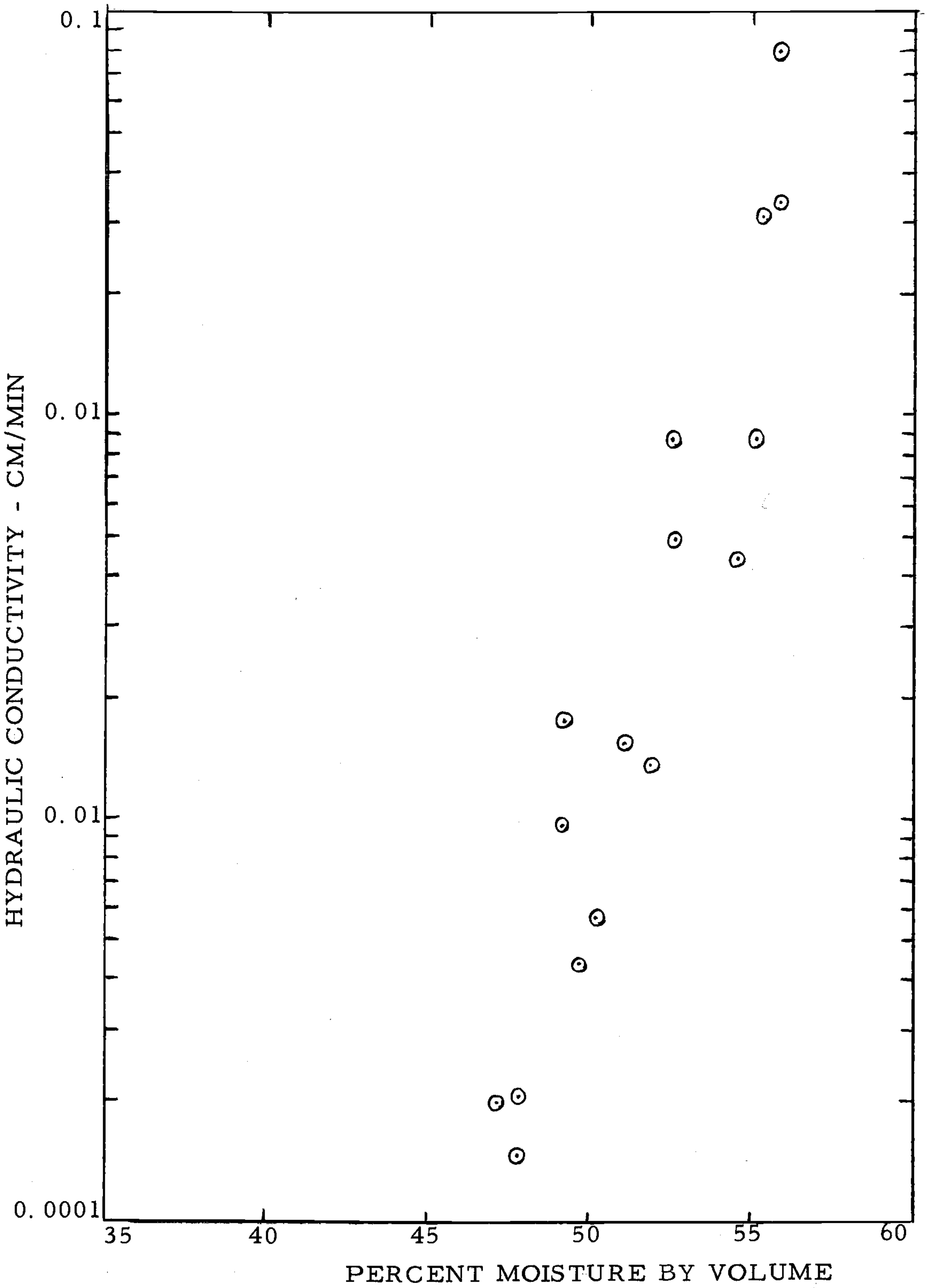

Figure 20. Hydraulic conductivity vs. θ for Amity silt loam with clay added, column two. Calculations by Kunze and Kirkham method.

Table 5. Comparison of $D(\frac{cm^2}{min})$ for the different soil textures. All calculations were by the Kunze and Kirkham method.

Tension cm.	Clay Soil Columns One	Clay Soil Columns Two	Natural Soil Columns One	Natural Soil Columns Two	Silt Soil Columns One	Silt Soil Columns Two
5			169.2	146.3		
10			31.26	52.87		
15	5.876	99.50	13.50	14.63	5.565	
20	3.249	6.846	6.499	4.310	1.692	1.597
25	0.761	1.233	3.056	2.530	1.655	1.021
30	0.932	3.417	8.029	4.931	1.233	
35	0.588	0.761			0.317	0.527
40	0.426	0.895	1.984	0.881	0.748	0.262
45	0.319	0.561			0.262	0.287
50	0.223	0.437		1.251	0.194	0.401
55					0.060	1.550
60		12.34	1.092	1.233		
65	1.161					0.669
70						0.659
75	0.328	0.347	0.863	0.731	0.241	
80						
85					0.227	0.342
90			0.523	0.690		
95					0.301	
100						
105		0.311	0.308	0.654	0.278	5.076

Table 6. Comparison of K($\frac{cm}{min}$) for the different soil textures. All calculations were by the Kunze and Kirkham method.

Tension cm.	Clay Soil Column One	Clay Soil Column Two	Natural Soil Column One	Natural Soil Column Two	Silt Soil Column One	Silt Soil Column Two
5			0.08900	0.08659		
10			0.04775	0.02410		
15	0.01311	0.0328	0.06098	0.01954	0.0104	
20	0.01521	0.00891	0.02376	0.02470	0.00188	0.00333
25	0.00224	0.00484	0.01284	0.01205	0.00331	0.00152
30	0.00155	0.00499	0.00451	0.00355	0.00166	
35	0.00091	0.00118			0.00047	0.00076
40	0.00057	0.00156	0.00388	0.00204	0.00127	0.00032
45	0.00039	0.00057			0.00035	0.00027
50	0.00027	0.00044		0.00296	0.00024	0.00029
55					0.000123	0.00386
60			0.00133	0.00176		
65	0.0091	0.01073				0.00040
70						
75	0.00042	0.00021	0.00080	0.00081	0.00011	0.00027
80						
85					0.00022	0.00018
90			0.00029	0.00046		
95					0.00012	
100						
105		0.00015	0.00018	0.00032	0.00013	0.0034

SUMMARY AND CONCLUSIONS

The transient outflow methods of Gardner (6), Elrick (5), Rijtema (15), Kunze and Kirkham (12), and Crank's square-root-of-time method for measuring diffusivity and calculating unsaturated hydraulic conductivity were compared. Also the steady state method of Elrick's of calculating unsaturated hydraulic conductivity was compared to the transient methods. Not all the methods mentioned apply to the experimental data that was obtained from the two soil columns of Amity natural silt loam. The methods of Gardner, Elrick, and Rijtema require the intercept of their plots to be ln $8/\pi^2$ or greater. This limitation was not met in 23 out of 24 pressure increments. Had it been possible to use more than one term of equation 4

$$Q/Q_0 = (1-8/\pi^2) \sum_{n=0}^{\infty} \frac{\exp[-(2n+1)^2\pi^2 Dt/4L^2]}{(2n+1)^2}$$

the intercept might have been greater than ln $8/\pi^2$. It appears that development of a computer program for more than the first term of equation 4 would be required for solutions of D and K by these methods. The use of Kunze and Kirkham's method gave results that appear good in the range from saturation to about 47% moisture content by volume. The outflow curves in the drier ranges did not fit the theoretical curves as well. It appears that non-constant diffusivity becomes a hindrance with this method in the drier ranges. The square-root-of-time method appears to give reasonable results in the drier range, below 47% moisture content, but the values measured in the range between 47% and saturation (57%), appear to be low. This is probably due to the assumption of negligible plate impedance.

The effect of clay content on measured values of diffusivity and unsaturated hydraulic conductivity was investigated. By dividing an Amity silt loam sample into three parts, an attempt was made to subtract clay particles from one part and add them to one of the other two, making three soil textures from one. This effort met with partial success. The clay particles added to a natural soil gave the desired increase in percent clay. The sand and silt sample however had an apparent breakdown in soil structure. The clay removed was 4% of the original sample, but the 2 to 20μ silt fraction decreased ten percentage points, the 20 to 50μ silt increased eight points and the sand and clay fractions remained about the same. This made a soil with a high silt content that acted more like a clay loam than the coarser texture soil it was supposed to be. The natural soil had higher (about twice as high) D and K values than either of the altered soils. Further work with other soil textures and repeated experiments are needed to substantiate these measurements.

The differences between the corresponding values of hydraulic conductivity of the two natural Amity silt loam soil columns were studied. The replication between soil columns showed the K values of one column to range from nearly two times larger to four times smaller than the other column.

It is recommended that alternate steady state and transient outflow measurements not be made on the same soil column because this seemed to be associated with plate failure. Difficulty was encountered with the inflow plates admitting air at some point less than 100 cm tension, thus causing a loss of measurements and time during an experimental run. Taking small increments of pressure to validate the assumption of nearly constant diffusivity appears successful in this work. The results obtained indicate small increments are very desirable in the range from saturation to about 100 cm of water tension. This range includes most of the freely moving moisture that will be affected by drainage systems.

It should be pointed out that any conclusions drawn from this investigation can only be tentative, based on results of only a few experimental measurements. However it appears that usage of the Kunze and Kirkham method works very well to the point where the outflow curves flatten out because of non-constant diffusivity. From this point on it is the thought of the author that the square-root-of-time method yields better results. In the comparison of experimental measured moisture contents for a 100 cm vertical soil column to the computed solutions using D and K values of the Kunze and Kirkham and the Crank methods, it appears that the Kunze and Kirkham values fit better than the Crank values.

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APPENDIX

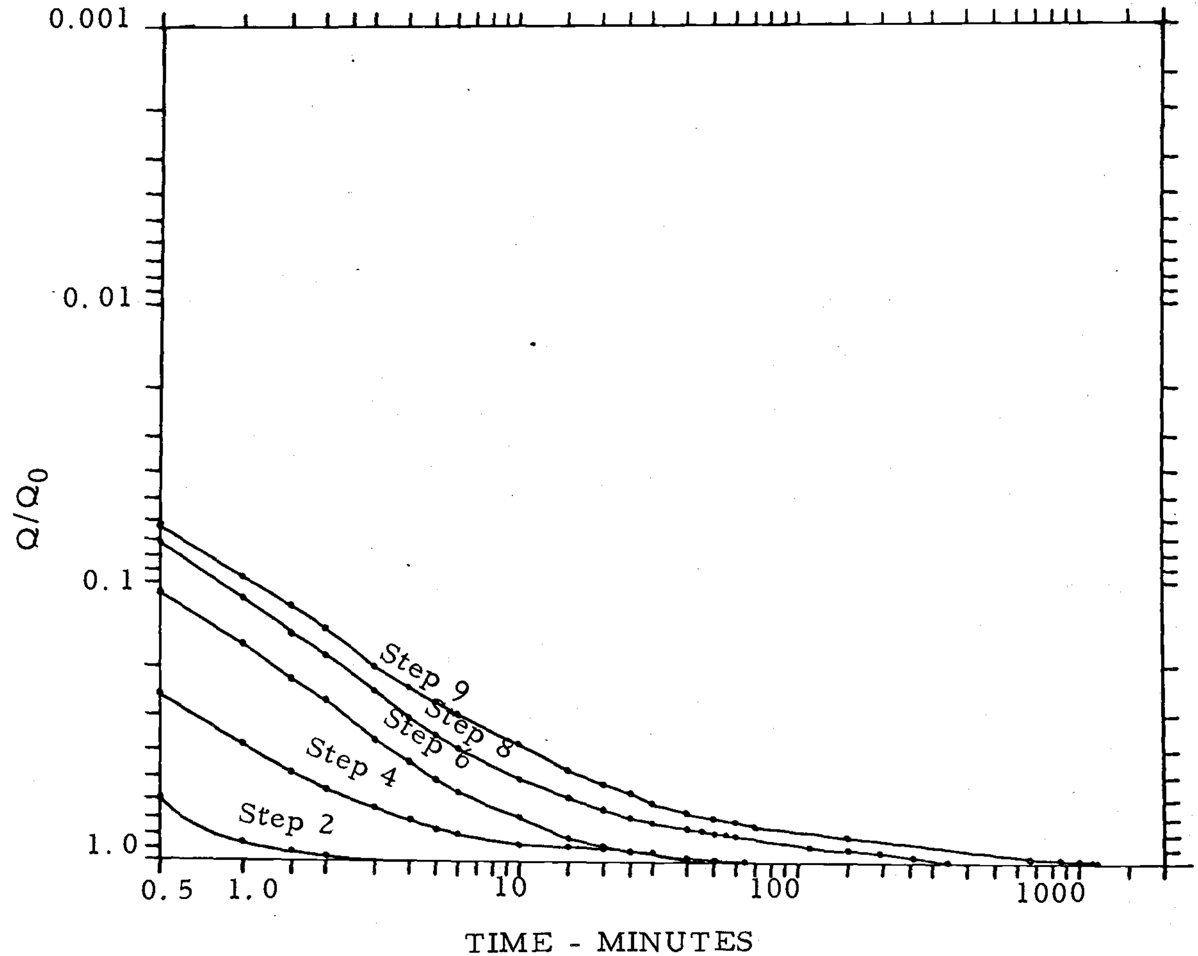

Figure 21. Outflow curves of Amity silt loam, soil column one. Step numbers refer to pressure increments, step two was first 5 cm increment from saturation.

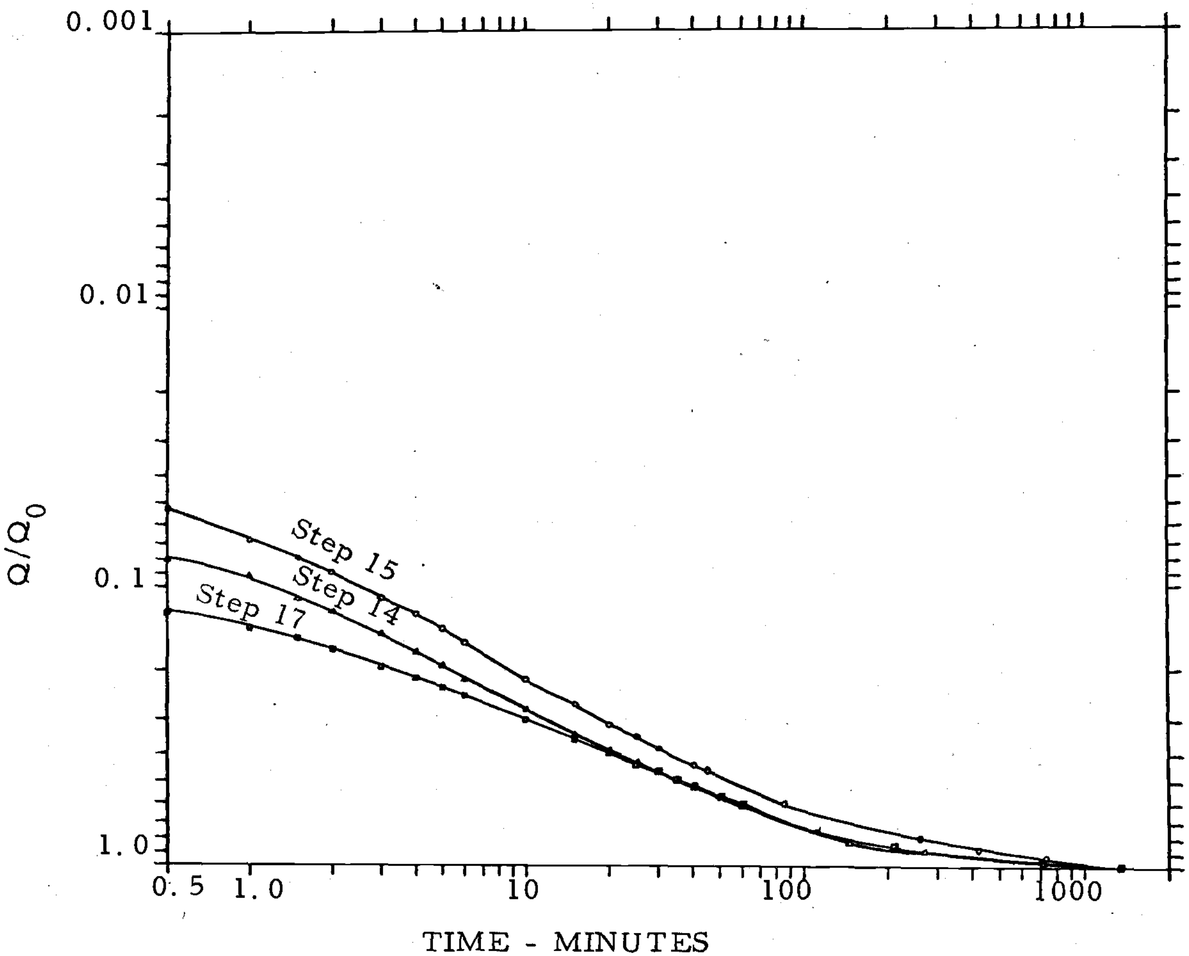

Figure 22. Outflow curves of Amity silt loam, column two. Steps 14, 15, 17 are in the dry range with 17 being the driest.

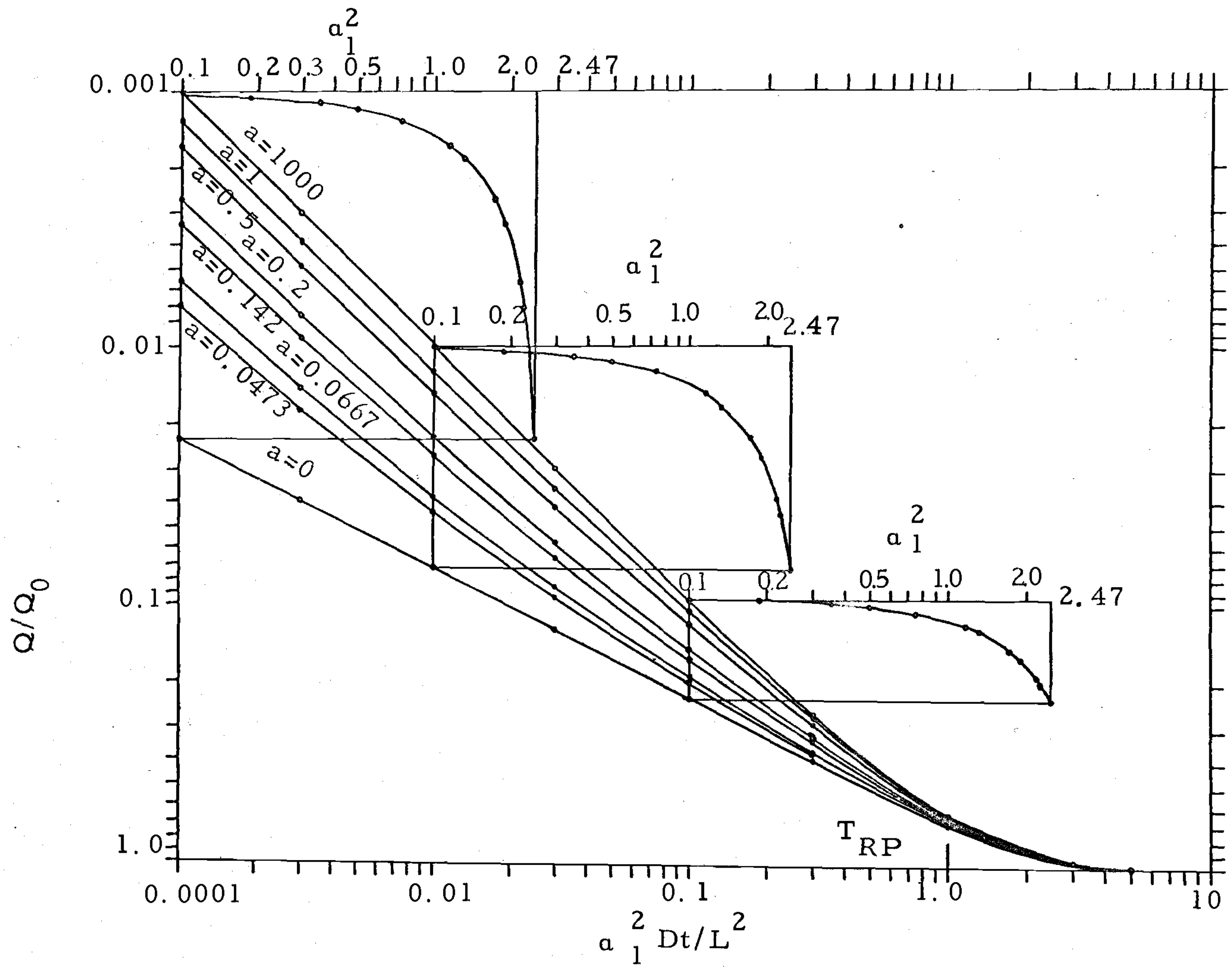

Figure 23. Theoretical curves used to calculate diffusivity from outflow data, from Kunze and Kirkham (12).

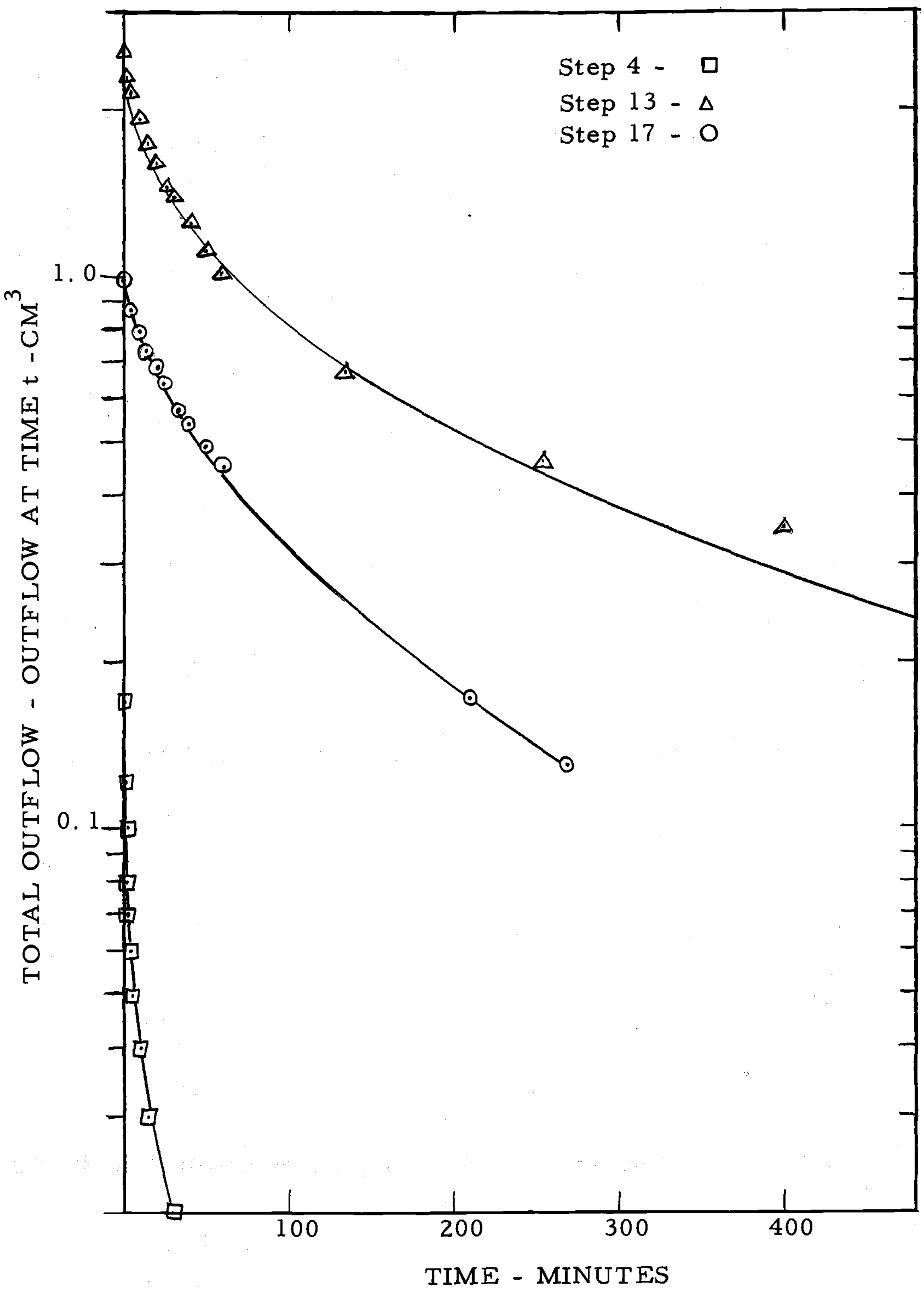

Figure 24. Outflow versus time according to the Gardner method. Amity silt loam, soil column two.

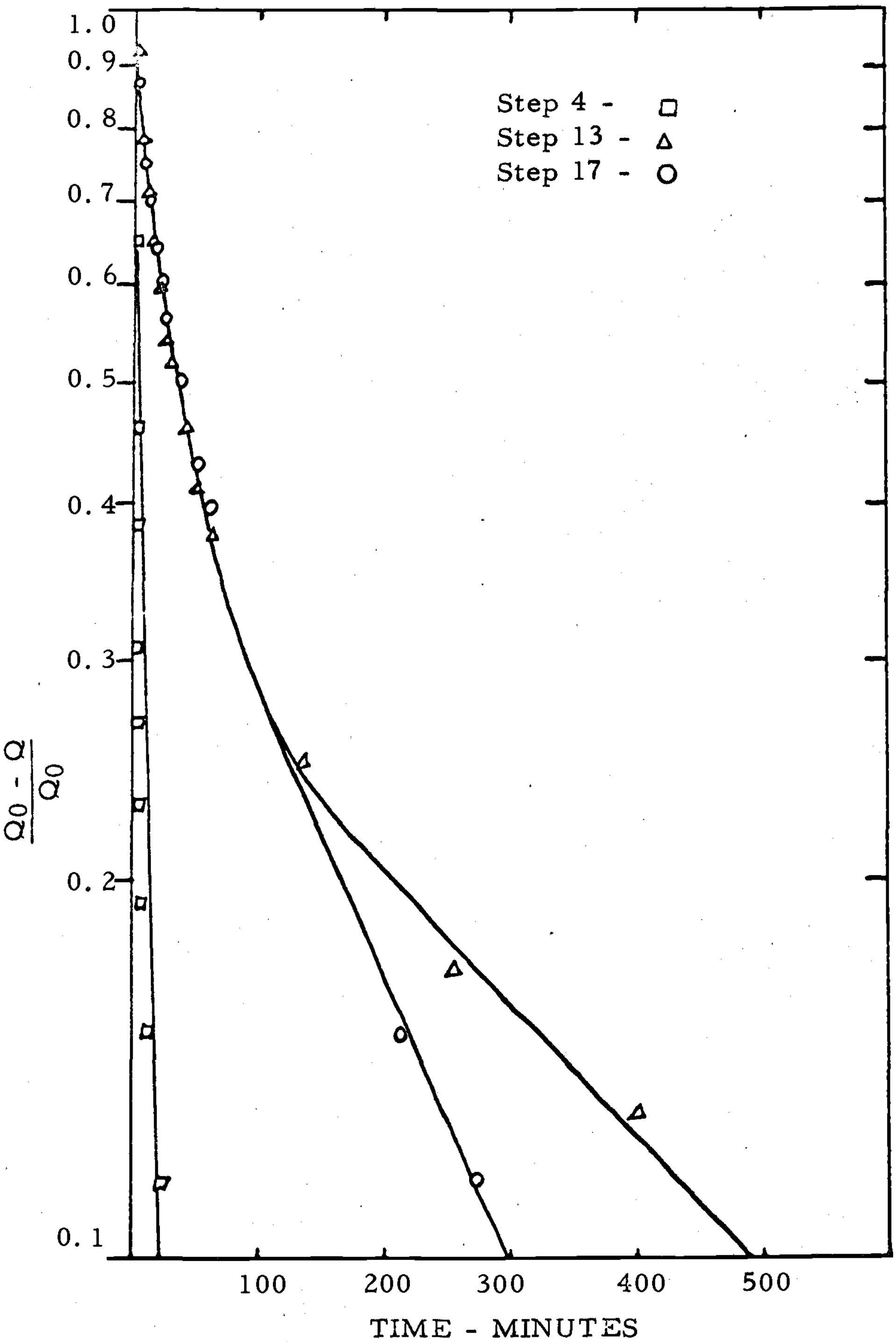

Figure 25. Outflow versus time according to the Elrick method. Amity silt loam, soil column two.

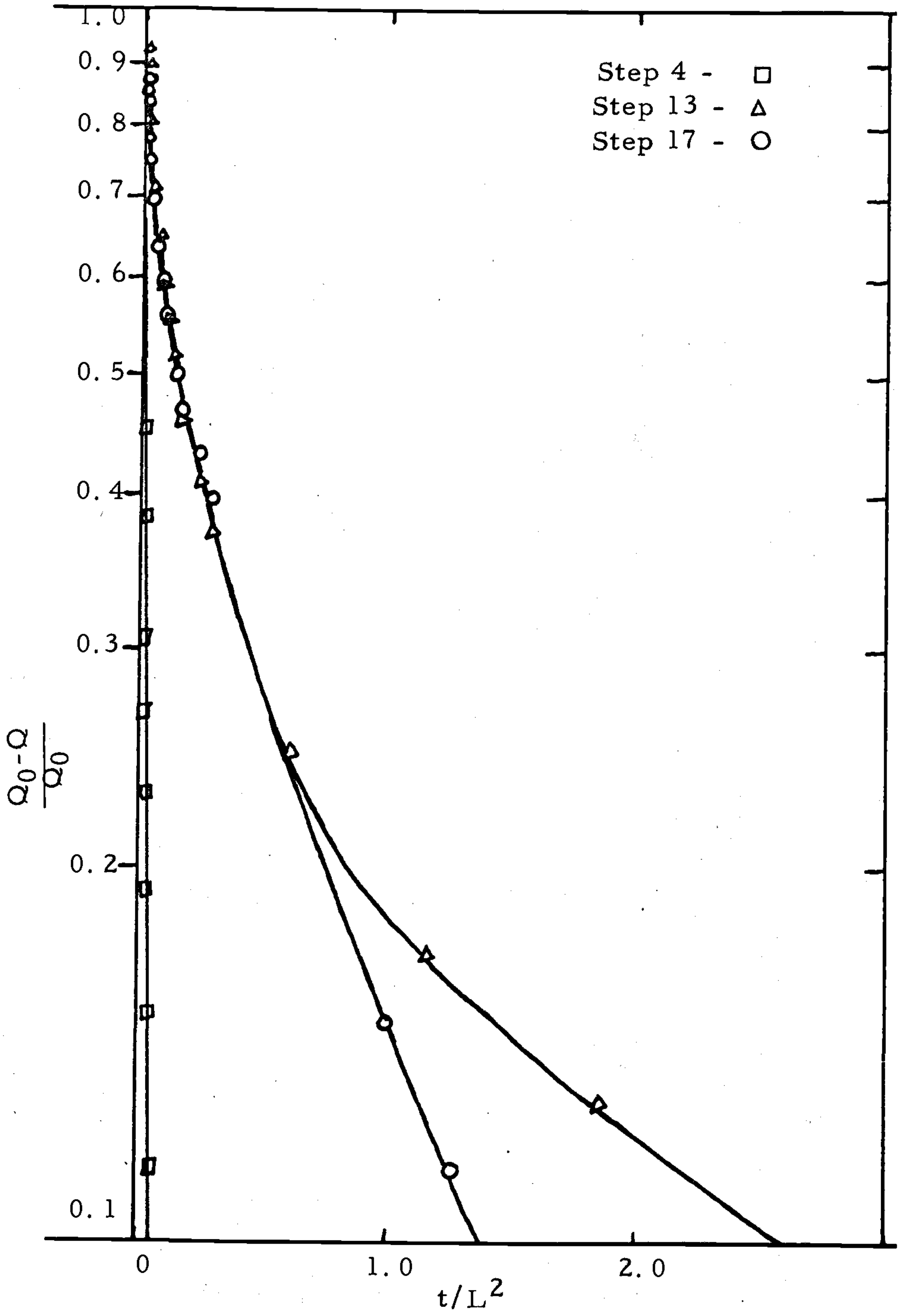

Figure 26. Outflow versus time over length squared according to the Rijtema method. Amity silt loam, soil column two.

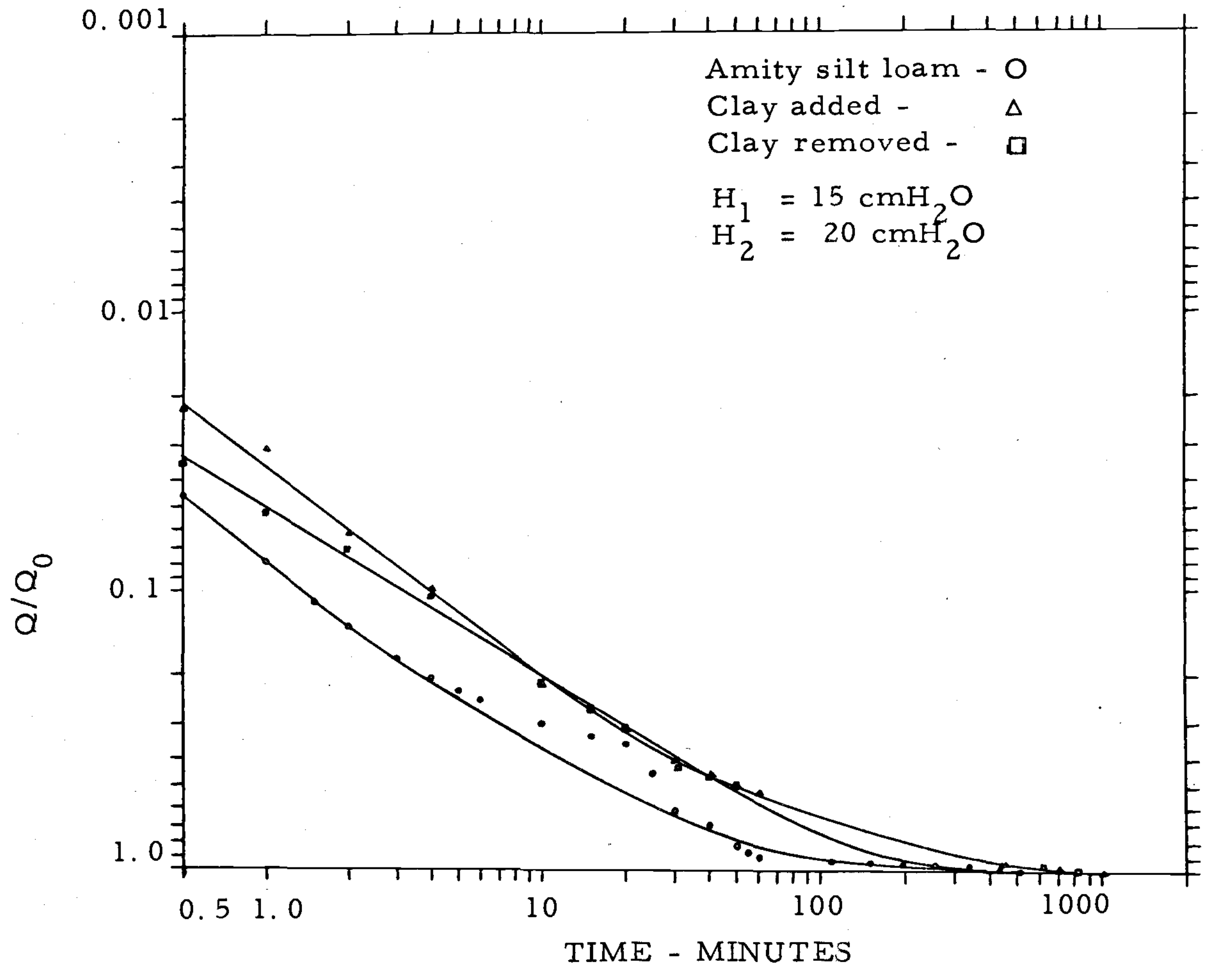

Figure 27. Outflow versus time showing the difference of soil texture on outflow at the same tension. Moisture content about 2 percentage points less than saturation.

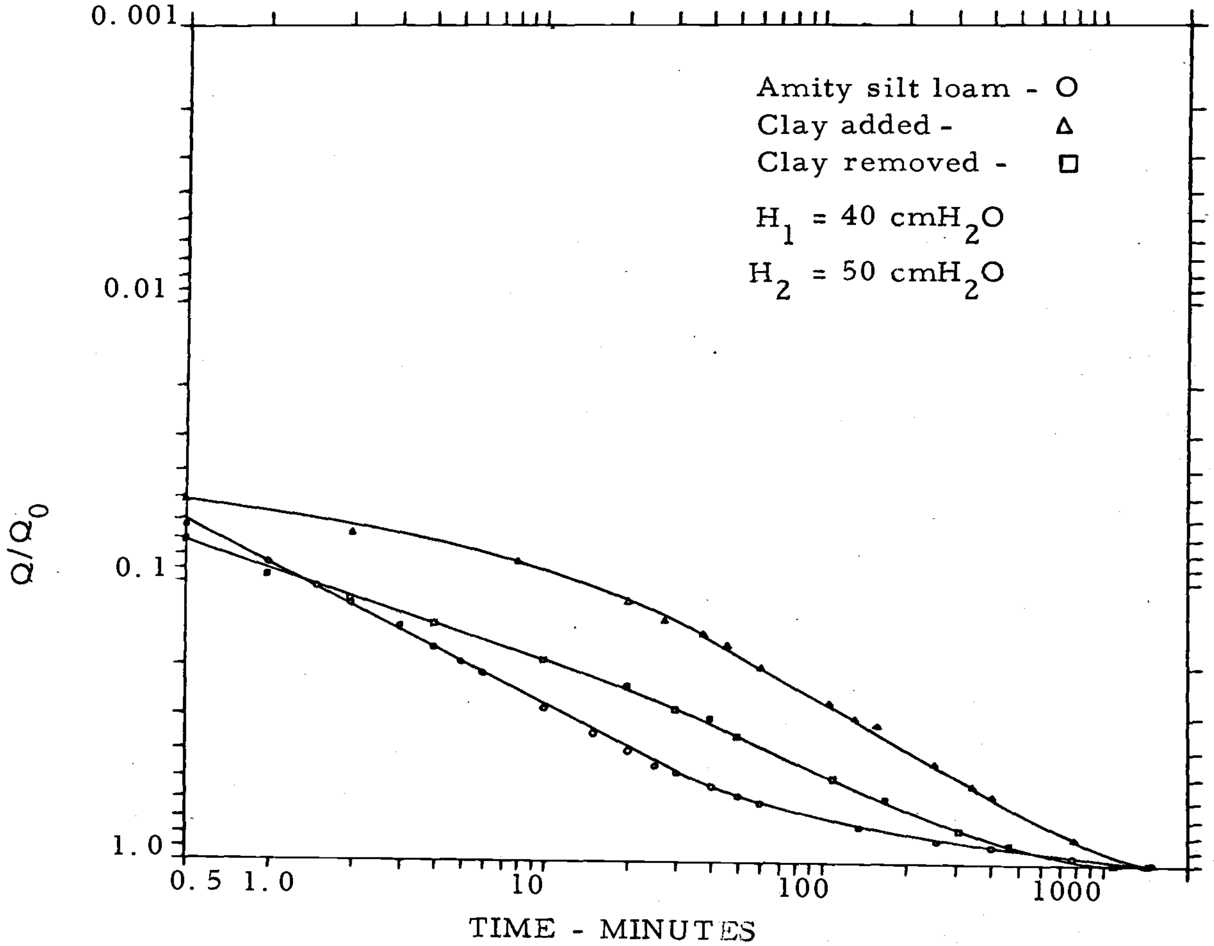

Figure 28. Outflow versus time showing the difference of soil texture on outflow at the same tension with a moisture content about 9 percentage points less than saturation.

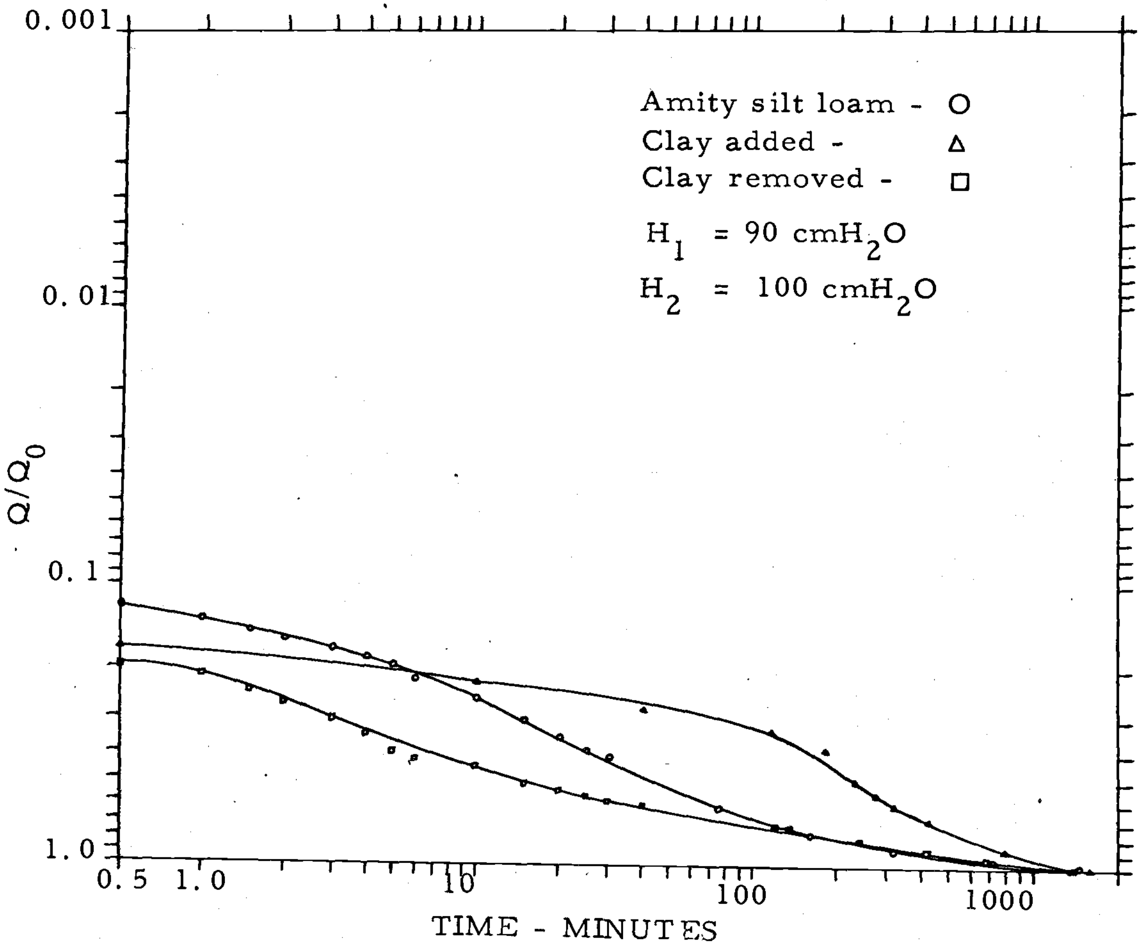

Figure 29. Outflow versus time showing the difference of soil texture on outflow at the same tension with a moisture content about 15 percentage points less than saturation.

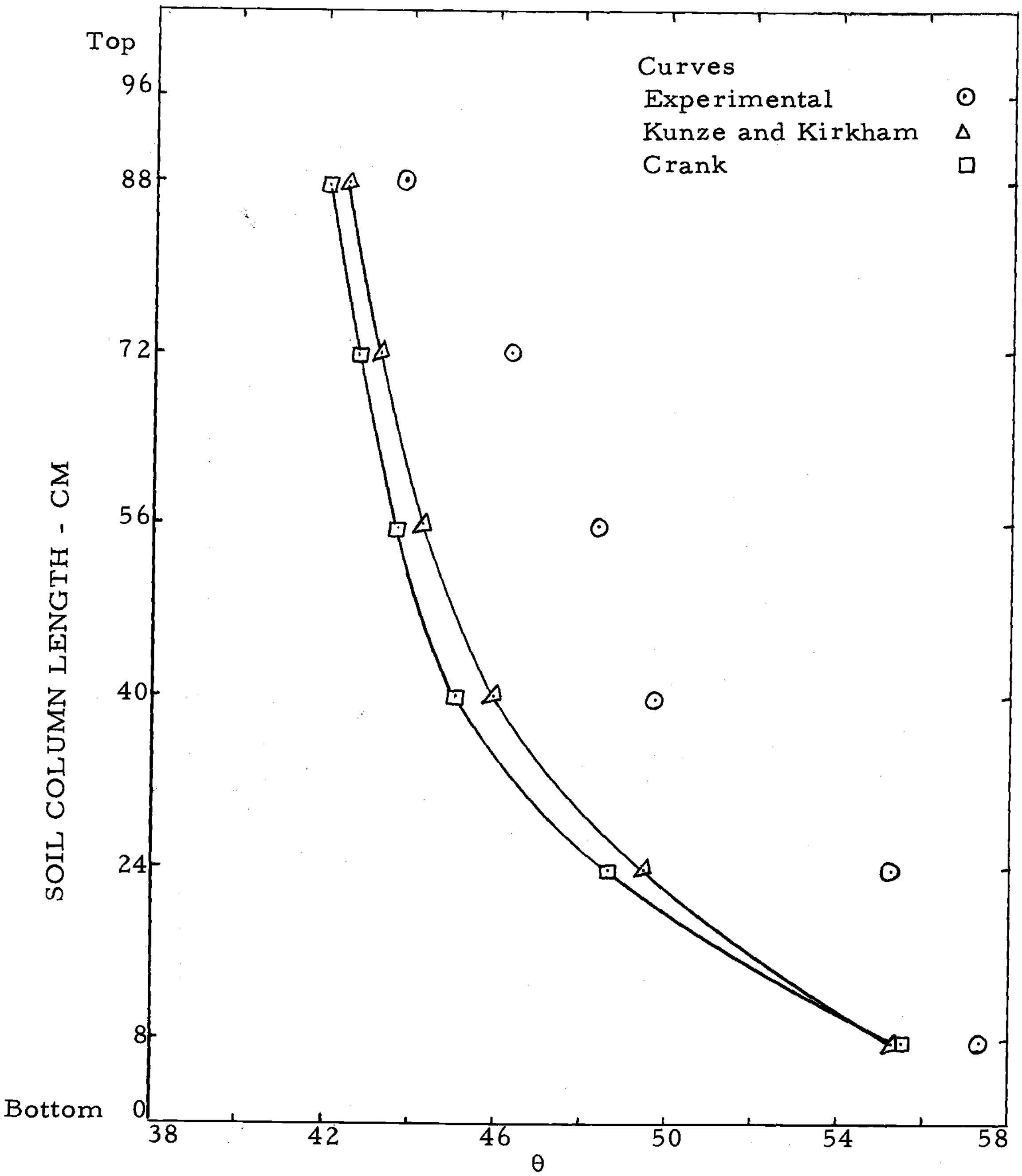

Figure 30. Plot of computer solutions of equilibrium moisture content of a 100 cm soil column using Crank method and Kunze and Kirkham method D and K values for Amity silt loam soil, column one.