CAN SELF-THINNING BE USED AS A PRODUCTION CONTROL MECHANISM? THE CASE OF MUSSEL FARMING IN QUÉBEC

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ABSTRACT

A bio-economic model, based on Faustmann rotation, was developed to analyze the relevance of removing thinning activities which are part of the dominant technology used in blue mussel production in Canada. Removing these density control activities (thinning) seemed a profitable option to producers. It was thought that this collector line technology would allow a part-time production, which would help start a small local industry in Eastern Québec. Biological and economic literature show the importance of density relationships upon mussel growth and harvest. Many advantages, such as restocking thinned mussel and improved growth rate, would therefore be lost. These dynamics, collectively called self-thinning in the biological literature, could be used to make this technology profitable if properly framed. Samples were taken on 140 foot-long lines used for collecting mussels of age 0+ to 3+ from local producers, as well as financial information. Analysis of population dynamics under this technology showed differences in yield and variability of samples taken at deep and shallow portions of the lines. Gompertz growth functions were fitted to these two cases. A Weibull CDF was then estimated to evaluate risk of fall-off for deep and shallow samples, determining the proportion of harvestable lines at a given time. Initial setting seemed more important then biomass growth to explain the large variability in yield. Results show that maximum revenue is attained after 4 years rather then 3, which is the current practice by local producers. In post-optimality analyses, the optimal rotation period seemed insensitive to most production or financial changes. Parameters considered reasonable to producers yielded negative profit while sub-standard subsidies allowed profitability. This function was more sensitive to changes on economic variables while reducing fall-off rate is also significant. Such changes could allow economic profit for producers.

Keywords: Self-thinning, mussel, optimal rotation period, Faustmann, production control activities

CONTEXT

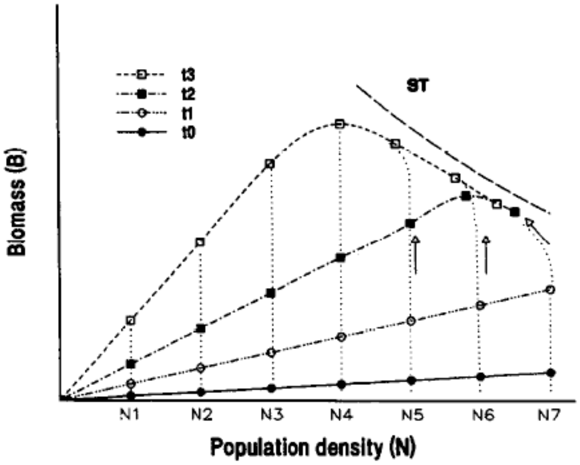
In regions characterized by a high unemployment rate, one option frequently chosen by the inhabitants is to turn to exploitation of natural resources such as fisheries. However, the effects of overexploitation in North American fisheries limits this option. Fishermen have seen their quotas decrease, and public management as well as cost recovery is already subject to a rigorous management by the State. Mussel and other invertebrate production offers an interesting alternative to fishing, although a full-time activity may not be possible. Conventional mussel production, such as practiced in the east of Canada, is generally in four stages. The second stage consists in controlling the density of the population, by taking the small first year mussels which settle and grow on lines placed in the water, and inserting them into net sleeves. Several important advantages arise from such a practice. However, some aquaculture producers from the region of Gaspé have tried for some years to develop a technology which would not need to pass by this stage, which would eliminate an important cost element. Mussel production of this type, possibly run on a cooperative basis, using only these line “collectors” would offer part-time alternative employment to producers at an interesting income. However, although the production costs using this simpler technology are largely decreased, so are the expected profits. The profitability of this alternative production method is a controversial issue among the local producers in Québec, and with diffusion of this technology world-wide over time, its adoption in the Canadian economic context may well turn out not to be profitable. This is the reason why we present a financial analysis which explores the viability of this technique of production.

The main purpose of this study is to determine the economic feasibility of the line method, which relies exclusively on the population dynamics of the natural set of the mussel population, and which includes a phenomenon known as "self-thinning". However, other questions related to this subject are explored to better understand the problem. In order to arrive at our results, we developed a methodology which fit the data most reasonably and which took into account the appropriate characteristics of this technology based upon the technique. Moreover, the majority of the necessary theory, both biological and economic, comes from forestry. These ideas were borrowed both at the micro-level, such as explanations of the growth of the individuals, and also at the population level. It is important to describe these characteristics, so as to produce an explanatory model which is appropriate for mussel production using this technology of line "collectors".

A detailed explanation of this line technique for mussel production is necessary, accompanied with a theoretical explanation of how spat collectors work from the ecological standpoint of self-thinning dynamics [5]. A bio-economic model is then developed from field data from a working mussel aquaculture firm using this technique, and is analyzed using optimal rotation notions first developed by Faustmann [4,16]. We investigated a number of suppositions and phenomena, not the least of which is the consideration of uncertainty, and the risk of "natural disasters" associated with the technology [10, 14, 15]. One such possible "disaster" is that self-thinning dynamics results in an increased risk through time that sections of the line will slough or drop off, leaving them bare. On the other hand, even with these sloughing events which we call "disasters", the results obtained by optimizing the rotation with respect to the expected value of the profit suggest that producers should wait an additional year before harvesting at 48 months, rather than the customary practice of waiting for 36 months. Producers argue that such a production is not possible due to the fact that the market presently requires the size of the individuals at 36 months which is between 50 mm and 75 mm. In either case, this production is not profitable on its own. The technology is still in an experimental stage, and therefore improvements in the flotation gear, and the rigging of the lines could result in positive profit.

Production Method

Besides the general biological knowledge necessary to understand how a mussel farm works, this technique of production also requires detailed knowledge of the population dynamics of the species. These dynamics are heavily influenced by density dependent growth and the fact that once fixation occurs, the individuals do not move, or move only a little bit with much difficulty. The mechanism of self-thinning therefore has a determining impact on the growth of the population. Self-thinning was a phenomenon first observed by plant biologists [18], notably in forestry. It is defined in aquaculture as the elimination (by dropping off) of least favored individuals through space and food competition, which then favors the growth of those who are left by reducing both food and space constraints. This elimination is the result of competition among the individuals. To use an analogy with the economics of competition, competition in this sense does not appear before the total demand on factor resources by the population faces a supply of these factors by nature which imposes increasing costs on the individuals. It is therefore possible to see the evolution of the relation between average biomass of the individual and the density at time (BN) as a manifold of trajectories through time, which are enveloped by a self-thinning frontier production [1] (Figure 1).



Source: Fréchette 2006

Figure 1. Self-thinning boundary

This phenomenon occurs naturally but the line technology based upon self-thinning accentuates the effect. An increasing population will sink the collectors to a level where the environmental constraints are more important and will therefore accelerate the falling off mechanism. When enough individuals fall off, the lines will be able to go back up in the column of water. Therefore, the control of the density, normally done by a labor intensive stage of placing juvenile mussels in net sleeves, is completely eliminated.

The bio-economic literature regarding the importance of density reveals that even though various practices of harvest were applied, the sleeving stage would not only increase the growth rate, but also permit the use of mussels that would have been eliminated by the competition by saving them for grow-out, therefore offering better returns [8, 9]. This "net sleeve" technique essentially supposes that the labor and materials costs of placing individuals in net sleeves are lower than the opportunity cost of these individuals being lost to the ecosystem. This is an important advantage of the net sleeve technology which the line collector technique does not have. Furthermore, since density cannot be controlled using collector lines, the growth rate is usually lower than that of the sleeve technique. On the other hand, line techniques are not labor intensive, requiring little care throughout the growing cycle.

Using Bjorndal (1988) as a point of departure, we suppose that mussel aquaculture of this type can only use the time variable to control production because density cannot be controlled and it is impossible to control the contribution of natural food. The determination of the period of optimal rotation, which will maximize the returns on production, becomes the crucial element of this analysis. Faustmann's model allows the producer to determine the period of growth which will maximize profits while taking account of the opportunity cost of the substrate, the opportunity cost of the time of the producer, and the opportunity cost of capital.

Faustmann's model is considered to be the real method to estimate the value of the land on which a producer wishes to develop a natural resource. The literature of the last few years [12, 13] do, however, agree that the integration of uncertainty is essential to obtain more robust results. Not taking account this effect causes the overestimation of the value of the production. Therefore, uncertainty has been overviewed on three levels: biological growth, evolution of prices and the risk of natural disasters.

MODELING

This work is based on data collected in 2001, 2002 and 2003, by one of the authors (Fréchette) and his team, which studies density dependent growth on collectors of an aquaculture firm of the Baie des Chaleurs[i]. About 140 samples were made over a period of 41 months (Figure 2). Other pieces of information were collected as well, such as weight, density, commercial weight, commercial density, and the incidence of fall-off. These variables allow for the observation of other factors, such as the biomass-density relations, and the analysis of two important functions for the study of growth and fall-off. Samples taken at the top and in the bottom of the collectors reveal that the growing environment was not uniform, and environmental constraint were different. Because of this, it is necessary to divide the data set into two parts and to model production by taking this situation into account.

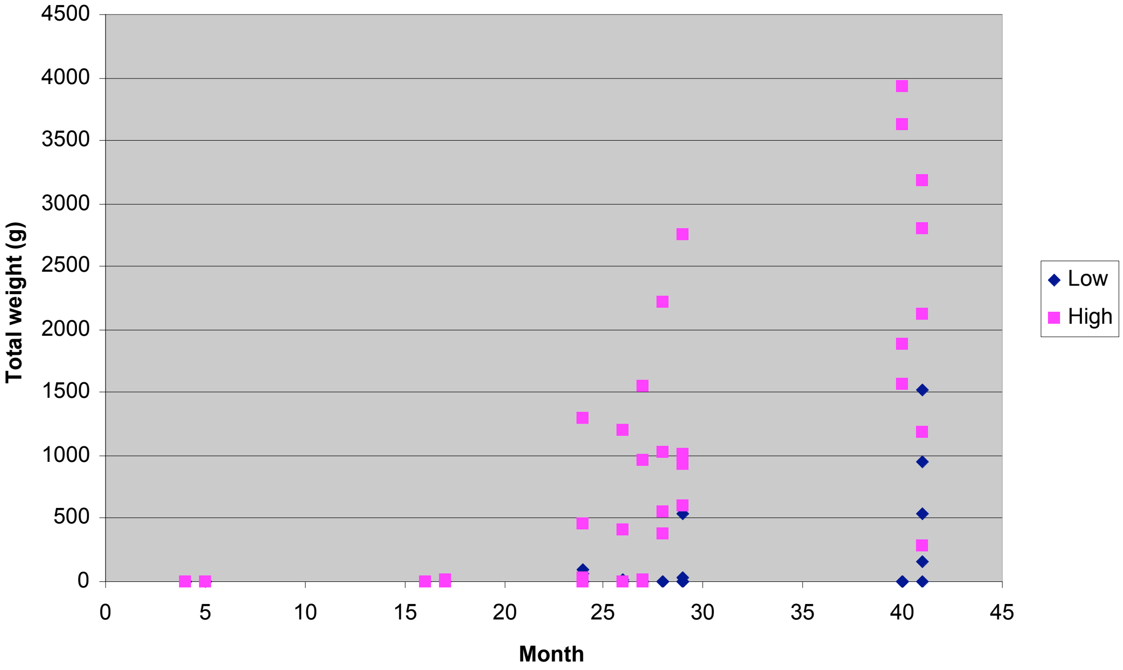


Figure 2 : The distribution of commercial biomass (individuals of 5cm or greater) in grams per sampled foot of line over time, taking account of the samples made high in the water column and low in the water column.

The analysis of the information collected allowed us to determine that it will not be possible to model true risk at the level of the growth of the biomass, in spite of the importance of this variable. Uncertainty arises mainly from the first fixation or "set" of spat, which is highly variable from season to season as well as according to depth. Growth rate, on the other hand, seems to be rather stable according to depth of the set (Figure 3), which is one of the predictions of self-thinning theory. This figure shows that the percentage of commercial biomass does not have as much variability as standard data and seems to follow a production frontier quite robust. The data that would be necessary for modeling the primary fixation, however, were not collected. The best growth curve fits, for the collectors in both high and the low part of the water column, assumed a Gompertz's growth function (Eq. 3)[ii]. Data presenting signs of fall-off were not included into the growth functions, using SPSS 12.0.1 for Windows. For the low part of the collectors, coefficients b and c are respectively 3.899 and 1.069 with an R_ of 0.88433[iii]. The coefficients of the growth function from the higher part of the collectors are 3.484 and 1.210 for an R_ of 0.65986.

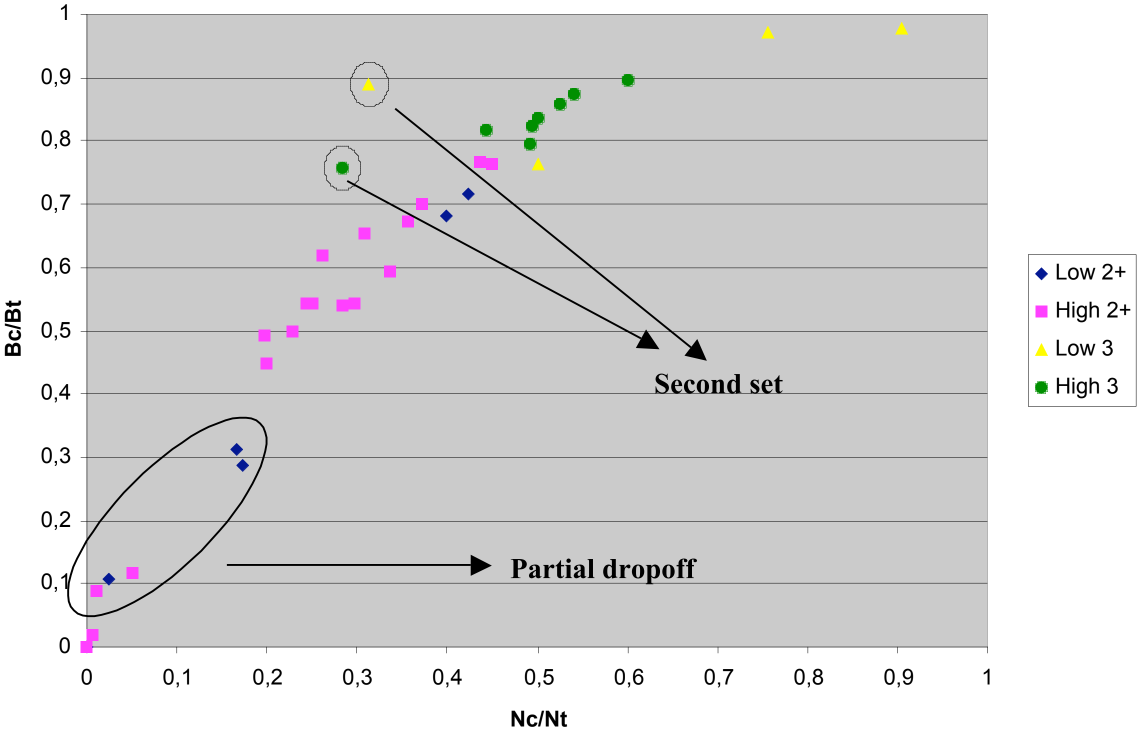


Figure 3 : The ratio between commercial biomass to total biomass, compared to the density of commercial sized individuals compared to total density; and the impact of the second set phenomenon.

Financial data obtained from the same producer allow us to define the costs connected with the functioning of the company. Three categories were determined: the costs of preparation (annual), the costs of harvest (by harvest rotation) and the fixed costs which have a life expectancy of twenty years. Information supplied by the producer indicated that prices have been very stable for several years. An invariable price of 1,28$ / kg was used since it did not seemed necessary to model uncertainty at this level, and the current tax incidence for a company of this size is 20 %. The opportunity cost of capital, or the required rate of return on investment for this analysis was 10 %, considering the risks involved and the fact that it is a new industry. The relative lack of financial alternatives in this region and this branch of industry played a role in our choice of this rate. We considered that the opportunity cost for the time of the producer is 15$ / hour.

The consideration of the possibility that a "disaster" like fall-off may arise remains very important in this analysis, even though losing mussels over part of a collector line does not rise to the same level of drama and impressiveness as a forest fire. This falling-off event, when complete clusters get loose from collectors, was modeled in the analysis in order to determine the probability of occurrence over time. The main cause of fall-off seems to be the friction of the collectors on the sea floor, self-thinning also seems to play some role over this. A cumulative probability density function (CDF) following Weibull's law, was fitted to determine the proportion of line that should be colonized after a certain period of time, assuming that as time increases, the probability that these falls will arise on a foot of collector line approaches 1. The data presenting a total density lower than forty individuals and those where size and density data showed a BN relationship of a previous year, indicating re-colonization by younger individuals of a subsequent set, were used to evaluate this function[iv] (Figure 4 and 5). Coefficients _ and _ for the lower collectors are respectively 42.1866 and 1.77242 with an R_ of 0.87579. For the collectors higher in the water column, the coefficients are 145.201 and 1.03962 with an R_ of 0.87872 (Eq. 4). These data were estimated using “AvSim+ 9.0.2 DEMO version” software. A review of the bio-economic literature concerning the risk of natural disasters convinced us that the addition of the supplementary information would be useful. Net actual value will be obtained with Eq. 1, which will also determine the optimal rotation period.

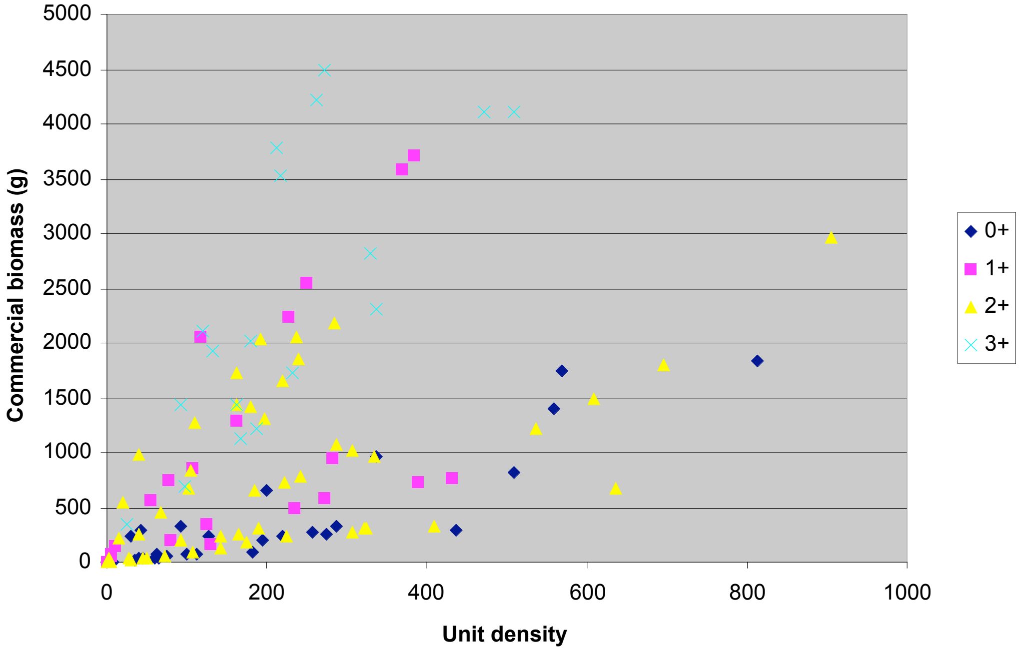


Figure 4 : Biomass density plots by age group with “second set”

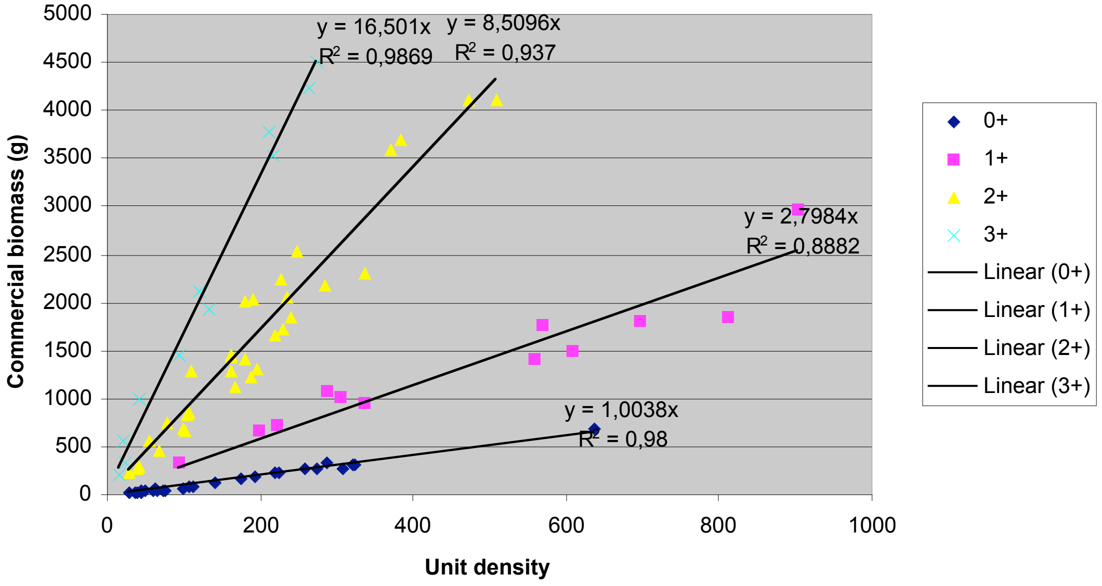


Figure 5 : Biomass density plots by age group without “second set”

$$V = [\{e^{-\delta I}(P*Q(t) - C_2) - C_1\}*\{ e^{\delta I}/(e^{\delta I}-1)\}]*(1-T) - (C_3 - A)*(e^{240\delta}/(e^{240\delta}-1)) \qquad \text{(Eq.1)}$$

V = The capitalized value of the resource

δ = The opportunity cost of capital, monthly

I = Production interval

P = Price of the product per kilogram

C_1 = Preparation and set-up costs

C_2 = Harvest costs

C_3 = Fixed costs

Q(t) = Total biomass produced in Time *t*

$$= W(t)_{hi} * [N/2*\{1-K(t)_{hi}\}] + W(t)_{lo} * [N/2*\{1-K(t)_{lo}\}] \qquad \text{(Eq.2)}$$ [v]

N = Total size of collectors to harvest, in feet

$W(t)_{lohi}$ = Gompertz's function of biomass growth

= W_∞ _ $\exp^{[(-\exp(b-ct)]}$ (Eq.3)

W = biomass in grams at time *t*

W_∞ = maximum weight[vi]

b et c : endogenously estimated coefficients

$K(t)_{lohi}$ = Cumulative distribution function of the probability of failure, using a Weibull distribution where K is the probability of mussel fall-off at time *t*

= 1- exp(–(t / _)$^{-}$) (Eq.4)

_ et _ = coefficients to estimate without any meaning

T = Tax rate

A = Capital gains after depreciation

RESULTS AND DISCUSSION

The results obtained suggest that at the selling price, the period of optimal rotation would be of 46.23 months, contrary to the current practice (± 37 months). This period is surprisingly resistant to the variations in basic suppositions. Table I presents the optimal valued obtained for Eq. 2.

Table I : Yield and expected fall-off at 37 and at 46.23 months

46,23 month	High	Low		37 month	High	Low
Kg / foot	*3,3097*	*2,0166*		Kg / foot	*2,0634*	*0,7243*
% dropoff	*0,2623*	*0,6915*		% dropoff	*0,2145*	*0,5473*

However, the opinion of the producers is that a harvest made in the fourth year would result in a lower selling price, because of a decrease in the quality of the product. A lack of reliable information on this issue does not, however, allow us to make a solid diagnosis of this issue. This is an important difference between mussel production and most forestry, where the continuous growth of the species is not possible. For either rotation policy, production would not allow the producers to make profits, at an opportunity cost of capital of 10 %. If we think of these results in terms of government subsidies for agriculture, a four-year cycle, if there was a market for the product, would cost a subsidizing government less than the status quo. For four year production cycles, the subsidy needed by the company to cover all it's costs and to break even would be 17 % of break-even, which is well below the standard subsidy level in Canada of 30 %. A production in 37 months would need a subsidy of 31,1 % to become profitable.

It is the economic variables which have the most impact on profits, and efforts should especially be made to decrease the costs of the infrastructure in the water and the purchase of the boat. The number of lines in production had a much smaller effect than expected, probably due to the fact that we cannot see an economy of scales effect working with one producer, and the environment is not necessarily a constraining factor. As for biological variables, an improvement of the grow-out infrastructure would probably reduce the incidence of fall-off. The distribution of the number of mussels between the lines in the high versus the low water column had the most impact on yields; it is very unlikely to increase return in commercial biomass. Out of 45 different scenarios studied, only the most optimistic (strong prices, low fall-off, and low interest rates) suggest financial profitability. For the scenarios with subsidies, even the most pessimistic would show a positive income with a subsidy that is 30% of break even. The uncontrollable variables are the interest rate, as well as the individual growth dynamics that occur at different levels of the water column, which could also be taken as an indicator of the importance of biological return when there are variations in the set from one season to the next. In other words, yields on the lower lines might be similar to a result with a poor set year. For purposes of simulation, we varied price and fall-off rate, within a range of 10% of actual values.

Tables II-IV give an overall view of our analysis, based upon the model assumptions we used. The numbers in the left hand column of Figure 6 are scenarios based on interest rate and % of units in weak environmental constraint, and the columns represent different scenarios of price and fall-off rates examined. A more detailed look at these variables can be found in Figure 8. What is clear from Figures 6 and 7 is that either prices will have to be very good, or the fall-off very low (or strong, for strong set) for producers using a similar technology to make a profit without government assistance. In addition, the opportunity cost of capital must be somewhat on the low side as well. It is possible to make a modest profit assuming that a producer has a low opportunity cost of capital under the status quo conditions. However, the producer exposes himself to losses if the market price is weak, even if sets are considered strong. This situation obviously improves when government steps in to subsidize the firm.

It is important to point out that that all opportunity costs of productive factors have been covered; the producer receives a salary for his services, the amortization of all capital goods are considered, and, in using optimal rotation methods, the opportunity cost of the land is taken into account. Therefore, the losses we see here are literally net present values of profit over the life of the project. In addition, there are other ways of looking at the opportunity cost of capital. A producer may be willing to accept a much lower rate of return if aquaculture in rural Québec is really what the person wants to do.

Table II : Different scenarios of optimized net present value of the profit at the optimal rotation of 46.23 months, without subsidies at 30% of break-even. [vii]

	Weak/Weak	Weak/Strong	Normal	Strong/Weak	Strong/Strong	Average
12:45	-$269 981	-$217 629	-$206 201	-$200 692	-$136 504	-$206 201
12:50	-$247 850	-$196 571	-$182 166	-$173 559	-$110 686	-$182 166
12:55	-$225 720	-$175 513	-$158 132	-$146 425	-$84 867	-$158 132
10:45	-$214 972	-$150 305	-$136 189	-$129 384	-$50 096	-$136 189
10:50	-$187 636	-$124 293	-$106 500	-$95 867	-$18 204	-$106 500
10:55	-$160 300	-$98 282	-$76 811	-$62 351	$13 688	-$76 811
08:45	-$132 256	-$49 041	-$30 877	-$22 120	$79 909	-$30 877
08:50	-$97 080	-$15 570	$7 327	$21 010	$120 948	$7 327
08:55	-$61 903	$17 902	$45 531	$64 139	$161 988	$45 531
Average	-$177 522	-$112 145	-$93 780	-$82 806	-$2 647	

Table III : Different scenarios of optimized net present value of the profit at the optimal rotation of 46.23 months, with subsidies at 30% of break-even.

	Weak/Weak	Weak/Strong	Normal	Strong/Weak	Strong/Strong	Average
12:45	-$87 501	-$35 150	-$23 722	-$18 213	$45 975	-$23 722
12:50	-$65 371	-$14 092	$313	$8 921	$71 793	$313
12:55	-$43 241	$6 966	$24 348	$36 054	$97 612	$24 348
10:45	-$25 836	$38 831	$52 947	$59 752	$139 040	$52 947
10:50	$1 500	$64 843	$82 636	$93 269	$170 932	$82 636
10:55	$28 836	$90 854	$112 325	$126 785	$202 824	$112 325
08:45	$66 905	$150 120	$168 284	$177 042	$279 070	$168 284
08:50	$102 082	$183 592	$206 489	$220 171	$320 110	$206 489
08:55	$137 258	$217 064	$244 693	$263 301	$361 149	$244 693
Average	$12 737	$78 114	$96 479	$107 453	$187 612	

Table IV : Sensitivity analysis without subsidies

VARIABLES	VARIATION	VALUES	VARIATION IN V	
Sale price	-10%	1.15 $	-155 965 $	46.45%
	+10%	1.41 $	-57 036 $	-46.45%
% high units	-10%	45%	-136 189 $	27.88%
	+10%	55%	-76 811 $	-27.88%
Number of lines	-10%	75.60	-120 328 $	12.98%
	+10%	92.40	-92 672 $	-12.98%
Interest rate	-10%	0.09%	-55 949 $	-47.47%
	+10%	0.11%	-147 801 $	38.78%
% of fall-off; high	-10%	24%	-92 561 $	-13.09%
	+10%	29%	-120 440 $	13.09%
% of fall-off;low	-10%	62%	-85 188 $	-20.01%
	+10%	76%	-127 812 $	20.01%
Fixed cost	-10%	431 767.22 $	-58 526 $	-45.05%
	+10%	527 715.49 $	-154 474 $	45.05%

There is much about this particular technology that might be improved. Different rigs that place lines higher up in the water column might by tried, as well as flotation devices that would more easily keep the lines from touching bottom could improve profitability. A serious exploration should be done of the market potential of both larger mussels nearer the Faustmann rotation recommendation, and smaller mussels, which are preferred on the European market. Finally, marketing and market structure considerations may be as important as the production economics considerations we have presented here. Profitability may well ultimately be limited by the fact that small producers from Québec may not have the same type of market access as other firms do. Important technological improvements, as well as a reduction of capital costs could allow producers using this technique to realize a profit. A group of producers, who could share the costs of boats and other equipment, may be a partial solution to the profitability problem. Alternatively, a reorientation of fishermen towards part-time aquaculture using very simple techniques such as an improved line technology might be a way to decrease these costs. However, these proposed measures raise a number of important policy questions. What are the benefits of government subsidizing production in an industry that is still in it's experimental phases, compared to subsidizing improvements in technology? One might argue that there are long run advantages to be had by encouraging producers to "learn by doing", and that production is after all the objective of the exercise. It may also be true that subsidizing production will ultimately lead to technological improvements through experimentation by members of the industry, and that this experimentation by entrepreneurs is preferable to similar efforts in government laboratories. Yet our analyses suggest that left completely to themselves, members of this experimental industry in Québec seem to have a very small window of opportunity, even under optimized conditions; and it is not clear if the situation is improving or not.

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ENDNOTES

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[i] Samples were taken on a length 30,5 cm, or a foot of length. The installation contains 54 lines. A line contains 80 loops situated diagonally in the water column. Each length of the loop measures 19,8 feet, and every loop is supported with a buoy. The area can support 84 lines, The analysis will be base on a farm of 84 lines.

[ii] There isn’t any particular interpretation to those coefficients, while the initial value used was 1 for every coefficient. Every one was significant based on a Student test at 0,05, while the total regressions were also significant at 0,05 on a Fisher test.

[iii] There were only 5 observations with numbers greater than zero.

[iv] For sampled lines having between 0 and 40 individuals, these were considered to be subject to a partial fall-off, which is the same here as having a density of 0.

[v] Flotation buoys collect spat as well, and are harvested. Considering this fact, sample counts in the upper column were adjusted upwards by 1 per buoy.

[vi] W_∞ is generally exogenous, and defined from a data series. In our case, it may be a result of technological constraints. For example, this value may be affected by the fact that the buoys were not able to bear a weight more than 4kg per foot. Taking account of the presence of holes on the collectors and the observation of data, W_∞ is estimated at 4,5 kg per foot.

[vii] The first value is price and the second value is the fall-off rate. For prices, a strong price is 1,41/kg and a weak price is 1,15/kg. For fall-off , strong is 24% for the high section and 62% for the low section, whereas weak is respectively 29% and 76%.