AN ABSTRACT OF THE THESIS OF

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The purpose of this study has been to investigate the effects of an activity approach to teaching geometry in certain high schools of Afghanistan. A brief review of the historical background of mathematics education especially in Afghanistan has been presented in this study. Students in the activity approach were involved in a learning process using solution keys and practical activities to supplement lecture and textbook presentation. This approach was compared to traditional methods which consist of lecture, use of a textbook, and recitation based on memorization only.

Participants in this study were students in two high schools in Afghanistan. Eight teachers were involved and a total of 602 students were randomly selected to participate in the study. Selected students were divided into experimental and control groups by means of an "even and odd" procedure. There were seven classes in each group,

with each teacher teaching one or two classes in one of the groups.

The activity approach consists of 48 activities (24 activities for each of the ninth and tenth grades) which were introduced as learning modules supplementing the presently used traditional approach.

Six hypotheses were stated claiming no difference between the two approaches in common learning outcomes for students learning geometry, such as over-all achievement in understanding concepts, creative thinking, ability to recall concepts, ability to solve problems, ability to explain facts, and ability to set up step-by-step proofs for various theorems. The hypotheses were all tested statistically and were rejected in favor of their alternatives.

Since there was no standardized test appropriate for testing the content of high school geometry programs in Afghanistan, three intermediate tests and a comprehensive final examination were constructed by a committee consisting of the experimental as well as the control group teachers. The tests and the final examination were administered to both groups at the same time. The tests together with the final examination were designed to measure the six specific learning outcomes in geometry mentioned earlier.

The experimental design employed was a "post-test only control group design." The design was supplemented by three intermediate tests administered every other month during the 32 weeks (one academic year) duration of this study.

The statistical analysis which consisted of computation of related mean scores for each one of the six specific learning outcomes in geometry and for each test including the final examination was computer processed. Finally a Student-t-test was used to draw conclusions related to each of the learning outcomes.

Conclusions

The following conclusions were drawn from the analysis of the data and from testing of the hypotheses. In comparison to the traditional approach, the use of activity approach:

1. Significantly improves a student performance in overall understanding of geometry.
2. Helps students achieve higher levels in creative thinking.
3. Helps students develop greater ability to explain geometric concepts.
4. Helps students improve their ability in solving geometric problems.
5. Helps students develop the ability to recall geometric concepts better.
6. Helps students to develop greater ability in setting up complete proofs for geometric theorems.

Some Effects of an Activity Approach to Teaching Geometry in the High Schools in Afghanistan

by

Mohammad Ibrahim Monier

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APPROVED:

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Professor of Science Education

in charge of major

Redacted for Privacy

Chairman of Department of Science Education

Redacted for Privacy

Dean of Graduate School

Date thesis is presented May 2, 1977

Typed by Clover Redfern for Mohammad Ibrahim Monier

VITA

Name: Mohammad Ibrahim Monier

Birthdate and place: September 12, 1940, Kabul, Afghanistan

Parents: Mr. and Mrs. Mohammad Ismail Monier

Education:

1977	Ph. D. in Mathematics Education, Oregon State University, Corvallis, Oregon U.S.A.
1968	M.A. in Teaching Mathematics, Rhode Island College, Providence, Rhode Island U.S.A.
1963	B.S. in Mathematics and Physics, Faculty of Science, Kabul University, Kabul, Afghanistan
1959	Baccalaureate Degree, Habibia High School, Kabul, Afghanistan

Professional Experience:

1974-77	Assistant Professor of Mathematics Education, Faculty of Science, Kabul University
1970-73	Chairman of the Mathematics Department and Director of Academic Relations, Faculty of Education, Kabul University
1968-70	Chairman of the Mathematics and Science Department, Institute of Education, Kabul University
1963-65	Counterpart to Mathematics Specialist, Institute of Education, Kabul University

Publications:

1. Mathematics for Tenth, Eleventh and Twelfth Grades
2. Geometry for Tenth and Eleventh Grades
3. Analytic Geometry for Sophomores in Faculty of Education

4. Foundation of Mathematics for College Seniors

5. Teaching Mathematics for College Seniors

6. Calculus I and II, for College Sophomores and Juniors

7. An article on "Elliptic Geometry." Published in Tarbiat. Faculty of Education, Kabul University

8. Twenty-four Experiments for Ninth Grade Geometry

9. Twenty-four Experiments for Tenth Grade Geometry

10. Solution Key for Ninth Grade Geometry

11. Solution Key for Tenth Grade Geometry

The publications listed above are available in Persian from the author through the Ministries of Education or Higher Education, Afghanistan.

Research:

1. The methodology and Upgrading of Curricula in Mathematics, National Institute for Educational Research, Tokyo, Japan

2. A survey of Professional and Educational Difficulties of Mathematics Teachers in the High Schools of Afghanistan, 1973, Kabul University

3. Some Effects of an Activity Approach to Teaching Geometry in High Schools in Afghanistan, Ph. D. Dissertation, Oregon State University, Corvallis, Oregon U.S.A. 1977

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I. INTRODUCTION

In this work the activity approach is evaluated as an aid in teaching geometry to high school students. The activity approach is defined as active student involvement in the learning process, using solution keys and practical activities to supplement lecture and textbook presentations. The purpose of this approach is to help students relate concepts to practical applications.

An examination of the literature reveals no previous attempts to determine if an activity-based approach to teaching and learning geometry enhances the high school student's ability to think creatively, to understand geometric concepts adequately, and to apply such knowledge effectively.

The study was conducted in Afghanistan between January and December, 1976. Afghanistan provides an ideal environment for conducting the evaluation, because the Afghan educational system is formalized, stressing rote memorization. Teaching methods almost never include applied activities and solution keys in mathematics with school students of any age.

Statement of the Problem

At present, the nineteenth century approach (lecture, textbook and rote) of teaching and learning geometry is used in all schools in Afghanistan. This situation results from the closed educational system in which teachers teach as they were taught. The preparation of students in such a system is insufficient to meet the goals of a rapidly developing country, particularly in the area of science and technology. This study was undertaken with the intent of improving the present teaching system, and introducing and evaluating a new dimension in teaching and learning geometry in the high schools.

Based on certain learning theories, Jean Piaget (1951), Z. P. Dienes (1963), Jerome S. Brumer (1960) and the findings presented in literature and on reasons which will be discussed in detail in various parts of this study, one can conclude that there is an urgent need for an experimental study of the activity approach in teaching and learning geometry at the high school level. It is assumed that the elementary grades of Afghanistan will experience this opportunity through the Columbia University affiliated Curriculum and Textbook Project,[1] in the near future.

[1] The Curriculum and Textbook Project represents a major effort by the Ministry of Education of Afghanistan to modernize the national primary school curriculum through the development of a new course of study and the preparation of new instructional materials.

The only work of a similar nature in the past two decades of the history of mathematics education in the secondary schools of Afghanistan was the introduction (on an experimental basis) of new mathematics textbooks into the tenth, eleventh and twelfth grades of two high schools through a project called the Mathematics Science Project (Carl C. Tranberg, Jr., 1966). The textbooks produced through the project contained a modest change in content and approach, as compared to the presently used traditional textbooks. The project was discontinued in 1971 largely because limited funds and limited technical resources made it impossible to prepare enough teachers to teach the content of the newly developed textbooks.

In order to help upgrade the quality of secondary mathematics education in Afghanistan, it was considered timely to design and introduce the activity approach to teaching geometry in high schools; the present study evaluates the results of the new approach. One of the aims of this investigation was to demonstrate that certain changes in the methodology of teaching geometry designed to achieve national goals based on present Afghanistan educational reform are desirable.

Assumptions

The following assumptions are basic to this study:

1. The proposed tests and final examinations are valid and reliable instruments (limited by the validity and reliability of procedures used in this study) for measuring the student's ability to understand and recall geometric concepts, think creatively, use problem solving techniques effectively and provide a logical sound geometric proof.
2. The sampled schools are representative of high schools in Afghanistan which has a centralized system of education.
3. The sample students in the experimental schools are representative of the ninth and tenth grade student populations of Afghanistan.

Hypotheses

The major purpose of this study was to investigate the question of whether or not there is a significant difference in the achievement level of students taught geometry by means of an activity-based approach, over those taught by the "traditional" method. To achieve this purpose the following null hypotheses were tested against their alternatives.

Hypothesis 1 ($H_0 1$): There is no significant difference in overall achievement in geometry between experimental and control groups.

Hypothesis 2 ($H_0 2$): There is no significant difference between the two groups of students in creative thinking in geometry.

Hypothesis 3 ($H_0$3): There is no significant difference between the two groups of students in their ability to explain geometric concepts correctly in their own words.

Hypothesis 4 ($H_0$4): There is no significant difference between the two groups of students in their ability to recall geometric concepts.

Hypothesis 5 ($H_0$5): There is no significant difference between the two groups of students in their ability to solve geometric problems that are original to them.

Hypothesis 6 ($H_0$6): There is no significant difference between the two groups of students in their ability to set up complete step-by-step proofs for certain geometric theorems.

The learning phenomena identified in these hypotheses are interpreted by means of subsets of the test items found in Appendix B. Items related to each hypothesis are identified in Tables 2-9.

Limitations

The study was subject to the following limitations:

1. The study was limited to the content and topics of ninth and tenth grade geometry programs as listed in the general syllabuses and geometry textbook used in all secondary schools of Afghanistan.
2. The study was limited to a series of three intermediate tests and a final examination which was given at the end of the

academic year. The statistical evaluation and the comparison of these groups is based on the raw scores derived from the tests and the final examination. The conclusions derived from the study are limited by the reliability of the tests and the final examination.

3. The study was limited to one academic year of 32 weeks.

Delimitations

1. During the course of this investigation no attempt was made to compare experimental and control groups of students by their sex, socio-economic class, or ethnic origin.
2. This study makes no attempts to investigate the use of an activity approach to teaching in areas not included in the geometry program for ninth and tenth grades.

Need For and Significance of the Study

This study represents one of the first attempts to document the impact of an activity approach upon the teaching and learning processes in high school geometry. Although recent advances in the technology of teaching have emphasized activities and manipulatives, very little research has been done to justify the use of such technology. The recently developed educational plans for Afghanistan are favorable to the implementation of changes in teaching methods and provide opportunity for testing and research into the effectiveness of an activity approach to teaching high school geometry. In addition, the Ministry of Education of the Republic of Afghanistan is committed to improving the quality of education, and particularly mathematics education, in order to insure the nations place in the modern technological world.

This study, then, serves a two-fold purpose--to research the activity approach to the teaching and learning of geometry and to develop teaching methods and materials better suited to the needs of the people of Afghanistan.

Many teachers of geometry have experienced frustration because of the lack of motivation their students display during the ninth and tenth grades, as geometric theories become increasingly abstract. As a consequence the classroom atmosphere usually becomes a methodic routine through which the students and teacher must suffer. In order to remedy this deficiency, the students might be involved in activities. The mathematics educators might help direct them to learn by themselves, use trial and error methods and refine their thinking through experience in appropriate activities in geometry. As described by Joe Phillips (1966):

> Gone forever are the days when the mathematics classroom were a book, a pencil and a ream of paper. Multisensory aids to learning, often times commonly found in the ordinary classroom environment, are essential to a successful contemporary program.

Many students are unable to comprehend abstract geometric concepts, except through relating these concepts to something concrete. It is the purpose of the activity approach to provide needed, meaningful concept applications. Too often, classroom drill calls forth a rote response. Instead, the teacher should guide the learner to abstract generalizations by presenting them to him in different ways. The more experience the learner has with different ways in which a principle can be applied, the deeper will be his understanding of it. Instead of expecting students to store and retrieve information, the teacher should direct the students to build and test theories and ideas for themselves by the use of open-ended questions and experiments.

The completion of an experiment or activity usually results in an individual product or independent discovery which furnishes to the student tangible evidence of progress. However, a partially incorrect student answer does not necessarily mean that the student has not attained partial understanding of the idea. Hence, one should accept incomplete responses until more experience develops more complete understanding.

Each student must do some independent thinking as he works on an activity, collects data, or plays a game. Each student's active involvement is the key to successful learning (Z. P. Dienes, 1963).

It is very important to motivate the student to discover many applications of geometry, to furnish the student with a knowledge of useful geometric concepts necessary for further studies, and to impress upon the students the role geometry will play in his or her future.

Afghanistan has embarked on a comprehensive developmental plan designed to improve and expand the general economy. It is intended that this plan will raise the general standard of living, will make employment practices more equitable, and will provide increasing opportunities to a better educated and better skilled citizenary.

The achievement of these objectives calls for a greatly enriched educational program. The need for a much larger number of competent technical and professional persons is at once obvious. It is known from the experiences of other nations that the achievement of a modern economy requires widespread literacy, calling for wide-spread primary education and sound and competent instruction especially in mathematics science and technology. To meet the requirements for the rapid development of mathematics, the quality of teaching and learning in high schools must be improved as quickly as possible.

In order to accomplish this task, there is an urgent need for modern instructional materials for the teaching of the main concepts of geometry. It is important for the students in the high schools to have the opportunity to learn geometric concepts and their applications through practical activities. The use of the activity approach will result in greater student involvement in the entire teaching-learning process and will elicit increased student creativity and increased independent discovery. By doing practical activities students will be able to apply the concepts of geometry more generally and authoritatively in their work in other subject areas. Also it is possible that students involved with these modern teaching and learning processes will gain a better basic education as well as a broader, and stronger residual knowledge of geometry. To date geometry has not been taught by means of practical application in Afghanistan, and students have not been given the chance to use laboratory materials in any branch of mathematics. The only exception has been the groups of students involved in this experimental study.

It is impossible to find a foreign curriculum or teaching procedure tailor-made to fulfill the aims and needs of a developing nation. As I.N. Thut and D. Adams (1964) have stressed:

> One of the imperative educational needs in the under-developed countries is the application of a more scientific and experimental approach to such problems as the writing of textbooks and other instructional materials . . .

As stated earlier, Afghanistan is in a stage of rapid economic development and cultural change. Improvement in the curriculum and textbook content as well as in teaching methods in mathematics at this stage requires the development of culturally dependent material for integration into the existing system.

Developed countries have an obligation to help developing countries achieve their educational goals, and to help them lay the foundations for other desired changes. In order to introduce change into mathematics as a basis for future technological improvements, all plausible approaches should be evaluated. Afghans however should develop their own educational system. Educated citizens of developing countries should use their knowledge, experience, ingenuity, and creativity to the maximum possible extent in order to develop the kind of technology needed. By following the patterns traced by the educational leaders in developed countries the task is simplified, but it takes the educated people of the country, to make it a desirable success.

The needs and significance may be summarized as follows:

1. To provide for much needed research into the relative merits of activity and traditional teaching methods in geometry.
2. To introduce activity-based learning experiences in place of the traditional "textbook and rote" approach which has proven insufficient in providing high school students with an adequate level of the knowledge of geometry.

3. To train students to a level of mathematical maturity sufficient to meet the scientific and technological requirements of the developmental programs which Afghanistan has recently undertaken to upgrade the social and economic status of her people.

II. BACKGROUND AND REVIEW OF RELATED LITERATURE

This chapter is organized into the following five sections: (1) Historical background of activities in mathematics education in Afghanistan, (2) Historical development of the activity-based approach to teaching and learning mathematics, (3) Learning theories pertaining to this study, (4) Review of the related literature, and (5) Summary.

Historical Background of Activities in Mathematics Education in Afghanistan

Formal secondary education in Afghanistan began in 1903 (Afghan Ministry of Education, 1965). Until recent years there was no attempt to define, or formulate a specific set of objectives for the teaching of mathematics, as well as other subjects, for the secondary schools. Most of the curricular content had been simply borrowed and translated, with slight changes, from foreign (mostly European) curriculums. The few changes that were occasionally introduced were made by translators who were not professional educators or specialists in the field.

Some attempts to improve the science and mathematics content of high school curriculums were initiated in 1964 through the Institute of Education, Kabul University, with the aid of a team from Teachers' College, Columbia University affiliated with Kabul University (Kayhan

Hussein and Carl C. Tranberg, Jr., 1964). Even then new textbooks were experimented with and introduced only on a small scale. Due to the presence of various customs and barriers, no attempts were made to make use of practical and contemporary laboratory approaches in teaching basic mathematical concepts via practice and exercises, as are used in Europe, the United States, and some other developed and developing countries.

At present the Curriculum and Textbook Project (Comparison of the New and Old Curriculum Materials, 1976), established in 1966 with the Afghan Ministry of Education, is working on the preparation of new curriculum and textbook materials for the elementary schools. The first paragraph of the mathematics section, from "Comparison of the New and Old Curriculum Materials" is quoted below:

> The Mathematics Section of the Curriculum and Textbook Project has undertaken an extensive revision and updating of the mathematics curriculum of the primary schools of Afghanistan. This effort began in 1966 with the defining of the aims and objectives for primary education by a national commission of educators. Soon after the new aims and objectives were published, a study of the existing mathematics curriculum and textbooks of Afghanistan was begun. The results of this study indicated that there were numerous shortcomings in both the curriculum and the teaching materials.

Among the major curricular shortcomings listed on page 94 of the study are the following:

1. The curriculum itself was outdated and did not reflect recent research and trends in content and methodology.

2. No guides were available for teachers and there was insufficient explanation for them in the pupil's textbook. Often the aim of the lesson was not clear, and the teachers were in doubt about what and how to teach.
3. Emphasis throughout the programs was on drill, not understanding.
4. Textbooks for various grades were written at different times by different authors and lacked consistency. Different procedures and methods were used in each grade which tends to confuse both students and teachers.

The job of elementary textbook revision for the curriculum and textbook project, with a limited number of members, is onerous. Consequently progress is slow. After about five years of work the project is only half finished. It will take a long time to complete the elementary level curriculum and textbook revision in mathematics, especially if practical activities are introduced. The development of quality teaching methodologies together with improved textbooks for mathematics can not keep up with the expected rate of change in the country if attempts in innovation for education are limited to their present slow rate.

Afghanistan has a centralized system of education. The Ministry of Education is the core of all educational administration and is the source of academic decisions for all schools. A uniform curriculum and syllabus are followed by high schools throughout the nation. All high school graduates are expected to receive equivalent training.

Since the establishment of republican form of government in Afghanistan on July 17, 1973, extensive educational planning has taken place as part of a basic educational reform policy. One of the objectives of this reform is preparation of better educated children and university students who, it is hoped, will help meet the nation's needs for more qualified scientists and technologists. The plans of the Ministry of Education are directed toward providing students with an education that will insure a better life and a brighter future for the people of Afghanistan.

Kabul University, the highest educational institution in the nation, provides training for teachers and supplies educational leadership for high school. Although other institutions have recently been established for training elementary school teachers, most of the students in these institutions are part-time teachers, who are taking in-service college level courses to improve their professional status.

The present goal of the reform in mathematics education is to train Afghan students to make use of geometric, algebraic and other mathematical concepts and relationships in their future life situations. A secondary goal is to develop in students general understanding of the scope and structure of mathematics, and an appreciation of the importance of its applications. The expectation behind the educational reform in mathematics is that future students will be able to take their places more effectively as productive and creative members of a rapidly developing nation. The program is expected to lay a stronger and longer lasting foundation for those who will continue their education beyond the secondary level and who will need mathematics as a basic instrument for the mastery of other related fields.

Historical Development of Activity-Based Approaches to Teaching and Learning Mathematics

For a long time certain mathematics educators have believed that the activity approach to teaching mathematics facilitated the learning process in all branches of mathematics, especially geometry, which has more direct relations with objects and reality. Johann Pestalozzi (1746-1827), a swiss educator, believed that in the learning process in mathematics the concrete must precede the abstract. Barnard Richard Paul (1972) states that:

> Experience has shown that those very children, who had acquired the first elements in the concrete and familar method described, had two great advantages over others. First, they were perfectly aware, not only of what they were doing, but also of the reason why. They were acquainted with the principal on which the solution depended; they were not merely following a formula by rote . . .

The study of Barnard Richard Paul (1972), concluded that before the twentieth century, reactions to the laboratory methods for secondary mathematics were varied. Around the turn of this century, A. W. Myers (1903) favored the use of mathematics laboratories at the secondary school level and advocated a flexibility of method. For him the mathematics laboratory meant:

> . . . Work-work on the pupil's part--or better still, method of getting work done by the pupil on his own initiative, under the impulse of his natural interests, and largely under the guidance of his own intelligence.

He went on to say that:

> . . . an advantage that can hardly be overestimated of the laboratory procedure with mathematical classes is that pupils sense the difficulties to be overcome as real and natural, actually needing to be resolved and demanding a knowledge of the mathematical tool as a means of their resolution.

Between 1919 and 1924 William A. Austin (1924), a teacher of geometry at the high school in Fresno, California, wrote several articles on the laboratory approach to the teaching of geometry. G. H. Ropes (1972) explains Austin's laboratory course in geometry as follows:

> The student was first introduced to geometry in a concrete form. The student was given a manual from which he was to work independently. The manual was so written that the student could prepare drawings and demonstrations, and learn definitions and theorems, with little or no assistance from the teacher. The student was required to make a drawing according to the directions in the manual. From his drawings he would infer a theorem, deduce a proof, and then classify his discovery.

R.M. McDill (1931) introduced a program in geometry based on the laboratory method. He emphasized experimental work to go along with geometry lessons at the primary level. J. A. Ramseyer (1935) was concerned with the fact that mathematics students left the schools with sufficient mathematics information, but without the ability to apply their knowledge. He believed that students learned better when they could see practical applications for mathematics. His students went on field trips to see the place of mathematics in society. He felt that if students could see the practical applications of mathematics on field trips, they could also see it in a properly-equipped classroom. Ramseyer, therefore, set up and equipped a mathematics laboratory in which the students were encouraged to use all the equipment. The conclusion was that better learning took place when students saw the meaning and the practical applications of the computations they learned to perform.

Following World War II progress was made in defining and recommending laboratory materials for all grade levels in school mathematics including geometry. The 1945 yearbook of the National Council of Teachers of Mathematics included a long, annotated bibliography related to the mathematics classroom and its laboratory materials.

H. F. Fehr (1947), in his article "The Place of Multisensory Aids in the Teacher Training Program," suggested the use of the laboratory in the training of teachers, so that they in turn would use the laboratory in their classes:

> . . . What I am saying is that laboratory teaching is one answer to giving reality to our subject without the loss of its abstract and theoretical aspects. There is not a single topic in grade or high school mathematics that can not be exemplified and put to work in a mathematical laboratory. But so static and traditional have we been in our teaching, that it has taken the armed forces and industry to show us

> the values of laboratory techniques in aiding the teaching of mathematics. We must not open ourselves to attacks such as were made on the teaching of our subject before the war. We must not delay in making our teaching vital, real, and necessary.

It took quite a long time and considerable effort for mathematicians and educators to create a renewed interest in the laboratory approach after the war.

A considerable part of the twenty-second yearbook of the National Council of Teachers of Mathematics (1954) was devoted to mathematics learning aids. Most of the articles in this section had to do with activities in mathematics teaching and with utilization of teacher-made and student-made laboratory materials in teaching mathematics.

After the Russians launched Sputnik I in 1957, a sense of national pride and security forced American educators to look for better ways to teach mathematics. Reforms in mathematics instruction were strengthened by large sums of money set aside by the United States Government for improvements in the training of mathematics teachers. Several research and activity projects provided for the better training of students, for more useful and pratical curriculums, and for rigorous research programs in mathematics education, e.g., The Madison Project, University of Illinois Arithmetic Project, Greater Cleveland Mathematics Project, School Mathematics Study Group, and Minnemath.

By 1965 the activity approach and the introduction of laboratory programs in the teaching of mathematics were widely accepted. The National Science Foundation and other groups found that students learn more effectively with the aid of activities. Thus, the need for creating new materials and activities and various approaches to make extensive use of them was obvious. This widespread interest in the teaching and learning of mathematics together with related activities has continued to develop.

G.H. Ropes (1972) concludes that more work on the laboratory approach has been done since 1965, than throughout the entire previous history of mathematics education. In the last decade, educators paid more attention to designing a method which would preserve the unity of mathematical concepts for the so-called new curriculums while matching the cognitive development of elementary school children. Programs related to teacher education paid more attention to the use of manipulative and concrete materials, and introduced the laboratory approach as a significant part of teaching methodology.

Edith E. Biggs (1970) of Britain greatly influenced elementary mathematics instruction. She came to the United States in 1967 and conducted a series of workshops on laboratory learning. She also conducted a workshop in 1970 to familiarize teachers and principals with the preparation of techniques for conducting classroom experiments in mathematics.

According to W.M. Fitzgerald (1969), teacher education institutions began to include laboratory training in their undergraduate and graduate programs. Writers included the practical approach in their textbooks on pedagogy of mathematics instruction. Three universities that began to incorporate laboratory techniques into their pre-service mathematics programs were, Michigan State University, State University College at New Paltz, and Madison College.

W. A. Ewbank (1971) wrote an article in dialogue form describing the mathematics laboratory as he had used it in the Taylor University Mathematics program and in a public school. The entire December, 1971, issue of The Arithmetic Teacher was devoted to the mathematics laboratory. G. Matthews and J. Camber (1971) discussed their experiences with mathematics laboratories in the Nuffield Project in England. They described the approach, the materials and the effective ways they developed for insuring active student involvement in learning mathematics.

During the last five years, mathematics educators (especially in England, France, Germany and the United States of America) broadly introduced newly developed concrete instructional materials along with the traditional textbook, paper, pencil, and chalkboard in their mathematics classrooms in most primary schools, some junior high schools and a very few senior high schools.

G.H. Ropes (1972), in his study states:

> The use of concrete objects in mathematics education has a long history. For centuries those who propounded the use of concrete materials were few in number and won few converts. During the twentieth century, however, the advocates of manipulative materials have become more numerous, and today constitute a vital force in the development of mathematics education.

Between 1973 and 1976 a large number of educators became involved in promoting the activity and laboratory approaches for teaching mathematics, but it seems that almost all the attempts were focused on the elementary grade levels.

Marilyn N. Suydam and Jon L. Higgins (1976), in a final report "Review and Synthesis of Studies of Activity-Based Approach to Mathematics Teaching" submitted to the National Institute of Education, were as comprehensive as possible in compiling the list of all available studies related to mathematics teaching in recent years. Suydam (1976) in Appendix A of her review paper presents a lengthy (47 pages) annotated list of references. But no research was cited on the activity approach to teaching geometry at the high school level. However, the overall results of Suydam's review study indicate that students using activity-oriented programs can generally be expected to achieve at least as well as or better than students using programs not activity-oriented.

Learning Theories Pertaining to This Study

Considering methodology of learning and teaching mathematics in Afghanistan, one can say that not enough attention has been given to modernizing the teaching-learning processes, based on the educational and philosophical techniques and related theories of learning followed by developed nations. To meet the objectives of this study, it is appropriate to review the educational theories suggested by leading educators and researchers in the field.

It has been clearly obvious to educators that one often forgets what he has read or heard, that after a while he forgets what he has seen, but that he always remembers and retains longer that which he has had an opportunity to handle, manipulate, bend, cut, paste, measure, make, or draw. These activities increase the student's achievements while he is learning various mathematical concepts.

Irving Adler (1966) puts Jean Piaget's ideas about activity learning in the following way:

> The child is a self-acting organism that grows. Learning is not a mere accumulation of knowledge but is a process of growth with action and practices . . .

In another part of the same article, emphasizing the importance of the activity approach in teaching mathematics, Adler, summarizes Piagetian theory as follows:

> Physical action is one of the bases of learning. To learn effectively, the child must be a participant in events, not merely a spectator. To develop his concepts of number and space, it is not enough that he look at things. He must also touch things, move them, turn them, put them together, and take them apart.

Also, Clair W. McClure (1971) states:

> Piaget has worked with children of all ages, attempting to open the doors of their intricate minds and discover how children learn. The individualistic nature of the child's learning experience, as emphasized by Piaget's reports, makes the laboratory methods even more important as it may provide the children with easy opportunity to choose experiments appropriate to their particular stage of development.

Jean Piaget (1951) believes that confronting children with problems relating to practical situations often ends the indifference they may display when asked to handle less practical problems. His concern for self-learning and self-motivated activity is made clear in his statement,

> The real aim of education is to induce the student to win truth for himself, even at the risk of losing time and taking all the devious routes implicit in real personal initiative.

Z.P. Dienes (1963) considers usage of laboratory as the fundamental and basic step in learning science and mathematics. In his article, "On the Learning of Mathematics" he points out the importance of using concrete material in teaching mathematics:

> The use of concrete materials in the classroom has for one of its purposes the building up of mathematical imagery. Such imagery, once built up, can be manipulated without the aid of any concrete object. It's these image-manipulations or better still, structure-manipulations that should be symbolized by the symbols or using objects . . .

To Dienes, the Piagetian notion of cognitive development (comprising three stages, sensorimotor, concrete operational, and formal operational) is mirrored in the formation of every concept. This cycle of concept formation with its three stages is necessary before a mathematical concept becomes operational for students.

Dienes contends, as does Jean Piaget, that a concept is usually formed roughly in three stages: (a) The first is a play stage in which the subject plays with the elements relating to the concept in concrete situations, (b) The next stage involves the slow realization of a direction along which experience of the subject is ordered into a meaningful whole, and (c) The last stage involves the moment of understanding as the subject apprehends the concept. This view of concept formation is utilized at the three levels of mathematics awareness which are: (1) construction, (2) translation from construction to analysis, and (3) analysis. Pervading all three levels mentioned above is the dynamic principle which states that all abstraction, especially in mathematics, is based on experience and practical action and on that learning process involving the cycle of concept formation as the basis of learning.

In his article entitled "On the Learning of Mathematics"[2] Jerome S. Bruner (1960) of Harvard University, in stating his theories

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[2] A paper presented before the National Council of Teachers of Mathematics, Salt Lake City, Utah, August, 1960.

relative to the transmission of knowledge in mathematics and science to the learner, pointing out the importance of physical embodiment and the usage of concrete materials, saying:

> To a young student who is used to thinking of things that either exist or do not exist, it is hard to tell the truth in answer to his question of whether pressure "really" exist. We wish to transmit the idea that there are observables that have regularities and constructs that are used for conserving and representing these regularities, that both, in different senses, exist and the constructs are not fantasies like gremlins or fairies, that is the structure.

In another part of his article he points out the importance of the discovery method in learning mathematics.

> With the active attitude that an emphasis on discovery can stimulate, with greater emphasis (or fewer restraints) on intuition in our students, and with a courteous and ingenious effort to translate organizing ideas into the available thought forms of our students, we are in a position to construct curricula that have continuity and depth and that carry their own reward in giving a sense of increasing mastry over powerful ideas and concepts that are worth knowing, not because they are interesting in a trivial sense but because they give the ultimate delight of making the world more predictable and less complex.

According to Rechard R. Skemp (1971), concepts are formed over previous experiences and this takes more mental power with the opportunity to experiment, think, and draw further abstractions using previous experiences, together with objects and ideas. Skemp calls this stage in the process, "the second order abstraction" or the next step in concept formation. In order for students to learn geometry, one of the basic branches of mathematics, it is important and very useful for them to have the opportunity to form the fundamental stages by using concrete objects. It is also important for them to think creatively about the interrelationships of the objects, apply their knowledge to various situations, solve problems, state the facts in their own words, be able to give examples in a creative manner, relate them to physical situations and show interest in the subject. Having these opportunities they will learn more meaningfully, retain the concepts, and make use of them in further stages of their development.

As was mentioned earlier in this study, within the existing system of teaching mathematics in Afghanistan there has not been an opportunity to provide materials needed for a more effective way of learning than lecture and textbook with rote memorization on the part of students. Modern mathematics educators agree that the activity approach reflects a philosophy of pedagogy which is quite different from the traditional authoritarian approach practiced in Afghan schools. It is likely that many teachers are already using this more informal style of teaching in some areas of the curriculum other than mathematics, e.g., the science, arts, and physical education. But mathematics more than any other area, seems often to stagnate in a rote memorization method. The learning theories of Jean Piaget, Z.P. Dienes, J.S. Bruner and R.R. Skemp supported by the results of this study hopefully will provide impetus for implementing methodological innovation in mathematics education.

Review of Related Literature

Reported experiences and some research studies cited in available indexes concerning the activity approach to teaching mathematics suggest the ideas pursued in this study.

Large scale movements such as the Nuffield Project and the School Mathematics Project in England are focused on laboratory approaches to elementary mathematics and are thus not considered closely related to this study. The literature more relevant to this study is directed toward guidelines for practical and activity-based approaches to teaching and learning mathematics, especially geometry in secondary schools. Current thinking covering laboratory technique in mathematics education in the United States is illustrated by Math-Lab-Matrix (1975):

> When physical models are used to illustrate geometric ideas, there is a nice opportunity to integrate science and mathematics. The work with mirrors, measurement and solid structures provides illustrations of these possibilities. . . . Mathematics laboratory aids are not the total answer but they do provide a vehicle for active involvement of both the students and teachers in explaining the basic ideas of geometry.

Writing about the effect of the mathematics laboratory Marilyn N. Suydam (1975) states:

> The findings of a study by Ropes are similar to those of some other studies on the strategy. A group of sixth graders and a group of second graders each spent one 45-minute period per week for fourteen weeks in a mathematics laboratory, working in small groups with a variety of manipulative materials and activity sheets. Compared with students not given a mathematics laboratory experience, these pupils had no significant change in over-all attitude toward mathematics, although they did develop a greater awareness of the enjoyment to be derived from mathematics and an increased likeing for it. On achievement tests, they scored as well as pupils in regular classes despite the 20 percent less time that laboratory students spent in regular mathematics lessons.

Clair Wylie McClure (1971) studied the laboratory techniques for elementary schools and recommended that the same techniques be studied for junior and senior high schools. He states:

> Experiences which are good for elementary students may also be good for junior and senior high school students. If laboratory techniques increase the achievement levels of elementary classes, there is no real reason to doubt that the same thing will happen at the eighth grade level. At present it appears that the elementary schools are leading the secondary schools in their use of laboratory techniques. However, the idea of the discovery approach can definitely be enhanced at all levels, kindergarten through graduate school, through effective use of the mathematics laboratory.

P.L. Spencer and M. Brydeguard (1952) have been complimentory in their description of mathematics laboratories and their beneficial influence in the mathematics teaching processes:

> The classroom for mathematics should be a learning laboratory. This does not imply that equipment or fancy gadgets in a room, or things children build make a "learning laboratory" rather a classroom becomes a learning-laboratory when it produces mental and physical activity that results in experimentation; this in turn should lead to formulation of procedures and to generalizations based upon reliable and

> sufficient information. The materials for laboratory are within the reach of every teacher. The materials consist, for the most part, of things that children and teachers bring into the classroom for the lessons under consideration. Cups, glasses, bottles, cans, jars, boxes, labels, cartons, string, measuring sticks, and innumerable other things to use in experiments with measuring should be a part of every mathematics classroom.

R.E. Reys (1971) wrote a lengthy article describing the use of manipulative materials. He discussed the rationale behind the use of the materials and their selection criteria. He felt that the manipulative, concrete materials would serve as aids, not as a substitute, for the good teacher in mathematics.

In the summer of 1963, 29 mathematicians and scientists met in Cambridge, Massachusetts to review school mathematics and to establish goals for mathematics education. The participants were bound only by their imaginations as they looked toward the twenth-first century. The conference resulted in a bulletin "Goals for School Mathematics" commonly called The Cambridge Report. It included proposals for mathematics curriculums for schools and set guidelines to serve as a basis for further discussions and experimentation. The report stated:

> Setting aside time for a mathematical laboratory is another way of stimulating interest and a creative approach. In elementary school several hours each month should be made available for mathematics games, special topics, experiments with apparatus, such as needles and lines, thumbtacks, and computers, etc. This provides a means of reaching many students not responding well to the regular classroom instruction. It gives regular opportunity for

> progress through experimentation, with the curriculum and with the pedagogical techniques. As many such laboratory sessions cut across grade and ability levels several classes may be handled together. This gives opportunity for several mathemitics teachers in the same school to pool their time and talent in the design of those sessions and in their supervision.

J. D. Rowlett (1966) found that a discovery approach was more effective in retention when measured over a long period of time.

In May, 1968, E. E. Biggs discussed the mathematics revolution in Britain. She said that the discovery method of teaching shifted emphasis from teaching to learning and from the adult's world to the child's world. The discovery method gives children the opportunity to think for themselves so that learning becomes an active, creative process. Biggs said that although the method did not start in the field of mathematics, some of the greatest and most exciting changes were taking place there. When children learned by active methods, they were constantly learning. The effect of the method on the children was to awaken a lively interest in discovery making for an increased sense of responsibility, and versatility.

P.S. Jones (1970) believed that the most effective mathematics instruction occurred when mathematics was taught in correlation with other subjects and when the laboratory approach was included in the instruction.

In searching through the literature, no previous research measuring the effect of activity or practical approach in teaching geometry was found. There have been a few laboratory approach studies in elementary levels related to the one this study concentrated upon, such as: A Study of the Effect of Mathematics Laboratories on Mathematical Achievement and Attitude of Elementary Schools, by Loye Y. Hollis (1974). A Study of Laboratory Approach and Guided Discovery in the Teaching Learning of Mathematics by Children and Prospective Teachers by Esther R. Unkel (1972) and Effects of Mathematics Laboratories for Eighth Grades by Clair Wylie McClure (1971). Marilyn N. Suydam has prepared a bulletin "Mathematics Education (ME) published by ERIC/SMEAC[3] (1974-1975), in which she has indicated that:

> Bernard traced the historical development of the laboratory approach between 1966 and 71, such an approach was used in more programs, discussed in more publications, and advocated by more educators than at any previous time. The literature was primarily devoted to philosophical discussions of the merits of the laboratory approach, and most writers spoke enthusiastically about their success with the laboratory approach, but no reports of research to test the alleged superiority of the laboratory approach to school mathematics were found in the literature.

Summary

The existing mathematical educational system of Afghanistan can be characterized as relying too much on rote memorization, while

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[3] ERIC is an abbreviation for Educational Resource Information Center and SMEAS stands for Science and Mathematics Education Analysis Center.

current theories of Jean Piaget, Z. P. Dienes J. Bruner and others emphasize more meaningful learning can take place where whole student activities play a role in the teaching-learning process.

No research in mathematics education related to laboratory approach has been done in Afghanistan and very little in secondary mathematics education, laboratory techniques and activity approaches elsewhere. Also no related studies could be found concerning laboratory techniques in geometry education in the secondary level. Hence a great need for a study of this type was inferred, and it was hoped that it would be of value for mathematics instruction in Afghanistan.

III. THE STUDY

This chapter is divided into the following major sections: (1) Overview, (2) Experimental Design, (3) Method of Random Sampling, (4) Implementation of the Activity Approach, (5) The Evaluation Instruments and Procedures Used in Collecting Data, and (6) Summary.

Overview

The purpose of this study is to determine if students following an activity approach to learning geometry show significantly better achievement than those who follow the traditional lecture approach.

In November 1975 the author was officially introduced by Kabul University to the Afghan Ministry of Education to administer an experiment involving the activity approach to teaching geometry in some selected high schools in Kabul. Permission was granted by the Ministry of Education, and the authorities kindly helped in the random selection of two high schools (Aesha-Dorrani and Shahi-Doshamshera) as representative of high schools in the area. The author was introduced to the schools as a mathematics specialist authorized to present his approach to teaching geometry in those schools. The objectives were described to the principals and heads of mathematics departments in both of the schools. They agreed to help randomly select four teachers in each school from among those mathematics teachers who had similar educational experience and academic background. Twelve teachers in one school and eight in the other were determined to meet the criteria of having similar background and experience. Two teachers were further selected from among the four previously selected ones in each of the schools, to use the activity approach as they taught geometry in two classes in each of the ninth and the tenth grades (the experimental classes). The remaining two teachers of the four in each school were to follow the traditional approach in two geometry classes in each of the ninth and tenth grades (the control classes).[4] For a summary of teachers, classes and students in both schools see Table 1 on page 52.

During the winter vacation (January-March 1976), the four experimental teachers met twice a week for two hours to gain familiarity in the use of the materials and understand the objectives and the methods of presentation of the activity approach to their students. Also they were instructed in learning how to help their students to discover phenomena in the physical world related to the concepts in geometry and how to prepare their students to solve and enjoy related home assignments. By the beginning of the school year (March 22) guidelines for 48 activities (24 for each grade) related to the topics in

[4]For the procedures used to select students and classes see the section on Methods of Random Sampling in this chapter.

the curriculum were prepared and duplicated for use by those teachers in the experimental classes. Six selected activity guides (three for each grade) similar in form and style to the other 42, have been translated from Persian into English and are displayed in Appendix A. All activity guides are available from the author.

In order not to place the group of control teachers at a possible disadvantage as compared to the group of experimental teachers, a parallel inservice program (reinforcement program) was held twice weekly for two hours for them as well. These meetings dealt with questions related to the concepts and contents of the ninth and tenth grade geometry textbooks and the traditional instruction thereof. Solution keys for the textbooks were prepared by the author at this time and all problems and exercises were worked out in detail with the teachers. A major product of this reinforcement program was the two complete solution keys for the ninth and tenth grade geometry textbooks. Such keys had never before been produced. The keys were mimeographed and placed in the school libraries for use of all staff and students of the two high schools.

Prior to the beginning of the academic year, 602 students were randomly selected from among 1540 students enrolled in the ninth and tenth grades of the two schools. These students were then randomly divided into 14 sections as follows: four ninth grade experimental classes, four ninth grade control classes, three tenth grade experimental classes and three tenth grade control classes. The corresponding teachers were then assigned to these classes.

The students in the experimental classes were taught the same material as the control students with the modification that approximately once a week an activity module was presented to them for 20 minutes. The activity modules were presented following the lecture and related illustrations, and these activities replaced the traditional subsequent recitation.

Three intermediate tests (given every other month) and a final examination were given to both groups during the academic year. The scores from these tests and the final examination were used as data for the purposes of comparison in this study.[5] English translation of all test instruments are given in Appendix B.

Experimental Design

The research design for this study is "the posttest only control group design," described by Campbell and Stanley (1963). Since introducing a new set of concepts of geometry and student achievement in particular, were among factors to be studied in this investigation, a pretest in the ordinary sense was not possible.

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[5] For details about the randomization in sampling, nature of activity approach, the evaluation instruments and procedures used in collecting and analyzing data, see related sections.

According to Campbell and Stanley (1963):

> . . . , many problems exist for which pretests are unavailable, inconvenient, or likely to be reactive, and for such purposes the legitimacy of design 6 still needs emphasis in many quarters. In addition to studies of the mode of teaching novel subject materials, a large class of instances remains in which (1) the X and post test O can be delivered to students or groups as a single natural package, and (2) a pretest would be awkward. Such settings frequently occur in research on testing procedures themselves, as in studies of different instructions different answersheet formats. . . . Where student anonymity must be kept, Design 6 is usually the most convenient.

The same authors point out on page 33:

> . . . Multiple O's should be an orthodox requirement in any study of teaching methods. At the simplest level both essay and objective examinations should be used.

Engelhart (1972) states:

> In the case of experimentation concerned with treatments or methods of instruction, a posttest may provide the crucial factor in the acquisition of concepts or skills.

The design used for this investigation is illustrated below.

$$R_1 \quad X \quad O_1 \quad O_2 \quad O_3 \quad O_4$$

$$R_2 \quad\quad\;\; O_1 \quad O_2 \quad O_3 \quad O_4$$

This design indicates the use of two groups randomly selected. The experimental group (R_1) received the experimental (activity approach) treatment (X). The control group (R_2) received the same treatment with the exception that the activity modules in the classroom and outside were withheld. Three intermediate posttests (O_1, O_2, O_3)

and a final examination (O_4) were given to both groups on the same days.

For the purpose of this investigation the students in the population were selected and divided into two groups by "their names drawn from a covered box" and "even and odd" procedures respectively. Two hundred fifty-two out of 300 students in R_1 and 242 out of 302 students in R_2 were able to stay in school until the end of the school year.

After the tests were taken, they were graded by the teachers and were passed by the teacher to a different teacher for duplicate checking. The scores were then recorded and the test papers were returned to the students to be sure they agreed upon the scores and understood how to correct their mistakes.

For proper statistical survey for the design used in this study, Campbell and Stanley (1963) suggest a standard t-test to conclude the significance of outcomes. Since the alternate hypothesis expresses the direction of difference, a one-tailed t-test is appropriate.

Methods of Random Sampling

There were four stages in this study that required randomization: (1) selection of schools, (2) selection of teachers, (3) selection of students for participation in the study, and (4) assignment of the students in the study to either the experimental or control groups.

Random sampling of schools was achieved in the following manner: Names of the 25 high schools in Kabul, were written on a list and numbered. A covered box with 25 pieces of paper each representing a number between zero and 26 was used. Two pieces of paper with the numbers representing Shahi-Doshamshera and Aesha-Dorrani high schools in the list of schools were drawn.

The random selection of teachers in each of the two schools was done in the same manner. Boxes with pieces of paper each bearing a mathematics teacher's name were used. Four mathematics teachers out of 12 having similar academic standing and teaching experience in Aesha-Dorrani and four teachers out of eight with similar preparation and experience in Shahi-Doshamshera high school were selected as the eight participating teachers for the research program. By means of the same selection procedure, two teachers from among the participating four in each school were further selected to teach the experimental classes. Thus four teachers were identified as teachers of the control classes.

Before the schools opened in March, 188 students from among 517 in the ninth grade and 140 students out of 446 students in the tenth grade were randomly selected in Aesha-Dorrani high school. In Shahi-Doshamshera high school, 184 students from among 270 students in the ninth grade and 90 students out of 307 students in the tenth grade were randomly selected. Randomization in both schools was of the simplest type, "using the box." As the pieces of paper with student's names, accompanied by their father's names, were drawn from the box, the names were listed in chronological order. Then, following an "even and odd" procedure along the list the students were assigned into one of the two groups R_1 and R_2 known as "the activity approach group" and "the control group," respectively.

There were four ninth grade classes with a total of 186 students, and three tenth grade classes with a total of 114 students in the activity approach group in the combined populations. The same number of classes with a total of 186 students in the ninth grade, and a total of 116 students in the tenth grades in the traditional approach control group in the combined populations were involved in the study.

Letters indicating section as well as the location of each of the classrooms[6] were assigned to each of the ninth and tenth grade classes by the principal's office in each school. The purpose was to locate the experimental and control group students as far apart physically from each other as possible in the school to avoid student contact between the two groups.

[6]Students of each class were assigned to the classrooms and the teachers moved from classroom to classroom.

The Activity Approach

During the school year (March 22-December 15, 1976) the students in the experimental classes were taught geometry by means of the activity approach which is defined as active student involvement in the learning process using solution keys and practical activities to supplement lecture and textbook presentations. The students in the control classes followed the traditional lecture and textbook method. The content presented to each of the two groups in each grade was limited to the curriculum topics prescribed in the textbooks for geometry for those grades in high schools in Afghanistan. The experimental and control classes had the normal 2 and 3 45-minute periods of geometry per week for the ninth and tenth grades respectively.

Meetings were scheduled with the members of each group of teachers during the school year. These meetings were held for at least 45 minutes twice each week at the end of the school day. Each of the groups met separately. The purpose of these meetings was to maintain communication and review various pedagogical aspects of the activity approach along with the application of related materials, to provide guidance in developing evaluation instruments, and to help the teachers improve their background in the field.

To avoid a possible Hawthorne effect, the teachers involved in the study were all treated as participants in an innovative program with differences only in classroom procedures. The same amount of time was spent with each group developing their professional expertise. The students in the program were not informed that a study was being conducted. Obviously they were aware that the two classes taught by one teacher in each grade were using a different approach, but they were told that it was their teacher's decision to try a different method with their classes and they were not made aware of the existence of a special experimental study. The author was always behind the scene. During his occasional visits to a given class for observation he was considered as an inspector from the Ministry of Education.

In all of the experimental sections the activity modules followed the lectures, and were scheduled for not more than 20 minutes each. They were presented only at those times that the class was ready for them, i.e., usually towards the end of the class period, after the lecture and at a time when some illustrative problems related to the lecture would normally have been presented. To perform each activity the students were given the materials and activity guide sheets. The major parts included in the guide sheets were: The topic of the activity which was usually the same topic as the lesson covered in the lectures for that week or possibly the week before, the objective involved in the activity, a list of materials needed for the performance of the activity, the guidelines for performance and completion of the activity, some questions related to the concept considered in the activity and at the end a set of related home assignments. Two sets of three activity guide sheets were randomly selected and are presented in Appendix A. The questions on the activity guide sheets were arranged in such a way as to guide the students through situations which hopefully would motivate the students to try things on their own and experiment for themselves.

Activity materials such as waxed paper, onion skin paper, square and circular geoboards, tangram pieces, measuring tapes, small rectangular-shaped mirrors, narrow straight sticks for building models, wires, compasses, rulers, cutters, pins, nails, paper plates, thread, glue, protractors and other drawing instruments were either made available or procurred by the students. All the materials mentioned above were available locally.

Each of the activities was presented to the students as a discovery lesson. They were also instructed in the use of the materials. Some oral instructions were given by the teachers prior to each activity. Most guideline sheets also presented problems which required that the students search the literature or ask questions of others in school or at home. Oftentimes they were encouraged to find a solution or application of the concepts in physical situations, in nature, or in the arts. During the class lectures in the experimental classes many students showed interest in alternative approaches to concepts other than those available in their textbooks. They also appeared anxious to apply that which they had learned in theory to real situations. The role of the teacher in the presentation of the activities was to give proper guidance in initiating the activity; the rest of the activity was done by the students to develop the geometric concepts involved. Students were not expected to complete all the activities contained in a guideline sheet, by the end of one class period. They willingly took the materials needed (except for the guideline sheets which were not to be taken from classroom) home with them to continue with the activity and to practice with them at home.

Written homework or reports were brought to school along with related materials produced by those who were able to do so. Students' creative productions were examined by the teachers and any comments or criticisms were made directly to the student. In case the contribution was considered illustrative or useful in clarifying the concept involved, the student was asked to describe it either to the members of his working group of four to six students or to all his classmates.

In the control sections, recitation was stressed as the usual manners in the traditional approach. Home assignments were given, but the activities and the related concept applications were not made available. The solution keys for the textbook exercises placed in the school libraries were used more often by the activity group students, and rarely by students in the control groups, simply because the control group students already had most of the solutions presented to them in the classroom as part of the lecture. The activity group students used the solution keys to solve those textbook problems not presented in class and consulted other references in the library to gain insight into the problems posed in the activities.

The Evaluation Instruments and Collecting the Data

As the activity approach was used during the academic year 1976, four teacher-made, nonstandardized, noncommercial tests were given to both the control and the experimental groups in both schools. The purpose of each of these tests was to measure variables related to the hypotheses of this study as well as to provide normal high school evaluation data. It was necessary to train the teachers to construct these tests. During the inservice seminars, the author presented to both groups of teachers a brief description of various techniques for effective evaluation and test construction as suggested by leading experts such as Norman E. Grondlund (1971), Benjamin S. Bloom (1963), Robert L. Ebel (1972), Robert F. Mager (1968) and others. Some standardized American tests such as, The Cooperative Plane Geometry Test (form Z), the Shaycoft Plane Geometry Test, the Seattle Plane Geometry Test and the Illinois (Chicago) Plane Geometry Test, were used as examples of quality tests during these presentations to the teachers.

Each of the tests was constructed in a joint committee meeting of both the control and experimental groups of teachers, who were involved in teaching the ninth and tenth grade in both schools. The teachers who taught the classes were responsible both for content coverage of the geometry syllabi and for constructing the tests. Each suggestion in the committee was fully discussed and the results were incorporated in the test items. Translations of all eight tests (four for each of the ninth and tenth grade) are given in Appendix B. The tests were limited to 18 items (16 multiple choice and 2 essay) and were given during a 50 minute period. Each item was selected on the basis of its relationship to the concepts being taught. The committee members also considered the list of hypotheses for this study given in Chapter I as they were discussing the items for the tests in order to make sure that appropriate items were included which could be used to test these hypotheses.

The tests included no geometric topics which had not been covered by both control and experimental groups of students in each grade. Neither a new term nor any unfamiliar concept or symbolism was used in order not to place either group at a disadvantage. Time was not a factor in as much as more than 90 percent of the students finished each test in the allotted time. Scoring was on a 100-point basis, each multiple choice item was worth five points and the two essays worth up to ten points each. The scoring criteria were based on the theory that "on a power test omissions and incorrect answers probably mean inability to master the item" (J. P. Guilford, 1965).

A pilot version of each test was given to a class of the same grade students but which was neither an experimental nor a control class. The pilot tests were monitored by a mathematics teacher who was not involved in the study and was given at least two or three days before the test was administered to the students involved in the study. The purpose of these pilot runs was to identify problems with too high a difficulty index or with too low a discrimination index for maintaining test reliabilities. Such items were replaced by substitute problems made by the committee.

The validity of each test was checked by a jury of four experts; two chairmen of the mathematics departments in both schools, and two mathematics professors (Mr. Ebrahimzada of Kabul University and Dr. Barry D. Vogeli of Teacher's College, Columbia University Team in Kabul).

Each test was monitored by at least two teachers, plus some school administrative personnel. Maximum spacing of students during tests insured test security in both schools. All classes in both groups of the same grade in both schools were tested on the same day.

The test papers for each class were graded by the teachers who were in charge of each class. All teachers in charge of classes in both control and experimental groups of the same grade compared their keys for the tests prior to grading the test, in order to agree upon rules for grading the essay problems, and to keep the scores as consistent as possible. To insure maximum accuracy in scoring the tests, each teacher was expected to pass the test papers to another teacher in the same grade to double check the scoring.

All the teachers were supposed to solve the problems on the chalk board and give the test papers back to the students, to see if the students agreed with their scores. Then each teacher recorded the scores on an official class list for the student's record files. The official class lists were turned over to the teachers by the school office, signed and stamped by the principal of each school.

The author then borrowed the test papers from the teachers to record the scores for each item as the main data for this study. The papers then were handed back to the teachers to return to their students.

Two other evaluation instruments for this study consisted of an opinionnaire and some recommendation papers from persons not directly involved in the study. Some of these persons were fully aware of the effects of the study in the schools, such as the principals, while others were authorities in the field and professionally able to judge and comment upon the value of the expected outcomes of the study.

The opinionnaire consisted of 30 statements, each purposely phrased positively or negatively by the author. The opinionnaires were given to those teachers who were directly involved in this study and to the chairmen of mathematics departments of each of the two high schools. A copy of the opinionnaire, quotations of the collected comments and a table summarizing the results from the opinionnaire are given in Appendix C.

Copies of the recommendation papers from: the two principals of both Aesha-Dorrani and Shahi-Doshamshera high schools; Mr. Ahmad Ebrahimzada, assistant professor of mathematics education in the Faculty of Science, Kabul University; Dr. Lennart L. Nilson, UNESCO expert in curriculum, mathematics and science teaching aids assigned to Afghanistan; Dr. George W. Glidden, educational consultant of the Nebraska University Team at the Kabul University; Dr. Barry D. Vogeli, mathematics advisor, Teachers College Columbia University Team Curriculum and Textbook Project, of the Afghan Ministry of Education are given in Appendix D.

The purpose of adding the recommendations from individuals other than those directly involved in this study is to convey some expert views to the reader of this study and to illustrate the expected results and the importance of the investigation.

Summary

The study was conducted during the 1976 school year (March 22-December 15). Ninth and tenth grade students in two high schools in Kabul city were selected at random to participate in the study.

The teachers and students were assigned to the classes by the school principal's offices at random from among the group of teachers available for teaching mathematics to those levels and from among the total enrollement list of pupils for each grade in both schools. Table 1 summarizes the distribution and number of schools, teachers, randomization groups, classes and students in each group in both schools.

Table 1. Teachers, classes and students in both schools.

Schools	Aesha-Dorrani				Shahi-Doshamshera				Total
Teachers	1	2	3	4	5	6	7	8	8
Randomization groups	R_1	R_2	R_1	R_2	R_1	R_2	R_1	R_2	2
Classes	10D 10H	10A 10B	9A 9C	9T 9Z	10C	10A	9A 9B	9C 9D	14
Number of pupils in (R_1) experimental group	68		93		46		93		300
Number of pupils in (R_2) control group		72		95		44		91	302

During the winter vacation (January-mid-March 1976) the teachers participating in the study were presented with appropriate inservice programs to help them perform their roles as experimental and control group teachers. These programs were extended through the academic year.

The design for this study was a true design described as the "Posttest Only Control Group Design" and is illustrated by

$$\begin{array}{cccccc} R_1 & X & O_1 & O_2 & O_3 & O_4 \\ R_2 & & O_1 & O_2 & O_3 & O_4 \end{array}$$

It indicates the use of two randomly selected groups, the experimental (R_1) and the control (R_2) groups. The experimental group receives the activity approach treatment (X) while the control group receives the traditional lecture treatment, which is considered the same in all its aspects as the X, except that the activity portion is withheld. The O_i ($i = 1, 2, 3, 4$) represent O_1, O_2 and O_3, the three intermediate tests, and O_4, the final examination. Each test was given to both groups of students in each grade on the same day in both schools. The tests were constructed by committees consisting of both the traditional and the activity-approach teachers of each grade in both schools. The tests were considered as substitutes for standardized geometry tests, because none of the available standardized tests are appropriate for high schools of Afghanistan. Each one of the three tests and final examination consisted of 16 multiple-choice and two essay items. Their validity was checked by a jury of four experts.

The test papers were graded by the teachers and double-checked by another teacher, then the scores were recorded. The teachers reviewed the solutions in the classrooms and returned the test papers to the students.

Other evaluation instruments used for this study were a teacher opinionnaire and statements of recommendations written by individuals not directly involved in the study, but who were however, in a position to comment upon its expected learning outcomes authoritatively and professionally.

IV. ANALYSIS OF DATA AND FINDINGS

The purpose of a statistical analysis of the data given in this chapter is to evaluate the effectiveness of the activity approach model in teaching and learning geometry in the ninth and tenth grade as compared to the non-activity model (i. e., traditional lecture and rote).

The following hypotheses were tested against their alternatives:

($H_0$1): There is no significant difference between the experimental and control groups of students in overall achievement in learning geometry.

($H_0$2): There is no significant difference between the two groups of students in creative thinking in geometry.

($H_0$3): There is no significant difference between the two groups of students in their ability to explain geometric concepts correctly in their own words.

($H_0$4): There is no significant difference between the two groups of students in their ability to recall geometric concepts.

($H_0$5): There is no significant difference between the groups of students in their ability to solve geometric problems that are original to them.

($H_0$6): There is no significant difference between the two groups of students in their ability to set up complete step by step proofs for certain geometric theorems.

The data for this study were processed and analyzed through the Statistical Interactive Programming System (SIPS) on a CDC-3300 computer at Oregon State University. The mean scores of the paired groups were computed and the "Student"-t-test was used to draw conclusions as to the difference between the two groups of students involved. The statistical analysis of student achievement in learning geometry indicated in the hypotheses are analyzed separately for intermediate tests and collectively for final examination.

Each experimental group was compared with a paired control group by their raw score on the three tests and the final examination. The tests were given at least two months apart to find out if there was any difference in the class means for each group of students, over the two months' academic period. The final examination was administered at the end of the academic year in order to compare overall achievement for the two groups in each grade.

In this chapter, first, the data relating to each one of the three tests will be studied. Second, the final examination scores will be analyzed, and finally the results and findings will be discussed to determine the existence of significant differences between students' achievements in each one of the aspects of learning and teaching geometry specified by the hypotheses previously stated.

There were 16 multiple-choice and two essay type items in each test and in the final examination. Each item included in the test contributed to measuring one of the five aspects of learning geometry corresponding to one of the hypotheses, and were used collectively to determine overall achievement tested in the first hypothesis. All of the six hypotheses were tested against their alternatives. The results are shown in Tables (2-9) which present the processed data for each test.

Analysis of the Test Scores

Scores derived from administering the tests were analyzed to determine student achievement for the experimental groups of students versus the control groups. The objective of these tests was to measure if there was a significant difference in the mean scores from each of the tests over each of the two months' periods of testing. The results presented by the tables below indicate that almost all aspects of learning geometry tested were improved substantially in the experimental groups as compared to the control groups.

Analysis of the First Test Scores

Since there were two groups of students in each of the ninth and tenth grades, the findings from the first tests for different grades are analyzed and presented in Tables 2 and 3.

Table 2. Data analysis for ninth grade first test.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp. Con.	53.13 43.08	15.47 13.30	1.27 1.11	5.89
Creative thinking	9, 16	Exp. Con.	6.35 4.40	3.60 3.63	0.30 0.30	4.55
Explaining concepts	5, 8	Exp. Con.	4.75 4.74	3.55 3.55	0.29 0.28	0.02
Recalling concepts	1, 4, 12 13, 14	Exp. Conc.	7.08 5.04	6.27 4.76	0.52 0.39	3.10
Problem solving	2, 3, 6, 7 10, 11, 15, 17	Exp. Con.	28.8 23.7	8.24 7.99	0.68 0.66	5.29
Setting proofs	18	Exp. Con.	6.11 5.13	2.24 2.19	0.18 0.18	3.73

Number of students in Exp. group = 144
Number of students in Con. group = 143
t-table value at .99 = 2.57 $\alpha_1 = 0.1$ $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96 $\alpha_2 = 0.5$ $1 - \alpha_2 = 95\%$

Comparing the computed t's with its table values all the hypotheses except number three were rejected in favor of their alternatives and concluded that there is a significant difference between achievement levels of the two groups. (A possible reason that hypothesis number three was not rejected will be given later when the results are generally discussed.) The experimental students performed significantly higher than the control group at the 0.99 level, based on testing the hypotheses (other than number three) including overall achievement.

Table 3. Data analysis for tenth grade first test.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp. Con.	65.98 48.41	14.86 14.36	1.43 1.42	8.65
Creative thinking	3, 5, 10 11, 15	Exp. Con.	18.54 13.86	.56 .54	5.80 5.46	5.98
Explaining concepts	6, 7, 17	Exp. Con.	12.45 9.06	4.07 4.13	.39 .41	5.95
Recalling concepts	9, 14, 16	Exp. Con.	10.21 7.83	4.52 4.36	.43 .43	3.85
Problem solving	1, 2, 4 8, 12, 13	Exp. Con.	19.05 12.48	6.61 6.03	.63 .60	7.47
Setting proofs	18	Exp. Con.	5.71 5.16	2.29 2.08	.22 .20	1.78

Number of students in Exp. group = 107
Number of students in Con. group = 101
t-table value at .99 = 2.57, $\alpha_1 = 0.01$, $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96, $\alpha_2 = 0.05$, $1 - \alpha_2 = 95\%$

From the entries in the column headed Student-t when compared with t-table values, the null hypotheses was rejected and conclusions were drawn that tenth grade experimental students performed significantly better than the control students. All computed t's were significant at the .99 level except for the last hypothesis which was not significant at the .95 level, but at the .90 level.

Analysis of the Second Test Scores

The second test was analyzed using the same processes as the first test and the results are given in Tables 4 and 5.

Table 4. Data analysis for ninth grade second test.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp. Con.	54.37 45.22	14.23 14.42	1.18 1.21	5.40
Creative thinking	2, 4, 5, 12 13, 15, 16	Exp. Con.	17.67 11.61	7.09 6.53	.58 .54	7.52
Explaining concepts	8	Exp. Con.	2.07 1.76	2.13 2.06	.17 .17	1.27
Recalling concepts	10, 14, 18	Exp. Con.	15.47 14.71	9.25 5.46	.77 .45	0.84
Problem solving	1, 3, 6 7, 9, 11	Exp. Con.	17.15 13.71	6.34 6.12	.52 .51	4.67
Setting proofs	17	Exp. Con.	6.35 5.09	2.18 2.08	.18 .17	4.99

Number of students in Exp. group = 145
Number of students in Con. group = 142
t-table value at .99 = 2.57, $\alpha_1 = 0.01$, $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96, $\alpha_2 = 0.05$, $1 - \alpha_2 = 95\%$

Comparing the data for experimental and control groups on the second test, and using t-test leads one to the rejection of the hypotheses in favor of their alternatives, and one can conclude that the experimental group performed significantly higher than the control group at the .99 level on the aspects related to hypotheses 1, 2, 5 and

6 and at the .80 level on the aspects related to hypotheses 3 and 4.

Table 5. Data analysis for tenth grade second test.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp. Con.	68.00 52.78	13.76 15.60	1.33 1.56	7.43
Creative thinking	2, 7, 9 11	Exp. Con.	12.55 9.20	4.76 4.77	.46 .47	5.04
Explaining concepts	5, 12, 17	Exp. Con.	11.24 9.16	4.67 4.59	.45 .46	3.21
Recalling concepts	13, 15	Exp. Con.	7.42 6.46	3.52 3.86	.34 .38	1.87
Problem solving	1, 3, 4, 6 8, 10, 14, 16	Exp. Con.	29.77 21.74	6.06 7.29	.58 .73	8.61
Setting proofs	18	Exp. Con.	7.00 6.21	2.14 2.18	.20 .21	2.64

Number of students in Exp. group = 107
Number of students in Con. group = 99
t-table value at .99 = 2.57, $\alpha_1 = 0.01$, $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96, $\alpha_2 = 0.05$, $1 - \alpha_2 = 95\%$

Conducting a test of the hypotheses using t-test reveals that all the hypotheses can be rejected in favor of their alternatives and one can conclude that the tenth grade experimental group students showed significantly better achievement than the control group on the second test. All the hypotheses were rejected at the .99 level, except No. 4 which was rejected at the .90 level.

Analysis of the Third Test Scores

The scores from the third test were analyzed in the same way as the scores from the first two tests. The results and the conclusions are presented in Tables 6 and 7 and the descriptions following each table.

Table 6. Data analysis for ninth grade third test.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp. Con.	51.47 38.53	14.07 13.46	1.17 1.13	7.89
Creative thinking	2, 6, 7 8, 14	Exp. Con.	12.61 9.66	5.93 5.94	.49 .50	4.17
Explaining concepts	5, 10	Exp. Con.	5.79 4.05	3.53 2.94	.29 .24	4.47
Recalling concepts	3, 12, 16	Exp. Con.	8.60 4.39	4.57 3.95	.38 .33	8.27
Problem solving	1, 4, 9, 11 13, 15, 18	Exp. Con.	18.51 15.13	7.15 7.06	.59 .59	3.99
Setting proofs	17	Exp. Con.	5.95 5.28	2.41 2.42	.20 .20	2.33

Number of students in Exp. group = 143
Number of students in Con. group = 140
t-table value at .99 = 2.57, $\alpha_1 = .01$, $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96, $\alpha_2 = .05$, $1 - \alpha_2 = 95\%$

Application of Student-t test on the data obtained from the third test administered to the experimental and control groups in ninth grades, reveals that one can reject all the hypotheses in favor of their alternatives and conclude that the experimental group performed significantly better than the control group. The level of significance had a probability of error at .99 for the first five hypotheses and .95 for the last one.

Table 7. Data analysis for tenth grade third test.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp.	67.71	12.42	1.21	6.60
		Con.	56.32	12.03	1.21	
Creative thinking	4, 6, 10 11, 14	Exp.	18.62	5.58	.54	6.01
		Con.	14.11	5.04	.50	
Explaining concepts	5, 12 13, 15	Exp.	11.63	4.82	.47	3.53
		Con.	9.34	4.32	.43	
Recalling concepts	18	Exp.	6.63	2.21	.21	.07
		Con.	6.61	2.09	.21	
Problem solving	1, 2, 3, 7 8, 9, 16	Exp.	24.10	6.07	.59	3.79
		Con.	20.76	6.41	.64	
Setting proofs	17	Exp.	6.71	2.11	.20	3.77
		Con.	5.48	2.47	.25	

Number of students in Exp. group = 104
Number of students in Con. group = 98
t-table value at .99 = 2.57, $\alpha_1 = .01$, $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96, $\alpha_2 = .05$, $1 - \alpha_2 = 95\%$

By comparing the group means and applying the t-test, one can reject all the hypotheses in favor of their alternatives except No. 4 and conclude that there is a significant difference in scores between the two groups. The experimental group means were significantly higher than the control groups at the .99 level concerning hypotheses other than No. 4, on the third test.

Generally speaking, the result from the test statistics for analysis of the scores on the three tests provides enough evidence to conclude that both in the ninth and tenth grades, the experimental groups achieved significantly higher levels of learning geometry related to overall geometric concepts, creativity, problem solving and setting step-by-step proofs for geometric theorems.

On the first test for ninth grades the difference between the group means was small, and the t-test showed no significant difference between the experimental and control groups in explaining geometric concepts, which were measured by items numbered five and eight in the test. A possible reason for this result is that this was the first time that these students had been given this particular type of true-false item on a test (see Appendix B). They apparently did not understand how to explain correctly the geometric facts presented in false statements. Similarly styled items in later tests showed significant differences between the two groups of students.

The data on measuring students ability to recall concepts, though in favor of the experimental group, did not reveal a difference as significant as the data measuring other concepts. A reason for this may be that the activity approach does not emphasize rote memorization quite as much as the traditional approach.

Analysis of the Final Examination Scores

The final examinations were designed to test most of the major concepts involved in the curriculum content for ninth and tenth grade geometry. The scores from the final examinations were used in this study to measure the degree of success in learning geometry via using the activity approach in teaching as compared with the currently followed traditional approach. Based on the intermediate test results it was assumed that an improvement in learning basic concepts of high school geometry would be reflected in the final score. A summary of the processed final data is presented in Tables 8 and 9.

Table 8. Data analysis for ninth grade final examination.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp. Con.	58.88 44.29	13.16 11.65	1.09 .97	9.95
Creative thinking	5, 8, 14	Exp. Con.	5.42 4.28	4.46 3.59	.37 .30	2.38
Explaining concepts	2, 3	Exp. Con.	5.51 2.20	3.85 2.88	.31 .24	8.25
Recalling concepts	6, 9, 12 13, 16, 17	Exp. Con.	21.72 16.55	6.06 6.29	.50 .52	7.10
Problem solving	1, 4, 7 10, 11, 15	Exp. Con.	19.71 15.26	5.81 5.79	.48 .48	6.50
Setting proofs	18	Exp. Con.	6.49 5.98	2.19 2.22	.18 .18	1.96

Number of students in Exp. group = 145
Number of students in Con. group = 143
t-table value at .99 = 2.57, $\alpha_1 = .01$, $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96, $\alpha_2 = .05$, $1 - \alpha_2 = 95\%$

Table 9. Data analysis for tenth grade final examination.

Hypotheses	Related Item No.	Group Tested	Group Mean	Group St. Dev.	St. Err. Mean	Student t
Overall achievement	1-18	Exp. Con.	75.63 54.66	14.79 14.46	1.43 1.45	10.24
Creative thinking	3, 10, 16	Exp. Con.	11.01 6.44	4.49 4.25	.43 .42	7.47
Explaining concepts	2, 4	Exp. Con.	7.60 4.87	3.06 3.35	.29 .33	6.07
Recalling concepts	8, 13, 14 15, 18	Exp. Con.	20.23 15.24	8.45 6.18	.82 .62	4.79
Problem solving	1, 5, 6, 7 9, 11, 12	Exp. Con.	26.62 21.81	7.18 7.08	.69 .71	4.81
Setting proofs	17	Exp. Con.	10.15 6.28	10.20 2.71	.99 .27	3.65

Number of students in Exp. group = 106
Number of students in Con. group = 99
t-table value at .99 = 2.57, $\alpha_1 = .01$, $1 - \alpha_1 = 99\%$
t-table value at .95 = 1.96, $\alpha_2 = .05$, $1 - \alpha_2 = 95\%$

Conducting Student-t test as a statistical measurement for the ninth grade final examination scores, we are able to reject all the six hypotheses and conclude that their alternatives are true. Hence we can say that the experimental group performed significantly better than the control group on the final examination. The significant level of the test-statistic used was greater than .95 for all hypotheses tested.

Based on the comparison of the scores on the tenth grade final examination, and by applying Student-t test, all the hypotheses were rejected in favor of their alternatives and we can conclude that the difference in performance between the two groups is significant at the highest, .99 level. The study reveals that the activity approach helped the experimental groups of students achieve markedly higher learning levels in all tested aspects involved in learning geometry.

Summary of the Results

The purpose of this chapter was to compare the test scores between the experimental groups of students who were taught geometry by the activity approach and control groups of students who were taught geometry by the traditional approach.

Mean scores for each group in each one of the three tests and in the final examination were computed and compared using the Student-t test as test statistics.

All six hypotheses were tested and were rejected in favor of their alternatives for most tests and the final examinations.

The results from the statistical testing and comparison reveals that the level of student skill in learning geometric concepts as is measured by the tests and the final examination are significantly raised through the activity approach.

The scores of the experimental groups from each test and the final examination had larger means than the scores of the control groups.

The results of the analysis of data from the final examinations were used as a more general guide to student performance than the other tests.

The difference in overall achievement level between the experimental and control groups involved in this study were highly significant at the 0.99 level. Hence there is less than 1% probability that introduction of the activity approach in high schools would fail to help students' achievement in geometry.

V. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This chapter is divided into three major sections. The first section presents a summary of the study including a review of the problem, the experimental design and procedures, and the statistical analysis with the results. The second section presents the conclusions which may be drawn from the findings of this study. The third section is devoted to recommendations for further studies.

Summary

The problem of this study was to compare the effects of an activity approach which consists of student involvement in learning processes using solution keys and practical activities to supplement lecture presentation, with a traditional classroom, textbook, and rote learning method of teaching geometry.

It was expected that the results of this study would provide worthwhile information for administrative personnel in education, among them mathematics educators, teachers and others interested and concerned with curriculum development, teacher training and upgrading learning procedures as well as for persons interested in practical approaches to teaching geometry and its applications at the high school level, especially in Afghanistan.

This study addressed the problem students and teachers face in high school as theories become too abstract to be easily comprehended. It is assumed that the introduction of concrete materials can help teachers teach more readily and students understand the concepts and their application more easily. One need for this study was the lack of research about the effect of the activity approach in teaching and learning geometry at the high school level. A special need for the study was to provide information about ways to help upgrade the teaching and learning of geometry as a branch of mathematics in Afghanistan, and to meet the desired goals of recent plans for educational reform.

The learning theories and the literature reviewed in Chapter II support the need and importance of the study.

The study was limited to the contents of ninth and tenth grade programs in geometry for high schools of Afghanistan. The study was further limited to one academic year, three tests, a final examination and by the validities and reliabilities of the testing instruments.

The model presented and tested in this study involved 48 activity modules related to the concepts in the ninth and tenth grade geometry textbooks generally used in all high schools of Afghanistan.

Randomization was followed to select (372) ninth grade students and (230) tenth grade students from among (787) ninth grade students and (753) tenth grade students of two randomly selected high schools in Kabul. The selected students were further divided into two groups, using even and odd procedure in each school as experimental (activity approach) and control (traditional) groups of students.

During the nine-month academic year (March 22 to December 15) 1976, all students in the experimental and control groups in each grade took three tests and a final examination. The tests and examinations were constructed by a joint committee of the teachers of the experimental and control groups for each grade in both schools. The test items were trial-tested with a section of students in the same grade who were not involved in the study, for the purpose of establishing the test validity and reliability. Items with low discrimination power or high difficulty indexes were replaced by the committee before administering the test to the students involved in the study.

The experimental design used to test the hypotheses was the post-test only control group design described by Campbell and Stanley (1963). Six null hypotheses were tested against their alternatives, to evaluate the activity approach versus the traditional model. All hypotheses were rejected in favor of their alternatives. Based on the reliability of the measurements used it was concluded that the activity approach was instrumental in helping students perform at higher levels in all hypothesized aspects when compared with the traditional approach.

Conclusions

It was concluded that the activity approach in teaching and learning will significantly improve student achievement in understanding facts of geometry.

It was concluded that the activity approach helps students achieve higher levels in creative thinking as compared with the students taught by the traditional method.

It was concluded that the activity approach helps students develop greater ability to explain geometric concepts than that acquired by the students taught by the traditional methods.

It was concluded that the activity approach to learning and teaching geometry helps students improve their ability in solving geometric problems better than the traditional method does.

It was concluded that the activity approach helps students develop the ability to recall geometric facts better than do traditional methods.

Finally it is concluded that the activity approach helps students to develop greater ability to set up complete step-by-step proofs for geometric theorems than is achieved when the traditional methods are used.

Justification for the conclusions stated above rested on the rejection of all null hypotheses given on page 55. All the hypotheses were tested against their alternatives and were rejected at least at the 0.95 level. In other words the probability was at least 95 percent that the alternatives to the hypotheses were true.

The conclusions stated above are restricted by the limitations for this study, but they are markedly valid for 1976 students in ninth and tenth grades of the two schools participating in the study. The conclusions are further restricted by their limitation to the subject geometry, at the high school level.

The probability is very high that the introduction of the activity approach to teaching and learning geometry could be as successfully iterated in all high schools of Afghanistan as in the two high schools taken as a sample for this experimental investigation. It would be desireable that the outcomes from this study be generalized throughout all the high schools in the nation.

Recommendations

Based on the literature reviewed and on the experience gained during the administration of the activity approach to teaching and learning geometry, the following recommendations are presented.

A follow-up study is recommended to appraise if there is a significant difference between the achievement levels of those students who have been taught using the activity approach in their ninth or tenth grades, compared to the ones who followed the traditional method in a year following this study.

Studies should be made to evaluate the effects of an activity approach in teaching other branches of mathematics, e.g., algebra, trigonometry and other related subjects, in high schools.

It is further recommended that a comparable study should be made to investigate what specific teacher competencies are needed to stimulate and improve student achievement in mathematics through individualized and self-directed learning.

It is recommended that a study be undertaken to compare the levels of achievements in geometry between two groups of students--one group provided during class period with the solutions to the text-book problems and the other group having access to a solution manual only in the school library.

A study should be made to measure teachers' and learners' attitudes towards an activity approach in learning mathematics in general.

Studies are recommended to develop standardized evaluation material to measure and compare students' achievement in learning mathematics at the high school level in Afghanistan.

Studies are recommended to compare the success in teaching mathematics as between two groups of teachers, one of which is provided with information about how to teach using an activity approach, while the other group is not so provided.

Studies are recommended to compare student achievement between two groups of students--the one of which is provided with textbooks on geometry written with consideration of the practical and activity approach to learning while the other group is provided with textbooks based on the theoretical approach only.

Since the results of this study indicate that many gains could be achieved by more activity oriented lessons in mathematics education in Afghanistan, it is recommended that the activity approach to teaching and learning mathematics be introduced throughout the national educational system.

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APPENDICES

APPENDIX A

Randomly Selected Activities

Activity No. 6 for Ninth Grade: Practical Application of Thales Theorem on Trapezoids

Objective: To learn about the application and uses of Thales' theorem on trapezoids.

Materials Needed: Geoboard with colored rubberbands, graduated rulers, measuring tapes, pencil and paper.

Procedure: Since by definition a trapezoid has two sides not parallel, if extended they intersect. Their extension can be considered as two intersecting lines in Thales theorem. Make a trapezoid on a geoboard using a large rubberband. Place a second rubberband of another color on the geoboard, such that it is parallel to the parallel sides of the trapezoid and intersects the non-parallel sides or their extensions. Identify all ratios of the distance between the various intersections on one non-parallel side, and compare them to the related ratios of the distance on the other non-parallel side. Are these ratios equal? Does any line parallel to the bases of a trapezoid intercept proportional segments on its non-parallel sides? Is it always true that any line which intercepts proportional segments of the non-parallel sides of a trapezoid, is parallel to its bases? Show this fact on the geoboard. Furthermore, as a homework assignment, can you design some experiments to explain this fact to your classmates to expand their knowledge and to clarify this fundamental concept? Submit your written report to the teacher.

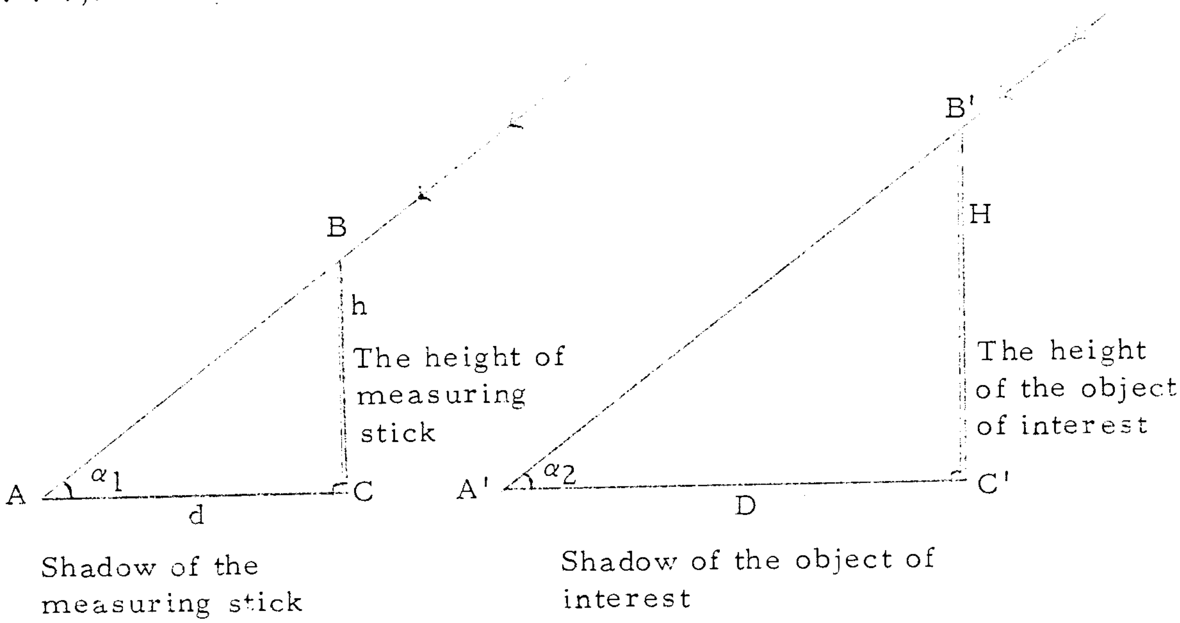
Activity No. 11 for Ninth Grade: Using Triangle Similarities to Find Heights of Buildings, Poles, etc.

Objective: To learn some applications of triangular similarities in routine physical problems.

Materials Needed: Measuring stick, 15 to 30 cm long. A source of light which shines from a distance to produce parallel beam, e.g., sunlight. Measuring tape. Paper and pencil for taking notes.

Procedure: The two similar triangles shown below describe how the height of an object of interest will be determined in this experiment. As is known, sunlight or light from a distant source has nearly parallel beams. The angle of incidence of the sun's beam on the surface of the earth at a given time and location is unique. In other words $\alpha_1 = \alpha_2$.

Place a measuring stick perpendicular to the ground. The measuring stick, its shadow, and the sunlight beam joining the top of the measuring stick to the end point of its shadow, represents a triangle. Similarly there is another triangle represented by the height of the object of interest; its shadow and the sunlight beam joining the top of the object to the end point of its shadow represents a triangle similar to the first triangle. Hence using the figure and the information about similar triangle with proportional sides you can write $\frac{h}{d} = \frac{H}{D}$. You can easily measure h (the length of your measuring stick), d (the length of the shadow of measuring stick), and D (the length of the shadow of the object of interest) and calculate H (the height of the object of interest, i. e., building, flag poles, trees . . .).



Notice: . In case of using the sunbeam, the measurements must be taken at nearly the same time, because the angle of its incident may change.

Using the triangle similarities students can easily think of some experiments to measure distances which are difficult to measure directly, i.e., the width of a river, the height of a mountain, etc.

As homework try to create an experiment to measure some length of your interest and present your experiment describing your procedure and finding to your groupmates and the teacher.

Activity No. 15 for Ninth Grade: The Ratio K Between the Length of a Segment and the Length of its Projection on a Given Line Depends on the Angle between the Segment and the Projection Line

Objective: To understand the meaning of projection, and the relationship between the length of a segment and its projection with respect to the angle between the segment and its projection.

Materials Needed: A square piece of cardboard, 40 x 40 cm^2. A piece of stiff straight wire about 20 cm long. A small weight. Thread. Measuring tape, Protractor. Paper and pencil for taking notes.

Procedure: Using the center of the cardboard as the origin, draw two axes perpendicular to each other and to the edges of the cardboard. Bend the wire in a right angle about 1 cm from the end, and stick the short end into the cardboard at the origin. Tie a thread about 10 cm long on the wire near the opposite end. Tie the small weight to the other end of the thread like a plumb bob. Stand the cardboard on one of its edges and perpendicular to the surface of your table. The wire segment from the origin to the position where the thread was tied on the wire can be selected as desired. As the angle α, between the wire and the horizontal line is varied between 0° and 180°, the length of the projection of the wire segment onto the horizontal line (the distance between the origin and the point where the thread crosses the horizontal line) is measured. One can see that a fixed wire segment will produce different ratios between the length of the wire segment and its projection on the horizontal line as the angle between the wire and the axis is changed from zero to 180 degrees.

As a home assignment you can draw the graphs to indicate the relationship between the lengths of the projection and the size of the angle between the segment and the horizontal line. You may use the protractor to measure the angle. Bring your graph and be ready to explain the findings of your experiment to your classmates.

<u>Activity No. 6 for Tenth Grade: Angles and the Ordinary Scales Used for Measuring Them</u>

<u>Objective</u>: To learn about angles, including the extent of rotation of the second side of an angle relative to the first side, taken to be the positive x-axis, and the various systems of angle

measurement, the basis of each system, and the ways they compare with each other.

Materials Needed: A prepared cardboard disc containing three concentric graduated scales which are used for angle measurements. Two movable straightedges are attached at the center of the concentric scales, and are used to construct angles.

Procedure: The teacher may explain the concept of angle measurement as the extent of rotation of a side from one direction to another direction, and that the measure of an angle is a real number, which is different from the angle itself defined as the union of two rays (its sides). Then the class is given some positive and negative angle measurements and is asked to construct angles corresponding to the measurements using the apparatus for this experiment.

This experiment originally consists of three parts treated as separate sub-experiments namely, radian scale of angle measurements, degree scale of angle measurement and grad scale of angle measurement. Since the treatment for introducing each of the scales is similar, it is considered sufficient for the purpose of this appendix to present only one out of three scales.

Grad Scale of Angle Measurement: The grad scale is usually used in those technical affairs dealing with the metric system of measurement. The unit of angle measurement in this scale is called the grad, which is 1/400 of a full revolution. The smaller components of this scale (like other metric scales) are each ten times smaller than the previous ones and are decigrads, centigrads and milligrads, i.e., 1 grad = 10 decigrad, and 1 decigrad = 10 centigrad. Any angle can be measured using the grad scale. Also the angle measurements with this scale can be compared with angle measurements in other scales. If R represents angle measurements in radians, d represents angle measurements in degrees and g represents angle measurements in grads, then we can use a portion of the following formula to convert measurements from one scale to the other:

$$\frac{g}{400} = \frac{d}{360} = \frac{R}{2\pi}$$

As you may notice the formula indicates a full revolution in each of the angle measurement scales. The same formula has been simplified on page 25 in the textbook of tenth grade geometry as:

$$\frac{g}{200} = \frac{d}{180} = \frac{R}{\pi}$$

± 1 revolution in the grad system is equal to ± 400 g.

$\pm \frac{1}{2}$ revolution in the grad system is equal to $\pm \frac{1}{2} \times 400 = \pm 200$ g.

$\pm \frac{1}{4}$ revolution in the grad system is equal to $\pm \frac{1}{4} \times 400 = \pm 100$ g.

$\pm \frac{1}{6}$ revolution in the grad system is equal to $\pm \frac{1}{6} \times 400 = \pm 66.6$ g.

$\pm \frac{1}{8}$ revolution in the grad system is equal to $\pm \frac{1}{8} \times 400 = \pm 50$ g.

$\pm \frac{1}{12}$ revolution in the grad system is equal to $\pm \frac{1}{12} \times 400 = \pm 33.3$ g.

and finally $\pm 1 \frac{1}{2}$ revolution in this system indicates:

$$\pm 1.5 \times 400 = \pm 600 \text{ g}.$$

As an assignment, using the apparatus for this experiment, read and note the equivalent measurements in the grad scale for 360°, 180°, 100°, 45°, 30° and $\frac{\pi}{2}$, $\frac{\pi}{3}$, $\frac{3\pi}{2}$ radians. Finally using the formula 7 on page 25 of your textbook find the measurements of each of the angles given below in two other scales of measurements. You can test your results using the apparatus for this experiment. The angles are; 25° to be measured in grads and radians, -2 radians to be measured in grads and degrees, 150 grads to be measured in radians and degrees.

Activity No. 11 for Tenth Grade: Introducing Various Kinds of Symmetry

Objective: To help the students understand the meaning of point symmetry, line symmetry, and finally the symmetry which exists in natural objects. To learn about the use of symmetry in art and design.

Materials Needed: Waxed paper or onion skin paper. Small rectangular mirror. Writing ink. Graduated ruler. Protractor. Pencil and pad.

Procedure: Prior to the experiment, the teacher explains the meaning and various kinds of symmetry. Also the teacher discusses with the class the symmetries of lines, angles, and some other geometric figures with respect to a point and a line. Then to practice the concept of point symmetry, students are asked to chose a point on a piece of paper and draw point symmetric figures for an angle, a semi-circle, a triangle, etc. They should be cautioned that in point symmetry every point in the symmetric figure is symmetrically related to a point in the original figure, a segment in a symmetric figure is symmetric to a segment in the original figure and finally an angle in a symmetric figure is congruent but with opposite direction to an angle in the original figure.

To demonstrate that symmetric figure drawn in a plane are exactly as a line symmetry of that figure in a mirror, with respect to a given axis, the student is asked to draw the line symmetric figure for a selected figure, and then place the edge of the rectangular mirror along the line of symmetry. See if he can get the same figure in the mirror as he has drawn on the paper.

Now that students understand the meaning of symmetry, can they locate the line of symmetry between two parallel lines? If students need help in this matter they should be asked to draw a segment perpendicularly joining two parallel lines and then draw the perpendicular bisector of the segment. Can they test if the perpendicular bisector they have drawn is the line of symmetry? They may be asked to tell their groupmates of their method.

As homework assignments students may try to solve problems on page 71 in the textbook, using the practical ideas learned in this experiment. Furthermore, each student should try to find examples of point and line symmetry in art or nature. Then write a report or arrange an experiment to show how symmetry is used in the object or figure in which the students have noticed symmetry. Students are to submit their report to the teacher or present the experiment they arranged to their classmates.

Activity No. 15 for Tenth Grade: Demonstration of Stewart's Theorem

Objective: To learn the relationships between the lengths of different Cevians of a triangle, its sides, and the length of segments on the base of the triangle divided by the intersection of the Cevian and the base of the triangle.

Materials Needed: Graduated ruler, paper and pencil.

Procedure: Using your ruler and pencil draw as large a triangle as the paper permits. Then draw a Cevian in the triangle. Indicate the foot of the Cevian on the base of the triangle by an F. Name the segment to the right and left of F on the base of the trangle n and m respectively. Now in order to demonstrate that Stewart's theorem is correct, use actual measurements for the length of the sides of the triangle a, b, c, the lengths of the segments on the base of the triangle m, n and finally the length of the Cevian $\overline{AF}$. Substitute the values measured in Stewart's formula

$$a \cdot \overline{AF}^2 = m \cdot b^2 + nc^2 - mna$$

and see if the equality holds. Do you agree that the formula is always true for all triangles and all Cevians? If yes why? If no why? If you select a median as a Cevian by choosing $m = n$, how do you calculate the length of a median if you are given the length of its base using Stewart's formula? Can you find the length of an angle bisector of a triangle if you are given the length of its sides and the segments on its base? Try to find it both graphically and by use of Stewart's formula and explain it to your groupmates.

As an optional exercise, students may try to read the related sections in the textbook for tenth grade geometry and try to discover another formula for either the law of sines, the law of cosines, the radii of circumscribed or inscribed circle of triangles or the area and altitude of a triangle, given its sides. The student may write a report and explain how he reached his discovery and present it to the teacher.

APPENDIX B

Tests and Final Examinations

Geometry for Ninth Grade: First Test

Name______________Fathers Name_______________Section_____Date_______

Explanations: This test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answers to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. Segment AB is 12 cm long, point M divides the segment into two parts with the ratio $\frac{MA}{MB} = \frac{1}{3}$, find MA = ________________.

2. A line parallel to the bases of a trapezoid intersects _______________ segments on the two intersecting sides.

3. The line which intercepts proportional segments on two or more transversal lines are ______________ to each other.

4. If $\overline{AB} = 18$ cm and $\overline{CD} = 2$ cm, their geometric mean is equal to _______________ cm.

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statement is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false but fail to write a corrected version, you will only receive three points for the problem.

T F 5. The relation $\overline{AB} = 3\overline{CD}$ says that $\overline{CD}$ is longer than $\overline{AB}$.

T F 6. The ratio of a segment divided by an exterior and an interior point are of the same sign.

T F 7. The Thales theorem is applicable to the trapezoid.

T F 8. If a transversal line intercepts proportional segments of the two sides of a triangle, it is not parallel to the third side.

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

Notice: Each possible answer in the right hand column may be used once, more than once, or not at all.

___ 9. How many points, divide the segment $\overline{AB}$ into other segments with a ratio of $\frac{2}{3}$?

___ 10. Given a pair of non-parallel lines with orderly related pairs of points dividing the lines into proportional segments, what is the relationship of line segments joining the pairs of points?

___ 11. When a line parallel to one of its sides divides the other two sides of a triangle, what is the relationship of the segments of the two sides to each other?

___ 12. The point N divides the segment $\overline{AB}$ externally, such that NB = 20 and NA = 15 units, what is the length for $\overline{AB}$?

1. Proportional
2. Equal
3. Non-parallel
4. Parallel
5. Five
6. Two
7. One
8. Thirty-five

Part I. Section D: For each of the problems (13-16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. Which one of the following represents a whole ratio between the lengths 24 m and 6 m?

<u>a</u> $\frac{1}{4}$. <u>b</u> -4. <u>c</u> 4. <u>d</u> $\frac{6}{24}$. <u>e</u> 4 m.

14. What is the geometric mean of the two segments 3 cm and 12 cm long?

<u>a</u> 15. <u>b</u> 7.5. <u>c</u> 6. <u>d</u> $\frac{12}{3}$. <u>e</u> 36.

15. If in a triangle ABC, a line parallel to the base BC intersects the remaining two sides at the points E and F, according to the Thales theorem, which one of the following is true?

<u>a</u> $\frac{AE}{AC} = \frac{AB}{AF}$. <u>b</u> $\frac{AE}{AB} = \frac{AF}{AC}$. <u>c</u> $\frac{AB}{BC} = \frac{EF}{FC}$. <u>d</u> $\frac{AC}{AE} = \frac{AF}{FC}$.

<u>e</u> None of the above.

16. How many points exterior to a segment can divide the segment in a fixed ratio?

<u>a</u> Very many. <u>b</u> Two points. <u>c</u> One point.
<u>d</u> Four points. <u>e</u> None of them.

Part II. Write your answer in the space provided.

First Problem: State the Thales theorem and its converse.

Second Problem: Prove one of the conditions of the Thales theorem applicable to a triangle.

Geometry for Ninth Grade: Second Test

Name_______________Fathers Name________________Section____Date____

Explanations: This test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answer to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. Triangles A'B'C' and ABC are similar. If triangle DEF is congruent to the triangle A'B'C', it is also __________ to the triangle ABC.

2. If three non-parallel lines are intercepted by parallel lines so that the segments of the parallel lines are proportional, the three lines will __________ at one point.

3. A line joining the midpoints of the two parallel sides of a trapezoid, passes through the point of intersection of the two non-parallel sides, and through the intersection of its __________.

4. If a segment joining the midpoints of the two parallel sides of a trapezoid, is extended, the extension would intersect the two non-parallel sides of it at __________ point(s).

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statement is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false but fail to write a corrected version, you will only receive three points for the problem.

T F 5. If P is midpoint of $\overline{CD}$ then PC = 2CD.

T F 6. Given $\frac{AF}{m} = \frac{FG}{n}$ we can write AF + n = m + FG.

T F 7. Two triangles are called similar if their corresponding sides are congruent and corresponding angles are proportional.

T F 8. In a triangle ABC if we draw its interior angle bisector for the angle B and it intersects the opposite side at a point D, then by Thales Theorem we can write: $\frac{AB}{AC} = \frac{DA}{DB}$.

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

Notice: Each possible answer in the right hand column may be used once, more than once, or not at all.

____ 9. A line parallel to a side of a triangle intercepts segments of the other two sides of the triangle which

____ 10. Given three lengths a = 2, b = 3 and c = 5 find the fourth proportional length.

____ 11. An exterior angle bicetor of a triangle intersects the side opposite to the corresponding interior angle at point M. What kind of relation exists between the intercepted segments and the other sides of the triangle.

____ 12. How many points divide a given segment AB into other segments with a fixed ratio, if you do not consider the direction of the segment.

1. Are equivalent
2. Are proportional
3. Are equal
4. Two
5. $\frac{15}{2}$
6. $\frac{8}{5}$
7. Very many
8. None

Part I. Section D: For each of the problems (13-16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. A given angle of a triangle has how many angle bisectors?

a Very many angle bisectors. b Two angle bisectors.
c One angle bisector. d Four angle bisectors.
e None of them.

14. In triangle ABC the line DE is parallel to the base BC. If AB = 72 cm, BC = 45 cm, AD = 24 cm and EC = 12 cm how long is DE?

a 28 cm. b 35 cm. c 15 cm. d 30 cm.
e None of them.

15. Given two similar right triangles ABC and A'B'C', if the sides of their right angles are AC = 9 cm, BC = 6 cm and A'C' = 3 cm how long is B'C'?

a 18 cm. b 6 cm. c 2 cm. d 27 cm. e 3 cm.

16. If two parallel lines intercept proportional segments on many non-parallel lines the non-parallel lines:

a Have two points in common. b Are parallel.
c Have one point in common. d Have all points in common.
e None of the above.

Part II. Write your answer in the space provided.

First Problem: Prove that all the lines passing over a common point 0 would intercept segments on two parallel lines Δ and Δ' which are proportional to each other.

Second Problem: Given a trapezoid ABCD with shorter base AB = 28 cm and longer base CD = 35 cm. Point M is chosen on AD such that AD = 4/3 MD, a line through M and parallel to the bases intersects BD in p and BC in N. How long are $\overline{MP}$, $\overline{PN}$ and $\overline{MN}$?

Geometry for Ninth Grade: Third Test

Name______________Fathers Name_______________Section____Date____

Explanations: This test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answer to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. The ratio between the two corresponding altitudes of __________ triangles equals the ratio between their perimeters.

2. The projection of a perpendicular segment on a line is a __________ on the line.

3. If $\sin B = \frac{1}{2}$ is given, the value of $\cos B$ is ____________.

4. The projection of a segment parallel to a line is a __________ of the line.

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statement is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false but fail to write a corrected version, you will only receive three points for the problem.

T F 5. The numerical value for the cosine of a $100°$ angle has a negative sign.

T F 6. It is possible for a right triangle to be equilateral too.

T F 7. If two sides of one triangle are congruent with the two sides of another triangle, the two triangles are called similar to each other.

T F 8. If one of the trigonometric functions of an angle is not given, we are not able to find other trigonometric function for the given angle.

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

Notice: Each possible answer in the right hand column may be used once, more than once, or not at all.

___ 9. What trigonometric function would you get if you divide cosine of an angle by the sine of that angle?

___ 10. What is the numerical value for the tangent of a zero degree angle?

___ 11. What is the name sometimes used interchangeably for the tangent of an angle?

___ 12. What is the numerical value of the cosine of an angle whose sides are of opposite direction?

1. Tangent
2. Zero
3. 0°
4. Cotangent
5. Non-parallel
6. Tangent line
7. Perpendicular to
8. Negative one

Part I. Section D: For each of the problems (13-16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. Two equilateral triangles are always similar to each other because,

a Their sides are not equal.
b Their sides are different from each other.
c Their sides are all equal in length.
d Their sides are proportional. e All of the above are true.

14. For which value of A does sin A equal the cos A?

a $A + A = 100°$. b $A = 30°$. c $A = 90°$. d $A = 45°$.
e None of the above.

15. On a given line how long is a projection of a segment perpendicular to that line?

a One unit long. b As long as the given line.
c As long as the segment. d Zero units. e It doesn't exist.

16. Given a right triangle DEF such that $D = 90°$, $E = 27°$, what is a measure for its third angle?

a 27°. b 36°. c 63°. d 73°. e It is a right angle.

Part II. Write your answer in the space provided.

First Problem: Draw the segments representing the sines of two supplementary angles and prove that they are congruent.

Second Problem: Draw an angle with the numerical value for its sine of $-(0.50)$ and explain how you drew it.

Geometry for Ninth Grade: Final Examination

Name________________Fathers Name__________________Section_____Date_____

Explanations: This test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answer to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. The trigonometric ratio $\frac{\cos a}{\sin a}$ is called ____________.

2. A circle is circumscribed about a polygon if it contains the ____________ of the polygon.

3. The projection of a given segment on an axis is a segment ____.

4. Parallel lines intercept ____________ segments on two or more transversals.

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statement is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false but fail to write a corrected version, you will only receive three points for the problem.

T F 5. Two supplementary angles have common sines.

T F 6. If a square has a side (a) units long, its diagonal must be ($a\sqrt{2}$) units long.

T F 7. The projection of a zigzag line on a line is another zigzag line.

T F 8. Given any scalene triangle, the perpendicular bisectors of its sides contains their opposite vertex.

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

Notice: Each possible answer in the right hand column may be used once, more than once, or not at all.

____ 9. If we cut the circumference of a circle into 9 congruent arcs and then join the end points of an arc to the center of the circle, what is the measure of the central angle drawn?	1. 40° 2. 360° 3. One
____ 10. Given a segment on a line, how many points divide that segment into segments with the ratio of $\left\|\frac{2}{3}\right\|$?	4. Two 5. Sine 6. 12 cm
____ 11. Cosecant is a function reciprocal of what other function?	7. Cosine
____ 12. Given three lengths, a = 4 cm, b = 6 cm, c = 8 cm, what is the fourth length that completes their proportion?	8. 3 cm

Part I. Section D: For each of the problems (13-16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. What is the fourth rational number which completes the proportional sequence $\frac{6}{2}$, $\frac{8}{3}$, and $\frac{9}{4}$?

a 6. b 12. c 72. d 2. e None of them.

14. If right triangle ABC has a 45° angle, and one of its perpendicular sides is 6 m long, how long is the other perpendicular side?

a 3 m. b 12 m. c $6\sqrt{2}$ m. d 6 m. e None of them.

15. Which of the products given below is equivalent to the product of the perpendicular sides of a right triangle?

a Altitude x median. b Hypotenuse x altitude.
c Base x altitude.
d Angle bisector x perpendicular bisector of side.
e None of the given products.

16. If x is an acute angle, what is the value of x in the equation $\sin x = \cos x$?

a $x = 0°$. b $x = 22°30'$. c $x = 45°$. d $x = 60°$.
e None of the above.

Part II. Write your answer in the space provided.

First Problem: Calculate the trigonometric functions of a 45° angle.

Second Problem: Define and prove the Pythagorean Theorem.

Geometry for Tenth Grade: First Test

Name______________Fathers Name_______________Section____Date____

Explanations: This test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answer to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. All equilateral triangles are __________ to each other.

2. Two lines perpendicular to the same line are __________ to each other.

3. In a regular hexagon, the measure of its angles are __________ to each other.

4. Parallel lines intercept __________ segments on two intersecting lines.

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statment is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false but fail to write a corrected version, you will only receive three points for the problem.

T F 5. If a triangle is equilateral, it is also equiangular.

T F 6. An angular measure of 1/8 revolution is equivalent to 45 grads.

T F 7. An angle is inscribed in an arc if the vertex of the angle is a point on the circle and the sides of the angle are its diameters.

T F 8. The sum of the measures of two non adjacent angles in an inscribed quadrilateral is (3.1416) radians.

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

Notice: Each possible answer in the right hand column may be used once, more than once, or not at all.

___ 9. The sum of $30°49'$ and $59°11'$ is

___ 10. What is the measure of the angle between two parallel lines having the same direction?

___ 11. If an angle of a parallelogram inscribed in a circle is a right angle, the measure for each of its remaining angles is

___ 12. The sides of two similar polygons are said to be

1. Zero degrees
2. $\frac{\pi}{2}$ R
3. Obtuse angle
4. 180°
5. 110 grads
6. Are equal
7. Proportional
8. Half a right angle

Part I. Section D: For each of the problems (13-16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. The area of a triangle is equal to

a Length x width. b. Base, multiplied by the altitude.
c Base, multiplied by the altitude divided by two.
d Two times base multiplied by the altitude.
e None of the above.

14. If one of the angles of a quadrilateral inscribed in a circle measures 145°, its opposite angle is equal to

a 45°. b 35°. c 215°. d 180° e 90°.

15. The points which have numerals with the same sign for their abscissa and ordinate values are located in the following quadrants:

a First and second. b Third and fourth.
c First and third. d Second and fourth.
e First and fourth.

16. What is the length of AB, if the coordinates for A and B are given as A(2,2) and B(-2,5)?

a $-\sqrt{25}$. b 25. c $\sqrt{9}$. d 5. e $\sqrt{5}$

Part II. Write your answer in the space provided.

First Problem: Define: central angle, inscribed angle and tangential angle.

Second Problem: Prove that in the same circle, inscribed angles corresponding to the same arc or equivalent arcs are of the same measure.

Geometry for Tenth Grade: Second Test

Name ______________ Fathers Name _______________ Section ____ Date ____

Explanations: This test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answer to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. The system of angle measurement in which its units are compared with each other by multiples of ten is called the __________ system.

2. The center of a circle is at the same time its center of __________.

3. The result of symmetry of a line with respect to a point is another line __________ the given line.

4. If x is given $a < x < b$, the set of the possible values of x is called a __________ interval.

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statement is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false but fail to write a corrected version, you will only receive three points for the problem.

T F 5. 120 grads is the measure of a quarter of a full revolution while $3\pi/2$ represents a measure half as large.

T F 6. Every angle is symmetric with respect to its interior angle bisector.

T F 7. The point P(-3,5) is in the second quandrant and the point Q (0,4) is on the x axis.

T F 8. An equilateral triangle has three axes of symmetry.

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

___ 9. If the measure of a tangential angle corresponding to an intercepted arc AB is 90°, what is the measure of a central angle corresponding to arc AB?	1. 45° 2. Is less than
___ 10. How can you compare the number of symmetric axes of an equilateral triangle with those of a circle?	3. π radian 4. Is more than
___ 11. A point symmetric for a triangle is another triangle, such that there exists a one-to-one correspondence between the points on each triangle. These triangles are having the same	5. Coordinates of vertices 6. Is equal to
___ 12. If triangles ABC and DEF are symmetric with respect to a line Δ, and a triangle PQR is symmetric to the triangle DEF with respect to another line Δ', we can conclude that triangles ABC and PQR are related to each other as	7. Area 8. Similar

Part I. Section D: For each of the problems (13-16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. An angle has a measure of 270°. What is its equivalent measure in radians?

a $\frac{4\pi}{3}$. b $\frac{3\pi}{2}$. c $\frac{\pi}{4}$. d $\frac{\pi}{8}$. e $\frac{\pi}{2}$

14. How many axes of symmetry does a triangle have?

a Only one. b Very many. c Three. d None. e Two.

15. The measure of $\frac{1}{12}$ of a revolution is equivalent to

a 45°. b $\frac{\pi}{6}$ radians. c 33 grad. d $\frac{\pi}{4}$ radians. e 60°.

16. How many axes of symmetry does a circle have?

a Only one. b Very many. c Three. d None. e Two.

Part II. Write your answer in the space provided.

First Problem: Triangles ABC and A'B'C' are symmetric with respect to their axis of symmetry Δ, and the area of triangle ABC is known to be 25 m^2. If triangle A'B'C' has a base 12 m long, find the length of its related altitude (a figure is needed).

Second Problem: Prove that a line symmetric of a line which intersects its axes of symmetry passes through the point of intersection for the first line and its axes of symmetry.

Geometry for Tenth Grade: Third Test

Name_____________Fathers Name_________________Section____Date____

Explanations: The test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answer to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. The locus is ___________ as it moves under a given set of conditions.

2. In order to determine the radius of a circle circumscribed about a triangle, we use the formula R = ___________.

3. The sum of the measures of the angles around a point is ___________ grads.

4. The median and altitude of a triangle are also called its ___________.

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statement is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false, but fail to write corrected version, you will only receive three points for the problem.

T F 5. We can formulate Stewart's theorem as:

$$\overline{AF}^2 = b^2 - c^2 + m \cdot n$$

T F 6. The points between the center of a circle and its circumference and coplanar with the circle are at the same time on the circle.

T F 7. The angle between the tangent and the radius of a circle is pre-determined.

T F 8. The measure of an angle is not dependent upon the length of its sides.

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

Notice: Each possible answer in the right hand column may be used once, more than once, or not at all.

___ 9. The locus of points for which the ratio of the undirected distances to two fixed points is constant is a geometric figure which is called,

___ 10. What is another name for a circle?

___ 11. What is the maximum number of circles that contains the three given non-collinear points?

___ 12. What is a point symmetric of a segment?

1. Is a plane
2. Perpendicular bisector of a segment
3. Appolonius circle
4. Is a line
5. Too many circles
6. A segment
7. Two circles
8. One circle

Part I. Section D: For each of the problems (13 - 16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. A segment is called a Cevian if it is,

a A line which contains the vertex.
b Perpendicular biscetor of a side of a triangle.
c Perpendicular to the median.
d An altitude. e None of them.

14. In what kind of triangle are, an altitude, a median and an angle bisector collinear?

a A right triangle. b An isosceles triangle.
c An equilateral triangle. d An obtuse triangle.
e None of them.

15. The locus of the points equidistant from the two fixed given points is called

a A circle.
b A perpendicular bisector of a segment which joins the given points.
c An angle bisector. d An Appolonius circle.
e None of them.

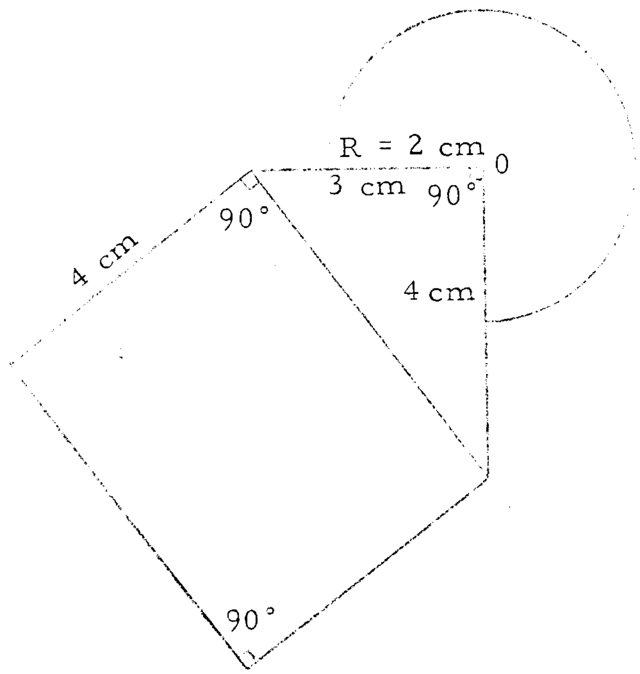
16. The locus of the points for which the sums of the squares of their distances from two fixed points are constant is a circle with a center on

a The segment joining the fixed points.
b The mid point of the segment joining the fixed points.
c A circle.
d The perpendicular bisector not on the segment joining the points.
e None of them.

Part II. Write your answer in the space provided.

First Problem: Prove the law of cosines.

Second Problem: Find the total area of the figure shown below.



Geometry for Tenth Grade: Final Examination

Name_____________Fathers Name________________Section____Date____

Explanations: This test consists of two parts:

Part I. Consists of sixteen (16) problems, which are divided into four sections A, B, C and D. Explanations related to each section are given at the beginning of each section. Your answer to the problems should appear in the space provided.

Part II. Consists of two (2) essay type problems. You are required to show all your work in the spaces provided.

Part I. Section A: Read each of the problems (1-4) carefully and fill the blanks with geometric terms which correctly completes it.

1. The part on the interior region of a circle bounded by a given arc and two radii which joins the end points of the arc to the center of the circle is called ___________.

2. If all sides of a polygon are tangents to a given circle, then that circle is called an ___________ circle.

3. A regular quadrilateral inscribed in a circle is called a __________.

4. The distance from the center of a regular polygon to each of the sides is called the ___________ of the polygon.

Part I. Section B: In problems (5-8) read the statement carefully and draw a circle around the letter (T) if the statement is true. Draw a circle around the letter (F) if the statement is false. If a statement is false write a corrected version, that is a statement which is true, in the space provided. If you correctly mark a statement false but fail to write a corrected version, you will only receive three points for the problem.

T F 5. The point p(1, -4) is in the second quadrant.

T F 6. The sum of the measures of two opposite angles of a quadrilateral inscribed in a circle is 400 grads.

T F 7. The interval [a, b] is called a closed interval.

T F 8. Stewart's theorem may be written as:

$$b \cdot \overline{BF}^2 = ma^2 + nc^2 + mnb.$$

Part I. Section C: In problems (9-12) read each problem carefully, then place in the blank provided the number of the best possible answer given in the right hand column.

Notice: Each possible answer in the right hand column may be used once, more than once, or not at all.

___ 9. What is the measure of the angle formed by a radius which intersects a tangent at the point of tangency?

___ 10. If the measure of a tangential angle corresponding to an intercepted arc AB is 90°, what would be the measure of a central angle corresponding to $\widehat{AB}$?

___ 11. The region bounded by an arc of a circle and the chord of that arc is called a

___ 12. How many axes of symmetry does a circle have?

1. A quarter
2. A right angle
3. π radian
4. One
5. Segment
6. Sector
7. Very many
8. Zero degrees

Part I. Section D: For each of the problems (13-16) you are given five possible answers. Read each problem carefully, try to solve it, then draw a circle around the letter which corresponds to its correct answer.

13. If the length of the radius of a circle is 7 m and the length of its arc AB is 42 m, what is the measure of the central angle corresponding to the arc AB?

<u>a</u> 3 radians. <u>b</u> π radians. <u>c</u> 2π radians. <u>d</u> 6 radians.
<u>e</u> None of the measures given.

14. If the lengths of sides of a triangle are given as $a = 7$ cm, $b = 8$ cm, $c = 5$ cm what is the area of the triangle?

a $(10 \cdot \sqrt{3})\text{cm}^2$. b $(3\sqrt{10})\text{cm}^2$. c $(10)\text{cm}^2$. d $(\frac{15}{2})\text{cm}^2$.
e $280\ \text{cm}^2$.

15. If the radius of a circle is $\frac{1}{3}$ units, what is the circumference of the circle?

a 2π. b $\frac{\pi}{3}$. c $\frac{2\pi}{3}$. d $\frac{\pi}{9}$. e None of them.

16. What comparison can be made between the length of a side of a regular hexagon and the length of the radius of its circumscribed circle?

a Side is $\frac{1}{2}$ the radius. b Side is shorter. c Side is longer.
d Side has the same length as radius. e Not comparable.

Part II. Write your answer in the space provided.

First Problem: Find the area of a regular hexagon when the radius of its circumscribed circle is 6 m.

Second Problem: The altitude of a triangle is shorter than its corresponding base by (3) meters. The area of the triangle is $90\ \text{m}^2$. Find the lengths of its base and the corresponding altitude.

APPENDIX C

Opinionnaire

On the effect of the activity approach to teaching geometry:

1. Male___________ Female___________.

2. Age___________.

3. Marital status: Single____ Married____ Separated____ Widowed____

4. Higher degree held ___________.

5. Year of teaching experience ___________.

6. School you are assigned to at present_____________.

The following statements are purposely phrased positively or negatively. Your agreement or disagreement will be determined on the basis of your particular convictions. Kindly check your position on the scale as the statement first impresses you. Indicate what you believe, rather than what you think you should believe.

a. I strongly agree.

b. I agree

c. I am undecided.

d. I disagree.

e. I strongly disagree.

	a	b	c	d	e
1. The mathematical and professional standing of the teachers who followed the activity approach to teaching geometry was not considerably increased.					
2. This approach encourages teachers to think creatively.					
3. Teachers should get better preparations for more effective discussion with students about various applications of geometric concepts by using this approach.					
4. If the same materials (handouts and activity guides) were available, math teachers could be trained in a four week seminar to use this method.					
5. I have not clearly noticed any improvement in students creativity toward learning geometry by using activity approach.					
6. I have not noticed any significant change in my student attitude toward learning geometry by using this approach.					
7. I found my geometry students very enthusiastic in searching for relations between geometric concepts and facts in the physical world.					
8. My students have not found any reason why they should spend their time out of class in learning geometry.					
9. In order to make all the geo-lab materials needed for an average size high school, we need around 6000 Afs or $100.00.					
10. If the Ministry of Education decides to generalize this method in all schools it would face no academic problems.					

	a	b	c	d	e
11. Parents would be appreciative seeing their children taught geometry using this approach.					
12. With the solution keys being available we did not need more time in class using the activity approach compared with the traditional one.					
13. Using solution keys we were easily saved enough time in class to get the experiment started.					
14. Only ten to fifteen minutes once a week is enough time to get students to understand what they are expected to do practically for an assignment.					
15. Using the solution keys the students could see how they should attempt to solve geometric problems.					
16. Placing solution keys in the library helped students to go there and use other related books.					
17. The tests jointly constructed by experimental and control groups teachers, were not actually measuring what they were designed for.					
18. The tests were not useful in predicting successful student performance in geometry.					
19. The objective parts of tests, being independent of personal judgment of the scorer, were very interesting and enjoyable to my students who had not taken this type of test before.					
20. The objective parts of tests yield a clear score value for each performance.					

	a	b	c	d	e
21. It would be very hard to expand this approach to other branches of math. (Algebra, trig., Stat. . . .) under the present school conditions in Afghanistan.					
22. It would be a big problem to train teachers to proceed with this method effectively in their classes.					
23. I have noticed change in my students and my own attitude toward geometry, because with this method we could understand its applications.					
24. Having the same amount of time as the control sections do, we have covered more of the text in the experimental classes using the activity approach, than the control classes.					
25. The materials needed for teaching geometry through the activity approach, could not easily be prepared everywhere in the country.					
26. The handouts prepared for applications of the activity approach to teaching geometry were adequate.					
27. The materials utilized in the activity approach to teaching geometry in your school were adequate.					
28. The supplies prepared for the activity approach to teaching geometry in your school were adequate.					
29. Most of the topics presented by Mr. Monier were new to you and very useful.					
30. Mr. Monier did a good job as a consultant and/or adviser to you this year.					

Your comments:

Comments Made by the Teachers and Chairpersons of Mathematics Departments Quoted from the Opinionnaires

I personally want to apply the activity approach to teaching geometry in all of the sections in tenth grades which I will teach. Students and myself found the approach very effective in learning and teaching the fundamental concepts of geometry. By this approach students are encouraged to find for themselves, that it's necessary to know geometry and they enjoyed its application in life and science.

(The tenth grade math teacher in Aesha-Dorrani High School.)

I enjoyed seeing my students getting involved in learning geometry through activity approach. They all showed interest in learning a subject they used to think that it was very hard to learn and enjoy.

Based on my experience with this learning and teaching approach, I suggest that the Ministry of Education should make use of the findings from the research study done, and try to get the way paved for generalizing it as soon as possible.

(The ninth grade math teacher in Aesha-Dorrani High School.)

As the chairperson of mathematics department, I found these research activities very interesting and quite beneficial to our students and teachers in this school. I am looking forward to seeing the applications of its findings in all our classes in the ninth and tenth grades.

(Chairperson of department of mathematics in Aesha-Dorrani High School.)

I wish that similar approach could have been used long ago, that myself and others in my generation and the ones before us, could have had the same opportunity to increase retention, interest and learn to achieve as substantial improvement as my students experienced with the activity approach this year. As a teacher, using this approach I definitely benefited from it and will use it in the future with all my students.

(The tenth grade math teacher in Shahi-Doshamshera High School.)

As a teacher and the chairman of mathematics department in Shahi-Doshamshera high school. I am impressed and feel quite happy to see that the activity approach to teaching geometry helps students enjoy learning the subject. I noticed that this approach encourages student, as well as the teacher to search and discover some applications of geometric concepts in routine life and physical situations. I hope that in the near future we would be able to expand the use of the approach to all other schools and to the other branches of mathematics.

The table below summarizes responses from teachers and chairpersons of mathematics departments in Aesha-Dorrani and Shahi-Doshamishera high schools.

Item Number	Choice a	b	c	d	e
* 1				1	4
2	5				
3	2	3			
4	1	4			
* 5				2	3
* 6				3	2
7	3	2			
* 8					5
9	2	3			
10	2	2	1		
11	5				
12		5			
13	2	3			
14	1	4			
15	4	1			
16	5				
*17				1	4
*18				1	4
19	5				
20	1	4			
*21				3	2
*22				2	3
23	4	1			
24	3	2			
*25				4	1
26	4	1			
27	4	1			
28	3	2			
29	5				
30	5				

*Items indicated by an * are items purposely phrased negatively. The choice e for * items will represent the most favorable opinion.

APPENDIX D

Recommendations

To Whom It May Concern:

I have known Mr. M. Ibrahim Monier for more than eight years as my colleague. Working in the same department with him has always been my great pleasure because of his superior personality and his creative talent in teaching and doing research in mathematics and related topics.

I again met Mr. Monier when he returned from the United States after almost two years of doctoral study. At that time he told me about his research project on the activity approach to teaching geometry. Being a staff member of the Mathematics Department of the Faculty of Science at Kabul University and a member of Curriculum Development Committee of the Ministry of Education for the secondary schools program, I found his proposal for the research project very interesting. I have followed Mr. Monier's research activities step by step and have become very familiar with his objectives and procedures and I consider it very valuable information to me.

In a relatively short time, Mr. Monier has produced two solution keys to the high school geometry texts for the ninth and the tenth grades which have been needed for a long time. Then using his knowledge and creativity he wrote the 48 experiments related to the outlines of the two texts. I found the experiments very facinating especially in that they used inexpensive and locally available materials. This would make it possible for almost every high school in our country to supply them easily. The relationships between the geometric concepts and the physical world that he emphasizes in the experiments makes learning geometry interesting and more attractive than every before for children in Afghanistan. It definitely increases the learners' ability and understanding and develops their creativity.

After working with the teachers and preparing the handouts and materials needed, Mr. Monier was ready to apply his approach in two high schools, Aesha Dorrani a girls school and Shahe Doshamshera a boys school, at the begining of the academic year 1975-76. On his request both of the schools chose students for the experimental and control groups at random. Throughout the project Mr. Monier was supervising the experimental group teachers, presenting new and up to date topics to both experimental and control group teachers and observing and studying the reactions of the students to the new method. The results of semi-standardized tests for experimental and control groups have shown in practice what was logically reasonable in theory.

At this time Mr. Monier has completed what he had planned to do. He has introduced and applied a new method of teaching geometry for high schools in our country and hopefully this methodolgy can be expanded to other subjects in mathematics at an experimental level. This method enables students to relate the fundamental concepts in mathematics with life, which in turn increases their curiosity and builds creativity. I believe Mr. Monier's work has been a great success and has shown value in upgrading teaching, understanding and application in mathematics for Afghan students.

-3-

It is my pleasure to write about what I have observed and what I believe concerning the value of the system introduced by Mr. Monier in mathematical pedagogy. There is a strong relationship between Mr. Monier's dissertation and the objectives of the Faculty of Science of Kabul University where he will continue teaching and extend his academic contributions.

Sincerely,

Redacted for Privacy

Ahamd Ibrahimzada
Assistant Professor
of Mathematics
Faculty of Science
Kabul University

To Whom It May Concern:

About a year ago I was officially informed by the Ministry of Education, that Mr. Ibrahim Monier, an Assistant Professor of Mathematics, Faculty of Science, Kabul University was going to contact Aesha Dorrani High School, to apply his method called "An Activity Approach to Teaching Geometry." I was told that he would collect related data during the academic year, for the purpose of analyzing the data, drawing conclusions and if possible, generalizing his method to all other high schools through his Ph.D. dissertation. Before the school year started, Mr. Monier came and gave a full explanation about the objectives and procedures of his method. I found all his statements logically sound and very interesting.

I have an M.A. in Educational Administration and Supervision from San Francisco State University with thirteen years of experience as a teacher and administrator. I believe that in order to be able to introduce educational change, we need to upgrade our teacher's knowledge, provide students with better and up-to-date materials, and apply varieties of methods which takes into consideration individual differences. Students must be encouraged to do creative thinking and guided to gain the ability to relate knowledge and fundamental concepts for analysis and problem solving. If we can accomplish this as educators we will also increase natural curiosity and enhance the ability to use logic in practical applications of information to the environment and various life situations. Fortunately, I found that all the points mentioned above were taken into consideration in Mr. Monier's approach to teaching geometry as a fundamental subject.

As the principal and the person responsible for Aesha Dorrani High School, I agreed to the applications of Mr. Monier's method in our school. We first selected students for experimental and control sections in each of our ninth and tenth grades. This was a random selection from among all students registered for ninth and tenth grades in our school. Then, following an even and odd procedure, we chose students from the randomly selected group for two experimental and two control groups in each grade.

Since the beginning of the project, I have visited the experimental classes during the regular class session as well as during examination periods. I have also examined the material prepared for the 48 experiments and sat in on the seminars given by Mr. Monier for our mathematics teachers.

Now that the school year is coming to an end, I am very happy to say that I have gathered considerable evidence to show the progress and success of the experiment.

a. Various types of tests were jointly prepared by the teachers of the experimental and control groups and given at intervals of two months. The result of these tests clearly show improvement in experimental students' performance in geometry.

b. Students homework, written essays, models and materials produced have been evaluated and are stored at the school.

c. I have contacted teachers using the experimental material regularly, and have found them enthusiastic. They seem to feel the value of what they are receiving in terms of professional growth and in terms of what they feel they can offer their students for a better and longer lasting grasp of the fundamental concepts of geometry.

d. I have contacted as many parents as possible and have asked them particularly about their impressions about their childrens involvement in home assignments in geometry. I was particularly interested to know if they had noticed change in their childrens learning patterns or reasoning processes. Briefly I can summarize their statements below:

They feel that such a start could help their children develop an attraction to the physical sciences, in which geometry is an essential part. They feel it has a potential for long range benefits as this could help to relieve the nations most urgent need: The need for engineers, technologists and experts in physical science. If this method of teaching were generalized, the children would develop the ability to see the relations between the physical, natural and social sciences and could appreciate the value of mathematics within their daily life situations.

e. From a comparison betwen class averages for the experimental and control classes in subjects related to mathematics, I conclude that their attitudes are gradually changing and success in acquiring these new techniques gives the learners increased self-confidence. This attitude almost concedes the victory to their problems in science as well. When the change in attitude occurs their self-confidence in being able to related things also begins to improve. They do the work methodically and carefully, keeping everything under control, and that makes it less likely that errors will occur.

I am sure that by presentation and generalization of his method Mr. Monier will be able to develop the details and work them out to best advantage over a period of time. It is too late for those of us who have already graduated to benefit from such an educational change in learning mathematics in a direct way, but we can benefit indirectly as our children and schools benefit directly.

The total effect has been reawakened interest in geometry and related subjects. This revitalization is very important in addition to all the practical results of Mr. Monier's system. It is our hope that Afghan people will receive the full benefit of these opportunities, both particular and general. We are convinced that the activity approach to teaching geometry originated and introduced by Mr. Monier will be more widely applied as as time goes on.

Sincerely,
Redacted for Privacy

Zabiha Maqusoud
Principal of
Aesha Dorrani
High School.

To Whom It May Concern:

As the principal of Shahe doshamshera High School, I would like to express my opinions about some of the effects of the activity approach to teaching geometry in the experimental classes of ninth and tenth grades in my high school.

Historically speaking, this activity began three months before the academic year 1975-76 started, when Mr. M. Ibrahim Monier gave instructions to our mathematics teachers during winter vacation. The handouts, guide-notes for 48 experiments and related materials (inexpensive in cost and locally available), prepared by Professor Monier for the activity approach to teaching geometry are fascinating even for me as principal, as well as other teachers in the school. However, I believe that the value of this new approach is most apparent and appreciable to those who teach and learn geometry in the experimental grades and those who are in close contact with them.

Being the person responsible for academic activities in the school, I have visited classes regularly and asked the teachers and students about the effect of activity approach in their classes. I was informed not only by them but also by some parents who contacted me and eagerly expressed their appreciation about the results of this method of teaching. They appreciated the kind of practical and meaningful learning habits their children were being introduced to and were interested in seeing it continued. I think that with this approach our students are no longer compelled to dull the natural eagerness of their minds with many combinations of ready-made statements. With the activity approach the subject is much more interesting and as the year goes by they can keep the mathematical concepts not only alive but very productive.

I can tell by visiting classes and seeing the materials produced by the students for their home assignments and talking with the teachers, that the natural and practical sides of the subject with this approach can easily carry the students' academic interest and eagerness. This keeps them going ahead instead of being repelled by monotomy. They are attracted by the diversity of the ideas in geometry and their relations with other mathematical and physical subjects, as well as their use in daily life. I clearly remember that while talking about the system of teaching geometry presented by Mr. Monier a math teacher said that, "One of the important points about this method is that the students were attracted and their interest kept alive and active by their own successful achievements".

Not only did the experimental students do well in upgrading learning concepts in geometry by this approach, but they have improved in related subjects as well. I can tell by contacting the teachers and examining the lists of test scores for related subjects such as, physics, chemistry, and biology that the experimental students have done much better than their equivalent groups of students (control classes) under very similar conditions in our school.

As far as my math staff and I are concerned this approach is the right way to start and I strongly believe that it will succeed. We would like to see this sort of thing happen in all subjects and schools throughout the nation. It is true, of course, that for many reasons changes on a large scale must be made slowly. No one can reasonably expect any important changes in a nation's education methodology and pedago~y to occur in a short time. But if these changes can be made, even gradually, what a boon it would be! The students themselves would be freed from the burden of their worst drugery, and the subject that they consider most difficult would become lighter and much more interesting. For many students, truly fascinating possibilities would begin to open up.

I would like to congratualte Mr. Monier for originating the great educational movement he has introduced via his doctoral research. I am looking forward to seeing him generalize his ideas through his disseration, which I am sure will be a valuable one.

Sincerely,

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A. Quader Saddiqi
Principal of Shahe doshamshera
High School

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

Science Teaching & Audio - Visual Aids Centre
P. O. BOX 5
KABUL
AFGHANISTAN

Telephone: 40007
Cable address: UNDEVPRO - KABUL

No:..................................

Date 5th Dec., 76.

TO WHOM IT MAY CONCERN

Mr. Ibrahim Monier has requested me to give some comments on his "Activity Approach to Teaching Geometry in High Schools in Afghanistan".

I have met Mr. Monier and have had some general discussions on a couple of occasions. Furthermore he has given me the opportunity to look at the thesis proposal, the instructions for the teachers and the opinianaire.

Having spent only a couple of months in Afghanistan and not having seen Mr. Monier's project in a classroom situation I would like to restrict myself to more general comments.

The educational system in Afghanistan has been very static and still is so especially the secondary level. The educational reform of 1975 has had little practical meaning so far. No functioning system for curriculum development has been in operation. About ten years ago new mathematics material was produced for the high schools with the aid of a Columbia University Team. The material was trial tested in a few schools but never got any further.

Mr. Monier was attached to the team and this way he came in contact with ideas on how new material could be produced and the need for a different approach to the teaching of mathematics.

The need to get away from rote learning cannot be over emphasized. There are considerable merits in Mr. Monier's work just by the fact that some-one is trying to do something. In selecting topics for his experimental approach Mr. Monier has been obliged to follow the existing traditional syllabus. In my opinion he has tried to make the most out of the situation. He has given me the impression of knowing what he is talking about, being aware of the problems and generally having a realistic attitude to these problems and being determined to help in solving them. Even if Mr. Monier's efforts so far are pieces in the over all pattern of an educational reform they clearly fall within the framework of relevant development of the mathematics education in Afghanistan.

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Lennart L. Nilsson
Expert in Curriculum, Mathematics
and Science Teaching Aids.

NEBRASKA UNIVERSITY TEAM

American Embassy (USAID)
Kabul, Afghanistan

University of Nebraska
Representatives to
Kabul University, Afghanistan

To Whom It May Concern:

For the past year I have functioned as Mr. A. Monier advisor while he has been in Kabul, Afghanistan collecting data for his doctoral dissertation. This data collection has involved the preparation of materials, inservice education of teachers, and evaluation procedures to implement his activity approach to teaching geometry in experimental classes in the ninth and tenth grades in two high schools.

I have been extremely impressed by the way that Mr. Monier has approach this task. The time and effort that he has placed into this project is worthy of the compliment that can be paid to him. Afghanistan is not an easy country to work in it is even more difficult to conduct research in of the type that Mr. Monier has engaged in. It is do to his skill and hard work that he has been able to carry his research problem out to a successful conclusion.

I have formd Mr. Monier to be most cooperative and completely responsive to the suggestions that were made during our frequent conferences. Mr. Monier continuously showed an inter-discipline to carry out his doctoral research plan. Not once during the year of our association did I have to encourage him to get to work. He had his task outline and follow through completely on his own. I can think of no higher complement or description of his ability than this.

Sincerely,

Redacted for Privacy

George W. Glidden
Educational Consultant

Teachers College, Columbia University Team
USAID, % AMERICAN EMBASSY, KABUL, AFGHANISTAN

To Whom It May Concern:

In my role as mathematics advisor to the Curriculum and Textbook Project in the Ministry of Education of Afghanistan, I have become acquainted with Mr. Mohammad Ibrahim Monier. Mr. Monier, who is a professor at Kabul University, is one of the leading figures in mathematics education in Afghanistan. He has been associated with various curriculum projects and shows a keen interest in improving education in his country.

I was most happy when Mr. Monier received the opportunity to return to the United States to pursue his doctoral studies. There was no doubt in my mind that he would make the best of this opportunity. The additional training and experience that he is receiving will greatly benefit Kabul University and mathematics in Afghanistan in general.

Mr. Monier's concern for improving mathematics education in his country is evidenced by the fact that he has chosen to return to Afghanistan to undertake his doctoral research. The topic that he has selected is quite relevant to the educational needs of the country and should produce much useful information. Independent research of any type, especially in mathematics, is quite extraordinary in Afghanistan. Moreover, to propose an activity oriented approach to the teaching of geometry is nothing short of revolutionary, when the traditional teaching method involves lectures and memorization with little or no active student participation.

Numerous obstacles were encountered and overcome during the course of his research. There was initial skepticism and resistance on the part of teachers and school officials which was soon converted to enthusiasm and support. Of course, materials are necessary to an activity orientated approach and in Afghanistan suitable materials are often difficult to find. However, Mr. Monier did an excellent job of acquiring local materials for the numerous activities carried out by his experimental classes. Throughout the project, students displayed a level of interest and enthusiasm which is not normally found in the traditional geometry class. Naturally this was a welcome change for both students and teachers. However, in addition to this change in attitude about learning geometry, students also showed considerably better achievement.

In my opinion, Mr. Monier's entire research project was well planned, carefully executed and quite valuable. I am certain that the considerable information resulting from his research will be of benefit to mathematics education in Afghanistan.

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Barry D. Vogeli
Mathematics Advisor
Teachers College, Columbia University Team
Curriculum and Textbook Project
Ministry of Education
Kabul, Afghanistan