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Certain fluids exhibit an increase in flow resistance when subjected to an electric field. This electroviscous response, known as the Winslow effect, has been the basis for development of a unique control medium. Applying a strong potential across the fluid causes it to "stiffen", even to the point of ceasing to flow; this provides a valve action.

Contained herein, is an analysis of a hydraulic system which employs the valving technique described above. A mathematical model of the so-called "electrohydraulic valve" is incorporated into the system through the use of hydraulic-electrical analogues. Requiring that the model describe linear operation, the system is reduced to an equivalent circuit; for relatively small signal variations, a method is realized for investigating system characteristics.

A LINEAR MODEL OF AN ELECTROHYDRAULIC VALVE AND ITS INTEGRATION INTO SYSTEM ANALYSIS AND DESIGN

by

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LIST OF SYMBOLS

A	Orifice area of conventional hydraulic valve
A_A	Area of output piston face
A_V	Effective area ($w\epsilon$) of an EHV
C_d	Discharge coefficient
C_o	A number varying from 2 to 3, dependent upon flow rate and applied field strength; assumed to be 2.5 for calculations
D	Characteristic dimension with respect to Reynolds number
e	Signal voltage
e_{co}	Magnitude of e necessary to realize short-circuited cutoff
E	Electric field strength
EHV_n	Indicates a particular electrohydraulic valve (n=1, 2, 3, or 4)
F_A	Force applied to piston
F_x	Force applied to control volume centered inside passage of EHV
L	Length of EHV flow passage along direction of flow
m	Mass of the load
N_R	Reynolds number
p	Absolute pressure
P	Incremental pressure drop across an EHV
P_o	Pressure drop with respect to an orifice
P_S	Supply pressure
P_{wn}	Incremental pressure drop across EHV_n due to the Winslow effect

P_{wco}	Pressure drop across an open-circuited EHV
q	Volume flow rate
q_A	Volume flow rate through the actuator
q_{co}	Volume flow rate for open-circuited cutoff
Q_S	Supply fluid flow
R	Hydraulic resistance of an EHV
S	Fluid shear stress due to applied field
u	Velocity of fluid in EHV
V	Applied voltage
V_b	Bias voltage
w	EHV width perpendicular to applied field and to flow direction
x	Output displacement
y	Altitude of the control volume centered inside the passage of an EHV
y_c	Magnitude of y where relative motion begins between adjacent lamina of fluid.
Y	$\frac{y}{\epsilon/2}$
Y_c	$\frac{y_c}{\epsilon/2}$
α	Fluid gain parameter equal to S/E^2
ϵ	EHV gap in the direction of applied field
μ	Intrinsic or ordinary viscosity
μ_o	Viscosity at atmospheric pressure
ρ	Fluid density

τ Shear stress in the fluid (sum of the ordinary viscosity effect and the Winslow effect)

τ_m Magnitude of τ at the boundary of the EHV flow passage

ϕ Signal voltage expressed as a fraction of V_b

ϕ_{co} Magnitude of ϕ necessary to realize short-circuited cutoff

$\hat{\phi}_{co}$ Magnitude of ϕ_{co} with respect to maximum q_{co}

Increments in quantities such as V are noted by ΔV, differentials are noted as dV; i.e. $\Delta V \rightarrow dV$

$\Delta \rightarrow 0$

A LINEAR MODEL OF AN ELECTROHYDRAULIC VALVE AND ITS INTEGRATION INTO SYSTEM ANALYSIS AND DESIGN

INTRODUCTION

To design a system, or to predict its performance, it is necessary to understand and be able to apply the fundamental laws and basic principles peculiar to each component and device. Frequently a system which will do its job fairly well can be produced by trial and error, and with no analysis. Nevertheless, an optimum design can be achieved economically only by the application of existing theoretical and empirical knowledge. Few systems, however, are pure enough or simple enough to conform very closely to basic theoretical concepts. To at least "breakthrough" the barrier of impurity and complexity, simplifying assumptions justified by empirical data provide a beginning for analyses.

In this treatise a novel hydraulic system, that of a bridge configuration employing field-sensitive valving, is analyzed from the point of view of determining (1) a realistic model of valving technique and (2) theoretical formulae which describe system behavior. System capability is constrained with respect to small-signal operation; the magnitude of investigation is reduced by two simplifying assumptions: the system is linear and hydraulic resistance is ohmic.

Hydraulic-electrical analogies are used throughout this writing; the lumped parameters of hydraulic networks are represented by symbols of analogous electrical-parameters (voltage source for pressure pump, resistor for hydraulic resistance, etc.). Throughout the derivations and results, any consistent set of units may be used for finding numerical values.

I. THE FLUID

The operation of a fluid-power device necessitates the use of a physical fluid; it is the fluid which provides a medium for the transmission of power. While it is not intended that this treatise encompass or editoralize hydraulic fluids per se, it does seem requisite to expose and clarify those peculiarities of a fluid which make possible the valving technique discussed herein. Not only must this fluid have the characteristics of conventional hydraulic media, it must, in addition, possess the property of the so called "electric-fluids".

The Winslow Effect

W. M. Winslow (13, 15, 16) began a series of investigations in 1939 which initiated the development of electric-fluids (E-fluids) and related devices.[1] It has been found that specially prepared semi-conductive-fluids will exhibit an increase in apparent viscosity when acted upon by an electric field. This non-magnetic phenomenon, now known as the Winslow effect, is realized by a capacitor configuration: the E-fluid, a dielectric, fills the gap or energy-storage region between electrodes; by impressing a voltage across the electrodes and, thus, creating a potential gradient through the fluid, it is possible to

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[1] Elisha Gray, Thomas Edison, Johnson, Rabbek, and Fitch are reported (9) to have worked with field-controlled force-transmitting devices using wet and dry powders.

"stiffen" the fluid and mechanically couple the electrodes; if one electrode moves parallel and relative to the other, the applied force necessary to maintain motion is a function of the impressed voltage.

Figure 1 illustrates the relation between shear stress and electric-field strength for a typical E-fluid (12). The curve approximates the expression

$$S = \alpha E^2 \tag{1}$$

where S is the induced shear stress, E is the applied electric-field, and α is a constant of proportionality; for the E-fluid of Figure 1, $\alpha = 6.9(10)^{-9}$ newton/(volt)^2.

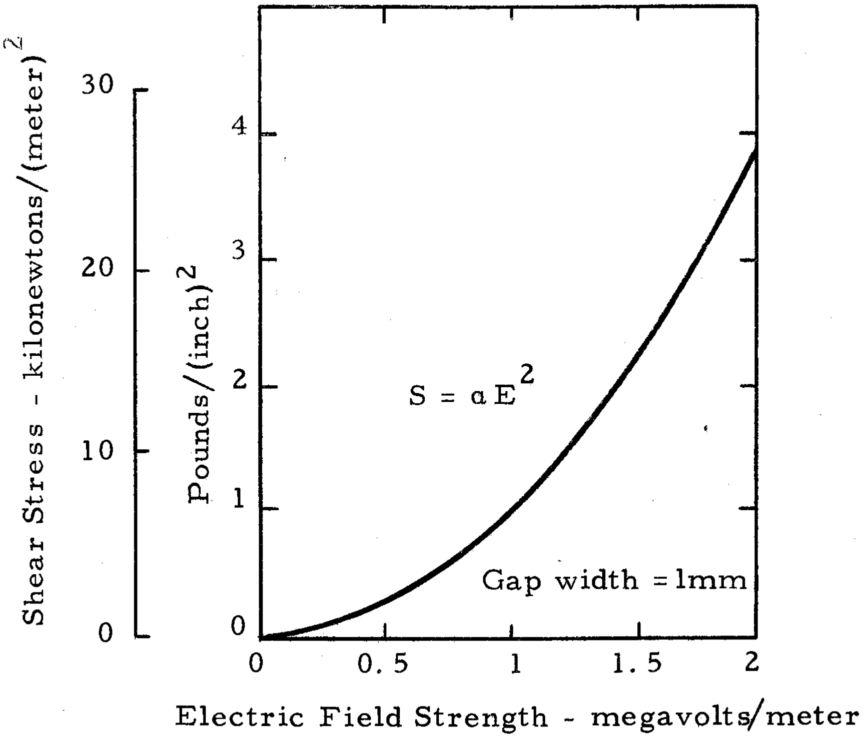

Figure 1. Electrohydraulic Shear Stress vs. Electric Field Strength.

Winslow framed an experiment (14) that upheld an earlier hypothesis, Equation (1): the apparent viscosity of an E-fluid is related to the electric field such that the induced shear stress is proportional to the square of the field. Two conducting members, an inner cylinder and a relatively low-inertia outer shell, are concentrically located and insulated from one another by an E-fluid. The inner cylinder is revolved at constant speed while the outer shell, coupled by a string to the cone of a loudspeaker, transmits the torque developed across the fluid. Impressing a 60 cycle voltage between the cylinders results in an output signal, audible from the speaker, with a frequency twice that of the input signal. The basic arrangement for the experiment is shown in Figure 2.

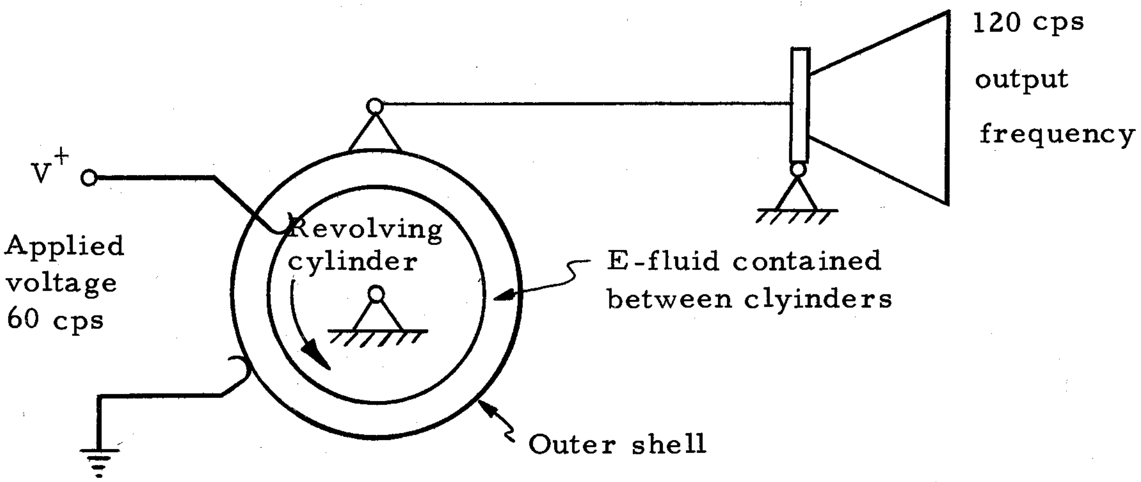

Figure 2. Winslow's Frequency Experiment.

The experiments performed by Winslow and the results obtained by recent investigators (3, 4, 5) have shown that E-fluids do behave in accordance with Equation (1); the constant of proportionality, a, is found to be a function of each fluid -- its substance, contamination, etc. -- and is independent of geometry. Equation (1) should, however, be recognized as an approximation: experimental data (5, p. 13) shows a scattering of points about the parabolic curve of Figure 1, and the data diverges from the square-law relation for relatively high electric-field intensities.

The Mechanism

The salient property of E-fluids, field-sensitive viscosity, is independent of the attracting force that exists between the electrodes of a capacitor; in practice, the frictional force due to the Winslow effect increases more rapidly than the force due to electrode attraction (8, p. 111). Also, the Winslow effect is not involved with the "tug" or "push" that can be experienced when inserting or removing a dielectric through the energy-storage region of a capacitor; the E-fluid, whether flowing through the field or contained *en toto* within the field, always occupies and completely fills the gap of the capacitor configuration; the dielectric constant of the E-fluid may change as a function of applied voltage, but the change is uniformly distributed, and energy is transferred thermally and not mechanically.

Winslow postulates that the shear stress wrought-up by the field sensitivity of certain fluids arises from the components of force along the fluid film due to the tension of inclined fibers, i.e., the E-fluid forms conducting chains parallel to the applied electric-field. A similar phenomenon occurs in the so-called "magnetic fluids": iron filings mixed with an oil-type vehicle will exhibit a rearrangement of an otherwise random orientation and align with an impressed magnetic-field; this induced alignment results in a thickening of the oil or an apparent increase in viscosity. Winslow has used finely-ground silica gel dispersed in a kerosene fraction of relatively low dielectric-constant. Like the iron filings of magnetic fluids, the silica gel particles -- activated with respect to their water-carrying property -- polarize and align with the applied electric-field; this polarization of particles within the colloidal E-fluid couples the lamina of fluid and impedes their slippage and shear rate: hence, the idea of conducting chains and induced fibration.

From elementary theory of electromechanical devices, the particles of either magnetic fluids or E-fluids must assume a configuration of minimum energy or, equivalently, of minimum reluctance. Attempts have been made to calculate the shear stress by assuming that the particles are identical and disk-shaped (10). The results of such calculations have not correlated with experimental findings; some E-fluids have no particles at all and, from microscope studies, show no conducting chains (5, p. 22). Winslow's hypothesis of "induced fibration" may very well be erroneous.

II. THE ELECTROHYDRAULIC VALVE

Although the physical mechanisms that underlie the Winslow effect are not fully understood, the physical E-fluid does exist,[2] and its unique characteristics can be put to use in the development of electrohydraulic, force-transmitting devices. One such device is the electrohydraulic valve.

The Valve

The purpose of a valve is to vary the restriction it places in that part of a system in which it is installed. In hydraulic systems, conventional or sliding-type valves are designed so that a variable orifice or outlet is achieved. By controlling the effective area of the valve's channel, mass flow and/or pressure gradient within the valve-domain is impeded or esculated with respect to a desired performance. Thus, conventional hydraulic-valving requires that valve geometries be of themselves variable quantities and, consequently, the afore-mentioned valve has moving parts.

The electrohydraulic valve (EHV) has no moving components. The geometries of an EHV, once selected and dimensioned, are fixed. As previously implied, an EHV constitutes a capacitor configuration,

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[2] The Warner Electric Brake and Clutch Co. manufactures an E-fluid under the trade name "Electro-Fluid".

i.e., the valve is designed so that fluid must pass through an applied voltage-gradient in order to traverse the valve domain. No valving is realized, of course, unless the fluid is field-sensitive. Thus, the Winslow effect constitutes the control mechanism of an EHV; by controlling the voltage impressed across the channel of an EHV, flow through the valve is restricted in accordance with Equation (1).

The Pressure Equation

Experimental data has supported a hypothesis (3, p. 15) as to the relation which might exist between the pressure gradient through an EHV and the corresponding electric-field. For an incompressible fluid, the pressure drop through an EHV is thought to have the form:

$$P = Rq + C_o \frac{L}{\epsilon} S \tag{2}$$

where

P = pressure drop through an EHV

R = laminar flow-resistance,

q = volume flow rate

C_o = a constant (assumed to be 2.5)

$\frac{L}{\epsilon}$ = length to gap ratio, and

S = fluid shear-stress due to the applied electric field.

Figure 3 shows the mathematical model of an EHV from which

Equation (2) is derived; the complete derivation is contained in Appendix A.

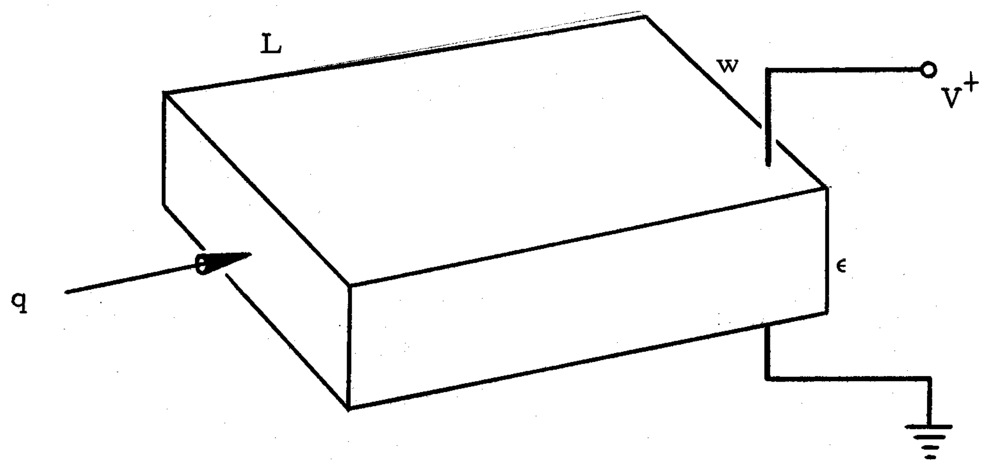

Figure 3. Theoretical Model of an EHV.

As illustrated in Figure 3, an EHV is represented by a pair of parallel flat plates, L long by w wide, and ϵ apart; a voltage, V, is impressed across the plates. In practice, an EHV is constructed with concentric cylinders (similar to the configuration of Figure 2) in which the fluid is constrained to flow through the annular separation. The cylinders, like the plates, constitute a capacitor. An entire cylindrical EHV is illustrated in Appendix B.

Assuming the shear-field relation to be parabolic (Figure 1), from Equations (1) and (2) it follows that

$$P = Rq + KV^2 \tag{3}$$

where

$V =$ the applied voltage, and

$K = 2.5 \dfrac{L}{\epsilon^3} \alpha$.

From Figure 3, the electric field strength, $E \equiv V/\epsilon$.

Analogies

Equation (3) expresses the pressure drop through an EHV as the sum of two different pressures: the RQ drop and the pressure due to the Winslow effect, KV^2. This relationship of pressures lends itself to hydraulic-electrical analogies: Equation (3) may be assumed a sum of voltages, i.e., a product of electrical resistance and current (Rq) and a voltage source (KV^2).

J.F. Blackburn (2, p. 180) discusses the limitations imposed upon hydraulic-electrical analogies:

> ... the analogue is reasonably good with respect to switching operations, and to the correspondence of pressure to voltage, inertia to inductance, and compressibility to capacitance, but the resistance analogue must be used with care. Most electrical resistances obey Ohm's law: the current proportional to the applied voltage. In the hydraulic field, the analagous law is obeyed for laminar flow, but the very large temperature coefficient of viscosity of all practical liquid hydraulic media must always

> be kept in mind; the case is as if electrical resistors were made not of metal but of high-coefficient thermistor material... .

Pressure also affects the viscosity of fluid (2, p. 21). Hydraulic resistance cannot be considered ohmic unless flow is laminar, temperature constant, and the pressure gradient bounded;[3] without ohmic resistance, hydraulic-electrical analogues are of superficial use.

Assumptions and Rational

Again, it is intended that this project develop a simplified but valid model of an EHV. The magnitude of investigation is lessened if hydraulic-electrical analogues are used: by the very nature in which the analogues are employed, the EHV and related systems reduce to electrical-type networks with the necessary constraints required by simplifying-assumptions. The assumptions, stated somewhat implicitly in the preceding sections, are now weighed and evaluated.

Incompressibility. To a first approximation, hydraulic fluids

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[3] The derivation of Equation 3 (Appendix A) shows that hydraulic resistance, R, as related to the configuration in Figure 3 is

$$R = \frac{12\ L}{w\epsilon^3}\ \mu$$

where μ is the ordinary or intrinsic viscosity of an E-fluid. Since the geometries of an EHV are fixed, R varies as μ only.

are incompressible; Equation (1) was derived upon this premise, and experimental data have shown that the assumption is realistic (3, 4, 5). Assuming that the E-fluid is incompressible, a resonant hydraulic-circuit can be avoided if entrained air within the fluid is eliminated, and system elements are structurally short and stiff.

If the design of an EHV provides for deaeration of the fluid and the containing walls of the valve are rigid, the E-fluid as used in the analysis herein is considered incompressible,[4] i. e., the density of the fluid is assumed independent of pressure.

Temperature. Figure 4 shows the effect of temperature on the activated (Winslow effect) viscosity of an E-fluid (12). Except for high field-strengths, the temperature does not adversely alter the activated viscosity. However, it is known that the intrinsic or ordinary viscosity of a fluid is considerably affected by temperature. In order that viscosity remain virtually temperature-independent, the E-fluid must be temperature controlled and regulated.

Pressure. Conventional or sliding-type hydraulic valves have, by the characteristic of their control mechanism, regions within the valve-domain where relatively high pressure-gradients are localized. Consider, for example, the valve-controlled hydraulic actuator as

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4 Average hydraulic fluids have a bulk modulus of compressibility of about $14(10)^8$ newtons/(meter)^2. The bulk modulus of water is approximately $25(10)^9$ newtons/(meter)^2 .

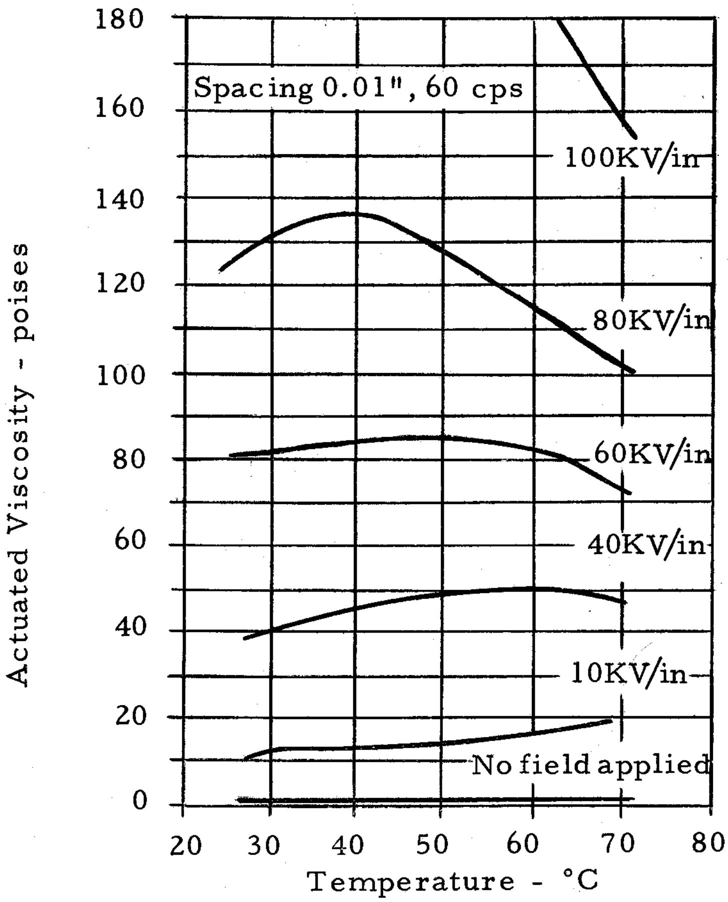

Figure 4. The Effect of Temperature on the Activated Viscosity of an Electric Fluid.

illustrated in Figure 5. This valving scheme is used in many applications as a power amplifier; very little power is required to position the valve, but a large power output is controlled. It can be seen that the pressure gradient within the regions of orifice variation (sumps and inlets) may surmount the pressure gradient across the valve housing; for the valve position of Figure 5, the localized pressure-gradient of each orifice is at a maximum. When the pressure gradient is high and the possibility of turbulent flow exists, hydraulic resistances are considered nonohmic and follow at least approximately to the orifice

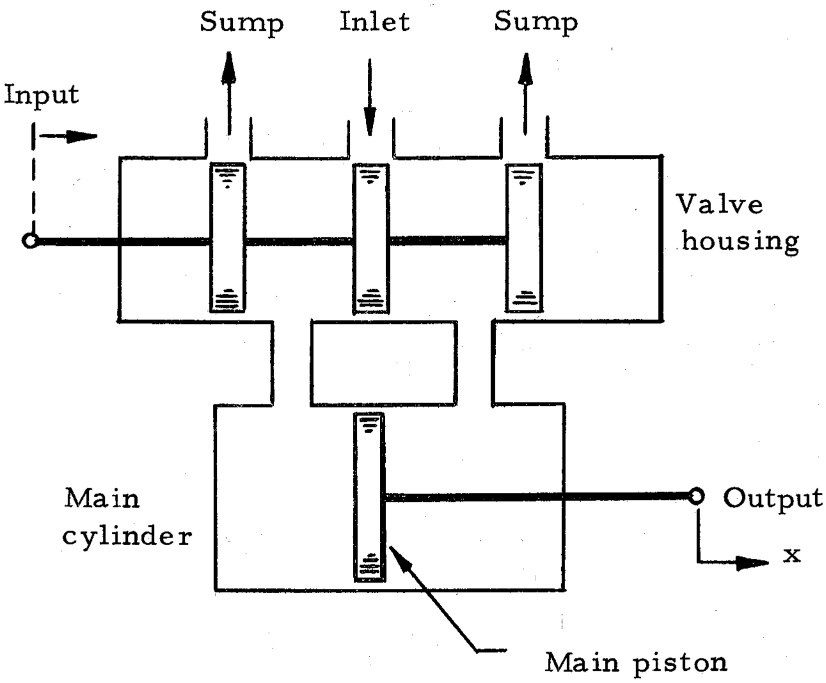

Figure 5. The Valve-Controlled Hydraulic Actuator.

equation (2, p. 181):

$$q = C_d A \sqrt{2P_o/\rho} \tag{4}$$

where

q = rate of flow

C_d = discharge coefficient

A = area of valve orifice

P_o = pressure drop, and

ρ = density of liquid.

The discharge coefficient, C_d, depends both upon the characteristics of the fluid and the orifice geometry. Since the performance of an orifice is related to its shape and edge topology, emperical data must be ascertained as to the valve's pressure-flow relationship.

Unlike its conventional counterpart, the EHV maintains a constant pressure-gradient throughout the valve-domain. Once an incremental volume of E-fluid enters the EHV, it is subjected to a linear decrease in pressure as it traverses to the outlet-terminal.[5] Since the geometry of an EHV constitutes a trivial configuration (see Figure 3), its effective area is easily obtained; by anticipating the maximum velocity of the fluid within the valve, the possibility of turbulent flow can be directly determined from the following equation:

$$N_R = \frac{2u\epsilon\rho}{\mu} \tag{5}$$

where

N_R = Reynolds number

ρ = density of the E-fluid

μ = intrinsic viscosity of the E-fluid

u = velocity of the E-fluid, and

ϵ = width of the gap.[6]

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[5] This assumes that the channel of an EHV consists of a virtual, parallel-plate configuration.

[6] From elementary fluid mechanics, Reynolds number is formulated by the following equation:

$$N_R = \frac{\text{inertia force}}{\text{friction force}} = \frac{Du\rho}{\mu}$$

In order to insure laminar flow,

$$N_R \leq 2000 \tag{6}$$

If the above inequality is satisfied and sustained, hydraulic resistance is ohmic with respect to fluid-flow conditions.

Viscosity, like volume flow rate, is a function of pressure. If the hydraulic-electrical analogues are to be applicable, not only must the flow be laminar, but the viscosity must be virtually independent. For viscosity to be independent of pressure, the magnitude of the pressure gradient through an EHV must be bounded. The viscosity of a liquid increases with increasing pressure approximately according to the expression (7):

$$\log_{10} \frac{\mu}{\mu_o} = 7 \times 10^{-5} \Delta p \tag{7}$$

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where D is the characteristic dimension. By definition,

$$D = \frac{4(\text{cross-sectional area of the orifice})}{2(\text{perimeter of the orifice inlet})} .$$

From Figure 3,

$$D = \frac{4(w\epsilon)}{2(w+\epsilon)}$$

For $\epsilon \ll w$, it follows that

$$D = 2\epsilon .$$

where

μ_o = intrinsic viscosity of a liquid at atmospheric pressure

μ = intrinsic viscosity of the liquid above atmospheric pressure, and

Δp = incremental pressure (expressed in psi) corresponding to μ.

If μ is allowed a maximum variation of ten percent, then, from Equation (7),

$$\Delta p \leqq 650 \text{ psi}$$

where 650 psi = $4.54(10)^6$ newton/meter^2. Therefore, if the pressure drop through an EHV does not exceed, say, 500 psi, μ will be relatively constant and R may be considered independent of pressure.

Figure 6 illustrates the changes which occur in pressure and viscosity as an incremental volume of E-fluid traverses the length of an EHV. It can be seen that an EHV is a "gentle" valve, i.e., no abrupt or localized pressure-gradient is realized within the valve as a result of the valving mechanism.

The Model

By adhering to the preceding assumptions and constraints, R is virtually ohmic. In accordance with Equation (3), the proposed EHV-model is shown in Figure 7. The model represents an EHV with

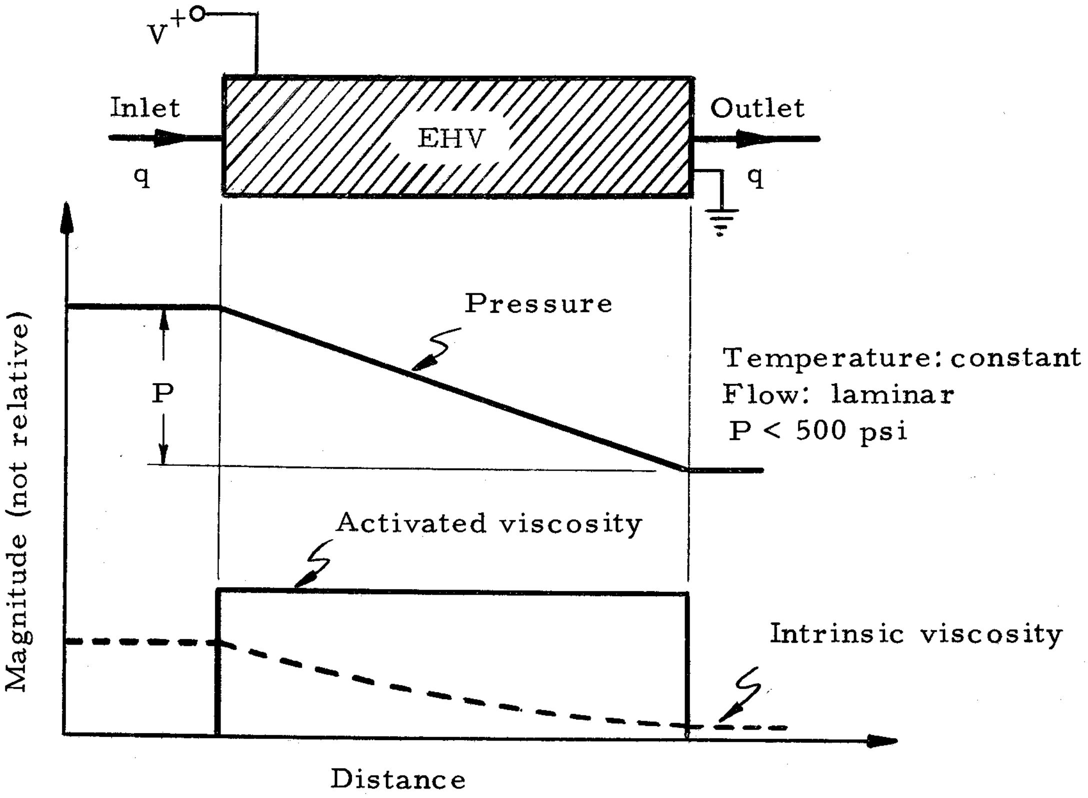

Figure 6. Variations in Pressure and Viscosity as an Incremental Volume of E-Fluid Traverses the Valve-Domain.

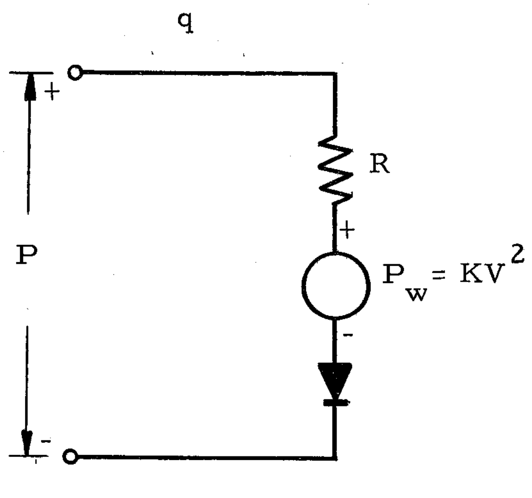

Figure 7. The EHV-Model.

respect to its developed pressures: the RQ drop and the Winslow effect, $P_w = KV^2$. The unilateral element, the diode, implies that the active element, P_w, is an artifice, i. e., an EHV is not a pump; a similar model applies to the nonlinear operation of a vacuum-tube triode.

So far, no mention has been made to the mass of an E-fluid. Assuming that the EHV is to be incorporated into one system or another, the quantity of fluid contained in an EHV will be relatively small. If the mass of the fluid should contribute appreciably to the performance of a particular system, it may be lumped with similar parameters in a more convenient location, e.g., the fluid in the channel between the pump and EHV.

III. THE ELECTROYHYDRAULIC BRIDGE

Rather than reduce the network of Figure 7 to an equivalent linear-model, it proves somewhat expedient to achieve linearization by investigating a system in which an EHV is used. This roundabout approach to the problem not only exemplifies the role of an EHV as a system component but, also, illustrates a logical development of design equations that are formulated through the correlation of system parameters and the linear EHV.

The System

A research project (10) has led to the design and construction of a novel exciter-system. Schematically represented in Figure 8, this bridge configuration functions as a force-transmitting device through the principle of the Winslow effect. The actuator (piston-cylinder assembly) is driven by hydraulic power, controlled by four EHV's. The EHV's consist of concentric cylinders, insulated from each other, and separated by a narrow gap through which E-fluid is pumped; voltages applied to the cylinders control the flow of fluid by their direct action upon the fluid. While the aforementioned project was primarily an investigation into the feasibility of shock-vibration simulation, the resulting "electrohydraulic bridge" is readily adapted to EHV control and the generation of related systems beyond the

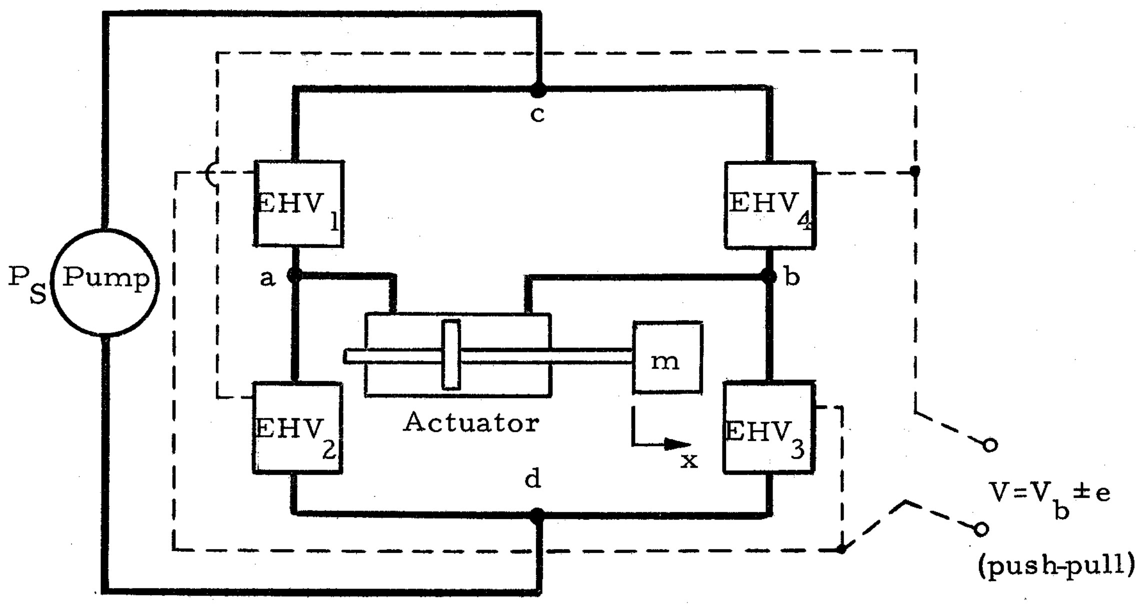

Figure 8. Schematic View of Exciter System.

exciter prototype.

The schematic view of Figure 8 shows the fluid flow (heavy line) with respect to the four EHV's and the actuator assembly; also shown are the electrical circuits (dashed line) by which the control voltages are applied. The EHV's -- operated in push-pull fashion with a steady bias-voltage, V_b -- control flow from a constant pressure-source, P_S, in response to a signal voltage, e. The mass, m, is displaced by a pressure differential developed across the "null" of actuator branch, a-b.

Figure 9 shows the EHV model of Figure 7 integrated into the bridge configuration. The channels, or "conductors", connecting system elements are assumed to be short in length and to have, like the actuator, large diameters; therefore, as an additional simplification, hydraulic resistance is considered to be a property only of the EHV's. Voltage, applied push-pull, is distributed in such a manner that the bridge is magnitude-symmetric. As illustrated in Figure 9, the bridge is represented as a lattice network with each branch of the lattice constituting one EHV; with respect to the actuator, q_A and P_A are the volume flow-rate and pressure drop respectively.

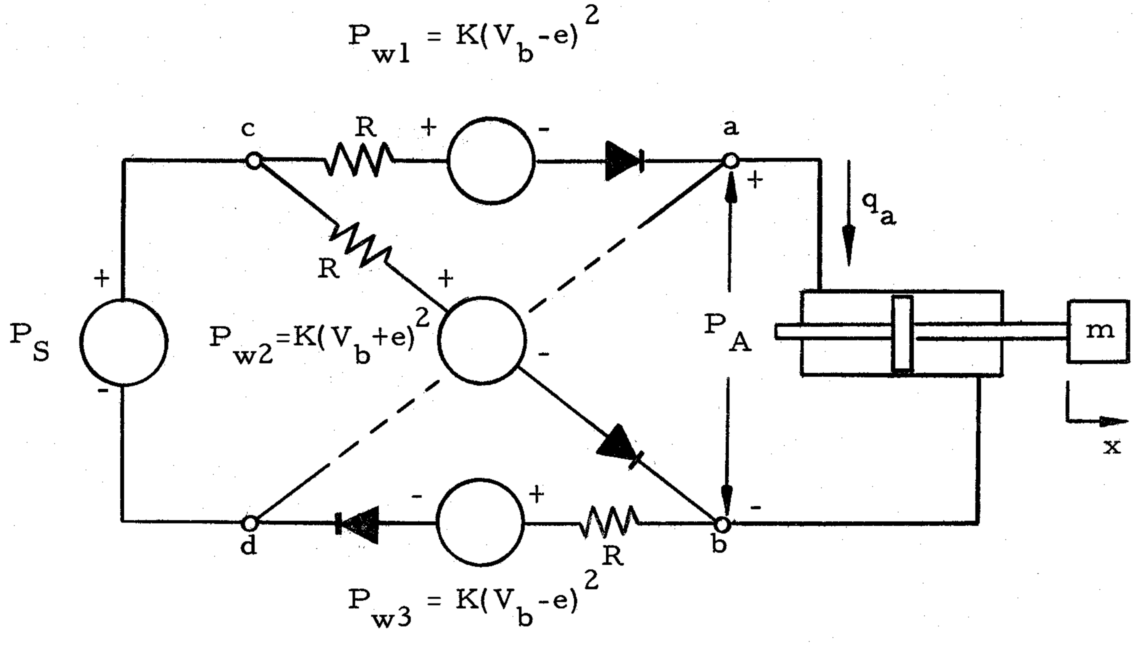

Figure 9. Electrohydraulic Bridge with EHV Models.

Again, the electrical symbols shown in the figures represent analogous, hydraulic elements. In order to better visualize the mechanism of piston displacement, x, in relation to fluid flow (fluid does not flow through the piston), the actuator is represented in the figures by its piston-cylinder configuration. With respect to system response, the actuator represents a restraining force of inertia which, for hydraulic-electrical analogues, may be inductive reactance.[7]

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[7] With no leakage in the system,

$$q_A = A_A \frac{dx}{dt}$$

where A_A is the effective, cross-sectional area of the actuator. From mechanics (assuming the actuator dissipates no energy, and the piston has negligible mass),

$$F_A = m \frac{d^2x}{dt^2}$$

where F_A is the magnitude of applied force due to fluid acting on the piston. In relation to the pressure,

$$P_A = \frac{F_A}{A_A} = \frac{m}{A_A} \frac{d^2x}{dt^2} .$$

But

$$\frac{dq_A}{dt} = A_A \frac{d^2x}{dt^2} .$$

Therefore,

$$P_A = \frac{m}{A_A^2} \frac{dq_A}{dt} .$$

With respect to hydraulic-electrical analogues, the actuator is an inductance of m/A_A^2 henrys.

The Equivalent System

The contraction of a circuit to its Thévenin and Norton equivalent usually involves the reduction of the non-varying part. For the circuit of Figure 9, however, it proves advantageous to reduce the lattice -- the EHV branches -- to an equivalent with respect to the actuator; the symmetry characteristic of the bridge-configuration allows that the pressures P_{w1} and P_{w2} be simply expressed in the reduced network. The Thévenin equivalent system is shown in Figure 10.

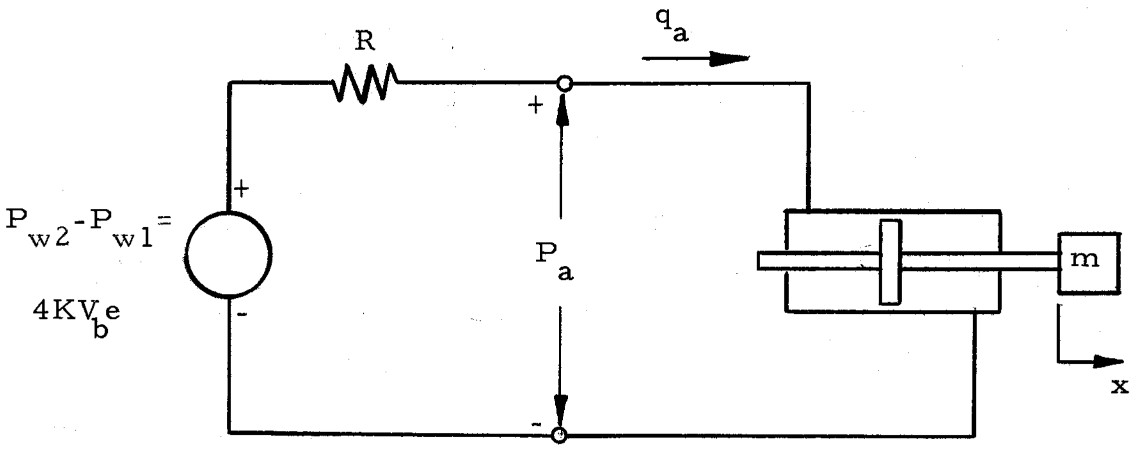


Figure 10. The Equivalent Bridge Configuration.

The Model

Since the EHV's are operated push-pull, it follows that

$$P_{wn} = KV_n^2 = K(V_b \pm e)^2 \tag{9}$$

where

P_{wn} = pressure drop through EHV_n due to the Winslow effect ($n = 1, 2, 3,$ or 4)

V_n = voltage applied to EHV_n

V_b = bias voltage, and

e = input signal voltage (the sign of e is chosen such that the symmetric lattice of Figure 9 is preserved).

Therefore, if fluid flow is impeded with respect to EHV_2 and, correspondingly, EHV_4 -- then, from Equation (9),

$$P_{w2} = P_{w4} = K(V_b^2 + 2V_b e + e^2) \tag{10}$$

and

$$P_{w1} = P_{w3} = K(V_b^2 - 2V_b e + e^2) \,. \tag{11}$$

From Equations (10) and (11), it is seen that for relatively large magnitudes of e, the incremental change of pressure through one pair of EHV's is not equal to the incremental change in the other. This difference is a result of the square-law relationship as expressed in Equation (1). In order that the developed pressures, P_{wn}, follow the input more closely, and, thus, assist the quasi-linear effect of push-pull, the magnitude of e must be small with respect to V_b.

According to Equation (9), the bias pressure, P_{wb}, may be expressed as follows:

$$P_{wb} = KV_b^2 \tag{12}$$

where $P_{wn} = P_{wb}$ for $e = 0$. Assuming that small-signals are applied, the differential of Equation (12) may be used as an approximation to the incremental value of P_{wn}, i.e.,

$$\Delta P_{wn} = \Delta P_{wb} \cong dP_{wb} \tag{13}$$

provided ΔV_b is sufficiently small. Therefore,

$$\Delta P_{wb} \cong 2KV_b \Delta V_b \tag{14}$$

if $\Delta V_b << V_b$. Since Equation (14) is an expression for the change in P_{wn} corresponding to the change in applied voltage,

$$P_{wn} \cong KV_b^2 \pm 2KV_b \Delta V_b \,. \tag{15}$$

Again: ΔV_b is considered relatively small; the choice of sign (±) must preserve the symmetry of the bridge configuration.

Suppose the magnitude of signal voltage, e, is small, e.g., $e << V_b$. From Equation (15),

$$P_{wn} \cong KV_b^2 \pm 2KV_b e \tag{16}$$

where $e = \Delta V_b$. Similarly,

$$P_{wn} = KV_b^2 (1 + 2\phi) \tag{17}$$

where ϕ is some fraction of the magnitude of V_b, i.e. $\phi V_b = \Delta V_b$. The equality relation, as implied by Equation (17), will be used in the proceeding derivations and formulae; it is understood that $\phi \ll 1$.

By applying Equation (17) to the equivalent system of Figure 10, the pressure drop through the actuator is expressed as follows:

$$P_A = 4KV_b^2\phi - Rq_A \, . \tag{18}$$

As long as an EHV is not "open-circuited",[8] Equation (17) insures that the pressures, P_{wn}, vary linearly with the applied signal-voltage, $e = \phi V_b$; hence, the pressure source of Figure 10 follows the applied voltage without appreciable distortion. The linear or small-signal model of the equivalent system is shown in Figure 11.

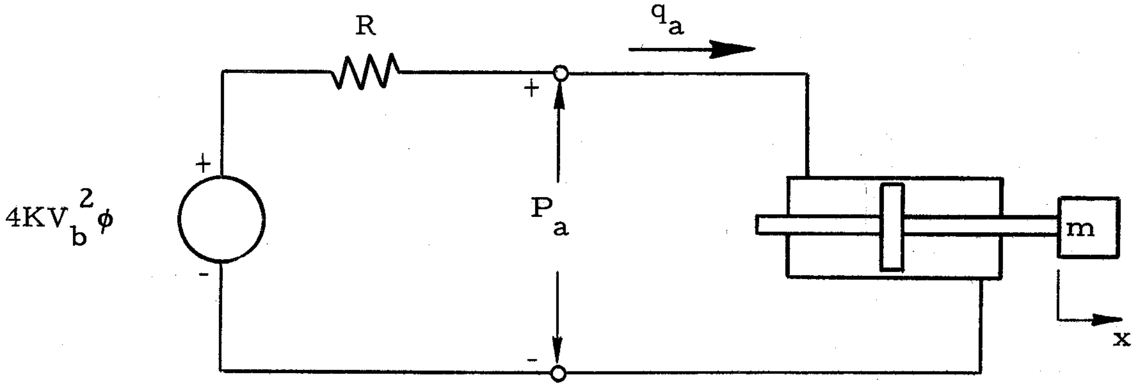


Figure 11. Linear Model of the Bridge Configuration.

[8] An open-circuited EHV (or "cutoff"condition) is realized when applied voltage induces the E-fluid to stiffen to the point of ceasing to flow.

The Design Equations

The linear model of Figure 11 is of little use unless some criteria is set down which correlates ϕ and V_b into an interrelationship that is physically realizable. The magnitude of the Thévenin pressure-source, $4KV_b^2\phi$, is implicitly dependent upon P_S, and-- since R is assumed ohmic -- fluid flow must be constrained within the limit of Equation (6). In search of a possible design criteria, the network of Figure 9 is again analyzed.

Two modes of operation for the electrohydraulic bridge are shown in Figure 12. For the operating mode of Figure 12-a, the fluid flow through any one EHV is at a maximum if the actuator is short-circuited. Furthermore, this short-circuited, cutoff condition demands a greater fluid flow through an EHV than does the bias or

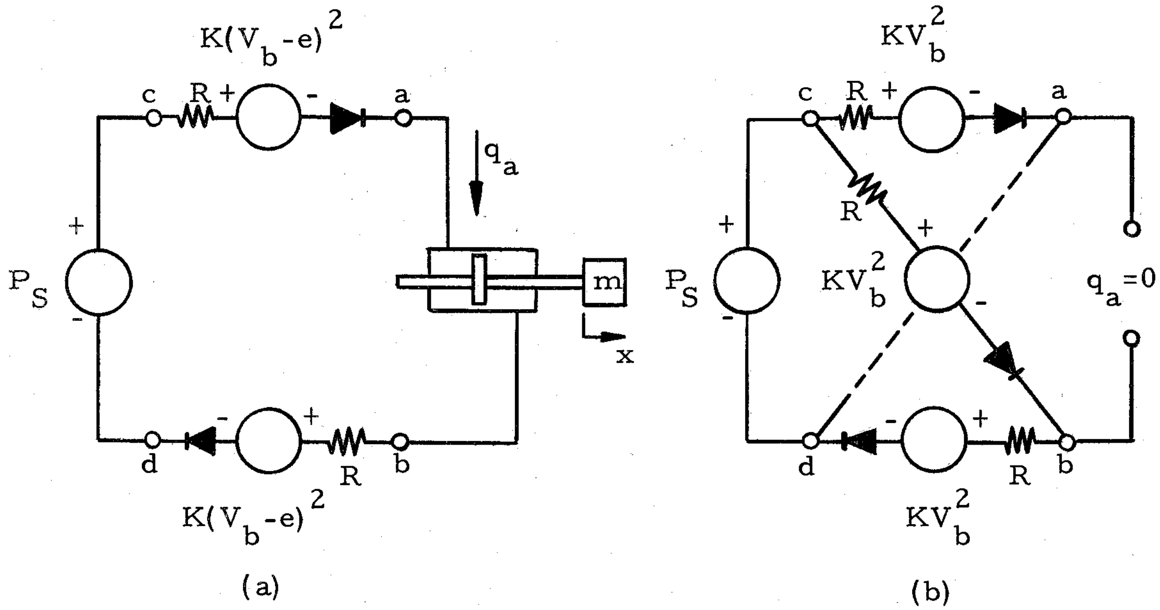


Figure 12. Modes of Operation: (a) Cutoff, and (b) Quiescence.

quiescent mode of Figure 12-b.[9] Assuming that the transient of the systems (resulting from the effective mass and hydraulic resistance) decays to negligible magnitude during the displacement of m, the actuator will short-circuit. If an EHV-pair (EHV_1-EHV_3 or EHV_2-EHV_4) has an applied voltage $V_b + e_{co}$ which just open-circuits that pair when the actuator "shorts-out", the corresponding signal level, e_{co} is unique: short circuited cutoff is realized while the pressures, P_{wn}, continuously follow e for $e \leqq e_{co}$; also, e_{co} has a corresponding volume flow-rate, q_{co}, which can easily be related to system parameters. The intermediate levels of fluid flow, those between bias (q_b) and cutoff (q_{co}), are difficult to determine analytically.

[9] The system model is considered a non-resonant circuit, i.e., fluid is incompressible, system components are stiff, and leakage is negligible; the system response, therefore, exhibits no overshoot, and the maximum volume flow-rate is limited as follows:

At quiesence (Figure 12-b), the fluid flow through an EHV, q_b, is

$$q_b = \frac{P_S - 2KV_b^2}{2R}$$

At short-circuited cutoff (figure 12-a with the actuator short-circuited), the fluid flow through an EHV, q_c is

$$q_c = \frac{P_S - 2K(V_b - e)^2}{2R}$$

where the magnitude of e is at some fixed level which sustains cutoff. Since $V_b > V_b - e$, it follows that $q_c > q_b$.

For the aforementioned mode of operation, the magnitude of fluid flow may be obtained from the analysis of Figure 11: at cutoff,

$$q_A = q_{co} ; \tag{19}$$

also,

$$q_{co} = \frac{4KV_b^2 \phi_{co}}{R} . \tag{20}$$

From Equation (5),

$$q_{co} = \frac{N_R \mu A_V}{2\epsilon \rho} \tag{21}$$

where A_V is the effective area of an EHV, i.e., $A_V = w\epsilon$.[10] By combining Equation (20) with Equation (21), ϕ_{co} may be evaluated as follows:

$$\phi_{co} = \frac{1}{8} \frac{A_V \mu R N_R}{\epsilon \rho K V_b^2} , \tag{22}$$

---

10 Volume flow rate, q, is related to N_R in the following manner

$$q = uA_V$$

where u is the velocity of the fluid perpendicular to the effective area of the valve. Since

$$N_R = \frac{Du\rho}{\mu}$$

then

$$q = \frac{N_R \mu A_V}{D\rho}$$

or

$$\phi_{co} = \frac{3}{5} \frac{\mu^2 N_R}{\rho \alpha V_b^2} \quad . \tag{23}$$

As expressed above, ϕ_{co} is a dimensionless quantity -- which it should be. ϕ_{co} is dependent only upon fluid parameters and the magnitude of the bias voltage.

From Equation (17), the pressure drop through an open-circuited, EHV pair is

$$P_{wco} = KV_b^2 (1 + 2\phi_{co}) \tag{24}$$

where ϕ_{co} corresponds, in magnitude, to some fractional part of V_b; $\phi_{co} = e_{max}$. Alternately,[11]

---

[11] Consider Figure 12-a with the actuator short circuited; let

$$V_{min} = V_b - e_{co} \; .$$

The pressure drop through the terminals c to a (or d to b) is

$$P_{ca} = P_{db} = P_{wco} \; .$$

From circuit analysis,

$$P_{wco} = P_S - (\frac{P_S - 2KV_{min}^2}{2R} R + KV_m^2),$$

or

$$P_{wco} = \frac{P_S}{2} \; .$$

$$P_{wco} = \frac{P_S}{2} \quad . \tag{25}$$

From Equations (24) and (25)

$$P_S = 2KV_b^2 (1 + 2\phi_{co}). \tag{26}$$

In order for the actuator of Figure 12-a to achieve a maximum response (maximum velocity), the magnitudes of P_S, V_b, and ϕ_{co} must be selected so that fluid flow persists without turbulence. For a Reynolds number of 2000, Equation (23) is expressed as follows:

$$\hat{\phi}_{co} = 12 \frac{\mu^2}{\rho a V_b^2} (10)^2 \tag{27}$$

where $\hat{\phi}_{co}$ is that magnitude of signal voltage (equal to some percent of V_b) which must be sustained so that q_{co} is maximum.

From the preceding analysis, the Thévenin pump of Figure 11 is constrained to operate within the following interval:

$$\text{Thévenin pressure} \leqq |4KV_b^2 \hat{\phi}_{co}| \; .$$

For $\hat{\phi}_{co} << 1$, the system is virtually linear and its performance -- corresponding to maximum flow -- may be investigated through the cut-and-try procedure implied by Equations (26) and (27).

It should be noted that the aforementioned equations simplify design or analysis not only through linearization, but, also, by their apparent independence with respect to hydraulic resistance, R; w is theoretically limited only by its physical size and, therefore, provides additional freedom in the dimensioning of system parameters.

CONCLUSION

If the analysis herein is upheld (or modified) by experimentation, then a new look should be taken at ways to control fluid flow with the better-understood EHV's. Needless to say, electrohydraulic valving is not restricted to linear operation. System behavior with respect to wide temperature variation, high pressure, and turbulent flow require more sophisticated methods of investigation.

Succeeding analyses, whether linear or not, would be on a firmer basis if a quantitative understanding of the Winslow effect was achieved. A basic study could be aimed toward such goals as (1) being able to predict the strength of the effect from known properties of the fluid, (2) finding whether dielectric strength can be increased without weakening the Winslow effect, (3) predicting an optimum fluid and estimating how far present ones are from it. Long term stability of the E-fluids should also be considered.

BIBLIOGRAPHY

1. Angermayer, Karl. Structural aluminum design. Richmond, Va., Reynolds Metal Company, 1962. 233 p.

2. Blackburn, J. F. et al. Fluid power control. New York, Wiley, 1962. 710 p.

3. Eige, John J. and Edward C. Frazer. Feasibility study of vibration and shock exciter using electric field modulation of hydraulic power. Wright-Patterson Air Force Base, Ohio, Wright Air Development Division, 1961. 27 p. (WADD Technical Report 61-185)

4. ________________. Vibration and shock exciter using electric field modulation of hydraulic power. Wright-Patterson Air Force Base, Ohio, Aeronautical Systems Division, 1962. 69 p. (Technical Documentary Report no. ASD-TDR-62-536)

5. Eige, John J. and John Peschon. Vibration shock system. Wright-Patterson Air Force Base, Ohio, Wright Air Development Division, 1960. 45 p. (Interim Scientific Report 1)

6. Demos, William R., Sales Correspondent, Warner Electric Brake and Clutch Co. Personal communication. Beloit, Wis. May, 1965.

7. Hersey, Mayo D. and Richard F. Hopkins. Viscosity of lubricants under pressure. New York, American Society of Mechanical Engineers, 1954. 87 p.

8. Hix, C. F. and R.A. Alley. Physical laws and effects. New York, Wiley, 1958. 112 p.

9. Mason, J. A. Introduction to electrostatically influenced force transmitting fluids. New York, 1960. n. p. (American Institute of Electrical Engineers Conference Paper no. CP60-387)

10. Menzel, D. H. Professor of Astrophysics, Harvard University. Theory of the Winslow Electrostatic Clutch. Personal communication. Cambridge, Mass. Sept., 1964. (processed)

11. Motionless valves. Research for Industry 15(1):21-22. Jan.-Feb. 1963.

12. Warner Electric Brake and Clutch Co. Electro fluid. Beloit, Wis., n.d. 12 sheets (processed)

13. Winslow, W. M. Method and means of translating electrical impulses into mechanical force. U.S. patent no. 2,417,850. March 25, 1947. (Abstracted in U.S. Patent Office Official Gazette, vol. 596) (Microfilm)

14. _______________. Induced fibration of suspensions. Journal of Applied Physics 20:1137-1140. 1949.

15. _______________. Field controlled hydraulic devices. U.S. patent no. 2,661,596. Dec. 8, 1953. (Abstracted in U.S. Patent Office Official Gazette, vol. 677) (Microfilm)

16. _______________. Electric motor with a field response clutch. U.S. patent no. 2,663,809. Dec. 22, 1953. (Abstracted in U.S. Patent Office Official Gazette, vol. 677) (Microfilm)

APPENDICES

APPENDIX A

DERIVATION OF BASIC PRESSURE EQUATION FOR AN ELECTROHYDRAULIC VALVE

Figure 3 shows the theoretical model of an EHV. With the assumptions as detailed in Part II, the derivation of the pressure equation for an EHV is as follows:

Consider the forces due to fluid flow acting on the model of Figure 3. Assuming fully-developed laminar-flow, the pressure is constant throughout any cross section perpendicular to the flow direction, and the velocity profile is the same in all directions.

Since there is no net momentum flux through the ends of the control volume, the momentum equation of a control volume centered inside the passage (see Figure A-1) reduces to a balance of forces:

$$\Sigma F_x = 0$$

$$P\,2yw - 2\tau Lw = 0 \qquad \text{(A-1)}$$

where τ is the shear stress in the fluid (as a function of y). Evaluate Equation (A-1) at $y = \epsilon/2$:

$$P\epsilon w - 2\tau_m Lw = 0 \qquad \text{(A-2)}$$

where τ_m is the value of shear stress at the boundary. Divide Equation (A-1) by Equation (A-2) to find the shear stress distribution:

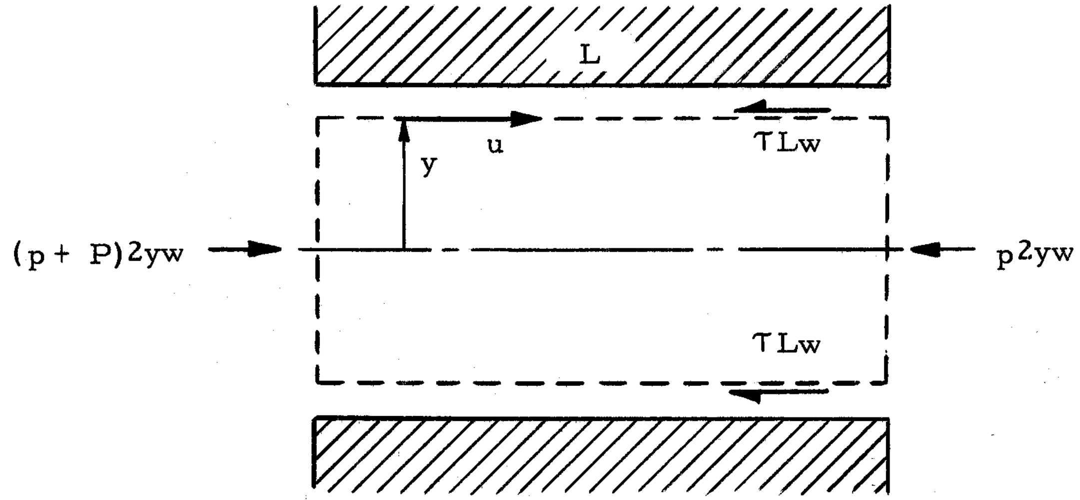


Figure A-1. Cross Section of Model EHV Showing Control Volume.

$$\tau = \frac{y}{\epsilon/2}\,\tau_m \tag{A-3}$$

The above says that shear stress increases linearly from zero at the center to a maximum, τ_m, at each wall. This distribution is shown in Figure A-2.

To find the velocity profile, assume that the shear stress at any point in the fluid is the sum of the ordinary viscosity effect and the electric field effect, S -- that is:

$$\tau = -\mu\frac{\partial u}{\partial y} + S$$

or

$$\frac{\partial u}{\partial y} = -\frac{\tau - S}{\mu} \tag{A-4}$$

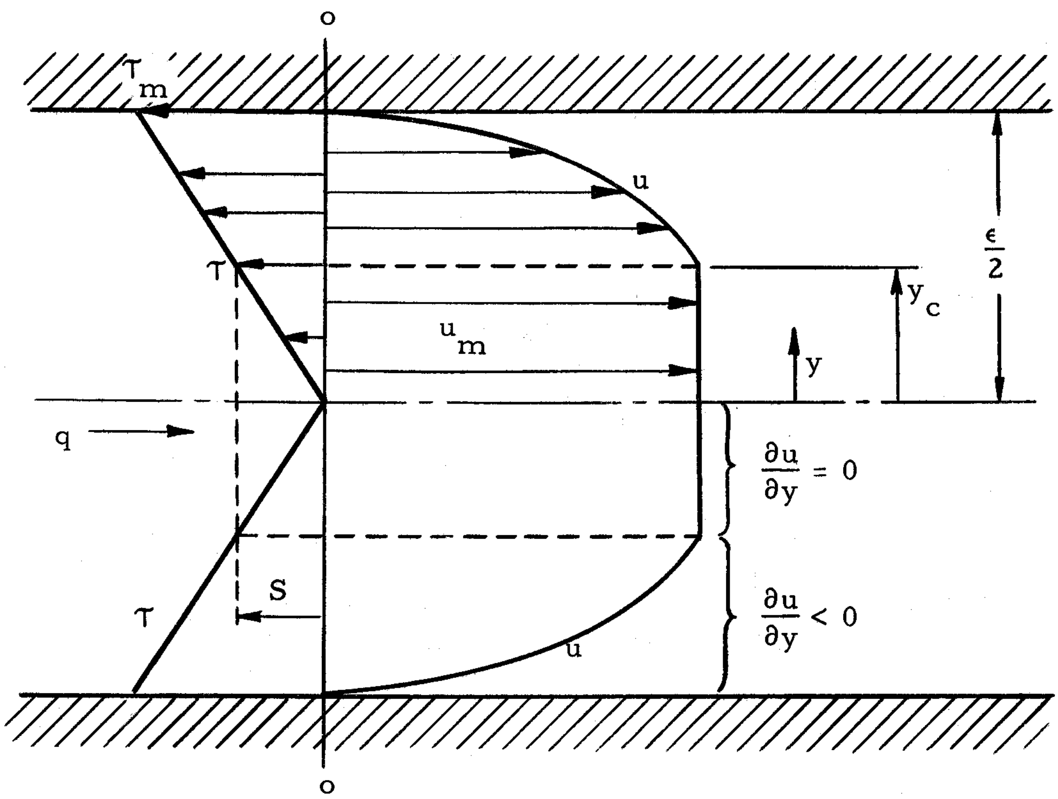


Figure A-2. Shear Stress and Velocity Profiles in Model EHV.

Further assume that there can be no relative motion between adjacent lamina of fluid as far as the shear stress (which increases linearly out from the center line) can be supported by the electric field effect. After τ exceeds S, slip occurs between all lamina further out toward the walls. The velocity gradient, $\partial u/\partial y$, must then be zero in some center portion of the passage and negative in the outer portion where τ is greater than S.

Substitute τ from Equation (A-3) in Equation (A-4), and integrate to find the velocity profile in the outer region, as follows:

$$-\mu \int_{y}^{\epsilon/2} \frac{\partial u}{\partial y}\, dy = \int_{y}^{\epsilon/2} \left(\frac{\tau_m}{\epsilon/2}\, y - S\right) dy$$

$$\mu u = \frac{\tau_m}{\epsilon/2}\left[\frac{(\epsilon/2)^2}{2} - \frac{y^2}{2}\right] - S\left(\frac{\epsilon}{2} - y\right) .$$

Let:

$$Y \equiv \frac{y}{\epsilon/2} \quad .$$

Then:

$$\mu u = \frac{\tau_m \epsilon}{4}\,(1 - Y^2) - \frac{S\epsilon}{2}\,(1 - Y) \tag{A-5}$$

The maximum velocity, u_m, occurs at the critical distance, y_c, where $\tau = S$ and $\partial u/\partial y = 0$. If

$$Y_c \equiv \frac{y_c}{\epsilon/2} , \tag{A-6}$$

then u_m is expressed by substituting Y_c for Y in Equation (A-5). The velocity profile is seen to be flat at the center region of the gap and parabolic in the outer regions, as shown in Figure A-2.

The velocity can now be integrated across the passage to find the flow rate, q, as follows:

$$q = 2w \int_0^{\epsilon/2} u dy$$

$$= 2wu_m y_c + 2w \int_{y_c}^{\epsilon/2} u dy$$

$$q = wu_m Y_c \epsilon + w\epsilon \int_{Y_c}^{1} u dY \, . \tag{A-7}$$

Y_c can be evaluated by solving Equation (A-7) at $y = y_c$ and $\tau = S$, giving

$$Y_c = \frac{S}{\tau_m} \tag{A-8}$$

An alterante expression can be found by solving Equation (A-6) for τ_m, giving

$$\tau_m = \frac{\epsilon/2}{L} P \tag{A-9}$$

and by combining Equation (A-9) with Equation (A-8), as follows:

$$Y_c = \frac{2LS}{\epsilon P} \tag{A-10}$$

Equation (A-7) can now be evaluated:

$$q = \frac{1}{R}[P - (3 - Y_c^2) \frac{L}{\epsilon} S] \tag{A-11}$$

where

$$R = \frac{12\mu L}{\epsilon^3 w} . \tag{A-12}$$

Solving for P gives

$$P = Rq + (3 - Y_c^2) \frac{L}{\epsilon} S \tag{A-13}$$

The term in parentheses varies from 2 to 3 depending on flow conditions. For the limiting case where field effects are strong compared to viscous effects, Y_c approaches unity. Corresponding to Equation (2),

$$C_o = 3 - Y_c^2 .$$

APPENDIX B

THE DESIGN OF A CYLINDRICAL ELECTROHYDRAULIC VALVE

The electrodes of a cylindrical EHV are illustrated by a two-view projection in Figure B-1 (for clarity, the separation between electrodes, ϵ, is exaggerated). It is desired that the valve be used primarily for experimental purpose, i. e., to investigate the characteristics of E-fluids and an EHV. Data obtained by the Warner Electric Brake and Clutch Company (12) show than an appreciable variation in apparent viscosity can be realized by varying the electric field from $100(10)^3$ volts/inch to $600(10)^3$ volts/inch (deaerated E-fluids have been found to sustain a field intensity of $5(10)^6$ volts/inch before exhibiting dielectric breakdown). In order to eliminate the need for extremely high voltages, ϵ will be selected as 0.01 inch; the maximum applied potential will be 600 volts.

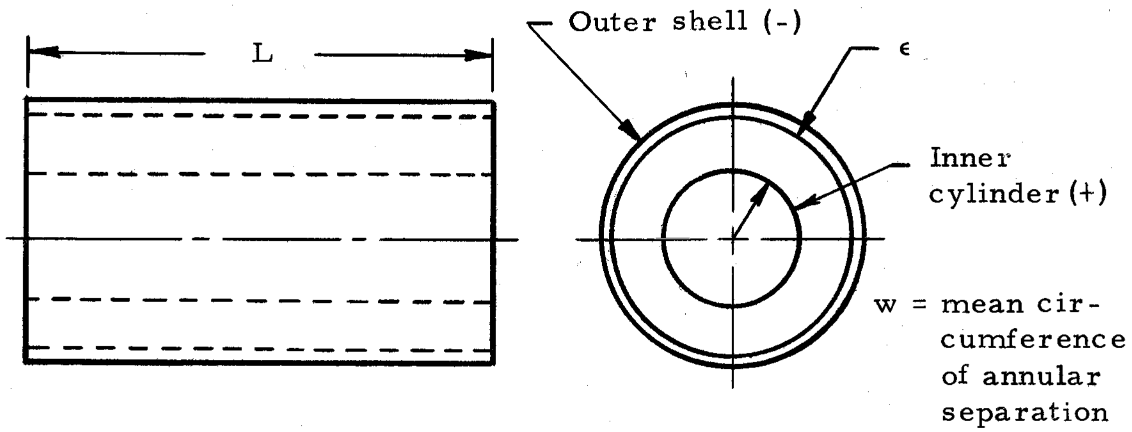


Figure B-1. Electrodes of Cylindrical EHV.

Dynamic Characteristics

Figure B-2 shows a schematic view of the proposed test system. The E-fluid is pumped through the EHV at $(2)^4$ gal/min from a constant flow source, Q_S. The EHV is designed so that the ohmic drop, $Q_S R$, is negligible at maximum pressure.

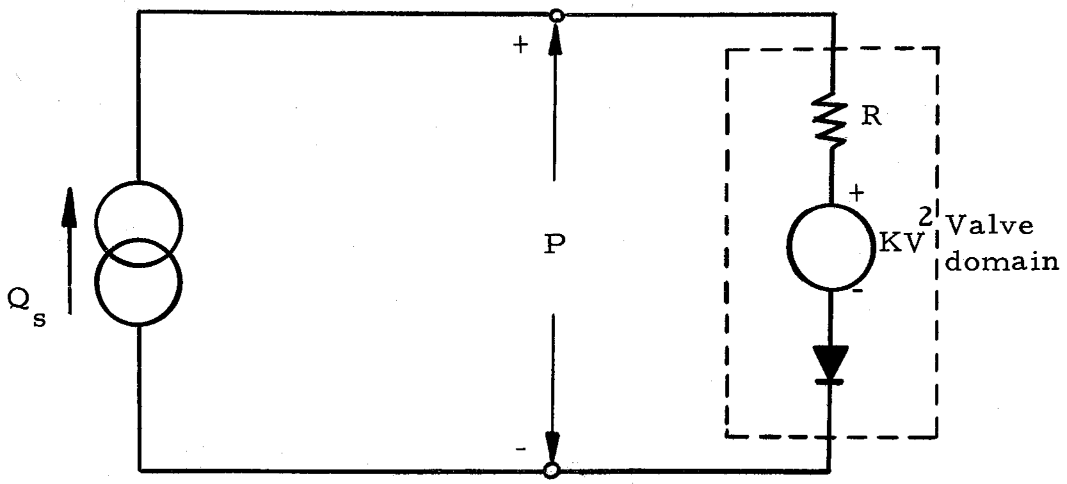


Figure B-2. Proposed Test System.

From Equation (3),

$$P_{max} = KV_{max}^2 , \tag{B-1}$$

or

$$P_{max} = 2.5 \frac{L\alpha V_{max}^2}{\epsilon^3} . \tag{B-2}$$

Suppose that a P_{max} of 1000 psi is desired; as previously stated, V_{max} = 600 volts and ϵ = 0.01 inch. If $\alpha = 2.25(10)^{-10}$ lbf/(volt)^2 (a conservative magnitude for α), then, from Equation (B-2),

$$L \cong 5 \text{ inches} \qquad \text{(B-3)}$$

To determine the effective width, w, of the EHV, the ohmic drop will be chosen as five percent of maximum pressure, i.e.,

$$RQ_S = 0.05 P_{max} \qquad \text{(B-4)}$$

Combining Equations (B-2) and (B-4),

$$w = 240 \frac{\mu L Q_S}{P_{max} \epsilon^3}, \qquad \text{(B-5)}$$

or

$$w \cong 13 \text{ inches} \qquad \text{(B-6)}$$

Thus, from the valve dimensions cited above,

$$R = 6.75 \text{ lbf-sec/(inch)}^5. \qquad \text{(B-7)}$$

It is assumed that the EHV can be temperature controlled (water-cooled, etc.); a maximum pressure drop through the EHV of 1000 psi does not appreciably exceed the constraint of Equation (8) (a variation in R of 15 percent is anticipated. A check should be made, however, to determine whether laminar flow has been preserved: from Equation (5),

$$N_R = \frac{2u\epsilon\rho}{\mu} \tag{B-8}$$

where, expressed in units more commonly found in the literature,

$N_R \leqq 2000$ for laminar flow

$\epsilon = 2.54(10)^{-4}$ meters

$\mu = 0.01$ dekapoise

$\rho = (10)^3$ kilograms/(meter)^3, and

$u = 1.53$ meters/sec.

The ordinary viscosity, μ, and fluid density, ρ, are assumed to be that of a typical hydraulic fluid (6). Velocity, u, is determined from the relation

$$Q_S = uw\epsilon \ . \tag{B-9}$$

For the proposed experiment, $N_R = 80$. Q_S, with respect to valve and fluid parameters, is well within the range of laminar flow.

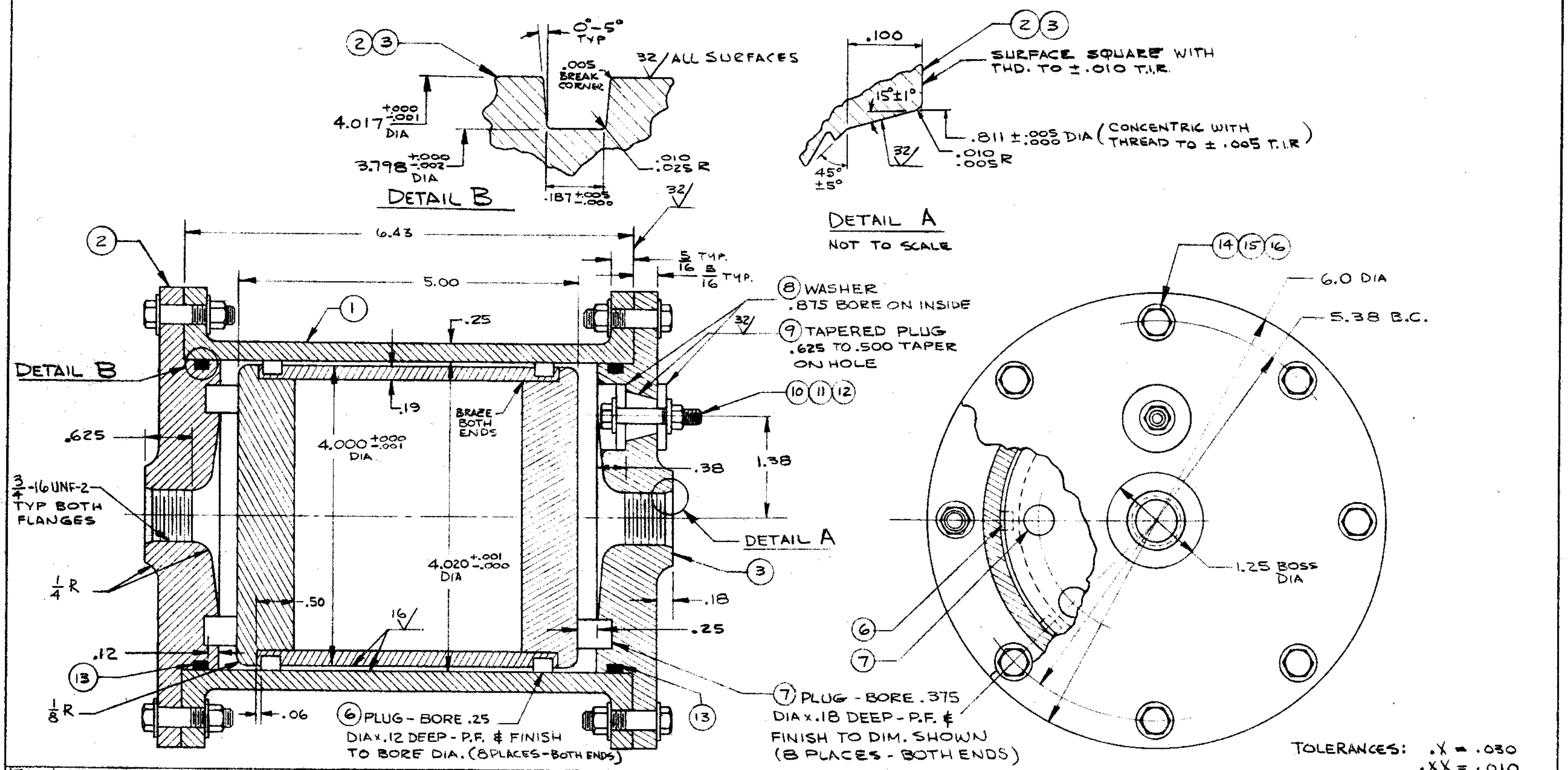
Structural Considerations

The electrodes of the EHV are to be made of aluminum. The metal is lightweight, stiff, and economical and has good conductivity;

because of its weldability and corrosion resistance, 6061-T6 aluminum is to be used. The characteristics of 6061-T6 and the design formulae by which the thickness of each electrode-cylinder may be determined is contained in a structural aluminum handbook compiled and published by the Reynolds Metal Company (1).

An assembly drawing of the proposed EHV is shown in Figure B-3. The thickness of metal components has been calculated with respect to bursting pressure, elongation, etc. The valve should easily withstand a P_{max} of 1000 psi; the variation in ϵ is negligible.

As shown in Figure B-3, the inner electrode is made hollow to reduce its mass; the end plates of both electrodes are thick so as to avoid low-frequency resonances which may interfere with a smooth over-all frequency response; the insulating spacers are to be short for the same reason. Phenolic is chosen for the spacers because of its good stiffness, dielectric strength, absence of brittleness, and availability in rod form.



TOLERANCES: .X = .030
.XX = .010
.XXX = .005

ITEM NO.	NAME	NO. REQ.	STOCK SIZE & MATL	ITEM NO.	NAME	NO. REQ.	STOCK SIZE & MATL
1	BODY	1	6.0 DIA x 6.0 - 6061-T6 AL.	11	NUT	2	#10-32 BRASS
2	FLANGE	1	6.0 DIA x 1.0 - 6061-T6 AL.	12	WASHER	2	#10 BRASS
3	FLANGE	1	6.0 DIA x 1.0 - 6061-T6 AL.	13	O-RING	2	PARKER # 2-240
4	SLEEVE	1	4.0 DIA x 4.5 - 6061-T6 AL.	14	BOLT	16	1/4-28 x 1.0 STL
5	PLATE	2	4.0 DIA x .75 - 6061-T6 AL.	15	NUT	16	1/4-28 STL
6	PLUG	16	.25 DIA x .25 PHENOLIC	16	WASHER	32	1/4 STL
7	PLUG	16	.38 DIA x .50 PHENOLIC				
8	WASHER	2	.87 DIA x .125 FIBER				
9	TAPER PLUG	1	.625 DIA x .50 TEFLON				
10	BOLT	1	10-32 x 1.25 BRASS				

ELECTROHYDRAULIC VALVE
SCALE: NONE
DRAWN BY W.R. SHRADER
8 JANUARY 1965

Figure B-3. Assembly Drawing of Proposed EHV.