


AN ABSTRACT OF THE THESIS OF

STUART LESLIE CATO for the M. S. in Structural Engineering
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Title WEB BUCKLING FAILURE OF BUILT-UP GIRDERS WITH
RECTANGULAR HOLES

Abstract approved 
(Major professor)

This thesis investigated the methods of failure in the webs of built up girders. The beams had rectangular holes five and one-half inches wide and seven and one-half inches high with rounded corners in a web fourteen inches deep between flanges. The longitudinal position of the hole edge varied from four inches to sixteen inches from the support.

The results of the test were compared to analytical analysis for failure by column action, plate buckling and shear in the web. Failure occurred by local crippling in the web in the test beams with holes four and eight inches from the support. The test beams with holes 12 inches and 16 inches from the support failed by yielding of the web under the applied load.

The test results indicated that further study is needed of local stress effects at the edge of the holes.

WEB BUCKLING FAILURE OF BUILT-UP
GIRDERS WITH RECTANGULAR HOLES

by

STUART LESLIE CATO

A THESIS

submitted to

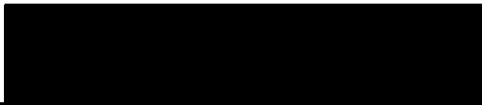
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Professor of Civil Engineering

In Charge of Major



Head of Department of Civil Engineering



Dean of Graduate School

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Nomenclature

- a Same as h .
- b Length of loaded edge of plate in inches.
- h Clear distance between flanges in inches.
- k A constant which depends on the manner in which the edges of a plate are supported, the ratio of plate length to plate width and upon the nature of loading.
- s_{cr} Critical stress for plate buckling in pounds per square inch.
- t Thickness of plate in inches.
- u Poisson's ratio dimensionless.
- v Shear stress in pounds per square inch.
- w Thickness of web in inches.
- A Gross area in square inches.
- Aw Area of web in square inches.
- E Modulus of elasticity of steel (29,000,000 pounds per square inch).
- Fy Specified minimum yield point of type of steel being used in pounds per square inch.
- I Moment of inertia in inches to the fourth power (in^4).
- K Effective length factor for Eulers equation dimensionless.

WEB BUCKLING FAILURE OF BUILT-UP GIRDERS WITH RECTANGULAR HOLES

INTRODUCTION

The investigation of I beam web buckling has been very limited. To the authors knowledge, no investigations have been done on buckling of beam webs with holes where vertical shear is the primary concern. Some research has been done by the Texas Engineering Experiment Station, Texas A and M College, on beams with holes. This research was primarily concerned with stress concentration around the holes and the requirement for reinforcement around the holes.

Some studies have been made on strength of I beams. Early tests (7) were concerned with strength of I beams in flexure. These tests led to further tests of web strength of I beams and girders (6) and web buckling in steel beams (4). The beams tested for web strength all failed by shear rather than buckling, except for some failure over bearing blocks. A further result of the web buckling tests was that an I beam with an h/t of less than 70 will tend to fail by shear rather than web instability. Buckling of the beam web can be expected at an h/t greater than 80.

OBJECT OF TEST

The object of this test is to determine the effect of holes cut in I beam webs on the buckling strength of the beam web. It was anticipated that the beam would fail by buckling of the web section between the hole and the support as shown in Figure 1.

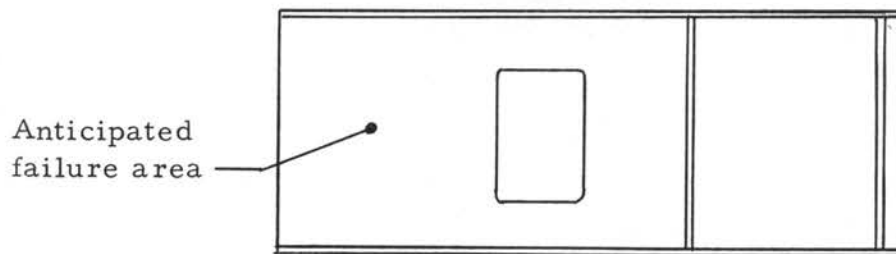


Figure 1. Typical beam.

Present practice is to limit the location of the hole to no closer than three feet to the support. This test will investigate specimens with the edge of the holes four inches, eight inches, twelve inches and sixteen inches from the support.

TEST SPECIMENS

Description of Specimen

The beams chosen were fabricated by machine welding of 1/2" x 4" flange plates to 1/4" x 14" web plates as shown in Figure 2. The specimens were chosen to coincide in size and shape with existing test beams being used to investigate the effect of holes on long specimens which are subject primarily to flexural stresses. These tests are presently being conducted by W. Landers of the Oregon State University Engineering Experiment Station. The over-all specimen is fifteen inches high and three feet three inches long as shown in Figure 2. The holes are seven inches high and five and one-half inches wide with the corners rounded to a one inch radius. Beam 1 has the center of the hole six and three-quarters inches from the support resulting in four inches of material between the edge of the hole and the support. Beam 2 has 8 inches of material, beam 3 has 12 inches of material, and beam 4 has 16 inches of material between the hole and support. In order to clear the hole when loading beam 4 the loading was applied at the third point. Stiffeners were added under the load and at the nearer support to prevent the larger shear in the left (Figure 2) part of the beam from failing the web in that area.

Properties of Specimen

All beam material is ASTM A-36 steel.

Modulus of elasticity of steel (assumed)	$E = 29 \times 10^6$ psi
Modulus of elasticity of specimen (by test)	$E = 26.86 \times 10^6$ psi
Moment of inertia of section	$I = 273.91$ in ⁴
Area of section	$A = 7.5$ in ²
Area of web	$A_w = 3.5$ in ²
Web thickness	$w = 1/4$ in.
Clear distance between flanges	$h = 14$ in.

Instrumentation of Specimen

As stated previously, this investigation is for web failure by buckling. The location and number of gages was picked assuming the failure would occur by direct compression and buckling of the portion of the beam between the left edge of the holes and left support as shown in Figure 2. SR-4 strain gages were used for instrumentation as shown in Figure 3. The gages were placed vertically in a line over the support and adjacent to the hole. Two gages were placed at the center line of the web and numbered 2 and 6. Two each were centered on a line running horizontal and coinciding with the top and bottom of the holes and numbered 1, 3, 5, and 7. A single gage, numbered 4, was placed horizontally and centered on a line coinciding with the

bottom of the hole. After preliminary tests were run it was found that additional instrumentation was needed. Gages were added to beams 1 and 2 on the opposite side of the web from existing gages 5 and 7 as shown in Figures 3(b) and 3(d). The gages used for beams 1 and 2 were SR-4 type A5-S6 with a gage length of one and one-half inches. Beams 3 and 4 used SR-4, type A1-S6 gages with a gage length of two inches. As the primary concern was web buckling, no strain gages were put on the flanges and no measurements of deflection were taken for the span to compute bending stress. When bending stresses are below 20 percent of the critical value for pure bending they lower the shearing stress that may induce buckling by approximately four percent of the critical value for pure shear (3, p. 194, 196). Because of the very small effect of bending stresses, they were neglected for these tests.

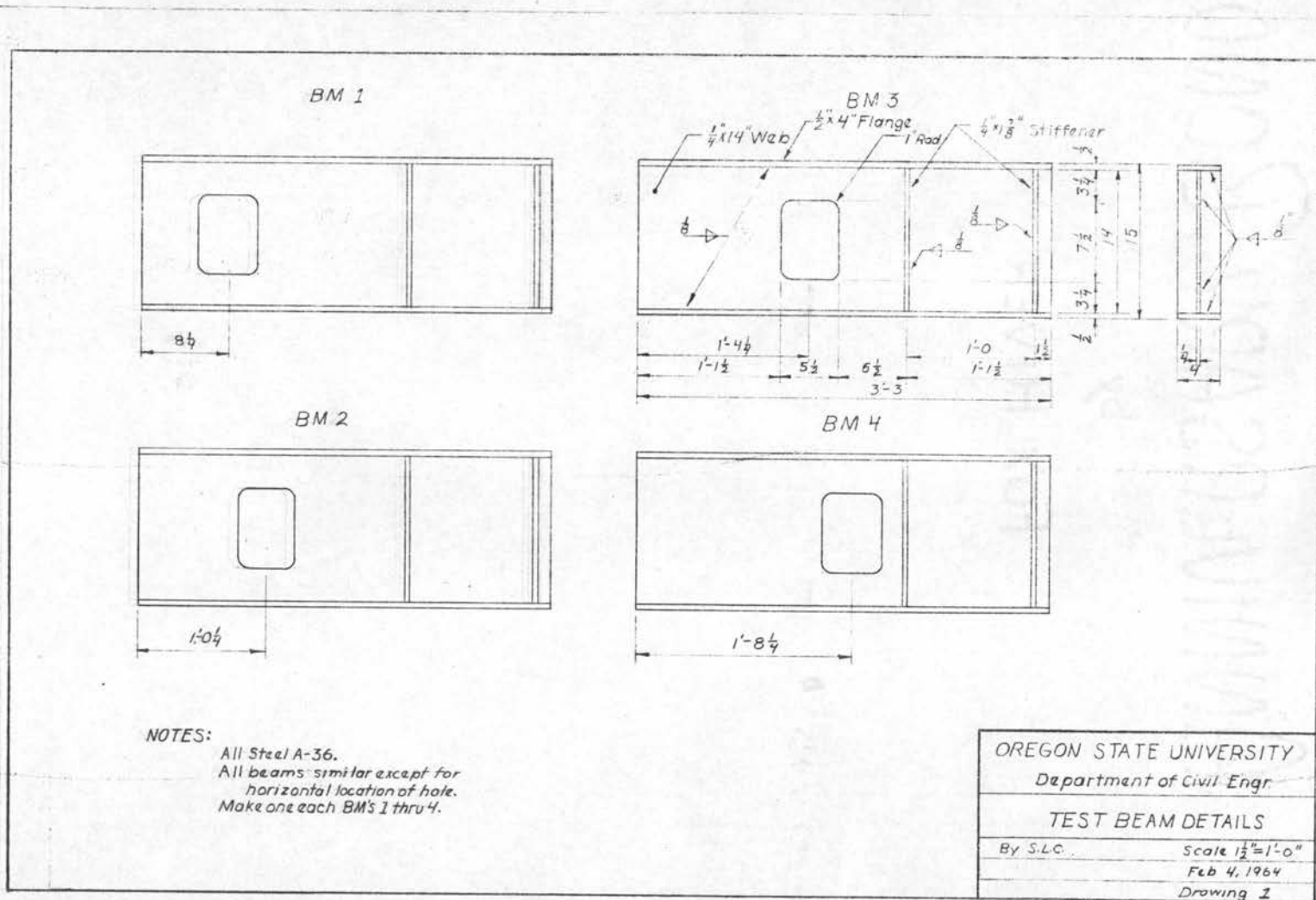
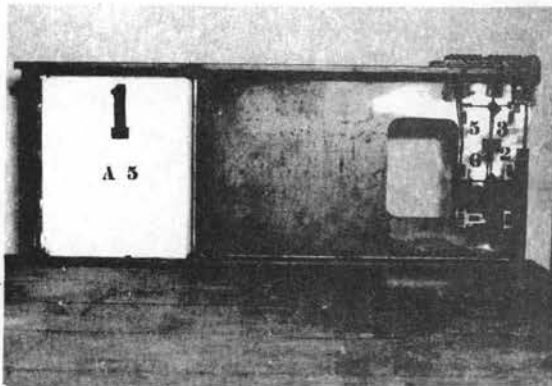
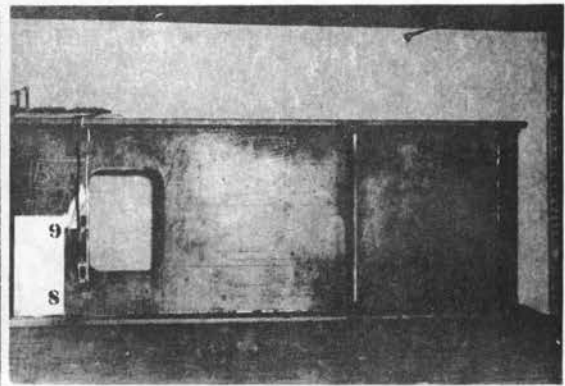


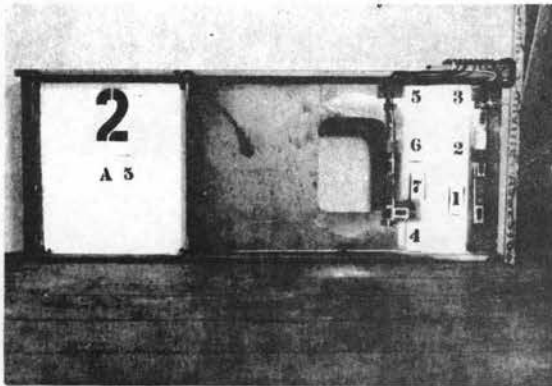
Figure 2. Test beam shop drawing.



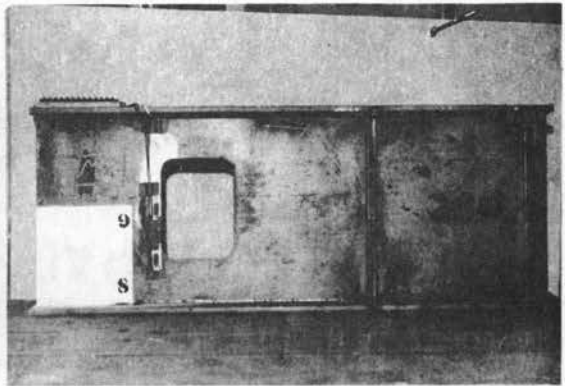
(a) Beam 1



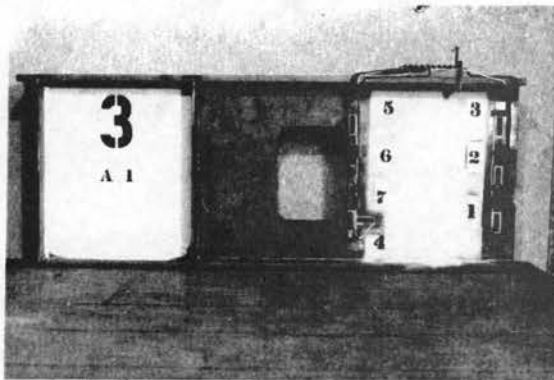
(b) Beam 1



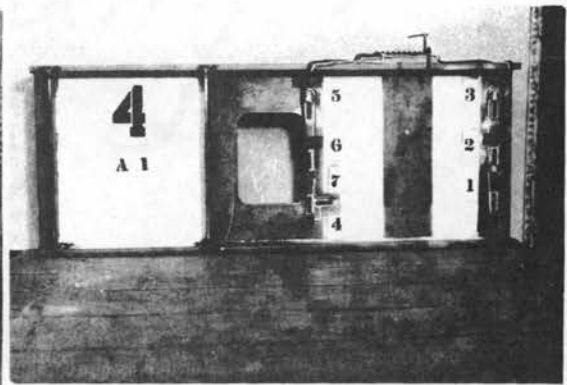
(c) Beam 2



(d) Beam 2



(e) Beam 3



(f) Beam 4

Figure 3. Instrumentation of test beams showing location of strain gages.

APPARATUS

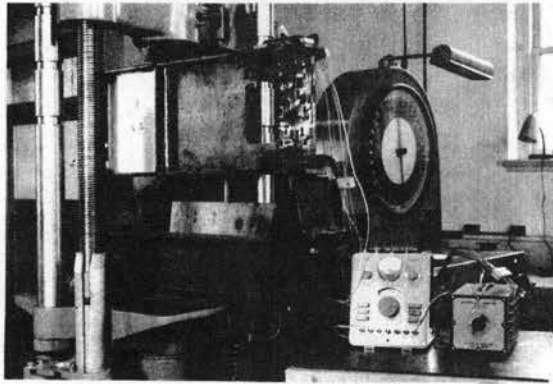
Preliminary Test

The beams were supported on either end by fixed bearing blocks with rounded heads. The beams were too long to fit the plate of the hydraulic testing machine so a section of eight inch bearing pile (8x8 BP 36) was cut to act as a support beam for the bearing blocks. The support beam had to cantilever over the bearing plate of the testing machine as shown in Figure 4(a) in order to position the test specimen so it could be loaded at the third point. The testing machine used was a 60 Kip Baldwin-Southwark hydraulic machine as shown in Figure 4(b). The loading rate used was variable between periods of sustained loadings when strain gage readings were taken. The load was applied through a bearing block with a one inch diameter curve surface. No lateral support was provided as it was not deemed necessary.

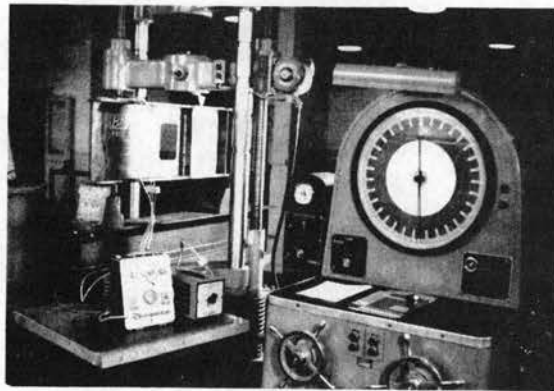
Final Test

The second test was necessary to fail the beams. A Rhiele 150 Kip beam testing machine was used as shown in Figure 4(c). The test specimen was again supported on fixed bearing blocks with rounded tops. The load was applied through a bearing block with

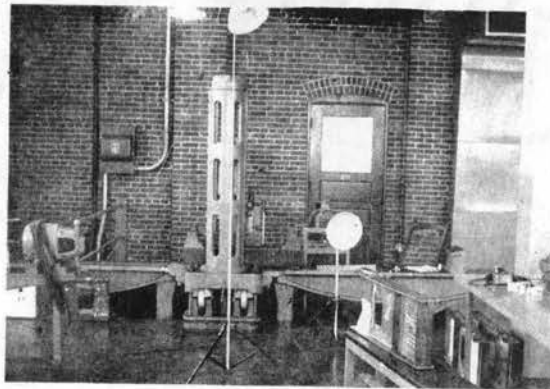
rounded edges with a one-quarter inch plate, four inches wide, between the block and the flange of the beam for beams 2, 3 and 4. A three-quarter inch plate was used for beam 1. The plate was added to distribute the load application over the top flange.



(a)



(b)



(c)

Figure 4. Testing apparatus.

TEST PROCEDURE

Trial Test

A trial run was taken on beam 4 to establish desirable increments of loading. The beam was loaded in 1,000 pound increments up to 10,000 pounds then loaded in 2,000 pound increments up to 30,000 pounds. The results of the test showed that larger increments in this range of loading were desirable and 3,000 pound increments were chosen.

Preliminary Test

The specimens were placed in the testing machine and zero readings were taken for the strain gages. The loading was applied in increments of 3,000 pounds. The loads were held constant between increments and strain gage readings were taken. The specimen was also observed for flaking of mill scale, bowing of the web and translation of the top flange over the support. The maximum accurate loading was 54,000 pounds which gave a total shear of 18 Kips in the web section. The specimen was then unloaded and zero load readings were taken for the strain gages to determine if the cross section had returned to its original state.

Final Test

The final test was run similar to the preliminary test. The load increments were 15,000 pounds which resulted in increasing increments in total shear by three Kips in the portion of beam under investigation. The loading was again held between increased increments to observe the strain gage readings, flaking of mill scale, bowing of the web, and translation of the top flange over the support. Zero readings of the strain gages were not taken for the specimens.

RESULTS

Preliminary Tests

The results of the strains are shown in Figure 7 through 10. None of the beams failed during the preliminary tests. Beam webs that were bowed under loading returned to original shape upon unloading. The area of the top flange directly under the loading head showed some slight crushing on all specimens but the top flange itself did not deform inelastically on the preliminary tests. The beam webs were initially straight upon loading. The top flange of all beams had a slight curvature toward the web as shown in Figure 5.

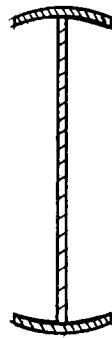


Figure 5. Exaggerated curvature of beam flange.

Beam 1. At a load of 39 Kips the web had a bow from the vertical of three sixty-fourths inch at mid-height of the web. This bow of the web was maintained to a load of 54 Kips where it increased to one-sixteenth inch. At a load of 42 Kips there was slight flaking of mill scale between the web stiffeners. All gages show a linear strain

in Figure 7 up to a load of 24 Kips. At this load gages 4 and 5 show a slight inelastic action of the web. The area of the web at gages 7 and 8 show tensions of the same magnitude on both sides of the web and therefore would indicate no bowing of the web in this area. Gages 6 and 9 located at the center line of the web are of opposite sign and indicate that bowing is present at the center line. No rational explanation of the tension effect at the bottom of the hole is available. The effect of stress concentration may be the cause of the tension. Additional tests with more instrumentation would be required to study this condition of tension at the location of gages 7 and 8. Gage 7 showed erratic behavior up to a load of 24 Kips then indicated a linear strain distribution. Gage 8 had linear strain to 30 Kips then changed slope and remained linear. Both gages 7 and 8 gave linear strain readings in the 0 to 50 Kip range in the second tests. This would indicate that some residual strains were present around the hole in the first tests and that they were overcome above a load of 30 Kips. No failure of the beam was indicated in the test. All of the gages returned fairly close to their original readings upon unloading (Table 1) except gages 7 and 8.

Beam 2. The gages show evidence of bowing from zero to six Kips loading (See Figure 8). Then the compressive loading overcame the tension in the web caused by bowing over the support. This

was possibly an effect of the transverse flange curvature and the beam settling firmly onto the support. The strains were fairly linear throughout the 0 to 48 Kip loading range. Above 48 Kips slight inelastic action is shown in gages 1, 4, 6 and 7. The same tension effect at gage 7 is present as in beam 1. Gage 9, on the opposite side of the web from gage 6, did not operate properly so there is no indication of bowing of the web by comparing gages. The web started to show visual evidence of bowing at a load of 50 Kips. The bow was one thirty-second of an inch at 42 Kips and remained there until a load of 56 Kips was reached when it had increased to three sixty-fourths of an inch. The strain gages returned exceptionally well upon unloading as shown in Table 1.

Beam 3. The beam strains were linear in all gages through the entire loading range except for slight inelastic action of gage 7 at loads exceeding 48 Kips. The strains in Figure 9 show tension over the support to a load of six Kips. This effect might be caused by the curvature of the flanges, mentioned previously, causing tension until the beam settles on the support or it may be an effect of local stress concentrations.

Beam 4. The strains were linear to a load of 36 Kips as shown in Figure 10. Above a load of 36 Kips the material shows some sign of yielding adjacent to the lower portion of the hole as shown by strains of gages 4, 5 and 7. The beam had mill scale

cracking between stiffeners and over the hole at a load of 56 Kips as shown in Figure 6. The first yield occurs at a load of 37 Kips in the web.

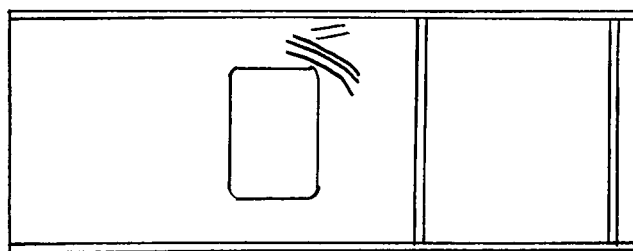
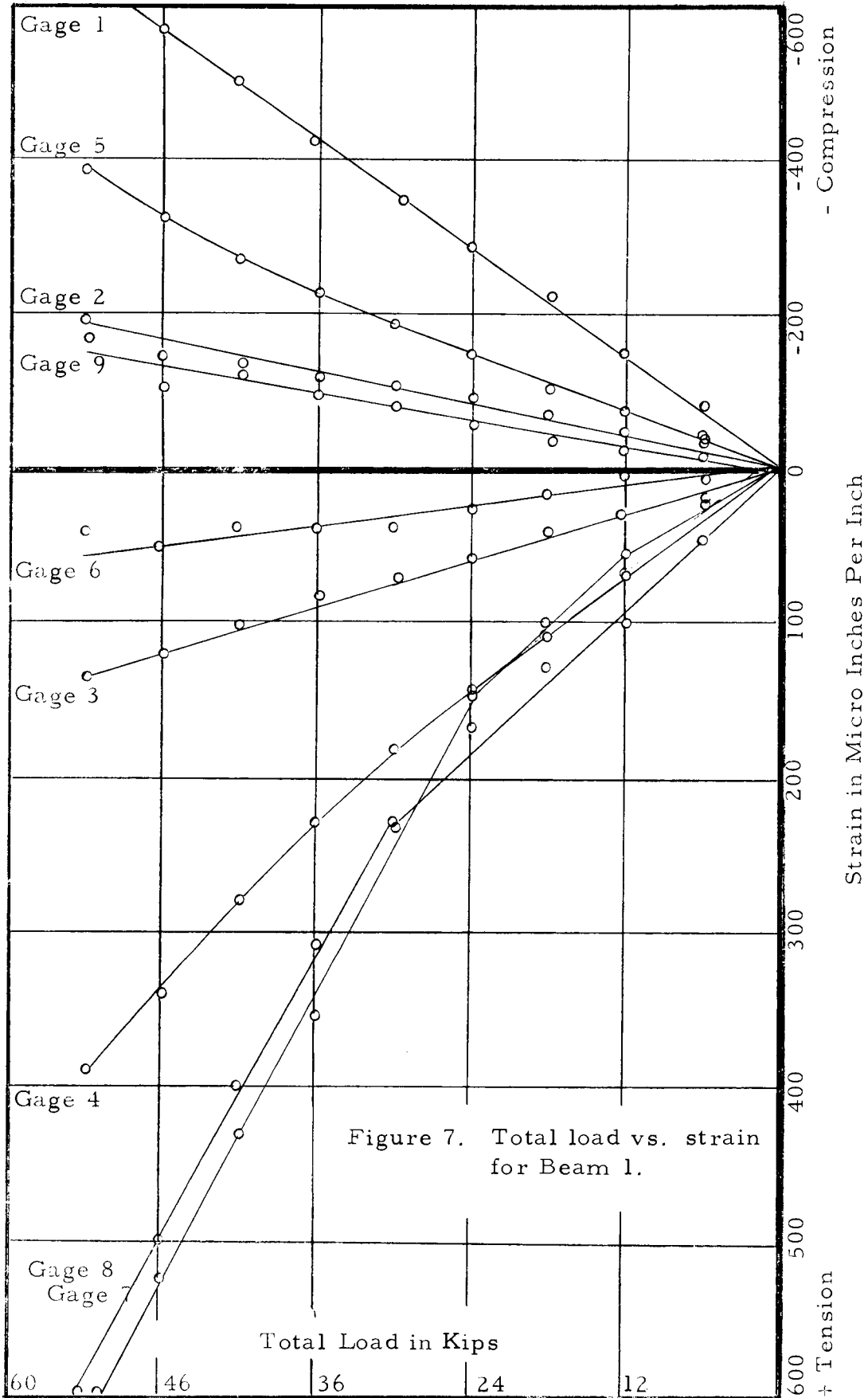


Figure 6. Mill scale cracking on beam 4.

Table 1. Residual strain upon unloading in micro inches per inch.

Gage Number	1	2	3	4	5	6	7	8	9
Beam 1	0	+10	+20	+90	-50	-20	+305	+270	-15
Beam 2	-10	+330	-10	+15	0	0	+80	+150	+10
Beam 3	-15	-15	0	+30	+130	+90	+45		
Beam 4*	-5	-5	-10	-5	-10	+5			

* Note readings for beam 4 were taken when indicator had been re-connected and are only indicative of differences between gages.



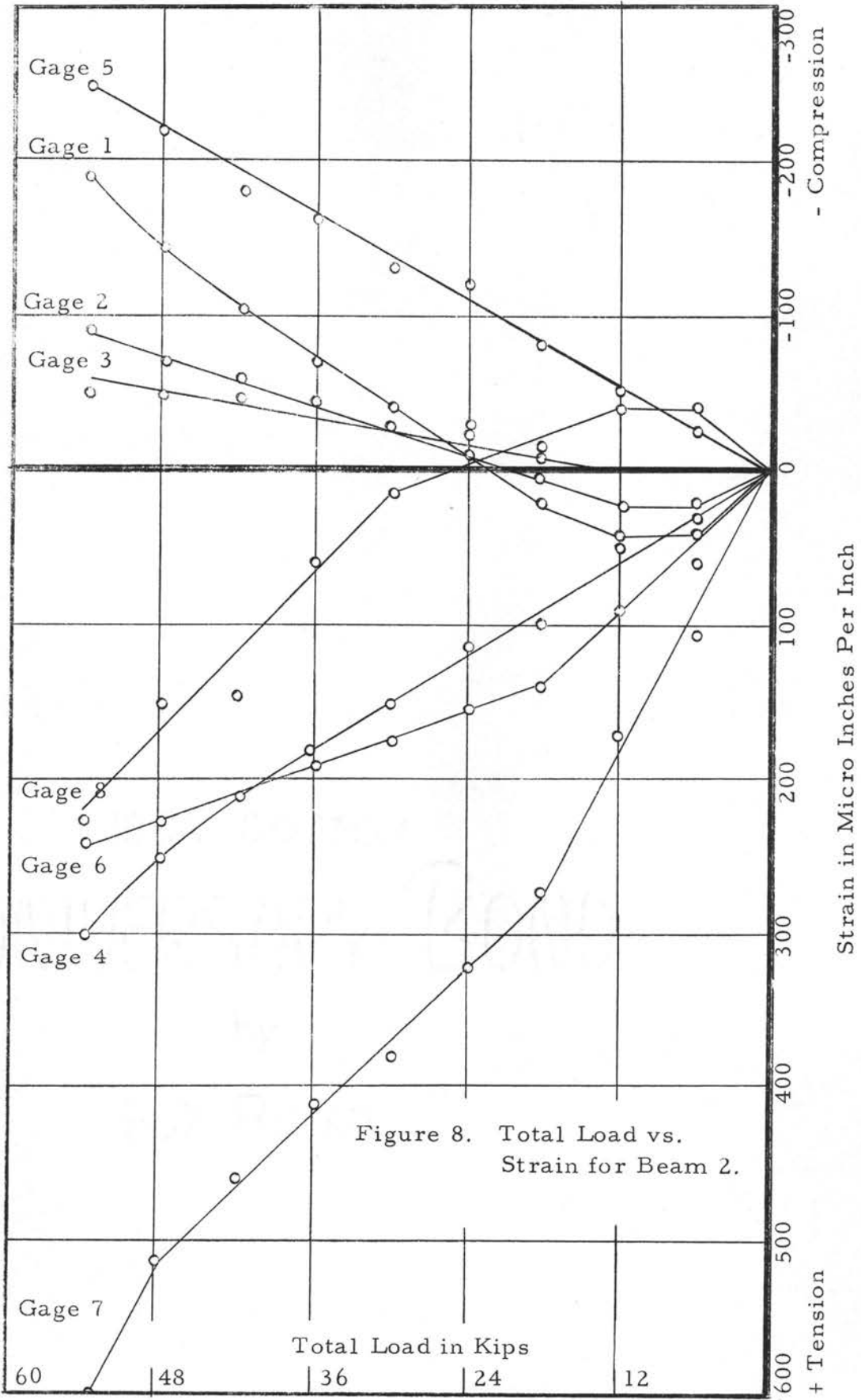
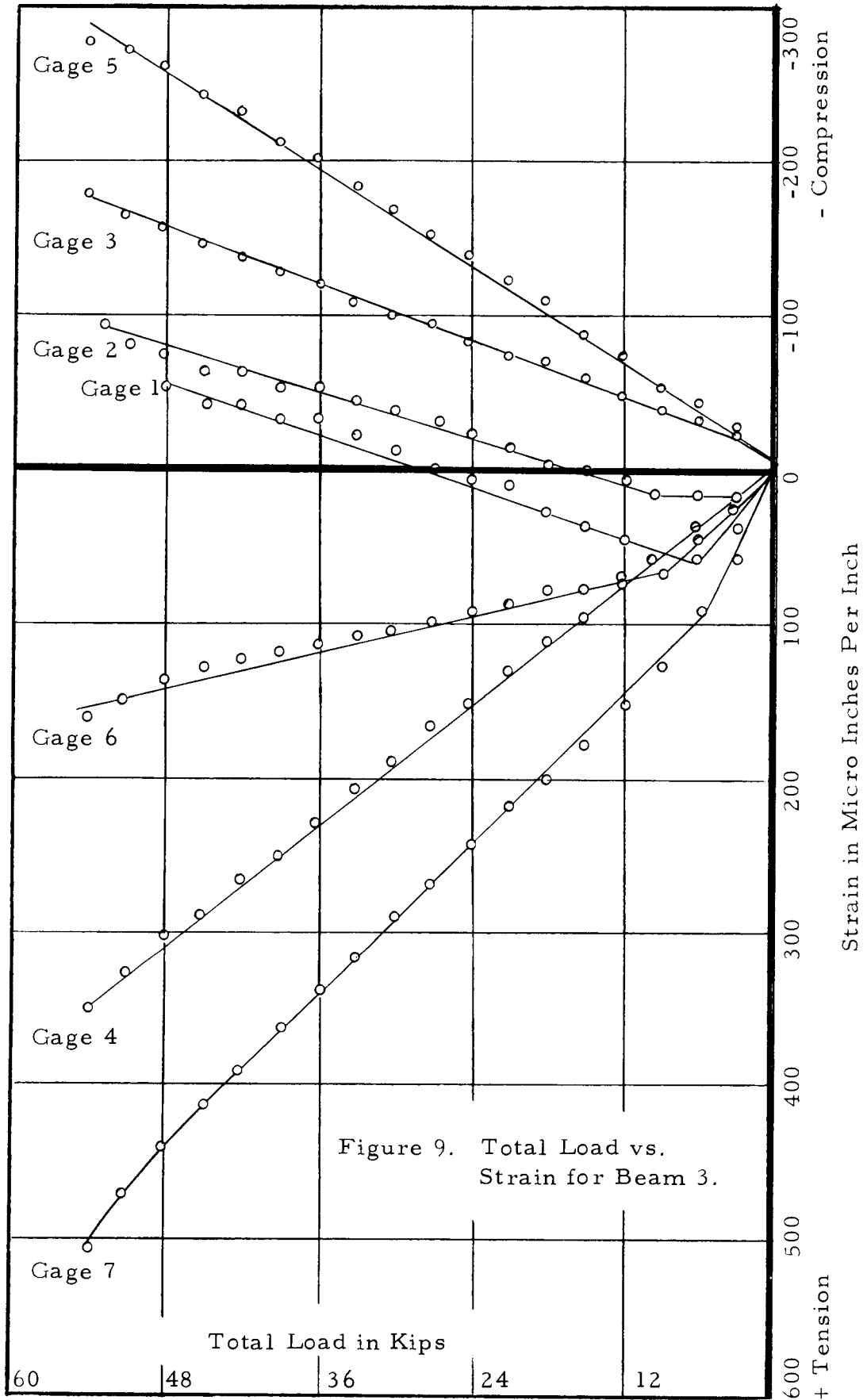
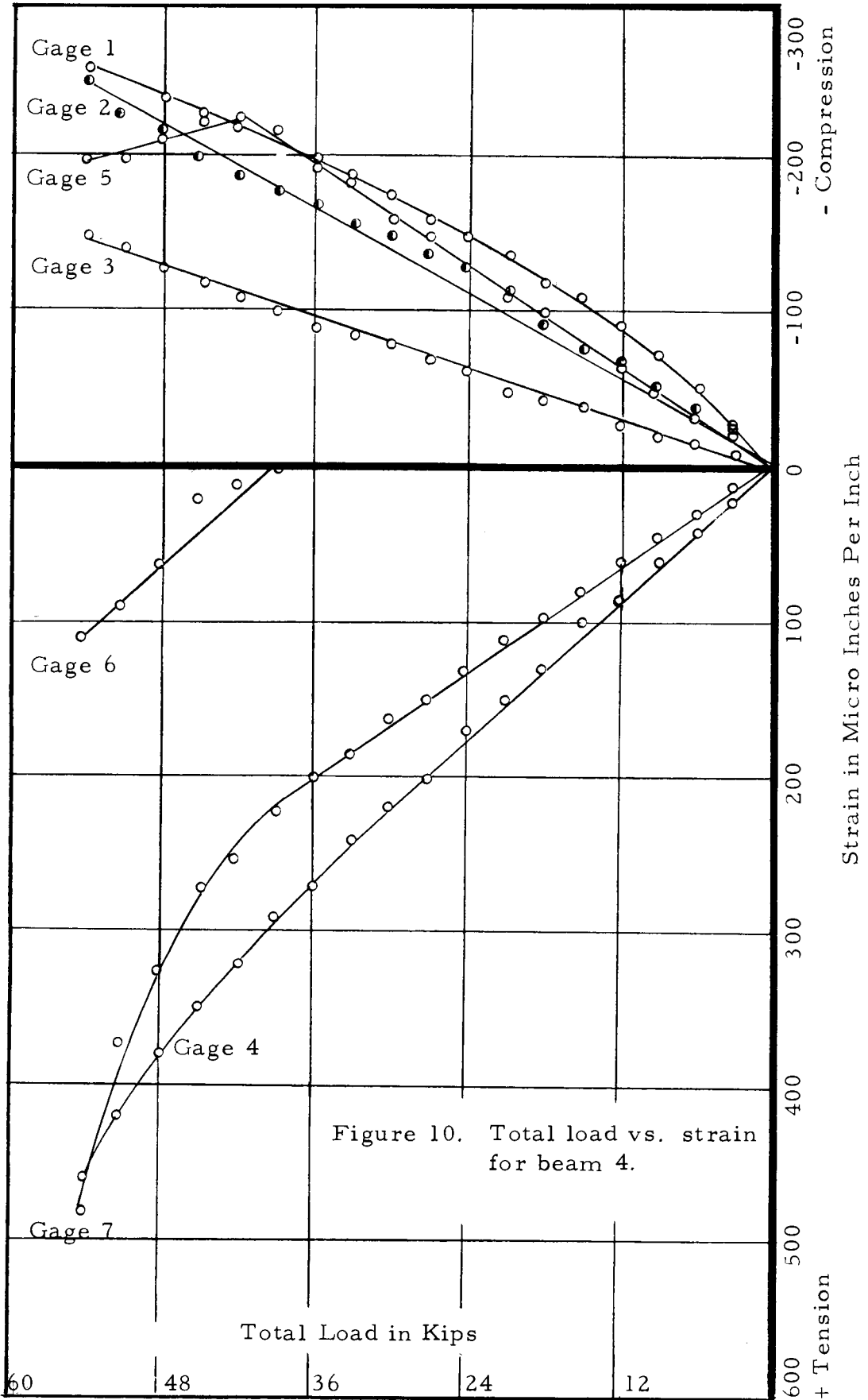


Figure 8. Total Load vs. Strain for Beam 2.

- Compression

+ Tension





Final Results

The final tests resulted in local crippling of the beam webs over the support at an applied loading of 90 Kips and 120 Kips for beams 1 and 2 respectively. Beams 1 and 2 also had inelastic yielding of the web at the stiffeners. Beams 3 and 4 failed by inelastic yielding at an applied load of approximately 115 Kips. The results of the strain readings are shown in Figures 15 through 18. All beams showed first apparent signs of local inelastic yielding at a load of 60 Kips in Figures 15 through 18. The average web stress at this load was 5.7 Ksi.

Beam 1. Beam 1 maintained linear strains up to a load of 60 Kips for all gages but number 5 which had signs of inelastic yielding at a load of 30 Kips. A visible bowing of the web of one thirty-second inch was noticeable at a load of 15 Kips and remained until a load of 75 Kips when it increased to three sixty-fourths inch. Flaking of mill scale was first observed between the stiffeners at a loading of 60 Kips. The flaking occurred until failure. The beam failed by local crippling over the support as shown in Figure 13(c), and by yielding of the web at the stiffener as shown in Figure 13(b). The failure by crippling was sudden and not preceded by any visible signs of scaling or excessive bowing. Local yielding had caused the loading rate to drop off at around 75 Kips as is indicated by the load

strain curve of Figure 15. The loading had been stopped at 97 Kips to observe if any flaking of mill scale was apparent. While under this constant load the web failed. The failure occurred at 9.25 Ksi of average shear stress. Inelastic yielding was first noticeable when the average shear stress was 5.7 Ksi.

Beam 2. Beam 2 maintained linear strains (Figure 16) to a loading of approximately 48 Kips. The gages that in the first series of tests showed abrupt change in slope (Figure 8) of load strain curves at a load of six Kips, did not have abrupt changes at that load during the test. Gage 5 had an abrupt change in linear strain at a load of 15 Kips. At a load of 30 Kips the web had a slight visible bow to the left of vertical of one thirty-second of an inch. This bow in the web remained up to a load of 60 Kips. At a load of 51 Kips, the top flange over the support moved left approximately one-tenth of an inch. The bow was three sixty-fourths of an inch at 60 Kips and remained there until failure. Flaking of the mill scale started to appear between the stiffeners at a load of 66 Kips. At a load of 90 Kips flaking of scale appeared at the corner of the hole as shown in Figure 11(a). Flaking of mill scale appeared above and below the hole at a load of 105 Kips as shown in Figure 11(b). At 75 Kips a slight depression was noted under the loading block on the top flange. Yielding was noticeable when increasing from a load of 90 Kips to a load of 105 Kips. As shown in the load strain curve in

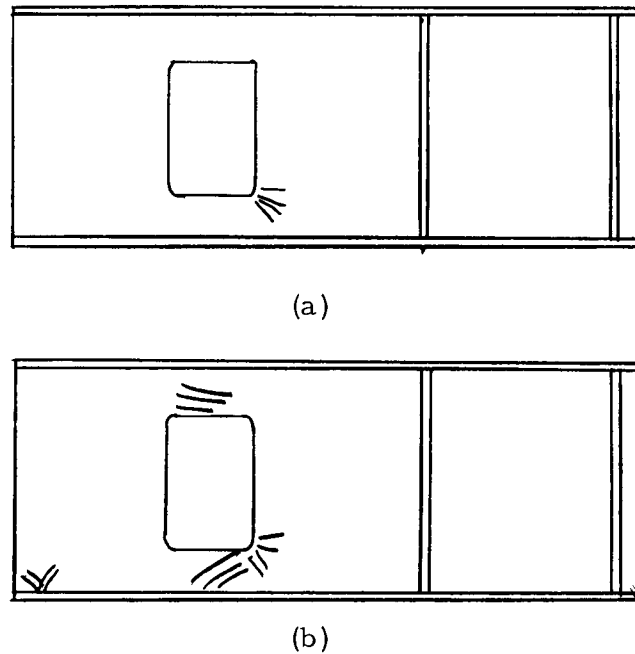


Figure 11. Mill scale lines at (a) 90 K (b) 105 K load

Figure 16, inelastic strains are very obvious above a 60 Kip load in gages 1, 7 and 8. Yielding was quite noticeable when the load was increased from 105 Kips to 120 Kips. The beam failed as shown in Figure 13(d) by local crippling of the web over the support at a sustained loading of 120 Kips, and yielding of the web at the stiffener as shown in Figure 13(e). The failure was sudden. The average web shear stress was 11.4 Ksi at failure and 5.7 Ksi when inelastic yielding was first noticeable.

Beam 3. Beam 3 had linear strain distribution up to a load of 60 Kips as shown in Figure 17. There was no curvature in the beam web until a load of 45 Kips. The top flange moved transversely one-eighth of an inch to the right at this load. The top moved right

another one-eighth of an inch at a load of 51 Kips and remained to 66 Kips. At a load of 66 Kips the top flange moved to the left. This reversal in the direction of movement of the top flange probably accounts for the change in slope of the strain plots of Figure 17 above a load of 60 Kips. At a load of 75 Kips the top flange had returned to its original unloaded position. The web had a bow of one-sixteenth inch at a load of 75 Kips. At a load of 90 Kips flaking of mill scale appeared as shown in Figure 12(a). At a load of 105 Kips scaling increased as shown in Figure 12(b) and there was a slight crushing under the applied load resulting in the load falling off slightly. General yielding was apparent at a load of 111 Kips and 115 Kips and was accompanied by heavy flaking of mill scale as shown in Figure 12(c). The beam failed by crushing under the point of application of the load as shown in Figure 14(a) and 14(b). Due to the reversal of movement of the beam flange the plot of the load vs. strain in Figure 17 does not clearly indicate at what load inelastic yielding first took place. The apparent first yielding at 105 Kips took place at an average shear of 10 Ksi.

Beam 4. Beam 4 maintained linear strains up to a load of 45 Kips as shown in Figure 18. The strain at gage seven showed inelastic yielding was excessive. The web did not bow during the test. At a load of 105 Kips there was extreme flaking of mill scale (Figure 14(d)) over the hole and yielding of the flange under the load.

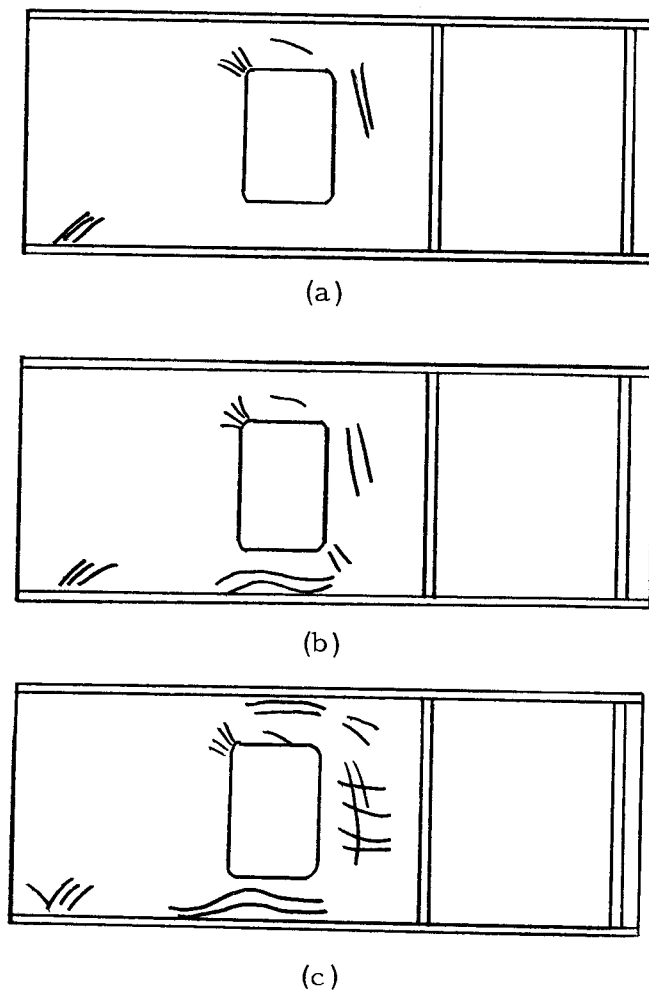
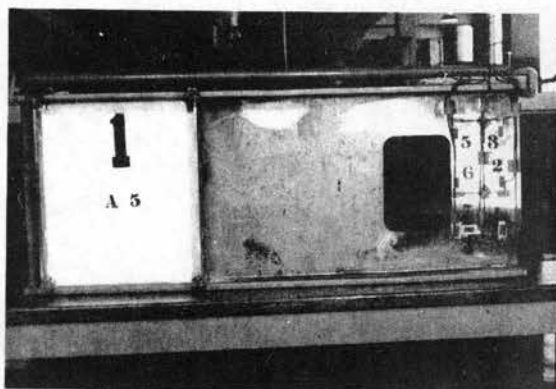


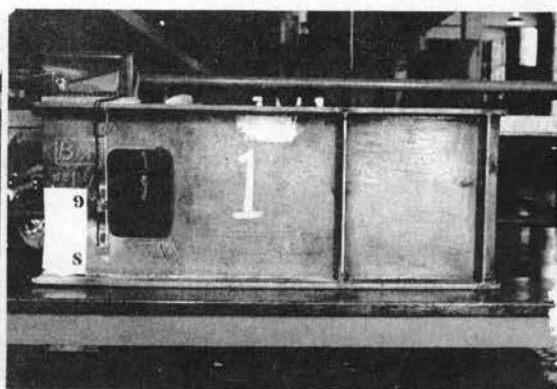
Figure 12. Mill scale lines for Bm-3 (a) at 90 K (b) at 105 K load (c) at 115 K load.

The beam could not be loaded above this range. The beam failed by yielding in the top flange under the load as shown in Figure 14(c).

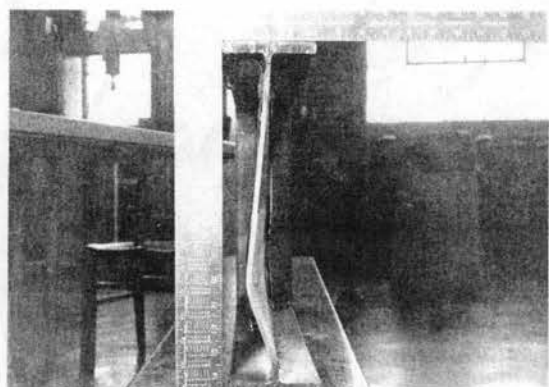
The average shear in the web was 10 Ksi at maximum loading. The average shear at first sign of inelastic yielding, at a load of 60 Kips, was 5.7 Ksi.



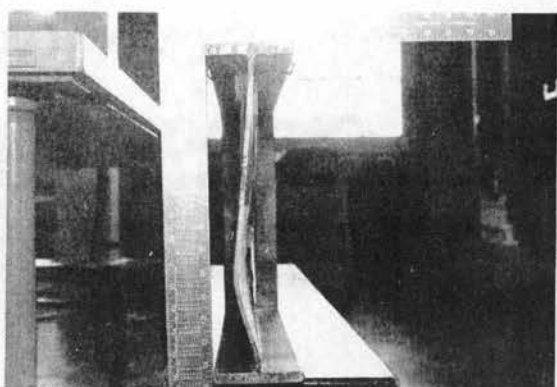
(a) Beam 1



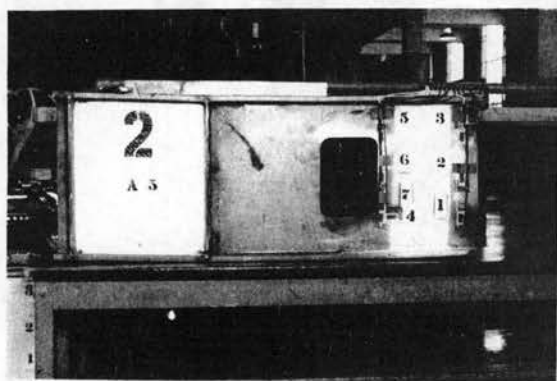
(b) Beam 1



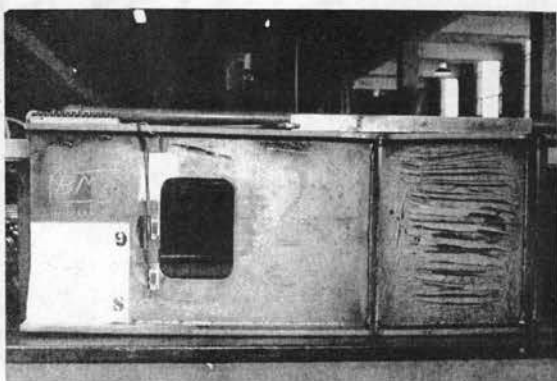
(c) Beam 1



(d) Beam 2

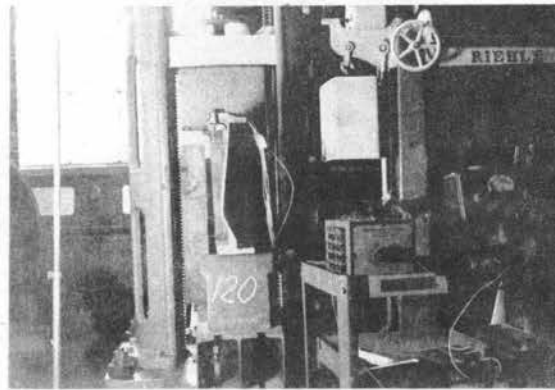


(e) Beam 2

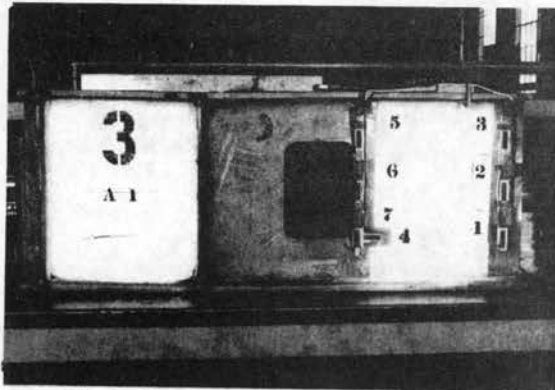


(f) Beam 2

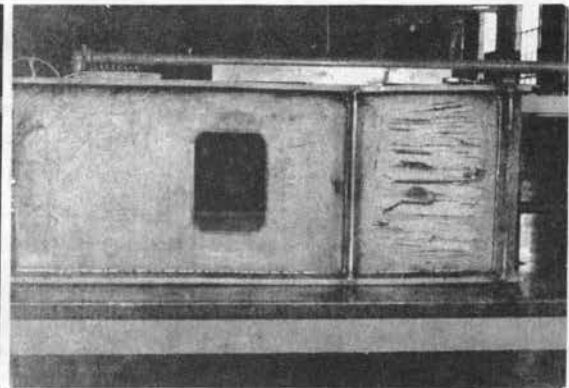
Figure 13. Beams 1 and 2 after failure.



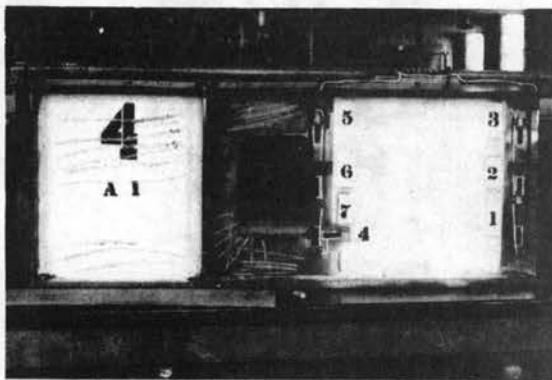
(a) Beam 2



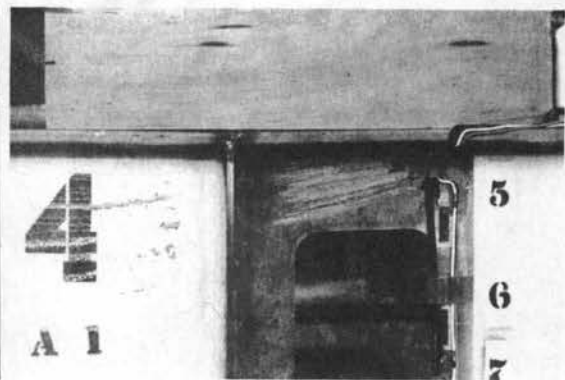
(b) Beam 3



(c) Beam 3

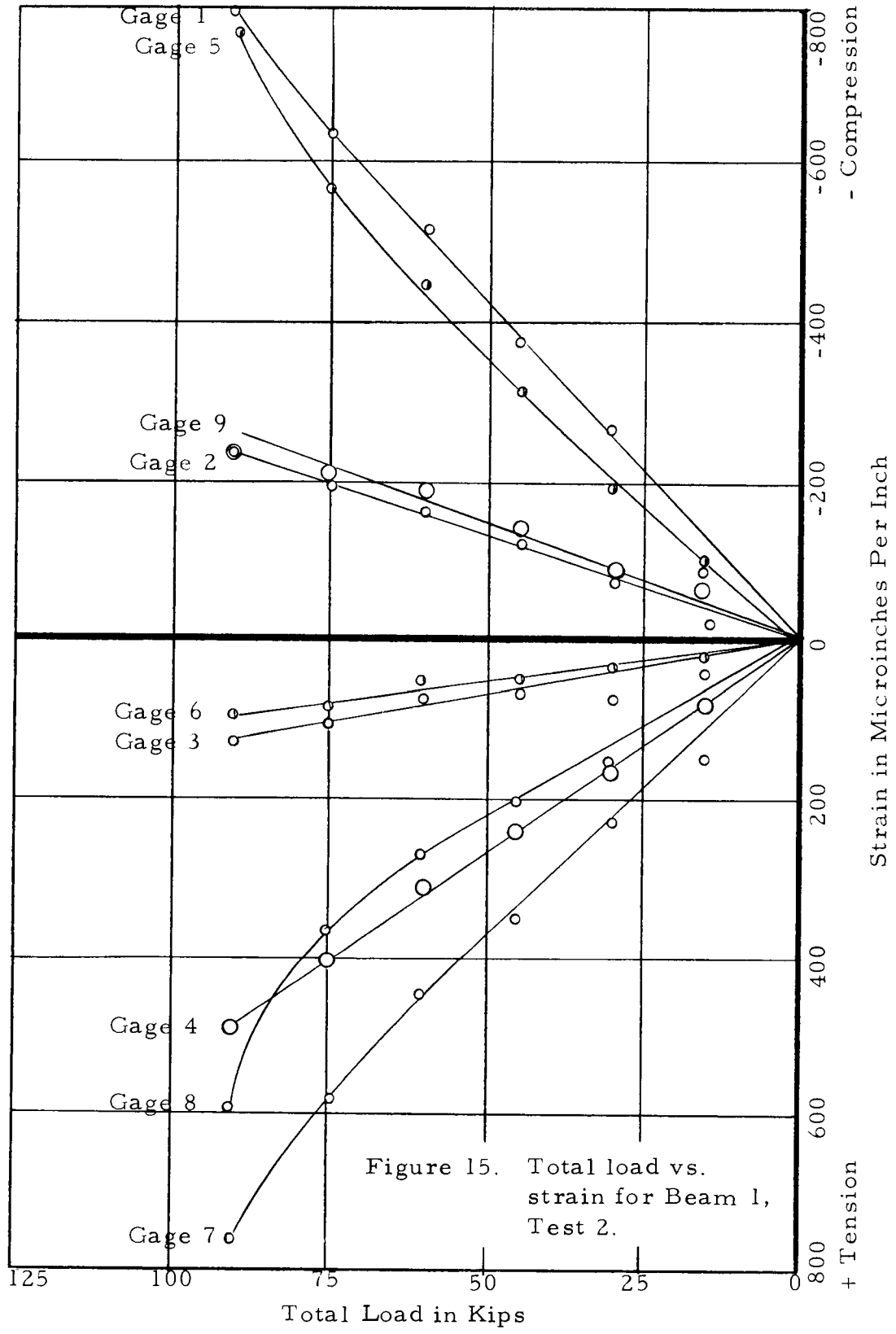


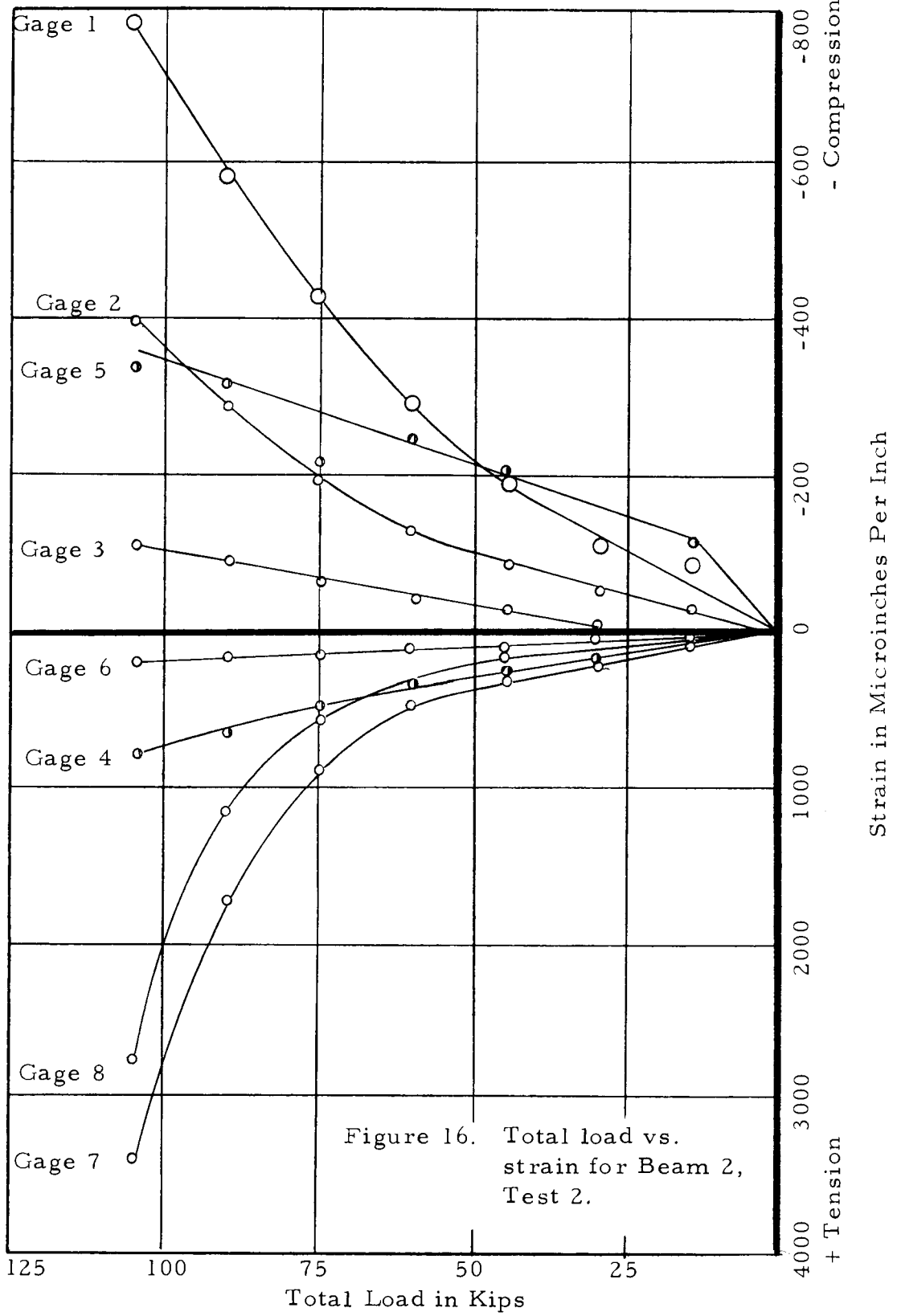
(d) Beam 4



(e) Beam 4

Figure 14. Beams 2, 3 and 4 after failure.





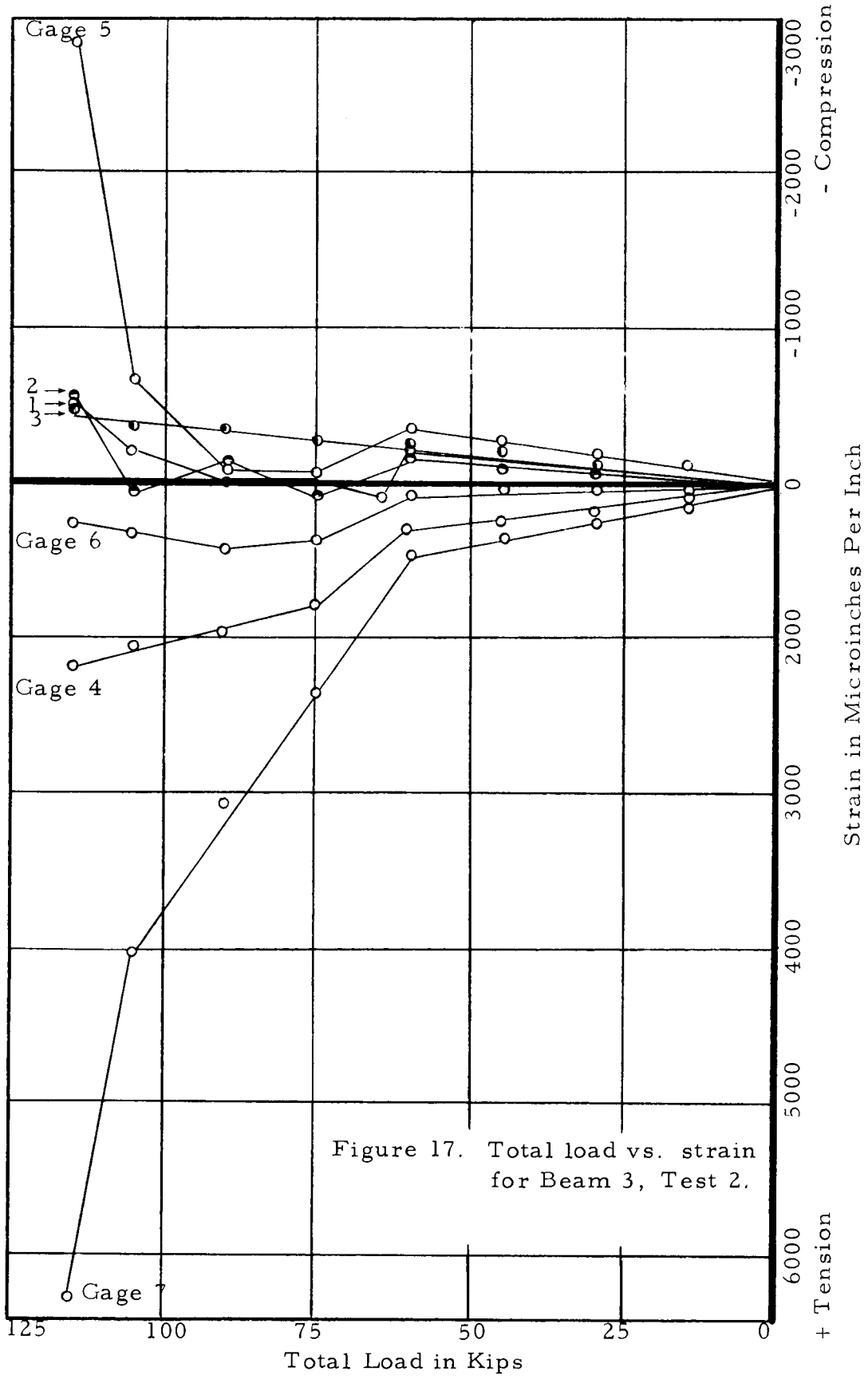


Figure 17. Total load vs. strain for Beam 3, Test 2.

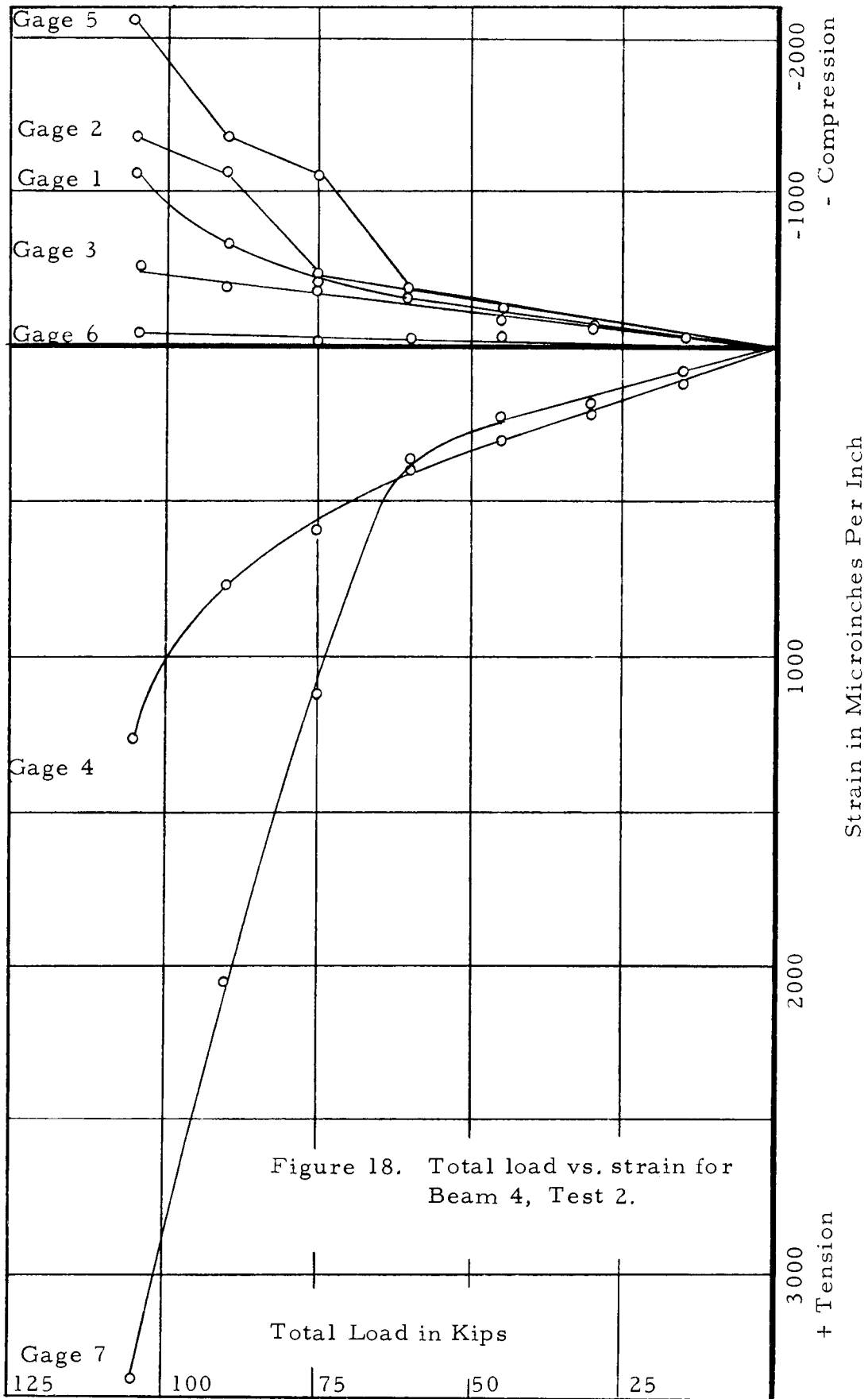


Figure 18. Total load vs. strain for Beam 4, Test 2.

Analytical Results

The resulting failure of the beams were to be compared to theoretical failure by column action of the web, plate buckling of the portion between the hole and support and shear failure.

Column action. Beams 1 and 2 were analyzed for column buckling of the portion of the web between the hole and support. It is probable, by noting the mode of failure (Figure 13) in beams 1 and 2, that the failure was by local crippling and not buckling. Calculations were made using Euler's column equation to determine a ratio of effective column height to the actual height of the web. The calculation (Appendix A) gave an effective length factor (K) of 0.567 and 0.661 for beams 1 and 2.

Plate buckling. Beams 1 and 2 were analyzed for plate buckling (Appendix B) assuming that the portion of web between the hole and support was simply supported on three edges and free on the fourth edge (3, p. 155). Beam 1 had a computed critical shear of 13.75 Ksi by the plate buckling equation and an average computed shear of 23.5 Ksi across the section. Beam 2 had a computed critical shear by the plate buckling equation of 16.2 Ksi. Beam 2 gave comparable results for plate buckling failure. The computed average shearing stress across the section between the hole and support was 16.8 Ksi. Beam 1 failed at a much higher shear than

the computed critical shear for plate buckling. It is interesting to note that at a failure the product of $E\epsilon$ for gage 7 on beams 1 and 2 was approximately 22 Ksi which is slightly higher than the critical shearing stress of steel as computed by Von Mises shear equation.

Shear failure. Beams 3 and 4 failed at a load of approximately 105 Kips. The failure was by yielding of the web at the stiffener and bending of the flanges under the load. The average shearing stress between the stiffeners at this load was 23.8 Ksi which is above the value of critical shear for steel as computed by the Von Mises shear equation, $\rho v_{cr} = Fy/\sqrt{3}$ (2, p. 218).

CONCLUSIONS AND RECOMMENDATIONS

The critical mode of failure is controlled by shear in the unstiffened web of the beam when the edge of the holes are 12 inches or more from the support and h/t is ≤ 55 . This would follow the existing design criteria for I-beams with a $h/t \leq 70$ as stated in the AISC Steel Manual (1).

The failure of I-beams with holes in the web may be dictated by plate buckling and/or local shear failure of the unstiffened web when the edge of the hole is closer than 12 inches to the support and $h/t \leq 55$.

Additional tests should be run on I-beams with holes in the webs. The tests should include beams that have holes located within 12 inches from the support. More instrumentation is needed. Strain gage rosettes should be used to obtain lines of principal strains at the corners of the holes, over the supports and at the lower portion of the web below the hole. Vertical gages should be put on both sides of the web to determine bowing. Tests should also be run with the top flange secured to prevent lateral movement. The beam should rest on a flat rocker plate rather than the support used and the load should be applied through a plate.

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APPENDIX A

Calculations for Eulers buckling load

$$\text{Euler Equation} \quad P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$K = \frac{\pi^2 EI}{P_{cr} L^2} \frac{1}{2}$$

$$\text{Moment of Inertia} \quad I = \frac{bh^3}{12}$$

$$\begin{aligned} & \text{Beam 1} \\ I &= \frac{(5.5) (1/4)^3}{12} \end{aligned}$$

$$= \frac{0.00716 \text{ in}^3}{12}$$

$$K_1 = \left[\frac{(\pi^2) (29 \times 10^6) (0.00716)}{(32,333) (14)^2} \right]^{1/2}$$

$$\underline{\underline{K_1 = 0.567}}$$

$$\begin{aligned} & \text{Beam 2} \\ I &= \frac{(9.5) (1/4)^3}{(12)} \end{aligned}$$

$$= \frac{0.0125 \text{ in}^3}{(12)}$$

$$K_2 = \left[\frac{(\pi^2) (29 \times 10^6) (0.0125)}{(40,000) (14)^2} \right]^{1/2}$$

$$\underline{\underline{K_2 = 0.661}}$$

APPENDIX B

Calculations for plate buckling

$$\text{Plate buckling equation (2, p. 113)} \quad \text{scr} = k \frac{\pi^2 E}{12(1-u^2) (b/t)^2}$$

The mode of loading was assumed as case 4 in Gaylord (2, p. 115).

Beam 1

$$\text{vave} = \frac{P}{A}$$

$$= \frac{32,300 \text{ Kip}}{1,375 \text{ in}^2}$$

$$= \underline{23.5 \text{ Ksi}}$$

$$\text{scr} = \frac{(0.255) (\pi^2) (29 \times 10^6) \text{ Ksi}}{12 (1 - 0.25^2) \left(\frac{5.5}{1/4}\right)^2 \left(\frac{\text{in}}{\text{in}}\right)^2}$$

$$= \underline{\underline{13.75 \text{ Ksi}}}$$

$$\text{vave} = \frac{P}{A}$$

$$= \frac{40,000 \text{ Kip}}{2.38 \text{ in}^2}$$

$$= \underline{16.81 \text{ Ksi}}$$

$$\text{scr} = \frac{(0.894)(\pi^2)(29 \times 10^6) \text{ Ksi}}{12(1 - 0.25^2) \left(\frac{5.5}{1/4}\right)^2 \left(\frac{\text{in}}{\text{in}}\right)^2}$$

$$= \underline{\underline{16.2 \text{ Ksi}}}$$