MECHANICAL DETERMINATION OF HORIZONTAL RADIATION PATTERNS FOR TWO AND THREE ELEMENT VERTICAL ANTENNAS

by

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PARTS

I. Introduction ........................................ 1
II. List of Symbols ..................................... 4
III. An Instrument for Calculating Horizontal Radiation Patterns .................. 7

   Mechanical Synthesis of Antenna Directivity, p. 7; Description of Radiation Pattern Calculator, p. 14; Instructions for Operating Radiation Pattern Calculator, p. 21.

IV. Examples Illustrating Mechanical Determination of Patterns .................. 28

   Possible Patterns from KOAC Array, p. 28; Specifications for Directive Antenna, p. 32; Influence of Increasing Effective Spacing on KOAC Pattern, p. 33; Design of a Ten-Kilowatt Pattern for KOAC, p. 38.

V. Conclusion ........................................... 44

   Practical Limitations, p. 44; Suggested Improvements, p. 45.

VI. Appendix ............................................. 46

   Useful Mathematical Relationships Pertaining to Patterns Drawn by the Radiation Pattern Calculator, p. 46; A Collection of Three-Element Patterns, p. 54.
ILLUSTRATIONS

Figure

1. Fundamental Equation and Vector Diagram of Directivity ................. 8
2. Tower Configuration and Modified Vector Diagram ......................... 10
3. Essential Mechanism of Horizontal Polar Pattern Calculator .............. 12
4. Top View Photograph of Horizontal Polar Pattern Calculator .............. 15
5. Bottom View Photograph of Horizontal Polar Pattern Calculator .......... 16
6. Close-Up of Vectors ............................................ 17
7. Tower Configuration with Corresponding Pattern ........................... 22
8. Examples Illustrating Accuracy of Horizontal Polar Pattern Calculator .... 27
9. Horizontal Radiation Patterns Available from KOAC Directional Array .... 30
10. Effect of Increasing Tower Spacing on Horizontal Pattern of KOAC Five-KW Transmitter .......................... 34
11. Patterns Considered in the Search for a Ten-Kilowatt Pattern for KOAC ...... 39
12. Calculating Chart for Function \(2\pi K \cos \phi\) .................................. 50
13. Curve of Bessel Function of the First Kind of Zero Order \(J_0\) with Degree Arguments .................. 51

A Collection of Three-Element Antenna Patterns .......................... 56-75
TABLES

I  Effect of Increasing Tower Spacing on Horizontal Pattern of KOAC Five-KW Transmitter . . . . . 35

II  A Table of Values for the Function $A_1 = (1 + K_2^2 + K_3^2)\pi$ . 49
I. INTRODUCTION

Use of Directional Antennas Increasing

The use of directional antennas in radio broadcasting has spread to approximately 150 stations in the United States. The primary reason is the prevention of interference between stations allocated the same frequency. A second reason is economy from the broadcaster's point of view. A station proposing to operate on a certain frequency must consider the directions in which lie other stations operating on this same frequency and must provide for suppressed radiation in those directions. Also, the broadcaster is interested in using the power from his transmitter to serve effectively as many people as possible.

Pattern Selection First Problem in Directional Antenna Design

The nature of a coverage and interference problem dictates at once the approximate shape of the polar pattern representing the relative field strengths in the

various directions about the transmitting antenna. The task of determining the constants for an antenna system to provide an approximation of the needed pattern becomes the engineer's first problem. Usually a trial and error process follows in which known patterns\(^2\) are studied and then modified by changing some of the constants. The calculations involve considerable time and labor, especially for the non-symmetrical patterns of arrays having more than two elements. Analytical determination of a single pattern may require several hours, and many patterns may be discarded in the search for a suitable solution.

**Pattern Shapes Determined Mechanically**

A quicker and easier means for computing field patterns for directional arrays will aid in extending the range of known patterns and will simplify the making of final adjustments in a pattern under consideration.

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An instrument for calculating mechanically the relative field patterns for two and three element arrays has been developed and is described herein. Three problems based on the KOAC array at Granger, Oregon, are worked out to illustrate its use. An appendix contains a collection of three-element patterns for tower spacings up to one wavelength.
II. LIST OF SYMBOLS

The symbols listed are used throughout this work with the meanings given below. Any others used will be defined where they appear.

\( K_1 \) field ratio for tower one; is taken equal to unity and represents one inch graphically on patterns drawn by the pattern calculator.

\( K_2 \) field ratio for tower two; is equal to the ratio of current in tower two to the current in tower one for both towers having the same height; is equal to or less than unity; represents a fraction of an inch graphically on patterns drawn by the calculator.

\( K_3 \) field ratio for tower three; is equal to the ratio of current in tower three to the current in tower one for both towers having the same height; is equal to or less than unity; represents a fraction of an inch graphically on patterns drawn by the calculator.

\( S_2 \) spacing in degrees between centers of towers one and two; is equal to the distance between the centers of the towers in meters divided by the wavelength in meters and multiplied by 360°.

\( S_3 \) spacing in degrees between centers of towers one and three.
$S_{2-3}$ spacing in degrees between the centers of towers two and three.

$\Theta$ horizontal angle lying in the plane of the bases of the towers and measured in a positive direction counterclockwise from the line from tower one to tower two; line is zero degrees in the direction of tower two.

$\beta$ angle measured in a positive direction counterclockwise between the line from tower one and tower two and the line from tower one to tower three; is the included angle between sides $S_2$ and $S_3$ of the triangular configuration of the three towers.

$\phi_2$ phase angle between currents in tower one and tower two; is positive for current in tower two leading current in tower one.

$\phi_3$ phase angle between currents in tower one and tower three; is positive for current in tower three leading current in tower one.

$\alpha_2$ phase angle between voltage induced in a receiving antenna at a remote point P from the array due to current flowing in tower one and the voltage induced due to current flowing in tower two; is equal to $S_2 \cos \Theta + \phi_2$. 
\( \phi_3 \) phase angle between voltage in the receiving antenna due to current flowing in tower one and voltage due to current flowing in tower three; is equal to \( S_3 \cos (\theta - \phi) + \phi_3 \).

\( r \) radius of relative horizontal polar pattern drawn by the calculator and is measured in inches.

\( A \) area of relative horizontal polar pattern drawn by the calculator and is measured in square inches.

\( A_1 \) is area component of \( A \) depending only on \( K_2 \) and \( K_3 \).

\( A_2 \) is area component of \( A \) depending only on \( K_2 \), \( \phi_2 \), and \( S_2 \).

\( A_3 \) is area component of \( A \) depending only on \( K_3 \), \( \phi_3 \), and \( S_3 \).

\( A_2 - A_3 \) is area component of \( A \) depending only on product of \( K_2 \) and \( K_3 \), difference between \( \phi_2 \) and \( \phi_3 \), and the distance between towers two and three, \( S_{2-3} \).

\( J_0(S) \) is Bessel Function of the first kind of zero order with tower spacing in degrees for arguments.
III. AN INSTRUMENT FOR CALCULATING HORIZONTAL RADIATION PATTERNS

Mechanical Synthesis of Antenna Directivity

The discussion to follow gives briefly the nature of horizontal directivity of antenna arrays of more than one element and how a mechanical device was made to follow the same mathematical law.

The signal voltage, $E_{P0}$, induced in a receiving antenna at a remote point, P, removed from a multi-element array is the vector sum of the voltages each of which is due to radiation resulting from currents flowing in each of all the elements of the array.

Figure 1 gives the fundamental equation and the vector diagram for the three-element case. The values of the individual voltages, $E_i$, are proportional to the respective figures of merit of the radiating antennas. For antennas of different heights, the figure of merit in each case must be determined. The voltage, $E_1$, is taken to represent the signal arriving from the reference tower which induces the largest signal and has the largest figure of merit. The angles between this reference voltage and the other two change as P moves around the array changing $\theta$.

---

\[ E_{p\theta} = E_1 + E_2 e^{j\alpha_2} + E_3 e^{j\alpha_3} \]  \hspace{1cm} (1)

FIGURE 1

Fundamental Equation and Vector Diagram of Directivity
The magnitude of the angle, $\alpha$, between the reference voltage, $E_1$, and either of the other voltages is determined by two things. Figure 2A shows one which is the difference in distance traveled by the signal from tower one and the distance traveled by the signal from either of the other towers. This difference in distance introduces an angular displacement between $E_1$ and either of the other voltages equal to this distance expressed in degrees. The assumption leading to this conclusion is that the lines drawn from the three towers to P are substantially parallel.

$$\Psi_2 = S_2 \cos \theta$$

$$\Psi_3 = S_3 \cos (\theta - \alpha)$$

The other component of $\alpha$ is the initial phasing of the current flowing in one of the towers with respect to the current flowing in the reference tower.

$$\alpha_2 = \Psi_2 + \phi_2$$

$$\alpha_3 = \Psi_3 + \phi_3$$

A different form of the vector diagram of Figure 1 is shown in Figure 2B. The voltage, $E_3$, is simultaneously reversed and subtracted vectorially, which is equivalent to vector addition. This form of the vector diagram is the basis of the operating principle of the horizontal polar pattern calculator.
REFERENCE TOWER

(A)

(B)

FIGURE 2

Tower Configuration and Modified Vector Diagram
As point, P, moves around the antenna array, \( \lambda \) varies between the limits \((S + \phi)\) and \((S - \phi)\) as shown in Figure 3A. The instantaneous displacement between the ends of vectors, \( E_2 \) and \( E_3 \) is equal to the instantaneous magnitude of \( E_P \). The vectors \( E_2 \) and \( E_3 \) do not swing in synchronism between these limits except in the special case of three towers in a straight line, and swinging of these vectors is simple harmonic motion. The mean direction about which each vector swings is determined by the phasing for the current in the particular tower.

The mechanism diagrammed in Figure 3B serves to deflect \( E_2 \) in simple harmonic motion. The disc sketched underneath the sliding plate carries the adjustable cam, whose distance from the center determines the length of excursion of the sliding plate, and hence, the limits of deflection swung through by \( E_2 \). The relationships between the adjustment, \( S \), the diameter, \( d \), of the drum upon which \( E_2 \) is mounted, the excursion of \( \lambda \), and \( \phi \) are:

\[
S = \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{2} \quad (6)
\]

\[
\phi = \lambda_{\text{max}} - S \quad (7)
\]

\[
\phi = \lambda_{\text{min}} + S \quad (7a)
\]

\[
S = \frac{s'}{2\pi d} \quad (8)
\]

\((s' \) is distance from center of disc out to center of cam in the same units as \( d \)).
FIGURE 3

Essential Mechanism of Horizontal Polar Pattern Calculator
Any phasing, $\phi$, is introduced by releasing the tension on the cable about the drum and deflecting vector, $V_2$, any desired amount from zero degrees with the cam directly above the center of the disc. Positive angles are counterclockwise.

Placing two mechanisms of this kind one above the other so that the drums for turning the vectors are displaced a distance equal to the length of the reference vector provide the nucleus for a three-element calculator. Zero degrees for the lower mechanism is, of course, in line with zero degrees for the one shown in Figure 3B, but it is directed in the opposite direction to conform to the vector diagram in Figure 3A.

Synchronized motion between the upper and lower discs provides for the proper simple harmonic motion of the movable vectors. The angle, $\beta$, is introduced as an initial angular displacement between the two discs before they are locked together. Zero degrees for both is taken in the direction of the cam, and clockwise rotation of the discs simulates counterclockwise movement of P about the array which is in the direction of increasing horizontal angle. To introduce a positive $\beta$ angle between the two discs, the lower disc is held stationary with its zero to the right, and the upper disc is turned clockwise until the zero of the lower disc is opposite the proper angle
scribed on the edge of the upper disc corresponding to the
desired $\phi$. The angles marked off on the upper disc are
positive counterclockwise so that as the disc rotates
clockwise, a stationary indicator at the right shows
increasing $\theta$.

A means provided to record the instantaneous dis-
placement of the ends of the vectors as a function of $\theta$
is the only remaining essential requirement. The method
employed by the horizontal polar pattern calculator is
shown in the close-up photograph of Figure 6.

**Description of Radiation Pattern Calculator**

To follow the principles of operation described in
the previous section, the instrument shown in the top view
photograph reproduced in Figure 4 and the bottom view in
Figure 5 was designed and built.\(^4\) The function of the
various working parts labeled and their uses will be
described in detail.

The vectors, $E_2$ and $E_3$, appearing near the center of
the photograph rotate in parallel horizontal planes about
axes displaced two inches as shown. Figure 6 gives a
close-up view of the vectors. The instantaneous angular
direction assumed by each in any position is given by the

\[ \theta = \text{constant} \]

\[^4\text{Working drawings were prepared by the author in May, 1941, and are on file with the Electrical Engineering Department at Oregon State College.} \]
Top View Photograph of
HORIZONTAL POLAR PATTERN CALCULATOR
Bottom View Photograph of
HORIZONTAL POLAR PATTERN CALCULATOR
CLOSE-UP PHOTOGRAPH OF VECTORS

FIGURE 6
corresponding protractor calibrated with positive angles on the inner circle and negative angles on the outer circle. The set screws retain the sliding vectors' blocks at the proper length setting read on the calibrated scale. The length of vector $E_2$ is read through the conical hole in the top of the vector, and the length of vector $E_3$ is read on the calibrations on the face of the lower vector.

Power to operate the instrument is supplied by hand through the drive crank and ring gear attached to disc two shown in the top view photograph. Disc three receives its power through the central shaft. The lock ring secures disc two to the central shaft. Releasing the lock ring with a special wrench inserted into the milled notches permits angular displacement between the discs. Tightening the ring locks disc two securely to the central shaft.

The cam guides permit the cams to be secured any distance out from the centers of the discs within their lengths. As the discs rotate, the cams slide from side to side in the cam slots and impart reciprocating simple harmonic motion to the sliding plates mounted in the slide plate guides.

5. Disc two is the upper and disc three the lower. Numbering corresponds to the actuated vectors. There is no disc one.
The phosphor bronze drive cables attached to the sliding plates pass over the three pulleys and are wrapped around the vector drums. Springs inserted in the long sides$^6$ of the cables aid to prevent slipping. Stretching the springs toward the right releases tension on the cables around the vector drums so that the latter can be turned easily when desired.$^7$

The recording table together with the disc upon which it is mounted turns in synchronism with discs two and three. Power is transmitted by means of the phosphor bronze belt around disc two and the recording table disc. Tension can be released by loosening the nut shown below the recording table in the bottom view. The washer shown under the nut covers a slot that allows the bolt, disc, and table all to slide slightly to the left, slackening the belt. Proper operation of the instrument, however, demands that the belt length shall be adjusted to give good driving tension with the bolt as far to the right in the slot as it will go. If the adjustment is not correct, the unpainted surface of the cross-piece under the washer shows. Clips attached to the top of the recording table hold the paper in place for drawing a horizontal radiation pattern.

---

6. The three pulleys and the long side of the cable are shown more clearly in the diagram of Figure 3B.

7. Forcefully turning the vectors damages the drum surface.
The instantaneous displacement of the ends of the vectors, $E_2$ and $E_3$, is measured by means of a thread secured by a knot in the hole of vector, $E_3$, and allowed to slide through the hole in vector $E_2$ back up the bearing volt, and out the top as shown in Figure 6. After it is looped around the pulley in the scribing block, the thread is attached to the thread length adjusting block. The proper adjustment of the thread length adjusting block for a given length of $E_2$ is such that the scriber is situated a calibrated distance out from the center of the recording table equal to the sum of the calibrated lengths of vectors $E_2$ and $E_3$ and one when both vectors are placed in their zero degree positions. The scriber calibrations are marked on top of the scriber guide.

The scriber block permits the penholder to be easily removed for cleaning, filling, or changing the pen and changing papers upon which patterns are drawn.

The counter-weight suspended by the string attached to the scriber block keeps the string taut. The pin attaching the pulley section of the scriber block may be removed, allowing the block to be locked at any radius needed for drawing circles.

---

8. The pulley inserts a two to one ratio between the displacement of the ends of the vectors and the actual distance the scribing block rests out from the center.
The foregoing material has described all of the working parts of the horizontal polar pattern calculator.

**Instructions for Operating Radiation Pattern Calculator**

Detailed instructions for setting up the constants and drawing the pattern for a directional array will now be given.

The tower configuration and the minimum number of parameters to determine the pattern for this example are shown in Figure 7. All operations given refer to Figure 4 except where noted.

1. Loosen lock ring and adjust the angle between the lines of towers, $\beta$, to $150^\circ$ by holding disc three with its zero calibration opposite the indicator supported on the drive crank holder and turning disc two until its indicated reading is $150^\circ$. Tighten lock ring without changing relative positions of the discs.

2. Adjust the field ratios by setting the corresponding vector block to the proper lines calibrated and re-tightening the setscrews.

   $$\begin{align*}
   K_2 &= 0.7 \\
   K_3 &= 0.5
   \end{align*}$$

3. Adjust measuring string length by setting both
FIGURE 7

Tower Configuration with Corresponding Pattern
vectors on their zeros (angles) and setting the string length adjusting block so that it retains the scriber block at a position equal to the sum of the three field ratios.

\[ \text{Block position} = 0.7 + 0.5 + 1.0 = 2.2 \]

4. Remove any previous phasing settings in the vectors by setting each disc in turn on its 90° mark and returning each vector arm to its zero degree position.

5. Adjust cams for spacings, \( S_2 \) and \( S_3 \), by setting each disc in turn on its zero mark and placing the cams so that the vectors read 95° and 160° respectively. A check for precise adjustment is usually needed. On setting each disc in turn on 180° the vectors should read -95° and -160° respectively. A few trials are sometimes needed together with slight slipping of the vector drums to remove last traces of residual phasing. Equations (6), (7), and (7a) are useful in setting the spacing by taking advantage of the fact that one-half the total angular deflection of the vector is equal to the spacing for which the cam is set. By this method it is not necessary to remove the residual phasing. Maximum and minimum values for \( \alpha \) occur at the zero and 180° positions respectively,
of the discs. Care must be used in noting the proper algebraic sign for $\alpha_{\text{min}}$, because it is negative for $S$ greater than $\phi$, and addition of the two angles is required to find the spacing.

6. Introduce the phasing to each vector arm in turn by setting each disc to zero degrees and slipping the vector until it reads the sum of the spacing plus the desired phasing. The readings for these adjustments are:

$$S_2 + \phi_2 = 35^\circ$$
$$S_3 + \phi_3 = 280^\circ$$

7. After returning disc two to the zero degree position, the instrument is ready for drawing the pattern. Place a piece of smooth finish bond paper ($8\frac{1}{2} \times 11$) on the recording table under the clips and centered slightly to the right of the recording table center, since the zero degree line points in the direction toward the operator.

8. The recording pen filled with a few drops of black drawing ink should be carefully placed in the scriber block and the string pulled to move the block a few millimeters to draw a zero degree line. Releasing the string allows the block to return to its original
9. Trace the pattern by turning the crank slowly (about four or five seconds per revolution) in the direction of increasing $\theta$. As each ninety degree point is reached, a short line similar to the one for zero degrees should be drawn. On reaching zero degrees the crank is reversed and the pattern traced in the opposite direction this time, omitting the marks.\(^{10}\)

10. Before the pattern is taken from the recording table, it is very important that the upper right hand corner of the sheet shall have an identifying mark to show the proper direction for zero degrees. The upper right hand corner of the sheet is in the nearer left hand corner of the recording table when the angle, $\theta$, is set on zero degrees.

The values of both the parameters and maximum and minimum excursions for $\theta$ should be recorded on each pattern sheet. The values of spacing and phasing may be

\(^{9}\) A slight amount of tension applied by hand to aid the counterweight to return scriber block when the radius $r$ is decreasing results in superior quality patterns.

\(^{10}\) The marks at each quadrant supply rectangular coordinates to establish the origin without which any pattern is worthless.
readily checked from $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$ to supply evidence that the proper values of spacing and phasing were set on the instrument.

The pattern resulting from the constants given in this example is shown below the tower configuration in Figure 7 reproduced exactly as drawn by the machine and after the axes were extended, through the center, and labeled.

To indicate roughly the accuracy that can be expected from patterns drawn by the instrument, Figure 8 shows two upon which calculated points are superimposed.
Examples Illustrating Accuracy of Horizontal Polar Pattern Calculator
IV. EXAMPLES ILLUSTRATING MECHANICAL DETERMINATION OF PATTERNS

Solutions to the following three problems, based on the KOAC two-element array at Granger, Oregon, have been worked out utilizing the horizontal polar pattern calculator. These show only a few of the several types of directivity problems involving two or three elements that can be readily solved by the instrument. A maximum of one wavelength spacing for towers two and three is the only limitation in the choice of operating parameters. This value of spacing is well beyond any that may be employed in practical coverage and interference problems.

Problem 1

Possible Patterns from KOAC Array

The objective of this study was to present a chart indicating all possible field patterns available from the present KOAC antenna array. The chart was intended to include patterns available only at the assigned frequency of 550 kilocycles. The parameters were to be taken in sufficiently small increments that changes in the field pattern encountered between any two patterns may be clearly understood.

Field ratios were taken in twenty-five percent steps and phasings in forty-five degree steps. Thirty-two
patterns were drawn to cover the entire range of possibilities available from the two towers spaced 135° and lying on a line azimuth 295°.

Figure 9 shows the patterns all reduced to the same area and arranged in order of the changing increments of phasing and field ratio. The root-mean-square circle accompanies each pattern and represents the relative field of a single vertical radiator of the same height supplied with the same power.\textsuperscript{11} The vertical line through each pattern represents true North above the pattern, and the pair of perpendicular axes represent the line of towers which is directed in the north-westerly direction and its perpendicular bisector. For the phasings and field ratios given on the pattern sheet, the direction of zero degrees for each pattern is north-westerly in the line of towers. The angle \( \theta \) is positive counterclockwise.

Changes produced in any pattern by varying either field ratio of phasing to values intermediate to those shown will result in a pattern that has directional properties lying between those of the original pattern and the next one in the series. If the sheet is imagined to be placed in the shape of a right cylinder with the

HORIZONTAL RADIATION PATTERNS AVAILABLE FROM K O A C DIRECTIONAL ARRAY

PHASING OF WEST TOWER

-180°  -135°  -90°  -45°  0°  45°  90°  135°

CURRENT IN WEST TOWER CURRENT IN EAST TOWER

0.25  0.50  0.75  1.0

FIGURE 9
right vertical column of patterns adjacent and left of the left column, the effect of changing the phasing of the west tower for a given field ratio causes the pattern to assume shapes successively given around the cylinder.

If desired, an intermediate pattern drawn on the horizontal pattern calculator employed in this investigation will give the directional characteristics for the parameters selected.
SPECIFICATIONS FOR DIRECTIVE ANTENNA

K O A C, CORVALLIS? OREGON

FIVE KILOWATTS ON 550 KILOCYCLES

Number of Elements--Two

Type of Elements--Self-supporting shunt excited towers with grounded base. Base width not over 5.6% of height.

Antenna Height--325 feet.

Height Above Sea Level--550 feet (at top of tower).

Orientation of Array--On a line bearing North 115° East

Spacing of Elements--670.6 feet (135°)

Phasing of Towers--Current in East tower lags the current in the West tower by 50°.

Current Ratio--Current in West tower 0.7 of that in East tower.

Ground System--Radials two feet deep and spaced three degrees apart, length one-half wavelength. Radials common to both towers bonded at their junctions.
Problem 2

Influence of Increasing Effective Spacing on KOAC Pattern

Experience with shunt excited vertical antennas has recently indicated that the shunt leads tend to increase the effective spacing of the towers as evidenced by the effect on the horizontal radiation pattern when these shunt leads are placed on opposite sides of the array from each other. Since this is the situation at KOAC's directional array, a knowledge of the effect of small increases in spacing was desired. The purpose of this investigation was to determine the change in signal strength in the directions of interference requiring protection by increasing the spacing in small increments. These three directions of interference are Tongue Point, Oregon; KOY, Phoenix, Arizona; and KFYR, Bismarck, North Dakota.

By increasing the spacing of the towers in increments of two or three degrees at a time, seven successive patterns were drawn on the calculator beginning with the present spacing of 135° and holding all other parameters of the array unchanged. Only six of the patterns, however, are shown in Figure 10.

Table I shows the measured resulting signals in the directions of Tongue Point, KFYR, and KOY, respectively, for the first five patterns. The range covered represents
EFFECT OF INCREASING TOWER SPACING ON HORIZONTAL PATTERN OF KOAC FIVE-KW TRANSMITTER

**Pattern 1**  
$S = 135^\circ$

**Pattern 2**  
$S = 138^\circ$

**Pattern 3**  
$S = 140^\circ$

**Pattern 4**  
$S = 142^\circ$

**Pattern 5**  
$S = 145^\circ$

**Pattern 6**  
$S = 149^\circ$

**FIGURE 10**
Effect of Increasing Tower Spacing on Horizontal Pattern of KOAC Five-KW Transmitter

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Tower Spacing Increase in Spacing over present 135° or 670.6 ft.</th>
<th>Signal Strength Levels in Decibels Above or Below rms Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Degrees</td>
<td>degrees feet</td>
</tr>
<tr>
<td>1</td>
<td>135</td>
<td>0 0</td>
</tr>
<tr>
<td>2</td>
<td>138</td>
<td>3 15</td>
</tr>
<tr>
<td>3</td>
<td>140</td>
<td>5 25</td>
</tr>
<tr>
<td>4</td>
<td>142</td>
<td>7 35</td>
</tr>
<tr>
<td>5</td>
<td>145</td>
<td>10 50</td>
</tr>
</tbody>
</table>

TABLE I
as much increase in effective spacing as could be anticipated. The signal strengths are given in decibels above or below zero level which was taken as the unattenuated signal from a single antenna of the same height radiating the same power.

\[ \text{Decibels} = 20 \log_{10} \frac{r \text{ of directional array}}{r \text{ of single antenna}} \]  

The patterns drawn for this investigation are reproduced in Figure 10. Light pencil lines were used for axes and radii to give accuracy in the graphical determination of the values in Table I.\(^{12}\) Radii were laid in the directions of interference and the root-mean-square circles drawn. With the circle as reference, radii to the

\[ 12. \text{ The following mathematical approach was considered in this investigation:} \]

Equation (11) (Appendix) differentiated with respect to \( S_2 \) after \( K_3 \) was allowed to become zero resulted in:

\[ \frac{\partial r}{\partial S_2} = \frac{-K_2}{r} \cos \theta \sin \phi \]

Substituting the angles corresponding to the directions of interference for \( \theta \) and the other corresponding parameters of the KOAC array in the above relationship gave negative rates of change of \( r \) with respect to \( S_2 \), but an inspection of equations (12) and (13) (Appendix) showed that the area was also decreasing in the \( J_0(S_2) \) term. Much labor was saved by drawing the patterns and making the measurements. On the other hand, if \( \text{Area} \) had been increasing or changing very slowly, the use of the partial derivative would have been sufficient to assure that the signal strengths in the directions of interference were not increasing.
pattern outline gave relative signal strengths. Equation (9) gave the decibel levels.

The data obtained in Table I show that very favorable pattern modifications can be expected if the shunt leads should increase the effective spacing. The changes in decibel levels show that increasing the effective spacing from 135° to not more than an additional 10° did not increase the horizontal signal strength in the three directions of interference considered. In several instances the signal strengths were reduced as shown.

An inspection of the patterns in Figure 10 shows that the reduction of signal strength in the easterly and westerly directions is compensated by an increase in signal strength in the directions of the lobes. The patterns become more slender with increasing spacing, and the small lobe in the westerly direction of the line of towers is seen to enlarge.
Problem 3

Design of a Ten-Kilowatt Pattern for KOAC

The objective of this study was to obtain the operating constants for a directional array for KOAC to permit doubling the power input\(^{13}\) without increasing the horizontal radiation in the directions of Tongue Point, KOY, and KFYR. It was also desired that the radiation toward the Pacific coast would remain suppressed, and the field strength would be increased in the northerly and the southern directions along the Willamette Valley. Utilization of the two towers already erected was required.

An inspection of the possible patterns available from KOAC's two-element array shown in Figure 9 showed that the desired protection could be afforded in either the direction of Tongue Point or in the directions of KOY and KFYR. It was too much to expect protection for all three with only a two-element array.

Figure 11 shows the patterns considered in the search for a solution to this problem. Although the solution to a problem of this kind is the result of a trial and error process, an organized and straightforward

\[^{13}\text{Minor changes in the Western Electric 405 B2 transmitter permits of operating at ten kilowatts.}\]
FIGURE 11

Patterns Considered in the Search for a Ten-Kilowatt Pattern for KOAC
procedure is imperative.

The first pattern shows the directive characteristics for the five-kilowatt two-tower array with the root-mean-square circle indicated. The directions of interference were laid out and the corresponding signal strengths measured in decibels below the ten-kilowatt circle taken as zero level.

<table>
<thead>
<tr>
<th>Location</th>
<th>Signal Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tongue Point</td>
<td>-7 db</td>
</tr>
<tr>
<td>KOY</td>
<td>-2 db</td>
</tr>
<tr>
<td>KFYR</td>
<td>-1.2 db</td>
</tr>
</tbody>
</table>

The specifications for the ten-kilowatt pattern include the above limitations together with requirements for northerly and southern directivity. A rough free-hand sketch to be used as a guide in the search for a suitable pattern is usually helpful at this point in a problem of this nature.

A search was made in the file of three-element patterns having spacings in the region corresponding to that of the present KOAC array.14 The second and third

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14. This file of three-element patterns is in the process of preparation by the author and his assistant, Mr. Donald Halfhill, a junior in Electrical Engineering at Oregon State College. The collection was one-third complete on April 1, 1942, and will represent in a collection of 2268 patterns the possible three-element patterns with the parameters taken in 60° steps.
patterns in the first row were selected as a starting point. These represent 120 degree spacings, each with $\beta$ of 60° and phasings of 60° for tower two and -120° and 180° respectively for the phasings of the third tower. Splitting the difference between these two resulted in the symmetrical pattern on the left in the second row, and it has the same parameters as the other two with the exception that $\phi_3$ becomes -150°. The next step was to correct $S_2$ to 135° shown by the slight non-symmetry of the second pattern in the second row. This pattern, however, has been inverted together with its tower configuration about the zero to 180 degree axis, so that $\beta$ becomes 300°.

A few trial adjustments showed that the origin of this pattern could not be pushed in the direction of the Pacific coast (left) by any means involving only moderate adjustments in the parameters.

The last pattern in the second row is the result of rotating the previous one about the 90 to 270 degree axis. Taking the West tower as the new reference resulted in the following parameters:

- $S_2$ remained 135 degrees
- $\phi_2$ became -60 degrees
- $\phi_3$ became 150 degrees
- $S_3$ became 130 degrees
- $\beta$ became 305 degrees
The short lines projecting out from this pattern indicate the directions of interference.

The process from here on consisted in using the settings on the calculator of the above pattern as an approximate solution and making small trial and error adjustments based on visual observations of the effects produced by making changes one at a time in the parameters. The points on disc two corresponding to the directions of interference and coverage were marked clearly for definite identification. As the disc was turned slowly, the moving vectors were watched together with the movement of the scribing pen and disc two. No attempt was made to draw patterns after each adjustment, but the total effects in the directions of interest were studied visually before each adjustment was made. It was found that adjustments on some of the parameters were favorable in some directions but unfavorable in others. Compromises were made where necessary.

It was seen that an increase in $\beta$ to 325° and to reduce $K_3$ to 0.85 together with setting its spacing to 71° and making small adjustments in phasings of the currents in the two towers ($\phi_2$ to -42° and $\phi_3$ to 177°) a null could be produced in the direction of Tongue Point simultaneously with protecting K0Y and KFYR and retaining desired lobes. The pattern on the bottom row (left) shows
the progress up to this point.

The disadvantages of the above pattern included unbalanced radiation North and South in addition to the large lobe in the West. Further adjustments in small amounts gave the second pattern in the last row. \( \phi_2 \) became \(-26^\circ\), \( K_3 \) increased to 0.9, \( \phi_3 \) became \( 179^\circ \), and \( S_3 \) reduced to \( 63^\circ \).

It was anticipated that further increase in \( \phi \) offered possibilities; the final pattern resulting from the adjustments given below is the last one in Figure 11.

\[
\begin{align*}
\phi &= 15 \text{ degrees} \\
\phi_2 &= -19 \text{ degrees} \\
\phi_3 &= 157 \text{ degrees} \\
S_2 &= 135 \text{ degrees} \\
S_3 &= 69 \text{ degrees} \\
K_2 &= 1.0 \\
K_3 &= 0.9
\end{align*}
\]

Measurements with a decibel scale indicated that this pattern came within the limitations imposed at the outset.
V. CONCLUSION

It has been shown that the determination of horizontal radiation patterns for two and three element vertical antennas has been simplified with the aid of the mechanical device described. Thus, a means has been provided for easily extending the knowledge of possible two and three element patterns in addition to a means for determining the optimum two or three element pattern for any coverage and interference problem. A check on the accuracy of introducing the operating constants of the array is always provided in the maximum and minimum angles swept through by the vectors. The result of the development of this instrument is the saving of much time and labor.

This instrument has made possible a new method for modifying the shape of a pattern as has been described. Adjustments of the parameters are governed by the observed influence they make on the shape of the pattern as contrasted to the method of trial and error adjustments of the parameter where a pattern is required after each adjustment. This method eliminates the calculation of several intermediate patterns.

Practical Limitations

Determination of two and three element patterns requires an instrument having only the relatively simple
mechanical construction herein described. Extension of the principle to the determination of patterns for arrays having more than three elements leads to the need for mounting a moving vector on the end of another moving vector. Provisions for actuating these additional moving vectors calls for a much more complicated mechanism. This could be accomplished with the aid of Selsyn motors. For this reason, the original design was intentionally limited to determination of three element patterns.

Suggested Improvements

Experience in the use of the horizontal polar pattern calculator described has shown that incorporation of some mechanical refinements found in well known precision instruments can certainly yield greater accuracy and ease of operation.

The following features are suggested as possible improvements in design:

1. All metal construction.
2. Rack and pinion adjustments for spacing and field ratio.
3. Clutch inside vector drum to facilitate phasing adjustment.
4. Fine phosphor bronze chain passing over swivel pulleys to replace measuring string.
5. Replacement of lock ring with friction lock to facilitate adjustment of $\beta$ angle.

Others considered include electric motor drive and relocation of the scribe guide to give clearer view and greater accessibility of the pattern. It is believed, however, that the five tabulated improvements can contribute most to the utility of the instrument.
VI. APPENDIX

1. Useful Mathematical Relationships Pertaining to Patterns Drawn by the Radiation Pattern Calculator

The components of the radius, $E_{P\theta}$, of the horizontal polar pattern have been given in equation (1). If the value of each $E$ is measured in millivolts per meter at a mile, the graph of $E_{P\theta}$ as a function of $\theta$ is the familiar horizontal field intensity pattern for three towers. If the components of the radius are arbitrary constants proportional to the respective currents flowing in the towers, the graph becomes the relative horizontal field intensity pattern. The List of Symbols define the constants ($K$).

Rewriting equation (1) with field ratio constants inserted and replacing $E_{P\theta}$ with $r$:

$$r = 1 + K_2 e^{j\phi_2} + K_3 e^{j\phi_3}$$  \hspace{1cm} (10)

$e = \text{base of natural logarithms}$

Since the magnitude of $r$ as a function of $\theta$ is all that is required for the polar diagram, equation (10) is broken into its real and imaginary components. The square root of the sum of the squares of these two components yields $r = f(\theta)$ for the field ratios, the spacings, the phasings, and the $\beta$ angle all taken as parameters:
\[
\sqrt{1 + K_2^2 + K_3^2
+ 2K_2 \cos(S_2 \cos \theta + \phi_2)
+ 2K_3 \cos(S_3 \cos(\theta - \beta) + \phi_3)
+ K_2 K_3 \cos[(S_2 \cos \theta + \phi_2) - (S_3 \cos(\theta - \beta) + \phi_3)]}
\]

(11)

Only the positive value of the square root is used. The above equation can serve for the relative field pattern of a two-element array by allowing \( K_3 \) to become zero.

The area of this polar graph is the integral from zero to two \( \pi \) of one-half the radius squared with respect to \( \theta \). This operation removes the square-root sign from the right hand side of equation (11), multiplies through by one-half, and integrates the four terms separately:

\[
A = \frac{1}{2} \int_{0}^{2\pi} r^2 \, d\theta = A_1 + A_2 + A_3 + A_{2-3}
\]

(12)

The result of the integration is:

\[
A_1 = (1 + K_2^2 + K_3^2)\pi
\]

(13a)

\[
A_2 = 2\pi K_2 \cos\phi_2 \, J_0(S_2)
\]

(13b)

\[
A_3 = 2\pi K_3 \cos\phi_3 \, J_0(S_3)
\]

(13c)

\[
A_{2-3} = 2\pi K_2 K_3 \cos(\phi_2 - \phi_3) \, J_0(S_{2-3})
\]

(13d)

The value of \( S_{2-3} \) may be obtained graphically or by the law of cosines:

\[
S_{2-3} = \sqrt{S_2^2 + S_3^2 - 2S_2 S_3 \cos \beta}
\]

(14)

The area of a pattern for a two-element array results by allowing \( K_3 \) to become zero.
Table II gives values for equation (13a) computed for integral tenths of the parameters $K_2$ and $K_3$.

Values for equations (13b), (13c), and (13d) may be computed with the aid of the chart in Figure 12 to give the first part of each of these terms. A straight edge, a line scratched on a celluloid triangle, or a thread stretched from the origin to the proper angle, $\theta$, intersects the arcs indicated by values of $K$ at the value of the product $2\pi K \cos \theta$ marked off along the abscissa. Figure 13, a curve of the zero order Bessel function of the first kind, covers the range of spacing possible for the calculator. Bessel functions are read from the center of the line. Both charts were carefully drawn to a large scale and reduced photostatically to retain accuracy.

The areas of the patterns may also be determined with a polar planimeter, but it is necessary to have the pattern drawn before the area can be measured. In some instances\textsuperscript{15} it is desirable to determine the area of a pattern before it is drawn. The charts and table presented were prepared to facilitate these computations.

\textsuperscript{15} A part of the work connected with the use of the radiation pattern calculator consisted in preparing a collection of three-element patterns. In order that the line width of the pattern could be made proportional to the size of the pattern, it was very useful to have the areas determined in advance. After photostatic reduction all patterns had approximately the same line density.
A TABLE OF VALUES FOR THE FUNCTION: $A_1 = (1 + K_2 + K_3 )^\pi$

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<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
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<td>5.15</td>
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<td>3.46</td>
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<td>3.96</td>
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<td>4.81</td>
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<td>6.49</td>
<td>6.81</td>
<td>7.21</td>
<td>7.70</td>
<td>8.24</td>
<td>8.82</td>
<td>9.42</td>
</tr>
</tbody>
</table>

TABLE II
FIGURE 12

CALCULATING CHART FOR THE FUNCTION $2\pi K \cos \phi$
FIGURE 13

CURVE OF BESSEL FUNCTION OF THE FIRST KIND OF ZERO ORDER ($J_0$) WITH DEGREE ARGUMENTS.
Equation (11) differentiated with respect to $\theta$ yields an expression that becomes equal to zero as $r$ becomes a maximum or a minimum. The expression is positive for values of $\theta$ for $r$ increasing with increasing $\theta$ and negative for values of $r$ decreasing with increasing $\theta$.

$$\frac{\partial r}{\partial \theta} = \frac{K_2 K_3}{r} \left[ S_2 \sin \theta \left\{ \frac{\sin \alpha_2}{K_3} + \sin (\alpha_2 - \alpha_3) \right\} + S_3 \sin (\theta - \beta) \left\{ \frac{\sin \alpha_3}{K_2} + \sin (\alpha_3 - \alpha_2) \right\} \right]$$ \hspace{1cm} (15)

In the directions of complete nulls, $r$ becomes zero in the ideal case of filamentary antennas and equation (15) becomes infinite.

A corresponding relation for the two-element case is given below:

$$\frac{\partial r}{\partial \theta} = \frac{K_2 K_3}{r} \sin \theta \sin \alpha_2$$ \hspace{1cm} (16)

Equation (16) gives that for the two-element case there will always be a maximum or a minimum point for $r$ in the line of towers. It may also be used to solve for the direction of a maximum or a minimum, because in these directions $\alpha_2$ will be equal to $0^\circ$ or $180^\circ$.

$$\theta \text{ (for a max or min } r) = \cos^{-1} \frac{n \pi - \phi_2}{S_2}$$ \hspace{1cm} (17)

$n$ is any positive or negative integer, or zero.
To determine the rate of change of \( r \) with respect to any of the parameters other than \( \theta \) is without meaning unless the behavior of that area of the pattern is known. For example, if it is found that the rate of change of \( r \) with respect to a parameter is positive, it means that to increase the parameter will increase \( r \) in the direction \( \theta \) is selected for the test, but it does not mean that the signal strength in the direction of \( \theta \) is increasing unless the area is not increasing fast enough to offset the gain made by \( r \). An inspection of equation (13) may indicate whether the area is increasing or decreasing with the proposed change in the parameter.
2. A Collection of Three-Element Patterns

Extensive collections of field patterns for directive arrays of several kinds have appeared in the literature.\textsuperscript{16} Among those of most value to the broadcast antenna engineer have been the two-element patterns. Three-element patterns, and especially those of the non-symmetrical types have been published only occasionally.

This collection contains 320 three-element patterns, the first half of which represents three vertical towers standing in a right angle configuration.\textsuperscript{17} The second half represents three towers in a line. Each sheet contains sixteen patterns indicating the variations possible resulting from changing the phasings of towers two and three. The spacings were taken in ninety degree steps and are constant for a given sheet of patterns.

It will be seen that for a given value of $S_3$, $S_2$ was taken first equal to $S_3$ and then increases from sheet to sheet until it became $360^\circ$. Then $S_3$ was moved out one step and $S_2$ returned equal to this new value of $S_3$, apparently leaving out a possible tower configuration. This

\textsuperscript{16} See references in Footnote 2 on page 2.

\textsuperscript{17} The angle $\phi$ for this group is $270^\circ$ but is given as $-90^\circ$. 
was done intentionally to avoid further pattern shape duplication. The general rule for obtaining the patterns for these "omitted" configurations is:

Rotate the **pattern** clockwise through an angle equal to $+\theta$ about an axis perpendicular to the paper and through the origin of the pattern. This operation places tower three in the position held by tower two. Next, rotate the **paper** about this new zero axis and hold the paper up to the light. The original tower two is now tower three and visa versa; therefore, interchanging the subscripts in the margins gives the correct phasings. The subscripts associated with the spacings also are interchanged.

A general rule for obtaining the pattern for a tower configuration in which $\theta$ is taken in the opposite direction from zero degrees is:

Rotate the paper on the zero to one hundred eighty degree axis and hold it up to the light. The tower numbers are the same as before, and the subscripts as shown are valid.

Although this collection of patterns is very brief, it encompasses a very large variety of pattern shapes, and it is not difficult to follow changes that occur in the pattern as one of the parameters changes through the variations shown.
THREE-ELEMENT ANTENNA PATTERNS

\[ K_2 = 1 \quad K_3 = 1 \quad \beta = 90^\circ \quad S_2 = 90^\circ \quad S_3 = 90^\circ \quad S_{2-3} = 127.2^\circ \]

\[ \phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \]

\[ \phi_3 \]

\[ 180^\circ \]

\[ 90^\circ \]

\[ 0^\circ \]

\[ -90^\circ \]
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1$  $K_3 = 1$  $\beta = -90^\circ$  $S_2 = 180^\circ$  $S_3 = 90^\circ$  $S_{2-3} = 201.5^\circ$

$\phi_2 \rightarrow 180^\circ$  $90^\circ$  $0^\circ$  $-90^\circ$

$\phi_3$

$180^\circ$

$90^\circ$

$0^\circ$

$-90^\circ$
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1 \quad K_3 = 1 \quad \beta = -90^\circ \quad S_2 = 270^\circ \quad S_3 = 90^\circ \quad S_{2-3} = 284.5^\circ$

$\phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ$

$\phi_3$

180°

90°

0°

-90°
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1$  $K_3 = 1$  $\beta = 90^\circ$  $S_2 = 360^\circ$  $S_3 = 90^\circ$  $S_{2-3} = 371^\circ$

$\phi_2 \rightarrow 180^\circ$  $90^\circ$  $0^\circ$  $-90^\circ$

$\phi_3$

$180^\circ$

$90^\circ$

$0^\circ$

$-90^\circ$
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1 \quad K_3 = 1 \quad B = -90^\circ \quad S_2 = 180^\circ \quad S_3 = 180^\circ \quad S_{2-3} = 254^\circ$

$\phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ$

$\phi_3$

180°

90°

0°

-90°
THREE-ELEMENT ANTENNA PATTERNS

\( k_2 = 1 \quad k_3 = 1 \quad \beta = -90^\circ \quad \theta_2 = 270^\circ \quad \theta_3 = 180^\circ \quad \theta_{2-3} = 324^\circ \)

\( \phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \)

\( \phi_3 \)

\( 180^\circ \)

\( 90^\circ \)

\( 0^\circ \)

\( -90^\circ \)
THREE-ELEMENT ANTENNA PATTERNS

\[
\begin{align*}
K_2 &= 1 & K_3 &= 1 & \beta &= 90^\circ & S_2 &= 360^\circ & S_3 &= 180^\circ & S_{2-3} &= 402^\circ \\
\phi_2 &\rightarrow 180^\circ & 90^\circ & 0^\circ & -90^\circ \\
\phi_3 &\downarrow & & & \\
180^\circ & & & & \\
90^\circ & & & & \\
0^\circ & & & & \\
-90^\circ & & & & 
\end{align*}
\]
THREE-ELEMENT ANTENNA PATTERNS

\[ \begin{align*}
K_2 &= 1 & K_3 &= 1 & \beta &= 90^\circ & S_2 &= 270^\circ & S_3 &= 270^\circ & S_{2-3} &= 382^\circ \\
\phi_2 &\to 180^\circ & 90^\circ & 0^\circ & -90^\circ \\
\phi_3 &\downarrow & & & \\
180^\circ & & & & \\
90^\circ & & & & \\
0^\circ & & & & \\
-90^\circ & & & &
\end{align*} \]
THREE-ELEMENT ANTENNA PATTERNS

\[
K_2 = 1 \quad K_3 = 1 \quad \beta = 90^\circ \quad S_2 = 360^\circ \quad S_3 = 270^\circ \quad S_{2-3} = 450^\circ
\]

\[
\Phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ
\]

\[
\Phi_3
\]

\[
180^\circ
\]

\[
90^\circ
\]

\[
0^\circ
\]

\[
-90^\circ
\]
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1 \quad K_3 = 1 \quad \beta = -90^\circ \quad S_2 = 360^\circ \quad S_3 = 360^\circ \quad S_{2-3} = 509^\circ$

$\phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ$

$\phi_3$

$180^\circ$

$90^\circ$

$0^\circ$

$-90^\circ$
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1 \quad K_3 = 1 \quad \beta = 180^\circ \quad S_2 = 90^\circ \quad S_3 = 90^\circ \quad S_{2,3} = 180^\circ$

$\phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ$

$\phi_3$

\[\begin{array}{cccc}
180^\circ & & & \\
90^\circ & & & \\
0^\circ & & & \\
-90^\circ & & & \\
\end{array}\]
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1 \quad K_3 = 1 \quad \beta = 180^\circ \quad S_2 = 180^\circ \quad S_3 = 90^\circ \quad S_{2-3} = 270^\circ$

$\phi_2 \quad 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ$

$\phi_3$

180°

90°

0°

-90°
THREE-ELEMENT ANTENNA PATTERNS

\[ K_2 = 1 \quad K_3 = 1 \quad \beta = 180^\circ \quad S_2 = 270^\circ \quad S_3 = 90^\circ \quad S_{23} = 360^\circ \]

\[ \phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \]

\( \phi_3 \)

180°

90°

0°

-90°
THREE-ELEMENT ANTENNA PATTERNS

\[ K_2 = 1 \quad K_3 = 1 \quad \beta = 180^\circ \quad S_2 = 360^\circ \quad S_3 = 90^\circ \quad S_{2-3} = 450^\circ \]

\[ \phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \]

\[ \phi_3 \]

\[ 180^\circ \]

\[ 90^\circ \]

\[ 0^\circ \]

\[ -90^\circ \]
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1$  $K_3 = 1$  $\beta = 180^\circ$  $S_2 = 180^\circ$  $S_3 = 180^\circ$  $S_{2-3} = 360^\circ$

$\phi_2 \rightarrow 180^\circ$  $90^\circ$  $0^\circ$  $-90^\circ$

$\phi_3$

$180^\circ$

$90^\circ$

$0^\circ$

$-90^\circ$
THREE-ELEMENT ANTENNA PATTERNS

\[ K_2 = 1 \quad K_3 = 1 \quad B = 180^\circ \quad S_2 = 270^\circ \quad S_3 = 180^\circ \quad S_{2-3} = 450^\circ \]

\[ \phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \]

\[ \phi_3 \]

\[ 180^\circ \]

\[ 90^\circ \]

\[ 0^\circ \]

\[ -90^\circ \]
THREE-ELEMENT ANTENNA PATTERNS

\[ K_2 = 1 \quad K_3 = 1 \quad \beta = 180^\circ \quad S_2 = 360^\circ \quad S_3 = 180^\circ \quad S_{2-3} = 540^\circ \]

\[ \phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \]

\[ \phi_3 \]

\[ 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \]
THREE-ELEMENT ANTENNA PATTERNS

$K_2 = 1$  $K_3 = 1$  $\beta = 180^\circ$  $S_2 = 270^\circ$  $S_3 = 270^\circ$  $S_{2-3} = 540^\circ$

$\phi_2 \rightarrow 180^\circ$  $90^\circ$  $0^\circ$  $-90^\circ$

$\phi_3$

$180^\circ$

$90^\circ$

$0^\circ$

$-90^\circ$
THREE-ELEMENT ANTENNA PATTERNS

\[
\begin{array}{c}
K_2 = 1 \\
K_3 = 1 \\
\beta = 180^\circ \\
S_2 = 360^\circ \\
S_3 = 270^\circ \\
S_{2-3} = 630^\circ \\
\phi_2 \rightarrow 180^\circ \\
90^\circ \\
0^\circ \\
-90^\circ \\
\phi_3 \\
180^\circ \\
90^\circ \\
0^\circ \\
-90^\circ \\
\end{array}
\]
THREE-ELEMENT ANTENNA PATTERNS

\[ K_2 = 1 \quad K_3 = 1 \quad \beta = 180^\circ \quad S_2 = 360^\circ \quad S_3 = 360^\circ \quad S_{2,3} = 720^\circ \]

\[ \phi_2 \rightarrow 180^\circ \quad 90^\circ \quad 0^\circ \quad -90^\circ \]

\[ \phi_3 \]

\[ 180^\circ \]

\[ 90^\circ \]

\[ 0^\circ \]

\[ -90^\circ \]