AN ELECTRONIC PARALLEL INVERTER WITH POLYPHASE SINUSOIDAL OUTPUT VOLTAGE

by

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INTRODUCTION

This thesis will cover three main points. The first point is to give a general idea of the operation of the various types of electronic inverter circuits, mainly the parallel and series circuits, and to show how each can be made to be separately or self-excited.

The second point will consist of an analysis of the parallel inverter circuit with resistive load and the design of an inverter. The analysis and design of an electronic control circuit for a three-phase inverter will also be covered.

The last point to be covered will be the testing of the experimental inverter under various load conditions. The output wave forms obtained will be compared with those of a typical alternating current generator and a practical power system.

The control system for this inverter will be unusual in that it will be driven by a single-phase oscillator instead of the usual procedure of using an existing polyphase circuit. This type of control was required because of the desire to have the output frequency of the inverter very stable instead of having the normal variation that is common to electrical machinery or power systems.

THE OPERATION OF ELECTRONIC INVERTERS

There are two distinct types of electronic inverters: (1) those which are separately excited and (2) those which are self-excited. Separately excited inverters are of the nature of amplifiers, as they require a small amount of alternating current power for their excitation but are capable of producing a much larger amount of alternating current power in the output circuit. Self-excited inverters are more of the nature of oscillators, as they supply their own excitation losses from the alternating current output of the circuit.

SEPARATELY EXCITED INVERTERS. A typical circuit of a separately excited inverter is shown in figure 1. All of the energy taken from the output terminals is drawn from the direct current power supply indicated in the lower part of the figure. The alternating current excitation must be supplied from an available low power source such as a vacuum-tube oscillator.

The operation of this inverter may be studied by first considering the half cycle of excitation voltage during which the grid of tube 1 is positive and assuming that tube 2 is not drawing plate current. Plate current begins to flow in tube 1 as soon as the grid becomes more positive than the starting voltage, and as this current increases an emf will be induced in the output transformer which will be impressed across the condenser C, causing it to charge with a negative polarity toward the plate of tube 1. An emf will also be induced into the secondary of the output transformer and so be impressed on the load.

At the start of the next half cycle the grid of tube 1 will become negative, but this will have no effect upon the flow of plate current. However, the grid of tube 2 will become positive, and this tube will fire causing current to flow from the direct current source through the lower half of the output transformer. The voltage drop between anode and cathode of tube 2 is very low during conduction so that the lower terminal of condenser C will be virtually connected to the negative terminal of the direct current supply. Since it had been previously charged by tube 1 with negative polarity on its upper plate, the full negative charge of this condenser will momentarily be applied between the plate and cathode of tube 1, permitting its grid to resume control. If the grid of tube 1 were not negative at the time this action takes place, both tubes would conduct after condenser C discharged, thus short-circuiting the direct current supply.

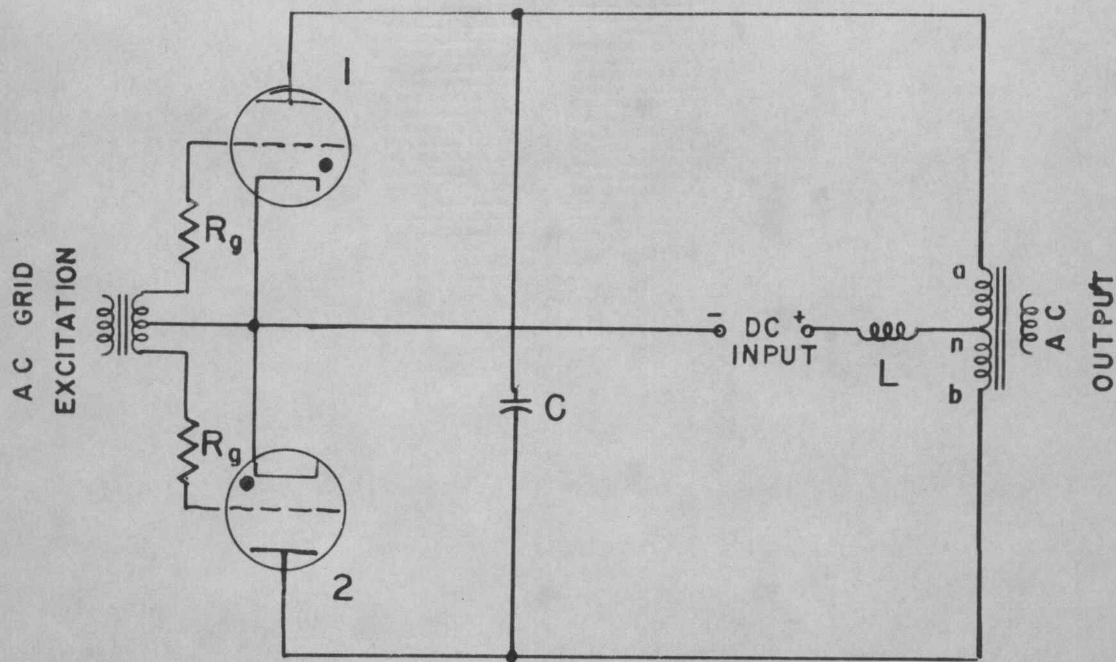


FIGURE 1. SEPARATELY EXCITED INVERTER, PARALLEL TYPE

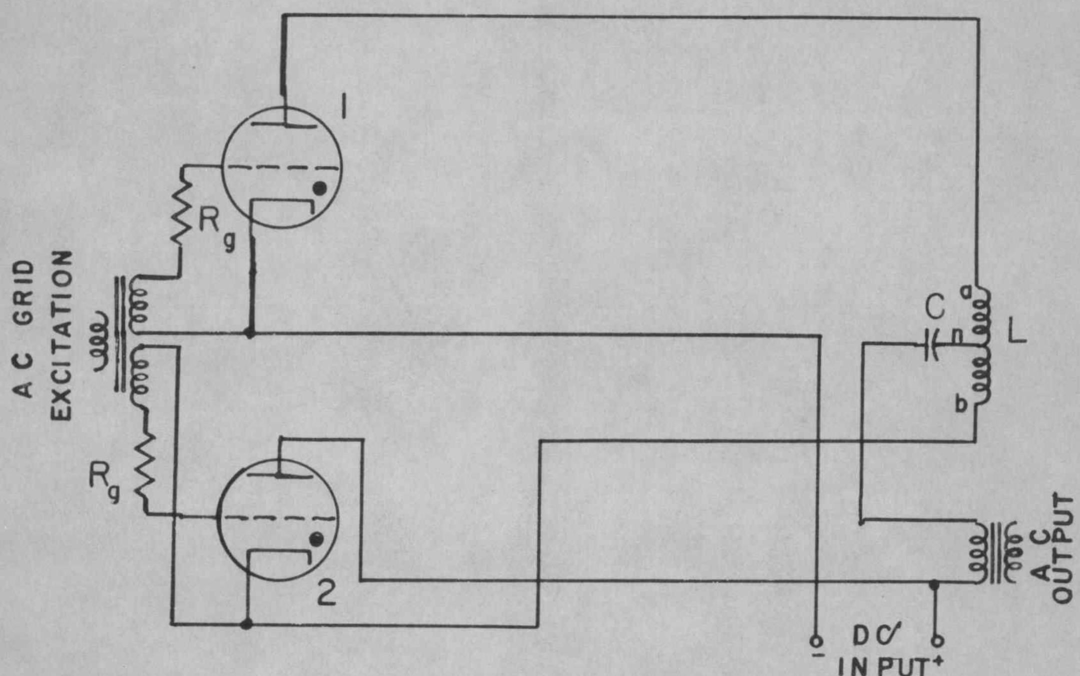


FIGURE 2. SEPARATELY EXCITED INVERTER, SERIES TYPE

The action described in the preceding paragraph continues to repeat itself; the action of the two tubes being interchanged at the beginning of successive half cycles.

Condenser C is known as a commutating condenser because it is provided solely for the purpose of commutating the direct current from one tube to the other at the end of each half cycle of the exciting voltage. Its size must be such as to provide a negative charge of sufficient duration to enable the grid to assume control.

A leading power-factor load will obviously serve the same purpose as the commutating condenser C. On the other hand, a lagging power-factor load will tend to reduce the effect of Condenser C, so that a much larger condenser must be used in an inverter required to handle an inductive load. Thus it is important to know the type of load that is to be supplied before designing the inverter.

The two thyratrons may be operated in a series type of circuit as compared with the parallel arrangement of figure 1. A typical circuit of this type is that of figure 2. Consider that tube 1 is conducting and that tube 2 is idle. Current flows from the positive terminals through the primary of the output transformer T, the upper half of the choke L, and tube 1, charging the condenser C, with positive polarity on its left terminal.

As the polarity of the grid excitation voltage changes, tube 2 will fire, causing the condenser to discharge, thereby inducing a voltage of opposite polarity in the output transformer and inducing a high negative voltage between terminals "a" and "n" of the choke L, sufficient to extinguish the arc in tube 1 and permit its negative grid to assume control again.

On the next half cycle of the excitation voltage tube 1 will again fire causing the condenser to charge. A large voltage is again built up across the inductance and a positive voltage appears between "b" and "n" which is applied to the cathode of tube 2. Since this will have the same effect as applying a negative voltage to the plate, plate current will cease to flow and the grid can again assume control. This cycle is repeated as the polarity of the excitation voltage reverses--thus producing an a-c output across the secondary of transformer T of the same frequency as the excitation voltage but of much higher power.

Since the same polarity of voltage across L is required to extinguish either tube 1 or tube 2, it might seem that both tubes should cease to conduct simultaneously. That they do not is due to the difference in the charge on the condenser C, this condenser being fully charged when tube 2 fires and at a minimum charge when tube 1 fires. The potential applied to the plate of tube 1 is

$$E_{b1} = E_o - E_c - E_t$$

where E_o is the supply voltage

E_t is the voltage across half coil L plus voltage across T

E_c is the voltage condenser C

While the plate voltage of tube 2 is

$$E_b{}^2 = E_c - E_t$$

When tube 2 fires, E_c is very nearly equal to E_o and E_t is somewhat less than E_c. The above equations therefore show that tube 1 will have a negative plate voltage while that of tube 2 is somewhat positive. When tube 1 fires, condenser C will have discharged and its voltage will be quite small; thus, the equations show that the polarity applied at that time to tube 1 will be somewhat positive while that applied to tube 2 will be negative. These are the necessary conditions for commutation of the current from one tube to the other.

Series inverters are generally preferred to the parallel type owing to the reduced probability of short circuiting the direct current supply through the failure of a tube to commutate. This improvement is due to the maintenance of negative voltage on the plate of the thyratron at the end of its conducting period for a longer period of time in the series type than in the parallel type, thus increasing the time for the tube to deionize.

SELF-EXCITED INVERTERS. Either the parallel or the series type of inverter may be made self-excited by supplying the grid excitation from the alternating current output terminals through suitable circuits. Figure 3 shows such an application to the parallel type of inverter. T_1 is the output transformer, and C_1 is the commutating condenser, as in the separately excited inverter. The grids of the two tubes are excited by means of transformer T_2 which is energized from the output transformer as shown.

When the direct current circuit is first closed to start the inverter, both tubes would start conducting at once if a means were not provided to discriminate against one or the other. Condenser C_3 performs this function, being charged through R_1 and L_1, with the return circuit through R_3 and half of transformer T_2. The charging current of this condenser induces a positive voltage in that side of transformer T_2 supplying the grid of tube 1 and consequently induces a negative voltage on the grid of tube 2. Therefore, tube 1 will be the first to conduct and tube 2 will be held inoperative.

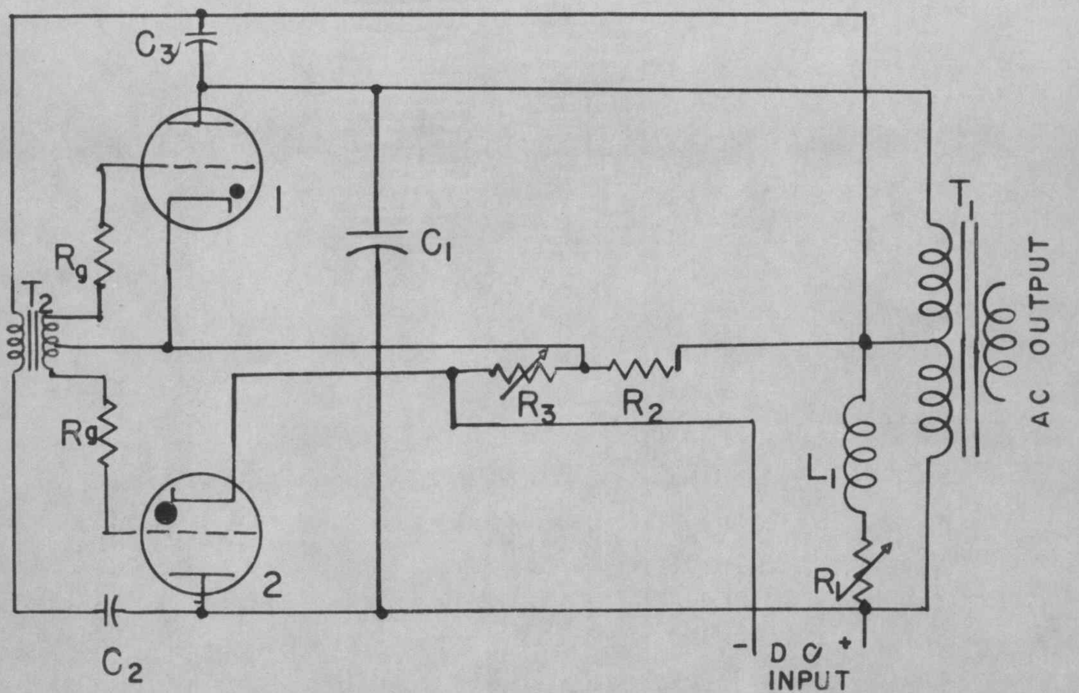


FIGURE 3. SELF EXCITED INVERTER, PARALLEL TYPE

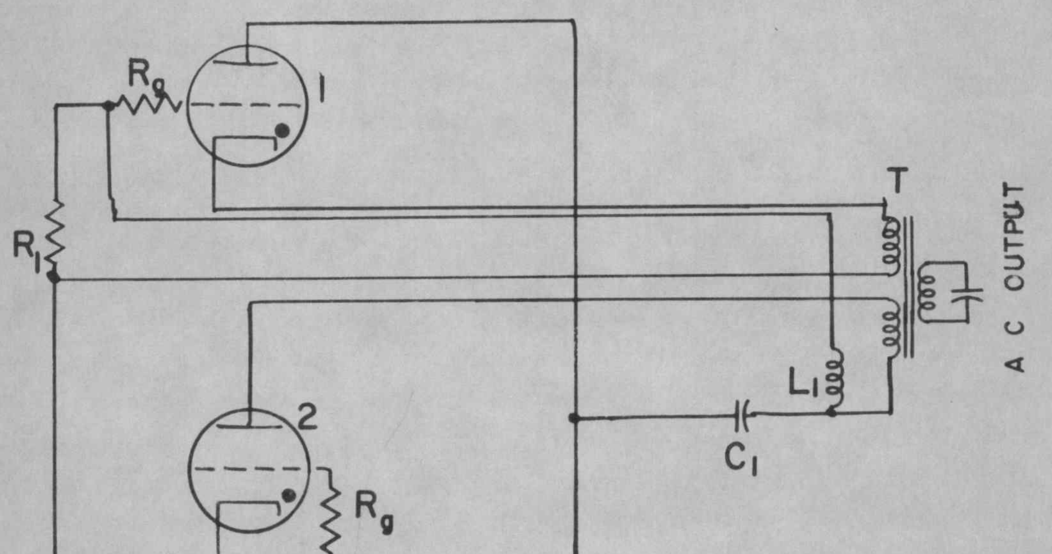


FIGURE 4. SELF EXCITED INVERTER, SERIES TYPE

At the same time the plate current drawn by tube 1 through the upper half of transformer T_1 will induce a negative voltage on the upper terminal of that half. The lower half of the transformer will therefore impress a positive voltage on the plate of tube 2 which is a function of the charging period of condenser C_1. Eventually tube 2 will become conducting as the induced voltage in transformer T_2 falls off at the end of the charging period of condenser C_1 and allows the grid to go positive. When this occurs, condenser C_1 will apply a high negative voltage on the plate of tube 1 to stop the flow of plate current as previously discussed for the separately excited inverter.

The impedance of transformer T_1 prevents the instantaneous discharge of condenser C_1 thus maintaining its negative voltage sufficiently long for the simultaneously induced voltage in transformer T_2 to swing the grid of tube 1 negative and prevent reestablishment of current through this tube for another half cycle.

The sequence of events just described continues to occur with the action of the two tubes interchanged, the frequency of the output being determined primarily by the constants of the output circuit.

A series type of inverter is shown in figure 4. If tube 1 is conducting, current will flow from the direct current source, charging condenser C_2. The current also passes through the resistance R_2 and so maintains a negative voltage on the grid of tube 2 which prevents that tube from firing. As condenser C_2 becomes charged, the current flow ceases, causing tube 1 to stop conducting and removing the negative voltage from the grid of tube 2. Firing of tube 2 follows, and condenser C_2 then discharges, setting up a negative voltage across R_1 which is applied to the grid of tube 1. Also, the drop across coil L_1 applies a negative voltage to the anode of tube 1 through the condenser C_1, which has zero voltage across its terminals at the time when tube 2 fires, since it was discharged by tube 1 while C_2 was charging. The negative potential remains on the anode for a sufficient length of time to permit the tube to deionize and so permit the grid to resume control. After condenser C_2 has discharged and C_1 charged, the current flow through tube 2 ceases and tube 1 fires. In this case the discharge of condenser C_1 sets up a voltage across L_1 of the opposite polarity, thus applying a negative voltage to the anode of tube 2 which permits it to deionize. This cycle is then repeated at a frequency determined by the circuit constants.

THE PARALLEL INVERTER WITH RESISTIVE LOAD

In general, two distinct modes of operation are recognizable. These two modes are dependent upon the relative values of the circuit constants and arise from the physical limitations of the tubes which permit current conduction in one direction only. Under certain circumstances, the current in the direct current side of the inverter flows in pulses, two for each cycle of the control frequency, with periods of current zero intervening. Under other circumstances, the current in the direct current side is continuous without reaching zero. The calculations of current and voltage for the former case is much more involved than that for the latter. However, it so happens that the most important case for which the current flows in pulses is that in which the periods of current zero approach zero. But this may be regarded also as a limiting case for which the current on the direct current side is continuous. No attempt will be made, therefore, to derive expressions for the case in which the direct current flows in pulses.

In the following development, these simplifications are made:

1. The magnetizing current and the leakage reactance of the transformer are neglected.

2. All three windings have the same number of turns. If this is not the case, the load impedance can be reduced readily to an equivalent impedance.

3. The resistance of the impedance in the direct current side is neglected. This can be corrected for by subtracting a constant voltage drop from the applied continuous voltage.

4. The voltage drops in the tubes are neglected. Since this is essentially constant regardless of current, it can be corrected by subtracting the drop from the applied voltage.

The circuit under consideration is shown schematically in figure 5, in which the symbols are self-explanatory. Part or all of the capacitor C may be connected in parallel with the load. It may also be desirable that some of the capacitance be switched on simultaneously with the load.

The general plan of attack will be to set up the differential equations for that portion of the circuit involved while one tube is conducting and the other tube is open circuited. Because of certain symmetry in the circuit, definite terminal conditions may be set up. Thus the load current at the beginning of a half period must be equal and opposite to the load current at the end of the half period. Similarly, the charge q on the

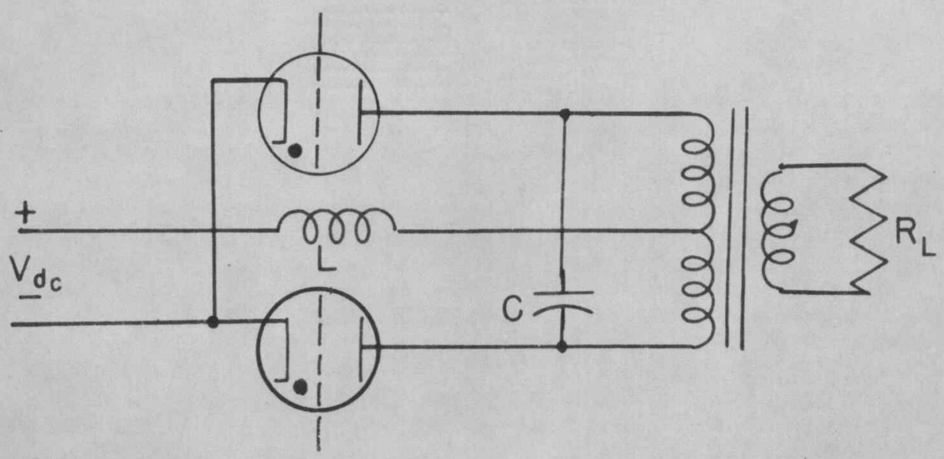


FIGURE 5. PARALLEL INVERTER WITH RESISTIVE LOAD

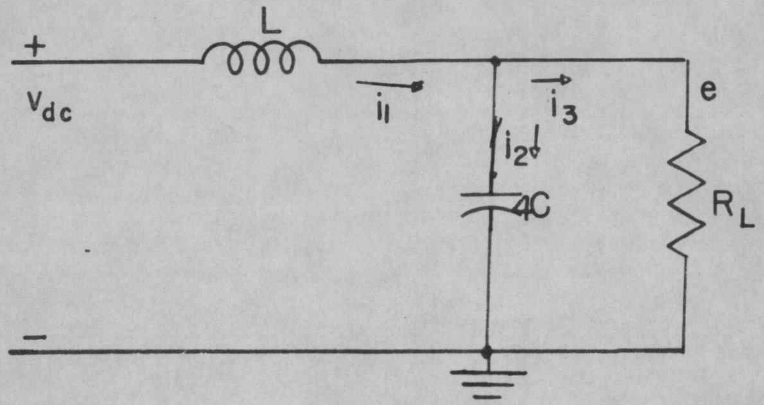


FIGURE 6. EQUIVALENT CIRCUIT OF INVERTER WITH RESISTIVE LOAD

commutating capacitor at the beginning of a half period must be equal and opposite to the charge at the end of the half period. Calling the period T, these relations may be expressed by the following:

$$i_b(\text{for } t = 0) = i_b(\text{for } t = T/2)$$

$$q(\text{for } t = 0) = -q(\text{for } t = T/2)$$

The foregoing assumptions permit the elimination of the transformer for the purpose of setting up the differential equations, thus reducing the equivalent circuit to that shown in figure 6. The output voltage E in the alternating current side will be evaluated in terms of the direct current voltage V and the constants of the circuit.

From the figure shown, the following differential equations may be written:

$$i_1 = \frac{1}{L}\int(V - E)\,dt$$

$$i_2 = 4C\,dE/dt$$

$$i_3 = E/R$$

By use of the Laplace transform these equations become

$$I_{1(s)} = \frac{(V_{(s)} - E_{(s)}}{L_s} + \frac{I_o}{s}$$

$$I_{2(s)} = 4CSE + 4CE_o$$

$$I_{3(s)} = E_{(s)} / R$$

where

I_o is the initial current flowing in the inductor.

E_o is the initial voltage appearing across the terminals of the condenser C

Since in an electrical circuit the sum of the currents at a point must be equal to zero,

$$I_1 = I_2 + I_3$$

and

$$\frac{V_s - E}{LS} + \frac{IO}{s} = 4CSE_s + 4CE_o + E/R$$

Assuming an applied voltage of a step function as would be the case in the firing of a thyratron, the equation becomes:

$$\frac{V}{LS^2} + \frac{I_o}{S} = \frac{E_{(s)}}{LS} + \frac{E_{(s)}}{R} + 4CSE_{(s)} + 4CE_o$$

which can be reduced as follows:

$$E_{(s)} = \frac{V + I_oLS + 4CE_oS^2}{LS^2\left(\frac{1}{LS} + \frac{1}{R} + 4CS\right)}$$

$$E_{(s)} = \frac{V + I_oLS + 4CE_oLS^2}{S(1 + \frac{LS}{R} + 4CLS^2)}$$

$$E_s = \frac{(V + I_oLS + 4CE_oLS^2)\,\frac{1}{4CL}}{S(S^2 + \frac{S}{4CR} + \frac{1}{4CL})}$$

In order to solve this equation it is necessary to find the roots of the following auxiliary equation:

$$S(S^2 + \frac{S}{4CR} + \frac{1}{4CL}) = 0 \qquad \text{Eq1}$$

These roots are

$$S = 0$$

and

$$S = -\frac{1}{8CR} \pm \sqrt{\frac{1}{64C^2R^2} - \frac{1}{4CL}}$$

These roots may be either complex or real. In the former case the solution consists of damped sinusoids, but in the latter of exponentials. Since the output of this inverter is to be sinusoidal only the case in which the roots are complex will be considered.

For the case in which the roots are complex, let them be designated as

$$S = (-a \pm jb)$$

in which

$$a = \frac{1}{8CR}$$

$$b = \sqrt{\frac{1}{4CL} - \frac{1}{64C^2R^2}}$$

therefore

$$E_{(s)} = \frac{V/4CL + I_oS/4C + E_oS^2}{S(S + a + jb)\,(S + a - jb)}$$

This equation may now be solved by breaking it into three parts:

$$E_{(s)} = \frac{V}{4CL}\left[S(S+a+jb)(S+a-jb)\right]^{-1}$$

$$+ \frac{I_o}{4C}\left[(S+a+jb)(S+a-jb)\right]^{-1}$$

$$+ E_o \quad \frac{S}{(S+a+jb)(S+a-jb)}$$

Now let

$$E_{1(s)} = \frac{V}{4CL}\left[S(S+a+jb)(S+a-jb)\right]^{-1}$$

$$E_{2(s)} = \frac{I_o}{4C}\left[(S+a+jb)(S+a-jb)\right]^{-1}$$

$$E_{3(s)} = E_o \quad \frac{S}{(s+a+jb)(s+a-jb)}$$

Taking the inverse transforms of these equations

$$E_1 = V - \frac{V^{-at}}{2jb}\left[-ae^{-jbt} + ae^{+jbt} + jbe^{-jbt} + be^{+jbt}\right]$$

$$E_2 = \frac{I_o}{8jb}e^{-at}\left[-e^{-jbt} + e^{+jbt}\right]$$

$$E_3 = \frac{E_o e^{-at}}{2jb}\left[-ae^{-at} + ae^{+jbt} - jbe^{-jbt} - jbe^{+jbt}\right]$$

These equations can be reduced to the following circular functions:

$$E_1 = V - Ve^{-at}\left(\frac{a \sin bt}{b} + \cos bt\right)$$

$$E_2 = \frac{I_o e^{-at}}{4bC} \sin bt$$

$$E_3 = E_o e^{-at} \left(\frac{a \sin bt}{b} - \cos bt \right)$$

since

$$E = \mathcal{L}^{-1} E_{1s} + \mathcal{L}^{-1} E_{2s} + \mathcal{L}^{-1} E_{3s}$$

$$E = V + e^{-at}\left\{\left[-\frac{Va}{b} + \frac{I_o}{4Cb} + \frac{E_o a}{b}\right] \sin bt + \left[-E_o - V\right] \cos bt\right\}$$

Substituting initial values which are

$$E_o = V$$

$$I_o = V/R$$

gives

$$E = V + Ve^{-at}\left\{\frac{1}{4CbR} \sin bt - 2 \cos bt\right\}$$

or

$$E = V + Ve^{-\frac{t}{8CR}}\left\{\frac{1 \sin bt}{4CR\left(\frac{1}{4CL} - \frac{1}{64C^2R^2}\right)^{\frac{1}{2}}} - 2 \cos bt\right\}$$

$$= V + Ve^{-t/8CR}\left\{\frac{1 \sin bt}{\left(4\frac{CR^2}{L} - \frac{1}{4}\right)^{\frac{1}{2}}} - 2 \cos bt\right\}$$

$$= V + Ve^{-t/8CR}\left\{2\sqrt{L}\,\frac{\sin bt}{(16CR^2 - L)^{\frac{1}{2}}} - 2 \cos bt\right\}$$

$$= V + Ve^{-t/8CR}\left(\frac{4L}{16CR^2-L} + 4\right)^{\frac{1}{2}} \sin\left[bt + \tan^{-1} -\left(\frac{16CR-L}{L}\right)^{\frac{1}{2}}\right]$$

$$= V + Ve^{-t/8CR}\left(\frac{64CR^2}{16CR^2-L}\right)^{\frac{1}{2}} \sin\left(bt + \tan^{-1} - \sqrt{\frac{16CR^2-L}{L}}\right) \qquad \text{Eq2}$$

In the design of an inverter, it is, as already mentioned, necessary to know the approximate impedance of the load. The term $(64CR^2/16CR^2-L)^{\frac{1}{2}}$ in the previous equation determines the ratio of the maximum alternating current output voltage to the direct current voltage. The term $e^{-t/8CR}$ is the rate of damping of the alternating current voltage. It is necessary that the product of RC be large if the inverter is to have a sinusoidal output. The sine term consists of two parts; the first varies with time and determines the correct operating frequency for the inverter, while the second term determines the angular delay between the firing of the thyratrons and the output voltage.

Since it is desired that this inverter have an output that is sinusoidal, it is necessary that the term bt in equation 2 be equal to ωt for the frequency at which the inverter is to be operated. In this case, where the frequency is 60 cycles the term must equal 377t.

In order to accomplish this, the term bt must be set equal to ωt, and the following equations result:

$$\omega = b$$

but

$$b = \frac{1}{4CL} - \frac{1}{64C^2R^2}$$

$$\omega^2 = \frac{1}{4CL} - \frac{1}{64C^2R^2}$$

$$0 = -16CR^2 + L + 64C^2R^2L^2\omega^2$$

$$C = \frac{1}{8L^2\omega^2} \pm \frac{1}{8L^2\omega^2}\sqrt{1 - \frac{L^2\omega^2}{R}}$$

For a frequency of 60 cycles per second and the inductance available for this project of 0.12 henry:

$$C = 7.35\left(1 \pm \sqrt{1 - \frac{2045}{R^2}}\right) \text{ microfarads}$$

The thyratrons to be used were type 3C23 which have an average current rating of 1.5 amperes. It was desired to have the tubes not operate in excess of rating and also to have an output voltage of 200 volts. Therefore, the inverter was designed to supply a load of 150 ohms. It was found by using this value of resistance in the previous equation that the commutating capacitor should be 12 microfarads. Substituting these values of R, L, C, and ω in equation 2 gives the following formula for the output voltage:

$$E = V + 2.1Ve^{-70t} \sin (377 + 88^{\circ})$$

The voltage in the above formula is the voltage across the terminals of the transformers on the direct current side. The voltage on the alternating current side would be

$$E_{a-c} = 2.1Ve^{-70t} \sin (377t + 88^{\circ})$$

if the turns ratio of the transformer is one as assumed.

THE CONTROL CIRCUIT

The control circuit is probably the most important part of the inverter, for without it the inverter could not operate at all. The control circuit is to blame for many of the shortcomings of a normal inverter for if it is not operated properly, the direct current source of energy can be short circuited. It is the control circuit that determines (1) the frequency of the inverter; (2) the amount of power the inverter supplies if connected to an alternating current bus with a source of power; (3) and to a large extent the amount of harmonic distortion in the inverter.

In order for the inverter to work properly, it is required that the control circuit have the following properties: (1) Fire the six thyratrons at intervals of 60 degrees. In order to accomplish this it is required that the control be adjustable for the individual tube characteristics. (2) Advance or retard the firing of the thyratrons in order to control the amount of power supplied by the inverter to a system. (3) Be reliable in operation.

The control system for this project did not have to fulfill the second requirement above because it was not desired to have the inverter supply power to a bus with several sources of power. This control did, however, have to receive its signal from that of a low frequency signal generator.

There are several ways to control thyratrons. For this project it was decided to use the phase-shift method. By use of this method it was required to shift the output of the audio oscillator by 60° and 120°. Since the oscillator to be used had a balanced output with negligible distortion between the output terminal and ground, it was not necessary to use a phase inverter to get the control for the other tubes.

The actual circuit for the control system is shown in figure 7. The circuit in block notation is in figure 8. In order to represent the system by the block diagram, it was necessary to determine the transfer function of each of the four basic circuits used. The determination of the transfer function was made on the basis that the loading of one element on the other was negligible. While this is not true the error involved is very small.

The first basic circuit to be analyzed is that shown in figure 9. Since there is no loading on the circuit, i_1 is equal to i_2. Therefore

$$C \frac{de_{in}}{dt} - C \frac{de_{out}}{dt} = \frac{e_{out}}{R}$$

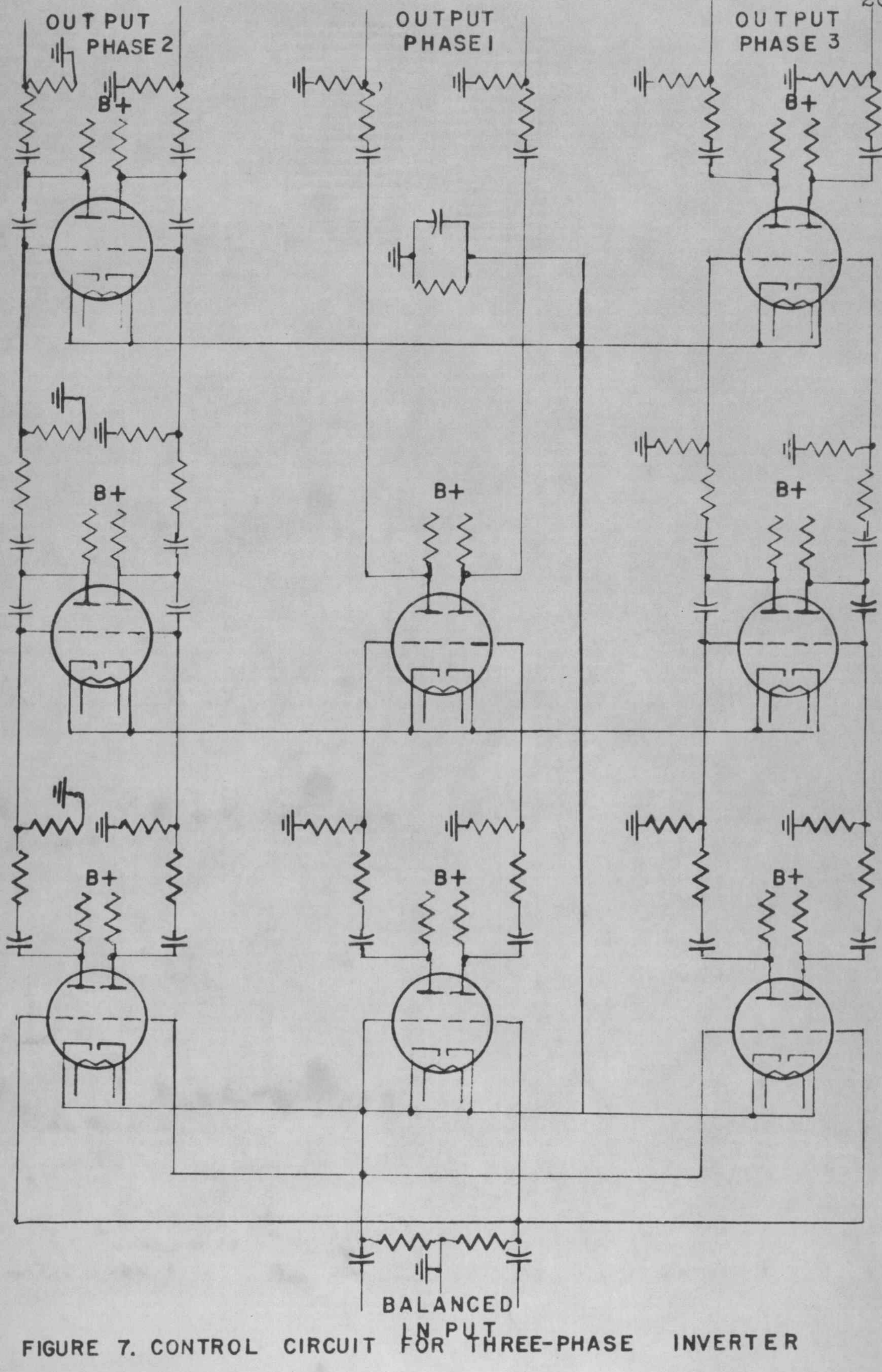


FIGURE 7. CONTROL CIRCUIT FOR THREE-PHASE INVERTER

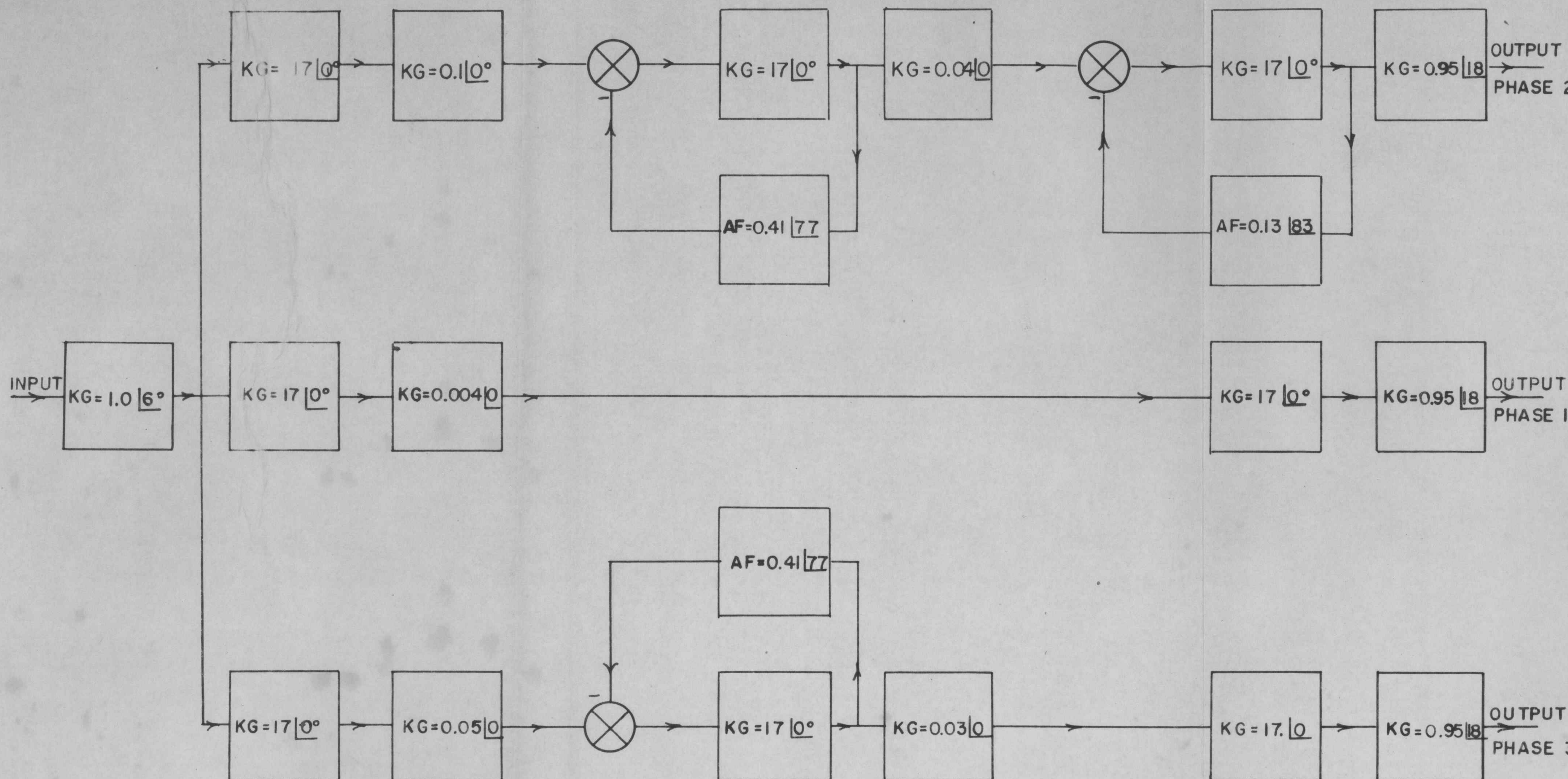


FIGURE 8. CONTROL CIRCUIT IN BLOCK NOTATION

By use of the Laplace transform

$$CS(E_{sin} - E_{sout}) = \frac{Eout}{R}$$

or

$$\frac{E_{out_s}}{E_{in}} = \frac{CS}{CS + R}$$

Assuming a sinusoidal input this becomes

$$\frac{E_{out}}{E_{in}}(j\omega) = 1/(1 - j\ 1/\omega CR)$$

The second circuit to be analyzed is that of a voltage amplifier. A typical voltage amplifier is shown in figure 10A with its equivalent circuit in figure 10B. In this circuit

$$\frac{e_{out}}{RL} = \mu\frac{e_{in} - e_{out}}{r_p}$$

This reduces to the following transfer function

$$\frac{e_{out}}{e_{in}} = \mu\frac{R_L}{R_L + rp}$$

The next circuit is that of a voltage divider shown in figure 11,

$$\frac{e_{in} - e_{out}}{R-RK} = e_{out}/RK$$

which reduces to

$$e_{out}/e_{in} = K$$

The last circuit to be analyzed is that shown in figure 12A. This is a feedback amplifier in which the feedback is not equal to one. Using normal block

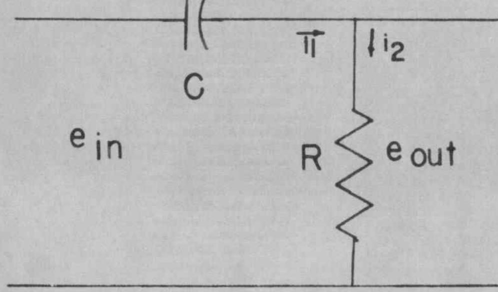
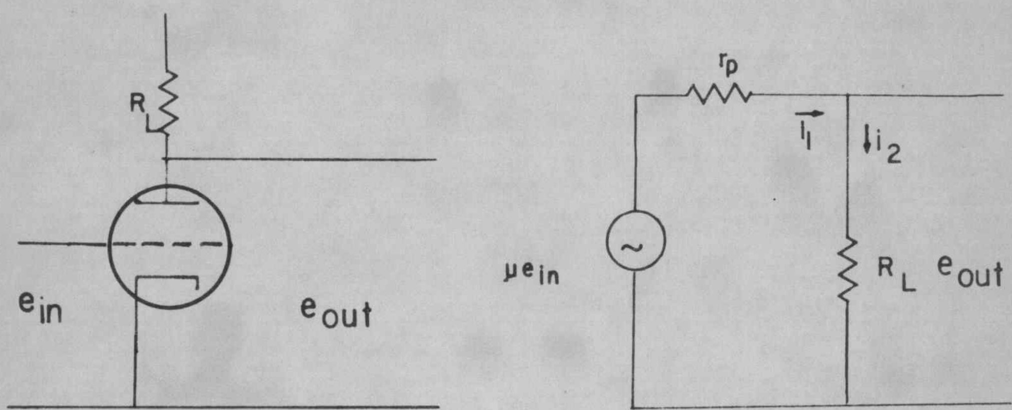


FIGURE 9. PHASE-SHIFT NETWORK



A. ACTUAL CIRCUIT

B. EQUIVALENT CIRCUIT

FIGURE 10. VOLTAGE AMPLIFIER CIRCUIT

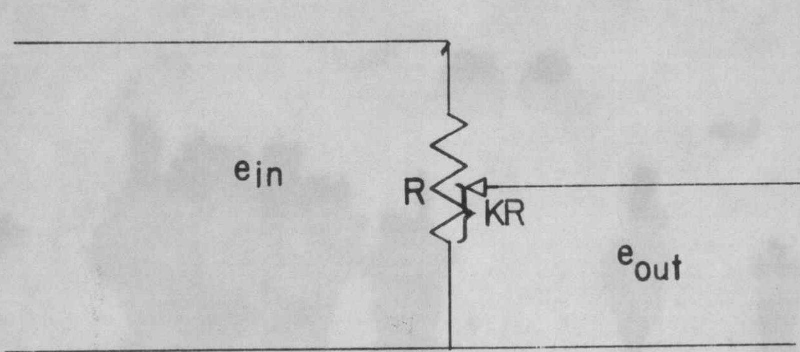


FIGURE 11. POTENTIOMETER CIRCUIT

notation this circuit can be represented as shown in figure 12B, in which e_{in} is the input voltage, e_{out} is the output voltage, e_f is the feedback voltage, e_s is the difference between e_{in} and e_f, AF is the transfer function of the feedback network and KG is the transfer function of the amplifier. For the circuit shown the following equations may be written:

$$\frac{e_{out}}{e_s} j\omega = KG(j\omega) \qquad 1$$

$$\frac{e_f\ j\omega}{e_{put}} = AF(j\omega) \qquad 2$$

$$e_s = e_{in} - e_f \qquad 3$$

Substituting equation 3 in equation 1 gives

$$\frac{e_{out}}{e_{in}-e_f} j\omega = KG(j\omega) \qquad 4$$

and substituting 2 in 4 gives

$$\frac{e_{out}}{e_{in} - e_{out}(AFj\omega)} j\omega = KG(j\omega)$$

$$e_{out} = (KGj\omega)\ e_{in}-e_{out}(AFj\omega)$$

$$= (KGj\omega)e_{in} - (KGj\omega e_{out}AFj\omega)$$

From this

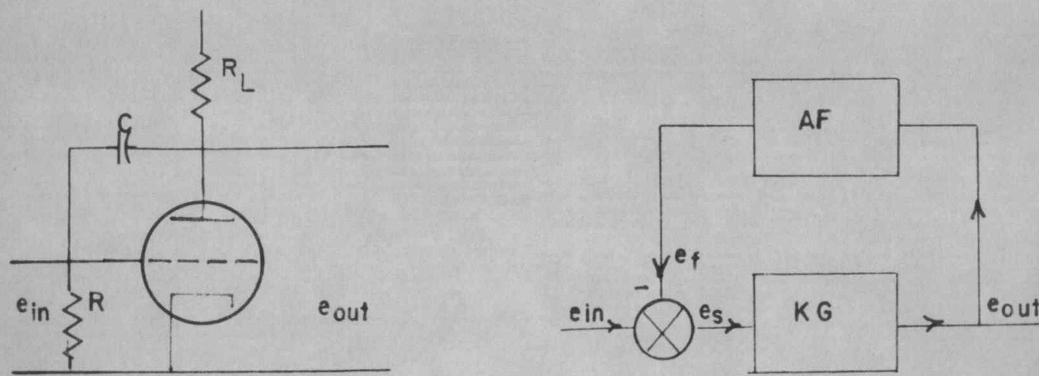
$$e_{out}(1 + KGj\omega AFj\omega) = e_{in}KGj\omega$$

and

$$\frac{e_{out}}{e_{in}} j\omega = \frac{KG(j\omega)}{1 + KGj\omega AFj\omega}.$$

The feedback amplifiers were designed so that each amplifier would have 60 degrees phase shift when operated at 60 cycles per second. In order that the three inverters would operate satisfactorily together it was necessary that the three balanced control circuits have the same gain and that the phase shift across the control differ by 60 and 120 degrees. As can be seen from figure 13 the gain in the three circuits was the same and the phase shift differed by the required amount.

In the actual wiring diagram, figure 19, it can be seen that potentiometers were used in order to control the phase shift and amplitude. This was necessary due to the assumption made in the design of the circuit, frequency error, tolerance in the parts involved, and variation in the firing characteristics of the tubes involved.



A. ACTUAL CIRCUIT B. BLOCK NOTATION

FIGURE 12. FEEDBACK AMPLIFIER WITH FEEDBACK ≠ UNITY

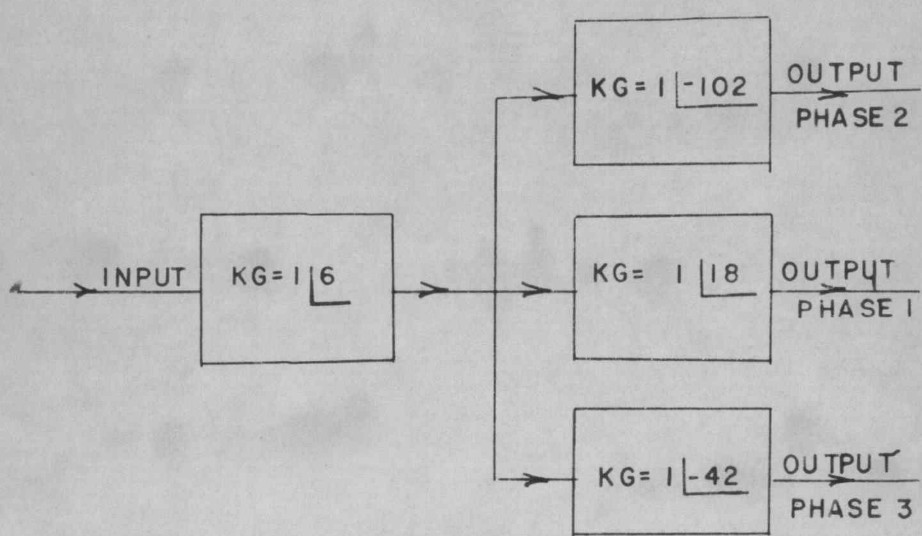


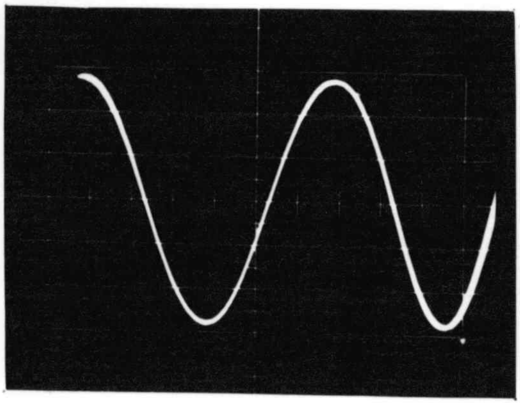
FIGURE 13. EQUIVALENT BLOCK NOTATION FOR CONTROL CIRCUIT

TESTING OF THE INVERTER

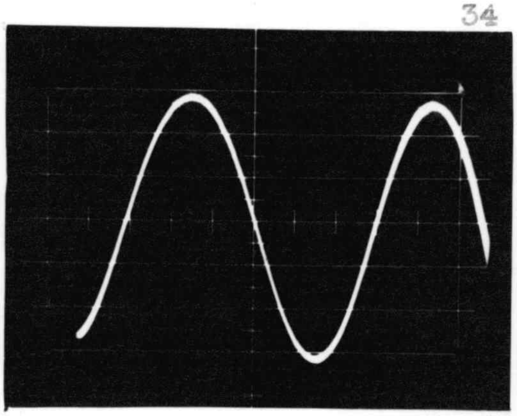
The inverter was constructed in accordance with the preceding two sections. Upon completion tests were performed to see if the inverter performed in accordance with the circuit analysis previously performed.

The control circuit acted satisfactorily as designed. The output of the control circuits are shown in figure 14. This figure shows only three of the six control voltages; the other three voltages are 180 degrees out of phase with those shown. In figure 14E, it can be seen that the phase shift between input and output for the phase with no phase shift feedback amplifiers was approximately 20 degrees. This compares very favorably with the calculated value.

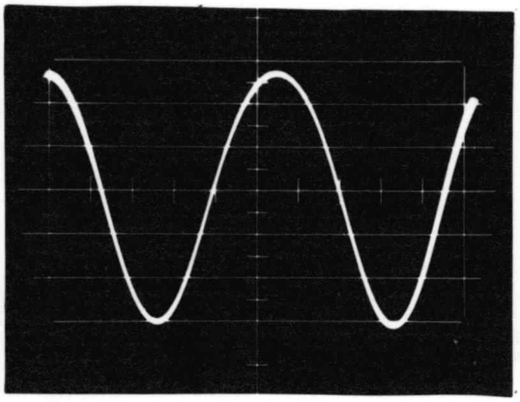
The adjustments of the control system were very critical as the slightest error in the firing of the thyratrons would cause the harmonics in the output waveform to increase greatly. As was stated before the firing of the thyratrons was dependent upon the magnitude and the phase shift of the control voltage. It was found also that any distortion in this control voltage could cause the tubes to fire either before or behind time. The change in firing time is caused by the above factors causing the time of the control voltage crossing



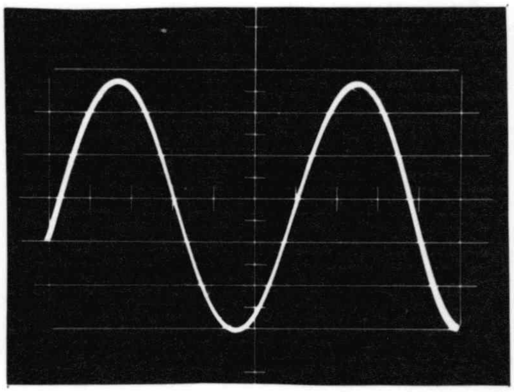
(a) Input



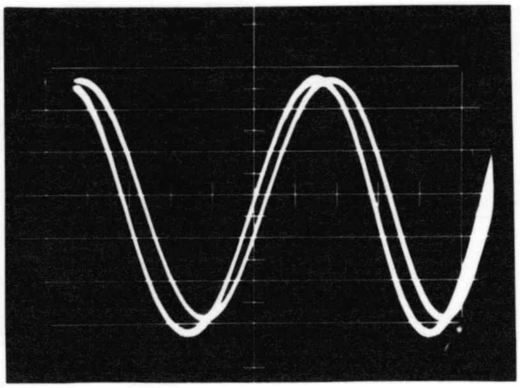
(b) Output phase one



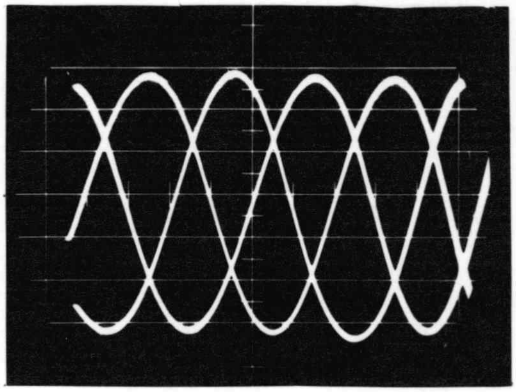
(c) Output phase two



(d) Output phase three



(e) Input and output phase 1



(f) Three-phase output

Figure 14. Typical voltage wave forms of control circuit.

the thyratron critical grid characteristic curve to change. The thyratrons used in this project had directly heated cathodes. It was found that if the frequency of the inverter varied from that of the alternating current power supplied to the filaments the time of firing would be changed slightly and the harmonic content of the output of the inverter would be increased. In order to avoid this trouble it is possible to do many things. Several ways are (1) increase the gain of the control circuit so that the magnitude of the control voltage is increased; (2) use thyratrons with indirectly heated cathodes; (3) supply the voltage for the filaments of the tubes from a separate direct current voltage source; (4) supply the filaments of the tubes from the output of the inverter after it started operating.

In this as in many thyratron circuits, the presence of grid current had to be taken into account in the construction of the control system because if the thyratron grid resistor was too small there would be a breakdown between the grid and cathode and poor control would result; on the other hand, if the grid resistor was too large the control circuit would lose control because there would be no path for the grid current to flow. It was found that the grid current was reversed if the grid resistor was reduced from 100,000 to 10 ohms. In the first case the electron flow was from the grid to the cathode. When the grid resistor was 10 ohms the grid would act like the plate of a rectifier and a rectified current would flow through the grid resistor. This could be seen by the presence of a blue gas discharge between the grid and cathode.

As would be expected by the use of phase shifting networks, this type of control can only be used where it is desired to have a fixed output frequency.

The inverter operated in accordance with the circuit analysis previously developed as far as output waveform is concerned. However, the ratio of the output alternating current voltage to the input direct current voltage was much larger than anticipated. This ratio also varied with the resistance of the load instead of remaining almost constant. This was due to the transformer having been assumed to be an ideal transformer. This, however, was not true. This assumption would be justified if the full load rating of the transformer was the same as the full load rating of the inverter. In this case, however, the full load rating of the transformer was 1.5 KVA per phase while that of the inverter was found to be 0.15 KW per phase at unity power factor.

Since in this case the magnetizing current and the leakage reactance of the transformer could not be neglected, the equivalent circuit should have been that of figure 15. As can be seen, this circuit would have been very difficult to analyze. Because of the complexity of this circuit and the fact that in any practical circuit the transformer would not have a rating which is ten times the rating of the thyratrons used in the circuit. A qualitative explanation of what occurred will be made instead of a quantitative.

Mr. C. F. Wagner (7, pp.970-981) analyzed the circuit for the inverter with an inductive load but he neglected the effect of the leakage reactance and magnetizing current. His equivalent circuit is shown in figure 16. For this circuit the following equations were derived:

$$\frac{E}{V} = \frac{1}{W}\left\{\frac{2\pi}{K} e^{-2\pi a \frac{t}{T}}\left[A \cos 2 b\frac{t}{T} - B\sin 2\pi b\frac{t}{T} + PF\right]\right\}$$

where

$$W = \frac{2}{K(a^2+B^2)}\left\{\varepsilon^{-\pi a}\left[1-aA+bB\right]\cos\pi b + (bA+aB\sin\pi B) - (-aA+bB) + PF\right\}$$

$$A = \frac{(4RY)(bm-an)+n}{\pi b(m^2+n^2)}$$

$$B = \frac{-m+(4RY)(am+bn)}{\pi b(m^2+n^2)}$$

PF is the power factor of the load

T is the period

t is the time

X is the reactance at control frequency of the inductor

Y is the admittance at control frequency of the condenser

Z is the impedance at control frequency of the Load

$$a = \frac{R}{2X}$$

$$b = \sqrt{\frac{1}{4YR}\left(\frac{R}{X}\right) \cdot \left(1 - \frac{1}{4}\frac{R}{X}4YR\right)}$$

$$k = 4YZ$$

$$m = (-a \pm jb)\omega$$

$$m + jn = 1 + e^{(-a+jb)\pi}$$

From Mr. Wagner's equation it can be seen that the alternating current output voltage to direct current input voltage ratio increases as the impedance of the load is increased. Experimentally as can be seen from figure 17, the ratio increased almost as the square root of the load resistance.

As can be seen from the variation of the input voltage while maintaining a constant output voltage in figure 17 the voltage regulation of this circuit was very poor.

The efficiency of the inverter is also plotted in figure 17. As can be seen the efficiency of the inverter

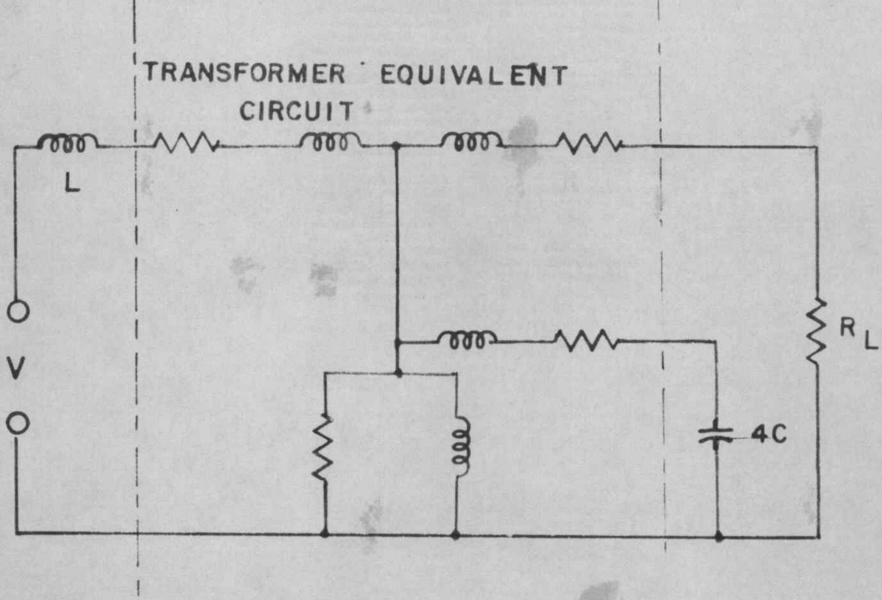


FIGURE 15. REVISED EQUIVALENT CIRCUIT OF PARALLEL INVERTER WITH RESISTIVE LOAD

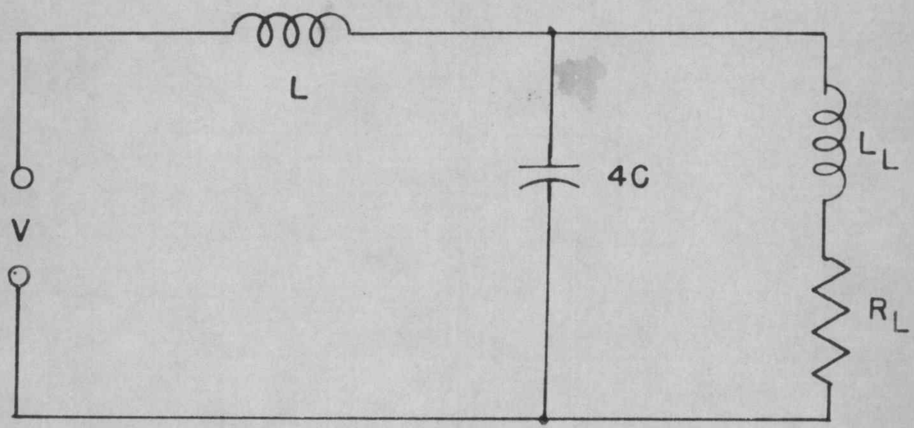


FIGURE 16. EQUIVALENT CIRCUIT OF PARALLEL INVERTER WITH INDUCTIVE LOAD

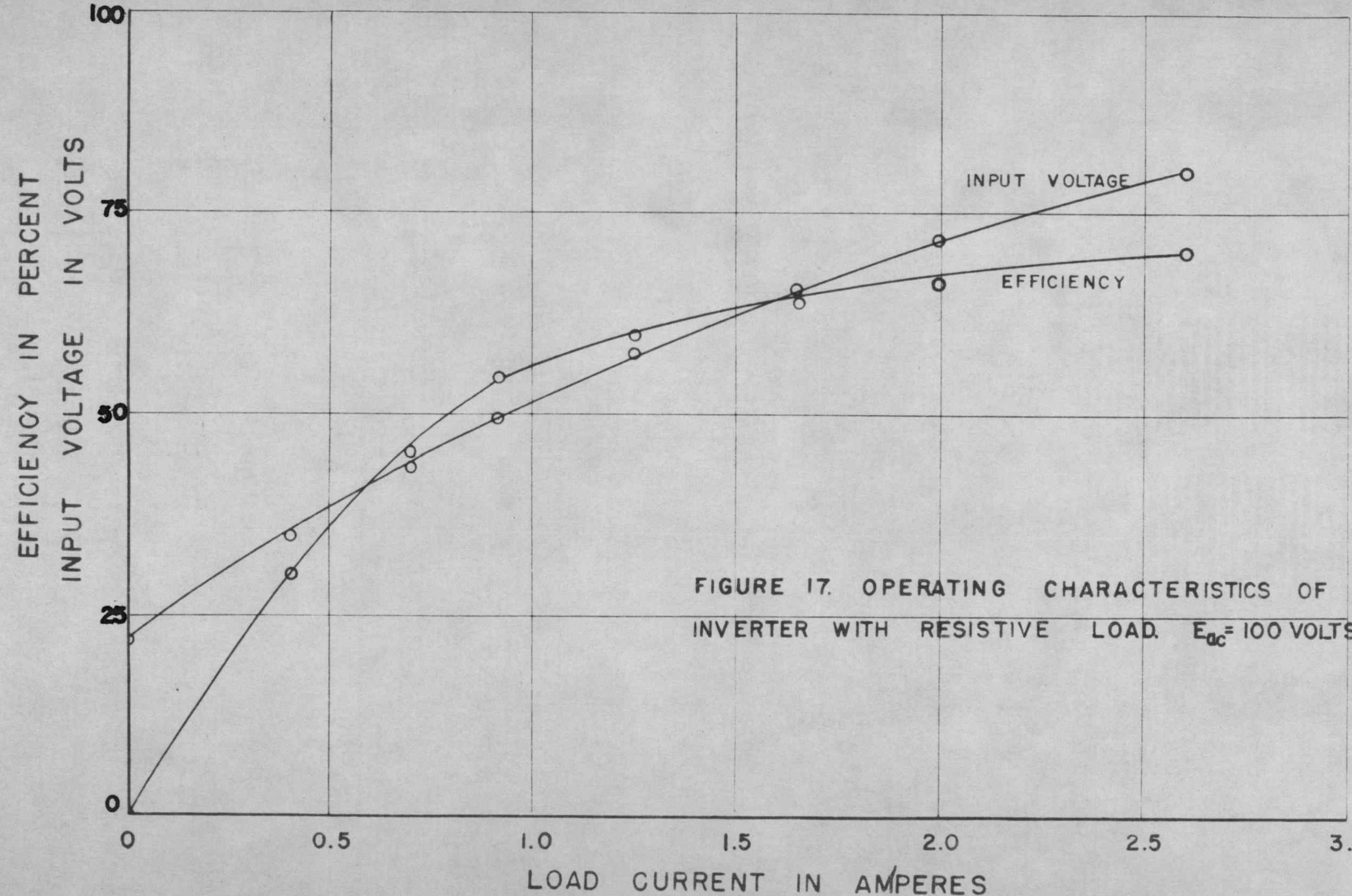


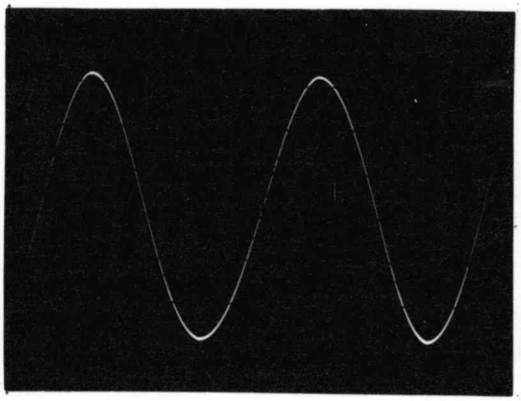
FIGURE 17. OPERATING CHARACTERISTICS OF INVERTER WITH RESISTIVE LOAD. E_{ac} = 100 VOLTS

at full load was approximately 70 per cent. The power consumed in the control circuit was neglected in figuring the efficiency of the inverter. The efficiency of the circuit would have been increased if the transformer were of the proper size for an inverter of this rating.

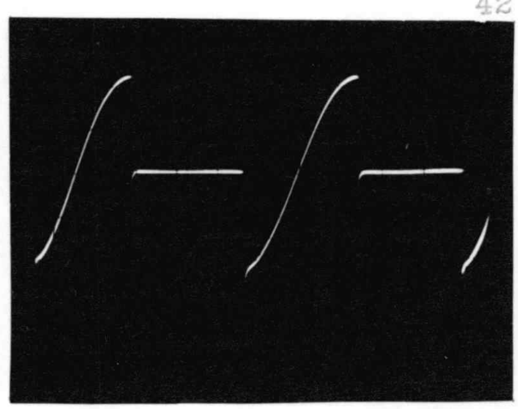
In an electronic inverter there is always the possibility of one of the tubes failing to commutate. If this happens the direct current source of power is short circuited. To prevent any damage being done when this occurs it is necessary to have a fast operating breaker in the direct current side of the inverter that will open a highly inductive arc.

If this inverter were to be used to supply power to an alternating current bus with several sources, it would be necessary to have the control circuit receive its excitation from some source other than a signal generator. As long as the inverter receives its signal from a signal generator the inverter will have a constant output frequency while if it is connected in parallel with an alternating current generator the generator will, with addition of load, tend to decrease the frequency. As can be seen this situation would become very unstable.

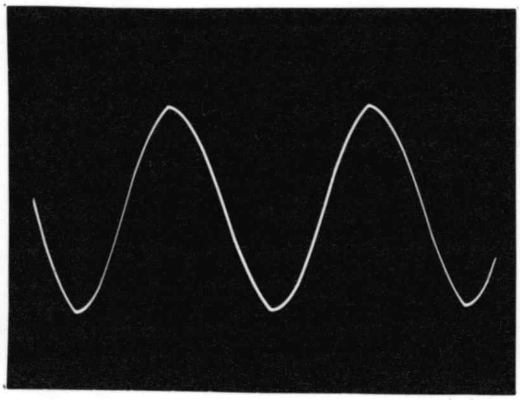
Waveforms of the output voltage of the inverter under various conditions are shown in figure 18. The



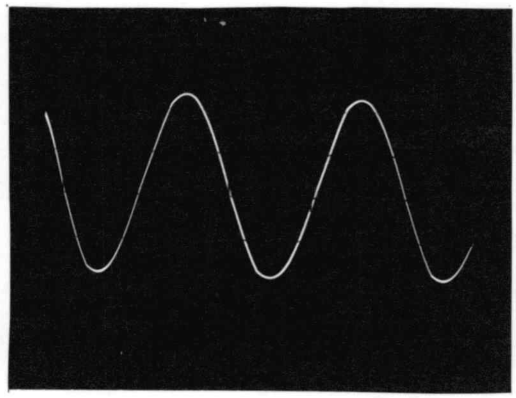
(a) Inverter with light resistive load.



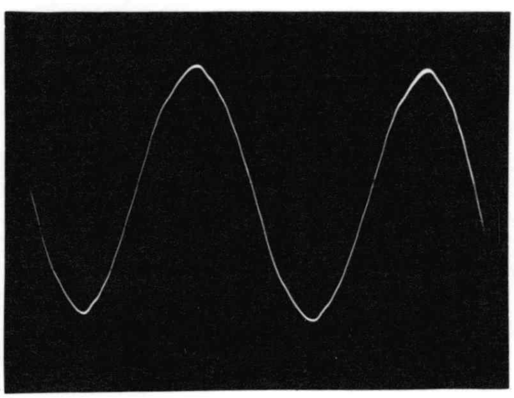
(b) Plate to cathode voltage in inverter.



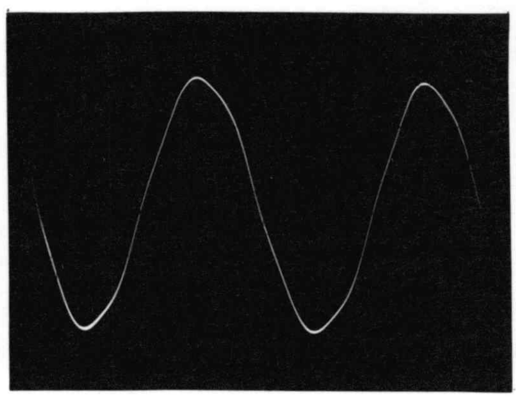
(c) Inverter with 1/6 hp. motor load.



(d) Inverter with 50% resistive load.



(e) Pacific Power and Light



(f) Motor-generator set.

Figure 18. Typical voltage wave forms of energy sources.

waveforms were as the circuit analysis predicted. At no load the harmonic distortion of the inverter was less than 1 per cent, at half load it was 1.3%, and at full load, 4.1%. The amount of distortion is mainly dependent on the value of the damping factor in equation 2. This damping factor is $e^{\frac{-t}{CR}}$. The larger the product of the capacitance and resistance in this factor the closer the output will be to a sinusoid. Since the value of capacitance is fixed by the frequency of operation of the inverter, it is only possible to change the load resistance. In order to have the load resistance appear as large as possible it is necessary to operate the inverter at as high a direct current supply voltage as practical and use an output transformer in the inverter with the proper turns ratio to give the required output voltage.

In the three-phase inverter it was possible to connect the three inverters in Delta or Wye. In order to reduce the third harmonic voltage to a minimum in the output of the inverter the delta connection was used. This in effect short circuited the third harmonic current in the secondaries of the transformers.

A small three-phase motor was connected to the output of the inverter and the waveform of figure 18C was obtained. It was necessary to connect this motor to the inverter through series condensers in order to supply the reactive requirements of the motor and to keep the impedance of the load large enough so that the inverter could operate it.

The resistance of the load must be large enough to prevent the roots of equation 1 from becoming real. In order to prevent this it is necessary that the value of resistance be

$$R_L > (L/16C)^{\frac{1}{2}}$$

or for this inverter load resistance greater than 25 ohms.

In the inverter it was found that the value of the commutating condenser should change with changes in load. This can be seen from the following equation which was previously developed:

$$V_{ac} = Ve^{-\frac{t}{8CR}} \left\{ \frac{64CR^2}{16CR^2L} \sin\left(\sqrt{\frac{1}{4CL} - \frac{1}{64C^2R^2}} + \tan^{-1} - \sqrt{\frac{16CR^2-L}{L}} \right) \right\}$$

in which $(1/4CL-1/64C^2R^2)^{\frac{1}{2}}$ must equal ωt if the output is to be a sine wave. In order to change the value of capacitance it is possible to switch in fixed condensers or to use a synchronous condenser. The latter would have the advantage of having very smooth control while the former had to be added in steps.

The harmonic distortion of the output of the inverter was under most conditions less than that of a typical motor-generator set as long as the load on the inverter did not exceed approximately half the rating of the inverter. The distortion of the inverter was also less than that of a typical power system up to a load of approximately 75 per cent. The harmonic distortion would have been less if the transformer of the inverter had a different turns ratio than the 2 to 1 that it had.

The wiring diagram that was used in this experiment is shown in figure 19. One of the main elements found missing in this circuit was the lack of a radio frequency noise filter. The R-F energy radiated by this circuit would cause poor radio reception in the vicinity of the inverter. This would have to be rectified if the inverter were to be used any place where radio receivers were being used.

NOTE: B+ is 300 volts

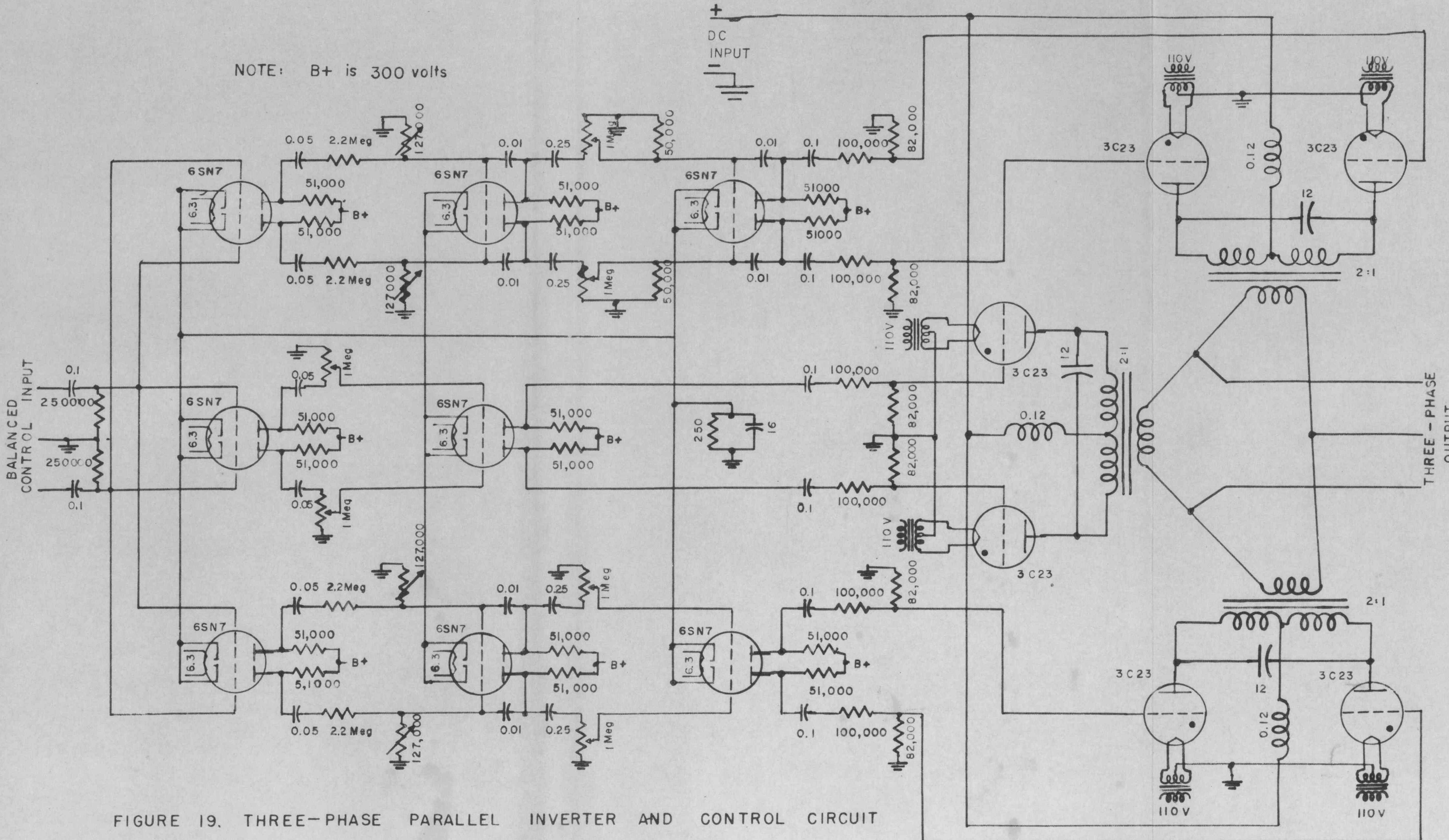


FIGURE 19. THREE-PHASE PARALLEL INVERTER AND CONTROL CIRCUIT

FUTURE WORK

There are many possibilities for future work to be done on the control system for the inverter. If the control system had used pulses to control the firing of the thyratrons, the trouble caused by the filament voltage and the difficulty of controlling the precise firing of the thyratrons would have been corrected.

In this as in many control applications the possibility of using magnetic cores instead of vacuum tubes to improve the reliability of the control circuit is always important. Because wherever there is a vacuum tube in the control circuit there is a possibility of the direct current supply being short circuited when the tube wears out.

The development of a control circuit that could be used with an inverter to supply three-phase voltages at various frequencies would be very helpful for the variable speed control of an alternating current motor or as a variable frequency source of power. To perform the latter task, the control circuit would also have to control the value of the commutating condenser.

CONCLUSIONS

The electronic inverter was designed and built in accordance with the equations developed in this thesis. Within the practical limitation of components available the operation was entirely satisfactory.

It was possible to obtain a three-phase sinusoidal voltage for the operation of an induction motor, a very difficult task for an inverter.

The waveforms predicted by the theoretical development were obtained in the inverter.

The three-phase control circuit operated as predicted.

The phase-shift method of thyratron control exhibited its usual characteristic of having poor control over the thyratron firing time.

The output frequency was constant and independent of the amount of load placed on the inverter.

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APPENDIX

TABLE I

WAVE ANALYSIS

Frequency	Pacific Power and Light	Motor generator
60	125.0 volts	115.0 volts
120	0.6 volts	0.6 volts
180	0.1 volt	0.1 volt
240	0.1 volt	0.1 volt
300	4.0 volts	1.5 volts
360	0.1 volt	Neg.
420	0.6 volt	1.2 volts
480	Neg.	Neg.
540	Neg.	Neg.
600	Neg.	Neg.
Harmonic distortion	3.3%	1.8%

TABLE II

WAVE ANALYSIS OF INVERTER OUTPUT VOLTAGE

Frequency	No Load	Load Current 2.5 amperes	Load Current 1.25 amperes	Motor Load*
60	100.0volts	100.0volts	100.0 volts	100.0
120	0.4volt	1.5volts	0.2 volt	0.7
180	0.7volt	2.5volts	0.7 volt	0.8
240	0.1volt	0.3volt	0.2 volt	0.1
300	0.3volt	2.5volts	0.9 volt	1.0
360	Neg.	0.5volt	0.3 volt	0.6
420	0.1volt	1.2volts	0.3 volt	0.8
480	Neg.	Neg.	Neg.	0.1
540	Neg.	0.5	0.1	0.2
600	Neg.	Neg.	Neg.	Neg.
Harmonic distortion	0.9%	4.1%	1.3%	1.8%

*
Series Condenser 21 microfarads per phase
1/6 horsepower, three-phase, 110volt motor