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The demand for transistor circuitry to perform within more exacting specifications has created the need for a method to accurately predict transistor circuit performance. A method utilizing an electronic digital computer for the analysis of direct-coupled transistor amplifiers is explained in this paper.

The dc bias levels and the mid-band ac voltage amplification of a circuit are discussed. The dependence of these parameters upon changes in value of circuit components is investigated. The method for analysis generated a mathematical model consisting of a system of simultaneous equations for the physical model of the circuit. From this system of equations the values of the node voltages and the loop currents are found by solving these equations with a digital computer.

The method of analysis provides a means to predict the circuit

performance without physically constructing it. The predicted theoretical values and experimental results showed good correspondence. Where a digital computer is unavailable, the application of signal flow graph techniques may be used as illustrated herein; however, the algebraic manipulations may be awkward to handle.

# A METHOD FOR ANALYSIS OF DIRECT-COUPLED TRANSISTOR AMPLIFIERS

by

STANLEY ROBERT BISHOP

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To my advisor, Professor Leonard Weber, special appreciation is due for his encouragement and resourceful ideas which ultimately resulted in the use of a digital computer in this thesis.

The need for investigation in the area of this thesis was aroused in me while engaged in work at the Development Laboratory of the Hewlett Packard Company, Palo Alto, California. The circuit illustrated in the analysis was part of a system developed under the supervision of Dr. Leonard Cutler.

I would like to take this opportunity to express my deepest appreciation to my wife for her understanding and encouragement during my years at graduate school.

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# A METHOD FOR ANALYSIS OF DIRECT-COUPLED TRANSISTOR AMPLIFIERS

## INTRODUCTION

The analysis of direct-coupled, transistor amplifiers using negative bias feedback for operating point and ac amplification stabilization can be a difficult problem. The complexity of this problem is greatly increased as the number of transistors in the circuit is increased. Methods to overcome this difficulty will have to be devised because the transistor is rapidly displacing the vacuum tube in many electronic applications. Along with this trend is the need for circuitry to perform within more exacting specifications. A necessary step toward achieving these goals is to be able to accurately analyze the parameters of a direct-coupled, transistor amplifier in order to predict its performance capabilities.

The method described here utilizes an electronic digital computer. To accomplish these analyses a mathematical model of a transistor circuit is generated. This model is rearranged into a format acceptable to the digital computer for computation. The result of these computations gives values of voltage and current for each part of the amplifier circuit. This is done for both the dc biasing levels and ac voltage amplification of the circuit. These calculated values are then compared with values obtained by evaluating the circuit experimentally.

## LITERATURE REVIEW

The methods for analyzing circuits with negative feedback have appeared in the literature. These involve the basic feedback equation according to Hurley (2, p. 119-138) and Millman (8, p. 441).

$$\text{Gain} = \frac{\text{output}}{\text{input}} = \frac{A}{1 + \rho A}$$

$A$ is the forward amplification factor and $\rho$ is the negative feedback expressed as a fraction of the fed-back signal to the total output. Due to the bilateral properties of transistors, the quantities $A$ and $\rho$ may be difficult to determine; furthermore, the presence of interaction between transistor stages in the form of multiple feedback paths could compound the complexity of the analysis. Therefore, the above expression for gain with negative feedback would be difficult to apply.

"Signal Flow Graph Analysis", which is a method for analysis of electrical networks, was published by Mason (5, p. 1144-1156) and later by Truxal (11, p. 88-160). Essentially, this technique consists of a combination of loop and node analysis which follow the assumed path of the signal flow through the circuit from input to output. By appropriately representing the transistor with an equivalent circuit, signal flow graph analysis can also be applied to include transistor circuits. Mason's method (7, p. 92-174) is adequate for the less involved circuits; however, in more complex circuits the use of Mason's formula for signal flow graph reduction becomes confounded by tedious amounts of algebra. Ways to circumvent this problem are presented in this paper.

The use of digital computers in solving transistor circuits of this kind has not been found in the literature.

# METHOD OF ANALYSIS

This section will explain how the physical model of the circuit will be transformed into a mathematical one. From these equations of the mathematical model, the values of the node voltages and loop currents are found.

The object of analysis here is to express the response at some point in the circuit to the excitation or input applied at some other point in the circuit in terms of the circuit elements. This is done by expressing a mesh current or node voltage in terms of the circuit elements and known voltages or currents. For small-signal, piece-wise linear operation of the transistor, the "T" equivalent circuit may be used to represent the transistor in the circuit. Equations are written for every possible path of signal flow beginning from the point of application of the excitation signal toward the point of output of the response. The result of this is the generation of a mathematical model consisting of a system of equations which collectively represent the complete network. These equations are solved to give the transfer function of the network from input to output. Here the input and output may be any node or loop designated in the network. The solution of this mathematical model may be obtained by the direct solution of the systems of equations by an electronic digital computer.

In general, there will be a large number of equations in this mathematical model. For example, a one transistor amplifier may require five equations and a three transistor amplifier may require 13 equations. The actual number of equations required depends entirely upon the complexity of the circuit. However, it must be emphasized that the manual solution of a system of equations for circuits with more than one transistor becomes rapidly infeasible. The logical procedure then, is the solution of these equations of the mathematical model by digital computer techniques. These simultaneous equations should be arranged to equate independent variable terms or the excitation function to the dependent variable terms or the response function. In this system of simultaneous equations, the coefficients of both the independent variables and the dependent variables may be represented by a vector and a matrix respectively.

Digital computer programs are available for the solution of linear simultaneous equations where the matrix is read into the computer and stored for future use. The vector for the first set of excitation conditions is read next. The computer then solves for the dependent variables and writes out the answers. Next, the computer accepts the second vector combining it with the stored matrix to compute the answers which are then written out. This process continues until all the vectors have been processed and their respective answers written out.

An alternate method for obtaining the solution to the system of equations of the mathematical model is possible. However, this method is considered feasible only for the less complex circuits because the number of terms to be manipulated becomes prohibitively large very rapidly for all but the simplest circuits.

This alternate method involves the creation of a signal flow graph from the system of equations of the mathematical model. The transfer function from input to output may be obtained by a systematic reduction of the flow graph according to Truxal (11, p. 101) or by the direct application of Mason's formula (10, p. 18-23), or by a preliminary reduction of the flow graph before the application of Mason's formula.

## Application of Method of Analysis

A three-stage audio frequency transistor amplifier with direct coupling and negative dc and ac feedback is analyzed by the methods discussed above. This amplifier circuit is shown in Figure 1.

## DC Bias Level Analysis

The dc equivalent circuit for the amplifier, shown in Figure 2, is used for determining all the dc operating levels of the circuit. From Figure 2, equations may be written expressing the dependent variables in terms of the independent variables, beginning at the excitation signal and working toward the output. These equations are

Figure 1. Circuit Diagram of Direct-Coupled AC Amplifier

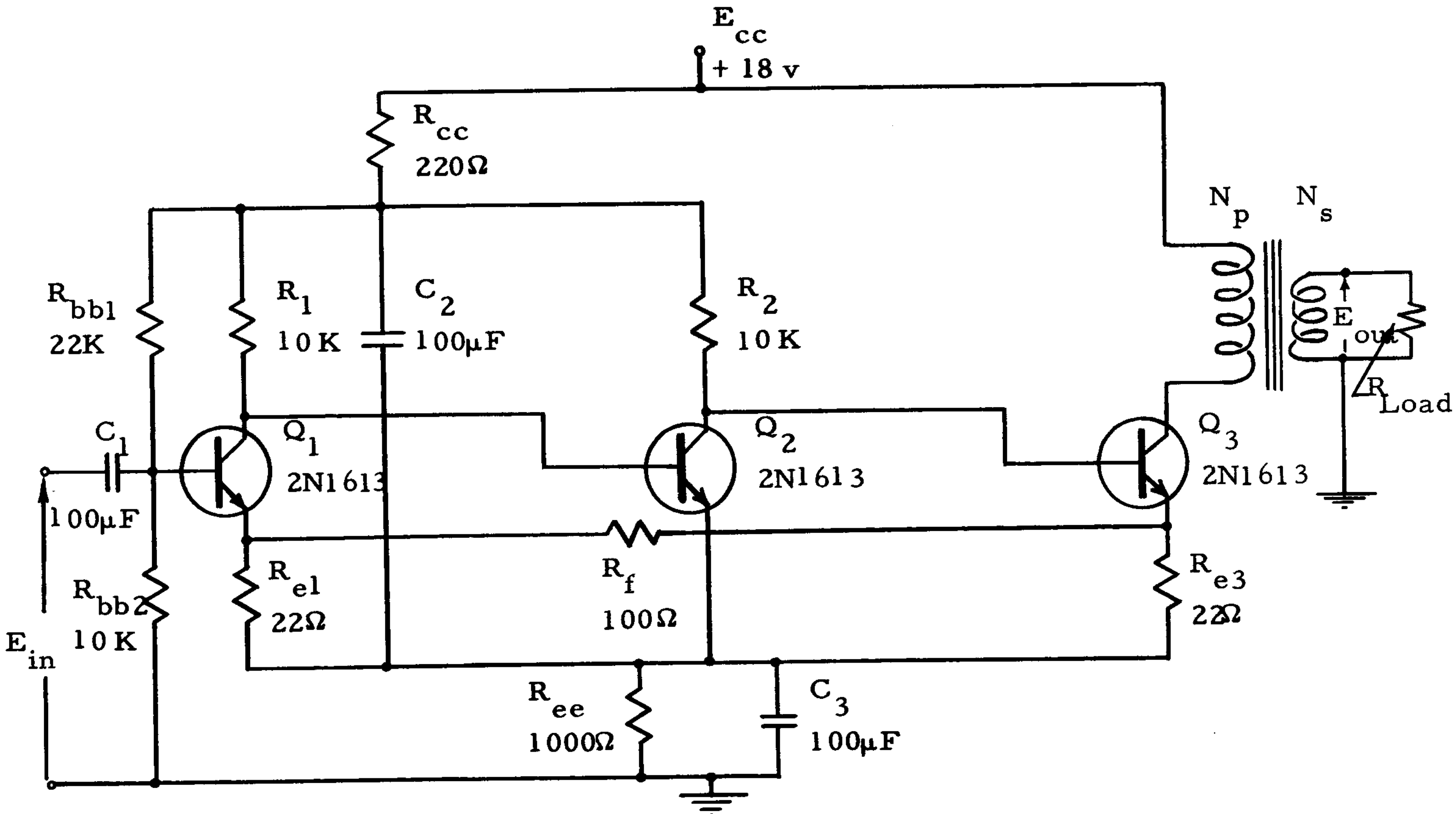
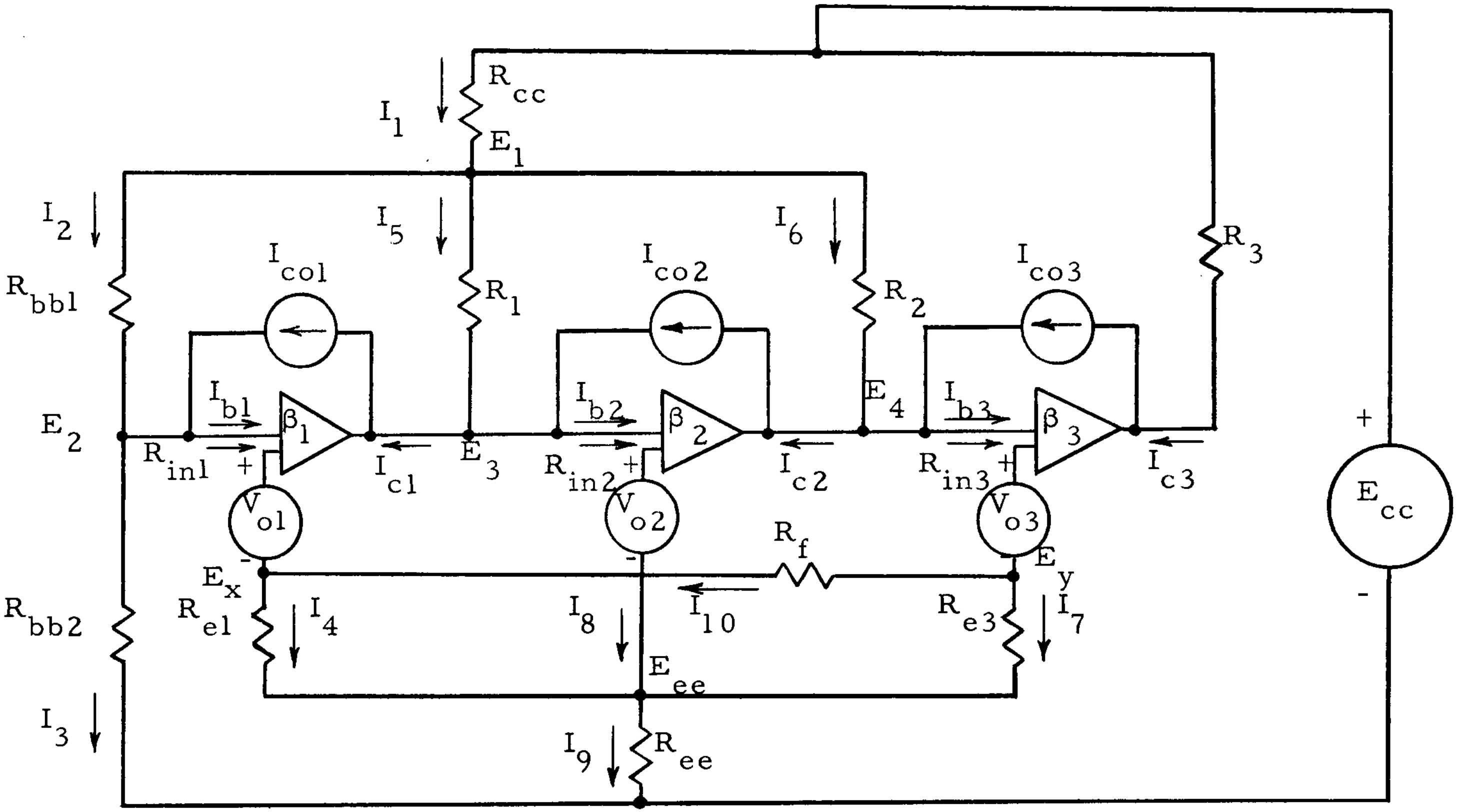


Figure 2. DC Equivalent Circuit for the Amplifier



found in Table 1.

Table 1. System of Equations Representing the DC Equivalent Circuit

For a given $E_{cc}$:

| | |
|---|---|
| $E_1 = E_{cc} - R_{cc} I_1$ | $E_4 = E_1 - R_2 I_6$ |
| $I_1 = I_2 + I_5 + I_6$ | $I_{b3} = (E_4 - E_y - V_{o3})/R_{in3}$ |
| $I_2 = I_3 + I_{b1} - I_{co1}$ | $I_{c3} = \beta_3 I_{b3} + I_{co3}$ |
| $E_2 = E_1 - R_{bb1} I_2$ | $I_4 = I_{10} + I_{b1} + I_{c1} - I_{co1}$ |
| $I_3 = E_2/R_{bb2}$ | $I_7 = I_{b3} + I_{c3} - I_{10} - I_{co3}$ |
| $I_{b1} = (E_2 - E_x - V_{o1})/R_{in1}$ | $I_8 = I_{b2} + I_{c2} - I_{co2}$ |
| $I_{c1} = \beta_1 I_{b1} + I_{co1}$ | $I_9 = I_4 + I_7 + I_8$ |
| $I_5 = I_{c1} + I_{b2} - I_{co2}$ | $E_{ee} = R_{ee} I_9$ |
| $E_3 = E_1 - R_1 I_5$ | $E_x = E_{ee} + R_{e1} I_4$ |
| $I_{b2} = (E_3 - E_{ee} - V_{o2})/R_{in2}$ | $E_y = E_{ee} + R_{e3} I_7$ |
| $I_{c2} = \beta_2 I_{b2} + I_{co2}$ | $I_{10} = (E_y - E_x)/R_f$ |
| $I_6 = I_{c2} + I_{b3} - I_{co3}$ | |

Here, the dc supply voltage ($E_{cc}$), the collector to base reverse leakage current ($I_{co}$), and the base to emitter forward voltage drop ($V_o$) are considered to be the excitation because their variations cause changes in the output: that is, the bias currents for each transistor. In a transistor circuit, variations in $E_{cc}$, $I_{co}$, and $V_o$ may be expected.

$E_{cc}$ variations may be related to the dc power supply stability. $I_{co}$ and $V_o$ variations may be related primarily to the transistor junction temperature and to a lesser degree to age and radio-active radiation. For silicon transistors with junction temperatures near 25° C, $I_{co}$ increases by approximately 15 percent and $V_o$ decreases by approximately two millivolts per degree centigrade temperature rise.

The signal flow graph appearing in Figure 3 was constructed from the equations in Table 1. Because of the complexity of this signal flow graph it will not be reduced to a transfer function by Mason's formula here. Instead, an electronic digital computer will be used to evaluate the set of equations shown in Table 1. By rearranging and combining terms in these equations, the system of equations in Table 2 are obtained. This system of 13 linear simultaneous equations can be readily solved by a digital computer. By substituting the values of the circuit elements for their respective symbols of the dependent variable terms in Table 2 and rearranging rows into columns, the matrix is rewritten as it appears in Table 3 which is in the format acceptable to the ALWAC III digital computer.

The input condition or excitation to the dc equivalent circuit consists of the independent variables which are $E_{cc}$, $I_{co}$, and $V_o$ for each of the transistors. As each independent variable takes on a different value, a new input condition is established. The independent variable terms of the simultaneous equations in Table 2 define a 13 dimensional

Figure 3. Signal Flow Graph of the DC Equivalent Circuit

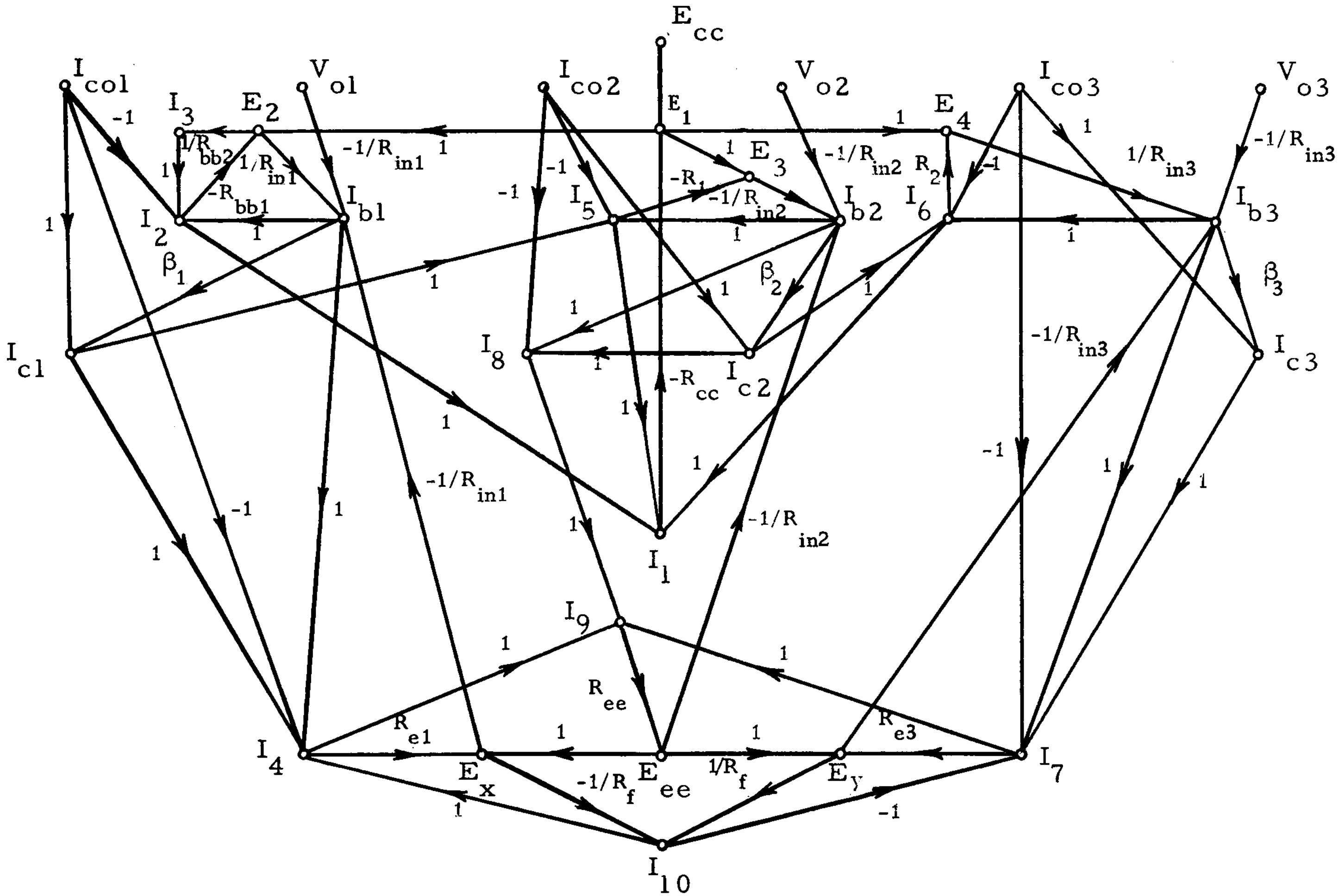


Table 2. Equations Defining DC Equivalent Circuit

| Equation Number | Dependent Variables | Independent Variables |
|---|---|---|
| 1 | $E_1/R_{cc} + E_2/R_{bb2} + I_{b1} + I_{c1} + I_{b2} + I_{c2} + I_{b3}$ | $= E_{cc}/R_{cc} + I_{co1} + I_{co2} + I_{co3}$ |
| 2 | $-E_1/R_{bb1} + E_2(1/R_{bb1} + 1/R_{bb2}) + I_{b1}$ | $= I_{co1}$ |
| 3 | $E_2 - I_{b1}R_{in1} - E_x$ | $= V_{o1}$ |
| 4 | $-I_{b1}\beta_1 + I_{c1}$ | $= I_{co1}$ |
| 5 | $-E_1/R_1 + I_{c1} + E_3/R_1 + I_{b2}$ | $= I_{co2}$ |
| 6 | $E_3 - I_{b2}R_{in2} - E_{ee}$ | $= V_{o2}$ |
| 7 | $-I_{b2}\beta_2 + I_{c2}$ | $= I_{co2}$ |
| 8 | $-E_1/R_2 + I_{c2} + E_4/R_2 + I_{b3}$ | $= I_{co3}$ |
| 9 | $E_4 - I_{b3}R_{in3} - E_y$ | $= V_{o3}$ |
| 10 | $-I_{b3}\beta_3 + I_{c3}$ | $= I_{co3}$ |
| 11 | $I_{b1} + I_{c1} + I_{b2} + I_{c2} + I_{b3} + I_{c3} - E_{ee}/R_{ee}$ | $= I_{co1} + I_{co2} + I_{co3}$ |
| 12 | $I_{b1} + I_{c1} + E_{ee}/R_{e1} - E_x(1/R_{e1} + 1/R_f) + E_y/R_f$ | $= I_{co1}$ |
| 13 | $I_{b3} + I_{c3} + E_{ee}/R_{e3} - E_y(1/R_{e3} + 1/R_f) + E_x/R_f$ | $= I_{co3}$ |

Table 3. Matrix of DC Equivalent Circuit

| Dependent Variable | Equation Number (rows from Table 2) 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $E_1$ | .004, 629, 6 | -.000, 042, 735 | 0 | 0 | -.000, 101, 823 | 0 | 0 | -.000, 104, 81 | 0 | 0 | 0 | 0 | 0 |
| $E_2$ | .000, 104, 167 | .000, 146, 902 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $I_{b1}$ | 1 | 1 | -1480 | -60 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| $I_{c1}$ | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| $E_3$ | 0 | 0 | 0 | 0 | .000, 101, 823 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $I_{b2}$ | 1 | 0 | 0 | 0 | 1 | -1570 | -62 | 0 | 0 | 0 | 1 | 0 | 0 |
| $I_{c2}$ | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| $E_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .000, 104 81 | 1 | 0 | 0 | 0 | 0 |
| $I_{b3}$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -430 | -80 | 1 | 0 | 1 |
| $I_{c3}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| $E_{ee}$ | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | -.001, 075, 3 | .045, 454, 5 | .045, 454, 5 |
| $E_x$ | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -.055, 454, 5 | .01 |
| $E_y$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | .01 | -.055, 454, 5 |

This matrix is obtained from Table 2 by arranging rows into columns and substituting circuit values for their symbols.

vector representing the input condition to the dc equivalent circuit. Hence, for each input condition a different vector results. The vector of Table 2 is rearranged by transposing each column into a row. Numerical values of the independent variables for each input condition are substituted into the vector. Any number of input conditions may be created; however, in this investigation ten were created with the result that ten corresponding vectors were generated. These are presented in Table 4 together with the definition of the ten corresponding input conditions. The solution for the values of the dependent variables of the matrix for each input condition of the independent variables of the vector were found with the ALWAC III digital computer. These solutions are presented in Table 5.

## AC Voltage Amplification Analysis

The ac equivalent circuit for this amplifier is shown in Figure 4. Here, only the mid-band frequency operation of the transistor is considered and the parameters of the transistors are assumed to be real. With this assumption, the mid-band ac voltage amplification of the circuit is determined. Referring to Figure 4 equations expressing the dependent variable in terms of the independent variable may be written beginning at the excitation signal and working toward the output. This system of equations is presented in Table 6. From these equations the signal flow graph may be constructed; see Figure 5.

Table 4. Vector of Input Conditions for DC Equivalent Circuit

| Vector<br>Equation No. (rows from Table 2)<br>1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | Input Conditions<br>Independent variables have nominal* values except as noted below |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .083, 332, 8 | 0 | .65 | 0 | 0 | .64 | 0 | 0 | .66 | 0 | 0 | 0 | 0 | Nominal Values |
| .069, 440, 0 | 0 | .65 | 0 | 0 | .64 | 0 | 0 | .66 | 0 | 0 | 0 | 0 | $E_{cc} = 15\,v$ |
| .097, 221, 6 | 0 | .65 | 0 | 0 | .64 | 0 | 0 | .66 | 0 | 0 | 0 | 0 | $E_{cc} = 21\,v$ |
| .083, 335, 8 | $10^{-6}$ | .65 | $10^{-6}$ | $10^{-6}$ | .64 | $10^{-6}$ | $10^{-6}$ | .66 | $10^{-6}$ | $3 \times 10^{-6}$ | $10^{-6}$ | $10^{-6}$ | $I_{co1} = 1\mu A$; $I_{co2} = 1\mu A$; $I_{co3} = 1\mu A$ |
| .083, 332, 8 | 0 | 1.05 | 0 | 0 | .64 | 0 | 0 | .66 | 0 | 0 | 0 | 0 | $V_{o1} = 1.05\,v$ |
| .083, 332, 8 | 0 | .65 | 0 | 0 | 1.04 | 0 | 0 | .66 | 0 | 0 | 0 | 0 | $V_{o2} = 1.04\,v$ |
| .083, 332, 8 | 0 | .65 | 0 | 0 | .64 | 0 | 0 | 1.06 | 0 | 0 | 0 | 0 | $V_{o3} = 1.06\,v$ |
| .083, 572, 8 | .00024 | .65 | .00024 | 0 | .64 | 0 | 0 | .66 | 0 | .00024 | .00024 | 0 | $I_{co1} = 240\mu A$ |
| .084, 332, 8 | 0 | .65 | 0 | .001 | .64 | .001 | 0 | .66 | 0 | .001 | 0 | 0 | $I_{co2} = 1000\,\mu A$ |
| .085, 332, 8 | 0 | .65 | 0 | 0 | .64 | 0 | .002 | .66 | .002 | .002 | 0 | .002 | $I_{co3} = 2000\,\mu A$ |

*The nominal input conditions were as follows: $E_{cc} = 18\,v$, $V_{o1} = 0.65\,v$, $V_{o2} = 0.64\,v$, $V_{o3} = 0.66\,v$, $I_{co1} = 0$, $I_{co2} = 0$, and $I_{co3} = 0$.

The value of one independent variable was changed from its nominal value to create each additional input condition.

Table 5. Computed Values of Transistor Collector Current

| Transistor Collector Current | | | Input Condition |
| --- | --- | --- | --- |
| $I_{c1}$ (ma) | $I_{c2}$ (ma) | $I_{c3}$ (ma) | Independent variables have nominal* values except as noted below. |
| 1.250,09 | 1.279,28 | 1.844,99 | Nominal Values |
| 1.041,77 | 1,067,67 | 1.454,30 | $E_{cc}$ = 15v |
| 1.458,34 | 1.490,83 | 2.315,56 | $E_{cc}$ = 21v |
| 1.250,43 | 1.279,44 | 1.894,92 | $I_{co1}$ = 1µ A, $I_{co2}$ = 1µ A, $I_{co2}$ = 1µ A |
| 1.285,18 | 1.323,91 | 1.413,00 | $V_{o1}$ = 1.05v |
| 1.214,40 | 1.278,49 | 1.930,42 | $V_{o2}$ = 1.04v |
| 1.251,50 | 1.239,44 | 1.925,69 | $V_{o3}$ = 1,06v |
| 1.088,00 | 1.068,85 | 4.327,67 | $I_{co1}$ = 240µ A |
| 2.28667 | 1.301,11 | 1.632,20 | $I_{co2}$ = 1000 µ A |
| 1.209,29 | 3.309.95 | 1.902,86 | $I_{co3}$ = 2000 µ A |

*The nominal input conditions were as follows: $E_{cc}$ = 18v, $V_{o1}$ = 0.65 v, $V_{o2}$ = 0.64v, $I_{co1}$ = 0, $I_{co2}$ = 0, and $I_{co3}$ = 0. The value of one independent variable was changed from its nominal value to create each additional input condition.

Figure 4. AC Equivalent Circuit for the Amplifier

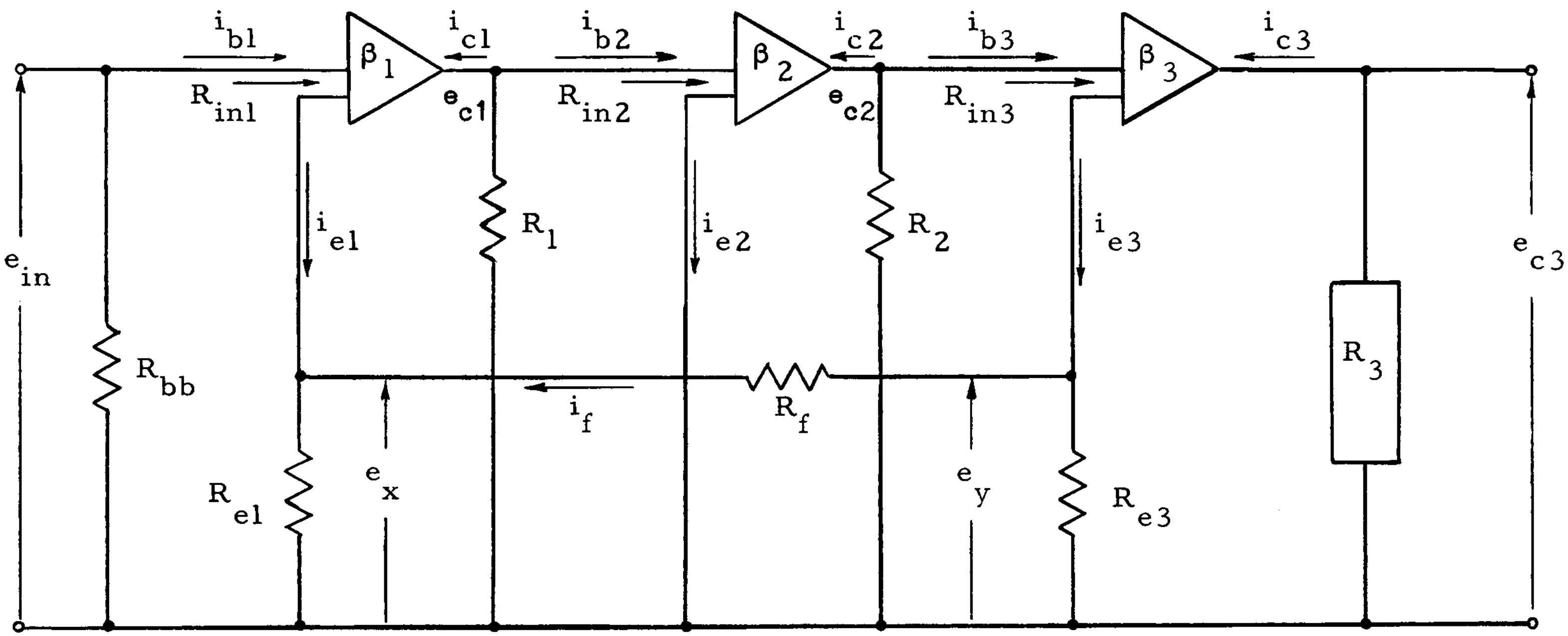


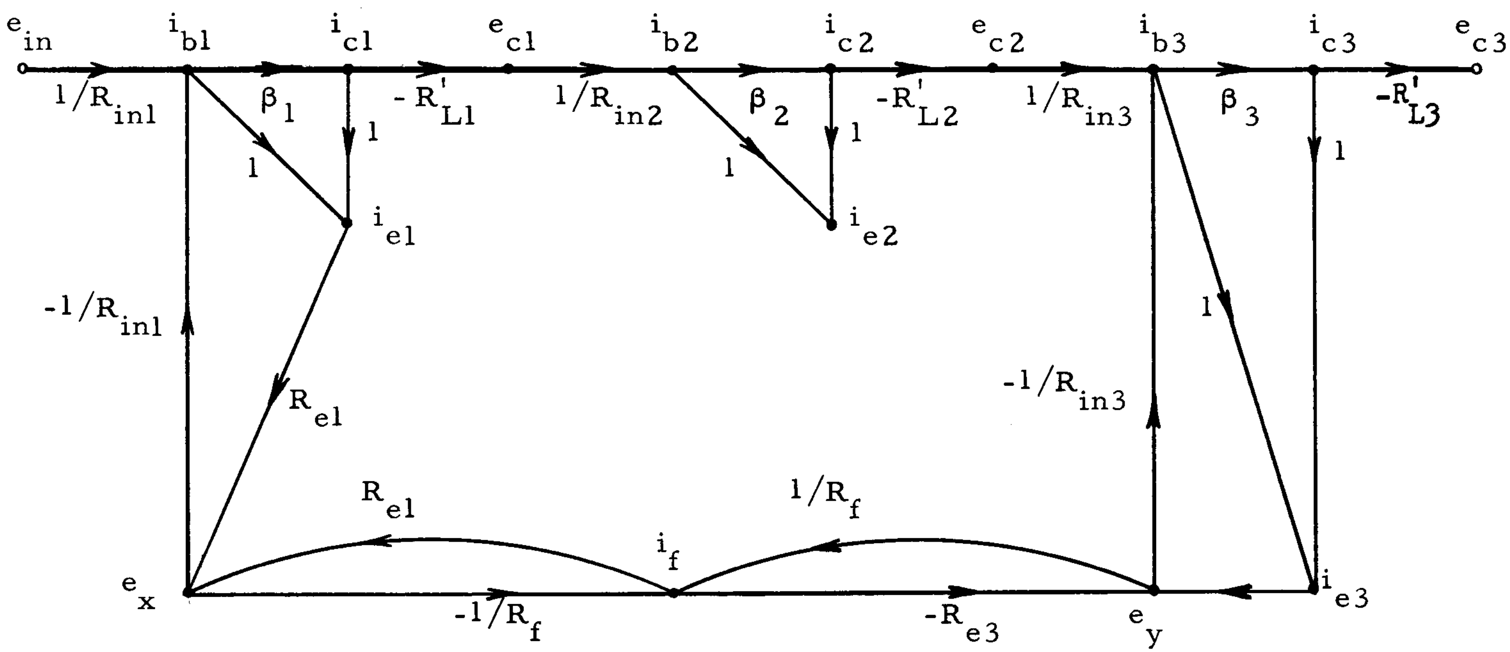
Table 6. System of Equations Representing the Mid-band AC Equivalent Circuit

For a given $e_{in}$ :

1. $i_{b1} = (e_{in} - e_x)/R_{in1}$
2. $i_{c1} = i_{b1}\beta_1$
3. $i_{e1} = i_{b1} + i_{c1}$
4. $e_{c1} = -i_{c1} R'_{L1}$
5. $i_{b2} = e_{c1}/R_{in2}$
6. $i_{c2} = i_{b2}\beta_2$
7. $i_{e2} = i_{b2} + i_{c2}$
8. $e_{c2} = -i_{c2} R'_{L2}$
9. $i_{b3} = (e_{c2} - e_y)/R_{in3}$
10. $i_{c3} = i_{b3}\beta_3$
11. $i_{e3} = i_{b3} + i_{c3}$
12. $e_{c3} = -i_{c3} R'_{L3}$
13. $e_y = (i_{e3} - i_f) R_{e3}$
14. $e_x = (i_{e1} + i_f) R_{e1}$
15. $i_f = (e_y - e_x)/R_f$

The values for $R_{in1}$, $R_{in2}$, $R_{in3}$, $R'_{L1}$, $R'_{L2}$, and $R'_{L3}$ were calculated and appear in the Appendix.

Figure 5. Signal Flow Graph of the AC Equivalent Circuit



The arrows indicate the assumed direction of signal flow. Since this graph is much simpler than the one for the dc equivalent circuit, signal flow graph techniques will be used for the purpose of illustrating an alternate means of solving the system of equations. The transfer function from input to output for the circuit can be obtained with a modification of Mason's formula by Anderson (1) presented as follows:

$$\frac{X_{output}}{X_{input}} = \frac{\sum [\text{paths } \{\Sigma \text{ non-touching (loops)}\}]}{\Sigma \text{(loops)}}$$

| | |
|---|---|
| Path | = the product of transmittances joining $X_{output}$ and $X_{input}$ without recrossing itself |
| $\Sigma$(loops) | = 1 - $\Sigma$ first order loops + $\Sigma$ second order loops - $\Sigma$ third order loops + .......... $(-1)^n$ $\Sigma$nth order loop |
| non-touching (loops) | = where each loop in the $\Sigma$ (loops) does not touch the path which is a product with it |
| first order loop | = the product of transmittances forming a loop with no recrossing |
| second order loop | = the product of two non-touching first order loops |
| third order loop | = the product of a triplet of non-touching first order loops |

Therefore, from Figure 5 the first order loops are as follows:

$$\frac{R_{e1}}{R_{in1}},\ \frac{R_{e1}\beta_1}{R_{in1}},\ \frac{R_{e1}}{R_f},\ -\frac{R_{e3}}{R_f},\ -\frac{R_{e3}}{R_{in3}},\ \frac{-R_{e3}\beta_3}{R_{in3}},$$

$$\frac{-\beta_1 R'_{L1}\beta_2 R'_{L2} R_{e3} R_{e1}}{R_{in2} R_{in3} R_f R_{in1}},\ \frac{-\beta_1 R'_{L1}\beta_2 R_{L2}\beta_3 R_{e3} R_{e1}}{R_{in2} R_{in3} R_f R_{in1}}.$$

The second order loops are as follows:

$$\left(\frac{-R_{e1}}{R_{in1}}\right)\left(\frac{-R_{e3}}{R_f}\right),\ \left(\frac{-R_{e1}}{R_{in1}}\right)\left(\frac{-R_{e3}}{R_{in3}}\right),\ \left(\frac{-R_{e1}}{R_{in1}}\right)\left(\frac{-\beta_3 R_{e3}}{R_{in3}}\right),$$

$$\left(\frac{-\beta_1 R_{e1}}{R_{in1}}\right)\left(\frac{-R_{e3}}{R_f}\right),\ \left(\frac{-\beta_1 R_{e1}}{R_{in1}}\right)\left(\frac{-R_{e3}}{R_{in3}}\right),\ \left(\frac{-\beta_1 R_{e1}}{R_{in1}}\right)\left(\frac{-\beta_3 R_{e3}}{R_{in3}}\right),$$

$$\left(\frac{-R_{e1}}{R_f}\right)\left(\frac{-R_{e3}}{R_{in3}}\right),\ \left(\frac{-R_{e1}}{R_f}\right)\left(\frac{-\beta_3 R_{e3}}{R_{in3}}\right).$$

There are no third and higher order loops.

Using the modified Mason's formula, the ac voltage amplification of the circuit is obtained:

$$\frac{e_{c3}}{e_{in}} = \left[\frac{\left\{\dfrac{\beta_1(-R'_{L1})\beta_2(-R'_{L2})\beta_3(-R'_{L3})}{R_{in1}R_{in2}R_{in3}}\left[1+\dfrac{R_{e1}}{R_f}+\dfrac{R_{e3}}{R_f}\right]\right\}}{1-\left[\dfrac{-R_{e1}}{R_{in1}}\ \dfrac{-\beta_1 R_{e1}}{R_{in1}}\ \dfrac{-R_{e1}}{R_f}\ \dfrac{+R_{e3}}{R_f}\ \dfrac{+R_{e3}}{R_{in3}}\ \dfrac{+\beta_3 R_{e3}}{R_{in3}}\right.}\right.$$

$$+\left\{\frac{R_{e1}(-R_{e3})\beta_3(-R'_{L3})}{(-R_f)(-R_{in3})(R_{in1})}(1-0)\right\}$$

$$\overline{+\frac{\beta_1\beta_2 R_{e1}R_{e3}R'_{L1}R'_{L2}}{R_{in1}R_{in2}R_{in3}R_f}+\frac{\beta_1\beta_2\beta_3 R_{e1}R_{e3}R'_{L1}R'_{L2}}{R_{in1}R_{in2}R_{in3}R_f}\Bigg]}$$

$$+\left\{\frac{\beta_1 R_{e1}(-R_{e3})\beta_3(-R'_{L3})}{(-R_f)(-R_{in3})(R_{in1})}(1-0)\right\}$$

$$\overline{+\Bigg[\frac{R_{e1}R_{e3}}{R_{in1}R_f}+\frac{R_{e1}R_{e3}}{R_{in1}R_{in3}}+\frac{\beta_3 R_{e1}R_{e3}}{R_{in1}R_{in3}}+\frac{\beta_1 R_{e1}R_{e3}}{R_{in1}R_f}}$$

$$\overline{+\frac{\beta_1 R_{e1}R_{e3}}{R_{in1}R_{in3}}+\frac{\beta_1\beta_3 R_{e1}R_{e3}}{R_{in1}R_{in3}}+\frac{R_{e1}R_{e3}}{R_{in3}R_f}+\frac{\beta_3 R_{e1}R_{e3}}{R_{in3}R_f}\Bigg]}\Bigg]$$

After collecting terms and multiplying both the numerator and denominator by $(R_{in1}R_{in2}R_{in3}R_f)$, the following expression is obtained:

$$\frac{e_{c3}}{e_{in}}=\frac{\Big[(1+\beta_1)\beta_3 R_{e1}R_{e3}R'_{L3}R_{in2}-\beta_1\beta_2\beta_3 R'_{L1}R'_{L2}R'_{L3}(R_f+R_{e1}+R_{e3})\Big]}{\begin{aligned}\Big\{&R_{in1}R_{in2}R_{in3}R_f+(1+\beta_1)R_{e1}R_{in2}R_{in3}R_f\\&+(R_{e1}+R_{e3})R_{in1}R_{in2}R_{in3}+(1+\beta_3)R_{e3}R_{in1}R_{in2}R_f\\&+(1+\beta_3)\beta_1\beta_2 R_{e1}R_{e3}R'_{L1}R'_{L2}+(1+\beta_1+\beta_1\beta_3+\beta_3)R_{e1}R_{e3}R_{in2}R_f\\&+(1+\beta_1)R_{e1}R_{e3}R_{in2}R_{in3}+(1+\beta_3)R_{e1}R_{e3}R_{in1}R_{in2}\Big\}\end{aligned}}$$

After a further rearrangement of terms, the exact equation for the ac voltage amplification form $e_{in}$ to $e_{c3}$ is obtained and presented below.

$$\frac{e_{c3}}{e_{in}} = \frac{(A - B)}{(C + D + E + F + G + H + I + J)}$$

where:

$A = (1 + \beta_1)\beta_3 R_{e1} R_{e3} R'_{L3} R_{in2}$

$B = \beta_1 \beta_2 \beta_3 R'_{L1} R'_{L2} R'_{L3} (R_f + R_{e1} + R_{e3})$

$C = R_{in1} R_{in2} R_{in3} R_f$

$D = (1 + \beta_1)\, R_{e1} R_{in2} R_{in3} R_f$

$E = (R_{e1} + R_{e3}) R_{in1} R_{in2} R_{in3}$

$F = (1 + \beta_3) R_{e3} R_{in1} R_{in2} R_f$

$G = (1 + \beta_3)\beta_1 \beta_2 R_{e1} R_{e3} R'_{L1} R'_{L2}$

$H = (1 + \beta_1 + \beta_1 \beta_3 + \beta_3) R_{e1} R_{e3} R_{in2} R_f$

$I = (1 + \beta_1) R_{e1} R_{e3} R_{in2} R_{in3}$

$J = (1 + \beta_3) R_{e1} R_{e3} R_{in1} R_{in2}$

The values for $R_{in}$ and $R'_L$ were calculated in the Appendix for this circuit. Using these values, the terms of $e_{c3} / e_{in}$ are found to have the following values:

$A = 51 \times 50 \times 22 \times 22 \times 2.97 \times 10^5 \times 1.57 \times 10^3 = 5.75 \times 10^{13}$

$B = 50^3 \times 1360 \times 413 \times 2.973 \times 10^5 \times 144 = 3.0 \times 10^{17}$

$C = 1480 \times 1570 \times 430 \times 100 = 10^{11}$

$D = 51 \times 22 \times 1570 \times 430 \times 100 = 7.57 \times 10^{10}$

$E = (22 + 22) \times 1480 \times 1570 \times 430 = 4.4 \times 10^{10}$

$F = 51 \times 22 \times 1480 \times 1570 \times 100 = 2.61 \times 10^{11}$

$G = 51 \times 50 \times 50 \times 22 \times 22 \times 1360 \times 413 = 3.47 \times 10^{13}$

$H = (1 + 50 + 50^2 + 50) \times 22 \times 22 \times 1570 \times 100 = 1.978 \times 10^{11}$

$I = 51 \times 22 \times 22 \times 1570 \times 430 = 1.67 \times 10^{10}$

$J = 51 \times 22 \times 22 \times 1480 \times 1570 = 5.73 \times 10^{10}$

Since $B >> A$, and $G >> (C + D + E + F + H + I + J)$, then for a first approximation, only terms B and G need be considered. This gives $e_{c3}/e_{in} \simeq (-B/G)$ which is roughly two percent greater than the exact value for $e_{c3}/e_{in}$. Therefore,

$$\frac{e_{c3}}{e_{in}} \simeq \left(-\frac{B}{G}\right) = \frac{-\beta_3}{1 + \beta_3}\left\{\frac{R'_{L3}(R_f + R_{e1} + R_{e3})}{R_{e1}R_{e3}}\right\}$$

A second approximation may also be made.

For $\beta_3 \geq 50$, then $\beta_3 \simeq (1 + \beta_3)$

$$\text{and} \quad \frac{\beta_3}{1 + \beta_3} \simeq 1.$$

Through these two approximations, the approximate equation for the ac voltage amplification $e_{c3}/e_{in}$ is obtained and presented below:

$$\frac{e_{c3}}{e_{in}} \simeq \frac{R'_{L3}(R_f + R_{e1} + R_{e3})}{R_{e1}R_{e3}}$$

Each of the two approximations made above has contributed to increase the error by two percent, with the result that their combined effect is to cause a maximum error of four percent at the nominal value of the circuit elements considered here.

To obtain the overall ac voltage amplification $e_{out}/e_{in}$ of the amplifier, the turns ratio $(N_s/N_p)$, of the output coupling transformer must be considered. Therefore,

$$e_{out}/e_{in} = (N_s/N_p)\, e_{c3}/e_{in}.$$

Through the use of the approximate equation, the ac voltage amplification $e_{out}/e_{in}$ of the circuit may be found as a function of each of the amplification determining circuit elements with all other circuit elements at their nominal values. These calculated values are shown in Tables 7, 8, and 9 and are presented as curves in Figure 8.

Table 7. AC Voltage Amplification vs Load Resistance $R_{Load}$ (from approximate equation)

| Load Resistance $R_L$ (ohms) | AC Voltage Amplification (decibels) |
|---|---|
| 10 | 39.68 |
| 20 | 45.53 |
| 40 | 51.23 |
| 80 | 56.61 |
| 150 | 61.09 |
| 300 | 65.31 |
| 600 | 68.57 |
| 1200 | 70.75 |
| 2400 | 72.25 |
| 4800 | 72.97 |
| 10,000 | 73.39 |

Table 8. AC Voltage Amplification *vs* Feedback Resistor $R_f$ (from approximate equation)

| Feedback Resistor $R_f$ (ohms) | AC Voltage Amplification (decibels) |
|---|---|
| 0 | 58.27 |
| 1 | 58.46 |
| 10 | 60.03 |
| 50 | 64.86 |
| 100 | 68.57 |
| 400 | 78.34 |
| 1000 | 85.78 |
| 4000 | 97.52 |
| 10,000 | 105.43 |

Table 9. AC Voltage Amplification *vs* Feedback Resistors $R_{e1}$ or $R_{e3}$ (from approximate equation)

| Feedback Resistors $R_{e1}$ or $R_{e3}$ (ohms) | AC Voltage Amplification (decibels) |
|---|---|
| 1 | 93.96 |
| 5 | 80.33 |
| 10 | 74.63 |
| 22 | 68.56 |
| 40 | 64.38 |
| 80 | 60.28 |
| 160 | 57.13 |
| 300 | 55.19 |
| 600 | 53.80 |
| 1000 | 53.20 |
| 2000 | 52.76 |

The solution for the ac voltage amplification using the exact equation was obtained. Since the arithmetic becomes rather redious, an electronic digital computer, the ALWAC III, was used to compute the values of amplification for different values of circuit components. The computed values of amplification are presented in Table 10. In each case the nominal values of the circuit elements were used except for the value of the circuit element noted. A comparison of these results with those obtained by the approximate equation is made in Table 14.

Table 10. AC Voltage Amplification for Various Values of Circuit Components (from exact equation)

| Resistance Varied and its value in ohms | | AC Voltage Amplification (decibels) |
|---|---|---|
| $R_{Load}$ | 10 | 39.51 |
| | 40 | 51.05 |
| | 150 | 60.92 |
| | 600 | 68.39 |
| | 2400 | 72.09 |
| | 10,000 | 73.23 |
| $R_f$ | 1 | 58.29 |
| | 10 | 59.87 |
| | 50 | 64.69 |
| | 100 | 68.39 |
| | 400 | 78.16 |
| | 4000 | 97.31 |
| $R_{e1}$ | 1 | 93.85 |
| | 5 | 80.17 |
| | 22 | 68.39 |
| | 80 | 60.12 |
| | 300 | 55.04 |
| | 2000 | 52.59 |
| $R_{e3}$ | 1 | 93.86 |
| | 5 | 80.17 |
| | 22 | 68.39 |
| | 80 | 60.12 |
| | 300 | 55.04 |
| | 2000 | 52.59 |

The computation for the ac voltage amplification shown here was performed on the ALWAC III digital computer at Oregon State University.

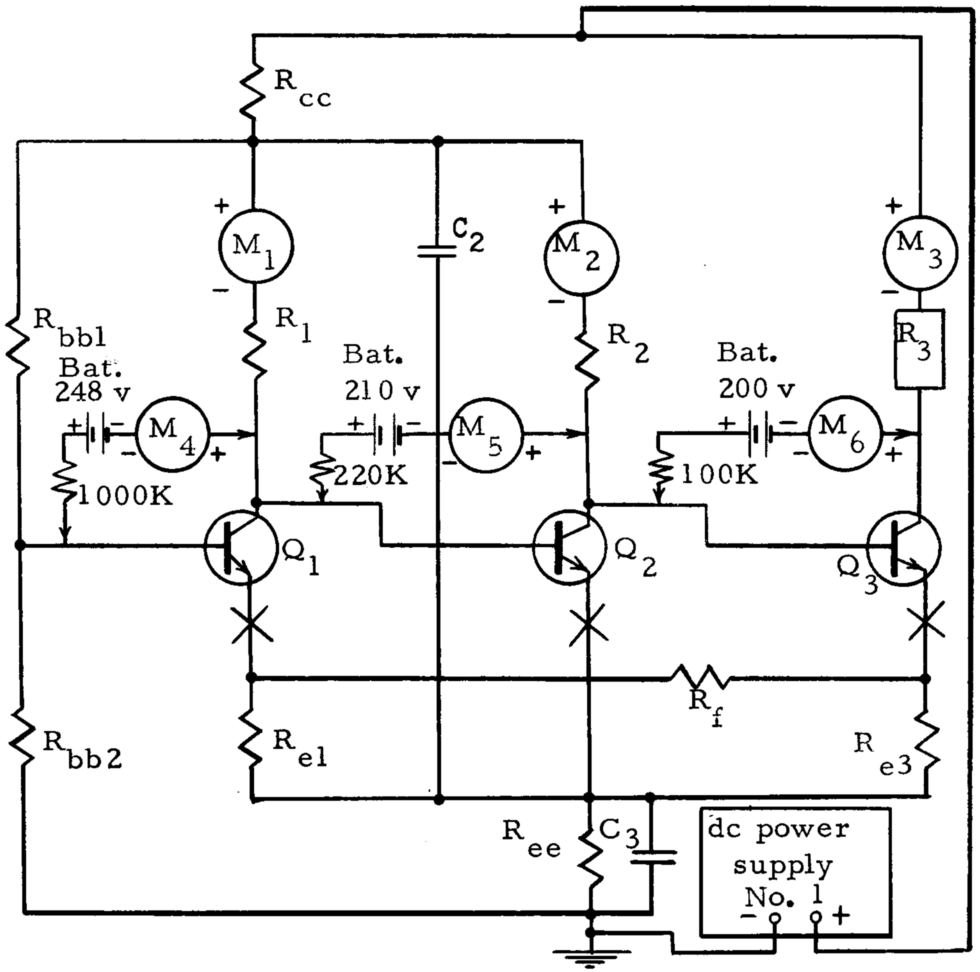
# EXPERIMENTAL VERIFICATION OF ANALYSIS

Methods for analysis of the transistor amplifier have been presented. In order to determine the validity of these analyses some means for verification is necessary. This may be done experimentally. Since there were two general regions of analysis, each of them should be experimentally verified. The values for the dc bias levels of the amplifier circuit will be verified first, and those for the ac voltage amplification of the amplifier circuit will be considered second.

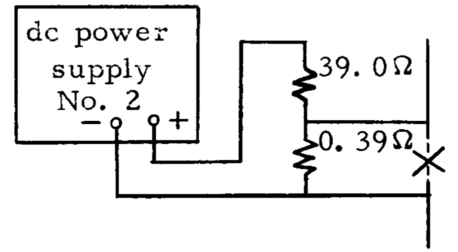
## DC Bias Level Determination

The dc bias levels for each transistor of the amplifier were measured with no ac signal input. However, both the input and output of the amplifier were terminated in 600 ohm resistances. The instrumentation for these measurements is shown in the circuit diagram in Figure 6. The bias levels were determined by measuring the collector current for each transistor with the series connected d'Arsonval milliammeters. The collector currents were measured at different values of $E_{cc}$, $I_{co}$, and $V_o$. Different values of $E_{cc}$ were obtained by directly varying the supply voltage. Different values for $I_{co}$ and $V_o$ were obtained by varying the externally injected current and voltage to simulate $I_{co}$ and $V_o$ respectively. The injected quantity

Figure 6. Physical Set-up for DC Bias Level Determination




Note: Circuit opened at "X" and voltage source inserted.




Apparatus Used:

D.C. power supply No. 1, transistor, regulated (OSU).

D.C. power supply No. 2, (Hewlett-Packard, 721A)

"Bat." series connected dry cells

| | |
|---|---|
| $M_1$ | 0 - 3 milliammeter |
| $M_2$ | 0 - 3 milliammeter |
| $M_3$ | 0 - 5 milliammeter |
| $M_4$ | 0 - 1 milliammeter |
| $M_5$ | 0 - 3 milliammeter |
| $M_6$ | 0 - 3 milliammeter |

for $I_{co}$ and $V_o$ were synthesized by a current source and a voltage source respectively. The experimentally determined bias levels appear in Table 11.

Table 11. Experimental Values for the DC Bias Levels of the Amplifier

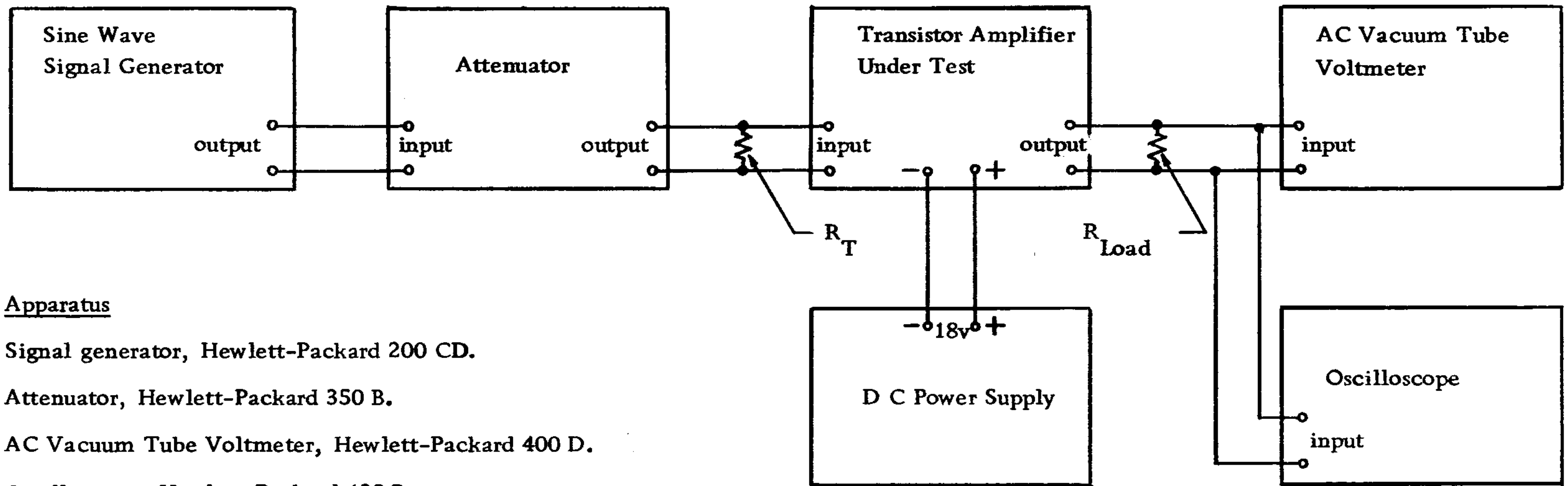
| Input Condition Varied* | Transistor Collector Current (ma) | | |
|---|---|---|---|
| | $I_{c1}$ | $I_{c2}$ | $I_{c3}$ |
| DC Supply Voltage $E_{cc}$ (volts) | | | |
| 15 | 1.00 | 1.06 | 1.40 |
| 18 | 1.20 | 1.26 | 1.80 |
| 21 | 1.40 | 1.46 | 2.20 |
| Base-emitter Voltage (volts) | | | |
| $V_{o1}$ = 1.05 | 1.23 | 1.30 | 1.32 |
| $V_{o2}$ = 1.04 | 1.16 | 1.26 | 1.83 |
| $V_{o3}$ = 1.06 | 1.20 | 1.23 | 1.82 |
| Collector to Base Leakage Current (μA) | | | |
| $I_{co1}$ = 240 | 1.06 | 1.08 | 4.30 |
| $I_{co2}$ = 1000 | 2.22 | 1.23 | 1.60 |
| $I_{co3}$ = 2000 | 1.19 | 3.30 | 1.94 |

*The nominal values of the input conditions are as follows: $E_{cc}$ = 18v, $V_{o1}$ = 0.65v, $V_{o2}$ = 0.64v, $V_{o3}$ = 0.66v, $I_{co1}$ = 0, $I_{co2}$ = 0, and $I_{co3}$ = 0. The value of one independent variable was changed from its nominal value to create each additional input condition.

## AC Voltage Amplification Determination

The ac voltage amplification of the amplifier was measured at its output transformer resonance frequency which was approximately two kilocycles. The instrumentation used for these measurements is shown in the circuit diagram in Figure 7. A sine wave of a known amplitude from the signal generator was fed into an attenuator. The attenuated sine wave signal then drives the transistor amplifier. The output of this amplifier is connected to a 600 ohm resistive load. The amplifier and the signal generator outputs were maintained at 1.00 volt r.m.s. Therefore, the ac voltage amplification of the amplifier would be equal to the amount of attenuation of the attenuator. This quantity could be read directly off the dials of the attenuator. The ac voltage amplification was measured for various values of $R_{Load}$, $R_f$, $R_{e1}$, and $R_{e3}$. Only one of these parameters was varied at a time and the other parameters were kept at their nominal values. These experimental results appear in Table 12.

Figure 7. Physical Set-up for AC Voltage Amplification Determination




Apparatus

Signal generator, Hewlett-Packard 200 CD.

Attenuator, Hewlett-Packard 350 B.

AC Vacuum Tube Voltmeter, Hewlett-Packard 400 D.

Oscilloscope, Hewlett-Packard 130 B.

DC Power Supply, Transistor Regulated, O. S. U.

$R_T$ = 638Ω, in parallel with $R_{in}$ of amplifier provides 600Ω Termination to attenuator.

$R_{Load}$ = 600Ω (nominal value) is load resistance for amplifier.

Table 12. Experimental Values for AC Voltage Amplification

| Circuit Element Varied and Its Value in ohms | | AC Voltage Amplification (decibels) |
|---|---|---|
| $R_{Load}$ | 10 | 39.9 |
| | 40 | 51.4 |
| | 150 | 61.2 |
| | 600 | 68.5 |
| | 2,400 | 72.2 |
| | 10,000 | 73.3 |
| | Open Circuit | 73.8 |
| $R_f$ | 1 | 61.1 |
| | 10 | 62.0 |
| | 50 | 65.3 |
| | 100 | 68.5 |
| | 400 | 78.3 |
| | 4,000 | 93.0 |
| $R_{e1}$ | 1 | 92.1 |
| | 5 | 79.8 |
| | 22 | 68.5 |
| | 80 | 60.3 |
| | 300 | 55.1 |
| | 2,000 | 53.6 |
| $R_{e3}$ | 1 | 92.3 |
| | 5 | 79.8 |
| | 22 | 68.5 |
| | 80 | 60.3 |
| | 300 | 55.1 |
| | 2,000 | 52.6 |

# RESULTS AND DISCUSSION

A comparison of theoretical and experimental results of both the ac voltage amplification and the dc operating conditions was made. This comparison revealed that the theoretical and experimental results were very similar. This similarity seems to verify that the method of analysis is correct and the theoretical model appears to represent the physical system.

## DC Bias Level

The theoretically and experimentally determined values of the dc bias levels of each of the three transistors of the amplifier are compared in Table 13 at different values of $E_{cc}$, $I_{co}$, and $V_o$. These data show, in general, that there is good agreement between the theoretical and experimental results. Numerical values of some of the transistor parameters necessary for these calculations were not measured for the particular transistors used in the experimental verification. For the analysis, the manufacturers published typical values were used for $r_b$, $r_e$, and $r_c$ of the transistor. However, the common emitter forward current gain $\beta$ and the base to emitter voltage $V_o$ were measured for each transistor and these were used in the analysis. The accuracy of the theoretical results depends on the exactness of the values used for $r_b$, $r_e$, and $r_c$ of the transistors.

Table 13. Theoretical and Experimental Values of the DC Bias Levels

| Input Condition* Varied and Its Value | Transitor Q1 Experimental | Transitor Q1 Theoretical | Transistor Q2 Experimental | Transistor Q2 Theoretical | Transistor Q3 Experimental | Transistor Q3 Theoretical |
|---|---|---|---|---|---|---|
| $E_{cc}$ = 18v | $I_{c1}$ = 1.20ma | $I_{c1}$ = 1.25ma | $I_{c2}$ = 1.26ma | $I_{c2}$ = 1.28ma | $I_{c3}$ = 1.80ma | $I_{c3}$ = 1.89ma |
| | $\Delta I_{c1}/I_{c1}$ | $\Delta I_{c1}/I_{c1}$ | $\Delta I_{c2}/I_{c2}$ | $\Delta I_{c2}/I_{c2}$ | $\Delta I_{c3}/I_{c3}$ | $\Delta I_{c3}/I_{c3}$ |
| $E_{cc}$ = 15v | -16.7% | -16.8% | -15.9% | -16.4% | -22.2% | -22.8% |
| $E_{cc}$ = 21v | +16.7% | +16.8% | +15.9% | +16.4% | +22.2% | +22.8% |
| $V_{o1}$ = 1.05v | 2.5% | 2.4% | 3.2% | 3.9% | -26.8% | -25.0% |
| $V_{o2}$ = 1.04v | -3.3% | -3.2% | 0% | +0.8% | 1.7% | +2.7% |
| $V_{o3}$ = 1.06v | 0% | 0% | -2.5% | -2.3% | 1.1% | 2.1% |
| $I_{co1}$ = 240μ A | -11.7% | -13.6% | -14.3% | -16.4% | +138.0% | +130.0% |
| $I_{co2}$ = 1000μA | +1.7% | +83.3% | +1.6% | +1.6% | -11.1% | -13.4% |
| $I_{co3}$ = 2000μA | -.8% | -3.3% | +3.2% | +158% | +7.8% | +1.3% |

*The nominal value of input conditions are as follows: $E_{cc}$ = 18v, $V_{o1}$ = 0.65v, $V_{o2}$ = 0.64v, $V_{o3}$ = 0.65v, $I_{co1}$ = 0, $I_{co2}$ = 0, and $I_{co3}$ = 0.

In spite of this possible source of error, there is good agreement between the experimental results and the results of the theoretical method of analysis.

## AC Voltage Amplification

Both the theoretical and experimental values of the ac voltage amplification for the variations of each of the parameters $R_{Load}$, $R_f$, $R_{e1}$, and $R_{e3}$ are shown graphically in Figure 8. A comparison of the results obtained from the exact theoretical equation, the approximate theoretical equation, and the experimental evaluation appear in Table 14. These data show that there is good agreement between the theoretical and experimental results. It can be seen that the theoretical curve follows the experimental curve for regions close to the operating point of the parameter under consideration. For regions greatly removed from the operating point of the parameter, there appears to be a divergence of the two curves. This is caused by the use of a simplified model to represent the physical system; that is, constant instead of variable values were used to represent $R_{in}$, $r_c$, and $\beta$ for each transistor. This simplification introduces no error when these circuit parameters are at their nominal values, However, when $R_f$, $R_{e1}$ and $R_{e3}$ are altered from their nominal values, the dc bias levels of the transistors in the circuit are also altered from their nominal value. This in turn affects the values of $R_{in}$, $R_{out}$, and $\beta$ for each transistor in the circuit. Consequently, the results from

Figure 8. Theoretical and Experimental Values of AC Voltage Amplification

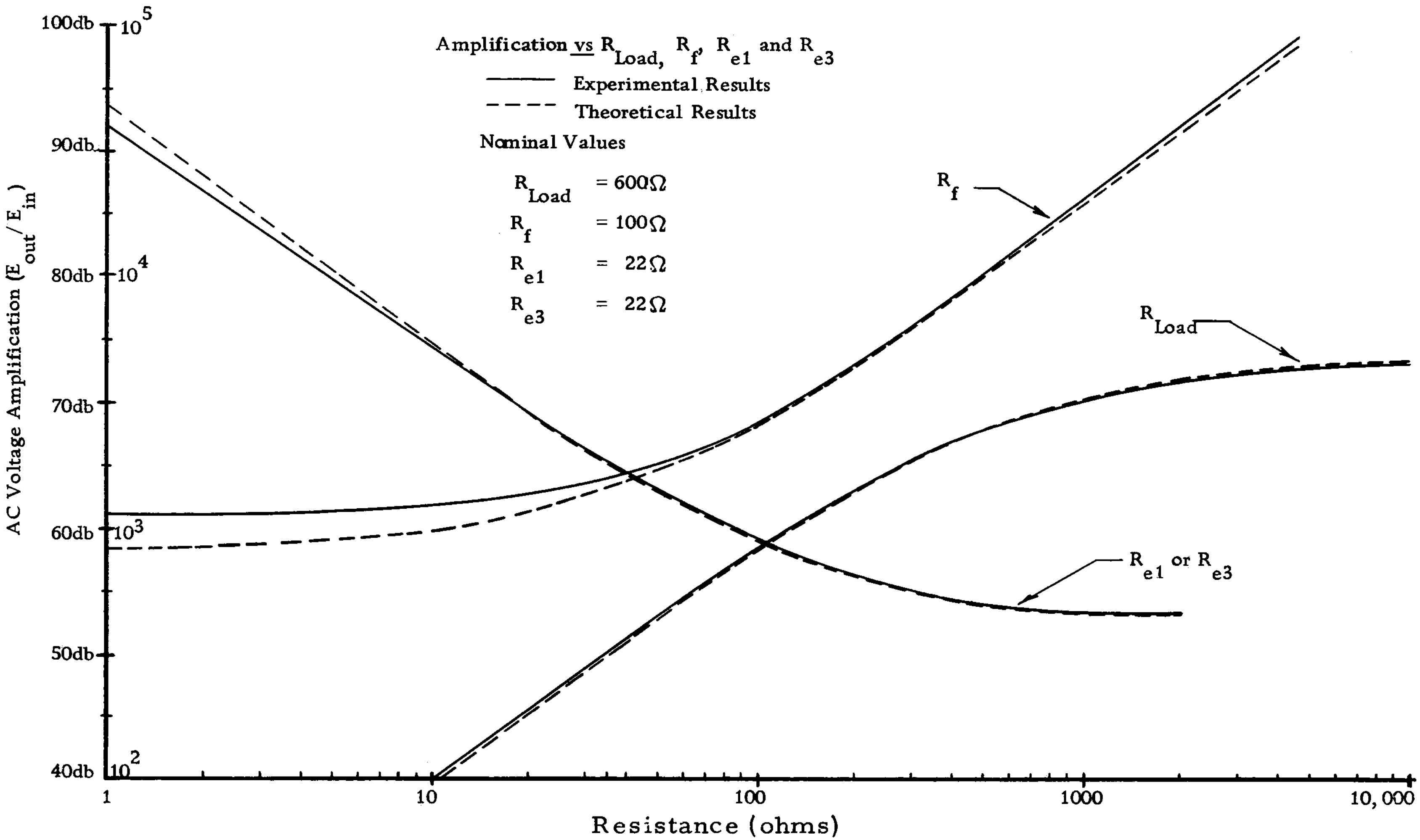


Table 14. Comparison of Results for the Three Means of Determining the AC Voltage Amplification

| Circuit Element Varied and Its Value in (ohms) | | AC Voltage Amplification (decibels) Experiment | Theoretical | Theoretical (Approx.) |
|---|---|---|---|---|
| $R_{Load}$ | 10 | 39.9 | 39.506 | 39.68 |
| | 40 | 51.4 | 51.050 | 51.23 |
| | 150 | 61.2 | 60.920 | 61.09 |
| | 600 | 68.5 | 68.390 | 68.57 |
| | 2400 | 72.2 | 72.090 | 72.25 |
| | 10,000 | 73.3 | 73.230 | 73.39 |
| $R_f$ | 1 | 61.1 | 58.289 | 58.45 |
| | 10 | 62.0 | 59.872 | 60.03 |
| | 50 | 65.3 | 64.686 | 64.86 |
| | 100 | 68.5 | 68.390 | 68.57 |
| | 400 | 78.3 | 78.167 | 78.34 |
| | 4000 | 98.0 | 97.312 | 97.52 |
| $R_{e1}$ | 1 | 92.1 | 93.853 | 93.96 |
| | 5 | 79.8 | 80.166 | 80.33 |
| | 22 | 68.5 | 68.390 | 68.56 |
| | 80 | 60.3 | 60.117 | 60.28 |
| | 300 | 55.1 | 55.036 | 55.19 |
| | 2000 | 53.6 | 52.586 | 52.76 |
| $R_{e3}$ | 1 | 92.3 | 93.860 | 93.96 |
| | 5 | 79.8 | 80.167 | 80.33 |
| | 22 | 68.5 | 68.390 | 68.56 |
| | 80 | 60.3 | 60.117 | 60.28 |
| | 300 | 55.1 | 55.036 | 55.19 |
| | 2000 | 52.6 | 52.586 | 52.76 |

this method of analysis would tend to be most accurate in the region of the nominal value of the circuit elements $R_f$, $R_{e1}$, and $R_{e3}$. This fact is supported by the graph in Figure 8.

Discussion of Experimental Accuracy

Special precautions were taken to reduce measurement errors. Whenever possible, the unknown quantity was measured by a device containing only passive elements, since they usually retain their calibration better than active elements. For this reason, measurement of the ac voltage amplification of the transistor amplifier was made with an attenuator containing only passive elements. An ac vacuum tube voltmeter was used merely to establish the reference input voltage to the attenuator and a constant output voltage from the amplifier. The specified accuracy of the attenuators and consequently the accuracy of the ac voltage amplification measurements is ± 0.375 db or ± five percent. Measurement of the dc bias levels; that is, the transistor collector current, was achieved by using d'Arsonval milliampere meter movements with an accuracy of ± five percent.

## SUMMARY

A method for analysis of multiple-transistor, direct-coupled amplifiers with multiple feedback loops was investigated. This method involved the generation of a mathematical model consisting of a system of equations that described the amplifier circuit. The method for solving these equations of the mathematical model employed the use of an electronic digital computer.

The dc bias levels and ac voltage amplification of a three-transistor, direct-coupled amplifier were analyzed. The results of these analyses were compared with the experimental results. This comparison revealed a close agreement between the theoretical and experimental values.

# CONCLUSIONS

1. The method of analysis presented in this paper shows that the theoretical mathematical model of a transistor amplifier may be used to predict some aspects of the circuit performance without physically constructing it. Consequently, transistor circuits, in general, can be evaluated by the method described here.

2. The generation of the system of equations for this method of analysis did not require the application of feedback control theory. It was merely necessary to follow a systematic procedure of writing equations to express the signal voltage or current at nodes and loops within the amplifier circuit.

3. This mathematical model consisting of a system of equations was readily adaptable to automatic data processing techniques. Therefore, a direct solution of the system of equations was obtained by the application of an electronic digital computer.

4. The variation of the dc bias currents for each transistor could be found for changes in amplifier dc supply voltage, collector to base reverse leakage current, and base to emitter forward voltage drop for the transistor. Since changes in the last two factors are closely related to the temperature of the transistor, the effect of temperature changes upon the dc bias currents for each transistor of the circuit may be determined.

5. The sensitivity of the ac voltage amplification to changes in value of the circuit parameters was revealed by plotting values of ac voltage amplification as a function of the circuit parameters.

6. In cases where a digital computer is not available, the application of signal flow graph techniques may be used; however the number of terms in the transfer function that are obtained may be large and awkward to handle. To remedy this condition, simplification of the transfer function is possible by approximation techniques. This is done by neglecting the smaller and insignificant terms of the transfer function.

7. This approximate transfer function would reveal which circuit parameters strongly influenced the circuit response and in what manner each of these parameters affected this response.

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# APPENDIX

## Calculations for Input and Load Resistance for Each Transistor Stage

For the transistors used, the following data applies. The typical values of the 2N1613 transistor parameters based on manufacturers specifications are presented below.

| Symbol | Parameter | Typical Value | Test Condition $I_c$(ma) | $V_{ce}$(volts) |
|---|---|---|---|---|
| $r_c$ | collector resistance | 625 K ohms | 1 | 5 |
| | | 545 K ohms | 2 | 5 |
| $r_b$ | base resistance | 440 ohms | 1 | 5 |
| | | 421 ohms | 2 | 5 |
| $r_e$ | emitter resistance | 20 ohms | 1 | 5 |
| | | 4.4 ohms | 2 | 5 |
| $\alpha$ | CB current gain | 0.983 | 1 | 5 |
| | | 0.985 | 2 | 5 |

For a common-emitter connected transistor, the input resistance is

$$R_{in} = r_b + \left(\frac{r_e}{1 - \alpha}\right) \frac{1 + R'_L/r_e}{1 + (r_e + R'_L)/r_e(1 - \alpha)}$$

where $R'_L$ is the equivalent load resistance. $R'_L$ may be found as follows:

$$\frac{1}{R'_L} = \frac{1}{R_L} + \frac{1}{R_{in}} + \frac{1}{r_c}\,, \quad \text{where } R_L = \text{load resistance}$$

# APPENDIX

$R_{in}$ = input resistance of following transistor

$r_c$ = collector resistance of the transistor

$R_{L3}$ is the load resistance reflected to the primary of the output coupling transformer, where

$$R_{L3} = R_{Load} \left(\frac{N_s}{N_p}\right)^2 = 600 \times 1090 = 654 \times 10^3 \text{ohms}$$

$\left(\frac{N_s}{N_p}\right)$ = transformer secondary to primary turns ratio.

$$\frac{1}{R'_{L3}} = \frac{1}{R_{L3}} + \frac{1}{R_{in4}} + \frac{1}{r_{c3}} = \frac{1}{R_{L3}} + \frac{1}{r_{c3}} = 3.28 \times 10^{-6} \text{mho}$$

The values of $R'_{L3}$, $R_{in3}$, $R'_{L2}$, $R_{in2}$, $R'_{L1}$, and $R_{in1}$ in this order were calculated using the expressions for $R_{in}$ and $R'_L$ and the data above. These calculated values are shown below.

$R'_{L3} = 305\ K\Omega$, $R_{in3} = 430\Omega$

$R'_{L2} = 413\ \Omega$, $R_{in2} = 1570\Omega$

$R'_{L1} = 1360\Omega$, $R_{in1} = 1480\Omega$