AN ABSTRACT OF THE THESIS OF

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Title TIME-DEPENDENT DEFLECTION OF DOUGLAS FIR BEAMS IN BENDING AND SHEAR MODES

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Creep in wood has been observed since 1833 but only recently has it been studied quantitatively. The aim of this paper is to study the time-dependent deflections of Douglas fir, Coast Region beams in bending and shear modes. The general method of approach used is that of large scale observation.

Three straight-grained clear wood specimens were used for testing. The beams were simply supported and loaded at the quarter span points, thus, each beam had a section of pure bending moment and sections of constant shear with linearly varying bending moment. The beams had a 40 inch span with a width of 1-1/2 inches and a depth of 3-1/2 inches. The maximum stresses induced were $f = 1370$ psi and $v = 120$ psi. The moisture content of the wood was brought down to approximately seven percent and the beams were placed in a testing environment of approximately uniform relative humidity and temperature.

Deflection data were collected for the quarter span and mid-span points. These measurements were made for 70 days at which time the creep rate had decreased to a very small value. The time-dependent deflections were plotted versus log time in days.

These curves showed, as expected, that creep varies linearly with the log of time. The final values of creep were 14 percent of the initial elastic deflections. The concept of effective moduli was used to compare the bending and shear contributions to the creep behavior. The elastic shear deflection of beam #1 appears to have been partially recovered with time. Creep due to shear in beam #2 remained in the approximate ratio of its contribution to the elastic deflection. This lack of correlation made definite conclusions unattainable. The results suggest that the shear and bending modes of creep behavior are dependent upon different variables of the internal structure.

It is obvious to the author that more sophisticated and complex methods must be used to thoroughly investigate the time-dependent deflections of wood.

TIME-DEPENDENT DEFLECTION OF DOUGLAS FIR BEAMS IN BENDING AND SHEAR MODES

by

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A THESIS

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TIME-DEPENDENT DEFLECTIONS OF DOUGLAS FIR BEAMS IN BENDING AND SHEAR MODES

INTRODUCTION

The French engineer Vicat was the first person to make qualitative statements about creep (3, p. 2-3). In the construction of suspended bridges he noticed that the cables underwent deflections not predicted by the material's elastic properties. In 1831 Vicat made a series of tests with wires under different loading and from the results hypothesized that there was a limiting stress under which no creep would occur. Another French engineer Tresca formulated basic laws of plastic flow of solids in the 1870's (3,p. 2-3). Many investigations have been made since in studying time-dependent deflections but they have been directed largely towards creep of metals at high temperature. Creep in wood has been observed since 1833 but only recently has it been treated quantitively (9).

Throughout all the investigations to date no new properties have been found that can adequately describe creep behavior. Hence, elastic properties are used to predict this behavior with varying degrees of inaccuracy. It is fortunate that in almost all cases these parameters do provide an adequate base for engineering analysis. Today, however, the functions of a structure are becoming more complex and at the same time the designs are becoming more refined. This in general tends to elevate in importance the effects of elastic and time-dependent deflections.

The complexity of creep varies directly with the complexity of the internal structure.

> Wood is a very heterogeneous material consisting of elongated hollow cells having walls built up of fibrils wound at various angles in different layers and consisting of chain molecules of cellulose with hydrogen bonds between the chains (Roelofsen, 1959). ... Because of this heterogeneity it is unlikely that physical constants can be more than a rough average for the material but these constants are still of considerable interest ... (6, p. 228).

It has been shown that the plastic behavior of wood is very similar to that of high polymer materials (6).

The two general approaches used in the investigation of creep are molecular or micro-level studies and large scale or macro-level studies. All studies at the molecular level encounter many complex experimental problems but it is from this type of study that properties of the material can be found that actually control the creep behavior. Large scale studies on the other hand do not present difficult experimental problems but the information that is gained can only be analyzed with reservations already set aside.

The purpose of this paper is to study the time-dependent deflections of Douglas fir beams in bending and shear modes. The general approach was made from the macro-level (the wood was considered homogeneous and continuous and all observations were large scale in nature). The following are definitions of creep as used in this paper. Creep is the name given to time-dependent deformation of solid materials; basic creep is time-dependent deformation without exchange of moisture; and drying or wetting creep is creep due to moisture exchange.

THEORY

Classical mechanics of deformable materials is developed on the assumption that the material exhibits a linear relation between elastic stress and strain, which is expressed by Hooke's law $\sigma = Ee$ and $\tau = G\gamma$. In recent work by Ivanov, a Russian, it is contended that the stress-strain relationship for wood is curvilinear from the very beginning of loading (5). Be this as it may, the elastic parameters are assumed to be entirely valid and are used in analysis and prediction of the time-dependent behavior in terms of empirical relationships. Classical theory can provide only approximate relationships for creep behavior since its fundamental assumptions are no longer valid. The stress-strain relationship no longer obeys Hooke's law.

Initial elastic deflections can be calculated using energy methods. For the beam shown in Figure 1, the elastic deflections are found as follows.

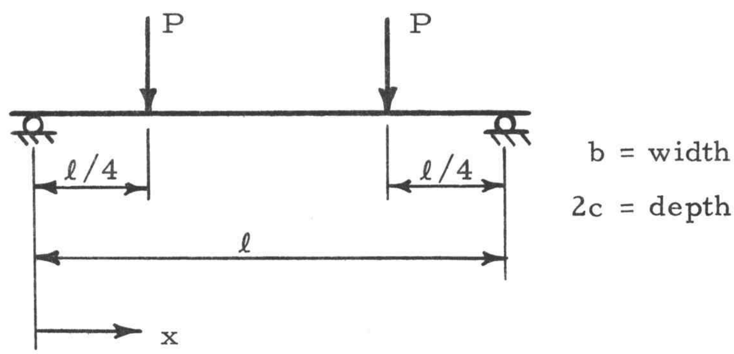

Figure 1. Beam loaded at quarter spans.

Equating the external energy with the internal energy one obtains

$$P\Delta_{\ell/4} = \int_0^{\ell} \frac{M^2}{2EI}\,dx + \int_0^{\ell} \frac{V^2c^2}{5GI}\,dx \tag{1}$$

where

$$\frac{M^2}{2EI}\,dx = \text{internal bending energy of an elemental section}$$

and

$$\frac{V^2c^2}{5GI}\,dx = \text{internal shear energy of an elemental section.}$$ [1]

Evaluating this equation for the quarter span deflection gives

$$\Delta_{\ell/4} = \frac{P\ell^3}{48EI} + \frac{P\ell c^2}{10GI} \quad . \tag{2}$$

Similarly, using the dummy unit load method for the midspan deflection gives

$$\Delta_{℄} = \frac{11P\ell^3}{384\,EI} + \frac{P\ell c^2}{10GI} \quad . \tag{3}$$

By substitution and rearrangement of the above two deflection formulas, equations for both the modulus of elasticity (E) and the modulus of rigidity (G) are obtained.

$$E = \frac{0.0078\,P\ell^3}{I(\Delta_{℄} - \Delta_{\ell/4})} \quad . \tag{4}$$

---

[1] Refer to appendix for development.

$$G = \frac{P\ell c^2}{10I\left[\Delta_{\mathbb{C}} - \frac{11P\ell^3}{384EI}\right]} \quad . \tag{5}$$

PRELIMINARY CONSIDERATIONS

There are three main factors which affect creep: stress level; time; moisture content and temperature. For stress levels up to about 50 percent of the ultimate strength of wood, creep appears to vary linearly with stress. At higher stress levels creep begins to increase non-linearly (6).

The rate of creep decreases with time. When creep is plotted versus the log of time, it generally varies linearly until the final stage of creep begins (for practical purposes it is assumed that there is a final value of creep). In the final stage, usually referred to as the curvilinear phase, the creep increases non-linearly with increase in log of time. Figure 2 shows a typical creep curve.

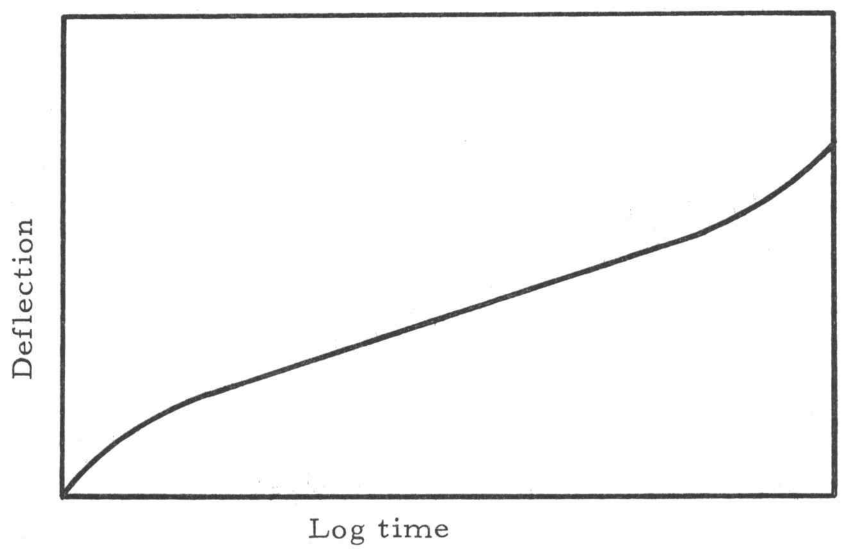

Figure 2. Typical deflection versus log time curve.

The effects of moisture content and temperature on creep involve many variables and relationships that are not fully understood. Generally, increased temperatures will increase the rate of creep. During moisture content exchanges (regardless of direction), the creep rate increases (2, 4). The size and shape of the member also affects creep to a limited extent.

The following are some of the more common experimental equations used to describe and predict creep behavior.

Hyperbolic

$$\epsilon = \frac{t\epsilon_{\infty}}{N+t}$$

where ϵ = strain, ϵ_{∞} = final value of strain, N = time for $1/2\epsilon_{\infty}$, and t = time.

Exponential

$$\epsilon = \epsilon_{\infty}[1-e^{-(At+B)}]$$

where A and B are empirical constants .

Logarithmic

$$\epsilon = A \log \frac{t}{t_o}$$

where A is an empirical constant and t_o is time at start of observation (7).

The hyperbolic equation is probably the simplest to use both for estimating the final value of creep and for calculating the creep at intermediate ages. The hyperbolic and exponential equations yield almost identical results. By and large, however, the majority of creep data is represented by log time plots, hence, all the creep data obtained in this research are plotted on logarithmic scales. The logarithmic form illustrates that creep increases forever at an ever decreasing rate.

It was paramount that the amount of elastic deflection due to shear be recognizable. The magnitude of elastic shear deflection depends largely on the span to depth ratio and also the E/G ratio. For large span/depth ratios (10 or more) shear flexure is insignificant; for small span/depth ratios (5 to 10) both moment and shear flexure are significant; and for small span/depth ratios (5 or less) shear flexure predominates (8). Thus, in most cases, the elastic deflection due to shear will be small but there are some cases where it will be significant.

EXPERIMENTAL APPARATUS AND PROCEDURE

Since the effects of both bending and shear stress were desired, the loading was designed so there was a section of pure bending and sections of constant shear with linearly varying bending moment. The beam loading, shear diagram and moment diagram are shown in Figure 3.

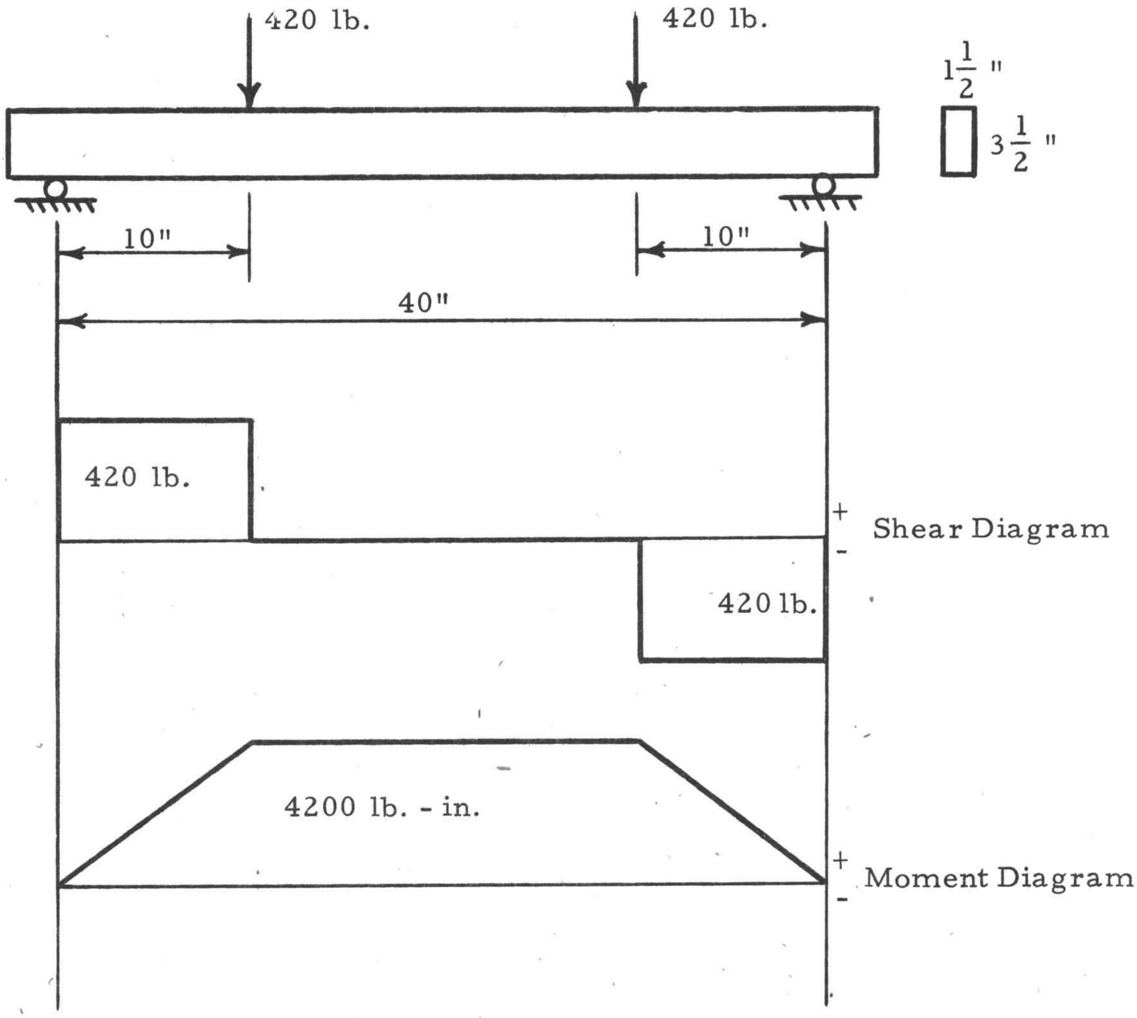

Figure 3. Beam specimen with shear and moment diagrams.

The beams were simply supported and loaded with equal loads at the quarter span points. Three beams were tested. For economy in loading the beams were placed on top of each other. Figures 4, 5 and 6 show the beams set up in the testing lab. The bottom beam is labeled #1; the middle beam #2; and the top beam #3. The beams were supported on the upper flange of a wide flanged steel beam.

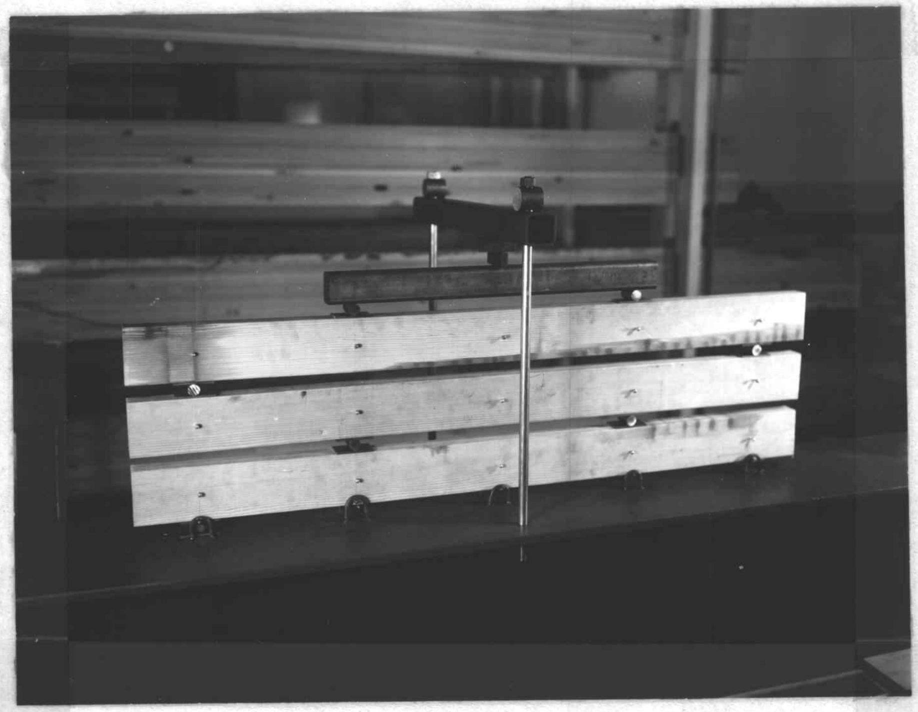

Figure 4. Front view of beams during testing.

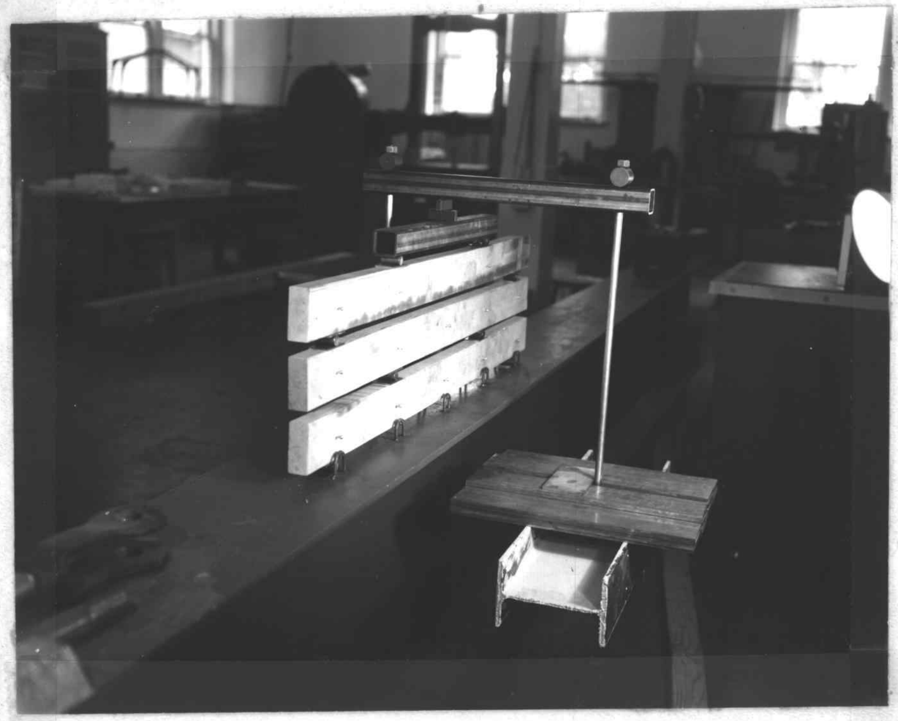

Figure 5. Back view of beams during testing.

Figure 6. Example of deflection reading.

The beam specimens were straight-grained clear wood of Douglas fir, Coast Region - Dense Select Structural. Beams #1 and #2 were cut from the same timber; beam #3 was alien. These specimens were kiln dried and obtained with a moisture content of approximately 12 percent. It was found necessary to place the beams into a kiln for further length of time to bring the moisture content down to 7 percent (the average moisture content of the wood that had been in the surrounding testing environment for almost a year). The maximum induced bending and shear stresses were respectively 1370 psi and 120 psi compared with f all'd = 2050 psi and v all'd = 120 psi (10, p. 5).

The deflection measurement was accomplished by means of spherical head screws and micrometers. The screws were placed at the neutral axis at the supports, quarter span and midspan points. Since the beams were expected to experience some twisting due to either imperfections of the wood or slight eccentricity of loading, deflection measurements were made on both sides. Two micrometers with an accuracy of 1/1000 inch were used to measure the deflections. All measurements were referenced from the supporting steel WF.

Data were taken at one day intervals for the first ten days, then five day intervals were used and finally ten day intervals. In order to convert the experimental data into actual deflections of the quarter and midspan points, it was first necessary to correct for the compression of the wood that occurred at the supports. The actual deflections were then obtained by taking the mean of the readings on either side of the beams.

RESULTS

From the initial elastic deflections the values of modulus of elasticity and the modulus of rigidity were calculated using equations 4 and 5. At the end of loading and for the duration of loading, effective moduli were calculated using the same equations. Table 1 represents these results below. The omittance of beam #3's data will be explained in the discussion of the results.

Table 1. Elastic and effective moduli (E and G).

Beam	Elastic Moduli psi	Effective Moduli at t = 70 days psi	E/G	Eeff/ Geff at t = 70 days	Eeff/Geff average for entire duration of load
#1	$E = 2.38 \times 10^6$ $G = 7.9 \times 10^4$	$Eeff = 1.9 \times 10^6$ $Geff = 11.9 \times 10^4$	30	16	19
#2	$E = 2.15 \times 10^6$ $G = 13.8 \times 10^4$	$Eeff = 1.89 \times 10^6$ $Geff = 12.1 \times 10^4$	15.6	15.6	16.8

Using the values given in Table 1, the bending moment and shear components of both the elastic deflections and the time-dependent deflections were determined and are summarized in Table 2.

Table 2. Shear and bending moment contributions to elastic and time-dependent deflections of the midspan.

Beam	Total Creep / Total Elastic Δ	Elastic Shear Δ / Total Elastic Δ	Shear Creep / Elastic Shear Δ	Bending Creep / Elastic Bending Δ	Shear Creep / Total Creep
#1	14%	17%	-33%	25%	-36%
#2	14%	10%	14%	14%	11%

The curves on the following pages show the time-dependent deflections of the quarter span and the midspan points for both beams. Figure 9 shows the variation of temperature and relative humidity of the surrounding environment that took place during testing. The temperature and relative humidity data were recorded by Bristol's Thermo-Humidigraph.

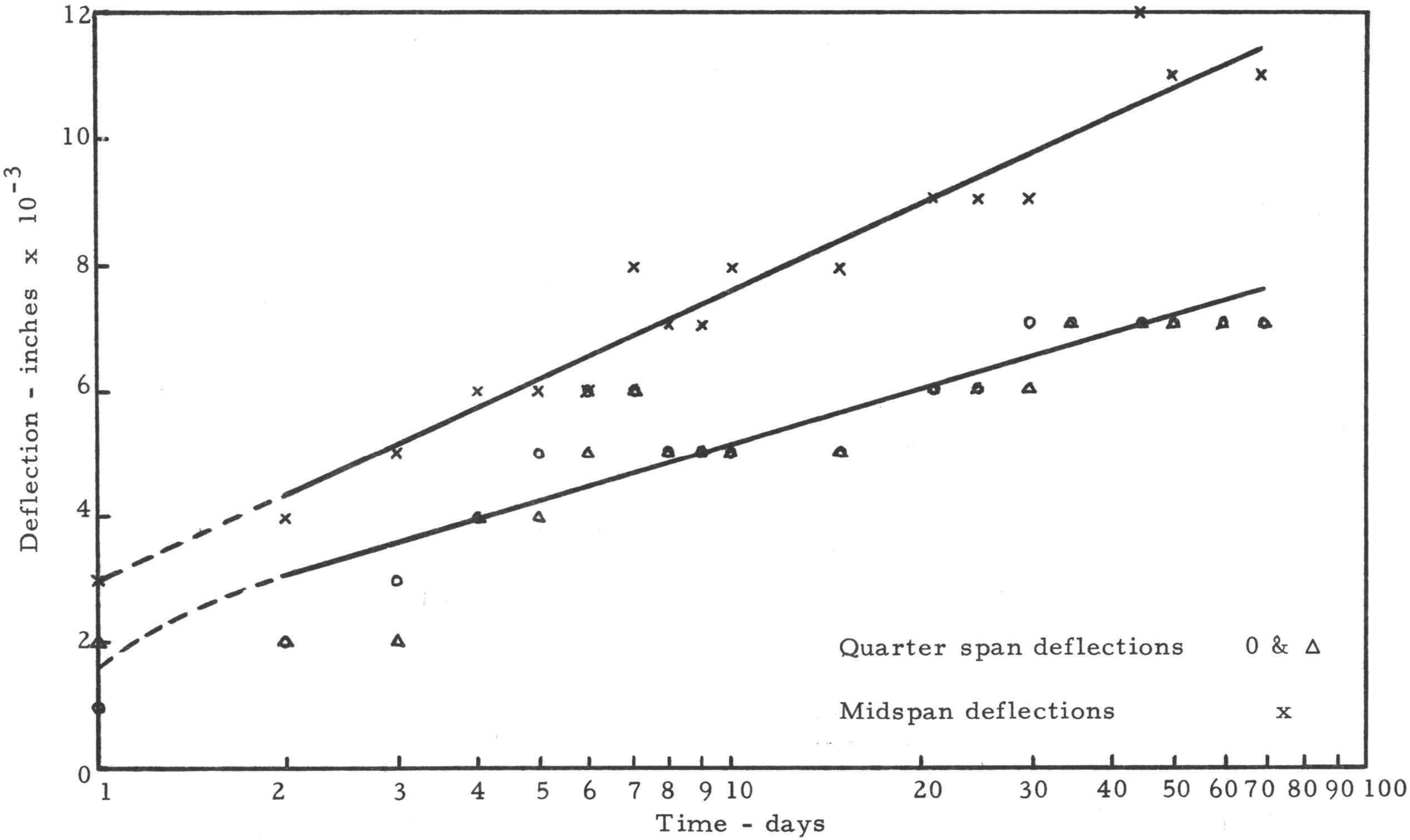

Figure 7. Time-dependent deflections of beam #1.

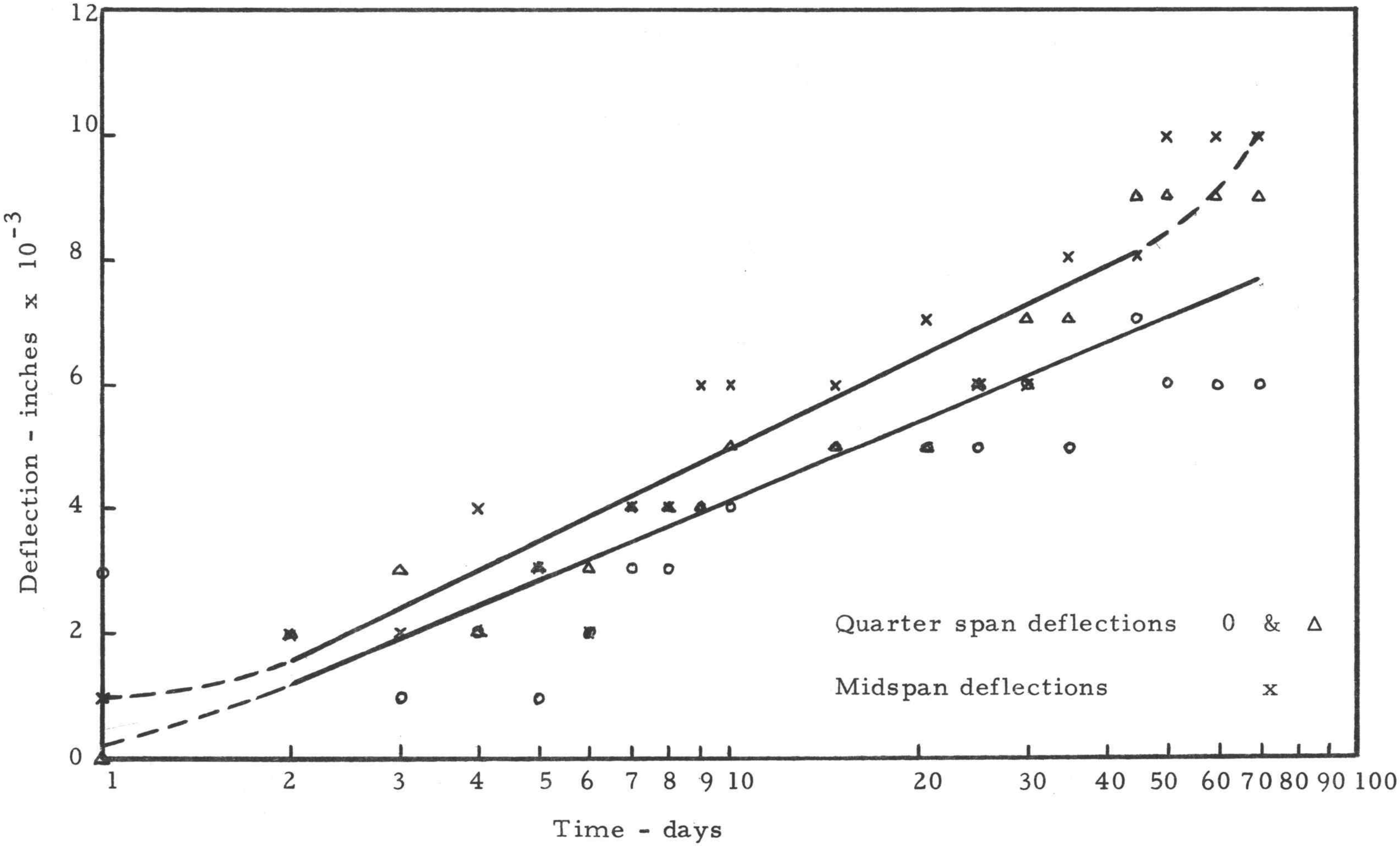

Figure 8. Time-dependent deflections of beam #2.

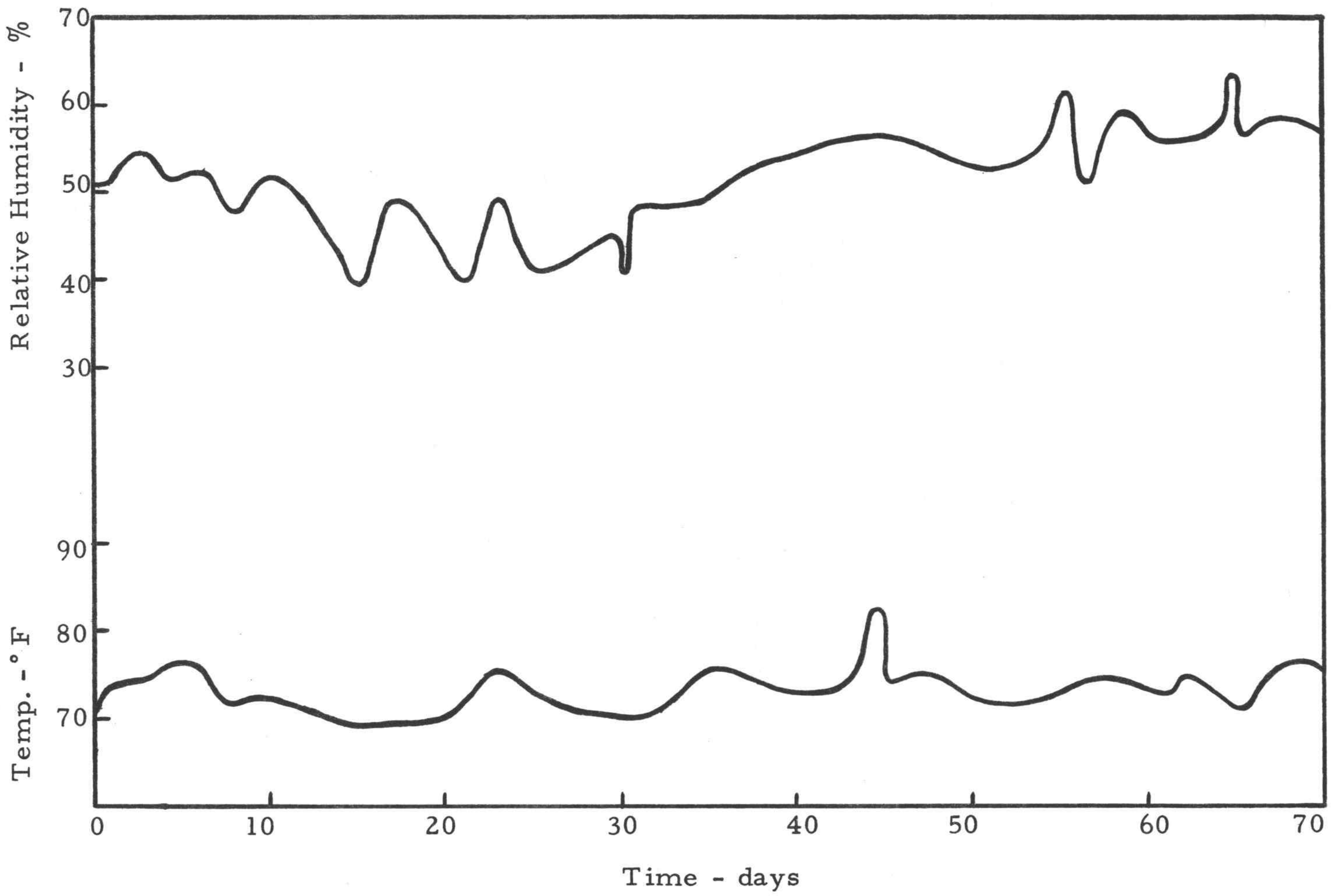

Figure 9. Relative humidity and temperature of testing environment versus time.

DISCUSSION OF RESULTS

Beam #3's data could not be presented due to an error in the design of the measurement system. After the beams were loaded, two measurements were impossible to make since the micrometers could not be placed between the screws. These two measurements pertained to beam #2, thus, the measurement system was altered at the expense of beam #3's readings in order that all beam #2's readings could be made. At the time the author was not particularly concerned because little support compression and twisting of the beams was expected. However, this was not the case and without these readings the rest of the data could not be used.

All the time-dependent deflection curves varied linearly with the log of time as anticipated. The midspan deflection of beam #2 increased non-linearly as t approached t_{70}; this is characteristic of the final stage of wood creep curves.

The underlying assumption that the results are based upon is that creep in the normal mode varies linearly with stress. By observing the time-dependent behavior of the section in pure bending, the bending moment contribution to the quarter span deflections can then be determined. The use of effective moduli E and G to describe the creep behavior is entirely empirical in nature. Effective moduli are the most practical and straightforward means of assessing long term creep behavior. It is not to be implied that these parameters give a precise picture of the time-dependent behavior but they do furnish good estimates of the final creep values.

The considerable scattering of data that occurred is to a certain degree the usual rule for wood creep data. Beam #2's data were considerably more dispersed than beam #1's. This leads the author to suspect that the accumulation of experimental error contained in beam #1 and #2's readings may be significant. Reasonable correlation between the two beams' behavior was expected since they were cut from the same timber. A definite difference in their behavior was exhibited, however, both from their initial elastic deflections and from the creep behavior.

Other tests with Douglas fir specimens show an average E/G ratio of 29.29 (1). The calculated E/G ratio of beam #2 is significantly below this value. The values of modulus of elasticity for both beams are well within the range that wood exhibits.

Table 2 summarizes the shear and bending contributions to the elastic and time-dependent midspan deflections. An explanation of the negative values in Table 2 is by no means apparent. These values could indicate that part of the initial elastic shear deflection for beam #1 was recovered with time, however, this is very unlikely. It could be evidence that the creep in the normal mode did not vary linearly with stress as assumed. It could also be evidence that the data obtained in this experiment are unreliable. Perhaps the explanation lies entirely within the wood's internal structure for which no information has been obtained. Beam #2's shear creep contribution to the total creep remained in the approximate ratio of its contribution to the elastic deflection.

It is difficult to ascertain the experimental errors that are present in the data. The accuracy in using micrometers is primarily dependent upon human feel. Hence, the author became well acquainted with their use before testing began. However, due to the small amount of creep that occurred, the results are very sensitive to small errors in the data.

The effects of moisture exchanges and temperature changes that occurred during testing are considered to be completely negligible. Significant alterations in the environment must occur before recognizable wetting or drying creep commences. Therefore, the creep that took place is considered to be basic creep due only to the loading.

It has been advised in many articles that one must be very bold in extracting meaningful information from wood creep curves. It must be kept in mind that wood is non-homogeneous, discontinuous and anisotropic, thus, a high degree of correlation between various wood specimens will normally not occur.

No statements can be made with regards to the nature of the creep itself. Creep is an elastic-viscous time-dependent deformation.

It is usually broken down into primary creep, which is recoverable upon unloading, and secondary creep, which is irrecoverable and usually referred to as genuine creep.

SUMMARY

No definite findings can be concluded until many specimens are tested and statistical methods are used to summarize the results. The lack of correlation between the beams and particularly the negative values in Table 2 raise questions which have no satisfactory answers. The results tend to indicate that the shear and bending modes of time-dependent deflections do differ in behavior. This difference may be due to shear creep's higher sensitivity to irregularities in the wood's internal structure.

The use of effective moduli is recommended for predicting the long term creep deflections. In any subsequent investigations of shear and bending components of creep it is suggested that large members be used for testing so that the final magnitudes of creep are large.

It is apparent that for a full understanding of the time-dependent deflection of wood, new parameters must be found which actually control this behavior. In all investigations, such as this, all seeming unimportant properties of the wood and testing environment must be observed so that more insight can be obtained in terms of the small scale or molecular level of behavior.

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APPENDIX

OREGON STATE UNIVERSITY
CE Structural Lab

ELASTIC DEFLECTION DATA

Beam #1

	FRONT					BACK				
Time	1	2	3	4	5	1	2	3	4	5
Before Loading	2.051	1.784	2.050	2.066	1.753	1.800	1.633	1.964	1.772	1.640
After Loading	2.051	1.726	1.978	2.010	1.753	1.800	1.580	1.894	1.720	1.640
		58	72	56			53	70	52	
					Elastic deflections		$\ell/4$	℄	$\ell/4$	
							.056	.071	.054	

Beam #2

Before Loading	4.861	4.892	4.861	4.850	4.895	4.962	5.011	5.000	4.892	5.015
After Loading	4.756	4.892	4.893	4.850	4.785	4.853	5.011	5.036	4.892	4.899
	105		32		110	109		36		116
					Elastic deflections		$\ell/4$	℄	$\ell/4$	
							.051	.073	.059	

Beam #3

Before Loading	4.782	4.725	4.833	4.795	4.796	4.490	4.505	4.512	4.701	4.663
After Loading	4.782	4.661	4.674	4.730	4.796		4.507		4.645	4.663

OREGON STATE UNIVERSITY
C. E. Structural Lab

DEFLECTION DATA - BEAM #1

	FRONT					BACK				
Time	1	2	3	4	5	1	2	3	4	5
0	2.051	1.726	1.978	2.010	1.753	1.800	1.580	1.894	1.720	1.640
1	2.051	1.725	1.974	2.008	1.753	1.799	1.578	1.891	1.718	1.640
2	2.051	1.724	1.973	2.008	1.753	1.800	1.578	1.890	1.717	1.640
3	2.051	1.722	1.973	2.008	1.753	1.800	1.578	1.8895	1.717	1.639
4	2.051	1.721	1.970	2.007	1.753	1.800	1.577	1.889	1.715	1.640
5	2.051	1.720	1.970	2.0065	1.753	1.800	1.576	1.889	1.715	1.640
6	2.051	1.719	1.970	2.005	1.7525	1.799	1.5755	1.889	1.715	1.640
7	2.051	1.718	1.9695	2.0045	1.7525	1.799	1.575	1.887	1.714	1.640
8	2.0505	1.719	1.969	2.005	1.752	1.799	1.5755	1.888	1.715	1.640
9	2.049	1.719	1.9675	2.004	1.751	1.798	1.5755	1.888	1.714	1.640
10	2.049	1.7185	1.966	2.003	1.7505	1.798	1.573	1.886	1.715	1.640
15	2.048	1.7175	1.965	2.002	1.7495	1.798	1.574	1.8855	1.714	1.639
21	2.048	1.7165	1.965	2.001	1.750	1.7975	1.573	1.866	1.713	1.640
25	2.048	1.7165	1.964	2.001	1.7495	1.798	1.573	1.866	1.7135	1.6395
30	2.047	1.714	1.963	2.000	1.7485	1.797	1.572	1.884	1.713	1.638
35	2.047	1.7145	1.963	1.9995	1.7485	1.797	1.572	1.8835	1.712	1.6395
45	2.047	1.714	1.960	1.999	1.7485	1.797	1.572	1.8825	1.712	1.6395
50	2.047	1.713	1.961	1.999	1.748	1.797	1.5715	1.8825	1.712	1.6385
60	2.047	1.713	1.960	1.999	1.748	1.797	1.5715	1.884	1.712	1.639
70	2.047	1.713	1.960	1.999	1.748	1.797	1.5715	1.883	1.712	1.639

b = 1.499 in.

d = 3.531 in.

I = 5.5 in.^{4}

OREGON STATE UNIVERSITY
C. E. Structural Lab

DEFLECTION DATA - BEAM #2

	FRONT					BACK				
Time	1	2	3	4	5	1	2	3	4	5
0	4.756	4.892	4.893	4.850	4.785	4.853	5.011	5.036	4.892	4.899
1	4.753	4.892	4.892	4.850	4.783	4.849	5.011	5.036	4.892	4.898
2	4.7525	4.892	4.891	4.850	4.780	4.849	5.010	5.0385	4.892	4.896
3	4.752	4.892	4.893	4.850	4.779	4.848	5.011	5.040	4.891	4.895
4	4.750	4.892	4.8945	4.850	4.778	4.847	5.010	5.039	4.892	4.894
5	4.750	4.892	4.894	4.849	4.777	4.846	5.010	5.040	4.892	4.893
6	4.748	4.892	4.894	4.850	4.775	4.845	5.010	5.040	4.892	4.8925
7	4.747	4.892	4.893	4.850	4.774	4.844	5.010	5.040	4.891	4.891
8	4.746	4.891	4.892	4.848	4.773	4.844	5.010	5.039	4.8905	4.891
9	4.7445	4.890	4.892	4.847	4.771	4.843	5.010	5.0405	4.890	4.890
10	4.744	4.8895	4.891	4.846	4.769	4.8435	5.010	5.0405	4.891	4.890
15	4.742	4.889	4.890	4.8455	4.7685	4.841	5.009	5.039	4.889	4.8875
21	4.741	4.888	4.890	4.8455	4.7675	4.8395	5.008	5.039	4.889	4.8865
25	4.740	4.888	4.8905	4.8455	4.7665	4.839	5.007	5.039	4.889	4.886
30	4.736	4.886	4.889	4.844	4.764	4.837	5.007	5.039	4.889	4.885
35	4.737	4.886	4.889	4.844	4.763	4.837	5.007	5.041	4.8895	4.8845
45	4.736	4.8855	4.888	4.844	4.760	4.835	5.007	5.041	4.890	4.883
50	4.736	4.885	4.889	4.844	4.7595	4.836	5.006	5.041	4.8885	4.883
60	4.736	4.885	4.890	4.844	4.758	4.835	5.006	5.041	4.888	4.883
70	4.736	4.885	4.890	4.844	4.758	4.835	5.006	5.042	4.888	4.883

$b = 1.468$ in.

$d = 3.531$ in.

$I = 5.4$ in.^{4}

OREGON STATE UNIVERSITY
C. E. Structural Lab

DEFLECTION DATA - BEAM #3

	FRONT					BACK				
Time	1	2	3	4	5	1	2	3	4	5
0	4.782	4.661	4.674	4.730	4.796		4.507		4.645	4.663
1	4.782	4.658	4.673	4.729	4.796		4.505		4.643	4.663
2	4.781	4.658	4.672	4.727	4.796		4.504		4.641	4.6625
3	4.781	4.657	4.672	4.727	4.796		4.503		4.640	4.662
4	4.781	4.656	4.669	4.726	4.797		4.502		4.640	4.662
5	4.781	4.656	4.669	4.7255	4.797		4.501		4.638	4.662
6	4.781	4.655	4.668	4.725	4.797		4.500		4.637	4.662
7	4.781	4.655	4.667	4.7245	4.796		4.4995		4.636	4.661
8	4.780	4.6545	4.667	4.724	4.796		4.4995		4.636	4.661
9	4.780	4.651	4.663	4.721	4.7955		4.497		4.635	4.6615
10	4.779	4.650	4.663	4.721	4.795		4.497		4.635	4.661
15	4.7785	4.650	4.6615	4.7205	4.7945		4.496		4.633	4.660
21	4.778	4.6485	4.6615	4.7195	4.7945		4.496		4.633	4.660
25	4.7785	4.6485	4.661	4.719	4.7945		4.495		4.633	4.660
30	4.777	4.645	4.6575	4.716	4.793		4.493		4.629	4.659
35	4.778	4.645	4.657	4.7155	4.794		4.494		4.631	4.660
45	4.778	4.645	4.656	4.714	4.794		4.493		4.629	4.660
50	4.778	4.645	4.655	4.714	4.794		4.492		4.628	4.659
60	4.778	4.6445	4.655	4.715	4.795		4.492		4.628	4.660
70	4.778	4.644	4.655	4.714	4.795		4.492		4.628	4.660

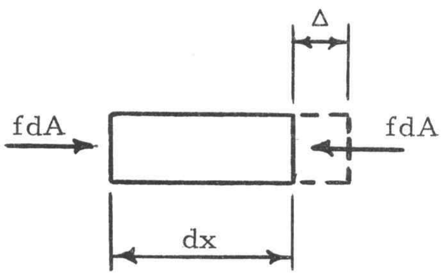
b = 1.479 in.

d = 3.561 in.

I = 5.78 $in.^4$

DERIVATION OF EQUATIONS FOR BENDING MOMENT AND SHEAR DEFLECTIONS USING ELASTIC ENERGY METHODS

Normal Stress



Given the element shown on the left with

Δ = normal deflection

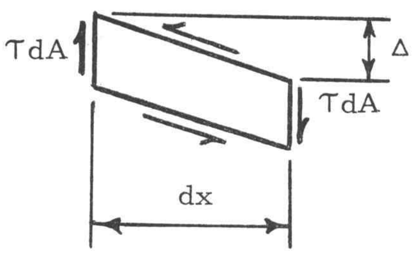
dA = area of face

E = modulus of elasticity $= f/\epsilon$

$$\epsilon = \frac{\Delta}{dx} \quad \text{or} \quad \Delta = \epsilon\, dx = \frac{f dx}{E}$$

$$\text{Elastic energy of the element} = \frac{f dA}{2}\,\Delta = \frac{f^2}{2E}\, dA dx$$

Shear Stress



Given the element shown on the left and

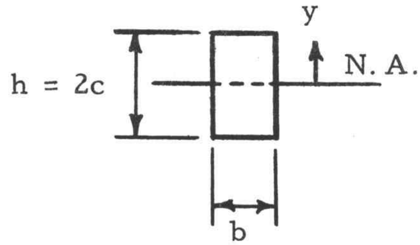
G = modulus of rigidity $= \dfrac{\tau}{\gamma}$

$$\gamma = \frac{\Delta}{dx} \quad \text{or} \quad \Delta = \gamma dx = \frac{\tau dx}{G}$$

$$\text{Elastic energy of the element} = \frac{\tau dA\, \Delta}{2} = \frac{\tau^2}{2G} dA dx$$

$$\therefore \quad \text{Total elastic energy} = W_i = \int_0^A \int_0^\ell \frac{f^2}{2E}\, dA dx + \int_0^A \int_0^\ell \frac{\tau^2}{2G}\, dA dx$$

where $f = \dfrac{My}{I}$ and $\tau = \dfrac{V}{2I}(c^2 - y^2)$



Making these substitutions one obtains

$$W_i = \int_0^A \int_0^\ell \frac{M^2 y^2}{2EI^2}\, dA dx + \int_0^A \int_0^\ell \frac{V^2}{8GI^2}(c^4 - 2c^2 y^2 + y^4)\, dA dx$$

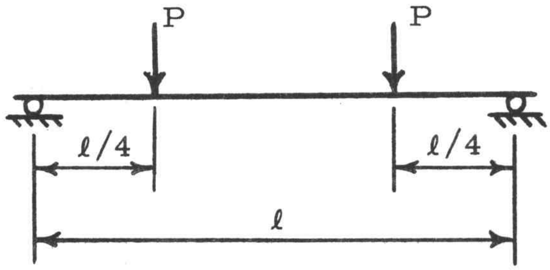
$$\int_0^A \frac{M^2 y^2}{2EI^2}\, dA dx = b \int_{-h/2}^{h/2} \frac{M^2 y^2}{2EI^2}\, dy dx = \frac{M^2 b h^3}{24EI^2}\, dx = \frac{M^2}{2EI}\, dx$$

Similarly

$$\int_0^A \frac{V^2}{8GI^2}(c^4 - 2c^2 y^2 + y^4)\, dA dx = \frac{V^2 c^2}{5GI}\, dx$$

$$\therefore \quad W_i = \int_0^\ell \frac{M^2 dx}{2EI} + \int_0^\ell \frac{V^2 c^2}{5GI}\, dx$$

Deflection at quarter span points



$$\frac{W_i}{2} = \int_0^{\ell/4} \frac{P^2x^2}{2EI}dx + \int_{\ell/4}^{\ell/2} \frac{P^2\ell^2}{32EI}dx + \int_0^{\ell/4} \frac{P^2c^2}{5GI} dx$$

$$\frac{W_i}{2} = \frac{P^2\ell^3}{384EI} + \frac{P^2\ell^3}{128EI} + \frac{P^2\ell c^2}{20\ GI}$$

$$W_i = \frac{P^2\ell^3}{48EI} + \frac{P^2\ell c^2}{10GI}$$

Equating the external energy with internal energy one obtains

$$P\Delta_{\ell/4} = \frac{P^2\ell^3}{48EI} + \frac{P^2\ell c^2}{10\,GI}$$

$$\therefore \quad \Delta_{\ell/4} = \frac{P\ell^3}{48EI} + \frac{P\ell c^2}{10GI}$$

SAMPLE CALCULATIONS

Determination of E and G for Beam #1

2

$$E = \frac{0.0078\ P\ell^3}{I(\Delta_{\text{℄}} - \Delta_{\ell/4})} = \frac{0.0078 \times 420 \times 40^3}{5.5(\Delta_{\text{℄}} - \Delta_{\ell/4})} = \frac{3.81 \times 10^4}{\Delta_{\text{℄}} - \Delta_{\ell/4}}$$

$$E = \frac{3.81 \times 10^4}{0.071 - 0.055} = \frac{3.81 \times 10^4}{.016} = 2.38 \times 10^6 \text{ psi}$$

and

$$G = \frac{P\ell c^2}{10 I \left[\Delta_{\text{℄}} - \frac{11 P\ell^3}{384\, EI} \right]} = \frac{420 \times 40 \times 1.725^2}{55 \left[\Delta_{\text{℄}} - \frac{11 \times 420 \times 40^3}{384 \times 5.5 \times E} \right]}$$

$$= \frac{951}{\Delta_{\text{℄}} - \frac{1.4 \times 10^5}{E}}$$

$$G = \frac{951}{.071 - \frac{1.4 \times 10^5}{2.38 \times 10^6}} = \frac{951}{.071 - .059} = \frac{951}{.012} = 7.9 \times 10^4 \text{ psi}$$

Shear contribution to elastic midspan deflection for Beam #1

Midspan deflection = 0.071 "

Shear component of deflection $= \dfrac{P\ell c^2}{10\, GI} = \dfrac{420 \times 40 \times 1.765^2}{10 \times 7.9 \times 10^4 \times 5.5}$

$= 0.012''$

$$\frac{\text{Elastic shear deflection}}{\text{Elastic } \Delta_{\text{℄}}} \times 100 = \frac{0.012}{0.071} \times 100 = 17 \text{ percent.}$$