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In many statistical applications an interval is needed that will contain the values of all J future observations with some preassigned probability. For example, suppose twenty rockets have been fired in a test program and three have failed. If two more test programs are to be conducted, an interval that will, with probability $1-\alpha$, contain the maximum number of failures in either of the two programs is called an α confidence level prediction interval.

In this thesis a general procedure is given for predicting future observations when there is one unknown parameter and other conditions are satisfied. The normal and the gamma distributions are used as examples to illustrate the procedure in the continuous case. It is shown that Poisson random variables can be predicted using the negative multinomial distribution. Tables of negative multinomial probabilities are provided and approximation procedures are suggested. It is also shown that negative binomial random variables can be

predicted using the multivariate beta negative binomial and binomial random variables can be predicted using the multivariate negative hypergeometric distribution.

The prediction intervals given in this thesis can also be used for simultaneous hypothesis testing for the Poisson, negative binomial and binomial distributions.

Statistical Prediction Intervals

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STATISTICAL PREDICTION INTERVALS

I. INTRODUCTION

This thesis is concerned with the development of statistical prediction intervals for well known distributions. A statistical prediction interval is defined as a random interval that depends on past samples and that will contain all of a given number of future observations with a specified probability.

Statistical prediction intervals are frequently confused with other types of intervals. In order to draw a distinction, one prediction problem will be viewed in three different ways. In the first case there is enough data to assume that the distribution of the future samples is known. In the second, a tolerance interval is described and in the third a statistical prediction interval is described.

Suppose telephone switchboard operators have recorded the number of incoming telephone calls at a factory each working day for the past year. Assume the number of calls follows a Poisson distribution with mean λ and let $\bar{x}$ be the average number of incoming calls each day. If the variance of $\bar{x}$ is small, then $\bar{x}$ is very close to λ with probability near one. An interval that would contain the number of incoming calls for 95% of the future days could easily be calculated by substituting $\bar{x}$ for λ. The fact that $\bar{x}$ is only an estimate of λ would not appreciably affect the results.

Often there is not enough data to assume the parameter is essentially known. If records on incoming calls have only been kept for the past ten days then an interval that contains 95% or more of the number of future incoming calls per day with 90% certainty is called a tolerance interval.

In many cases only the next few events or observations are of interest. For example suppose the ten days of records on incoming telephone calls must be used to find an interval that will, with probability .95, contain the number of incoming calls for each of the next five days. An interval like the one just described contains the value of a fixed number of future observations with a specified probability and is known as a prediction interval.

This thesis considers the following prediction interval problem. Suppose $X_1, X_2, \ldots, X_I, Y_1, Y_2, \ldots, Y_J$, are independent observations from a family of distributions with one common unknown parameter. It will be assumed that the sum of the random variables is sufficient for the one unknown parameter and certain other conditions are satisfied. If $X_1, X_2, \ldots, X_I$ are observed before $Y_1, Y_2, \ldots, Y_J$ then how can $X_1, X_2, \ldots, X_I$ be used to find a prediction region for $Y_1, Y_2, \ldots, Y_J$? Specific distributions considered are the Poisson, the negative and positive binomial, the normal with known variance and the gamma distribution with an unknown scale parameter.

Considering the importance of prediction intervals, surprisingly little work has been done until recently. Several authors such as Aitchison and Sculthorpe (1965), Chew (1968), Faulkenberry (1972), Fisher (1956), Hahn (1969, 1970), Hewett (1968), Nelson (1970a, 1970b), Shah (1969), Thatcher (1964), and Weiss (1955) have given solutions for specific distributions or suggested methods for finding prediction intervals. Hahn and Nelson (1972) have recently surveyed the literature and provided an excellent summary. Hora and Bueler (1967) have discussed the uses of fiducial theory for prediction.

The work in this thesis is based on the paper by Weiss (1955) and extensions by Faulkenberry (1972).

II. GENERAL PREDICTION RESULTS

A. The Problem

Prediction intervals are similar to confidence intervals in that the probability statements are valid only before the data are taken. Suppose $X_1, X_2, \ldots, X_I$ is a random sample from a normal distribution with mean μ and known variance 1. Before the data are taken a probability statement of the form below is valid.

$$P[\mu \leq \overline{X} + Z_{1-\alpha}/\sqrt{I}] = 1 - \alpha$$

where

$$\overline{X} = \frac{1}{I} \sum_{i=1}^{I} X_i$$

and $Z_{1-\alpha}$ is the $1-\alpha$ percentage point of the standard normal distribution. However after the data are taken μ is either above or below the fixed value $\overline{x} + Z_{1-\alpha}/\sqrt{I}$. The probability statement is no longer valid and the $1-\alpha$ is referred to as a confidence level. The same type of statements are given for prediction intervals. The probability statements associated with prediction intervals are only valid in the usual probability sense before sampling. Like the confidence interval, the prediction interval, in the long run, will only fail to give correct results 100α percent of the time.

To clarify the type of statements to be made, consider a special case. Suppose X and Y are independent identically distributed random variables. The independence implies $P[Y \leq K/X = x] = P[Y \leq K]$. In this context knowing x gives no information about possible values of Y. However suppose there exists a function $h_\alpha(X)$ such that the joint probability statement of the form $P[Y \leq h_\alpha(X)] = 1-\alpha$ holds. In repeated sampling with $X = x$ and $Y = y$, y will be in the interval $(-\infty, h_\alpha(x)]$ $100(1-\alpha)$ percent of the time. Therefore, having observed $X = x$ a $1-\alpha$ confidence level would be associated with $(-\infty, h_\alpha(x)]$ as a prediction interval for Y. As with the confidence interval, strict probability statements would be valid only before the data are taken, but the statement would be meaningful after $X = x$ has been observed. For any given value of the unknown parameter in the joint distribution of X and Y there may be many functions $h_\alpha(X)$ such that $P[Y \leq h_\alpha(X)] = 1-\alpha$. It will be shown that in some special cases there is one function $h_\alpha(X)$ that works for all parameter values.

Definition 1. Let X and Y be random variables and suppose there exists a family of functions $h_\alpha(x)$ such that $P[Y \leq h_\alpha(X)] = 1-\alpha$. If for an observed x, there exists a unique α' such that $h_{\alpha'}(x) = K$, then the predictive probability that $Y \leq K$ having observed $X = x$ is defined to be $1-\alpha'$ and denoted by

$P[Y \leq K; x] = 1-\alpha'$.

In Section IIB, the family of functions $h_\alpha(x)$ with the required properties will be constructed.

If X and Y come from a discrete distribution and the function $h_\alpha(x)$ exists then there may not be a value K such that $P[Y \leq K; x] = 1-\alpha$ for some specified α. In this case K will be viewed as a random variable taking on one of two consecutive values. For example, if X and Y are independent identically distributed Poisson random variables, then it will be shown that $P[Y \leq 7; X = 3] = .9281$ and $P[Y \leq 8; X = 3] = .9673$. If a statement of the form $P[Y \leq K_{.95}; X = 3] = .95$ is required then $K_{.95}$ must be a random variable of the form

$$K_{.95} = \begin{cases} 7 & \text{with probability } .4413 \\ 8 & \text{with probability } .5587 . \end{cases}$$

The notation used for $K_{.95}$ will be 7.5587.

In the discrete case, the function $h_\alpha(x)$ will be a random variable for each observed x. This leads to the following definition.

Definition 2. A random function, $g(Z)$, is a function of Z and is a random variable for each observed value of $Z = z$.

If it is possible for each x and α $(0 < \alpha < 1)$ to find a value K such that $P[Y \leq K; x] = 1-\alpha$ then a predictive cummulative distribution function (cdf) exists for each x. The notation used for the cdf will be $F(y;x)$. If the corresponding density function exists, it will be referenced using $f(y;x)$. If several previous observations have been taken say $X_1, X_2, \ldots, X_I$ and $X = \sum_{i=1}^{I} X_i$ is sufficient for the unknown parameter then the notation and concepts just described above remain unchanged except X is the sum of I observations rather than one observation.

Before developing a solution, a final comment is in order. Suppose $h_\alpha(X)$ is found such that $P[Y \leq h_\alpha(X)] = 1-\alpha$ and when X and Y are observed, $y > h_\alpha(x)$. Then either an event with probability at most α has occurred or one of the assumptions has been violated. If X and Y are independent and from the same family of distributions, then the assumption violated would be that the unknown parameter for X is the same as the unknown parameter for Y. It follows that prediction intervals can be used for hypothesis testing. For example suppose X, Y_1, and Y_2 are Poisson distributed with parameters λ_X, λ_{Y_1} and λ_{Y_2} respectively. A prediction interval of the form $[0 \leq Y_1 < K,\ 0 \leq Y_2 \leq K]$ could be used to test the hypothesis $H: \lambda_X = \lambda_{Y_1} = \lambda_{Y_2}$ vs. $A: \lambda_X < \lambda_{Y_1}$ or $\lambda_X < \lambda_{Y_2}$. Multivariate prediction intervals can be used to test simultaneous hypothesis about unknown parameters.

B. The Theory

Suppose that the random variables $X_1, X_2, \ldots, X_I$, $Y_1, Y_2, \ldots, Y_J$ are independent from a joint distribution with one unknown parameter and the sum of the observations is sufficient for the parameter. Let $X = \sum_{i=1}^{I} X_i$, $Y = \sum_{j=1}^{J} Y_j$ and $T = X + Y$. Then X is sufficient for the distribution of $X_1, X_2, \ldots, X_I$, Y is sufficient for $Y_1, Y_2, \ldots, Y_J$, and T is sufficient for the joint distribution of X and Y.

The random variable T has a probability distribution with an unknown parameter. The distributions to be considered will be restricted to those whose sets of positive measure do not depend on the unknown parameter. In this way, the statement "for almost all t" will refer to any distribution of T, irrespective of the unknown parameter.

The problem is then to find the family of functions $h_\alpha(x)$ such that $P[Y \leq h_\alpha(X)] = 1-\alpha$ and such that for each K in the sample space of Y, there exists a unique α such that $h_\alpha(x) = K$. This will be done by first constructing a (random) function $k_\alpha(t)$ such that $P[Y \leq k_\alpha(T)] = 1-\alpha$. Theorem 1 below shows it is possible to construct such a function.

<u>Theorem 1</u>. Suppose X and Y are independent random

variables with a joint distribution with one unknown parameter and the sum of the observations, $T = X + Y$, is sufficient for the parameter. If the joint distribution of X and Y is continuous then $k_\alpha(t)$, a function of t, can be constructed such that

$$P[Y \leq k_\alpha(T)] = 1-\alpha . \tag{1}$$

If the joint distribution of X and Y is discrete then a random function $k_\alpha(t)$ of t can be constructed such that (1) holds.

Proof: Since T is sufficient, the distribution of Y given T has no unknown parameters. It is then possible for each t to find K_t (a random variable as described in the previous section in the discrete case) such that $P(Y \leq K_t/T = t) = 1-\alpha$. Let $k_\alpha(t) = K_t$ for each t and let

$$\phi(Y, T) = \begin{cases} 1 & \text{if} \quad Y \leq k_\alpha(T) \\ 0 & \text{if} \quad Y > k_\alpha(T) . \end{cases}$$

Then

$$\begin{aligned} P[Y \leq k_\alpha(T)] &= E[\phi(Y, T)] = E[E(\phi(Y, T)/T=t)] \\ &= E[P[Y \leq k_\alpha(T)/T=t)]] = E(1-\alpha) = 1-\alpha . \end{aligned}$$

That is

$$P[Y \leq k_\alpha(T)] = 1-\alpha .$$

Under conditions to be given, the set of y in the interval

$(-\infty, h_\alpha(t)]$ maps onto an interval $(-\infty, h_\alpha(x)]$ and it is possible to find the α value associated with the interval $(-\infty, h_\alpha(x)]$. Let $g_\alpha(t) = t - k_\alpha(t)$. In the continuous case, a sufficient condition is that $g_\alpha(t)$ is a strictly increasing function. In the discrete case, a sufficient condition is for $g_\alpha(t)$ to be a stochastically strictly increasing random function.

<u>Definition 3.</u> A function $g_\alpha(t)$ is said to be stochastically strictly increasing if for almost all $t_1 < t_2$, $P[g_\alpha(t_1) \leq x_0] > P[g_\alpha(t_2) < x_0]$ whenever $P[g_\alpha(t_1) \leq x_0] \neq 0$ or $P[g_\alpha(t_2) \leq x_0] \neq 1$.

<u>Theorem 2.</u> Let $g_\alpha(t) = t - k_\alpha(t)$. If $g_\alpha(t)$ is a strictly increasing function of t when the distributions of X and Y are continuous or a stochastically strictly increasing random function of t when the distributions of X and Y are discrete then $P[Y \leq K; x] = P[Y \leq K/T = x+K] = 1-\alpha$.

Proof: First consider the continuous case. In Theorem 1 a function of $k_\alpha(t)$ was constructed for each α between zero and one. It is possible to choose α by selecting the value of $g_\alpha(\cdot)$ say $g_\alpha(t) = x$ provided that there is some α corresponding to the choice. The value of α is uniquely chosen if $0 < \alpha < 1$, as will be shown. For each fixed t,

$$1-\alpha = P[Y \leq k_\alpha(T)/T = t] = P[Y \leq k_\alpha(X+Y)/X+Y = t]$$

$$= P[X \geq X+Y - k_\alpha(X+Y)/X+Y = t] = P[X \geq g_\alpha(T)/T = t].$$

Since the distribution of X given $T = t$ is free of unknown parameters, assigning $g_\alpha(t_0) = x_0$ implies $1-\alpha = P[X \geq x_0/T = t_0]$ and therefore a unique α can be calculated.

The fact that $g_\alpha(t)$ is strictly increasing implies that if $g_\alpha(t) = x$ then there is only one value of x such that $g_\alpha^{-1}(x) = t$. Let $t_\alpha(x) = g_\alpha^{-1}(x)$ where $t_\alpha(x)$ is defined for all x for which a value t exists such that $g_\alpha(t) = x$. Then

$$P[X \geq g_\alpha(T)] = P[g_\alpha^{-1}(X) \geq T] = P[t_\alpha(X) \geq T] .$$

So

$$P[Y \leq k_\alpha(T)] = P[Y \leq k_\alpha(X+Y)] = P[X \geq X+Y - k_\alpha(X+Y)]$$

$$= P[X \geq T - k_\alpha(T)] = P[X \geq g_\alpha(T)] = P[t_\alpha(X) \geq T]$$

$$= P[X+Y \leq t_\alpha(X)] = P[Y \leq t_\alpha(X)-X] = P[Y \leq t_\alpha(x)-x; x].$$

The fact that $g_\alpha(t)$ is increasing makes it possible for the equality, $P[Y \leq t_\alpha(X)-X] = P[Y \leq t_\alpha(x)-x; x]$ to hold. For if $g_\alpha(t)$ were not monotone, $t_\alpha(x)$ would not be monotone and the set of values of t such that $t \leq t_\alpha(x)$ would not necessarily be an interval. This means that the set of y where $y \leq t(x)-x$ would not necessarily be an interval. Therefore the equality would not necessarily hold.

However, since $g_\alpha(t)$ is strictly increasing, $P[Y \leq k_\alpha(T)] = P[Y \leq t_\alpha(x)-x; x]$ and the function $h_\alpha(x)$ can be defined as $h_\alpha(x) = t_\alpha(x)-x$.

A value of α will now be selected by setting $g_\alpha(t) = x + K$. Since $g_\alpha(t) = t - k_\alpha(t)$, $g_\alpha(t) = x + K$ implies that $k_\alpha(x+K) = K$, $t_\alpha(x) = x + K$ and $h_\alpha(x) = K$.

The proof of the theorem in the continuous case now follows in a few short steps.

$$P[Y \leq K/T = x+K] = P[Y \leq k_\alpha(x+K)/T=x+K] = P[Y \leq k_\alpha(T)]$$
$$= P[Y \leq t_\alpha(x)-x; x] = P[Y \leq h_\alpha(x); x]$$
$$= P[Y \leq K; x] \; .$$

In the discrete case, for each fixed t, $g_\alpha(t)$ is a random variable, so that it is not clear what would be meant by $g_\alpha^{-1}(x)$. However, it is possible to define a random function $t_\alpha(x)$ which plays a role similar to that played by $g_\alpha^{-1}(x)$ in the continuous case. Once $t_\alpha(x)$ is defined the proof follows a similar line to that of the continuous case.

Note

$$1-\alpha = P[g_\alpha(T) \leq X/T=t] = \sum_t \sum_x P[g_\alpha(t) \leq x]P[X = x/T = t] \; .$$

Then $t_\alpha(x)$ can be defined by $P[t_\alpha(x) \leq t] = P[g_\alpha(t) \leq x]$ which gives

$$P[X \geq g_\alpha(T)] = \sum_t \sum_x P[x \geq g_\alpha(t)]P[X = x/T = t]P[T=t]$$

$$= \sum_t \sum_x P[t_\alpha(x) \geq t]P[X=x/T=t]P[T=t] = P[t_\alpha(X) \geq T] \, .$$

It is possible to find the probability distribution associated with $t_\alpha(x)$. First choose x and t_1 and let t_0 be the largest value of t such that $t_0 < t_1$. Since $g_\alpha(t)$ is stochastically strictly increasing,

$$P[t_\alpha(x) \geq t_1] = P[g_\alpha(t_1) \leq x] \leq P[g_\alpha(t_0) \leq x] = P[t_\alpha(x) \geq t_0] \, .$$

Therefore

$$P[t_\alpha(x) = t_0] = P[t_\alpha(x) \geq t_0] - P[t_\alpha(x) \geq t_1] \geq 0 \, .$$

This result implies $t_\alpha(x)$ is a random function and $(-\infty, t_\alpha(X)-X]$ is a random interval.

Define $h_\alpha(x) = t_\alpha(x) - x$ and note that

$$\{(x, y); y \leq k_\alpha(x+y)\} = \{(x, y); y \leq h_\alpha(x)\} \, .$$

Then choose α by selecting k_α such that $P[k_\alpha(x_0+K) = K] = 1$. This implies that $P[h_\alpha(x_0) = K] = 1$. For the chosen α, the equalities below prove the theorem in the discrete case.

$$P[Y \leq K/T = x_0+K] = P[Y \leq k_\alpha(x_0+K)/T = x_0+K] = P[Y \leq k_\alpha(T)]$$

$$= P[g_\alpha(T) \leq X] = P[t_\alpha(X) \geq T]$$

$$= P[X+Y \leq t_\alpha(X)] = P[Y \leq t_\alpha(X)-X]$$

$$= P[Y \leq h_\alpha(X)] = P[Y \leq h_\alpha(x_0); x_0]$$

$$= P[Y \leq K; x_0] .$$

This proves the theorem and provides a method of finding $P[Y \leq K; x]$ by using the distribution of Y given $T = x + K$ to find $1-\alpha$. It should be pointed out that choosing the value of g_α or k_α at a point does not constrain the form of the function.

C. Special Classes of Distributions

Showing that the (random) function $g_\alpha(t)$ is (stochastically) strictly increasing when the other conditions for Theorem 2 are satisfied is not always easy. Three cases will be considered and examples of each will be given.

1. Conditional Location Parameter Families

Theorem 3. Suppose the conditions for Theorem 1 are satisfied and let $F_{X|t}(x)$ be the conditional cdf of X given t. Then

a) if $\ell(t)$ is a location parameter for the distribution of X given t, $(F_{X|t}(x) = F(x-\ell(t))$ where $\ell(t)$ is a strictly increasing function of t for almost all t

b) and in the continuous case if for every K_1 and K_2 such that $F(K_1) \neq 1$ and $F(K_2) \neq 0$, $K_1 < K_2$ implies $F(K_1) < F(K_2)$,

c) then $g_\alpha(t)$ is a (stochastically) strictly increasing (random) function of t.

In addition

d) if $\ell'(t)$ is a location parameter for the distribution of Y given t where $\ell'(t)$ is a strictly increasing function of t for almost all t,

e) then $P[Y \leq K; x] = G(K-\ell'(x+K))$ where $G(\cdot)$ is the cdf of $Y - \ell'(t)$.

Before Theorem 3 is proved it should be noted that Theorem 5 below is similar. The main difference is that Theorem 5 assumes $\ell(t)$ and $\ell'(t)$ are scale parameters. The proofs for both these theorems are essentially the same so Theorem 3 will be proved for the discrete case and Theorem 5 will be proven for the continuous case.

Proof for the Discrete Case: 1) $1 - F_{X|t}[g_\alpha(t)] = F_{Y|t}[k_\alpha(t)]$ for almost all t. To prove this statement let the density function for X given t be $f_{X|t}(x)$ and for Y given t be $f_{Y|t}(y) = f_{X|t}(t-y)$. Then a change of variable gives

$$\sum_{y_\alpha(t) \le x < \infty} f_{X|t}(x) = \sum_{-\infty < y \le k_\alpha(t)} f_{Y|t}(y)$$

or

$$1 - F_{X|t}(g_\alpha(t)) = F_{Y|t}(k_\alpha(t)) \;.$$

Since $k_\alpha(t)$ is a random variable, $F_{Y|t}(k_\alpha(t))$ would also be a random variable. But to avoid introducing additional notation, $F_{Y|t}(k_\alpha(t))$ will be taken as the expected value. That is, suppose

$$k_\alpha(t) = \begin{cases} a & \text{with probability } p \\ a+1 & \text{with probability } 1-p \,. \end{cases}$$

Then

$$F_{Y|t}[k_\alpha(t)] = \sum_{-\infty < y \le k_\alpha(t)} f_{Y|t}(y) = \sum_{-\infty < y \le a} f_{Y|t}(y) + (1-p) f_{Y|t}(a+1)$$

2) $F_{Y|t_1}[k_\alpha(t_1)] = F_{Y|t_2}[k_\alpha(t_2)]$ for almost all t_1 and t_2.

This follows since $k_\alpha(\cdot)$ is defined so that $P[Y \le k_\alpha(t) \mid T = t] = 1-\alpha$ for almost all t.

3) Combining 1) and 2) gives

$$1 - F_{X|t_1}[g_\alpha(t_1)] = F_{Y|t_1}[k_\alpha(t_1)] = F_{Y|t_2}[k_\alpha(t_2)] = 1 - F_{X|t_2}[g_\alpha(t_2)] \;.$$

4) For almost all t_1 and t_2 such that $t_1 < t_2$ $F_{X|t_1}[g_\alpha(t_1)] > F_{X|t_2}[g_\alpha(t_1)]$. Since $\ell(t)$ is a location parameter, $F_{X|t_1}[g_\alpha(t_1)] = F[g_\alpha(t_1)-\ell(t_1)]$ and $F_{X|t_2}[g_\alpha(t_1)] = F[g_\alpha(t_1)-\ell(t_2)]$. Let $f(x)$ be the density function corresponding to $F(\cdot)$.

The expression $g_\alpha(t_1)-\ell(t_1)$ and $g_\alpha(t_1)-\ell(t_2)$ need clarification. For $0 \le p < 1$

$$g_\alpha(t_1) = \begin{cases} a & \text{with probability } p \\ a+1 & \text{with probability } 1-p \end{cases}$$

so that

$$g_\alpha(t_1)-\ell(t_2) = \begin{cases} a-\ell(t_2) & \text{with probability } p \\ a-\ell(t_2)+1 & \text{with probability } 1-p \end{cases}$$

Then

$$F_{X|t_2}[g_\alpha(t_1)] = F[g_\alpha(t_1)-\ell(t_2)]$$

$$= \sum_{-\infty < x \le a-\ell(t_2)} f(x) + (1-p) \times \sum_{a-\ell(t_2) < x \le a-\ell(t_2)+1} f(x)$$

and

$$F_{X|t_1}[g_\alpha(t_1)] = F[g_\alpha(t_1)-\ell(t_1)]$$

$$= \sum_{-\infty < x \le a-\ell(t_1)} f(x) + (1-p)f(a-\ell(t_1)+1) = \alpha > 0.$$

Note that the point $a-\ell(t_1)+1$ has positive probability since it is a translation of the point "a+1" which has positive probability with respect to the distribution of X given $T = t_1$. Recall that $\ell(t)$ is a strictly increasing function of t so that $a-\ell(t_2)+1 < a-\ell(t_1)+1$. Therefore the probability at $a-\ell(t_1)+1$ is not included in the probability summed to get $F(g_\alpha(t_1)-\ell(t_2))$. The probability at $a-\ell(t)+1$ is included at least in part $[(1-p) > 0]$ when finding $F[g_\alpha(t_1)-\ell(t_1)]$. Therefore

$$F_{X|t_1}[g_\alpha(t_1)] > F_{X|t_2}[g_\alpha(t_2)] \, .$$

5) Item 3) implies $1 - F_{X|t_1}[g_\alpha(t_1)] = 1 - F_{X|t_2}[g_\alpha(t_2)]$ and item 4) implies $1 - F_{X|t_1}[g_\alpha(t_1)] < 1 - F_{X|t_2}[g_\alpha(t_1)]$. Therefore $g_\alpha(t_1) < g_\alpha(t_2)$ when $t_1 < t_2$ for almost all t_1 and t_2 and $g_\alpha(\cdot)$ is a stochastically strictly increasing function of t.

To show part d) note the condition that $g_\alpha(t)$ is strictly increasing gives the condition needed to use Theorem 2. So

$$P[Y \leq K; x] = P[Y \leq K | T = x+K] = F_{Y|T=x+K}[K] = G[K-\ell(x+K)].$$

Corollary 1. If $\ell'(t) = bt$, $b > 0$, is a location parameter for the distribution of Y given t, then for some a, $k_\alpha(t) = a + bt$.

Proof: Choose $\Delta > 0$ and let $F_{Y|t}[\cdot]$ be the cdf for the distribution of Y given $T = t$. Then

$$F_{Y|t}[k_\alpha(t)] = G[k_\alpha(t)-\ell'(t)] = G[k_\alpha(t)+\ell'(\Delta)-\ell'(\Delta)-\ell'(t)]$$

$$= G[k_\alpha(t)+\ell'(\Delta)-\ell'(t+\Delta)] = F_{Y|T=t+\Delta}[k_\alpha(t)+\ell'(\Delta)] = 1 - \alpha .$$

But

$$F_{Y|t}[k_\alpha(t)] = F_{Y|T=t+\Delta}[k_\alpha(t+\Delta)]$$

so

$$F_{Y|T=t+\Delta}[k_\alpha(t)+\ell'(\Delta)] = F_{Y|T=t+\Delta}[k_\alpha(t+\Delta)] .$$

Then $k_\alpha(t)+\ell'(\Delta) = k_\alpha(t+\Delta)$. But $\ell(\Delta) = b\Delta$, where $b > 0$ so $k_\alpha(0) + b\Delta = k_\alpha(\Delta)$. Letting $a = k_\alpha(0)$ gives $k_\alpha(t) = a + bt$.

2. Example: The Normal Distribution

Suppose $X_1, X_2, \ldots, X_I, Y_1, Y_2, \ldots, Y_J$ are independent normally distributed random variables with unknown mean μ and known variance σ^2. Let $N = I + J$, $X = \sum_{i=1}^{I} X_i$, $Y = \sum_{j=1}^{J} Y_j$ and $T = X + Y$. The conditional distribution of X given t is used to show $\ell(t) = \frac{J}{N}t$ is a location parameter and the conditional distribution of Y given t is used to find the predictive distribution. The distribution of Y given t is $N(\frac{J}{N}t, \sigma^2 \frac{IJ}{N})$ and of X given t is $N(\frac{I}{N}t, \sigma^2 \frac{IJ}{N})$. It follows that $\ell(t) = \frac{I}{N}t$ is a location parameter for the distribution of X given t. Therefore $g_\alpha(t)$ is a strictly increasing function of T and Theorem 2 could be applied to find the predictive probability for Y.

Since $\ell'(t) = \frac{I}{N}t$ is a location parameter for the distribution of Y given t, Theorem 3 can be applied to find the predictive probability. First note the distribution of $Y - \ell'(t) = Y - \frac{J}{N}t$ given $T = t$ is normally distributed with mean zero and variance $\sigma^2 \frac{IJ}{N}$ and cdf $G(\cdot)$. Let $H(\cdot)$ be the cdf of the standard normal distribution. Then

$$P[Y \leq K; x] = G[K - \frac{J}{N}(x+K)] = G[\frac{IK-Jx}{N}] = H[\frac{IK-Jx}{N} \frac{\sqrt{N}}{\sigma\sqrt{IJ}}]$$

$$= H[\frac{K/J - \bar{x}}{\sigma\sqrt{N/IJ}}]$$

where $\bar{x} = x/I$.

The predictive distribution of Y when $J = 1$ and X has been observed to be x is normal with mean $\bar{x}$ and variance $\sigma^2 \frac{N}{I}$.

The fact that $\ell'(t) = \frac{J}{N}t$ is a location parameter implies that $k_a(t) = a + (J/N)t$. For $t = 0$, the value of a can easily be found. Then

$$P[Y \leq k_a(T)] = P[Y \leq a+(J/N)T] = P[Y \leq a+(X+Y)\frac{J}{N}] = P[Y \leq \frac{N}{I}a + J\bar{X}] .$$

The function $h_a(x)$ is linear. This suggests that if X and Y are from distributions which tend to be normal, as I and J become large, $h_a(x)$ tends to be linear.

3. Conditional Scale Parameter Family

Theorem 4. Let $F_{X|t}(x)$ be the conditional cdf of X given t and $F_{Y|t}(y)$ be the conditional cdf of Y given t.

a) If the strictly increasing function $\ell(t)$ is a scale parameter for the distribution of X given t $(F(\frac{K}{\ell(t)}) = F_{X|t}(K))$,

b) and in the continuous case if for every K_1 and K_2, such that $F(K_1) \neq 1$ and $F(K_2) \neq 0$, $K_1 < K_2$ implies $F(K_1) < F(K_2)$.

c) then $g_\alpha(t)$ is a (stochastically) strictly increasing (random) function of t.

d) If the strictly increasing function $\ell'(t)$ is a scale parameter for the distribution of Y given t,

e) then $P[Y \leq K; x] = G(\frac{K}{\ell'(x+K)})$, where $G(\cdot)$ is the cdf of $Y/\ell'(t)$.

Proof for the continuous case: 1) $1 - F_{X|t}[g_\alpha(t)] = F_{Y|t}[k_\alpha(t)]$ for almost all t. This equation is equivalent to

$$\int_{g_\alpha(t)}^{\infty} dF_{X|t}(x) = \int_{-\infty}^{k_\alpha(t)} dF_{Y|t}(y),$$

where $y = t - x$ and $F_{X|t}(t-y) = F_{Y|t}(y)$. The result comes from a change of variable and noticing $g_\alpha(t) = t - k_\alpha(t)$, so $g_\alpha(t) \leq x < \infty$ if and only if $-\infty < y \leq k_\alpha(t)$.

2) $F_{Y|t_1}[k_\alpha(t_1)] = F_{Y|t_2}[k_\alpha(t_2)]$. This equality follows from the definition of $k_\alpha(t)$.

3) Combining 1) and 2) gives

$$1 - F_{X|t_1}[g_\alpha(t_1)] = F_{Y|t_1}[k_\alpha(t_1)] = F_{Y|t_2}[k_\alpha(t_2)]$$

$$= 1 - F_{X|t_2}[g_\alpha(t_2)] \; .$$

4) Since for $K_1 < K_2$, $F(K_1) < F(K_2)$ and since $\ell(t)$ is a scale parameter,

$$F_{X|t_1}[g_\alpha(t_1)] = F[\frac{g_\alpha(t_1)}{\ell(t_1)}] > F[\frac{g_\alpha(t_1)}{\ell(t_2)}] = F_{X|t_2}[g_\alpha(t_1)] \; ,$$

for almost all $t_1 < t_2$.

5) Item 4) states $F_{X|t_1}[g_\alpha(t_1)] > F_{X|t_2}[g_\alpha(t_1)]$ and item 3) implies $F_{X|t_1}[g_\alpha(t_1)] = F_{X|t_2}[g_\alpha(t_2)]$. This means $g_\alpha(t_1) < g_\alpha(t_2)$ for almost all $t_1 < t_2$ or equivalently $g_\alpha(t)$ is a strictly increasing function of t.

Since the condition that $g_\alpha(t)$ is a strictly increasing function of t gives the final required condition for Theorem 2 to hold,

$$P[Y \leq K; x] = P[Y \leq K | T = x+K] = F_{Y|T=x+K}[K] = G[\frac{K}{\ell'(x+K)}] \; .$$

This proves part d) of Theorem 5.

4. Example: The Gamma Distribution

Suppose the waiting time for an event to occur follows an exponential distribution with expected waiting time $1/\beta$. Let X_i be the waiting time for α events to occur. Then X_i is distributed according to a gamma distribution with density function,

$$f(x_i) = \left(\frac{\beta^{\alpha}}{\gamma(\alpha)}\right) x_i^{\alpha-1} e^{-x_i\beta}, \quad 0 < x_i < \infty.$$

Suppose $X_1, X_2, \ldots, X_I$, $Y_1, Y_2, \ldots, Y_J$ are independent identically distributed random variable from the gamma described above. Let

$$X = \sum_{i=1}^{I} X_i, \quad Y = \sum_{j=1}^{J} Y_j \quad \text{and} \quad N = I + J .$$

Since the gamma distribution is reproductive, X, Y and T have gamma distributions with parameters $(I\alpha, \beta)$, $(J\alpha, \beta)$ and $(N\alpha, \beta)$ respectively. The density functions are

$$f_X(x) = \frac{\beta^{I\alpha}}{\gamma(I\alpha)} x^{I\alpha-1} e^{-x\beta} \qquad 0 < x < \infty,$$

$$f_Y(y) = \frac{\beta^{J\alpha}}{\gamma(J\alpha)} y^{J\alpha-1} e^{-y\beta} \qquad 0 < y < \infty,$$

$$f_T(t) = \frac{\beta^{N\alpha}}{\gamma(N\alpha)} t^{N\alpha-1} e^{-t\beta} \qquad 0 < t < \infty.$$

Then

$$f_{X|t}(x) = \frac{f_{XT}(x,t)}{f_T(t)} = \frac{f_{XY}(x,y)}{f_T(t)}$$

$$= \frac{\gamma(N\alpha)}{\gamma(I\alpha)\gamma(J\alpha)} \left(\frac{x}{t}\right)^{I\alpha-1}\left(\frac{t-x}{t}\right)^{J\alpha-1}\frac{1}{t}$$

and

$$f_{Y|t}(y) = \frac{\gamma(N\alpha)}{\gamma(I\alpha)\gamma(J\alpha)} \left(\frac{y}{t}\right)^{J\alpha-1}\left(\frac{t-y}{t}\right)^{I\alpha-1}\frac{1}{t} .$$

To show that $\ell(t) = t$ is a scale parameter for the distribution of X given t let $z = x|t$. Then

$$F_{X|t}(K|t) = F\left(\frac{K}{t}\right) = \int_0^{K/t} \frac{\gamma(N\alpha)}{\gamma(J\alpha)\gamma(I\alpha)} z^{I\alpha-1}(1-z)^{J\alpha-1}dz .$$

In a similar way it can be shown that $\ell'(t) = t$ is a scale parameter for the distribution of Y given t. Theorem 4 implies that

$$P[Y \leq K; x] = G\left(\frac{K}{x+K}\right) = \int_0^{K/x+K} \frac{\gamma(N)}{\gamma(I\alpha)\gamma(J\alpha)} z^{J\alpha-1}(1-z)^{I\alpha-1}dz .$$

A value K such that $P[Y \leq K; x] = .95$ can be found by using tables of incomplete beta functions to find a value K_0 such that $G(K_0) = .95$. Then $K = K_0x/(1-K_0)$.

5. Conditional Monotone Likelihood Ratio Family

Definition 4. A real parameter family of distributions is said to have a monotone likelihood ratio if the densities exist such that whenever $t_1 < t_2$ the likelihood ratio

$$\frac{f_{X|t_1}(x)}{f_{X|t_2}(x)}$$

is a nondecreasing function of x in the set of existence, that is, for x in the set of points for which at least one $f_{X|t_1}(x)$ and $f_{X|t_2}(x)$ is positive.

Theorem 5.

a) If the distribution of X given t has density $f_{X|t}(x)$ which has a monotone likelihood ratio and cdf $F_{X|t}(x)$ and

b) if for almost all $t_1 < t_2$ there exists a measurable set A in the sample space such that

$$\int_A dF_{X|t_1}(x) \neq \int_A dF_{X|t_2}(x) ,$$

c) then $g_a(t)$ is a (stochastically) strictly increasing (random) function of t.

Proof: 1) As in the case of Theorems 3 and 4 we have

$$F_{X|t_1}[g_\alpha(t_1)] = F_{X|t_2}[g_\alpha(t_2)] .$$

This follows since

$$F_{Y|t}[k_\alpha(t)] = 1 - F_{X|t}[g_\alpha(t)]$$

by a change of variable and since for almost all t_1 and t_2 and

$$F_{Y|t_1}[k_\alpha(t_1)] = F_{Y|t_2}[k_\alpha(t_2)] = 1 - \alpha$$

by the definition of $k_\alpha(\cdot)$.

2) It will be shown that for $t_1 < t_2$

$$F_{X|t_1}(g_\alpha(t_1)) > F_{X|t_2}(g_\alpha(t_1)).$$

This will be done by considering t in the distribution of X given t as a parameter and testing $H_0: T = t_1$ against $H_a: T = t_2$. To find a best test of size $\alpha \neq 0$ the Neyman-Pearson Lemma can be used. The form of the best α-level test is

$$\phi_\alpha(x) = \begin{cases} 1 & \text{if } f_{X|t_2}(x)/f_{X|t_1}(x) > K_0 \\ \beta(x) & \text{if } f_{X|t_2}(x)/f_{X|t_1}(x) = K_0 \\ 0 & \text{if } f_{X|t_2}(x)/f_{X|t_1}(x) < K_0 \end{cases}$$

where $0 \leq \beta(x) < 1$. Since $f_{X|t}(x)$ has a monotone likelihood ratio, $f_{X|t_2}(x)/f_{X|t_1}(x)$ is an increasing function of x. Then $\phi_\alpha(x)$ can also be written in the form

$$\phi_\alpha(x) = \begin{cases} 1 & \text{if} \quad x > x_0 \\ \beta & \text{if} \quad x = x_0 \\ 0 & \text{if} \quad x < x_0 \end{cases} .$$

Let $x_1 = x_0$ in the continuous case and $x_1 = x_0$ with probability β and the next smaller value of x with probability $1-\beta$ in the discrete case. Then

$$E_{t_1}\phi_\alpha(X) = \int_{x_1}^{\infty} dF_{X|t_1}(x) = \alpha = \int_{g_\alpha(t_1)}^{\infty} dF_{X|t_1}(x).$$

Since ϕ_α is unique (Ferguson, 1968), $g_\alpha(t) = x_1$ and

$$E_{t_2}\phi_\alpha(X) = \int_{x_1}^{\infty} dF_{X|t_2}(x) = \int_{g_\alpha(t_1)}^{\infty} dF_{X|t_2}(x)$$

Let $\phi_0(x) = \alpha$. Then the test $\phi_0(x)$ has power α and it is seen that $E_{t_1}\phi_\alpha(X) \leq E_{t_2}\phi_\alpha(X)$, since $\phi_\alpha(x)$ is a best test of size α and $E_{t_2}\phi_0(X) = \alpha$. If $E_{t_1}\phi_\alpha(X) = E_{t_2}\phi_\alpha(X)$ then $\phi_0(x) = \alpha$ is a best of size α and $\phi_\alpha(x)$ must have the form

$$\phi_\alpha(x) = \begin{cases} 1 & \text{if } f_{X|t_2}(x) > Kf_{X|t_1}(x) \\ \beta & \text{if } f_{X|t_2}(x) = Kf_{X|t_1}(x) \\ 0 & \text{if } f_{X|t_2}(x) < Kf_{X|t_1}(x) \end{cases}$$

Then $\beta = \alpha$ and $f_{X|t_2}(x) = Kf_{X|t_1}(x)$ almost everywhere and this implies $K = 1$. But this contradicts condition b) of the theorem. Therefore $E_{t_1}\phi_\alpha(X) < E_{t_2}\phi_\alpha(X)$ or

$$\int_{g_\alpha(t_1)}^{\infty} dF_{X|t_1}(x) < \int_{g_\alpha(t_1)}^{\infty} dF_{X|t_2}(x)$$

so

$$F_{X|t_1}(g(t_1)) > F_{X|t_2}(g(t_1)) .$$

3) Items 1) and 2) established $F_{X|t_1}[g_\alpha(t_1)] = F_{X|t_2}[g_\alpha(t_2)]$ and $F_{X|t_1}[g_\alpha(t_1)] > F_{X|t_2}[g_\alpha(t_1)]$. Therefore $g_\alpha(t_1) < g_\alpha(t_2)$ and $g_\alpha(\cdot)$ must be a (stochastically) strictly increasing (random) function of t.

Theorems 2 and 5 will be used to predict observations from the Poisson, the binomial and negative binomial distributions. In each of these cases it will be shown that the distribution of X given t is distributed according to some member of the exponential family which has a monotone likelihood ratio.

III. THE POISSON DISTRIBUTION

Theorems 2 and 5 will be used to solve a special case of the problem stated below. This result will then be extended to solve the more general problem. Suppose $X_1, X_2, \ldots, X_I$ are independent Poisson random variables observed for periods $\ell_1, \ell_2, \ldots, \ell_I$ respectively. The expected number of observations per unit of time, λ, is unknown. Suppose J more observations $Y_1, Y_2, \ldots, Y_J$ will be taken for periods of length $m_1, m_2, \ldots, m_J$ respectively. Let $X = \sum_{i=1}^{I} X_i$ and $L = \sum_{i=1}^{I} \ell_i$. The problem is then to find random variables K_j such that

$$K_j = \begin{cases} a_j & \text{with probability} \quad p_j \\ a_j+1 & \text{with probability} \quad 1-p_j \end{cases}$$

where a_j is an integer and

$$P[Y_1 \le K_1, Y_2 \le K_2, \ldots, Y_J \le K_J;\ X = x] = 1 - \alpha .$$

A. Predicting One Future Poisson Observation

Let $Y = \sum_{j=1}^{J} Y_j$, $T = X + Y$ and $M = \sum_{j=1}^{J} m_j$. Then X, Y, and T are Poisson distributed with parameters $L\lambda$, $M\lambda$ and $(L+M)\lambda$ respectively. Since T is sufficient for the distribution of

X and Y, if $g_\alpha(t)$ is a stochastically monotone strictly increasing random function of t, then $P[Y \leq K; x] = P[Y \leq K | T = x + K]$. The condition will be satisfied if the distribution of X given t has a monotone likelihood ratio.

The distribution of X given t and Y given t are easily found. Since X and Y are independent

$$f_{XY}(x, y) = \frac{e^{-\lambda L}(\lambda L)^x}{x!} \frac{e^{-\lambda M}(\lambda M)^y}{y!}$$

The density function for t is given by

$$f_T(t) = \frac{e^{-\lambda(L+M)}\lambda^t(L+M)^t}{t!} \qquad t = 0, 1, 2, \ldots .$$

Then

$$f_{X|t}(x) = \frac{f_{XT}(x, t)}{f_T(t)} = \frac{f_{X,Y}(x, y)}{f_T(t)} = \frac{t!}{X!Y!}\left(\frac{L}{L+M}\right)^x\left(\frac{M}{L+M}\right)^y$$

$$= \binom{t}{x} p^x (1-p)^{t-x} ,$$

where $p = \frac{L}{L+M}$. Similarly

$$f_{Y|t}(y) = \binom{t}{y} p^{t-y}(1-p)^y .$$

The conditional distribution of X given t is binomial. Since the binomial distribution has a monotone likelihood ratio (Ferguson, 1968) Theorem 5 implies $g_\alpha(t)$ is a stochastically monotone increasing random function of t.

Theorem 2 then implies

$$P[Y \le K; x] = P[Y \le K | T = x+K] = \sum_{y=0}^{K} \binom{x+K}{y} p^{x+K-y}(1-p)^y .$$

For example if $L = M$ and $J = 1$ then

$$P[Y \le 7; 3] = \sum_{y=0}^{7} \binom{10}{y} (\tfrac{1}{2})^{10-y} (\tfrac{1}{2})^y = .9281$$

and

$$P[Y \le 8; 3] = \sum_{y=0}^{8} \binom{11}{y} (\tfrac{1}{2})^{11-y} (\tfrac{1}{2})^y = .9673.$$

To find a value $K_{.95}$ such that $P[Y \le K_{.95}; 3] = .95$ use $K_{.95} = 7$ with probability .4413 and use $K_{.95} = 8$ with probability .5587 (denoted as 7.5587).

It is now possible to find the predictive density function $f(y; x)$.

<u>Theorem 6.</u> Let Z be a random variable with a negative binomial distribution with parameters x and $q = \frac{M}{L+M}$. Then

$$P[Y \le K; x] = P[Z \le K] .$$

Proof: The conclusion follows from the two well known results given by Beyer (1966).

1) $$\sum_{j=0}^{K} \binom{j+x-1}{x-1} q^x p^j = \sum_{j=x}^{x+K} \binom{x+K}{j} q^j p^{K+x-j}$$

2) $$\sum_{j=x}^{x+K} \binom{K+x}{j} q^j p^{K+x-j} = \sum_{j=0}^{K} \binom{K+x}{j} p^j q^{K+x-j} .$$

From 1) and 2) it follows that

$$P[Z \leq K] = \sum_{j=0}^{K} \binom{j+x-1}{x-1} q^x p^j = \sum_{j=0}^{K} \binom{K+x}{j} p^j q^{K+x-j}$$

$$= P[Y \leq K \mid T = x+K] = P[Y \leq K; x] .$$

Theorem 6 makes it possible to set up a one-sided or two-sided prediction interval or any other sort of prediction region and find the probability that the future observation will be in that interval or region.

B. Predicting J Future Poission Observations

The predictive probability of J future observations, $P[Y_1 = K_1, \ldots, Y_I = K_J; x]$ when Y is sufficient for the distribution of $Y_1, Y_2, \ldots, Y_J$, is given by the predictive probability that $Y = K.$ times the probability $Y_1 = K_1, \ldots, Y_J = K_J$ given $Y = K$. That is

$$P[Y_1, \ldots, Y_J = K_J; x] = P[Y = K. ; x] P[Y_1 = K_1, \ldots, Y_J = K_J / Y = K] .$$

It is now easy to extend the above results to the case where $1 < J < \infty$.

Theorem 7. If $X, Y_1, Y_2, \ldots, Y_J$ are independent Poisson random variables with parameters $\lambda L, \lambda m_1, \lambda m_2, \ldots, \lambda m_J$ respectively, $(L, m_1, m_2, \ldots, m_J$ known) then

$$P[Y_1 = K_1, \ldots, Y_J = K_J; X = x] = \binom{K. + x - 1}{K_1, \ldots, K_J}\left(\frac{L}{L+M}\right)^x \left(\frac{m_1}{L+M}\right)^{K_1} \cdots \left(\frac{m_J}{L+M}\right)^{K_J},$$

where $M = \sum_{j=1}^{J} m_j$ and $K. = \sum_{j=1}^{J} K_j$. That is, the multivariate predictive distribution of $Y_1, Y_2, \ldots, Y_J$ when $X = x$ is a negative multinomial (Johnson and Kotz, 1969).

Proof: First observe

$$P[Y_1 = K_1, Y_2 = K_2, \ldots, Y_J = K_J] = \binom{K.}{K_1, \ldots, K_{J-1}}\left(\frac{m_1}{M}\right)^{K_1} \cdots \left(\frac{m_J}{M}\right)^{K_J}$$

That is, the conditional distribution of Poisson random variables given their total follows a multinomial distribution. This can be shown as follows:

$$P[Y_1=K_1,\ldots,Y_J=K_J/Y=K.] = \frac{P[Y_1=K_1,\ldots,Y_J=K_J]}{P[Y=K.]}$$

$$= \frac{\dfrac{e^{-\lambda M}\lambda^{K.} m_1^{K_1}\cdots m_J^{K_J}}{K_1!\ldots K_J!}}{\dfrac{e^{-\lambda M}\lambda^{K.} M^{K.}}{K.!}}$$

$$= \binom{K.}{K_1,\ldots,K_{J-1}}\left(\frac{m_1}{M}\right)^{K_1}\cdots\left(\frac{m_J}{M}\right)^{K_J} .$$

Then

$$P[Y_1=K_1,\ldots,Y_J=K_J;X=x]$$

$$= P[Y=K.;X=x]P[Y_1=K_1,\ldots,Y_J=K_J/Y=K.]$$

$$= \left[\binom{K.+x-1}{x-1}\left(\frac{L}{L+M}\right)^x\left(\frac{M}{L+M}\right)^{K.}\right]\left[\binom{K.}{K_1,\ldots,K_{J-1}}\left(\frac{m_1}{M}\right)^{K_1}\cdots\left(\frac{m_J}{M}\right)^{K_J}\right]$$

$$= \frac{\gamma(K.+x)}{\gamma(x)K_1!\ldots K_J!}\left(\frac{L}{L+M}\right)^x\left(\frac{m_1}{M}\right)^{K_1}\cdots\left(\frac{m_J}{M}\right)^{K_J}$$

which is the density function of a negative multinomial.

As in the case of one future observation it is possible to use the predictive density to assign probability to various types of prediction regions. One particular type of special interval is

$[Y_1, Y_2, \ldots, Y_J; 0 \le Y_j \le K \quad \text{for} \quad j = 1, 2, \ldots, J]$.

C. Computing Form of the Negative Multinomial

Values of K have been tabled in the Appendix for 2, 3, 4 and 5 future observations for $\alpha = .1$, $.05$ and $.01$. The values in the tables are random variables as described in Section IIA. In order to keep the size of the tables reasonable, only one decimal of K has been tabled. The values have been truncated rather than rounded. Let L be the period of time X was observed and M be the period of time each Y_j will be observed.

The problem of computing tables is then to find K such that $P[Y_1 \leq K_1, \ldots, Y_J \leq K; x] = 1 - \alpha$ using

(1) $$P[Y_1 \leq K, \ldots, Y_J \leq K; x]$$

$$= \sum_{y_1=0}^{K} \cdots \sum_{y_J=0}^{K} \frac{(y_1+\ldots+y_J+x-1)!}{y_1!\ldots y_J!(x-1)!} p^x q^{y_1+y_2+\ldots+y_J}$$

where $p = \dfrac{L}{L+JM}$ and $q = \dfrac{M}{L+JM}$.

Direct computation of K for large values of x requires excessive amounts of computer time except when p is near 1. However a transformation makes possible the computation of the tables using a reasonable amount of computer time. Let

$$z_1 = y_1, z_2 = y_1+y_2, \ldots, z_J = y_1+y_2+\ldots+y_J .$$

Then $y_1 = z_y, y_2 = z_2 - z_1, \ldots, y_J = z_J - z_{J-1}$ and

(2) $P[Y_1 \leq K, \ldots, Y_J \leq K; x]$

$$= \frac{p^x}{(x-1)!} \sum_{z_J=0}^{JK} (z_J+x-1)!q^{z_J} \sum_{z_{J-1}=\max[0,\, z_J-K]}^{\min[z_J,\, (J-1)K]} \frac{1}{(z_J-z_{J-1})!} \cdots$$

$$\times \sum_{z_1=\max[0,\, z_2-K]}^{\min[z_2,\, K]} \frac{1}{z_1!(z_2-z_1)!} .$$

Computational form (2) requires considerably fewer arithmetic operations than form (1). In addition computations used for tables of J future observations, can be used for tables of future $J+1$ observations. To illustrate this define

$$S_{2,\, z_2,\, K} = \sum_{z_1=\max[0,\, z_2-K]}^{\min[z_2,\, K]} \frac{1}{z_1!(z_2-z_1)!}$$

and

$$S_{J,\, z_J,\, K} = \sum_{z_{J-1}=\max[0,\, z_J-K]}^{\min[z_J,\, (J-1)K]} \frac{1}{(z_J-z_{J-1})!} S_{J-1, z_{J-1}, K}.$$

Then

$$P[Y_1 \le K, Y_2 \le K; x] = \frac{p^x}{(x-1)!} \sum_{z_2=0}^{2K} q^{z_2} (z_2+x-1)! S_{2,z_2,K}$$

and

$$P[Y_1 \le K, Y_2 \le K, \ldots, Y_J \le K; x] = \frac{p^x}{(x-1)!} \sum_{z_J=0}^{JK} q^{z_J} (z_J+x-1)! S_{J,z_J,K} \, .$$

Perhaps as important as the fact that computations used for J future observations can be used for calculations for $J+1$ future observations, is the fact that $S_{J,z_J,K}$ is independent of p, q and x. Therefore one set of $S_{J,z_J,K}$ values can be used to develop the entire table for J future observations. Computational form (2) requires storing $(z_J+x-1)!$ in memory. For the CDC 3300, at Oregon State University (z_J+x-1) must be less than 171. If larger values are needed, an alternate computing form is available. Note that

$$\frac{1}{z_1!(z_2-z_1)!\ldots(z_J-z_{J-1})!}$$

$$= \frac{z_2! \qquad z_3! \qquad \ldots \qquad z_J!}{z_1!(z_2-z_1)! \; z_2(z_3-z_2)!\ldots z_{J-1}!(z_J-z_{J-1})!z_J!}$$

This observation can be used to show

(3)

$$P[Y_1 \leq K, \ldots, Y_J \leq K; x]$$

$$= p^x \sum_{z_J=0}^{JK} q^{z_J} \binom{z_J+x-1}{z_J} \sum_{z_{J-1}=\max[0,z_J-K]}^{\min[z_J,(J-1)K]} \binom{z_J}{z_{J-1}} \cdots \sum_{z_1=\max[0,z_2-K]}^{\min[z_2,K]} \binom{z_2}{z_1} .$$

Then by defining

$$C_{2,z_2,K} = \sum_{z_1=\max[0,z_2-K]}^{\min[z_2,K]} \binom{z_2}{z_1}$$

and

$$C_{J,z_J,K} = \sum_{z_{J-1}=\max[0,z_J-K]}^{\min[z_J,(J-1)K]} \binom{z_J}{z_{J-1}} C_{J-1,z_{J-1},K} ,$$

it can be seen that computing form (3) has the same advantages as does form (2). However, for form (3) the largest number stored is $\binom{z_J+x-1}{W}$ where W is $(z_J+x)/2$ or $(z_J+x-1)/2$ whichever is an integer. The term $\binom{z_J+x-1}{W}$ is much smaller than $(z_J+x-1)!$. For example $\binom{1028}{514} = 7.1560 \times 10^{307}$ which may be stored in the CDC 3300. But $170! = 7.2574 \times 10^{306}$ and is the largest factorial that can be stored. So form (2) can be used when $(z_J+x-1) \leq 170$ and form (3) can be used when $(z_J+x-1) \leq 1028$.

But form (3) also has a difficulty which can partly be overcome.

In order to compute $C_{J, z_J, K}$ for various J and K, it would be reasonable to start by computing a two-dimensional array containing $\binom{a+b}{a}$ for all required values a and b. However two-dimensional arrays are more difficult to access, causing method 3 to use more computer time. Also two-dimensional arrays become large very fast. For example if $z_J+x-1 = 170$, the required array could not be stored in the CDC 3300. However both of these computational problems can be overcome at the expense of some additional computing time. At each computational step, a series of combinations of the form $\binom{z_{J+1}}{z_J}$ are needed, where z_{J+1} is constant and $\max[0, z_{J+1}-K] \leq z_J \leq \min[z_{J+1}, JK]$. These combinations can be computed and stored in a one-dimensional array. In the next computational loop, combinations of the form $\binom{z_{J+1}+1}{z_J}$ are needed. These are easily computed since

$$\binom{z_{J+1}+1}{z_J} = \frac{z_{J+1}+1}{z_{J+1}-z_J} \binom{z_{J+1}}{z_J} .$$

Because the one-dimensional array requires little storage, memory space is no longer a problem. There is additional computer time required, since the combinations will be recomputed many times.

The values of $C_{J, z_J, K}$ tend to get too large and the values of $S_{J, z_J, K}$ tend to get too small. In special applications, it is easy to combine the two methods so that overflow or underflow will not be a problem.

Equation (1) was coded for $J = 3$ $K = 0, 1, 2, \ldots, 19$ and $x = 1, 2, \ldots, 20$. About ten minutes of computer time was used to calculate the probabilities with $q = .2$. Identical results were obtained using both the computing forms. The computer time used when forms (2) and (3) were used was less than 15 seconds in each case.

D. Approximating the Negative Multinomial

The tables in the Appendix are bulky and do not include all cases that may be of interest. C.G. Khatri and Sujit Kumar Mitra (1969) give methods for approximating the negative multinomial distribution. In this thesis, another approach will be taken.

Suppose Y has a negative binomial distribution with parameters x and p. Then $Y = \sum_{i=1}^{x} z_i$ where z_i has a geometric distribution with parameter p. Since the variance of z_i is constant for all i, the central limit theorem implies that as x tends to infinity, Y tends to be normally distributed.

The situation is similar in the multivariate case. The vector $(Y_1, Y_2, \ldots, Y_J)$ is the sum of x multivariate geometric random variables with mean vector $(q/p, q/p, \ldots, q/p)$ and a positive definite covariance matrix $\| \delta_{ij}\, q/p + q/p \|$ where δ_{ij} is the

Kronecker delta. The central limit theorem as given by Samuel S. Wilks (1962) states that $Y_1, Y_2, \ldots, Y_J$ has a asymptotic distribution which is multivariate normal with mean vector $x(q/p, \ldots, q/p)$ and covariance matrix xV where

$$V = \begin{bmatrix} r+r^2 & r^2 & \cdots & r^2 \\ r^2 & r+r^2 & \cdots & r^2 \\ \vdots & \vdots & & \vdots \\ r^2 & r^2 & \cdots & r+r^2 \end{bmatrix}$$

and $r = q/p$.

If it is assumed that $Y_1, Y_2, \ldots, Y_J$ are approximately multivariate normal then

$$P[Y_j \leq N(x+c)+x^{1/2}(N(N+1))^{1/2}W(J, \rho, \gamma) \text{ for } j = 1,2,\ldots,J] \simeq \gamma ,$$

where $N = M/L$, c is a correction term, $W(J, \rho, \gamma)$ is the 100γ percent point of the distribution of the maximum of J joint unit normal random variable with correlation $\rho = M/(L+M)$. This follows since for the negative multinomial $E(Y_j) = x(q/p) = x(M/L)$, $\mathrm{var}(Y_j) = x(q/p)(1+q/p) = x(M/L)(1+M/L)$ and $\mathrm{cov}(Y_j, Y_i) = x(q/p)^2 = x(M/L)^2$.

If the value K is approximated using

$$\hat{K} = Nc + x^{1/2}(N(N+1))^{1/2}W(J, \rho, \gamma) + Nx$$

the difference $\hat{K}-K$ will be small when x is large. The values of $W(J, \rho, \gamma)$ have been tabled by Gupta (1963) and Milton (1963) for some values of ρ.

The tabled values of K were fitted to the equation $\hat{K} = A + Bx^{1/2} + Nx$ using regression techniques for values of x between 21 and 40. The largest residual was less than .2. This was somewhat surprising since the tabled values were truncated after the first decimal. It was observed that A and B were functions of $N = M/L$. In fact fitting A and B to the equations

$$A = C_0 + C_1 N$$

$$B = d_0 + d_1 N + d_2 N^{-1}$$

usually gave values of A and B to within .1 of the values obtained from the tables and often much closer.

The values of A and B can be calculated using Table 1. The three examples illustrate the accuracy of the method. They also indicate that $\hat{K}-K$ is small even when x is small.

<u>Example 1</u>. Two Future Observations. For $\gamma = .90$, $L = M = 1$, $\hat{A} = .0710$, $\hat{B} = 2.2889$, and $\hat{K} = .0710 + 2.2889X + X$.

X	$\hat{K}$	Tabled K
1	3.36	3.0
4	8.65	8.5
9	15.94	15.7
16	25.17	25.0
25	36.52	36.2
36	49.80	49.9

<u>Example 2</u>. Three Future Observations. For $\gamma = .95$, $L = 4$, $M = 1$, $\hat{A} = .2832$, $\hat{B} = 1.1798$, $\hat{K} = .2832 + 1.1798X + .25X$.

X	$\hat{K}$	Tabled K
1	1.71	1.6
4	3.64	4.1
9	6.07	6.0
16	9.00	8.9
25	12.43	12.4
36	16.36	16.4

<u>Example 3.</u> Five Future Observations. For $\gamma = .90$, $L = 10$, $M = 1$, $\hat{A} = .0736$, $\hat{B} = .7000$, and $\hat{K} = .0736 + .7X + .1X$.

X	$\hat{K}$	Tabled K
1	.87	.7
4	1.87	1.8
9	3.07	2.9
16	4.47	4.3
25	6.07	5.9
36	7.87	7.7

Table 1. One-sided Poisson prediction intervals. $P(Y_1 \leq K, \ldots, Y_J \leq K; x) = \gamma$, $\hat{K} = \hat{A} + \hat{B}X + NX$, $N = M/L$.

Two Future Observations

$\gamma = .90$	$\hat{A} = -.2166 + .2876N$ $\hat{B} = +.5103 + 1.7916N - .01301N^{-1}$
$\gamma = .95$	$\hat{A} = -.0448 + .7058N$ $\hat{B} = +.5967 + 2.1991N - .01445N^{-1}$
$\gamma = .99$	$\hat{A} = +.3764 + 1.8174N$ $\hat{B} = +.7890 + 2.9167N - .0959N^{-1}$

Three Future Observations

$\gamma = .90$	$\hat{A} = -.1307 + .4355N$ $\hat{B} = .5752 + 1.9594N - .01470N^{-1}$
$\gamma = .95$	$\hat{A} = .0578 + .9016N$ $\hat{B} = .6585 + 2.3450N - .01625N^{-1}$
$\gamma = .99$	$\hat{A} = .4606 + 3.3391N$ $\hat{B} = .8323 + 3.0639N - .02012N^{-1}$

Four Future Observations

$\gamma = .90$	$\hat{A} = -.0575 + .5966N$ $\hat{B} = .6174 + 2.0622N - .01549N^{-1}$
$\gamma = .95$	$\hat{A} = .0985 + 1.1033N$ $\hat{B} = .7133 + 2.4233N - .01814N^{-1}$
$\gamma = .99$	$\hat{A} = .6296 + 3.5035N$ $\hat{B} = .8533 + 3.1716N - .02028N^{-1}$

Five Future Observations

$\gamma = .90$	$\hat{A} = .0120 + .6159N$ $\hat{B} = .6613 + 2.1347N - .01752N^{-1}$
$\gamma = .95$	$\hat{A} = .1831 + 1.1198N$ $\hat{B} = .7338 + 2.5223N - .01852N^{-1}$
$\gamma = .99$	$\hat{A} = .5712 + 2.4376N$ $\hat{B} = .8886 + 3.2185N - .02110N^{-1}$

IV. THE NEGATIVE BINOMIAL DISTRIBUTION

The development of prediction intervals for the negative binomial random variables is similar to that for the Poisson random variables. Suppose $X_1, X_2, \ldots, X_I$ are independent negative binomial random variables with an unknown parameter p and known parameters $r_1, r_2, \ldots, r_I$ respectively. Suppose $Y_1, Y_2, \ldots, Y_J$ are independent observations each from a negative binomial distribution with unknown parameter p and known parameters $s_1, s_2, \ldots, s_J$ respectively. Let

$$X = \sum_{i=1}^{I} X_i, \quad \alpha = \sum_{i=1}^{I} r_i, \quad Y = \sum_{j=1}^{J} Y_j, \quad \beta = \sum_{j=1}^{J} s_j \quad \text{and} \quad T = X + Y.$$

The problem is then to find random variables K_j such that

$$K_j = \begin{cases} a_j & \text{with probability } p_j \\ a_j + 1 & \text{with probability } 1-p \end{cases},$$

where a_j is an integer and

$$P[Y_1 \le K_1, Y_2 \le K_2, \ldots, Y_J \le K_J; x] = \gamma .$$

A. Predicting One Future Negative Binomial Observation

The random variables X, Y and T have a negative binomial distribution with unknown parameter p and known parameters α, β and $\alpha+\beta$ respectively.

Since T is sufficient for the distribution of X and Y

$$P[Y \leq K; x] = P[Y \leq K \mid T = x+K]$$

if $g_\alpha(t)$ is a stochastically strictly increasing function of t. This condition will be satisfied if the distribution of X given t has a monotone likelihood ratio.

The density functions for X, Y and T are

$$f_X(x) = \binom{\alpha+x-1}{\alpha-1} p^{\alpha}(1-p)^{x}, \quad x = 0, 1, 2, \ldots,$$

$$f_Y(y) = \binom{\beta+y-1}{\beta-1} p^{\beta}(1-p)^{y}, \quad y = 0, 1, 2, \ldots, \quad \text{and}$$

$$f_T(T) = \binom{\alpha+\beta+t-1}{\alpha+\beta-1} p^{\alpha+\beta}(1-p)^{t}, \quad t = 0, 1, 2, \ldots \quad \text{respectively.}$$

Then

$$f_{X|t}(x) = \frac{\binom{\alpha+x-1}{\alpha-1}\binom{\beta+t-x-1}{\beta-1}}{\binom{\alpha+\beta+t-1}{\alpha+\beta-1}}$$

$$= \binom{t}{x} \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \frac{\gamma(t-x+\beta)\gamma(x+\alpha)}{\gamma(t+\alpha+\beta)} .$$

This is a beta negative binomial density as described by Thomas S. Ferguson (1968) and Johnson and Kotz (1969). Similarly

$$f_{Y|t}(y) = \binom{t}{y} \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \; \frac{\gamma(y+\beta)\gamma(t-y+\alpha)}{\gamma(t+\alpha+\beta)} .$$

In order to show $g_\alpha(t)$ is a stochastically strictly increasing function of t, it will be shown that $f_{X|t}(x)$ has a monotone likelihood ratio. The ratio is

$$\frac{f_{X|t+i}(x)}{f_{X|t}(x)} = C \frac{(t-x)!}{(t-x+\beta-1)!} \; \frac{(t+i-x+\beta-1)!}{(t+i-x)!}$$

where C is a constant for fixed α, β and t. When $t < x \leq t+i$, the ratio is of the form $f(x/t+i)/0$. If $\beta = 1$ then the likelihood ratio is constant for values of $x \leq t$. In this case, the likelihood ratio is monotone. If $\beta > 1$, then

$$\frac{f_{X|t+1}(x)}{f_{X|t}(x)} = C \frac{(t+i-x+\beta-1)(t+i-x+\beta-2)\ldots(t+i-x+1)}{(t-x+\beta-1)(t-x+\beta-2)\ldots(t-x+1)}$$

where there are $\beta-1$ factors in both the numerator and denominator. The ratio of the factors is of the form $\frac{(t+i-x+\beta-j)}{(t-x+\beta-j)}$ where $j = 1, 2, \ldots, \beta-1$. Each of these ratios is increasing as x increases when $0 \leq x \leq t$. Therefore the likelihood ratio is monotone and $g_\alpha(t)$ is stochastically strictly increasing. Then Theorem 2 implies

(1) $$P[Y \leq K; x] = \sum_{y=0}^{K} \binom{x+K}{y} \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \; \frac{\gamma(x+K-y+\alpha)\gamma(y+\beta)}{\gamma(x+K+\alpha+\beta)} .$$

For any value of K Equation (1) makes it possible to find the predictive probability.

Theorem 8. Let Z be a random variable with a beta-negative binomial distribution, with parameters α, β, and x, that is,

$$f_Z(z) = \binom{z+x-1}{x-1} \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \frac{\gamma(x+\alpha)\gamma(z+\beta)}{\gamma(x+z+\alpha+\beta)} .$$

Then $P[Y \leq K; x] = P[Z \leq K]$.

Proof: Two well known results will be used.

a) $$\int_0^1 z^{\alpha-1}(1-z)^{\beta-1}dz = \frac{\gamma(\alpha)\gamma(\beta)}{\gamma(\alpha+\beta)}$$

b) $$\sum_{j=0}^{K} \binom{x+j-1}{x-1} q^x p^j = \sum_{j=x}^{x+K} \binom{x+K}{j} q^j p^{K+x+j} = \sum_{j=0}^{K} \binom{K+x}{j} p^j q^{K+x-j} .$$

Then

$$P[Z \leq K] = \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \sum_{j=0}^{K} \frac{\gamma(\alpha+x)\gamma(j+\beta)}{\gamma(\alpha+\beta+x+j)} \binom{j+x-1}{x-1}$$

$$= \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \sum_{j=0}^{K} \int_0^1 z^{\alpha+x}(1-z)^{j+\beta}dz \quad \binom{j+x-1}{x-1}$$

$$= \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \int_0^1 z^{\alpha}(1-z)^{\beta} \sum_{j=0}^{K} \binom{j+x-1}{x-1} z^x(1-z)^j dz \quad =$$

$$= \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \int_0^1 z^{\alpha}(1-z)^{\beta} \sum_{j=0}^{K} \binom{K+x}{j} z^{K+z-j}(1-z)^{j} \; dz$$

$$= \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \sum_{j=0}^{K} \binom{K+x}{j} \int_0^1 z^{\alpha+K+x-j}(1-z)^{j+\beta} dz$$

$$= \sum_{j=0}^{K} \frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \frac{\gamma(K+x-j+\alpha)\gamma(j+\beta)}{\gamma(K+x+\alpha+\beta)} \binom{K+x}{j}$$

$$= P[Y \leq K; x] \; .$$

Theorem 8 makes is possible to set up a one- or two-sided prediction interval or any sort of prediction region and find the probability that it contains a future observation.

Since the beta positive and negative binomial distributions are not well known a comment is in order. Thomas S. Ferguson (1968) points out that if U has a beta distribution and if the conditional distribution of V given $U = u$ is binomial then the distribution of V is beta-binomial. It will be shown that if the conditional distribution of V given u is a negative binomial distribution rather than a binomial then the marginal distribution of V is beta-negative binomial. The density functions of V given u and of U are

$$f_{V|u}(N) = \binom{r+v-1}{v-1} u^{r}(1-u)^{v}, \quad v = 0, 1, 2, \ldots,$$

and

$$f_U(u) = \frac{\gamma(\alpha)\gamma(\beta)}{\gamma(\alpha+\beta)} u^{\alpha-1}(1-u)^{\beta-1} \qquad 0 \leq u \leq 1$$

respectively. Then

$$f_V(v) = \frac{\gamma(\alpha)\gamma(\beta)}{\gamma(\alpha+\beta)} \binom{r+v-1}{v-1} \int_0^1 u^{\alpha+r-1}(1-u)^{v+\beta-1} du$$

$$= \frac{\gamma(\alpha)\gamma(\beta)}{\gamma(\alpha+\beta)} \binom{r+v-1}{v-1} \frac{\gamma(\alpha+r)\gamma(v+\beta)}{\gamma(x+r+\alpha+\beta)} \qquad v = 0, 1, 2, \ldots \ .$$

Johnson and Kotz (1969) give additional details and references for these distributions.

B. Predicting J Future Negative Binomial Observations

The results for one future observation can be extended to J future observations using methods similar to those used in the extension for Poisson random variables.

Theorem 9. If $X_1, X_2, \ldots, X_I, Y_1, Y_2, \ldots, Y_J$ are independent negative binomial random variables each with unknown parameter p and known parameters $r_1, r_2, \ldots, r_I, s_1, s_2, \ldots, s_J$ respectively, then

$$P[Y_1=K_1, Y_2=K_2, \ldots, Y_J=K_J; X=x] =$$

$$= \frac{\gamma(\alpha+\beta)}{\gamma(\alpha) \prod_{j=1}^{J} \gamma(s_j)} \frac{\gamma(s_1+y_1)\gamma(s_2+y_2)\cdots\gamma(s_j+y_j)\gamma(x+\alpha)}{\gamma(\alpha+x+\beta+y)} \binom{y+x-1}{y_1, y_2, \ldots, y_J}$$

where

$$x = \sum_{i=1}^{I} x_i, \quad \alpha = \sum_{i=1}^{I} r_i, \quad y = \sum_{j=1}^{J} y_j \quad \text{and} \quad \beta = \sum_{j=1}^{J} s_j .$$

This is the density function of the multivariate beta-negative binomial distribution.

Proof: Let $K. = \sum_{j=1}^{J} K_j$ and $Y = \sum_{j=1}^{J} Y_j$. Then

$$P[Y_1=K_1, \ldots, Y_J=K_J \mid Y=K.] = \frac{\binom{s_1+K_1-1}{K_1} \cdots \binom{s_J+K_J-1}{K_J}}{\binom{\beta+K.-1}{K.}} .$$

This follows since

$$P[Y_1=K_1, \ldots, Y_J=K_J] = \binom{s_1+K_1-1}{K_1} \cdots \binom{s_J+K_J-1}{K_J} p^{\beta} q^{K.}$$

and

$$P[Y=K.] = \binom{\beta+K.-1}{K.} p^{\beta} q^{K.} .$$

Then

$$P[Y_1=K_1, \ldots, Y_J=K_J \mid Y=K.] = \frac{P[Y_1=K_1, \ldots, Y_J=K_J]}{P[Y=K.]} .$$

The theorem follows from the equalities below.

$$P[Y_1=K_1, \ldots, Y_J=K_J; X=x]$$

$$= P[Y=K.\,; X=x] P[Y=K_1, \ldots, Y_J=K_J \mid Y=K.\,]$$

$$= \left[\frac{\gamma(\alpha+\beta)}{\gamma(\alpha)\gamma(\beta)} \; \frac{\gamma(x+\alpha)\gamma(K.+\beta)}{\gamma(x+K+\alpha+\beta)} \binom{K.+x-1}{K.}\right] \frac{\binom{s_1+K_1-1}{K_1} \cdots \binom{s_J+K_J-1}{K_J}}{\binom{K.+\beta-1}{K.}}$$

$$= \frac{\gamma(\alpha+\beta)}{\gamma(\alpha) \prod_{j=1}^{J} \gamma(s_j)} \; \frac{\gamma(s_1+K_1) \cdots \gamma(s_J+K_J)\gamma(x+\alpha)}{\gamma(x+K+\alpha+\beta)} \binom{K.+x-1}{K_1, K_2, \ldots, K_J} .$$

This is the density function for the multivariate beta-negative binomial. It can be used to find the predictive probability associated with any prediction region.

V. THE BINOMIAL DISTRIBUTION

The development of prediction intervals for binomial random variables is similar to that for the Poisson and negative binomial random variables. Suppose $X_1, X_2, \ldots, X_I$ are independent binomial random variables with common unknown parameter p and known parameters $m_1, m_2, \ldots, m_I$ respectively. Suppose $Y_1, Y_2, \ldots, Y_J$ are independent observations each to be taken from a binomial distribution with unknown parameter p and known parameters $n_1, n_2, \ldots, n_J$ respectively. Let

$$X = \sum_{i=1}^{I} X_i, \quad M = \sum_{i=1}^{I} m_i, \quad Y = \sum_{j=1}^{J} Y_j, \quad N = \sum_{j=1}^{J} n_j \quad \text{and} \quad T = X + Y .$$

The problem is then to find random variables K_j such that

$$K_j = \begin{cases} a_j & \text{with probability } p_j \\ a_j+1 & \text{with probability } 1-p_j \end{cases}$$

where a_j is an integer and $P[Y_1 \le K_1, \ldots, Y_J \le K_J; X=x] = \gamma$.

A. Predicting One Future Binomial Observation

The random variables X, Y and T have binomial distributions with common unknown parameter p and known parameters M, N, and $(M+N)$ respectively.

Since T is sufficient for the distribution of X and Y,

$$P[Y \leq K; x] = P[Y \leq K \mid T = x+K]$$

if $g_\alpha(t)$ is a stochastically strictly increasing function of t. This condition will be satisfied if the distribution of X given t has a monotone likelihood ratio.

The density functions for X, Y and T are

$$f_X(x) = \binom{M}{x} p^x (1-p)^{M-x}, \quad x = 0, 1, 2, \ldots, M,$$

$$f_Y(y) = \binom{N}{y} p^y (1-p)^{N-y}, \quad y = 0, 1, 2, \ldots, N \quad \text{and}$$

$$f_T(t) = \binom{M+N}{t} p^t (1-p)^{M+N-t}, \quad t = 0, 1, 2, \ldots, M+N$$

respectively. Then

$$f_{X|t}(x) = \frac{\binom{M}{x}\binom{N}{t-x}}{\binom{M+N}{t}}, \quad \max[0, t-N] \leq x \leq \min[M, t].$$

The conditional distribution of X given t is hypergeometric. Since this distribution is known (Ferguson, 1968) to have a monotone likelihood ratio, it follows that

$$P[Y \leq K; x] = P[Y \leq K \mid T=x+K] = \sum_{y=\max[0,K+x-M]}^{K} \frac{\binom{M}{K+x-y}\binom{N}{y}}{\binom{M+N}{x+K}},$$

$$K \leq \min[N, x+K].$$

Theorem 10. Let Z be a random variable with a negative hypergeometric distribution with density function

$$f_Z(z) = \frac{\binom{M}{x-1}\binom{N}{z}\,M-x+1}{\binom{M+N}{x+z-1}\,M+N-z-x+1}\,, \qquad 0 \leq z \leq M+N-x\,.$$

Then

$$P[Y \leq K; x] = P[Z \leq K]\,.$$

Proof: It will be shown that for

$$\max[0, K+x-M] \leq K \leq \min[N, x+K]\,,$$

$$\sum_{y=\max[0,K+x-M]}^{K} \frac{\binom{M}{K+x-y}\binom{N}{y}}{\binom{M+N}{x+K}} = \sum_{y=0}^{K} \frac{\binom{M}{x-1}\binom{N}{y}\,M-x+1}{\binom{M+N}{x+y-1}\,M+N-x-y+1}\,.$$

To show this equality note that

$$\sum_{y=\max[0,K+x-M]}^{K} \frac{\binom{M}{K+x-y}\binom{N}{y}}{\binom{M+N}{x+K}} = \binom{M+N}{N}^{-1} \sum_{y=\max[0,K+x-M]}^{K} \binom{M+N-x+K}{N-y}\binom{x+K}{y}$$

and

$$\sum_{y=0}^{K} \frac{\binom{M}{x-1}\binom{N}{y}\,M-x+1}{\binom{M+N}{x+y-1}\,M+N-x-y+1} = \sum_{y=0}^{K} \frac{M!N!(x+y-1)!(M+N-x+y)!}{(x-1)(M-x)y!(N-y)!(M+N)!}$$

$$= \binom{M+N}{N}^{-1} \sum_{y=0}^{K} \binom{x+y-1}{y}\binom{M+N-x-y}{N-y}\,.$$

Since Lieberman and Owen (1961) and Thatcher (1964) have shown

$$\binom{M+N}{N}^{-1} \sum_{y=\max[0,K+x-M]}^{K} \binom{M+N-x-K}{N-y}\binom{x+K}{y} = \binom{M+N}{N}^{-1} \sum_{y=0}^{K} \binom{x+y-1}{y}\binom{M+N-x-y}{N-y}$$

the results of the theorem follow.

The negative hypergeometric distribution arises when sampling without replacement in the same way the negative binomial distribution arises when sampling with replacement. Suppose an urn contains M black balls and N white balls and sampling without replacement is continued until X black balls are obtained. If Y is the number of white balls obtained, then Y has a negative hypergeometric distribution.

As in the case of the other discrete distribution, the theorem makes it possible to set up a one- or two-sided prediction interval or any sort of prediction region and find the probability that it contains a future observation.

B. Predicting J Future Binomial Observations

The above results can be extended to J future observations using methods similar to those used in the preceding sections.

Theorem 11. If $X_1, X_2, \ldots, X_I$, $Y_1, Y_2, \ldots, Y_J$ are independent binomial random variables, each with unknown parameter p and known parameters $m_1, m_2, \ldots, m_I$, $n_1, n_2, \ldots, n_J$ then

$$P[Y_1=K_1, Y_2=K_2, \ldots, Y_J=K_J; X=x] = \frac{(M-x+1)\quad \binom{M}{x-1}\binom{n_1}{K_1}\binom{n_2}{K_2}\cdots\binom{n_J}{K_J}}{(M+N-x-K.+1)\quad \binom{M+N}{x+K.-1}}$$

where

$$x = \sum_{i=1}^{I} x_i, \quad M = \sum_{i=1}^{I} m_i, \quad N = \sum_{j=1}^{J} n_j, \quad K. = \sum_{j=1}^{J} K_j .$$

Proof: Let $Y = \sum_{j=1}^{J} Y_j$ and note that

$$P[Y_1=K_1, Y_2=K_2, \ldots, Y_J=K_J \mid Y=K.] = \frac{\binom{n_1}{K_1}\binom{n_2}{K_2}\cdots\binom{n_J}{K_J}}{\binom{N}{K.}} .$$

Then

$$P[Y_1=K_1, \ldots, Y_J=K_J; X=x]$$

$$= P[Y_1=K.; X=x]P[Y_1=K_1, \ldots, Y_J=K_J \mid Y=K.]$$

$$= \left[\frac{(M-x-1)}{(M+N-x-K.+1)} \frac{\binom{M}{x-1}\binom{N}{K.}}{\binom{M+N}{x+K.-1}}\right] \frac{\binom{n_1}{K_1}\binom{n_2}{K_2}\cdots\binom{n_J}{K_J}}{\binom{N}{K.}}$$

$$= \frac{(M-x+1)}{(M+N-x-K.+1)} \frac{\binom{M}{x-1}\binom{n_1}{K_1}\binom{n_2}{K_2}\cdots\binom{n_J}{K_J}}{\binom{M+N}{x+K.-1}} .$$

This is the density function of the negative multivariate hypergeometric distribution. It can be used to find the predictive probability associated with any prediction region.

Finally it should be noted that exact probabilities for any x and M are easily computed when $K_1 = K_2 = \ldots = K_J$ and $n_1 = n_2 = \ldots = n_J$ using

$$\frac{(M-x+1)\binom{M}{x-1}}{(M+N-x-K+1)\binom{M+N}{x+K.-1}} \sum_{y_1=0}^{K_1} \binom{n_1}{y_1} \cdots \sum_{y_J=0}^{K_J} \binom{n_J}{y_J}$$

$$= \frac{(M-x+1)\binom{M}{x-1}}{(M+N-x-K+1)\binom{M+N}{x+K.-1}} \left[\sum_{y=0}^{K_1} \binom{n_1}{y_1} \right]^J .$$

VI. COMMENTS AND SUGGESTED TOPICS FOR FURTHER STUDY

The result in Theorem 2, that under given conditions,

$$P[Y \leq K; x] = P[Y \leq K | T = x+K],$$

can be expanded for J future observations at least for the normal distribution.

Theorem 12. If $X_1, X_2, \ldots, X_I, Y_1, Y_2, \ldots, Y_J$ are N independent normally distributed random variables with a common unknown mean and known variance σ^2 and if

$$T = \sum_{i=1}^{I} X_i + \sum_{j=1}^{J} Y_j \quad \text{and} \quad t_0 = \sum_{i=1}^{I} x_i + JK$$

then

$$P[Y_1 \leq K, Y_2 \leq K, \ldots, Y_J \leq K | T = t_0]$$

$$= P[Y_1 \leq K, Y_2 \leq K, \ldots, Y_J \leq K; \sum_{i=1}^{I} X_i = \sum_{i=1}^{I} x_i]$$

The proof is long and not very interesting or useful. It depends on the special properties of the normal distribution. However, the theorem does suggest it may be possible to extend Theorm 2 for use in predicting J future observations directly.

The idea of conditioning on the sum of the observations can be used when there is more than one unknown parameter. For example, consider predicting one future observation for the normal distribution when both parameters are unknown. It can be shown that the distribution of $X_1, X_2, \ldots, X_J$ given $T = t$ is multivariate normal with mean vector $(t/(I+1), t/(I+1), \ldots, t/(I+1))'$ and covariance matrix

$$\frac{\sigma^2}{I+1}\begin{bmatrix} I & -1 & \cdots & -1 \\ -1 & I & \cdots & -1 \\ \vdots & & & \\ -1 & -1 & \cdots & I \end{bmatrix} .$$

Then t may be treated as a parameter and a uniformly most accurate invariant or unbiased confidence interval, $I(x)$, may be found for t. Subtracting $\sum_{i=1}^{I} x_i$ from each point in $I(x)$ gives a prediction interval for y. This interval is the same one given by Hahn (1969). The idea of conditioning of the sum of the observations and then removing remaining parameters by requiring invariance or unbiasness may be useful for other distributions.

BIBLIOGRAPHY

Aitchison, J. and Sculthorpe, D. (1965). Some problems of statistical prediction. Biometrika 52, 469-483.

Beyer, W.H. (Ed.). (1966). Handbook of Tables for Probability and Statistics, Cleveland. The Chemical Rubber Co.

Chew, V. (1968). Simultaneous prediction intervals. Technometrics 10, 323-330.

Faulkenberry, G.D. (1972). Some results on prediction intervals. Technical Report No. 14, Department of Statistics, Oregon State University.

Ferguson, T.S. (1968). Mathematical Statistics, A Decision Theory Approach, New York. Academic Press.

Fisher, R.A. (1956). Statistical Methods and Scientific Inferences, Edinburg, Oliver and Boyd.

Gupta, S.S. (1963). Probability integrals of multivariate normal and multivariate t. Ann. Math. Statis. 34, 793-828.

Hahn, G.J. (1969). Factors for calculating two-sided prediction intervals for samples from a normal distribution. J. Amer. Statist. Asso. 64, 878-888.

Hahn, G.J. (1970). Calculating prediction intervals from a normal distribution. J. Amer. Statist. Asso. 65, 1668-1676.

Hahn, G.J. and Nelson, W.B. (1972). A survey of prediction intervals and their application. General Electric Company Corporate Research and Development Report No. 72CRD027. (Available from the General Electric Company, Corporate Research and Development Distribution, P.O. Box 43, Bldg. 5, Schnectady, N.Y. 12301).

Hewett, J.E. (1968). A note on prediction intervals based on partial observations in certain life test experiments. Technometrics 10, 850-853.

Hora, R.B. and Buchler, R.J. (1967). Fiducial theory and invariant prediction. Ann. Math. Statist. 38, 795-801.

Johnson, N. L. and Kotz, S. (1969). Discrete Distributions, Boston. Houghton Mifflin Co.

Khutri, C.G. and Mitra, S.K. (1969). Some identities and approximation concerning positive and negative multinomial distributions. Multivariate Analysis-II, Proceedings of the Second International Symposium on Multivariate Analysis. Daton (1968) New York. Academic Press. 241-260.

Lieberman, G.J. and Owen, D.B. (1961). Tables of the Hypergeometric Probability Distribution, Stanford, California. Stanford University Press.

Milton, R.C. (1963). Tables of the equally correlated multivariate normal probability integral. Technical Report No. 27, Department of Statistics, University of Minnesota.

Mosimann, J.E. (1963). On the compound negative multinomial distribution and correlations among inversely sampled pollen counts. Biometrika 50, 47-54.

Nelson, W. (May 1970a). Confidence intervals for the ratio of two Poisson means and Poisson predictor intervals. IEEE Transactions on Reliability, 42-49.

Nelson, W. (Nov. 1970b). A statistical prediction interval for availability. IEEE Transactions on Reliability, 179-182.

Shan, B.V. (Nov. 1969). On predicting failures in a future time period from known observations. IEEE Transactions on Reliability, 203-204.

Thatcher, A.R. (1964). Relationships between Bayesian and confidence limits for predictions. J.R. Statist. Soc. B. 26, 176-192.

Weiss, L. (1955). A note on confidence sets for random variables. Ann. Math. Statist 26, 142-144.

Wilks, S.S. (1965). Mathematical Statistics, New York. John Wiley and Sons Inc.

APPENDIX

APPENDIX

Table of Negative Multinomial for Predicting Poisson Observations

Section IIIC described the computational aspects of this table. The table can be used to find K such that

$$P[Y_1 \leq K, Y_2 \leq K, \ldots, Y_J \leq K; x] = 1 - \alpha$$

Recall that K in the discrete case is a random variable. The fractional part of the tabled values are the truncated probabilities that K take the integer part plus one of the tabled value. The tabled values are as described in Section IIA. To find conservative prediction intervals choose the smallest integer greater than the tabled value of K. Values have been tabled for 2, 3, 4 and 5 future observations, for $= .1, .05$ and $.01$ and for x taking on values one through forty. The value L is the number of periods of time x was observed and M is the number of periods of time each Y_j will be observed. Note the tables are only a function of L/M.

90% PREDICTION INTERVAL - TWO FUTURE OBSERVATIONS

L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	3.0	5.1	6.8	8.5	10.0	11.5	12.9	14.4	15.7	17.1	18.5	19.8	21.1	22.4	23.7	25.0	26.3	27.5	28.8	30.0
6	5	1.20	2.6	4.3	5.8	7.2	8.5	9.8	11.0	12.2	13.4	14.5	15.7	16.8	17.9	19.0	20.1	21.2	22.3	23.3	24.4	25.5
5	4	1.25	2.5	4.2	5.6	6.9	8.2	9.4	10.6	11.8	12.9	14.0	15.1	16.2	17.3	18.3	19.4	20.4	21.5	22.5	23.5	24.5
4	3	1.33	2.4	3.9	5.3	6.6	7.8	8.9	10.0	11.1	12.2	13.2	14.3	15.3	16.3	17.3	18.3	19.3	20.3	21.2	22.2	23.1
7	5	1.40	2.3	3.8	5.1	6.3	7.5	8.6	9.6	10.7	11.7	12.7	13.7	14.6	15.6	16.6	17.5	18.5	19.4	20.3	21.2	22.1
3	2	1.50	2.1	3.6	4.8	5.9	7.0	8.0	9.0	10.0	11.0	11.9	12.8	13.8	14.7	15.6	16.5	17.4	18.2	19.1	19.9	20.8
8	5	1.60	1.9	3.4	4.6	5.6	6.6	7.6	8.6	9.5	10.4	11.3	12.1	13.0	13.9	14.7	15.6	16.4	17.2	18.0	18.8	19.7
5	3	1.67	1.9	3.3	4.4	5.4	6.4	7.4	8.3	9.1	10.0	10.9	11.7	12.6	13.4	14.2	15.0	15.8	16.6	17.4	18.2	18.9
7	4	1.75	1.8	3.1	4.2	5.2	6.1	7.0	7.9	8.8	9.6	10.4	11.2	12.0	12.8	13.6	14.4	15.1	15.9	16.7	17.4	18.1
9	5	1.80	1.8	3.0	4.1	5.0	5.9	6.9	7.7	8.6	9.4	10.2	10.9	11.7	12.5	13.3	14.0	14.8	15.5	16.2	16.9	17.7
2	1	2.00	1.7	2.8	3.7	4.6	5.5	6.3	7.0	7.8	8.6	9.3	10.0	10.7	11.4	12.1	12.8	13.5	14.1	14.8	15.5	16.1
11	5	2.20	1.6	2.6	3.5	4.3	5.0	5.8	6.5	7.2	7.9	8.6	9.2	9.8	10.5	11.1	11.8	12.4	13.0	13.6	14.2	14.8
9	4	2.25	1.5	2.5	3.4	4.2	4.9	5.7	6.4	7.0	7.7	8.4	9.0	9.7	10.3	10.9	11.5	12.1	12.7	13.4	13.9	14.5
7	3	2.33	1.5	2.5	3.3	4.0	4.8	5.5	6.2	6.8	7.5	8.1	8.8	9.4	9.9	10.6	11.2	11.8	12.4	12.9	13.5	14.1
12	5	2.40	1.4	2.4	3.2	3.9	4.7	5.4	6.0	6.7	7.3	7.9	8.6	9.1	9.7	10.3	10.9	11.5	12.0	12.6	13.2	13.7
5	2	2.50	1.4	2.3	3.1	3.8	4.6	5.2	5.8	6.5	7.0	7.7	8.3	8.8	9.4	9.9	10.5	11.1	11.6	12.2	12.7	13.3
13	5	2.60	1.3	2.2	2.9	3.7	4.4	5.0	5.7	6.3	6.8	7.4	7.9	8.6	9.1	9.7	10.2	10.7	11.3	11.8	12.3	12.8
8	3	2.67	1.3	2.2	2.9	3.7	4.3	4.9	5.5	6.1	6.7	7.3	7.8	8.4	8.9	9.5	9.9	10.5	11.0	11.5	12.0	12.6
14	5	2.80	1.2	2.0	2.8	3.5	4.1	4.7	5.3	5.9	6.4	6.9	7.5	8.0	8.6	9.0	9.6	10.0	10.6	11.0	11.5	12.0
3	1	3.00	1.1	1.9	2.7	3.3	3.9	4.5	5.0	5.6	6.0	6.6	7.0	7.6	8.0	8.6	9.0	9.5	9.9	10.4	10.9	11.3
16	5	3.20	1.0	1.8	2.6	3.1	3.7	4.2	4.7	5.3	5.7	6.2	6.7	7.2	7.6	8.1	8.5	8.9	9.4	9.8	10.3	10.7
10	3	3.33	.9	1.8	2.5	2.9	3.6	4.1	4.6	5.0	5.6	6.0	6.5	6.9	7.4	7.8	8.2	8.7	9.1	9.5	9.9	10.4
17	5	3.40	.9	1.8	2.4	2.9	3.5	4.0	4.5	4.9	5.5	5.9	6.4	6.8	7.3	7.7	8.1	8.5	8.9	9.4	9.8	10.2
7	2	3.50	.9	1.7	2.4	2.9	3.5	3.9	4.4	4.9	5.3	5.8	6.2	6.7	7.0	7.5	7.9	8.3	8.7	9.1	9.5	9.9
18	5	3.60	.9	1.7	2.3	2.8	3.4	3.8	4.3	4.8	5.2	5.7	6.0	6.5	6.9	7.3	7.7	8.1	8.5	8.9	9.3	9.7
11	3	3.67	.9	1.7	2.3	2.8	3.3	3.8	4.2	4.7	5.1	5.6	5.9	6.4	6.8	7.2	7.6	8.0	8.4	8.8	9.2	9.6
15	4	3.75	.9	1.7	2.2	2.7	3.2	3.7	4.2	4.6	5.0	5.5	5.8	6.3	6.7	7.0	7.5	7.8	8.2	8.6	8.9	9.4
19	5	3.80	.9	1.6	2.2	2.7	3.2	3.7	4.1	4.6	4.9	5.4	5.8	6.2	6.6	6.9	7.4	7.8	8.1	8.5	8.9	9.3
4	1	4.00	.8	1.6	2.0	2.6	3.0	3.5	3.9	4.4	4.8	5.2	5.6	5.9	6.3	6.7	7.0	7.5	7.8	8.2	8.5	8.8
17	4	4.25	.8	1.5	1.9	2.5	2.9	3.4	3.8	4.1	4.6	4.9	5.3	5.7	6.0	6.4	6.7	7.0	7.4	7.8	8.1	8.4
13	3	4.33	.8	1.5	1.9	2.5	2.8	3.3	3.7	4.1	4.5	4.8	5.2	5.6	5.9	6.3	6.6	6.9	7.3	7.6	7.9	8.3
9	2	4.50	.8	1.4	1.9	2.4	2.8	3.2	3.6	3.9	4.4	4.7	5.0	5.4	5.7	6.0	6.4	6.7	7.0	7.4	7.7	8.0
14	3	4.67	.8	1.4	1.8	2.3	2.7	3.1	3.5	3.8	4.2	4.6	4.9	5.2	5.6	5.9	6.2	6.6	6.8	7.2	7.5	7.8
5	1	5.00	.7	1.3	1.8	2.1	2.6	2.9	3.3	3.7	3.9	4.3	4.6	4.9	5.3	5.6	5.9	6.2	6.5	6.8	7.0	7.4
16	3	5.33	.7	1.2	1.7	2.0	2.5	2.8	3.1	3.5	3.8	4.0	4.4	4.7	4.9	5.3	5.6	5.8	6.1	6.4	6.7	6.9
11	2	5.50	.7	1.1	1.6	1.9	2.4	2.7	3.0	3.4	3.7	3.9	4.3	4.6	4.8	5.1	5.5	5.7	5.9	6.3	6.6	6.8
6	1	6.00	.7	1.0	1.5	1.8	2.2	2.6	2.8	3.1	3.5	3.7	3.9	4.3	4.6	4.8	5.0	5.3	5.6	5.8	6.0	6.3
19	3	6.33	.6	.9	1.5	1.8	2.1	2.5	2.7	2.9	3.3	3.6	3.8	4.0	4.4	4.6	4.8	5.1	5.4	5.6	5.8	6.0
13	2	6.50	.6	.9	1.5	1.8	2.0	2.4	2.7	2.9	3.2	3.5	3.7	3.9	4.3	4.5	4.7	4.9	5.2	5.5	5.7	5.9
20	3	6.67	.6	.9	1.4	1.7	1.9	2.4	2.6	2.9	3.1	3.4	3.7	3.9	4.2	4.4	4.7	4.9	5.1	5.4	5.6	5.8
7	1	7.00	.6	.9	1.4	1.7	1.9	2.2	2.5	2.8	3.0	3.3	3.6	3.8	3.9	4.2	4.5	4.7	4.9	5.1	5.4	5.6
15	2	7.50	.6	.9	1.3	1.6	1.8	2.1	2.4	2.7	2.8	3.1	3.4	3.6	3.8	3.9	4.2	4.5	4.7	4.8	5.0	5.3
8	1	8.00	.5	.8	1.2	1.5	1.8	1.9	2.3	2.5	2.7	2.9	3.2	3.4	3.6	3.8	3.9	4.2	4.4	4.6	4.8	4.9
17	2	8.50	.5	.8	1.0	1.5	1.7	1.9	2.1	2.4	2.6	2.8	2.9	3.2	3.5	3.6	3.8	3.9	4.2	4.4	4.6	4.8
9	1	9.00	.5	.8	.9	1.4	1.6	1.8	2.0	2.3	2.5	2.7	2.9	3.0	3.3	3.5	3.7	3.8	3.9	4.2	4.4	4.6
19	2	9.50	.4	.8	.9	1.3	1.6	1.8	1.9	2.2	2.4	2.6	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.2	4.4
10	1	10.00	.4	.7	.9	1.2	1.5	1.7	1.9	2.0	2.3	2.5	2.7	2.8	2.9	3.2	3.4	3.6	3.7	3.8	4.0	4.2
11	1	11.00	.3	.7	.8	1.1	1.4	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.6	3.7	3.8
12	1	12.00	.3	.7	.8	.9	1.3	1.5	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.8	2.9	3.0	3.2	3.4	3.5	3.6
13	1	13.00	.2	.6	.8	.9	1.1	1.4	1.6	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.1	3.3	3.4
14	1	14.00	.2	.6	.8	.9	1.0	1.3	1.5	1.6	1.8	1.9	1.9	2.2	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.2
15	1	15.00	.1	.6	.7	.8	.9	1.2	1.4	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.7	2.8	2.9	3.0
16	1	16.00	.1	.5	.7	.8	.9	1.1	1.3	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.7	2.8	2.9
17	1	17.00	0	.5	.7	.8	.9	1.0	1.2	1.4	1.6	1.7	1.8	1.8	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.8
18	1	18.00	0	.5	.7	.8	.9	.9	1.1	1.3	1.5	1.6	1.7	1.8	1.9	1.9	2.1	2.2	2.4	2.5	2.6	2.7
19	1	19.00	0	.4	.6	.8	.8	.9	1.1	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.9	2.1	2.2	2.4	2.5	2.6
20	1	20.00	0	.4	.6	.7	.8	.9	1.0	1.2	1.4	1.5	1.6	1.7	1.8	1.8	1.9	1.9	2.1	2.2	2.4	2.5

90% PREDICTION INTERVAL - TWO FUTURE OBSERVATIONS

OBSERVED NUMBER OF COUNTS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1	1.00	31.3	32.5	33.8	35.0	36.2	37.5	38.7	39.9	41.1	42.3	43.5	44.7	45.9	47.1	48.3	49.5	50.7	51.8	53.0	54.2
6	5	1.20	26.5	27.6	28.6	29.6	30.7	31.7	32.7	33.7	34.7	35.8	36.8	37.8	38.8	39.8	40.8	41.8	42.8	43.8	44.8	45.8
5	4	1.25	25.5	26.6	27.6	28.6	29.5	30.5	31.5	32.5	33.5	34.5	35.4	36.4	37.4	38.3	39.3	40.3	41.2	42.2	43.1	44.1
4	3	1.33	24.1	25.0	26.0	26.9	27.8	28.8	29.7	30.6	31.6	32.5	33.4	34.3	35.2	36.1	37.0	37.9	38.8	39.7	40.6	41.5
7	5	1.40	23.0	23.9	24.9	25.8	26.7	27.5	28.4	29.3	30.2	31.1	31.9	32.8	33.7	34.6	35.4	36.3	37.1	38.0	38.8	39.7
3	2	1.50	21.7	22.5	23.4	24.2	25.0	25.9	26.7	27.5	28.4	29.2	30.0	30.8	31.6	32.4	33.3	34.1	34.9	35.7	36.5	37.3
8	5	1.60	20.5	21.3	22.0	22.8	23.6	24.4	25.2	26.0	26.7	27.5	28.3	29.1	29.8	30.6	31.4	32.1	32.9	33.6	34.4	35.1
5	3	1.67	19.7	20.5	21.3	22.0	22.8	23.5	24.3	25.0	25.8	26.5	27.3	28.0	28.7	29.5	30.2	30.9	31.7	32.4	33.1	33.8
7	4	1.75	18.9	19.6	20.4	21.1	21.8	22.5	23.2	23.9	24.7	25.4	26.1	26.8	27.5	28.2	28.9	29.6	30.3	31.0	31.7	32.4
9	5	1.80	18.4	19.1	19.8	20.6	21.3	21.9	22.7	23.4	24.0	24.7	25.5	26.1	26.8	27.5	28.2	28.9	29.6	30.2	30.9	31.6
2	1	2.00	16.8	17.4	18.1	18.7	19.4	20.0	20.6	21.3	21.9	22.5	23.1	23.8	24.4	25.0	25.6	26.3	26.9	27.5	28.1	28.7
11	5	2.20	15.4	16.0	16.6	17.2	17.8	18.4	18.9	19.5	20.1	20.7	21.3	21.8	22.4	22.9	23.5	24.1	24.7	25.2	25.8	26.4
9	4	2.25	15.1	15.7	16.3	16.9	17.5	18.0	18.6	19.1	19.7	20.3	20.8	21.4	21.9	22.5	23.1	23.6	24.2	24.7	25.3	25.8
7	3	2.33	14.7	15.2	15.8	16.3	16.9	17.5	18.0	18.6	19.1	19.6	20.2	20.7	21.3	21.8	22.3	22.9	23.4	23.9	24.5	25.0
12	5	2.40	14.3	14.8	15.4	15.9	16.5	17.0	17.6	18.1	18.6	19.2	19.7	20.2	20.7	21.3	21.8	22.3	22.8	23.3	23.8	24.4
5	2	2.50	13.8	14.3	14.8	15.4	15.9	16.4	16.9	17.5	17.9	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5
13	5	2.60	13.3	13.8	14.4	14.8	15.4	15.9	16.4	16.8	17.4	17.8	18.3	18.8	19.3	19.8	20.3	20.8	21.3	21.7	22.2	22.7
8	3	2.67	13.0	13.6	14.0	14.5	15.0	15.5	16.0	16.5	16.9	17.5	17.9	18.4	18.9	19.4	19.9	20.3	20.8	21.3	21.7	22.2
14	5	2.80	12.5	12.9	13.5	13.9	14.4	14.9	15.3	15.8	16.3	16.7	17.2	17.6	18.1	18.6	19.0	19.5	19.9	20.4	20.8	21.3
3	1	3.00	11.8	12.2	12.7	13.1	13.6	14.0	14.5	14.9	15.3	15.7	16.2	16.6	17.0	17.5	17.9	18.3	18.7	19.2	19.6	20.0
16	5	3.20	11.1	11.6	12.0	12.4	12.8	13.2	13.7	14.1	14.5	14.9	15.3	15.7	16.1	16.5	16.9	17.3	17.7	18.1	18.5	18.9
10	3	3.33	10.8	11.2	11.5	11.9	12.4	12.8	13.2	13.6	13.9	14.4	14.8	15.1	15.6	15.9	16.3	16.7	17.1	17.5	17.8	18.2
17	5	3.40	10.6	10.9	11.4	11.8	12.2	12.6	12.9	13.4	13.7	14.1	14.5	14.9	15.3	15.7	16.0	16.4	16.8	17.2	17.6	17.9
7	2	3.50	10.3	10.7	11.1	11.5	11.9	12.3	12.6	13.0	13.4	13.8	14.1	14.5	14.9	15.3	15.6	16.0	16.4	16.7	17.1	17.5
18	5	3.60	10.1	10.5	10.8	11.2	11.6	11.9	12.4	12.7	13.1	13.5	13.8	14.2	14.5	14.9	15.3	15.6	15.9	16.3	16.7	17.0
11	3	3.67	9.9	10.3	10.7	11.0	11.4	11.8	12.1	12.5	12.9	13.2	13.6	13.9	14.3	14.7	15.0	15.4	15.7	16.1	16.4	16.8
15	4	3.75	9.7	10.1	10.5	10.8	11.2	11.6	11.9	12.3	12.6	12.9	13.3	13.7	14.0	14.4	14.7	15.1	15.4	15.8	16.1	16.5
19	5	3.80	9.6	10.0	10.4	10.7	11.0	11.4	11.8	12.1	12.5	12.8	13.2	13.5	13.9	14.2	14.6	14.9	15.2	15.6	15.9	16.3
4	1	4.00	9.2	9.6	9.9	10.3	10.6	10.9	11.3	11.6	11.9	12.3	12.6	12.9	13.3	13.6	13.9	14.3	14.6	14.9	15.2	15.6
17	4	4.25	8.8	9.1	9.4	9.7	10.0	10.4	10.7	11.0	11.4	11.7	11.9	12.3	12.6	12.9	13.2	13.5	13.8	14.1	14.5	14.8
13	3	4.33	8.6	8.9	9.3	9.6	9.9	10.2	10.5	10.8	11.2	11.5	11.8	12.1	12.4	12.7	13.0	13.3	13.6	13.9	14.2	14.5
9	2	4.50	8.4	8.7	8.9	9.3	9.6	9.9	10.2	10.5	10.8	11.1	11.4	11.7	12.0	12.3	12.6	12.9	13.2	13.5	13.8	14.0
14	3	4.67	8.1	8.4	8.7	9.0	9.3	9.6	9.9	10.2	10.5	10.8	11.0	11.4	11.6	11.9	12.2	12.5	12.8	13.0	13.3	13.6
5	1	5.00	7.7	7.9	8.2	8.5	8.8	9.0	9.3	9.6	9.9	10.2	10.4	10.7	10.9	11.2	11.5	11.8	12.0	12.3	12.6	12.8
16	3	5.33	7.3	7.5	7.8	8.0	8.3	8.6	8.8	9.1	9.4	9.6	9.9	10.1	10.4	10.7	10.9	11.1	11.4	11.7	11.9	12.1
11	2	5.50	7.0	7.3	7.6	7.8	8.1	8.4	8.6	8.9	9.1	9.4	9.6	9.9	10.1	10.4	10.6	10.8	11.1	11.4	11.6	11.8
6	1	6.00	6.6	6.8	7.0	7.3	7.6	7.8	8.0	8.3	8.5	8.7	8.9	9.2	9.4	9.6	9.8	10.1	10.3	10.6	10.8	10.9
19	3	6.33	6.3	6.5	6.8	6.9	7.2	7.5	7.7	7.9	8.1	8.3	8.6	8.8	8.9	9.2	9.4	9.7	9.8	10.1	10.3	10.5
13	2	6.50	6.1	6.4	6.6	6.8	7.0	7.3	7.5	7.7	7.9	8.1	8.4	8.6	8.8	9.0	9.2	9.5	9.7	9.9	10.0	10.3
20	3	6.67	6.0	6.3	6.5	6.7	6.9	7.1	7.4	7.6	7.8	7.9	8.2	8.4	8.6	8.8	9.0	9.2	9.5	9.7	9.8	10.0
7	1	7.00	5.8	6.0	6.2	6.5	6.7	6.8	7.0	7.3	7.5	7.7	7.8	8.0	8.3	8.5	8.7	8.8	9.0	9.3	9.5	9.7
15	2	7.50	5.5	5.7	5.9	6.0	6.3	6.5	6.7	6.8	7.0	7.3	7.5	7.6	7.8	8.0	8.2	8.4	8.6	8.7	8.9	9.1
8	1	8.00	5.2	5.4	5.6	5.8	5.9	6.1	6.3	6.5	6.7	6.8	7.0	7.2	7.4	7.6	7.8	7.9	8.1	8.3	8.5	8.6
17	2	8.50	4.9	5.1	5.3	5.5	5.7	5.8	6.0	6.2	6.4	6.6	6.7	6.9	7.0	7.2	7.4	7.6	7.7	7.9	8.0	8.2
9	1	9.00	4.7	4.9	5.0	5.2	5.4	5.6	5.7	5.9	6.0	6.2	6.4	6.6	6.7	6.9	7.0	7.2	7.4	7.5	7.7	7.8
19	2	9.50	4.6	4.7	4.8	5.0	5.2	5.4	5.5	5.7	5.8	5.9	6.1	6.3	6.5	6.6	6.7	6.9	7.0	7.2	7.4	7.5
10	1	10.00	4.4	4.5	4.7	4.8	4.9	5.1	5.3	5.5	5.6	5.7	5.9	6.0	6.2	6.3	6.5	6.6	6.8	6.9	7.0	7.2
11	1	11.00	4.0	4.2	4.3	4.5	4.6	4.7	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	6.1	6.3	6.4	6.5	6.7
12	1	12.00	3.8	3.9	4.0	4.1	4.3	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	6.0	6.2
13	1	13.00	3.6	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.5	4.7	4.8	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8
14	1	14.00	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.3	5.4	5.5
15	1	15.00	3.1	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.2
16	1	16.00	2.9	3.1	3.2	3.3	3.5	3.6	3.7	3.7	3.9	3.9	4.0	4.1	4.2	4.4	4.5	4.6	4.6	4.7	4.8	4.9
17	1	17.00	2.8	2.9	3.0	3.1	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7
18	1	18.00	2.7	2.8	2.9	2.9	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5
19	1	19.00	2.6	2.7	2.8	2.9	2.9	3.0	3.2	3.3	3.4	3.5	3.6	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.3
20	1	20.00	2.6	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.7	3.8	3.8	3.9	4.0	4.1

95% PREDICTION INTERVAL - TWO FUTURE OBSERVATIONS

L	M	L/M	OBSERVED NUMBER OF COUNTS 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1.00	4.1	6.4	8.3	10.0	11.7	13.3	14.8	16.3	17.8	19.2	20.6	22.0	23.4	24.8	26.2	27.5	28.8	30.2	31.5	32.8
6	5	1.20	3.5	5.5	7.0	8.6	9.9	11.3	12.6	13.9	15.1	16.4	17.6	18.8	19.9	21.1	22.2	23.4	24.5	25.6	26.7	27.8
5	4	1.25	3.4	5.3	6.8	8.3	9.6	10.9	12.2	13.4	14.6	15.8	16.9	18.1	19.2	20.4	21.5	22.6	23.6	24.7	25.8	26.8
4	3	1.33	3.2	4.9	6.5	7.8	9.1	10.4	11.6	12.7	13.8	14.9	16.0	17.1	18.2	19.2	20.3	21.3	22.3	23.4	24.4	25.4
7	5	1.40	3.1	4.8	6.2	7.5	8.7	9.9	11.0	12.2	13.3	14.3	15.4	16.4	17.4	18.4	19.4	20.4	21.4	22.4	23.3	24.3
3	2	1.50	2.9	4.5	5.8	7.1	8.3	9.4	10.5	11.5	12.5	13.5	14.5	15.5	16.4	17.4	18.3	19.2	20.1	21.0	21.9	22.8
8	5	1.60	2.8	4.3	5.6	6.7	7.8	8.9	9.9	10.9	11.8	12.8	13.7	14.6	15.5	16.4	17.3	18.2	19.0	19.9	20.8	21.6
5	3	1.67	2.7	4.1	5.4	6.5	7.6	8.6	9.6	10.5	11.4	12.4	13.2	14.1	15.0	15.8	16.7	17.6	18.4	19.2	20.0	20.8
7	4	1.75	2.6	3.9	5.2	6.3	7.3	8.2	9.2	10.1	10.9	11.8	12.7	13.6	14.4	15.2	16.0	16.8	17.6	18.4	19.2	19.9
9	5	1.80	2.6	3.9	5.0	6.1	7.1	8.0	8.9	9.8	10.7	11.6	12.4	13.2	14.0	14.8	15.6	16.4	17.2	17.9	18.7	19.5
2	1	2.00	2.3	3.6	4.7	5.6	6.5	7.4	8.2	9.0	9.8	10.6	11.4	12.1	12.8	13.6	14.3	15.0	15.7	16.4	17.1	17.8
11	5	2.20	2.1	3.3	4.3	5.2	6.0	6.8	7.6	8.4	9.0	9.8	10.5	11.2	11.8	12.5	13.2	13.8	14.5	15.1	15.8	16.4
9	4	2.25	2.0	3.3	4.2	5.1	5.9	6.7	7.5	8.2	8.9	9.6	10.3	10.9	11.6	12.3	12.9	13.6	14.2	14.8	15.5	16.1
7	3	2.33	1.9	3.1	4.1	4.9	5.7	6.5	7.2	7.9	8.7	9.3	9.9	10.6	11.3	11.9	12.6	13.2	13.8	14.4	15.0	15.6
12	5	2.40	1.9	3.0	3.9	4.8	5.6	6.4	7.0	7.8	8.5	9.1	9.8	10.4	11.0	11.7	12.3	12.8	13.5	14.0	14.7	15.2
5	2	2.50	1.9	2.9	3.8	4.7	5.5	6.1	6.8	7.5	8.2	8.8	9.4	10.0	10.7	11.3	11.8	12.4	13.0	13.6	14.1	14.7
13	5	2.60	1.8	2.9	3.8	4.6	5.3	5.9	6.6	7.3	7.9	8.5	9.1	9.7	10.3	10.9	11.5	12.0	12.6	13.1	13.7	14.2
8	3	2.67	1.8	2.8	3.7	4.5	5.1	5.8	6.5	7.1	7.7	8.4	8.9	9.5	10.1	10.7	11.2	11.8	12.3	12.9	13.4	13.9
14	5	2.80	1.8	2.7	3.6	4.3	4.9	5.6	6.3	6.8	7.5	8.0	8.6	9.1	9.7	10.3	10.8	11.3	11.8	12.4	12.9	13.4
3	1	3.00	1.7	2.6	3.4	4.0	4.7	5.3	5.9	6.5	7.0	7.6	8.1	8.7	9.2	9.7	10.2	10.7	11.2	11.7	12.1	12.6
16	5	3.20	1.6	2.5	3.2	3.8	4.5	5.0	5.6	6.1	6.7	7.2	7.7	8.2	8.7	9.2	9.7	10.1	10.6	11.0	11.5	11.9
10	3	3.33	1.6	2.4	3.0	3.7	4.4	4.9	5.5	5.9	6.5	6.9	7.5	7.9	8.4	8.9	9.4	9.8	10.3	10.7	11.1	11.6
17	5	3.40	1.6	2.4	3.0	3.7	4.3	4.8	5.4	5.8	6.4	6.8	7.4	7.8	8.3	8.7	9.2	9.7	10.1	10.5	10.9	11.4
7	2	3.50	1.5	2.3	2.9	3.6	4.2	4.7	5.2	5.7	6.2	6.7	7.2	7.7	8.1	8.6	8.9	9.4	9.8	10.3	10.7	11.1
18	5	3.60	1.5	2.3	2.9	3.6	4.0	4.6	5.1	5.6	6.1	6.6	7.0	7.5	7.9	8.4	8.8	9.2	9.6	10.0	10.5	10.9
11	3	3.67	1.5	2.2	2.8	3.5	3.9	4.6	5.0	5.6	5.9	6.5	6.9	7.4	7.8	8.2	8.7	9.0	9.5	9.9	10.3	10.7
15	4	3.75	1.4	2.2	2.8	3.4	3.9	4.5	4.9	5.5	5.9	6.4	6.8	7.2	7.7	8.1	8.5	8.9	9.3	9.7	10.1	10.5
19	5	3.80	1.4	2.1	2.8	3.4	3.9	4.5	4.9	5.4	5.8	6.3	6.7	7.1	7.6	7.9	8.4	8.8	9.2	9.6	10.0	10.4
4	1	4.00	1.4	2.0	2.7	3.3	3.8	4.3	4.7	5.2	5.6	6.0	6.5	6.8	7.3	7.7	8.0	8.5	8.8	9.2	9.6	9.9
17	4	4.25	1.3	1.9	2.6	3.0	3.6	4.0	4.5	4.9	5.4	5.8	6.1	6.6	6.9	7.3	7.7	8.0	8.4	8.8	9.1	9.5
13	3	4.33	1.2	1.9	2.6	3.0	3.6	3.9	4.5	4.8	5.3	5.7	6.0	6.5	6.8	7.2	7.6	7.9	8.3	8.7	8.9	9.4
9	2	4.50	1.2	1.9	2.5	2.9	3.5	3.9	4.3	4.7	5.1	5.5	5.9	6.3	6.6	6.9	7.4	7.7	8.0	8.4	8.7	9.0
14	3	4.67	1.1	1.8	2.4	2.9	3.4	3.8	4.2	4.6	4.9	5.4	5.7	6.1	6.5	6.8	7.1	7.5	7.8	8.1	8.5	8.8
5	1	5.00	1.0	1.8	2.3	2.8	3.2	3.6	3.9	4.4	4.7	5.0	5.5	5.8	6.1	6.5	6.8	7.1	7.4	7.7	8.0	8.4
16	3	5.33	.9	1.7	2.1	2.7	2.9	3.5	3.8	4.1	4.5	4.8	5.2	5.5	5.8	6.1	6.5	6.7	7.0	7.4	7.6	7.9
11	2	5.50	.9	1.7	2.1	2.6	2.9	3.4	3.7	4.0	4.4	4.7	5.0	5.4	5.7	5.9	6.3	6.6	6.8	7.1	7.5	7.7
6	1	6.00	.9	1.6	1.9	2.4	2.8	3.1	3.5	3.8	4.1	4.5	4.7	4.9	5.3	5.6	5.8	6.1	6.4	6.7	6.9	7.2
19	3	6.33	.9	1.5	1.9	2.3	2.7	2.9	3.4	3.7	3.9	4.2	4.6	4.8	5.0	5.4	5.7	5.9	6.1	6.4	6.7	6.9
13	2	6.50	.9	1.5	1.8	2.3	2.7	2.9	3.3	3.6	3.8	4.1	4.5	4.7	4.9	5.3	5.6	5.8	6.0	6.3	6.6	6.8
20	3	6.67	.9	1.5	1.8	2.2	2.6	2.9	3.2	3.5	3.8	4.0	4.4	4.7	4.9	5.1	5.4	5.7	5.9	6.2	6.4	6.7
7	1	7.00	.8	1.4	1.8	2.1	2.5	2.8	3.0	3.4	3.7	3.9	4.2	4.5	4.7	4.9	5.2	5.5	5.7	5.9	6.2	6.4
15	2	7.50	.8	1.3	1.7	1.9	2.4	2.7	2.9	3.2	3.5	3.7	3.9	4.2	4.5	4.7	4.9	5.2	5.4	5.7	5.8	6.0
8	1	8.00	.8	1.2	1.7	1.9	2.3	2.6	2.8	3.0	3.3	3.6	3.8	4.0	4.3	4.5	4.7	4.9	5.1	5.4	5.6	5.8
17	2	8.50	.8	1.1	1.6	1.8	2.1	2.5	2.7	2.9	3.2	3.5	3.7	3.8	4.0	4.3	4.5	4.7	4.9	5.1	5.3	5.6
9	1	9.00	.8	1.0	1.5	1.8	1.9	2.4	2.6	2.8	2.9	3.3	3.5	3.7	3.9	4.1	4.3	4.6	4.7	4.9	5.1	5.3
19	2	9.50	.7	.9	1.5	1.7	1.9	2.2	2.5	2.7	2.9	3.1	3.4	3.6	3.8	3.9	4.1	4.4	4.6	4.7	4.9	5.0
10	1	10.00	.7	.9	1.4	1.7	1.9	2.1	2.4	2.7	2.8	2.9	3.2	3.5	3.7	3.8	3.9	4.2	4.4	4.6	4.7	4.9
11	1	11.00	.7	.9	1.3	1.6	1.8	1.9	2.2	2.5	2.7	2.8	2.9	3.2	3.4	3.6	3.7	3.9	4.0	4.2	4.4	4.6
12	1	12.00	.7	.9	1.2	1.5	1.7	1.9	2.0	2.3	2.5	2.7	2.8	2.9	3.1	3.4	3.5	3.7	3.8	3.9	4.1	4.3
13	1	13.00	.6	.9	1.0	1.4	1.7	1.8	1.9	2.1	2.4	2.6	2.7	2.8	2.9	3.1	3.3	3.5	3.6	3.8	3.9	3.9
14	1	14.00	.6	.8	.9	1.3	1.6	1.7	1.9	1.9	2.2	2.4	2.6	2.7	2.8	2.9	3.1	3.3	3.4	3.6	3.7	3.8
15	1	15.00	.6	.8	.9	1.2	1.5	1.7	1.8	1.9	2.0	2.3	2.5	2.6	2.7	2.8	2.9	3.1	3.2	3.4	3.5	3.7
16	1	16.00	.5	.8	.9	1.1	1.4	1.6	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.4	3.5
17	1	17.00	.5	.8	.9	1.0	1.4	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.3
18	1	18.00	.5	.8	.9	.9	1.3	1.5	1.6	1.8	1.8	1.9	2.1	2.3	2.4	2.6	2.7	2.8	2.8	2.9	3.0	3.2
19	1	19.00	.5	.7	.9	.9	1.2	1.4	1.6	1.7	1.8	1.9	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.8	2.9	3.0
20	1	20.00	.4	.7	.8	.9	1.1	1.4	1.5	1.7	1.8	1.8	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.8	2.9

95% PREDICTION INTERVAL - TWO FUTURE OBSERVATIONS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	34.1	35.4	36.7	37.9	39.2	40.5	41.8	43.0	44.3	45.5	46.8	48.0	49.3	50.5	51.7	52.9	54.2	55.4	56.6	57.8
6	5	1.20	28.9	30.0	31.1	32.2	33.3	34.3	35.4	36.5	37.5	38.6	39.6	40.7	41.7	42.8	43.8	44.8	45.8	46.9	47.9	48.9
5	4	1.25	27.9	28.9	30.0	31.0	32.1	33.1	34.1	35.1	36.2	37.2	38.2	39.2	40.2	41.2	42.2	43.2	44.2	45.2	46.2	47.1
4	3	1.33	26.4	27.3	28.3	29.3	30.3	31.3	32.2	33.2	34.1	35.1	36.0	37.0	37.9	38.9	39.8	40.7	41.7	42.6	43.5	44.5
7	5	1.40	25.2	26.2	27.1	28.0	29.0	29.9	30.8	31.8	32.7	33.6	34.5	35.4	36.3	37.2	38.1	39.0	39.9	40.8	41.7	42.5
3	2	1.50	23.7	24.6	25.5	26.4	27.3	28.1	29.0	29.9	30.7	31.6	32.4	33.3	34.1	34.9	35.8	36.6	37.5	38.3	39.1	40.0
8	5	1.60	22.5	23.3	24.1	24.9	25.8	26.6	27.4	28.2	29.0	29.8	30.6	31.4	32.2	33.0	33.8	34.6	35.4	36.2	36.9	37.7
5	3	1.67	21.7	22.5	23.3	24.0	24.8	25.6	26.4	27.2	28.0	28.9	29.5	30.3	31.1	31.8	32.6	33.4	34.1	34.8	35.6	36.4
7	4	1.75	20.8	21.5	22.3	23.0	23.8	24.6	25.3	26.0	26.8	27.6	28.3	29.0	29.7	30.5	31.2	31.9	32.7	33.4	34.1	34.8
9	5	1.80	20.3	21.0	21.7	22.5	23.2	23.9	24.7	25.4	26.1	26.9	27.6	28.3	29.0	29.7	30.4	31.1	31.8	32.6	33.3	33.9
2	1	2.00	18.5	19.2	19.8	20.5	21.2	21.8	22.5	23.2	23.8	24.5	25.1	25.8	26.5	27.1	27.7	28.4	29.0	29.7	30.3	30.9
11	5	2.20	17.0	17.7	18.3	18.9	19.5	20.1	20.7	21.4	21.9	22.6	23.1	23.7	24.3	24.9	25.5	26.1	26.7	27.3	27.8	28.4
9	4	2.25	16.7	17.2	17.9	18.6	19.1	19.7	20.3	20.9	21.5	22.1	22.7	23.3	23.8	24.4	25.0	25.6	26.2	26.7	27.3	27.9
7	3	2.33	16.2	16.8	17.4	17.9	18.6	19.1	19.7	20.3	20.8	21.4	22.0	22.6	23.1	23.7	24.2	24.8	25.4	25.9	26.5	27.0
12	5	2.40	15.8	16.4	16.9	17.6	18.1	18.7	19.2	19.8	20.4	20.9	21.5	22.0	22.6	23.1	23.7	24.2	24.7	25.3	25.8	26.4
5	2	2.50	15.3	15.8	16.4	16.9	17.5	18.0	18.6	19.1	19.7	20.2	20.7	21.3	21.8	22.3	22.8	23.4	23.9	24.4	24.9	25.4
13	5	2.60	14.8	15.3	15.8	16.4	16.9	17.5	17.9	18.5	19.0	19.5	20.0	20.6	21.0	21.6	22.0	22.6	23.1	23.6	24.1	24.6
8	3	2.67	14.5	15.0	15.5	16.0	16.6	17.1	17.6	18.1	18.6	19.1	19.6	20.1	20.6	21.1	21.6	22.1	22.6	23.0	23.6	24.0
14	5	2.80	13.9	14.4	14.9	15.4	15.9	16.4	16.9	17.4	17.8	18.3	18.8	19.3	19.8	20.2	20.7	21.2	21.6	22.1	22.6	23.0
3	1	3.00	13.1	13.6	14.0	14.5	14.9	15.5	15.9	16.4	16.8	17.3	17.7	18.2	18.6	19.1	19.5	19.9	20.4	20.8	21.3	21.7
16	5	3.20	12.4	12.9	13.3	13.8	14.2	14.6	15.0	15.5	15.9	16.4	16.8	17.2	17.6	18.0	18.5	18.9	19.3	19.7	20.1	20.6
10	3	3.33	12.0	12.5	12.9	13.3	13.7	14.1	14.6	14.9	15.4	15.8	16.2	16.6	17.0	17.5	17.8	18.2	18.7	19.0	19.5	19.8
17	5	3.40	11.8	12.2	12.7	13.1	13.5	13.9	14.3	14.7	15.1	15.6	15.9	16.4	16.7	17.1	17.6	17.9	18.3	18.7	19.1	19.5
7	2	3.50	11.6	11.9	12.4	12.8	13.2	13.6	13.9	14.4	14.8	15.2	15.6	15.9	16.4	16.7	17.1	17.5	17.9	18.3	18.7	19.0
18	5	3.60	11.3	11.7	12.1	12.5	12.9	13.3	13.7	14.0	14.5	14.8	15.2	15.6	15.9	16.4	16.7	17.1	17.5	17.8	18.2	18.6
11	3	3.67	11.1	11.5	11.9	12.3	12.7	13.0	13.5	13.8	14.2	14.6	14.9	15.4	15.7	16.1	16.5	16.8	17.2	17.6	17.9	18.3
15	4	3.75	10.9	11.3	11.7	12.0	12.5	12.8	13.2	13.6	13.9	14.3	14.7	15.0	15.4	15.8	16.1	16.5	16.9	17.2	17.6	17.9
19	5	3.80	10.8	11.2	11.6	11.9	12.3	12.7	13.0	13.4	13.8	14.2	14.5	14.9	15.3	15.6	15.9	16.3	16.7	17.0	17.4	17.7
4	1	4.00	10.4	10.7	11.1	11.5	11.8	12.2	12.5	12.9	13.2	13.6	13.9	14.3	14.6	14.9	15.3	15.7	15.9	16.3	16.7	17.0
17	4	4.25	9.8	10.2	10.6	10.9	11.2	11.6	11.9	12.3	12.6	12.9	13.2	13.6	13.9	14.2	14.6	14.9	15.2	15.5	15.8	16.2
13	3	4.33	9.7	10.0	10.4	10.7	11.0	11.4	11.7	12.0	12.4	12.7	13.0	13.4	13.7	14.0	14.3	14.6	14.9	15.3	15.6	15.9
9	2	4.50	9.4	9.7	10.0	10.4	10.7	11.0	11.4	11.7	12.0	12.3	12.6	12.9	13.3	13.6	13.9	14.2	14.5	14.8	15.1	15.4
14	3	4.67	9.1	9.5	9.8	10.1	10.4	10.7	11.0	11.4	11.7	11.9	12.3	12.6	12.8	13.2	13.5	13.8	14.0	14.4	14.7	14.9
5	1	5.00	8.7	8.9	9.3	9.6	9.8	10.1	10.5	10.7	11.0	11.3	11.6	11.9	12.2	12.5	12.7	13.0	13.3	13.6	13.8	14.1
16	3	5.33	8.2	8.5	8.8	9.0	9.4	9.6	9.9	10.2	10.5	10.7	10.9	11.3	11.6	11.8	12.0	12.3	12.6	12.8	13.1	13.4
11	2	5.50	8.0	8.3	8.6	8.8	9.1	9.4	9.7	9.9	10.2	10.5	10.7	10.9	11.3	11.5	11.8	12.0	12.3	12.6	12.8	13.0
6	1	6.00	7.5	7.7	7.9	8.3	8.5	8.8	9.0	9.3	9.5	9.8	9.9	10.3	10.5	10.7	10.9	11.2	11.5	11.7	11.9	12.1
19	3	6.33	7.2	7.4	7.7	7.9	8.1	8.4	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.2	11.4	11.6
13	2	6.50	7.0	7.3	7.5	7.8	7.9	8.2	8.5	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.2	11.4
20	3	6.67	6.9	7.1	7.4	7.6	7.8	8.0	8.3	8.5	8.7	8.9	9.2	9.4	9.7	9.8	10.0	10.3	10.5	10.7	10.9	11.2
7	1	7.00	6.7	6.8	7.1	7.3	7.6	7.8	7.9	8.2	8.4	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.1	10.3	10.5	10.7
15	2	7.50	6.3	6.5	6.7	6.9	7.1	7.4	7.6	7.8	7.9	8.2	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	9.9	10.1
8	1	8.00	5.9	6.2	6.4	6.6	6.8	6.9	7.2	7.4	7.6	7.8	7.9	8.1	8.4	8.6	8.7	8.9	9.1	9.3	9.5	9.7
17	2	8.50	5.7	5.9	6.1	6.3	6.5	6.7	6.8	7.0	7.2	7.4	7.6	7.8	7.9	8.1	8.3	8.5	8.7	8.8	9.0	9.2
9	1	9.00	5.5	5.7	5.8	6.0	6.2	6.4	6.6	6.7	6.9	7.1	7.3	7.5	7.6	7.8	7.9	8.1	8.3	8.5	8.6	8.8
19	2	9.50	5.3	5.5	5.6	5.8	5.9	6.1	6.3	6.5	6.7	6.8	6.9	7.1	7.3	7.5	7.6	7.8	7.9	8.1	8.3	8.4
10	1	10.00	5.0	5.2	5.4	5.6	5.7	5.9	6.0	6.2	6.4	6.6	6.7	6.8	6.9	7.2	7.3	7.5	7.7	7.8	7.9	8.1
11	1	11.00	4.7	4.8	4.9	5.2	5.4	5.5	5.7	5.8	5.9	6.0	6.2	6.4	6.5	6.7	6.8	6.9	7.1	7.2	7.4	7.5
12	1	12.00	4.5	4.6	4.7	4.8	4.9	5.1	5.3	5.5	5.6	5.7	5.8	5.9	6.1	6.2	6.4	6.5	6.7	6.8	6.9	7.0
13	1	13.00	4.1	4.3	4.5	4.6	4.7	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8	5.9	6.1	6.2	6.4	6.5	6.6
14	1	14.00	3.9	4.0	4.2	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.3
15	1	15.00	3.8	3.8	3.9	4.1	4.2	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.3	5.4	5.5	5.6	5.7	5.8	5.9
16	1	16.00	3.6	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.1	5.2	5.4	5.5	5.6	5.7
17	1	17.00	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.1	5.2	5.3	5.4
18	1	18.00	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.9	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.0	5.2
19	1	19.00	3.1	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.4	4.5	4.6	4.6	4.7	4.8	4.9	4.9
20	1	20.00	2.9	3.1	3.3	3.4	3.5	3.6	3.7	3.7	3.8	3.9	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.6	4.7	4.8

99% PREDICTION INTERVAL - TWO FUTURE OBSERVATIONS

			OBSERVED NUMBER OF COUNTS																			
L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1.00	6.5	9.1	11.3	13.3	15.2	16.9	18.7	20.4	21.9	23.6	25.1	26.7	28.2	29.7	31.1	32.6	34.0	35.5	36.9	38.3
6	5	1.20	5.6	7.8	9.7	11.5	13.0	14.6	16.0	17.5	18.8	20.2	21.5	22.8	24.1	25.4	26.6	27.8	29.1	30.3	31.5	32.7
5	4	1.25	5.5	7.6	9.5	11.0	12.6	14.1	15.5	16.8	18.2	19.5	20.8	22.0	23.3	24.5	25.7	26.9	28.1	29.2	30.4	31.6
4	3	1.33	5.2	7.2	8.9	10.5	11.9	13.4	14.7	15.9	17.2	18.5	19.7	20.8	22.0	23.2	24.3	25.5	26.6	27.7	28.8	29.8
7	5	1.40	4.9	6.9	8.6	10.1	11.5	12.8	14.1	15.4	16.6	17.7	18.9	20.0	21.1	22.3	23.4	24.4	25.5	26.5	27.6	28.6
3	2	1.50	4.7	6.6	8.2	9.6	10.9	12.1	13.4	14.5	15.7	16.8	17.8	18.9	19.9	21.0	22.0	23.0	24.0	25.0	26.0	27.0
8	5	1.60	4.5	6.3	7.8	9.1	10.4	11.6	12.7	13.8	14.8	15.9	16.9	17.9	18.9	19.9	20.9	21.8	22.8	23.7	24.6	25.6
5	3	1.67	4.4	6.0	7.5	8.8	10.0	11.2	12.3	13.3	14.4	15.4	16.4	17.4	18.3	19.2	20.2	21.1	22.0	22.9	23.8	24.7
7	4	1.75	4.2	5.8	7.2	8.5	9.7	10.7	11.8	12.8	13.8	14.8	15.7	16.7	17.6	18.5	19.4	20.3	21.1	22.0	22.8	23.7
9	5	1.80	4.1	5.7	7.0	8.3	9.5	10.5	11.6	12.6	13.5	14.5	15.4	16.3	17.2	18.0	18.9	19.8	20.7	21.5	22.3	23.1
2	1	2.00	3.8	5.3	6.6	7.7	8.7	9.7	10.6	11.6	12.4	13.3	14.1	14.9	15.8	16.6	17.4	18.1	18.9	19.7	20.5	21.2
11	5	2.20	3.6	4.9	6.1	7.1	8.0	8.9	9.8	10.7	11.5	12.3	13.1	13.8	14.6	15.4	16.1	16.8	17.5	18.2	18.9	19.6
9	4	2.25	3.5	4.8	5.9	6.9	7.9	8.8	9.7	10.5	11.3	12.1	12.8	13.6	14.4	15.1	15.8	16.5	17.2	17.9	18.6	19.3
7	3	2.33	3.5	4.7	5.8	6.8	7.7	8.6	9.5	10.2	10.9	11.8	12.5	13.2	13.9	14.7	15.4	16.0	16.7	17.4	18.0	18.7
12	5	2.40	3.4	4.7	5.7	6.7	7.6	8.4	9.2	9.9	10.8	11.5	12.2	12.9	13.7	14.3	15.0	15.7	16.4	16.9	17.6	18.3
5	2	2.50	3.2	4.5	5.6	6.5	7.4	8.1	8.9	9.7	10.5	11.1	11.8	12.6	13.2	13.9	14.5	15.2	15.8	16.5	17.0	17.7
13	5	2.60	3.1	4.4	5.4	6.3	7.1	7.9	8.7	9.4	10.1	10.8	11.5	12.1	12.8	13.5	14.1	14.7	15.3	15.9	16.6	17.1
8	3	2.67	3.0	4.3	5.3	6.2	6.9	7.8	8.5	9.2	9.9	10.6	11.3	11.9	12.6	13.2	13.8	14.4	15.0	15.6	16.2	16.8
14	5	2.80	2.9	4.1	5.1	5.9	6.8	7.5	8.2	8.9	9.6	10.2	10.8	11.5	12.1	12.7	13.3	13.9	14.5	15.0	15.6	16.1
3	1	3.00	2.8	3.9	4.8	5.7	6.5	7.1	7.8	8.5	9.0	9.7	10.3	10.9	11.5	12.0	12.6	13.1	13.7	14.2	14.8	15.3
16	5	3.20	2.8	3.8	4.7	5.5	6.1	6.8	7.5	8.0	8.7	9.2	9.8	10.4	10.9	11.5	11.9	12.5	13.0	13.6	14.0	14.6
10	3	3.33	2.7	3.7	4.6	5.3	5.9	6.6	7.2	7.8	8.4	8.9	9.5	10.0	10.6	11.1	11.6	12.1	12.6	13.1	13.6	14.1
17	5	3.40	2.7	3.7	4.5	5.2	5.8	6.5	7.1	7.7	8.3	8.8	9.4	9.9	10.5	10.9	11.5	11.9	12.5	12.9	13.4	13.8
7	2	3.50	2.6	3.6	4.4	5.0	5.8	6.4	6.9	7.6	8.1	8.7	9.2	9.7	10.2	10.7	11.2	11.7	12.1	12.6	13.1	13.6
18	5	3.60	2.6	3.5	4.3	4.9	5.7	6.3	6.8	7.4	7.9	8.5	8.9	9.5	9.9	10.5	10.9	11.5	11.9	12.4	12.8	13.3
11	3	3.67	2.6	3.5	4.2	4.9	5.6	6.1	6.8	7.3	7.8	8.4	8.8	9.4	9.8	10.4	10.8	11.3	11.7	12.2	12.7	13.1
15	4	3.75	2.5	3.4	4.1	4.8	5.5	6.0	6.7	7.2	7.7	8.2	8.7	9.2	9.7	10.1	10.6	11.0	11.6	11.9	12.4	12.8
19	5	3.80	2.5	3.4	4.1	4.8	5.5	5.9	6.6	7.1	7.7	8.1	8.7	9.1	9.6	10.0	10.5	10.9	11.4	11.8	12.3	12.7
4	1	4.00	2.4	3.3	3.9	4.7	5.2	5.8	6.4	6.8	7.4	7.8	8.3	8.8	9.2	9.7	10.1	10.6	10.9	11.4	11.8	12.2
17	4	4.25	2.3	3.1	3.8	4.5	4.9	5.6	6.0	6.6	7.0	7.5	7.9	8.4	8.8	9.3	9.7	10.1	10.5	10.9	11.3	11.7
13	3	4.33	2.2	3.0	3.8	4.4	4.9	5.5	5.9	6.5	6.9	7.4	7.8	8.3	8.7	9.1	9.6	9.9	10.4	10.7	11.1	11.5
9	2	4.50	2.1	2.9	3.7	4.3	4.8	5.4	5.8	6.3	6.8	7.2	7.7	8.0	8.5	8.9	9.3	9.7	10.0	10.5	10.8	11.2
14	3	4.67	2.0	2.9	3.5	4.1	4.7	5.2	5.7	6.1	6.6	6.9	7.5	7.8	8.3	8.7	9.0	9.4	9.8	10.2	10.6	10.9
5	1	5.00	1.9	2.8	3.5	3.9	4.5	4.9	5.5	5.8	6.3	6.7	7.0	7.5	7.8	8.2	8.6	8.9	9.3	9.7	9.9	10.4
16	3	5.33	1.9	2.7	3.3	3.8	4.3	4.8	5.2	5.6	5.9	6.4	6.8	7.1	7.5	7.8	8.2	8.6	8.9	9.2	9.6	9.8
11	2	5.50	1.9	2.7	3.2	3.8	4.2	4.7	5.0	5.5	5.8	6.3	6.7	6.9	7.4	7.7	7.9	8.4	8.7	8.9	9.4	9.7
6	1	6.00	1.8	2.6	2.9	3.6	3.9	4.4	4.8	5.1	5.6	5.9	6.2	6.6	6.9	7.2	7.6	7.8	8.1	8.5	8.8	9.0
19	3	6.33	1.8	2.5	2.9	3.5	3.8	4.2	4.7	4.9	5.4	5.7	5.9	6.4	6.7	6.9	7.3	7.6	7.8	8.1	8.5	8.7
13	2	6.50	1.8	2.4	2.9	3.4	3.8	4.1	4.6	4.9	5.2	5.6	5.9	6.2	6.6	6.8	7.1	7.5	7.7	7.9	8.3	8.6
20	3	6.67	1.8	2.4	2.8	3.3	3.7	4.0	4.5	4.8	5.1	5.5	5.8	6.1	6.5	6.7	6.9	7.3	7.6	7.8	8.1	8.4
7	1	7.00	1.7	2.3	2.8	3.2	3.7	3.9	4.4	4.7	4.9	5.3	5.6	5.9	6.2	6.5	6.8	7.0	7.3	7.6	7.8	8.1
15	2	7.50	1.7	2.1	2.7	2.9	3.5	3.8	4.1	4.5	4.8	5.0	5.4	5.7	5.9	6.2	6.5	6.7	6.9	7.2	7.5	7.7
8	1	8.00	1.6	1.9	2.6	2.9	3.3	3.7	3.9	4.3	4.6	4.8	5.1	5.4	5.7	5.9	6.2	6.5	6.7	6.9	7.1	7.4
17	2	8.50	1.6	1.9	2.5	2.8	3.2	3.6	3.8	4.0	4.4	4.7	4.9	5.2	5.5	5.7	5.9	6.1	6.4	6.7	6.8	7.0
9	1	9.00	1.5	1.9	2.4	2.8	2.9	3.4	3.7	3.9	4.2	4.5	4.8	4.9	5.2	5.5	5.7	5.9	6.1	6.4	6.6	6.8
19	2	9.50	1.4	1.9	2.3	2.7	2.9	3.3	3.6	3.8	4.0	4.4	4.6	4.8	5.0	5.3	5.6	5.7	5.9	6.1	6.4	6.6
10	1	10.00	1.4	1.8	2.2	2.6	2.9	3.1	3.5	3.7	3.9	4.2	4.5	4.7	4.9	5.0	5.3	5.6	5.8	5.9	6.1	6.4
11	1	11.00	1.2	1.8	2.0	2.5	2.8	2.9	3.3	3.6	3.8	3.9	4.1	4.4	4.6	4.8	4.9	5.2	5.4	5.6	5.8	5.9
12	1	12.00	1.1	1.7	1.9	2.3	2.7	2.8	2.9	3.3	3.6	3.8	3.9	4.1	4.4	4.6	4.7	4.9	5.0	5.3	5.5	5.7
13	1	13.00	1.0	1.7	1.9	2.2	2.5	2.8	2.9	3.1	3.4	3.6	3.8	3.9	4.1	4.3	4.5	4.7	4.8	4.9	5.1	5.4
14	1	14.00	.9	1.6	1.8	2.0	2.4	2.7	2.8	2.9	3.2	3.5	3.6	3.8	3.9	4.0	4.3	4.5	4.7	4.8	4.9	5.0
15	1	15.00	.9	1.5	1.8	1.9	2.3	2.6	2.7	2.9	2.9	3.3	3.5	3.7	3.8	3.9	4.0	4.3	4.5	4.6	4.7	4.8
16	1	16.00	.9	1.5	1.8	1.9	2.1	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.0	4.2	4.4	4.6	4.7
17	1	17.00	.9	1.4	1.7	1.9	2.0	2.4	2.6	2.7	2.8	2.9	3.2	3.4	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.5
18	1	18.00	.9	1.3	1.7	1.8	1.9	2.2	2.5	2.7	2.8	2.9	2.9	3.2	3.4	3.6	3.7	3.8	3.9	3.9	4.2	4.4
19	1	19.00	.9	1.3	1.6	1.8	1.9	2.1	2.4	2.6	2.7	2.8	2.9	3.0	3.3	3.5	3.6	3.7	3.8	3.9	3.9	4.1
20	1	20.00	.9	1.2	1.6	1.8	1.9	2.0	2.3	2.5	2.7	2.8	2.9	2.9	3.1	3.3	3.5	3.6	3.7	3.8	3.9	3.9

99% PREDICTION INTERVAL - TWO FUTURE OBSERVATIONS

			OBSERVED NUMBER OF COUNTS																			
L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1	1.00	39.7	41.1	42.5	43.9	45.2	46.6	47.9	49.3	50.6	51.9	53.3	54.6	55.9	57.2	58.6	59.8	61.2	62.5	63.7	65.0
6	5	1.20	33.8	35.0	36.2	37.4	38.5	39.7	40.8	41.9	43.1	44.2	45.3	46.4	47.6	48.7	49.8	50.8	51.9	53.0	54.1	55.2
5	4	1.25	32.7	33.8	34.9	36.0	37.2	38.3	39.4	40.5	41.6	42.6	43.7	44.8	45.9	46.9	48.0	49.0	50.1	51.2	52.2	53.3
4	3	1.33	30.9	32.0	33.0	34.1	35.1	36.2	37.2	38.3	39.3	40.3	41.3	42.3	43.3	44.3	45.3	46.3	47.3	48.3	49.3	50.3
7	5	1.40	29.7	30.7	31.7	32.7	33.7	34.7	35.7	36.7	37.7	38.6	39.6	40.6	41.5	42.5	43.5	44.4	45.4	46.3	47.2	48.2
3	2	1.50	27.9	28.9	29.8	30.8	31.7	32.7	33.6	34.6	35.5	36.4	37.3	38.2	39.1	40.0	40.9	41.8	42.7	43.6	44.5	45.3
8	5	1.60	26.5	27.4	28.3	29.2	30.0	30.9	31.8	32.7	33.6	34.4	35.3	36.1	36.9	37.8	38.7	39.5	40.4	41.2	42.0	42.8
5	3	1.67	25.6	26.5	27.3	28.2	29.0	29.9	30.7	31.6	32.4	33.2	34.0	34.9	35.7	36.5	37.3	38.1	38.9	39.8	40.6	41.4
7	4	1.75	24.6	25.4	26.2	27.0	27.8	28.7	29.5	30.3	31.1	31.9	32.7	33.5	34.2	35.0	35.8	36.6	37.4	38.1	38.9	39.7
9	5	1.80	23.9	24.8	25.6	26.4	27.2	27.9	28.8	29.6	30.3	31.1	31.9	32.7	33.4	34.2	34.9	35.7	36.5	37.2	37.9	38.7
2	1	2.00	21.9	22.7	23.5	24.2	24.9	25.6	26.3	27.0	27.7	28.5	29.2	29.8	30.6	31.2	31.9	32.6	33.3	34.0	34.7	35.4
11	5	2.20	20.3	20.9	21.7	22.3	23.0	23.7	24.3	24.9	25.6	26.3	26.9	27.6	28.2	28.8	29.5	30.1	30.7	31.4	31.9	32.6
9	4	2.25	19.9	20.6	21.3	21.9	22.6	23.2	23.9	24.5	25.1	25.8	26.4	27.0	27.7	28.3	28.9	29.5	30.1	30.8	31.4	31.9
7	3	2.33	19.4	20.0	20.6	21.3	21.9	22.6	23.2	23.8	24.4	25.0	25.6	26.2	26.8	27.5	28.0	28.6	29.2	29.8	30.4	31.0
12	5	2.40	18.9	19.6	20.2	20.8	21.4	22.0	22.6	23.2	23.8	24.5	25.0	25.6	26.2	26.8	27.4	27.9	28.6	29.1	29.7	30.3
5	2	2.50	18.3	18.9	19.5	20.1	20.7	21.3	21.9	22.5	23.0	23.6	24.2	24.8	25.4	25.9	26.5	27.0	27.6	28.1	28.7	29.3
13	5	2.60	17.7	18.3	18.9	19.5	20.0	20.6	21.2	21.8	22.3	22.9	23.4	23.9	24.5	25.1	25.6	26.2	26.7	27.3	27.8	28.3
8	3	2.67	17.4	17.9	18.5	19.1	19.7	20.2	20.8	21.3	21.8	22.4	22.9	23.5	24.0	24.6	25.1	25.6	26.1	26.7	27.2	27.7
14	5	2.80	16.7	17.3	17.8	18.4	18.9	19.4	19.9	20.5	21.0	21.5	22.0	22.6	23.1	23.6	24.1	24.6	25.1	25.6	26.1	26.6
3	1	3.00	15.8	16.4	16.8	17.4	17.9	18.4	18.9	19.4	19.8	20.4	20.8	21.3	21.8	22.3	22.8	23.3	23.7	24.2	24.7	25.1
16	5	3.20	15.0	15.5	16.0	16.5	16.9	17.5	17.9	18.4	18.8	19.3	19.8	20.3	20.7	21.2	21.6	22.0	22.5	22.9	23.4	23.8
10	3	3.33	14.6	15.0	15.5	15.9	16.5	16.9	17.4	17.8	18.3	18.7	19.1	19.6	20.0	20.5	20.9	21.4	21.8	22.2	22.7	23.0
17	5	3.40	14.4	14.8	15.3	15.7	16.2	16.6	17.1	17.5	17.9	18.4	18.8	19.3	19.7	20.1	20.6	21.0	21.5	21.8	22.3	22.7
7	2	3.50	14.0	14.5	14.9	15.4	15.8	16.3	16.7	17.1	17.6	17.9	18.4	18.8	19.3	19.7	20.1	20.5	20.9	21.4	21.8	22.2
18	5	3.60	13.7	14.2	14.6	15.0	15.5	15.9	16.3	16.8	17.2	17.6	18.0	18.4	18.8	19.3	19.7	20.0	20.5	20.9	21.3	21.7
11	3	3.67	13.5	13.9	14.4	14.8	15.3	15.7	16.1	16.5	16.9	17.4	17.8	18.2	18.6	18.9	19.4	19.8	20.2	20.6	20.9	21.4
15	4	3.75	13.3	13.7	14.1	14.6	14.9	15.4	15.8	16.2	16.6	17.0	17.5	17.8	18.2	18.6	19.0	19.4	19.8	20.2	20.6	20.9
19	5	3.80	13.1	13.6	13.9	14.4	14.8	15.2	15.7	16.0	16.5	16.8	17.3	17.7	18.0	18.5	18.8	19.2	19.6	19.9	20.4	20.8
4	1	4.00	12.7	13.0	13.5	13.8	14.3	14.7	15.0	15.4	15.8	16.2	16.6	16.9	17.3	17.7	18.0	18.5	18.8	19.2	19.6	19.9
17	4	4.25	12.0	12.5	12.8	13.2	13.6	13.9	14.4	14.7	15.1	15.5	15.8	16.2	16.5	16.9	17.2	17.6	17.9	18.3	18.6	18.9
13	3	4.33	11.9	12.3	12.7	13.0	13.4	13.8	14.1	14.5	14.8	15.2	15.6	15.9	16.3	16.6	16.9	17.3	17.7	18.0	18.4	18.7
9	2	4.50	11.6	11.9	12.3	12.7	13.0	13.4	13.7	14.0	14.5	14.8	15.1	15.5	15.8	16.1	16.5	16.8	17.1	17.5	17.8	18.1
14	3	4.67	11.3	11.6	11.9	12.3	12.7	13.0	13.4	13.7	14.0	14.4	14.7	15.0	15.4	15.7	16.0	16.4	16.7	16.9	17.3	17.6
5	1	5.00	10.7	11.0	11.4	11.7	12.0	12.4	12.7	12.9	13.3	13.6	13.9	14.3	14.6	14.9	15.2	15.5	15.8	16.1	16.4	16.7
16	3	5.33	10.2	10.5	10.8	11.1	11.5	11.8	12.0	12.4	12.7	12.9	13.3	13.6	13.8	14.1	14.5	14.7	15.0	15.3	15.6	15.9
11	2	5.50	9.9	10.3	10.6	10.9	11.2	11.5	11.8	12.1	12.4	12.7	12.9	13.3	13.6	13.8	14.1	14.4	14.7	14.9	15.2	15.5
6	1	6.00	9.4	9.7	9.9	10.2	10.5	10.8	11.0	11.4	11.6	11.9	12.1	12.4	12.7	12.9	13.2	13.5	13.7	13.9	14.3	14.5
19	3	6.33	8.9	9.3	9.6	9.8	10.0	10.4	10.6	10.9	11.1	11.4	11.7	11.9	12.2	12.5	12.7	12.9	13.2	13.5	13.7	13.9
13	2	6.50	8.8	9.1	9.4	9.7	9.9	10.1	10.5	10.7	10.9	11.2	11.5	11.7	11.9	12.2	12.5	12.7	12.9	13.2	13.4	13.7
20	3	6.67	8.7	8.9	9.2	9.5	9.7	9.9	10.2	10.5	10.7	10.9	11.2	11.5	11.7	11.9	12.2	12.5	12.7	12.9	13.1	13.4
7	1	7.00	8.4	8.7	8.9	9.1	9.4	9.7	9.8	10.1	10.4	10.6	10.8	11.0	11.3	11.6	11.8	11.9	12.2	12.5	12.7	12.9
15	2	7.50	7.9	8.2	8.5	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.3	10.6	10.8	10.9	11.2	11.4	11.6	11.8	12.0	12.3
8	1	8.00	7.7	7.8	8.0	8.3	8.6	8.8	8.9	9.2	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.8	11.0	11.3	11.5	11.7
17	2	8.50	7.3	7.6	7.8	7.9	8.2	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	9.9	10.2	10.4	10.6	10.8	10.9	11.2
9	1	9.00	6.9	7.2	7.5	7.7	7.8	8.0	8.3	8.5	8.7	8.8	9.0	9.2	9.4	9.6	9.8	9.9	10.2	10.4	10.6	10.7
19	2	9.50	6.8	6.9	7.1	7.4	7.6	7.8	7.9	8.1	8.3	8.5	8.7	8.9	9.0	9.2	9.5	9.6	9.8	9.9	10.1	10.3
10	1	10.00	6.6	6.7	6.9	7.1	7.3	7.5	7.7	7.8	7.9	8.2	8.4	8.6	8.7	8.9	9.0	9.3	9.5	9.6	9.8	9.9
11	1	11.00	6.1	6.3	6.5	6.7	6.8	6.9	7.2	7.4	7.6	7.7	7.8	7.9	8.2	8.4	8.5	8.7	8.8	8.9	9.1	9.3
12	1	12.00	5.8	5.9	6.1	6.3	6.5	6.6	6.8	6.9	7.0	7.2	7.4	7.6	7.7	7.8	7.9	8.1	8.3	8.5	8.6	8.7
13	1	13.00	5.5	5.7	5.8	5.9	6.0	6.3	6.4	6.6	6.7	6.8	6.9	7.1	7.3	7.5	7.6	7.7	7.8	7.9	8.1	8.3
14	1	14.00	5.2	5.4	5.6	5.7	5.8	5.9	6.0	6.2	6.4	6.6	6.7	6.8	6.9	7.0	7.2	7.3	7.5	7.6	7.7	7.8
15	1	15.00	4.9	5.1	5.3	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.4	6.5	6.6	6.7	6.8	6.9	7.1	7.2	7.4	7.5
16	1	16.00	4.9	4.9	5.0	5.2	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.4	6.5	6.6	6.7	6.8	6.9	6.9	7.1
17	1	17.00	4.7	4.8	4.8	4.9	5.1	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.5	6.6	6.7	6.8	6.8
18	1	18.00	4.5	4.6	4.7	4.8	4.9	4.9	5.1	5.3	5.4	5.6	5.7	5.8	5.8	5.9	6.0	6.2	6.3	6.4	6.5	6.6
19	1	19.00	4.3	4.5	4.6	4.7	4.8	4.9	4.9	5.0	5.2	5.4	5.5	5.6	5.7	5.8	5.3	5.9	6.0	6.1	6.3	6.4
20	1	20.00	4.1	4.3	4.4	4.6	4.7	4.7	4.8	4.9	4.9	5.1	5.3	5.4	5.5	5.6	5.7	5.8	5.8	5.9	6.0	6.1

90% PREDICTION INTERVAL - THREE FUTURE OBSERVATIONS

L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	3.5	5.6	7.4	9.1	10.7	12.2	13.7	15.2	16.6	18.0	19.4	20.8	22.1	23.5	24.8	26.1	27.4	28.7	30.0	31.3
6	5	1.20	2.9	4.8	6.4	7.8	9.1	10.5	11.7	12.9	14.2	15.4	16.5	17.7	18.8	19.9	21.1	22.2	23.3	24.4	25.5	26.6
5	4	1.25	2.8	4.6	6.1	7.5	8.8	10.1	11.3	12.5	13.7	14.8	15.9	17.0	18.2	19.3	20.3	21.4	22.5	23.5	24.6	25.6
4	3	1.33	2.7	4.4	5.8	7.1	8.4	9.6	10.7	11.8	12.9	14.0	15.1	16.1	17.2	18.2	19.2	20.2	21.2	22.2	23.2	24.2
7	5	1.40	2.6	4.2	5.6	6.8	8.0	9.2	10.3	11.4	12.4	13.4	14.5	15.5	16.5	17.4	18.4	19.4	20.3	21.3	22.2	23.1
3	2	1.50	2.5	3.9	5.3	6.5	7.6	8.7	9.7	10.7	11.7	12.7	13.6	14.6	15.5	16.4	17.3	18.2	19.1	20.0	20.9	21.8
8	5	1.60	2.4	3.8	4.9	6.1	7.2	8.2	9.2	10.1	11.0	11.9	12.9	13.8	14.7	15.5	16.4	17.2	18.1	18.9	19.8	20.6
5	3	1.67	2.3	3.7	4.8	5.9	6.9	7.9	8.8	9.8	10.7	11.6	12.5	13.3	14.1	15.0	15.8	16.6	17.5	18.3	19.0	19.8
7	4	1.75	2.1	3.5	4.7	5.7	6.7	7.6	8.5	9.4	10.3	11.1	11.9	12.8	13.6	14.4	15.2	15.9	16.7	17.5	18.3	19.0
9	5	1.80	2.1	3.5	4.6	5.6	6.5	7.4	8.3	9.2	10.0	10.8	11.7	12.5	13.2	14.0	14.8	15.6	16.3	17.1	17.8	18.6
2	1	2.00	1.9	3.1	4.2	5.1	5.9	6.8	7.6	8.4	9.2	9.9	10.7	11.4	12.1	12.8	13.5	14.2	14.9	15.6	16.3	16.9
11	5	2.20	1.8	2.9	3.8	4.7	5.5	6.3	7.0	7.8	8.5	9.2	9.8	10.5	11.2	11.8	12.5	13.1	13.7	14.4	15.0	15.6
9	4	2.25	1.8	2.8	3.8	4.6	5.4	6.2	6.9	7.6	8.3	8.9	9.7	10.3	10.9	11.6	12.2	12.8	13.5	14.1	14.7	15.3
7	3	2.33	1.7	2.8	3.7	4.5	5.3	5.9	6.7	7.4	8.0	8.7	9.4	10.0	10.6	11.3	11.8	12.5	13.1	13.7	14.3	14.8
12	5	2.40	1.7	2.7	3.6	4.4	5.1	5.8	6.6	7.2	7.9	8.5	9.1	9.8	10.4	10.9	11.6	12.2	12.8	13.4	13.9	14.5
5	2	2.50	1.6	2.6	3.5	4.3	4.9	5.7	6.4	6.9	7.6	8.2	8.8	9.5	10.0	10.6	11.2	11.8	12.3	12.9	13.5	14.0
13	5	2.60	1.6	2.6	3.4	4.1	4.8	5.5	6.1	6.8	7.4	7.9	8.6	9.1	9.7	10.3	10.8	11.4	11.9	12.5	13.0	13.6
8	3	2.67	1.6	2.5	3.3	4.0	4.7	5.4	6.0	6.6	7.2	7.8	8.4	8.9	9.5	10.0	10.6	11.1	11.7	12.2	12.7	13.3
14	5	2.80	1.5	2.4	3.2	3.9	4.6	5.2	5.8	6.4	6.9	7.5	8.0	8.6	9.1	9.7	10.2	10.7	11.2	11.7	12.2	12.7
3	1	3.00	1.4	2.3	2.9	3.7	4.3	4.9	5.5	6.0	6.6	7.1	7.6	8.1	8.6	9.1	9.6	10.1	10.6	11.0	11.5	12.0
16	5	3.20	1.3	2.1	2.8	3.5	4.0	4.7	5.2	5.7	6.2	6.7	7.2	7.7	8.2	8.7	9.1	9.6	10.0	10.5	10.9	11.4
10	3	3.33	1.3	2.0	2.8	3.4	3.9	4.5	5.0	5.6	6.0	6.5	6.9	7.5	7.9	8.4	8.8	9.3	9.7	10.1	10.6	10.9
17	5	3.40	1.3	2.0	2.7	3.3	3.9	4.5	4.9	5.5	5.9	6.4	6.9	7.3	7.8	8.2	8.7	9.1	9.6	9.9	10.4	10.8
7	2	3.50	1.2	1.9	2.7	3.3	3.8	4.3	4.8	5.3	5.8	6.3	6.7	7.2	7.6	8.0	8.5	8.9	9.3	9.7	10.1	10.6
18	5	3.60	1.2	1.9	2.6	3.2	3.7	4.2	4.7	5.2	5.7	6.1	6.6	6.9	7.5	7.8	8.3	8.7	9.1	9.5	9.9	10.3
11	3	3.67	1.1	1.9	2.6	3.1	3.7	4.2	4.7	5.1	5.6	6.0	6.5	6.9	7.3	7.7	8.1	8.6	8.9	9.4	9.8	10.1
15	4	3.75	1.1	1.9	2.6	3.0	3.6	4.1	4.6	5.0	5.5	5.9	6.4	6.8	7.2	7.6	8.0	8.4	8.8	9.2	9.6	9.9
19	5	3.80	1.1	1.8	2.5	3.0	3.6	4.0	4.6	4.9	5.4	5.8	6.3	6.7	7.1	7.5	7.9	8.3	8.7	9.1	9.5	9.8
4	1	4.00	1.0	1.8	2.4	2.9	3.5	3.9	4.4	4.8	5.2	5.6	6.0	6.5	6.8	7.2	7.6	7.9	8.4	8.7	9.1	9.5
17	4	4.25	.9	1.7	2.3	2.8	3.3	3.7	4.1	4.6	4.9	5.4	5.7	6.1	6.5	6.8	7.2	7.6	7.9	8.3	8.7	8.9
13	3	4.33	.9	1.7	2.3	2.8	3.2	3.7	4.0	4.5	4.9	5.3	5.7	6.0	6.4	6.8	7.1	7.5	7.8	8.2	8.5	8.8
9	2	4.50	.9	1.7	2.2	2.7	3.1	3.6	3.9	4.4	4.7	5.1	5.5	5.8	6.2	6.6	6.9	7.3	7.6	7.9	8.3	8.6
14	3	4.67	.9	1.6	2.1	2.6	3.0	3.5	3.8	4.2	4.6	4.9	5.4	5.7	6.0	6.4	6.7	7.0	7.4	7.7	8.0	8.4
5	1	5.00	.9	1.6	1.9	2.5	2.8	3.3	3.7	3.9	4.4	4.7	5.0	5.4	5.7	6.0	6.4	6.7	6.9	7.3	7.6	7.9
16	3	5.33	.8	1.5	1.9	2.4	2.7	3.1	3.5	3.8	4.1	4.5	4.8	5.1	5.5	5.7	6.0	6.3	6.6	6.9	7.2	7.5
11	2	5.50	.8	1.5	1.8	2.3	2.7	3.0	3.4	3.7	4.0	4.4	4.7	4.9	5.3	5.6	5.9	6.2	6.5	6.7	7.0	7.3
6	1	6.00	.8	1.3	1.8	2.1	2.5	2.8	3.2	3.5	3.8	4.0	4.4	4.7	4.9	5.2	5.5	5.8	6.0	6.3	6.6	6.8
19	3	6.33	.8	1.3	1.7	2.0	2.4	2.7	3.0	3.4	3.7	3.9	4.2	4.5	4.7	4.9	5.3	5.6	5.8	6.0	6.3	6.5
13	2	6.50	.8	1.2	1.7	1.9	2.4	2.7	2.9	3.3	3.6	3.8	4.1	4.4	4.7	4.9	5.1	5.4	5.7	5.9	6.1	6.4
20	3	6.67	.8	1.2	1.7	1.9	2.3	2.7	2.9	3.2	3.5	3.8	4.0	4.3	4.6	4.8	5.0	5.3	5.6	5.8	6.0	6.3
7	1	7.00	.7	1.1	1.6	1.9	2.2	2.6	2.8	3.1	3.4	3.7	3.9	4.1	4.4	4.7	4.8	5.1	5.4	5.6	5.8	6.0
15	2	7.50	.7	1.0	1.5	1.8	2.1	2.4	2.7	2.9	3.2	3.5	3.7	3.9	4.1	4.4	4.6	4.8	5.0	5.3	5.5	5.7
8	1	8.00	.7	.9	1.5	1.7	1.9	2.3	2.6	2.8	3.0	3.3	3.5	3.7	3.9	4.2	4.4	4.6	4.8	5.0	5.2	5.5
17	2	8.50	.7	.9	1.4	1.7	1.9	2.2	2.5	2.7	2.9	3.1	3.4	3.6	3.8	3.9	4.2	4.4	4.6	4.8	4.9	5.2
9	1	9.00	.6	.9	1.3	1.6	1.8	2.0	2.4	2.6	2.8	2.9	3.2	3.5	3.6	3.8	3.9	4.2	4.4	4.6	4.8	4.9
19	2	9.50	.6	.9	1.2	1.6	1.8	1.9	2.2	2.5	2.7	2.8	3.0	3.3	3.5	3.7	3.8	4.0	4.2	4.4	4.6	4.8
10	1	10.00	.6	.8	1.2	1.5	1.7	1.9	2.1	2.4	2.6	2.8	2.9	3.1	3.4	3.6	3.7	3.9	4.0	4.2	4.4	4.6
11	1	11.00	.5	.8	1.0	1.4	1.6	1.8	1.9	2.2	2.4	2.6	2.8	2.9	3.1	3.3	3.5	3.6	3.8	3.9	4.0	4.3
12	1	12.00	.5	.8	.9	1.3	1.5	1.7	1.8	2.0	2.3	2.5	2.6	2.8	2.9	3.0	3.2	3.4	3.6	3.7	3.8	3.9
13	1	13.00	.5	.8	.9	1.2	1.5	1.6	1.8	1.9	2.1	2.3	2.5	2.6	2.8	2.9	3.0	3.2	3.4	3.5	3.6	3.7
14	1	14.00	.4	.7	.9	1.0	1.4	1.6	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.1	3.3	3.4	3.6
15	1	15.00	.4	.7	.8	.9	1.3	1.5	1.6	1.8	1.9	1.9	2.2	2.4	2.5	2.6	2.7	2.8	2.9	3.1	3.2	3.4
16	1	16.00	.3	.7	.8	.9	1.2	1.4	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.2
17	1	17.00	.3	.7	.8	.9	1.1	1.3	1.5	1.6	1.8	1.8	1.9	2.1	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
18	1	18.00	.3	.6	.8	.9	1.0	1.3	1.5	1.6	1.7	1.8	1.9	1.9	2.1	2.3	2.4	2.5	2.6	2.7	2.8	2.9
19	1	19.00	.2	.6	.8	.9	.9	1.2	1.4	1.5	1.7	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.7	2.7	2.8
20	1	20.00	.2	.6	.7	.8	.9	1.1	1.3	1.5	1.6	1.7	1.8	1.9	1.9	2.0	2.2	2.3	2.5	2.6	2.7	2.7

90% PREDICTION INTERVAL - THREE FUTURE OBSERVATIONS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	32.5	33.8	35.0	36.3	37.6	38.8	40.0	41.3	42.5	43.7	44.9	46.2	47.4	48.6	49.8	51.0	52.2	53.4	54.6	55.8
6	5	1.20	27.6	28.7	29.7	30.8	31.8	32.9	33.9	35.0	36.0	37.0	38.1	39.1	40.1	41.2	42.2	43.2	44.2	45.2	46.2	47.2
5	4	1.25	26.6	27.7	28.7	29.7	30.7	31.7	32.7	33.7	34.7	35.7	36.7	37.7	38.7	39.7	40.6	41.6	42.6	43.6	44.5	45.5
4	3	1.33	25.1	26.1	27.1	28.0	29.0	29.9	30.9	31.8	32.8	33.7	34.6	35.6	36.5	37.4	38.3	39.3	40.2	41.1	42.0	42.9
7	5	1.40	24.1	25.0	25.9	26.8	27.8	28.7	29.6	30.5	31.4	32.3	33.1	34.0	34.9	35.8	36.7	37.6	38.4	39.3	40.2	41.0
3	2	1.50	22.7	23.5	24.4	25.2	26.1	26.9	27.8	28.6	29.5	30.3	31.2	32.0	32.8	33.7	34.5	35.3	36.1	36.9	37.8	38.6
8	5	1.60	21.4	22.2	23.0	23.9	24.7	25.5	26.3	27.0	27.8	28.6	29.4	30.2	31.0	31.8	32.5	33.3	34.1	34.8	35.6	36.4
5	3	1.67	20.7	21.4	22.2	23.0	23.8	24.6	25.3	26.1	26.8	27.6	28.4	29.1	29.9	30.6	31.4	32.1	32.8	33.6	34.3	35.1
7	4	1.75	19.9	20.6	21.3	22.0	22.8	23.5	24.3	25.0	25.7	26.4	27.2	27.9	28.6	29.3	30.0	30.7	31.5	32.2	32.9	33.6
9	5	1.80	19.3	20.0	20.8	21.5	22.2	22.9	23.7	24.4	25.1	25.8	26.5	27.2	27.9	28.6	29.3	30.0	30.7	31.4	32.0	32.7
2	1	2.00	17.6	18.3	18.9	19.6	20.3	20.9	21.6	22.2	22.8	23.5	24.1	24.8	25.4	26.0	26.7	27.3	27.9	28.6	29.2	29.8
11	5	2.20	16.2	16.8	17.5	18.0	18.7	19.3	19.8	20.5	21.0	21.6	22.2	22.8	23.4	23.9	24.5	25.1	25.7	26.3	26.8	27.4
9	4	2.25	15.9	16.5	17.1	17.7	18.3	18.9	19.5	20.0	20.6	21.2	21.8	22.4	22.9	23.5	24.0	24.6	25.2	25.7	26.3	26.8
7	3	2.33	15.4	16.0	16.6	17.2	17.7	18.3	18.8	19.4	20.0	20.6	21.1	21.7	22.2	22.7	23.3	23.8	24.4	24.9	25.5	26.0
12	5	2.40	15.0	15.6	16.2	16.7	17.3	17.8	18.4	18.9	19.5	20.0	20.6	21.1	21.7	22.2	22.7	23.3	23.8	24.3	24.8	25.4
5	2	2.50	14.6	15.1	15.6	16.2	16.7	17.2	17.8	18.3	18.8	19.4	19.9	20.4	20.9	21.4	21.9	22.5	22.9	23.5	24.0	24.5
13	5	2.60	14.1	14.6	15.1	15.6	16.2	16.7	17.2	17.7	18.2	18.7	19.2	19.7	20.2	20.7	21.2	21.7	22.2	22.7	23.2	23.7
8	3	2.67	13.8	14.3	14.8	15.3	15.8	16.3	16.8	17.3	17.8	18.3	18.8	19.3	19.8	20.3	20.7	21.2	21.7	22.2	22.7	23.1
14	5	2.80	13.2	13.7	14.2	14.7	15.2	15.7	16.1	16.6	17.1	17.6	18.0	18.5	18.9	19.4	19.9	20.4	20.8	21.3	21.7	22.2
3	1	3.00	12.5	12.9	13.4	13.8	14.3	14.8	15.2	15.7	16.1	16.6	17.0	17.4	17.8	18.3	18.7	19.2	19.6	20.0	20.5	20.9
16	5	3.20	11.8	12.3	12.7	13.1	13.6	13.9	14.4	14.8	15.2	15.7	16.1	16.5	16.9	17.3	17.7	18.1	18.6	18.9	19.4	19.8
10	3	3.33	11.4	11.8	12.3	12.7	13.1	13.5	13.9	14.3	14.7	15.1	15.5	15.9	16.3	16.7	17.1	17.5	17.9	18.3	18.7	19.1
17	5	3.40	11.2	11.7	12.0	12.5	12.9	13.3	13.7	14.1	14.5	14.9	15.3	15.7	16.0	16.5	16.8	17.2	17.6	18.0	18.4	18.7
7	2	3.50	10.9	11.4	11.8	12.2	12.6	12.9	13.4	13.7	14.1	14.5	14.9	15.3	15.7	16.0	16.4	16.8	17.2	17.6	17.9	18.3
18	5	3.60	10.7	11.1	11.5	11.9	12.3	12.7	13.0	13.4	13.8	14.2	14.6	14.9	15.3	15.7	16.0	16.4	16.8	17.1	17.5	17.8
11	3	3.67	10.6	10.9	11.3	11.7	12.1	12.5	12.8	13.2	13.6	13.9	14.3	14.7	15.0	15.4	15.8	16.1	16.5	16.8	17.2	17.6
15	4	3.75	10.4	10.7	11.1	11.5	11.8	12.2	12.6	12.9	13.3	13.7	14.0	14.4	14.8	15.1	15.5	15.8	16.2	16.6	16.9	17.2
19	5	3.80	10.2	10.6	10.9	11.4	11.7	12.1	12.5	12.8	13.2	13.6	13.9	14.3	14.6	14.9	15.3	15.7	16.0	16.4	16.7	17.0
4	1	4.00	9.8	10.2	10.5	10.9	11.2	11.6	11.9	12.3	12.6	12.9	13.3	13.7	14.0	14.3	14.7	15.0	15.3	15.7	16.0	16.3
17	4	4.25	9.4	9.7	10.0	10.4	10.7	11.0	11.4	11.7	12.0	12.3	12.7	12.9	13.3	13.6	13.9	14.3	14.6	14.9	15.2	15.5
13	3	4.33	9.2	9.5	9.8	10.2	10.5	10.8	11.2	11.5	11.8	12.1	12.5	12.8	13.1	13.4	13.7	14.0	14.3	14.6	14.9	15.3
9	2	4.50	8.9	9.2	9.6	9.9	10.2	10.5	10.8	11.1	11.5	11.8	12.0	12.4	12.7	12.9	13.3	13.6	13.9	14.2	14.5	14.8
14	3	4.67	8.7	8.9	9.3	9.6	9.9	10.2	10.5	10.8	11.1	11.4	11.7	12.0	12.3	12.6	12.9	13.2	13.5	13.8	14.0	14.3
5	1	5.00	8.2	8.5	8.8	9.0	9.4	9.6	9.9	10.2	10.5	10.8	11.0	11.3	11.6	11.9	12.2	12.5	12.7	12.9	13.3	13.5
16	3	5.33	7.8	8.0	8.3	8.6	8.9	9.1	9.4	9.7	9.9	10.2	10.5	10.7	11.0	11.3	11.5	11.8	12.0	12.3	12.6	12.8
11	2	5.50	7.6	7.8	8.1	8.4	8.7	8.9	9.2	9.5	9.7	9.9	10.2	10.5	10.7	10.9	11.2	11.5	11.7	12.0	12.2	12.5
6	1	6.00	7.0	7.3	7.6	7.8	8.0	8.3	8.6	8.8	9.0	9.3	9.5	9.8	9.9	10.2	10.5	10.7	10.9	11.2	11.4	11.6
19	3	6.33	6.8	7.0	7.3	7.5	7.7	7.9	8.2	8.4	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.2	10.5	10.7	10.9	11.1
13	2	6.50	6.6	6.8	7.1	7.3	7.6	7.8	8.0	8.3	8.5	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.2	10.5	10.7	10.9
20	3	6.67	6.5	6.7	6.9	7.2	7.4	7.7	7.8	8.1	8.3	8.5	8.7	8.9	9.2	9.4	9.6	9.8	10.0	10.2	10.5	10.7
7	1	7.00	6.3	6.5	6.7	6.9	7.1	7.4	7.6	7.8	7.9	8.2	8.4	8.6	8.8	9.0	9.2	9.5	9.6	9.8	10.0	10.2
15	2	7.50	5.9	6.1	6.4	6.6	6.7	6.9	7.1	7.4	7.6	7.8	7.9	8.1	8.4	8.6	8.7	8.9	9.1	9.3	9.5	9.7
8	1	8.00	5.6	5.8	6.0	6.2	6.4	6.6	6.8	6.9	7.2	7.4	7.6	7.7	7.9	8.1	8.3	8.5	8.7	8.8	9.0	9.2
17	2	8.50	5.4	5.6	5.7	5.9	6.1	6.3	6.5	6.7	6.8	7.0	7.2	7.4	7.6	7.7	7.9	8.0	8.2	8.4	8.6	8.8
9	1	9.00	5.1	5.3	5.5	5.7	5.8	6.0	6.2	6.4	6.6	6.7	6.9	7.0	7.2	7.4	7.6	7.7	7.8	8.0	8.2	8.4
19	2	9.50	4.9	5.1	5.3	5.5	5.6	5.8	5.9	6.1	6.3	6.5	6.6	6.8	6.9	7.0	7.2	7.4	7.6	7.7	7.8	8.0
10	1	10.00	4.7	4.9	5.0	5.2	5.4	5.6	5.7	5.8	6.0	6.2	6.3	6.5	6.7	6.8	6.9	7.1	7.2	7.4	7.6	7.7
11	1	11.00	4.4	4.6	4.7	4.8	4.9	5.1	5.3	5.5	5.6	5.7	5.8	6.0	6.1	6.3	6.5	6.6	6.7	6.8	6.9	7.1
12	1	12.00	4.1	4.3	4.4	4.6	4.7	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.8	5.9	6.0	6.1	6.3	6.4	6.6	6.7
13	1	13.00	3.8	3.9	4.1	4.3	4.4	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.4	5.6	5.7	5.8	5.9	6.0	6.1	6.3
14	1	14.00	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.6	4.7	4.9	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8	5.9
15	1	15.00	3.5	3.6	3.7	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.4	5.5	5.6
16	1	16.00	3.3	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3
17	1	17.00	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.9	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.0
18	1	18.00	2.9	3.1	3.3	3.4	3.5	3.6	3.7	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.8
19	1	19.00	2.9	2.9	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7
20	1	20.00	2.8	2.9	2.9	3.0	3.2	3.3	3.4	3.5	3.6	3.7	3.7	3.8	3.9	3.9	4.0	4.1	4.2	4.3	4.4	4.5

95% PREDICTION INTERVAL - THREE FUTURE OBSERVATIONS

L	M	L/M	OBSERVED NUMBER OF COUNTS 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1.00	4.6	6.9	8.8	10.7	12.4	14.0	15.6	17.1	18.6	20.1	21.6	23.0	24.4	25.8	27.2	28.6	29.9	31.3	32.6	33.9
6	5	1.20	3.9	5.9	7.6	9.1	10.6	12.0	13.4	14.7	15.9	17.2	18.4	19.6	20.8	22.0	23.2	24.4	25.5	26.7	27.8	28.9
5	4	1.25	3.8	5.7	7.4	8.8	10.3	11.6	12.9	14.2	15.4	16.6	17.8	18.9	20.1	21.3	22.4	23.5	24.6	25.7	26.8	27.9
4	3	1.33	3.6	5.5	6.9	8.4	9.7	10.9	12.2	13.4	14.6	15.7	16.8	17.9	19.0	20.1	21.2	22.2	23.3	24.3	25.3	26.4
7	5	1.40	3.5	5.2	6.7	8.0	9.4	10.6	11.7	12.9	13.9	15.1	16.1	17.2	18.3	19.3	20.3	21.3	22.3	23.3	24.3	25.3
3	2	1.50	3.3	4.9	6.4	7.6	8.8	9.9	11.1	12.1	13.2	14.2	15.2	16.2	17.2	18.2	19.1	20.1	21.0	21.9	22.9	23.8
8	5	1.60	3.1	4.7	6.0	7.2	8.4	9.5	10.5	11.5	12.5	13.5	14.4	15.4	16.3	17.2	18.1	19.0	19.9	20.8	21.6	22.5
5	3	1.67	2.9	4.6	5.8	6.9	8.1	9.1	10.1	11.1	12.1	13.0	13.9	14.8	15.7	16.6	17.5	18.4	19.2	20.0	20.9	21.7
7	4	1.75	2.9	4.4	5.6	6.7	7.8	8.8	9.8	10.7	11.6	12.5	13.4	14.3	15.1	15.9	16.8	17.6	18.4	19.2	20.0	20.8
9	5	1.80	2.8	4.3	5.5	6.6	7.6	8.6	9.6	10.5	11.4	12.2	13.1	13.9	14.8	15.6	16.4	17.2	18.0	18.8	19.6	20.4
2	1	2.00	2.6	3.9	5.0	6.0	6.9	7.9	8.8	9.6	10.4	11.2	11.9	12.8	13.5	14.3	15.0	15.7	16.5	17.2	17.9	18.6
11	5	2.20	2.5	3.7	4.7	5.6	6.5	7.3	8.1	8.9	9.6	10.4	11.1	11.8	12.5	13.2	13.8	14.6	15.2	15.8	16.5	17.2
9	4	2.25	2.4	3.6	4.6	5.5	6.4	7.2	7.9	8.7	9.5	10.2	10.9	11.6	12.3	12.9	13.6	14.3	14.9	15.6	16.2	16.8
7	3	2.33	2.3	3.5	4.5	5.4	6.2	6.9	7.7	8.5	9.2	9.9	10.6	11.3	11.9	12.6	13.2	13.8	14.5	15.1	15.7	16.4
12	5	2.40	2.3	3.5	4.4	5.3	6.0	6.8	7.6	8.3	8.9	9.7	10.3	10.9	11.6	12.3	12.9	13.5	14.1	14.8	15.4	15.9
5	2	2.50	2.2	3.3	4.3	5.1	5.8	6.6	7.3	8.0	8.7	9.4	9.9	10.6	11.3	11.9	12.5	13.1	13.7	14.3	14.8	15.4
13	5	2.60	2.1	3.2	4.1	4.9	5.7	6.4	7.1	7.8	8.5	9.0	9.7	10.3	10.9	11.5	12.1	12.7	13.2	13.8	14.4	14.9
8	3	2.67	2.0	3.1	4.0	4.8	5.6	6.3	6.9	7.6	8.3	8.9	9.5	10.1	10.7	11.3	11.8	12.4	12.9	13.5	14.1	14.6
14	5	2.80	1.9	3.0	3.9	4.7	5.4	6.0	6.7	7.4	7.9	8.6	9.1	9.7	10.3	10.8	11.4	11.9	12.5	13.0	13.5	14.0
3	1	3.00	1.9	2.9	3.7	4.5	5.1	5.7	6.4	6.9	7.6	8.1	8.7	9.2	9.7	10.3	10.8	11.3	11.8	12.3	12.8	13.3
16	5	3.20	1.8	2.8	3.6	4.2	4.8	5.5	6.0	6.6	7.1	7.7	8.2	8.7	9.2	9.7	10.2	10.7	11.2	11.7	12.1	12.6
10	3	3.33	1.8	2.7	3.5	4.0	4.7	5.3	5.8	6.4	6.9	7.5	7.9	8.5	8.9	9.4	9.9	10.4	10.8	11.3	11.7	12.2
17	5	3.40	1.8	2.7	3.4	3.9	4.7	5.2	5.8	6.3	6.8	7.4	7.8	8.3	8.8	9.3	9.7	10.2	10.7	11.1	11.6	12.0
7	2	3.50	1.7	2.6	3.3	3.9	4.6	5.1	5.7	6.2	6.7	7.2	7.7	8.1	8.6	9.0	9.5	9.9	10.4	10.8	11.3	11.7
18	5	3.60	1.7	2.6	3.2	3.8	4.5	4.9	5.6	6.0	6.6	7.0	7.5	7.9	8.4	8.8	9.3	9.8	10.2	10.6	11.0	11.5
11	3	3.67	1.7	2.5	3.2	3.8	4.4	4.9	5.5	5.9	6.5	6.9	7.4	7.8	8.3	8.7	9.2	9.6	10.0	10.5	10.9	11.3
15	4	3.75	1.7	2.5	3.1	3.8	4.3	4.8	5.4	5.8	6.4	6.8	7.3	7.7	8.1	8.6	9.0	9.5	9.8	10.3	10.7	11.1
19	5	3.80	1.7	2.5	3.1	3.7	4.3	4.8	5.3	5.8	6.3	6.7	7.2	7.7	8.0	8.5	8.9	9.4	9.8	10.2	10.6	10.9
4	1	4.00	1.6	2.4	2.9	3.6	4.1	4.6	5.1	5.6	6.0	6.5	6.9	7.4	7.8	8.2	8.6	8.9	9.4	9.8	10.1	10.6
17	4	4.25	1.5	2.2	2.8	3.5	3.9	4.5	4.9	5.3	5.8	6.2	6.6	6.9	7.4	7.8	8.2	8.6	8.9	9.3	9.7	10.0
13	3	4.33	1.5	2.2	2.8	3.4	3.8	4.4	4.8	5.3	5.7	6.1	6.5	6.9	7.3	7.7	8.0	8.5	8.8	9.2	9.6	9.9
9	2	4.50	1.5	2.1	2.8	3.3	3.8	4.2	4.7	5.1	5.5	5.9	6.3	6.7	7.0	7.5	7.8	8.2	8.6	8.9	9.3	9.6
14	3	4.67	1.4	2.0	2.7	3.2	3.7	4.1	4.6	4.9	5.4	5.8	6.1	6.5	6.9	7.3	7.6	7.9	8.3	8.7	8.9	9.3
5	1	5.00	1.4	1.9	2.6	2.9	3.5	3.9	4.4	4.7	5.1	5.5	5.8	6.2	6.6	6.9	7.2	7.6	7.9	8.2	8.5	8.8
16	3	5.33	1.3	1.9	2.5	2.9	3.4	3.7	4.1	4.5	4.8	5.2	5.6	5.9	6.2	6.6	6.8	7.2	7.5	7.8	8.1	8.4
11	2	5.50	1.2	1.8	2.4	2.8	3.3	3.7	4.0	4.4	4.8	5.1	5.5	5.8	6.0	6.4	6.7	7.0	7.3	7.6	7.9	8.2
6	1	6.00	1.1	1.8	2.2	2.7	3.0	3.5	3.8	4.1	4.5	4.8	5.0	5.4	5.7	5.9	6.3	6.6	6.8	7.1	7.4	7.7
19	3	6.33	1.0	1.7	2.1	2.6	2.9	3.3	3.7	3.9	4.3	4.6	4.9	5.2	5.5	5.8	6.0	6.3	6.6	6.8	7.1	7.4
13	2	6.50	.9	1.7	2.0	2.6	2.9	3.3	3.6	3.9	4.2	4.5	4.8	5.0	5.4	5.7	5.9	6.2	6.5	6.7	6.9	7.2
20	3	6.67	.9	1.7	2.0	2.5	2.8	3.2	3.6	3.8	4.1	4.5	4.7	4.9	5.3	5.6	5.8	6.0	6.4	6.6	6.8	7.1
7	1	7.00	.9	1.6	1.9	2.4	2.8	3.0	3.4	3.7	3.9	4.3	4.6	4.8	5.0	5.4	5.6	5.8	6.1	6.4	6.6	6.8
15	2	7.50	.9	1.6	1.9	2.3	2.7	2.9	3.2	3.6	3.8	4.0	4.4	4.6	4.8	5.0	5.4	5.6	5.8	6.0	6.3	6.5
8	1	8.00	.9	1.5	1.8	2.2	2.6	2.8	3.0	3.4	3.7	3.9	4.1	4.4	4.6	4.8	5.0	5.3	5.6	5.8	5.9	6.2
17	2	8.50	.9	1.4	1.8	2.0	2.5	2.7	2.9	3.2	3.5	3.7	3.9	4.2	4.5	4.7	4.8	5.0	5.3	5.5	5.7	5.9
9	1	9.00	.9	1.4	1.7	1.9	2.3	2.6	2.8	3.0	3.4	3.6	3.8	3.9	4.3	4.5	4.7	4.8	5.0	5.3	5.5	5.7
19	2	9.50	.8	1.3	1.7	1.9	2.2	2.5	2.8	2.9	3.2	3.5	3.7	3.8	4.0	4.3	4.5	4.7	4.9	5.0	5.3	5.5
10	1	10.00	.8	1.2	1.6	1.9	2.1	2.5	2.7	2.9	3.0	3.4	3.6	3.7	3.9	4.1	4.4	4.6	4.7	4.9	5.0	5.3
11	1	11.00	.8	1.1	1.6	1.8	1.9	2.3	2.5	2.7	2.9	3.0	3.3	3.5	3.7	3.8	4.0	4.2	4.4	4.6	4.7	4.9
12	1	12.00	.8	.9	1.5	1.7	1.9	2.1	2.4	2.6	2.8	2.9	3.0	3.3	3.5	3.7	3.8	3.9	4.1	4.3	4.5	4.6
13	1	13.00	.7	.9	1.4	1.6	1.8	1.9	2.2	2.5	2.6	2.8	2.9	3.1	3.3	3.5	3.6	3.8	3.9	4.0	4.2	4.4
14	1	14.00	.7	.9	1.3	1.6	1.8	1.9	2.0	2.3	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.6	3.7	3.8	3.9	4.1
15	1	15.00	.7	.9	1.2	1.5	1.7	1.8	1.9	2.2	2.4	2.6	2.7	2.8	2.9	3.1	3.3	3.4	3.6	3.7	3.8	3.9
16	1	16.00	.7	.9	1.1	1.4	1.6	1.8	1.9	2.0	2.3	2.5	2.6	2.7	2.8	2.9	3.1	3.3	3.4	3.6	3.7	3.8
17	1	17.00	.7	.9	.9	1.4	1.6	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.8	2.8	2.9	3.1	3.3	3.4	3.5	3.6
18	1	18.00	.6	.8	.9	1.3	1.5	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.7	2.8	2.8	2.9	3.1	3.2	3.4	3.5
19	1	19.00	.6	.8	.9	1.2	1.5	1.6	1.8	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	2.9	3.1	3.2	3.4
20	1	20.00	.6	.8	.9	1.1	1.4	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.9	2.9	3.1	3.2

95% PREDICTION INTERVAL - THREE FUTURE OBSERVATIONS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	35.3	36.6	37.9	39.2	40.5	41.8	43.1	44.4	45.6	46.9	48.2	49.4	50.7	51.9	53.2	54.4	55.7	56.9	58.2	59.4
6	5	1.20	30.0	31.1	32.2	33.3	34.4	35.5	36.6	37.7	38.8	39.8	40.9	41.9	43.0	44.1	45.1	46.2	47.2	48.3	49.3	50.3
5	4	1.25	28.9	30.0	31.1	32.2	33.2	34.3	35.3	36.3	37.4	38.4	39.4	40.5	41.5	42.5	43.5	44.5	45.5	46.5	47.5	48.5
4	3	1.33	27.4	28.4	29.4	30.4	31.4	32.4	33.3	34.3	35.3	36.3	37.2	38.2	39.1	40.1	41.0	42.0	42.9	43.9	44.8	45.8
7	5	1.40	26.2	27.2	28.1	29.1	30.0	31.0	31.9	32.9	33.8	34.7	35.7	36.6	37.5	38.4	39.3	40.2	41.1	42.0	42.9	43.8
3	2	1.50	24.7	25.6	26.5	27.4	28.3	29.2	30.0	30.9	31.8	32.7	33.5	34.4	35.3	36.1	37.0	37.8	38.7	39.5	40.4	41.2
8	5	1.60	23.4	24.2	25.0	25.9	26.7	27.6	28.4	29.2	30.0	30.9	31.7	32.5	33.3	34.1	34.9	35.7	36.5	37.3	38.1	38.9
5	3	1.67	22.6	23.4	24.2	25.0	25.8	26.6	27.4	28.2	29.0	29.8	30.6	31.4	32.1	32.9	33.7	34.5	35.2	36.0	36.8	37.5
7	4	1.75	21.6	22.4	23.2	23.9	24.7	25.5	26.3	27.0	27.8	28.6	29.3	30.0	30.8	31.5	32.3	33.0	33.8	34.5	35.2	35.9
9	5	1.80	21.1	21.9	22.6	23.4	24.1	24.9	25.6	26.4	27.1	27.8	28.6	29.3	30.0	30.8	31.5	32.2	32.9	33.6	34.4	35.1
2	1	2.00	19.3	20.0	20.7	21.4	22.0	22.7	23.4	24.1	24.8	25.4	26.1	26.8	27.4	28.1	28.7	29.4	30.0	30.7	31.3	31.9
11	5	2.20	17.8	18.5	19.1	19.7	20.4	20.9	21.6	22.2	22.8	23.4	24.0	24.7	25.3	25.8	26.5	27.0	27.7	28.2	28.8	29.4
9	4	2.25	17.5	18.1	18.7	19.4	19.9	20.6	21.2	21.8	22.4	23.0	23.6	24.2	24.8	25.4	25.9	26.5	27.1	27.7	28.3	28.8
7	3	2.33	16.9	17.5	18.2	18.8	19.4	19.9	20.5	21.1	21.7	22.3	22.8	23.4	24.0	24.6	25.1	25.7	26.3	26.8	27.4	27.9
12	5	2.40	16.6	17.1	17.7	18.3	18.9	19.5	20.0	20.6	21.2	21.8	22.3	22.9	23.5	24.0	24.6	25.1	25.7	26.2	26.7	27.3
5	2	2.50	16.0	16.6	17.1	17.7	18.3	18.8	19.4	19.9	20.5	21.0	21.6	22.1	22.6	23.2	23.7	24.2	24.8	25.3	25.8	26.4
13	5	2.60	15.5	16.0	16.6	17.1	17.7	18.2	18.7	19.3	19.8	20.3	20.8	21.4	21.9	22.4	22.9	23.5	23.9	24.5	24.9	25.5
8	3	2.67	15.2	15.7	16.2	16.8	17.3	17.8	18.4	18.9	19.4	19.9	20.4	20.9	21.4	21.9	22.5	22.9	23.5	23.9	24.5	24.9
14	5	2.80	14.6	15.1	15.6	16.1	16.6	17.1	17.6	18.1	18.6	19.1	19.6	20.1	20.6	21.0	21.5	22.0	22.5	22.9	23.5	23.9
3	1	3.00	13.8	14.3	14.7	15.2	15.7	16.2	16.6	17.1	17.6	18.0	18.5	18.9	19.4	19.8	20.3	20.8	21.2	21.7	22.1	22.6
16	5	3.20	13.0	13.5	13.9	14.4	14.9	15.3	15.8	16.2	16.7	17.1	17.5	17.9	18.4	18.8	19.2	19.7	20.1	20.5	20.9	21.4
10	3	3.33	12.6	13.1	13.5	13.9	14.4	14.8	15.3	15.7	16.1	16.5	16.9	17.4	17.8	18.2	18.6	19.0	19.4	19.8	20.2	20.6
17	5	3.40	12.5	12.9	13.3	13.7	14.2	14.6	15.0	15.4	15.8	16.3	16.7	17.1	17.5	17.9	18.3	18.7	19.1	19.5	19.9	20.3
7	2	3.50	12.1	12.6	12.9	13.4	13.8	14.2	14.7	15.0	15.5	15.8	16.3	16.7	17.0	17.5	17.8	18.3	18.7	19.0	19.4	19.8
18	5	3.60	11.9	12.3	12.7	13.1	13.5	13.9	14.3	14.7	15.1	15.5	15.9	16.3	16.7	17.0	17.5	17.8	18.2	18.6	18.9	19.4
11	3	3.67	11.7	12.1	12.5	12.9	13.3	13.7	14.1	14.5	14.9	15.3	15.7	16.0	16.4	16.8	17.2	17.6	17.9	18.3	18.7	19.0
15	4	3.75	11.5	11.9	12.3	12.7	13.0	13.5	13.8	14.2	14.6	15.0	15.4	15.7	16.1	16.5	16.8	17.2	17.6	17.9	18.4	18.7
19	5	3.80	11.4	11.8	12.2	12.6	12.9	13.3	13.7	14.1	14.5	14.9	15.2	15.6	15.9	16.3	16.7	17.0	17.4	17.8	18.1	18.5
4	1	4.00	10.9	11.3	11.7	12.0	12.4	12.8	13.1	13.5	13.8	14.2	14.6	14.9	15.3	15.7	16.0	16.4	16.7	17.0	17.4	17.7
17	4	4.25	10.4	10.8	11.1	11.5	11.8	12.2	12.5	12.8	13.2	13.6	13.9	14.2	14.6	14.9	15.2	15.6	15.9	16.2	16.6	16.8
13	3	4.33	10.3	10.6	10.9	11.3	11.7	11.9	12.3	12.7	12.9	13.3	13.7	14.0	14.3	14.7	14.9	15.3	15.6	15.9	16.3	16.6
9	2	4.50	9.9	10.3	10.6	10.9	11.3	11.6	11.9	12.3	12.6	12.9	13.3	13.6	13.9	14.2	14.5	14.8	15.1	15.5	15.8	16.1
14	3	4.67	9.7	9.9	10.3	10.7	10.9	11.3	11.6	11.9	12.3	12.6	12.9	13.2	13.5	13.8	14.1	14.4	14.7	15.0	15.3	15.6
5	1	5.00	9.1	9.5	9.8	10.1	10.4	10.7	10.9	11.3	11.6	11.9	12.2	12.5	12.8	13.0	13.4	13.6	13.9	14.2	14.5	14.8
16	3	5.33	8.7	8.9	9.3	9.6	9.9	10.2	10.5	10.7	11.0	11.3	11.6	11.8	12.1	12.4	12.7	12.9	13.2	13.5	13.7	14.0
11	2	5.50	8.5	8.8	9.0	9.4	9.7	9.9	10.2	10.5	10.7	11.0	11.3	11.6	11.8	12.1	12.4	12.6	12.9	13.1	13.4	13.7
6	1	6.00	7.9	8.2	8.5	8.8	9.0	9.3	9.6	9.8	10.0	10.3	10.6	10.8	11.0	11.3	11.6	11.8	12.0	12.3	12.5	12.8
19	3	6.33	7.7	7.9	8.1	8.4	8.7	8.9	9.1	9.4	9.6	9.8	10.1	10.4	10.6	10.8	11.0	11.3	11.5	11.8	11.9	12.2
13	2	6.50	7.5	7.7	7.9	8.2	8.5	8.7	8.9	9.2	9.5	9.7	9.9	10.1	10.4	10.6	10.8	11.0	11.3	11.5	11.7	11.9
20	3	6.67	7.4	7.6	7.8	8.0	8.3	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.2	10.4	10.6	10.8	11.0	11.3	11.5	11.7
7	1	7.00	7.0	7.3	7.6	7.8	7.9	8.2	8.5	8.7	8.9	9.1	9.4	9.6	9.8	9.9	10.2	10.5	10.7	10.8	11.0	11.3
15	2	7.50	6.7	6.9	7.1	7.4	7.6	7.8	8.0	8.2	8.5	8.7	8.8	9.0	9.3	9.5	9.7	9.9	10.1	10.3	10.5	10.7
8	1	8.00	6.4	6.6	6.8	7.0	7.2	7.5	7.7	7.8	8.0	8.2	8.5	8.7	8.8	9.0	9.2	9.4	9.6	9.8	9.9	10.2
17	2	8.50	6.1	6.3	6.6	6.7	6.9	7.1	7.3	7.5	7.7	7.8	8.0	8.3	8.5	8.6	8.8	8.9	9.1	9.4	9.5	9.7
9	1	9.00	5.8	6.0	6.3	6.5	6.6	6.8	6.9	7.2	7.4	7.6	7.7	7.9	8.0	8.3	8.4	8.6	8.8	8.9	9.1	9.3
19	2	9.50	5.7	5.8	5.9	6.2	6.4	6.6	6.7	6.9	7.0	7.2	7.4	7.6	7.7	7.9	8.0	8.3	8.4	8.6	8.7	8.9
10	1	10.00	5.5	5.6	5.8	5.9	6.1	6.3	6.5	6.7	6.8	6.9	7.1	7.3	7.5	7.6	7.8	7.9	8.1	8.3	8.4	8.6
11	1	11.00	5.0	5.2	5.4	5.6	5.7	5.8	6.0	6.2	6.4	6.5	6.7	6.8	6.9	7.1	7.2	7.4	7.6	7.7	7.8	7.9
12	1	12.00	4.8	4.9	5.0	5.2	5.4	5.5	5.7	5.8	5.9	6.0	6.2	6.4	6.5	6.7	6.8	6.9	7.0	7.2	7.4	7.5
13	1	13.00	4.5	4.7	4.8	4.9	5.0	5.2	5.4	5.5	5.6	5.7	5.8	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	7.0
14	1	14.00	4.3	4.4	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.5	6.6	6.7
15	1	15.00	4.0	4.2	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.4	5.6	5.7	5.8	5.8	5.9	6.1	6.2	6.4
16	1	16.00	3.8	3.9	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
17	1	17.00	3.7	3.8	3.9	4.0	4.2	4.3	4.4	4.6	4.7	4.7	4.8	4.9	5.0	5.1	5.3	5.4	5.5	5.6	5.7	5.8
18	1	18.00	3.6	3.7	3.8	3.9	3.9	4.1	4.2	4.4	4.5	4.5	4.7	4.8	4.8	4.9	5.0	5.1	5.3	5.4	5.5	5.6
19	1	19.00	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1	5.2	5.4
20	1	20.00	3.4	3.5	3.6	3.7	3.8	3.8	3.9	3.9	4.1	4.2	4.3	4.5	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1

99% PREDICTION INTERVAL - THREE FUTURE OBSERVATIONS

L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
									OBSERVED NUMBER OF COUNTS													
1	1	1.00	6.9	9.7	11.9	13.9	15.9	17.7	19.5	21.2	22.8	24.4	26.0	27.6	29.1	30.7	32.1	33.6	35.1	36.6	38.0	39.4
6	5	1.20	6.0	8.4	10.3	12.0	13.7	15.2	16.7	18.2	19.6	20.9	22.3	23.7	24.9	26.2	27.5	28.8	30.0	31.2	32.5	33.7
5	4	1.25	5.9	8.1	9.9	11.7	13.2	14.7	16.2	17.6	18.9	20.3	21.6	22.8	24.1	25.4	26.6	27.8	29.0	30.2	31.4	32.5
4	3	1.33	5.6	7.7	9.5	11.1	12.6	13.9	15.4	16.7	17.9	19.2	20.5	21.7	22.8	24.0	25.2	26.3	27.5	28.6	29.7	30.8
7	5	1.40	5.4	7.5	9.1	10.7	12.1	13.5	14.8	16.0	17.3	18.5	19.6	20.8	21.9	23.0	24.2	25.3	26.4	27.4	28.5	29.5
3	2	1.50	5.1	7.0	8.7	10.1	11.5	12.7	13.9	15.2	16.3	17.5	18.6	19.7	20.7	21.8	22.8	23.8	24.8	25.9	26.9	27.8
8	5	1.60	4.9	6.7	8.2	9.6	10.9	12.1	13.3	14.4	15.5	16.6	17.6	18.6	19.7	20.7	21.6	22.6	23.6	24.5	25.5	26.4
5	3	1.67	4.7	6.5	7.9	9.3	10.6	11.7	12.8	13.9	14.9	16.0	17.0	18.0	19.0	19.9	20.9	21.8	22.8	23.7	24.6	25.5
7	4	1.75	4.6	6.3	7.7	8.9	10.1	11.3	12.4	13.4	14.4	15.4	16.4	17.3	18.3	19.2	20.1	21.0	21.9	22.8	23.6	24.5
9	5	1.80	4.5	6.1	7.6	8.8	9.9	11.0	12.1	13.1	14.1	15.0	16.0	16.9	17.8	18.8	19.7	20.5	21.4	22.2	23.1	23.9
2	1	2.00	4.1	5.7	6.9	8.1	9.2	10.2	11.1	12.0	12.9	13.8	14.7	15.6	16.4	17.2	18.0	18.8	19.6	20.4	21.2	21.9
11	5	2.20	3.8	5.3	6.5	7.6	8.6	9.5	10.4	11.2	12.0	12.9	13.7	14.5	15.2	15.9	16.7	17.5	18.2	18.9	19.6	20.3
9	4	2.25	3.8	5.2	6.4	7.5	8.4	9.3	10.2	11.0	11.8	12.7	13.5	14.2	14.9	15.7	16.4	17.1	17.8	18.6	19.3	19.9
7	3	2.33	3.7	5.0	6.2	7.2	8.2	9.0	9.9	10.7	11.6	12.3	13.0	13.8	14.6	15.3	15.9	16.7	17.4	18.0	18.7	19.4
12	5	2.40	3.7	4.9	6.1	7.0	7.9	8.8	9.7	10.5	11.3	12.0	12.8	13.5	14.2	14.9	15.6	16.3	16.9	17.6	18.3	18.9
5	2	2.50	3.6	4.8	5.9	6.9	7.8	8.6	9.4	10.2	10.9	11.7	12.4	13.1	13.8	14.5	15.1	15.8	16.5	17.0	17.7	18.4
13	5	2.60	3.5	4.7	5.8	6.7	7.6	8.4	9.1	9.9	10.6	11.3	12.0	12.7	13.4	14.0	14.7	15.3	15.9	16.6	17.2	17.8
8	3	2.67	3.4	4.7	5.7	6.6	7.4	8.2	8.9	9.7	10.4	11.1	11.8	12.5	13.1	13.7	14.4	14.9	15.6	16.2	16.8	17.4
14	5	2.80	3.3	4.5	5.5	6.4	7.1	7.9	8.7	9.4	10.0	10.7	11.4	11.9	12.6	13.2	13.8	14.5	15.0	15.6	16.2	16.8
3	1	3.00	3.1	4.3	5.2	6.0	6.8	7.6	8.2	8.9	9.6	10.2	10.8	11.4	11.9	12.6	13.1	13.7	14.3	14.8	15.4	15.9
16	5	3.20	2.9	4.0	4.9	5.8	6.5	7.2	7.8	8.5	9.1	9.7	10.3	10.8	11.4	11.9	12.5	13.0	13.6	14.1	14.6	15.1
10	3	3.33	2.9	3.9	4.8	5.6	6.3	6.9	7.7	8.2	8.8	9.4	9.9	10.6	11.0	11.6	12.1	12.7	13.1	13.7	14.1	14.7
17	5	3.40	2.9	3.9	4.8	5.6	6.2	6.9	7.6	8.1	8.7	9.3	9.8	10.4	10.9	11.5	11.9	12.5	12.9	13.5	13.9	14.4
7	2	3.50	2.8	3.8	4.7	5.5	6.1	6.8	7.4	7.9	8.6	9.0	9.6	10.1	10.7	11.2	11.7	12.2	12.7	13.1	13.7	14.1
18	5	3.60	2.8	3.8	4.6	5.3	5.9	6.7	7.2	7.8	8.4	8.9	9.5	9.9	10.5	10.9	11.5	11.9	12.4	12.9	13.4	13.8
11	3	3.67	2.8	3.8	4.6	5.3	5.9	6.6	7.1	7.7	8.3	8.8	9.3	9.8	10.3	10.8	11.3	11.8	12.2	12.7	13.2	13.6
15	4	3.75	2.7	3.7	4.5	5.2	5.8	6.5	6.9	7.6	8.1	8.7	9.1	9.7	10.1	10.7	11.1	11.6	12.0	12.5	12.9	13.4
19	5	3.80	2.7	3.7	4.5	5.1	5.8	6.4	6.9	7.5	8.0	8.6	9.0	9.6	10.0	10.6	10.9	11.5	11.9	12.4	12.8	13.3
4	1	4.00	2.6	3.6	4.3	4.9	5.6	6.1	6.7	7.2	7.8	8.3	8.7	9.2	9.7	10.1	10.6	11.0	11.5	11.9	12.3	12.8
17	4	4.25	2.6	3.4	4.1	4.8	5.4	5.9	6.5	6.9	7.5	7.9	8.4	8.8	9.3	9.7	10.1	10.6	10.9	11.4	11.8	12.2
13	3	4.33	2.5	3.4	4.0	4.7	5.3	5.8	6.4	6.8	7.4	7.8	8.3	8.7	9.1	9.6	9.9	10.4	10.8	11.2	11.6	11.9
9	2	4.50	2.5	3.3	3.9	4.6	5.1	5.7	6.2	6.7	7.1	7.6	8.0	8.5	8.9	9.3	9.7	10.1	10.5	10.9	11.3	11.7
14	3	4.67	2.4	3.2	3.8	4.5	4.9	5.6	5.9	6.5	6.9	7.4	7.8	8.3	8.7	9.0	9.5	9.8	10.2	10.6	10.9	11.4
5	1	5.00	2.2	2.9	3.7	4.3	4.8	5.3	5.8	6.2	6.7	7.0	7.5	7.8	8.3	8.7	8.9	9.4	9.7	10.1	10.5	10.8
16	3	5.33	2.1	2.9	3.6	4.0	4.6	5.0	5.6	5.9	6.4	6.8	7.1	7.6	7.9	8.3	8.6	8.9	9.3	9.7	9.9	10.3
11	2	5.50	2.0	2.9	3.5	3.9	4.6	4.9	5.4	5.8	6.2	6.6	6.9	7.4	7.7	8.0	8.5	8.8	9.1	9.5	9.8	10.0
6	1	6.00	1.9	2.8	3.3	3.8	4.3	4.7	5.0	5.5	5.8	6.2	6.6	6.9	7.3	7.6	7.9	8.2	8.6	8.8	9.2	9.5
19	3	6.33	1.9	2.7	3.2	3.7	4.0	4.6	4.9	5.3	5.7	5.9	6.4	6.7	6.9	7.3	7.7	7.9	8.2	8.6	8.8	9.1
13	2	6.50	1.9	2.7	3.1	3.7	3.9	4.5	4.8	5.2	5.6	5.9	6.2	6.6	6.9	7.2	7.5	7.8	8.0	8.4	8.7	8.9
20	3	6.67	1.9	2.6	3.0	3.6	3.9	4.4	4.8	5.1	5.5	5.8	6.1	6.5	6.8	7.0	7.4	7.7	7.9	8.2	8.6	8.8
7	1	7.00	1.8	2.6	2.9	3.5	3.8	4.3	4.7	4.9	5.3	5.7	5.9	6.2	6.6	6.8	7.1	7.4	7.7	7.9	8.2	8.5
15	2	7.50	1.8	2.4	2.9	3.3	3.7	4.0	4.5	4.8	5.0	5.4	5.7	5.9	6.3	6.6	6.8	7.0	7.4	7.6	7.8	8.1
8	1	8.00	1.8	2.3	2.8	3.1	3.6	3.9	4.2	4.6	4.8	5.1	5.5	5.7	5.9	6.3	6.6	6.8	6.9	7.3	7.6	7.8
17	2	8.50	1.7	2.2	2.7	2.9	3.5	3.8	4.0	4.4	4.7	4.9	5.2	5.5	5.8	5.9	6.3	6.5	6.8	6.9	7.2	7.5
9	1	9.00	1.7	2.0	2.7	2.9	3.3	3.7	3.9	4.2	4.6	4.8	4.9	5.3	5.6	5.8	5.9	6.3	6.5	6.7	6.9	7.1
19	2	9.50	1.6	1.9	2.6	2.9	3.2	3.6	3.8	4.0	4.4	4.7	4.8	5.1	5.4	5.6	5.8	5.9	6.3	6.5	6.7	6.9
10	1	10.00	1.6	1.9	2.5	2.8	3.0	3.5	3.7	3.9	4.2	4.5	4.7	4.9	5.2	5.5	5.7	5.8	6.0	6.3	6.5	6.7
11	1	11.00	1.5	1.9	2.3	2.7	2.9	3.2	3.6	3.8	3.9	4.2	4.5	4.7	4.8	5.0	5.3	5.5	5.7	5.9	6.0	6.3
12	1	12.00	1.4	1.8	2.1	2.6	2.8	2.9	3.3	3.6	3.8	3.9	4.2	4.5	4.7	4.8	4.9	5.2	5.4	5.6	5.8	5.9
13	1	13.00	1.3	1.8	1.9	2.5	2.7	2.9	3.1	3.4	3.7	3.8	3.9	4.2	4.4	4.6	4.8	4.9	5.1	5.3	5.5	5.7
14	1	14.00	1.2	1.8	1.9	2.3	2.6	2.8	2.9	3.2	3.5	3.7	3.8	3.9	4.2	4.4	4.6	4.7	4.9	4.9	5.2	5.4
15	1	15.00	1.1	1.7	1.9	2.2	2.5	2.7	2.9	3.0	3.3	3.6	3.7	3.8	3.9	4.2	4.4	4.6	4.7	4.8	4.9	5.1
16	1	16.00	1.0	1.7	1.9	2.0	2.4	2.7	2.8	2.9	3.1	3.4	3.6	3.7	3.9	3.9	4.2	4.4	4.6	4.7	4.8	4.9
17	1	17.00	.9	1.6	1.8	1.9	2.3	2.6	2.8	2.9	2.9	3.3	3.5	3.6	3.8	3.9	3.9	4.2	4.4	4.5	4.7	4.8
18	1	18.00	.9	1.6	1.8	1.9	2.2	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	3.9	4.2	4.4	4.5	4.6
19	1	19.00	.9	1.5	1.8	1.9	2.1	2.4	2.6	2.8	2.9	2.9	3.2	3.4	3.6	3.7	3.8	3.9	3.9	4.2	4.3	4.5
20	1	20.00	.9	1.5	1.7	1.9	1.9	2.3	2.6	2.7	2.8	2.9	3.0	3.3	3.5	3.6	3.7	3.8	3.9	3.9	4.1	4.3

99% PREDICTION INTERVAL - THREE FUTURE OBSERVATIONS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	40.8	42.3	43.7	45.0	46.5	47.8	49.2	50.6	51.9	53.3	54.6	55.9	57.3	58.6	59.9	61.2	62.6	63.9	65.2	66.5
6	5	1.20	34.9	36.1	37.3	38.4	39.6	40.8	41.9	43.1	44.2	45.4	46.5	47.6	48.7	49.9	51.0	52.1	53.2	54.3	55.4	56.5
5	4	1.25	33.7	34.8	35.9	37.1	38.2	39.4	40.5	41.6	42.7	43.8	44.9	45.9	47.0	48.1	49.2	50.3	51.3	52.4	53.5	54.5
4	3	1.33	31.9	32.9	34.0	35.1	36.2	37.2	38.3	39.3	40.4	41.4	42.4	43.5	44.5	45.5	46.5	47.5	48.5	49.5	50.5	51.5
7	5	1.40	30.6	31.6	32.6	33.7	34.7	35.7	36.7	37.7	38.7	39.7	40.7	41.6	42.6	43.6	44.6	45.5	46.5	47.5	48.4	49.4
3	2	1.50	28.8	29.8	30.8	31.7	32.7	33.7	34.6	35.5	36.5	37.4	38.3	39.2	40.1	41.0	41.9	42.9	43.8	44.7	45.6	46.5
8	5	1.60	27.3	28.2	29.1	30.0	30.9	31.8	32.7	33.6	34.5	35.4	36.3	37.1	38.0	38.8	39.7	40.6	41.4	42.3	43.1	43.9
5	3	1.67	26.4	27.3	28.2	29.0	29.9	30.8	31.6	32.5	33.3	34.2	35.0	35.8	36.7	37.5	38.3	39.2	39.9	40.8	41.6	42.4
7	4	1.75	25.4	26.2	27.0	27.9	28.7	29.5	30.4	31.2	31.9	32.8	33.6	34.4	35.2	36.0	36.8	37.6	38.4	39.1	39.9	40.7
9	5	1.80	24.8	25.6	26.4	27.2	28.0	28.8	29.7	30.5	31.2	32.0	32.8	33.6	34.4	35.1	35.9	36.7	37.4	38.2	38.9	39.7
2	1	2.00	22.7	23.5	24.2	24.9	25.7	26.4	27.1	27.9	28.5	29.3	30.0	30.7	31.5	32.1	32.8	33.5	34.2	34.9	35.6	36.3
11	5	2.20	21.0	21.7	22.4	23.1	23.8	24.4	25.1	25.8	26.4	27.1	27.7	28.4	29.0	29.7	30.3	30.9	31.6	32.2	32.8	33.5
9	4	2.25	20.6	21.3	21.9	22.7	23.3	23.9	24.6	25.3	25.9	26.6	27.2	27.8	28.5	29.1	29.8	30.4	31.0	31.6	32.3	32.8
7	3	2.33	20.0	20.7	21.4	22.0	22.7	23.3	23.9	24.6	25.2	25.8	26.4	27.0	27.7	28.3	28.9	29.5	30.1	30.7	31.3	31.9
12	5	2.40	19.6	20.2	20.9	21.5	22.1	22.8	23.4	23.9	24.6	25.2	25.8	26.4	27.0	27.6	28.2	28.8	29.4	29.9	30.6	31.1
5	2	2.50	18.9	19.6	20.2	20.8	21.4	22.0	22.6	23.2	23.8	24.4	24.9	25.6	26.1	26.7	27.3	27.8	28.4	28.9	29.6	30.1
13	5	2.60	18.4	18.9	19.6	20.2	20.7	21.3	21.9	22.5	23.0	23.6	24.2	24.7	25.3	25.8	26.4	26.9	27.5	28.0	28.6	29.1
8	3	2.67	18.0	18.6	19.2	19.8	20.3	20.9	21.5	22.0	22.6	23.1	23.7	24.2	24.8	25.3	25.8	26.4	26.9	27.5	28.0	28.6
14	5	2.80	17.4	17.9	18.5	19.0	19.6	20.1	20.7	21.2	21.7	22.2	22.8	23.3	23.8	24.4	24.8	25.4	25.9	26.4	26.9	27.4
3	1	3.00	16.4	16.9	17.5	17.9	18.5	19.0	19.6	20.0	20.6	21.0	21.6	22.0	22.5	23.0	23.5	23.9	24.5	24.9	25.4	25.9
16	5	3.20	15.6	16.1	16.6	17.1	17.6	18.1	18.6	19.0	19.5	19.9	20.5	20.9	21.4	21.8	22.3	22.8	23.2	23.7	24.1	24.6
10	3	3.33	15.1	15.6	16.1	16.6	17.0	17.5	17.9	18.5	18.9	19.4	19.8	20.3	20.7	21.1	21.6	22.0	22.5	22.9	23.4	23.8
17	5	3.40	14.9	15.4	15.8	16.3	16.8	17.2	17.7	18.1	18.6	19.0	19.5	19.9	20.4	20.8	21.3	21.7	22.1	22.6	22.9	23.4
7	2	3.50	14.6	15.0	15.5	15.9	16.4	16.8	17.3	17.7	18.2	18.6	19.0	19.5	19.9	20.4	20.8	21.2	21.6	22.0	22.5	22.9
18	5	3.60	14.3	14.7	15.2	15.6	16.0	16.5	16.9	17.4	17.8	18.2	18.7	19.0	19.5	19.9	20.3	20.7	21.1	21.6	21.9	22.4
11	3	3.67	14.0	14.5	14.9	15.4	15.8	16.3	16.7	17.1	17.6	17.9	18.4	18.8	19.2	19.6	20.0	20.4	20.8	21.2	21.7	22.0
15	4	3.75	13.8	14.3	14.7	15.1	15.6	15.9	16.4	16.8	17.2	17.7	18.0	18.5	18.8	19.3	19.7	20.0	20.5	20.8	21.3	21.7
19	5	3.80	13.7	14.1	14.6	14.9	15.4	15.8	16.2	16.7	17.0	17.5	17.8	18.3	18.7	19.0	19.5	19.8	20.3	20.7	21.0	21.4
4	1	4.00	13.2	13.6	13.9	14.4	14.8	15.2	15.6	15.9	16.4	16.8	17.2	17.6	17.9	18.3	18.7	19.0	19.5	19.8	20.2	20.6
17	4	4.25	12.6	12.9	13.4	13.8	14.1	14.5	14.9	15.3	15.7	16.0	16.4	16.7	17.1	17.5	17.8	18.2	18.6	18.9	19.3	19.6
13	3	4.33	12.4	12.8	13.2	13.6	13.9	14.3	14.7	15.0	15.4	15.8	16.1	16.5	16.8	17.2	17.6	17.9	18.3	18.6	18.9	19.3
9	2	4.50	12.0	12.5	12.8	13.2	13.6	13.9	14.3	14.6	14.9	15.3	15.7	16.0	16.4	16.7	17.0	17.4	17.7	18.1	18.4	18.8
14	3	4.67	11.7	12.1	12.5	12.8	13.2	13.6	13.9	14.2	14.6	14.9	15.3	15.6	15.9	16.3	16.6	16.9	17.3	17.6	17.9	18.2
5	1	5.00	11.1	11.5	11.8	12.2	12.5	12.8	13.2	13.5	13.8	14.1	14.5	14.8	15.1	15.4	15.7	16.0	16.4	16.7	16.9	17.3
16	3	5.33	10.7	10.9	11.3	11.6	11.9	12.3	12.6	12.8	13.2	13.5	13.8	14.1	14.4	14.7	14.9	15.3	15.6	15.8	16.1	16.5
11	2	5.50	10.4	10.7	11.0	11.4	11.7	11.9	12.3	12.6	12.9	13.2	13.5	13.8	14.0	14.4	14.7	14.9	15.2	15.5	15.8	16.0
6	1	6.00	9.8	10.0	10.4	10.7	10.9	11.2	11.5	11.8	12.0	12.4	12.7	12.9	13.2	13.5	13.7	13.9	14.3	14.5	14.8	15.0
19	3	6.33	9.4	9.7	9.9	10.3	10.6	10.8	11.0	11.4	11.6	11.9	12.1	12.4	12.7	12.9	13.2	13.5	13.7	13.9	14.2	14.5
13	2	6.50	9.2	9.6	9.8	10.0	10.4	10.6	10.8	11.1	11.4	11.7	11.9	12.2	12.5	12.7	12.9	13.2	13.5	13.7	13.9	14.2
20	3	6.67	9.0	9.4	9.6	9.9	10.1	10.4	10.7	10.9	11.2	11.5	11.7	11.9	12.2	12.5	12.7	12.9	13.2	13.4	13.7	13.9
7	1	7.00	8.8	9.0	9.3	9.6	9.8	10.0	10.3	10.6	10.8	11.0	11.3	11.6	11.8	11.9	12.2	12.5	12.7	12.9	13.2	13.4
15	2	7.50	8.4	8.6	8.8	9.0	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.2	11.4	11.7	11.8	12.0	12.3	12.6	12.7
8	1	8.00	7.9	8.2	8.5	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.1	11.3	11.6	11.7	11.9	12.1
17	2	8.50	7.7	7.9	8.1	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	10.0	10.2	10.5	10.7	10.8	11.0	11.2	11.5	11.7
9	1	9.00	7.4	7.6	7.8	7.9	8.2	8.5	8.7	8.8	9.0	9.2	9.5	9.7	9.8	9.9	10.2	10.4	10.6	10.8	10.9	11.1
19	2	9.50	7.1	7.3	7.6	7.7	7.9	8.1	8.3	8.6	8.7	8.9	9.0	9.3	9.5	9.7	9.8	9.9	10.2	10.4	10.6	10.7
10	1	10.00	6.8	7.0	7.3	7.5	7.7	7.8	7.9	8.2	8.4	8.6	8.8	8.9	9.1	9.3	9.5	9.7	9.8	9.9	10.2	10.4
11	1	11.00	6.5	6.7	6.8	6.9	7.2	7.4	7.6	7.7	7.8	8.0	8.2	8.4	8.6	8.7	8.9	9.0	9.2	9.4	9.6	9.7
12	1	12.00	6.0	6.3	6.5	6.6	6.8	6.9	7.0	7.3	7.5	7.6	7.8	7.9	8.0	8.2	8.4	8.6	8.7	8.8	8.9	9.1
13	1	13.00	5.8	5.9	6.1	6.3	6.5	6.6	6.8	6.9	6.9	7.2	7.4	7.5	7.7	7.8	7.9	8.0	8.2	8.4	8.5	8.7
14	1	14.00	5.6	5.7	5.8	5.9	6.1	6.3	6.5	6.6	6.7	6.8	6.9	7.1	7.3	7.4	7.6	7.7	7.8	7.9	8.0	8.2
15	1	15.00	5.3	5.5	5.6	5.7	5.8	5.9	6.1	6.3	6.5	6.6	6.7	6.8	6.9	7.0	7.2	7.4	7.5	7.6	7.7	7.8
16	1	16.00	5.0	5.2	5.4	5.5	5.7	5.8	5.9	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	6.9	7.1	7.3	7.4	7.5
17	1	17.00	4.9	4.9	5.1	5.3	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.8	6.9	7.0	7.2
18	1	18.00	4.7	4.8	4.9	5.0	5.2	5.4	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.3	6.4	6.5	6.6	6.7	6.8	6.9
19	1	19.00	4.6	4.7	4.8	4.9	4.9	5.1	5.3	5.4	5.6	5.7	5.8	5.8	5.9	6.0	6.1	6.3	6.4	6.5	6.6	6.7
20	1	20.00	4.5	4.6	4.7	4.8	4.9	4.9	5.0	5.2	5.4	5.5	5.6	5.7	5.8	5.8	5.9	6.0	6.2	6.3	6.4	6.5

90% PREDICTION INTERVAL - FOUR FUTURE OBSERVATIONS

L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	3.7	5.9	7.8	9.6	11.2	12.7	14.3	15.8	17.2	18.6	20.0	21.4	22.8	24.2	25.5	26.8	28.2	29.5	30.8	32.1
6	5	1.20	3.2	5.1	6.7	8.2	9.6	10.9	12.2	13.5	14.7	15.9	17.1	18.3	19.5	20.6	21.7	22.8	24.0	25.1	26.2	27.3
5	4	1.25	3.1	4.9	6.5	7.9	9.3	10.6	11.8	13.0	14.2	15.4	16.5	17.7	18.8	19.9	21.0	22.0	23.1	24.2	25.3	26.3
4	3	1.33	2.9	4.7	6.2	7.5	8.8	10.0	11.2	12.3	13.5	14.6	15.6	16.7	17.8	18.8	19.8	20.8	21.9	22.9	23.9	24.9
7	5	1.40	2.8	4.5	5.9	7.2	8.4	9.6	10.7	11.8	12.9	13.9	15.0	16.0	17.0	18.0	19.0	20.0	20.9	21.9	22.9	23.8
3	2	1.50	2.7	4.3	5.6	6.8	7.9	9.0	10.1	11.2	12.2	13.2	14.1	15.1	16.0	17.0	17.9	18.8	19.7	20.7	21.6	22.4
8	5	1.60	2.6	4.0	5.3	6.5	7.6	8.6	9.6	10.6	11.5	12.5	13.4	14.3	15.2	16.1	16.9	17.8	18.7	19.5	20.4	21.2
5	3	1.67	2.5	3.9	5.1	6.3	7.3	8.3	9.3	10.2	11.1	12.0	12.9	13.8	14.7	15.5	16.4	17.2	18.0	18.8	19.7	20.5
7	4	1.75	2.4	3.8	4.9	6.0	7.0	7.9	8.9	9.8	10.7	11.6	12.4	13.3	14.1	14.9	15.7	16.5	17.3	18.1	18.9	19.6
9	5	1.80	2.4	3.7	4.8	5.9	6.8	7.8	8.7	9.6	10.5	11.3	12.1	12.9	13.7	14.6	15.3	16.1	16.9	17.7	18.4	19.2
2	1	2.00	2.1	3.4	4.5	5.4	6.3	7.2	7.9	8.8	9.6	10.4	11.1	11.8	12.6	13.3	14.0	14.7	15.5	16.1	16.8	17.5
11	5	2.20	1.9	3.1	4.1	5.0	5.8	6.6	7.4	8.1	8.8	9.6	10.3	10.9	11.6	12.3	12.9	13.6	14.3	14.9	15.5	16.2
9	4	2.25	1.9	3.1	4.0	4.9	5.7	6.5	7.3	7.9	8.7	9.4	10.1	10.7	11.4	12.0	12.7	13.4	13.9	14.6	15.2	15.8
7	3	2.33	1.9	2.9	3.9	4.8	5.6	6.3	7.0	7.8	8.5	9.1	9.8	10.4	11.1	11.7	12.3	12.9	13.6	14.2	14.8	15.4
12	5	2.40	1.8	2.9	3.8	4.7	5.5	6.2	6.9	7.6	8.3	8.9	9.6	10.2	10.8	11.5	12.0	12.7	13.3	13.8	14.4	15.0
5	2	2.50	1.8	2.8	3.7	4.6	5.3	5.9	6.7	7.4	7.9	8.6	9.3	9.8	10.5	11.0	11.7	12.2	12.8	13.4	13.9	14.5
13	5	2.60	1.8	2.8	3.6	4.4	5.1	5.8	6.5	7.1	7.7	8.4	8.9	9.6	10.1	10.7	11.3	11.8	12.4	12.9	13.5	14.0
8	3	2.67	1.7	2.7	3.6	4.3	5.0	5.7	6.4	6.9	7.6	8.2	8.8	9.4	9.9	10.5	11.0	11.6	12.1	12.7	13.2	13.7
14	5	2.80	1.7	2.6	3.4	4.1	4.8	5.5	6.1	6.7	7.3	7.8	8.5	8.9	9.6	10.1	10.6	11.1	11.7	12.2	12.7	13.2
3	1	3.00	1.6	2.5	3.2	3.9	4.6	5.2	5.8	6.4	6.9	7.5	7.9	8.5	9.0	9.5	10.0	10.5	11.0	11.5	11.9	12.5
16	5	3.20	1.5	2.4	3.0	3.7	4.4	4.9	5.5	6.0	6.6	7.0	7.6	8.0	8.6	9.0	9.5	9.9	10.5	10.9	11.4	11.8
10	3	3.33	1.5	2.3	2.9	3.6	4.2	4.8	5.3	5.8	6.4	6.8	7.4	7.8	8.3	8.7	9.2	9.7	10.1	10.6	11.0	11.4
17	5	3.40	1.5	2.3	2.9	3.6	4.1	4.7	5.2	5.8	6.3	6.7	7.2	7.7	8.1	8.6	9.0	9.5	9.9	10.4	10.8	11.2
7	2	3.50	1.4	2.2	2.9	3.5	4.0	4.6	5.1	5.6	6.1	6.6	7.0	7.5	7.9	8.4	8.8	9.3	9.7	10.1	10.6	10.9
18	5	3.60	1.4	2.1	2.8	3.5	3.9	4.5	4.9	5.5	5.9	6.5	6.9	7.4	7.8	8.2	8.7	9.1	9.5	9.9	10.3	10.7
11	3	3.67	1.4	2.1	2.8	3.4	3.9	4.5	4.9	5.4	5.9	6.4	6.8	7.2	7.7	8.1	8.5	8.9	9.4	9.8	10.2	10.6
15	4	3.75	1.3	2.0	2.7	3.3	3.8	4.4	4.8	5.3	5.8	6.2	6.7	7.1	7.6	7.9	8.4	8.8	9.2	9.6	9.9	10.4
19	5	3.80	1.3	2.0	2.7	3.3	3.8	4.3	4.8	5.3	5.7	6.2	6.6	7.0	7.5	7.9	8.3	8.7	9.1	9.5	9.9	10.3
4	1	4.00	1.2	1.9	2.6	3.1	3.7	4.1	4.6	5.0	5.5	5.9	6.4	6.8	7.2	7.6	7.9	8.4	8.7	9.1	9.5	9.8
17	4	4.25	1.2	1.9	2.5	2.9	3.5	3.9	4.4	4.8	5.3	5.7	6.0	6.5	6.8	7.2	7.6	7.9	8.3	8.7	9.0	9.4
13	3	4.33	1.1	1.8	2.5	2.9	3.5	3.9	4.4	4.8	5.2	5.6	5.9	6.4	6.7	7.1	7.5	7.8	8.2	8.6	8.9	9.2
9	2	4.50	1.1	1.8	2.4	2.9	3.4	3.8	4.2	4.7	5.0	5.4	5.8	6.2	6.6	6.9	7.3	7.6	7.9	8.3	8.6	8.9
14	3	4.67	1.0	1.8	2.3	2.8	3.3	3.7	4.1	4.5	4.9	5.3	5.7	5.9	6.4	6.7	7.0	7.4	7.7	8.0	8.4	8.7
5	1	5.00	.9	1.7	2.2	2.7	3.1	3.5	3.9	4.3	4.7	4.9	5.4	5.7	6.0	6.4	6.7	6.9	7.3	7.6	7.9	8.2
16	3	5.33	.9	1.6	2.0	2.6	2.9	3.4	3.7	4.0	4.4	4.8	5.0	5.4	5.7	6.0	6.4	6.7	6.9	7.3	7.8	7.8
11	2	5.50	.9	1.6	2.0	2.5	2.9	3.3	3.7	3.9	4.3	4.7	4.9	5.3	5.6	5.9	6.2	6.5	6.8	7.0	7.4	7.7
6	1	6.00	.9	1.5	1.9	2.4	2.7	3.0	3.4	3.7	4.0	4.4	4.7	4.9	5.2	5.5	5.8	6.0	6.4	6.6	6.9	7.1
19	3	6.33	.8	1.5	1.8	2.3	2.6	2.9	3.3	3.6	3.9	4.2	4.5	4.7	5.0	5.3	5.6	5.8	6.0	6.4	6.6	6.8
13	2	6.50	.8	1.4	1.8	2.2	2.6	2.9	3.2	3.5	3.8	4.1	4.4	4.7	4.9	5.2	5.5	5.7	5.9	6.2	6.5	6.7
20	3	6.67	.8	1.4	1.8	2.1	2.6	2.8	3.1	3.5	3.7	3.9	4.3	4.6	4.8	5.1	5.4	5.6	5.8	6.1	6.4	6.6
7	1	7.00	.8	1.3	1.7	2.0	2.5	2.7	2.9	3.3	3.6	3.8	4.1	4.4	4.7	4.9	5.1	5.4	5.7	5.8	6.1	6.4
15	2	7.50	.8	1.3	1.7	1.9	2.3	2.6	2.9	3.1	3.5	3.7	3.9	4.2	4.5	4.7	4.9	5.1	5.4	5.6	5.8	5.9
8	1	8.00	.8	1.2	1.6	1.9	2.2	2.5	2.8	2.9	3.3	3.5	3.8	3.9	4.2	4.5	4.7	4.8	5.1	5.3	5.5	5.7
17	2	8.50	.7	1.1	1.6	1.8	2.0	2.4	2.7	2.9	3.1	3.4	3.6	3.8	3.9	4.2	4.5	4.7	4.8	5.0	5.3	5.5
9	1	9.00	.7	1.0	1.5	1.8	1.9	2.3	2.6	2.8	2.9	3.2	3.5	3.7	3.8	4.0	4.3	4.5	4.7	4.8	5.0	5.2
19	2	9.50	.7	.9	1.4	1.7	1.9	2.2	2.5	2.7	2.8	3.0	3.3	3.5	3.7	3.9	4.1	4.3	4.5	4.7	4.8	5.0
10	1	10.00	.7	.9	1.4	1.7	1.8	2.0	2.4	2.6	2.8	2.9	3.2	3.4	3.6	3.8	3.9	4.1	4.3	4.5	4.7	4.8
11	1	11.00	.6	.9	1.2	1.6	1.8	1.9	2.2	2.4	2.6	2.8	2.9	3.1	3.4	3.5	3.7	3.8	3.9	4.2	4.4	4.5
12	1	12.00	.6	.9	1.1	1.5	1.7	1.8	1.9	2.3	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.6	3.8	3.9	4.0	4.2
13	1	13.00	.6	.8	1.0	1.4	1.6	1.8	1.9	2.1	2.3	2.5	2.7	2.8	2.9	3.1	3.3	3.4	3.6	3.7	3.8	3.9
14	1	14.00	.5	.8	.9	1.3	1.5	1.7	1.8	1.9	2.2	2.4	2.5	2.7	2.8	2.9	3.0	3.2	3.4	3.5	3.7	3.8
15	1	15.00	.5	.8	.9	1.2	1.5	1.6	1.8	1.9	2.0	2.2	2.4	2.6	2.7	2.8	2.9	3.0	3.2	3.4	3.5	3.6
16	1	16.00	.5	.8	.9	1.1	1.4	1.6	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.3	3.5
17	1	17.00	.5	.7	.9	1.0	1.3	1.5	1.7	1.8	1.9	1.9	2.2	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.3
18	1	18.00	.4	.7	.8	.9	1.2	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.9	2.9	3.1
19	1	19.00	.4	.7	.8	.9	1.1	1.4	1.6	1.7	1.8	1.9	1.9	2.1	2.3	2.4	2.5	2.6	2.7	2.8	2.9	2.9
20	1	20.00	.4	.7	.8	.9	1.1	1.3	1.5	1.6	1.7	1.8	1.9	1.9	2.1	2.3	2.4	2.6	2.6	2.7	2.8	2.9

90% PREDICTION INTERVAL - FOUR FUTURE OBSERVATIONS

OBSERVED NUMBER OF COUNTS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1	1.00	33.4	34.6	35.9	37.2	38.5	39.7	41.0	42.2	43.5	44.7	45.9	47.2	48.4	49.6	50.8	52.1	53.3	54.5	55.7	56.9
6	5	1.20	28.4	29.5	30.5	31.6	32.7	33.7	34.8	35.8	36.9	37.9	39.0	40.0	41.0	42.1	43.1	44.1	45.2	46.2	47.2	48.2
5	4	1.25	27.4	28.4	29.5	30.5	31.5	32.5	33.6	34.6	35.6	36.6	37.6	38.6	39.6	40.6	41.6	42.5	43.5	44.5	45.5	46.5
4	3	1.33	25.8	26.8	27.8	28.8	29.8	30.7	31.7	32.6	33.6	34.5	35.5	36.4	37.4	38.3	39.2	40.1	41.1	42.0	42.9	43.8
7	5	1.40	24.8	25.7	26.7	27.6	28.5	29.4	30.3	31.3	32.2	33.1	33.9	34.9	35.8	36.7	37.5	38.4	39.3	40.2	41.1	41.9
3	2	1.50	23.3	24.2	25.1	25.9	26.8	27.7	28.6	29.4	30.3	31.1	31.9	32.8	33.6	34.5	35.3	36.1	36.9	37.8	38.6	39.5
8	5	1.60	22.0	22.9	23.7	24.5	25.4	26.2	26.9	27.8	28.6	29.4	30.2	30.9	31.8	32.6	33.3	34.1	34.9	35.7	36.5	37.2
5	3	1.67	21.3	22.1	22.9	23.7	24.5	25.2	26.0	26.8	27.6	28.3	29.1	29.9	30.6	31.4	32.2	32.9	33.7	34.4	35.2	35.9
7	4	1.75	20.4	21.2	21.9	22.7	23.5	24.2	24.9	25.7	26.4	27.2	27.9	28.6	29.4	30.1	30.8	31.5	32.2	32.9	33.7	34.4
9	5	1.60	19.9	20.7	21.4	22.1	22.9	23.6	24.3	25.0	25.8	26.5	27.2	27.9	28.6	29.3	30.0	30.7	31.5	32.1	32.8	33.5
2	1	2.00	18.2	18.9	19.6	20.2	20.9	21.6	22.2	22.9	23.5	24.2	24.8	25.5	26.1	26.7	27.4	28.0	28.7	29.3	29.9	30.6
11	5	2.20	16.8	17.4	18.0	18.6	19.2	19.8	20.5	21.0	21.7	22.3	22.8	23.5	24.0	24.6	25.2	25.8	26.4	26.9	27.5	28.1
9	4	2.25	16.5	17.1	17.7	18.3	18.9	19.5	20.1	20.7	21.2	21.8	22.4	23.0	23.6	24.1	24.7	25.3	25.9	26.4	27.0	27.6
7	3	2.33	15.9	16.6	17.1	17.7	18.3	18.9	19.5	20.0	20.6	21.2	21.7	22.3	22.8	23.4	23.9	24.5	25.0	25.6	26.2	26.7
12	5	2.40	15.6	16.2	16.7	17.3	17.9	18.4	19.0	19.6	20.1	20.7	21.2	21.7	22.3	22.8	23.4	23.9	24.5	25.0	25.5	26.0
5	2	2.50	15.0	15.6	16.2	16.7	17.3	17.8	18.4	18.9	19.4	19.9	20.5	21.0	21.5	22.0	22.6	23.1	23.6	24.1	24.6	25.1
13	5	2.60	14.6	15.1	15.7	16.2	16.7	17.2	17.7	18.3	18.8	19.3	19.8	20.3	20.8	21.3	21.8	22.3	22.8	23.3	23.8	24.3
8	3	2.67	14.3	14.8	15.3	15.8	16.4	16.8	17.4	17.9	18.4	18.9	19.4	19.9	20.4	20.8	21.4	21.8	22.3	22.8	23.3	23.8
14	5	2.80	13.7	14.2	14.7	15.2	15.7	16.2	16.7	17.1	17.6	18.1	18.6	19.0	19.5	20.0	20.5	20.9	21.4	21.9	22.4	22.8
3	1	3.00	12.9	13.4	13.9	14.4	14.8	15.3	15.7	16.2	16.6	17.1	17.5	17.9	18.4	18.9	19.3	19.7	20.2	20.6	21.0	21.5
16	5	3.20	12.3	12.7	13.1	13.6	14.0	14.5	14.9	15.3	15.8	16.2	16.6	17.0	17.5	17.8	18.3	18.7	19.1	19.5	19.9	20.4
10	3	3.33	11.8	12.3	12.7	13.1	13.6	13.9	14.4	14.8	15.2	15.6	16.0	16.5	16.8	17.3	17.7	18.0	18.5	18.8	19.3	19.7
17	5	3.40	11.7	12.1	12.5	12.9	13.4	13.8	14.2	14.6	14.9	15.4	15.8	16.2	16.6	16.9	17.4	17.8	18.1	18.6	18.9	19.3
7	2	3.50	11.4	11.8	12.2	12.6	13.0	13.4	13.8	14.2	14.6	15.0	15.4	15.8	16.2	16.6	16.9	17.3	17.7	18.1	18.5	18.8
18	5	3.60	11.1	11.6	11.9	12.3	12.7	13.1	13.5	13.9	14.3	14.7	15.0	15.4	15.8	16.2	16.6	16.9	17.3	17.7	18.0	18.4
11	3	3.67	10.9	11.4	11.8	12.1	12.6	12.9	13.3	13.7	14.0	14.5	14.8	15.2	15.6	15.9	16.3	16.7	17.0	17.4	17.8	18.1
15	4	3.75	10.8	11.2	11.6	11.9	12.3	12.7	13.0	13.4	13.8	14.2	14.6	14.9	15.3	15.6	16.0	16.4	16.7	17.1	17.4	17.8
19	5	3.80	10.7	11.0	11.4	11.8	12.2	12.6	12.9	13.3	13.7	14.0	14.4	14.7	15.1	15.5	15.8	16.2	16.5	16.9	17.2	17.6
4	1	4.00	10.2	10.6	10.9	11.3	11.7	12.0	12.4	12.7	13.1	13.5	13.8	14.1	14.5	14.8	15.2	15.5	15.8	16.2	16.5	16.8
17	4	4.25	9.7	10.1	10.4	10.8	11.1	11.5	11.8	12.1	12.5	12.8	13.1	13.5	13.8	14.1	14.4	14.7	15.0	15.4	15.7	16.0
13	3	4.33	9.6	9.9	10.3	10.6	10.9	11.3	11.6	11.9	12.3	12.6	12.9	13.2	13.6	13.8	14.2	14.5	14.8	15.1	15.5	15.8
9	2	4.50	9.3	9.6	9.9	10.3	10.6	10.9	11.3	11.6	11.9	12.2	12.5	12.8	13.1	13.5	13.7	14.0	14.4	14.7	14.9	15.3
14	3	4.67	9.0	9.4	9.7	9.9	10.3	10.6	10.9	11.2	11.6	11.8	12.1	12.5	12.7	13.0	13.4	13.6	13.9	14.2	14.5	14.8
5	1	5.00	8.6	8.8	9.1	9.5	9.7	10.0	10.3	10.6	10.9	11.2	11.5	11.8	12.0	12.3	12.6	12.9	13.2	13.5	13.7	14.0
16	3	5.33	8.1	8.4	8.7	8.9	9.3	9.5	9.8	10.1	10.4	10.6	10.9	11.2	11.5	11.7	11.9	12.2	12.5	12.8	13.0	13.3
11	2	5.50	7.9	8.2	8.5	8.8	9.0	9.3	9.6	9.8	10.1	10.4	10.6	10.9	11.1	11.4	11.7	11.9	12.2	12.5	12.7	12.9
6	1	6.00	7.4	7.7	7.9	8.2	8.4	8.7	8.9	9.2	9.4	9.7	9.9	10.1	10.4	10.6	10.9	11.1	11.4	11.6	11.8	12.0
19	3	6.33	7.1	7.4	7.6	7.8	8.0	8.3	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.2	10.4	10.7	10.8	11.1	11.3	11.6
13	2	6.50	6.9	7.2	7.5	7.7	7.9	8.1	8.4	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.2	10.4	10.7	10.8	11.1	11.3
20	3	6.67	6.8	7.0	7.3	7.5	7.8	7.9	8.2	8.5	8.7	8.9	9.1	9.4	9.6	9.8	9.9	10.2	10.4	10.7	10.8	11.0
7	1	7.00	6.6	6.8	7.0	7.3	7.5	7.7	7.9	8.1	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	10.0	10.2	10.5	10.7
15	2	7.50	6.2	6.5	6.7	6.8	7.0	7.3	7.5	7.7	7.9	8.1	8.3	8.5	8.7	8.9	9.1	9.3	9.5	9.7	9.9	10.0
8	1	8.00	5.9	6.1	6.4	6.6	6.7	6.9	7.1	7.3	7.5	7.7	7.9	8.1	8.3	8.5	8.7	8.8	9.0	9.2	9.4	9.6
17	2	8.50	5.7	5.8	6.0	6.2	6.5	6.6	6.8	6.9	7.2	7.4	7.6	7.7	7.9	8.0	8.3	8.4	8.6	8.8	8.9	9.1
9	1	9.00	5.4	5.6	5.8	5.9	6.1	6.3	6.5	6.7	6.8	7.0	7.2	7.4	7.6	7.7	7.9	8.0	8.2	8.4	8.6	8.7
19	2	9.50	5.2	5.4	5.6	5.7	5.9	6.0	6.2	6.4	6.6	6.7	6.9	7.0	7.2	7.4	7.6	7.7	7.9	8.0	8.2	8.4
10	1	10.00	4.9	5.2	5.4	5.5	5.7	5.8	5.9	6.2	6.3	6.5	6.7	6.8	6.9	7.1	7.3	7.4	7.6	7.7	7.9	8.0
11	1	11.00	4.7	4.8	4.9	5.1	5.3	5.5	5.6	5.7	5.9	6.0	6.2	6.3	6.5	6.6	6.8	6.9	7.0	7.2	7.3	7.5
12	1	12.00	4.4	4.5	4.7	4.8	4.9	5.0	5.2	5.4	5.5	5.7	5.8	5.9	6.0	6.2	6.3	6.5	6.6	6.7	6.8	6.9
13	1	13.00	4.1	4.3	4.4	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.5	6.6
14	1	14.00	3.9	3.9	4.1	4.3	4.4	4.6	4.7	4.8	4.9	5.0	5.1	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.2
15	1	15.00	3.7	3.8	3.9	4.0	4.2	4.3	4.5	4.6	4.7	4.8	4.9	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8	5.9
16	1	16.00	3.6	3.7	3.8	3.9	3.9	4.1	4.2	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.2	5.3	5.4	5.5	5.6
17	1	17.00	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1	5.3	5.4
18	1	18.00	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1
19	1	19.00	3.1	3.2	3.4	3.5	3.6	3.7	3.7	3.8	3.9	3.9	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9
20	1	20.00	2.9	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8

95% PREDICTION INTERVAL - FOUR FUTURE OBSERVATIONS

			OBSERVED NUMBER OF COUNTS																			
L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1.00	4.8	7.2	9.3	11.1	12.8	14.5	16.1	17.7	19.2	20.7	22.2	23.7	25.1	26.5	27.9	29.3	30.7	32.0	33.4	34.8
6	5	1.20	4.2	6.3	7.9	9.6	11.0	12.5	13.8	15.2	16.5	17.7	19.0	20.2	21.5	22.7	23.8	25.0	26.2	27.3	28.5	29.6
5	4	1.25	4.0	6.0	7.7	9.3	10.7	12.0	13.4	14.7	15.9	17.1	18.4	19.6	20.7	21.9	23.0	24.1	25.3	26.4	27.5	28.6
4	3	1.33	3.8	5.8	7.4	8.8	10.1	11.4	12.7	13.9	15.1	16.2	17.4	18.5	19.6	20.7	21.8	22.8	23.9	24.9	26.0	27.0
7	5	1.40	3.7	5.6	7.0	8.5	9.7	10.9	12.2	13.4	14.5	15.6	16.7	17.8	18.8	19.8	20.9	21.9	22.9	23.9	24.9	25.9
3	2	1.50	3.6	5.2	6.7	7.9	9.2	10.4	11.5	12.6	13.7	14.7	15.7	16.8	17.6	18.7	19.7	20.7	21.6	22.6	23.5	24.4
8	5	1.60	3.4	4.9	6.4	7.6	8.8	9.8	10.9	11.9	12.9	13.9	14.9	15.9	16.8	17.7	18.7	19.6	20.5	21.4	22.2	23.1
5	3	1.67	3.3	4.8	6.1	7.4	8.5	9.6	10.6	11.6	12.6	13.5	14.4	15.4	16.3	17.1	18.0	18.9	19.8	20.6	21.5	22.3
7	4	1.75	3.1	4.7	5.9	7.0	8.1	9.2	10.2	11.1	12.0	12.9	13.8	14.7	15.6	16.5	17.3	18.1	18.9	19.8	20.6	21.4
9	5	1.80	3.0	4.6	5.8	6.9	7.9	8.9	9.9	10.8	11.8	12.7	13.6	14.4	15.3	16.1	16.9	17.7	18.6	19.3	20.1	20.9
2	1	2.00	2.8	4.2	5.4	6.4	7.4	8.3	9.1	9.9	10.8	11.6	12.4	13.2	13.9	14.7	15.5	16.2	16.9	17.7	18.4	19.1
11	5	2.20	2.7	3.9	4.9	5.9	6.8	7.7	8.5	9.3	10.0	10.8	11.5	12.2	12.9	13.7	14.3	15.0	15.7	16.4	17.0	17.7
9	4	2.25	2.6	3.8	4.9	5.8	6.7	7.5	8.3	9.1	9.8	10.6	11.3	12.0	12.7	13.4	14.1	14.7	15.4	16.0	16.7	17.4
7	3	2.33	2.6	3.7	4.8	5.7	6.5	7.3	8.1	8.8	9.6	10.3	10.9	11.7	12.4	13.0	13.7	14.3	14.9	15.6	16.2	16.8
12	5	2.40	2.5	3.7	4.7	5.6	6.4	7.1	7.9	8.7	9.4	10.0	10.7	11.4	12.0	12.7	13.4	13.9	14.6	15.2	15.8	16.5
5	2	2.50	2.4	3.6	4.5	5.4	6.2	6.9	7.7	8.4	9.0	9.7	10.4	11.0	11.7	12.3	12.9	13.5	14.1	14.7	15.3	15.9
13	5	2.60	2.4	3.5	4.4	5.2	5.9	6.7	7.5	8.1	8.8	9.5	10.0	10.7	11.3	11.9	12.5	13.1	13.7	14.3	14.8	15.4
8	3	2.67	2.3	3.4	4.3	5.1	5.9	6.6	7.3	7.9	8.6	9.3	9.9	10.5	11.1	11.7	12.3	12.8	13.4	13.9	14.6	15.1
14	5	2.80	2.2	3.3	4.1	4.9	5.7	6.4	7.0	7.7	8.3	8.9	9.5	10.1	10.7	11.2	11.8	12.4	12.9	13.5	13.9	14.5
3	1	3.00	2.0	3.1	3.9	4.7	5.4	6.0	6.7	7.3	7.8	8.5	8.9	9.6	10.1	10.7	11.2	11.7	12.2	12.7	13.2	13.7
16	5	3.20	1.9	2.9	3.8	4.5	5.1	5.8	6.4	6.9	7.5	8.0	8.6	9.1	9.6	10.1	10.6	11.1	11.6	12.1	12.6	13.0
10	3	3.33	1.9	2.9	3.7	4.4	4.9	5.6	6.1	6.7	7.3	7.8	8.3	8.8	9.3	9.8	10.3	10.8	11.2	11.7	12.1	12.6
17	5	3.40	1.9	2.8	3.6	4.3	4.9	5.5	6.0	6.6	7.1	7.7	8.2	8.7	9.2	9.7	10.1	10.6	11.0	11.5	11.9	12.4
7	2	3.50	1.9	2.8	3.6	4.2	4.8	5.4	5.9	6.5	6.9	7.5	7.9	8.5	8.9	9.5	9.9	10.4	10.8	11.3	11.7	12.1
18	5	3.60	1.8	2.7	3.5	4.1	4.7	5.3	5.8	6.4	6.8	7.4	7.8	8.3	8.8	9.2	9.7	10.1	10.6	11.0	11.5	11.8
11	3	3.67	1.8	2.7	3.4	4.0	4.7	5.2	5.7	6.3	6.8	7.2	7.7	8.2	8.7	9.1	9.6	9.9	10.4	10.8	11.3	11.7
15	4	3.75	1.8	2.7	3.4	3.9	4.6	5.1	5.7	6.1	6.7	7.1	7.6	8.0	8.5	8.9	9.4	9.8	10.2	10.7	11.0	11.5
19	5	3.80	1.8	2.7	3.3	3.9	4.6	5.0	5.6	6.0	6.6	7.0	7.5	7.9	8.4	8.8	9.3	9.7	10.1	10.6	10.9	11.4
4	1	4.00	1.7	2.6	3.2	3.8	4.4	4.8	5.4	5.8	6.3	6.8	7.2	7.7	8.1	8.5	8.9	9.3	9.7	10.1	10.5	10.9
17	4	4.25	1.7	2.5	3.0	3.7	4.1	4.7	5.1	5.6	6.0	6.5	6.9	7.3	7.7	8.1	8.5	8.9	9.3	9.7	10.0	10.4
13	3	4.33	1.7	2.4	2.9	3.6	4.1	4.6	5.0	5.6	5.9	6.4	6.8	7.2	7.6	7.9	8.4	8.8	9.1	9.5	9.9	10.3
9	2	4.50	1.6	2.4	2.9	3.5	3.9	4.5	4.9	5.4	5.8	6.2	6.6	6.9	7.4	7.8	8.1	8.5	8.9	9.3	9.6	9.9
14	3	4.67	1.6	2.3	2.8	3.4	3.9	4.4	4.8	5.2	5.7	6.0	6.5	6.8	7.2	7.6	7.9	8.3	8.7	8.9	9.4	9.7
5	1	5.00	1.5	2.1	2.8	3.3	3.7	4.1	4.6	4.9	5.4	5.7	6.1	6.5	6.8	7.2	7.6	7.8	8.2	8.6	8.8	9.2
16	3	5.33	1.5	2.0	2.7	3.0	3.6	3.9	4.4	4.8	5.1	5.5	5.8	6.2	6.5	6.8	7.2	7.5	7.8	8.1	8.5	8.7
11	2	5.50	1.4	1.9	2.6	2.9	3.5	3.9	4.3	4.7	4.9	5.4	5.7	6.0	6.4	6.7	6.9	7.3	7.7	7.9	8.3	8.6
6	1	6.00	1.3	1.9	2.5	2.8	3.3	3.7	3.9	4.4	4.7	5.0	5.4	5.7	5.9	6.3	6.6	6.8	7.1	7.5	7.7	7.9
19	3	6.33	1.2	1.8	2.4	2.8	3.1	3.6	3.8	4.2	4.6	4.8	5.1	5.5	5.7	6.0	6.3	6.6	6.8	7.1	7.4	7.7
13	2	6.50	1.2	1.8	2.3	2.7	3.0	3.5	3.8	4.1	4.5	4.8	5.0	5.4	5.7	5.9	6.2	6.5	6.7	6.9	7.3	7.6
20	3	6.67	1.2	1.8	2.3	2.7	2.9	3.4	3.7	4.0	4.4	4.7	4.9	5.3	5.6	5.8	6.0	6.4	6.6	6.9	7.1	7.4
7	1	7.00	1.1	1.8	2.1	2.6	2.9	3.3	3.6	3.9	4.2	4.6	4.8	5.0	5.4	5.6	5.8	6.1	6.4	6.7	6.9	7.1
15	2	7.50	1.0	1.7	2.0	2.5	2.8	3.1	3.5	3.7	3.9	4.3	4.6	4.8	5.0	5.4	5.6	5.8	6.0	6.3	6.6	6.8
8	1	8.00	.9	1.6	1.9	2.4	2.7	2.9	3.3	3.6	3.8	4.1	4.4	4.6	4.6	5.1	5.4	5.6	5.8	5.9	6.3	6.5
17	2	8.50	.9	1.6	1.9	2.3	2.6	2.8	3.1	3.5	3.7	3.9	4.2	4.5	4.7	4.9	5.1	5.4	5.6	5.8	5.9	6.2
9	1	9.00	.9	1.5	1.8	2.1	2.5	2.8	2.9	3.3	3.6	3.8	3.9	4.3	4.5	4.7	4.9	5.1	5.4	5.6	5.7	5.9
19	2	9.50	.9	1.5	1.8	2.0	2.4	2.7	2.9	3.2	3.5	3.7	3.8	4.0	4.3	4.6	4.7	4.9	5.1	5.4	5.6	5.7
10	1	10.00	.9	1.4	1.8	1.9	2.4	2.6	2.8	3.0	3.3	3.6	3.8	3.9	4.1	4.4	4.6	4.8	4.9	5.1	5.3	5.5
11	1	11.00	.9	1.3	1.7	1.9	2.1	2.5	2.7	2.9	3.0	3.3	3.6	3.7	3.9	4.0	4.3	4.5	4.7	4.8	4.9	5.1
12	1	12.00	.8	1.2	1.6	1.8	1.9	2.3	2.6	2.7	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.2	4.4	4.6	4.7	4.8
13	1	13.00	.8	1.0	1.5	1.8	1.9	2.1	2.4	2.6	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.1	4.3	4.5	4.6
14	1	14.00	.8	.9	1.5	1.7	1.8	1.9	2.3	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.0	4.2	4.4
15	1	15.00	.8	.9	1.4	1.6	1.8	1.9	2.1	2.4	2.6	2.7	2.8	2.9	3.1	3.3	3.5	3.6	3.8	3.9	3.9	4.1
16	1	16.00	.8	.9	1.3	1.6	1.8	1.9	2.0	2.3	2.5	2.6	2.8	2.9	2.9	3.1	3.3	3.5	3.6	3.7	3.8	3.9
17	1	17.00	.7	.9	1.2	1.5	1.7	1.8	1.9	2.1	2.4	2.5	2.7	2.8	2.9	2.9	3.2	3.3	3.5	3.6	3.7	3.8
18	1	18.00	.7	.9	1.1	1.5	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.7	2.8	2.9	3.0	3.2	3.3	3.5	3.6	3.7
19	1	19.00	.7	.9	1.0	1.4	1.6	1.8	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.3	3.5	3.6
20	1	20.00	.7	.9	.9	1.3	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.7	2.8	2.8	2.9	3.0	3.2	3.3	3.5

95% PREDICTION INTERVAL - FOUR FUTURE OBSERVATIONS

			OBSERVED NUMBER OF COUNTS																			
L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1	1.00	36.1	37.4	38.8	40.1	41.4	42.7	44.0	45.3	46.6	47.8	49.1	50.4	51.7	52.9	54.2	55.5	56.7	57.9	59.2	60.5
6	5	1.20	30.8	31.9	33.0	34.1	35.2	36.3	37.4	38.5	39.6	40.7	41.7	42.8	43.9	44.9	46.0	47.1	48.1	49.2	50.2	51.3
5	4	1.25	29.7	30.8	31.8	32.9	34.0	35.0	36.1	37.1	38.2	39.2	40.3	41.3	42.3	43.4	44.4	45.4	46.4	47.4	48.4	49.5
4	3	1.33	28.1	29.1	30.1	31.1	32.1	33.1	34.1	35.1	36.1	37.0	38.0	39.0	40.0	40.9	41.9	42.9	43.8	44.8	45.7	46.7
7	5	1.40	26.9	27.9	28.8	29.8	30.8	31.7	32.7	33.6	34.6	35.5	36.4	37.4	38.3	39.2	40.1	41.0	42.0	42.9	43.8	44.7
3	2	1.50	25.4	26.3	27.2	28.1	29.0	29.9	30.8	31.7	32.6	33.4	34.3	35.2	36.0	36.9	37.8	38.6	39.5	40.4	41.2	42.0
8	5	1.60	24.0	24.8	25.7	26.6	27.4	28.3	29.1	29.9	30.8	31.6	32.4	33.2	34.1	34.9	35.7	36.5	37.3	38.1	38.9	39.7
5	3	1.67	23.2	24.0	24.8	25.7	26.5	27.3	28.1	28.9	29.7	30.5	31.3	32.1	32.9	33.7	34.5	35.2	36.0	36.8	37.6	38.3
7	4	1.75	22.2	23.0	23.8	24.6	25.4	26.2	26.9	27.7	28.5	29.2	30.0	30.8	31.5	32.3	33.0	33.8	34.5	35.2	36.0	36.7
9	5	1.80	21.7	22.5	23.3	24.0	24.8	25.6	26.3	27.0	27.8	28.5	29.3	30.0	30.7	31.5	32.2	32.9	33.7	34.4	35.1	35.8
2	1	2.00	19.8	20.6	21.3	21.9	22.7	23.4	24.0	24.7	25.4	26.1	26.7	27.4	28.1	28.7	29.4	30.1	30.7	31.4	32.0	32.7
11	5	2.20	18.4	19.0	19.6	20.3	20.9	21.6	22.2	22.8	23.4	24.0	24.7	25.3	25.9	26.5	27.1	27.7	28.3	28.9	29.5	30.1
9	4	2.25	18.0	18.6	19.3	19.9	20.5	21.1	21.8	22.4	22.9	23.6	24.2	24.8	25.4	26.0	26.6	27.2	27.8	28.4	28.9	29.5
7	3	2.33	17.5	18.1	18.7	19.3	19.9	20.5	21.1	21.7	22.3	22.9	23.5	24.0	24.6	25.2	25.8	26.4	26.9	27.5	28.0	28.6
12	5	2.40	17.0	17.7	18.3	18.8	19.5	20.0	20.6	21.2	21.8	22.3	22.9	23.5	24.0	24.6	25.2	25.7	26.3	26.8	27.4	27.9
5	2	2.50	16.5	17.1	17.7	18.2	18.8	19.4	19.9	20.5	21.0	21.6	22.1	22.7	23.2	23.8	24.3	24.8	25.4	25.9	26.5	27.0
13	5	2.60	15.9	16.6	17.1	17.7	18.2	18.7	19.3	19.8	20.4	20.9	21.4	21.9	22.5	23.0	23.5	24.0	24.6	25.1	25.6	26.1
8	3	2.67	15.7	16.2	16.7	17.3	17.8	18.4	18.9	19.4	19.9	20.5	20.9	21.5	22.0	22.5	23.0	23.5	24.0	24.6	25.0	25.6
14	5	2.80	15.0	15.6	16.1	16.6	17.1	17.6	18.1	18.6	19.1	19.6	20.1	20.6	21.1	21.6	22.1	22.6	23.1	23.6	24.0	24.5
3	1	3.00	14.2	14.7	15.2	15.7	16.2	16.7	17.1	17.6	18.1	18.6	19.0	19.5	19.9	20.4	20.8	21.3	21.8	22.2	22.7	23.1
16	5	3.20	13.5	13.9	14.4	14.9	15.4	15.8	16.3	16.7	17.1	17.6	18.0	18.5	18.9	19.4	19.8	20.2	20.6	21.0	21.5	21.9
10	3	3.33	13.0	13.5	13.9	14.4	14.8	15.3	15.7	16.1	16.6	17.0	17.5	17.8	18.3	18.7	19.1	19.5	19.9	20.4	20.8	21.2
17	5	3.40	12.8	13.3	13.7	14.2	14.6	15.0	15.5	15.9	16.3	16.7	17.1	17.6	17.9	18.4	18.8	19.2	19.6	20.0	20.5	20.8
7	2	3.50	12.6	13.0	13.4	13.8	14.3	14.7	15.1	15.5	15.9	16.4	16.8	17.2	17.6	17.9	18.4	18.8	19.2	19.6	19.9	20.4
18	5	3.60	12.3	12.7	13.1	13.6	13.9	14.4	14.8	15.2	15.6	15.9	16.4	16.8	17.2	17.6	17.9	18.3	18.7	19.1	19.5	19.9
11	3	3.67	12.1	12.5	12.9	13.4	13.7	14.1	14.6	14.9	15.4	15.7	16.1	16.5	16.9	17.3	17.7	18.0	18.5	18.8	19.2	19.6
15	4	3.75	11.9	12.3	12.7	13.1	13.5	13.9	14.3	14.7	15.0	15.5	15.8	16.2	16.6	16.9	17.4	17.7	18.1	18.5	18.8	19.2
19	5	3.80	11.8	12.2	12.6	12.9	13.4	13.8	14.1	14.5	14.9	15.3	15.7	16.0	16.4	16.8	17.2	17.6	17.9	18.3	18.7	19.0
4	1	4.00	11.3	11.7	12.0	12.5	12.8	13.2	13.6	13.9	14.3	14.7	15.0	15.4	15.8	16.1	16.5	16.8	17.2	17.5	17.9	18.2
17	4	4.25	10.8	11.1	11.5	11.8	12.2	12.6	12.9	13.3	13.6	13.9	14.3	14.7	15.0	15.4	15.7	16.0	16.4	16.7	17.0	17.4
13	3	4.33	10.6	10.9	11.4	11.7	12.0	12.4	12.7	13.1	13.4	13.8	14.1	14.5	14.8	15.1	15.5	15.8	16.1	16.4	16.7	17.1
9	2	4.50	10.3	10.7	11.0	11.4	11.7	12.0	12.4	12.7	13.0	13.4	13.7	14.0	14.3	14.7	14.9	15.3	15.6	15.9	16.3	16.6
14	3	4.67	10.0	10.4	10.7	11.0	11.4	11.7	12.0	12.4	12.7	12.9	13.3	13.6	13.9	14.2	14.6	14.8	15.2	15.5	15.8	16.1
5	1	5.00	9.5	9.8	10.1	10.5	10.8	11.0	11.4	11.7	11.9	12.3	12.6	12.9	13.2	13.5	13.8	14.0	14.4	14.7	14.9	15.2
16	3	5.33	9.0	9.4	9.7	9.9	10.2	10.6	10.8	11.1	11.4	11.7	11.9	12.3	12.5	12.8	13.1	13.4	13.6	13.9	14.2	14.5
11	2	5.50	8.8	9.1	9.4	9.7	9.9	10.3	10.6	10.8	11.1	11.4	11.7	11.9	12.2	12.5	12.8	13.0	13.3	13.6	13.8	14.1
6	1	6.00	8.3	8.6	8.8	9.1	9.4	9.6	9.9	10.1	10.4	10.7	10.9	11.2	11.4	11.7	11.9	12.2	12.4	12.7	12.9	13.2
19	3	6.33	7.9	8.2	8.5	8.7	8.9	9.2	9.5	9.7	9.9	10.2	10.5	10.7	10.9	11.2	11.5	11.7	11.9	12.1	12.4	12.6
13	2	6.50	7.8	8.0	8.3	8.6	8.8	9.0	9.3	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.2	11.5	11.7	11.9	12.1	12.4
20	3	6.67	7.7	7.9	8.1	8.4	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.3	10.6	10.8	10.9	11.2	11.5	11.7	11.9	12.1
7	1	7.00	7.4	7.6	7.8	8.1	8.3	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.1	10.4	10.6	10.8	11.0	11.2	11.5	11.7
15	2	7.50	6.9	7.2	7.5	7.7	7.9	8.1	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.8	11.0
8	1	8.00	6.7	6.9	7.1	7.3	7.6	7.7	7.9	8.1	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	9.9	10.1	10.4	10.5
17	2	8.50	6.4	6.6	6.8	6.9	7.2	7.4	7.6	7.8	7.9	8.2	8.4	8.6	8.8	8.9	9.1	9.3	9.5	9.7	9.8	10.0
9	1	9.00	6.1	6.4	6.6	6.7	6.9	7.1	7.3	7.5	7.7	7.8	8.0	8.2	8.4	8.6	8.7	8.9	9.1	9.3	9.5	9.6
19	2	9.50	5.9	6.0	6.3	6.5	6.7	6.8	6.9	7.2	7.4	7.6	7.7	7.9	8.0	8.2	8.4	8.6	8.7	8.9	9.0	9.2
10	1	10.00	5.7	5.8	6.0	6.2	6.4	6.6	6.8	6.9	7.0	7.3	7.5	7.6	7.8	7.9	8.0	8.3	8.4	8.6	8.7	8.9
11	1	11.00	5.3	5.5	5.7	5.8	5.9	6.1	6.3	6.5	6.6	6.8	6.9	7.0	7.2	7.4	7.6	7.7	7.8	7.9	8.1	8.3
12	1	12.00	4.9	5.1	5.3	5.5	5.6	5.8	5.9	6.0	6.2	6.4	6.5	6.7	6.8	6.9	7.0	7.2	7.4	7.5	7.6	7.8
13	1	13.00	4.7	4.8	4.9	5.1	5.3	5.5	5.6	5.7	5.8	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	7.0	7.2	7.3
14	1	14.00	4.5	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.5	6.6	6.7	6.8	6.9
15	1	15.00	4.3	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.3	6.4	6.5	6.6
16	1	16.00	4.0	4.2	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.8	5.9	6.0	6.2	6.3
17	1	17.00	3.9	4.0	4.2	4.3	4.4	4.6	4.7	4.8	4.8	4.9	5.0	5.2	5.3	5.4	5.6	5.6	5.7	5.8	5.9	6.0
18	1	18.00	3.8	3.9	3.9	4.1	4.2	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.0	5.2	5.3	5.4	5.5	5.6	5.7	5.8
19	1	19.00	3.7	3.8	3.8	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.0	5.2	5.3	5.4	5.5	5.6
20	1	20.00	3.6	3.7	3.8	3.8	3.9	3.9	4.1	4.3	4.4	4.5	4.6	4.7	4.7	4.8	4.9	4.9	5.1	5.2	5.3	5.4

99% PREDICTION INTERVAL - FOUR FUTURE OBSERVATIONS

L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	7.3	10.0	12.4	14.4	16.4	18.2	20.0	21.7	23.4	25.0	26.7	28.2	29.8	31.3	32.8	34.3	35.8	37.3	38.8	40.2
6	5	1.20	6.4	8.7	10.7	12.5	14.1	15.7	17.2	18.7	20.1	21.5	22.9	24.2	25.6	26.8	28.1	29.4	30.7	31.9	33.1	34.4
5	4	1.25	6.2	8.5	10.4	12.0	13.7	15.2	16.7	18.1	19.5	20.8	22.1	23.4	24.7	25.9	27.2	28.4	29.6	30.8	32.0	33.2
4	3	1.33	5.9	8.0	9.8	11.5	12.9	14.5	15.8	17.1	18.5	19.7	20.9	22.2	23.4	24.6	25.8	26.9	28.0	29.2	30.3	31.4
7	5	1.40	5.7	7.8	9.5	11.0	12.5	13.9	15.2	16.5	17.7	18.9	20.1	21.3	22.5	23.6	24.7	25.8	26.9	28.0	29.1	30.2
3	2	1.50	5.4	7.4	8.9	10.5	11.8	13.1	14.4	15.6	16.8	17.9	19.0	20.1	21.2	22.3	23.4	24.4	25.4	26.5	27.5	28.5
8	5	1.60	5.1	6.9	8.6	9.9	11.3	12.5	13.7	14.8	15.9	17.0	18.1	19.1	20.1	21.2	22.2	23.1	24.1	25.1	26.0	26.9
5	3	1.67	4.9	6.8	8.3	9.7	10.9	12.1	13.3	14.4	15.5	16.5	17.5	18.5	19.5	20.5	21.5	22.4	23.3	24.3	25.2	26.1
7	4	1.75	4.8	6.6	7.9	9.3	10.5	11.7	12.8	13.8	14.8	15.8	16.8	17.8	18.7	19.7	20.6	21.5	22.4	23.3	24.2	25.0
9	5	1.80	4.7	6.5	7.8	9.1	10.3	11.4	12.5	13.5	14.5	15.5	16.5	17.4	18.3	19.2	20.1	21.0	21.9	22.8	23.6	24.5
2	1	2.00	4.4	5.9	7.3	8.5	9.5	10.6	11.5	12.5	13.4	14.3	15.1	16.0	16.8	17.7	18.5	19.3	20.1	20.9	21.7	22.5
11	5	2.20	4.0	5.6	6.8	7.8	8.8	9.8	10.7	11.6	12.5	13.3	14.0	14.8	15.7	16.4	17.2	17.9	18.7	19.4	20.1	20.8
9	4	2.25	3.9	5.5	6.7	7.7	8.7	9.7	10.6	11.4	12.2	13.0	13.8	14.6	15.4	16.1	16.8	17.6	18.3	19.0	19.7	20.5
7	3	2.33	3.9	5.4	6.5	7.6	8.5	9.4	10.3	11.0	11.9	12.7	13.5	14.2	14.9	15.7	16.4	17.1	17.8	18.5	19.2	19.8
12	5	2.40	3.8	5.2	6.4	7.4	8.3	9.2	10.0	10.8	11.7	12.4	13.2	13.9	14.6	15.3	16.0	16.7	17.4	18.1	18.7	19.4
5	2	2.50	3.6	5.0	6.2	7.1	8.0	8.9	9.7	10.6	11.3	12.0	12.8	13.5	14.2	14.8	15.6	16.2	16.8	17.5	18.1	18.8
13	5	2.60	3.7	4.9	5.9	6.9	7.8	8.7	9.5	10.2	10.9	11.7	12.4	13.0	13.8	14.4	15.0	15.7	16.4	16.9	17.6	18.2
8	3	2.67	3.6	4.8	5.9	6.8	7.7	8.5	9.3	10.0	10.8	11.5	12.1	12.8	13.5	14.1	14.8	15.4	16.0	16.7	17.3	17.8
14	5	2.80	3.5	4.7	5.7	6.6	7.5	8.2	8.9	9.7	10.4	11.0	11.7	12.4	12.9	13.6	14.2	14.8	15.5	16.0	16.6	17.2
3	1	3.00	3.4	4.5	5.5	6.3	7.0	7.8	8.6	9.2	9.8	10.5	11.1	11.7	12.4	12.9	13.5	14.0	14.7	15.2	15.8	16.3
16	5	3.20	3.2	4.3	5.2	5.9	6.8	7.5	8.1	8.8	9.4	9.9	10.6	11.2	11.8	12.3	12.8	13.4	13.9	14.5	14.9	15.5
10	3	3.33	3.0	4.2	5.0	5.8	6.6	7.3	7.9	8.6	9.1	9.7	10.3	10.8	11.4	11.9	12.5	12.9	13.5	14.0	14.6	15.0
17	5	3.40	2.9	4.1	4.9	5.8	6.5	7.1	7.8	8.4	8.9	9.6	10.1	10.7	11.2	11.8	12.3	12.8	13.3	13.8	14.3	14.8
7	2	3.50	2.9	3.9	4.9	5.7	6.4	6.9	7.7	8.2	8.8	9.4	9.9	10.5	10.9	11.6	12.0	12.6	13.0	13.5	13.9	14.5
18	5	3.60	2.9	3.9	4.3	5.6	6.2	6.9	7.5	8.0	8.7	9.2	9.7	10.3	10.8	11.3	11.8	12.3	12.8	13.2	13.7	14.2
11	3	3.67	2.9	3.9	4.8	5.5	6.1	6.8	7.4	7.9	8.6	9.0	9.6	10.1	10.7	11.1	11.6	12.1	12.6	13.0	13.6	13.9
15	4	3.75	2.9	3.8	4.7	5.4	6.0	6.7	7.3	7.8	8.4	8.9	9.5	9.9	10.5	10.9	11.5	11.9	12.4	12.8	13.3	13.8
19	5	3.80	2.8	3.8	4.7	5.4	5.9	6.7	7.2	7.8	8.3	8.8	9.4	9.8	10.4	10.8	11.3	11.8	12.3	12.7	13.2	13.6
4	1	4.00	2.8	3.7	4.5	5.2	5.8	6.4	6.9	7.5	7.9	8.6	9.0	9.5	9.9	10.5	10.9	11.4	11.8	12.3	12.7	13.1
17	4	4.25	2.7	3.6	4.3	4.9	5.6	6.1	6.7	7.2	7.7	8.2	8.7	9.1	9.6	9.9	10.5	10.8	11.3	11.7	12.1	12.6
13	3	4.33	2.7	3.6	4.3	4.9	5.6	6.0	6.6	7.1	7.6	8.0	8.6	8.9	9.5	9.8	10.3	10.7	11.1	11.6	11.9	12.4
9	2	4.50	2.6	3.5	4.1	4.8	5.4	5.9	6.5	6.9	7.4	7.8	8.3	8.8	9.2	9.6	9.9	10.5	10.8	11.2	11.6	11.9
14	3	4.67	2.6	3.4	4.0	4.7	5.2	5.8	6.3	6.8	7.2	7.7	8.1	8.6	8.9	9.4	9.8	10.1	10.6	10.9	11.3	11.7
5	1	5.00	2.5	3.2	3.9	4.5	4.9	5.6	5.9	6.5	6.9	7.3	7.7	8.1	8.6	8.9	9.3	9.7	10.0	10.4	10.8	11.1
16	3	5.33	2.3	3.0	3.8	4.3	4.8	5.3	5.8	6.1	6.6	6.9	7.4	7.8	8.1	8.6	8.9	9.2	9.6	9.9	10.3	10.6
11	2	5.50	2.3	2.9	3.7	4.2	4.7	5.2	5.7	6.0	6.5	6.8	7.2	7.6	7.9	8.4	8.7	9.0	9.4	9.7	10.0	10.4
6	1	6.00	2.1	2.9	3.5	3.9	4.5	4.9	5.3	5.7	6.0	6.5	6.8	7.2	7.6	7.8	8.2	8.6	8.8	9.1	9.5	9.8
19	3	6.33	1.9	2.8	3.4	3.8	4.3	4.8	5.1	5.6	5.9	6.2	6.6	6.9	7.3	7.6	7.9	8.2	8.5	8.8	9.1	9.4
13	2	6.50	1.9	2.8	3.3	3.8	4.2	4.7	4.9	5.5	5.8	6.1	6.5	6.8	7.1	7.5	7.8	8.0	8.4	8.7	8.9	9.2
20	3	6.67	1.9	2.8	3.3	3.8	4.1	4.6	4.9	5.4	5.7	5.9	6.4	6.7	6.9	7.3	7.7	7.9	8.2	8.5	8.8	9.0
7	1	7.00	1.9	2.7	3.1	3.7	3.9	4.5	4.8	5.1	5.6	5.8	6.1	6.5	6.8	7.0	7.4	7.7	7.9	8.2	8.5	8.8
15	2	7.50	1.9	2.6	2.9	3.5	3.9	4.3	4.7	4.9	5.3	5.6	5.9	6.2	6.5	6.8	6.9	7.3	7.6	7.8	8.1	8.4
8	1	8.00	1.8	2.5	2.9	3.4	3.8	4.0	4.5	4.8	5.0	5.4	5.7	5.9	6.2	6.5	6.8	6.9	7.3	7.6	7.8	7.9
17	2	8.50	1.8	2.4	2.8	3.2	3.7	3.9	4.3	4.6	4.8	5.1	5.5	5.7	5.9	6.2	6.5	6.7	6.9	7.2	7.5	7.7
9	1	9.00	1.8	2.3	2.8	3.0	3.5	3.8	4.0	4.5	4.7	4.9	5.2	5.6	5.6	5.9	6.3	6.5	6.7	6.9	7.2	7.4
19	2	9.50	1.7	2.2	2.7	2.9	3.4	3.7	3.9	4.3	4.6	4.8	5.0	5.3	5.6	5.8	5.9	6.3	6.5	6.7	6.9	7.1
10	1	10.00	1.7	2.0	2.6	2.9	3.3	3.6	3.8	4.1	4.5	4.7	4.9	5.1	5.4	5.7	5.8	6.0	6.3	6.5	6.7	6.9
11	1	11.00	1.6	1.9	2.5	2.8	3.0	3.4	3.7	3.9	4.1	4.5	4.7	4.8	5.0	5.3	5.6	5.7	5.9	6.0	6.3	6.5
12	1	12.00	1.6	1.9	2.4	2.7	2.9	3.2	3.5	3.8	3.9	4.1	4.4	4.7	4.8	4.9	5.2	5.4	5.6	5.8	5.9	6.1
13	1	13.00	1.5	1.9	2.2	2.6	2.8	2.9	3.4	3.6	3.8	3.9	4.2	4.4	4.6	4.8	4.9	5.1	5.3	5.5	5.7	5.8
14	1	14.00	1.4	1.8	2.0	2.5	2.8	2.9	3.1	3.5	3.7	3.8	3.9	4.2	4.4	4.6	4.8	4.9	5.0	5.2	5.5	5.6
15	1	15.00	1.3	1.8	1.9	2.4	2.7	2.8	2.9	3.3	3.5	3.7	3.8	3.9	4.2	4.4	4.6	4.7	4.8	4.9	5.2	5.4
16	1	16.00	1.2	1.8	1.9	2.3	2.6	2.8	2.9	3.1	3.4	3.6	3.7	3.9	3.9	4.2	4.4	4.6	4.7	4.8	4.9	5.1
17	1	17.00	1.1	1.7	1.9	2.1	2.5	2.7	2.8	2.9	3.2	3.5	3.6	3.8	3.9	3.9	4.2	4.4	4.6	4.7	4.8	4.9
18	1	18.00	1.0	1.7	1.9	2.0	2.4	2.7	2.8	2.9	3.0	3.3	3.5	3.7	3.8	3.9	3.9	4.2	4.4	4.6	4.7	4.8
19	1	19.00	.9	1.6	1.8	1.9	2.3	2.6	2.7	2.8	2.9	3.2	3.4	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.6	4.7
20	1	20.00	.9	1.5	1.8	1.9	2.2	2.5	2.7	2.8	2.9	3.0	3.3	3.5	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.5

99% PREDICTION INTERVAL - FOUR FUTURE OBSERVATIONS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	41.6	43.0	44.5	45.9	47.3	48.7	50.0	51.4	52.8	54.1	55.5	56.8	58.2	59.5	60.9	62.2	63.5	64.8	66.2	67.5
6	5	1.20	35.6	36.8	37.9	39.2	40.4	41.5	42.7	43.8	45.0	46.2	47.3	48.5	49.6	50.7	51.8	52.9	54.1	55.2	56.3	57.4
5	4	1.25	34.4	35.5	36.7	37.8	38.9	40.1	41.2	42.3	43.5	44.6	45.7	46.7	47.8	48.9	50.0	51.1	52.2	53.3	54.3	55.4
4	3	1.33	32.5	33.6	34.7	35.8	36.8	37.9	39.0	40.0	41.1	42.1	43.2	44.2	45.2	46.3	47.3	48.3	49.3	50.3	51.3	52.3
7	5	1.40	31.2	32.3	33.3	34.3	35.4	36.4	37.4	38.4	39.4	40.4	41.4	42.4	43.4	44.4	45.3	46.3	47.3	48.2	49.2	50.2
3	2	1.50	29.5	30.5	31.4	32.4	33.4	34.3	35.3	36.2	37.1	38.1	39.0	39.9	40.9	41.8	42.7	43.6	44.5	45.5	46.4	47.3
8	5	1.60	27.9	28.8	29.8	30.7	31.6	32.5	33.4	34.3	35.2	36.0	36.9	37.8	38.7	39.6	40.4	41.3	42.1	43.0	43.8	44.7
5	3	1.67	26.9	27.9	28.5	29.7	30.5	31.4	32.3	33.1	33.9	34.8	35.7	36.5	37.4	38.2	39.0	39.8	40.7	41.5	42.3	43.1
7	4	1.75	25.9	26.8	27.6	28.5	29.3	30.1	30.9	31.8	32.6	33.4	34.2	35.0	35.8	36.7	37.5	38.2	39.0	39.8	40.6	41.4
9	5	1.80	25.3	26.2	26.9	27.8	28.6	29.5	30.3	31.0	31.8	32.7	33.5	34.2	35.0	35.8	36.6	37.4	38.1	38.9	39.7	40.4
2	1	2.00	23.2	24.0	24.7	25.5	26.3	26.9	27.7	28.5	29.2	29.9	30.6	31.3	32.0	32.8	33.5	34.2	34.9	35.6	36.3	36.9
11	5	2.20	21.5	22.2	22.9	23.6	24.3	24.9	25.6	26.3	26.9	27.7	28.3	28.9	29.6	30.3	30.9	31.6	32.2	32.8	33.5	34.1
9	4	2.25	21.1	21.8	22.5	23.2	23.8	24.5	25.2	25.8	26.5	27.1	27.8	28.4	29.0	29.7	30.4	30.9	31.6	32.2	32.8	33.5
7	3	2.33	20.5	21.2	21.8	22.5	23.1	23.8	24.5	25.1	25.7	26.4	26.9	27.6	28.2	28.8	29.5	30.0	30.7	31.3	31.9	32.5
12	5	2.40	20.0	20.7	21.4	22.0	22.6	23.3	23.9	24.5	25.1	25.7	26.4	26.9	27.6	28.2	28.8	29.4	29.9	30.6	31.1	31.7
5	2	2.50	19.4	20.0	20.7	21.3	21.9	22.5	23.1	23.7	24.3	24.9	25.5	26.1	26.7	27.3	27.8	28.4	28.9	29.6	30.1	30.7
13	5	2.60	18.8	19.5	20.0	20.6	21.2	21.8	22.4	22.9	23.6	24.1	24.7	25.3	25.8	26.4	26.9	27.5	28.0	28.6	29.2	29.7
8	3	2.67	18.5	19.0	19.7	20.2	20.8	21.4	21.9	22.5	23.1	23.7	24.2	24.8	25.3	25.8	26.4	26.9	27.5	28.0	28.6	29.1
14	5	2.80	17.8	18.4	18.9	19.5	20.0	20.6	21.1	21.7	22.2	22.7	23.3	23.8	24.3	24.8	25.4	25.9	26.4	26.9	27.5	27.9
3	1	3.00	16.8	17.4	17.9	18.5	18.9	19.5	19.9	20.5	21.0	21.5	22.0	22.5	23.0	23.5	24.0	24.5	24.9	25.5	25.9	26.5
16	5	3.20	16.0	16.6	17.0	17.6	18.0	18.5	19.0	19.5	19.9	20.5	20.9	21.4	21.8	22.3	22.8	23.3	23.7	24.2	24.7	25.1
10	3	3.33	15.6	16.0	16.5	16.9	17.5	17.9	18.4	18.9	19.4	19.8	20.3	20.7	21.2	21.6	22.0	22.5	22.9	23.4	23.8	24.3
17	5	3.40	15.3	15.8	16.3	16.7	17.2	17.7	18.1	18.6	19.0	19.5	19.9	20.4	20.8	21.3	21.7	22.2	22.6	23.0	23.5	23.9
7	2	3.50	14.9	15.5	15.9	16.4	16.8	17.3	17.7	18.2	18.6	19.0	19.5	19.9	20.4	20.8	21.2	21.7	22.1	22.5	22.9	23.4
18	5	3.60	14.7	15.1	15.6	16.0	16.5	16.9	17.4	17.8	18.2	18.7	19.0	19.5	19.9	20.4	20.8	21.2	21.6	22.0	22.5	22.8
11	3	3.67	14.5	14.9	15.4	15.8	16.2	16.7	17.1	17.6	17.9	18.4	18.8	19.2	19.7	20.0	20.5	20.9	21.3	21.7	22.1	22.5
15	4	3.75	14.2	14.7	15.1	15.5	15.9	16.4	16.8	17.2	17.7	18.0	18.5	18.9	19.3	19.7	20.1	20.5	20.9	21.3	21.7	22.1
19	5	3.80	14.0	14.5	14.9	15.4	15.8	16.2	16.7	17.0	17.5	17.9	18.3	18.7	19.1	19.5	19.9	20.3	20.7	21.1	21.5	21.9
4	1	4.00	13.6	13.9	14.4	14.8	15.2	15.6	15.9	16.4	16.8	17.2	17.6	17.9	18.4	18.7	19.1	19.5	19.9	20.3	20.7	21.0
17	4	4.25	12.9	13.4	13.7	14.1	14.5	14.9	15.3	15.7	16.0	16.4	16.8	17.1	17.5	17.9	18.3	18.6	18.9	19.4	19.7	20.0
13	3	4.33	12.8	13.1	13.6	13.9	14.3	14.7	15.0	15.5	15.8	16.2	16.6	16.9	17.3	17.6	17.9	18.4	18.7	19.0	19.4	19.7
9	2	4.50	12.4	12.8	13.1	13.6	13.9	14.3	14.7	14.9	15.4	15.7	16.0	16.4	16.8	17.1	17.5	17.8	18.1	18.5	18.8	19.2
14	3	4.67	12.0	12.5	12.8	13.2	13.6	13.9	14.2	14.6	14.9	15.3	15.6	15.9	16.3	16.7	16.9	17.3	17.7	17.9	18.3	18.7
5	1	5.00	11.5	11.8	12.2	12.5	12.8	13.2	13.6	13.8	14.2	14.5	14.8	15.2	15.5	15.8	16.1	16.5	16.8	17.0	17.4	17.7
16	3	5.33	10.9	11.3	11.6	11.9	12.3	12.6	12.9	13.2	13.6	13.8	14.1	14.5	14.8	15.0	15.4	15.7	15.9	16.3	16.6	16.8
11	2	5.50	10.7	11.0	11.4	11.7	11.9	12.3	12.6	12.9	13.2	13.6	13.8	14.1	14.4	14.7	14.9	15.3	15.6	15.9	16.2	16.5
6	1	6.00	10.0	10.4	10.7	10.9	11.3	11.6	11.8	12.1	12.4	12.7	12.9	13.3	13.6	13.8	14.0	14.4	14.6	14.9	15.1	15.4
19	3	6.33	9.7	9.9	10.3	10.6	10.8	11.1	11.4	11.7	11.9	12.2	12.5	12.7	12.9	13.3	13.5	13.8	14.0	14.3	14.6	14.8
13	2	6.50	9.6	9.8	10.0	10.4	10.7	10.9	11.2	11.5	11.7	11.9	12.2	12.5	12.8	12.9	13.3	13.5	13.8	14.0	14.3	14.5
20	3	6.67	9.4	9.7	9.9	10.2	10.5	10.7	10.9	11.2	11.5	11.8	11.9	12.3	12.6	12.8	13.0	13.3	13.5	13.8	13.9	14.3
7	1	7.00	9.0	9.3	9.6	9.8	10.1	10.4	10.6	10.8	11.1	11.4	11.6	11.8	12.0	12.4	12.6	12.8	13.0	13.3	13.5	13.8
15	2	7.50	8.7	8.9	9.1	9.4	9.6	9.8	10.1	10.4	10.6	10.8	11.0	11.3	11.5	11.7	11.9	12.2	12.4	12.7	12.8	13.0
8	1	8.00	8.3	8.5	8.7	8.9	9.2	9.5	9.7	9.9	10.1	10.4	10.6	10.8	10.9	11.2	11.5	11.7	11.8	12.0	12.3	12.5
17	2	8.50	7.9	8.1	8.4	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.1	10.3	10.6	10.7	10.9	11.1	11.4	11.6	11.8	11.9
9	1	9.00	7.7	7.8	8.0	8.3	8.5	8.7	8.9	9.1	9.3	9.6	9.7	9.9	10.1	10.3	10.5	10.7	10.9	11.0	11.3	11.5
19	2	9.50	7.4	7.6	7.8	7.9	8.2	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	9.9	10.1	10.3	10.5	10.7	10.8	11.0
10	1	10.00	7.1	7.3	7.6	7.7	7.9	8.0	8.3	8.5	8.7	8.8	9.0	9.2	9.4	9.6	9.8	9.9	10.1	10.3	10.5	10.7
11	1	11.00	6.7	6.8	7.0	7.2	7.5	7.6	7.8	7.9	8.1	8.3	8.5	8.7	8.8	8.9	9.1	9.3	9.5	9.7	9.8	9.9
12	1	12.00	6.3	6.5	6.7	6.8	6.9	7.2	7.4	7.6	7.7	7.8	7.9	8.1	8.3	8.5	8.7	8.8	8.9	9.0	9.2	9.4
13	1	13.00	5.9	6.1	6.4	6.5	6.7	6.8	6.9	7.1	7.3	7.5	7.6	7.7	7.8	7.9	8.2	8.3	8.5	8.6	8.8	8.9
14	1	14.00	5.7	5.9	5.9	6.2	6.4	6.5	6.7	6.8	6.9	7.0	7.2	7.4	7.5	7.7	7.8	7.9	8.0	8.2	8.3	8.5
15	1	15.00	5.5	5.7	5.8	5.9	6.0	6.2	6.4	6.5	6.7	6.8	6.9	6.9	7.1	7.3	7.5	7.6	7.7	7.9	7.9	8.0
16	1	16.00	5.3	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.4	6.5	6.7	6.8	6.8	6.9	7.1	7.3	7.4	7.5	7.7	7.8
17	1	17.00	5.0	5.2	5.4	5.5	5.7	5.8	5.8	5.9	6.1	6.3	6.4	6.5	6.7	6.8	6.8	6.9	7.0	7.2	7.4	7.5
18	1	18.00	4.9	4.9	5.1	5.3	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1
19	1	19.00	4.8	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.3	6.4	6.5	6.6	6.7	6.8	6.9
20	1	20.00	4.7	4.8	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.8	5.9	6.0	6.2	6.3	6.4	6.5	6.6	6.7

90% PREDICTION INTERVAL - FIVE FUTURE OBSERVATIONS

L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
									OBSERVED NUMBER OF COUNTS													
1	1	1.00	3.9	6.2	8.1	9.9	11.5	13.1	14.7	16.2	17.7	19.1	20.5	21.9	23.3	24.7	26.0	27.4	28.7	30.0	31.4	32.7
6	5	1.20	3.4	5.4	7.0	8.5	9.9	11.3	12.6	13.9	15.1	16.4	17.6	18.7	19.9	21.1	22.2	23.4	24.5	25.6	26.7	27.8
5	4	1.25	3.3	5.2	6.8	8.2	9.6	10.9	12.2	13.4	14.6	15.8	16.9	18.1	19.2	20.4	21.5	22.6	23.7	24.7	25.8	26.9
4	3	1.33	3.1	4.9	6.4	7.8	9.1	10.3	11.5	12.7	13.8	14.9	16.0	17.1	18.2	19.3	20.3	21.3	22.4	23.4	24.4	25.4
7	5	1.40	3.0	4.7	6.2	7.5	8.7	9.9	11.1	12.2	13.3	14.4	15.4	16.4	17.5	18.5	19.5	20.5	21.5	22.4	23.4	24.4
3	2	1.50	2.9	4.5	5.8	7.1	8.3	9.4	10.5	11.5	12.5	13.6	14.5	15.5	16.5	17.4	18.4	19.3	20.2	21.1	22.0	22.9
8	5	1.60	2.7	4.3	5.6	6.7	7.8	8.9	9.9	10.9	11.9	12.8	13.8	14.7	15.6	16.5	17.4	18.3	19.1	20.0	20.8	21.7
5	3	1.67	2.7	4.1	5.4	6.5	7.6	8.6	9.6	10.6	11.5	12.4	13.3	14.2	15.1	15.9	16.8	17.6	18.5	19.3	20.1	20.9
7	4	1.75	2.6	3.9	5.2	6.3	7.3	8.3	9.2	10.1	11.0	11.9	12.8	13.6	14.5	15.3	16.1	16.9	17.7	18.5	19.3	20.1
9	5	1.80	2.5	3.9	5.0	6.1	7.1	8.1	9.0	9.9	10.8	11.7	12.5	13.3	14.1	14.9	15.7	16.5	17.3	18.1	18.8	19.6
2	1	2.00	2.3	3.6	4.7	5.7	6.6	7.5	8.3	9.1	9.9	10.7	11.5	12.2	12.9	13.7	14.4	15.1	15.8	16.6	17.3	17.9
11	5	2.20	2.1	3.4	4.4	5.2	6.1	6.9	7.7	8.4	9.2	9.9	10.6	11.3	11.9	12.7	13.3	13.9	14.6	15.3	15.9	16.6
9	4	2.25	2.1	3.3	4.3	5.1	5.9	6.8	7.6	8.3	9.0	9.7	10.4	11.1	11.8	12.4	13.1	13.7	14.4	15.0	15.6	16.3
7	3	2.33	2.0	3.2	4.1	4.9	5.8	6.6	7.3	8.0	8.7	9.4	10.1	10.8	11.4	12.0	12.7	13.3	13.9	14.6	15.2	15.8
12	5	2.40	1.9	3.1	4.0	4.9	5.7	6.5	7.2	7.8	8.6	9.2	9.9	10.5	11.1	11.8	12.4	13.0	13.6	14.2	14.8	15.4
5	2	2.50	1.9	3.0	3.9	4.7	5.5	6.2	6.9	7.6	8.3	8.8	9.6	10.2	10.8	11.4	11.9	12.6	13.2	13.7	14.3	14.9
13	5	2.60	1.9	2.9	3.8	4.6	5.4	6.0	6.7	7.4	8.0	8.7	9.3	9.8	10.5	11.0	11.6	12.2	12.7	13.3	13.8	14.4
8	3	2.67	1.8	2.9	3.7	4.5	5.2	5.9	6.6	7.2	7.8	8.5	9.0	9.7	10.2	10.8	11.4	11.9	12.5	13.0	13.6	14.1
14	5	2.80	1.8	2.8	3.6	4.4	5.0	5.7	6.4	6.9	7.6	8.1	8.7	9.3	9.8	10.4	10.9	11.5	12.0	12.5	13.0	13.6
3	1	3.00	1.7	2.7	3.5	4.1	4.8	5.4	6.0	6.6	7.2	7.7	8.3	8.8	9.3	9.8	10.4	10.8	11.4	11.8	12.3	12.8
16	5	3.20	1.6	2.6	3.3	3.9	4.6	5.1	5.7	6.3	6.8	7.4	7.8	8.4	8.8	9.4	9.8	10.3	10.8	11.2	11.7	12.1
10	3	3.33	1.6	2.5	3.1	3.8	4.4	4.9	5.6	6.0	6.6	7.1	7.6	8.1	8.6	9.0	9.5	9.9	10.4	10.9	11.3	11.8
17	5	3.40	1.6	2.4	3.1	3.8	4.4	4.9	5.5	5.9	6.5	6.9	7.5	7.9	8.5	8.9	9.4	9.8	10.3	10.7	11.1	11.6
7	2	3.50	1.6	2.4	3.0	3.7	4.3	4.8	5.4	5.8	6.4	6.8	7.3	7.8	8.2	8.7	9.1	9.6	10.0	10.5	10.9	11.3
18	5	3.60	1.5	2.3	2.9	3.6	4.2	4.7	5.2	5.7	6.2	6.7	7.1	7.6	8.0	8.5	8.9	9.4	9.8	10.2	10.6	11.0
11	3	3.67	1.5	2.3	2.9	3.6	4.1	4.7	5.1	5.7	6.1	6.6	7.0	7.5	7.9	8.4	8.8	9.2	9.7	10.0	10.5	10.9
15	4	3.75	1.5	2.2	2.9	3.5	4.0	4.6	5.0	5.6	6.0	6.5	6.9	7.4	7.8	8.2	8.7	9.0	9.5	9.9	10.3	10.7
19	5	3.80	1.5	2.2	2.8	3.5	3.9	4.6	5.0	5.5	5.9	6.4	6.8	7.3	7.7	8.1	8.6	8.9	9.4	9.8	10.2	10.6
4	1	4.00	1.4	2.1	2.6	3.4	3.8	4.4	4.8	5.3	5.7	6.2	6.6	7.0	7.4	7.8	8.2	8.6	9.0	9.4	9.8	10.1
17	4	4.25	1.3	1.9	2.7	3.2	3.7	4.1	4.6	5.0	5.5	5.9	6.3	6.7	7.1	7.5	7.8	8.2	8.6	8.9	9.3	9.7
13	3	4.33	1.3	1.9	2.6	3.1	3.7	4.1	4.6	4.9	5.4	5.8	6.2	6.6	6.9	7.4	7.7	8.1	8.5	8.8	9.2	9.5
9	2	4.50	1.2	1.9	2.6	3.0	3.6	3.9	4.5	4.8	5.3	5.7	6.0	6.4	6.8	7.1	7.5	7.8	8.2	8.6	8.9	9.3
14	3	4.67	1.2	1.9	2.5	2.9	3.5	3.9	4.3	4.7	5.1	5.5	5.8	6.2	6.6	6.9	7.3	7.7	7.9	8.3	8.7	8.9
5	1	5.00	1.1	1.8	2.4	2.8	3.3	3.7	4.0	4.5	4.8	5.2	5.6	5.9	6.3	6.6	6.9	7.3	7.6	7.9	8.2	8.5
16	3	5.33	1.0	1.7	2.3	2.7	3.1	3.6	3.9	4.3	4.6	4.9	5.3	5.6	5.9	6.3	6.6	6.9	7.2	7.5	7.8	8.1
11	2	5.50	.9	1.7	2.2	2.7	3.0	3.5	3.8	4.2	4.5	4.8	5.2	5.5	5.8	6.1	6.5	6.7	7.0	7.3	7.6	7.9
6	1	6.00	.9	1.6	2.0	2.5	2.8	3.2	3.6	3.9	4.2	4.6	4.8	5.1	5.5	5.7	6.0	6.3	6.6	6.8	7.1	7.4
19	3	6.33	.9	1.6	1.9	2.4	2.8	3.1	3.5	3.8	4.0	4.4	4.7	4.9	5.2	5.5	5.8	6.0	6.3	6.6	6.8	7.1
13	2	6.50	.9	1.6	1.9	2.4	2.7	3.0	3.4	3.7	3.9	4.3	4.6	4.8	5.1	5.4	5.7	5.9	6.2	6.5	6.7	6.9
20	3	6.67	.9	1.5	1.9	2.3	2.7	2.9	3.3	3.6	3.9	4.2	4.5	4.8	5.0	5.3	5.6	5.8	6.0	6.3	6.6	6.8
7	1	7.00	.9	1.5	1.8	2.2	2.6	2.9	3.2	3.5	3.8	4.0	4.4	4.6	4.8	5.1	5.4	5.6	5.8	6.1	6.4	6.6
15	2	7.50	.8	1.4	1.8	2.1	2.5	2.8	3.0	3.4	3.6	3.8	4.1	4.4	4.6	4.8	5.1	5.4	5.6	5.8	5.9	6.2
8	1	8.00	.8	1.3	1.7	1.9	2.4	2.7	2.9	3.2	3.5	3.7	3.9	4.2	4.4	4.7	4.8	5.0	5.3	5.5	5.7	5.9
17	2	8.50	.8	1.2	1.7	1.9	2.2	2.6	2.8	2.9	3.3	3.6	3.8	3.9	4.2	4.5	4.7	4.8	5.0	5.3	5.5	5.7
9	1	9.00	.8	1.2	1.6	1.8	2.1	2.5	2.7	2.9	3.1	3.4	3.6	3.8	4.0	4.3	4.5	4.7	4.8	5.0	5.3	5.5
19	2	9.50	.8	1.1	1.6	1.8	2.0	2.4	2.6	2.8	2.9	3.3	3.5	3.7	3.9	4.0	4.3	4.5	4.7	4.8	5.0	5.2
10	1	10.00	.7	1.0	1.5	1.8	1.9	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.8	3.9	4.1	4.3	4.5	4.7	4.8	5.0
11	1	11.00	.7	.9	1.4	1.7	1.8	2.0	2.4	2.6	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.2	4.4	4.6	4.7
12	1	12.00	.7	.9	1.3	1.6	1.8	1.9	2.2	2.4	2.6	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.1	4.3	4.4
13	1	13.00	.7	.9	1.2	1.5	1.7	1.8	2.0	2.3	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.6	3.7	3.8	3.9	4.1
14	1	14.00	.6	.8	1.1	1.4	1.6	1.8	1.9	2.1	2.4	2.5	2.7	2.8	2.9	3.0	3.3	3.4	3.6	3.7	3.8	3.9
15	1	15.00	.6	.8	.9	1.3	1.6	1.7	1.8	1.9	2.2	2.4	2.6	2.7	2.8	2.9	3.0	3.2	3.4	3.5	3.7	3.8
16	1	16.00	.6	.8	.9	1.3	1.5	1.7	1.8	1.9	2.0	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.4	3.5	3.6
17	1	17.00	.5	.8	.9	1.2	1.5	1.6	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.4	3.5
18	1	18.00	.5	.8	.9	1.1	1.4	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.3
19	1	19.00	.5	.8	.9	1.0	1.3	1.5	1.7	1.8	1.8	1.9	2.1	2.3	2.4	2.6	2.7	2.8	2.8	2.9	3.0	3.2
20	1	20.00	.5	.7	.9	.9	1.2	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.3	2.5	2.6	2.7	2.8	2.8	2.9	3.0

90% PREDICTION INTERVAL - FIVE FUTURE OBSERVATIONS

L	M	L/M	OBSERVED NUMBER OF COUNTS 21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1	1.00	34.0	35.3	36.6	37.8	39.1	40.4	41.7	42.9	44.2	45.4	46.7	47.9	49.2	50.4	51.6	52.8	54.1	55.3	56.5	57.7
6	5	1.20	28.9	30.0	31.1	32.2	33.3	34.4	35.4	36.5	37.5	38.6	39.6	40.7	41.7	42.8	43.8	44.8	45.9	46.9	47.9	48.9
5	4	1.25	27.9	29.0	30.0	31.1	32.1	33.1	34.2	35.2	36.2	37.2	38.2	39.2	40.2	41.2	42.2	43.2	44.2	45.2	46.2	47.2
4	3	1.33	26.4	27.4	28.4	29.4	30.3	31.3	32.3	33.2	34.2	35.1	36.1	37.0	38.0	38.9	39.9	40.8	41.8	42.7	43.6	44.5
7	5	1.40	25.3	26.3	27.2	28.1	29.1	30.0	30.9	31.8	32.8	33.7	34.6	35.5	36.4	37.3	38.2	39.1	40.0	40.9	41.8	42.6
3	2	1.50	23.8	24.7	25.6	26.5	27.4	28.2	29.1	29.9	30.8	31.7	32.5	33.4	34.2	35.1	35.9	36.8	37.6	38.4	39.3	40.1
8	5	1.60	22.6	23.4	24.2	25.0	25.9	26.7	27.5	28.3	29.1	29.9	30.7	31.6	32.4	33.1	33.9	34.7	35.5	36.3	37.1	37.9
5	3	1.67	21.6	22.6	23.4	24.2	24.9	25.8	26.6	27.3	28.1	28.9	29.7	30.5	31.2	32.0	32.7	33.5	34.3	35.0	35.8	36.5
7	4	1.75	20.9	21.7	22.4	23.2	23.9	24.7	25.5	26.2	26.9	27.7	28.4	29.2	29.9	30.6	31.4	32.1	32.8	33.6	34.3	35.0
9	5	1.80	20.4	21.1	21.9	22.6	23.4	24.1	24.8	25.6	26.3	27.0	27.7	28.5	29.2	29.9	30.6	31.3	32.0	32.7	33.4	34.1
2	1	2.00	18.6	19.3	20.0	20.7	21.4	22.0	22.7	23.4	24.0	24.7	25.3	26.0	26.6	27.3	27.9	28.6	29.2	29.8	30.5	31.1
11	5	2.20	17.2	17.9	18.5	19.1	19.7	20.3	20.9	21.5	22.1	22.7	23.3	23.9	24.5	25.1	25.7	26.3	26.9	27.5	28.1	28.7
9	4	2.25	16.9	17.5	18.1	18.7	19.3	19.9	20.5	21.1	21.7	22.3	22.9	23.5	24.0	24.6	25.2	25.8	26.4	26.9	27.5	28.1
7	3	2.33	16.4	16.9	17.6	18.1	18.7	19.3	19.9	20.5	21.0	21.6	22.2	22.8	23.3	23.9	24.5	25.0	25.6	26.1	26.7	27.2
12	5	2.40	16.0	16.6	17.1	17.7	18.3	18.8	19.4	20.0	20.6	21.1	21.7	22.2	22.8	23.3	23.9	24.4	24.9	25.5	26.0	26.6
5	2	2.50	15.5	16.0	16.6	17.1	17.7	18.2	18.8	19.3	19.8	20.4	20.9	21.5	22.0	22.5	23.0	23.6	24.1	24.6	25.1	25.7
13	5	2.60	14.9	15.5	16.0	16.6	17.1	17.6	18.2	18.7	19.2	19.7	20.2	20.8	21.3	21.8	22.3	22.8	23.3	23.8	24.3	24.8
8	3	2.67	14.7	15.2	15.7	16.2	16.7	17.3	17.8	18.3	18.8	19.3	19.8	20.3	20.8	21.3	21.8	22.3	22.8	23.3	23.8	24.3
14	5	2.80	14.0	14.6	15.1	15.6	16.1	16.6	17.1	17.6	18.0	18.5	19.0	19.5	19.9	20.5	20.9	21.4	21.9	22.4	22.8	23.3
3	1	3.00	13.3	13.8	14.3	14.7	15.2	15.7	16.1	16.6	17.0	17.5	17.9	18.4	18.8	19.3	19.7	20.2	20.6	21.1	21.5	21.9
16	5	3.20	12.6	13.0	13.5	13.9	14.4	14.8	15.3	15.7	16.1	16.6	17.0	17.4	17.8	18.3	18.7	19.1	19.6	19.9	20.4	20.8
10	3	3.33	12.2	12.6	13.0	13.5	13.9	14.4	14.8	15.2	15.6	16.0	16.5	16.8	17.3	17.7	18.1	18.5	18.9	19.3	19.7	20.1
17	5	3.40	12.0	12.4	12.8	13.3	13.7	14.1	14.5	14.9	15.4	15.8	16.2	16.6	16.9	17.4	17.8	18.2	18.6	18.9	19.4	19.8
7	2	3.50	11.7	12.1	12.6	12.9	13.4	13.8	14.2	14.6	15.0	15.4	15.8	16.2	16.6	16.9	17.4	17.7	18.1	18.5	18.9	19.3
18	5	3.60	11.5	11.8	12.3	12.7	13.1	13.5	13.8	14.3	14.7	15.0	15.4	15.8	16.2	16.6	16.9	17.3	17.7	18.1	18.5	18.8
11	3	3.67	11.3	11.7	12.1	12.5	12.9	13.3	13.7	14.0	14.4	14.8	15.2	15.6	15.9	16.3	16.7	17.0	17.5	17.8	18.2	18.6
15	4	3.75	11.1	11.5	11.9	12.3	12.7	13.0	13.4	13.8	14.2	14.6	14.9	15.3	15.7	16.0	16.4	16.7	17.1	17.5	17.8	18.2
19	5	3.80	10.9	11.4	11.7	12.1	12.5	12.9	13.3	13.6	14.0	14.4	14.7	15.1	15.5	15.8	16.2	16.6	16.9	17.3	17.6	18.0
4	1	4.00	10.5	10.9	11.3	11.6	12.0	12.4	12.7	13.1	13.5	13.8	14.1	14.5	14.8	15.2	15.5	15.9	16.2	16.6	16.9	17.3
17	4	4.25	10.0	10.4	10.7	11.1	11.4	11.8	12.1	12.5	12.8	13.1	13.5	13.8	14.1	14.5	14.8	15.1	15.4	15.8	16.1	16.4
13	3	4.33	9.9	10.2	10.6	10.9	11.3	11.6	11.9	12.3	12.6	12.9	13.3	13.6	13.9	14.2	14.6	14.9	15.2	15.5	15.8	16.1
9	2	4.50	9.5	9.9	10.3	10.6	10.9	11.3	11.6	11.9	12.2	12.5	12.8	13.2	13.5	13.8	14.1	14.4	14.7	15.0	15.4	15.7
14	3	4.67	9.3	9.7	9.9	10.3	10.6	10.9	11.2	11.6	11.8	12.2	12.5	12.8	13.1	13.4	13.7	14.0	14.3	14.6	14.9	15.2
5	1	5.00	8.8	9.1	9.5	9.7	10.0	10.4	10.6	10.9	11.2	11.5	11.8	12.1	12.4	12.7	12.9	13.2	13.5	13.8	14.1	14.4
16	3	5.33	8.4	8.7	8.9	9.3	9.6	9.8	10.1	10.4	10.7	10.9	11.2	11.5	11.8	12.0	12.3	12.6	12.8	13.1	13.4	13.6
11	2	5.50	8.2	8.5	8.8	9.0	9.3	9.6	9.8	10.1	10.4	10.7	10.9	11.2	11.5	11.7	11.9	12.3	12.5	12.8	13.0	13.3
6	1	6.00	7.7	7.9	8.2	8.5	8.7	8.9	9.2	9.5	9.7	9.9	10.2	10.5	10.7	10.9	11.2	11.4	11.7	11.9	12.1	12.4
19	3	6.33	7.4	7.6	7.8	8.1	8.4	8.6	8.8	9.0	9.3	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.2	11.4	11.6	11.8
13	2	6.50	7.2	7.5	7.7	7.9	8.2	8.4	8.7	8.9	9.1	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.2	11.4	11.6
20	3	6.67	7.0	7.3	7.6	7.8	8.0	8.3	8.5	8.7	8.9	9.2	9.4	9.6	9.8	10.0	10.3	10.5	10.7	10.9	11.2	11.4
7	1	7.00	6.8	7.0	7.3	7.5	7.7	7.9	8.2	8.4	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.1	10.3	10.6	10.7	10.9
15	2	7.50	6.5	6.7	6.9	7.1	7.3	7.6	7.7	7.9	8.2	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	9.9	10.2	10.4
8	1	8.00	6.1	6.4	6.6	6.8	6.9	7.2	7.4	7.6	7.8	7.9	8.1	8.4	8.6	8.7	8.9	9.1	9.3	9.5	9.7	9.8
17	2	8.50	5.9	6.0	6.3	6.5	6.7	6.8	7.0	7.2	7.4	7.6	7.8	7.9	8.1	8.3	8.5	8.7	8.8	9.0	9.2	9.4
9	1	9.00	5.7	5.8	5.9	6.2	6.4	6.6	6.7	6.9	7.1	7.3	7.5	7.6	7.8	7.9	8.1	8.3	8.5	8.7	8.8	8.9
19	2	9.50	5.4	5.6	5.8	5.9	6.1	6.3	6.5	6.7	6.8	6.9	7.1	7.3	7.5	7.7	7.8	7.9	8.1	8.3	8.5	8.6
10	1	10.00	5.2	5.4	[illegible]	5.7	5.9	6.0	6.2	6.4	6.6	6.7	6.9	7.0	7.2	7.4	7.5	7.7	7.8	7.9	8.1	8.3
11	1	11.00	4.8	4.9	5.2	5.4	5.5	5.7	5.8	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	7.1	7.3	7.4	7.6	7.7
12	1	12.00	4.6	4.7	4.8	4.9	5.1	5.3	5.5	5.6	5.7	5.8	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	7.0	7.2
13	1	13.00	4.3	4.5	4.6	4.7	4.8	4.9	5.1	5.3	5.4	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.4	6.6	6.7	6.8
14	1	14.00	4.0	4.2	4.4	4.5	4.6	4.7	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.5
15	1	15.00	3.9	3.9	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.2	5.3	5.5	5.6	5.7	5.8	5.9	5.9	6.1
16	1	16.00	3.7	3.8	3.9	4.0	4.2	4.3	4.4	4.6	4.7	4.7	4.8	4.9	5.0	5.2	5.3	5.4	5.5	5.6	5.7	5.8
17	1	17.00	3.6	3.7	3.8	3.9	3.9	4.1	4.2	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1	5.3	5.4	5.5	5.6
18	1	18.00	3.5	3.6	3.7	3.7	3.8	3.9	4.0	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1	5.2	5.4
19	1	19.00	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.9	4.0	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.1
20	1	20.00	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9

95% PREDICTION INTERVAL - FIVE FUTURE OBSERVATIONS

			OBSERVED NUMBER OF COUNTS																			
L	M	L/M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1.00	5.1	7.5	9.6	11.5	13.2	14.9	16.6	18.1	19.7	21.2	22.7	24.2	25.6	27.0	28.5	29.9	31.3	32.6	34.0	35.4
6	5	1.20	4.4	6.5	8.3	9.9	11.4	12.8	14.2	15.6	16.9	18.2	19.4	20.7	21.9	23.1	24.3	25.5	26.7	27.9	29.0	30.2
5	4	1.25	4.3	6.3	8.0	9.6	11.0	12.4	13.7	15.0	16.3	17.6	18.8	20.0	21.2	22.3	23.5	24.6	25.8	26.9	28.0	29.1
4	3	1.33	4.0	5.9	7.6	9.1	10.5	11.8	13.0	14.3	15.5	16.6	17.8	18.9	20.0	21.1	22.2	23.3	24.4	25.5	26.5	27.6
7	5	1.40	3.9	5.8	7.3	8.7	10.0	11.3	12.5	13.7	14.8	15.9	17.1	18.2	19.2	20.3	21.4	22.4	23.4	24.4	25.4	26.4
3	2	1.50	3.7	5.5	6.9	8.3	9.5	10.7	11.8	12.9	14.0	15.1	16.1	17.2	18.2	19.2	20.1	21.1	22.1	23.0	23.9	24.9
8	5	1.60	3.6	5.2	6.6	7.8	9.0	10.2	11.3	12.3	13.3	14.3	15.3	16.3	17.2	18.2	19.1	20.0	20.9	21.8	22.7	23.6
5	3	1.67	3.5	5.0	6.4	7.6	8.8	9.8	10.9	11.9	12.9	13.8	14.8	15.7	16.7	17.6	18.5	19.3	20.2	21.1	21.9	22.8
7	4	1.75	3.3	4.8	6.1	7.3	8.4	9.5	10.5	11.5	12.4	13.3	14.2	15.1	16.0	16.8	17.7	18.6	19.4	20.2	21.0	21.9
9	5	1.80	3.2	4.8	6.0	7.2	8.2	9.3	10.2	11.2	12.1	13.0	13.9	14.8	15.6	16.5	17.3	18.1	18.9	19.8	20.6	21.4
2	1	2.00	2.9	4.4	5.6	6.6	7.6	8.5	9.4	10.3	11.1	11.9	12.8	13.6	14.4	15.1	15.9	16.6	17.4	18.1	18.8	19.6
11	5	2.20	2.8	4.1	5.2	6.1	7.0	7.9	8.7	9.6	10.3	11.1	11.8	12.6	13.3	13.9	14.7	15.4	16.0	16.7	17.4	18.1
9	4	2.25	2.8	4.0	5.1	6.0	6.9	7.8	8.6	9.4	10.1	10.9	11.6	12.3	13.0	13.7	14.4	15.1	15.8	16.5	17.1	17.7
7	3	2.33	2.7	3.9	4.9	5.9	6.8	7.6	8.4	9.1	9.8	10.6	11.3	11.9	12.7	13.4	14.0	14.7	15.3	15.9	16.6	17.2
12	5	2.40	2.7	3.8	4.8	5.8	6.6	7.4	8.2	8.9	9.7	10.4	11.0	11.7	12.4	13.0	13.7	14.3	14.9	15.6	16.2	16.8
5	2	2.50	2.6	3.7	4.7	5.6	6.4	7.2	7.9	8.7	9.4	10.0	10.7	11.4	11.9	12.6	13.3	13.9	14.5	15.1	15.7	16.3
13	5	2.60	2.5	3.7	4.6	5.5	6.2	6.9	7.7	8.4	9.0	9.7	10.4	11.0	11.6	12.2	12.8	13.5	14.0	14.6	15.2	15.8
8	3	2.67	2.5	3.6	4.5	5.3	6.1	6.8	7.6	8.2	8.9	9.6	10.2	10.8	11.4	11.9	12.6	13.2	13.8	14.3	14.9	15.5
14	5	2.80	2.4	3.5	4.4	5.1	5.9	6.6	7.3	7.9	8.6	9.2	9.8	10.4	10.9	11.6	12.1	12.7	13.2	13.8	14.3	14.8
3	1	3.00	2.2	3.3	4.1	4.9	5.6	6.3	6.9	7.5	8.1	8.7	9.3	9.8	10.4	10.9	11.5	12.0	12.5	13.0	13.6	14.0
16	5	3.20	2.1	3.1	3.9	4.7	5.4	5.9	6.6	7.2	7.7	8.3	8.8	9.4	9.9	10.4	10.9	11.4	11.9	12.4	12.9	13.4
10	3	3.33	2.0	2.9	3.6	4.6	5.2	5.8	6.4	6.9	7.5	8.0	8.6	9.0	9.6	10.0	10.6	11.0	11.5	11.9	12.5	12.9
17	5	3.40	1.9	2.9	3.8	4.5	5.1	5.7	6.3	6.8	7.4	7.9	8.5	8.9	9.5	9.9	10.4	10.9	11.4	11.8	12.3	12.7
7	2	3.50	1.9	2.9	3.7	4.4	4.9	5.6	6.1	6.7	7.2	7.7	8.3	8.7	9.2	9.7	10.2	10.7	11.1	11.6	11.9	12.5
18	5	3.60	1.9	2.9	3.7	4.3	4.9	5.5	6.0	6.6	7.0	7.6	8.0	8.6	9.0	9.5	9.9	10.4	10.8	11.3	11.7	12.2
11	3	3.67	1.9	2.8	3.6	4.2	4.8	5.4	5.9	6.5	6.9	7.5	7.9	8.5	8.9	9.4	9.8	10.3	10.7	11.1	11.6	11.9
15	4	3.75	1.9	2.8	3.6	4.1	4.8	5.3	5.8	6.4	6.8	7.4	7.8	8.3	8.8	9.2	9.7	10.1	10.5	10.9	11.4	11.8
19	5	3.80	1.9	2.8	3.5	4.1	4.7	5.3	5.8	6.3	6.8	7.3	7.8	8.2	8.7	9.1	9.6	9.9	10.4	10.8	11.3	11.7
4	1	4.00	1.8	2.7	3.4	3.9	4.6	5.0	5.6	6.0	6.6	6.9	7.5	7.9	8.4	8.8	9.2	9.6	10.0	10.4	10.8	11.2
17	4	4.25	1.8	2.6	3.2	3.8	4.4	4.8	5.4	5.8	6.3	6.7	7.1	7.6	7.9	8.4	8.8	9.2	9.6	9.9	10.3	10.7
13	3	4.33	1.8	2.6	3.2	3.8	4.3	4.8	5.3	5.7	6.2	6.6	7.0	7.5	7.8	8.3	8.7	9.0	9.4	9.8	10.2	10.6
9	2	4.50	1.7	2.5	3.0	3.7	4.2	4.7	5.1	5.6	5.9	6.5	6.8	7.2	7.7	8.0	8.4	8.8	9.1	9.5	9.9	10.2
14	3	4.67	1.7	2.5	2.9	3.6	4.0	4.6	4.9	5.5	5.8	6.3	6.7	7.0	7.5	7.8	8.2	8.6	8.9	9.3	9.6	9.9
5	1	5.00	1.6	2.3	2.9	3.4	3.9	4.4	4.8	5.2	5.6	5.9	6.4	6.7	7.0	7.4	7.8	8.1	8.5	8.8	9.1	9.5
16	3	5.33	1.6	2.2	2.8	3.3	3.7	4.1	4.6	4.9	5.3	5.7	6.0	6.4	6.7	7.0	7.4	7.7	8.0	8.4	8.7	8.9
11	2	5.50	1.5	2.1	2.7	3.2	3.7	4.0	4.5	4.8	5.2	5.6	5.9	6.3	6.6	6.9	7.2	7.6	7.8	8.2	8.5	8.8
6	1	6.00	1.5	1.9	2.6	2.9	3.5	3.8	4.2	4.6	4.9	5.2	5.6	5.8	6.2	6.5	6.8	7.0	7.4	7.7	7.9	8.2
19	3	6.33	1.4	1.9	2.5	2.9	3.3	3.7	3.9	4.4	4.7	4.9	5.4	5.7	5.9	6.2	6.6	6.8	7.1	7.4	7.7	7.9
13	2	6.50	1.4	1.9	2.5	2.8	3.2	3.7	3.9	4.3	4.7	4.9	5.3	5.6	5.8	6.1	6.4	6.7	6.9	7.2	7.5	7.8
20	3	6.67	1.3	1.9	2.4	2.8	3.2	3.6	3.9	4.2	4.6	4.8	5.1	5.5	5.7	5.9	6.3	6.6	6.8	7.1	7.4	7.6
7	1	7.00	1.3	1.8	2.3	2.7	3.0	3.5	3.8	4.0	4.4	4.7	4.9	5.3	5.6	5.8	6.0	6.4	6.6	6.8	7.1	7.4
15	2	7.50	1.2	1.8	2.2	2.6	2.9	3.3	3.6	3.9	4.2	4.5	4.8	4.9	5.3	5.6	5.8	6.0	6.3	6.6	6.8	6.9
8	1	8.00	1.1	1.7	2.0	2.5	2.8	3.1	3.5	3.7	3.9	4.3	4.6	4.8	5.0	5.3	5.6	5.8	5.9	6.2	6.5	6.7
17	2	8.50	1.0	1.7	1.9	2.4	2.7	2.9	3.3	3.6	3.8	4.0	4.4	4.6	4.8	5.0	5.3	5.6	5.8	5.9	6.2	6.4
9	1	9.00	.9	1.5	1.9	2.3	2.7	2.9	3.2	3.5	3.7	3.9	4.2	4.5	4.7	4.8	5.0	5.3	5.6	5.7	5.9	6.1
19	2	9.50	.9	1.6	1.9	2.2	2.6	2.8	3.0	3.4	3.6	3.8	3.9	4.3	4.5	4.7	4.9	5.1	5.3	5.6	5.7	5.9
10	1	10.00	.9	1.5	1.8	2.1	2.5	2.7	2.9	3.2	3.5	3.7	3.9	4.1	4.4	4.6	4.7	4.9	5.1	5.3	5.6	5.7
11	1	11.00	.9	1.4	1.8	1.9	2.3	2.6	2.8	2.9	3.2	3.5	3.7	3.8	4.0	4.3	4.5	4.7	4.8	4.9	5.1	5.4
12	1	12.00	.9	1.3	1.7	1.9	2.1	2.5	2.7	2.8	2.9	3.3	3.5	3.7	3.8	3.9	4.2	4.4	4.6	4.7	4.8	4.9
13	1	13.00	.8	1.2	1.6	1.8	1.9	2.3	2.6	2.7	2.9	3.0	3.3	3.5	3.7	3.8	3.9	4.1	4.3	4.5	4.6	4.8
14	1	14.00	.8	1.1	1.6	1.8	1.9	2.2	2.5	2.6	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	4.0	4.2	4.4	4.6
15	1	15.00	.8	1.0	1.5	1.7	1.9	2.0	2.3	2.5	2.7	2.8	2.9	3.1	3.3	3.5	3.7	3.8	3.9	3.9	4.2	4.3
16	1	16.00	.8	.9	1.4	1.7	1.8	1.9	2.2	2.4	2.6	2.7	2.8	2.9	3.1	3.3	3.5	3.6	3.8	3.8	3.9	4.1
17	1	17.00	.8	.9	1.4	1.6	1.8	1.9	2.0	2.3	2.5	2.7	2.8	2.9	2.9	3.2	3.4	3.5	3.6	3.7	3.8	3.9
18	1	18.00	.8	.9	1.3	1.6	1.7	1.9	1.9	2.2	2.4	2.6	2.7	2.8	2.9	3.0	3.2	3.4	3.5	3.6	3.7	3.8
19	1	19.00	.8	.9	1.2	1.5	1.7	1.8	1.9	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.0	3.2	3.4	3.5	3.6	[illegible].7
20	1	20.00	.7	.9	1.1	1.5	1.7	1.8	1.9	1.9	2.2	2.4	2.6	2.7	2.8	2.8	2.9	3.0	3.2	3.4	3.5	[illegible]

95% PREDICTION INTERVAL - FIVE FUTURE OBSERVATIONS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
			OBSERVED NUMBER OF COUNTS																			
1	1	1.00	36.7	38.1	39.4	40.7	42.0	43.4	44.7	46.0	47.3	48.6	49.8	51.1	52.4	53.7	54.9	56.2	57.5	58.7	60.0	61.2
6	5	1.20	31.3	32.5	33.6	34.7	35.8	36.9	38.0	39.1	40.2	41.3	42.4	43.5	44.6	45.6	46.7	47.8	48.8	49.9	50.9	52.0
5	4	1.25	30.2	31.3	32.4	33.5	34.6	35.6	36.7	37.8	38.8	39.9	40.9	41.9	43.0	44.0	45.0	46.1	47.1	48.1	49.1	50.1
4	3	1.33	28.6	29.6	30.6	31.7	32.7	33.7	34.7	35.7	36.7	37.7	38.7	39.6	40.6	41.6	42.6	43.5	44.5	45.5	46.4	47.4
7	5	1.40	27.4	28.4	29.4	30.4	31.3	32.3	33.2	34.2	35.1	36.1	37.0	37.9	38.9	39.8	40.8	41.7	42.6	43.5	44.5	45.4
3	2	1.50	25.9	26.8	27.7	28.6	29.5	30.4	31.3	32.2	33.1	34.0	34.9	35.8	36.6	37.5	38.4	39.2	40.1	40.9	41.8	42.7
8	5	1.60	24.5	25.3	26.2	27.1	27.9	28.8	29.6	30.5	31.3	32.1	33.0	33.8	34.6	35.5	36.3	37.1	37.9	38.7	39.5	40.3
5	3	1.67	23.6	24.5	25.3	26.1	26.9	27.8	28.6	29.4	30.2	31.0	31.8	32.6	33.4	34.2	35.0	35.8	36.6	37.4	38.1	38.9
7	4	1.75	22.7	23.5	24.3	25.1	25.9	26.7	27.4	28.2	29.0	29.8	30.5	31.3	32.0	32.8	33.6	34.3	35.1	35.8	36.6	37.3
9	5	1.80	22.1	22.9	23.7	24.5	25.3	26.0	26.8	27.6	28.3	29.0	29.8	30.6	31.3	32.0	32.8	33.5	34.2	34.9	35.7	36.4
2	1	2.00	20.3	21.0	21.7	22.4	23.1	23.8	24.5	25.2	25.9	26.6	27.2	27.9	28.6	29.3	29.9	30.6	31.3	31.9	32.6	33.2
11	5	2.20	18.7	19.4	20.0	20.7	21.4	21.9	22.6	23.3	23.9	24.5	25.1	25.7	26.4	26.9	27.6	28.2	28.8	29.4	30.0	30.6
9	4	2.25	18.4	19.0	19.7	20.3	20.9	21.6	22.2	22.8	23.4	24.0	24.7	25.3	25.9	26.5	27.1	27.7	28.3	28.8	29.5	30.0
7	3	2.33	17.8	18.5	19.1	19.7	20.3	20.9	21.5	22.1	22.7	23.3	23.9	24.5	25.1	25.7	26.3	26.8	27.4	28.0	28.6	29.1
12	5	2.40	17.5	18.0	18.7	19.3	19.8	20.5	21.0	21.6	22.2	22.8	23.4	23.9	24.5	25.1	25.6	26.2	26.8	27.3	27.9	28.5
5	2	2.50	16.9	17.5	18.0	18.6	19.2	19.8	20.3	20.9	21.5	22.0	22.6	23.1	23.7	24.2	24.9	25.3	25.8	26.4	26.9	27.5
13	5	2.60	16.4	16.9	17.5	18.0	18.6	19.1	19.7	20.2	20.8	21.3	21.8	22.4	22.9	23.5	23.9	24.5	25.0	25.6	26.0	26.6
8	3	2.67	16.0	16.6	17.1	17.7	18.2	18.7	19.3	19.8	20.4	20.9	21.4	21.9	22.4	22.9	23.5	23.9	24.5	25.0	25.5	26.0
14	5	2.80	15.4	15.9	16.5	16.9	17.5	18.0	18.5	19.0	19.6	20.0	20.6	21.0	21.6	22.0	22.5	23.0	23.5	24.0	24.5	24.9
3	1	3.00	14.6	15.0	15.6	16.0	16.6	17.0	17.5	17.9	18.5	18.9	19.4	19.9	20.4	20.8	21.3	21.7	22.2	22.7	23.1	23.6
16	5	3.20	13.8	14.3	14.8	15.2	15.7	16.2	16.6	17.0	17.5	17.9	18.4	18.8	19.3	19.7	20.2	20.6	21.0	21.5	21.9	22.4
10	3	3.33	13.4	13.8	14.3	14.7	15.2	15.6	16.1	16.5	16.9	17.4	17.8	18.2	18.7	19.1	19.5	19.9	20.4	20.8	21.2	21.6
17	5	3.40	13.2	13.6	14.0	14.5	14.9	15.4	15.8	16.3	16.7	17.1	17.5	17.9	18.4	18.8	19.2	19.6	20.0	20.5	20.8	21.3
7	2	3.50	12.9	13.3	13.8	14.2	14.6	15.0	15.5	15.9	16.3	16.7	17.1	17.5	17.9	18.4	18.7	19.1	19.6	19.9	20.4	20.7
18	5	3.60	12.6	13.0	13.5	13.9	14.3	14.7	15.1	15.5	15.9	16.3	16.7	17.1	17.6	17.9	18.3	18.7	19.1	19.5	19.9	20.3
11	3	3.67	12.4	12.8	13.3	13.7	14.1	14.5	14.9	15.3	15.7	16.1	16.5	16.9	17.3	17.7	18.0	18.5	18.8	19.2	19.6	19.9
15	4	3.75	12.2	12.6	13.0	13.4	13.8	14.2	14.6	15.0	15.4	15.8	16.2	16.6	16.9	17.4	17.7	18.1	18.5	18.8	19.2	19.6
19	5	3.80	12.1	12.5	12.9	13.3	13.7	14.1	14.5	14.8	15.3	15.7	16.0	16.4	16.8	17.2	17.6	17.9	18.3	18.7	19.0	19.4
4	1	4.00	11.6	11.9	12.4	12.8	13.1	13.5	13.9	14.3	14.7	15.0	15.4	15.7	16.1	16.5	16.8	17.2	17.6	17.9	18.3	18.6
17	4	4.25	11.0	11.5	11.8	12.2	12.6	12.9	13.3	13.6	13.9	14.3	14.7	15.0	15.4	15.7	16.0	16.4	16.7	17.0	17.4	17.7
13	3	4.33	10.9	11.3	11.6	11.9	12.4	12.7	13.0	13.4	13.7	14.1	14.5	14.8	15.1	15.5	15.8	16.1	16.5	16.8	17.1	17.5
9	2	4.50	10.6	10.9	11.3	11.7	11.9	12.3	12.7	13.0	13.4	13.7	14.0	14.4	14.7	14.9	15.3	15.7	15.9	16.3	16.6	16.9
14	3	4.67	10.3	10.7	10.9	11.3	11.7	11.9	12.3	12.7	12.9	13.3	13.6	13.9	14.3	14.6	14.9	15.2	15.5	15.8	16.1	16.5
5	1	5.00	9.8	10.1	10.4	10.7	11.0	11.4	11.7	11.9	12.3	12.6	12.9	13.2	13.5	13.8	14.1	14.4	14.7	14.9	15.3	15.6
16	3	5.33	9.3	9.6	9.9	10.2	10.5	10.8	11.1	11.4	11.7	11.9	12.3	12.6	12.8	13.1	13.4	13.7	13.9	14.2	14.5	14.8
11	2	5.50	9.1	9.4	9.7	9.9	10.3	10.6	10.9	11.1	11.4	11.7	11.9	12.3	12.5	12.8	13.0	13.4	13.6	13.9	14.1	14.4
6	1	6.00	8.5	8.8	9.0	9.4	9.6	9.9	10.1	10.4	10.7	10.9	11.2	11.5	11.7	11.9	12.2	12.5	12.7	12.9	13.2	13.5
19	3	6.33	8.2	8.5	8.7	8.9	9.2	9.5	9.7	9.9	10.3	10.5	10.7	10.9	11.2	11.5	11.7	11.9	12.2	12.5	12.7	12.9
13	2	6.50	8.0	8.3	8.6	8.8	9.0	9.3	9.6	9.8	10.0	10.3	10.6	10.8	11.0	11.3	11.5	11.7	11.9	12.2	12.5	12.7
20	3	6.67	7.9	8.1	8.4	8.7	8.9	9.1	9.4	9.6	9.8	10.1	10.4	10.6	10.8	11.0	11.3	11.5	11.7	11.9	12.2	12.4
7	1	7.00	7.6	7.8	8.1	8.4	8.6	8.8	9.0	9.3	9.5	9.7	9.9	10.2	10.4	10.7	10.8	11.0	11.3	11.5	11.7	11.9
15	2	7.50	7.2	7.5	7.7	7.9	8.1	8.4	8.6	8.8	9.0	9.2	9.5	9.7	9.9	10.1	10.3	10.5	10.7	10.9	11.1	11.4
8	1	8.00	6.9	7.1	7.4	7.6	7.8	7.9	8.2	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	10.0	10.2	10.4	10.6	10.8
17	2	8.50	6.6	6.8	6.9	7.2	7.5	7.7	7.8	8.0	8.2	8.5	8.6	8.8	8.9	9.2	9.4	9.6	9.8	9.9	10.1	10.3
9	1	9.00	6.4	6.6	6.7	6.9	7.1	7.3	7.5	7.7	7.9	8.0	8.3	8.5	8.6	8.8	8.9	9.2	9.4	9.6	9.7	9.9
19	2	9.50	6.1	6.3	6.5	6.7	6.8	7.0	7.2	7.4	7.6	7.8	7.9	8.1	8.3	8.5	8.7	8.8	8.9	9.1	9.3	9.5
10	1	10.00	5.9	6.0	6.3	6.5	6.6	6.8	6.9	7.1	7.3	7.5	7.7	7.8	7.9	8.1	8.3	8.5	8.7	8.8	8.9	9.1
11	1	11.00	5.5	5.7	5.9	5.9	6.2	6.4	6.5	6.7	6.8	6.9	7.1	7.3	7.5	7.6	7.8	7.9	8.0	8.2	8.4	8.5
12	1	12.00	5.2	5.4	5.5	5.7	5.8	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	7.1	7.3	7.5	7.6	7.7	7.8	7.9
13	1	13.00	4.9	5.0	5.2	5.4	5.5	5.7	5.8	5.9	6.0	6.2	6.4	6.5	6.6	6.8	6.9	6.9	7.1	7.3	7.4	7.6
14	1	14.00	4.7	4.8	4.9	5.0	5.2	5.4	5.5	5.7	5.8	5.9	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	7.0	7.1
15	1	15.00	4.5	4.6	4.7	4.8	4.9	5.1	5.2	5.4	5.5	5.6	5.7	5.8	5.9	6.1	6.2	6.4	6.5	6.6	6.7	6.8
16	1	16.00	4.3	4.4	4.6	4.7	4.8	4.9	4.9	5.1	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.4	6.5
17	1	17.00	4.0	4.2	4.4	4.5	4.6	4.7	4.8	4.9	4.9	5.1	5.3	5.4	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.2
18	1	18.00	3.9	4.0	4.2	4.3	4.4	4.6	4.7	4.7	4.8	4.9	5.0	5.1	5.3	5.4	5.5	5.6	5.7	5.8	5.9	5.9
19		19.00	3.8	3.9	3.9	4.1	4.3	4.4	4.5	4.6	4.7	4.8	4.8	4.9	5.0	5.2	5.3	5.4	5.5	5.6	5.7	5.8
20		20.00	3.7	3.8	3.9	3.9	4.0	4.2	4.3	4.5	4.6	4.6	4.7	4.8	4.9	4.9	5.0	5.2	5.3	5.4	5.5	5.6

99% PREDICTION INTERVAL - FIVE FUTURE OBSERVATIONS

L	M	L/M	OBSERVED NUMBER OF COUNTS 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1.00	7.6	10.4	12.7	14.8	16.7	18.6	20.4	22.1	23.8	25.5	27.1	28.7	30.3	31.8	33.4	34.9	36.4	37.8	39.3	40.8
6	5	1.20	6.6	8.9	10.9	12.9	14.5	16.0	17.6	19.1	20.5	21.9	23.3	24.7	26.0	27.3	28.6	29.9	31.1	32.4	33.7	34.9
5	4	1.25	6.4	8.7	10.7	12.4	14.0	15.6	17.0	18.5	19.8	21.2	22.5	23.8	25.1	26.4	27.7	28.9	30.1	31.3	32.5	33.7
4	3	1.33	6.1	8.3	10.1	11.8	13.3	14.8	16.2	17.5	18.8	20.1	21.4	22.6	23.8	25.0	26.2	27.4	28.5	29.7	30.8	31.9
7	5	1.40	5.9	7.9	9.8	11.4	12.8	14.2	15.6	16.8	18.1	19.4	20.6	21.7	22.9	24.0	25.2	26.3	27.4	28.5	29.6	30.6
3	2	1.50	5.6	7.6	9.3	10.8	12.1	13.5	14.7	15.9	17.1	18.3	19.4	20.6	21.6	22.7	23.8	24.8	25.9	26.9	27.9	28.9
8	5	1.60	5.4	7.3	8.8	10.2	11.6	12.8	14.0	15.1	16.3	17.4	18.5	19.5	20.6	21.6	22.6	23.6	24.5	25.5	26.5	27.4
5	3	1.67	5.2	7.0	8.6	9.9	11.2	12.4	13.6	14.7	15.8	16.8	17.8	18.9	19.9	20.8	21.8	22.8	23.7	24.7	25.6	26.5
7	4	1.75	4.9	6.8	8.3	9.6	10.8	11.9	13.0	14.1	15.2	16.2	17.2	18.1	19.1	20.0	20.9	21.9	22.8	23.7	24.6	25.5
9	5	1.80	4.9	6.7	8.0	9.4	10.6	11.7	12.8	13.8	14.8	15.8	16.8	17.8	18.7	19.6	20.5	21.4	22.3	23.2	24.0	24.9
2	1	2.00	4.6	6.2	7.5	8.7	9.8	10.8	11.8	12.8	13.7	14.6	15.5	16.4	17.2	18.0	18.8	19.7	20.5	21.3	22.0	22.8
11	5	2.20	4.3	5.8	6.9	8.0	9.1	10.0	10.9	11.8	12.7	13.6	14.4	15.2	15.9	16.7	17.5	18.3	18.9	19.7	20.5	21.2
9	4	2.25	4.2	5.7	6.9	7.9	8.9	9.9	10.8	11.7	12.5	13.3	14.1	14.9	15.7	16.5	17.2	17.9	18.7	19.4	20.1	20.8
7	3	2.33	4.0	5.6	6.7	7.8	8.7	9.7	10.5	11.4	12.2	12.9	13.7	14.5	15.3	15.9	16.7	17.5	18.1	18.8	19.5	20.2
12	5	2.40	3.9	5.5	6.6	7.6	8.6	9.5	10.3	11.1	11.9	12.7	13.5	14.2	14.9	15.7	16.4	17.0	17.7	18.4	19.1	19.8
5	2	2.50	3.9	5.3	6.4	7.4	8.3	9.1	9.9	10.8	11.6	12.3	13.0	13.8	14.5	15.2	15.8	16.5	17.2	17.8	18.5	19.1
13	5	2.60	3.5	5.1	6.2	7.2	8.0	8.9	9.7	10.5	11.2	11.9	12.7	13.4	14.0	14.7	15.4	16.0	16.7	17.3	17.9	18.6
8	3	2.67	3.8	5.0	6.1	7.0	7.9	8.8	9.6	10.3	11.0	11.7	12.5	13.1	13.8	14.5	15.1	15.7	16.4	16.9	17.6	18.2
14	5	2.80	3.7	4.9	5.9	6.8	7.7	8.5	9.2	9.9	10.7	11.3	11.9	12.7	13.3	13.9	14.6	15.1	15.7	16.4	16.9	17.5
3	1	3.00	3.5	4.7	5.7	6.5	7.3	8.0	8.9	9.5	10.1	10.8	11.4	11.9	12.6	13.2	13.8	14.4	14.9	15.5	16.0	16.6
16	5	3.20	3.4	4.5	5.4	6.2	6.9	7.7	8.4	8.9	9.7	10.3	10.8	11.5	12.0	12.6	13.1	13.7	14.2	14.8	15.3	15.8
10	3	3.33	3.2	4.4	5.3	6.0	6.8	7.5	8.1	8.8	9.4	9.9	10.6	11.1	11.7	12.2	12.8	13.3	13.8	14.3	14.8	15.3
17	5	3.40	3.2	4.3	5.2	5.9	6.7	7.4	7.9	8.7	9.2	9.8	10.4	10.9	11.5	12.0	12.6	13.0	13.6	14.1	14.6	15.1
7	2	3.50	3.1	4.2	5.0	5.8	6.6	7.2	7.8	8.5	9.0	9.6	10.2	10.7	11.3	11.8	12.3	12.8	13.3	13.8	14.3	14.8
18	5	3.60	3.0	4.1	4.9	5.9	6.5	7.0	7.7	8.3	8.8	9.5	9.9	10.5	11.0	11.6	12.0	12.6	13.0	13.5	13.9	14.5
11	3	3.67	2.9	4.0	4.9	5.7	6.4	6.9	7.6	8.2	8.8	9.3	9.8	10.4	10.9	11.4	11.9	12.4	12.8	13.4	13.8	14.3
15	4	3.75	2.9	3.9	4.8	5.6	6.3	6.9	7.5	8.0	8.6	9.1	9.7	10.2	10.7	11.2	11.7	12.2	12.7	13.1	13.6	14.0
19	5	3.80	2.9	3.9	4.8	5.6	6.2	6.8	7.5	7.9	8.6	9.0	9.6	10.1	10.6	11.1	11.6	12.0	12.6	12.9	13.5	13.9
4	1	4.00	2.9	3.8	4.7	5.4	5.9	6.6	7.1	7.7	8.2	8.9	9.3	9.8	10.2	10.7	11.1	11.6	12.0	12.5	12.9	13.4
17	4	4.25	2.8	3.7	4.5	5.1	5.8	6.4	6.8	7.4	7.9	8.4	8.8	9.4	9.8	10.2	10.7	11.1	11.6	11.9	12.4	12.8
13	3	4.33	2.8	3.7	4.5	5.0	5.7	6.3	6.8	7.3	7.8	8.3	8.8	9.2	9.7	10.1	10.6	10.9	11.4	11.8	12.2	12.6
9	2	4.50	2.7	3.6	4.3	4.9	5.6	6.0	6.6	7.1	7.6	8.0	8.6	8.9	9.4	9.8	10.3	10.7	11.0	11.5	11.9	12.3
14	3	4.67	2.7	3.6	4.2	4.8	5.5	5.9	6.5	6.9	7.4	7.8	8.3	8.8	9.2	9.6	9.9	10.4	10.8	11.2	11.6	11.9
5	1	5.00	2.6	3.4	3.9	4.7	5.1	5.7	6.1	6.7	7.0	7.5	7.9	8.4	8.7	9.1	9.6	9.9	10.3	10.7	10.9	11.4
16	3	5.33	2.5	3.2	3.9	4.5	4.9	5.5	5.9	6.4	6.8	7.2	7.6	7.9	8.4	8.7	9.1	9.5	9.8	10.2	10.6	10.8
11	2	5.50	2.4	3.1	3.8	4.4	4.8	5.4	5.8	6.2	6.7	6.9	7.5	7.8	8.2	8.6	8.9	9.3	9.6	9.9	10.3	10.7
6	1	6.00	2.3	2.9	3.7	4.1	4.7	4.9	5.5	5.9	6.3	6.7	6.9	7.4	7.7	8.0	8.4	8.7	9.0	9.4	9.7	9.9
19	3	6.33	2.1	2.9	3.6	3.9	4.5	4.9	5.3	5.7	6.0	6.5	6.8	7.1	7.5	7.8	8.0	8.4	8.7	8.9	9.4	9.7
13	2	6.50	2.1	2.9	3.5	3.9	4.4	4.8	5.2	5.6	5.9	6.3	6.7	6.9	7.3	7.7	7.9	8.3	8.6	8.8	9.1	9.5
20	3	6.67	2.0	2.8	3.4	3.8	4.3	4.8	5.1	5.5	5.8	6.2	6.6	6.8	7.2	7.5	7.8	8.1	8.5	8.7	8.9	9.3
7	1	7.00	1.9	2.8	3.3	3.8	4.2	4.6	4.9	5.4	5.7	5.9	6.4	6.7	6.9	7.3	7.6	7.8	8.1	8.5	8.7	8.9
15	2	7.50	1.9	2.7	3.1	3.7	3.9	4.5	4.9	5.0	5.5	5.8	6.0	6.4	6.7	6.9	7.2	7.5	7.8	8.0	8.3	8.6
8	1	8.00	1.9	2.6	2.9	3.5	3.8	4.2	4.6	4.9	5.2	5.6	5.8	6.0	6.4	6.7	6.9	7.2	7.5	7.7	7.9	8.2
17	2	8.50	1.9	2.5	2.9	3.4	3.8	4.0	4.5	4.7	4.9	5.3	5.6	5.8	6.1	6.4	6.7	6.9	7.1	7.4	7.7	7.9
9	1	9.00	1.8	2.5	2.8	3.2	3.7	3.9	4.3	4.6	4.8	5.1	5.4	5.7	5.9	6.1	6.5	6.7	6.9	7.1	7.4	7.6
19	2	9.50	1.8	2.4	2.8	3.1	3.6	3.8	4.0	4.5	4.7	4.9	5.2	5.5	5.7	5.9	6.2	6.5	6.7	6.9	7.1	7.4
10	1	10.00	1.8	2.2	2.7	2.9	3.5	3.7	3.9	4.3	4.6	4.8	4.9	5.3	5.6	5.8	5.9	6.2	6.5	6.7	6.8	7.0
11	1	11.00	1.7	2.0	2.6	2.9	3.2	3.6	3.8	3.9	4.3	4.6	4.8	4.9	5.2	5.5	5.7	5.8	6.0	6.3	6.5	6.7
12	1	12.00	1.6	1.9	2.5	2.8	2.9	3.4	3.7	3.8	4.0	4.3	4.6	4.8	4.9	5.1	5.4	5.6	5.8	5.9	6.1	6.3
13	1	13.00	1.6	1.9	2.4	2.7	2.9	3.1	3.5	3.7	3.9	4.0	4.4	4.6	4.8	4.9	5.0	5.3	5.5	5.7	5.8	5.9
14	1	14.00	1.5	1.9	2.2	2.6	2.8	2.9	3.3	3.6	3.8	3.9	4.1	4.4	4.6	4.7	4.9	4.9	5.2	5.4	5.6	5.7
15	1	15.00	1.4	1.8	2.0	2.5	2.8	2.9	3.1	3.4	3.7	3.8	3.9	4.1	4.4	4.6	4.7	4.8	4.9	5.1	5.4	5.5
16	1	16.00	1.4	1.8	1.9	2.4	2.7	2.8	2.9	3.3	3.5	3.7	3.8	3.9	4.1	4.4	4.6	4.7	4.8	4.9	5.1	5.3
17	1	17.00	1.3	1.3	1.9	2.3	2.6	2.8	2.9	3.1	3.4	3.6	3.7	3.8	3.9	4.2	4.4	4.6	4.7	4.8	4.9	5.0
18	1	18.00	1.2	1.7	1.9	2.2	2.5	2.7	2.9	2.9	3.2	3.5	3.7	3.8	3.9	3.9	4.2	4.4	4.6	4.7	4.8	4.9
19	1	19.00	1.1	1.7	1.9	2.0	2.5	2.7	2.8	2.9	3.1	3.4	3.6	3.7	3.8	3.9	3.9	4.2	4.4	4.6	4.7	4.8
20	1	20.00	1.1	1.7	1.9	1.9	2.4	2.6	2.8	2.9	2.9	3.2	3.4	3.6	3.7	3.8	3.9	4.0	4.2	4.4	4.6	4.7

99% PREDICTION INTERVAL - FIVE FUTURE OBSERVATIONS

OBSERVED NUMBER OF COUNTS

L	M	L/M	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1	1.00	42.2	43.7	45.1	46.5	47.9	49.3	50.7	52.1	53.5	54.8	56.2	57.6	58.9	60.2	61.6	62.9	64.3	65.6	66.9	68.2
6	5	1.20	36.1	37.3	38.5	39.7	40.9	42.1	43.3	44.5	45.6	46.8	47.9	49.0	50.2	51.3	52.5	53.6	54.7	55.8	56.9	58.1
5	4	1.25	34.9	36.0	37.2	38.4	39.5	40.7	41.8	42.9	44.0	45.1	46.3	47.4	48.5	49.6	50.6	51.7	52.8	53.9	54.9	56.0
4	3	1.33	33.0	34.1	35.2	36.3	37.4	38.5	39.5	40.6	41.7	42.7	43.7	44.8	45.8	46.8	47.9	48.9	49.9	50.9	51.9	52.9
7	5	1.40	31.7	32.8	33.8	34.8	35.9	36.9	37.9	38.9	39.9	40.9	41.9	42.9	43.9	44.9	45.9	46.9	47.9	48.8	49.8	50.8
3	2	1.50	29.9	30.9	31.9	32.9	33.8	34.8	35.8	36.7	37.7	38.6	39.6	40.5	41.4	42.4	43.3	44.2	45.1	46.0	46.9	47.8
8	5	1.60	28.4	29.3	30.2	31.1	32.0	32.9	33.9	34.8	35.7	36.6	37.5	38.3	39.2	40.1	40.9	41.8	42.7	43.6	44.4	45.3
5	3	1.67	27.4	28.3	29.2	30.1	31.0	31.9	32.7	33.6	34.5	35.3	36.2	37.0	37.9	38.7	39.6	40.4	41.2	42.0	42.9	43.7
7	4	1.75	26.4	27.2	28.0	28.9	29.8	30.6	31.5	32.3	33.1	33.9	34.7	35.6	36.4	37.2	37.9	38.8	39.6	40.4	41.1	41.9
9	5	1.80	25.7	26.6	27.4	28.3	29.1	29.9	30.7	31.5	32.3	33.1	33.9	34.7	35.5	36.3	37.1	37.8	38.6	39.4	40.2	40.9
2	1	2.00	23.6	24.4	25.2	25.9	26.7	27.4	28.1	28.9	29.6	30.4	31.1	31.8	32.5	33.2	33.9	34.7	35.4	36.0	36.8	37.5
11	5	2.20	21.9	22.6	23.3	23.9	24.7	25.4	26.0	26.7	27.4	28.0	28.7	29.4	30.0	30.7	31.4	32.0	32.7	33.3	33.9	34.6
9	4	2.25	21.5	22.2	22.9	23.6	24.2	24.9	25.6	26.2	26.9	27.6	28.2	28.8	29.5	30.1	30.8	31.4	32.0	32.7	33.3	33.9
7	3	2.33	20.9	21.6	22.2	22.9	23.6	24.2	24.8	25.5	26.1	26.8	27.4	28.0	28.7	29.3	29.9	30.5	31.1	31.7	32.3	32.9
12	5	2.40	20.4	21.1	21.7	22.4	23.0	23.7	24.3	24.9	25.5	26.1	26.8	27.4	28.0	28.6	29.2	29.8	30.4	31.0	31.6	32.2
5	2	2.50	19.9	20.4	21.0	21.7	22.3	22.9	23.5	24.1	24.7	25.3	25.9	26.5	27.1	27.7	28.3	28.8	29.4	29.9	30.6	31.1
13	5	2.60	19.2	19.8	20.4	20.9	21.6	22.2	22.8	23.4	23.9	24.5	25.1	25.7	26.2	26.8	27.4	27.9	28.5	29.0	29.6	30.1
8	3	2.67	18.8	19.4	19.9	20.6	21.2	21.7	22.3	22.9	23.5	24.0	24.6	25.1	25.7	26.3	26.8	27.4	27.9	28.5	28.9	29.5
14	5	2.80	18.1	18.7	19.2	19.8	20.4	20.9	21.5	22.0	22.6	23.1	23.7	24.2	24.7	25.2	25.8	26.3	26.8	27.4	27.8	28.4
3	1	3.00	17.1	17.7	18.2	18.8	19.3	19.8	20.4	20.8	21.4	21.9	22.4	22.9	23.4	23.9	24.4	24.9	25.4	25.8	26.4	26.8
16	5	3.20	16.3	16.8	17.4	17.8	18.4	18.8	19.4	19.8	20.3	20.8	21.3	21.8	22.2	22.7	23.2	23.6	24.1	24.6	25.0	25.5
10	3	3.33	15.8	16.3	16.3	17.3	17.8	18.3	18.7	19.2	19.7	20.1	20.6	21.0	21.5	21.9	22.5	22.9	23.3	23.8	24.2	24.7
17	5	3.40	15.6	16.0	16.6	17.0	17.5	17.9	18.5	18.9	19.4	19.8	20.3	20.7	21.2	21.7	22.1	22.5	22.9	23.4	23.8	24.3
7	2	3.50	15.3	15.7	16.2	16.7	17.1	17.6	18.0	18.5	18.9	19.4	19.8	20.3	20.7	21.1	21.6	22.0	22.5	22.9	23.3	23.7
18	5	3.60	14.9	15.4	15.9	16.3	16.8	17.2	17.7	18.1	18.6	18.9	19.4	19.8	20.3	20.7	21.1	21.6	21.9	22.4	22.8	23.2
11	3	3.67	14.7	15.2	15.7	16.1	16.6	16.9	17.4	17.8	18.3	18.7	19.1	19.6	19.9	20.4	20.8	21.2	21.7	22.0	22.5	22.9
15	4	3.75	14.5	14.9	15.4	15.8	16.3	16.7	17.1	17.6	17.9	18.4	18.8	19.2	19.6	20.0	20.5	20.8	21.3	21.7	22.0	22.5
19	5	3.80	14.4	14.8	15.2	15.7	16.1	16.5	16.9	17.4	17.8	18.2	18.6	19.0	19.5	19.8	20.3	20.7	21.0	21.5	21.8	22.2
4	1	4.00	13.8	14.2	14.7	15.0	15.5	15.9	16.3	16.7	17.1	17.5	17.9	18.3	18.7	19.0	19.5	19.8	20.2	20.6	20.9	21.4
17	4	4.25	13.2	13.6	13.9	14.4	14.8	15.2	15.6	15.9	16.4	16.7	17.1	17.5	17.8	18.2	18.6	18.9	19.3	19.7	20.0	20.4
13	3	4.33	12.9	13.4	13.8	14.2	14.6	14.9	15.4	15.7	16.1	16.5	16.8	17.2	17.6	17.9	18.3	18.7	19.0	19.4	19.7	20.0
9	2	4.50	12.7	13.0	13.5	13.8	14.2	14.6	14.9	15.3	15.7	15.9	16.4	16.7	17.0	17.5	17.8	18.1	18.5	18.8	19.2	19.5
14	3	4.67	12.3	12.7	13.0	13.5	13.8	14.1	14.5	14.8	15.2	15.6	15.9	16.3	16.6	16.9	17.3	17.6	17.9	18.3	18.7	18.9
5	1	5.00	11.7	12.0	12.5	12.8	13.1	13.5	13.8	14.1	14.5	14.8	15.1	15.5	15.8	16.1	16.4	16.7	17.0	17.4	17.7	17.9
16	3	5.33	11.2	11.5	11.8	12.2	12.5	12.8	13.1	13.5	13.8	14.1	14.4	14.7	15.0	15.4	15.7	15.9	16.3	16.6	16.8	17.1
11	2	5.50	10.9	11.3	11.6	11.9	12.2	12.6	12.8	13.2	13.5	13.8	14.1	14.4	14.7	14.9	15.3	15.6	15.9	16.2	16.5	16.7
6	1	6.00	10.3	10.6	10.9	11.2	11.5	11.8	12.0	12.4	12.7	12.9	13.2	13.5	13.8	14.0	14.4	14.6	14.9	15.1	15.5	15.7
19	3	6.33	9.9	10.2	10.5	10.8	11.0	11.4	11.7	11.9	12.2	12.5	12.7	12.9	13.3	13.5	13.8	14.0	14.3	14.6	14.8	15.0
13	2	6.50	9.8	9.9	10.3	10.6	10.8	11.1	11.4	11.7	11.9	12.2	12.5	12.8	12.9	13.3	13.6	13.8	14.0	14.3	14.6	14.8
20	3	6.67	9.6	9.8	10.1	10.4	10.7	10.9	11.2	11.5	11.7	11.9	12.3	12.5	12.8	13.0	13.3	13.6	13.8	14.0	14.3	14.5
7	1	7.00	9.3	9.6	9.8	10.0	10.3	10.6	10.8	11.0	11.4	11.6	11.8	12.0	12.4	12.6	12.8	13.0	13.3	13.6	13.8	13.9
15	2	7.50	8.8	9.0	9.4	9.6	9.8	10.0	10.3	10.6	10.8	11.0	11.3	11.5	11.7	11.9	12.2	12.5	12.7	12.9	13.1	13.3
8	1	8.00	8.5	8.7	8.9	9.2	9.4	9.7	9.8	10.1	10.4	10.6	10.8	10.9	11.2	11.5	11.7	11.8	12.0	12.3	12.5	12.7
17	2	8.50	8.1	8.4	8.6	8.8	8.9	9.3	9.5	9.7	9.9	10.1	10.4	10.6	10.8	10.9	11.2	11.4	11.6	11.8	11.9	12.2
9	1	9.00	7.8	7.9	8.3	8.5	8.7	8.9	9.1	9.3	9.6	9.7	9.9	10.1	10.4	10.6	10.7	10.9	11.1	11.3	11.5	11.7
19	2	9.50	7.6	7.8	7.9	8.2	8.4	8.6	8.8	8.9	9.2	9.4	9.6	9.8	9.9	10.1	10.4	10.6	10.7	10.9	11.0	11.3
10	1	10.00	7.3	7.5	7.7	7.9	8.0	8.3	8.5	8.7	8.8	9.0	9.2	9.5	9.6	9.8	9.9	10.1	10.4	10.6	10.7	10.8
11	1	11.00	6.8	6.9	7.2	7.5	7.6	7.8	7.9	8.1	8.3	8.5	8.7	8.8	8.9	9.2	9.4	9.6	9.7	9.8	9.9	10.2
12	1	12.00	6.5	6.7	6.8	6.9	7.2	7.4	7.6	7.7	7.8	7.9	8.2	8.4	8.5	8.7	8.8	8.9	9.1	9.3	9.5	9.6
13	1	13.00	6.1	6.4	6.5	6.7	6.8	6.9	7.1	7.3	7.5	7.6	7.8	7.9	8.0	8.2	8.4	8.6	8.7	8.8	8.9	9.0
14	1	14.00	5.9	5.9	6.2	6.4	6.6	6.7	6.8	6.9	7.0	7.3	7.4	7.6	7.7	7.8	7.9	8.0	8.2	8.4	8.6	8.7
15	1	15.00	5.7	5.8	5.9	6.0	6.2	6.4	6.6	6.7	6.8	6.9	7.0	7.2	7.4	7.5	7.7	7.8	7.8	7.9	8.1	8.3
16	1	16.00	5.5	5.6	5.7	5.8	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	6.9	7.2	7.3	7.5	7.6	7.7	7.8	7.9
17	1	17.00	5.2	5.4	5.6	5.7	5.8	5.9	5.9	6.1	6.3	6.5	6.6	6.7	6.8	6.9	6.9	7.1	7.3	7.4	7.5	7.7
18	1	18.00	4.9	5.2	5.3	5.5	5.6	5.7	5.8	5.9	6.0	6.2	6.3	6.5	6.6	6.7	6.8	6.9	6.9	7.1	7.2	7.4
19	1	19.00	4.9	4.9	5.1	5.3	5.4	5.6	5.7	5.8	5.8	5.9	6.0	6.2	6.4	6.5	6.6	6.7	6.8	6.9	6.9	7.0
20	1	20.00	4.8	4.8	4.9	5.0	5.2	5.4	5.5	5.6	5.7	5.8	5.9	5.9	6.1	6.2	6.4	6.5	6.6	6.7	6.8	6.8