AN ABSTRACT OF THE THESIS OF

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Title: Calibration of a Theoretically Derived Relationship between

Pan Evaporation and Evapotranspiration

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Richard H. Cuenca

Theoretical derivations to describe pan evaporation, $E_p$, and

evapotranspiration, $ET_r$, were developed based on a flow equation

expressed as the product of the driving force for vapor between the

evaporative surface and a plane of reference times a conductivity

coefficient of vapor through air. Under the assumptions of this

study, the driving force for both vapor flows was the same. A

thoretical relationship expressing the evapotranspiration as the

product of pan evaporation times the ratio of their conductivity

coefficients was obtained.

A data set containing 5800 daily observations for Coshocton, Ohio,

Davis, California, and Kimberly, Idaho, was available for testing the

theoretical models. The data included pan evaporation, standard

meteorological observations, and lysimeter measured evapotranspira-

tion. The theoretical models for $E_p$ and $ET_r$ were compared with

statistically developed models for the same flows. Using half of the

data set for each location, three variables of wind run, day-period

temperature and day-period relative humidity and twenty-four

transformations and interactions were included in a stepwise analysis.

The statistical models obtained included the triple interaction of

wind run, temperature and the logarithm of the relative humidity, that

was theoretically derived, plus a temperature-based correction factor.
To improve the calibration of the models, correction for autocorrelation of the error term and data filtering with a 5-day moving average were performed. A verification of the ET<sub>r</sub>/E<sub>p</sub> relationships was conducted with the second half of the data set. A double-mass curve was plotted for each location between estimated and measured evapotranspiration values. The pan evaporation-based model overestimated ET<sub>r</sub> for Coshocton and underestimated for Davis and Kimberly. An adjusting factor was obtained from the slope of the double mass-curves and the final function estimated the overall average of evapotranspiration with less than 10 percent error by use of a 5-day moving average of pan evaporation measurements.
CALIBRATION OF A THEORETICALLY DERIVED
RELATIONSHIP BETWEEN PAN EVAPORATION
AND EVAPOTRANSPIRATION

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CALIBRATION OF A THEORETICALLY DERIVED RELATIONSHIP BETWEEN PAN EVAPORATION AND EVAPOTRANSPIRATION

1. INTRODUCTION

1.1 Statement of the Problem.

Plant growth and development is the result of plant genetic potential interacting with the environment through a series of complex processes involving matter and energy exchanges. The main fluxes involved in the system are those of carbon dioxide, oxygen, water, nutrients, and radiant energy, (Loomis et al., 1971). Transformation of inorganic molecules to organic compounds through photosynthesis takes only a small part of the total radiant energy that plants receive. Therefore, most radiant energy is left to be dissipated to keep plant temperatures within appropriate limits.

Change of the physical state of water from liquid to vapor, evaporation, requires approximately 2.4 MJ/kg. This process is one of the most efficient ways to eliminate excess heat from the immediate neighborhood of plants. When a change of water from liquid within the plant to vapor at the plant leaves occurs, the process is called plant transpiration. When the change from liquid to vapor state occurs at the soil surface, it is called soil evaporation. Evaporation also can occur from the leaf surfaces when water is ponded on the leaves by rain or irrigation. The combined amount of water lost from plants and soil during a cycle is called evapotranspiration. This total vapor flow is determined by climatic, soil, and plant community characteristics.

Through the years, many models have been developed to estimate evapotranspiration. Among them are at least three different approaches; (a) aerodynamic methods, (b) energy-balance methods, and (c) combination methods, in which both energy-balance and aerodynamic terms appear in a single relationship.
Some widely used methods to estimate evapotranspiration, ET, worldwide are the Blaney-Criddle, Makkink radiation, Penman combination, and pan evaporation methods (Doorenbos and Pruitt, 1977). The first three methods have in common one or more meteorological variables to build the relationship, whereas pan evaporation, $E_p$, represents a direct and integrated meteorological variable whose behavior is in response to the combined effect of all other meteorological variables (Monteith, 1973; Mukammal, 1961; Christiansen and Mehta, 1964; Kohler, 1954). This characteristic of the $E_p$ method represents certain advantages when estimating ET. Among the advantages are: (a) simplicity in considering just one meteorological variable, (b) relatively reasonable cost to build and to operate a pan evaporation network compared to measuring numerous other meteorological variables.

The $E_p$ method is described by Doorenbos and Pruitt (1977) as:

\[
E_T = (K_p) E_p
\]  
(1)

where:

$E_T$ = reference evapotranspiration,

$E_p$ = pan evaporation,

$K_p$ = pan calibration coefficient.

For the method indicated by Doorenbos and Pruitt (1977), grass 8 to 15 cm high, fully shading the ground, and not stressed by water shortage, disease, or pests is the reference crop. For a particular crop:

\[
E_T = (K_c) E_T
\]  
(2)

where $E_T$ is the actual crop evapotranspiration and $K_c$ is the crop coefficient relative to grass.

When ET for a particular crop and crop stage is considered, the relationship could be written by combining equations (1) and (2) as:
\[ \text{ET}_c = (K_o) \, E_p \]  

(3)

where:

\[ K_o = K_p \, K_c \]  

(4)

There are two major practical problems associated with the pan evaporation method. First, it has been shown that the pan location and the installation conditions affect the evaporation rate, therefore \( K_p \) must be adjusted in accordance with the local environment (Thom et al., 1981; Doorenbos and Pruitt, 1977). An empirical relationship of \( K_p \) for different groundcover conditions and different levels of mean relative humidity and wind velocity is presented in the work by Doorenbos and Pruitt (1977).

The \( K_p \) values estimated by the method of Doorenbos and Pruitt (1977) were shown to vary from 0.35 to 0.85. This variation emphasizes the importance of selecting appropriate values to have an acceptable estimation of \( \text{ET}_r \).

The second type of problem associated with the pan evaporation method is the correct estimation of the crop coefficient, \( K_c \). \( K_c \) represents a calibration factor varying upon the particular crop and stage of crop development. Empirical relationships have been developed which change with location, cultural management, and cultivars. The associated problems with \( K_p \) and \( K_c \) for the pan evaporation method make it necessary to calibrate this method through extensive field experimentation for different sets of conditions (crop, location, season) (Tanner, 1967; Rosenberg, 1974).

Based on the physics of the vapor transfer process, a more theoretical approach to link \( \text{ET}_r \) and \( E_p \) was investigated. A detailed description of each phenomena in terms of a basic flow equation was used as a starting point to isolate the relevant environmental variables associated with \( \text{ET}_r \) and \( E_p \). This analysis allowed for calibration of a functional model for each process. The models for pan evaporation and evapotranspiration were combined into a single relationship expressing the linkage between vapor flows from the pan.
and from the plant. Though the complex nature of these processes may not allow for totally deterministic physical models, a better understanding of the system could be obtained and a theoretically based, as opposed to empirical, method to estimate ET$_r$ as a function of E$_p$ and actual crop characteristics could be developed.

1.2 Objectives.

The purpose of this project was to test models, theoretically derived, for pan evaporation and for evapotranspiration obtained from a physical description of the vapor flows from a pan evaporator and from the evaporative surfaces of the plant community to the atmosphere by use of the gas law and the logarithmic wind law. A second objective was to test a relationship to express ET$_r$ as a function of E$_p$ developed by combining the corresponding theoretical models.

Additionally, sensitivity analysis was conducted to determine critical factors of the ET$_r$ model and to ascertain if simplifications could be made to obtain a more practical and useful relationship. Through sensitivity analysis, the required precision of the input data was investigated.
2. DERIVATION OF THEORETICAL MODELS FOR PAN EVAPORATION AND EVAPOTRANSPIRATION

Theoretical background necessary to develop \( \text{ET}_r \) and \( \text{Ep} \) models is presented in the following sections. First, evaporation from a free-water surface is discussed and a pan evaporation model based on the flow equation is proposed. Second, evapotranspiration is analyzed and an \( \text{ET}_r \) model is derived analogous to the \( \text{Ep} \) relationship. Finally, an expression for \( \text{ET}_r \) as a function of pan evaporation is presented as the combination of both previously derived equations.

2.1 Derivation of the Pan Evaporation Model.

Evaporation from a free-water surface is the result of the mass transfer of water vapor from the liquid-gas interface to the atmosphere. Two major physical components are involved in determining the evaporation flux: (1) free energy of the system to transfer vapor from the evaporating surface to a reference plane at some height and, (2) conductance of the medium through which the transfer process takes place. These two factors are associated by the flow equation:

\[
\text{flux} = (\text{conductance}) (\text{driving force})
\]  

(5)

where the driving force is expressed by the gradient of total vapor potential (TVP). The TVP gradient is between the water-air interface and air at some reference plane, usually 2 or 3 m above the reference plane (Nobel, 1970; Krame, 1969; Slatyer, 1967). Conductance is associated with turbulent diffusivity of vapor in air, therefore it is related to the wind-profile structure near the ground (Penman et al., 1967).

2.1.1. Pan Evaporation Total Vapor Potential Gradient.

Nobel (1970) described total vapor potential (TVP) as the summation of two terms: a concentration term, or concentration
component, given by relative humidity and temperature, and a
gravitational term. The following equation (Nobel, 1970) expresses
this relation:

\[ H_{\text{wv}} = \left[ \frac{RT}{V_w} \right] \ln \left( \frac{e}{e_o} \right) + r_w g z \]  

(6)

where:

- \( H_{\text{wv}} \) = vapor potential in air, Pa
- \( R \) = universal gas constant, Pa(l)/mol(degree K)
- \( T \) = absolute temperature, degree K
- \( V_w \) = molar volume of liquid water, l/mol
- \( e \) = actual water vapor pressure in air, Pa
- \( e_o \) = saturated water vapor pressure in air, Pa
- \( r_w \) = liquid water density, kg/m^3
- \( g \) = acceleration of gravity, m/sec^2
- \( z \) = height from soil surface to reference plane, m

The magnitude of the gravitational term in the above equation is
small compared with concentration terms at both the water-air
interface and the reference plane. In equation (6), the gravitational
potential is 9.8 kPa/m. The concentration term is -46,000 kPa/m when
the relative humidity is 50 percent at 20°C, and -1,300 kPa/m when
relative humidity is 98 percent at the same temperature. Therefore,
the gravitational term may be neglected and equation (6) rewritten as:

\[ H_{\text{wv}} = \left[ \frac{RT}{V_w} \right] \ln \left( \frac{e}{e_o} \right) \]  

(7)

The TVP gradient between the water-air interface and the reference
plane, or the evaporation driving force, can be approximated by:

\[ \frac{dH}{dz} = \frac{H_p - H*_{\text{wv}}}{z} \]  

(8)

where:
\[ \frac{dH}{dz} \_p = \text{TVP gradient for pan evaporation}, \]

\[ H_p = \text{TVP at the water-air interface level}, \]

\[ H* \_wv = \text{TVP at the reference plane.} \]

By assuming that the interface is close to saturation (Penman et al., 1967), the TVP at the interface becomes zero \((H_p = 0)\), and equation (8) reduces to:

\[ \frac{dH}{dz} \_p = -H\_wv \]

or, in terms of equation (7):

\[ \frac{dH}{dz} \_p = -\frac{[RT / V_w]}{z} \ln \left( \frac{e}{e_0} \right) \]

where \(T\) is air the temperature and \((e/e_0)\) is the relative humidity, both at the reference plane.

2.1.2. Pan Evaporation Conductivity Coefficient.

In general, the conductance factor from the flow equation, equation (5), represents the ability of the fluid to move through a medium in accordance to the properties of the fluid and the medium. For evaporation, this coefficient represents the air conductivity for water vapor.

Two approaches generally have been used to study the conductivity associated with evaporation. First was the theoretical analysis based on the logarithmic wind law and Reynolds' analogy. Second were empirical approximations, or wind functions, on the basis of calibration coefficients. Pasquil (1943) and Sutton (1934) developed wind functions that included such field parameters as downwind length and field width. Penman (1948, 1956), used a wind function of the type:
where \( a \) and \( b \) are calibration coefficients and \( U \) the 24-hour wind run at 2 m height. Penman's (1948, 1956) original values for the constants have been modified to be used locally (Wright and Jensen, 1972), or to be applied with a wider range of climates as in the revised wind function proposed by Doorenbos and Pruitt (1977). Thom and Oliver (1977), Stigter (1980), and Thom et al. (1981) derived wind functions to be used with Penman's (1948, 1956) equation starting from theoretical considerations and statistical calibration.

A second common approach to analyze the conductivity coefficient for water vapor is based on momentum transfer by use of the logarithmic wind law and Reynolds' analogy principle (Thom, 1975).

Downward momentum flux can be expressed as:

\[
\frac{T^*}{p} = K_m \frac{\partial U}{\partial z}
\]  

where:

\( K_m \) = transport coefficient for momentum,
\( \frac{\partial U}{\partial z} \) = wind velocity gradient,
\( T^* \) = shearing stress in moving air,
\( p \) = air density.

Under neutral atmospheric conditions with a dry adiabatic lapse rate of 0.01°C/m, wind speed increases logarithmically with height over an extended uniform surface and the wind profile is represented by (Prandtl, 1932; Penman et al., 1967; Thom, 1975):

\[
U = \frac{U^*}{k} \ln \left( \frac{z-d}{z_0} \right)
\]

where:

\( U \) = wind velocity at height \( z \)
\( k \) = von Karman constant commonly taken as 0.04,
\( U^* \) = wind velocity at zero plane displacement,
\( d \) = height of the zero plane displacement,
\( z_0 \) = characteristic roughness length of the surface.
By definition:

$$U^* = \frac{T^*}{p}$$  \hspace{1cm} (14)

Integrating equation (12) between the soil surface and the plane of reference and combining with equations (13) and (14), we obtain (Penman et al., 1967):

$$K_m = \frac{k^2 U (z - d)}{\ln \left(\frac{z}{z_0}\right)}$$  \hspace{1cm} (15)

and by Reynolds' analogy (Penman et al., 1967):

$$K_v = K_m$$  \hspace{1cm} (16)

where $K_v$ is the transport coefficient for vapor transfer.

The atmosphere is seldom at neutral conditions so stages of either stability or instability are most common. When free convection caused by buoyancy forces are small compared with forced convection caused by inertial forces, the atmosphere is said to be in a stable condition, also termed inversion. Conversely, when free convection is strong compared with forced convection, the atmosphere is unstable, or at lapse rate (Monteith, 1973; Thom, 1975).

At non-neutral atmospheric conditions, the wind profile is no longer logarithmic and equation (15) therefore does not describe the momentum transfer constant exactly. This problem has been approached by introducing a correction factor, or stability function, to account for atmospheric departure from neutrality. The Richardson number, $\text{RI}$, and the Obukhov length, $L$, are two atmospheric stability criteria used by several researchers to statistically calibrate stability functions for momentum, vapor, and heat transfer (Pruitt et al., 1971; Thom, 1975; Brutsaert, 1982). The Richardson number, $\text{RI}$, is defined as:

$$\text{RI} = \frac{g \frac{\partial T_p}{\partial z}}{T_p \left(\frac{\partial U}{\partial z}\right)^2}$$  \hspace{1cm} (17)
where:
\( T_p \) = potential temperature,
\( g \) = acceleration of gravity,
\( \partial T_p/\partial z \) = potential temperature gradient,
\( \partial U/\partial z \) = wind velocity gradient.

Pruitt et al. (1971) found that the atmospheric stability function for vapor transfer under extremely stable conditions takes values of about 3 and under extremely unstable conditions has values of about 0.2, both for absolute values of RI greater than 0.9. Negative values for RI are obtained for unstable conditions, whereas positive values indicate a stable atmosphere.

Pruitt et al. (1974) also found that the assumption of equality of transport coefficients for momentum and water vapor on the basis of Reynolds' analogy is not strictly valid because of the differences in diffusion properties of each element in air. They suggested a conductivity coefficient for vapor about 13 percent higher than for momentum under neutral conditions.

Temperature gradients and wind velocity profiles, required in either the Richardson number or Obukhov length (Thom, 1975), are not standard meteorological measurements, therefore are not often available for an agricultural environment. The temperature of the water-air interface is difficult to determine but required to evaluate the appropriate temperature gradient. An estimation of that temperature can be obtained with an energy balance at the water surface that requires other difficult-to-estimate parameters such as net radiation of the water surface and heat flow. Wind-profile estimates also are difficult to obtain. Reliable measurements of wind velocity at a number of different heights are required to include stability effects. Conversely, even if estimates of RI or L were available, the stability function has to be estimated through empirical relationships such as proposed by Monin and Obukhov (1954) or Pruitt et al. (1971).

Correcting equation (15) by a stability coefficient for vapor flow, \( \phi_v \), to be obtained empirically, and specifying \( dp \) and \( z_{op} \) as pan
evaporation parameters, the following relation can be written for the
vapor conductivity coefficient for pan evaporation, $K_{vp}$:

$$K_{vp} = \frac{k^2 \cdot U}{\ln \left( \frac{z - d_p}{z_{op}} \right) \phi_v}$$  \hspace{1cm} (18)

2.2 Derivation of the Evapotranspiration Model.

Water losses from a vegetative cover to the atmosphere are
determined by the interaction of climatic conditions, plant
characteristics, soil physical properties, and water content in the
system (Holz, 1981). Under adequately watered conditions, ET is
controlled by meteorological factors, therefore the process occurs
at its maximum. When some degree of soil-water deficit is present,
plants can restrict water losses by decreasing the stomatal aperture
and ET occurs at a lower rate (Kramer, 1969; Hsiao, 1973). At any
particular time, reference evapotranspiration can be expressed as:

$$E_{Tr} = TR + E$$  \hspace{1cm} (19)

where TR represents plant transpiration and E soil evaporation.

The magnitude of contributions from plant transpiration and soil-
evaporation varies with many factors such as atmospheric evaporative
demand, crop type, stage of the crop development, soil cover, and soil
water content. Commonly, soil evaporation has been reported to be
about 10-12 percent of the annual evapotranspiration. These results
are from estimates made through field experimentation and by energy
balance methods (Rutter, 1975).

Liquid to vapor water change occurs in plants at the cell walls of
mesophyll cells, or evaporative surfaces. From there, vapor diffuses
through intercellular air spaces to stomata then to the atmosphere.
Alternatively, vapor might reach the atmosphere through the cuticula,
but such flow usually is insignificant (Hiaso, 1973).

Because vapor flow occurs through several structures, its
conductance or, inversely, its total resistance, is associated with
plant, canopy, and environmental characteristics. The total resistance corresponds to the summation of several series resistances along the vapor pathway (Nobel, 1970; Burrows and Milthorpe, 1976).

Water vapor for evaporation from the soil is supplied through the soil capillary system in both liquid and vapor phases to the soil surface where the change of state from the liquid fraction to vapor occurs. Water evaporating from the soil surface originates from either a water table or, more commonly, from storage in the soil profile (Brutsaert, 1982). Transport to the surface involves pressure, gravity, and often temperature and salt concentration gradients in a complex phenomena described by Philip (1957) and De Vries (1958).

Under conditions of high soil water content, the evaporation rate depends on the amount of energy reaching the soil surface and on atmospheric conditions. The ability of the soil profile to supply water to the surface decreases when the soil-water content decreases, therefore the limiting factor of the evaporation rate is the soil itself which controls the process thereafter (Brutsaert, 1982).

The pathway of vapor flow for evaporation from soil can be divided in two phases with different properties. First, vapor must be transported through the canopy to the plant height level. Second, vapor must be carried to the reference plane through the free atmosphere. Resistance to vapor flow through these two phases depends on different factors. The resistances involved are the same as that for vapor movement from plant transpiration.

2.2.1 Evapotranspiration Total Vapor Potential Gradient.

Evapotranspiration vapor flow occurs from stomata pores at plant leaves and soil pores at the soil surface to the atmosphere at the reference plane. The driving force of such a flow is the total vapor-potential gradient between the canopy evaporative surfaces, stomata and pores at the soil surface, and the reference plane in the atmosphere. The TVP gradient for evapotranspiration is approximated by:
\[
\frac{dH}{dz}_C = \frac{1}{z} (H_C - H_{wv})
\]  \hspace{1cm} (20)

where \( H_C \) is total water potential at the canopy level. Because in adequately watered plant communities, stomata and soil-surface pores are at or near saturation (Brutsaert, 1982; Hillel, 1980; Waggoner, 1975; Hsiao, 1973; Nobel, 1970), the term \( H_C \) in equation (20) could be assumed equal to zero. Small departures from saturation would produce a negligible change in the TVP gradient, considering the magnitude of vapor potential in the atmosphere already indicated. Under these conditions, equation (20) becomes:

\[
\frac{dH}{dz}_C = -\frac{H_{wv}}{z}
\]  \hspace{1cm} (21)

and analogously to equations (9) and (10):

\[
\frac{dH}{dz}_C = -\left[ \frac{RT}{V_w} \right] \frac{\ln(e/e_0)}{z}
\]  \hspace{1cm} (22)

The right hand side of equation (22) is the same expression used for pan evaporation in equation (10).

2.2.2 Evapotranspiration Conductivity Coefficient.

Slatyer (1967), Kramer (1969), and Nobel (1970) showed resistance networks for plant communities. In general, two groups of resistances to vapor flow can be found: a) intrinsic resistances associated with internal plant physiology or morphology whose major components are mesophyll, stomatal, and cuticular resistances; and b) extrinsic resistances associated with canopy architecture composed of the resistance for the unstirred air layer, aerodynamic canopy resistance, and atmospheric resistance.

Most formulas to estimate resistances require parameters seldom available such as stomatal-pore dimensions, number of stomata per unit
leaf surface, and leaf shape and distribution. Nobel (1970), proposed a relationship for total resistance to diffusion of vapor from the evaporative sites to the surrounding air that includes factors such as effective length of the cell-wall pores, effective length of the waxy layer of the mesophyll cells, depth, radius, and number of stomata per unit leaf area, thickness of the unstirred air layer, and others. On the basis of the hydrodynamic theory for flow adjacent to flat surfaces, Verma et al. (1976) studied unstirred-air-layer resistance and showed that the main factors affecting it are leaf morphology, with size more important than shape, and wind velocity at the leaf level.

Resistance to vapor flow within the plant community, or aerodynamic canopy resistance, is determined by wind speed inside the canopy which decreases exponentially with height from a maximum at plant-height level to a minimum at the soil surface. The extinction coefficient of wind speed depends on canopy architecture, i.e. plant flexibility, leaf-area index, and plant density (Cionco, 1972; Johns et al., 1983).

Atmospheric resistance represents the capacity of the medium to transfer vapor from the top of the vegetative cover to the atmosphere up to the reference plane. Major factors determining this resistance are wind speed and the roughness of the surface which involves plant height, vegetative-cover density, and plant flexibility (Thom, 1975; Bailey and Davies, 1981).

Nobel (1970) used the following expression to describe the total resistance to vapor flow from the evaporation sites at mesophyll cell walls to the air surrounding a leaf:

\[
R_v = \frac{1}{D_w} \left( \frac{d_{ias}}{n \alpha_{st}} + \frac{d_{st}}{r_{st}} + d_a \right)
\]  

(23)

where:

- \( R_v \) = total resistance to vapor flow
- \( d_{ias} \) = effective intercellular air space distance,
- \( d_{st} \) = depth of the stomatal pore,
\( r_{st} \) = radius of the stomatal pore,
\( \alpha_{st} \) = mean area of the stomatal pore,
\( n \) = number of stomata per unit leaf area,
\( D_w \) = diffusion coefficient of water vapor.

Thickness of the unstirred air layer can be expressed as:

\[
d_a = 0.4 \left[ \frac{l_1}{U} \right]^{1/2}
\] (24)

where:
\( l_1 \) = linear dimension of the leaf in the downwind direction,
\( U \) = wind speed.

Equation (23) does not include a term to account for aerodynamic components presented by Johns et al. (1983) through the following relationship for total vapor resistance, \( R_T \):

\[
R_T = R_a + R_c + R_l
\] (25)

where:
\( R_a \) = atmospheric resistance,
\( R_c \) = aerodynamic canopy resistance,
\( R_l \) = internal resistance.

The internal resistance, \( R_l \), corresponds to the combined effect of the resistances of mesophyll cell walls and stomata.

Considering the several plant factors involved in these formulas and the unavailability of sufficient data to estimate the vapor-flow resistance for plant communities, a method analogous to the one used to estimate the conductivity coefficient for pan evaporation was followed introducing appropriate canopy factors. The following relationship can be written for a canopy-conductivity coefficient, \( K_{vc} \):

\[
K_{vc} = \frac{k^2 U}{\ln \left( \frac{z - d_c}{z_{oc}} \right)} (z - d_c) \phi_v \tau
\] (26)
where:

\( d_c \) = zero plane displacement for the canopy,
\( z_{oc} \) = characteristics-roughness length for the canopy,
\( \pi \) = reference-crop coefficient, empirically estimated.

2.3 Theoretical Relationship Between Pan Evaporation and Evapotranspiration

Replacing the factors of equation (5) by the expression of equation (10) as the driving force and the expression of equation (18) as the conductance, a pan evaporation equation was derived:

\[
E_p = - \left( \frac{1}{\lambda} \right) \frac{k^2 u}{\ln \left( \frac{z-d_p}{z_{op}} \right) \phi_v} \frac{RT/V_w}{z} \ln \left( \frac{e}{e_0} \right)
\]

(27)

where \( \lambda \) is the heat of vaporization of water included in equation (27) as unit conversion factor. Combining equations (22) and (26) results in an analogous relationship for reference evapotranspiration:

\[
E_{Tr} = - \left( \frac{1}{\lambda} \right) \frac{k^2 u}{\ln \left( \frac{z-d_c}{z_{oc}} \right) \phi_v} \pi \frac{RT/V_w}{z} \ln \left( \frac{e}{e_0} \right)
\]

(28)

As can be seen by inspection of equations (27) and (28) and from the discussion in sections 2.1.1 and 2.2.1, the expression for the vapor potential gradient \([\left( RT/V_w \right) \lambda \ln(e/e_0)]\) is a common factor to both equations. These equations therefore can be combined in a relationship of \( E_{Tr} \) as function of pan evaporation, \( E_p \), as follows:

\[
E_{Tr} = \frac{K_{vc}}{K_{vp}} E_p
\]

(29)

or in expanded form:
\[ ET_r = \left[ \frac{(z-d_c) \ln \left( \frac{z-d_p}{z_{op}} \right)}{(z-d_p) \ln \left( \frac{z-d_c}{z_{oc}} \right)} \right] E_p \] (30)

where the term in brackets becomes an expression of the pan coefficient, \( K_p \), from equation (1).
3. PROCEDURE FOR DEVELOPMENT OF PAN EVAPORATION AND EVAPOTRANSPIRATION MODELS

Different approaches and scales can be chosen to build models for pan evaporation and evapotranspiration on the basis of equations (27) and (28). To introduce the atmospheric stability factor, as shown in sections 2.1.2 and 2.2.2., it is not possible to avoid an empirical calibration. This empirical part is equivalent to the wind function of Penman's equation (Penman, 1948; Doorenbos and Pruitt, 1977) and also to the empirical coefficient used by Monteith (1973) in his calibration of the Penman method.

The following steps were taken to attain the objectives proposed for this study. First, a statistical model was calibrated for pan evaporation by incorporating variables suggested by the theoretical background reviewed in previous sections. A second step was to develop a model for evapotranspiration through the same approach. Thirdly, conductivity coefficients for pan evaporation and evapotranspiration were estimated using these models and applied in a single relationship of ET$_r$ as a function of E$_p$. To verify the resulting evapotranspiration equation, unbiased ET$_r$ values were generated and compared with measured values. Sensitivity analysis was performed for each variable of the ET$_r$ model and the precision required for the input variables was estimated for each relationship.

3.1 Pan Evaporation Statistical Model.

The theoretical relationship for pan evaporation in equation (27) can be rewritten by grouping together all the universal (K, R, V$_w$) and local (z, d$_p$, z$_{op}$) constants:

\[ E_p = -C_p \phi_v U T \ln(e/e_0) \]  

(31)

where:
Factors $d_p$ and $z_{op}$ are of a complex nature and difficult to estimate. Paeschke (1937) indicates that:

$$z_o = 0.14 h_o$$

(33)

where:

$$h_o = \text{mean height of the roughness obstacles.}$$

The value of $z_o$ from Paeschke (1937), was confirmed by others such as Tanner and Pelton (1960) and Monteith (1973).

Brutsaert (1982) indicated that even when the problem is more complex, available formulas (Letttau, 1969; Seginer, 1974) that include factors such as density, flexibility, and frontal area of the roughness elements, are not practical because of the unavailability of necessary values. Stanhill (1969), indicates that:

$$d_o = 0.7 (h_o)^{0.98}$$

(34)

which is equivalent to:

$$\frac{d_o}{h_o} = 0.64$$

(35)

for an average $h_o$ of 0.66 m (Monteith, 1973; Munro and Oke, 1975). Munro and Oke (1975) indicated that $d_o$ is not as sensitive to surface roughness as $z_o$ and Stigter (1980) and Brutsaert (1982) indicated that for extremely sparse roughness elements, $d_o$ can be considered to be zero.

The only remaining unknown in equation (31) is the atmospheric stability function. Two alternative methods can be used to evaluate
this parameter. The first is to compute values for $\phi_v$ from equation (31), then calibrate a statistical model with $U$, $T$ and others as possible explanatory variables for $\phi_v$. A second approach would be to calibrate a statistical model for $E_p$ with explanatory variables used in equation (31), then compare it with the theoretical relationship. In this second method, $\phi_v$ would be included in the regression coefficients of the calibration equation. The later approach was chosen mainly because the first implies that all errors, coming either from the model development itself or from the data collection, would be included as part of the stability function, which may lead to unrealistic conclusions about that variable.

A regression model was calibrated to estimate $E_p$ for each location. To obtain an unbiased regression equation, a stepwise procedure was used to develop the following general regression model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \ldots$$

$$\ldots + \beta_4 X_1 X_2 + \ldots + \beta_7 X_1 X_2 X_3 + \ldots$$

$$\ldots + \beta_8 X_1^2 + \ldots + E$$

(36)

where:

$Y = E_p$

$X_1 = U$

$X_2 = T$

$X_3 = R$

$\beta_1 = \text{parameters to be estimated}$

$E = \text{random error}$

A previous analysis of the data set showed that the error term from equation (36) was autocorrelated in accordance with Durbin-Watson test (Neter and Wasserman, 1974). Autocorrelation of the error term violates the assumptions applied in the derivation of the regression equations. Therefore, a Gauss-Markov serial correlation scheme was used to estimate the corrected regression statistics which would not cause autocorrelation of the error. This correlation scheme was available through the Time Series Processor, TSP, computer package.
Because TSP does not provide a stepwise procedure, several models were run adding one variable at a time in the same order of variable entrance to the model from ordinary least-squares stepwise analysis. Models obtained were tested for significance of the regression with the F-tests, for significance of the individual regression coefficients with Student's t-test, and for homogeneity and model fitness by residual analysis. After an acceptable model was chosen for each location, a test for equality of regression equations was performed (Draper and Smith, 1966) to determine if local factor effects in the relationships were present.

3.2 Evapotranspiration Statistical Model.

A similar methodology as that used for pan evaporation was followed to develop the evapotranspiration model. In this instance the theoretical equation (28) was used as the starting point. Grouping the universal and local constants, equation (28) can be rewritten as:

\[
ET_r = - C_c \phi_v \pi U T \ln(e/e_0)
\]  

where:

\[
C_c = \frac{k^2 (z-d_c) R}{\ln(-d_c/z) V_w \lambda}
\]

The discussion concerning parameters \(d_p\) and \(z_{op}\) in the previous section applies to \(d_c\) and \(z_{oc}\). The atmospheric stability function, \(\phi_v\), and the crop reference factor, \(\pi\), are the unknowns in equation (37).

As for \(E_p\), a general model for linear regression was developed for \(ET_r\) applying stepwise analysis with ordinary least-squares:
\[
Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_1 X_2 + \ldots \\
\ldots + \beta_7 X_1 X_2 X_3 + \beta_8 X_1^2 + \ldots + E
\]  
(39)

where:

\[Y = ET_r\]
\[X_1 = U\]
\[X_2 = T_d\]
\[X_3 = R_d\]

\[\beta_1\] = parameters to be estimated
\[E\] = random error.

The error terms in equation (39) also were autocorrelated. To obtain the corrected parameters and statistics with the Gauss-Markov serial correlation scheme, several models were run with TSP adding one variable at a time in the same order that they were introduced in the ordinary least-square stepwise analysis. Models were tested for significance of the regression with the F-test and for significance of the individual regression coefficients with Student's t-test. Homogeneity of variance and general model fitness was tested by analysis of residuals. When a model was obtained for each location, a test for equality of the regression equations was used to determine if local effects were present.

3.3 Estimation of Conductivity Coefficients.

To assess a relationship between \(ET_r\) and \(E_p\) shown in equation (29), the following procedure was applied to estimate conductivity coefficients, \(K_{vp}\) and \(K_{vc}\), from the corresponding \(E_p\) and \(ET_r\) regression models.

3.3.1 Conductivity Coefficient for Pan Evaporation.

From equation (18):

\[K_{vp} = q_p U\]  
(40)
where:

\[ q_p = \frac{k^2 (z-d_p)}{\ln(z-d_p) / z_{op}} \]  \hspace{1cm} (41)

Combining equations (27) and (40):

\[ E_p = q_p U \frac{R T}{\lambda V_w z} \ln(e/e_0) \]  \hspace{1cm} (42)

Replacing \( E_p \) in the theoretically based model with \( \hat{E}_p \) from the regression model, \( (e/e_0) \) with relative humidity \( R \), values for \( q_p \) can be computed as follows:

\[ \hat{q}_{p1} = \frac{\hat{E}_{p1} \lambda V_w z}{U_1 T_1 \ln(R_1) R} \]  \hspace{1cm} (43)

\( i = 1, 2, 3, \ldots, \) number of observations for \( E_p \).

Provided that \( q_{p1} \) does not present particular trends:

\[ \hat{q}_p = \frac{1}{N} \sum \hat{q}_{p1} \]  \hspace{1cm} (44)

where:

\( N = \) total number of observations of \( E_p \)

therefore:

\[ \hat{k}_{vp} = \hat{q}_p U \]  \hspace{1cm} (45)
3.3.2 Conductivity Coefficient for Evapotranspiration.

The same methodology as for pan evaporation was followed to obtain the expression for the evapotranspiration conductivity coefficient in equation (26). This relation is similar to the one developed for pan evaporation in equation (40) except for the reference crop factor, \( \pi \), explained in equations (46) and (47).

From equation (26):

\[ K_{vc} = q_c U \]  

where:

\[ q_c = \frac{k^2 (z - d_c) \phi_v \pi}{\ln\left(\frac{z-d_c}{z_{oc}}\right)} \]  

combining equations (28) and (47):

\[ E_{Tr} = q_c U \frac{R}{\lambda V_w z} T \ln\left(\frac{e}{e_0}\right) \]  

and replacing the theoretically based \( E_{Tr} \) by \( \hat{E}_{Tr} \) from the statistically based regression model and \( (e/e_0) \) with relative humidity \( R \), values for \( q_c \) can be computed as follows:

\[ q_{ci} = \frac{\hat{E}_{Tr} \lambda V_w z}{U_i T_i \ln(R_i) R} \]  

\( i = 1, 2, 3, \ldots \), number of observations for \( E_{Tr} \).

As before, if \( q_{ci} \) does not have any particular trends:

\[ q_c = \frac{1}{N} \sum q_{ci} \]
therefore:

\[ \hat{k}_{VC} = \hat{\beta}_c U \]  

(51)

3.4 Model Verification.

To test the overall effectiveness of the resulting ET_r model, daily unbiased estimates were made. Estimated values were compared with measured ET_r for the corresponding days using both graphical and correlation analysis.

3.5 Sensitivity Analysis.

The resulting regression model for ET_r was analyzed in terms of the relative rate of change of the dependent variables with respect to unit changes in the independent variables, as shown in the following equation:

\[ S_j = \frac{\partial (Y)}{\partial x_i} \]  

(52)

j = pan, canopy; i = 1, 2, ..., variables in model

where:

- \( S_j \) = sensitivity coefficient of j,
- \( Y \) = dependent variables, i.e., ET_r,
- \( x_i \) = independent variable, T, U, ln(e/e_0), ...
4. METEOROLOGICAL AND EVAPOTRANSPIRATION DATA DESCRIPTION

4.1 Locations.

Empirical calibrations for evaporation are strongly affected by local climatic conditions. Therefore, the data used in the modeling process was selected carefully. For the purpose of this study, three U.S. sites with widely different climates were chosen among sites with available lysimeter evapotranspiration measurements (Table 1).

Table 1. Data set origin. Geographic situation and main climatic characteristics for three locations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Coshocton OHIO</th>
<th>Davis CALIFORNIA</th>
<th>Kimberly IDAHO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude, deg</td>
<td>40</td>
<td>38</td>
<td>42</td>
</tr>
<tr>
<td>Longitude, deg</td>
<td>81.86</td>
<td>121.74</td>
<td>114.37</td>
</tr>
<tr>
<td>Altitude, m</td>
<td>360</td>
<td>18</td>
<td>1195</td>
</tr>
<tr>
<td>Annual Precip, mm</td>
<td>1116</td>
<td>419</td>
<td>254</td>
</tr>
<tr>
<td>Annual Mean Temp, C</td>
<td>11.9</td>
<td>15.8</td>
<td>7.5</td>
</tr>
</tbody>
</table>

From an agricultural point of view, the climate of Coshocton represents low-elevation humid conditions, Davis a low-elevation semi-arid condition, and Kimberly a high-elevation arid condition.

4.2. Observations.

Meteorological measurements available at each location (Table 2), were collected through standard methods. Pan evaporation corresponds to the Class A pan of the U.S. Weather Service. Evapotranspiration was measured by lysimeters at the three sites. Table 2 also includes the original units of measurement and transformed units to the International System of Units (SI).
Table 2. Daily meteorological observations and units of measurement for three locations.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coshocton</th>
<th>Davis</th>
<th>Kimberly</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temperature</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>F</td>
<td>C</td>
<td>F</td>
<td>K</td>
</tr>
<tr>
<td>Minimum</td>
<td>F</td>
<td>C</td>
<td>F</td>
<td>K</td>
</tr>
<tr>
<td>Dew Point</td>
<td>-</td>
<td>-</td>
<td>F(^{1})</td>
<td>K</td>
</tr>
<tr>
<td><strong>Relative Humidity</strong></td>
<td>percent</td>
<td>percent</td>
<td>-</td>
<td>decimal</td>
</tr>
<tr>
<td>Maximum</td>
<td>percent</td>
<td>percent</td>
<td>-</td>
<td>decimal</td>
</tr>
<tr>
<td><strong>Solar Radiation</strong></td>
<td>ly/day</td>
<td>ly/day</td>
<td>ly/day</td>
<td>ly/day</td>
</tr>
<tr>
<td><strong>Wind Run</strong></td>
<td>mile</td>
<td>km</td>
<td>mile</td>
<td>cm</td>
</tr>
<tr>
<td><strong>Pan Evaporation</strong></td>
<td>in</td>
<td>mm</td>
<td>in</td>
<td>cm</td>
</tr>
<tr>
<td><strong>Evapotranspiration</strong></td>
<td>in</td>
<td>mm</td>
<td>mm</td>
<td>cm</td>
</tr>
</tbody>
</table>

\(^{1}\) Measured daily at 7 a.m.

Daily meteorological observations were available for the full year for Davis from 1965 to 1971. For Coshocton, (1978-1982), and Kimberly, (1965-1978), daily observations were available only for the growing season corresponding to the period from October to April (Table 3).

Table 3. Number of years and mean number of daily meteorological observations per year in the data set of each location.

<table>
<thead>
<tr>
<th>Location</th>
<th>No. Years</th>
<th>Obs/Year</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>5</td>
<td>150</td>
<td>750</td>
</tr>
<tr>
<td>Davis</td>
<td>7</td>
<td>365</td>
<td>2555</td>
</tr>
<tr>
<td>Kimberly</td>
<td>14(^{*})</td>
<td>180</td>
<td>2520</td>
</tr>
</tbody>
</table>

\(^{*}\) Lysimeter evapotranspiration data available for only 3 years (1969-1971).
4.3. Data Set Analysis.

To obtain a complete and consistent data set, it was necessary to perform some transformations and changes of scale for some of the observations. Also, the quality of the data was verified as described in the following sections.

4.3.1. Relative Humidity.

Because relative humidity measurements for Kimberly were not available (Table 2), an estimation was performed. Considering that dew point temperature, $T_{dp}$, measured early in the morning (0700 hrs) is considered to be a good daily average (Burman et al., 1983), maximum and minimum relative humidities can be computed by use of the expression by Bosen (1958):

$$\text{rh} = \left[ \frac{112 - 0.1 T + T_{dp}}{112 + 0.9 T} \right]^8$$

(53)

where:
- $\text{rh}$ = relative humidity,
- $T$ = air temperature,
- $T_{dp}$ = dew point temperature.

Maximum relative humidity is computed by use of minimum air temperature in equation (53) and minimum relative humidity with the corresponding maximum air temperature.

4.3.2. Day-Period Temperature and Relative Humidity.

Evaporation and evapotranspiration processes occur mainly during daylight hours following in general the daily variation of solar radiation (Brutsaert, 1982). In a preliminary study (Appendix A), evaporation showed stronger correlation with daily maximum temperature than with daily minimum temperature. Such results suggested that the
appropriate average temperature to be used in this study would be a weighted-average temperature corresponding to the day period rather than the daily-mean temperature computed as the simple average of maximum and minimum daily temperature. Because a 5-day data set with measurements at half-hour intervals from Davis was available, a method to estimate the day-period temperature, $T_d$, on the basis of maximum, $T_x$, and minimum, $T_m$, temperatures was developed as is shown by the equation:

$$T_d = \frac{1}{3} (2T_x + T_m)$$  \hspace{1cm} (54)

This expression showed the highest correlation with the average temperature for the daylight period from the actual data (Appendix A).

Analogously, the role of relative humidity was studied with a similar procedure. The minimum relative humidity, $(e/e_o)_m$, is more strongly related to evaporation than the maximum relative humidity, $(e/e_o)_x$. This suggested that a weighted-average relative humidity corresponding to the day-period would be a more appropriate value to use. The following equation expresses the day-period relative humidity, $R_d$, for the 5-day data set from Davis (Appendix A):

$$R_d = \frac{1}{3} \left[\frac{e}{e_o}_x + 2\frac{e}{e_o}_m\right]$$  \hspace{1cm} (55)

4.3.3. Wind Speed.

Wind run was measured at different heights at each location. At Coshocton wind was measured at 10 m, for Davis at 2 m and for Kimberly at 3.67 m above the ground. Because the Penman and other evapotranspiration equations require 2 m as the height of wind measurement, values from Coshocton and from Kimberly were transformed with the relationship below (Burman et al., 1983):
\[ U_2 = U_z \left( \frac{z}{2} \right)^{0.2} \]  

(56)

where \( U_2 \) is wind speed at 2 m height and \( U_z \) is wind speed actually measured at height \( z \).

4.4 Lysimeter.

The lysimeter at Coshocton is of the monolith type and its main use is in watershed-hydrology studies. At Davis and Kimberly, the lysimeters are for evapotranspiration studies and they are of the fill-in type. The main difference between these two types of lysimeters is that the monolith type is filled with a core of undisturbed soil whereas in the fill-in type the soil is disturbed but replaced keeping the same order of its layers (Aboukhaled et al., 1982). The lysimeters are of the weighing type with hourly recording equipment at all three locations.

4.4.1 Coshocton.

Soil at the lysimeter (Y-101) site is a Muskingun silt loam, permeable, with good drainage, and 240+ cm deep. Slope and soil profile are typical of the watershed. Cover is brome grass with no irrigation.

The dimensions are 1.90 m across, 4.27 m in length and 2.44 m deep. The lysimeter area is 8.22 m² or 0.008 ha and precision is ± 0.025 cm of water evaporated (Harrold and Dreibelbis, 1958; 1967).

4.4.2 Davis.

Soil at the lysimeter is a disturbed Yolo loam with no change in structure and without horizons within the first meter. These soil characteristics, and the filling techniques used, make the alterations of the soil profile negligible. Cover is alta fescue grass with
irrigation at 50% depletion of soil-moisture content (Pruitt and Angus, 1960).

Lysimeter dimensions are 6.1 m in diameter and 0.91 m deep. The area is 29.22 m² or 0.0029 ha. Precision for this lysimeter is ± 0.003 cm of water evaporated (Pruitt and Angus, 1960).

4.4.3 Kimberly.

The lysimeter is located in a 2.8-ha field and planted with alfalfa which is harvested as hay. The field is irrigated to keep soil-water content in the range of 20 to 60 kPa at 5 cm depth.

The lysimeter tank is a square of 1.83 m on a side and 1.22 m deep. The lysimeter area is 3.34 m² or 0.003 ha and precision is ± 0.05 mm of water evaporated (Wright and Jensen, 1972).

4.5 Utilization of the Data.

To have data available to perform an unbiased verification of the resulting models, data sets for each location were randomly split by years into two fractions. One group was used in model development and the other reserved to generate ETₚ values for model verification.
5. RESULTS OF PAN EVAPORATION AND EVAPOTRANSPIRATION
STATISTICAL MODEL DEVELOPMENT

Results of statistical model development are presented in the
following sections. Results from the pan evaporation modeling and
from the evapotranspiration modeling are reported and discussed
separately.

5.1 Pan Evaporation Statistical Model.

Because the original data sets from Coshocton and Davis included
an odd number of years, one more year was included for model
development than for verification at both locations. The data set
from Kimberly was split into two equal parts except for lysimeter
evapotranspiration for which only three years were available (Table 4).

Table 4. Sub-sets of data used in $E_p$ and $ET_r$ models
development and verification.\(^1\)

<table>
<thead>
<tr>
<th>Location</th>
<th>Development ($E_p$ and $ET_r$)</th>
<th>Verification ($ET_r$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>1979</td>
<td>1978</td>
</tr>
<tr>
<td></td>
<td>1980</td>
<td>1981</td>
</tr>
<tr>
<td></td>
<td>1982</td>
<td></td>
</tr>
<tr>
<td>Davis</td>
<td>1965</td>
<td>1966</td>
</tr>
<tr>
<td></td>
<td>1967</td>
<td>1968</td>
</tr>
<tr>
<td></td>
<td>1969</td>
<td>1970</td>
</tr>
<tr>
<td></td>
<td>1971</td>
<td></td>
</tr>
<tr>
<td>Kimberly</td>
<td>1968</td>
<td>1969</td>
</tr>
<tr>
<td></td>
<td>1969</td>
<td>1970</td>
</tr>
<tr>
<td></td>
<td>1970</td>
<td>1971</td>
</tr>
<tr>
<td></td>
<td>1974</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1975</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1976</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\) At Kimberly, lysimeter evapotranspiration was available
only for 1969, 1970 and 1971. All three years were used
for $ET_r$ model development and verification.
5.1.1 Stepwise Analysis.

To obtain an unbiased regression model, a stepwise-regression analysis was run with the day-period temperature, $T_d$, the day-period relative humidity, $R_d$, the 24-hour wind run, $U$, the corresponding logarithmic and square transformations and their interactions as independent variables. Twenty-seven independent variables (Appendix B) were included in the stepwise procedure with pan evaporation, $E_p$, as the dependent variable.

The variation of the adjusted $R^2$ (Neter and Wasserman, 1974) with the entering of the first five variables to the model was analyzed. After the second variable was in the model, little improvement of the adjusted $R^2$ was achieved (Figure 1). The first two variables were the ones which had a high degree of association with pan evaporation.

At Davis and Kimberly, the first variable entering the model was the day-period temperature, $T_d$, and the second was the triple interaction $U T_d ln R_d$. For Coshocton, however, the first variable in the model was the day-period relative humidity, $R_d$, and the second was $T_d$. For Coshocton, where the variable entering sequence was different from the other sites, $T_d$ and $U T_d ln R_d$ were still present among the first five variables in the model. Because the models for Davis and Kimberly included the term $U T_d ln R_d$, suggested by the theoretical equation (31), a model including those variables was run for Coshocton to ascertain the possibility of using a common model for the three sites. A comparison of two regression equations for Coshocton was run. One including the first two variables from the stepwise, $R_d$ and $T_d$, was compared with the one including $U T_d ln R_d$ and $T_d$. Both models, as indicated by the $F$-test, were significant in explaining part of the total variance of the dependent variable. However, the percentage of that variance explained, indicated by the adjusted $R^2$, is small in either instance with the first relationship being slightly better than the second (Table 5).
Figure 1. Ordinary least squares (OLS) stepwise analysis of pan evaporation model. Adjusted \( R^2 \) variation with the first five variables entering the model for (a) Coshocton, (b) Davis and (c) Kimberly.
Figure 1. Continued
Figure 1. Continued
Table 5. Statistics for two models of \( E_p \) for Coshocton.

<table>
<thead>
<tr>
<th>Variable in model</th>
<th>( R^2 )-adj</th>
<th>( F )</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_d, T_d )</td>
<td>0.16</td>
<td>41.64*</td>
<td>0.042</td>
</tr>
<tr>
<td>( UT_dlnR_d, T_d)</td>
<td>0.13</td>
<td>30.92*</td>
<td>0.044</td>
</tr>
</tbody>
</table>

* Significant with \( \alpha = 0.01 \).

Considering the above results and the fact that the values of the mean-squared error, MSE, for the models differed by less than 5 percent (Table 5), it was decided that the model containing the variables \( UT_dlnR_d \) and \( T_d \) could be used as a \( E_p \) model for Coshocton and for the other locations. This model was defined by the following equation:

\[
E_p = b_0 + b_1 \ UT_dlnR_d + b_2 T_d
\]  

(57)

5.1.2. Correction for Autocorrelation.

Data used in the above regression analysis were sequential in time. Consequently, the error term of the resulting model was tested for degree of autocorrelation. If autocorrelation existed, the statistics, especially the regression coefficients, would be inefficient although still unbiased (Draper and Smith, 1966). The Durbin-Watson statistical test was used to determine if the error term from the three regression models presented autocorrelation (Table 6).

Table 6. Autocorrelation test for \( E_p \) regression models for Coshocton, Davis and Kimberly.

<table>
<thead>
<tr>
<th>Location</th>
<th>Durbin-Watson*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>0.94</td>
</tr>
<tr>
<td>Davis</td>
<td>1.11</td>
</tr>
<tr>
<td>Kimberly</td>
<td>1.42</td>
</tr>
</tbody>
</table>

* Significant with \( \alpha = 0.01 \).
For all locations, the Durbin-Watson test was not significant which, in accordance with Neter and Wasserman (1974), meant that the autocorrelation coefficient of the error term, RHO, was not zero. As indicated in section 3.1, a correction procedure was used to improve the statistics obtained with the regression analysis. The correction, based on Gauss-Markov generalized least-squares estimation, was performed by adjusting iteratively the dependent and independent variables until the error terms became independent in a first-order serial-correlation scheme (Cochrane and Orcutt, 1949).

As is apparent in the case of Davis, the relatively high adjusted $R^2$ indicated that the resulting model was able to explain a large part of the total variance of the dependent variable (Table 7). Combined with the low mean-squared error, MSE, the model was considered acceptable. For Kimberly, the percentage of variance explained through the model after correction for autocorrelation was 64 percent (Table 7). This value could be viewed as acceptable considering the large number of observations included.

<table>
<thead>
<tr>
<th>Location</th>
<th>$R^2$-adj</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>0.14</td>
<td>0.0437</td>
</tr>
<tr>
<td>Davis</td>
<td>0.86</td>
<td>0.0139</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.64</td>
<td>0.0257</td>
</tr>
</tbody>
</table>

The MSE for Kimberly was almost twice as large as for Davis. This fact, together with the lower adjusted-$R^2$, made the model less efficient for Kimberly than for Davis. For Coshocton, however, the model did not have the capacity to explain a significant part of the variance, as is shown by the low adjusted-$R^2$ value (Table 7).

At this point, a decision was needed to further investigate the applicability of the theoretical model or search for an entirely different approach. Three main factors were considered in making the decision: (a) the proposed objective of this work, which was to
verify a specific theoretical model with the available data; (b) the results from the stepwise analysis which showed that the 2 variables considered in the model, of the initial 27, were the best combination for Davis and Kimberly (Figure 1) and they also were among the five best for Coshocton; and (c) the large amount of variation associated with the pan evaporation measurements for Coshocton compared with the other two sites (Appendix C). Furthermore, no statistical procedure could be expected to predict precisely the extreme values observed for Coshocton, as proven by the results of the corresponding stepwise analysis.

On the basis of the above considerations, it was decided to further investigate the possibilities of improving the application of the theoretically-based model even though some specific restrictions might be required.

5.1.3. Data Filtering.

A moving average was used as a filtering technique to smooth the severe effect of the extreme values in the data set. To retain the ability of comparing the results among the locations, the smoothing procedure was applied in the same manner to the three data sets. To ascertain the optimum number of days in the moving average, the model was run several times increasing the number of days in the moving average with each run. The regression model used, (57), included the variables UTdlnRd and Td corrected for autocorrelation of the residuals. The decrease in the MSE became negligible at a different number of days for each location (Figure 2). First was Davis with 3 days, second Kimberly with 4 days, and finally Coshocton with 5 days. To maintain the ability to compare the results among sites, a 5-day common moving average was chosen. The filtered values were computed as the mean of the observation of the corresponding day plus the previous 4 days.

The independent variables in the model again showed a significant association with the dependent variable in accordance with the values of the F-test (Table 8). However, after filtering, a larger part of
Figure 2. Relationship between the number of days included in the moving-average-filtering technique and the variation of the mean-squared error, MSE, for the $E_p$ regression model at each location.
the variance of the dependent variable is explained by the model, as indicated by the high adjusted-$R^2$ for all the locations under study.

Table 8. Coefficient of multiple determination, $R^2$-adj, and F-test from $E_p$ regression model corrected for autocorrelation with 5-day moving average.

<table>
<thead>
<tr>
<th>Location</th>
<th>$R^2$-adj</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>0.83</td>
<td>941*</td>
</tr>
<tr>
<td>Davis</td>
<td>0.99</td>
<td>67736*</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.96</td>
<td>18077*</td>
</tr>
</tbody>
</table>

* Significant with $\alpha = 0.01$.

5.2 Evapotranspiration Statistical Model.

The data sub-sets applied in the pan evaporation modeling also were used for evapotranspiration modeling except for Kimberly for which lysimeter measurements were available for only three years (Table 4). All three evapotranspiration data years, 1969, 1970, and 1971, were included in the model development phase for Kimberly. No attempt to reserve data for verification was made considering the limited data available.

Because of the great similarity of the $E_T_r$ modeling procedure with $E_p$ modeling, results will be reported emphasizing those aspects in which differences were found.

5.2.1 Stepwise Analysis.

For this statistical analysis, the same group of variables used for pan evaporation (Appendix B) were included for modeling $E_T_r$. Figure 3 shows the sequence of the first five variables entering the model for each location and the associated multiple coefficient of determination, adjusted-$R^2$. As for the pan evaporation model, the two first variables explained most of the variance in $E_T_r$. The variable entering sequence was different for each location. A model with the
Figure 3. Ordinary least squares (OLS) stepwise analysis of evapotranspiration model. Adjusted R² variation with the first five variables entering the model for (a) Coshocton, (b) Davis and (c) Kimberly.
Figure 3. Continued
Figure 3. Continued
same variables as for \( E_p \) was obtained only at Davis. These results from the stepwise analysis did not show agreement with either the theoretical equation for \( ET_r \), equation (37), or with the calibrated model for pan evaporation. Further investigation was required to analyze the applicability of the proposed theoretical model. A relationship that included the triple-interaction term from the theoretical analysis and the correction factor from the \( E_p \) calibration was computed for Coshocton and Kimberly. This relationship is given by the following equation:

\[
ET_r = b_0 + b_1 UT_d \ln R_d + b_2 T_d
\]  

(58)

No difference between the stepwise model and the model of equation (58) was observed for Kimberly (Table 9). The adjusted-\( R^2 \) with the first two variables (\( \ln T, \ln U \)) in the stepwise model was about 0.44 (Figure 3). For Coshocton, a lower percentage of the variance was explained with the model of equation (58) compared with the original stepwise model (Table 9). The adjusted-\( R^2 \) with the first two variables in the stepwise model was about 0.45 (Figure 3). This result suggested that the model of equation (58) did not describe \( ET_r \) adequately for Coshocton.

<table>
<thead>
<tr>
<th>Location</th>
<th>( R^2 )-adj</th>
<th>( F )</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>0.32</td>
<td>97.1*</td>
<td>0.016</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.43</td>
<td>240.6*</td>
<td>0.037</td>
</tr>
</tbody>
</table>

* Significant with \( \alpha = 0.01 \).

However, considering the improvements for the pan evaporation models achieved with the correction for autocorrelation and the filtering process, the Coshocton data-set was retained for the next steps to
further investigate application of the theoretically-based model at this site.

5.2.2 Correction for Autocorrelation.

Autocorrelation of the residuals from the model of equation (58) was examined for the three locations by the Durbin-Wason test (Table 10). No location showed a significant Durbin-Watson test, which means that the model for each location was not the most efficient. As discussed in section 5.1.2, a more efficient model could be obtained by using a serial-correlation correction.

Table 10. Statistical test for autocorrelation of ET<sub>r</sub> regression model.

<table>
<thead>
<tr>
<th>Location</th>
<th>Durbin-Watson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>1.24</td>
</tr>
<tr>
<td>Davis</td>
<td>0.77</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.57</td>
</tr>
</tbody>
</table>

For Davis and Kimberly, a high percentage of the total variance of the dependent variable was explained through the model corrected for autocorrelation (Table 11).

Table 11. Coefficient of multiple determination, R<sup>2</sup>-adj, and mean-squared error, MSE, from ET<sub>r</sub> regression models corrected for autocorrelation.

<table>
<thead>
<tr>
<th>Location</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;-adj</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>0.43</td>
<td>0.0134</td>
</tr>
<tr>
<td>Davis</td>
<td>0.79</td>
<td>0.0107</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.73</td>
<td>0.0175</td>
</tr>
</tbody>
</table>

For Coshocton, the adjusted-R<sup>2</sup> showed that only 43 percent of the corresponding variance was explained. The statistics related to ET<sub>r</sub>
(Table 11) showed higher values of adjusted-\(R^2\), except for Davis, and lower MSE for all locations compared to the \(E_p\) equivalents (Table 7). The exception at Davis was not of significant magnitude because the adjusted-\(R^2\) for that location decreased by less than 10 percent whereas the MSE decreased by more than 20 percent.

5.2.3 Data Filtering.

A number of different moving averages were tried to develop a model comparable to the \(E_p\) model. Figure 4 shows that with the 5-day moving average, about 90 percent of the MSE reduction was obtained for Coshocton and Kimberly and even more for Davis. Therefore, the \(ET_r\) regression model corrected for autocorrelation was computed for each location with filtered data.

Evapotranspiration was significantly represented by the variables included in the model, \(UT_d\ln R_d\) and \(T_d\), as shown by the high significance (\(\alpha = 0.01\)) of the F-test for the three locations (Table 12).

Table 12. Coefficient of multiple determination, \(R^2\)-adj, and F-test for \(ET_r\) regression model corrected for autocorrelation with 5-day moving average.

<table>
<thead>
<tr>
<th>Location</th>
<th>(R^2)-adj</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>0.89</td>
<td>1629*</td>
</tr>
<tr>
<td>Davis</td>
<td>0.99</td>
<td>83560*</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.96</td>
<td>8453*</td>
</tr>
</tbody>
</table>

* Significant with \(\alpha = 0.01\).

As expected with use of the 5-day moving average and the autocorrelation correction, a large part of the variance of the dependent variable, \(ET_r\), was explained by the regression model.
Figure 4. Relationship between the number of days included in the moving-average-filtering technique and the variation of the mean-squared error, MSE, for the ET<sub>r</sub> regression model at each location.
6. COMPARISON BETWEEN THEORETICAL AND STATISTICAL MODELS

6.1 Pan Evaporation Model.

To analyze the aptness of the Ep model, the assumptions of normal distribution and constant variance of the residuals were verified graphically as shown in Appendix D. The normal plot of the residuals approaches a straight line. When the residuals are plotted against Ep, the distribution about the mean is nearly constant. At no location were significant departures from the implicit assumptions of the linear regression model observed.

Because all the regression coefficients were significantly different from zero (Table 13), the regression equation for each location can be interpreted as the pan-evaporation response to the effect of the day-period temperature, \( T_d \), and to the triple interaction of wind run, day-period temperature, and the logarithm of the day-period relative humidity, \( R_d \). This relationship was expressed in equation (57).

Table 13. Regression coefficient, \( b_i \), and standard error, \( s_{b_i} \), for Ep models corrected for autocorrelation and with 5-day moving average.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Coshocton</th>
<th>Davis</th>
<th>Kimberly</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_0 )</td>
<td>-3.284*</td>
<td>-5.484*</td>
<td>-7.056*</td>
</tr>
<tr>
<td>( s_{b0} )</td>
<td>0.889</td>
<td>0.288</td>
<td>0.312</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>-8.145E-11*</td>
<td>-3.538E-11*</td>
<td>-3.849E-11*</td>
</tr>
<tr>
<td>( s_{b1} )</td>
<td>1.208E-11</td>
<td>0.078E-11</td>
<td>0.181E-11</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.012*</td>
<td>0.020*</td>
<td>0.026*</td>
</tr>
<tr>
<td>( s_{b2} )</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

* Significant with \( \alpha = 0.01 \).

The pan-evaporation response to the effect of the triple interaction \( UT_d \ln R_d \) is in agreement with the proposed theory, discussed in section 2.1.1 and represented by equation (27), or in its
reduced form by equation (31). Results in Table 13, however, indicate that pan evaporation also had a strong response to the day-period temperature. This factor is not explicit in equation (31) which could be considered as a lack of agreement between that theoretical equation and the experimental results. Another approach in interpreting the presence of \( b_2 T_d \) and the intercept of the regression model, \( b_0 \), can be obtained considering the following: (a) the atmospheric-stability function depends on wind speed and temperature profiles that were not available for this model, and (b) the atmospheric-stability function is the only factor completely unknown in equation (31). Therefore, it was expected that an indirect estimation of such a factor would occur through the model calibration. Consequently, the terms \( b_2 T_d \) and \( b_0 \) added together can be described as a correction factor that includes an expression of the atmospheric-stability function based on the day-period temperature. Even if such an interpretation can be useful in predicting pan evaporation, a partial lack of agreement between theory and results is still present. In equation (31), the stability function is included as a multiplicative factor of \( E_p \), but the regression model calibrated for \( E_p \) uses it additively.

6.2 Evapotranspiration Model.

The general aptness of the \( E_T \) model was examined in terms of the residual analysis. Normality of the residual distribution and homogeneity of the variance were verified graphically (Appendix D). At no location were departures from the implicit assumptions of the regression model found. This result together with the statistics on Table 12 makes the model acceptable to estimate \( E_T \).

The regression coefficients of the model for each location (Table 14) were significantly different from zero indicating that reference evapotranspiration responds significantly to the variables included. These were day-period temperature, \( T_d \), and the triple interaction of the day-period temperature, wind run, and the logarithm of the day-period relative humidity.
Table 14. Regression coefficients, \( b_i \), and standard error, \( s_{bi} \), for the \( ET_r \) regression models corrected for autocorrelation and with 5-day moving average.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Coshocton</th>
<th>Davis</th>
<th>Kimberly</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_0 )</td>
<td>-3.542*</td>
<td>-4.328*</td>
<td>-4.194*</td>
</tr>
<tr>
<td>( s_{b0} )</td>
<td>0.537</td>
<td>0.202</td>
<td>0.453</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>-9.601E-11*</td>
<td>-1.698E-11*</td>
<td>-1.785E-11*</td>
</tr>
<tr>
<td>( s_{b1} )</td>
<td>7.164E-12</td>
<td>5.416E-13</td>
<td>2.575E-12</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.013*</td>
<td>0.016*</td>
<td>0.016*</td>
</tr>
<tr>
<td>( s_{b2} )</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
</tbody>
</table>

* Significant with \( \alpha = 0.01 \).

As discussed for pan evaporation, the response of the evapotranspiration term to the triple interaction is in agreement with the theoretical equation (28) and its reduced expression in equation (37). The presence of \( T_d \) in the model can also be interpreted as a partial lack of agreement with the theoretical equation (37). The term obtained from the addition of \( b_0 \) and \( b_2 T_d \) of the regression equations can be interpreted as a correction factor representing the interaction of the atmospheric-stability function and the reference-crop coefficient which are the unknowns in equation (37).

6.2.1 Verification of Local Effects.

Evapotranspiration-regression equations for each location were compared with each other to verify if a unique equation could be found. The statistical procedure followed is described by Neter and Wasserman (1974). Ideally, the test applied would compare the mean squared error of the reduced model (a single equation for the three locations) against the sum of the mean-squared errors from the three separate equations. Because of limitations of the computer software available for the regression analysis of time series, it was not possible to obtain an estimation of the reduced model for the three locations.
Comparisons were able to be made between two locations at a time. No difference was detected between the ET$_r$ equations for Davis and for Kimberly. Yet the equations for Davis and Kimberly were different from the equation for Coshocton. This was shown by the F value computed as the ratio of the MSE of the reduced model over the MSE of the full model.

Table 15. Comparison of ET$_r$ regression models.

<table>
<thead>
<tr>
<th>Location</th>
<th>Compared with:</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davis</td>
<td>Coshocton</td>
<td>169.2*</td>
</tr>
<tr>
<td>Kimberly</td>
<td>Coshocton</td>
<td>52.8*</td>
</tr>
<tr>
<td>Davis</td>
<td>Kimberly</td>
<td>0.1</td>
</tr>
</tbody>
</table>

* Significant with $\alpha = 0.01$.

The difference detected for Coshocton can be explained either by presence of strong local effects at that site or possibly by an experimental error in the Coshocton data set. In the first instance, although the local effects are not explicitly included in equation (28), they can be reasoned to be represented by the parameters $z_0C$ and its associated $d_C$ related to wind speed and crop architecture. The characteristic-roughness length of the surface, $z_0$, is a function of wind speed for non-ocean conditions (Stigter, 1980). Monteith (1973) indicated that the behavior of $z_0$ and $d$ are associated strongly with the crop-architectural characteristics, including plant density, plant flexibility, height of the canopy, and leaf size (Brutsaert, 1982).

The reference-crop coefficient in equation (30) involves not only the architectural-crop characteristics and internal- and external-diffusion resistances, but also the general crop management at the lysimeter site. This latter factor constitutes another source of local variation. An example of different management methods is the irrigation criteria used at each location. As indicated in section 4.4, irrigation was applied at Davis after 50 percent depletion of
available soil moisture, at Kimberly when 60-kPa soil tension was reached, and no irrigation was applied at Coshocton. Evaluation of these practices on evapotranspiration without additional soil and plant information from the lysimeter sites was not possible.

An alternative to explain these resulting local differences was to analyze the experimental error of the data set. The main findings were that, at Coshocton, the experimental site conditions for the lysimeter and the meteorological station are different. The lysimeter and meteorological station sites at Davis and Kimberly are essentially the same. The most significant factor is that the meteorological station and the lysimeter sites at Coshocton are not in the same field and are exposed to different meteorological effects. The meteorological station is in the middle of a grass field on a flat area at the top of a hill with about 200 m of fetch. The lysimeter is in an area not as exposed to the wind with a general slope of 23 percent (Harrold and Dreibelbis, 1958). The lysimeter itself is on a 12-13 percent grade. These facts can explain a difference in consistency of data for Coshocton compared to Davis and Kimberly. Meteorological variables such as temperature and wind run are strongly associated with the landscape.

It seems reasonable to conjecture that the lack of consistency in some data from Coshocton contributed to the difference in the model for that location. This is especially so considering that no local effects were found between Davis and Kimberly even though these two sites have distinct geographical and climatological differences as can be observed in Table 1. Other meteorological phenomena associated with arid or semi-arid regions cannot be disregarded from affecting these results. Internal-boundary layers formed over the vegetative cover in arid areas can produce a sharp discontinuity in the vapor gradient. Humid sites like Coshocton also exhibit less extreme diurnal fluctuations in temperature and relative humidity than arid regions which gives a different meaning to any weighted average of temperature and relative humidity.
6.2.2 Sensitivity Analysis.

To estimate the precision required for each of the variables involved in the ET$_r$ model, an analysis of the sensitivity for each variable was performed. The sensitivity coefficient of equation (52) was developed for each variable in the ET$_r$ model represented by equation (58).

(a) Sensitivity coefficient for wind run, $S_u$:

$$S_u = \left( \frac{U}{ET_r} \right) b_1 T_d \ln R_d$$  \hspace{1cm} (59)

(b) Sensitivity coefficient for relative humidity, $S_r$:

$$S_r = \left( \frac{1}{ET_r} \right) b_1 U T_d$$  \hspace{1cm} (60)

(c) Sensitivity coefficient for temperature, $S_t$:

$$S_t = \left( \frac{T_d}{ET_r} \right) (b_1 U \ln R_d + b_2)$$  \hspace{1cm} (61)

The sensitivity coefficient represents the relative change in reference evapotranspiration with respect to relative change in each of the variables. Sensitivity coefficients were calculated for each location using the mean of daily values for the variables in the corresponding equation (Appendix E).

The highest sensitivity coefficient of the model corresponds to temperature (Table 18). This confirmed the results obtained in the stepwise analysis which indicated that temperature explained more than half of the total ET$_r$ variation.
Table 16. Sensitivity coefficients for the reference evapotranspiration model.

<table>
<thead>
<tr>
<th>Location</th>
<th>$S_u$</th>
<th>$S_r$</th>
<th>$S_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>1.39</td>
<td>-1.05</td>
<td>10.9</td>
</tr>
<tr>
<td>Davis</td>
<td>0.15</td>
<td>-0.27</td>
<td>13.0</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.17</td>
<td>-0.24</td>
<td>9.1</td>
</tr>
</tbody>
</table>

The second highest sensitivity coefficient corresponds to relative humidity for Davis and Kimberly which are in semi-arid or arid locations. For Coshocton, a humid site, wind run has the second highest sensitivity coefficient.

The computation of the required precision for each variable in the model at each location was made for an increment of 0.1 mm in reference evapotranspiration. The values of the corresponding variables were kept constant at their 24-hour mean. Because daily $ET_r$ is usually expressed in mm/day with one significant decimal place, the increment of 0.1 mm was chosen for this analysis. The results are indicated in Table 17.

Table 17. Precision required for each variable relative to 0.1 mm change in $ET_r$.

<table>
<thead>
<tr>
<th>Location</th>
<th>$U_{rd}$ (cm/day) x10^5</th>
<th>$R_d$ fraction</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>9.65</td>
<td>0.018</td>
<td>0.742</td>
</tr>
<tr>
<td>Davis</td>
<td>36.50</td>
<td>0.058</td>
<td>0.618</td>
</tr>
<tr>
<td>Kimberly</td>
<td>32.61</td>
<td>0.042</td>
<td>0.705</td>
</tr>
<tr>
<td>Mean</td>
<td>26.25</td>
<td>0.039</td>
<td>0.688</td>
</tr>
</tbody>
</table>
7. ANALYSIS OF EVAPOTRANSPIRATION/PAN EVAPORATION RELATIONSHIP

7.1 Conductivity Coefficients.

To estimate the conductivity coefficients for pan evaporation, $K_{vp}$, and for evapotranspiration, $K_{vc}$, equations (43) and (49) were applied. The resulting pan evaporation and evapotranspiration models for the three locations included a correction factor, $F_C$, given by:

$$F_{Ci} = b_{0i} + b_{2i} T_d$$  \hspace{1cm} (62)

where $i$ represents $E_p$ or $ET_r$. Equations (43) and (49) were modified by subtracting the correction factor from the numerator of each equation. Combining equation (43) and equation (62) for pan evaporation:

$$q_p = \left[ \frac{\lambda V_w z}{R} \right] \frac{\hat{E}_pi - b_0 - b_2 T_{di}}{U_1 T_{di} \ln R_{di}}$$  \hspace{1cm} (63)

$$= \left[ \frac{\lambda V_w z}{R} \right] \frac{b_1 U_1 T_{di} \ln R_{di}}{U_1 T_{di} \ln R_{di}}$$  \hspace{1cm} (64)

$$q_p = b_1 \left[ \frac{\lambda V_w z}{R} \right]$$  \hspace{1cm} (65)

Analogously, combining equations (49) and (62) for evapotranspiration:

$$q_c = \left[ \frac{\lambda V_w z}{R} \right] \frac{\hat{ET}_i - b_0 - b_2 T_{di}}{U_1 T_{di} \ln R_{di}}$$  \hspace{1cm} (66)

$$q_c = b_1 \left[ \frac{\lambda V_w z}{R} \right]$$  \hspace{1cm} (67)
where $b_1$ from equation (62) will be designated as $b_p$ to show that it is from the pan-evaporation regression model, and $b_1$ from equation (64) as $b_c$ to show that it is from the evapotranspiration-regression model.

The conductivity coefficient for pan evaporation, $\hat{k}_{vp}$, was obtained combining equation (45) and equation (65):

$$\hat{k}_{vp} = b_p \left[ \frac{\lambda}{R} \right] U$$  \hspace{1cm} (68)

The conductivity coefficient for evapotranspiration, $\hat{k}_{vc}$, was obtained combining equation (51) and equation (67):

$$\hat{k}_{vc} = b_c \left[ \frac{\lambda}{R} \right] U$$  \hspace{1cm} (69)

As indicated in previous discussion, the coefficients $b_p$ and $b_c$ for Davis and Kimberly were statistically equal and different from the corresponding coefficients of Coshocton for both pan evaporation and evapotranspiration (Table 18).

<table>
<thead>
<tr>
<th>Location</th>
<th>$b_p$ (x10^{-11})</th>
<th>$b_c$ (x10^{-11})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>-8.145a</td>
<td>-9.601c</td>
</tr>
<tr>
<td>Davis</td>
<td>-3.538b</td>
<td>-1.698d</td>
</tr>
<tr>
<td>Kimberly</td>
<td>-3.849b</td>
<td>-1.517d</td>
</tr>
</tbody>
</table>

1 Same letter indicates no difference, $\alpha = 0.01$. 
7.2 Evapotranspiration as Function of Pan Evaporation.

Equation (29) described the relationship between $E_T$ and $E_p$ vapor flows as the quotient of the corresponding conductivity coefficients. Substituting equations (68) and (69) into equation (29), the following expression is obtained:

$$E_T = \frac{b_c}{b_p} E_p$$  (70)

The values for the ratio $b_c/b_p$ were computed for each location. Simultaneously and as a comparison criteria, a linear regression model through the origin, corrected for autocorrelation, was calculated between evapotranspiration as the dependent variable and pan evaporation as the independent variable in accordance with the general model:

$$E_T = b^* E_p$$  (71)

where:

$b^*$ = regression coefficient.

The coefficient $b^*$ from equation (71) represents a statistical estimation of the ratio $b_c/b_p$ of equation (70). The same range of values was found between the estimated $b_c/b_p$ ratio and the $b^*$ for Davis and Kimberly (Table 19). The differences in wind exposure between the meteorological station and the lysimeter site can possibly explain some of the difference in results from Coshocton. The parameter $b_c$ obtained from the evapotranspiration model was developed with the 24-hour wind run from the meteorological station at Coshocton while the evapotranspiration was measured at the lysimeter site which has a different exposure.
Table 19. Coefficients of proportionality between $E_{Tr}$ and $E_p$.

<table>
<thead>
<tr>
<th>Location</th>
<th>$b_c/b_p$</th>
<th>$b^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>1.18</td>
<td>0.23</td>
</tr>
<tr>
<td>Davis</td>
<td>0.48</td>
<td>0.50</td>
</tr>
<tr>
<td>Kimberly$^1$</td>
<td>0.39</td>
<td>0.43</td>
</tr>
</tbody>
</table>

$^1$ Corrected for grass.

7.3 Verification of Evapotranspiration Function.

To determine the overall effectiveness of the evapotranspiration model for the three locations, reference evapotranspiration was estimated by use of equation (70) and the values in Table 18 for the coefficients $b_c$ and $b_p$. The pan-evaporation data used corresponded to the second half of the data set reserved for verification and not used in model calibration. Estimated reference evapotranspiration, $E_{Te}$, was compared with measured $E_{Tr}$. This procedure was not followed for Kimberly because data for only three years were available. At Kimberly, the model verification was made with the same data set already used for model calibration. This does not constitute an unbiased verification per se for Kimberly, but at least represents a test of the reasonableness of the model. For all sites, the comparison between estimated $E_{Te}$ and measured $E_{Tr}$ values was made by plotting cumulative values of $E_{Te}$ against the cumulative $E_{Tr}$ on a double-mass curve. The values compared are equal whenever the double-mass curve parallels a line making a 45 degree angle above the abscissa with a slope equal to 1.0 (Harrold and Dreibelbis, 1958). Figure 5, (a) through (c), shows the double-mass curves obtained for each location with data filtered with a 5-day moving average. At Coshocton, the slope of the double-mass curve is higher than 1.0 which means that the calculated reference evapotranspiration overestimated the measured values. The opposite result was obtained for Davis and Kimberly where the slopes of the double-mass curves were...
Figure 5. Double-mass curve for estimated reference evapotranspiration and lysimeter measured reference evapotranspiration.
Figure 5. Continued
Figure 5. Continued
lower than 1.0. The model for these latter two sites underestimated the actual evapotranspiration. Because the double-mass curves in the three cases are straight lines, the errors involved in the estimation are constants. In terms of the model of equation (70), the error comes from the ratio $b_c/b_p$. By use of the calibration-data set, a correction coefficient to adjust the $b_c/b_p$ ratio was computed. From equation (70), estimated evapotranspiration was given by:

$$ET_e = \frac{b_c}{b_p} E_p$$

(72).

From the double-mass curve on the average:

$$ET_e = m \ ET_r$$

(73)

where $m$ is the slope of the double-mass curve. Combining equations (72) and (73):

$$ET_r = \frac{1}{m} \ \left( \frac{b_c}{b_p} \right) E_p$$

(74)

Again by use of the calibration-data set, the slope $m$ for each double-mass curve and the $b_c/b_p$ ratios were calculated (Table 20). A weighted average for Davis-Kimberly also was computed because the equations for those two locations did not show significant differences. The adjusted ratio was weighted by the number of observations at each site. The values for the adjusted ratios were similar to each other and they were in the range of the values for the pan-evaporation coefficient given by Doorenbos and Pruitt (1977). The adjusted ratio is defined as the product of $1/m$ times $b_c/b_p$. 
Table 20. Slopes of the double-mass curve and adjusted ratios, $(b_c/b_p)_{adj}$

<table>
<thead>
<tr>
<th>Location</th>
<th>m</th>
<th>$(1/m)b_c/b_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>1.59</td>
<td>0.74</td>
</tr>
<tr>
<td>Davis</td>
<td>0.62</td>
<td>0.77</td>
</tr>
<tr>
<td>Kimberly</td>
<td>0.60</td>
<td>0.62</td>
</tr>
<tr>
<td>Davis-Kimberly</td>
<td>0.62</td>
<td>0.73</td>
</tr>
</tbody>
</table>

The fact that the $(b_c/b_p)_{adj}$ value for Coshocton is in the same range as that for the other locations is attributable to the averaging effect of the double mass-curve technique. Results of applying these $(b_c/b_p)_{adj}$ ratios to estimate reference evapotranspiration by equation (74) with a 5-day moving average is presented in Figure 6, (a) through (c).

Estimated values of reference evapotranspiration were of the same general magnitude as the measured values at each of the three locations as shown by the regression line between those two variables (Figure 6). A minimal dispersion of the values is observed at Davis. The largest dispersion occurred at Kimberly which contains some points on the upper left corner of the graph on Figure 6(c) that can be considered outliers. These outliers represent a small fraction of the 631 points on the graph. Table 21 presents statistics obtained through correlation between estimated reference ET$_e$ and measured reference ET$_r$ based on all data available for verification.

Although the estimated mean is statistically different from the mean of measured reference evapotranspiration for the three locations, their values are similar and the mean error of estimation of the mean ET$_r$ varies from an underestimation of 0.03 mm/day for Kimberly to an overestimation of 0.09 mm/day for Coshocton and of 0.06 mm/day at Davis.
Figure 6. Estimated reference evapotranspiration, ET', versus lysimeter measured reference evapotranspiration, ETr, for (a) Coshocton, (b) Davis and (c) Kimberly. NSE is the normalized standard error.
Figure 6. Continued

(b) DAVIS

ET = 1.0159 ET - 0.001
NSE = 0.14
Figure 6. Continued

(c) KIMBERLY

\[ E_T = 0.434 \, E_T^r + 0.200 \]

\[ NSE = 0.66 \]
Table 21. Measured mean $ET_r$, mean and confidence interval for estimated $ET_e$, and error of the estimation.

<table>
<thead>
<tr>
<th>Location</th>
<th>$ET_r$ mm/day</th>
<th>$ET_e$ mm/day</th>
<th>Confidence interval* mm/day</th>
<th>$ET_r - ET_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coshocton</td>
<td>3.60</td>
<td>3.69</td>
<td>3.75-3.63</td>
<td>0.025</td>
</tr>
<tr>
<td>Davis</td>
<td>3.68</td>
<td>3.74</td>
<td>3.77-3.69</td>
<td>-0.016</td>
</tr>
<tr>
<td>Kimberly**</td>
<td>4.21</td>
<td>4.18</td>
<td>4.20-4.15</td>
<td>0.007</td>
</tr>
</tbody>
</table>

* With $\alpha = 0.01$.
** $ET_r$ for grass.
8. SUMMARY AND CONCLUSIONS

8.1 Summary.

A theoretical derivation was performed to relate pan evaporation and evapotranspiration. Because the driving force for vapor transfer was the same for pan evaporation and evapotranspiration, similar models were developed for both vapor flows on the basis of the gas law and the logarithmic wind law with temperature, relative humidity and wind run as meteorological variables. The differences in the vapor flow from plants and from a free-water surface were included in the conductivity coefficient of vapor through the air. The theoretical relationship between the two vapor flows was a coefficient formed by the ratio of the conductivity coefficients from each flow.

To calibrate and verify the proposed models, a large data set was used. The data available were from three locations with different climatic conditions (arid, semi-arid and humid), with a total of 5800 days of daily meteorological observations. The data set included daily lysimeter-measured evapotranspiration.

A statistical procedure was followed to assure an unbiased calibration of the models. A model was obtained by use of stepwise analysis that contained not only the triple interaction of temperature, relative humidity and wind run as the theoretical derivation suggested, but also an additional term involving the temperature plus a constant for both pan evaporation and evapotranspiration. Additional terms in the model were considered to represent a correction factor accounting mainly for the atmospheric-stability function, a factor difficult to describe with the standard-meteorological-data set.

A time series procedure was applied to efficiently obtain estimated parameters after autocorrelation of the error term from the regression models was found. To decrease the mean-squared error of the regression equations, thereby improving the coefficient of determination, a 5-day moving average technique was applied to the
data. On the basis of the parameters of this improved model, the conductivity coefficients were estimated for pan evaporation and evapotranspiration. The precision required for the input variables in the evapotranspiration model was evaluated through a sensitivity analysis that demonstrated the strong temperature dependence of this model.

Finally, a verification of the ET$_T$ model was conducted. Comparisons of estimated-reference evapotranspiration with measured values was made with double-mass curves. An adjustment factor for the $b_C/b_p$ ratios was obtained. The final model estimated the mean evapotranspiration for all verification data taken together with less than 10 percent error. Jensen (1973) found that the error in estimating average seasonal evapotranspiration for 10 locations varied from 14 percent to 55 percent by use of 18 different methods. The lowest errors were found for the Papadakis and Penman methods. For the three locations of this study, the lowest average error found by Jensen was 14 percent with evapotranspiration estimated by the van Bavel-Businger method (Jensen, 1973). The seasonal averages would be expected to be more reliable than the daily-mean evapotranspiration results reported for this study. Corroboration of results reported by Jensen (1973) indicates the strength of the model developed in this project compared with previously applied models.

8.2 Description of the Model Application.

The final model, relating evaporation and pan evaporation, obtained through this study is described by equation (74).

$$ET_p = \frac{1}{m} \left( \frac{b_C}{b_p} \right) E_p$$

(74)

The values for the parameters $m$, $b_C$, and $b_p$ correspond to the model calibration for each location with filtered series of pan-evaporation observations as a 5-day moving average expressed in cm/day (Tables 18 and 20). In the calibration procedure, the moving averages
were computed as the mean of the observation of the corresponding day plus the previous 4 days. For application of this model, a 5-day moving average $E_p$ should be entered into equation (74) with the appropriate $m$, $b_c$, and $b_p$ factors. The results from equation (74) are then daily $E_{Tr}$ estimates starting at the fifth day in the series.

8.3 Conclusions.

The following specific conclusions were obtained from this study:

1. The theoretically derived models for pan evaporation and for evapotranspiration were found to adequately describe the corresponding vapor flow when: (1) a temperature-dependent-correction factor was included, (b) the models were calibrated locally, (c) the input-data set was filtered with a 5-day moving average technique and (d) the regression-parameter estimation was made with correction for autocorrelation of the error term.

2. The theoretical relationship between evapotranspiration and pan evaporation was represented by the ratio of conductivity coefficients for each flow, $K_c/K_p$. Use of the regression models indicated that the theoretical ratio corresponded to the ratio of the regression coefficients, $b_c/b_p$, associated with the triple interaction $UT_d\ln R_d$ from each model. The $E_{Tr}$ function was improved by adjusting the $b_c/b_p$ ratio with an empirical factor, $1/m$, obtained from the $E_{Tr}/E_p$ double-mass curve.

3. Evapotranspiration can be estimated on basis of pan evaporation by use of the derived function if filtered input data with a 5-day moving average are used. The function estimated the overall mean $E_{Tr}$ with less than 10 percent error compared to lysimeter measurements for the three locations of this study.

4. The variable with the highest sensitivity in the model to estimate evapotranspiration was temperature. The precision required for temperature measurements was on the average 0.7 °C to allow for an evapotranspiration estimate with 0.1 mm/day precision. The next most sensitive variable was wind run, for Coshocton, and
relative humidity for Davis and Kimberly. On the average, the precision required for wind-run measurements was 26 km/day (equivalent to 0.3 m/s) and that for relative humidity was 4 percent to estimate ET with 0.1 mm/day precision.

5. The 5-day moving average was the minimum number of days that reduced the mean-squared error of the regression models to a level such that any further reductions were negligible.

8.4 Recommendations for Future Research.

Three principal future research suggestions evolved from this study.

1. As indicated in the objectives, the scope of this study was development of a relationship between pan evaporation and evapotranspiration. In the model development process, expressions were found relating pan evaporation and evapotranspiration to meteorological variables either theoretically or statistically. No specific verification was made for these individual models as was done for the relationship between evapotranspiration and pan evaporation. An independent validation of the models for $E_p$ and $E_T$ as functions of the meteorological variables, therefore, is a next step.

2. The adjustment factor introduced into the $E_T$ model by the slope of the double mass-curve relating $E_T$ and $E_T$ is totally empirical. On the basis of the results of the investigation proposed in point 1, a better understanding might be obtained about how to incorporate the slope parameter into the theoretical relationship between pan evaporation and evapotranspiration.

3. The sensitivity analysis performed in this study was with the total mean values for the variables involved. Those variables change during the year giving a seasonal variation to the sensitivity of the model for each variable. Additional research is required to quantify the sensitivity of individual variables in determining $E_T$ on a seasonal basis.
BIBLIOGRAPHY


APPENDICES
APPENDIX A.

Day-Period Temperature and
Day-Period Mean Relative Humidity
### Day-Period Temperature Analysis

5-day half-hourly data set from Davis, CA

<table>
<thead>
<tr>
<th>Date</th>
<th>$T_X$</th>
<th>$T_m$</th>
<th>$\bar{T}_d$</th>
<th>$\bar{T}_n$</th>
<th>$T_d$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>020666</td>
<td>21.68</td>
<td>5.46</td>
<td>17.32</td>
<td>8.94</td>
<td>16.30</td>
<td>13.60</td>
</tr>
<tr>
<td>030666</td>
<td>23.70</td>
<td>6.45</td>
<td>18.91</td>
<td>10.94</td>
<td>18.00</td>
<td>15.10</td>
</tr>
<tr>
<td>130766</td>
<td>25.70</td>
<td>11.26</td>
<td>20.99</td>
<td>13.21</td>
<td>20.90</td>
<td>18.50</td>
</tr>
<tr>
<td>140766</td>
<td>26.10</td>
<td>11.14</td>
<td>21.09</td>
<td>15.18</td>
<td>21.20</td>
<td>18.70</td>
</tr>
<tr>
<td>040567</td>
<td>19.42</td>
<td>5.92</td>
<td>15.33</td>
<td>8.96</td>
<td>14.80</td>
<td>12.40</td>
</tr>
</tbody>
</table>

$T_X$ = Daily maximum temperature  
$T_m$ = Daily minimum temperature  
$\bar{T}_d$ = Day-period mean daily temperature  
$\bar{T}_n$ = Night-period mean daily temperature  
$T_d$ = Estimated day-period daily temperature  
$\bar{T}$ = Daily mean temperature

\[ T_d = \frac{1}{3} \left[ 2 \, T_X + T_m \right] \]  \hspace{1cm} (32)

\[ \bar{T} = \frac{1}{2} \left[ T_X + T_m \right] \]  \hspace{1cm} (75)
Day-Period Relative Humidity Analysis

5-day half-hourly data set from Davis, CA

<table>
<thead>
<tr>
<th>Date</th>
<th>$R_x$</th>
<th>$R_m$</th>
<th>$\bar{R}_d$</th>
<th>$\bar{R}_n$</th>
<th>$R_d$</th>
<th>$\bar{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>020666</td>
<td>0.84</td>
<td>0.31</td>
<td>0.46</td>
<td>0.74</td>
<td>0.49</td>
<td>0.58</td>
</tr>
<tr>
<td>030666</td>
<td>0.87</td>
<td>0.24</td>
<td>0.41</td>
<td>0.67</td>
<td>0.45</td>
<td>0.56</td>
</tr>
<tr>
<td>130766</td>
<td>0.89</td>
<td>0.41</td>
<td>0.58</td>
<td>0.80</td>
<td>0.57</td>
<td>0.65</td>
</tr>
<tr>
<td>140766</td>
<td>0.87</td>
<td>0.35</td>
<td>0.54</td>
<td>0.72</td>
<td>0.52</td>
<td>0.61</td>
</tr>
<tr>
<td>040567</td>
<td>1.00</td>
<td>0.49</td>
<td>0.68</td>
<td>0.88</td>
<td>0.66</td>
<td>0.75</td>
</tr>
</tbody>
</table>

$R_x$ = Daily maximum temperature

$R_m$ = Daily minimum temperature

$\bar{R}_d$ = Day-period mean daily temperature

$\bar{R}_n$ = Night-period mean daily temperature

$R_d$ = Estimated day-period daily temperature

$\bar{R}$ = Daily mean temperature

\[ R_d = \frac{1}{3} [R_x + 2R_m] \]  

(33)

\[ \bar{R} = \frac{1}{2} [R_x + R_m] \]  

(76)
APPENDIX B.

Independent Variables Included in Stepwise Analysis for Pan Evaporation and Evapotranspiration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_d$</td>
<td>Day-period temperature</td>
</tr>
<tr>
<td>$R_d$</td>
<td>Day-period relative humidity</td>
</tr>
<tr>
<td>$U$</td>
<td>Wind run at 2 m height</td>
</tr>
<tr>
<td>$T_d^2$</td>
<td></td>
</tr>
<tr>
<td>$\ln T_d$</td>
<td></td>
</tr>
<tr>
<td>$\ln R_d$</td>
<td></td>
</tr>
<tr>
<td>$\ln U$</td>
<td></td>
</tr>
<tr>
<td>$U^2$</td>
<td></td>
</tr>
<tr>
<td>$T_d\ U$</td>
<td></td>
</tr>
<tr>
<td>$T_d\ R_d$</td>
<td></td>
</tr>
<tr>
<td>$R_d\ U$</td>
<td></td>
</tr>
<tr>
<td>$U\ T_d\ R_d$</td>
<td></td>
</tr>
<tr>
<td>$T_d\ \ln U$</td>
<td></td>
</tr>
<tr>
<td>$T_d\ R_d^2$</td>
<td></td>
</tr>
<tr>
<td>$R_d\ \ln T_d$</td>
<td></td>
</tr>
<tr>
<td>$R_d^2$</td>
<td></td>
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<tr>
<td>$T_d\ U^2$</td>
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</tr>
<tr>
<td>$R_d\ \ln U$</td>
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</tr>
<tr>
<td>$R_d\ T_d^2$</td>
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</tr>
<tr>
<td>$R_d^2\ U^2$</td>
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</tr>
<tr>
<td>$U\ \ln R_d$</td>
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</tr>
<tr>
<td>$U\ T_d^2$</td>
<td></td>
</tr>
<tr>
<td>$U\ R_d^2$</td>
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</tr>
<tr>
<td>$T_d\ U\ R_d$</td>
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</tr>
<tr>
<td>$U\ T_d\ \ln R_d$</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX C.

Meteorological Variables at Coshocton, Davis and Kimberly versus Time

Calibration Data Set
PAN EVAPORATION - COSHOCTON

Apr - Oct, 79, 80, 82

Graph showing variations in evaporation.
MEAN TEMPERATURE  .COSHOCTON

Apr - Oct. 79, 80, 82

Observations
PAN EVAPORATION

Jon Dec. 66, 67, 69, 71

Observations

Jan - Dec. 66, 67, 69, 71
MEAN TEMPERATURE

APR - OCT, 68, 69, 70, 71, 72, 76

OBSERVATIONS
APPENDIX D.

Residual Analysis for Pan-Evaporation and Evapotranspiration Regression Models
PAN EVAPORATION COSHOCTON
EVAPOTRANSPERSION COSHOCTON
EVAPOTRANSPERSION  DAVIS

![Graph showing data points on a grid with axes labeled 'ETHAT' and '0.1' to '0.15' and '-0.10' to '0.1'.]
PAN EVAPORATION KIMBERLY
EVAPOTRANSPIRATION KIMBERLY
NORMAL PROBABILITY PLOT

RESIDUAL
EVAPOTRANSPIRATION KIMBERLY
APPENDIX E. Statistics for Meteorological Variables after 5-day Moving Average Filtering

<table>
<thead>
<tr>
<th>Statistic</th>
<th>ET&lt;sub&gt;r&lt;/sub&gt; cm/day</th>
<th>EP cm/day</th>
<th>T&lt;sub&gt;d&lt;/sub&gt; °K</th>
<th>Rd</th>
<th>U cm/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location: COSHOCTON</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>0.002</td>
<td>0.</td>
<td>280.57</td>
<td>0.343</td>
<td>0.210E+7</td>
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<tr>
<td>Maximum</td>
<td>0.912</td>
<td>1.295</td>
<td>303.53</td>
<td>0.936</td>
<td>3.021E+7</td>
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<tr>
<td>Mean</td>
<td>0.362</td>
<td>0.491</td>
<td>294.49</td>
<td>0.693</td>
<td>1.348E+7</td>
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<tr>
<td>St. Dev.</td>
<td>0.153</td>
<td>0.225</td>
<td>4.01</td>
<td>0.116</td>
<td>0.525E+7</td>
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<tr>
<td>Location: DAVIS</td>
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<tr>
<td>Minimum</td>
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<td>0.</td>
<td>276.43</td>
<td>0.143</td>
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<td>Maximum</td>
<td>1.080</td>
<td>1.730</td>
<td>309.99</td>
<td>1.000</td>
<td>8.960E+7</td>
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<tr>
<td>Mean</td>
<td>0.361</td>
<td>0.427</td>
<td>290.48</td>
<td>0.574</td>
<td>1.989E+7</td>
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<tr>
<td>St. Dev.</td>
<td>0.228</td>
<td>0.310</td>
<td>74.03</td>
<td>0.148</td>
<td>1.149E+7</td>
</tr>
<tr>
<td>Location: KIMBERLY</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
<td>0</td>
<td>273.35</td>
<td>0.223</td>
<td>0.187E+7</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.524</td>
<td>302.98</td>
<td>0.976</td>
<td>7.554E+7</td>
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<tr>
<td>Mean</td>
<td>0.675</td>
<td>290.61</td>
<td>0.498</td>
<td>2.333E+7</td>
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<tr>
<td>St. Dev.</td>
<td>0.268</td>
<td>6.24</td>
<td>0.107</td>
<td>1.050E+7</td>
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</tr>
</tbody>
</table>