AN ABSTRACT OF THE THESIS OF

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(Name) (Degree) (Major)

Date thesis is presented 5-12-60

Title Non-Saturating Amplifier Theory

Abstract approved [redacted]
(Major professor)

The performance of certain high-gain amplifiers, such as those used in null detectors, can be greatly improved by using a nonlinear element in a feedback loop. With the addition of nonlinear feedback, it is possible to obtain an amplifier that can handle a large dynamic range of input signals without saturating or blocking. The desired characteristic of the amplifier is a voltage gain that varies instantaneously and inversely with input signal amplitude. Thus, for small input signals, the voltage gain is maximum and as the input signal amplitude increases, the voltage gain decreases to unity or less.

The characteristics of a number of suitable nonlinear elements are investigated and a mathematical function is derived to describe them.

The variation in voltage gain of an amplifier with a single nonlinear feedback loop is investigated, both theoretically and experimentally, and the results are in good agreement.

Several single-loop nonlinear feedback amplifiers can be connected in cascade to obtain the equivalent of nonlinear elements that are not available. This method is used in an example of an approximation of a logarithmic function.

NON-SATURATING AMPLIFIER THEORY

by

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A THESIS

submitted to

OREGON STATE COLLEGE

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

June 1960

APPROVED:

[redacted]

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Date thesis is presented 5-12-60

Typed by Janette Crane

ACKNOWLEDGMENT

The work reported in this thesis was performed under the direction of L. N. Stone, Professor and Head of the Department of Electrical Engineering at Oregon State College. The author wishes to express his appreciation to Professor Stone for his suggestions, constructive criticisms, and advice.

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NON-SATURATING AMPLIFIER THEORY

INTRODUCTION

There are a number of special applications which require amplifiers that are capable of handling a large dynamic range of input signals. The voltage ratio of the largest to smallest signal may be on the order of 10^5 to 10^8. One such application is the logarithmic amplifier that is used with a neutron detector to measure the neutron concentration level in a nuclear reactor. Another application is the video pulse amplifier used in radar receivers. A third application is the null amplifier that is used as a detector in bridge circuits or as an error amplifier in servomechanisms.

Along with the required large dynamic range, these applications have another characteristic in common in that some distortion can be tolerated at large signal levels. Thus, these amplifiers should be easier to obtain than linear amplifiers with a large dynamic range.

Unfortunately, there is also a problem common to the above applications. If a very large signal is applied to the input of a high gain amplifier, successive stages will be either cutoff or saturated, causing the amplifier to block for a period following the large signal with the result that the amplifier cannot amplify small signals during that period (4, p. 113-122). This blocking effect arises when there is a change in the amount of energy stored in the coupling and bypass capacitors. The change in energy is due to nonlinear resistance phenomena which result from cutoff, saturation, or grid current flow in vacuum tubes, and cutoff or saturation in transistors. The problem of blocking can be eliminated by restricting the amplifier to a linear region of operation which is incompatible with a large dynamic range.

NONLINEAR FEEDBACK

A non-saturating amplifier with a large dynamic range can be obtained by incorporating nonlinear elements in one or more feedback paths in the amplifier. The desired result is an amplifier with an overall gain that varies instantaneously and inversely with input signal level. Usually the gain is maximum for small input signals and decreases to unity or even considerably less than unity at large signal levels. The exact manner in which the gain should vary depends upon the specific application. For a null amplifier, the gain should have a smooth exponential or logarithmic variation from maximum to minimum. For a pulse amplifier, the gain might be constant up to some particular signal level and then change drastically to less than unity to give a limiting action. In any case, the gain variation will be determined almost completely by the nonlinear functions of the elements in the feedback paths.

SINGLE-LOOP FEEDBACK AMPLIFIERS

Figure 1 shows the feedback circuit that is commonly used with operational amplifiers in analog computers. For this circuit, the voltage transfer function is approximately,

$$\frac{e_o}{e_i} = -\frac{Z_f}{Z_i} \qquad (1)$$

with the derivation shown in the appendix. If the input impedance (Z_i) is constant, the voltage ratio is directly proportional to the impedance in the feedback path (Z_f) as long as the assumptions in the derivation are true. The particular case investigated treats Z_i as a constant resistance and Z_f as a nonlinear resistance. The limits of $\frac{e_o}{e_i}$ as Z_f approaches zero or infinity are,

$$\lim_{Z_f \to \infty} \left[\frac{e_o}{e_i}\right] = -A \qquad (2)$$

$$\lim_{Z_f \to 0} \left[\frac{e_o}{e_i}\right] = 0 \qquad (3)$$

with $Z_i \neq 0$.

For the case where Z_i is small,

$$\lim_{\substack{Z_f \to 0 \\ Z_i \to 0}} \left[\frac{e_o}{e_i}\right] = -1 \qquad (4)$$

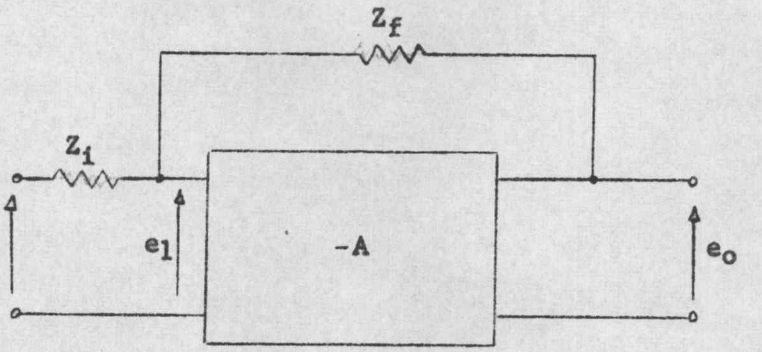


Figure 1

Basic Feedback Amplifier Circuit

Since the nonlinear resistance (Z_f) is a two-terminal device, it is usually described in terms of voltage and current. For example, an equation that describes a large proportion of the available nonlinear resistances is,

$$e = K i^n \tag{5}$$

where K is a constant and n may be either a constant or a variable. Since the standard definition of impedance is not applicable to nonlinear impedances, a suitable definition that will be used hereafter is,

$$\text{instantaneous impedance} = \frac{\text{instantaneous voltage}}{\text{instantaneous current}}$$

or

$$Z = \frac{e}{i} \tag{6}$$

If Equation (5) is rearranged to give,

$$i = \frac{e^{1/n}}{K^{1/n}} \tag{7}$$

and Equation (7) is substituted in Equation (6); then,

$$Z = \frac{K^{1/n}\ e}{e^{1/n}} \tag{8}$$

The voltage ratio of the basic feedback amplifier of Figure 1, including the nonlinear feedback impedance is found as follows:

$$Z_f = Z = \frac{K^{1/n}\ e}{e^{1/n}} \tag{9}$$

where $e = e_o$ since e_1 is approximately equal to zero. Then,

$$e_o = \frac{-Z_f}{Z_i}\ e_i \tag{10}$$

or

$$e_o = \frac{-K^{1/n}\ e_o}{Z_i\ e_o^{1/n}}\ e_i \tag{11}$$

simplifying,

$$e_o^{1/n} = \frac{-K^{1/n}}{Z_i}\ e_i \tag{12}$$

and raising both sides of the equation to the $n^{\underline{th}}$ power

$$e_o = \frac{-K}{Z_i^n}\ e_i^n = -K'\ e_i^n \tag{13}$$

where $K' = \dfrac{K}{Z_i^n}$.

Some caution must be observed in using Equation (13) since it is subject to the limitations of Equations (2), (3), (4) and the assumptions inherent in Equation (1). The most interesting feature of Equation (13) is that, except for the constant, it describes exactly the same curve as Equation (5). In other words, the gain of the feedback amplifier will have exactly the same characteristic curve as that of the nonlinear impedance in the feedback path. Consequently, one way of obtaining a particular desired gain characteristic is to find a nonlinear element with the same characteristic curve and insert it in the feedback path. The value of K' depends on Z_i and, thus, can be set arbitrarily by using the corresponding value of Z_i. If the value of Z_i is restricted, K' can also be varied by applying only a fraction of the amplifier output voltage to the feedback element.

NONLINEAR RESISTANCES

There are a number of electron devices whose characteristics can be described by Equation (5). Some examples are: semiconductor and vacuum-tube diodes, copper-oxide and selenium rectifiers, thermistors, and silicon-carbide or thyrite varistors. The voltage-current characteristics of a number of these devices are shown in Figure 2. The approximate value of n, which is the slope of the curve, is given on each curve. The value of K is

$$K = \frac{e}{i^n} \qquad (14)$$

and can be evaluated by substituting in the appropriate values of e, i, and n. If the characteristic is not a straight line, the value of K will only apply to the portion of the curve for which the value of n applies.

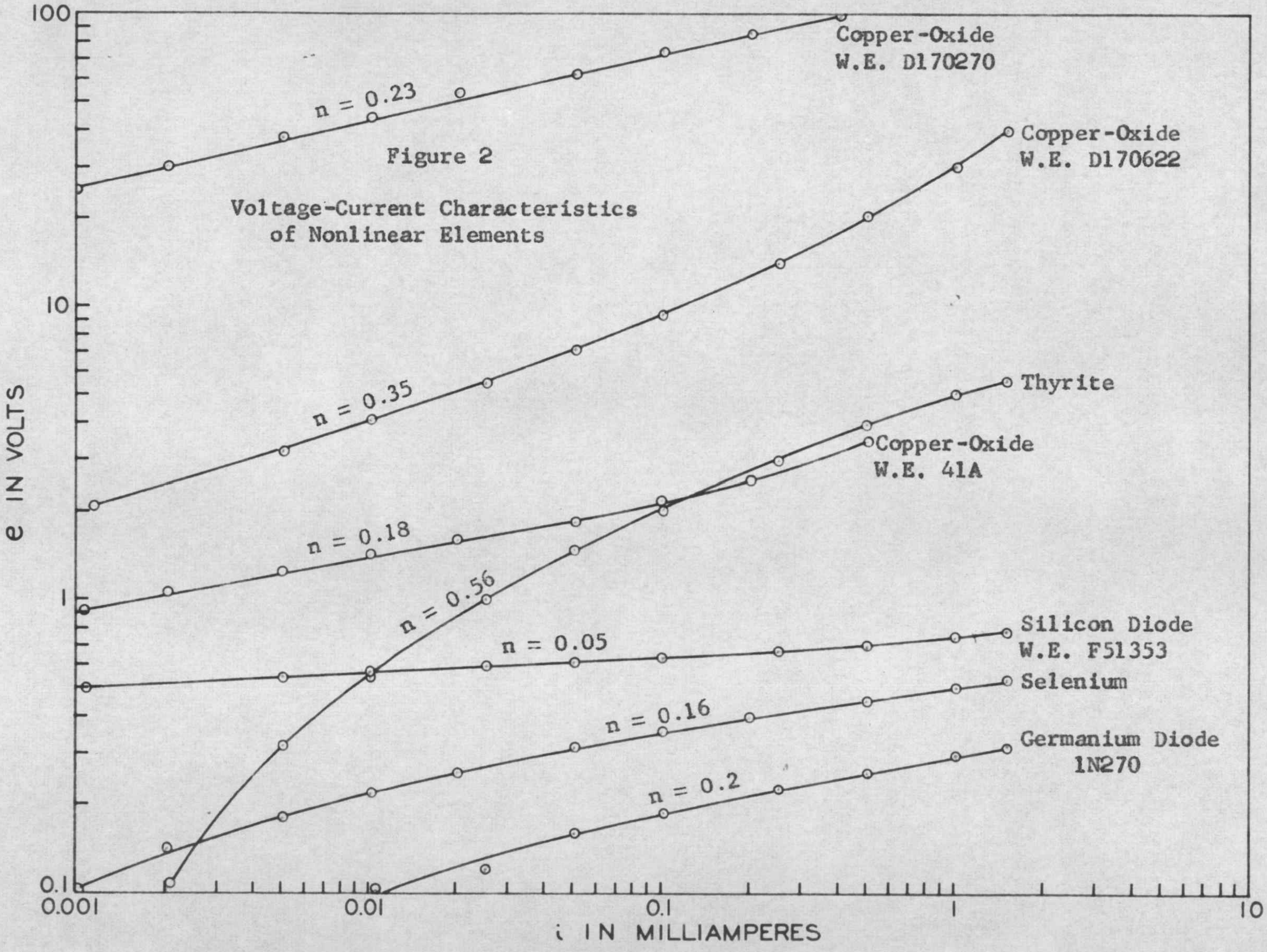


Figure 2
Voltage-Current Characteristics
of Nonlinear Elements
Copper-Oxide
W.E. D170270
n = 0.23
Copper-Oxide
W.E. D170622
n = 0.35
Thyrite
Copper-Oxide
W.E. 41A
n = 0.18
n = 0.56
n = 0.05
Silicon Diode
W.E. F51353
Selenium
n = 0.16
Germanium Diode
1N270
n = 0.2
100
10
1
0.1
e IN VOLTS
0.001
0.01
0.1
1
10
i IN MILLIAMPERES

EXPERIMENTAL SINGLE-LOOP AMPLIFIER

A single-loop feedback amplifier using one transistor was built to verify the above theories. The circuit diagram and voltage gain calculations are shown in the appendix. Figure 3 shows the voltage gain characteristics of the amplifier with two different nonlinear elements in the feedback path. The solid lines are the response as calculated from Equation (13) and the circles are the measured values. The line with a slope of unity in the left side of Figure 3 represents the constant maximum gain of the amplifier without any feedback. This is the limit given by Equation (2). The discrepancy between the calculated and measured values of the curves on the right side of Figure 3 is caused by the approximate value of the exponent, n, obtained from Figure 2. The experimental curves very closely resemble the corresponding regions of the respective experimental nonlinear device curves in Figure 2. Another possible source of error is the fact that the value of the forward gain,-A, of the amplifier is not very large as is assumed in the derivation of Equation 1.

A 60-cycle sinusoidal input was applied to the amplifier and the output waveforms for both the linear and nonlinear gain regions are shown in Figure 4. Although it may not be apparent from Figure 3, the instantaneous voltage gain varies from a maximum value of approximately 50 for

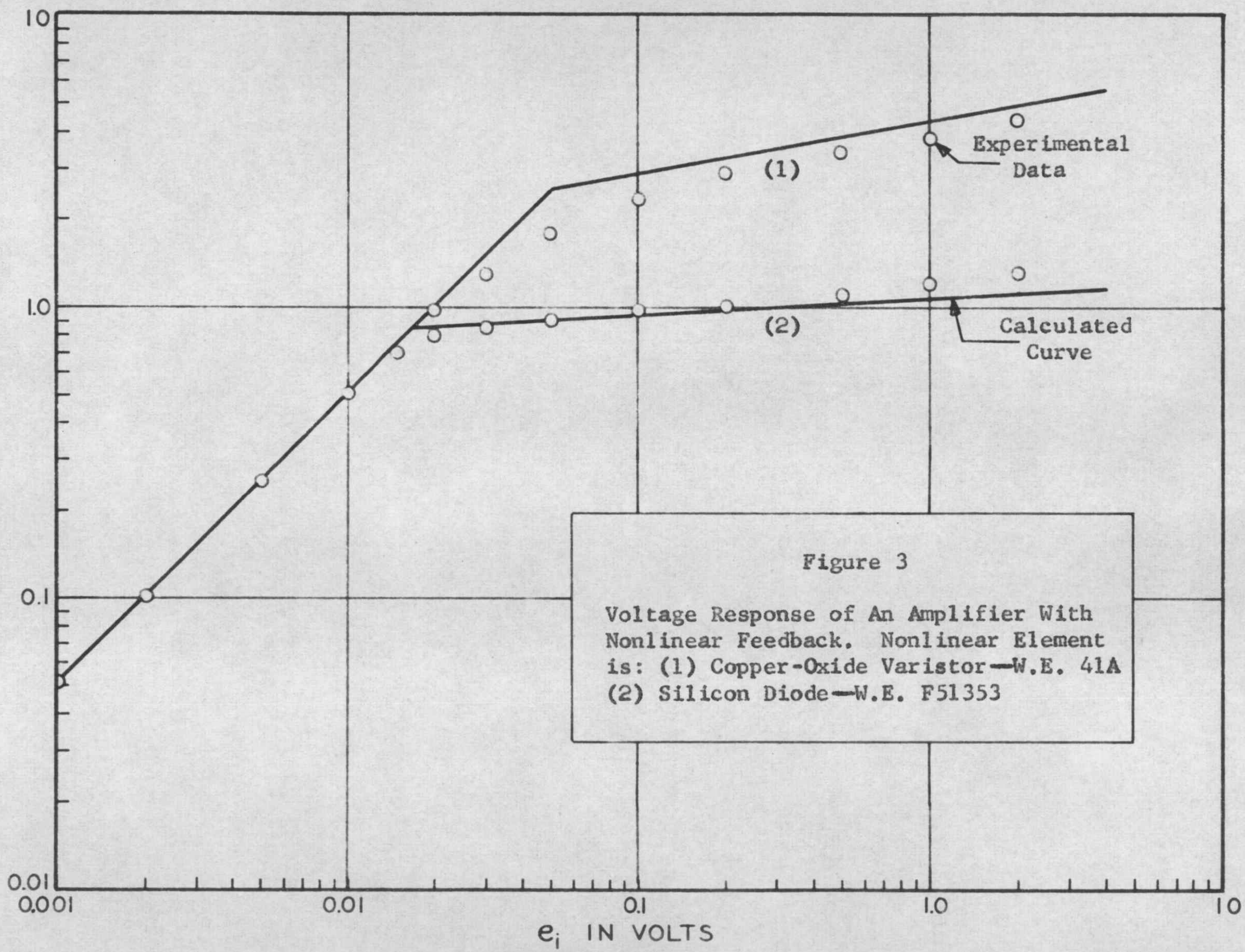
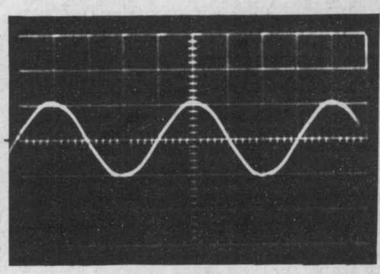


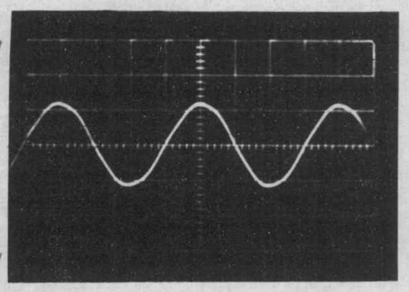
Figure 3

Voltage Response of An Amplifier With Nonlinear Feedback. Nonlinear Element is: (1) Copper-Oxide Varistor—W.E. 41A (2) Silicon Diode—W.E. F51353



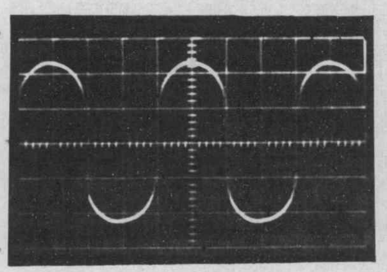
(a-1)

Output Waveform for
Input Signal of 2
millivolts peak-to-peak



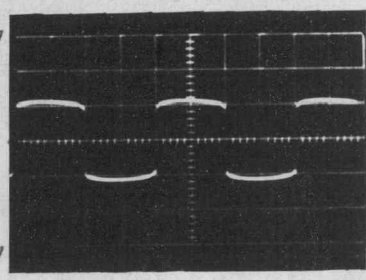
(b-1)

Output Waveform for
Input Signal of 2
millivolts peak-to-peak



(a-2)

Output Waveform for
Input Signal of 2
volts peak-to-peak



(b-2)

Output Waveform for
Input Signal of 2
volts peak-to-peak

Figure 4

Oscillograms of Output Waveforms of a Single-Stage Transistor Amplifier With Nonlinear Feedback. Nonlinear Element is (a) Copper-Oxide Varistor—W.E. D170622 (b) Silicon Diode—W.E. F51353.

Note: Horizontal Scale is 4 milliseconds per division, input signal is 60 cps sine wave.

small signals to something less than unity for large signals. Figure 5 shows the actual instantaneous gain variation corresponding to the data in Figure 3.

The two most important characteristics of this single-stage amplifier are the ability to handle; (1) a large dynamic range of input signals, and (2) large instantaneous variations in signal level without blocking. Figure 6 shows an oscillogram of the response of the amplifier as the input is switched from a very large input signal to a small input signal. The lower oscillogram shows the output of the amplifier with nonlinear feedback and the upper oscillogram shows the output without feedback. The upper oscillogram shows that the amplifier is actually blocked for approximately 12 seconds after the input voltage is switched to the low level condition. The amplifier with feedback shows no trace of blocking which is the desired result.

FOUR BASIC FEEDBACK CIRCUITS

Figure 7 shows the four basic feedback circuits that can be obtained with a single feedback loop (2, p. 38). The first word of the description of the circuit characterizes the input circuit connection and the second word of the description applies to the output circuit connection. For example, the term "parallel-voltage" means that the feedback signal is proportional to the output voltage and

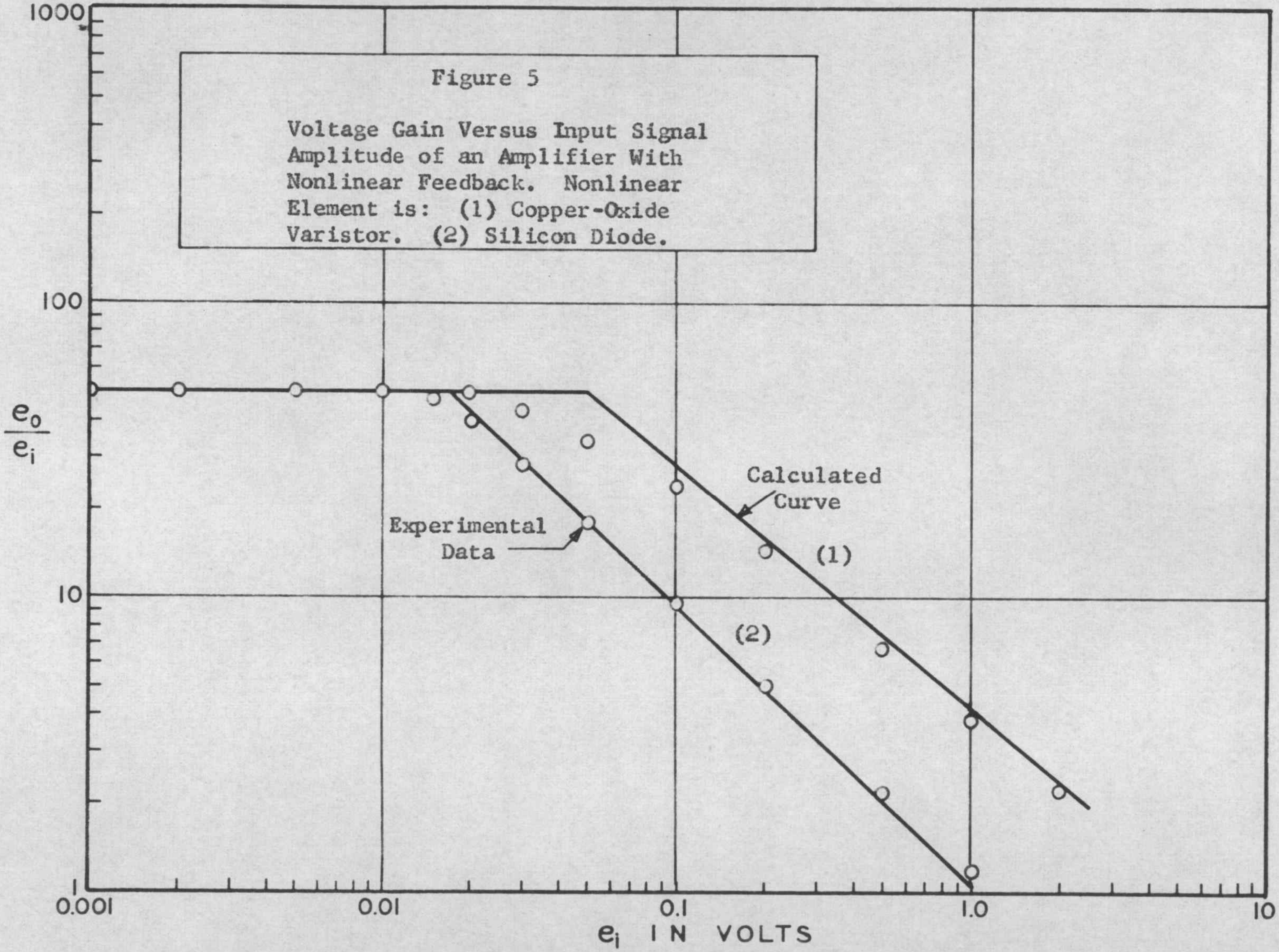
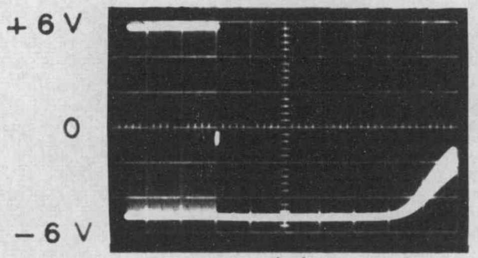
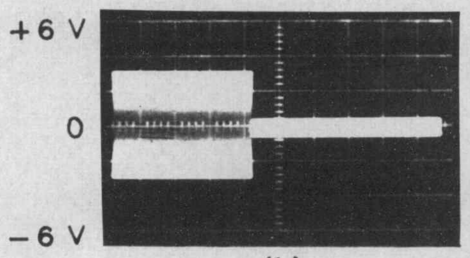


Figure 5
Voltage Gain Versus Input Signal Amplitude of an Amplifier With Nonlinear Feedback. Nonlinear Element is: (1) Copper-Oxide Varistor. (2) Silicon Diode.
1000
100
10
1
$\frac{e_o}{e_i}$
Calculated Curve
Experimental Data
(1)
(2)
0.001
0.01
0.1
1.0
10
e_i IN VOLTS



(a)

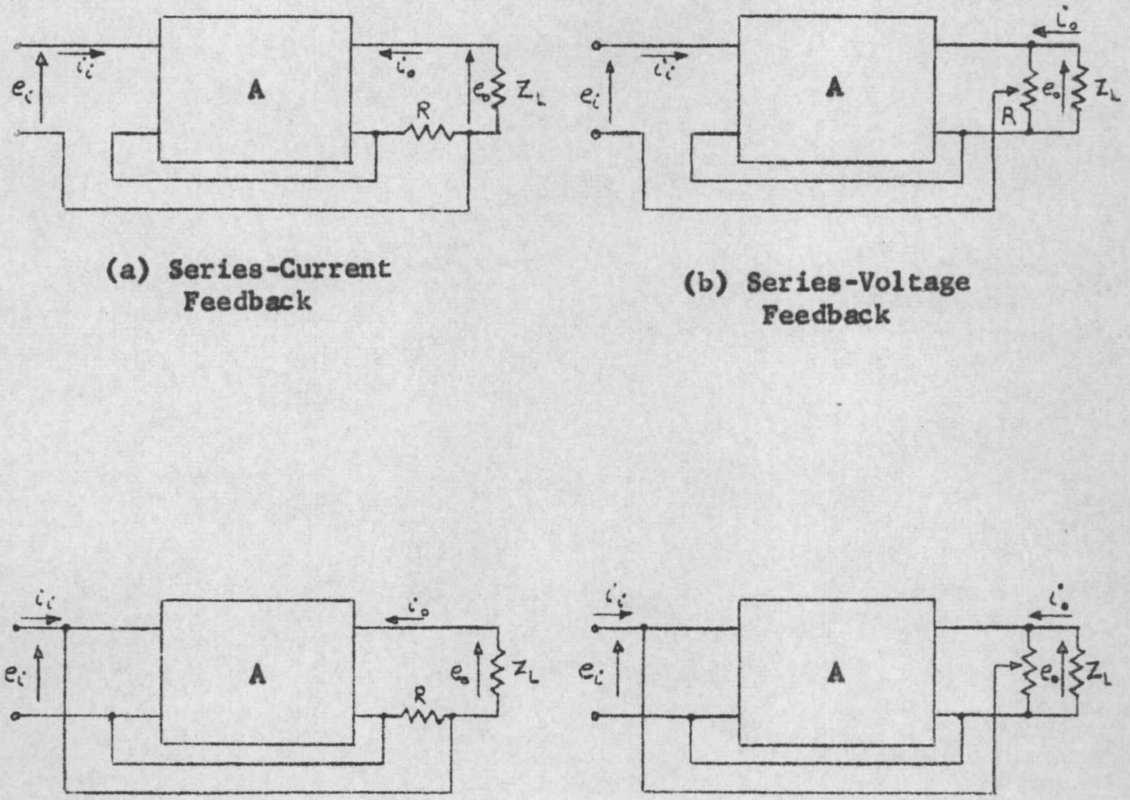


(b)

Figure 6

Oscillograms of The Output Signal of A Single-stage Transistor Amplifier. The Input Signal is Switched From 6 volts peak-to-peak to 20 millivolts peak-to peak. (a) Amplifier without feedback. (b) Amplifier with nonlinear feedback (Copper-Oxide Varistor--W.E. 41A)

Note: Horizontal Scale--2 seconds per major division. Input signal is a 60 cps sine wave.



(a) Series-Current Feedback

(b) Series-Voltage Feedback

(c) Parallel-Current Feedback

(d) Parallel-Voltage Feedback

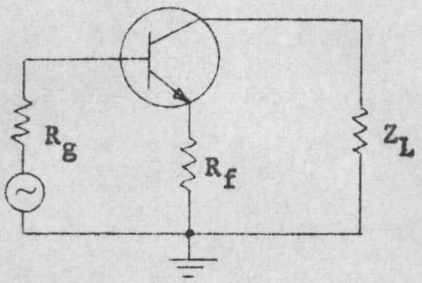
Figure 7

Four Basic Single-Loop Feedback Circuits

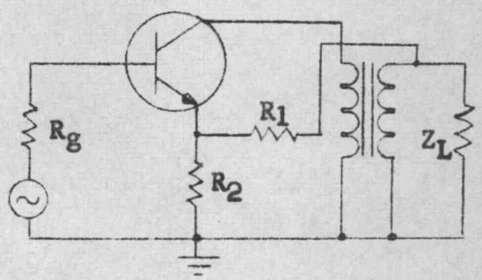
and is connected in parallel with the input signal. Figure 8 shows an example of each of the four basic circuits with a single transistor used as the amplifying element.

The properties of the amplifier depend upon the feedback configuration used. Table I shows how the amplifier characteristics are changed for each of the four types of feedback (2, p. 40). If a nonlinear element is used in the feedback path, the characteristics shown in Table I will vary with signal amplitude. For the case of the parallel-voltage feedback circuit, both the input and output impedance decrease as the signal level increases. If it is more desirable to have the input impedance increase for large signal levels, this characteristic may be obtained with the series-voltage feedback circuit. In general, however, the series-voltage circuit is not suitable for resistance-coupled single-stage amplifiers.

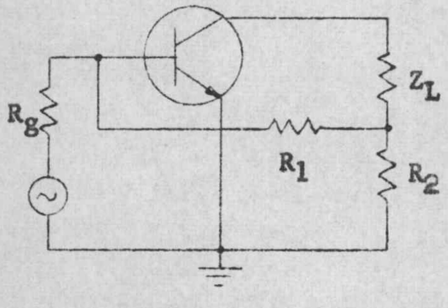
There is a minor problem that arises when a coupling capacitor is used in series with the nonlinear feedback element. When the supply voltage is initially turned on, the capacitor will have to charge to the d-c voltage from collector to base of the transistor. In order for the capacitor to acquire a charge, current must flow in the feedback path through the nonlinear resistance. Because of the feedback, the actual time constant is multiplied by the amplifier gain. Thus, the effective time constant can



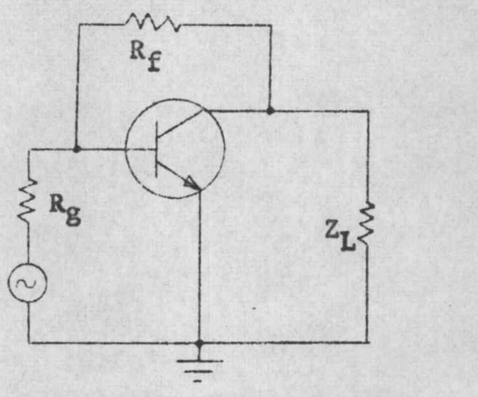
(a) Series-Current Feedback



(b) Series-Voltage Feedback



(c) Parallel-Current Feedback



(d) Parallel-Voltage Feedback

Figure 8

Examples of Four Basic Single-Loop Feedback Circuits

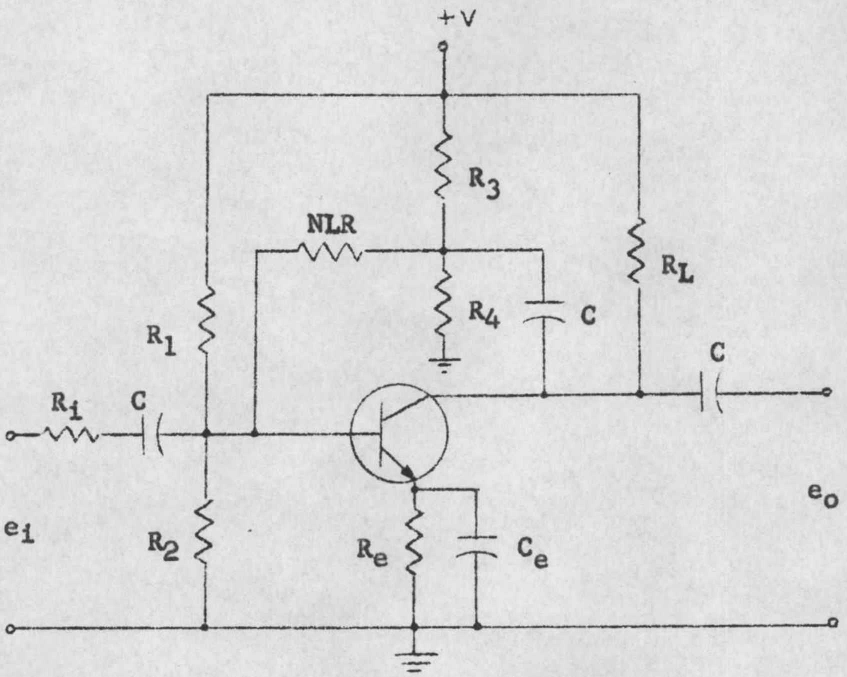
TABLE I

Circuit	Z_i	Z_o	A_i	A_v
Series Current Feedback	Increased	Increased	No Influence	Stabilized
Series Voltage Feedback	Increased	Decreased	No Influence	Stabilized
Parallel Current Feedback	Decreased	Increased	Stabilized	No Influence
Parallel Voltage Feedback	Decreased	Decreased	Stabilized	No Influence

be very large at small signal levels because the nonlinear resistance will be large. Consequently, there may be a long time interval before the amplifier attains its normal d-c operating point. This problem may be minimized by adding a voltage divider between the coupling capacitor and the nonlinear element as shown in Figure 9. Now the charging current for the capacitor can be supplied by the voltage divider instead of through the very high nonlinear resistance. The design of the voltage divider involves a compromise such that its impedance is high compared to the output impedance of the amplifier and yet low compared to the small signal impedance of the nonlinear element. The d-c voltage level supplied by the voltage divider should be designed so there will be no d-c voltage across the nonlinear element under normal operating conditions. It is usually undesirable to have a d-c voltage across the nonlinear element because this causes non-symmetry between the positive and negative halves of the output waveform.

SINGLE-LOOP FEEDBACK AMPLIFIERS IN CASCADE

An amplifier with a large dynamic range usually requires a number of stages of amplification in order to have a useful output for small input signals. Because of instability problems, feedback loops in practical amplifiers are usually limited to either one or two stages of amplification. A number of feedback stages may then be placed in



$R_1/R_3 = R_2/R_4$

NLR is Nonlinear Resistance

Figure 9

Single-Stage Transistor Amplifier
With Nonlinear Feedback

cascade in order to obtain a high amplification. The stages used in cascade may or may not be identical depending upon the specific application. Since the response of an amplifier with nonlinear feedback is

$$e_1 = K_1 e_i^{n_1} \qquad (15)$$

then, the output response of two stages in cascade would be

$$e_2 = K_2 e_1^{n_2} = K_2 (K_1 e_i^{n_1})^{n_2}$$

$$e_2 = K_2 K_1^{n_2} e_i^{n_1 n_2} \qquad (16)$$

If the stages are identical, then

$$e_2 = K^{n+1} e_i^{n^2} \qquad (17)$$

The output response of three stages in cascade would be

$$e_3 = K_3 e_2^{n_3} = K_3 (K_2 K_1^{n_2} e_i^{n_1 n_2})^{n_3}$$

$$e_3 = K_3 K_2^{n_3} K_1^{n_2 n_3} e_i^{n_1 n_2 n_3} \qquad (18)$$

For three identical stages,

$$e_3 = K^{1+n+n^2} e_i^{n^3} \qquad (19)$$

In general, for k identical stages,

$$e_o = K^{1+n+n^2+\ldots+n^{(k-1)}} e_i^{n^k} \qquad (20)$$

Basically, the exponent of the cascaded circuit is the product of the individual stage n values. This means that theoretically there are an infinite number of cascaded circuits that could be used to get a particular value of n. Practically, it means that the designer can use combinations of nonlinear elements that are available to obtain the characteristic of a nonlinear element that is not available.

An example of the response of two identical stages in cascade is shown in Figure 10. In this example each stage has a maximum gain of 20 db and an exponent of 0.5. The final slope of the output of the second stage is 0.25 which is the product of the individual stage exponents. Notice that as the input signal becomes small, the gain of the first stage reaches its maximum value of 20 db. It then becomes a linear amplifier with an $n = 1$. As the input is decreased below the breakpoint of the first stage, the second stage also reaches its maximum gain limit and becomes a linear amplifier. It is significant that the range of input signal between the breakpoints of the two stages is equal to the gain of the second stage. This would also be true if the stages were not identical.

This amplifier would satisfy a requirement for $n = 0.25$ over the last two decades of input signal.

The general shape of the curve of the second stage output suggests that it should be possible to get a piecewise approximation of a curve which has a variable n. Figure 11 shows how a logarithmic function might be

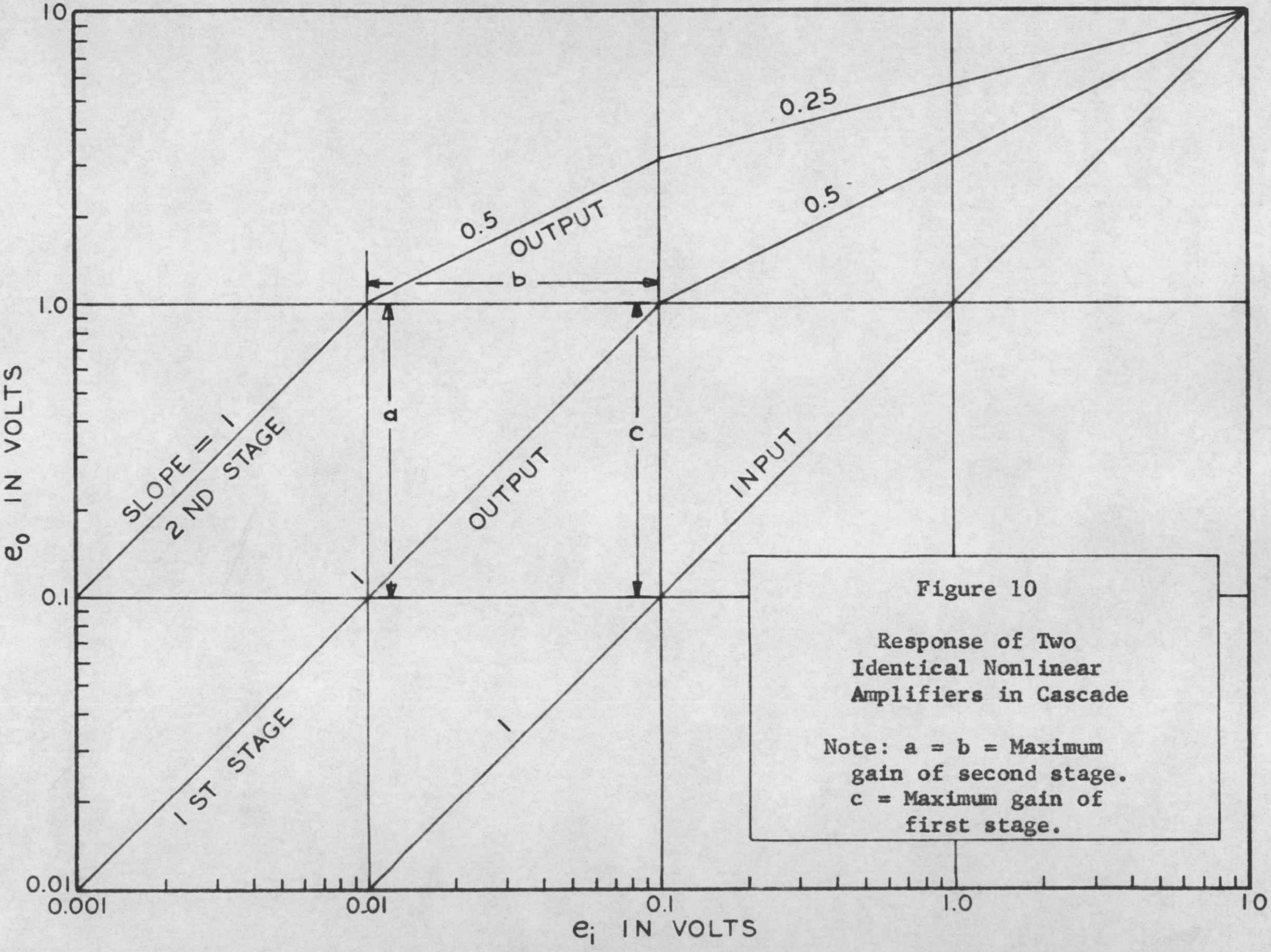


Figure 10

Response of Two Identical Nonlinear Amplifiers in Cascade

Note: a = b = Maximum gain of second stage. c = Maximum gain of first stage.

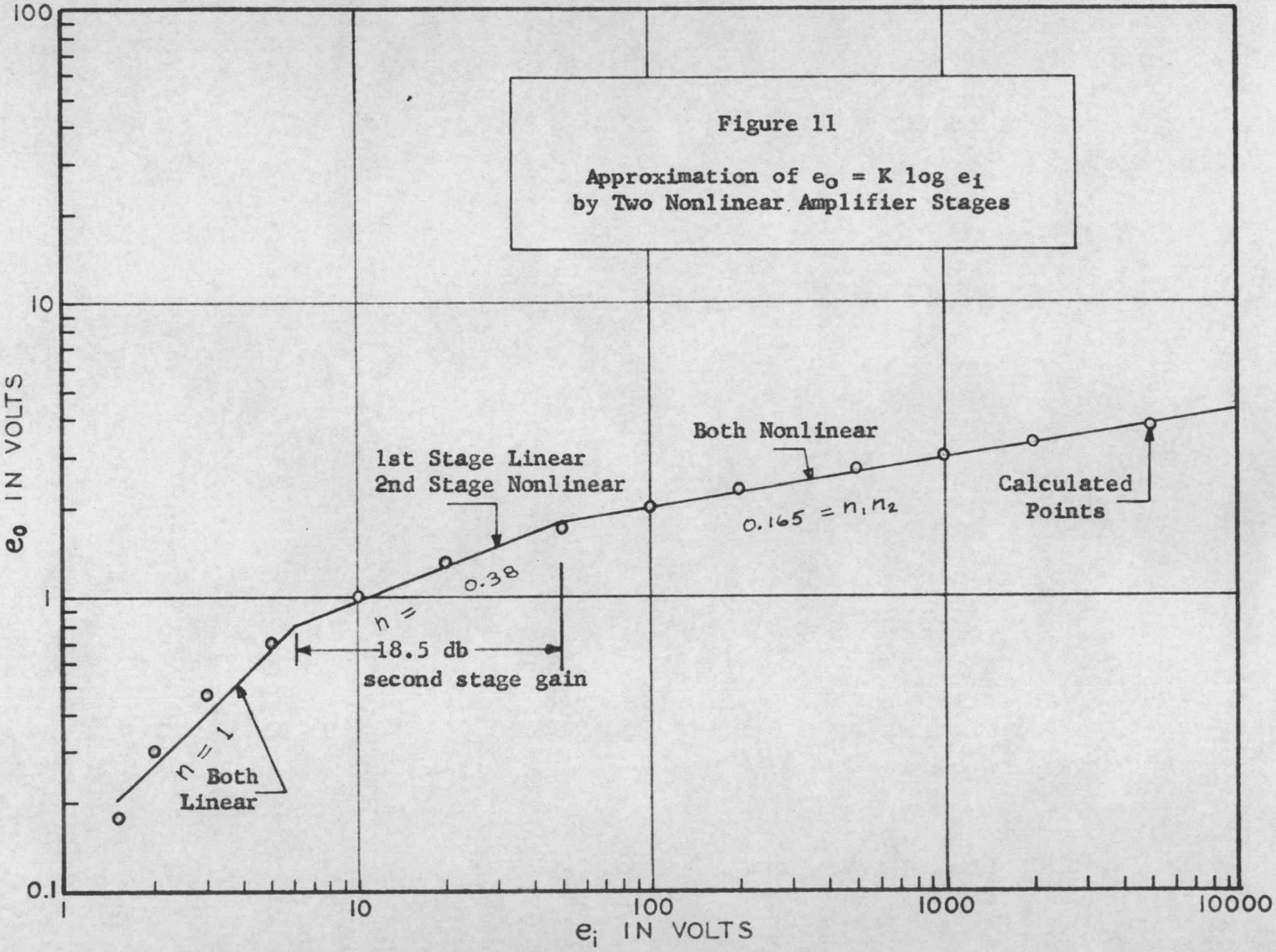
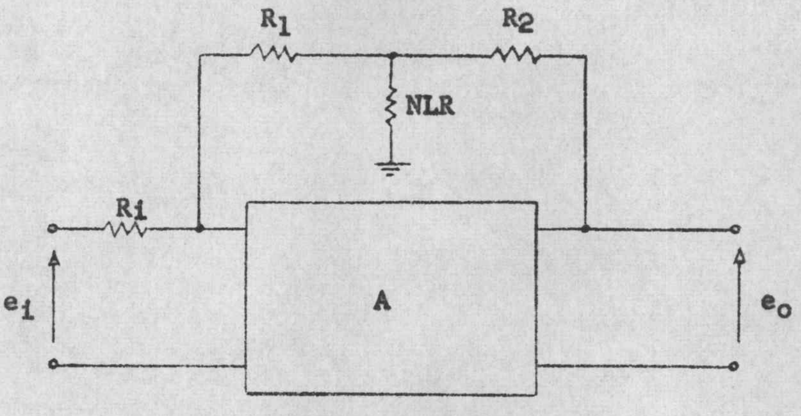


Figure 11

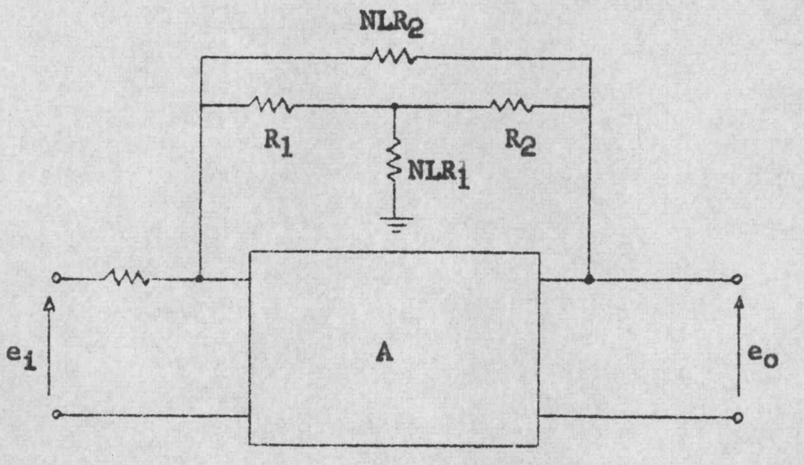
Approximation of $e_o = K \log e_i$ by Two Nonlinear Amplifier Stages approximated using two non-identical nonlinear feedback amplifiers. The characteristics of the amplifiers are found from the straight lines drawn to approximate the desired curve. For this example, the second stage n = 0.38 and the overall n = 0.16, so the first stage n = $\frac{0.16}{0.38}$ = 0.42. The required maximum gain of the second stage is 18.5 db which is the interval between the breakpoints. The maximum gain of the first stage depends upon the value of K and would be found from the region of the curve where both stages are linear. The gain for this region is simply the sum of the two individual maximum gains.

EXPONENTS GREATER THAN UNITY

The exponent of a nonlinear amplifier can be changed from less than unity to greater than unity by changing the position of the nonlinear element in the feedback loop. For example, when the nonlinear element is used in a shunt path in the feedback loop as shown in Figure 12(a), the result is an amplifier whose gain increases with increasing signal level. This type of feedback circuit can be used to obtain a square-law response. However, this circuit has a disadvantage in that it allows the amplifier to block on large input signals. The problem of blocking can be eliminated by adding another feedback loop with a series nonlinear element as shown in Figure 12(b). In order to be effective, the exponent of the series nonlinear element



(a)



(b)

Figure 12

(a) Amplifier With Shunt Nonlinear Element.
(b) Amplifier With Both Series and Shunt Nonlinear Elements.

must be smaller than the reciprocal of the effective exponent of the rest of the circuit.

A combination of both types of feedback loops offers a great deal of versatility in obtaining effective values of n that otherwise might not be available.

AVAILABILITY OF EXPONENTS

Figure 2 shows the characteristics of several nonlinear devices with various values of exponents. Since the characteristics are not straight lines, the values of n are not constant. Contrary to what might be expected, this curvature can be advantageous in finding specific values of n. The exponent value depends upon the region of operation on the characteristic curve and this can be changed by varying the fraction of output voltage applied to the feedback loop.

SUMMARY

Nonlinear feedback can be used in a voltage amplifier to prevent blocking and to obtain a gain characteristic that varies with signal amplitude. The characteristics of a number of nonlinear elements that would be suitable for use in a single-loop feedback amplifier have been shown.

Good results were obtained with an experimental amplifier using either of two different types of nonlinear elements in a feedback loop. A large dynamic range was obtained and blocking was eliminated.

It has been shown how nonlinear amplifier stages in cascade can be used to obtain a nonlinear function, either directly or with a piecewise exponential approximation.

More than one nonlinear function can be obtained with a single nonlinear element by changing the circuit configuration of the feedback loop. A combination of feedback loop circuits can be used to obtain exponent values that are not available in actual devices.

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APPENDIX

Derivation of the Transfer Function of Figure 1

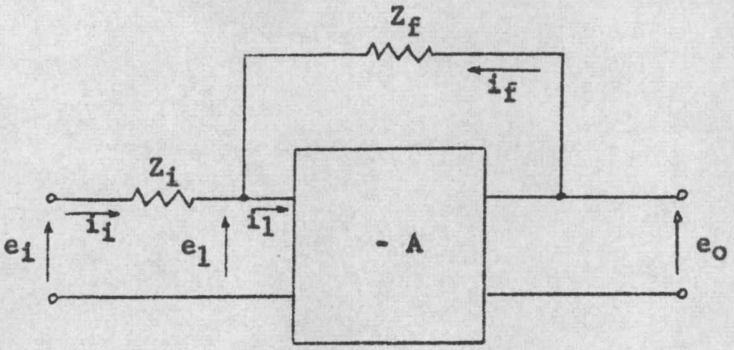


Figure 1

The relationship between e_o and e_1 is,

$$e_1 = -\frac{e_o}{A}$$

If $A \gg 1$, then $e_1 << 1$.

For this case,

$$e_o \cong i_f Z_f$$

and

$$e_o \cong i_i Z_i$$

which gives,

$$\frac{e_o}{e_i} \cong \frac{i_f Z_f}{i_i Z_i}$$

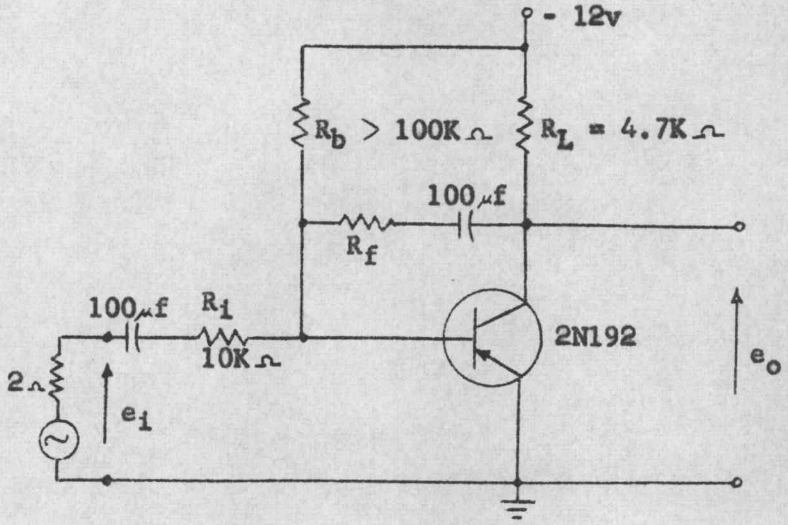
If $i_1 <<$ (i_i and i_f), then

$$i_i \cong - i_f$$

and

$$\frac{e_o}{e_i} \cong - \frac{Z_f}{Z_i}$$

Experimental Single-Stage Transistor Amplifier with Nonlinear Feedback



Measured h parameters at operating conditions of I_c = 1 milliampere, V_c = 5 volts.

$$h_{11} = 5500 \text{ ohms}$$
$$h_{12} = 5 \cdot 10^{-4}$$
$$h_{21} = 200$$
$$h_{22} = 3.4 \cdot 10^{-5} \text{mhos}$$

Voltage gain without feedback, $R_i = 0$.

$$A_v = - \frac{h_{21} R_L}{h_{11} + R_L(h_{11}h_{22} - h_{12}h_{21})}$$

$$= - \frac{200(4.7K)}{5500 + 4.7K(0.187 - 0.1)}$$

$$= \underline{158}$$

Measured Value = <u>165</u>

Voltage gain without feedback, R_i = 10K.

$$h_{11} = 5500 + 10K = 15.5K$$

$$A_v = - \frac{200(4.7K)}{15.5K + 4.7K(0.526 - 0.1)}$$

$$= \underline{53.7}$$

Measured Value = $\underline{54}$