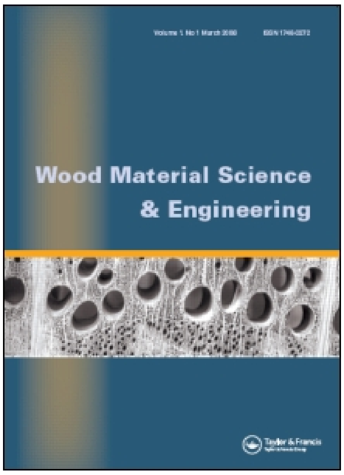

Wood Material Science and Engineering
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t741771155

Location of the neutral axis in wood beams: A preliminary study

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Online publication date: 20 November 2010

To cite this Article Betts, Sarah C. , Miller, Thomas H. and Gupta, Rakesh(2010) 'Location of the neutral axis in wood beams: A preliminary study', Wood Material Science and Engineering, 5: 3, 173 — 180
To link to this Article: DOI: 10.1080/17480272.2010.500060
URL: http://dx.doi.org/10.1080/17480272.2010.500060

Wood Material Science and Engineering, 2010; 5: 173–180

ORIGINAL ARTICLE

Location of the neutral axis in wood beams: A preliminary study

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Abstract

It is commonly accepted in the analysis of wood beams that the neutral axis coincides with the beam's centroid. However, wood is not an isotropic material, has different elastic properties in the tangential, radial and longitudinal directions, and is non-homogeneous, as it contains characteristics such as knots. Therefore, there is need for an analysis of the neutral axis for anisotropic, non-homogeneous materials, such as wood, to predict deformations and strains. Specifically, digital imaging correlation, a non-contact technique to measure deformation of an object's surface, is used to examine how knots influence neutral axis location. Output from digital imaging correlation software provides a clear image of the location of the neutral axis. The neutral axis in a clear wood beam remains close to the centroidal axis throughout loading, while the location of a knot determines the size of the compression and tension zones as well as the location of the neutral axis.

Keywords: *Anisotropic materials, inelastic behaviour, knots, neutral axis, wood.*

Introduction

The neutral axis for a beam is defined as the line in the cross-section where there is neither longitudinal compression nor tensile stress. It is commonly accepted in the analysis of wood beams that the neutral axis coincides with the centroid of the beam. Although this assumption is used in analysis, wood is not an isotropic material, as it has different elastic properties in the tangential, radial and longitudinal directions, nor is it homogeneous, as it contains natural characteristics, such as knots, at various locations. Therefore, there is a need for an analysis of the behaviour of the neutral axis for wood beams to predict deformations and strains, and to provide a better estimation of the mechanical properties of wood.

For beams where tension and compression zones are different, as in a wood member, the neutral axis will only lie on the centroidal axis when the beam is elastic. However, when the beam begins to show inelastic behaviour, the neutral axis will move up or down depending on whether the material has a higher tension or compression capacity. In addition, knots affect strength capacities of the beam even more depending on their location. Therefore, it can be expected that the neutral axis will move up or down depending on the location of the knot.

Clear wood is much stronger in tension than in compression, and according to the Forest Products Laboratory (1999), the typical tensile strength of clear wood parallel to the grain is approximately 100,000 kPa, while the compressive strength parallel to the grain is about 50,000 kPa. Thus, tensile stresses below the neutral axis of a simply supported wood beam subjected to bending may be much larger than the compressive stresses above the neutral axis at failure. Wood beams in bending most commonly fail in compression first. Kollmann and Cote (1968) explain that for an idealized trapezoidal distribution of the stresses in a wood beam in bending, the neutral axis location is given by eq. (1):

$$\frac{x}{h} = \frac{1 + m^2}{(1 + m)^2}, \text{ where } m = \frac{\sigma_{cb}}{\sigma_{tb}} \quad (1)$$

where x is the distance from the top of the beam to the neutral axis, h is the height of the beam, and m is the ratio of the compression to the tensile strength.

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(Received 12 February 2010; accepted 7 June 2010)

ISSN 1748-0272 print/ISSN 1748-0280 online © 2010 Taylor & Francis
DOI: 10.1080/17480272.2010.500060

For clear beams, the neutral axis is expected to be below the centroidal axis. From eq. (1), for a clear wood beam in bending where tensile strength is assumed to be twice the compressive strength, $m = 0.5$, and the neutral axis is at $x/h = 0.556$ (measured from the top). Using the idealized trapezoidal stress distribution for wood beams, eq. (2) holds true:

$$\varepsilon_{cb} = \frac{\sigma_{tb}}{E} \cdot \frac{1 + m^2}{2m} \quad \text{or} \quad \frac{\varepsilon_{cb}}{\varepsilon_{tb}} = \frac{1 + m^2}{2m} \tag{2}$$

where ε_{cb} and ε_{tb} are compressive and tensile strains in the beam, respectively, and σ_{cb} and σ_{tb} are the compressive and tensile stresses, respectively. This can also be used to determine the location of the neutral axis depending on the ratio between compression and tension strains.

Characteristics such as cross-grain cause the largest stresses to be at an angle to the grain, lowering the maximum allowable stress. Knots affect the mechanical properties of wood because the directions of the wood fibres around the knot are disrupted, causing cross-grain. Because of this disruption, stress concentrations develop and generally decrease the mechanical properties in beams containing knots compared with clear wood. Kollmann and Cote (1968) note that knots also reduce the elastic properties, such as the modulus of elasticity.

In a related study, Lopes and Bernardo (2004) studied the movement of the neutral axis in high-strength concrete beams in pure bending. Nineteen high-strength concrete beams were subjected to two equal concentrated loads placed symmetrically at one-third points of the span to determine whether the depth of the neutral axis at failure in high-strength concrete beams should be limited as for normal-strength concrete beams. Using strain gauges and linear transducers, strain was measured in the beam throughout the loading and load versus deflection curves were plotted. Tests showed that the depth of the neutral axis rises as the longitudinal tensile reinforcement ratio rises. The neutral axis rises away from the centroidal axis when cracks develop and when the tension steel begins to yield and the beam behaves plastically until failure. The neutral axis moves below the centroidal axis as beam ductility decreases.

This is related to the focus of this paper because like wood, reinforced concrete is an anisotropic material. In a clear wood beam subjected to pure flexure, the neutral axis is expected to fall below the centroidal axis when the beam behaves inelastically. Movement of the neutral axis during plastic behaviour is described by the Hoffman yield criteria (Bazant & Jirasek, 2002). However, the tension and compression zones will be different for wood than for reinforced concrete beams, and this will determine how far the neutral axis will move. The neutral axis will move depending on the capacity of the member in compression and tension. Because clear wood beams have approximately a 2:1 ratio of tensile to compressive strength, the neutral axis will fall below the centroidal axis at failure. In addition, just as the location of the neutral axis in concrete depends on the modular ratio, or the ratio of the modulus of elasticity of the concrete to that of the steel, the neutral axis for wood is expected to be dependent on the ratio of the modulus of elasticities for the compressive and tensile zones. Moreover, a lumber beam containing a small knot would have about 50% less tensile strength than a piece of clear wood. Although the knot's effect depends on its location within the beam and its size, this means that the neutral axis would move even further above the centroidal axis during bending.

This preliminary study will examine (1) the location of the neutral axis of a relatively clear, straight-grained piece of wood in pure bending, and then (2) how knots affect the location of the neutral axis in a non-homogeneous wood beam. Testing will be accomplished using digital image correlation (DIC) equipment to determine the strains and deformations of the beam. DIC takes a series of images of a small area during an applied loading. The images depict the deformation relative to the undeformed shape. Deformations can be used to determine the strains. Strain contour plots can be used to determine where the neutral axis is located.

Note that this preliminary study is not a detailed examination of the effect of knots on bending strength for use in determining design values, but rather a qualitative examination with just a few tests demonstrating the behaviour and movement of the neutral axis due to knots. Additional tests on small, clear and full-size beams are being conducted, and results will be published at a later date.

Materials and methods

Deformation testing and analysis were performed on four $25 \times 25 \times 406$ mm Douglas fir wood beams: one clear beam (Figure 1a), one containing a knot at the mid-height of the face (Figure 1b), one containing a knot in compression (Figure 1c), and one containing a knot in tension (Figure 1d). Note that the samples were selected to be straight-grained except around the knots. Densities were measured for the specimens at approximately 9% moisture content: 0.422 g cm^{-3} (clear), 0.478 g cm^{-3} (knot at centre), 0.485 g cm^{-3} (knot on compression side) and

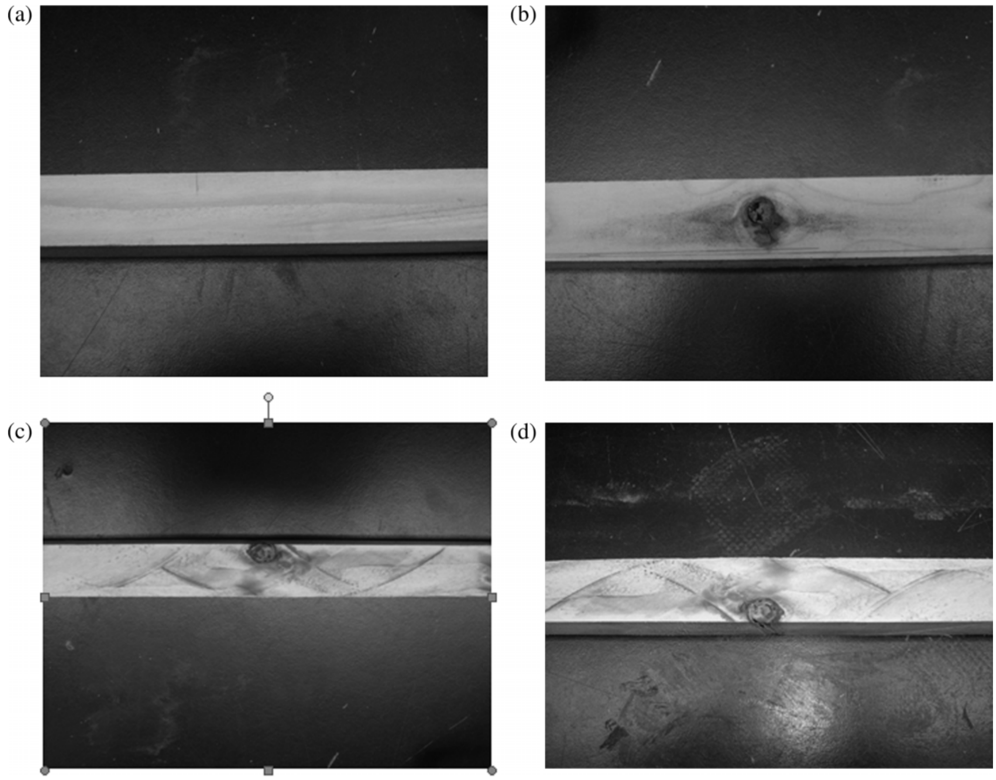

Figure 1. (a) Clear specimen; (b) specimen with a knot at the mid-height of the face; (c) specimen with a knot in compression; (d) specimen with a knot in tension.

0.468 g cm^{-3} (knot on tension side), for an average of 0.463 g cm^{-3}.

The beams were loaded to failure in symmetric four-point bending (at third points) to ensure pure flexure in the mid-section of the beam. The distance between load points on the beam was 191 mm and distance between supports and load heads was 82.5 mm.

Specific methods

Testing was performed using VIC-3D software and DIC equipment. DIC is a non-contact technique used to measure deformation of an object's surface. Two cameras are angled on the test object and capture a specified number of images every minute depending on loading rate. The VIC-3D software creates a mathematical correlation of the digital images taken during loading. From this correlation, the software can compute surface deformations and strains.

Test set-up

The testing equipment consists of two cameras with high-quality lenses. Cameras must be mounted on very stable tripods for measurement accuracy. In addition, the amount and angle of light on the object must be aligned with the f-stop of the camera to ensure accurate image correlation. The angle between the two cameras should ideally be between 30° and 60°. Much care in test set-up is required owing to the sensitivity of the cameras and accuracy of the test. For this experiment, cameras were set on one side of the tripod and flipped upside down to be in line with the specimen. The front of the camera lens was approximately 420 mm away from the beam. Camera lenses must be clean and free of dust, correctly focused on the object and not moved or touched during testing; tripods must be insensitive to floor vibrations; the lighting needs to be consistent and uniform throughout testing; and cameras must be set up at the correct angle for image collection during testing.

The test object must be sprayed with a random black speckle pattern over a white base, as this is important for accurate results. It is a trial and error process to determine the type of speckle pattern that will produce good results with the software. After the object surface has been sprayed, the cameras can track the pattern during loading. The cameras track this pattern within a specified area of interest (AOI). For this experiment, AOI was defined as a 25 × 50 mm rectangular area at the centre of the beam. Strains were examined at mid-section of the beam where the greatest stresses are located during bending.

Once all the test equipment has been set up, the VIC-3D software requires a calibration process particular to the test. Images from both cameras are taken of the calibration target at varying orientations. The software needs to recognize three dots on the surface to be tracked during deformation. The software will display the calibration report which shows the standard deviation for the images acquired, and it is important that this deviation is less than 0.1 for precise results. The threshold may need to be adjusted for accurate calibration, or the image may need to be deleted if its error is too large.

Once the cameras had been calibrated using the VIC-3D software, testing could commence. Beams were loaded at $4\,\mathrm{mm\,min}^{-1}$ and images taken every 2 s. At failure, cameras were stopped and images retrieved. The image before loading, or the reference image, and the images of the specimen under loading, or the deformed images, were selected. Given that the calibration was successful and the camera set-up remained the same for both calibration and testing, deformed images could be successfully correlated with calibration images.

Analysis

After testing and correlation of the deformed images, the software calculated principal, shear, horizontal and vertical strains from the partial derivatives of the displacement using LaGrange strain tensor equations (Correlated Solutions, 2005). Strain over the AOI was displayed as a contour plot. Statistics of the accuracy of the reference area were also calculated.

Results

Output data were initially checked for reasonableness by plotting deflections over time for each beam. Longitudinal strain in the beam was compared to the theoretical value based on the assumption that stresses are distributed linearly and symmetrically throughout an elastic cross-section. After this data check had been done for each beam, analysis commenced. Lines to approximate the location of the neutral axis for each beam were drawn. Then, the neutral axis height was calculated as a fraction of total beam height (Table I). Load versus deflection is plotted in Figure 2. Modulus of elasticity was determined for each of the samples using these data: 8.97 GPa for the clear beam, essentially the same 9.02 GPa for the centre knot beam, and reduced values of 5.72 GPa for the compression knot specimen and 3.91 GPa for the specimen with a tension side knot. Figure 3 shows longitudinal strain in each beam at the same scale in the elastic state at a total load of approximately 730 N.

Table I. Location[a] of neutral axis (NA) in area of interest (AOI).

	Location of NA from bottom of AOI[a]			
Beam	Left edge	Mid-section	Right edge	Average
Clear	0.474	0.457	0.457	0.462
Centre knot	0.500	0.612	0.457	0.564
Compression knot	0.205	0.231	0.239	0.222
Tension knot	0.655	0.733	0.733	0.715

Note: [a]Fraction of beam height just before failure.

Strain contour plots produced by the VIC-3D software gave a clear visual image of where the neutral axis lies in each loaded beam. Specifically, the contour plot of the longitudinal strain on the surface of the beam was the focus for this experiment. As cross-sections were assumed to remain plane and normal to the longitudinal axis, during deformation, longitudinal distances between cross-sections either lengthened or shortened except at the neutral axis. Normal strains were plotted initially in the contours as compression in purple, tension in red and green as neutral or no strain. The solid line labeled NA is the approximate neutral axis of the beam. Figure 4 shows that, just before failure, the neutral axis locations for the different beams varied significantly from each other. These comparisons between the clear specimen and the specimens with knots can be made because all of the specimens were obtained from the same nominal 2×4 (38×89 mm) board.

Clear beam

The neutral axis in the clear wood beam did not seem to move much away from the centroidal axis during loading, although it was expected the neutral axis would fall just below the centroidal axis after reaching the proportional limit. The contour plot of longitudinal strain just before failure (Figure 4a) and Table I show that the neutral axis for the clear beam ranged from 0.457 to 0.474 of the beam height from the bottom. The beam failed with a tension crack at a total load of approximately 4.08 kN. As shown in Figure 5, the neutral axis did not move away much from the centroidal axis even after it had reached the proportional limit.

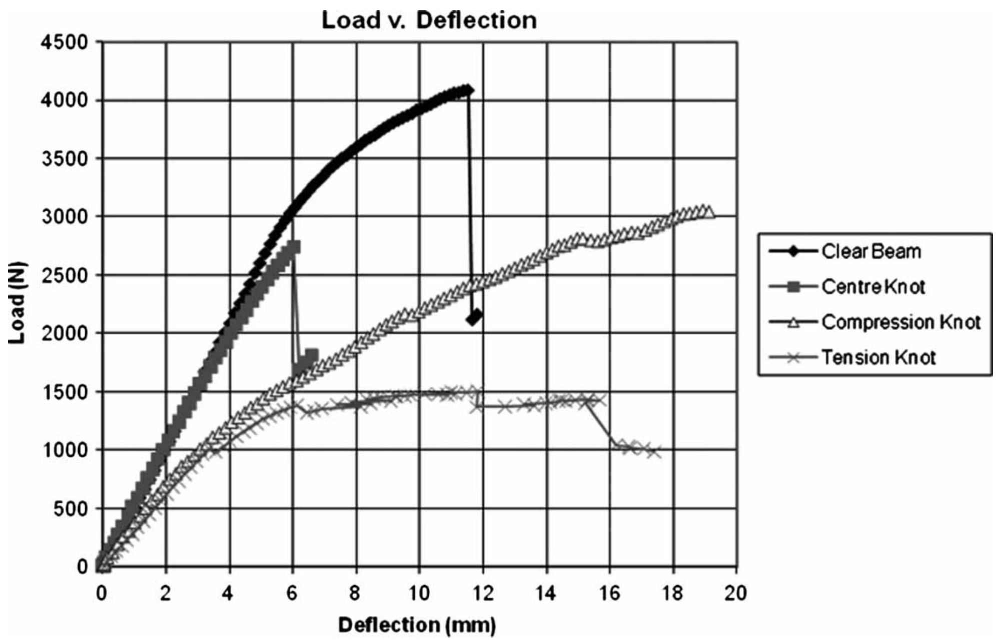

Figure 2. Plot of load versus deflection for each beam.

Knot at centre

Throughout the loading, the beam with a knot at the centre developed a tensile stress concentration around the knot (Figures 3b and 4b). This decreased load capacity, in comparison to the clear beam, by almost 33%. The beam failed with a tension crack at a total load of approximately 2.74 kN. It is also apparent from the contour plots that there was little stress in the middle of the knot because it was an encased knot and therefore slightly detached from the beam.

The average location of the neutral axis near failure was approximately 10% of the beam height above the neutral axis for the clear beam. However, variation in the neutral axis along the beam's length was very different. As can be seen in Figure 4(b) and Table I, the neutral axis began on the left side of the AOI at approximately the same location as for the clear beam. However, as the neutral axis moved towards the knot at the centre of the area, it rose above the knot to about 0.61 times the height of the beam from the bottom. Finally, towards the right

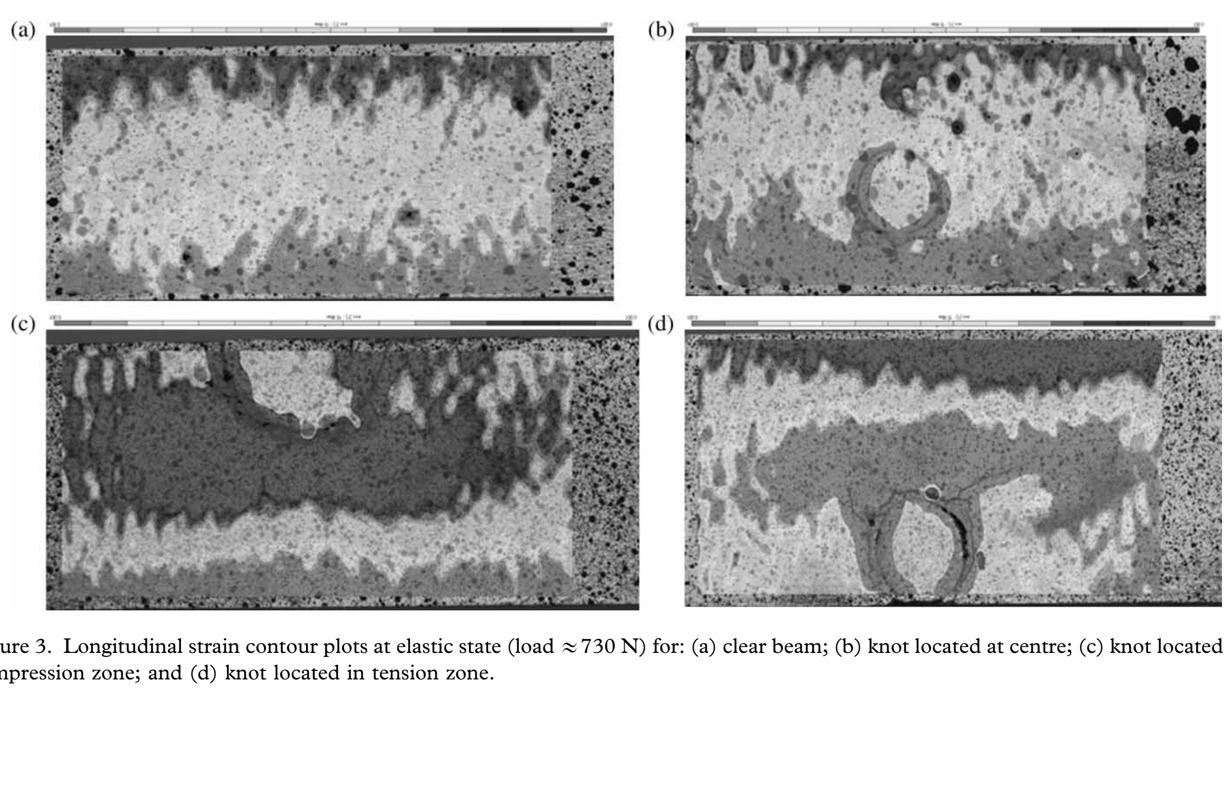

Figure 3. Longitudinal strain contour plots at elastic state (load ≈730 N) for: (a) clear beam; (b) knot located at centre; (c) knot located in compression zone; and (d) knot located in tension zone.

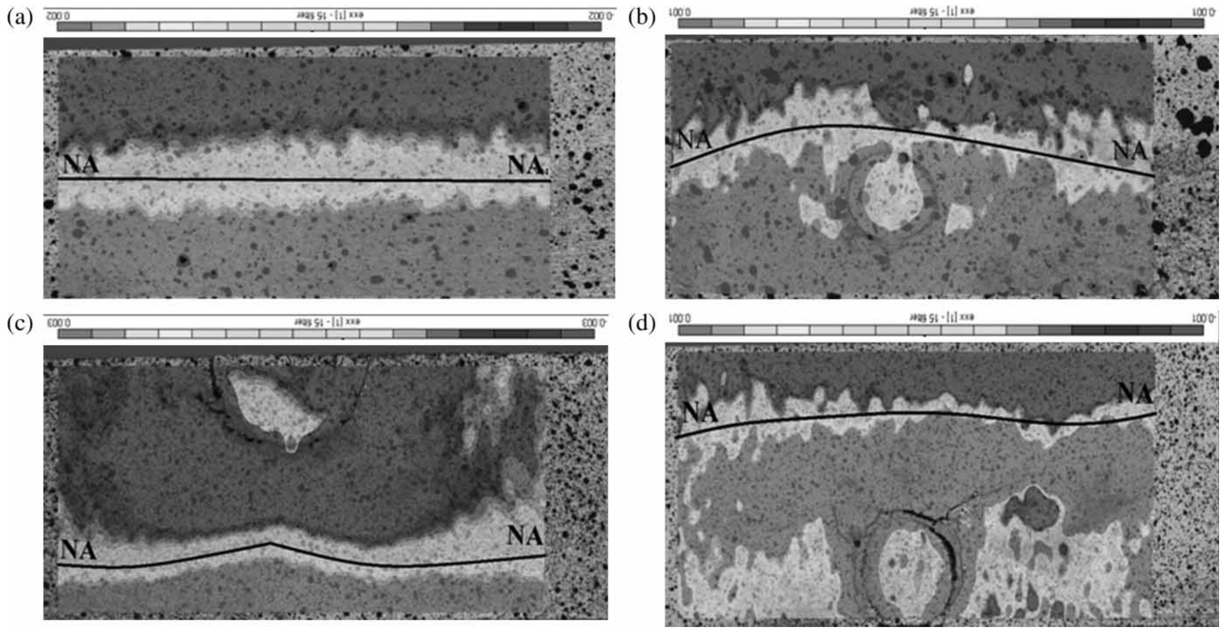

Figure 4. Longitudinal strain contour plots just before failure for: (a) clear beam; (b) knot located at centre; (c) knot located in compression zone; and (d) knot located in tension zone.

side of the area, the neutral axis moved back towards the centroidal axis and ended up at the same height as that of the clear beam at that location.

Knot in compression zone

Again, the beam developed a stress concentration around the knot throughout the loading; however, this knot seemed to be more connected to the beam because the stress distributed more evenly through the knot (Figures 3c, 4c). There was only a small section in the centre of the knot with almost no stress. Again, load capacity was significantly reduced. The beam with the knot in compression failed at a load of approximately 3.06 kN, a 25% reduction in strength compared to the clear beam.

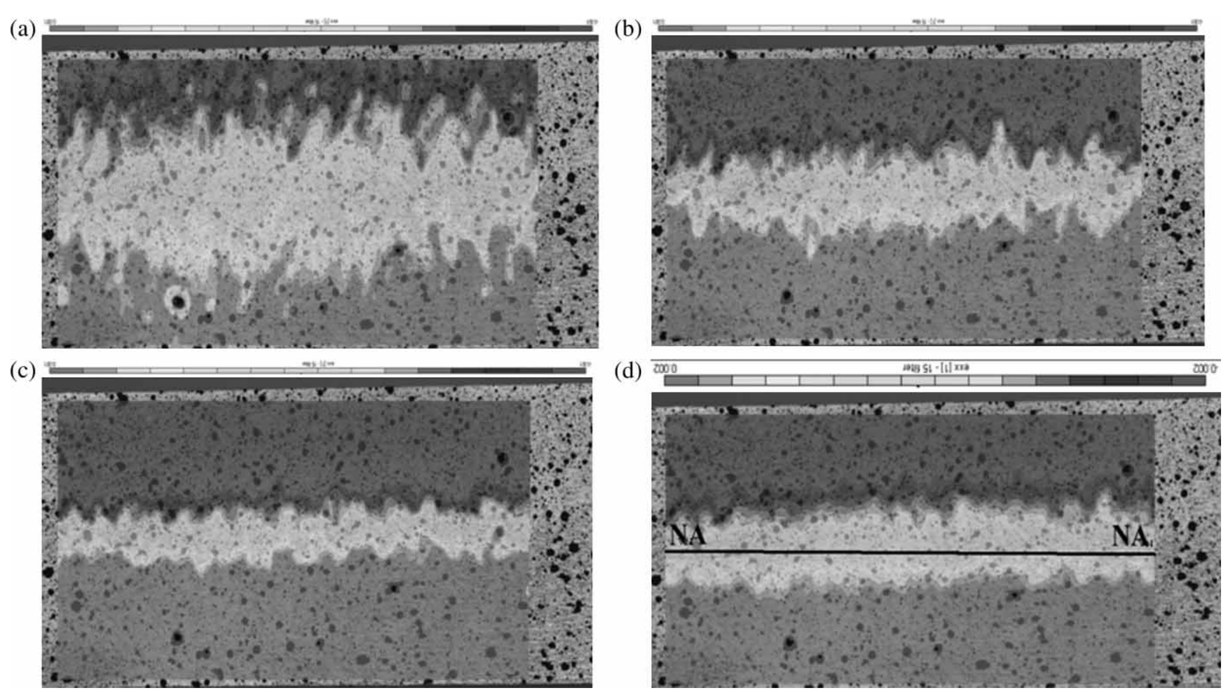

Figure 5. Longitudinal strain contour plots for a clear beam under (a) 0.997 kN load; (b) 2.01 kN load; (c) 3.03 kN load; and (d) failure load: 4.08 kN.

The beam failed at the knot, with a crack through the knot progressing through the cross-section of the beam in the compression zone.

As summarized in Table I and plotted in Figure 4(c), the neutral axis shifted significantly away from the centroidal axis as failure approached. The average location of the neutral axis was only 22% of the beam height from the bottom of the beam. The neutral axis started at approximately 21% of the height from the bottom at the left edge of the AOI and remained relatively straight along the beam's length. However, at the knot, the neutral axis actually moved up towards the knot, and was at 23% from the bottom.

Knot in tension zone

This beam behaved in nearly the opposite way to that with a knot in the compression zone. While the knot in the compression zone caused the neutral axis to move about 28% of the beam height below the centroidal axis, the knot in the tension zone caused the neutral axis to move about 22% above the centroidal axis. As shown in Figures 3(d) and 4(d), it also developed a stress concentration around the knot and much of the centre of the knot had little to no stress. This beam's strength was the most significantly reduced, failing at a load of only 1.40 kN, nearly a 66% reduction compared to the clear beam. Again, the beam failed at the knot, which nearly cracked out of the beam during failure.

Discussion

Output from the VIC-3D digital imaging correlation software provided a clear image of the location of the neutral axis. The neutral axis in the clear beam remained close to the centroidal axis throughout the loading. Even after reaching the proportional limit, the average location of the neutral axis remained at about 46% of the beam height from the bottom of the beam, very close to the centroidal axis. This closely follows the idealized trapezoidal stress distribution of Kollmann and Cote (1968), where the expected location of the neutral axis is 44% of the beam height from the bottom of the beam. However, from the strain contour plots, it is difficult to determine whether the neutral axis moved much during loading. The idealized distribution assumes elastic behaviour, and this small difference may be due to the inelastic behaviour of the clear beam just before failure.

Unfortunately, there is no idealized stress distribution for beams containing knots. From testing, the location of the knot determined the size of the compression and tension zones as well as the location of the neutral axis.

The neutral axis in the beam with a knot located at the centroid behaved like a clear beam away from the knot, but moved around the knot at the mid-section. The neutral axis moved towards the compression zone; however, the knot was slightly lower than the centroid (Figure 4b). The knot reduced the modulus of elasticity, causing tensile strain to increase, pushing the neutral axis above the centroidal axis. It would be interesting to look at a beam with the knot either at the exact location of the centroid or slightly above the centroid, to see the neutral axis locations for these cases.

For a beam with a knot in compression, the neutral axis moved approximately 28% of the beam height away from the centroidal axis towards the tension zone. This is different from the neutral axis for the beam with the knot at the centre where the neutral axis moved away from the knot at the mid-section. This may be because the knot lowered the effective modulus of elasticity locally. For the beam with the knot in compression, the knot reduced the modulus of elasticity causing compressive strain to increase, and the neutral axis to shift lower than the centroidal axis towards the tensile zone.

For a beam with a knot in tension, the neutral axis moved approximately 22% of the beam height away from the centroidal axis towards the compression zone. The neutral axis behaved very similarly to that of the beam with the knot in compression, moving considerably away from the knot as well. The average location of the neutral axis was at about 72% of the beam height above the bottom of the beam (Figure 4d and Table I). It remained relatively straight, starting at 66% of the beam height at the left side of the AOI and ending at 73% at the right side. The main difference was that at the location of the knot, the neutral axis did not seem to move away from or towards the knot. It did, however, drop towards the tension zone slightly after the mid-section.

Finally, the location of the knot also affected the strength of the beam. Knots located in the tension zone cause a greater strength reduction than those in compression.

Acknowledgements

The authors acknowledge the invaluable efforts of Milo Clauson and Arijit Sinha and the support provided by the Center for Wood Utilization Research grant.

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