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Title: MATHEMATICAL MODELING OF SELECTION AND CROSSBREEDING IN BEEF CATTLE

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A mathematical model was developed to simulate alternative selection and crossbreeding strategies for commercial beef production. Biological and economic constraints were selected to represent typical pasture production of calves in the Northwest. The crossbreeding study evaluated the relative economic efficiency of different 1) mating plans when sire breeds of several sizes were used, 2) maximum cow replacement ages, and 3) cow culling strategies based on measures of reproductive failure. Replacing open cows in the fall with a heifer increased economic efficiency, as did replacing cows without a live calf at the end of the calving season. Replacing cows for failure to wean a calf decreased economic efficiency. The use of a very large terminal breed in the criss-out-cross or the three-breed terminal cross surpassed the three-breed rotational cross in economic efficiency. The criss-out-cross was superior to the three-breed terminal cross at cow replacement ages less than twelve years. The cow replacement ages which maximized gross margin per cow were five, nine, and twelve years for three-breed rotational, criss-out-cross and three-breed terminal crossbreeding, respectively. This reflected the average utilization of individual and maternal heterosis for the various mating plans. The selection study evaluated the economic efficiency and selection response of alternative bull and cow culling strategies. Selection was practiced for forty years to increase weaning weight while holding birth weight constant. The bull use strategy which maximized selection gains was to keep bulls for four years while culling 75% of the two-year old bulls based on a progeny test. Economic efficiency was maximized by keeping bulls for only one year. Culling cows for being open or for not having a live calf at the end of the calving season improved economic efficiency to a greater degree than did changing the maximum cow replacement age or the proportion of cows culled based on progeny performance, both of which also improved economic efficiency. Weaning weight was increased either by decreasing the maximum cow replacement age or by increasing the proportion of cows culled for poor progeny performance, but not both, simultaneously.

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CHAPTER 1

INTRODUCTION

Efficient management of a commercial cow-calf enterprise requires decisions pertaining to selection and crossbreeding that can alter the biological and economic efficiency of the operation. However, proper comparisons of different mating plans, culling criteria, and selection methods are difficult because of the complex nature of the effects that the alternatives can have.

Mathematical modeling offers a method which utilizes relatively few resources and can make comparisons among complex alternatives. Long (1972) and Cartwright et al. (1975), using linear programming, compared different mating plans. Congleton and Goodwill (1980a, 1980b) compared mating plans and several culling criteria using dynamic programming. Notter et al. (1979) have also used mathematical modeling to evaluate mating plans. More recently, Clarke et al. (1982) investigated the interaction between mating plan and culling criteria related to reproductive failure.

This study was undertaken to explore the economic and biological consequences of management decisions regarding mating plans, breed sizes, cow culling strategies, and bull use strategies. Previous work (Clarke et al. 1982) indicated that the optimal culling strategy for a simulated spring-calving cow-calf operation was to replace all open cows with a heifer in the fall as well as all cows without a live calf at the end of the calving season. The number of cows and heifers was held constant in the breeding herd in that study. The first objective of this study was to determine the effects of culling criteria based on reproductive failure when the number of cows in the calving herd was held constant. In the previous study (Clarke et al., 1982), when the terminal breed was of the same size as the other breeds used, rotational crossbreeding was more economically efficient than terminal crossbreeding. The second objective of this study was to evaluate the relative efficiency, both biological and economic, of criss-out-crossing and terminal crossbreeding when the mature size, and hence, the growth rate, of the terminal breed was allowed to vary. Utilization of heterosis and complementarity in terminal crossing systems is dependent upon the frequency of different mating types in the herd. Cow replacement age, which will affect the frequency of different mating types, may be different in terminal crossing systems than in rotational crossing systems where heterosis utilization is independent of cow replacement age. The third objective of this study was to determine the optimum cow replacement ages for different mating plans.

Because of the long time periods involved in evaluating selection programs, biological and economic efficiency are not necessarily closely related. For any given period of time, biological efficiency of a selection program is often evaluated as the change in mean value of some measurable trait, such as weaning weight. But in an economic evaluation, the time value of money must be taken into consideration, and so the value of one dollar of profit in the first year of the selection program is not equivalent to one dollar of profit in the last year of the selection program. The fourth objective of this study was to evaluate the effects on economic and biological efficiency of various bull use strategies including the number of years a bull should be used and the value of progeny testing herd bulls. Culling cows for reproductive failure or at some maximum allowable age has large economic effects, but also alters the age structure of the cow herd and thereby affects the selection intensity and generation time, both of which are key factors in determining selection response. The fifth objective of this study was to investigate the effects on both biological and economic efficiency of both culling cows for poor progeny performance and the interactions between this and culling cows for various measures of reproductive failure and at various cow replacement ages.

CHAPTER 2

ECONOMICS OF BEEF COW REPLACEMENT[1]

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INTRODUCTION

A cow-calf operation derives its income from one source -- the sale of livestock of various ages. Recent work has shown that as much as 35% of the revenues may, at times, be derived from the sale of culled cows (Clarke et al., 1982). Beef cows are unusual assets in that they are capable of producing their own replacement, a heifer calf. They are also unique in that, as an asset, they vary tremendously from one to another, complicating replacement decisions. Because of the large amount of capital invested in beef cows, their proper management, as an asset, can have an important bearing on the financial well-being of the enterprise. To this end, theoretical asset replacement principles, in which all variables are functions of age and assets are replaced upon reaching a critical age, have been applied to beef cows in an attempt to determine the age at which beef cows should be replaced. There have also been several recent studies that investigated the culling of brood cows on criteria other than age. These studies have often varied as to biological and economic assumptions, and objective function. The objective of this paper is to review the conceptual framework of asset replacement and the empirical studies that specifically address the replacement of beef cows. Possible areas of future research will also be presented.

GENERAL ASSET REPLACEMENT PRINCIPLES

The choice of objective function to be maximized, time period of interest and discount rate are interrelated questions (Alpin et al., 1977). Early work (Jenkins and Halter, 1964) did not discount future revenues and expenses. This is equivalent to using a discount rate of zero in the computation of the present value of an investment. Some of the objective functions used to evaluate investments are the payback period, the return on investment, net present value, and the internal rate of return. The last two are widely accepted and the internal rate of return allows for investments with different lifespans to be compared. The two methods also allow (by increasing the discount rate) for the increasing uncertainty of returns in the future. Since the papers reviewed often differed in the objective function, time period, and the handling of risk, these areas will be addressed with each paper as necessary.

Definitions

Although beef cow replacement is not exactly a problem to be solved on a continuous-time basis, the conceptual framework will be presented as such, for the calculus simplifies the derivations. Discrete-time equivalents will be provided where appropriate. Some necessary definitions are:

i = annual interest rate for discounting future returns and expenses (this may be the interest rate for borrowing or lending, or a personal time preference rate, as appropriate);

$r = \ln(1+i)$, the equivalent continuous-time discount rate, such that $e^{rt} = (1+i)^t$;

$M(t)$ = market value of the asset at age t;

$R(t)$ = residual income (current revenue less current expenses) from an asset of age t;

s = age at which planned replacement occurs;

C_{bsv} = present value (PV) of the stream of residual earnings of a string of v assets acquired at age b and replaced at age s.

Replacement with an Identical Asset

The generally accepted replacement criterion is that an asset should be replaced so as to maximize the PV of an infinite string of identical assets. According to Perrin (1972), the PV of a single asset at the time of acquisition is

$$C_{bs1} = \int_b^s R(t)e^{-r(t-b)}dt + e^{-r(s-b)}M(s) - M(b) \qquad \text{Eq. 1}$$

To maximize the returns from a single asset, one would simply equate the first derivative of C_{bs1} (Eq. 1) to zero. This yields the replacement criterion

$$R(s) + \frac{\partial M(s)}{\partial s} = rM(s) \qquad \text{Eq. 2}$$

The value of s that satisfies Equation 2 is the age when the asset should be replaced. This would maximize the PV of a single asset.

The PV of an infinite string of assets is

$$C_{bs\infty} = C_{bs1} + e^{-r(s-b)}C_{bs1} + e^{-2r(s-b)}C_{bs1} + \ldots$$

$$= \frac{C_{bs1}}{1 - e^{-r(s-b)}} \qquad \text{Eq. 3}$$

This is the PV of a perpetual annuity of value C_{bs1} received every s years. The PV maximizing criterion for $C_{bs\infty}$ is

$$R(s) + \frac{\partial M(s)}{\partial s} = r[M(s) + C_{bs\infty}] \qquad \text{Eq. 4}$$

The difference between the right hand sides of Equation 2 and 4 is the opportunity cost of delaying earnings until future replacement cycles. In other words, this difference is the income foregone from the presently owned asset, in order to increase income from future assets. In the discrete-time case,

$$C_{bs\infty} = \frac{C_{bs1}}{1 - (1+i)^{-(s-b)}} \qquad \text{Eq. 5}$$

where

$$C_{bs1} = \sum_{t=b+1}^{s} (1+i)^{-(t-b)} R(t) + (1+i)^{-(s-b)} M(s) - M(b) \qquad \text{Eq. 6}$$

This yields the replacement criterion

$$R(s) + \Delta M(s+1) = \frac{iV(s)}{1 - (1+i)^{-(s-b)}} \qquad \text{Eq. 7}$$

where

$$V(s) = \sum_{t=b+1}^{s} (1+i)^{-(t-b)} R(t) + M(s) - M(b) \qquad \text{Eq. 8}$$

Burt (1965), working from the optimum situation that $C_{o(s-1)\infty} \leqslant C_{os\infty} \geqslant C_{o(s+1)\infty}$, derived the discrete-time replacement criterion

$$R(s+1) + \Delta M(s+1) \leqslant \frac{iV(s)}{1 - (1+i)^{-(s-b)}} \leqslant R(s) + (1+i)\Delta M(s) \qquad \text{Eq. 9}$$

Although Burt assumed an asset acquisition age of zero, Perrin (1972) later proved that acquisition age does not affect optimum replacement age. So Burt's replacement criterion is also applicable for non-zero acquisition ages. Unlike Equation 2, Equations 4, 7, and 9 are difficult to solve for s. As such, it is usually easier to maximize Equation 3 or 5 iteratively to determine the age when replacement should occur.

Replacement with Risk

Burt (1965), using the laws of expectation, derived criteria for optimal replacement when unplanned replacement occurs with some known probability. In beef cattle, this could be a cow that dies or is culled, based on other criteria that are stochastic in nature, such as, for failing to wean a calf. Included in the derivation was the possibility that the cost of replacement would differ between planned and unplanned replacement. Burt assumed the objective function was the PV of an infinite string of assets acquired at age zero. Additional definitions are:

$C(s) = M(b) - M(s)$, cost of planned replacement;

$D(t) = M(b) - M(t)$, cost of unplanned replacement at age t;

g_t = the probability of an asset not having undergone unplanned replacement prior to age t;

P_t = the conditional probability of an asset of age t remaining functional until age t+1;

$\tilde{R}(t) = P_t R(t) - (1-P_t)Dt$;

$L(s)$ = age at which replacement actually occurs.

Then, using the laws of expectation, it can be seen that

$$C_{os\infty} = \frac{E[C_{os1}]}{1 - E\left[e^{-rL(s)}\right]} = \frac{\int_0^s e^{-rt}\tilde{R}(t)g_t dt - g_s e^{-rs}C(s)}{r\int_0^s e^{-rx} g_x dx} \qquad \text{Eq. 10}$$

To determine s, the age at which planned replacement should occur, equate the first derivative to zero to get

$$\tilde{R}(s) - \frac{\partial C(s)}{\partial_s} - \frac{\frac{\partial g_s}{\partial_s} - rg_s}{g_s} C(s) = rCos\infty \qquad \text{Eq. 11}$$

As can be readily seen, the replacement criterion is as difficult to solve as the function to be maximized. So again, it is simpler to maximize the PV function iteratively. If $Q_t = \left(\frac{1}{1+i}\right)^t$, $P_o = 1$ and $P_s = 0$, then the discrete-time PV function is

$$C_{os\infty} \quad \frac{\sum_{i=0}^{s-1} Q_{i+1}\tilde{R}(i+1) \prod_{j=0}^{i} P_j + Q_s M(s) \prod_{j=0}^{s=1} P_j}{1 - \left[\sum_{i=0}^{s-1} Q_{i+1}(1-P_{i+1}) \prod_{j=0}^{i} P_j\right]} - M(o) \qquad \text{Eq. 12}$$

An alternative replacement decision model that is capable of accounting for production risk makes use of a recurrence relationship in the manner of dynamic programming (Jenkins and Halter, 1964). The model is capable of handling changes in parameters either by age of the asset, by the progression of time, or both. In this way, although the model was not designed with the intention of discounting future returns, it can be accomplished by weighting the future parameters accordingly. The recurrence relationship is:

$$A_{ytn} = MD(t_1)y + \underset{j=1}{\overset{J}{\text{Max}}} \left[-ME(t_2)y = R(t_2)y - \Delta_t = A_{y+1,\ t+1,\ n} \right] \qquad \text{Eq. 13}$$

where

A_{ytn} = the net profit from year y-n to year y starting with an asset of age t in year y and following the profit maximizing sequence of replacement;

$MD(t_1)y$ = the market value of the presently owned asset (age t_1) in the year y;

$ME(t_2)y$ = the market value of the potential replacement asset (age t_2), in year y;

$R(t_2)y$ = the residual income from an asset of t_2 in year y;

Δ_t = the transaction cost of replacing the presently owned asset with the potential replacement (equals 0 if presently owned unit is replaced by itself). This recurrence relationship could not easily be used to maximize the PV of an infinite string of replacements. However, Jenkins and Halter (1964) proposed that the final year of the expected life of the enterprise be the maximum y used. The model would then be constrained to sell the asset at that time. This is somewhat more realistic than assuming the enterprise will continue through infinity. This procedure has not been applied to beef cattle.

Replacement with a Technologically Improved Asset

As time passes, technological changes occur such that both R(t) and C(s) change. If the PV of a single asset is the objective function, this is of no importance. But if the objective function includes a period of time long enough for the asset to be replaced, technological improvements can become important. To determine the effect of a one-time technological improvement, Perrin (1972) introduced the notion of a Defender, which is the asset presently in use, as contrasted to the Challenger, the potential replacement. The objective function is maximization of D_{adv}, the PV of the stream of residual earnings of a Defender of age a, to be replaced at age d by a string of v Challengers.

$$D_{adv} = D_{ado} + e^{-r(d-a)}C_{os\infty} \qquad \text{Eq. 14}$$

The resulting replacement criterion is

$$R(d) + \frac{\partial M(d)}{\partial d} = r[M(d) + C_{os\infty}] \qquad \text{Eq. 15}$$

Perrin (1972) showed that the age a Defender is replaced is smaller when there is a technologically superior replacement available than when the Defender will be replaced with a technologically identical asset.

Replacement with Continual Technological Improvement

In 1980, Melton elaborated on asset replacement principles by including the possibility of continuous technological improvement. For most assets, technological improvement is not continuous, but discrete. However, when the asset produces its own replacement, as with livestock, technological improvement is accomplished through genetic selection in many small increments. For livestock, a severe complication is that the rate of technological progress is a function of the replacement rate of the herd, which itself is determined by the age at which the sires and dams are replaced. Let k equal the annual discrete-time growth rate of technology and h equal the equivalent continuous-time growth rate. Melton shows that

$$C_{bs\infty} = \frac{C_{bs1}}{1 - e^{(h-r)(s-b)}} \qquad \text{Eq. 16}$$

where C_{bs1} is obtained from Equation 1. In the discrete-time case,

$$C_{bs\infty} = \frac{C_{bs1}}{1 - (1+i)^{-(s-b)}(1+k)^{(s-b)}} \qquad \text{Eq. 17}$$

where C_{bs1} is obtained from Equation 6. Melton also included methods to determine k when selection was practiced on a closed herd of constant size.

The work of Burt (1965), Perrin (1972), and Melton (1980) all assumed that the values of future economic parameters (R(t) and M(t)) were known with certainty. One approach to account for the inherent uncertainty associated with future revenues and expenses is to increase the discount rate. This weights the early years more heavily when one has more confidence in the values of the economic parameters.

EMPIRICAL STUDIES OF BEEF COW REPLACEMENT

There have been several attempts to apply the previously discussed theoretical asset replacement principles to establish replacement criteria for beef cows. The information necessary to do this consists of:

1. R(t), the residual income from a cow of age t, the difference between gross revenue from the sale of the cow's calf and the variable costs (primarily nutritional) for the cow-calf pair;

2. M(t), the market value of a cow of age t;

3. D(t), the cost of unplanned replacement, the cost of a replacement heifer minus the market value of the cow at age t (if the cow dies, its market value is zero);

4. C(t), the cost of planned replacement, the cost of a replacement heifer minus the market value of the cow at replacement age; and

5. P_t, the conditional probability of a cow of age t remaining in the herd till age t+1. Ideally, the nature of changes that these parameters will undergo through time should also be known. Since

there is no way to predict these changes, the usual assumption that is made is that they will not change over time.

In 1972, Rogers attempted to establish beef cow replacement criteria by maximizing net returns per cow. The annual equivalent (A) of the PV of a cow was compared to the actual returns (R) at each age. The age at which A was first greater than R was the age at which a cow should be replaced. If R(t) is the net income per cow for age t, then

$$PV_t = Q_t R_t, \text{ and } A = \sum_{i=1}^{n} PV_i \frac{i(1+i)^n}{(1+i)^n - 1},$$

where n is the number of years of possible productive life left to the cow. This was then compared with R_t. Using typical eastern Washington beef cattle parameters, Rogers determined the optimum age to replace cows was at ten years. The higher returns to be achieved if cows were replaced at either three or four years of age were dismissed as infeasible, since the cow herd could not maintain itself in this case. Rogers assumed R(t) and M(t) did not change with time. However, M(t) declined with age and death loss of cows was taken into consideration.

Bentley _et al_. (1976) was the first to apply the objective function advocated by Perrin (1972) and Burt (1965) to beef cattle: to maximize the PV of an infinite stream of assets. The problem was handled in a discrete-time manner, and $C_{os\infty}$ was iteratively maximized. The cows were assumed to die at some estimated frequency, and were also culled for failing to wean a calf. To do this, Burt's derivations were used. Bentley _et al_. (1976) used several sets of production parameters, including those used by Rogers (1972).

Rogers' death loss probabilities were used in all cases. One conclusion reached by Bentley et al. (1976) was that the optimal replacement age was relatively insensitive to changes in calf price, discount rate and feed costs. The general recommendation was to replace cows at eight years of age. It was observed that the effect of lowering herd fertility was to lower the optimum age to replace cows to six years. As in Rogers (1972), the optimum decision to replace at two years of age was rejected as infeasible. Also, the effect of the change in cow worth with age ($M'(t)$) was found to be critical: if cow worth declined with age, the cows were culled at seven years instead of eight (again rejecting replacement at either two or three years as infeasible).

Kay and Rister (1977) examined the effects of marginal tax rate, discount rate, calf price, and replacement cost on the age at which to cull a cow, and whether to purchase or raise heifer replacements. The analysis ignored unplanned replacement and death loss. Production parameters were unlike those used in previous studies. All relevant tax laws (as of 1976) were included. The general conclusion was to raise replacements. Discount rate (5 vs. 10%) had little effect on the age at which to replace brood cows. In several instances, a 5% increase in discount rate decreased replacement age by one year. Changes in replacement cost had little effect. But at very low replacement costs, calf price became an important factor. With low replacement costs, doubling calf price decreased replacement age two to three years. Otherwise, calf price did not change optimum brood cow replacement age. Marginal tax rate (MTR) had a dramatic effect on replacement age. With a MTR of 22% the usual replacement age was ten years. With a 48% MTR, replacement age decreased to nine years. A 70% MTR yielded a biologically infeasible replacement age of three years.

Melton (1980) applied his derivations of optimum replacement under continuous technological improvement to typical Florida beef cattle parameters. It was not stated whether or not death loss and culling of cows that fail to wean a calf were included. The effect of a cow being genetically above or below average on replacement age was also investigated. For an average cow, the replacement age was eight years with genetic progress (i.e., selection) and eleven years without. For a cow one standard derivation (1 S.D.) above average, the age at replacement was eleven years with, and twelve years without accounting for genetic progress. For a cow 1 S.D. below average, replacement age was five years with, and ten years without genetic progress. Melton assumed a constant sized, closed herd for purposes of selection.

Other techniques have been applied to the question of appropriate criteria on which to base beef cow replacement decisions. These studies have tried to answer different questions than those posed in the conceptual asset replacement framework of the economist. These studies have not used the net present value approach. Instead, they have established equilibrium herd age structures and then compared single year net income or gross margin of the cow-calf enterprise under different culling regimes.

In 1972, Long made use of deterministic linear programming to examine beef production. In the model, cows were replaced at either nine or twelve years of age. The objective function was maximization of return to capital investment. There were two nutritional/environmental regimes in the study, a drylot environment with high feed costs and a pasture environment with relatively low feed costs. In the drylot regime, replacing a cow at nine years resulted in a greater net loss per cow than replacement at twelve years. However, in the pasture regime, replacing cows at nine years was superior to replacement at twelve years, increasing net income/cow fifty-six cents.

More recently, Clarke _et al_. (1982) simulated a Western range cow-calf operation with a deterministic simulation model. Eight different culling criteria were simulated in five different mating plans. The possible culling procedures all accounted for (1) death loss, using the same probabilities as Rogers (1972), (2) culling of one percent of the cows of each age group for unsoundness, and (3) culling all cows over fifteen years of age. Optional culling criteria were for failing to wean a calf, failing to have a live calf at side at the end of the calving season, failure to be pregnant when the previous year's calf crop was weaned, and for any combination of these criteria. For all mating plans, replacing cows that were not pregnant, and then replacing any pregnant cows that failed to have a live calf at side resulted in the greatest net income/cow (Table 1). Culling cows that failed to wean a calf had the lowest net income/cow in all mating plans. Replacement rates for the different culling criteria varied from 8 to 26%. Ranking of the other culling criteria varied depending on the particular mating plan, but in general: (1) culling open cows increased net income/cow, (2) culling cows without a live calf at side increased net income/cow, and (3) culling cows for failing to wean a calf decreased net income/cow. This study demonstrated the importance of criteria other than just age on which to base replacement decisions in beef cattle.

Congleton and Goodwill (1980a,b) used deterministic dynamic programming to study a cow-calf enterprise. Their objective was to determine the effect of culling cows after they failed to wean one _vs._ two consecutive calves. Evaluation was based on gross margin/cow. In all five mating plans there was a reduction in gross margin/cow when cows were culled after failing to wean only one calf when compared to the alternative of culling only after failing to wean two consecutive calves (Table 2). The average reduction was 3.7%. The average replacement rates for the two alternatives were 25.9 and 12.7% for failing to wean one or two consecutive calves, respectively.

Both Clarke _et_ _al_. (1982) and Congleton and Goodwill (1980a,b) pointed out the detrimental effect of increasing replacement rate on average herd production. As the replacement rate increases, the age distribution of the cows changes, increasing the proportion of young cows in the herd. This results in a decrease in average production because immature cows produce at a lower level than mature cows.

REPLACEMENT UNDER CYCLICAL PRICE VARIATION

The cyclical changes in beef cattle prices and inventory have been well documented (Franzmann, 1971). Two studies have attempted to characterize an optimal replacement pattern assuming a known price cycle.

Rogers (1971) used polyperiod linear programming to study replacement decisions in a cow-calf enterprise assuming a nine year known price cycle. The cycle was defined as having four parts, a three year bottom, and a two year upswing, top and downswing. In addition, it was assumed that the decision maker had prior knowledge of the future price changes in the cattle price cycle. One constraint was that the average replacement rate over the nine year cycle equal 15%. Biological parameters were the same as used by Rogers (1972). The optimum replacement pattern resulted in no replacements kept in year one of the upswing and in year two of the top, downswing and bottom. Other years had replacement rates from 23 to 31%. However, average net income/cow/year was only twenty-five cents higher than returns under a constant 15% replacement policy. Rogers concluded that because of the more consistent income, the constant 15% replacement policy was preferable, especially since perfect knowledge of future movements is not available.

In 1979, Trapp and King, assuming a twelve year price cycle (three years in each phase) also looked at replacement decisions. This model separated the culling of cows and adding replacement heifers to the herd. A cow was culled when its PV as a brood cow was less than the current market value. A heifer was added to the herd when its PV as a brood cow was greater than its current market value. It should be noted that the PV was for only the single cow or heifer in question, not an infinite string of assets. Birth rate of calves and calf death loss were the same as those used by Rogers (1972). As with Rogers (1971), it was assumed that the decision maker had perfect knowledge of the future cattle price changes.

Table 3 shows the changes that occur in the herd through the cattle cycle. Unfortunately, a comparison with a constant replacement rate was not included, so it is not possible to determine the relative merit of this replacement policy.

FUTURE RESEARCH

One area of immediate importance is a better characterization of the information used to determine the PV of an asset. Bentley *et al.* (1976) showed the importance of M(t), the market value of a cow, on the replacement decision. There is a need for a better understanding of how M(t) changes with age. Other information that is needed is the changes with age of the probabilities of a cow weaning a calf. Even better would be to break this down into its components as modeled by Clarke *et al.* (1982). Good estimates of the probability of a cow dying or being culled for unsoundness are now available (Greer *et al.*, 1980). Unfortunately, these differ considerably from the probabilities used in previous empirical studies.

Another area in need of further research deals with replacement of superior and inferior cows. Melton (1980) showed that genetically inferior cows should be culled at an early age. However, it is never possible to know, with certainty, the exact breeding value of a cow; it can only be estimated. Therefore, another area of research would be to incorporate this risk, perhaps using techniques similar to E-V frontiers, to evaluate the culling of cows from the herd. Present selection index theory can provide both the expectation and variance of the breeding value of an individual.

Another potential area of research is the determination of the effects of heterosis and its utilization on beef cow replacement. Of particular interest would be how the optimum replacement age changes in the first few generations of a rotational crossbreeding system. In the early generations, the utilization of direct and maternal heterosis differs. The result is that R(t) and M(t) would change, but not necessarily to the same extent, in each generation. This problem could be approached from theoretical asset replacement principles with a multistage technological improvement model.

Simulation studies such as Clarke et al. (1982) have shown that net income/cow can be increased by culling cows on the basis of a pregnancy test. A further step would be to determine the optimal combination of replacement based on a pregnancy test and age. Another important question is the actual monetary benefit of culling open heifers. Cattle sold that are under twenty-four months of age result in earned income; after twenty-four months, the money is considered capital gains, of which 60% is not taxed. As such, it may be that the rancher can receive more after-tax income by holding open heifers until they are eligible for consideration as capital gains. If so, it would be simpler not to pregnancy test any heifers; just wait until the calving season, and sell those heifers which fail to calve.

Running Head: Simulation of Cow-Calf Production

CHAPTER 3

MATHEMATICAL MODELING OF ALTERNATIVE CULLING AND CROSSBREEDING STRATEGIES IN BEEF PRODUCTION[1]

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SUMMARY

A 500-head spring-calving cow-calf enterprise was modeled in which the number of cows in the calving herd was held constant. The study was designed to evaluate the relative economic efficiency of (1) three mating plans (three-breed rotational, criss-out-cross or three-breed terminal) when terminal sire breeds of several sizes were used, (2) mandatory cow replacement ages between five and sixteen years, and (3) cow culling strategies based on measures of reproductive failure (not pregnant, no live calf at the end of the calving season or failing to wean a calf). Replacing open cows in the fall with a heifer increased economic efficiency, as did replacing cows without a live calf at the end of the spring calving season. Replacing cows for failure to wean a calf decreased economic efficiency. The use of a very large terminal breed in a criss-out-cross or a three-breed terminal cross surpassed the three-breed rotational cross in economic efficiency. The criss-out-cross was superior to the three-breed terminal cross at cow replacement ages less than twelve years. The cow replacement ages which maximized gross margin/cow were five, nine, and twelve years for three-breed rotational, criss-out-cross and three-breed terminal crossbreeding, respectively. This reflected the average utilization of individual and maternal heterosis for the various mating plans.

(Key Words: Beef Cattle, Modeling, Heterosis Utilization, Cow Replacement Age, Mating Plans, Culling Strategies.)

INTRODUCTION

Mathematical modeling allows one to investigate complex questions in a short time with a minimum of resources. This study was undertaken to explore the economic consequences of management decisions regarding mating plans, breed sizes and cow culling strategies. Previous work (Clarke et al., 1982) indicated that the optimal culling strategy for a simulated spring-calving cow-calf operation was to replace all open cows with a heifer in the fall as well as all cows without a live calf at the end of the spring calving season. In that study, the number of cows in the breeding herd was held constant. The first objective of this study was to determine the effects of various culling criteria based on reproductive failure when the number of cows in the calving herd was held constant. In the previous study (Clarke et al., 1982), rotational crossing was more economically efficient than terminal crossbreeding systems when the terminal breed was of the same size as the other breeds used in the mating plan. The second objective of this study was to evaluate the relative profitability of criss-out-crossing and terminal crossbreeding when mature size of the terminal breed was allowed to vary. Utilization of heterosis and complementarity, particularly in terminal crossbreeding systems, will depend upon the proportion of different mating types in the herd. Cow replacement age, which will affect the proportion of different mating types in the herd, may be higher in terminal crossbreeding systems than in rotational systems. The third objective of this study was to determine the optimum cow replacement ages for different mating plans.

MATERIALS AND METHODS

A spring-calving cow-calf enterprise was simulated using the deterministic model described by Clarke et al. (1982). The calving herd was held at 500 head, including heifers. Unless otherwise stated, biological and economic input parameters used to characterize the enterprise are the same as in Clarke et al. (1982).

Enterprise Characterization

Biological Constraints. Annual death loss in each age class of the cow herd was 2% year^{-1} (Greer et al., 1980), while heifers had a 1% death loss from weaning to the start of the breeding season. Cows two-, three-, and four-years-old at calving experienced unsoundness at a frequency of 1% year^{-1} and were culled. Two, three, and four percent of the cows aged five, six, and seven years, respectively, were culled for unsoundness while 5% of older cows were culled for unsoundness.

The conception rates for straightbred females were 88, 90, 92, and 94% for heifers, two-, three-, and four-year-old and older cows, respectively. Values for individual heterosis (H^I) and maternal heterosis (H^M) for conception rate were 4.3 and 7.4%, respectively (Wiltbank et al., 1967; Cundiff et al., 1974a; Spellbring et al., 1977b).

Of those cows conceiving, 90, 95, and 97% delivered a live calf during the calving season for two-, three-, and four-year-old and older cows at calving, respectively. This rate was decreased in heifers by an additional .88% for every kilogram increase in calf birth weight (Notter et al., 1978). Values for H^I and H^M for live calves born were 1.6 and 1.7%, respectively (Wiltbank et al., 1967; USDA, 1977; 1978).

Ninety-eight percent of all calves alive at the end of the calving season survived to weaning. Values for H^I and H^M for calf survival were 3.2% and .6%, respectively (Cundiff et al., 1974a; USDA, 1977; 1978).

Heterosis values were adjusted for the number of breeds in the rotation (Carmon et al., 1956) and to prevent reproductive percentages from exceeding 100% (Clarke et al., 1982). The resulting probabilities of a cow weaning a calf are presented in Table 4.

Three breed sizes were used in the study: (1) the Medium (M) breed, similar to Hereford or Angus, with birth weight and preweaning average daily gain (ADG) of 35.6 kg and .84 $kg \cdot day^{-1}$, respectively; (2) the Large (L) breed, similar to Limousin, with birth weight and ADG of 39.9 kg and .88 $kg \cdot day^{-1}$; and (3) the Very Large (VL) breed, similar to Charolais or Simmental, with birth weight and ADG of 44.3 kg and .92 $kg \cdot day^{-1}$, respectively (USDA, 1974).

All breeds in the three-breed rotation (3R) were M. Likewise, the two breeds in the rotational part of the criss-out-cross ((2R)T) and the first two breeds of the three-breed terminal cross (3T) were M. Mature cow weight for M was 500 kg. Mature bull weights were 850, 1000, and 1150 kg for M, L, and VL, respectively.

Multiplicative sex-of-calf and age-of-dam adjustments were used to produce calf weights for calves of different sex and from dams of different ages (Clarke et al., 1982). Values for H^I and H^M for birth weight were 3.9 and 3.0%, respectively (Sagebiel et al., 1973;

Cundiff et al., 1974b). Values for H^I and H^M for ADG were 5.8 and 5.4%, respectively (Cundiff et al., 1974b; Long and Gregory, 1974; Sagebiel et al., 1974). Heterosis for mature cow weight was 5% (Smith et al., 1976; Spellbring et al., 1977a; Gaines et al., 1978).

Economic Considerations. Livestock were simulated to graze irrigated pasture consisting of mixed alfalfa and orchard grass from April 9 to October 7. Cost of pasture production, which did not include return to land investment, was \$129.11 hectare^{-1} (Warnock and Carkner, 1981). It was further assumed that .54 hectares were needed per straightbred cow-calf pair for grazing, a typical value in the Pacific Northwest. Crossbred pairs required more grazing land in proportion to increased energy requirements because of greater weight. During the rest of the year, livestock received mixed alfalfa-orchard grass hay. Cost of hay production was \$34.98 metric ton^{-1}, which also did not reflect any return to land investment (Warnock and Carkner, 1981).

Fixed ranch costs were \$73.35 head^{-1} (Warnock and Carnker, 1981). Pregnancy testing cows cost \$2.00 head^{-1} (Dr. Jerry Reeves, personal communication). Each cow required 1.7 ampules of semen at a cost of \$8.85 ampule^{-1} (Dr. Joe Hillers, personal communication). Six clean-up bulls were used annually. These were purchased for \$1,300 each, kept three years and sold for slaughter. The owner-operator, who was budgeted to receive \$12,000 annually, provided up to 60 hours of labor per week. Any additional labor was acquired at \$5.00 hour^{-1}. Interest income and charges were computed biweekly for positive and negative cash flow at a rate of 15% (on a yearly basis).

Cattle prices were seasonally indexed (Clarke et al., 1982). The base prices were taken as the average of weekly prices for the first six months of 1981 in Washington. Prices for steer and heifer calves were \$1.63 and 1.39 kg^{-1}, respectively. Cull cows and cull bulls sold for \$.95 and 1.22 kg^{-1}, respectively, while open heifers were sold for \$1.29 kg^{-1}.

Simulations

A three-breed rotational cross was utilized to determine the effects of culling criteria based on reproductive failure. Culling options based on reproductive criteria were: M, only unsound cows and those which had reached the maximum allowable age were culled (these criteria were employed regardless of any additional culling that took place); P, culling open cows in the fall; L, culling all cows without a live calf at the end of the calving season; W, culling any cow that failed to wean a calf; and PL, culling open cows in the fall and then culling any cow that subsequently lost her calf in the spring. The maximum ages at which cows were replaced with a heifer were six, nine, or twelve years. Additional cow replacement ages were simulated as necessary to establish the age that maximized economic efficiency for a particular culling strategy. Cow replacement age could not be greater than sixteen years. An added restriction was that no more than 85% of the available heifer calves could be kept for replacement purposes.

Management alternatives were compared on gross margin per cow (GM/cow). Gross margin and GM/cow yield equivalent ranking of management alternatives since the number of cows was held constant.

Since gross margin and net income differ by a constant amount (fixed herd cost and owner salary), rankings for these would also be equivalent. Of the three criteria, GM/cow is somewhat more desirable as it eliminates herd size effects.

In order to determine the effect of mature size of the terminal breed on the efficiency of terminal crossing systems, two mating plans (3T and (2R)T), and the three sizes of terminal sire were compared in a two by three factorial arrangement. Open cows were culled in the fall as were cows without a calf at the end of the spring calving season. Cow replacement age was initially set at six, nine, or twelve years. Further simulations were used to determine the cow replacement age which maximized gross margins. This procedure also allowed evaluation of the effect of replacement rate on utilization of heterosis and complementarity.

RESULTS AND DISCUSSION

Rotational Crossbreeding

The effects on GM/cow of various culling strategies based upon reproductive failure are shown in Figure 1. In all simulations, net income was negative. These results were similar to those of Clarke et al. (1982) where the number of cows in the breeding herd was held constant. Culling open cows (P) resulted in an increase in GM/cow, as did culling cows which did not have a live calf at the end of the calving season (L). As in previous work (Clarke et al., 1982), culling cows which failed to wean a calf (W) decreased GM/cow. The combination culling scheme, PL, resulted in the highest GM/cow.

The effect of different maximum cow replacement ages on GM/cow was dependent upon the reproductive culling scheme. For M or W culling, GM/cow was maximized when cows were replaced at nine years. For P, L, and PL culling, it was maximized by replacing cows at five years. The results were caused by the effects of the various culling strategies on the number of calves produced, number of calves sold, number and timing of the sale of cows, the annual replacement rate and interest charges.

Culling open cows increased the number of calves produced (Figure 2). Culling open cows resulted in all cows in the calving herd being pregnant, which other culling criteria failed to do. In fact, L and W culling decreased the number of calves produced. The increase in calves produced as cow replacement age increased was the result of the increased probability that a mature cow would wean a calf relative to younger cows.

Culling open cows, while increasing the number of calves produced, did not increase the number of calves sold (Figure 3). The extra calves produced with P culling were needed as replacements for the open cows that were culled each fall. The other principle culling alternatives, L and W, both increased the replacement rate without increasing the number of calves produced, resulting in fewer calves sold. The number of calves sold increased as cow replacement age increased because fewer heifers were required for replacement purposes.

All reproductive culling schemes (P, L, and W) increased the annual replacement rate (Figure 4). Minimal culling resulted in the lowest annual replacement rate, followed by P which only removed open cows. The other culling alternatives occurred later in the reproductive cycle when there were more cows without a calf, resulting in higher annual replacement rates.

As shown earlier, P culling did not increase the number of calves sold. The economic advantage of P over M culling was derived from the increase in the number of cows sold (Figure 5), which paralleled the effects of culling on replacement rate.

The economic advantage of culling cows in the spring over M culling was due in part to the greater price received for cattle in the spring and to the decrease in annual interest charges incurred during the calendar year (Figure 6). Both P and W culling increased the interest costs because the increased replacement rate meant more heifers were kept and fed as replacements. But L culling provided income in the spring of the year which offset a portion of the negative cash flow incurred early in the year, resulting in a decrease in the annual interest cost.

The effect of cow replacement age on gross margins was minor when compared to the differences between reproductive culling criteria (Figure 1). With P, L, or PL culling, the optimum cow replacement age was at five years, which was the lower limit imposed by retaining not more than 85% of the available heifers. For M or W culling, the cow replacement age which maximized GM/cow was nine years, which was also the age which maximized the average sale weight of calves in 3R (Figure 7).

This, and the previous study (Clarke et al., 1982), show that regardless of whether the breeding herd or the calving herd is held to a constant size, culling open cows in the fall increases GM/cow, culling cows which do not have a live calf at the end of the calving season increases GM/cow and culling cows which fail to wean a calf decreases GM/cow. Furthermore, the cow replacement age which maximizes GM/cow is nine years for M or W culling and five years for P, L, and PL culling.

Terminal Crossbreeding

With (2R)T and 3T, the cow replacement age which maximized GM/cow depended upon the size of the terminal breed (Figure 8). Regardless of terminal breed size, 3T had larger GM than (2R)T for cow replacement ages of eleven or greater. Gross margin increased as the size of the terminal breed increased in 3T and (2R)T. The comparison of (2R)T and 3T was influenced by feed costs and receipts from the sale of cull cows and calves.

Herd feed costs decreased as cow replacement age increased

(Figure 9). As cow replacement age increased, fewer heifers were kept as replacements, decreasing feed costs for heifers. Cow feed costs also decreased since a greater proportion of the cows had reached mature weight and were no longer growing. As the size of the sire breed increased, calf feed costs increased because of the larger size of the calves. The (2R)T mating plan had greater feed requirements than 3T because all cows were crossbred and thus had higher daily energy requirements because of their larger size. Feed cost differences between (2R)T and 3T lessened as cow replacement age increased because the percentage of the cows in 3T that were crossbred, increased. This offset the decrease in cow and heifer feed costs due to decreasing replacement rate.

The average weight of calves sold (Figure 7) showed a pattern similar to GM (Figure 8). For 3T, average sale weight increased as cow replacement age increased. This resulted from an increase in the percentage of calves and cows that were crossbred. In (2R)T, the decrease in average sale weight as cow replacement age increased beyond nine years, lessened as terminal sire breed size increased. As cow replacement age increased beyond nine years, cows aged five through nine years comprised a smaller proportion of the cow herd. These cows produced calves which exhibited maximum weaning weight because the cows were in their prime producing years. The result of increasing cow replacement age beyond nine years was an observed decrease in average sale weight of calves. This was overshadowed in 3T by the increasing utilization of heterosis and complementarity as cow replacement age increased.

The total weight of calf sold, which was the product of the number of calves sold and the average sale weight, increased with cow replacement age (Figure 10). The (2R)T mating plan was superior to 3T at low cow replacement ages but did not maintain this superiority as cow replacement age increased, because heterosis and complementarity utilization increased more rapidly in 3T as cow replacement age increased.

The total live weight sold, which included cull cows and heifers, decreased as cow replacement age increased (Figure 11). The effect of cow replacement age on calf production was overshadowed by its effect on the number of cows culled.

Heterosis utilization was not affected by cow replacement age in 3R (Figure 12). Maternal heterosis (H^M) was lower in (2R)T, but also did not vary with replacement rate. As cow replacement age increased, the annual replacement rate decreased, and so fewer heifer calves were needed. This allowed a greater proportion of the cows to be bred to terminal breed sires, with a resulting increase in utilization of individual heterosis (H^I). Both H^I and H^M increased in 3T for the same reason.

Discussion

Cartwright et al. (1975), using linear programming, compared the (2R)T and 3T mating plans with a cow replacement age of twelve years in a pasture environment. Three-breed terminal crossing had a higher ($355) net income than (2R)T. This is similar to the $235 superiority of 3T over (2R)T using a Large terminal breed in this study. Notter et al. (1979), using deterministic programming, concluded that (2R)T was more economically efficient than 3T. In both of these studies, calves were finished in a feedlot, whereas weaned calves were sold in the present study.

Results from attempts to determine optimal cow replacement age using asset replacement principles have been variable. Optimal cow replacement ages of eight (Bentley et al., 1976), nine (Kay and Rister, 1977), ten (Rogers, 1972), and eleven (Melton, 1980) have been reported. Under certain biological or economic conditions, an optimal cow replacement age of three years was identified, although this was not feasible unless replacements were purchased (Rogers, 1972; Bentley et al., 1976; Kay and Rister, 1977). None of these studies involved crossbreeding or the culling of open cows. This study showed that the optimum cow replacement age depends upon the mating plan in use. The cow replacement ages which maximized GM/cow were five, nine, and twelve years for 3R, (2R)T, and 3T, respectively.

CHAPTER 4

MATHEMATICAL MODELING OF ALTERNATIVE SELECTION STRATEGIES FOR BEEF PRODUCTION[1]

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SUMMARY

A closed beef herd of 500 cows and heifers was simulated over forty years. Selection was practiced to increase weaning weight while holding birth weight constant. The bull use strategy which maximized selection gains was to keep bulls for four years while culling 75% of the two-year-old bulls based on a progeny test. Keeping bulls for two years maximized genetic gains in weaning weight if progeny testing was not practiced. Economic efficiency was maximized by keeping bulls for only one year. Culling cows for being open or for not having a live calf at the end of the calving season improved economic efficiency to a greater degree than did decreasing the maximum cow replacement age or increasing the proportion of cows culled based on progeny performance, both of which also improved economic efficiency. Weaning weight was increased either by decreasing the maximum cow replacement age or by increasing the proportion of cows culled for poor progeny performance, but not both simultaneously.

(Key Words: Beef Cattle, Modeling, Selection, Cow Replacement Age, Culling Strategies.)

INTRODUCTION

Crossbreeding and selection are two methods by which animal breeders can alter the biological and economic efficiency of a livestock production system. A recent paper evaluated by simulation the effects and interactions of several culling strategies in combination with several mating plans which utilized heterosis and breed complementarity to varying degrees (Clarke et al., 1983). The objectives of this study centered on the goals of maximizing selection progress in a closed straightbred herd and in maximizing economic gain. In particular, the objectives were to determine the optimum number of breeding seasons that bulls should be used, to determine the value of progeny testing bulls and to determine the value of repeated culling of cows based on progeny performance. The impact of this cow culling was evaluated alone and in combination with several reproductive culling strategies and with several maximum cow replacement ages.

MATERIALS AND METHODS

The alternative goals of maximizing genetic progress or economic gains were evaluated with a modified version of the deterministic model used previously to evaluate crossbreeding (Clarke et al., 1983). Two major modifications were required for simulation of genetic progress. First, the model was altered to allow simulation of multi-year periods. In so doing, age classes of livestock were assumed to be at their equilibrium number in the first year, as determined from the various age specific reproductive rates and selected culling criteria. Second, average genetic values and variances were generated for each age-sex class. The herd was closed and held at 500 cows and heifers at calving, with both heifer and bull replacements selected from within the calf crop.

Characterization of the cow-calf enterprise was the same as Clarke et al. (1983). Cattle initially were of the medium breed, which had a mature-cow, male-calf birth weight and preweaning average daily gain of 35.6 kg and .84 $\text{kg}\cdot\text{day}^{-1}$. Mature weights of bulls and cows were 850 and 500 kg, respectively.

Heifer and bull replacements were selected with a restricted selection index designed to hold birth weight constant while increasing weaning weight:

$$I = .2\ WW - .5\ BW,$$

where

I = resulting index value,

WW = age-of-dam and sex-of-calf adjusted 205-day weight, expressed as a deviation from the mean, and

BW = age-of-dam and sex-of-calf adjusted birth weight, expressed as a deviation from the mean.

Genetic and phenotypic parameters used to construct the index were weighted averages from Woldehawariat et al. (1977). The correlation between this index and the genetic values for weaning weight and birth weight were .37 and 0, respectively.

Weaning weights of calves from a specific age-of-dam class were assumed to be normally distributed. The mean genetic value for calves of each age-of-dam class was one-half of the mean genetic value of the herd bulls plus one-half of the mean genetic value of the dams of that class. When genetic differences between dams of different ages developed due to selection, proportionately more heifer and bull replacements were selected from the younger-aged dam groups, with the truncation points for calves from each age-of-dam class differing by the difference between their mean genetic values. Because of the small number of individuals chosen, selection intensities were adjusted downward by 1/4N, where N is the number of replacements selected (Van Vleck, 1979).

Because of the objectives dealing with subsequent culling of breeding stock based on progeny performance, genetic variance within the age classes of cows and bulls was decreased by selection:

$$\sigma_G^{2\,\prime} = \sigma_G^2 \{1 - r^2 i\ (i-t)\},$$

where

$\sigma_G^{2\,\prime}$ = additive genetic variance after selection,

σ_G^2 = additive genetic variance before selection,

r = correlation between index and genetic value for weaning weight,

t = truncation point on a standard normal distribution, and

i = selection intensity (Cochran, 1951).

It was assumed that segregation and independent assortment during meiosis and that recombination at fertilization reestablished the original level of additive genetic variance in each calf crop. It was further assumed that genetic variances and covariances did not change over time as the change would have been quite small (Fimland, 1979).

When cows and bulls were culled for poor progeny performance, the accuracy of selection was adjusted according to the number of offspring produced in that single year. In order to practice selection on the herd bulls, it was assumed that the sire of the calf was known, an assumption often not true in commercial production. Subsequent selection of bulls was based on progeny performance and occurred after their use as two-year-olds, when their yearling-sired progeny were weaned. Cow selection based on progeny performance was practiced repeatedly at each age.

The expected rate of inbreeding varied from .002 to .01 year^{-1}, depending on the specific breeding program (Emigh and Pollak, 1979). Inbreeding depression was ignored in the simulations because the averaging of the replacement heifer inbreeding coefficients, as would have been necessary because of overlapping generations, would have been invalid because of the nonlinear response of weaning weight to dam inbreeding, as demonstrated by Brinks and Knapp (1975).

The net present value (NPV) method of evaluating capital investments was used to compare alternative selection strategies economically because the cash flows were expected to vary over time (Alpin et al., 1977). A pre-tax discount rate of 15% was used to adjust year-end net cash flow per cow (NCF/cow) for each year back to present value. This was consistent with the within-year biweekly 15% rate (on a yearly basis) used to allocate interest income and expenses related to the timing of the cash flow during the year. Simulations ran for forty years.

Simulations

Two sets of simulations were conducted. The first set evaluated alternatives related to the number of years bulls should be kept and the value of progeny testing the bulls. Decisions related to cow culling were not varied. In the second set of simulations, the alternative cow culling strategies were investigated. These included changes in the maximum cow replacement age, the level of culling based on progeny performance and also the use of several criteria related to reproductive failure. The bull use strategies were not varied in this set of simulations.

To determine the optimal number of breeding seasons that bulls should be used, simulations were conducted with bulls kept for either one, two, three, or four breeding seasons. The number of cows a bull would service was age dependent (Ensminger, 1978) and evaluated at two levels (Table 5). Cows remained in the herd until nine years of age except that open cows were culled in the fall. To determine the value of progeny testing herd bulls, and indirectly, therefore, the value of knowing the sire of the calf, 25, 50, or 75% of the two-year-old bulls, after their second breeding season, were culled based upon the index values of their yearling-sired calf crop. The two alternative cow-servicing capabilities affected not only the number of herd bulls required, but also the correlation between the phenotypic predictor (average progeny index) and the bulls' true genetic value for weaning weight.

In the second set of simulations, the impact of culling cows based on progeny performance was determined by culling 0, 5, 10, 15, or 20% of each age-class each year. Culling was based on the index value of the calf produced that year. Most probable producing ability was not used since it included permanent environmental effects which are not transmitted to offspring and would, therefore, not improve selection gains. Maximum cow replacement age was five, six, nine, or twelve years. Cows were also culled for reproductive failure in order to investigate the interactions between these methods of culling cows. Options were no reproductive culling (M), open cows culled (P), or open cows culled in the fall, and then any cow left without a live calf at the end of the calving season also culled in the spring (PL). Bulls were used for two breeding seasons, assuming cow servicing capability schedule A (Table 5).

RESULTS AND DISCUSSION

Bull Use Strategies

Economic and genetic results with regard to the value of progeny testing bulls and the number of years bulls should be kept were similar for the two cow servicing capability schedules and so only the results of schedule A are presented.

The bull use strategy which maximized selection gains over forty years was to keep bulls for a maximum of four years but to cull 75% of the two-year-old bulls based on their yearling-sired progeny (Figure 13). If bulls were not progeny tested, using bulls for two years maximized selection gains, followed closely by using bulls for three years. Rankings among the strategies for genetic progress were identical after ten, twenty, and forty years of selection.

The yearly change in calf weaning weight was erratic in the first few years of the selection program, but stabilized by year twenty (Figure 14). The largest gains were made early in the selection program by keeping bulls for only one year because the other bull use strategies kept some of the original bulls, which had an average net breeding value of zero, in use for longer periods of time.

Response to selection for the various bull use strategies ranged from .06 to .08 σ year.$^{-1}$ Selection response from actual selection experiments with beef cattle include .05 σ year^{-1} for weaning weight (Koch et al., 1974), .08 σ year^{-1} for final weight (Nelms and Stratton, 1967) and .1 σ year^{-1} for yearling weight (Newman et al., 1973).

Keeping bulls for either two or three years resulted in the greatest genetic gains per year at the end of the forty year simulation if progeny testing was not practiced (Table 6). However, keeping bulls for four years and culling 75% as two-year-olds had the greatest selection gains per year, overall.

Keeping bulls for only one year resulted in the greatest NPV of NCF/cow (Figure 15). Rankings of bull use strategies were identical if NCF/cow was not discounted. The economic benefit of replacing bulls every year was due to the greater NCF/cow initially generated (Figure 16). In fact, keeping bulls for three or four years produced a situation where the cow-calf enterprise was operating at a loss in the early years of the selection program.

The greater economic efficiency that resulted from replacing bulls each year involved changes both in returns and in expenses. Fewer calves were sold when bulls were used only as yearlings since more bull calves were retained as replacements. In the early years of the selection program the income from the sale of cull yearling bulls more than offset the loss of income from the sale of weaner calves. In addition, the exclusive use of yearling bulls reduced feed costs because bulls were not kept after the end of the breeding season. These economic advantages of using yearling bulls exclusively decreased in the later years as the more rapid selection gains (due to increased selection intensity) that resulted from using bulls for two years increased the value of weaned calves. However, discounting future returns back to present value weighted the early years more heavily than later years, leaving the economic advantage in favor of using only yearling bulls.

Cow Culling Strategies

The biological effect of culling cows for poor progeny performance depended upon the cow replacement age and the reproductive culling criteria (Figures 17a, 17c, 17e). In general, as maximum cow replacement age increased, the percentage of cows culled for poor progeny performance which maximized selection gains also increased. In addition, the optimum level of culling for poor progeny performance was generally higher with M culling than with P or PL culling.

There were two reasons that the highest levels of culling failed to result in the highest selection gains, both of which were the result of the increased replacement rate (Figure 18). First, as the percentage of cows culled for any reason increased the proportion of heifer calves retained as replacements had to increase concommitantly. This decreased selection intensity in the females and thus lowered genetic gains. Second, the increased culling percentage decreased the average age of cows in the herd. Since reproductive rates were lower for heifers and young cows, even more heifers and cows were needed in the breeding herd to maintain the herd size of 500 head at the beginning of the calving season, especially when open cows were culled (P or PL culling). By increasing the size of the breeding herd, more bulls were required to service these cows, which in turn decreased the selection intensity for the males.

The effect on economic efficiency of culling cows was more straightforward (Figures 17b, 17d, 17f). In situations where NPV of NCF/cow was positive (P or PL culling), the higher the level of culling, either by decreasing the cow replacement age or by increasing the level of culling based on progeny performance, the greater the

NPV. The reason that culling cows based on progeny performance was economically advantageous was for the same reason that decreasing cow replacement age was advantageous. The sale value of cull cows more than offset the income lost by retaining a heifer replacement so long as herd size could be maintained. This result could change dramatically if the relative prices of cull cows and weaner calves were to change.

Discussion

Based on this study, it appears that strategies which maximize selection response do not necessarily maximize economic returns. In these simulations, the bull use strategy which maximized NPV of NCF/cow was that alternative which yielded the smallest gain in weaning weight. For commercial beef producers, there is little economic value in knowing the sire of the calf. But this knowledge is beneficial in increasing genetic progress.

In terms of biological efficiency (selection gains), decreasing cow replacement age is a simple alternative to culling cows based on poor progeny performance. Both methods reduce the average cow age so that a greater number of generations occur within a given period of time. The added genetic gains that result from culling cows on performance are of minor importance because of the effect on resulting selection intensity in replacement bulls and heifers and because such a small proportion of genetic progress is achieved as a result of female selection. In terms of economic efficiency, the optimum cow culling strategy is to cull open cows and cows without a live calf at the end of the calving season, decrease the maximum cow replacement age, and increase the culling of cows for poor progeny performance as much as possible while still maintaining herd size.

TABLE 1. Simulated average net income/cow for different culling criteria[a]

Culling Criteria[b]	Replacement rate, %[c]	Average net income/cow, $[c]
M	8	12.86
MP	15	40.25
ML	21	43.59
MW	20	-28.32
MPL	22	51.58
MPW	23	9.71
MLW	23	36.42
MPLW	24	43.39

[a] From Clarke et al., 1982.

[b] M = culling for unsoundness and for age (greater than 15 years) and death loss; P = culling open cows; L = culling cows without a live calf at side at the end of the calving season; W = culling cows for failing to wean a calf.

[c] Averaged across 8 different mating plans.

TABLE 2. Simulated gross margin/cow with different culling criteria[a]

Mating plan[b]	Cows are culled after failing to wean 1 calf	Cows are culled after failing to wean 2 consecutive calves
AA	128.85	130.44
HA	130.48	137.22
C-HA	128.57	135.52
HAX	136.49	141.32
CA	127.62	132.84

[a] From Congleton and Goodwill (1980a, b).

[b] First letter refers to breed of sire, second to breed of dam; A = Angus, H = Hereford, C = Charolais, X = rotational cross.

TABLE 3. Simulated optimal herd size and replacement and culling patterns under cyclical cattle prices[a]

Period	Herd size	Profit/ head	Replace- ments	Culls[b]	Optimal[c] culling age	Age of oldest cow in the herd	Average age of cows in the herd	Feeder[d] steer price
1	114.84	-4.46	26.45	22.65	12	12	6.35	27.70
2	118.64	-1.42	42.38	39.11	10	12	5.94	28.80
3	121.91	-1.74	28.14	27.08	9	10	4.23	31.03
4	122.97	10.78	20.40	22.39	8	9	4.03	33.00
5	120.98	42.58	36.67	40.09	6	8	4.29	36.36
6	117.56	49.82	23.50	28.43	6	6	3.74	38.04
7	112.63	51.64	8.94	12.92	9	6	3.92	38.37
8	108.65	51.32	7.96	11.35	13	7	4.74	37.27
9	105.26	54.37	10.17	10.98	13	8	5.48	35.04
10	104.45	40.30	14.39	12.14	13	9	6.06	32.27
11	106.70	22.56	17.94	13.90	13	10	6.28	29.69
12	110.74	6.49	20.62	16.52	12	11	6.39	28.02
Average	113.78	29.60	21.46	21.46	10.3	9	5.12	33.03

[a] From Trapp and King (1979).

[b] Culls include cows culled due to enforcement of the culling rule and due to failure to produce a calf.

[c] All cows are assumed to produce their first calf at age 2.

[d] Feeder steer prices are detrended and based at 1966 levels.

TABLE 4. Probability of a cow in the breeding herd weaning a calf when mated to a medium-sized sire

Cow age at breeding	S-S[a]	S-C	C-C	2R	(2R)T	3R
1	78.0	78.8	81.3	80.1	80.5	80.8
2	84.0	85.7	88.6	87.0	87.6	87.9
3	86.0	87.3	89.7	88.4	88.9	89.1
4-15	89.0	90.7	92.5	91.4	91.9	92.1

[a] S-S = straightbred cows producing straightbred calves;

S-C = straightbred cows producing crossbred calves;

C-C = crossbred cows producing crossbred calves;

2R = two-breed rotational cows producing two-breed rotational calves;

(2R)T = two-breed rotational cows producing terminal-cross calves;

3R = three-breed rotational cows producing three-breed rotational calves.

TABLE 5. Cow-servicing capabilities of various aged bulls

Age of bull	Schedule A	 B
Yearling	15[a]	10
2	30	25
3	45	40
4	45	40

[a] Number of cows a bull could service during the breeding season (Ensminger, 1978).

TABLE 6. Genetic gain in weaning weight (kgs) in the final year of the selection program

Number of years a bull is kept	Percentage of two-year-old bulls culled based on a progeny test			
	0	25	50	75
1	1.83			
2	2.03			
3	2.03	2.03	2.02	2.02
4	1.98	2.12	2.22	2.25

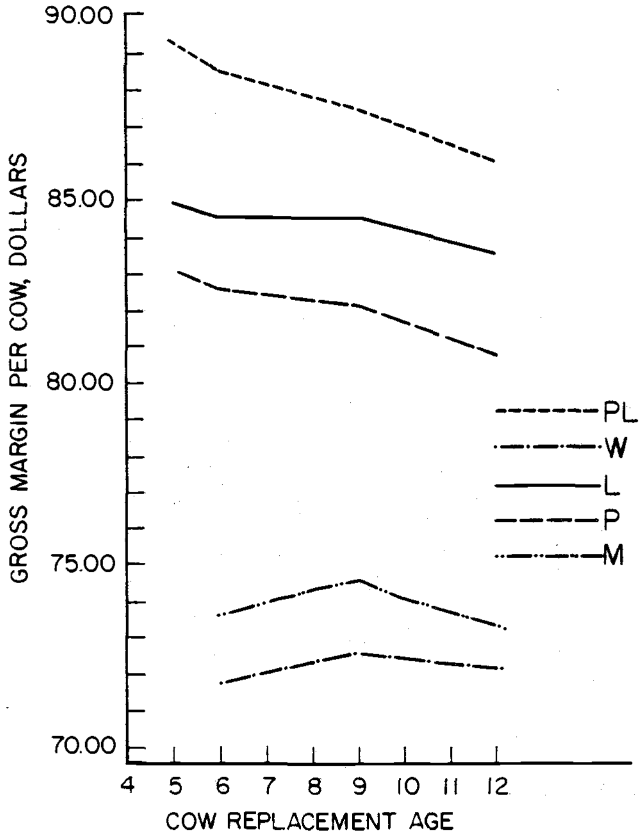


Figure 1. Effect of cow replacement age on economic efficiency with different culling criteria.

Legend: M = minimal culling; P = culling open cows; L = culling cows without a live calf at the end of the calving season; W = culling cows which fail to wean a calf.

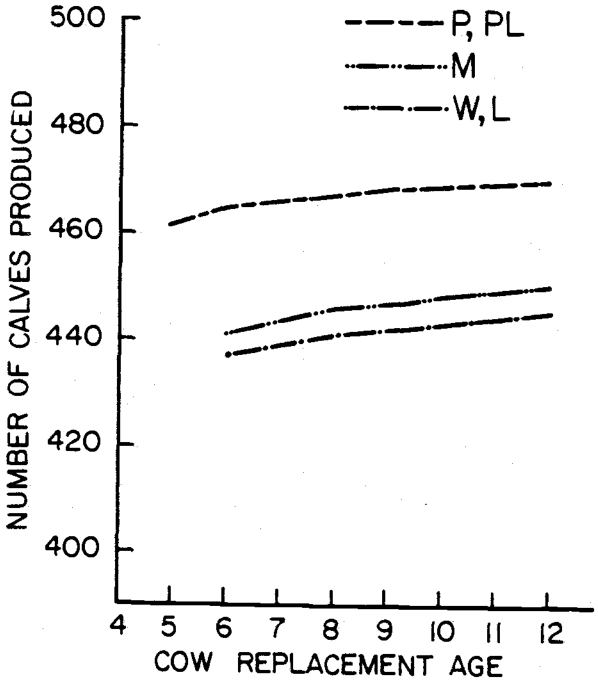


Figure 2. Effect of cow replacement age on the number of calves produced with different culling criteria.

Legend: See Figure 1.

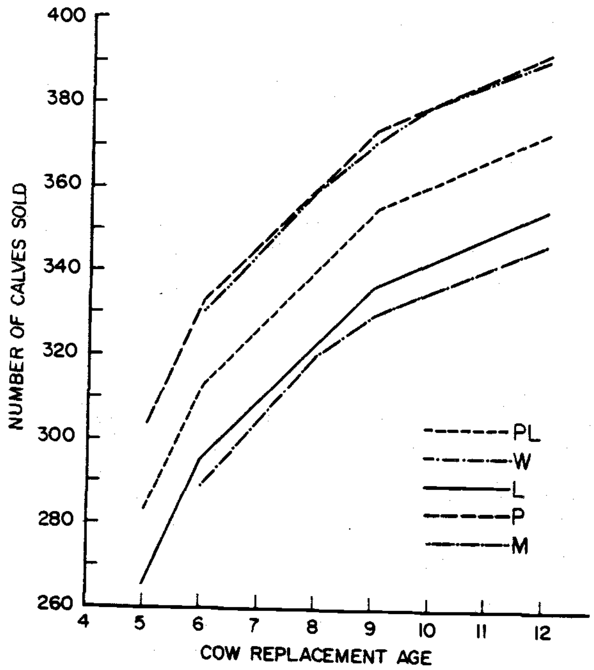


Figure 3. Effect of cow replacement age on the number of calves sold with different culling criteria.

Legend: See Figure 1.

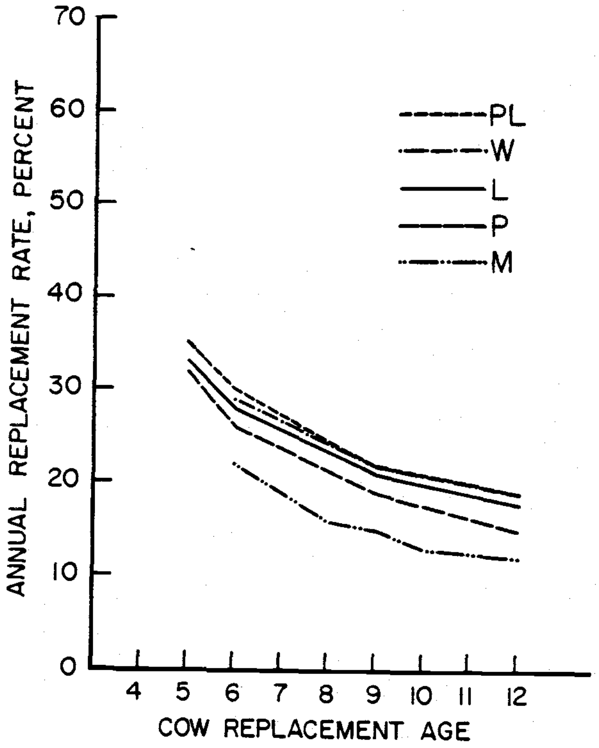


Figure 4. Effect of cow replacement age on the annual replacement rate with different culling criteria.

Legend: See Figure 1.

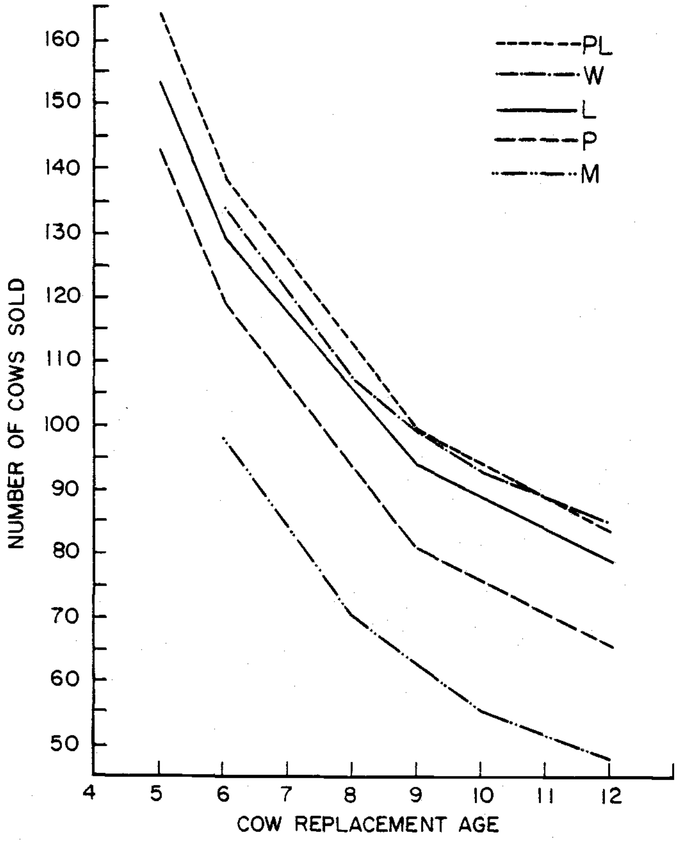


Figure 5. Effect of cow replacement age on the number of cows sold with different culling criteria.

Legend: See Figure 1.

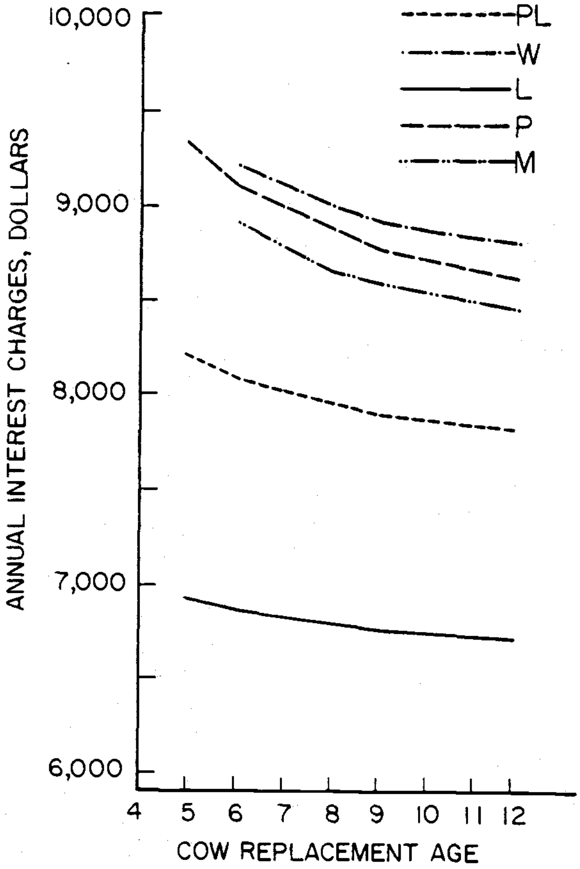


Figure 6. Effect of cow replacement age on annual interest charges with different culling criteria.

Legend: See Figure 1.

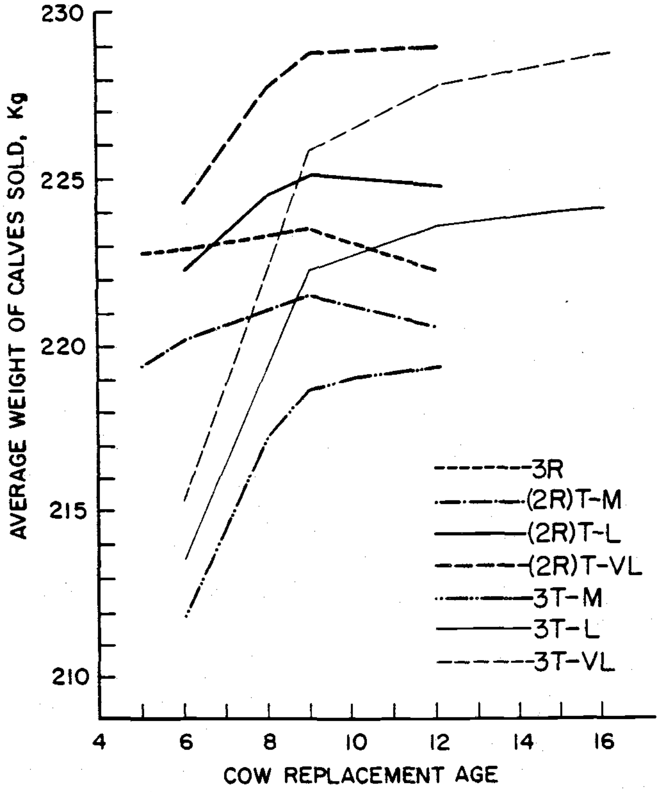


Figure 7. Effect of cow replacement age on average calf sale weight for different mating plans and breed sizes.

Legend: Mating plans: 3R = three-breed rotational; (2R)T = criss-out-cross; 3T = three-breed terminal.

Breed sizes: M = medium; L = large; VL = very large.

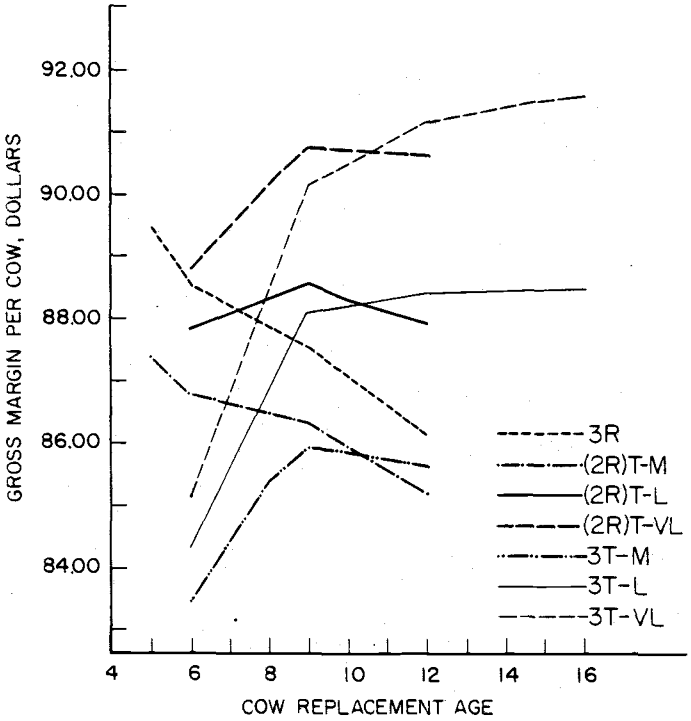


Figure 8. Effect of cow replacement age on economic efficiency for different mating plans and breed sizes.

Legend: See Figure 7.

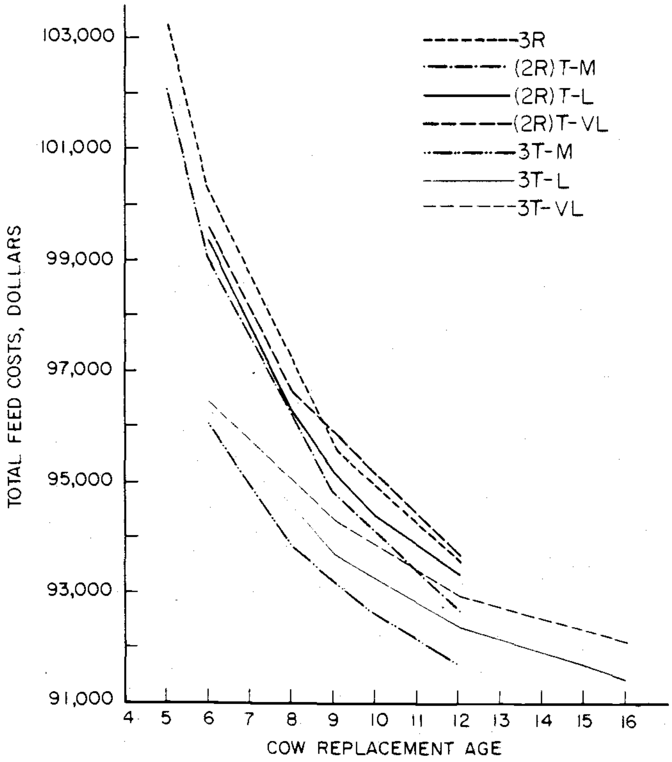


Figure 9. Effect of cow replacement age on total feed costs of a simulated 500-cow herd for different mating plans and breed sizes.

Legend: See Figure 7.

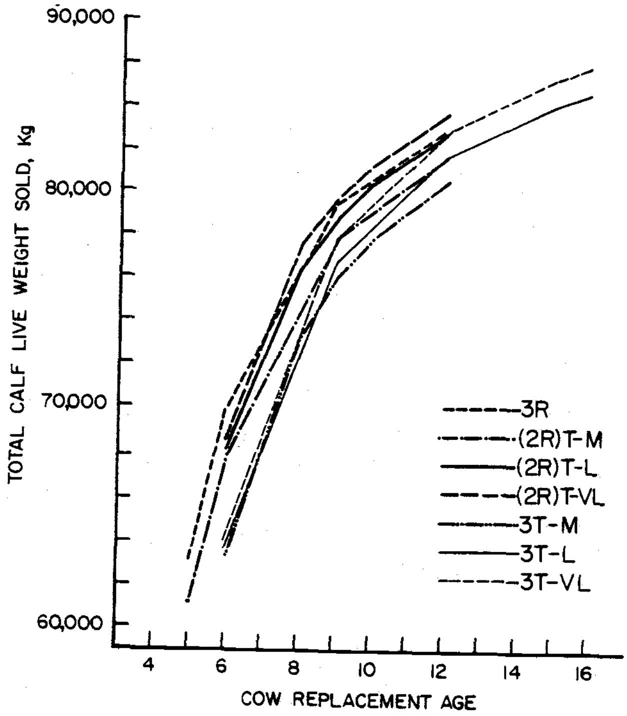


Figure 10. Effect of cow replacement age on total liveweight of calf sold for different mating plans and breed sizes.

Legend: See Figure 7.

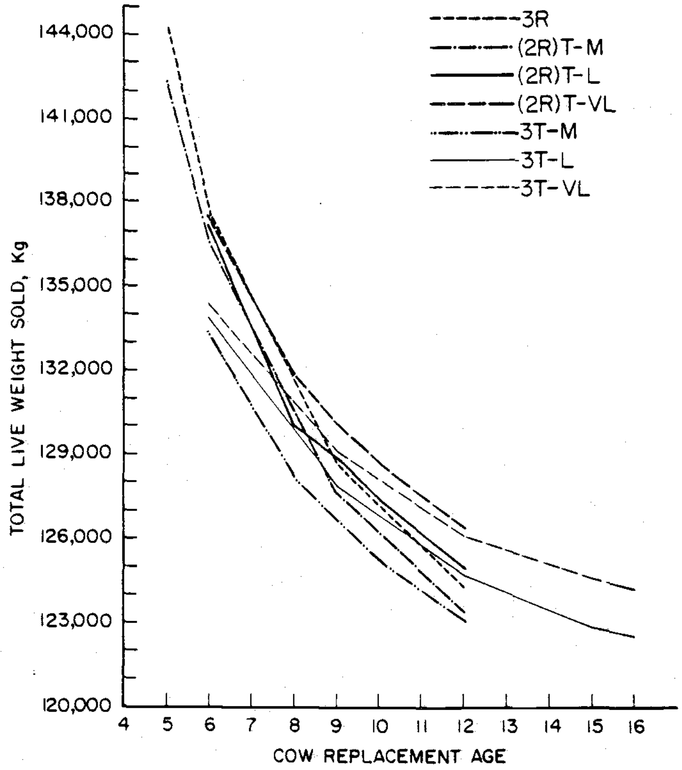


Figure 11. Effect of cow replacement age on total liveweight sold for different mating plans and breed sizes.

Legend: See Figure 7.

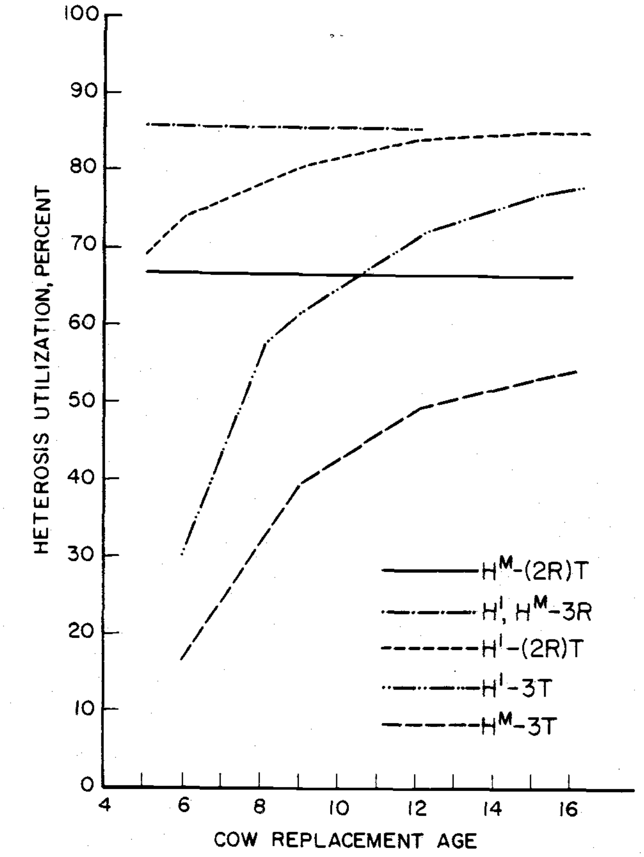


Figure 12. Effect of cow replacement age on individual heterosis (H^I) and maternal heterosis (H^M) for different mating plans.

Legend: See Figure 7.

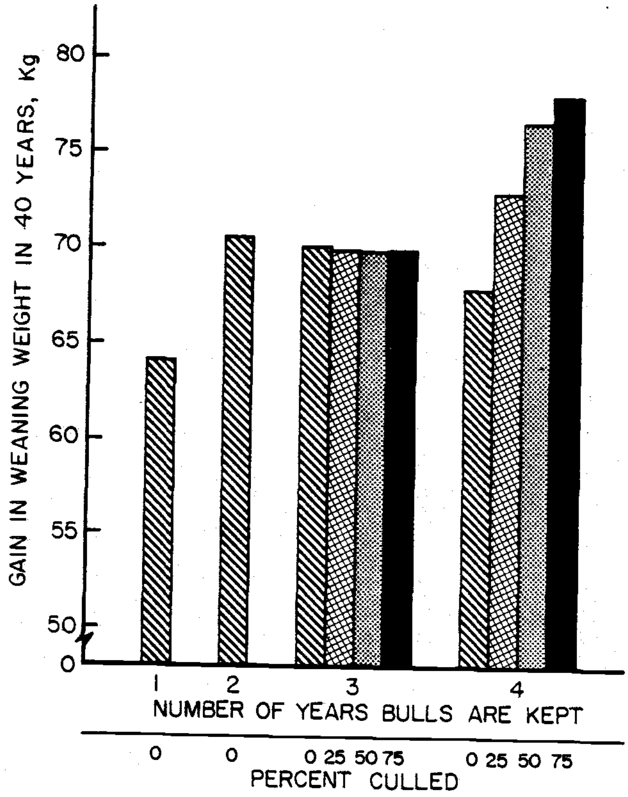


Figure 13. Increase in weaning weight for various bull use strategies.

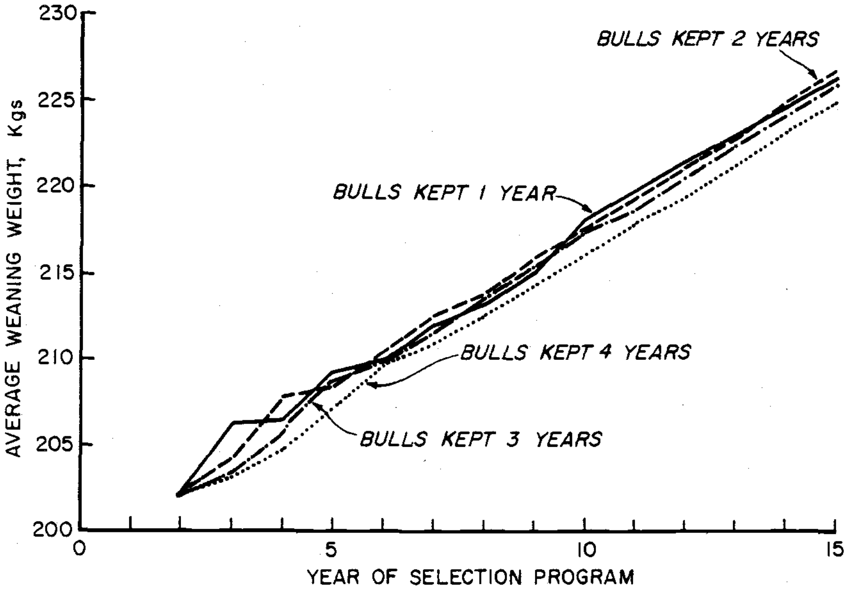


Figure 14. Average weaning weight in the early years of the selection program for several bull use strategies not employing progeny testing.

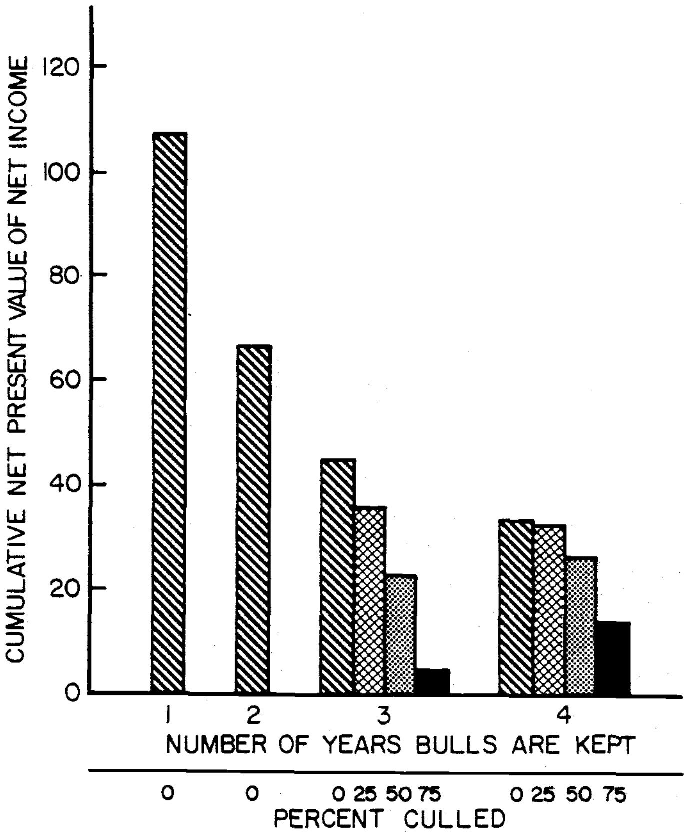


Figure 15. Net present value of net cash flow per cow for various bull use strategies.

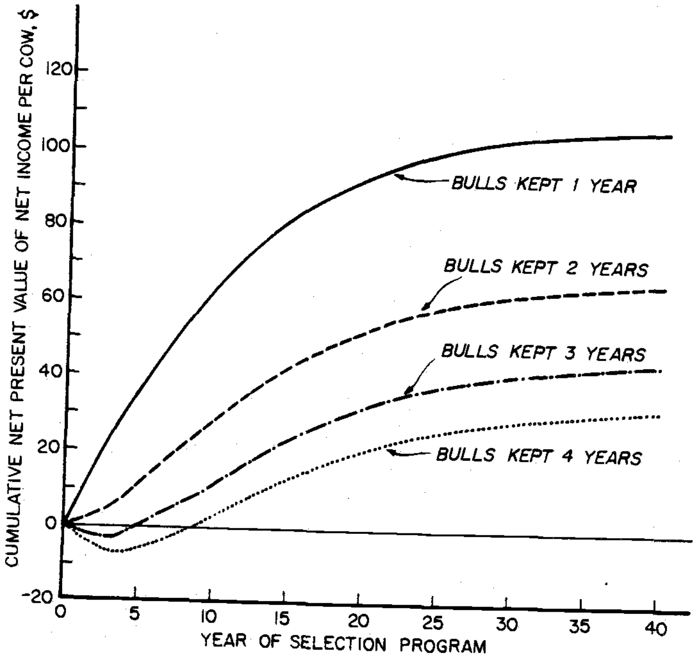
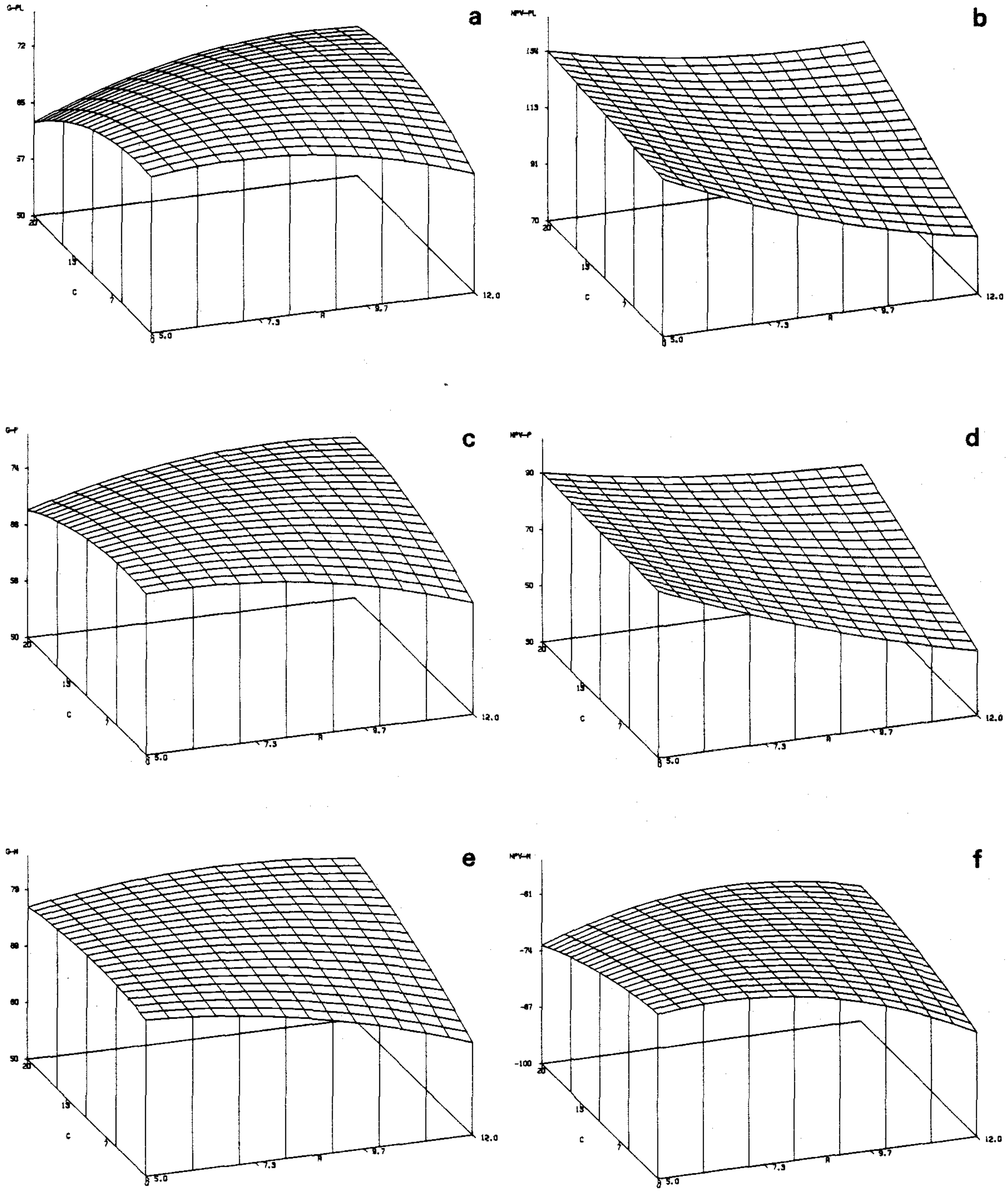


Figure 16. Net present value of net income per cow throughout the selection program when progeny testing was not practiced.

Figure 17. Genetic gains and net present value of net cash flow per cow with various reproductive culling criteria. C is the percentage of the cows culled based on progeny performance. A is the maximum cow replacement age. The response variable in parts a, c, and e is genetic gain in weaning weight over the forty year simulation. The response variable in parts b, d, and f is net present value of net income per cow. In parts a and b, open cows are culled as are cows without a live calf at the end of the calving season. In parts c and d, open cows are culled. In parts e and f, no reproductive culling is practiced.


G-PL
72
65
57
50
20
13
C
7
0
5.0
7.3
R
9.7
12.0
a
NPV-PL
134
113
91
70
20
13
C
7
0
5.0
7.3
R
9.7
12.0
b
G-P
74
66
58
50
20
13
C
7
0
5.0
7.3
R
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c
NPV-P
90
70
50
30
20
13
C
7
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9.7
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d
G-N
79
69
60
50
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5.0
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R
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-61
-74
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-100
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13
C
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Figure 17

Figure 18. Replacement percentage when open cows are culled and cows without a live calf at the end of the calving season are culled. C is the percentage of the cows culled based on progeny performance. A is the maximum cow replacement age.

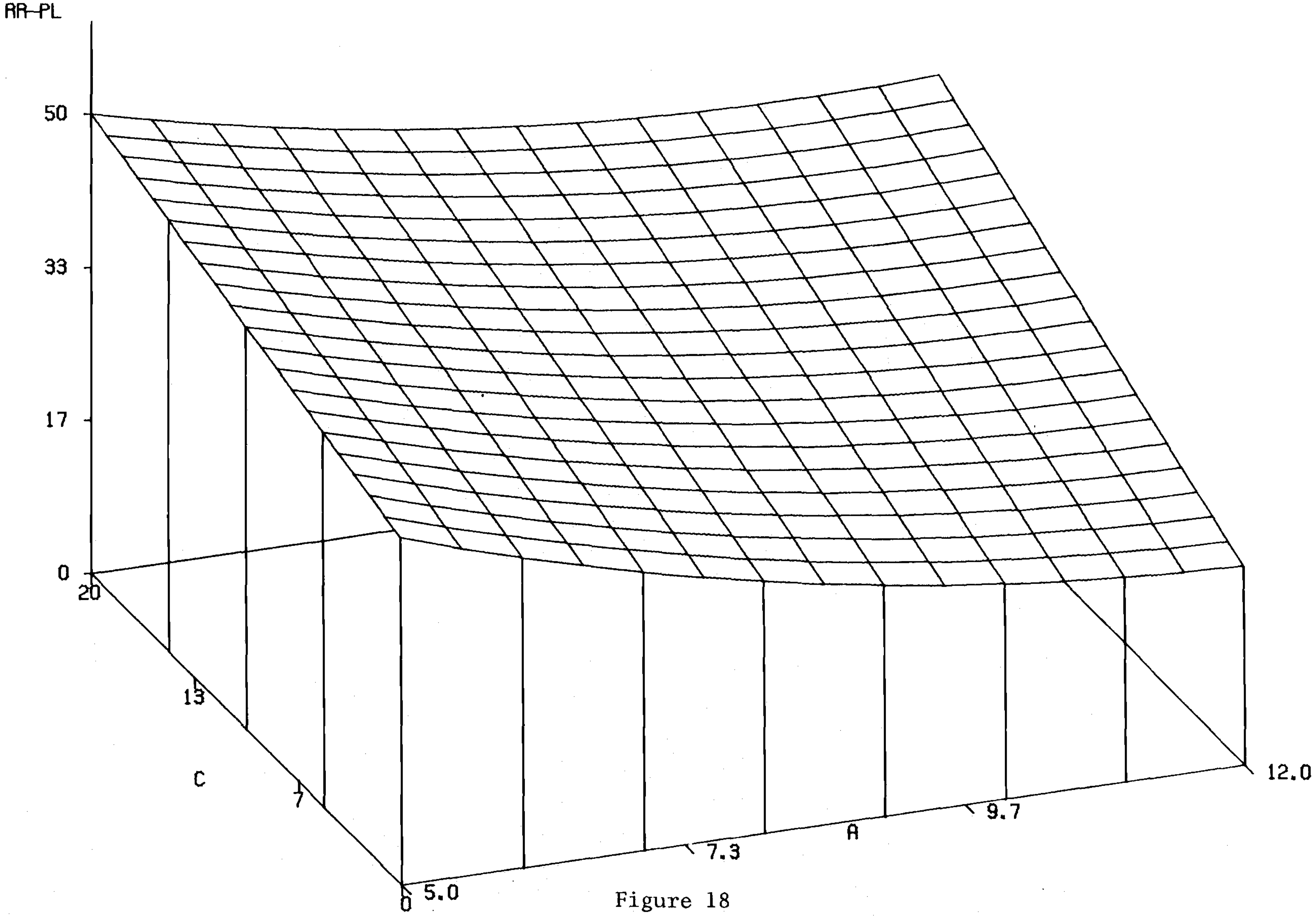

Figure 18

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