AN ABSTRACT OF THE THESIS OF

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Abstract Approved: ______________________
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A method is presented for appraising the hazards to nuclear power plants from missiles from surface traffic accident explosions. Due to the infrequency with which surface traffic accident explosions occur and the poor records kept concerning them, a probabilistic method is chosen to investigate the overall hazards potential to nuclear power plants of such events. An unacceptable accident is defined as one where one or more missiles from a surface traffic accident explosion anywhere along a traffic route running past a nuclear plant strike a safety related component at the plant or the walls of a structure housing such components and are able to penetrate these walls. Penetration of such walls is conservatively assumed to render such safety related equipment inoperative, thus constituting an "unacceptable" event.

The method of determining the annual probability of such an unacceptable accident incorporates modeling assumptions based on physical laws and conventional engineering concepts known to apply as

well as empirical relations used in similar studies in the past. The maximum range of missiles as a function of the amount of explosive involved is determined from an empirical relation. However, the determination of the impact characteristics (angle and velocity) of the missiles and their penetration characteristics is based on deterministic models for missile trajectories under the effect of aerodynamic drag. Plausible assumptions are made regarding the angular distribution, initial velocity, aerodynamic properties, and number distribution of the missiles. The equations describing this model are rather complex and require the use of numerical methods and computer techniques to obtain the desired probability of unacceptable events for specific cases. Several representative cases and parametric studies are presented.

In these reference studies it was found that the effects of drag were minimal. It is somewhat speculative to conclude that this is generally true; however, if it is, a much simpler drag-free model could be used to yield a conservative estimate of the desired probability. Drag effects begin to play an important role only in those cases where explosives yield missiles of high initial velocity and relatively small mass, as may be the case in munitions explosions.

The model is believed to be an improvement over past methods because of the increased confidence in results yielded by the application of physical laws to it. For the same reason, the model possesses flexibility such that it can be applied confidently to a wide range of conditions which may be encountered in actual cases. Finally, the model contributes to the better understanding of the problem at hand since its detailed structure permits identification and investigation of those factors which are most significant in the determination of the hazards potential.

The Missile Hazard of Surface Traffic Accidents to Nuclear Power Plants

by

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I. INTRODUCTION

A current issue of concern in nuclear power plant development is the possibly serious consequences of certain accidents involving the plant. Analyses must be performed of a number of such accidents (Design Basis Accidents) to show that the design is satisfactory and that the plant will respond to such accidental situations without endangering public health and safety. The initiating cause of severe accidents could be an internal event and be due to equipment failure or operator error. In addition, such accidents could be caused by external events such as earthquakes, tornadoes, or other of nature's fury, or man-made phenomena such as nearby explosion, fire, or similar occurrences. This paper deals with the issue of one such man-made external event--the hazards to the nuclear plant from missiles generated by a nearby explosion.

Since these accidents do not occur very often, it becomes necessary to assess the potential hazards of such accidents from external explosions by probabilistic methods. In most cases, explosive materials are found in sufficient quantity near nuclear plants only on transportation routes such as highways, waterways, or railways. Consequently, the analysis presented in this paper deals with the general case of such an accident occurring anywhere along the transportaion route near the nuclear plant. The missile hazard from potential explosions in a fixed location is covered as a special case.

Previously, as evidenced in the safety analysis reports of some nuclear plants, the problem of missile hazards from external explosions, when approached at all, was treated by the use of empirical relations based on statistical data obtained mainly from munitions tests which were then extrapolated to fit the case of concern.[1] The analysis presented here of the potential hazards to nuclear plants due to missiles from explosions approaches the issue by constructing a physical model of the accident based on physical laws, conventional engineering concepts, and reasonable assumptions; empirical and statistical relations are used where there is inadequate knowledge and understanding of the exact physical processes involved.

The following improvements can be expected from this approach over those based exclusively on empirical relations:

1. Increased Confidence in Results--Physical relations (e.g., trajectory information, conservation of mass and energy) are added to the empirical

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[1] Perhaps the most notable case of this type of analysis was used in the Safety Analysis Report for the Brunswick Steam Electric Plant (4). A detailed review was undertaken of past munitions tests where missile data was taken. The missile data was often expressed as a density function (missiles per unit area) and empirical formulas were derived to fit this data. The Brunswick analysis used these formulas after modifying parameters to fit the specific case of concern. The resulting probability was figured thus in terms of empirical missile area density at a radial distance from the explosion. No account was taken specifically of drag effects, plant parameters, etc.

relations and provide additional factual statements which are known to apply. Qualitative statements when supported by appropriate quantitative information can be expected to increase the confidence in the probabilistic hazards estimates.

2. Increased Model Flexibility--Since the model represents the physical processes at hand, it can be applied confidently to a wide range of conditions which may be encountered in actual cases. This makes the model a practical and flexible design tool, accounting for variable plant conditions, explosion types, and missile distributions.
3. Increased Insight and Understanding of Relevant Phenomena--Introduction of more detailed physical models leads to increased insight into phenomena and effects which are relevant towards creating a hazard at the nuclear plant. This understanding is essential in the subsequent effort to reduce such hazards by positive action.

Chapter II of this paper deals with the description of the basic model including a discussion of the assumptions made. Numerical methods employed to obtain quantitative results and the computer codes developed for this purpose are discussed in Chapter III.

Results and conclusions are presented in Chapter IV. Appendices contain amplifying information and derivations which are not considered essential to the main body of the text.

II. DESCRIPTION OF THE MODEL

A. Definition of Hazards Potential

It has become common practice in hazards analyses relating to nuclear plants to describe the potential hazards from accidents such as considered here in terms of the annual probability of the occurrence of an unacceptable event due to a certain cause and mechanism. In this instance, the cause is a transportation accident near the plant involving the potential occurrence of an explosion. The mechanism creating the unacceptable event is that of missiles, generated by the explosion, rendering inoperative safety related plant equipment.

The original cause and the probability of the explosion are assumed to be unrelated to actual properties of the explosion itself. This is expressed by the relation:

$$P = f_1 \cdot f_2$$

where: P = Annual probability of an unacceptable event

f_1 = Probability of an explosion along the transportation route per year

f_2 = Probability of an explosion causing unacceptable damage at the plant by missiles.

Explosions are assumed to occur anywhere along the transportation route or "track".[2] In general, f_1 may be a function of track position.

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[2] In the interest of concise expression, the "rail" case will be taken as the working example throughout the paper--hence, the term, "track"..

However, for simplicity and due to lack of specific information, f_1 will be assumed to be independent of the track coordinate and equal to the product of three factors as follows:

$$f_1 = f_t \cdot f_a \cdot f_e$$

where f_t = Annual frequency of shipments of explosive cargo

f_a = Accident rate per unit track length for any type of shipment

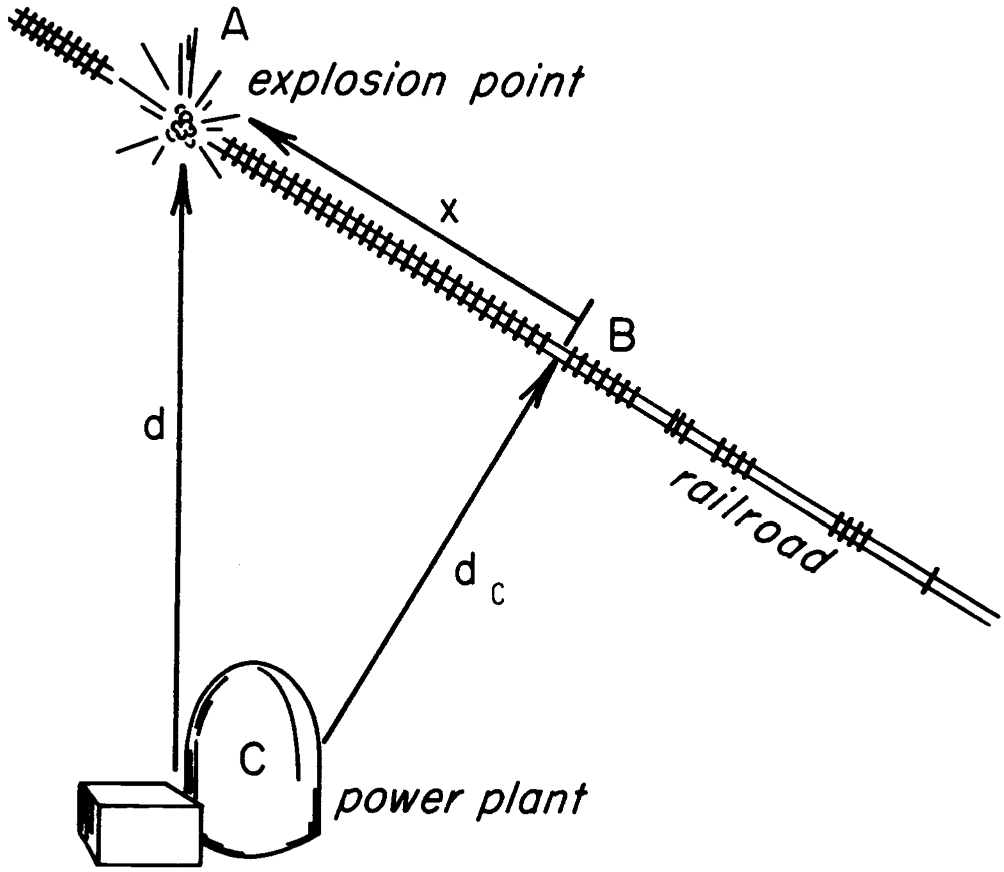
f_e = Probability of explosion, given an accident involving explosive cargo

For simplicity, all shipments are assumed to be of the same characteristics. As a result, the formulation of f_1 as the product of three factors as defined is based on the assumption that there is no causal relation between explosives shipments and the observed accident rate for the mode of transportation concerned.

The term, f_2, must be evaluated as a function of track position since the probability of an unacceptable accident due to missiles from an explosion is obviously dependent on the distance between the explosion and the plant. Referring to Fig. II-1, we note that the distance between the explosion and the plant, d, is a function of the track coordinate, x. We assume a straight track past the plant as a simple representative situation enabling us to demonstrate the effects of a variable plant-explosion distance. Cases other than straight track can be treated without formal difficulty by methods similar to those presented here. The term, f_2, can then be written:

$$f_2 = \int f_n \, dx$$

PLANT/TRAFFIC ROUTE LAYOUT



d_c -- distance CB

d -- distance CA

x -- distance BA

Fig. II-1 Plant/Traffic Route Layout

where: f_n = Probability of an unacceptable accident due to missiles from an explosion at point x (hence forth termed "point probability")

The term, f_2, can be interpreted as an equivalent critical track length over which unacceptable accidents can occur to the plant due to missiles from explosions.

B. Specification of the Unacceptable Accident Point Probability--f_n

In order to define f_n, additional modeling assumptions are introduced as follows:

1. All missiles have the same initial velocity, determined solely by the type and amount of explosive cargo involved.
2. All missiles have the same shape and material properties and possess no rotational motion during flight. Specifically, we assume a constant height-diameter ratio, cylindrical shape, constant drag coefficient C_d, constant missile material density and constant flight attitude.

The important implications of these assumptions are the following: First of all, the second assumption allows all missile characteristics to be expressed as a function of missile mass. This, in conjunction with the first assumption allows specific relations to be developed between missile mass, initial velocity and initial direction on one hand, and range of the missile, impact velocity, and impact angle on the other. The relations are a function of missile mass and initial velocity only. Since the initial velocity is assumed to be the same for missiles of all masses, the probability, f_n, pertaining to a given explosion coordinate, x, can be written as a summation over all missile masses. Thus:

$$f_n = \sum_i f_{m_i}$$

where: f_{m_i} = Probability per unit mass of unacceptable damage to the nuclear plant from missiles of mass, M_i, within the mass interval, ΔM_i, given the occurrence of the explosion at track coordinate, x.

$\sum_i$ = Summation over all mass intervals

It must be emphasized that missiles of different mass, M_i,will hit the plant by way of different trajectories even though the missiles have the same initial velocity. This is due to the effects of drag.

Now f_{m_i} may be written as follows:

$$f_{m_i} = 1 - (1-f_s \cdot f_b)^{N_i}$$

where: f_s = The probability of a missile of mass, M_i striking the plant from an explosion occurring at track position, x.

f_b = The probability given a missile strike of mass M_i from an explosion at track position, x, that some safety related equipment is rendered inoperative (i.e., the unacceptable accident occurs)

N_i = The (discrete) number of missiles of Mass, M_i within the i-th mass interval, ΔM_i.

and thus: $f_s \cdot f_b$ = The probability of an unacceptable accident from a single missile of mass, M_i, from an explosion at x.

$1\text{-}f_s \cdot f_b$ = The probability that a given missile of mass M does not cause an unacceptable accident.

$(1\text{-}f_s \cdot f_b)^{N_i}$ = The probability that none of the N_i missiles of mass M_i cause an unacceptable accident.

$1\text{-}(1\text{-}f_s \cdot f_b)^{N_i}$ = The probability that one or more of the N_i missiles of mass M_i cause an unacceptable accident.

For most cases of general interest here it is found that the product, $f_s \cdot f_b \cdot N_i$ is much less than unity. In this case, we can use the approximation:

$$f_{m_i} = 1\text{-}(1\text{-}f_s \cdot f_b)^{N_i} \simeq 1\text{-}(1\text{-}f_s \cdot f_b \cdot N_i) = f_s \cdot f_b \cdot N_i$$

and the relation for f_n can be written in the simple form:

$$f_n \simeq \sum_i f_s \cdot f_b \cdot N_i$$

We note that both f_s and f_b are functions of the missile mass and thus must remain under the summation sign. Replacing the discrete number of missiles, N_i, per mass interval, ΔMi, by a continuous missile number density distribution N, we can rewrite this relation in its integral form as:

$$f_n = \int f_s \cdot f_b \cdot N \cdot dM$$

Some comments are in order, concerning the approximation introduced above. It is seen that the approximate formulation for f_n can be interpreted as the mean number of damaging missile strikes from an explosion at x. This will be very nearly equal to the probability of one or more damaging missile strikes from the same event, as long as the probability of the damaging strike of a single missile is small compared to unity. (In this case, the probability of two or more missiles causing damaging strikes is small compared to the single damaging missile strike and can be neglected; also, the probability of a single missile strike then becomes approximately equal to the average number of missile strikes, which establishes the equality stated above.) There may be cases, however, where the explosion occurs very near the plant and a large number of energetic missiles may be generated, many of which may strike and damage the plant. In this case, the above approximation will be poor, since the average number of damaging missile strikes may be large (not limited by unity) while the probability of the unacceptable event (one or more damaging missile strikes) cannot exceed unity. However, it is noted that the approximate formulation always is conservative.

Since the evaluation of the above integral in general cannot be obtained analytically and numerical methods must be employed, we can in fact base the evaluation on either one of the summation formuli introduced above. Indeed, it is no more complicated to use the original formula, although we shall retain the continuous formulation of the missile number distribution, N. In this case, the number of missiles in the i-th mass interval is found approximately from:

$$N_i = N \cdot \Delta M_i$$

so that:

$$f_n = \sum_i (1-(1-f_s \cdot f_b)^{N \cdot \Delta M_i})$$

It must be noted however, that the exponent in general will no longer be an integer number.

The Probability of a Missile Strike--f_s

The assumption is made that all missiles are ejected with equal probability into any direction within the hemisphere surrounding the explosion (isotropic source). This is a reasonable assumption considering the actual missile directions for an average of a number of events would be random taking into account the diverse number of cases which could occur.

Consequently, the probability of a single missile strike on the plant is proportional to the solid angle which encompasses all possible initial velocity vectors of missile trajectories which result in a strike on the plant area (Fig. II-2). The probability of a single missile strike is simply then the ratio of this solid angle to that of the whole hemisphere, 2π.

For horizontal (H) plant areas then:

$$f_{sh} = \frac{1}{2\pi}\int d\Omega = \frac{1}{2\pi}\int \cos\alpha_0 d\alpha_0 d\phi = \frac{1}{2\pi}\int \frac{\cos\alpha_0}{d}\left|\frac{d\alpha_0}{dd}\right| \cdot d \cdot dd \cdot d\phi$$

where α_0, ϕ - spherical coordinates around the center of explosion describing the initial velocity vector (solid angle element $d\Omega = \cos\alpha_0 d\alpha_0 d\phi$).

d, ϕ - polar coordinates in the horizontal plane describing terminal points of missile trajectories (horizontal area element, $dA_H = d \cdot dd \cdot d\phi$)

If it is assumed that the dimensions of the plant area are small compared to the missile range, the Mean Value Theorem can be employed such that:

$$f_{sh} = \frac{1}{2\pi}\int \frac{\cos\alpha_0 dA_H}{d \cdot \frac{dd}{d\alpha_0}} \simeq \frac{1}{2\pi}\left[\frac{\cos\alpha_0}{d \cdot \frac{dd}{d\alpha_0}} \cdot \Delta A_H\right]$$

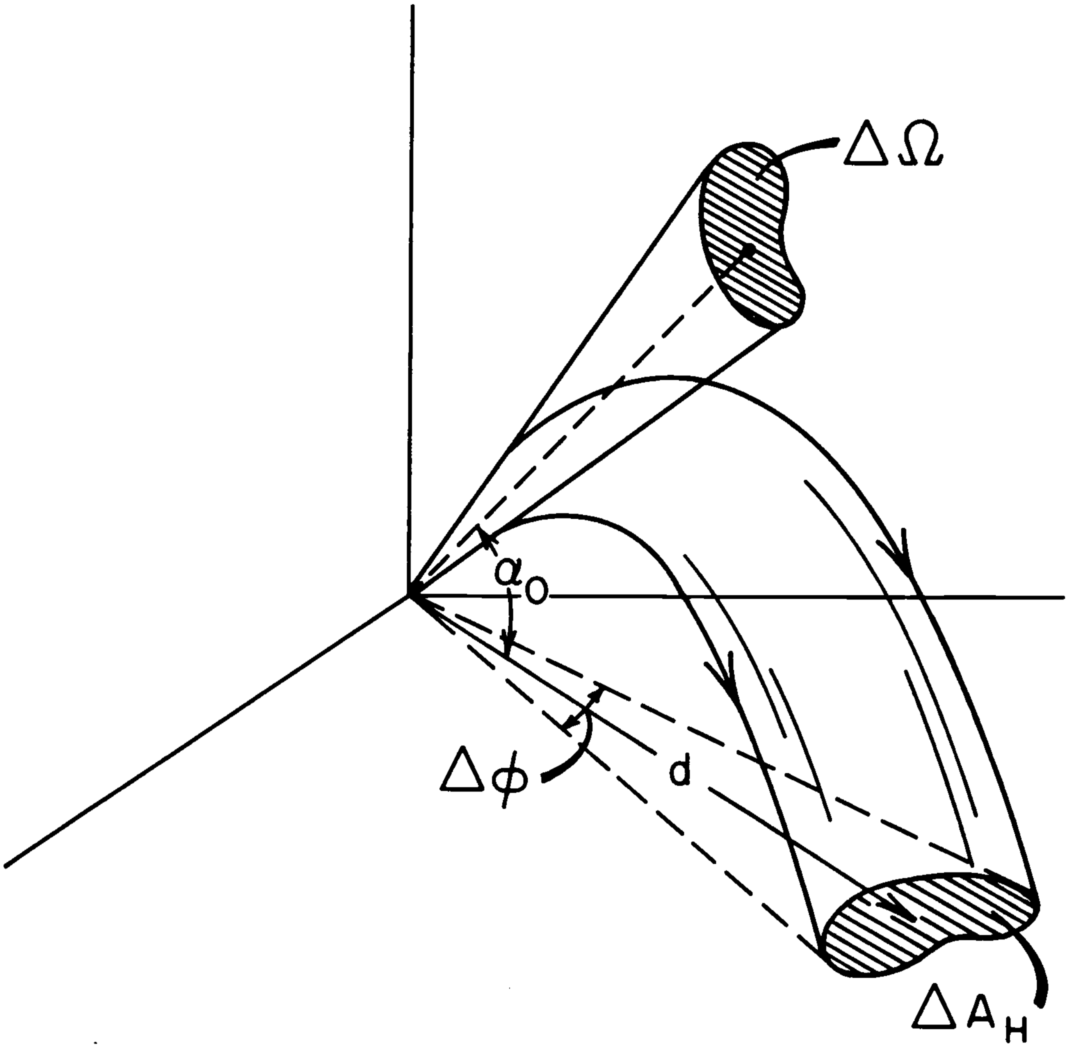


Fig. II-2 Missile Trajectory Perspective

In a similar fashion for vertical plant areas (V):

$$f_{sv} = \frac{1}{2\pi} \int \frac{\cos\alpha_0 dA_V}{d \cdot \frac{dH}{d\alpha_0}} \simeq \frac{1}{2\pi} \left[\frac{\cos\alpha_0 \cdot \Delta A_V}{d \cdot \frac{dd}{d\alpha_0} \cdot \frac{dH}{dd}} \right]$$

where H is the vertical coordinate (vertical area element, dA = $d \cdot d\phi \cdot dH$)

Noting the diagram (Fig. II-3) about the impact point of the missile defined in the above terms, it can be seen how $\frac{dH}{dd}$ is obtained for the vertical plant areas.

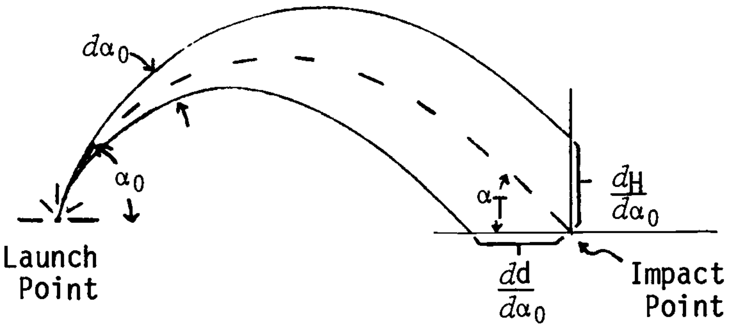


Figure II-3 Impact Diagram

It is apparent that: $\tan \alpha_T = \frac{\frac{dH}{d\alpha_0}}{\frac{dd}{d\alpha_0}} = \frac{dH}{dd}$

The terms, ΔA_V and ΔA_V, are the equivalent plant ares in the vertical and horizontal planes which are presented as targets for missiles. The vertical plant area is conservatively assumed always to face the explosion point (see footnote 3, p. 17). The strike probability per unit area is assumed constant over these areas as specified in the Mean Value Theorem.

The quantities α_T and $\frac{dd}{d\alpha_0}$ are obtained from solutions to standard ballistics equations including aerodynamic drag effects. These equations can be written as two coupled non-linear second order differential equations for which there are no known explicit solutions. To obtain the quantities listed above, particular solutions to the equations were obtained for a wide range of conditions using numerical integration techniques. Based on this array of specific solutions, general relaions as listed below were derived by suitable regression techniques and were used as approximations for the actual relations.

$$d = f(\alpha_0, v_0, M)$$

$$\alpha_T = f(\alpha_0, v_0, M)$$

$$\frac{dd}{d\alpha_0} = f(\alpha_0, v_0, M)$$

$$v_T = f(\alpha_0, v_0, M)$$

where:

α_0 = initial ejection angle

v_0 = initial missile velocity

M = missile mass

α_T = angle on impact

d = range

v_T = velocity on impact

Appendix A contains a detailed discussion of the derivation of these relations.

The Probability of an Unacceptable Accident in Case of Missile Strike-f_b

The unacceptable accident was described previously as one in which a missile strike renders a piece of safety related equipment inoperative. Since such systems are very different from one another and located in different areas of the plant, it is difficult to describe them separately as targets. It would appear to be conservative then to sum the exposed horizontal and vertical equivalent areas of buildings containing such systems to compose the areas ΔA_V and ΔA_H introduced previously in the definition of f_s. Another relevant characteristic is the equivalent thickness of the walls of such buildings which afford the safety related systems a degree of protection against damage by missiles. In light of this, it is reasonable to define the unacceptable accident as one where one (or more) missile(s) strike the walls of a structure housing safety related equipment and possess the proper characteristics to penetrate the wall or cause the spalling of large pieces from the wall's backside. The fact that penetration or spalling is achieved is considered conservatively to render the safety related equipment inoperative. The potential for penetration or spalling to occur is formulated as a "penetration criterion". If the penetration criterion is met, f_b is assumed to be unity; if it is not met, f_b is assumed to be zero.

The penetration criterion is based on a penetration formula commonly used in munitions work and known as the "Modified Petry Formula" (2). The formula as adopted for use in the model is:

$$Mc = \left[\frac{Tc \cdot k}{2\ K_1} \cdot \frac{1}{LOG_{10}\left(1 + \frac{V_s{}^2}{215000}\right)} \right]^3$$

where:

Mc = Mass which just penetrates the wall (lbs)

v_s = Velocity of the missile normal to the wall (ft/sec).[3]

vertical wall = $v_T \cos\alpha_T$

horizontal wall = $v_T \sin\alpha_T$

T_c = Equivalent wall thickness (in.)

K_1 = Concrete parameter (function of compressive strength ((2) (see footnote 5, p. 21)

k = Constant taking into account missile shape and material (see Appendix A for definition)

The formula yields the mass of a missile traveling at velocity V_s, which will just penetrate a wall of thickness, T_c. The spallation condition has not been incorporated in order to keep the model simple. In effect, a spallation condition would reduce T_c by a factor dependent on missile diameter (which, in turn, is a function of missile mass). Therefore, an appropriate reduction in T_c could be used to include spalling in an approximate manner.[4]

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[3] The vertical plant equivalent area is conservatively assumed always to face the explosion. In addition, the missile is conservatively assumed to strike the plant presenting its total cross-sectional area on impact even though the effective impact area is less due to the impact angle, α_T.

[4] The "Petry Formula" is commonly used when missiles of low impact velocities are concerned. Many other relations (continued)

By way of the ballistics relations introduced in the last section, it is seen that for a given set of the quantities, d and v_0, the impact parameters, α_T and v_T (and thus v_s), are specified and functions of missile mass, M, only. If the mass M_c calculated utilizing the penetration criterion by use of α_T and v_T is greater than M, f_b is one. If it is less, f_b is zero.

As with f_s, the determination of f_b must be made separately for horizontal and vertical plant areas since the velocity component normal to such areas is obviously different for a given α_T. As a result:

$$f_s \cdot f_b = f_{sv} \cdot f_{bv} + f_{sh} \cdot f_{bh}$$

where f_{sv}, (f_{sh}) = Probability of a missile of mass, M_i to strike a vertical (horizontal) plant area from an explosion at track position, x.

f_{bv} (f_{bh}) = Probability, given a missile strike on a plant area, that the missile of mass, M_i penetrates the wall representing that area.

It is noted that for a given track coordinate, x, initial piece velocity, v_0, and missile mass, M_i, each and every impact point can be reached by way of two different trajectories: a high trajectory and a low trajectory. (In the non-drag case, the two trajectories have initial angles which are symetric about 45°).

(footnote 4 continued) as given in reference (2) have been developed, mainly from tests of munitions explosions, and thus pertain to missiles of high impact velocities, which are believed to be a lesser concern in this context. However, these relations could be substituted in this model if needed.

Taking account of this circumstance, the product $f_s \cdot f_b$ must be broken down further as follows:

$$f_s \cdot f_b = f_{svl} \cdot f_{bvl} + f_{shl} \cdot f_{bhl} + f_{svh} \cdot f_{bvh} + f_{shh} \cdot f_{bhh}$$

where the additional subscript l refers to low trajectory missiles and the subscript h refers to high trajectory missiles.

Missile Distribution Function-N

To determine N, the number of missiles of Mass, M, per unit mass interval, a hypothetical distribution is chosen:

$$N = \frac{2}{M_T}\left[\left(\frac{M_T}{M}\right)\left(\frac{M}{M_T}\right)^{\frac{M_A}{M_T}} - 1\right] \quad \text{for } 0 < M < M_T$$

$$N = 0 \quad \text{for } M_T < M < 0$$

where: M_T = Total expected mass of all fragments

M_A = Most probable (expected) mass of a missile

It is seen that the function N obeys the limiting condition that the total mass of all the pieces together cannot be greater than M_T. Furthermore, the function, N incorporates a number of characteristics which appear to be reasonable for missile distributions, including the fact that N is monotonically decreasing over its entire range of definition, i.e., the number of missiles decreases with increasing mass. In particular, as the missile mass approaches the total, M_T, N becomes infintesimally small; and as M goes to zero, N approaches infinity. A complete derivation of N is given in Appendix C.

C. Summary Formulation of the Model

The model describes the hazard potential to a nuclear plant of missiles from possible explosions occurring along a transportation route (track). The hazard potential is defined in terms of P, the annual probability of an unacceptable accident due to missiles from explosions, which, in turn, can be calculated as the product of two quantities: f_1 and f_2. The term, f_1, is a quantity defined as the annual probability of an explosion per unit track length and is considered independent of the track coordinate. The term, f_2, is the equivalent length of track over which the explosion will constitute an unacceptable event. According to modeling assumptions presented in the preceding paragraphs, f_2 can be written (using the integral formulation) as:

$$f_2 = \int_X \int_M f_m dMdX$$

where all quantities have been defined earlier.

In turn, the quantity, f_m is given by:

$$f_m = (f_{svl} \cdot f_{bvl} + f_{shl} \cdot f_{bhl} + f_{svh} \cdot f_{bvh} + f_{shh} \cdot f_{bhh}) \cdot N$$

D. Principal Model Parameters

The principal model parameters can be divided into the following groupings: 1) parameters describing the plant 2) parameters describing the cargo, traffic, and track 3) parameters describing missile trajectories and 4) parameters describing the missile distribution. A brief description is given of the main parameters in each of these groupings below. Unless otherwise noted, values of the parameters are needed as input to the model. Representative values are given in the reference case described in Chapter IV.

Plant Parameters

a) ΔA_H, ΔA_V -- The equivalent horizontal and vertical plant areas describing the effective missile target areas presented by safety related systems, buildings, and equipment.

b) T_c -- The equivalent maximum wall thickness which missiles must penetrate to render safety related systems, buildings, and equipment inoperative.

c) K_1 -- A constant used in the Petry Formula related to the compressive strength of the concrete from which the equivalent wall is built.[5]

Traffic, Track and Cargo Parameters

a) f_t, f_a, f_e -- Three statistical parameters used in formulating f_1 as defined earlier

b) d_c -- The perpendicular (shortest) distance between the track and plant

c) W -- The equivalent tons of explosive cargo (tons TNT) involved in explosion

Missile Trajectory Parameters

a) C_d, ρ, γ -- The average missile drag coefficient, missile material density, and height-diameter ratio. These parameters essentially define the aerodynamic properties of the average missile.

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[5] A curve of the value of K_1 versus compressive strength of concrete is given on Figure 2-1 on page 2-6 of reference (2). The parameter K_p is plotted. $K_1 = 12 \cdot K_p$.

b) d_{max}, v_0 -- The maximum distance from the plant over which missiles have a finite probability of reaching the plant and the initial velocity of all missiles. These parameters are determined from basic assumptions in the model.

An empirical formula has been commonly used which relates maximum debris or missile distance to explosive yield in equivalent ons of TNT.[6]

$$LOG_{10}(d_{MAX}) = 2.96 + 0.317 \cdot LOG_{10}(W) - 0.0161 \cdot (LOG_{10}(W))^2$$

Since the formula takes into account many explosions of different types and yields, it is particularly suited for use in this analysis as it applies to an average of a great number of explosions with possibly a wide variety of characteristics.

With reference to v_0, as stated previously, the assumption is made that all missiles are ejected with the same initial velocity, v_0. Comments from ballistics experts give support to this assumption.[7] A valid estimation of the value of v_0 must be made with due regard to the nature of the explosion. Various modeling assumptions have been used in the literature leading to different results which appear to apply to different situations (e.g., weapons explosion, rupture of vessels under pressure). Since this study is aimed at

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[6] This formula was also referred to in the Brunswick Study (4). It appears to have fairly wide backing from experts in the explosives field.

[7] A letter received in Jan, 1975 from Capt. P.F. Klein, USN, Chairman of the Dept. of Defense Explosives Safety Board, states, "As to initial velocity, however, it can reasonably be argued that all fragments produced by a bomb, projectile, or rupturing vessel move at the velocity of case dilation immediately prior to breakup, regardless of their individual weights."

a description of conditions which constitute an average of a variety of possible situations, it appears adequate to derive a general formulation to obtain v_0 from the empirical relation between d_{MAX} and W presented above.

Given the maximum distance pieces travel from a given explosion, v_0, may be estimated by the following technique:

1) If aerodynamic drag is effective and under the assumption that mass is the only variable differentiating among missiles, the pieces with the largest mass have the least drag effect and fly the farthest. An estimate of v_0 can be made, then, by assuming the largest possible missile (mass, M_T) is found at d_{MAX}.

   This necessarily implies that no pieces are found beyond d_{MAX}.

2) Recalling one of the explicit relations describing a particular solution to the ballistics equations,

$$d = f(\alpha_0, v_0, M)$$

   it is found that d has a maximum when it is considered a function of α_0 only. The maximum distance, d_{MAX}, of course, is a function of v_0 and M. This function, when solved for v_0 can be written as:

$$v_0 = f(d_{MAX}, M)$$

   An approximate formulation of this function was developed on the basis of the numerical solution and regressions described earlier and is given in detail in Appendix A.

Consequently, given the maximum range, d_{MAX}, and the maximum possible piece mass, M_T, the initial velocity of this mass may be estimated. This velocity in turn is assumed to be the initial velocity v_0 of all the other missiles which, by definition, have a smaller mass.

Missile Distribution Parameters

a) M_T, M_A -- The total mass of all missiles and the most probable missile mass. These two parameters define the nature of the missile distribution function, N. The smaller the ratio M_T/M_A becomes, the smaller the total number of pieces and the larger the resulting most probable size of the pieces.

Effects of variation of many of these model parameters on the annual probability of an unacceptable event will be studied in detail in Section B of Chapter IV.

III. REALIZATION OF THE MODEL

A. The Numerical Approach

The quantity, f_1, the probability of an explosion per unit track length, is derived directly from statistical information as described in Chapter II. Representative values for the pertinent parameters will be given in Chapter IV. The quantity, f_2, the equivalent track length over which an unacceptable accident will occur given an explosion, however, is found by a double integration over mass and distance. Due to the transcendental form of the "penetration criterion" and the complicated nature of the integrand, analytical integration does not appear to be possible and particular solutions must be found by numerical integration methods.

The appropriate discrete formulation of the integration over the track coordinate is, taking advantage of symmetry:

$$f_2 \simeq 2 \cdot \sum_{i=0}^{K-1} f_n(X_i) \cdot \Delta X$$

where K = The number of discrete distance intervals into which the track distance is divided.

ΔX = Length of track distance interval ($\Delta X = \frac{X_{MAX}}{K}$)

(Note: $X_{MAX} = \sqrt{d^2_{MAX} - dc^2}$)

Now $f_n(X_i)$, the point probability of an unacceptable accident from missile strikes from an explosion at X_i, is written

$$f_n(X_i) = \sum_{j=0}^{L-1} f_m(X_i, M_j)$$

where f_m is given by the discrete formulation described previously:

$$f_m(X_i, M_j) = 1 - (1-f_s \cdot f_b)^{N \cdot \Delta M}$$

and N = Number of missiles per unit mass interval

ΔM = Mass interval

L = The number of mass intervals into which the piece distribution is divided

M_j = Mass at which f_m is evaluated

$$f_s \cdot f_b = f_{svl} \cdot f_{bvl} + f_{shl} \cdot f_{bhl} + f_{svh} \cdot f_{svh} + f_{shh} \cdot f_{bhh}$$

(as explained in Section IIB)

The limits of the mass integration could conservatively be chosen to be zero and M_T. As a result, ΔM and M_J would be written:

$$\Delta M = \frac{M_T}{L} \qquad M_j = \Delta M \cdot (j+½)$$

However, it is relatively easy to calculate a miminum mass that will not meet the penetration criterion in virtually all cases of practical interest. This is due to the fact that for these cases, the impact velocity, v_T, is limited by the free-fall velocity, v_f[8], which is defined as:

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[8] This conclusion was pointed out in reference (8) and later confirmed by the author's investigations.

$$v_f = \left(\frac{g}{\beta}\right)^{\frac{1}{2}}$$

where g = Acceleration of gravity (ft/sec^2)

β = Aerodynamic coefficient (ft^{-1})

Since all missile parameters are formulated as a function of missile mass only, β and thus v_f are only a function of missile mass, i.e., $v_f = aM^{1/6}$ where "a" is a constant. A complete definition of β and its derivation is given in Appendix A.

It can then be conservatively assumed that the quantity v_s, used in the Petry Formula is the free-fall velocity, v_f. Since v_f is a function of missile mass, M_T, the minimum mass, M_{min}, which just meets the penetration criterion can be calculated by iterative solution of the Petry Formula for M.

$$M = \left[\frac{Tc \cdot k}{2K_1} \cdot \frac{1}{LOG_{10}(1+\frac{v_s{}^2}{215000})}\right]^3 = \left[\frac{Tc \cdot k}{2K_1} \cdot \frac{1}{LOG_{10}(1+\frac{a^2M^{1/3}}{215000})}\right]^3$$

Thus, ΔM and M_j are reformulated as:

$$\Delta M = \frac{M_T - M_{MIN}}{L} \qquad M_j = M_{MIN} + \Delta M \cdot (j+\tfrac{1}{2})$$

The accuracy of the numerical method of integration is naturally dependent on the number of mass and distance intervals. Further comments on this and other limitations will be presented in Chapter IV.

B. Program EXPLOD

A computer program was written called EXPLOD which calculates

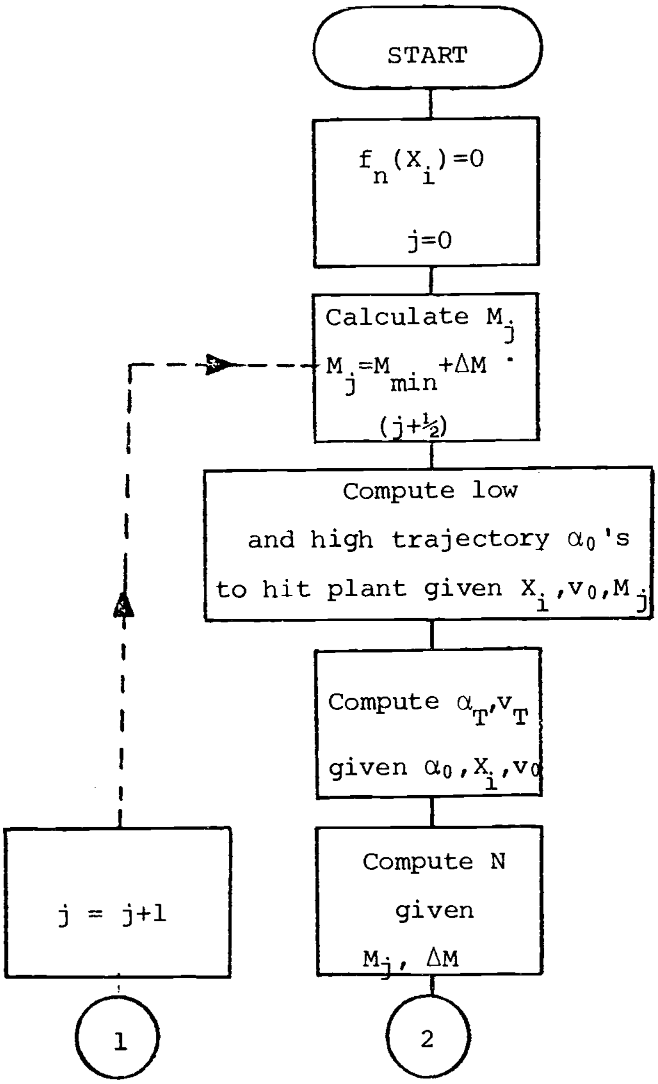


Fig. III-1 Program EXPLOD Flow Chart of $f_n(X_i)$ Calculation

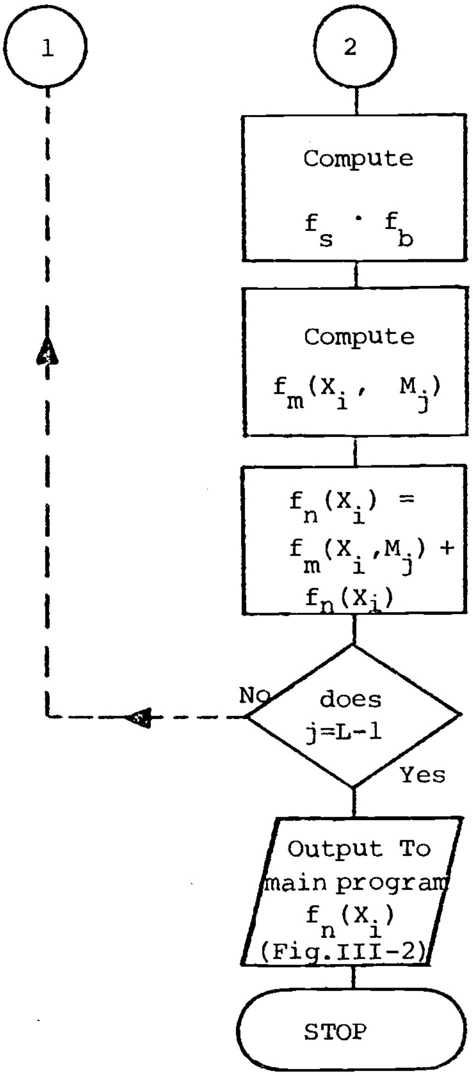
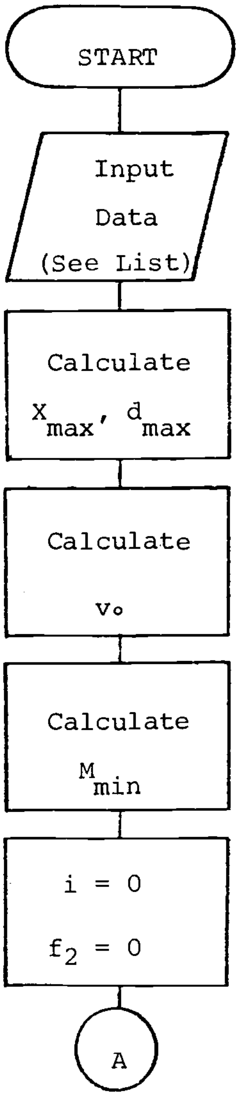


Fig. III-1 Cont.

the equivalent track length, f_2. Figure III-1 shows the logic involved in the calculation of the point probability of an unacceptable accident for a single mass and track coordinate, $f_m(X_i, M_j)$, and the summation of $f_m(X_i, M_j)$ over M to yield $f_n(X_i)$, the point probability of an unacceptable accident at X_i.

Figure III-2 shows the flow chart for the calculation of f_2 by integration over the track coordinate, x. Included in the chart also are the basic routines to calculate v_0, d_{MAX}, and M_{MIN} described previously. Required input information is as described in Section D of Chapter II.

More detailed information on Program EXPLOD is given in Appendix B.



Primary Input Data List

M_T, M_A, d_c, W, C_d,

ρ, γ, ΔA_H, ΔA_V

Fig. III-2 Program EXPLOD General Flowchart

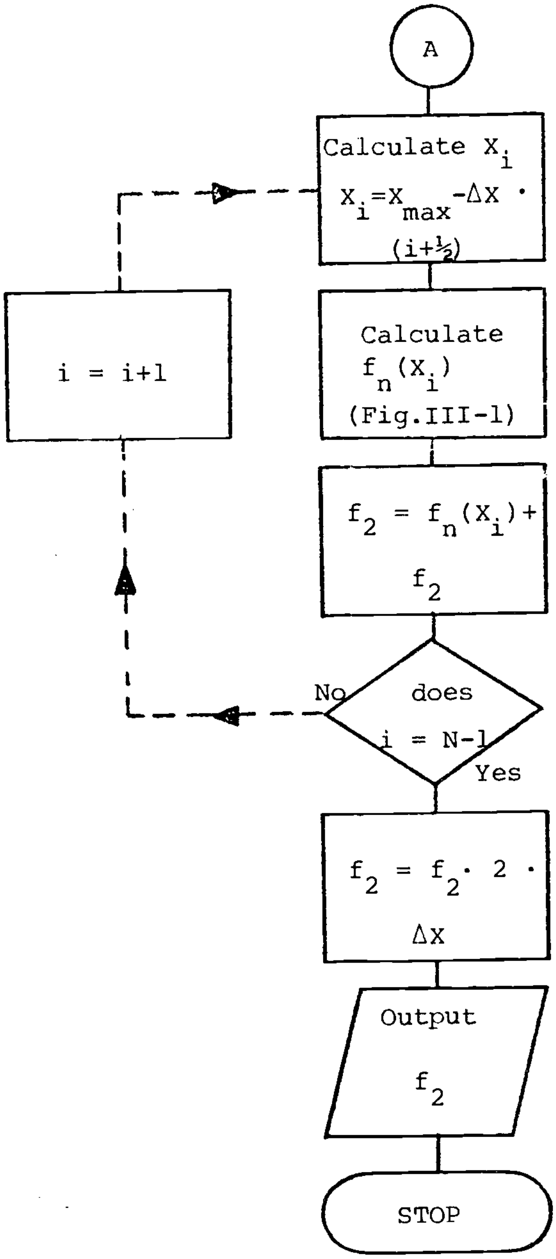


Fig. III-2 cont.

IV. RESULTS AND CONCLUSIONS

A. A Reference Case

To illustrate general use of the model a reference case is chosen for which data are shown in Table IV-1. M_T was chosen to be 50 tons which is equal to the typical cargo load of a rail car. M_A was chosen to be 10 tons implying that heavy items such as axles, wheels, couplings, etc., could be found among the missiles of largest masses. The ΔA_V and ΔA_H were estimated from approximate areas of the fuel services, control room, auxiliary, and diesel generator buildings of a typical nuclear plant as shown in Table IV-2.

Since the horizontal and vertical plant areas were of comparable magnitude, it was decided to make them equal in the reference case in order to facilitate comparisons in the subsequent parametric studies. The containment is considered separately since its wall thickness is substantially greater than that of the other buildings considered.

For this reference case, the following characteristic parameters are derived from the relations presented earlier and appear in the output:

d_{MAX} = 3185 ft.

x_{MAX} = 3145 ft.

v_0 = 320.3 ft/sec

M_{MIN} = 67 lbs.

f_2 = Equivalent track length = 394.5 ft.

TABLE IV-1

Reference Case Input Data for EXPLOD

Parameter			Units	
M_T	Total mass of all fragments	1×10^5	lbs	
M_A	Most probable expected mass of fragments	1×10^4	lbs	
K_1	Constant for concrete dependent on compressive strength	.03312	$\frac{\text{in ft}^2}{\text{lb}}$	For 4000 psi compressive strength concrete. Graph in Ref. (2)
ρ	Density of missile material	488	$\frac{\text{lbs}}{\text{ft}^3}$	Density of iron assumed
γ	Average height-diameter ratio of missiles	2	-----	
Cd	Average drag coefficient of missiles	1	-----	Picked in conjunction with γ
dc	Perpendicular distance between track and plant	500	ft.	
W	Amount of explosive considered (equivalent to TNT)	50	tons	
ΔA_V	Equivalent vertical plant area	64,200	ft^2	
ΔA_H	Equivalent horizontal plant area	64,200	ft^2	
T_c	Equivalent wall thickness	12	in	

TABLE IV-2

Estimated Equivalent Plant Areas

Bldg.	Horizontal Dimensions	ΔA_H	Vertical Dimensions	ΔA_V
Containment	Diameter ≃ 175'	7656 ft^2	Diameter ≃ 175' Height ≃ 200'	35000 ft^2
Fuel Services	Width ≃ 90' Length ≃ 310'	27900 ft^2	Diagonal ≃ 322' Height ≃ 100'	32200 ft^2
Control, Auxiliary Diesel Generator	Width ≃ 200' Length ≃ 250'	50000 ft^2	Diagonal ≃ 320' Height ≃ 100'	32000 ft^2

To find the annual probability of an unacceptable accident caused by missiles from such an explosion, we assume for the purpose of this reference case that 1) f_a, the annual probability per unit distance of an accident is 1×10^{-9} ft^{-1} (9) (approximate rail accident rate in U.S. based on 1972 data) 2) f_e, the probability of an explosion given an accident involving explosive cargo is .01 (3), (4), and 3) f_t -- the annual frequency of shipments involving explosive cargo is 10. Consequently, the annual probability, P, is found to be approximately 4×10^{-8} yr^{-1}. It is noted that this value of the probability, P, is about half of an order of magnitude less than the established limit for acceptable risks associated with severe nuclear plant accidents.[9]

It was found that convergence of the numerical integration process for the reference case was relatively fast--97.5% of the final answer was obtained using thirty mass intervals and thirty distance intervals. In general, for a given case, the rate of convergence is inversely proportional to W, the amount of explosive in the shipment. This will be discussed further in the parametric studies below.

B. Parametric Studies

The influence of parameters on the resulting hazards potential was studied in cases of those parameters which either are subject to great variations within the range of interest or for which the

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[9] A limit of 10^{-7} is commonly used and is based in part on the number of nuclear plants planned to be operating in the foreseeable future (1).

values are insufficiently determined due to lack of pertinent information. The parameters chosen reflect variance in the four primary parametric areas described in Section D of Chapter II.

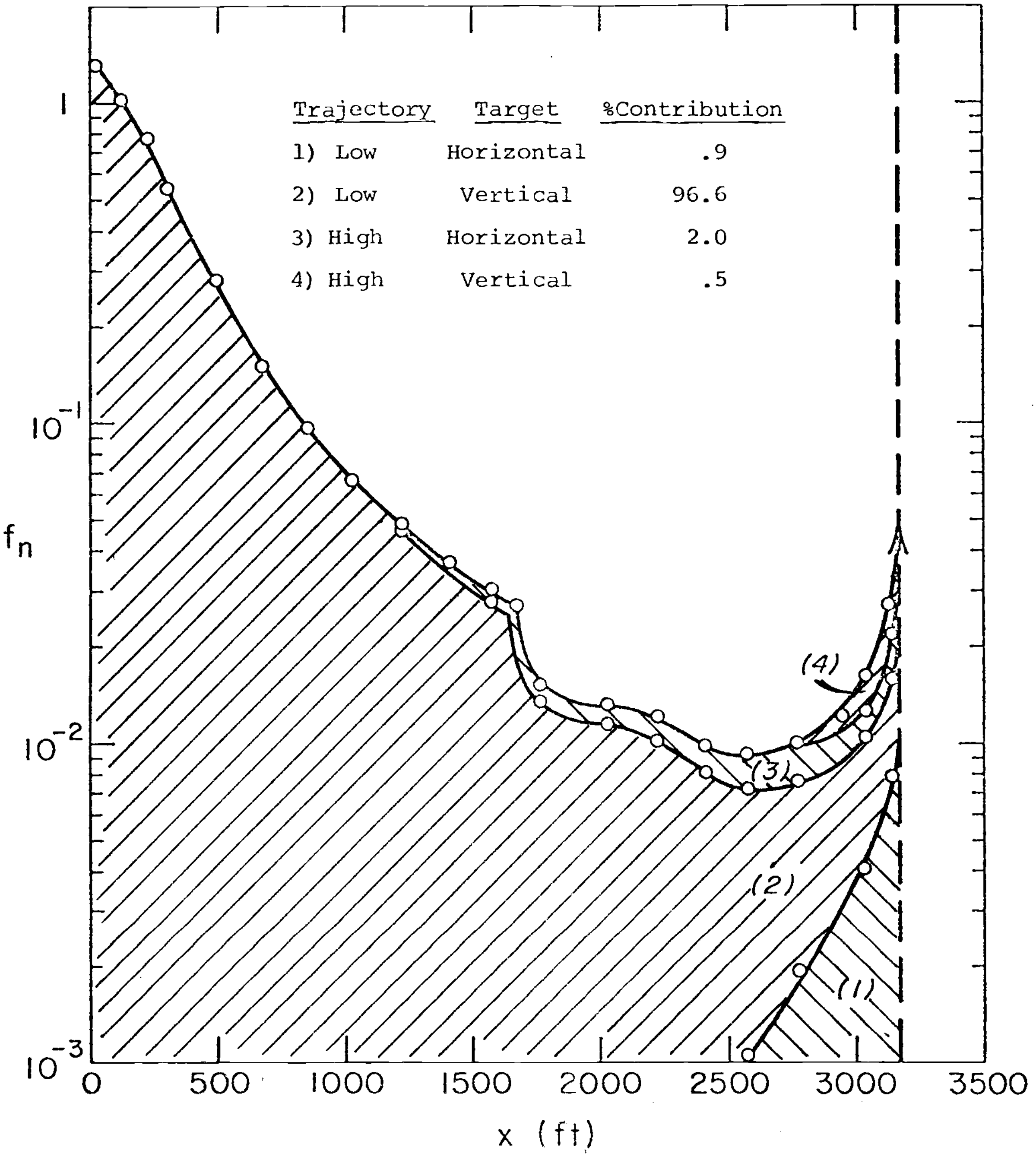
Variation of the Track-Plant Distance-d_c

Figures IV-1 through IV-8 show the variation of the point probability, f_n, with the track coordinate x, for various values of d_c. The quantity, f_n, as plotted in Figures IV-1 and 2 is broken down further into contributions from each of the target-trajectory combinations.

Figure IV-1 shows approximately the radial dependence of f_n since d_c is small compared to x_{MAX} and thus $d \simeq x$. As d_c becomes larger, however, succeeding figures portray this distribution along chords at successively increasing distances away from the plant.

The "hump" which occurs between 1000' and 2000' on graphs IV-1 through IV-5 reflects the increasing contribution to the point probability, f_n, made by the lighter and thus more plentiful missiles which may penetrate the vertical plant area by way of trajectories.

Figure IV-9 shows the relative contribution of the low trajectory missiles which penetrate vertical target walls in comparison to the contribution of all other combinations of trajectories and and target-wall orientations with reference to the 250 ft. case. It is interesting to note the latter contribution is negligible relative to the former contribution until d_c is approximately 1500 ft.



Trajectory	Target	%Contribution
1) Low	Horizontal	.9
2) Low	Vertical	96.6
3) High	Horizontal	2.0
4) High	Vertical	.5

Figure IV-1 Point Probability, f_n, Versus X, d_c=250'

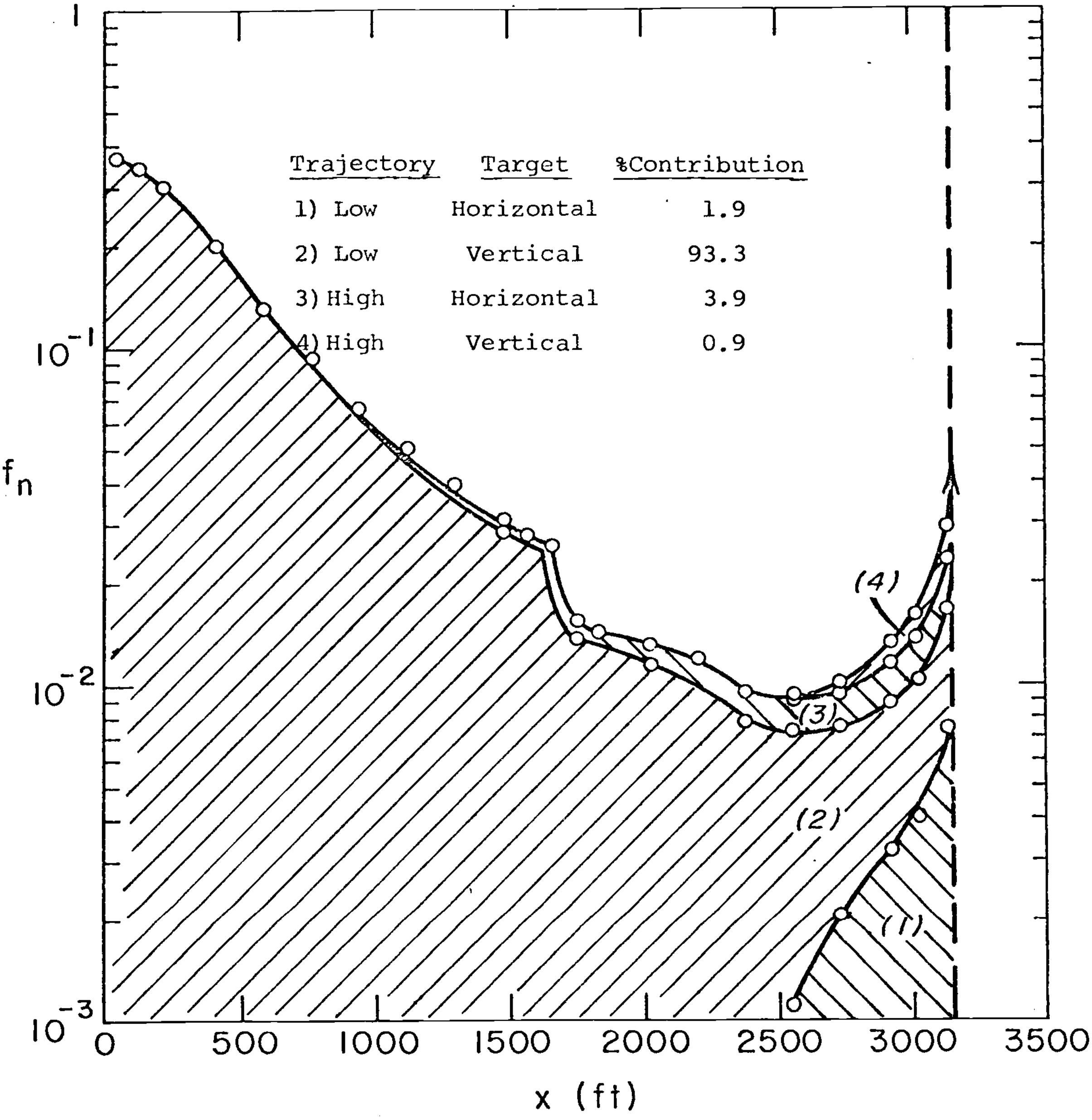
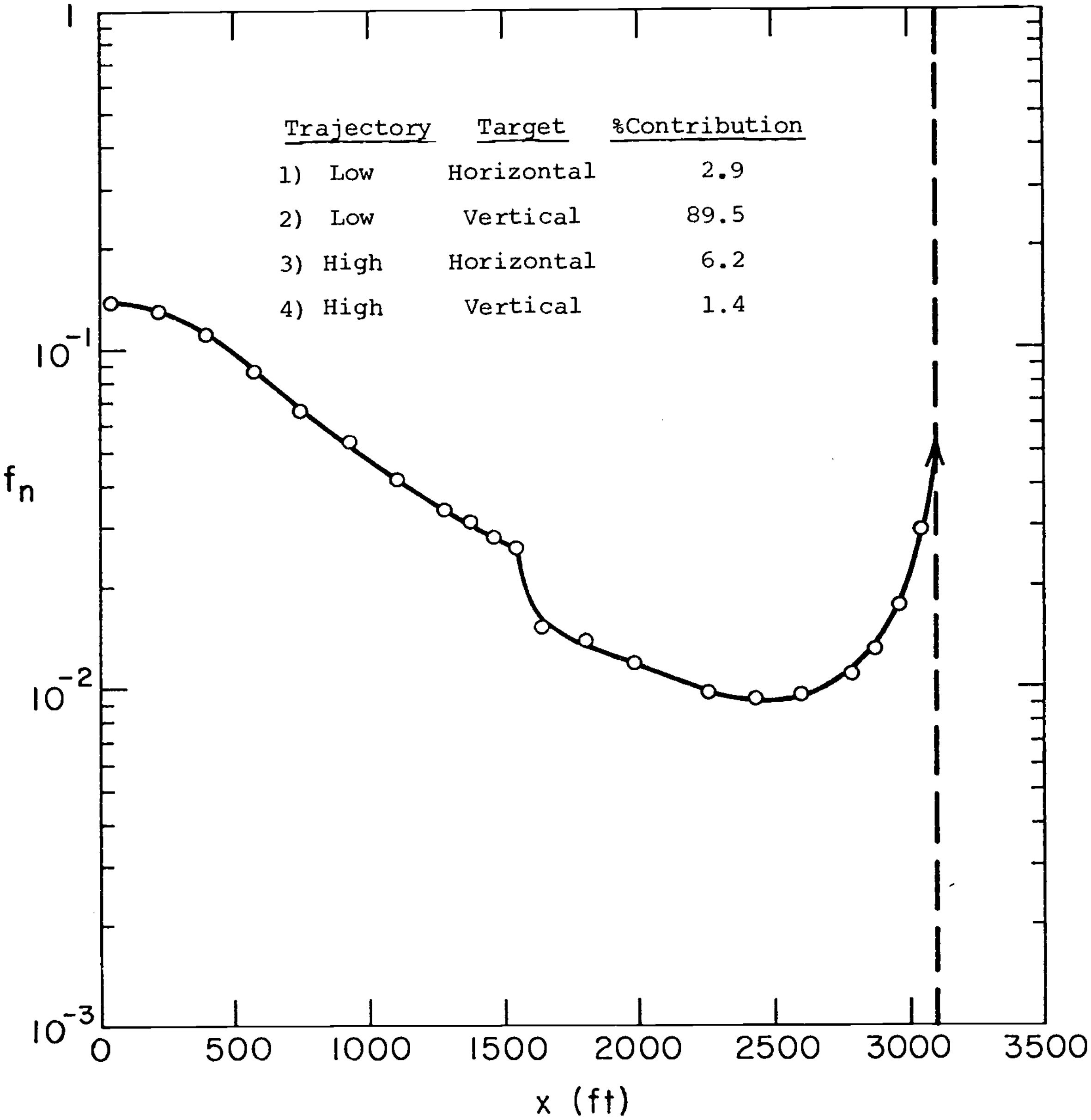
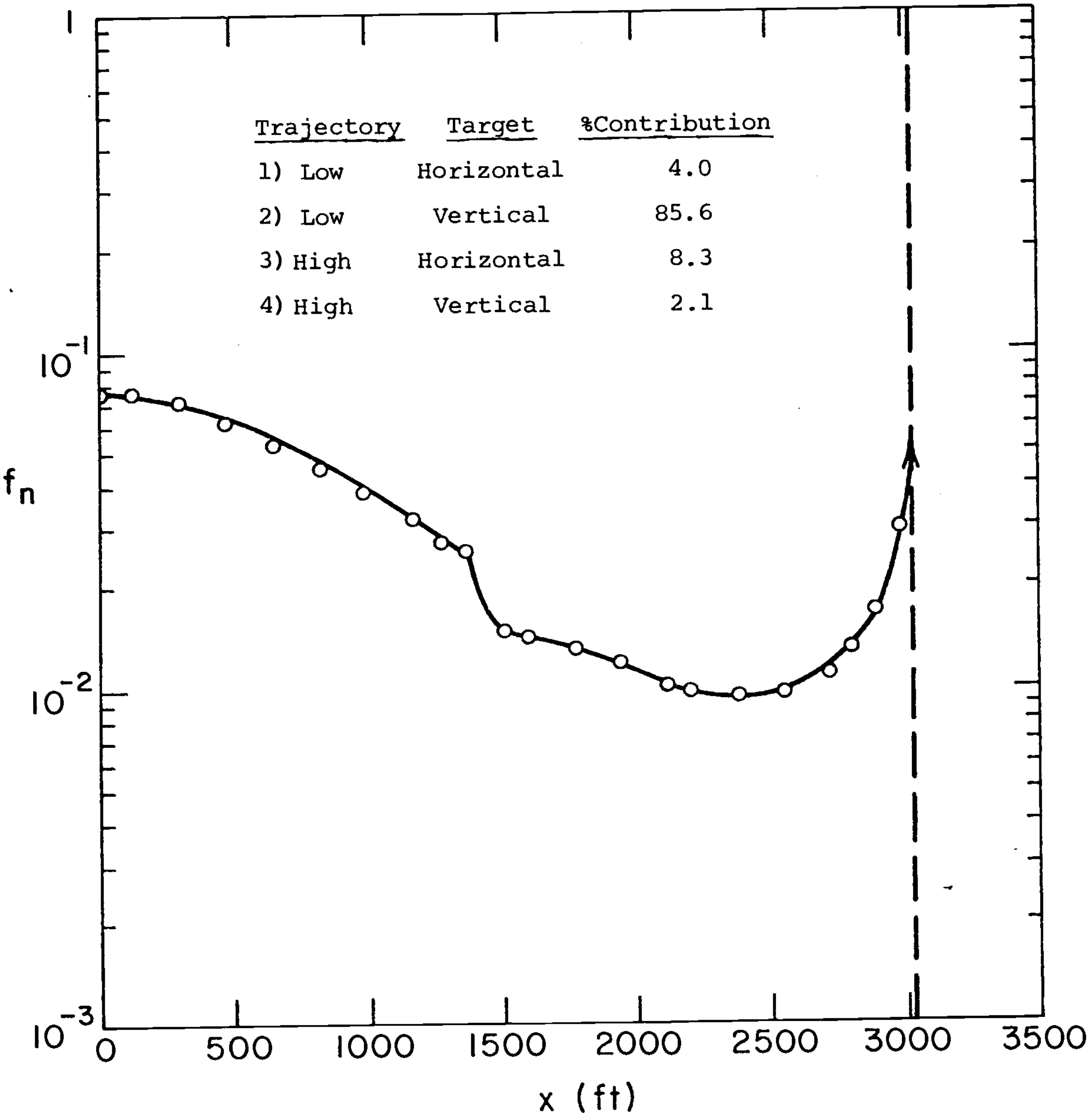


Figure IV-2 Point Probability, f_n, Versus X, d_c=500'



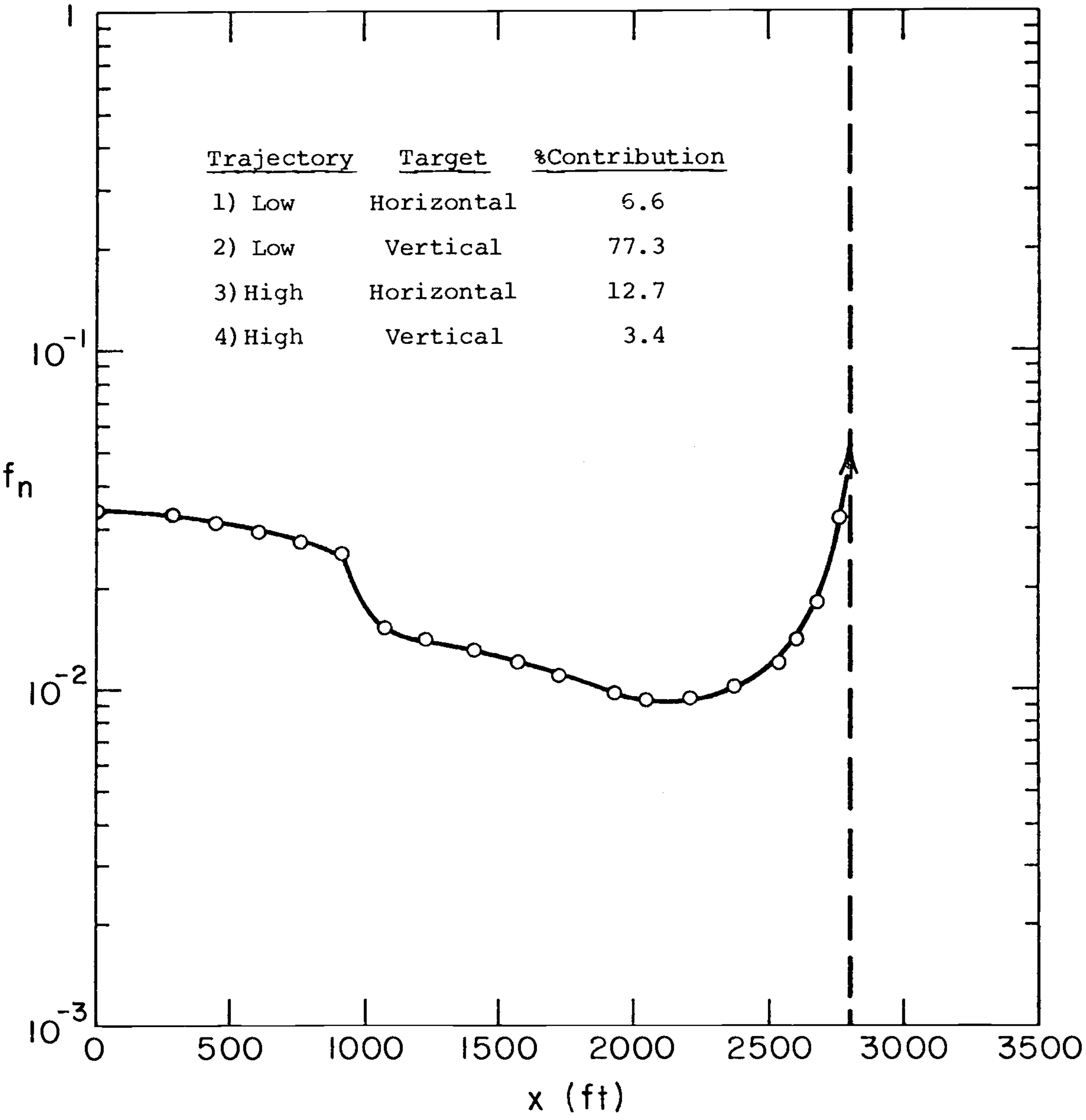
Trajectory	Target	%Contribution
1) Low	Horizontal	2.9
2) Low	Vertical	89.5
3) High	Horizontal	6.2
4) High	Vertical	1.4

Figure IV-3 Point Probability, f_n, Versus X, d_c=750'



Trajectory	Target	%Contribution
1) Low	Horizontal	4.0
2) Low	Vertical	85.6
3) High	Horizontal	8.3
4) High	Vertical	2.1

Figure IV-4 Point Probability, f_n, Versus X, d_c=1000'



Trajectory	Target	%Contribution
1) Low	Horizontal	6.6
2) Low	Vertical	77.3
3) High	Horizontal	12.7
4) High	Vertical	3.4

Figure IV-5 Point Probability, f_n, Versus X, d_c=1500'

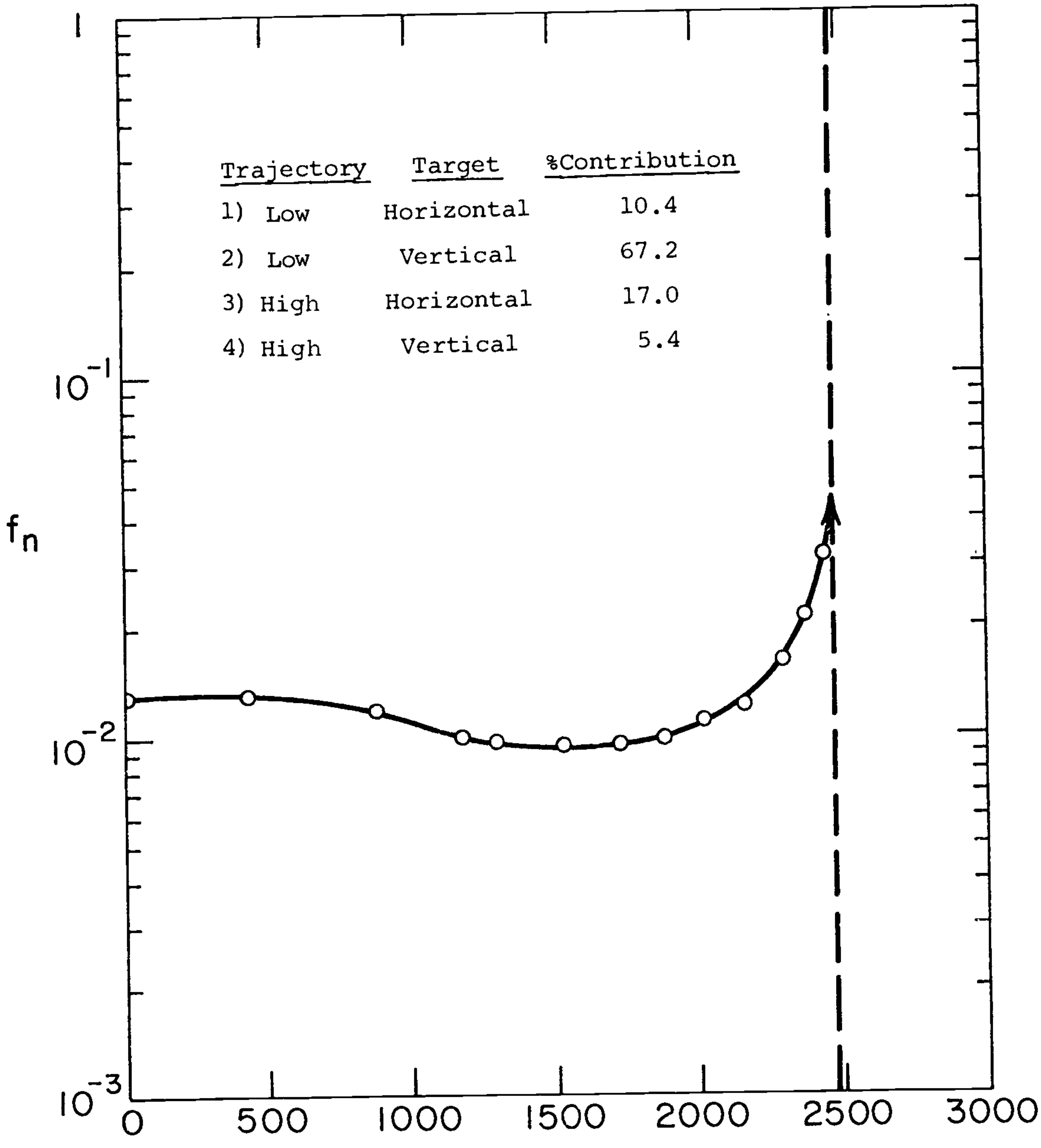
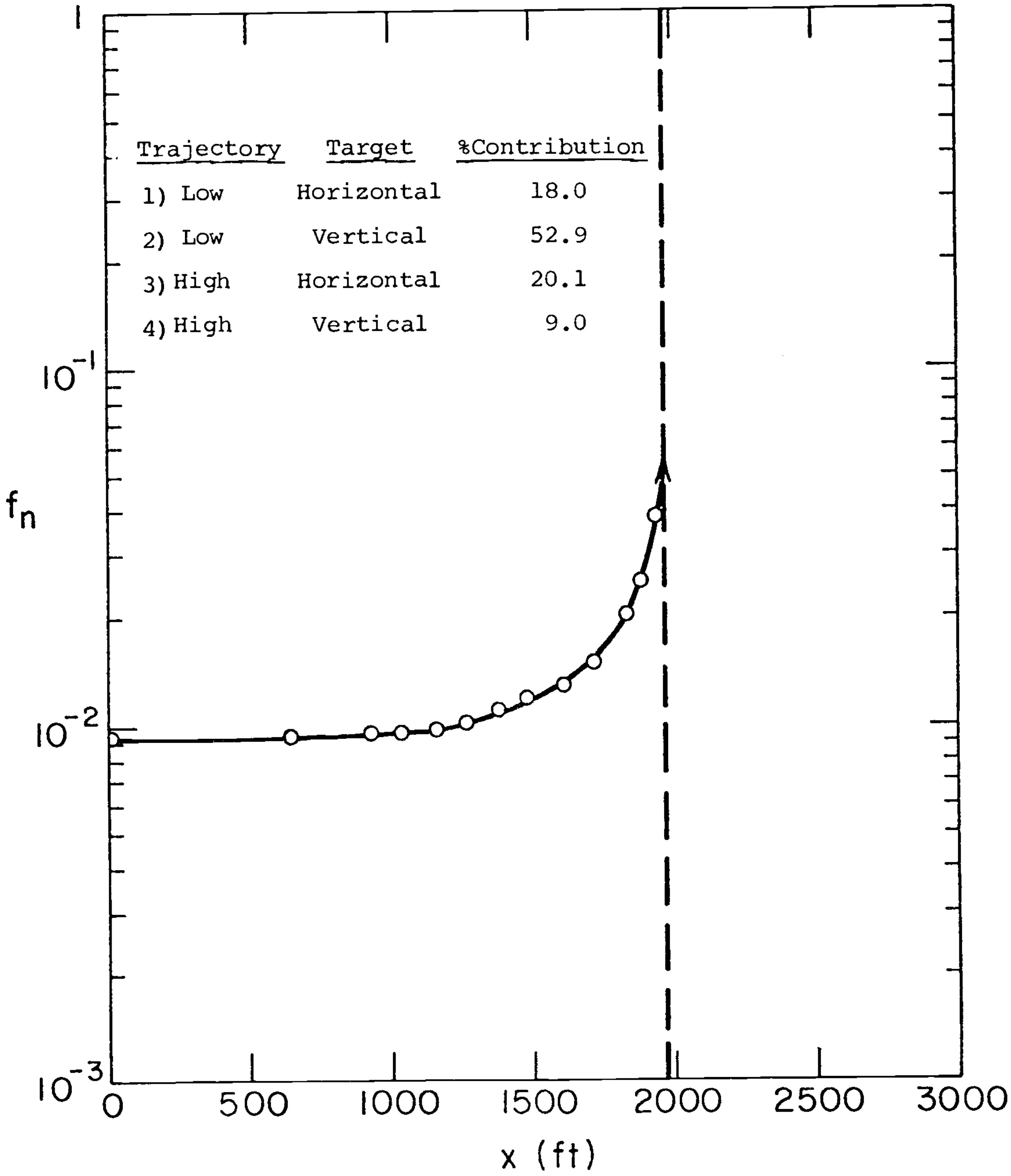
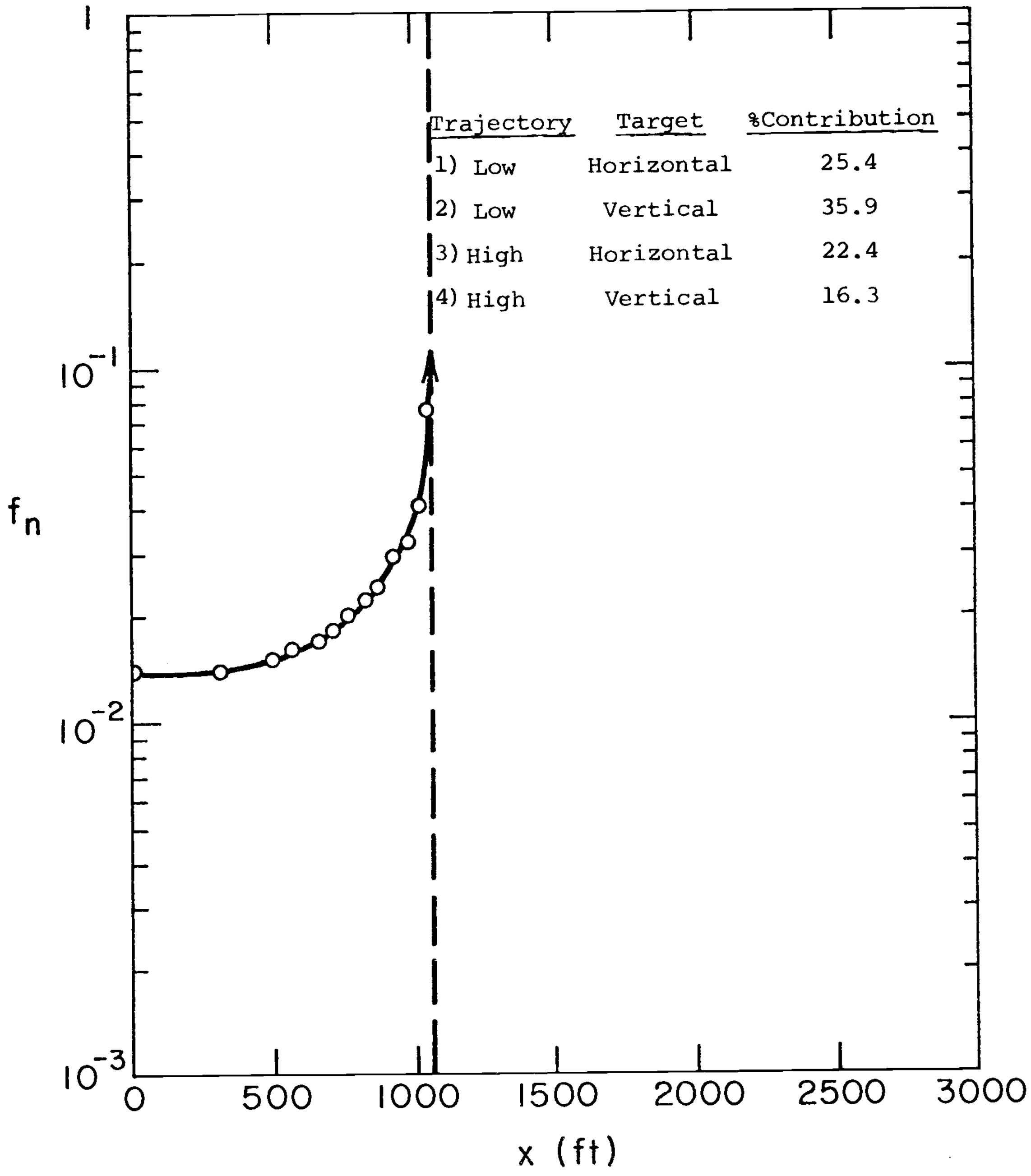


Figure IV-6 Point Probability, f_n, Versus X, d_c=2000'



Trajectory	Target	%Contribution
1) Low	Horizontal	18.0
2) Low	Vertical	52.9
3) High	Horizontal	20.1
4) High	Vertical	9.0

Figure IV-7 Point Probability, f_n, Versus X, d_c=2500'



Trajectory	Target	%Contribution
1) Low	Horizontal	25.4
2) Low	Vertical	35.9
3) High	Horizontal	22.4
4) High	Vertical	16.3

Figure IV-8 Point Probability, f_n, Versus X, d_c=3000'

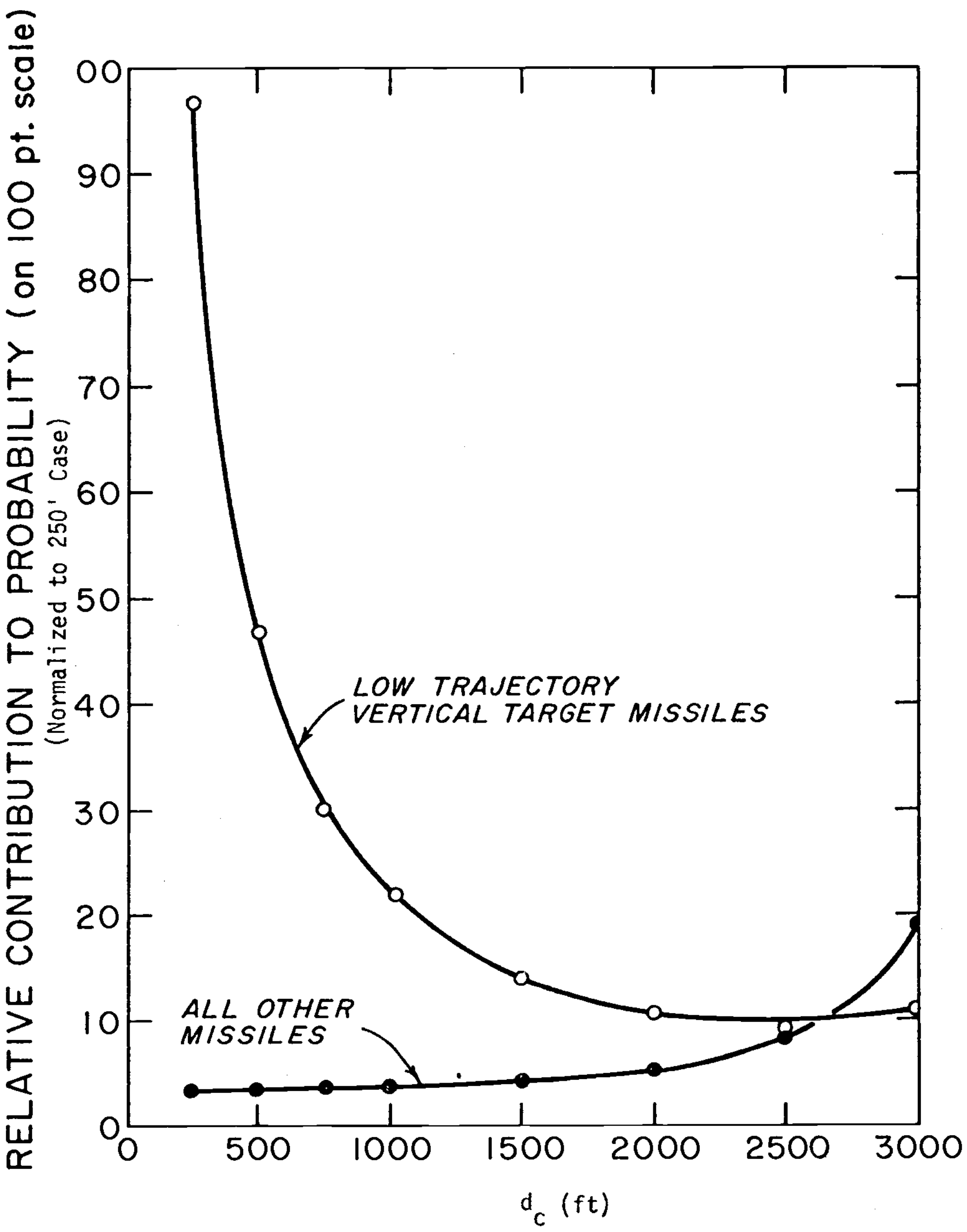


Figure IV-9 Relative Contribution to Probability Versus d_c Plant-Track Perpendicular Distances and then that the increasing contribution of the latter group is due almost exclusively to contributions from the low trajectory missiles penetrating horizontal target walls (as can be seen by the break down in Figs. IV-1, 2.) These missiles do not contribute until their trajectories get "high" enough so that horizontal plant areas are hit with a sufficiently large angle, α_T, to meet the penetration criterion.

Two difficulties arise in the numerical integration procedure in case of small values of d_c. One is due to the fact that the integrand f_n has a singularity (pole) at $d = 0$ and therefore small increments are required for accurate evaluation of the integral. The other difficulty is related to this singular property of f_n at $d = 0$ and also to the particular way the Mean Value Theorem is employed in computing the probability of a single missile strike, f_s. At distances close to the singularity, the error introduced by replacing the integration over the target area by the missile strike density function (evaluated at the representative target coordinate) times the target area becomes large. Both problems manifest themselves when values of the probability, f_s, greater than one are obtained, clearly in violation of the definition and interpretation of this quantity. However, since the error is always on the conservative side and since the magnitude of the error is large only in extreme cases of little practical interest, no further efforts were made to resolve this difficulty.

Another singularity occurs as shown in the figures at the maximum value of x, x_{MAX}. As x approaches x_{MAX} (and thus d approaches d_{MAX}), the quantity f_s goes to infinity as the term, $\frac{dd}{d\alpha_0}$, goes to zero. However, in this case the integral, f_n remains a finite quantity under all conditions.

Variation of Missile Parameters

The missile parameters essentially control the aerodynamic properties of the missile and consist of the average height-diameter ratio, γ, the average drag coefficient, C_d, and the average missile material density, ρ. Since these parameters occur together as one group, we choose to study their parametric effects by varying γ.

Variation of γ has two effects: 1) as γ increases, the aerodynamic drag decreases due to increasing slenderness of the missile. 2) as γ increases,the striking area of the missile becomes smaller allowing the missile to have more "penetrating" power.

The effects of varying γ on the equivalent track length, f_2, are shown in Fig. IV-10. The reason for the sudden jump around $\gamma = 1.25$ is due to the contribution of missiles in the lightest (and thus most plentiful) mass interval, ΔM_1, for which this value of γ is the threshold for penetration of vertical walls by way of low trajectories. It should be noted that there is a certain arbitrariness in the choice of the number of mass intervals and thus in the occurrence of these "humps" and break points (thresholds).

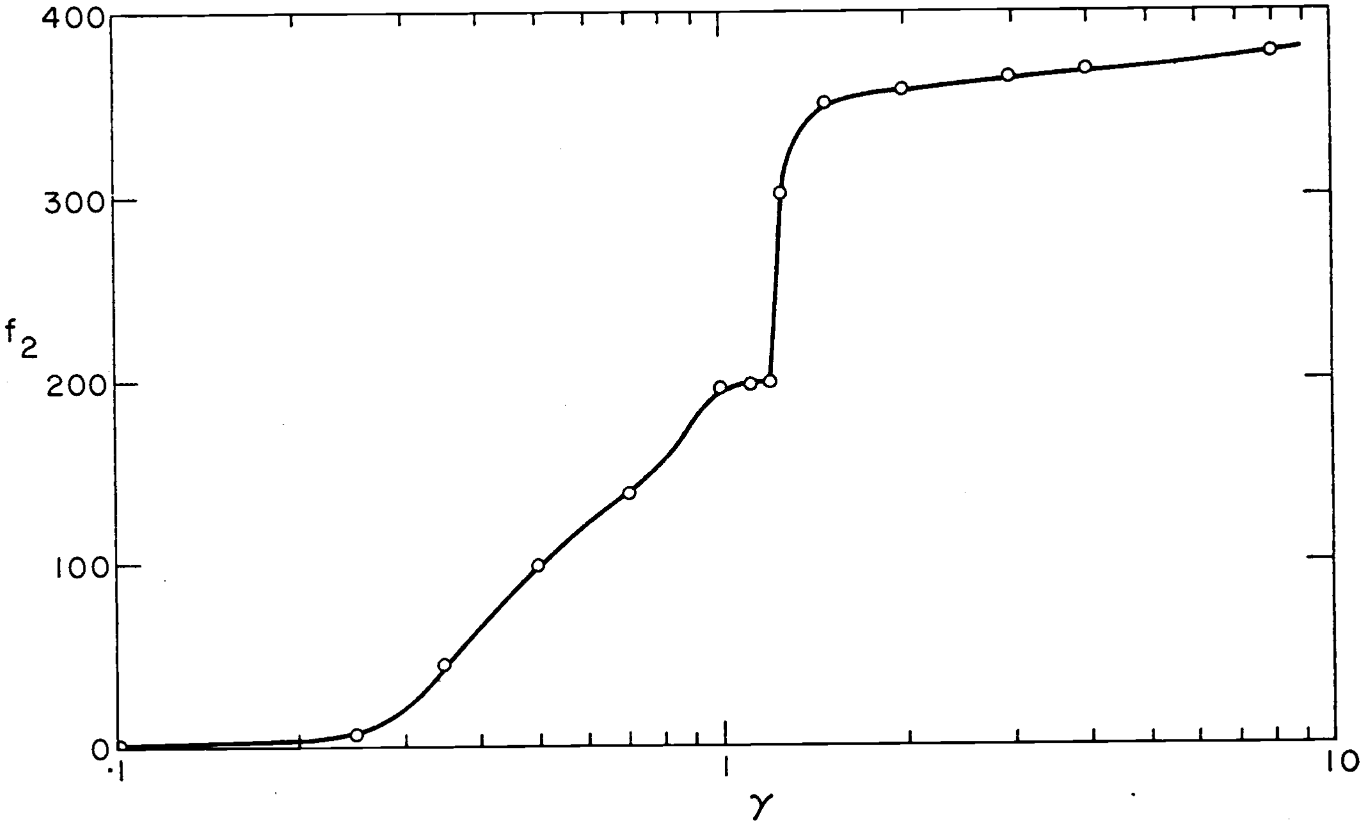


Figure IV-10 Equivalent Track Length, f_2, Versus Height/Diameter, γ

Increasing the number of mass intervals, especially in the lighter mass range, would tend to smooth out these break points, though the graph would still remain steep in this range. The rate of increase of the curve drops off rapidly as γ gets much larger than 1.5 for two reasons: 1) after missiles have reached a certain degree of slenderness due to increasing γ, drag free ballistics describe missile trajectories which make them independent of missile mass. 2) once even smallest missiles penetrate walls due to their high degree of slenderness, no further increase in the calculated probability expressed by f_2 is obtained through further increases of γ.

Variation of Equivalent Wall Thickness-T_c

Variation of the equivalent wall thickness, T_c, only affects the "penetration" criterion. Fig. IV-11 shows the effect of this variation on the equivalent track length, f_2. The "hump" occurring around 17" is due to penetration criterion being met at this point by missiles in the lightest (most plentiful) mass interval. It is noted that a tripling of wall thickness will result in a reduction in f_2 by one order of magnitude. This fact can be applied to estimate the value of f_2 associated with the containment. Giving due consideration to the smaller values of ΔA_H and ΔA_V for the containment, it can be seen that the containment adds little more to the total probability of an unacceptable accident.

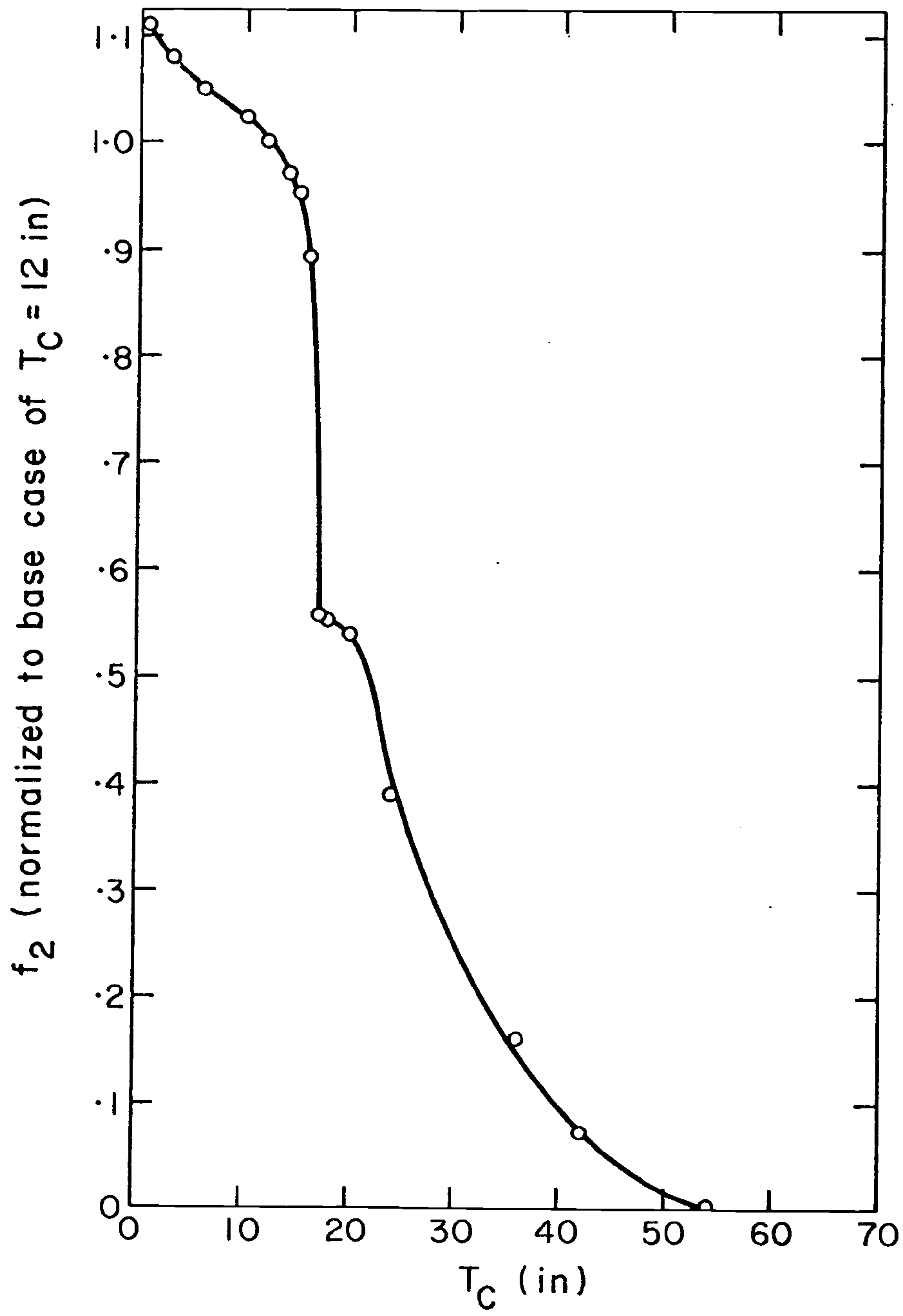


Figure IV-11 Normalized Equivalent Track Length Versus T_c

Variation of Missile Distribution Parameters-M_T/M_A

Variation of M_T/M_A affects the missile distribution per unit mass, N. As the ratio becomes larger, more pieces are implied as the most probable expected piece mass M_A becomes smaller in comparison to the total mass, M_T. The effect of this variation on equivalent track length f_2, is seen in Fig. IV-12. The curve levels off quickly beyond an M_T/M_A of approximately 100 due to the fact that the addition of more pieces is mostly in the small mass range where the penetration criterion is not met.

Variation of the Amount of Explosive-W

The amount of explosive, W enters into the model by way of the empirical relation for the maximum range of missiles introduced in Section D of Chapter II. In addition to this relation it appears reasonable to assume that the kinetic energy of all missiles constitutes a certain fraction of the total energy of the explosion with the remainder being taken up by generation of shock waves in the atmosphere and in the ground, heat, etc. For the reference case, it is found that 3.31% of the total energy of fifty tons of TNT goes into the kinetic energy of the missiles in the explosion. Keeping this percentage constant, the amount of explosive can be variedwhich results in a corresponding variation of M_T. The ratio M_T/M_A was kept constant at a value of ten. Fig. IV-13 shows M_T as a function of W, the number of equivalent tons of TNT explosive ,for fractional kinetic energies of 1%, 3.31% and 5%.

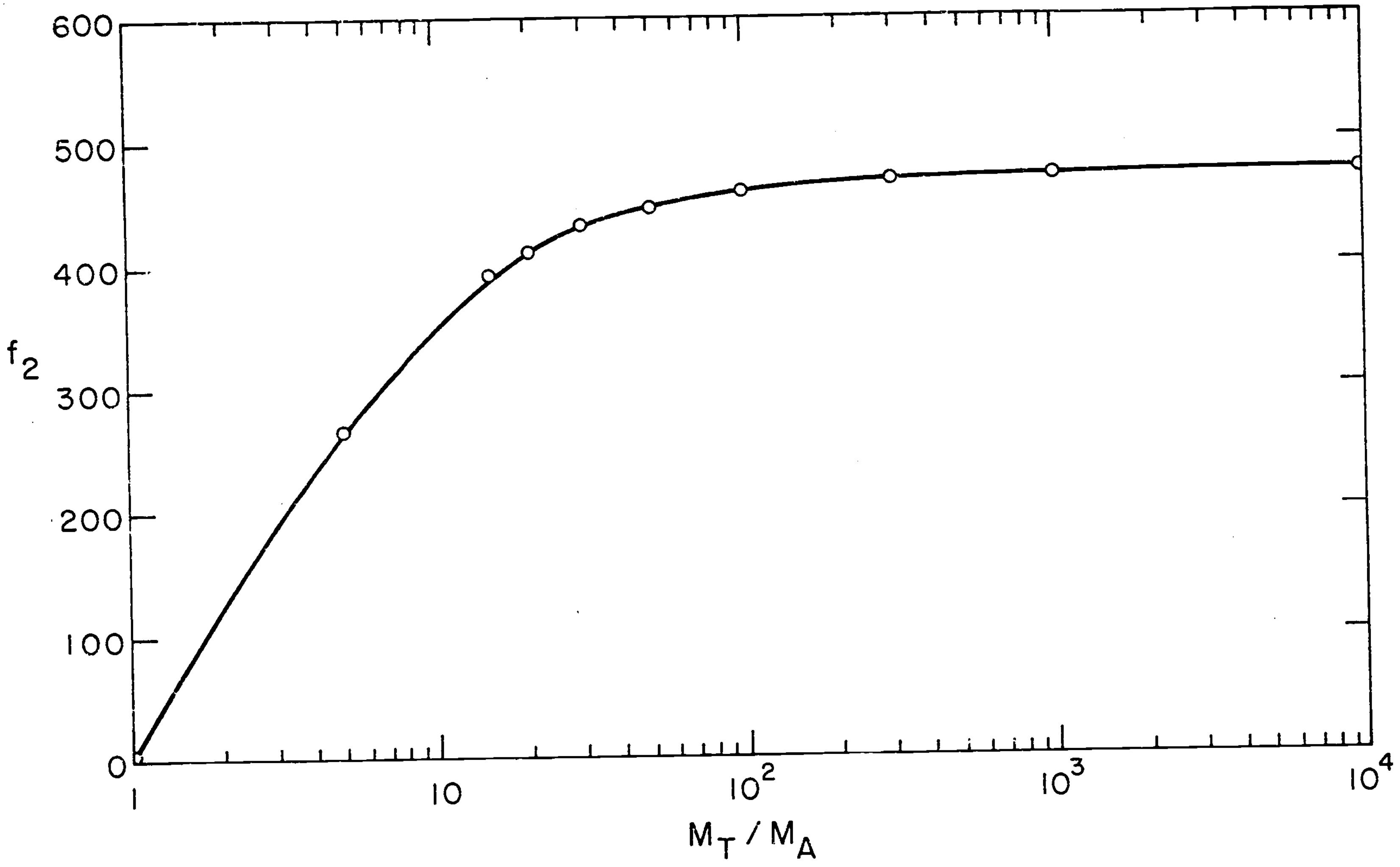


Figure IV-12 Equivalent Track Length Versus M_T/M_A

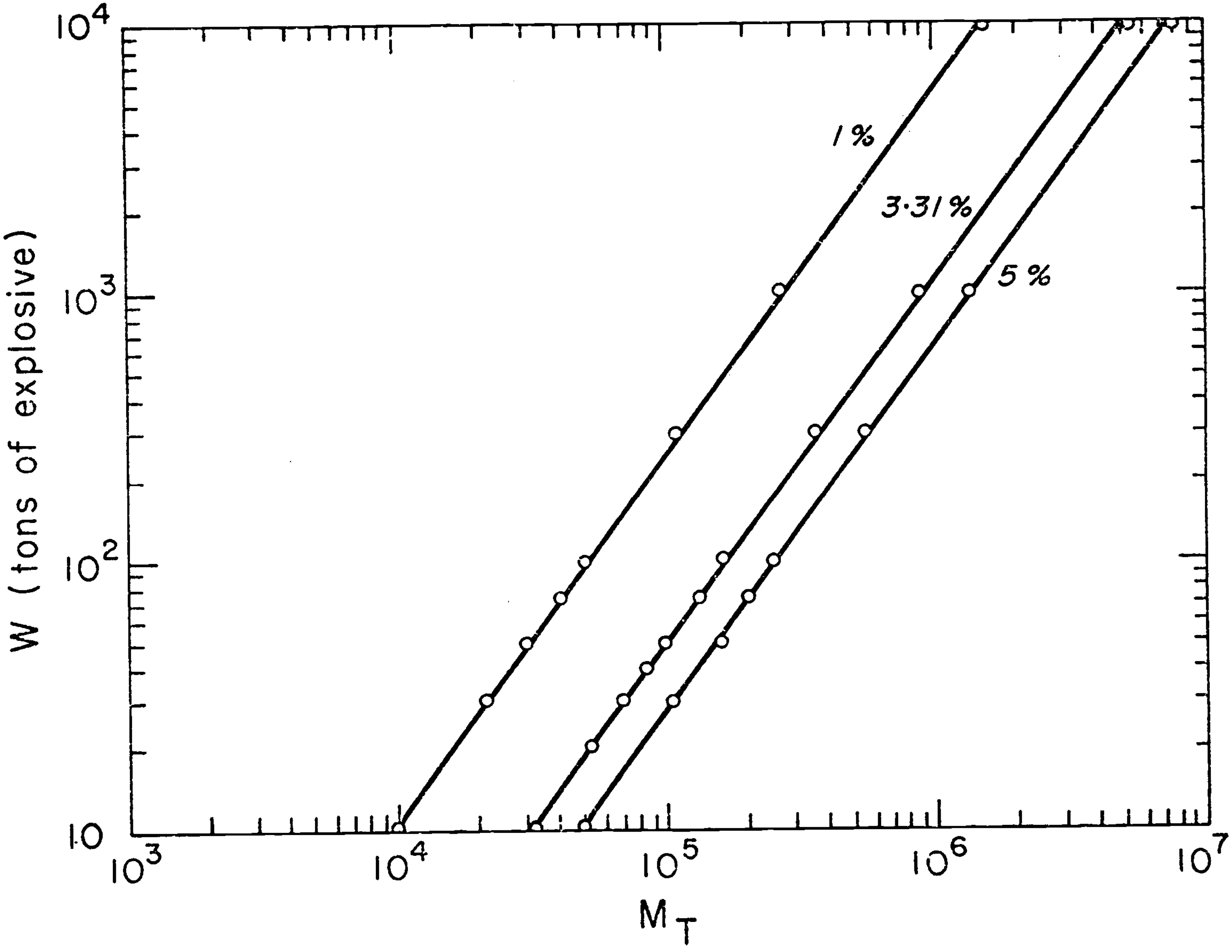


Figure IV-13 Tons of Explosive Versus M_T With M_T/M_A = 10

An increase of W under these constraints leads to an increase in the equivalent track length f_2 as shown in Fig. IV-14. Above W = 100, however, the rate of increase of f_2 quickly begins to taper off. Below $W \simeq 3$, f_2 is zero, i.e., there is no longer any hazard to the plant from missiles of such small explosions. The increase in f_2 with W is to be expected since it not only allows M_T to be increased thus allowing the missile distribution to shift towards more and heavier pieces, but also d_{MAX} is increased, thus permitting the plant to be vulnerable over a longer distance of the track.

C. CONCLUSIONS

Based on the detailed study of a representative reference case and the parametric variations from it, the following conclusions can be drawn: 1) For the reference case studied, drag was of little importance in the final result as characterized by the fact that $\gamma > 1$ for this case; consequently, a drag-free model would have given a conservative but close estimate of the annual probability of an unacceptable event. However, the relative importance of drag depends on certain assumptions which affect the initial velocity and size distribution of missiles and which are not thoroughly supported by evidence at this time. It is evident that drag becomes more important with smaller, lighter pieces. According to the assumed missile distribution, lighter pieces are more abundant--thus emphasizing the importance of drag effects up to the

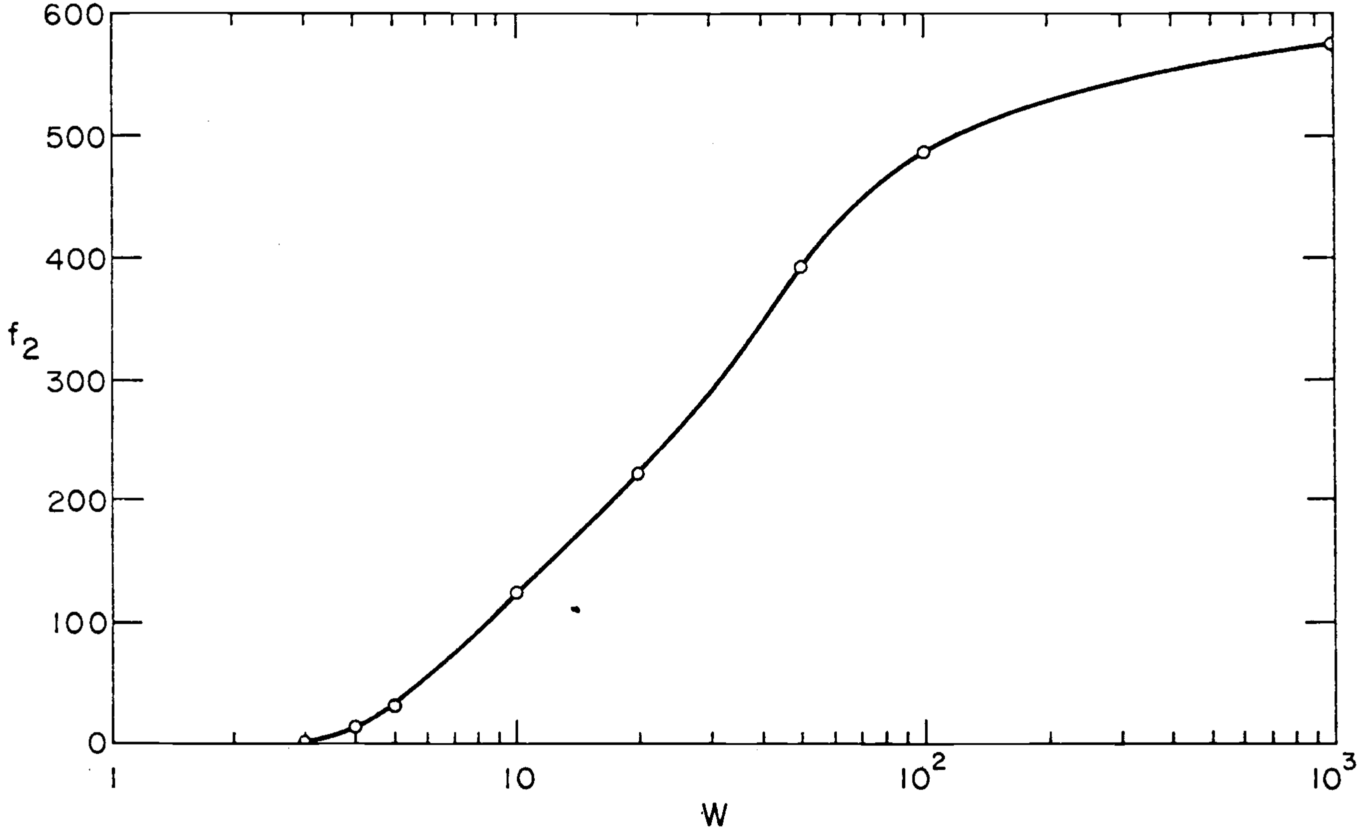


Figure IV-14 Equivalent Track Length, f_2, Versus Tons of Explosive, W.

point where penetration of the target wall no longer is achieved. The average height-diameter ratio of missiles, γ, was chosen conservatively high. Smaller and more realistic values of γ would make drag effects more significant. Actual piece distribution data and explosion characteristics information concerning amounts of energy actually propelling missiles would be needed to shed light on this problem. 2) The numerical integration process poses certain difficulties which were pointed out earlier. These difficulties are related to the singularities of the single missile point probability, f_S, at $d = 0$ and $d = d_{MAX}$. The simple integration schemes employed do not yield rapid convergence, expecially when $d_C << d_{MAX}$. Furthermore, it appears that the integration over mass in the lower mass ranges should be based on smaller mass increments. This is true especially when M_T is very large. The smaller mass ranges are relatively important in the calculation of f_2 due to the relative abundance of missiles in them. Smaller mass increments in this range would assist in smoothing the "jumps" occurring in some of the graphs from the parametric studies. 3) The model can be used to give a conservative estimate of the annual probability of an unacceptable accident to a nuclear plant due to missiles from surface traffic accident explosions. The model is readily adaptable to a wide variety of circumstances and results can be readily supported by physical interpretation due to the implementation of fundamental physical laws in the model. Although the reference case,considered to be typical,resulted in a probability half of an order of magnitude less than the currently acceptable limit of 10^{-7}, there appears to be a sufficiently wide range of realistic situations such that the missile hazard from surface traffic accident explosions cannot be categorically neglected.

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APPENDICES

APPENDIX A

Missile Trajectories

Missile trajectories are governed by the equations of motion. Besides the gravitational force a drag force also is considered which is assumed to be proportional to the square of the instantaneous velocity and acts in the direction opposite to the direction of motion. Additional assumptions made are constant density of air and zero wind speed.

$$f_d = \beta v^2$$

where f_d = drag force per unit mass

β = aerodynamic coefficient = $\frac{C_d \cdot W \cdot A}{2M}$

where C_d = drag coefficient

W = specific weight of air (lbs/ft^3)

A = fragment cross-sectional area (ft^2)

M = fragment weight (lbs)

If one assumes that all missiles are the same general shape as expressed by an equivalent cross-sectional area, A, and height-diameter ratio, γ, and that the missiles are made of the same material, it is found that:

$$A = k(M)^{2/3}$$

where $k = \left(\frac{\sqrt{\pi}}{2 \cdot \rho \cdot \gamma}\right)^{2/3}$

and ρ = density of missile material

thus $\beta = \frac{C_d \cdot W \cdot k}{2\ M^{1/3}}$

As will be seen, this parameter β is the only one which incorporates the effect of missile mass on the trajectory. The other factors in the definition of β are assumed to be constants.

For the purpose of making the equations of motion non-dimensional we introduce the quantity v_f defined as:

$$v_f = \left(\frac{g}{\beta}\right)^{\frac{1}{2}} = aM^{\frac{1}{6}}$$

$$\text{where } a = \left[\frac{2 \cdot g}{C_d \cdot W \cdot k}\right]^{\frac{1}{2}}$$

and interpreted as the asymptotic free fall velocity of the missile under consideration.

Referring to Figure A-1, and considering a fixed coordinate system, the equations of motion may be written:

x balance =

$$\frac{d^2x}{dt^2} + \beta v^2 \cos\alpha = 0$$

y balance =

$$\frac{d^2y}{dt^2} + g + \beta v^2 \sin\alpha = 0$$

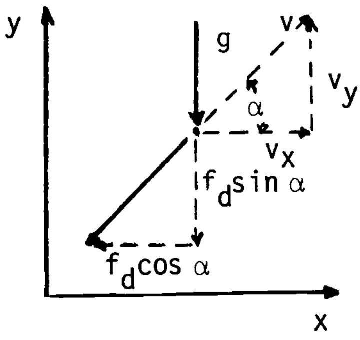

Fig. A-1 Trajectory Vector Diagram

From the figure:

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = v_x{}^2 + v_y{}^2$$

A general explicit solution is not known for the above set of coupled, second order, non-linear equations. As a result, the approach taken was to adopt a numerical integration scheme for the solution of these equations and to calculate the distance, d (x in the above coordinates), α, and v at the time of impact for a number of discrete values of the governing parameters covering the range of interest. From these specific solutions, relations between d, α_T, and v_T and M, v_0, and α_0 were developed by linear regression techniques.

To simplify this process, we introduce the following variables:

v_f = free fall velocity $= (\frac{g}{\beta})^{\frac{1}{2}}$

$D = \frac{v_f{}^2}{g}$

$\tau = \frac{D}{v_f}$

And define the following non-dimensional variables:

$$X^* = \frac{x}{D} \qquad Y^* = \frac{y}{D} \qquad Vy^* = \frac{v_y}{v_f} \qquad T^* = \frac{t}{\tau} \qquad V_x{}^* = \frac{v_x}{v_f}$$

Now rewriting the set of two second order equations as a set of four first order equations and making the appropriate substitutions to non-dimensionalize, we obtain:

1) Original form

$$\frac{dX}{dt} = v_x$$

Non-dimensional form

$$\frac{dX^*}{dT^*} = V_x^*$$

2) $$\frac{dV_x}{dt} = -\beta\,(v_x^2 + v_y^2)\cos\alpha$$

$$\cos\alpha = \frac{v_x}{\sqrt{v_x^2 + v_y^2}}$$

$$dV_x^* = -(V_x^{*2} + V_y^{*2})\cos\alpha$$

$$\cos\alpha = \frac{Vx^*}{\sqrt{V_x^{*2} + V_y^{*2}}}$$

3) $$\frac{dy}{dt} = v_y$$

$$\frac{dY^*}{dT^*} = Vy^*$$

4) $$\frac{dVy}{dt} = -g - \beta\,(v_x^2 + v_y^2)\sin\alpha$$

$$\sin\alpha = \frac{v_y}{\sqrt{v_x^2 + v_y^2}}$$

$$\frac{dVy^*}{dT^*} = -1 - (V_x^{*2} + V_y^{*2})\sin\alpha$$

$$\sin\alpha = \frac{Vy^*}{\sqrt{V_x^{*2} + V_y^{*2}}}$$

5) Initial conditions

$v_x = v_0 \cos\alpha_0$

$v_y = v_0 \sin\alpha_0$

$x = y = 0$

$V_x^* = V0^* \cos\alpha_0$

$V_y^* = V0^* \sin\alpha_0$

$X^* = Y^* = 0$

$V0^*$ is non-dimensional equivalent of v_0)

One notes that the non-dimensional formulation is independent of β (and thus independent of mass) such that the solutions of these equations are functions of α_0 and v_0 only.

A computer program using standard fourth order Runge-Kutta techniques was written to yield particular solutions to the equations. The impact characteristics V_T*, α_T*, and X_T* were obtained as characterized by the condition $Y_T* = 0$ corresponding to that point of the trajectory where the initial elevation is reached. (The subscript T refers to the point of impact, or "terminal point" of the trajectory.)

Graphs of various solutions are shown in Figs. A-2 through A-6 in the form in which they were obtained. We note that the range of negligible drag is characterized by the condition , $VO* << 1$. The range of VO* from 0 to 20 is believed sufficient to cover all plausible cases. The range from 0 to .3 is not plotted since simple drag-free ballistics were used for approximation in this range. Besides the numerical solutions shown as points, the graphs also depict the approximating continuous correlations which were developed by least squares fitting techniques. Tables A-1 through A-5 list these correlations.

As required by Program EXPLOD, the correlations for $X* = f(vO*, \alpha_0)$ must be inverted and put into the form $\alpha_0 = f(X*, VO*)$. This function is double-valued (two values of α_0 for each X*) corresponding to the low and high trajectory, respectively, which have the same impact location.

The additional correlation shown in Table A-6 was developed for use in the initial velocity calculation. It is the inverse of the relation for the maximum range $X*_{MAX}$ obtained for a given $VO*$: $VO* = f(X*_{MAX})$

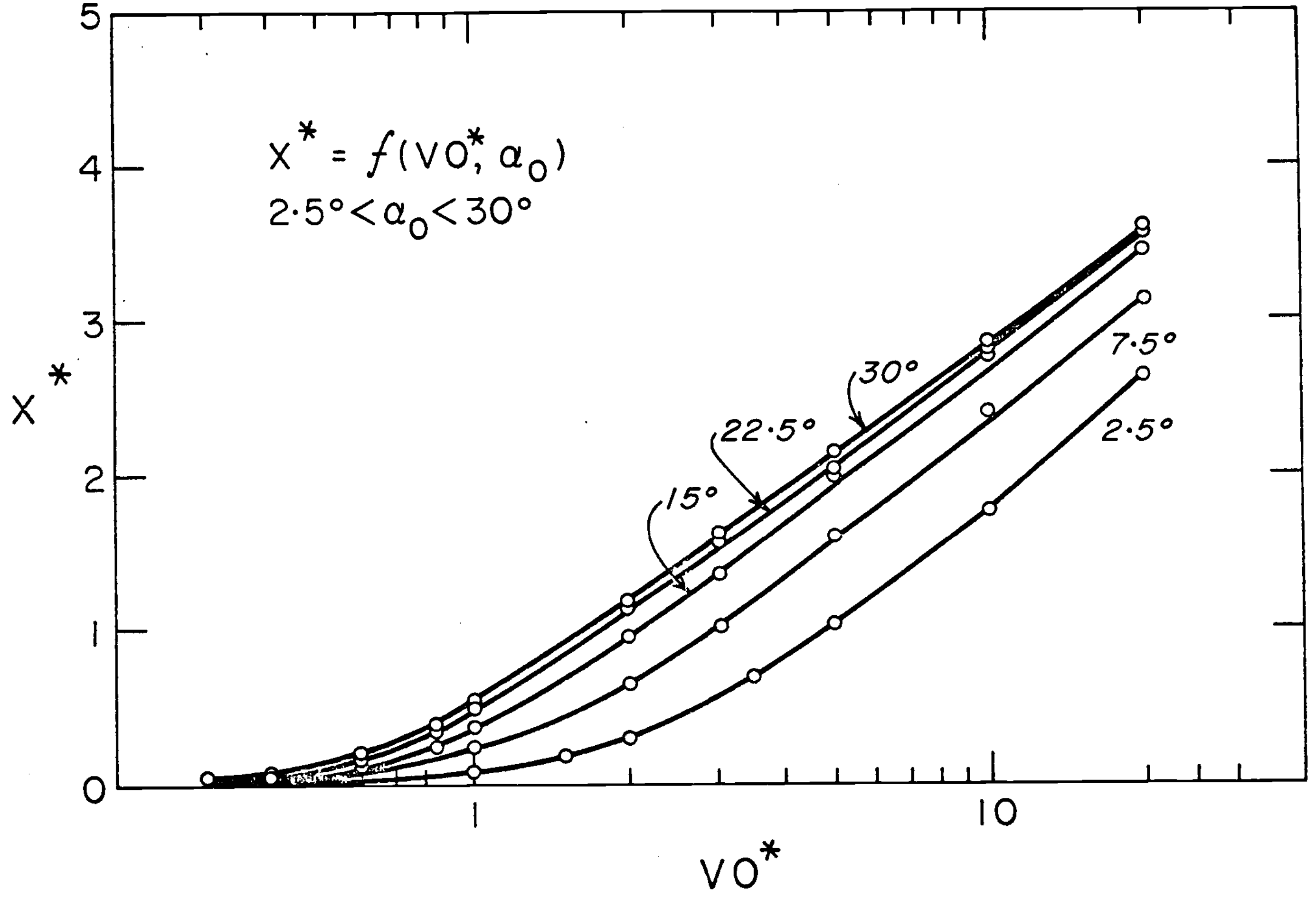

Figure A-2 X* Versus VO*, α_0, for 2.5° < α_0 < 30°

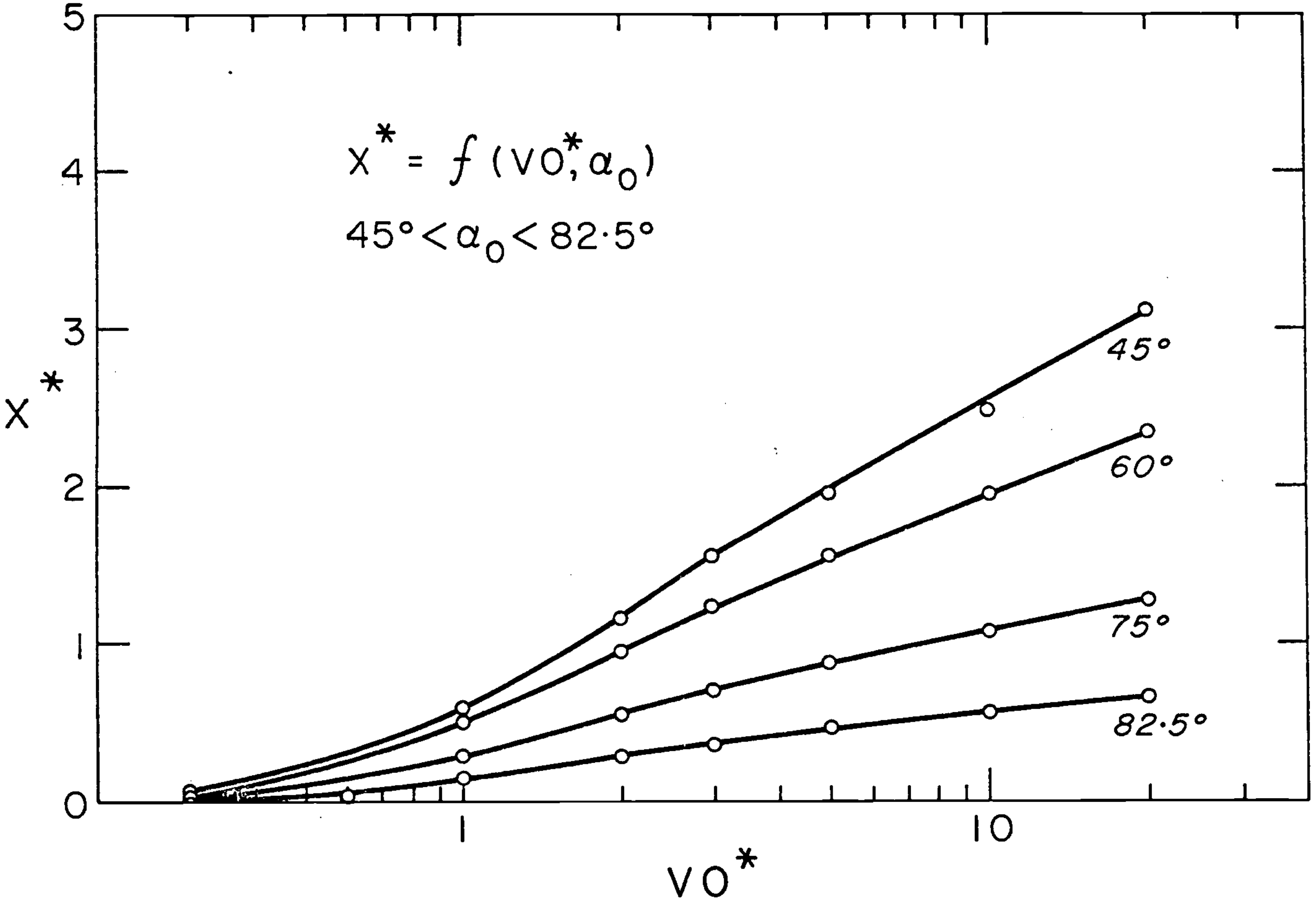

Figure A-3 X* Versus VO*, α_0, for 45° < α_0 < 82.5°

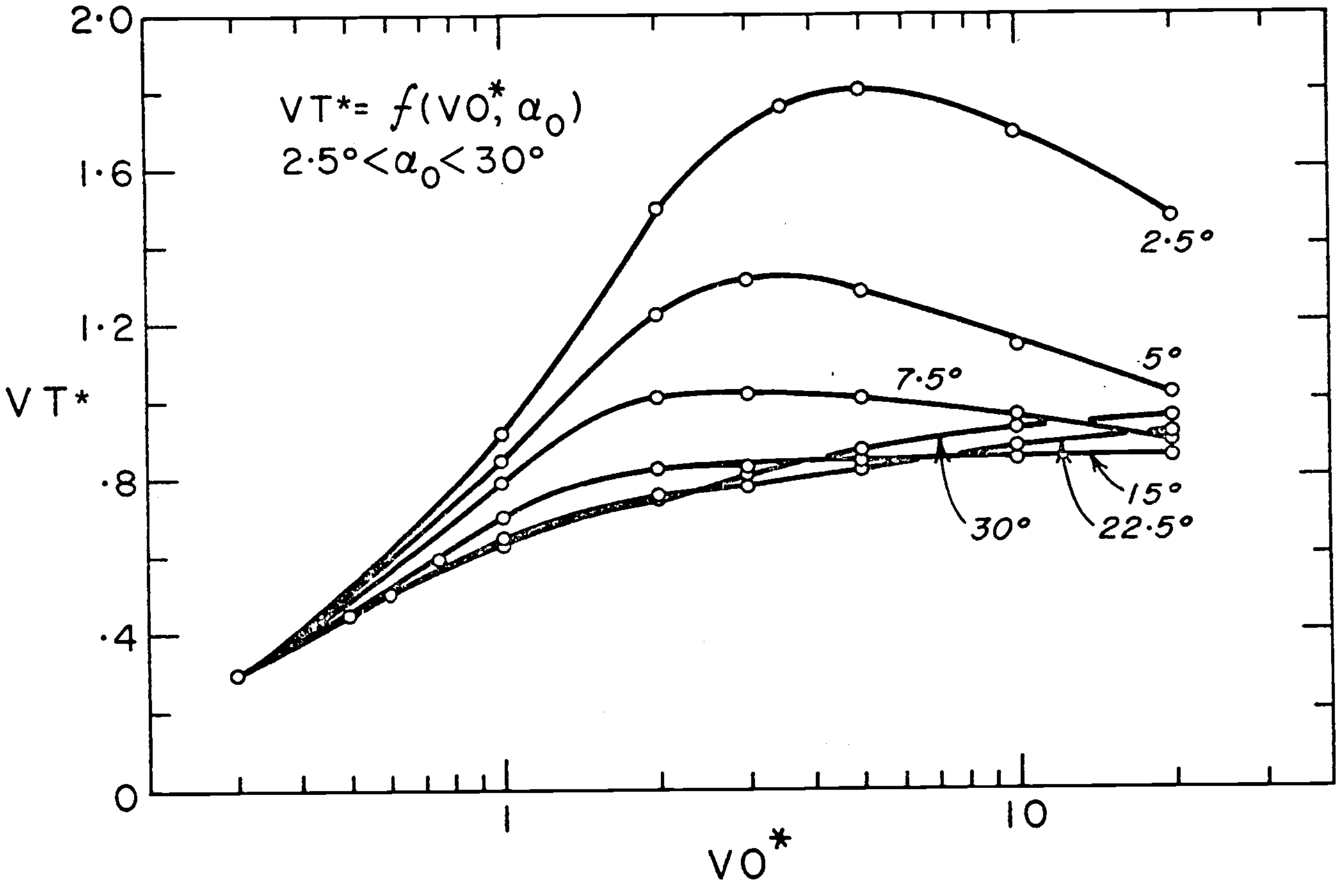

Figure A-4 V_T* Versus VO*, α_0, for 25° < α_0 < 30°

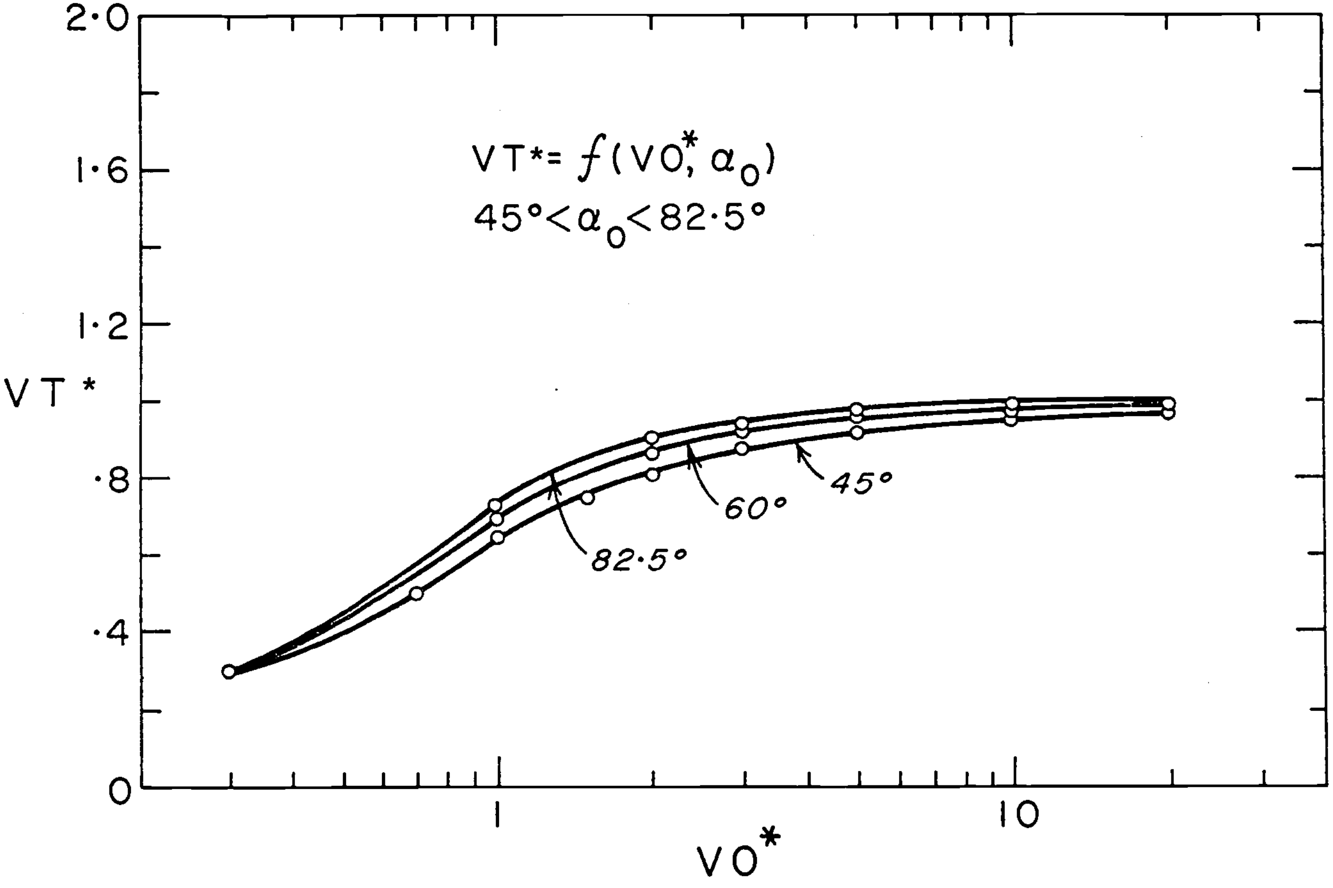

Figure A-5 V_T* Versus VO*, α_0, for 45° < α_0 < 82.5°

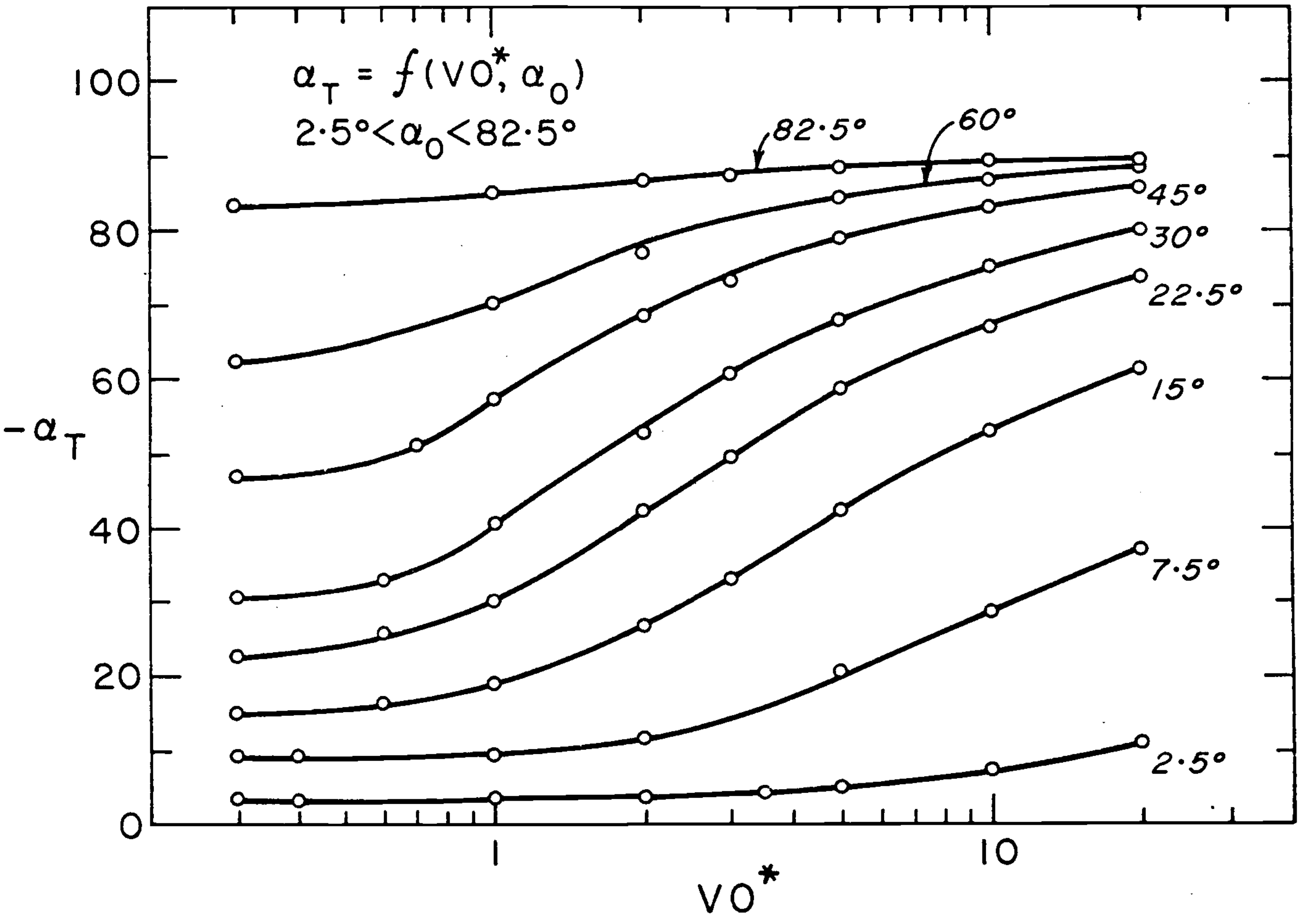

Figure A-6 $-\alpha_T$ versus VO*, α_0, for $2.5° < \alpha_0 < 82.5°$

TABLE A-1

CORRELATIONS FOR $X^* = f(VO^*, \alpha_0)$ FOR $\alpha_0 > 2.5°$

GENERAL FORM: $X^* = [A_{00}\alpha_0^2 + A_{01}\alpha_0 + A_{02}] \cdot f(VO^*) + [A_{10}\alpha_0^2 + A_{11}\alpha_0 + A_{12}]$

RANGE OF VO^*, α_0

FOR $VO^* > .3$

	$.3 < VO^* < 1$ $2.5 < \alpha_0 < 40°$	$1 < VO^* < 3$ $2.5° < \alpha_0 < 30°$	$3 < VO^* < 20$ $2.5° < \alpha_0 < 40°$
A_{00}	-4.845×10^{-4}	-9.995×10^{-4}	-1.299×10^{-4}
A_{01}	3.716×10^{-2}	4.860×10^{-2}	-8.810×10^{-4}
A_{02}	3.518×10^{-2}	3.854×10^{-1}	1.153
A_{10}	1.373×10^{-4}	-4.059×10^{-4}	-1.141×10^{-3}
A_{11}	-9.188×10^{-3}	3.122×10^{-2}	8.373×10^{-2}
A_{12}	-2.365×10^{-3}	-1.623×10^{-2}	-9.127×10^{-1}
$f(VO^*)$	VO^*	$\ln VO^*$	$\ln VO^*$

☒	$.3 < VO^* < 1$ $40° < \alpha_0 < 90°$	$1 < VO^* < 3$ $30° < \alpha_0 < 90°$	$3 < VO^* < 20$ $40° < \alpha_0 < 90°$
A_{00}	-2.588×10^{-4}	-2.055×10^{-4}	-1.299×10^{-4}
A_{01}	1.859×10^{-2}	8.576×10^{-3}	-8.810×10^{-4}
A_{02}	4.196×10^{-1}	8.919×10^{-1}	1.153
A_{10}	3.391×10^{-5}	-2.302×10^{-4}	-2.450×10^{-4}
A_{11}	-1.517×10^{-3}	1.800×10^{-2}	1.920×10^{-2}
A_{12}	-1.403×10^{-1}	2.389×10^{-1}	2.460×10^{-1}
$f(VO^*)$	VO^*	$\ln VO^*$	$\ln VO^*$

FOR $VO^* < .3$
AND ALL α_0 (drag free)

$$X^* = VO^{*2} \text{ SIN } 2\alpha_0$$

TABLE A-2

CORRELATIONS FOR $X^* = f(VO^*, \alpha_0)$ FOR $\alpha_0 < 2.5°$

GENERAL FORM: $X^* = A_1 \cdot \alpha_0 \cdot f(VO^*) + A_2 \cdot \alpha_0$

RANGE OF VO*

FOR $VO^* > .3$

	$.3 < VO^* < 1$	$1 < VO^* < 3$	$3 < VO^* < 20$
A_1	4.820×10^{-2}	1.938×10^{-2}	4.600×10^{-1}
A_2	$- 1.132 \times 10^{-2}$	1.348×10^{-2}	$- 3.330 \times 10^{-1}$
f(VO*)	VO*	ln VO*	ln VO*

FOR $VO^* < .3$
AND ALL α_0 (drag free)

$$X^* = VO^{*2} \sin 2\alpha_0$$

TABLE A-3

CORRELATIONS FOR $VT^* = f(VO^*, \alpha_0)$ FOR $\alpha_0 > 15°$

GENERAL FORM: $VT^* = [A_{00}\alpha_0^2 + A_{01}\alpha_0 + A_{02}] \cdot f(VO^*) + [A_{10}\alpha_0^2 + A_{11}\alpha_0 + A_{12}]$

RANGE OF VO^*, α_0

FOR $VO^* > .3$

	$.3 < VO^* < 2$ $15° < \alpha_0 < 30°$	$2 < VO^* < 8$ $15° < \alpha_0 < 40°$	$8 < VO^* < VO^*$ $15° < \alpha_0 < 30°$
A_{00}	3.020×10^{-4}	$- 2.440 \times 10^{-4}$	$- 1.991 \times 10^{-5}$
A_{01}	$- 1.600 \times 10^{-2}$	1.820×10^{-2}	9.680×10^{-4}
A_{02}	4.590×10^{-1}	$- 2.046 \times 10^{-1}$	$- 7.630 \times 10^{-3}$
A_{10}	2.489×10^{-4}	5.733×10^{-4}	2.933×10^{-4}
A_{11}	$- 1.467 \times 10^{-2}$	$- 3.543 \times 10^{-2}$	8.867×10^{-3}
A_{12}	8.190×10^{-1}	1.211	8.880×10^{-1}
$f(VO^*)$	ln VO*	ln VO*	ln VO*
☒	$.3 < VO^* < 2$ $30° < \alpha_0 < 90°$	$2 < VO^* < 8$ $40° < \alpha_0 < 90°$	$8 < VO^* < 20$ $30° < \alpha_0 < 90°$
A_{00}	$- 1.284 \times 10^{-5}$	1.849×10^{-5}	1.211×10^{-6}
A_{01}	2.818×10^{-3}	$- 3.496 \times 10^{-3}$	$- 1.935 \times 10^{-4}$
A_{02}	1.781×10^{-1}	2.536×10^{-1}	8.139×10^{-3}
A_{10}	$- 1.000 \times 10^{-5}$	$- 5.651 \times 10^{-5}$	$- 5.288 \times 10^{-5}$
A_{11}	2.822×10^{-3}	1.032×10^{-2}	7.823×10^{-3}
A_{12}	5.258×10^{-1}	3.681×10^{-1}	7.022×10^{-1}
$f(VO^*)$	ln VO*	ln VO*	ln VO*

FOR $VO^* < .3$
AND ALL α_0 (drag free)

$$VT^* = VO^*$$

TABLE A-4

CORRELATIONS FOR $VT^* = f(VO^*, \alpha_0)$ FOR $\alpha_0 < 15°$

GENERAL FORM: $VT^* = [A_{00}\alpha_0^2 + A_{01}\alpha_0 + A_{02}] \cdot f(VO^*) +$

$[A_{10}\alpha_0^2 + A_{11}\alpha_0 + A_{12}]$

RANGE OF VO^*, α_0

FOR $VO^* > .3$

	$.3 < VO^* < 1$ $0 < \alpha_0 < 15°$	$1 < VO^* < VO^*(\alpha)^1$ $0 < \alpha_0 < 15°$	$VO^*(\alpha)^1 < VO^* < 20$ $0 < \alpha_0 < 15°$
A_{00}	5.944×10^{-4}	3.662×10^{-3}	0
A_{01}	$- 2.437 \times 10^{-2}$	$- 1.117 \times 10^{-1}$	1.932×10^{-2}
A_{02}	5.687×10^{-1}	9.777×10^{-1}	$- 2.658 \times 10^{-1}$
A_{10}	7.943×10^{-4}	7.943×10^{-4}	1.019×10^{-2}
A_{11}	$- 3.149 \times 10^{-2}$	$- 3.149 \times 10^{-2}$	$- 2.796 \times 10^{-1}$
A_{12}	9.869×10^{-1}	9.869×10^{-1}	2.765
$f(VO^*)$	ln VO*	ln VO*	ln VO*

[1] $VO^*(\alpha) = (2.257 \times 10^{-2})\alpha_0^2 + (- 4.077 \times 10^{-1})\alpha_0 + (4.526)$

FOR $VO^* < .3$
AND ALL α_0 (drag free)

$$VT^* = VO^*$$

TABLE A-5

CORRELATIONS FOR $\alpha_T = f(VO^*, \alpha_0)$ FOR ALL α_0

GENERAL FORM: $\alpha_T = [A_{00}\alpha_0^2 + A_{01}\alpha_0 + A_{11}] \cdot f(VO^*) +$

$[A_{10}\alpha_0^2 + A_{11}\alpha_0 + A_{12}]$

RANGE OF VO*, α_0

FOR VO* > .3

	$.3 < VO^* < 1$ $0 < \alpha_0 < 30°$	$1 < VO^* < 5$ $0 < \alpha_0 < 20°$	$5 < VO^* < 20$ $0 < \alpha_0 < 15°$
A_{00}	-8.743×10^{-3}	0	8.811×10^{-2}
A_{01}	-1.879×10^{-1}	-9.853×10^{-1}	-2.265
A_{02}	9.500×10^{-1}	5.983×10^{-1}	3.400×10^{-1}
A_{10}	0	-1.981×10^{-2}	-1.744×10^{-1}
A_{11}	-1	-9.035×10^{-1}	1.260
A_{12}	0	-2.349×10^{-1}	-2.900×10^{-1}
f(VO*)	VO*	ln VO*	ln VO*
	$.3 < VO^* < 1$ $30° < \alpha_0 < 90°$	$1 < VO^* < 5$ $20 < \alpha_0 < 90°$	$5 < VO^* < 20$ $15° < \alpha_0 < 90°$
A_{00}	5.773×10^{-3}	1.458×10^{-3}	-2.174×10^{-3}
A_{01}	-4.619×10^{-1}	1.110×10^{-1}	4.036×10^{-1}
A_{02}	-4.447	-21.454	-19.065
A_{10}	0	7.079×10^{-3}	1.529×10^{-2}
A_{11}	-1	-1.649	-2.428
A_{12}	0	2.391	7.992
f(VO*)	VO*	ln VO*	ln VO*

FOR VO* < .3
AND ALL α_0 (drag free)

$$\alpha_T = -\alpha_0$$

TABLE A-6

CORRELATIONS FOR

$VO^* = f(x^*_{MAX})$

1. $\underline{0 < X^*_{MAX} < .09}$

$$VO^* = \sqrt{X^*_{MAX}}$$

2. $\underline{.09 < X^*_{MAX} < .595}$

$$VO^* = \frac{X^*_{MAX} + (.1264}{(.7214)}$$

3. $\underline{.595 < X^*_{MAX} < 1\text{-}6}$

$$\ln VO^* = \frac{X^*_{MAX} - (.5954)}{(.9148)}$$

4. $\underline{1.6 < X^*_{MAX}}$

$$\ln vO^* = \frac{X^*_{MAX} - (.4420)}{(1.054)}$$

APPENDIX B

Input, Output and Use Instructions
For Program EXPLOD

A copy of program EXPLOD is included as part of this Appendix. Program EXPLOD was written in standard FORTRAN IV for use on a CDC CYBER 73 computing system. It is specifically designed for on line use from remote terminals.

Program EXPLOD calculates the equivalent track length over which an explosion would cause an unacceptable accident due to missiles as described in the main body of the paper.

Table B-1 lists the input parameters as used by EXPLOD

Table B-2 lists the output as generated by EXPLOD.

Table B-3 lists the diagnostic error statements generated by EXPLOD.

Figure B-1 is an annotated sample data file and sample run with complete sample output for the reference case discussed in Chapter IV.

Figure B-2 gives annotated sample data files and runs showing the L = 1 and K = 1 options (see Table B-1 for definitions of L, K)

Figure B-3 is a copy of program EXPLOD.

TABLE B-1

Input for EXPLOD

PROGRAM VARIABLE	VARIABLE	READ IN FROM:	FORMAT	UNITS	DEFINITION
AMASSA	M_A	Tape 5	E 10.4	lbs	Average (expected) missile mass
AMASST	M_T	"	"	lbs	Total mass of all fragments
TC	T_c	"	"	in	Equivalent wall thickness
AK1	K_1	"	"	$\frac{in\text{-}ft^2}{lb}$	Parameter for concrete strength (2)
CD	C_d	"	"	---	Representative drag coefficient
W	w	"	"	lb/ft^3	Specific weight of air
RHO	ρ	"	"	lb/ft^3	Density of missile material
HD	γ	"	"	---	Representative height-diameter ratio for missles
DC	dc	"	"	ft	Perpendicular distance between traffic route and plant
WEIGHT	W	"	"	tons	Equivalent tons of TNT explosive
DELAH	ΔA_H	"	"	ft^2	Equivalent horizontal target area
DELAV	ΔA_V	"	"	ft^2	Equivalent vertical target area
K	--	"	I2	---	Program Mode Variable[1]
L	--	"	I2	---	Program Mode Variable[2]
N, M	--	Remote Terminal On Line	I4, 1X, I4	---	No. of mass and distance intervals desired respectively.[3]

1. If K is set to 1, initial velocity can be inputted in ft/sec. If K is set to any other value, initial velocity is calculated as described in the main text.
2. If L is set to 1, a specific distance can be inputted for which a specific point probability, f_n, will then be calculated. If L is set to any other value, the equivalent track length, f_2, is calculated.
3. If N is set to 0, the program terminates. Note: After calculation, program returns automatically to receive new N, M if desired.

TABLE B-2

Output for EXPLOD

PROGRAM VARIABLE	VARIABLE	OUTPUTTED TO:	FORMAT	UNITS	DEFINITION
DMAX	d_{max}	Remote Terminal	E11.4	ft	Maximum range of missiles
XMAX	x_{max}	"	"	ft	Section of traffic route over which potential missile hazard exists
VINIT	v_0	"	"	ft/sec	Calculated initial velocity of all pieces
AMASS1	M_{min}	"	"	lb	Calculated minimum mass to meet penetration criterion
FSFB	f_2, f_n	"	"	ft,---	Equivalent track length or, if L is set to 1, point probability for specified distance.
The following variables are outputted for every mass interval for detailed analysis if desired					
FSFB1, FSFB2	$f_{shl} \cdot f_{bhl}$ $f_{svl} \cdot f_{bvl}$	Tape 10	2E9.2	---	Point probability for a single missile strike and penetration for the various trajectory--wall orientation combinations
FSFB3, FSFB4	$f_{shh} \cdot f_{bhh}$ $f_{svh} \cdot f_{bvh}$	"		---	
FSUM	$f_s \cdot f_b$	"	E9.2	---	Sum of FSFB1 → FSFB4
ANT	N_j	"	"	lb^{-1}	No. of missiles per unit mass interval about mass, M_j
FSFB	f_n	"	"	---	Running sum of $1-(1-FSUM)^{ANT}$
ALPH 1, DALPH 1	$\alpha_0, \frac{dd}{d\alpha_0}$	Tape 15	2E9.2	radians, ft/radian	Low and high trajectory α_0's and $\frac{dd}{d\alpha_0}$, respectively
ALPH 2, DALPH 2	"	"	"		
ATL, ATH	α_T	"	"	radians	Low and high trajectory impact angles
v0	v0*	"	E9.2	---	Non-dimensional initial velocity
VTL, VTH	v_T	"	2E9.2	ft/sec	low and high trajectory impact velocities
AMASS	M_j	"	E11.4	lb	Missile mass at mid point of mass interval
DIST	d	"	"	ft	Distance between plant and explosion
The following variables are outputted for every distance interval for a less detailed analysis					
F1, F2, F3, F4	---	Tape 12	4E9.2	---	Sum of FSFB1, FSFB2, -- over mass intervals
FSFB	f_n	"	E9.2	---	Point probability at given distance
DIST	d	"	E11.4	ft	Same as above

TABLE B-3

Diagnostic Statements For EXPLOD

"DMAX IS LESS THAN DC"	Inputted d_c is greater than d_{max} $\therefore$ No missiles can hit the plant.
"MASLIM DIVERGING"	Iteration routine to calculate M_{min} is diverging. If this occurs, M_{min} should be inputted (conservative guess--$M_{min} = 0$).
"DALPH LOW OR FSUM > 1"	Plant is too close to explosion point such that model breaks down or the plant is too close to d_{max} such that $\frac{dd}{d\alpha_0} \to 0$ and $f_s \to \infty$. (see discussion in Section IV)

Reference Case Data File and Sample Run

```
   AMASSA         AMASST        TC          AK1           CD         W
              RHO          HD          DC          WEIGHT      DELAH       DELAV
                                                                                K    L
CØPY,TAPE5
    1.E04       1.E05      12.E00  3.312E-2      1.E00    8.08E-2
  4.88E02       2.E00       5.E02       5.E01   6.4E04     6.4E04

END ØF INFØRMATIØN ENCØUNTERED.
/REWIND,TAPE5,LGØ
$REWIND,TAPE5,LGØ.                N   M
/LGØ
 INPUT NØ. ØF MASS INTERVALS DESIRED AND NØ.
   ØF TRACK DISTANCE INTERVALS DESIRED IN I4,1X,I4 FØRMAT.
?  20   20
 DMAX=    .3185E+04 XMAX=    .3145E+04
 INTIAL VELØCITY=    .3203E+03
 MIN. CRITICAL PIECE MASS=    .6700E+02
 INTEGRATED PRØBABILITY =    .3561E+03
                                          New Value of N, M
?   35   35
 DMAX=    .3185E+04 XMAX=    .3145E+04
 INTIAL VELØCITY=    .3203E+03
 MIN. CRITICAL PIECE MASS=    .6700E+02
 INTEGRATED PRØBABILITY =    .3939E+03
                                          N = 0  Causes Program
?    0                                           Termination
    18.735 CP SECØNDS EXECUTIØN TIME
/
```

Sample Output Tape 12
(First 7 Lines of N = 20, M = 20 Case)
Total of 20 Lines for this file
for this Case

```
      F1        F2        F3        F4       FSFB        DIST

REWIND,TAPE12
$REWIND,TAPE12.
/EDIT,TAPE12
 BEGIN TEXT EDITING.
? L:7
   .33E-01   .43E-01   .28E-01   .21E-01   .19E-01   .3107E+04
   .19E-01   .34E-01   .15E-01   .87E-02   .11E-01   .2952E+04
   .13E-01   .35E-01   .12E-01   .47E-02   .15E-01   .2797E+04
   .77E-02   .36E-01   .10E-01   .22E-02   .14E-01   .2643E+04
   .93E-03   .39E-01   .89E-02   .21E-03   .14E-01   .2488E+04
  0.         .42E-01   .81E-02 0.          .15E-01   .2334E+04
  0.         .47E-01   .74E-02 0.          .16E-01   .2181E+04
```

Figure B-1 Reference Case Data File, Run, and Complete Output Sample

Sample Output Tape 10
(First 7 Lines of N = 20, M = 20 Case)
Total of 400 Lines for This File
for This Case

```
      FSFB1     FSFB2     FSFB3     FSFB4     FSUM      FSFB      ANT

 REWIND,TAPE10
$REWIND,TAPE10.
/EDIT,TAPE10
 BEGIN TEXT EDITING.
? L:7
  0.        0.        0.        0.        0.        0.        .26E+01
  0.        .23E-02   .15E-02  0.         .37E-02   .35E-02   .92E+00
   .18E-02  .23E-02   .15E-02   .12E-02   .67E-02   .71E-02   .55E+00
   .18E-02  .23E-02   .15E-02   .12E-02   .67E-02   .97E-02   .38E+00
   .18E-02  .23E-02   .15E-02   .12E-02   .68E-02   .12E-01   .28E+00
   .18E-02  .23E-02   .15E-02   .12E-02   .68E-02   .13E-01   .22E+00
   .18E-02  .23E-02   .15E-02   .12E-02   .68E-02   .14E-01   .17E+00
? END
 END TEXT EDITING.
$EDIT,TAPE10.
/
```

Sample Output Tape 15
(First 14 Lines of N = 20, M = 20 Case)
Total of 800 Lines for This File
for This Case

```
          VTL        VTH       AMASS     DIST

     ALPH1     DALPH1     ALPH2     DALPH2      ATL       ATH       VO

 REWIND,TAPE15
$REWIND,TAPE15.
/EDIT,TAPE15
 BEGIN TEXT EDITING.
? L:14
   .67E+00   .14E+04   .90E+00   .14E+04   .67E+00   .90E+00   .30E+00
    .32E+03   .32E+03   .2565E+04   .3107E+04
   .67E+00   .14E+04   .90E+00   .14E+04   .67E+00   .90E+00   .25E+00
    .32E+03   .32E+03   .7562E+04   .3107E+04
   .67E+00   .14E+04   .90E+00   .14E+04   .67E+00   .90E+00   .23E+00
    .32E+03   .32E+03   .1256E+05   .3107E+04
   .67E+00   .14E+04   .90E+00   .14E+04   .67E+00   .90E+00   .22E+00
    .32E+03   .32E+03   .1756E+05   .3107E+04
   .67E+00   .14E+04   .90E+00   .14E+04   .67E+00   .90E+00   .21E+00
    .32E+03   .32E+03   .2255E+05   .3107E+04
   .67E+00   .14E+04   .90E+00   .14E+04   .67E+00   .90E+00   .20E+00
    .32E+03   .32E+03   .2755E+05   .3107E+04
   .67E+00   .14E+04   .90E+00   .14E+04   .67E+00   .90E+00   .19E+00
    .32E+03   .32E+03   .3255E+05   .3107E+04
? END
 END TEXT EDITING.
$EDIT,TAPE15.
```

Figure B-1 cont.

K = 1 Option

K Set to 1

```
  LJ*
     1.E04      1.E05      12.E00   3.312E-2     1.E00    8.08E-2
   4.88E02      2.E00       5.E02      5.E01    6.4E04     6.4E04 1

  -END OF FILE-
 ? END
  END TEXT EDITING.
 $EDIT,TAPES.
 /REWIND,TAPES,EXBIN
 $REWIND,TAPES,EXBIN.
 /EXBIN
  INPUT NO. OF MASS INTERVALS DESIRED AND NO.
    OF TRACK DISTANCE INTERVALS DESIRED IN I4,1X,I4 FORMAT.
 ?   20   20
 DMAX=    .3185E+04 XMAX=    .3145E+04
  INPUT INITIAL VELOCITY IN E11.4 FORMAT.
 ?   4.0000E02
  MIN. CRITICAL PIECE MASS=    .6700E+02
  INTEGRATED PROBABILITY =    .3758E+03

 ?   0
      4.416 CP SECONDS EXECUTION TIME
```

L = 1 Option

L Set to 1

```
  LJ*
     1.E04      1.E05      12.E00   3.312E-2     1.E00    8.08E-2
   4.88E02      2.E00       5.E02      5.E01    6.4E04     6.4E04      1

  -END OF FILE-
 ? END
  END TEXT EDITING.
 $EDIT,TAPES.
 /REWIND,TAPES,EXBIN
 $REWIND,TAPES,EXBIN.
 /EXBIN
  INPUT DISTANCE IN F6.1 FORMAT.
 ?  500.0
  INPUT NO. OF MASS INTERVALS DESIPED AND NO.
    OF TRACK DISTANCE INTERVALS DESIRED IN I4,1X,I4 FORMAT.
 ?  20   20
  DMAX=    .3185E+04 XMAX=    .3145E+04
  INTIAL VELOCITY=    .3203E+03
  MIN. CRITICAL PIECE MASS=    .6700E+02
  INTEGRATED PROBABILITY =    .2370E+00
```

No of Distance Intervals
Superfluous Here

Actually, Point
Probability, f_n,
for 500'

Figure B-2 Sample Data Files and Runs with L = 1 or K = 1 Option

```
      PRØGRAM EXPLØD(INPUT,ØUTPUT,TAPE60=INPUT,TAPE61=ØUTPUT,
     1TAPE5,TAPE10,TAPE15,DEBUG=ØUTPUT,TAPE12)
      CØMMØN BRAVØ,AMASST,AMASSA,ZEBRA,SBRAVØ,N,DELAH,DELAV,FLAG
C
C************************INPUT*****************************
C
      READ(5,10)AMASSA,AMASST,TC,AK1,CD,W,RHØ,HD,DC,WEIGHT,DELAH,
     1DELAV,K,L
   10 FØRMAT(6E10.4,/,6E10.4,I2,1X,I2)
      IF(L .NE. 1) GØ TØ 13
      WRITE(61,11)
   11 FØRMAT(1H ,'INPUT DISTANCE IN F6.1 FØRMAT.')
      READ(60,12)DIST
   12 FØRMAT(F6.1)
   13 WRITE(61,15)
   15 FØRMAT(1H ,'INPUT NØ. ØF MASS INTERVALS DESIRED AND NØ.',/,
     1'   ØF TRACK DISTANCE INTERVALS DESIRED IN I4,1X,I4 FØRMAT.')
   16 READ(60,20)N,M
   20 FØRMAT(I4,1X,I4)
      IF(N .EQ. 0) GØ TØ 65
C
C******INPUT MANIPULATIØN
C
      AK=(1.77245/(2.*RHØ*HD))**.6667
      BRAVØ=64.4/(CD*W*AK)
      SBRAVØ=SQRT(BRAVØ)
      ZEBRA=TC*AK/(2.*AK1)
C
C******CALCULATE INITIAL DMAX,XMAX
C
      PØWER=2.96+.347*ALØG10(WEIGHT)-.0161*(ALØG10(WEIGHT))**2
      DMAX=10.**PØWER
      IF(DMAX .LE. DC) GØ TØ 63
      XMAX=SQRT(DMAX*DMAX-DC*DC)
      WRITE(61,30)DMAX,XMAX
   30 FØRMAT(1H ,'DMAX= ',E11.4,' XMAX= ',E11.4)
C
C******CALCULATE INITIAL VELØCITY
C
      IF(K .NE. 1)GØ TØ 34
      WRITE(61,31)
   31 FØRMAT(1H ,'INPUT INITIAL VELØCITY IN E11.4 FØRMAT.')
      READ(60,32)VINIT
   32 FØRMAT(E11.4)
      GØ TØ 38
   34 DBAR=DMAX*32.2/(AMASST**.3333*BRAVØ)
      VINIT=VZERØ(DBAR)*AMASST**.16667*SBRAVØ
      WRITE(61,35)VINIT
   35 FØRMAT(1H ,'INTIAL VELØCITY= ',E11.4)
C
C******CALCULATE MINIMUM MASS,DELTA X, INITIAL X
C
   38 DELX=XMAX/M
      X=XMAX-DELX/2.
      CALL MASLIM(AMASS1)
      WRITE(61,40)AMASS1
```

Figure B-3 Program EXPLOD

```
   40 FØRMAT(1H ,'MIN. CRITICAL PIECE MASS= ',E11.4)
C
C******INTEGRATE PRØBABILITY DØWN TRACK
C
      FTEMP=0.
      IF(L .NE. 1) GØ TØ 48
      CALL MASINT(VINIT,DIST,FSFB,AMASS1)
      GØ TØ 55
   48 CØNTINUE
      DØ 50 J=1,M
         DIST=SQRT(DC*DC+X*X)
         CALL MASINT(VINIT,DIST,FSFB,AMASS1)
         FTEMP=FSFB+FTEMP
         X=X-DELX
   50 CØNTINUE
      FSFB=FTEMP*DELX*2.
C
C******ØUTPUT THE INTEGRATED PRØBABILITY
C
   55 WRITE(61,60)FSFB
   60 FØRMAT(1H ,'INTEGRATED PRØBABILITY = ',E11.4,/)
      GØ TØ 16
   63 WRITE(61,64)
   64 FØRMAT(1H ,'DMAX IS LESS THAN DC.')
   65 STØP
      END
C
C
C
C******FUNCTIØN SUBPRØGRAM VZERØ******************************
C**CALCULATES VO GIVEN DBAR (NØN-DIMENSIØNED V AND D)
C
      FUNCTIØN VZERØ(DM)
      IF(DM .GT. 1.6) GØ TØ 10
      IF(DM .GT. .595) GØ TØ 20
      IF(DM .GT. .09) GØ TØ 30
      VZERØ=SQRT(DM)
      RETURN
   10 ARG=(1./1.054)*(DM-.442)
      VZERØ=EXP(ARG)
      RETURN
   20 ARG=(1./.9148)*(DM-.595)
      VZERØ=EXP(ARG)
      RETURN
   30 VZERØ=(1./.7214)*(DM+.1264)
      RETURN
      END
C
C
C******SUBRØUTINE MASLIM*************************************
C**CALCULATES MIN. CRITICAL MASS BY ITERATIØN ØF PETRY FØRMULA
C
      SUBRØUTINE MASLIM(AMASS1)
      CØMMØN BRAVØ,AMASST,AMASSA,ZEBRA,SBRAVØ,N,DELAH,DELAV,FLAG
      I=0
      D=BRAVØ/215000.
```

Figure B-3 cont.

```
      AM=AMASST/2.
   10 AM1=(ZEBRA/ALØG10(1.+D*AM**.3333))**3
      IF(ABS(AM1-AM) .LE.1.) GØ TØ 20
      AM=AM1
      I=I+1
      IF(I .GT. 100) GØ TØ 15
      GØ TØ 10
   15 WRITE(61,17)
   17 FØRMAT(1HO,'MASLIM DIVERGING.')
      AM=0.
   20 AMASS1=AINT(AM1)
      RETURN
      END
C
C
C******SUBRØUTINE MASINT***************************************
C**INTEGRATES PØINT PRØBABILITY ØVER ALL MASSES AT X
C
      SUBRØUTINE MASINT(VINIT,DIST,FSFB,AMASS1)
      CØMMØN BRAVØ,AMASST,AMASSA,ZEBRA,SBRAVØ,N,DELAH,DELAV,FLAG
C
C******CALCULATE MASS INTERVAL AND STARTING MASS
C
      DELM=(AMASST-AMASS1)/N
      AMASS=AMASS1+DELM/2.
C
C******BEGIN MASS INTEGRATIØN ØF PØINT PRØBABILITY
C
      F1=0. $ F2=0.
      F3=0. $ F4=0.
      FSFB=0.
      DØ 20 I=1,N
         FLAG=1.
C******NØN-DIMENSIØN DISTANCE,INITIAL VELØCITY
         CØN1=32.2/(AMASS**.3333*BRAVØ)
         DBAR=DIST*CØN1
         CØN2=AMASS**.16667*SBRAVØ
         VO=VINIT/CØN2
C******CALCULATE ALPHAO'S,DERIVATIVE TERMS,ALPHAT VT
C**L===LØW TRAJECTØRY   H===HIGH TRAJECTØRY
         CALL AOCALC(VO,DBAR,ALPH1,ALPH2,DALPH1,DALPH2)
         IF(FLAG .EQ. 0.)GØ TØ 30
         VTL=VT(ALPH1,VO)
         IF(VTL .GT. VO) VTL=VO
         ATL=ALPHAT(ALPH1,VO)
         IF(ATL .GT. 0. .ØR. ABS(ATL) .LT. ALPH1)ATL=-ALPH1
         VTH=VT(ALPH2,VO)
         ATH=ALPHAT(ALPH2,VO)
         IF(ABS(ATH) .GE. 90.)ATH=-89.9999
C******REDIMENSIØN AND CØNVERT ANGLES TØ +RADIANS
         ALPH1=ALPH1*.01745 $ ATL=-ATL*.01745
         ALPH2=ALPH2*.01745 $ ATH=-ATH*.01745
         VTL=VTL*CØN2 $ VTH=VTH*CØN2
         DALPH1=ABS((DALPH1/.01745)/CØN1)
         DALPH2=ABS((DALPH2/.01745)/CØN1)
         IF(DALPH1 .LE. .00001 .ØR. DALPH2 .LE. .00001)FLAG=0.
```

Figure B-3 cont.

```
          IF(FLAG .EQ. 0.) GØ TØ 25
C******CALCULATE SPECIFIC PØINT PRØBABILITIES
C**HØRIZØNTAL TARGET,LØW TRAJECTØRY
          FSFB1=FSH(ALPH1,DALPH1,ATL,DIST)*FB(AMASS,VTL*SIN(ATL))
C**VERTICAL TARGET,LØW TRAJECTØRY
          FSFB2=FSV(ALPH1,DALPH1,ATL,DIST)*FB(AMASS,VTL*CØS(ATL))
C**HØRIZØNTAL TARGET,HIGH TRAJECTØRY
          FSFB3=FSH(ALPH2,DALPH2,ATH,DIST)*FB(AMASS,VTH*SIN(ATH))
C**VERTICAL TARGET,HIGH TRAJECTØRY
          FSFB4=FSV(ALPH2,DALPH2,ATH,DIST)*FB(AMASS,VTH*CØS(ATH))
          F1=FSFB1+F1 $ F2=FSFB2+F2
          F3=FSFB3+F3 $ F4=FSFB4+F4
          FSUM=FSFB1+FSFB2+FSFB3+FSFB4
          IF(FSUM .GT. 1.)GØ TØ 25
          GØ TØ 40
   25     WRITE(61,27)
   27     FØRMAT(1H ,'DALPH LØW ØR FSUM > 1.')
          WRITE(61,28)FSUM
   28     FØRMAT(1H ,'FSUM= ',E11.4)
   30     FSUM=0.
C******CALCULATE TØTAL PØINT PRØBABILITY FØR N MISSILES
   40     ANT=AN(AMASS,DELM)
          FSFB=1.-(1.-(FSUM))**ANT+FSFB
C******INCREMENT MASS AND ØUTPUT
          WRITE(15,10)ALPH1,DALPH1,ALPH2,DALPH2,ATL,ATH,VO,VTL,VTH,
     1    AMASS,DIST
   10     FØRMAT(1H ,7E9.2,/,2X,2E9.2,2E11.4)
          AMASS=AMASS+DELM
          WRITE(10,15)FSFB1,FSFB2,FSFB3,FSFB4,FSUM,FSFB,ANT
   15     FØRMAT(1H ,7E9.2)
   20 CØNTINUE
      WRITE(12,12)F1,F2,F3,F4,FSFB,DIST
   12 FØRMAT(1H ,5E9.2,E11.4)
      RETURN
      END
C
C
C*********SUBRØUTINE AØCALC*********************************
C**CALCULATES ALPHAO'S AND DERIVATIVE TERMS
C
      SUBRØUTINE AOCALC(VO,DBAR,ALPH1,ALPH2,DALPH1,DALPH2)
      CØMMØN BRAVØ,AMASST,AMASSA,ZEBRA,SBRAVØ,N,DELAH,DELAV,FLAG
      IF(VO .GT. 3.) GØ TØ 10
      IF(VO .GT. 1.) GØ TØ 20
      IF(VO .GT. .3) GØ TØ 30
      ARG=DBAR/(VO*VO)
      IF(ARG .GT. 1.) GØ TØ 5
      ALPH1=.5*ASIN(ARG)
      ALPH2=1.5708-ALPH1
      DALPH1=2.*VO*VO*CØS(2.*ALPH1)/57.3
      DALPH2=2.*VO*VO*CØS(2.*ALPH2)/57.3
      ALPH1=ALPH1*57.3
      ALPH2=ALPH2*57.3
      RETURN
    5 FLAG=0.
      RETURN
```

Figure B-3 cont.

```
   10 AA=-1.299E-4  $  DD=8.3725E-2
      BB=-1.14118E-3  $  EE=1.1533
      CC=-8.8095E-4  $  FF=-.91271
      HH=AL0G(VO)  $  GG=1.
      IF(DBAR .GE. (1.15*HH-.833)) G0 T0 15
      ALPH1=DBAR/(.46*HH-.333)
      G0 T0 16
   15 ALPH1=ACALC(VO,DBAR,AA,BB,CC,DD,EE,FF,HH,GG)
   16 DALPH1=DCALC(AA,BB,CC,DD,HH,ALPH1)
      BB=-2.45E-4  $  FF=.246
      DD=1.92E-2  $  GG=-1.
      ALPH2=ACALC(VO,DBAR,AA,BB,CC,DD,EE,FF,HH,GG)
      DALPH2=DCALC(AA,BB,CC,DD,HH,ALPH2)
      RETURN
   20 AA=-9.955E-4  $  DD=3.1215E-2
      BB=-4.0593E-4  $  EE=.3854
      CC=4.86E-2  $  FF=-1.6231E-2
      HH=AL0G(VO)  $  GG=1.
      IF(DBAR .GE. (.4846*HH+.0337)) G0 T0 25
      ALPH1=DBAR/(.1938*HH+1.348E-2)
      G0 T0 26
   25 ALPH1=ACALC(VO,DBAR,AA,BB,CC,DD,EE,FF,HH,GG)
   26 DALPH1=DCALC(AA,BB,CC,DD,HH,ALPH1)
      AA=-2.0549E-4  $  DD=1.7997E-2
      BB=-2.30195E-4  $  EE=.89194
      CC=8.5759E-3  $  FF=.23891
      GG=-1.
      ALPH2=ACALC(VO,DBAR,AA,BB,CC,DD,EE,FF,HH,GG)
      DALPH2=DCALC(AA,BB,CC,DD,HH,ALPH2)
      RETURN
   30 AA=-4.7684E-4 $ DD=-8.3554E-3
      BB=1.4519E-4 $ EE=3.2E-2
      CC=3.6434E-2 $ FF=-8.3E-3
      HH=VO  $  GG=1.
      IF(DBAR .GE. (.1205*HH-.02831)) G0 T0 35
      ALPH1=DBAR/(4.82E-2*HH-1.1324E-2)
      G0 T0 36
   35 ALPH1=ACALC(VO,DBAR,AA,BB,CC,DD,EE,FF,HH,GG)
   36 DALPH1=DCALC(AA,BB,CC,DD,HH,ALPH1)
      AA=-2.588E-4  $  DD=-1.5174E-3
      BB=3.3913E-5  $  EE=.41963
      CC=1.859E-2  $  FF=-.14034
      GG=-1.
      ALPH2=ACALC(VO,DBAR,AA,BB,CC,DD,EE,FF,HH,GG)
      DALPH2=DCALC(AA,BB,CC,DD,HH,ALPH2)
      RETURN
      END
C
C
C**********FUNCTI0N SUBPR0GRAM ACALC AND DCALC**********************
C**CALCULATI0NAL FUNCTI0NS F0R SUBR0UTINE AOCALC
C
      FUNCTI0N ACALC(VO,DBAR,AA,BB,CC,DD,EE,FF,HH,GG)
      C0MM0N BRAV0,AMASST,AMASSA,ZEBRA,SBRAV0,N,DELAH,DELAV,FLAG
      GAM=AA*HH+BB
      BETA=CC*HH+DD
```

Figure B-3 cont.

```
      SCI=EE*HH+FF-DBAR
      TEMP=BETA*BETA-4.*GAM*SCI
      IF(TEMP .LT. 0.) GØ TØ 10
      ACALC=(1./(2.*GAM))*(-BETA+GG*SQRT(TEMP))
      IF(ACALC .LE. 0.)ACALC=0.
      RETURN
   10 FLAG=0.
      ACALC=0.
      RETURN
      END
C
C
      FUNCTIØN DCALC(AA,BB,CC,DD,HH,ANGLE)
      GAM=AA*HH+BB
      BETA=CC*HH+DD
      DCALC=2.*GAM*ANGLE+BETA
      RETURN
      END
C
C
C**************FUNCTIØN SUBPRØGRAM VT***********************
C**CALCULATES NØN-DIMENSIØNAL TERMINAL VELØCITY
C
      FUNCTIØN VT(AO,VO)
      IF(AO .GT. 15.) GØ TØ 50
      VOA=(2.257E-2)*AO*AO-4.0771E-1*AO+4.5264
      IF(VO .GT. VOA) GØ TØ 10
      IF(VO .GT. 1.) GØ TØ 20
      IF(VO .GT. .3) GØ TØ 30
      VT=VO
      RETURN
   10 VT=(1.9317E-2*AO-.2658)*ALØG(VO)+1.0189E-2*AO*AO-2.796E-1*
     1AO+2.765
      RETURN
   20 VT=(3.6617E-3*AO*AO-.1117*AO+.9777)*ALØG(VO)+7.943E-4*AO*
     1AO-3.1488E-2*AO+.9869
      RETURN
   30 VT=(5.9442E-4*AO*AO-2.4373E-2*AO+.5687)*ALØG(VO)+7.943E-4*
     1AO*AO-3.1488E-2*AO+.9869
      RETURN
   50 IF(VO .GT. 8.) GØ TØ 60
      IF(VO .GT. 2.) GØ TØ 70
      IF(VO .GT. .3) GØ TØ 80
      VT=VO
      RETURN
   60 IF(AO .GT. 30.) GØ TØ 55
      VT=(-1.991E-5*AO*AO+9.68E-4*AO-7.63E-3)*ALØG(VO)+2.933E-4*
     1AO*AO-8.8667E-3*AO+.888
      RETURN
   65 VT=(1.21119E-6*AO*AO-1.93496E-4*AO+8.1392E-3)*ALØG(VO)-
     15.28803E-5*AO*AO+7.8233E-3*AO+.70218
      RETURN
   70 IF(AO .GT. 40.) GØ TØ 75
      VT=(-2.44E-4*AO*AO+1.8201E-2*AO-.20466)*ALØG(VO)+
     15.7333E-4*AO*AO-3.5393E-2*AO+1.21145
      RETURN
```

Figure B-3 cont.

```
   75 VT=(1.8488E-5*A0*A0-3.4963E-3*A0+.25366)*ALOG(V0)-
     15.6505E-5*A0*A0+1.032E-2*A0+.36807
      RETURN
   80 IF(A0 .GT. 30.) GO TO 85
      VT=(3.02E-4*A0*A0-1.6E-2*A0+.459)*ALOG(V0)+2.4889E-4*
     1A0*A0-1.467E-2*A0+.819
      RETURN
   85 VT=(-1.2838E-5*A0*A0+2.818E-3*A0+.1781)*ALOG(V0)-
     11.0004E-5*A0*A0+2.8218E-3*A0+.52588
      RETURN
      END
C
C
C****************FUNCTION SUBPROGRAM ALPHAT*****************
C**CALCULATES TERMINAL STRIKE ANGLE
C
      FUNCTION ALPHAT(A0,V0)
      IF(V0 .GT. 5.) GO TO 10
      IF(V0 .GT. 1.) GO TO 20
      IF(V0 .GT. .3) GO TO 30
      ALPHAT=-A0
      RETURN
   10 IF(A0 .GT. 15.) GO TO 15
      ALPHAT=(8.8109E-2*A0*A0-2.2648*A0+.33977)*ALOG(V0)-
     1.1744*A0*A0+1.26*A0-.29
      RETURN
   15 ALPHAT=(-2.17385E-3*A0*A0+.403565*A0-19.065)*ALOG(V0)+
     11.52869E-2*A0*A0-2.42847*A0+7.99237
      RETURN
   20 IF(A0 .GT. 20.) GO TO 25
      ALPHAT=(-.9853*A0+.5983)*ALOG(V0)-1.9806E-2*A0*A0-
     1.90346*A0-.23485
      RETURN
   25 ALPHAT=(1.4576E-3*A0*A0+.11095*A0-21.4539)*ALOG(V0)+
     17.0789E-3*A0*A0-1.64938*A0+2.39058
      RETURN
   30 IF(A0 .GT. 30.) GO TO 35
      ALPHAT=(-8.7433E-3*A0*A0-1.87917E-1*A0+.94956)*V0-A0
      RETURN
   35 ALPHAT=(5.7726E-3*A0*A0-.46188*A0-4.4465)*V0-A0
      RETURN
      END
C
C
C*********FUNCTION SUBPROGRAMS FSH AND FSV************************
C**CALCULATES POINT PROBABILITY  FOR HORIZONTAL
C****AND VERTICAL TARGETS
C
      FUNCTION FSH(A0,DA0,AT,DIST)
      COMMON BRAVO,AMASST,AMASSA,ZEBRA,SBRAVO,N,DELAH,DELAV,FLAG
      CON=COS(A0)/(2.*3.1416*DIST)
      FSH=CON*DELAH/DA0
      RETURN
      END
```

Figure B-3 cont.

```
      FUNCTIØN FSV(AO,DAO,AT,DIST)
      CØMMØN BRAVØ,AMASST,AMASSA,ZEBRA,SBRAVØ,N,DELAH,DELAV,FLAG
      IF(AT .EQ. O.) GØ TØ 10
      CØN=CØS(AO)/(2.*3.1416*DIST)
      FSV=CØN*DELAV/(DAO*TAN(AT))
      RETURN
   10 FSV=0.
      RETURN
      END
C
C
C***********FUNCTIØN SUBPRØGRAM FB***************************
C**CALCULATES PRØBABILITY ØF PENETRATIØN GIVEN STRIKE
C
      FUNCTIØN FB(AMASS,VS)
      CØMMØN BRAVØ,AMASST,AMASSA,ZEBRA,SBRAVØ,N,DELAH,DELAV,FLAG
      IF(VS .EQ. O.) GØ TØ 10
      MC=(ZEBRA/ALØG10(1.+VS*VS/215000.))**3
      IF(MC .GE. AMASS) GØ TØ 10
      FB=1.0
      RETURN
   10 FB=0.
      RETURN
      END
C
C
C************FUNCTIØN AN***********************************
C**CALCULATES NØ. ØF MISSILES IN MASS INTERVAL
C
      FUNCTIØN AN(AMASS,DELM)
      CØMMØN BRAVØ,AMASST,AMASSA,ZEBRA,SBRAVØ,N,DELAH,DELAV,FLAG
      A=AMASSA/AMASST
      B=(AMASS/AMASST)**A
      AN=2.*DELM/AMASST*((AMASST/AMASS)*B-1.)
      RETURN
      END
 END ØF INFØRMATIØN ENCØUNTERED.
/
```

Figure B-3 cont.

APPENDIX C

Missile Fragmentation Model[10]

A model for the number distribution of missiles as a function of their mass was developed for use in the present analysis. The model is similar to one used in related studies (e.g., Brunswick PSAR (4)). Its formulation is based on parameters which can be interpreted in a direct manner, e.g., total mass of all missiles, M_T, the most probable mass, M_A, of the mass distribution, and the total number N_T of all missiles.

It is important that the distribution of missiles is understood as a statistical one, i.e., it represents the expected (average) distribution which may be obtained as the limiting case of a large number of explosions of equal characteristics. This, indeed, is desirable when considering the over-all objective of this work.

We shall use the following quantities and definitions:

M	(lb)	mass(independent variable)
M_T	(lb)	total mass of all missiles
M_A	(lb)	most probable mass of mass distribution
$N_{(m)}$	(--)	number of missiles with mass less than M
$N^*_{(m)}$	(--)	number of missiles with mass greater than M
N_T	(--)	total number of all missiles
$P_{N(m)}$	(lb^{-1})	number distribution of missiles per mass interval

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[10] This fragmentation model was developed by Dr. Karl Hornyik, Assistant Professor of Nuclear Engineering, Oregon State University.

$P_{M(M)}$ ....(--).........mass distribution of missiles per mass interval

The following basic relations must hold true:

$$P_{N(M)} \equiv 0, \text{ for } M > M_T \tag{1}$$

$$P_{M(M)} = P_{N(M)} \cdot M \tag{2}$$

$$N_{(M)} = \int_0^M P_{N(M)} \, dM \tag{3}$$

$$N_T = \int_0^\infty P_{N(M)} \, dM = N_{(M_T)} \tag{4}$$

$$N^*_{(M)} = \int_M^\infty P_{N(M)} \, dM = N_T - N_{(M)} \tag{5}$$

$$M_T = \int_0^\infty P_{N(M)} \cdot M \cdot dM = \int_0^\infty P_{M(M)} \cdot dM \tag{6}$$

The following additional properties of the number distribution function are postulated, strictly on the basis of judgement:

1) The distribution function shall be monotonically decreasing with increasing mass, i.e., the expected number of missiles decreases as their mass increases. This assumption appears reasonable especially for the range of intermediate and large masses, which are the only ranges of concern here, since small missiles will not penetrate the protective walls.

2) The distribution function shall be continuous and shall tend to zero as M approaches M_T, i.e., the probability that the entire mass accelerated by the explosion will be retained as a single missile is infinitesimally small.

The analytical formulation which is chosen for the number distribution function is that of an inverse power law, in accordance with earlier work (e.g., Brunswick PSAR (4)) and modified to accommodate the second one of the postulated properties; thus:

$$P_N = A \cdot \left[\left(\frac{M_T}{M}\right)^n - 1\right] \qquad \text{for } 0 < M < M_T \qquad (7)$$

$$\equiv 0 \qquad \text{for } M_T < M < 0$$

We note that this relation is meaningful only when $0 < n < 1$

The constant A is determined from the normalization condition, eqn.(6):

$$A = \frac{2}{M_T} \cdot \left(\frac{2}{n} - 1\right) \qquad (8)$$

The total number of missiles is determined from eqn. (4) as:

$$N_T = 2\,\frac{2 - n}{1 - n} \qquad (9)$$

and solving for n one obtains:

$$n = \frac{N_T - 4}{N_T - 2} \qquad (10)$$

The constant A also can be expressed in terms of N_T as: (eqn. (8) & (10))

$$A = \frac{2}{M_T} \cdot \frac{N_T}{N_T - 4} \qquad (11)$$

which leads to the formulation of the distribution function P_N:

$$P_N = \frac{2}{M_T} \cdot \frac{N_T}{N_T - 4} \cdot \left[\left(\frac{M_T}{M}\right)^{\frac{N_T - 4}{N_T - 2}} - 1\right] \qquad (12)$$

For a large number of missiles, i.e., $N_T >> 1$, this distribution approaches the asymptotic form:

$$P_N \to \frac{2}{M_T} \left[\left(\frac{M_T}{M}\right) - 1\right] \qquad (13)$$

Using this asymptotic form, one finds for the number of missiles greater than mass M from eqn. (5) and (13)

$$N^*{}_{(M)} = 2 \cdot \left[\frac{M}{M_T} - \ln \frac{M}{M_T} - 1\right] \qquad (14)$$

A graph of this function is shown in Fig. C-1 and shows that one can expect on the average one missile with a mass greater than 30% of the total mass of all missiles.

Finally, it is of interest to determine the peak of the differential mass distribution, which corresponds to the most probable mass, M_A. From eqn. (2) and (12) we find:

$$P_{M(M)} = \frac{2N_T}{N_T - 4} \cdot \left[\left(\frac{M}{M_T}\right)^{\frac{2}{N_T - 2}} - \left(\frac{M}{M_T}\right)\right] \qquad (15)$$

and with the condition:

$$\left.\frac{dP_N}{dM}\right|_{M = M_A} = 0$$

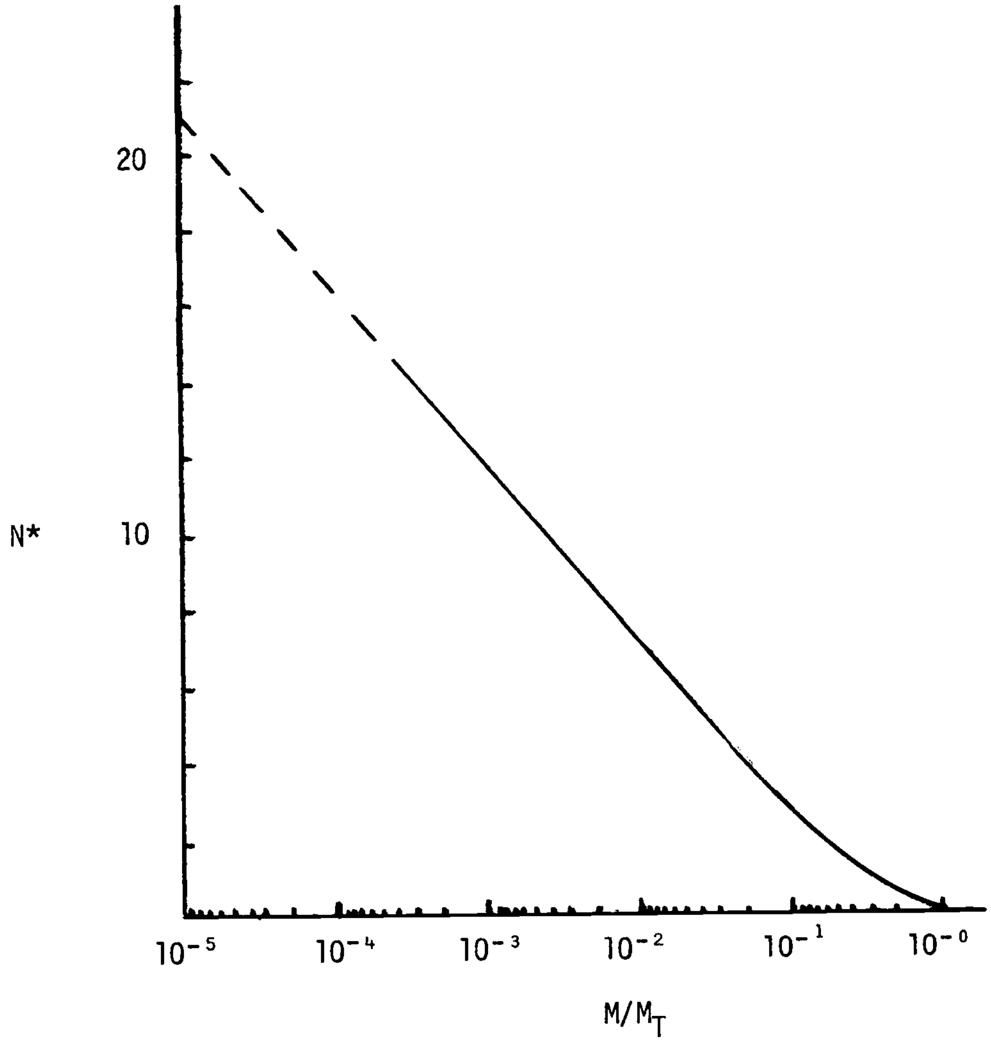

Fig. C-1 The Number of Missiles Greater than M Versus M/M_T

we get: $$M_A = M_T \cdot \left(\frac{2}{N_T - 2}\right)^{\frac{N_T - 2}{N_T - 4}} \qquad (16)$$

which, for large values of N_T, approaches:

$$M_A \rightarrow M_T \cdot \frac{2}{N_T} \qquad (17)$$

Introducing eqn. (17) into eqn. (12) and neglecting the fraction in front of the brackets we obtain an alternate formulation of P_N in terms of the ratio M_A/M_T:

$$P_N = \frac{2}{M_T} \left[\left(\frac{M_T}{M}\right)^{1 - \frac{M_A}{M_T}} - 1\right] \qquad (18)$$

It is noted that a variety of other formulations for P_N could be used. However, as long as one adheres to the fundamental relations, eqn. (1) - (6), and the two postulated properties of the distribution function, one will find that the distribution functions do not vary a great deal from each other and will have the general characteristics of the function used here, as plotted in Fig. C-1. Therefore, it is not expected that the details of the choice of this distribution function will have great influence on the estimated hazards potential under conditions as adopted above.