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In this thesis, we are concerned with the synthesis of a specified voltage ratio transfer function having a specified complex termination. A method for the active network synthesis with specified complex termination using resistive and capacitive elements and only one negative impedance converter is presented. The synthesis technique involves rearranging a transfer function and terminal function in which the denominator has restricted negative real roots, followed by the application of the Yanagisawa method. The first part of this paper considers the properties of the negative impedance converter and the general circuit realization theory which will be used in this synthesis method.

Synthesis of Networks with Complex Terminations Using Active Networks

by

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SYNTHESIS OF NETWORKS WITH COMPLEX TERMINATIONS USING ACTIVE NETWORKS

I. INTRODUCTION

Network synthesis using only resistors and capacitors is attractive in electronic circuits due to its compactness and relatively inexpensive production costs. However, the inherent loss associated with resistors and the restricted nature of the critical frequencies of the network function which can be realized by passive RC elements often makes this method of electronic circuitry inapplicable or impractical.

The use of an active device such as a negative impedance converter in combination with passive RC elements can overcome these defects. A negative impedance converter (NIC) is an active two-port device whose driving point impedance at one port is the negative of the load impedance connected at the other port. These simple characteristics tend to simplify the design problem of many active networks.

Various practical NIC's have been devised (6, 7, 8, 12) and also many synthesis techniques of RC-NIC circuits have been developed (1, 6, 12) during the last several years.

This paper describes an RC-Active network synthesis with complex termination. The active network synthesis with complex termination is obtained through rearranging the transfer function with the terminal function and the application of the Yanagisawa's configuration.

This thesis is an extension of the work done by Junior Akio Nagaki (9) to include active networks. By extending his idea for synthesis of networks with complex termination synthesis, it is possible to reduce the restrictions and increase the usefulness of the network.

II. NEGATIVE IMPEDANCE CONVERTER

Definition

A Negative Impedance Converter (NIC) is defined as a two-port device characterized by the property that the input impedance seen at either port is the negative (sometimes constant) of the impedance connected to the other port (namely, $Z_{in} = -Z_2$).

To establish these characteristics, consider a two-port network described in terms of its g parameters.

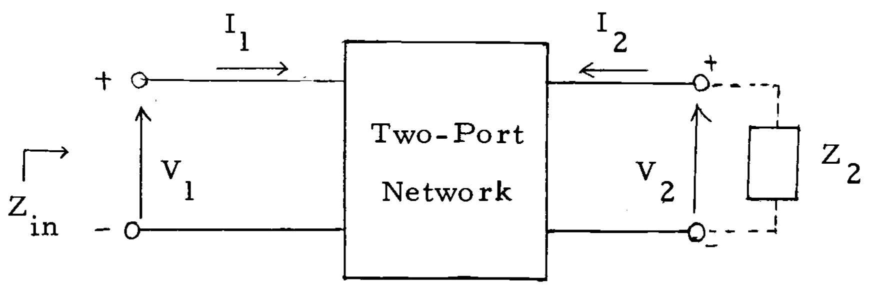


Figure 1. Typical two-port network

The pertinent equations are,

g parameters

$$\begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2 \\ V_2 &= g_{21}V_1 + g_{22}I_2 \end{aligned} \tag{1}$$

Terminating the two-port network in an impedance Z_2 adds the additional relationship $V_2 = Z_2(-I_2)$ to the above. Then we can solve the equations for the input impedance as

$$Z_{in} = \frac{V_1}{I_1} = \frac{1}{g_{11} - \frac{g_{12}g_{21}}{Z_2 + g_{22}}} \qquad (2)$$

Thus we see that to have an ideal NIC the two-port device must be characterized by

$$g_{11} = g_{22} = 0, \qquad g_{12} \cdot g_{21} = 1 \qquad (3)$$

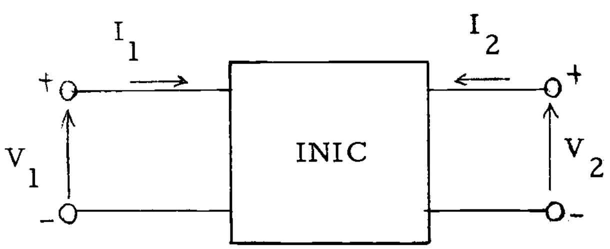
So

$$Z_{in} = -Z_2.$$

Any active two-port device satisfying the above condition is an ideal NIC. From this it follows that we can have two types of NIC's.

<u>One Type of NIC (INIC)</u>

In this case, the voltages at port 1 and at port 2 are identical while the current flowing into port 1 and the current flowing out of port 2 are equal and opposite. The parameter matrices are



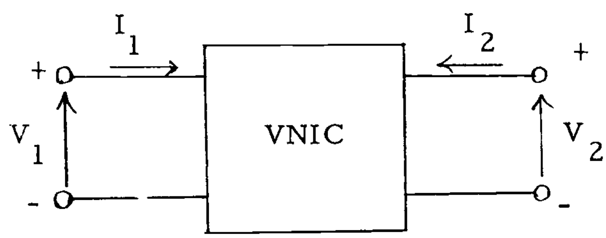
$V_1 = V_2 \qquad I_1 = I_2$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (4)$$

Another Type of NIC (VNIC)

The current in the two ports is the same current flowing through the device while the voltages across the two ports are equal and opposite. The parameter matrices are



$V_1 = -V_2 \quad I_1 = -I_2$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad (5)$$

Properties of NIC

Property I

The negative immittance conversion operation is bilateral.

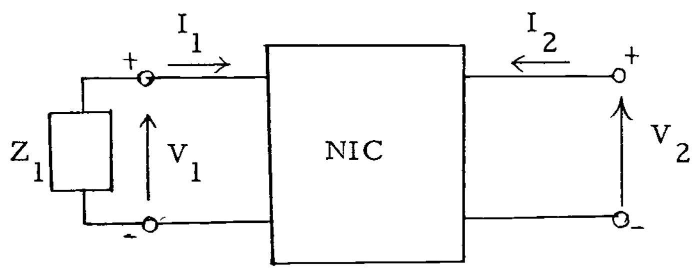


Figure 2. Two-port network terminated in Z_1 at port 1

Consider a two-port network which is terminated in an impedance Z_1 at port 1 as shown in Figure 2. This establishes the relationship $V_1 = Z_1(-I_1)$ between the variables V_1 and I_1. If we substitute this relationship in equation (3) and solve for the input impedance as seen looking into the terminals of port 2, we obtain

$$\frac{V_2}{I_2} = g_{22} - \frac{g_{12}g_{21}}{g_{11} + \frac{1}{Z_1}} \qquad (6)$$

Equation (6) also requires that the input immittance seen looking into port 2 is the negative of the immittance connected to port 1.

Property II

If an NIC is nonideal in the sense that either g_{11} or g_{22} is not equal to zero it may be compensated by an external impedance in such a manner that the resulting device is an ideal NIC.

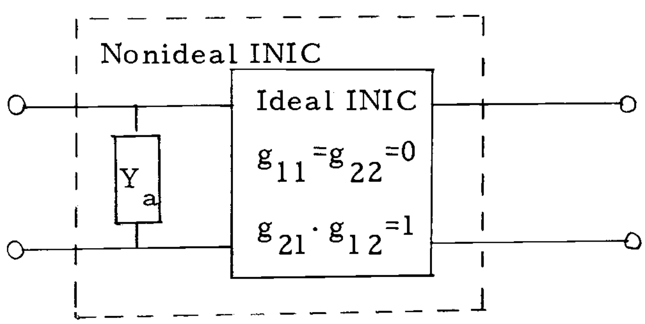


Figure 3. A nonideal INIC with $g_{11} \neq 0$.

Consider the case in which g_{11} (the open circuit admittance at port 1) is not equal to zero. Specifically, let $g_{11} = Y_a$. This situation is shown in Figure 3 where an ideal NIC which satisfies the conditions of (3) has an admittance Y_a connected in shunt with port 1. Then the resulting g parameters for the nonideal NIC (including Y_a) are

$$\begin{bmatrix} Y_a & g_{12} \\ g_{21} & 0 \end{bmatrix}$$

The nonidealness of this NIC may easily be compensated for by an external admittance $Y_a' = Y_a$ as shown in Figure 4. We may consider this as part of the load impedance Z_2. Equation (2) then becomes

$$\frac{V_1}{I_1} = \frac{1}{Y_a - \cfrac{1}{\cfrac{1}{Y_a' + Y_2} + 0}} = \frac{1}{Y_a - (Y_a' + Y_2)} \qquad (7)$$

$$= -\frac{1}{Y_2} \qquad (\text{if } Y_a = Y_a')$$

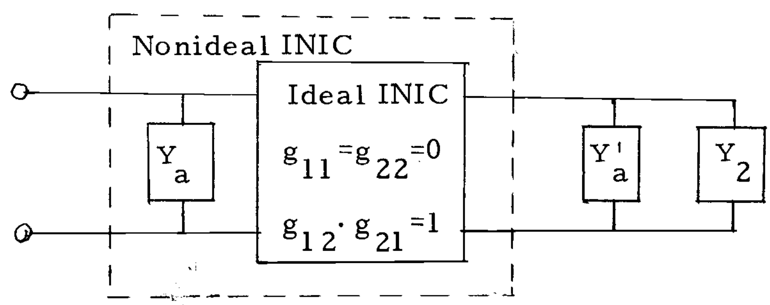


Figure 4. Compensating the INIC when $g_{11} \neq 0$.

In a similar manner we compensate an NIC having a nonzero g_{22}.

The process is illustrated in Figure 5. In this case the short-circuit impedance at port 2 is $Z_a = g_{22}$. The Z_1 of Figure 2 is now the series sum of the load impedance Z_1' and the compensating impedance Z'_a .

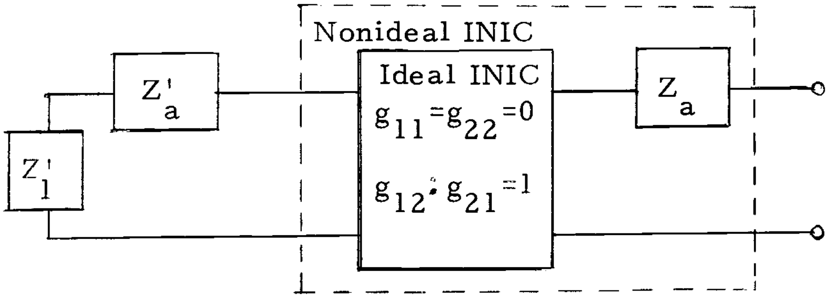


Figure 5. Compensating the NIC when $g_{22} \neq 0$.

$$\frac{V_2}{I_2} = Z_a - \frac{1}{\frac{1}{Z_1 + Z'_a} + 0} = Z_a - (Z_1 + Z'_a) \tag{8}$$

$$= -Z_1 \qquad (\text{if } Z_a = Z'_a)$$

Property III

A NIC represents an ideal transformer of turns ratio $\frac{1}{k}$ in cascade with an NIC except for isolation. This property of the NIC may be seen if we establish the conditions on a two-port network in which $g_{11} = g_{22} = 0$ and $g_{12} \cdot g_{21} = k^2$, where k is an arbitrary positive number. Under these conditions the input impedance of this device when terminated in an impedance Z_2 at port 2 may be found from Equation (2) as

$$\frac{V_1}{I_1} = -\frac{Z_2}{k^2} \tag{9}$$

Similarly, the input impedance at port 2 when the device is terminated in Z_1 at port 1 may be found from Equation (6) as

$$\frac{V_2}{I_2} = -k^2 Z_1 \quad . \tag{10}$$

Property IV

The determinant of the chain matrix (AD- BC) is equal to one. Therefore, when two identical and symmetric two-port networks are connected in tandem with a NIC, one at each side of the NIC, the composite system remains the same kind of NIC, since, for example

$$\begin{bmatrix} A & B \\ C & A \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A & B \\ C & A \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{11}$$

The General Theory of NIC Realizations

We shall now investigate the effects of combining these ideal two-port devices with various passive networks. When a two-port network is cascade connected to a NIC at either side there are four possible combinations, depending on whether the INIC or VNIC appears at the input or the output terminals of the resulting two-port. The combinations are shown in Figure 6 and the resulting two-port networks have been labeled as networks a, b, c and d as indicated. The effect the NIC's have in determining the properties of the overall two-port network can be seen by examining Table 1. This table lists the y, z, g, and h parameters of the various network combinations shown in Figure 6 (5, p. 118-124; 11, p. 36-46).

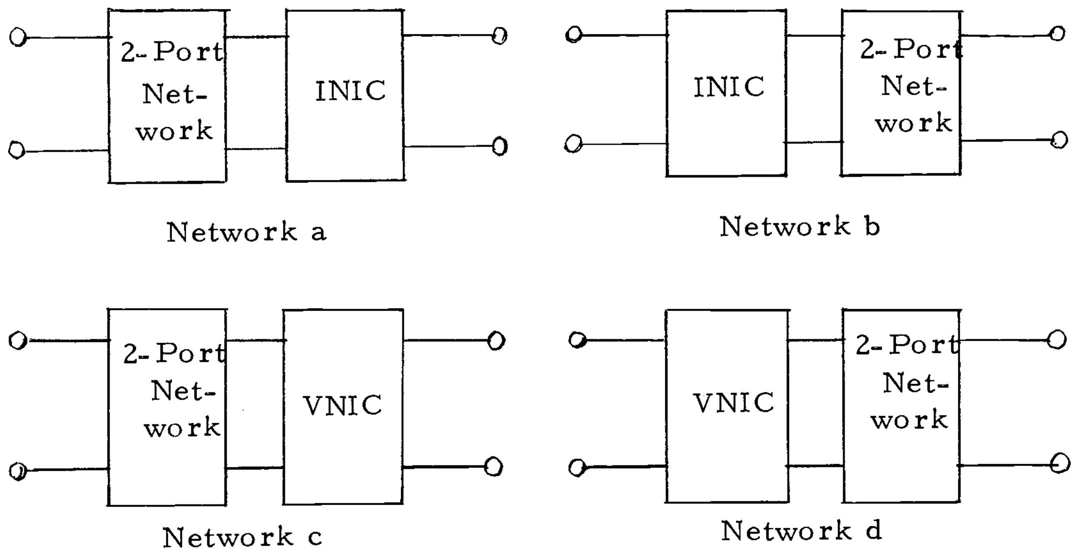


Figure 6. Various combinations of networks cascaded with NIC's.

Table 1. Parameters for the networks of Figure 6.

n. w.	z parameters	y parameters	g parameters	h parameters
a	$\begin{matrix} z_{11} & -z_{12} \\ z_{12} & -z_{22} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ -y_{21} & -y_{22} \end{matrix}$	$\begin{matrix} g_{11} & -g_{12} \\ g_{21} & -g_{22} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ -h_{21} & -h_{22} \end{matrix}$
b	$\begin{matrix} -z_{11} & z_{12} \\ -z_{21} & z_{22} \end{matrix}$	$\begin{matrix} -y_{11} & -y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} -g_{11} & -g_{12} \\ g_{21} & g_{22} \end{matrix}$	$\begin{matrix} -h_{11} & h_{12} \\ -h_{21} & h_{22} \end{matrix}$
c	$\begin{matrix} z_{11} & z_{12} \\ -z_{21} & -z_{22} \end{matrix}$	$\begin{matrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{matrix}$	$\begin{matrix} g_{11} & g_{12} \\ -g_{21} & -g_{22} \end{matrix}$	$\begin{matrix} h_{11} & -h_{12} \\ h_{21} & -h_{22} \end{matrix}$
d	$\begin{matrix} -z_{11} & -z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} -y_{11} & y_{12} \\ -y_{21} & y_{22} \end{matrix}$	$\begin{matrix} -g_{11} & g_{22} \\ -g_{21} & g_{22} \end{matrix}$	$\begin{matrix} -h_{11} & -h_{12} \\ h_{21} & h_{22} \end{matrix}$

We can consider the realization of an open-circuit voltage transfer function which may be expressed in terms of the y parameters of a given two-port network as

$$\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22}} \tag{12}$$

The numerator and denominator are each decomposed into a sum of two polynomials. This decomposition is such that the resulting polynomials are realizable as two RC networks. One of these polynomials is negative algebraic sign. The network realized from this polynomial is cascaded with a negative impedance converter. The resulting network is connected parallel to the network realized from the other polynomial

$$\frac{V_2}{V_1} = \frac{-y_{21} + y'_{21}}{y_{22} - y'_{22}} \tag{13}$$

where the y_{ij} parameters are those of the paralled network.

We require a network in which the y'_{21} and y'_{22} parameters have had their sign changed. From Table 1 the network of Figure 7(a) provides such a change.

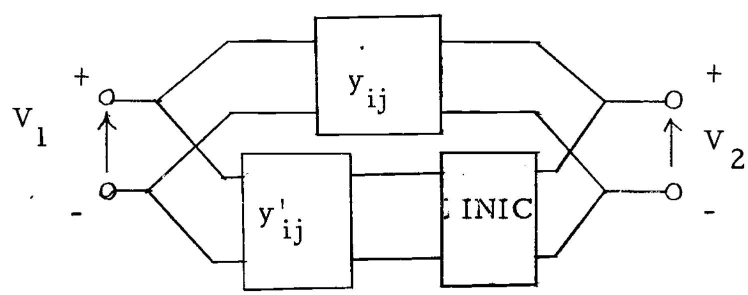
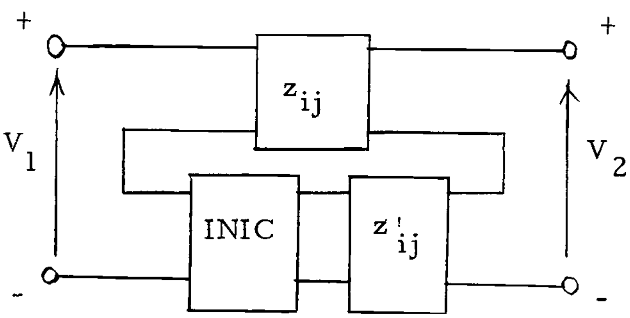


Figure 7(a). A realization for $\frac{V_2}{V_1}$ using network 6(a) with parallel connection.

In similar manner we may provide a different decomposition in terms of z parameters, the voltage transfer function is

$$\frac{V_2}{V_1} = \frac{z_{21}}{z_{11}} \tag{14}$$

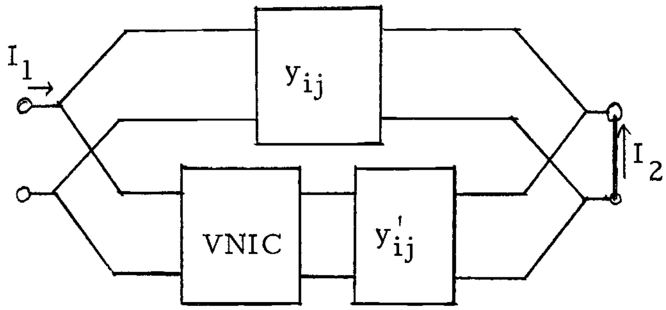
For the z parameters to combine additively the component networks must be connected in series. We find that network 6(b) has the parameters z_{21} and z_{11} and their signs are reversed. This network may be connected in series with an unaltered network as shown in Figure 7(b).



$$\frac{V_2}{V_1} = \frac{z_{21} - kz'_{21}}{z_{11} - kz'_{11}} \tag{15}$$

Figure 7(b). A realization for $\frac{V_2}{V_1}$ using network 6(b).

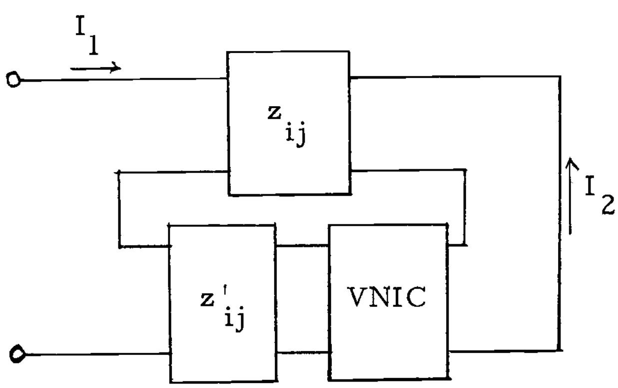
Similar procedures may be used for the realization of short-circuit current transfer functions. In terms of y parameters, the network of Figure 6(d) may be used to obtain the network of Figure 8(a).



$$\frac{I_2}{I_1} = \frac{y_{21} - ky'_{21}}{y_{11} - ky'_{11}} \qquad (16)$$

Figure 8(a) A realization for $\frac{I_2}{I_1}$ using network 6(d)

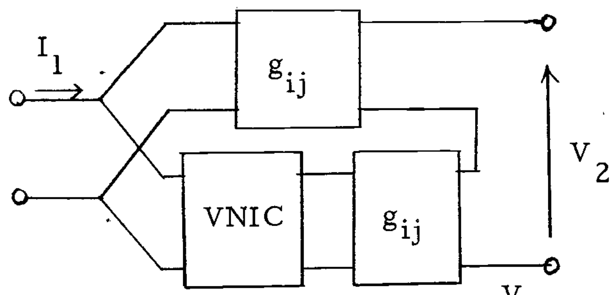
A realization using networks connected in series is possible with network of Figure 6(c). The overall network configuration is shown in Figure 8(b). For this figure we obtain



$$\frac{I_2}{I_1} = \frac{-z_{21} + kz'_{21}}{z_{22} - kz'_{22}} \qquad (17)$$

Figure 8(b). A realization for $\frac{I_2}{I_1}$, using network 6(c)

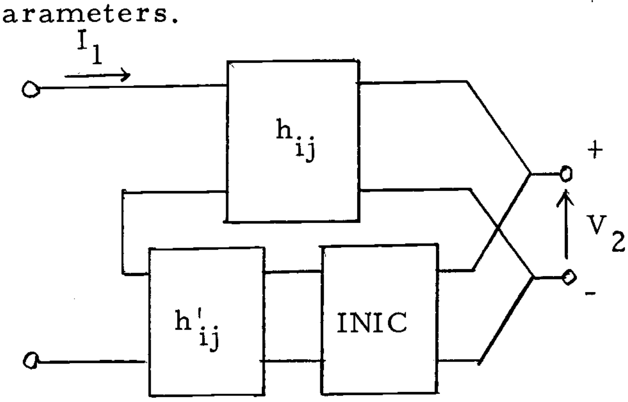
In a similar manner the properties of the g parameters and the associated circuit can be applied. The open-circuit transfer impedance can be realized by using network 6(d) with g parameters.



$$\frac{V_2}{I_1} = \frac{g_{21} - kg'_{21}}{g_{11} - kg'_{11}} \tag{18}$$

Figure 9(a). A realization for $\frac{V_2}{I_1}$, using network 6(d)

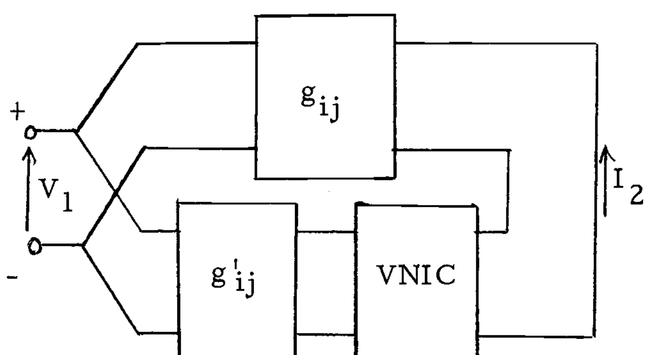
Similarly the properties of the parameter and the associated circuit can be used to realize the network of Figure 6(a) with h parameters.



$$\frac{V_2}{I_1} = \frac{-h_{21} + kh'_{21}}{h_{22} - kh'_{22}} \tag{19}$$

Figure 9(b). A realization for $\frac{V_2}{I_1}$, using network 6(a)

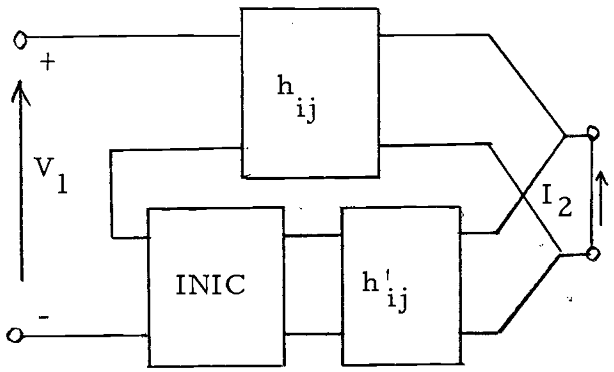
Realization of the short-circuit transfer admittance can be obtained using network 6(c) with the g parameters



$$\frac{I_2}{V_1} = \frac{-g_{21} + kg'_{21}}{g_{22} - kg'_{22}} \tag{20}$$

Figure 10(a). A realization for $\frac{I_2}{V_1}$, using network 6(c)

The realization of the short-circuit transfer admittance can be obtained using network 6(b) with the h parameters



$$\frac{I_2}{V_1} = \frac{h_{21} - kh'_{21}}{h_{11} - kh'_{11}} \qquad (21)$$

Figure 10(b). A realization for $\frac{I_2}{V_1}$, using network 6(b)

<u>Yanagisawa's Synthesis Method</u>

Yanagisawa's synthesis method (12) is a typical RC - NIC circuit. This circuit can be used in realizing any open-circuit voltage ratio, that is expressed as a rational function of s.

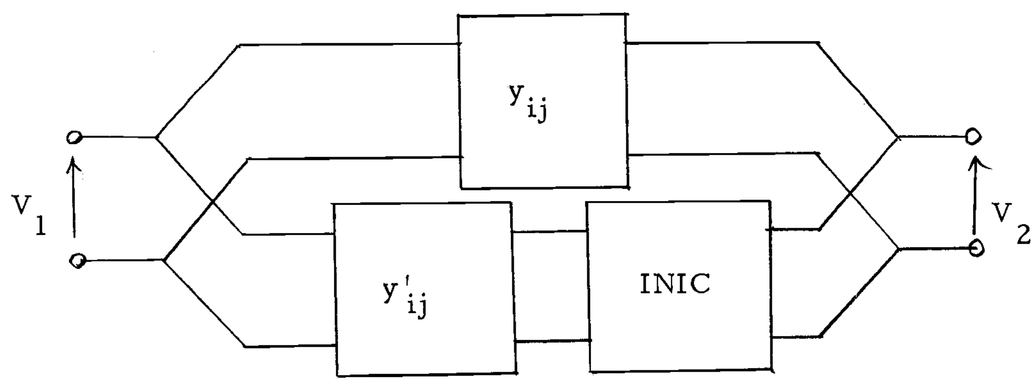


Figure 11. RC - Active Network

The INIC type with a conversion factor of k has a chain matrix equal to

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -k \end{bmatrix} . \tag{22}$$

Analysis will show that

$$T_1 = \frac{V_2}{V_1} = -\frac{y_{21} - ky'_{21}}{y_{22} - ky'_{22}} . \tag{23}$$

Let the given voltage ratio function be denoted by

$$T_1 = \frac{V_2}{V_1} = \frac{P(s)}{Q(s)} . \tag{24}$$

Choose an arbitrary polynomial $q_{(s)}$ having the following constraints:

1. The roots are negative-real and simple.
2. The degree is one less than the degree of $P(s)$ or $Q(s)$, whichever is higher degree.

Then

$$-(y_{21} - ky'_{21}) = \frac{P(s)}{q(s)} \tag{25a}$$

$$y_{22} - ky'_{22} = \frac{Q(s)}{q(s)} \tag{25b}$$

Practically, we can use an inverted L for each two-port as shown in Figure 12.

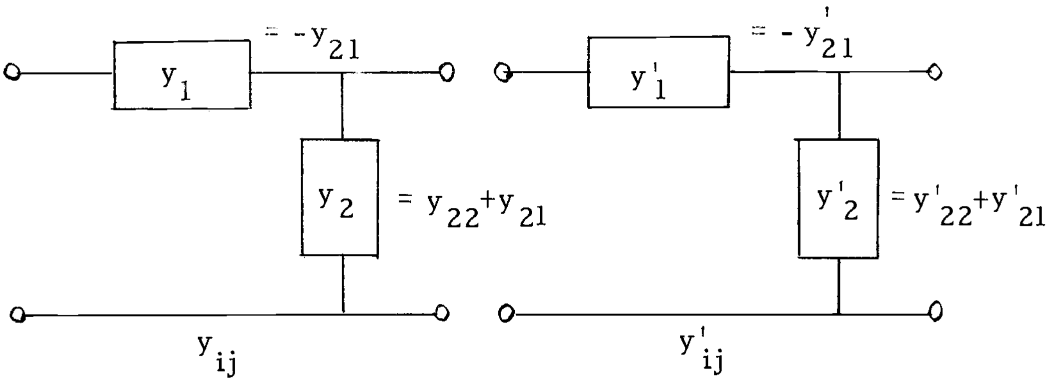


Figure 12. Inverted L configuration

The voltage transfer function becomes

$$\frac{V_2}{V_1} = \frac{y_1 - ky_1'}{y_1 + y_2 - k(y_1' + y_2')} \tag{26}$$

By comparing Equation (24) and Equation (27) the following identifications can be made

$$\frac{P(s)}{q(s)} = y_1 - ky_1' \tag{27a}$$

and

$$\frac{Q(s) - P(s)}{q(s)} = y_2 - ky_2' \tag{27b}$$

The partial fraction expansions of these functions are

$$\frac{P(s)}{q(s)} = k_{21}^{(\infty)} s + k_{21}^{(0)} + \sum_{\nu} \frac{k_{21}^{(\nu)} s}{s + \sigma_{\nu}} \tag{28a}$$

and

$$\frac{Q(s) - P(s)}{q(s)} = k_{22}^{(\infty)} s + k_{22}^{(0)} + \sum_{\nu} \frac{k_{22}^{(\nu)} s}{s + \sigma_{\nu}} \tag{28b}$$

By assigning the appropriate terms to different functions, all four admittances, y_1, y'_1, y_2 and y'_2 can be realized by RC elements.

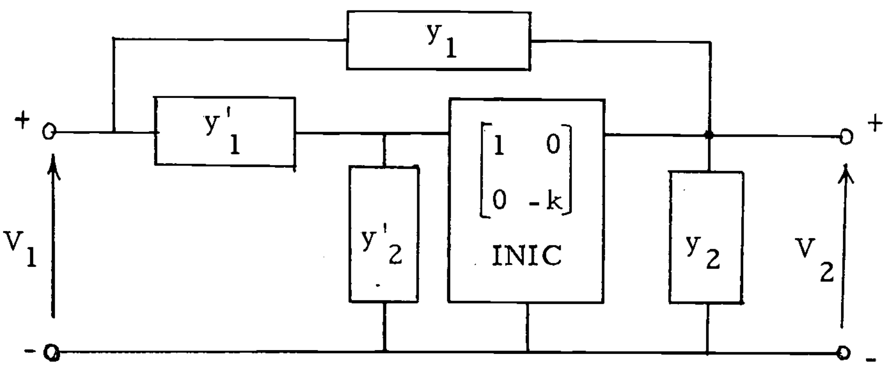


Figure 13. Yanagisawa's RC-active network

III. NETWORK SYNTHESIS WITH COMPLEX TERMINATION

Synthesis Procedure Using NIC

If a problem is given as a voltage ratio transfer function with a terminating immittance function the transfer function will be a ratio of polynomials in s, as follows

$$T(s) = \frac{P(s)}{Q(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0} \qquad (29)$$

The terminating immittance function is a two-terminal function (either admittance or impedance) and has the following characteristics:

1. It is a rational function of s.
2. The functions must be positive-real.
3. Complex poles or zeros must occur in conjugate pairs.
4. The degree of the highest-order-terms of N(s) and D(s) differ at most by unity.

This is represented as a ratio of polynomials in s

$$Y_L = \frac{N(s)}{D(s)} = \frac{\Pi(s+a_1)\,\Pi(s+a_2)(s+\bar{a}_2)}{\Pi(s+b_1)\,\Pi(s+b_2)(s+\bar{b}_2)} \qquad (30)$$

The first step is to simultaneously manipulate the transfer function and terminal immittance function to derive the desired form. (This can be done in two ways.)

Method I.

$$T(s) = \frac{P(s)}{Q(s)} \qquad\qquad Y_L(s) = \frac{N(s)}{D(s)} \tag{31}$$

$$= \frac{\frac{P(s)}{D(s)}}{\frac{Q(s)}{D(s)}}$$

$$= \frac{\frac{P}{D}}{\frac{Q-XN}{D} + \frac{XN}{D}}$$

$$= \frac{\frac{P}{XD}}{\frac{Q-XN}{XD} + \frac{N}{D}} \tag{32}$$

The following identifications can be made.

$$T(s) = \frac{\frac{P}{XD}}{\frac{Q-XN}{XD} + \frac{N}{D}} \rightleftarrows \frac{-y_{21}}{y_{22} + Y_L} \tag{33}$$

Apply Yanagisawa's circuit to this relationship as shown below.

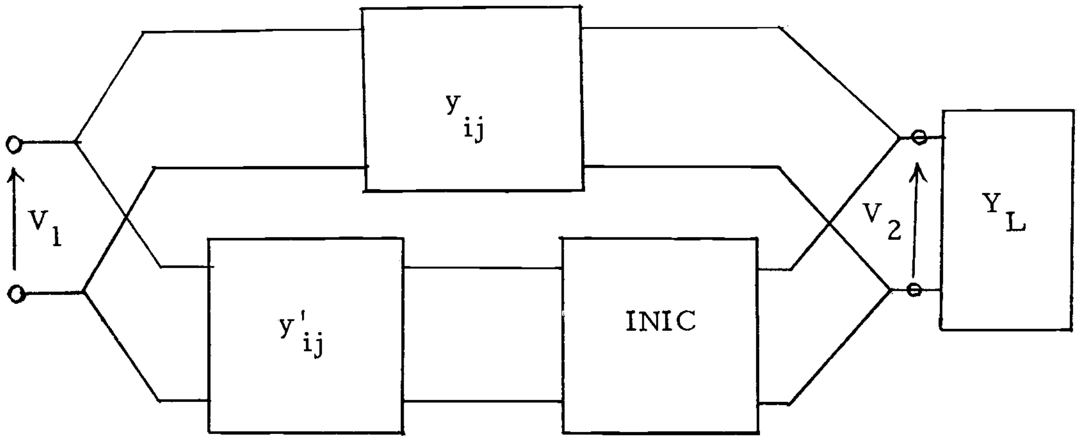


Figure 14. Realization for $\frac{V_2}{V_1}$ terminated in Y_L

From this circuit (Figure 14), any open-circuit voltage ratio which is expressed as a rational function of s can be realized. The INIC, with a conversion factor of k, has a chain matrix equal to

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -k \end{bmatrix} \tag{34}$$

From Equation (33) with the corresponds, respectively.

$$\begin{aligned} -y_{21} &= \frac{P}{XD} \rightleftarrows -(y_{21} - ky'_{21}) \\ y_{22} &= \frac{Q-XN}{XD} \rightleftarrows y_{22} - ky'_{22} \end{aligned} \tag{35}$$

The arbitrary polynomial $X(s)$ of Equations (32) and (33) has the following constraints:

1. The roots must be negative and real.

2. The degree of the $[D(s) \cdot X(s)]$ can differ from the degree of the $P(s)$ or $[Q(s) - X(s) \cdot N(s)]$ (whichever is higher) by not more than one.

In actual practice, we can use an inverted L network, for each two ports as shown below.

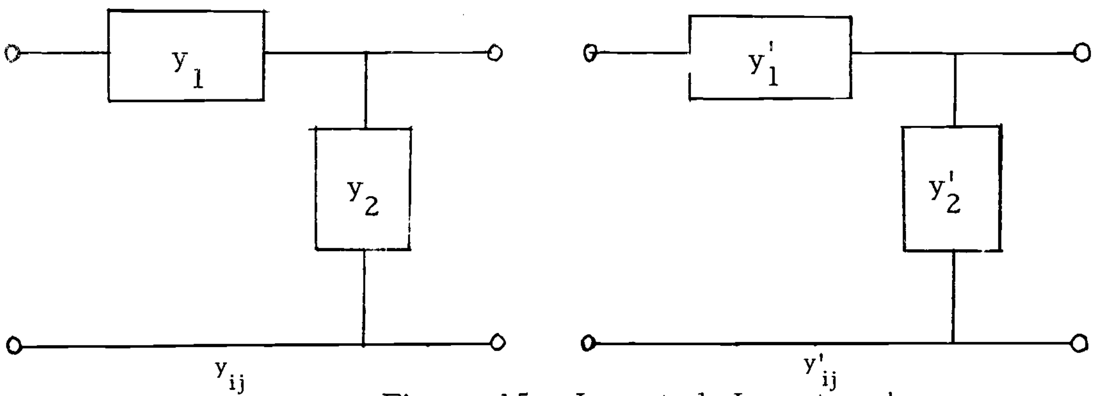


Figure 15. Inverted L networks

Then

$$\frac{\frac{P}{XD}}{\frac{Q-XN}{XD}} = -\frac{y_{21} - ky'_{21}}{y_{22} - ky'_{22}} = \frac{y_1 - ky'_1}{(y_1 - ky'_1) + (y_2 - ky'_2)} \tag{36}$$

The following identifications can be made.

$$\frac{P}{XD} = y_1 - ky'_1$$

$$\frac{(Q-XN) - P}{XD} = y_2 - ky'_2 \tag{37}$$

By obtaining the partial fraction expansions (Foster expansion) of both $\frac{P}{XD}$ and $\frac{(Q-XN)-P}{XD}$, and assigning the appropriate terms to different functions, all four admittances, y_1, y'_1, y_2, and y'_2, can be synthesized by the RC network.

$$\frac{P}{XD} = k_{21}^{(\infty)} s + k_{21}^{(0)} + \sum_{\nu} \frac{k_{21}^{(\nu)} s}{s+\sigma_\nu}$$

$$\frac{(Q-XN)-P}{XD} = k_{22}^{(\infty)} s + k_{22}^{(0)} + \sum_{\nu} \frac{k_{22}^{(\nu)} s}{s+\sigma_\nu} \tag{38}$$

Finally, the network is realized for the voltage transfer function with terminal immittance function.

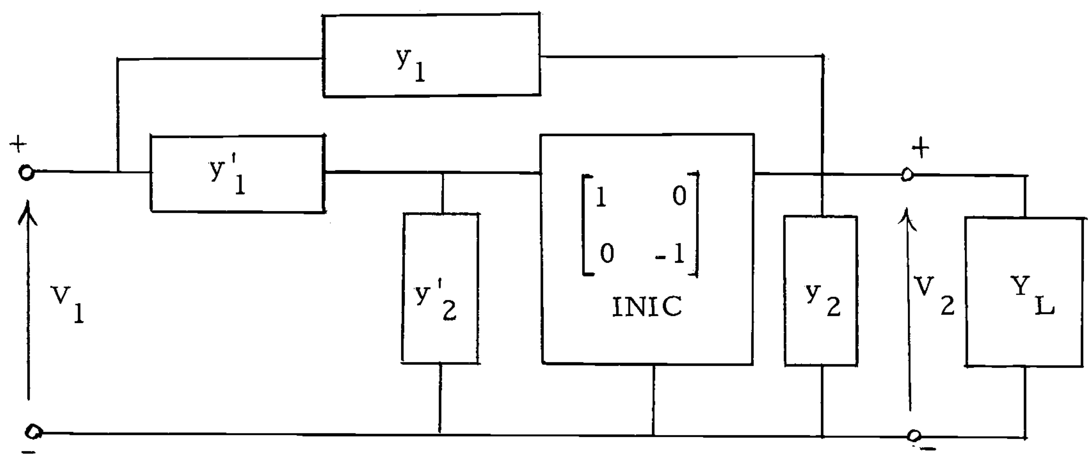


Figure 16. The use of L configuration for RC-NIC synthesis with terminal Y_L

Method 2.

$$T(s) = \frac{P(s)}{Q(s)} \qquad\qquad Y_L(s) = \frac{N(s)}{D(s)}$$

$$= \frac{\frac{P}{X} \cdot \frac{N}{D}}{\frac{Q}{X} \cdot \frac{N}{D}}$$

$$= \frac{\frac{P}{X} \cdot \frac{N}{D}}{\frac{QN - XN}{XD} + \frac{XN}{XD}}$$

$$= \frac{\frac{P}{X} \cdot \frac{N}{D}}{\frac{(Q-X)N}{XD} + \frac{N}{D}} \qquad (39)$$

We can identify terms in the same way as in method 1.

$$T(s) = \frac{\frac{P(s)\,N(s)}{X(s)\,D(s)}}{\frac{(Q-X)N}{XD} + \frac{N}{D}} \rightleftarrows \frac{-y_{21}}{y_{22} + Y_L} \qquad (40)$$

Then

$$\frac{PN}{XD} \rightleftarrows -(y_{21} - ky'_{21})$$

$$\frac{Q-XN}{XD} \rightleftarrows y_{22} - ky'_{22} \qquad (41)$$

Choosing $X(s)$ to have the same constraints as before:

1. The roots must be negative and real.
2. The degree of $[D(s) \cdot X(s)]$ can differ at most by one from the highest degree of PN or $(Q-X)N$.

Use inverted L network to obtain

$$\frac{\frac{PN}{XD}}{\frac{(Q-X)N}{XD}} = \frac{-(y_{21} - ky'_{21})}{y_{22} - ky'_{22}} = \frac{y_1 - ky'_1}{(y_1 - ky'_1) + (y_2 - ky'_2)} \tag{42}$$

Make the following identifications.

$$\frac{PN}{XD} = y_1 - ky'_1$$

$$\frac{(Q-X-P)N}{XD} = y_2 - ky'_2 \tag{43}$$

The first step in this synthesis of a network is to expand the given functions by partial fraction expansions.

$$\frac{PN}{XD} = k_{21}^{(\infty)} s + k_{21}^{(0)} + \sum_{\nu} \frac{k_{21}^{(\nu)} s}{s+\sigma_{\nu}} \tag{44a}$$

$$\frac{(Q-X-P)N}{XD} = k_{22}^{(\infty)} s + k_{22}^{(0)} + \sum_{\nu} \frac{k_{22}^{(\nu)} s}{s+\sigma_{\nu}} \tag{44b}$$

The positive terms of Equation (44a) can be identified as y_1 of Figure 16 and the negative terms can be identified as y_1' of Figure 16. In the same manner the positive and negative terms of Equation (44b) can be identified with y_2 and y'_2 respectively.

Constraints on the Polynomial X(s)

1. X(s) may be constant or restricted to roots that are negative real and simple.
2. X(s) can not contain the roots of D(s).

3. Degree of $X(s)$ is one less than the degree of either $Q(s) - D(s)$ or $P(s) - D(s)$, whichever is higher.

<u>Consideration of Terminal Function</u>

1. $\frac{N(s)}{D(s)}$ will be a driving-point immittance function.

2. In this synthesis method the roots of $D(s)$ are constrained to be negative-real and simple. The load admittance function, in general form, can be expressed as

$$Y_L = \frac{N(s)}{D(s)} = \frac{\Pi(s+\sigma_i)\ \Pi(s+\sigma_j)(s+\bar{\sigma}_j)}{\Pi(s+\sigma_\nu)} \qquad (45)$$

The partial fraction expansion of Equation (45) is

$$Y_L = k^{(0)} + k^{(\infty)}s + \frac{k^{(1)}}{s} + \Sigma\frac{k^{(m)}}{s+\sigma_{\nu_1}} + \Sigma\frac{k^{(n)}s}{s+\sigma_{\nu_2}} \qquad (46)$$

The terms of Equation (46) can be identified as follows: $k^{(0)}$ is resistance, $k^{(\infty)}s$ is capacitance, $\frac{k^{(1)}}{s}$ is inductance, $\frac{k^{(m)}}{s+\sigma_{\nu_1}}$ is series combination of capacitance and resistance and $\frac{k^{(n)}s}{s+\sigma_{\nu_2}}$ is the series combination of inductance and resistance. Realization of Y_L will result in the following general network.

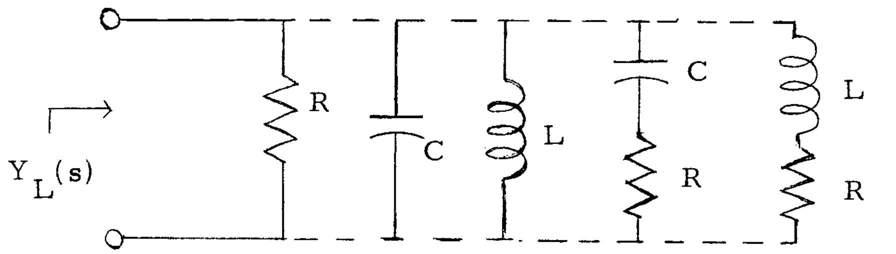


Figure 17. General form of terminating admittance

Restrictions for Realizability

Restrictions of Given Function

1. Roots of P(s), Q(s), and N(s), may have any s plane location. Complex roots must appear in conjugate pairs.
2. Roots of D(s) must be negative-real and simple.

Cases of realizability, even though D(s) has complex roots:

In Method 1

P(s) and Q(s) - X(s)N(s) can simultaneously have the complex roots of D(s).

In Method 2

P(s) and Q(s) - X(s) can simultaneously have the complex roots of D(s).

IV. EXAMPLES OF THE SYNTHESIS METHOD

Example 1. Synthesize a network with a third-order Butterworth low-pass filter with complex termination. The voltage gain would be

$$T(s) = \frac{1}{(s+1)\cdot(s^2+s+1)} \tag{47}$$

Let the complex termination be the RLC network shown.

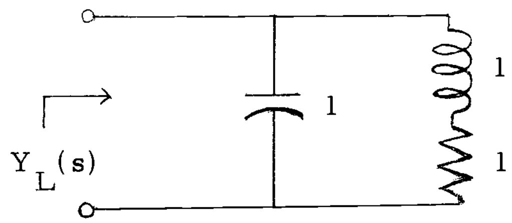


Figure 18. Specified terminating network of Example 1.

The admittance $Y_L(s)$ of this network is

$$Y_L(s) = \frac{s^2 + s + 1}{s + 1} \tag{48}$$

Using the steps of Method 1

$$T(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{\dfrac{1}{s+1}}{\dfrac{(s+1)(s^2+s+1)}{s+1}}$$

$$T(s) = \frac{\dfrac{1}{s+1}}{\dfrac{s^3+2s^2+2s+1-X(s)(s^2+s+1)}{s+1} + \dfrac{(s^2+s+1)\,X(s)}{s+1}}$$

$$= \frac{\dfrac{1}{X(s)} \quad \dfrac{1}{s+1}}{\dfrac{s^3+2s^2+2s+1-X(s)\cdot(s^2+s+1)}{X(s)\cdot(s+1)} + \dfrac{s^2+s+1}{s+1}}$$

Choose $X(s) = s+2$ according to the condition of $X(s)$, (p. 26).

Then

$$T(s) = \frac{\dfrac{1}{(s+2)(s+1)}}{\dfrac{(s^3+2s^2+2s+1)-(s+2)(s^2+s+1)}{(s+2)\,(s+1)} + \dfrac{s^2+s+1}{s+1}}$$

After simple calculation the equation can be written.

$$T(s) = \frac{\dfrac{1}{(s+1)(s+2)}}{\dfrac{(s^2+s+1)}{(s+1)(s+2)} + \dfrac{s^2+s+1}{s+1}} \rightarrow \frac{-y_{21}}{y_{22}+Y_L}$$

From this equation we can identify

$$\frac{P}{XD} = \frac{1}{(s+1)(s+2)} = \frac{1}{2} - \frac{\frac{1}{2}s^2+\frac{3}{2}s}{(s+1)(s+2)}$$

$$= \frac{1}{2} + \frac{\frac{1}{2}s}{s+2} - \frac{s}{s+1} \rightarrow y_1 - y'_1 \qquad (49)$$

and

$$\frac{Q - XN}{XD} = \frac{-(s^2 + s + 1)}{(s + 1)(s + 2)}$$

$$\frac{Q - XN - P}{XD} = \frac{-(s^2 + s + 2)}{(s + 1)(s + 2)}$$

$$= \frac{2s}{s + 1} - 1 - \frac{2s}{s + 2} \rightarrow y_2 - y'_2 \qquad (50)$$

Synthesis of Equation (49) and (50) gives the network configuration of Figure 19.

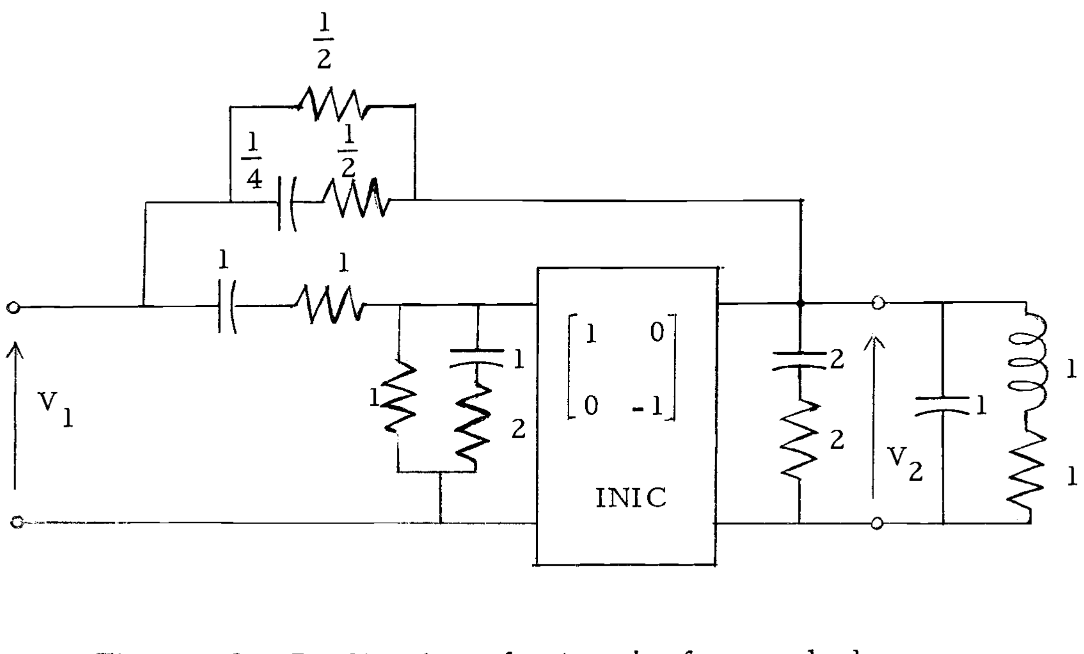


Figure 19. Realization of network of example 1. (Low Pass Filter)

Example 2. Synthesize a network with a third-order Butterworth high-pass filter with complex termination. The voltage gain would be

$$T_H(s) = \frac{s^3}{(s+1)(s^2+s+1)} \tag{51}$$

Let the complex termination be the RLC network shown

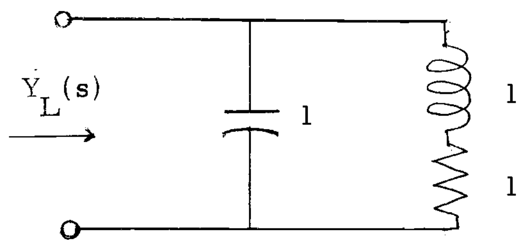


Figure 20. Specified terminating network of Example 2.

The admittance $Y_L(s)$ of this network is

$$Y_L(s) = \frac{s^2+s+1}{s+1} \tag{52}$$

Using the steps of Method 1

$$T_H(s) = \frac{s^3}{(s+1)(s^2+s+1)}$$

$$= \frac{\dfrac{s^3}{s+1}}{\dfrac{(s+1)(s^2+s+1)}{s+1}}$$

$$T_H(s) = \frac{\dfrac{s^3}{s+1}}{\dfrac{s^3+2s^2+2s+1-X(s)(s^2+s+1)}{X(s)\cdot(s+1)} + \dfrac{s^2+s+1}{s+1}}$$

$$= \frac{\left(\dfrac{1}{X(s)}\right)\cdot\left(\dfrac{s^3}{s+1}\right)}{\dfrac{s^3+2s^2+2s+1-X(s)(s^2+s+1)}{X(s)\cdot(s+1)} + \dfrac{s^2+s+1}{s+1}}$$

Choose $X(s) = s+2$ according to the condition of $X(s)$

Then

$$T_H(s) = \frac{\dfrac{s^3}{(s+1)(s+2)}}{\dfrac{(s^3+2s^2+2s+1)-(s+2)(s^2+s+1)}{(s+1)(s+2)} + \dfrac{s^2+s+1}{s+1}}$$

After simple calculation the equation can be written

$$T_H(s) = \frac{\dfrac{s^3}{(s+1)(s+2)}}{\dfrac{-(s^2+s+1)}{(s+1)(s+2)} + \dfrac{s^2+s+1}{s+1}} \rightarrow \frac{-y_{21}}{y_{22}+Y_L}$$

From this equation we can identify

$$\frac{P}{XD} = \frac{s^3}{(s+1)(s+2)} = s + \frac{s}{s+1} - \frac{4s}{s+2} \rightarrow y_1 - y'_1 \qquad (53)$$

And

$$\frac{Q - XN}{XD} = \frac{-(s^2 + s + 1)}{(s + 1)(s + 2)}$$

$$\frac{Q - XN - P}{XD} = \frac{-(s^3 + s^2 + s + 1)}{(s + 1)(s + 2)}$$

$$= \frac{\frac{5}{2} s}{s + 2} - s - \frac{1}{2} \rightarrow y_2 - y'_2 \tag{54}$$

Synthesis of Equation (53) and (54) gives the network configuration of Figure 21.

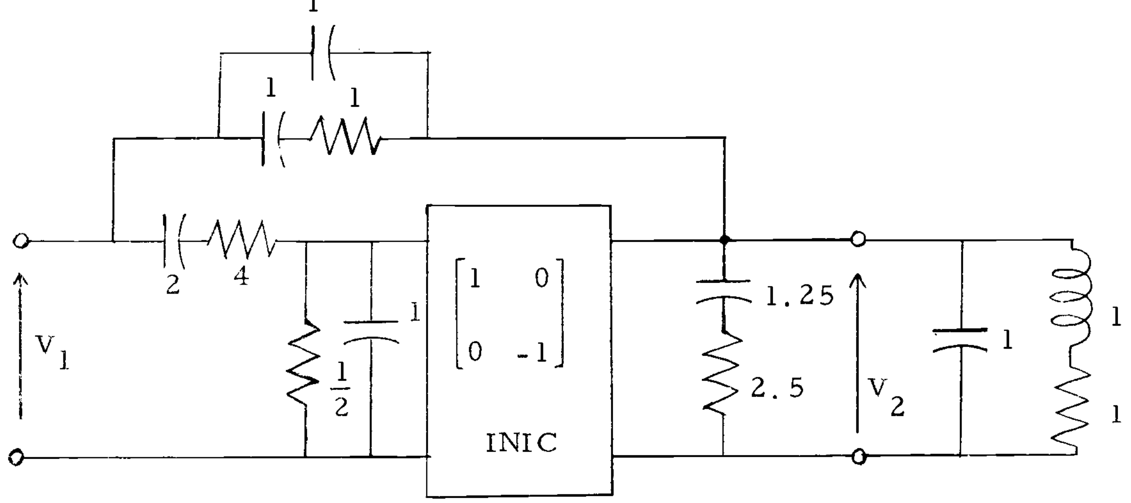


Figure 21. Realization of network of example 2. (High Pass Filter)

V. FUTURE CONSIDERATION

Further investigation of this problem would be to design a more perfect network synthesis procedure having no restrictions, not even a terminal immittance function. Another possibility would be to use a different active device such as a gyrator or operational amplifier with more practical consideration such as sensitivity and stability problem can be suggested.

VI. SUMMARY AND CONCLUSIONS

This thesis treats an aspect of an Active Network Synthesis. Investigation of the Active network was undertaken because of its great potential application in modern network synthesis.

The first part of this thesis reviews the theory of negative impedance converter and its properties as an active network element and general synthesis procedures employing negative impedance converters.

From this general synthesis procedure we proceed to the objective of this thesis, which is the development of a synthesis method for networks terminated in a specified complex impedance. A procedure has been developed for the synthesis of networks having the general form of Figure 16.

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