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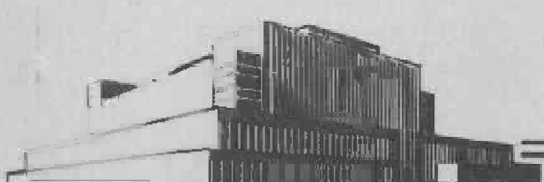

FOREST PRODUCTS LABORATORY
MADISON 5, WISCONSIN

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE

In Cooperation with the University of Wisconsin

EFFECT OF CIRCUMFERENTIAL STIFFENERS ON THE BUCKLING PROPERTIES OF THIN, CURVED PLYWOOD PANELS IN AXIAL COMPRESSION[1]

By

T. B. HEEBINK, Engineer
and
C. B. NORRIS, Engineer

Forest Products Laboratory,[2] Forest Service
U.S. Department of Agriculture

Summary

This report presents the results of tests performed at the Forest Products Laboratory on curved plywood panels stiffened with a single circumferential stiffener and loaded in axial compression. A relationship between panel stiffness and the effectiveness of the stiffeners makes it possible to select stiffeners of minimum size and maximum effectiveness. This relation was obtained by an empirical analysis of data from tests of 105 stiffened panels.

Introduction

The critical buckling stress of a curved plywood panel subjected to axial compression can be increased by gluing a single circumferential stiffener to the center of the panel. If the stiffener is sufficiently stiff, the panel may be considered as two panels, each having a length-to-width ratio half

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[1] The research here reported was begun during World War II in cooperation with the Army-Navy-Civil Committee on Aircraft Design Criteria and was completed by the Forest Products Laboratory because of its general significance to plywood design theory and practice. This report was originally issued in February 1950.

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

that of the entire panel. The critical stress of the stiffened panel is, therefore, equal to the larger critical stress of the half-panel.

It has been shown[3,4] that the critical stress of a curved plywood panel subjected to axial compression is approximately equal to the critical stress of a plywood cylinder of the same construction, axial dimension, and radius, unless the ratio of panel thickness to radius of curvature is very small. The critical stress of such a cylinder, and, therefore, that of a curved panel, can be computed by methods previously described.[5,6] The purpose of studies described in this report was to determine the minimum stiffness of a stiffener that will cause the curved panel to be broken up into two curved panels.

Thirteen groups of seven to nine similarly curved plywood panels, with stiffeners of various sizes attached, were tested in axial compression. Stiffeners were chosen so as to cover the range from no stiffness at all to stiffnesses much greater than that required to divide the panel into two panels. A study of the observed critical stresses and the size and type of buckle that occurred led to a criterion for the critical stiffness of stiffener.

Because previous work indicated that inconsistent results were obtained when critical stresses were greater than or near the proportional limit stress, panels in this study were so designed that all the critical stresses were much less than the proportional limit stresses. Consequently, the increases in critical stresses due to stiffening were rather small (less than 65 percent). For this reason, and because buckling was sometimes progressive making it difficult to determine the critical stresses accurately, the results are erratic. An analysis by statistical methods, however, yields a solution that is both clear-cut and conclusive.

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[3]Kuenzi, E. W. Buckling of Thin, Curved, Plywood Plates in Axial Compression. Forest Products Laboratory Report No. 1508. Reviewed and reaffirmed 1960.

[4]Werren, F., and Norris, C. B.. Effect of Axial Stiffeners on the Buckling Properties of Thin, Curved Plywood Plates in Axial Compression. Forest Products Laboratory Report No. 1567. Reviewed and reaffirmed 1961.

[5]March, H. W., Norris, C. B., and Kuenzi, E. W. Buckling of Long, Thin, Plywood Cylinders in Axial Compression. Forest Products Laboratory Report No. 1322, with Supplements 1322-A and 1322-B. Reviewed and reaffirmed 1956.

[6]Kuenzi, E. W. Effect of Length on the Buckling Stresses of Thin-walled, Plywood Cylinders in Axial Compression. Forest Products Laboratory Report No. 1514. Reviewed and reaffirmed 1959.

Preparation of Test Panels

Yellow birch veneer of aircraft grade was bag-molded into seven-, five-, or four-ply plywood of single curvature. The direction of the grain of each ply was perpendicular to that of adjacent plies except in the four-ply construction, in which the grain directions of the two inner plies were parallel to each other. The radii of curvature were 24 and 30 inches. The axial length varied from 9-3/4 to 21-3/8 inches, the width from 23 to 29 inches, and the thickness from 0.059 to 0.152 inch. The grain direction of the face plies of each plate was either axial or circumferential. Test specimens used to determine the elastic properties of the panels were cut from the four edges of each panel.

The stiffeners were constructed from 4-inch wide strips of 1/28-inch, quarter-sliced Sitka spruce veneer. The number of plies was varied so as to produce plywood thicknesses of 0.077 to 0.520 inch. The laminated strip was assembled in a special jig which molded it to the desired curvature. The 4-inch, uncurved section at each end was removed and cut up into test specimens for the determination of the modulus of elasticity. The curved section was then cut into stiffeners of desired width.

The curved plywood panels were trimmed to size and then nail-strip glued to end-support blocks (fig. 1). Then the stiffener was glued to the panel. The nailing strips were left on the panel during test. Just before testing the panel, each curved edge was run over a jointer to insure an even bearing in the testing machine. Side support rails having slots 1/4 inch deep and just wide enough to receive the thickness of plywood were slipped on the panel and held in place by the friction fit. These rails were approximately 1/4 inch shorter than the panel at both ends so that they did not bear against either loading head of the testing machine.

Method of Test

The panels were tested in axial compression in the 25,000-pound mechanical testing machine shown in figure 2. The specimen was placed on the center of a 1-inch thick, machined steel plate. The plate rested on a slab whose spherical base was supported by a seat on the lower head of the testing machine. The upper head was lowered slowly, and the spherical base was adjusted to obtain an even bearing on both loaded edges of the specimen. A small initial load was applied, and the plate was locked in position by means of a machine screw located at each corner (fig. 2).

The panel curvature along the stiffener was traced on a sheet of paper, and then the load was applied at a slow rate of speed until the panel buckled. If buckling was rapid, the movement of the testing machine head was stopped, and a second trace was made along the stiffener in its buckled condition. If buckling was slow or progressive, it was difficult to detect the buckling load exactly, and in these cases the load at the first noticeable indentation was recorded as the critical load. A typical set of traces is shown in figure 3. The curve D-1A was obtained from a buckled panel having no stiffener. The next trace, D-2B, was obtained from a panel having a light stiffener. The remaining curves show the buckling patterns of panels having progressively heavier stiffeners.

Explanation of Table 1

All pertinent data obtained from tests are presented in table 1. The first nine columns describe the stiffened plates. Column 10 lists the observed critical buckling stresses, (p_{cr}). These are the critical loads divided by the cross-sectional areas of the plates. The values in column 11 (p_{crp}) are the theoretical critical stresses for the plates, computed, as though no stiffeners were attached, by the formula[6]

$$p_{crp} = kE \, L\frac{h}{r} \qquad (1)$$

The coefficient k was obtained by the use of figures 4 and 5.

The ratios in column 12 are parameters involving the stiffness of the stiffener $(EI)_s$ as computed by methods in Forest Products Laboratory Report No. 1557,[7] the theoretical critical stress (p_{crp}), the radius of curvature (r), and the thickness (h) of the curved plywood plates. These ratios are the abscissas in figures 6 and 7. The ratios of the increase in critical stress due to a stiffener of less than the critical stiffness to the increase due to a very heavy stiffener are presented in column 13. The numerator is obtained by subtracting the observed critical stress of the plate without a stiffener (p_{cr0}) from that of the plate with a stiffener of a particular depth (p_{cr}). The denominator is obtained by subtracting the observed critical stress of the plate without a stiffener (p_{cr0}) from that of the plate with a very heavy stiffener (p_{crm}). These ratios are plotted as ordinates in figure 6.

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[7]Smith, C. B., Heebink, T. B., and Norris, C. B. The Effective Stiffness of a Stiffener Attached to a Flat Plywood Plate. Forest Products Laboratory Report No. 1557. Reviewed and reaffirmed 1956.

The values for the effective modulus of elasticity of the plywood in bending measured (1) parallel and (2) perpendicular to the unloaded edges of the plates (axial direction) are presented in columns 14 and 15, respectively. The bending specimens were cut from the edges of the panel before testing in axial compression. These specimens were 1 inch wide and were tested on a span length 48 or more times the thickness of the specimen when the face grain direction was parallel to the span, and 24 or more times the thickness when the face grain direction was perpendicular to the span. Thus, it was not necessary to correct for shear deformation. The specimens rested on end supports rounded to a radius of approximately 1/16 inch. The load was applied at the center with a bar approximately 1/8 inch in radius.

The values for the effective modulus of elasticity of the plywood in compression measured (1) parallel and (2) perpendicular to the loaded edges of the plate are listed in columns 16 and 17, respectively. The compression specimens were 1 inch wide and 4 inches long, with the face grain directions parallel and perpendicular to the direction of the load. Because the specimen was relatively thin, an apparatus consisting of thin spring fingers was used to give lateral support. The strain was measured over a 2-inch gage length with a Marten's mirror apparatus. Load-deformation curves were plotted as each test was run.

Column 18 contains values of E_L which can be computed from the relations

$$E_1 + E_2 = E_a + E_b = E_L + E_T = E_L (1 + C)$$

E_L was assumed to be $\dfrac{E_1 + E_2 + E_a + E_b}{2.10}$ for yellow birch.

The values for the modulus of elasticity of the stiffeners in compression are shown in column 19. These were obtained from tests of 1- by 4-inch specimens cut from the uncurved ends of the stiffeners.

Analysis of Experimental Results

In a study of stiffened, flat plywood plates[8] a successful analysis of the experimental results was accomplished by evaluating the ratio of the observed

[8] Heebink, T. B., March, H. W., and Norris, C. B. Buckling of Stiffened, Flat, Plywood Plates in Compression--A Single Stiffener Perpendicular to Stress. Forest Products Laboratory Report No. 1553. Reviewed and reaffirmed 1956.

increase in the critical stress due to a stiffener of less than the critical stiffness to the observed increase due to a heavy stiffener, and then plotting this ratio against the similar ratio derived from theory. In the case of flat plates, this method resulted in a plot from which it was possible to derive an equation for the critical size stiffener. However, there is no theory available for computing the critical stress of a stiffened, curved plywood plate or the stiffness of a curved stiffener. As an approach to the problem, the ratios of the observed increase in the critical stress due to a stiffener to the observed increase due to a heavy stiffener were plotted as ordinates against an empirical, nondimensional parameter (fig. 6). The parameter chosen was:

$$\frac{(EI)_s}{p_{crp}\, r^2\, h^2} \qquad (2)$$

which can also be written:

$$\frac{(EI)_s}{kE_L\, rh^3} \qquad (3)$$

Three requirements were considered in making the choice: (1) the parameter must be nondimensional, (2) the parameter must, essentially, be the ratio of the stiffness of the stiffener to that of the curved plywood, (3) the radius of curvature must enter the parameter in such a way that, when the data are plotted, the points will not separate into two groups, one group representing panels of one radius of curvature and the other group, panels of the other radius. It is evident that the first two requirements are satisfied by the form of the parameter chosen. That the third requirement is satisfied can be seen in figure 6, in which experimentally determined values are plotted against this parameter. The data for both radii of curvature intermingle in the plot.

It was expected that in figure 6 the points would group themselves about a line from the origin to some point on a horizontal straight line through the ordinate 1.0 and thence along that horizontal line.[8] An examination of figure 6 shows that, for values of abscissas up to about 0.5, the points are so widely scattered that it is impossible to represent them with reasonable accuracy by a single curve. The principal reason for this scatter is that the behavior of a plywood cylinder (and, therefore, a curved plate) buckling under axial compression is analagous to that of a column with a nonlinear elastic support. The buckling load of such a column is greatly affected by irregularities in shape or in material. This sensitivity is greater for a cylinder or curved plate than for a flat plate because there is an additional dimension in which these irregularities can manifest their effects.

In order to reduce the data statistically and use it to arrive at the critical size stiffener, figure 7 was prepared. The ordinates are the root mean squares of the deviations from unity, of the ratios of the observed p_{cr} to the maximum observed p_{crm} for all points having values of abscissas equal to or greater than the abscissa represented.

If the stiffener has a stiffness greater than the critical value, it will not buckle when the panel buckles. Thus the root mean square of $\left(\frac{p_{cr}}{p_{crm}}\right) - 1$ should be independent of the stiffness of the stiffener. However, if this stiffness is less than the critical value, the critical value of the stress becomes less than that for a panel adequately stiffened because the stiffener buckles with the panel. Thus the root mean square of $\left(\frac{p_{cr}}{p_{crm}}\right) - 1$ increases as the stiffness of the stiffener is reduced, and the value at which this increase begins is the critical stiffness of the stiffener. Figure 7 shows that this increase occurs at a value of the abscissa of approximately 0.4.

The critical stiffness of a circumferential stiffener, then, is given by

$$(EI)_{scr} = 0.4\, p_{crp}\, r^2 h^2 \qquad (4)$$

However, this equation is empirical and should not be used for ratios of plywood thickness to radius much outside the range of this ratio for the panels tested; that is, outside the range from 0.0019 to 0.005.

Conclusions

The critical stress of a curved plywood plate in axial compression can be increased appreciably by attaching a small, circumferential stiffener at the center of the plate. The critical size of such a stiffener can be computed from the relationship

$$\frac{(EI)_s}{P_{crp}\, r^2 h^2} = 0.4$$

This relationship was developed empirically after making several assumptions and approximations. However, the study was based on data from tests of 105 stiffened plates, so the relationship appears to be a safe criterion for design purposes if the range of the test data is not greatly exceeded.

The critical stress of a circumferentially stiffened, curved plywood panel is that of half the unstiffened panel if the stiffener attached has a stiffness at least that of the critical value given above.

Notation

The following symbols are used in this report. All quantities are in inch and pound units.

h = thickness of plywood.

k = coefficient in equation (1) determined from Forest Products Laboratory Report No. 1322 and No. 1514.

L = the length of an unloaded edge of the curved plywood plate, measured in the axial direction.

a = the length of a loaded edge of the curved plywood plate, measured along the circumference of the curve.

r = mean radius of curvature.

E_a = effective modulus of elasticity of plywood in compression measured in the circumferential direction of a curved plate.

E_b = effective modulus of elasticity of plywood in compression measured in the axial direction of a curved plate.

E_1 = effective modulus of elasticity of plywood in bending measured in the axial direction of a curved plate.

E_2 = effective modulus of elasticity of plywood in bending measured in the circumferential direction of a curved plate.

E_L = modulus of elasticity of wood in the direction parallel to the grain. (Assumed to be $\frac{E_1 + E_2 + E_a + E_b}{2.10}$ for yellow birch.)

E_s = modulus of elasticity of a Sitka spruce stiffener in the direction parallel to the grain, as determined from a compression test.

d = depth of stiffener (perpendicular to the plane of the plywood plate).

t = width of stiffener (parallel to the plane of the plywood plate).

p_{cr} = observed critical compressive stress for the buckling of a stiffened, curved plywood plate.

p_{cr0} = observed critical compressive stress for the buckling of an unstiffened, curved plywood plate.

p_{crm} = observed critical compressive stress for the buckling of a stiffened, curved plywood plate having a stiffener large enough to cause the plate to buckle as two independent plates.

p_{crp} = theoretical critical compressive stress for the buckling of an unstiffened, curved, plywood plate.

$(EI)_s$ = a measure of the stiffness of the stiffener, computed about a neutral axis, whose position is found by the following formula (Forest Products Laboratory Report No. 1557, equation 68).

$$Z_n = \frac{1}{2}\,\frac{d + h}{\dfrac{2ahE_a}{\pi\alpha\left(\dfrac{E_b}{E_a}\right)^{1/4} tdE_s} + 1 + \dfrac{E_b h}{E_s d}}$$

where

$$\alpha = \sqrt{k + \sqrt{k^2 - 1}}$$

and

$$k = \frac{\sqrt{E_a E_b}}{2\mu_{LT}} - \frac{\sigma_{TL} E_L}{\sqrt{E_a E_b}}$$

$(EI)_{scr}$ = the minimum stiffness of a stiffener that will cause a plywood plate to buckle as two independent plates.

z_n = distance from the center of the plywood to the neutral axis which has been shifted by attaching a stiffener to the plywood.

μ_{LT} = shear modulus in the LT plane as determined by the plate shear test.

σTL = Poisson's ratio of extension in the longitudinal direction to compression in the tangential direction due to a compressive stress in the tangential direction. (Assumed to be 0.02 for yellow birch.)

Table 1.--Buckling properties of circumferentially stiffened and unstiffened curved plywood plates in axial compression

Group and plate number	Plywood construction	Orientation of face grain to load	Dimensions of plate: Thickness (h)	Length (Axial) (L)	Width (Circumferential) (a)	Radius of curvature (r)	Dimension of stiffener: Width (t)	Depth (d)	Critical stresses: Observed (p_{cr})	Computed (p_{crm})	Ratios: $\frac{(EI)_s}{P_{crp}r^2h^2}$	$\frac{P_{cr}-P_{cr0}}{P_{crm}-P_{cr0}}$	Moduli of elasticity: E_1	E_2	E_a	E_b	E_L	E_a
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
			Inch	Inches	Inches	Inches	Inch	Inch	P.s.i.	P.s.i.			1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.
A-1A	1:1:1:1:1	Parallel	0.152	12.98	29.05	30.7	....	....	[1]1,699	1,724	0.0	0.0	1,724	527	1,066	1,465	2,277	....
2B	do	do	.152	13.00	29.06	30.5	0.242	0.114	1,524	1,793	.019	-.393	1,810	568	1,051	1,510	2,352	1,462
3A	do	do	.151	13.00	29.06	30.5	.239	.187	1,750	1,637	.055	.115	1,677	538	1,012	1,344	2,176	1,476
1B	do	do	.152	12.98	29.06	30.7	.242	.260	1,983	1,724	.115	.638	1,724	527	1,066	1,465	2,277	1,601
3B	do	do	.151	13.00	29.06	30.7	.247	.266	1,807	1,615	.129	.243	1,677	538	1,012	1,344	2,176	1,601
4B	do	do	.151	13.00	29.07	31.1	.250	.281	1,845	1,705	.127	.328	1,709	562	1,070	1,480	2,296	1,442
2A	do	do	.152	13.00	29.06	30.7	.242	.513	[2]2,119	1,782	.572	.944	1,810	568	1,051	1,510	2,352	1,668
4A	do	do	.150	13.01	29.06	30.7	.239	.520	[2]2,168	1,705	.621	1.054	1,709	562	1,070	1,480	2,296	1,668
B-1A	1:1:1:1:1:1:1	Parallel	.142	13.00	29.06	30.9	....	....	[1]1,672	1,735	.0	.0	1,691	842	1,186	1,494	2,483	....
2B	do	do	.143	13.00	29.06	30.9	.243	.114	1,579	1,755	.020	-.193	1,722	822	1,158	1,506	2,479	1,462
3A	do	do	.143	13.00	29.06	30.5	.240	.191	1,860	1,670	.061	.389	1,590	834	970	1,525	2,342	1,476
1B	do	do	.141	13.01	29.06	30.5	.241	.264	1,930	1,722	.133	.534	1,691	842	1,186	1,494	2,483	1,601
3B	do	do	.142	13.00	29.06	30.9	.243	.266	2,108	1,636	.138	.903	1,590	834	970	1,525	2,342	1,601
4B	do	do	.143	13.03	29.06	30.9	.248	.277	[2]1,877	1,725	.132	.424	1,635	832	1,153	1,500	2,438	1,442
2A	do	do	.144	13.00	29.06	30.9	.240	.515	[2]2,182	1,767	.620	1.056	1,722	822	1,158	1,506	2,479	1,668
4A	do	do	.142	13.00	29.06	30.7	.237	.517	[2]2,128	1,725	.655	.944	1,635	832	1,153	1,500	2,438	1,668
C-1A	1:2:1	Parallel	.121	13.00	29.04	30.7	....	....	[1]1,281	1,150	.0	.0	1,756	372	1,131	1,086	2,069	....
2B	do	do	.121	13.00	29.04	30.7	.242	.113	1,357	1,170	.034	.289	1,713	385	1,186	1,140	2,107	1,462
3A	do	do	.122	13.02	29.06	30.9	.239	.187	1,410	1,156	.096	.490	1,744	366	1,086	1,138	2,064	1,476
1B	do	do	.122	13.01	29.04	30.7	.240	.263	1,552	1,158	.224	1.030	1,756	372	1,131	1,086	2,069	1,601
3B	do	do	.121	13.00	29.04	31.1	.245	.266	1,520	1,140	.235	.909	1,744	366	1,086	1,138	2,064	1,601
4B	do	do	.121	12.99	29.06	30.5	.248	.270	[2]1,266	1,169	.229	-.057	1,768	352	1,118	1,180	2,104	1,442
4A	do	do	.122	13.01	29.04	30.9	.238	.507	[2]1,552	1,171	1.085	1.030	1,768	352	1,118	1,180	2,104	1,668
2A	do	do	.121	13.01	29.04	30.7	.240	.515	[2]1,537	1,170	1.158	.973	1,713	385	1,186	1,140	2,107	1,668
D-1A	1:1:1:1:1	Parallel	.101	12.98	29.03	30.5	....	....	[1]1,214	1,172	.0	.0	1,976	640	1,176	1,556	2,547	....
2B	do	do	.101	13.01	29.04	30.7	.239	.113	1,193	1,200	.040	.112	2,012	630	1,209	1,622	2,606	1,462
3A	do	do	.101	13.01	29.06	30.7	.242	.191	1,390	1,220	.127	.941	2,022	696	1,250	1,555	2,630	1,476
4B	do	do	.102	13.00	29.04	30.7	.250	.230	1,320	1,213	.200	.567	2,156	665	1,114	1,586	2,629	1,476
1B	do	do	.101	13.00	29.02	30.7	.241	.262	[2]1,296	1,164	.299	.439	1,976	640	1,176	1,556	2,547	1,601
3B	do	do	.102	13.00	29.04	30.5	.243	.265	[2]1,428	1,240	.291	1.144	2,022	696	1,250	1,555	2,630	1,601
4A	do	do	.101	13.01	29.06	30.7	.242	.510	[2]1,597	1,202	1.555	.979	2,156	665	1,114	1,586	2,629	1,668
2A	do	do	.100	13.00	29.04	30.7	.240	.519	[2]1,377	1,180	1.672	.872	2,012	630	1,209	1,622	2,606	1,668
E-1A	1:1:1:1:1	Perpendicular	.101	13.00	29.03	30.7	....	....	[1]955	1,125	.0	.0	678	2,022	1,631	1,236	2,650	....
2B	do	do	.102	13.00	29.06	30.7	.241	.114	1,046	1,134	.043	.717	670	2,035	1,614	1,238	2,646	1,462
3A	do	do	.103	13.01	29.06	30.5	.240	.190	1,019	947	.155	.504	538	1,534	1,312	1,110	2,140	1,476
4A	do	do	.103	13.01	29.05	30.5	.245	.196	[2]1,059	1,010	.161	.819	594	1,686	1,433	1,080	2,283	1,476
1B	do	do	.101	13.01	29.04	30.9	.242	.254	[2]1,111	1,126	.286	1.228	678	2,022	1,631	1,236	2,650	1,601
3B	do	do	.103	13.00	29.06	31.1	.248	.273	[2]1,016	929	.355	.480	538	1,534	1,312	1,110	2,140	1,442
4B	do	do	.103	13.01	29.05	31.1	.246	.339	[2]1,003	990	.577	.378	594	1,686	1,433	1,080	2,283	1,463
2A	do	do	.101	13.01	29.03	30.5	.240	.519	[2]1,197	1,130	1.745	1.906	670	2,035	1,614	1,238	2,646	1,668
F-1A	1:2:1	Perpendicular	.081	13.02	29.03	30.2	....	....	[1]663	737	.0	.0	428	1,814	1,162	1,326	2,252	....
2B	do	do	.082	13.01	29.03	30.9	.241	.115	609	715	.086	-.482	394	1,898	1,276	1,147	2,246	1,462
3B	do	do	.082	13.01	29.05	30.5	.249	.155	676	806	.168	.116	430	2,033	1,368	1,374	2,479	1,555
4A	do	do	.083	13.00	29.05	31.1	.239	.189	693	774	.241	.268	414	1,993	1,307	1,366	2,419	1,476
3A	do	do	.081	13.02	29.02	30.7	.242	.189	[2]730	791	.254	.598	430	2,033	1,368	1,374	2,479	1,476
4B	do	do	.082	13.00	29.04	30.7	.248	.238	[2]760	775	.449	.866	414	1,993	1,307	1,366	2,419	1,476
1B	do	do	.082	13.01	29.03	30.7	.241	.264	[2]756	734	.624	.830	428	1,814	1,162	1,326	2,252	1,601
2A	do	do	.081	13.00	29.03	30.5	.241	.519	[2]808	716	3.640	1.295	394	1,898	1,276	1,147	2,246	1,668
G-1A	1:2:1	Parallel	.060	13.00	29.02	30.7	....	....	[1]534	560	.0	.0	2,068	376	1,248	1,323	2,388	....
3B	do	do	.059	13.03	29.05	31.1	.248	.077	554	568	.070	.093	2,012	459	1,348	1,248	2,413	1,498
2B	do	do	.060	13.00	29.03	30.5	.245	.110	632	596	.143	.456	2,036	480	1,382	1,228	2,441	1,462
4A	do	do	.060	13.00	29.05	30.5	.248	.115	574	558	.170	.186	1,818	449	1,390	1,108	2,269	1,462
4B	do	do	.060	13.02	29.05	30.9	.247	.155	557	550	.352	.107	1,818	449	1,390	1,108	2,269	1,555
3A	do	do	.059	13.00	29.03	30.5	.241	.200	[2]716	579	.621	.847	2,012	459	1,348	1,248	2,413	1,476
1B	do	do	.060	13.01	29.03	30.5	.242	.266	[2]752	559	1.364	1.014	2,068	376	1,248	1,323	2,388	1,601
2A	do	do	.060	13.01	29.03	30.5	.239	.520	[2]746	595	6.900	.986	2,036	480	1,382	1,228	2,441	1,668

Z M 84121 F

Table 1.--Buckling properties of circumferentially stiffened and unstiffened curved plywood plates in axial compression (Continued)

Group and plate number	Plywood construction	Orientation of face grain to load	Dimensions of plate: Thickness (h)	Length (Axial) (L)	Width (Circumferential) (a)	Radius of curvature (r)	Dimension of stiffener: Width (t)	Depth (d)	Critical stresses: Observed (p_{cr})	Computed (p_{crp})	Ratios: $(EI)_s / p_{crp} r^2 h^2$	$\frac{p_{cr}-p_{cr0}}{p_{crm}-p_{cr0}}$	Moduli of elasticity: E_1	E_2	E_a	E_b	E_L	E_s
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
			Inch	Inches	Inches	Inches	Inch	Inch	P.s.i.	P.s.i.			1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.
I-6	1:1:1:1:1:1:1	Parallel	0.110	21.38	23.08	23.6			[1]1,308	1,290	0.0	0.0	1,470	775	1,063	1,304	2,196	
5	do	do	.108	21.42	23.10	23.5	0.140	0.078	1,331	1,346	.018	.082	1,566	817	1,070	1,428	2,325	1,601
2	do	do	.110	21.40	23.06	24.0	.253	.109	1,395	1,300	.058	.309	1,523	813	1,034	1,357	2,251	1,613
4	do	do	.109	21.36	23.08	23.9	.240	.150	1,411	1,283	.106	.365	1,524	791	1,004	1,368	2,232	1,551
1	do	do	.109	21.39	23.06	24.0	.250	.181	1,516	1,230	.170	.738	1,568	728	904	1,386	2,184	1,542
6X	do	do	.111	21.39	23.10	23.8	.238	.234	1,478	1,290	.246	.603	1,470	775	1,063	1,304	2,196	1,356
3	do	do	.108	21.41	23.06	24.0	.248	.276	[2]1,467	1,334	.383	.564	1,652	818	1,116	1,395	2,372	1,402
7	do	do	.110	21.39	23.10	23.9	.246	.336	[2]1,590	1,248	.667	1.000	1,482	738	995	1,285	2,143	1,507
J-4B	1:1:1:1:1	Perpendicular	.101	9.75	23.10	23.8			[1]1,252	1,354	.0	.0	627	1,822	1,466	1,046	2,363	
3B	do		.102	9.76	23.08	23.8			[1]1,427	1,315	.0	.0	590	1,812	1,422	1,024	2,308	
2B	do	do	.102	9.76	23.06	23.6	.130	.078	1,399	1,375	.017	.179	626	1,878	1,474	1,044	2,391	1,601
4A	do	do	.103	9.75	23.08	23.8	.130	.081	1,338	1,391	.017	-.006	627	1,822	1,466	1,046	2,363	1,601
1B	do	do	.103	9.76	23.09	23.8	.254	.111	1,373	1,242	.068	.100	492	1,656	1,395	1,022	2,174	1,613
2A	do	do	.101	9.76	23.10	23.8	.240	.152	1,552	1,350	.115	.642	626	1,878	1,474	1,044	2,391	1,551
1A	do	do	.101	9.76	23.06	23.8	.248	.190	[2]1,760	1,208	.214	1,273	492	1,656	1,395	1,022	2,174	1,542
3A	do	do	.101	9.76	23.08	23.9	.247	.267	[2]1,604	1,298	.402	.800	590	1,812	1,422	1,024	2,308	1,402
3BX	do	do	.102	9.75	23.10	23.8	.245	.338	[2]1,647	1,315	.730	.930	590	1,812	1,422	1,024	2,308	1,507
K-7	1:2:1	Parallel	.081	21.39	23.09	23.8			[1]675	907	.0	.0	1,984	437	1,266	1,306	2,378	
6	do	do	.081	21.39	23.08	23.5			[1]673	914	.0	.0	1,963	463	1,286	1,254	2,365	
5	do	do	.080	21.43	23.07	23.8	.130	.080	875	876	.034	.561	1,906	457	1,242	1,285	2,328	1,601
2	do	do	.081	21.39	23.06	24.0	.252	.108	1,015	862	.117	.953	1,946	404	1,191	1,335	2,322	1,613
4	do	do	.081	21.38	23.04	24.0	.242	.144	958	838	.219	.793	1,886	398	1,197	1,255	2,255	1,551
1	do	do	.081	21.41	23.06	23.8	.249	.189	1,011	874	.386	.941	2,094	419	1,173	1,214	2,333	1,542
3	do	do	.082	21.40	23.08	23.9	.253	.273	[2]1,032	863	.858	1.000	1,819	407	1,199	1,290	2,245	1,402
L-7	1:1:1:1:1	Parallel	.079	21.39	23.10	23.8			[1]912	878	.0	.0	1,624	612	1,077	1,316	2,204	
6	do	do	.078	21.40	23.08	23.6			[1]861	904	.0	.0	1,734	609	1,087	1,399	2,300	
6X	do	do	.078	21.38	23.08	23.8	.131	.080	905	897	.035	.306	1,734	609	1,087	1,399	2,300	1,601
5	do	do	.078	21.40	23.06	23.6	.130	.083	974	911	.037	1.419	1,802	598	1,128	1,378	2,336	1,601
2	do	do	.078	21.39	23.06	23.9	.254	.113	[2]891	754	.155	.008	1,542	472	916	1,253	1,992	1,613
4	do	do	.079	21.33	23.07	24.0	.239	.156	[2]960	808	.264	1.194	1,627	522	1,008	1,249	2,098	1,551
1	do	do	.079	21.40	23.06	24.0	.250	.189	[2]950	759	.449	1.032	1,472	454	938	1,274	1,971	1,542
3	do	do	.078	21.40	23.07	24.0	.250	.278	[2]934	712	1.058	.774	1,620	478	853	1,016	1,889	1,402
M-4B	1:1:1:1:1	Parallel	.077	9.74	23.08	23.9			[1]1,069	1,116	.0	.0	1,792	698	1,107	1,493	2,424	
3B	do	do	.078	9.77	23.06	23.9			[1]1,145	1,000	.0	.0	1,501	600	1,005	1,361	2,127	
2B	do	do	.078	9.74	23.06	23.9	.140	.078	1,279	912	.034	.427	1,488	560	860	1,196	1,954	1,601
4A	do	do	.076	9.74	23.08	23.8	.244	.081	1,511	1,107	.053	1.002	1,792	698	1,107	1,493	2,424	1,601
1B	do	do	.075	9.75	23.06	24.0	.252	.110	1,495	1,065	.108	.963	1,922	687	1,092	1,376	2,418	1,613
2A	do	do	.078	9.75	23.03	23.8	.237	.151	[2]1,447	909	.222	.844	1,488	560	860	1,196	1,954	1,551
1A	do	do	.075	9.75	23.04	23.9	.248	.190	[2]1,591	1,069	.356	1.201	1,922	687	1,092	1,376	2,418	1,542
3A	do	do	.078	9.73	23.03	24.0	.254	.271	[2]1,419	996	1.010	.774	1,501	600	1,005	1,361	2,127	1,402
3BX	do	do	.077	9.76	23.05	23.8	.244	.338	[2]1,521	991	1.422	1.027	1,501	600	1,005	1,361	2,127	1,507
N-7	1:2:1	Parallel	.061	21.38	23.10	23.8			[1]557	652	.0	.0	2,011	410	1,153	1,374	2,356	
6	do	do	.061	21.40	23.08	23.5			[1]508	697	.0	.0	2,128	439	1,314	1,342	2,487	
5	do	do	.060	21.41	23.06	23.5	.130	.077	629	670	.059	.610	2,130	398	1,340	1,332	2,476	1,601
2	do	do	.059	21.39	23.06	23.9	.251	.112	[2]649	663	.243	.736	2,206	452	1,266	1,301	2,488	1,613
4	do	do	.060	21.35	23.07	24.0	.243	.151	[2]704	700	.421	1.081	2,226	456	1,390	1,374	2,593	1,551
6X	do	do	.060	21.38	23.08	23.8	.249	.155	[2]639	677	.478	.673	2,128	439	1,314	1,342	2,487	1,551
1	do	do	.061	21.40	23.03	23.9	.251	.191	[2]709	649	.793	1.113	2,095	396	1,198	1,348	2,399	1,542
3	do	do	.060	21.39	23.06	24.0	.250	.272	[2]713	674	1.700	1.138	2,152	482	1,306	1,306	2,497	1,402

[1]Assumed to be p_{cr0}. When more than one unstiffened panel was tested, the average was taken as p_{cr0}.

[2]Assumed to be p_{crm}. Values of observed critical stress that appeared to reach a constant maximum were averaged and taken as p_{crm}.

Z M 84122 F

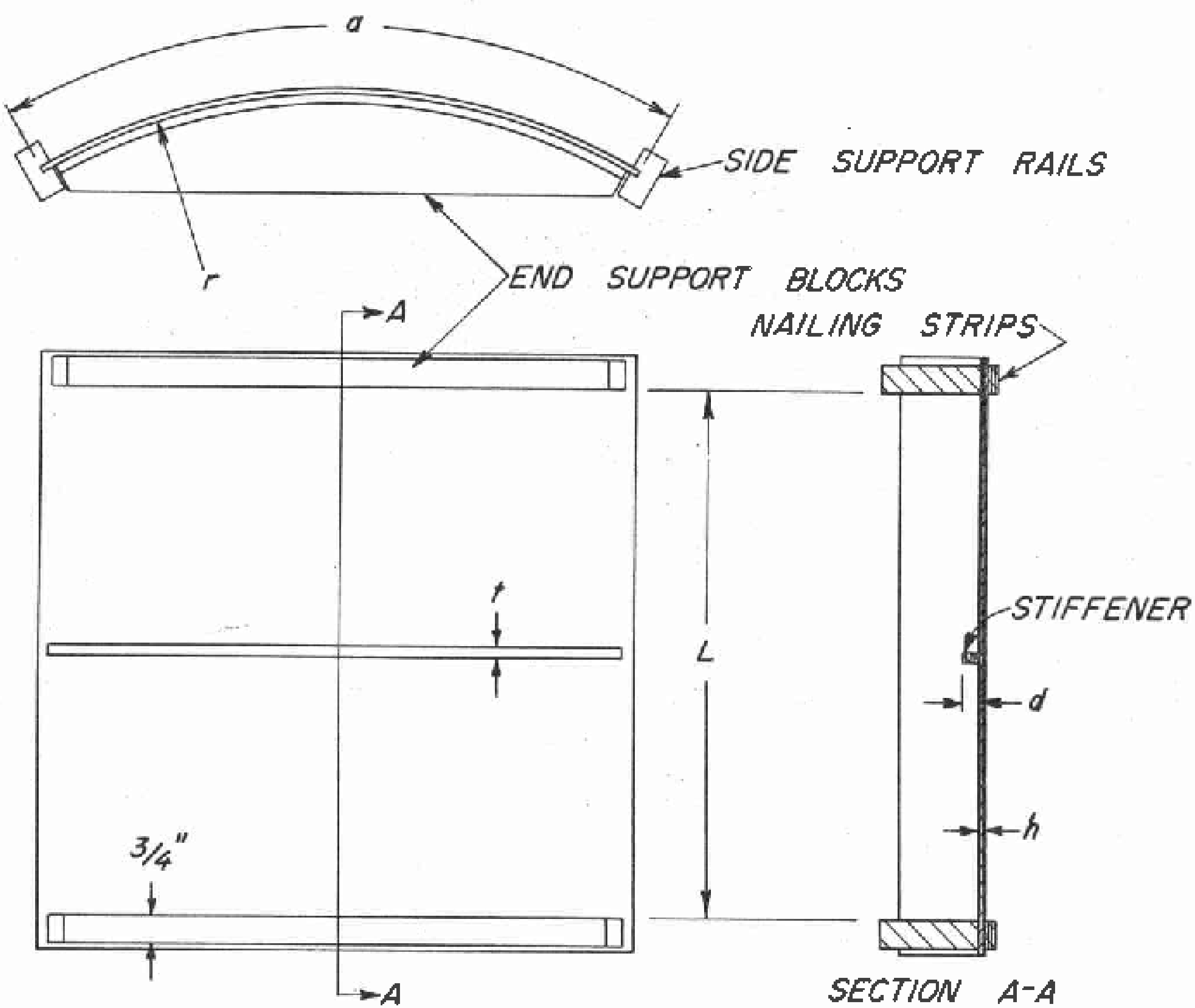


Figure 1.--Sketch showing construction and dimensions of a curved, circumferentially stiffened plywood plate to be tested in axial compression.

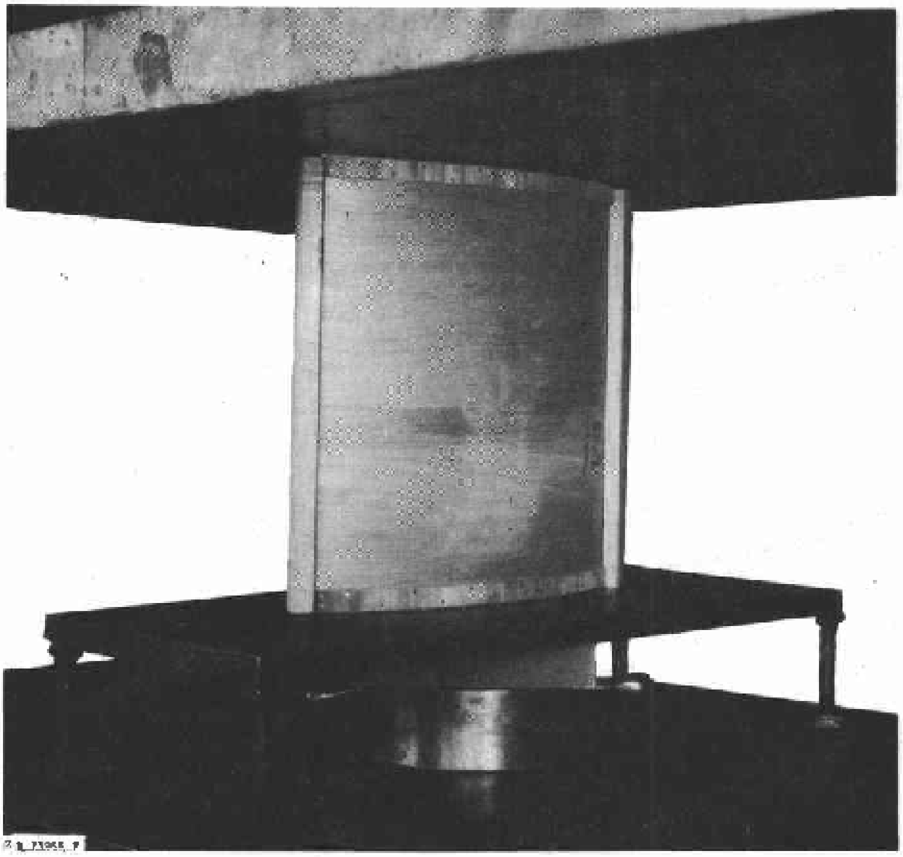


Figure 2. --Illustration of the method of test used on both stiffened and unstiffened panels.

ZM 73955 F

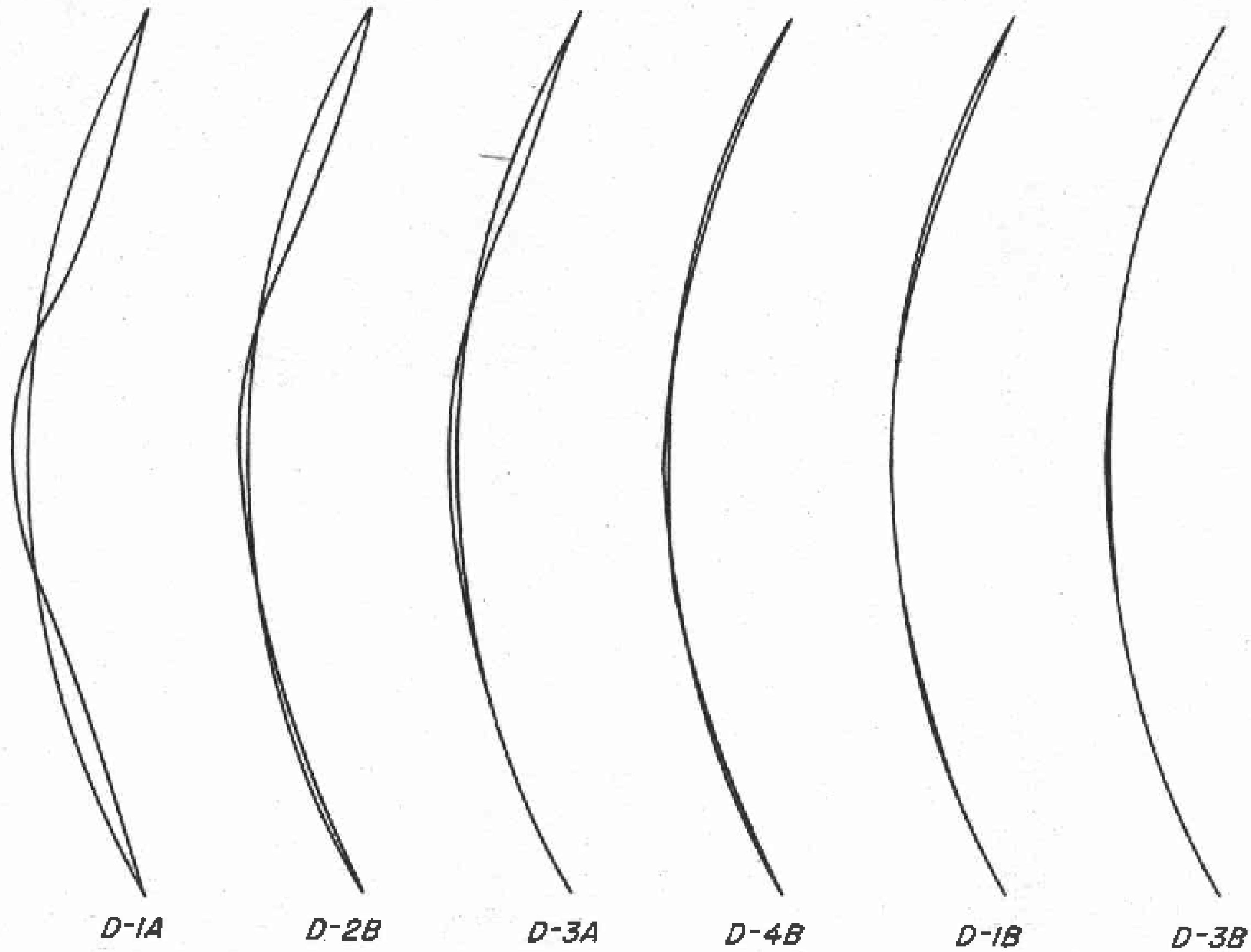


Figure 3.--Typical buckle patterns of circumferentially stiffened plates viewed in the axial direction, obtained by making a trace on the horizontal centerline of the plate in the testing machine.

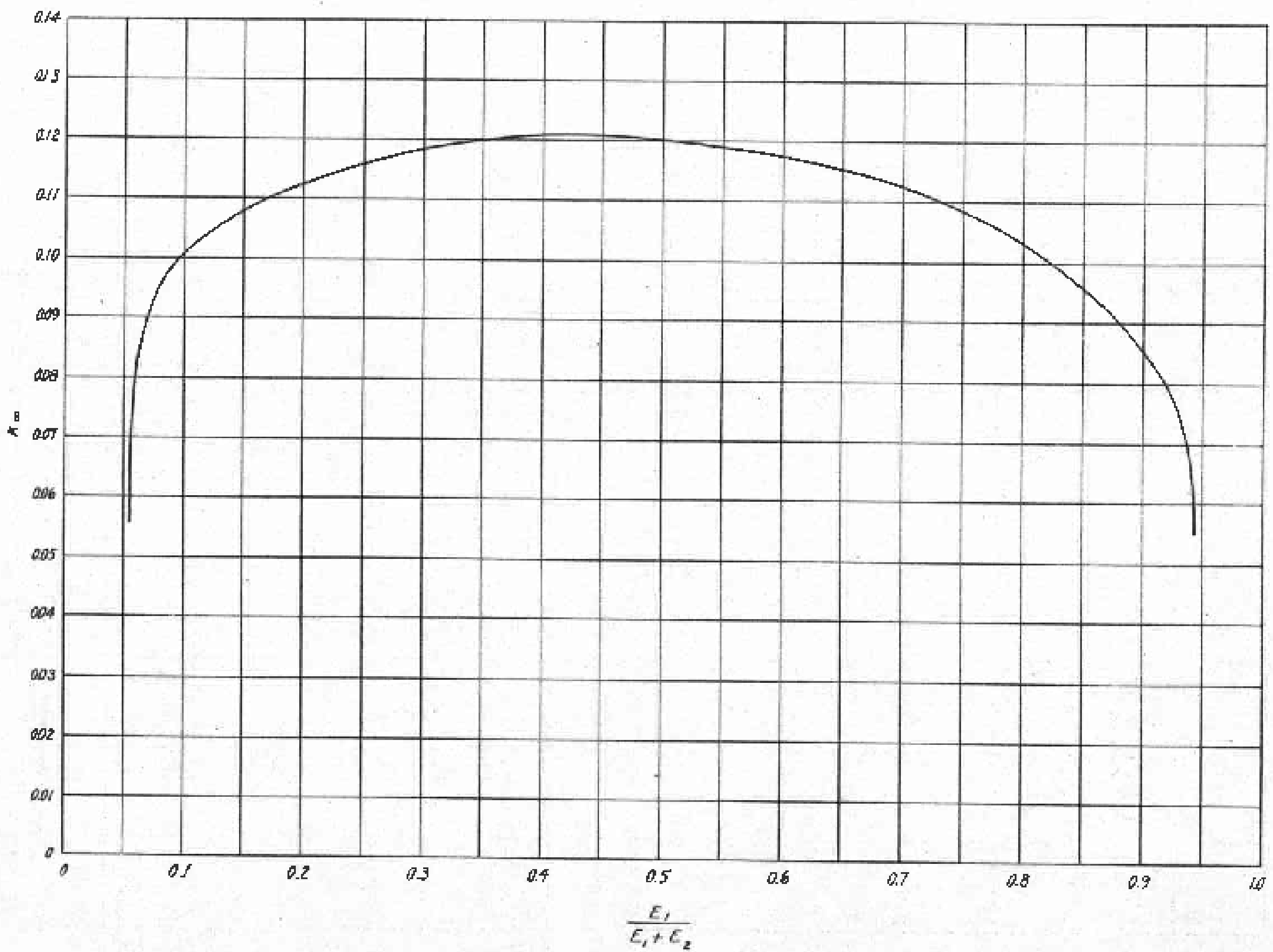


Figure 4.--Theoretical curve for long, thin, plywood cylinders in axial compression. The ordinates represent k in the formula $p = kE_L\, h/r$, where p is the buckling stress. The abscissas represent the ratio $E_1/(E_1 + E_2)$ where E_1 and E_2 are proportional to the stiffness of the plywood in the axial and circumferential directions, respectively.

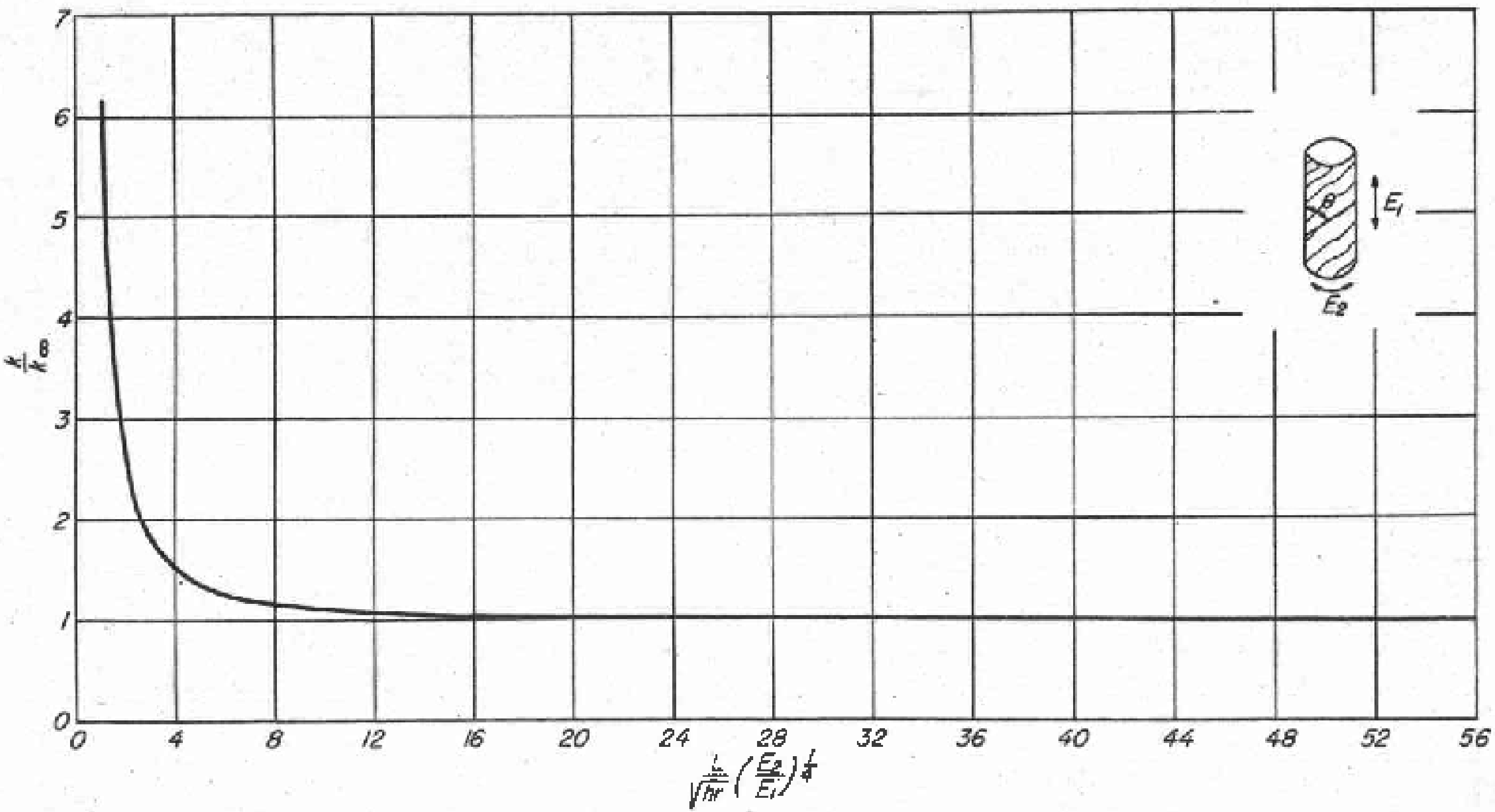
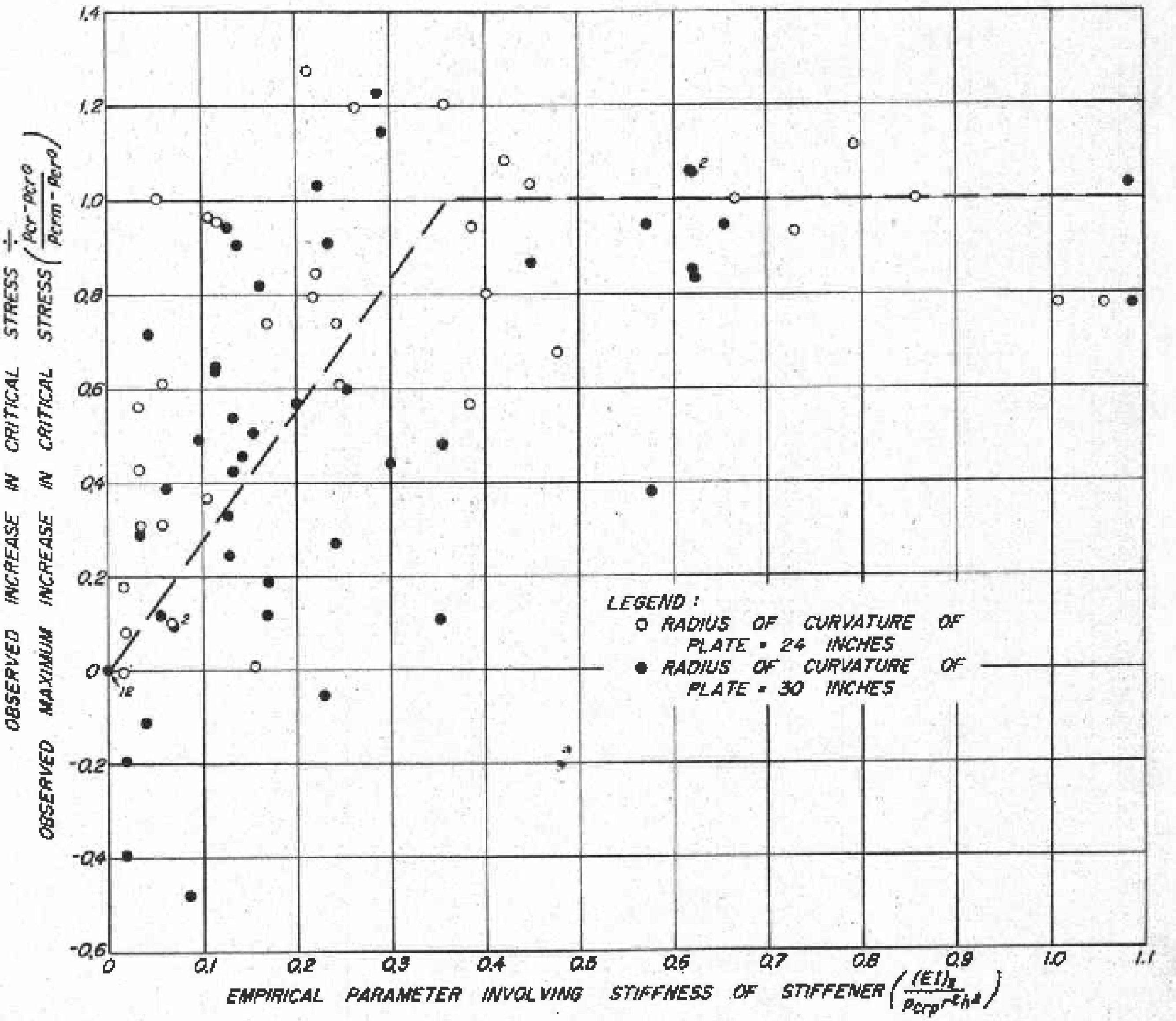


Figure 5.--The effect of length of specimen on the buckling constant for thin plywood cylinders in axial compression.



Z M 84290 F

Figure 6.--Comparison of ratios of the observed increase in critical stress to the observed maximum increase in critical stress with an empirical parameter involving stiffness of stiffener.

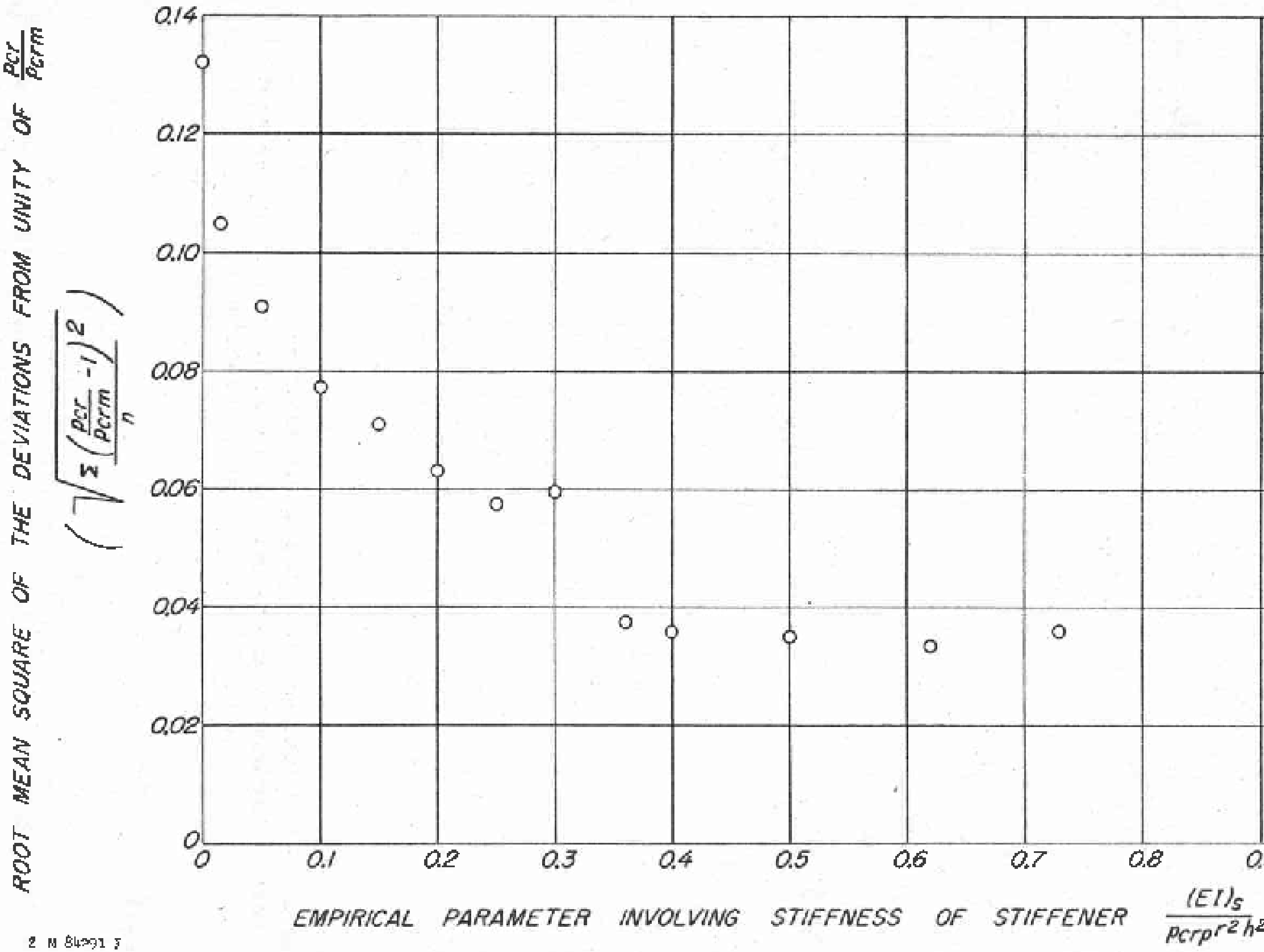


Figure 7.--Comparison of the root mean squares of the deviations from unity of the ratios of observed critical stress to observed maximum critical stress with an empirical parameter involving stiffness of stiffener.

Z M 84291 F

Heebink, Thomas Bruce
Effect of circumferential stiffeners on the buckling properties of thin, cruved plywood panels in axial compression, By T. B. Heebink and C. B. Norris. 3rd, ed. Madison, Wis., U.S. Forest Products Laboratory, 1961.
9 p., illus. (F.P.L. rpt. no. 1812)

Presents data from tests of 105 curved plywood panels stiffened with a single longitudinal stiffener and loaded in axial compression. From certain relationships, it is possible to select stiffeners of minimum size and maximum effectiveness.

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