AN ABSTRACT OF THE THESIS OF

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Title: ROTARY BISPECTRAL ANALYSIS OF THE WIND TO CURRENT ENERGY TRANSFER

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Comparisons are made between wind and current data taken at site D in the Atlantic and at the TOTEM buoy off Oregon using the rotary bispectral and energy transfer analysis techniques. The rotary cross bispectra of the two data sets showed similar topography. Also the energy transfer functions for the data of site D and that of TOTEM are similar in that for both there is non-linear transfer of energy at triplets of frequency throughout the frequency plane. However, the calculated energy transferred from local winds to currents at site D was a much smaller percentage (27%) of the existing total current spectrum than was the transfer at TOTEM (63%). The physical environment at the two sites was very different: at TOTEM the direction of winds and current was essentially constant throughout the record, while at site D the current seems to be dominated by a Gulf Stream Ring and the wind field
shows large variations on the time scale of a few days. From several possible explanations, it is proposed that the most probable explanation is that non-stationarity in the wind field accounted for the relatively small transfer of energy observed at site D.
Rotary Bispectral Analysis of the Wind to Current Energy Transfer

by

Edwin James Chapin Sobey

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I. INTRODUCTION

The rotary bispectral analysis and the non-linear energy transfer function techniques have been developed recently to investigate the transfer of energy from wind stress to ocean currents (Yao, 1974). The following research is a natural extension of the above mentioned research in that these techniques are applied to other sets of data in order 1) to confirm or contradict the results of the previous research and, 2) to add understanding to the physical phenomena of the energy transfer.

Inertial oscillations in the ocean are examined closely in any analysis of the energy transfer. This is due to, in part, the fact that they constitute one of the major modes of oceanic motion in the low to intermediate frequency region. Also, their periods of oscillation can be accurately predicted and thus the absence or presence of inertial waves is easily determined on a current spectrum. Most researchers seem to agree that these oscillations are directly created and destroyed by features of atmospheric motions (Sakou, 1970; Sakou and Neshyba, 1972; Perkins, 1972; Pollard, 1974; etc.). Thus this research will also examine closely the inertial currents in an attempt to better understand the energy transfer.
The statistical analysis techniques used in this study are presented in Yao (1974) and are only summarized here. Recent theories of energy transfer to the current field, and in particular to inertial currents, are also discussed.

A. Review of Energy Transfer Theories

Inertial oscillations in the ocean current field are the predominant non-tidal oscillations and are easily recognized. Pollard (1974) states that when energy is transferred to the ocean from atmospheric motions, the inertial currents constitute a high percentage of the total oceanic response. For these reasons, the generation of inertial oscillations has been heavily studied in order to understand the energy transfer process. Thus a review of the existing theories on energy transfer must be also a review of inertial oscillation theories.

Inertial oscillations are a type of unaccelerated motion in which the Coriolis force is balanced by the centrifugal force. The motion is nearly circular and has a period given by

\[ T = \frac{12 \text{ hours}}{\sin (\theta)} \]  

(1)

where \( \theta \) is the latitude of the observation (Newmann and Pierson, 1966). Since the Coriolis term is involved, the sense of rotation of inertial currents is determined for the hemisphere in which the
observation is taken: for the Northern Hemisphere, the motion is anticyclonic or cum-sole. All data used in this study were taken in the Northern Hemisphere.

The overwhelming majority of authors hold that inertial oscillations are caused by some local phenomena (Sakou, 1970; Webster, 1968; Pollard and Millard, 1970; Halpern, 1974.) However, beyond this point there is some disagreement on the particulars of inertial motion generation. These points will be addressed below:

1) For a time, it was believed that inertial currents were caused by tides (Knauss, 1962 and Reid, 1962). However, this idea later became unpopular as more observations of the currents were made (Webster, 1968). Fofonoff and Webster (1971) and Sakou and Neshyba (1972) examined this question and both indicated that there is no correlation between the tidal and inertial motions.

2) That inertial currents are intermittent has been well established (Webster, 1968). However, investigation into the spatial coherence of these currents has only recently been fully carried out. Webster (1969) found that low-frequency elements of the current field were coherent over a horizontal scale of three kilometers. Spatial coherence of 70 km was reported by Pollard (1974) for the inertial band of frequencies at a depth of 12 m. Measurements at greater depths and at other frequencies were not as coherent. Although several authors have found inertial currents at all depths
(Webster, 1968; Gonella, 1971; Webster, 1972; and Brekhovskikh, et al., 1971) there appears to be little vertical coherence between measurements (Webster, 1968; Fomin and Savin, 1973). Some of the most recent measurements (Pollard, 1974) indicated that coherence was found to exist between vertically adjacent current meters but not between other combinations of meters on the same mooring.

3) Several investigators have successfully attempted to correlate local atmospheric phenomena to ocean currents. Sakou (1970) found such a correlation from the wind to the inertial component of the current using linear regression techniques. Correlations between the wind and the entire current field, however, were very low. Sakou also showed that wind stress proved to be no better correlated than the wind. In making a test of a model proposed by Pollard (1970), Pollard and Millard (1970) observed inertial current accelerations in response to wind acceleration. They conclude that the features of the wind field that best control the formation of inertial currents are (i) a strong wind blowing in one direction for a few hours (up to half an inertial period) and (ii) a strong wind combined with a fairly sudden shift in its direction. In addition, a phase lock situation is postulated: the phase between the wind field and the current field determines whether inertial currents will be enhanced or destroyed. Once generated, the inertial current continues to oscillate, and only slowly decays by downward
dispersion unless destroyed by a new, out-of-phase wind input. Pollard (1974) adds further support to these ideas; he emphasized that local wind alone is required to forecast inertial current amplitude. Wind fronts, with clockwise rotating shifts of direction, and separated from subsequent fronts by one third an inertial period, are suggested as being the most effective phenomena in inertial current generation.

4) Storm activity has been found to be an effective inertial current generator (Day and Webster, 1965; Pollard and Millard, 1970). The most recent reporting of this is by Halpern (1974) who states that, in response to the passage of a storm front, the ocean current speed increased by 400% and much of this response was at the inertial frequency.

5) The generation of inertial waves by phenomena internal to the current field is an idea that is discussed very little in the literature. Fomin (1968) states that since inertial currents are superimposed on an existing current structure, they must draw energy from that structure. Due to the vertical inhomogeneity of inertial motion, Gonella (1971) concludes that the oscillations at different depths at the same location may have diverse origins and that some are probably transmitted by turbulence or boundary effects. Aside from these comments, no one has argued for the internal generation and no measurements to support its possibility
have been reported.

6) Resonance theories for the generation of inertial currents have some supporters (Endoh and Nitta, 1971; and Kraus, 1972). In 1967, Belyaev and Kolesnikov (1967) derived, from the equations of motion, a transmission function for the transfer of energy from the atmosphere to the ocean. Their transmission function had a distinct peak centered at the inertial frequency and they concluded that inertial frequency motion in the atmosphere is predominantly responsible for the transfer of energy to inertial currents and thus to the current field in the ocean. Further, theoretical work by the above mentioned authors (Endoh and Nitta, 1971) supports Belyaev and Kolesnikov, but no empirical support has yet been presented. Yao (1974), on the other hand, directly calculated the linear and non-linear energy transfer between wind stress and the current field. In that research, he found no linear transfer of energy to the inertial current, and in fact no linear transfer to any frequency in the intermediate frequency range. Twenty-three percent of the total non-linear energy transferred to the inertial frequency at 14 m depth did involve inertial frequencies in the wind (interacting with the mean component of the wind). Yao concludes that his findings contradict the resonance theory.

Investigators have recently turned their attention to the hydrographic conditions during current and wind observations. This is in
response to the qualitative findings by many that currents seem to be coherent over much smaller scales vertically than horizontally (Webster, 1972). Fofonoff and Webster (1971) noticed that coherence decreased if the current meters were separated by a region of abrupt change in density. The authors mention specifically that near-surface seasonal thermoclines inhibit vertical coherence. The idea that stratification controls the rate at which the energy of the inertial frequency band disperses downward is stated by Pollard and Millard (1970). Pollard (1974) mentions that in a well-mixed layer overlying a strong thermocline, surface stress momentum will spread throughout the layer in a time short compared with the inertial period. As mentioned above, Halpern (1974) found a large increase in current energy in the mixed layer in response to the passage of a storm front. Below the mixed layer, the response was delayed, of smaller amplitude, and was shorter in duration. From this experiment, Halpern concludes that stratification acts as a barrier to the downward penetration of momentum introduced at the sea surface. This idea is supported by Yao's (1974) measurements. While he was able to account for 63% of the current field energy at 14 m depth by transfer from the wind stress, he could account for only 7% of the energy at 34 m (below the well-mixed layer).

Thus inertial currents seem to be the dominant oceanic
response to atmospheric energy input. The mechanism by which they are generated or controlled by the wind, whether by resonance, phase lock, or by other means, is very much in question. They are locally generated by wind and their vertical extent is controlled by the existing hydrographic conditions. The nature of the internal interactions between currents at inertial and those at other frequencies, if in fact there are any, has not been studied or even postulated.

B. The Rotary Spectral and Bispectral Approach

Mooers (1970) first used rotary component analysis and since then several other investigators have employed this technique (Gonella, 1972; Perkins, 1972; Yao, 1974). The rotary spectral technique has been used instead of the rectangular Fourier analysis because of the following advantages that it offers:

1. Computed rotary spectral energies and coherences are independent of axis orientation and current meter orientation error.

2. Separation of spectral energy into positive and negative rotating components allows one to more clearly distinguish those natural processes which are characterized by rotation in one sense only.

3. Because some naturally occurring motions entail rotation in only one direction, often one of the rotary components can fully
describe the motion independent of the other rotary component.

This independence of the scalar components is a property which is, in general, not found in rectangular coordinate analysis.

4. With the rotary method, the number of bispectral planes is reduced from eight to four tri-frequency planes. This greatly reduced the difficulty in interpreting the bispectrums.

Because inertial oscillations are an important element in the analysis of this data, and since their motion is governed by the earth's rotation, analysis by rotary spectral techniques is especially beneficial in this study. Inertial peaks show up at the inertial frequency, but only in the clockwise rotating component (in the Northern Hemisphere).

In the rotary spectral technique, the rectangular scalar components of the vector random process are Fourier decomposed into two components: one having a positive and the other a negative angular velocity with the magnitude of both equaling the frequency of the particular motion. Thus periodic motion is represented as the interaction of two counter-rotating elements called rotors. The relative lengths of the rotors and the phases between the two rotors determine the nature of the oscillation at that frequency. For example, purely circular cyclonic flow would be represented by only the cyclonic rotor.  

---

1/Cyclonic angular frequencies are designated as being positive and anticyclonic frequencies are negative.
would be represented as the superposition of both the positive and negative rotors at that particular frequency; each rotor would have the same magnitude and the same phase. Situations between the two extremes of pure circular flow and non-rotating flow can be represented by some combination of lengths and phases of the rotary components.

1. **Rotary Spectrum**

The rotary spectral density, $P_{u_+ u_+}(0)$, for the vector random process $u(t)$, is given by Yao, (1974)

$$ P_{u_+ u_+}(0) \, \text{d}\omega = < U_+ (0) \, U^*_+ (0) > \tag{2} $$

where $<$ denote the averaging process, $^*$ denotes the complex conjugate and $U_+ (0)$ is the rotary Fourier coefficient of $u(t)$ and is given below.

$$ U_+ (\omega) = [A_1(\omega) + B_2(\omega)] + i [A_2(\omega) + B_1(\omega)] \tag{3} $$

$$ A_J(\omega) = 1/T \, \int_0^T u_J(t) \cos \omega t \, \text{d}t $$

$$ B_J(\omega) = 1/T \, \int_0^T u_J(t) \sin \omega t \, \text{d}t \tag{4} $$

Here $\omega$ is the angular velocity of the motion and $\omega$ is the positive or negative angular frequency defined by:
\[ U^+ (0) = U(\omega) \text{ if } \omega = 0 \]  
\[ U^- (0) = U(\omega) \text{ if } \omega = -0. \]  

In a similar manner, the rotary cross spectral density can be obtained and is

\[ P_{u^+ v^+} (0) = < U_{u^+} (0) U^*_{v^+} (0) > \]  

A more useful measure of the linear coexistence of two time series is the coherence. The rotary cross coherence is given by

\[ R_{coh u^+ v^+} = \frac{\left| P_{u^+ v^+} (0) \right|^2}{P_{u^+ v^+} (0) P_{v^+ v^+} (0)} \]  

The coherence has a magnitude less than or equal to the value (1.0) (Bendat and Piersol, 1966).

2. Rotary Bispectra

Because of the inherent difficulties in interpreting the (rectangular component) bispectrum, only the rotary bispectrum and rotary cross bispectrum of the various vector process were examined. The ordinary bispectrum can be thought of as the third order interaction between the frequency components of a process, just as the spectrum is the second order interaction (MacDonald, 1963). The cross bispectrum can be interpreted either as the interaction of three random processes or the quadratic interaction
of two processes (Yao, 1974).

Since there is a basic understanding of the physical processes in the energy transfer being examined by this technique, a cause and effect description of the transfer is made (Yao, 1974). Thus the cross bispectrum can measure the interaction of two input variables on the output (third variable) or it can be used to examine quadratic (or second power) interactions between one causing function and the response function. It is the latter application that will be used throughout this report.

The rotary bispectrum is defined in the same terms as the rotary spectrum:

\[
RB (\pm \sigma_1, \pm \sigma_2) \, d\sigma^2 = \langle U_+^*(\sigma_1) \, U_+^*(\sigma_2) \, U_+^*(\sigma_3) \rangle \tag{8}
\]

where \(\sigma_1, \sigma_2,\) and \(\sigma_3\) fit one of four constraining equations each of which require that the algebraic sum of \(\sigma_1\) and \(\sigma_2\) equals \(\sigma_3\). Eight combinations of this equation are possible, but only four represent independent processes (Yao, 1974). This symmetry allows an additional constraint of \(\sigma_2 > \sigma_1\).

The rotary cross bispectrum is defined under the same four constraining equations involving \(\sigma_1, \sigma_2,\) and \(\sigma_3\):
3. Energy Transfer Functions

The rotary cross spectrum and the rotary cross bispectrum show the energy that is shared mutually between the two component processes. They in general cannot be interpreted as interactions or energy transfer from the cause to the effect processes. In order to study the energy transfer, linear and non-linear transfer functions must be used. For rotary component analysis Yao (1974) defines these functions:

\[
T_\pm (0) = \frac{P^* S_\pm^* C_\pm (0)}{P S_\pm (0)}
\]  

(10)

\[
K_\pm (\pm \sigma_1, \pm \sigma_2) = \frac{RB^*_{SSC} (\pm \sigma_1, \pm \sigma_2)}{2 P_{SS (\pm \sigma_1)} P_{SS (\pm \sigma_2)}}
\]  

(11)

\[
- \frac{T (\pm \sigma_3) RB^*_{SSS} (\pm \sigma_1, \pm \sigma_2)}{2 P_{SS (\pm \sigma_1)} P_{SS (\pm \sigma_2)}}
\]
\[ V_{\pm} (\pm \sigma_1 \pm \sigma_2) = \frac{RB^{*}_{SSC} (\pm \sigma_1, \pm \sigma_2)}{2P_{SS}(\mp \sigma_1) P_{SS} (\pm \sigma_2)} \] (12)

\[ - \frac{T_{\pm}(\sigma_3) RB^{*}_{SSS} (\pm \sigma_1, \pm \sigma_2)}{2P_{SS}(\mp \sigma_1) P_{SS} (\pm \sigma_2)} \]

Subscripts S and C denote wind stress and currents, respectively.

\( T (0) \) is the linear transfer function, \( K \) and \( V \) are the non-linear transfer functions with \( K \) involving positive and negative summations of \( \sigma_1 \) and \( \sigma_2 \) and \( V \) involving the arithmetic differences between \( \sigma_1 \) and \( \sigma_2 \). To get at the estimate of the energy transferred from the cause field to the effect field, the following equation is employed:

\[ P_{CC} (\pm \sigma_3) s1\sigma = |T(\pm \sigma_3)|^2 P_{SS}(\pm \sigma_3) \, d\sigma + \] (13)

\[ \sum_{\sigma_1} \sum_{\sigma_2} |K(\pm \sigma_1, \pm \sigma_2)|^2 P_{SS} (\pm \sigma_1) P_{SS} (\pm \sigma_2) \, d\sigma^2 \]

\[ + \sum_{\sigma_1} \sum_{\sigma_2} |V(\pm \sigma_1, \pm \sigma_2)|^2 P_{SS} (\pm \sigma_1) P_{SS} (\pm \sigma_2) \, d\sigma^2 \]

where the four constraining equations apply throughout.
II DATA

A. Source and Description

All of the data used in this research were collected at site D by personnel of the Woods Hole Oceanographic Institute. Site D is located at 39° 20'N, 70° W in about 2640 m of water (Webster, 1969). This location has been used by Woods Hole for current meter measurements on a continuous basis for several years. The position is 50 km south of the continental shelf and about 175 km north of the mean axis of the Gulf Stream (Webster, 1969). Pollard and Millard (1970) state that the mean current at site D is strongly influenced by its proximity to both the Gulf Stream and to the continental shelf and Webster (1969) says that the Gulf Stream can reach site D. (Examples are cited in Schmitz, 1970; Magaard and McKee, 1973). In that study, Webster reports that significant variations in the intensity of physical oceanic processes occur over a wide range of time scales and he warns against using simplified statistics to describe the motions there. Progressive vector diagrams (PVDs) generated from the current data collected by Webster show a complicated pattern moving generally westerly. In another report (Fofonoff and Webster, 1971) it is stated that the current at all depths is to the west.
1. Results of Previous Site D Analysis.

Webster analyzed spectra obtained from data taken at site D in 1965 (Webster, 1969). The dominant peaks found were at the semi-diurnal period and at the inertial period: 18.9 hours. Other significant peaks were at the diurnal tide period, a 36 hour peak attributed by Webster to an inertial and semi-diurnal beat frequency, and a 60 hour peak of unknown origin.

A study of the semi-diurnal tides at site D was conducted by Magaard and McKee in 1973. They found the semi-diurnal tidal current to be very strong with a maximum flow of 1 cm/sec in the east-west component. The principal tide-generating element was the $M_2$ component. The authors also reported that a Gulf Stream eddy passed through site D during their observations.

Several authors have described both cyclonic (Richardson et al., 1973; Cheney and Richardson, 1974; and Fuglister, 1971) and anticyclonic eddies of the Gulf Stream (Saunders, 1971; Thompson and Gotthardt, 1971). Anticyclonic eddies or "rings" form on the north side of the Gulf Stream when meanders are pinched off and leave a lamina of clockwise rotating water on the shoreward side of the Gulf Stream. Fuglister (1971) estimates that between five and eight rings are generated on each side of the Stream each year. Saunders (1971) reports eddy speeds of 30 to 70
cm/sec with a much smaller advection rate: 10 cm/sec. These rings are traced by their temperature and salinity contrast to the surrounding waters. Obviously this phenomena could play an important and possibly a dominant effect on observed current measurements at side D.

2. Description of Data Analyzed

Two sets of data were analyzed during this research. By far, the more heavily analyzed data was from buoy 309, set in June 1969 and recovered in August 1969. Secondary to this was data from buoy 298 which recorded information from April, 1969 to August of that year. Only the first 38 days of data from 298 were used in the study. The choice of this section of the data was based on observations of PVDs for that record. During the time that mooring 309 was in place, 298 was subjected to essentially the same current and wind regimes. The separation between the two moorings was approximately nine nautical miles.

Mooring 309 was located at 39° 9.0' N, 70° 0.2' W. For this position, the expected inertial period is 18.9 hours (from equation 1). Slightly to the south was mooring 298, at 39° 1.1' N, 69° 59.0' W, and its calculated inertial period is 19.1 hours. Both buoy systems were of the surface mooring types described by Fofonoff and Webster (1971). Wind measurements were recorded
using a cup anemometer (Millard, 1971) supported on a toroid buoy. The current meter in the first data set was located at 13 m, and in the second set (298) at 14 m. Wind and current measurements were taken at 15 minute intervals for 309 and 30 minute intervals for 298. Burst sampling techniques (described by Fofonoff and Webster, 1971) were employed to remove high frequency noise. However orbital wave motion is believed to excite those speed and direction sensors located within 20 meters of the surface. Also horizontal displacements of the surface float are transmitted to a depth of at least 100 m and vertical displacements are thought to be transmitted even deeper (Fofonoff and Webster, 1971). Although Fofonoff and Webster comment that these induced motions degrade the quality of the measurements, Halpern (1974) feels that at least at the inertial frequency the motions have little effect. However in comparing the data obtained with this type of buoy with data from a more stable platform, these induced motions must be kept in mind.

B. Statistical Analysis Techniques

Preliminary to any data analysis, the data were block averaged to yield hourly values instead of 15 or 30 minute values. Thus the effective Nyquist frequency, adjusted to a rate of hourly observations, is 0.5 cycles per hour. The Nyquist frequency is given by (Bendat and Pierson, 1966)
\[ f = \frac{1}{2(SR)} \]  \hspace{1cm} (14)

where SR denotes the sampling rate.

Initial analysis included calculations of the elementary statistics (e.g., mean, variance, standard deviation, etc.) and the testing of the normality and stationarity hypothesis. Normality testing was done using \( \chi^2 \) test for goodness of fit, the reduced \( \chi^2 \) test, and the Cornu ratio and skewness tests. The former two techniques are outlined by Bevington (1969), and the latter is documented by Crew and Bodvarsson (1971). The normality and stationarity tests used in this study are given by Yao (1974); the stationarity test is outlined below.  

Haubrich (1965) described a procedure for testing the hypothesis of weak stationarity. For normal processes, the condition of weak stationarity ensures complete stationarity (Jenkins and Watts, 1968). As pointed out, the data are at least mildly non-Gaussian, however the testing of complete stationarity is impractical (Bendat and Piersol, 1966). For data set 309, the stationarity tested showed that the data were unstationary. As recommended by

\[ 2/ \] The results of the normality test indicated that the processes were not normal.
Jenkins and Watts (1968) the means of the component series were removed and testing was tried again with no better results. The linear trend was removed (as outlined by Bevington, 1969) but to no avail. The 309 series were then split into two equal length segments which shall be referred to as 309A for the first segment and 309B for the second. This procedure was effective in eliminating the non-stationarity. All subsequent calculations were run on each segment individually. Data set 298 proved to be stationary after removal of the mean.

Progressive vector diagrams (PVDs) were made for 309A and 309B and for a segment of data series 298 running from 26 April to 2 June, 1969. As a check, the PVD's were compared to plots furnished by the Woods Hole Oceanographic Institute; this check showed that no errors had arisen in the transit of the data. Plots were also made of the wind and current speed squared and of the daily averaged wind and current velocities (in scalar components).

Auto- and cross-correlations were generated following the form of Bendat and Pierson (1966). Spectral analysis of the rectangular components of the data series was completed, but since the same information is contained in the rotary spectral analysis in a form that is easier to interpret, the ordinary spectral analysis is not used in this report.
Complex demodulates were run at several frequencies on the various data sets. The techniques are outlined by Granger and Hatanaka (1964); and the computer program documentation is given in Ochs et al. (1970). One hundred nineteen weights (symmetric filter length) were used in the analysis of the current data and 69 (or 119) were used for the wind. The increased number of weights for the current field is required to control leakage from surrounding frequencies (especially the semi-diurnal tide). As a test, demodulation was run at the tidal frequency and this was compared to the demodulate at the inertial frequency. Visually it appeared that no contamination had occurred.

Comparisons between the wind and current complex demodulates are difficult to make. Because the analysis was carried out on rectangular coordinates, shifts in wind direction are indistinguishable with change in intensity when looking only at one component. By analyzing both of the wind demodulates with the current demodulate, this problem can be reduced. Also one might expect that the lag or response time of the ocean would be different in generation and decay. This would inhibit numerical analysis of the complex demodulates.

Pillsbury (1972) points out that lengthening of the filter array eliminates contamination but it also stretches out, in time domain, the phenomena at the frequency of interest. For example, he used
241 hours of weights and observed that the complex demodulate began to change five days before the actual event occurred and it continued for five days after the event had ceased. Since time domain analysis is much more susceptible to leakage than is frequency domain analysis, a compromise must be reached in regard to the length of the filter.

Rotary auto-spectrum were generated for wind and current velocities and accelerations and for wind stress. (The computer programs used for rotary spectral and bispectral analysis and for

\[ W_{si} = \left( W_1^2 + W_2^2 \right)^{1/2} W_i \rho C_D. \]

\( W_1 \) and \( W_2 \) are the rectangular components of the wind and the subscript \( i \) denotes the desired scalar component of the wind stress. \( \rho \) is the density of air

\( (1.25 \times 10^{-3} \text{ gm/cm}^3) \)

and \( C_D \) is the drag coefficient

\( (2.6 \times 10^{-3}). \)
the energy transfer functions were written by Yao (1974) and used in his thesis.) In order to achieve increased degrees of freedom, overlapping of the 309 data was done at one half of the segment length. The fast Fourier transform technique was used to calculate the spectrum and the most optimum segment length was found to be 128. At this length and with the overlapping of the data, the equivalent degrees of freedom are approximately 14. Data series 298 has 14 degrees of freedom and it was not overlapped.

Rotary cross spectral estimates were made between various pairs of data. Since the rotary cross coherence is easier to interpret, it and not the rotary cross spectrum will be presented. A test of significance usable for such coherences is given in Jenkins and Watts (1968) and is employed by Yao (1974). The test is based on the fact that a particular random variable, that is a function of the number of equivalent degrees of freedom and of the coherence estimate squared, is distributed as $F_{2, (EDF-2)}^2$. When this random variable is less than the Fisher function, that estimate of the coherence is not significantly greater than zero:

$$\frac{(EDF-2) \hat{\gamma}_{12}^2(\sigma)}{2(1-\hat{\gamma}_{12}^2(\sigma))} \leq f_{2, (EDF-2)} (1-\alpha) \quad (15)$$

where $\alpha$ is the desired significance level and $\sigma$ is the frequency of interest. $\hat{\gamma}_{12}^2(\sigma)$ is the coherence squared at frequency $\sigma$.
A significance test for auto- and cross-bispectral estimates is given by Yao (1974). The test involves the comparison of the quantity $6/EDF$ with the bicoherence squared. When the latter quantity is greater than the former, the bicoherence is significant at the five percent level. The test criteria is based on the normality of the bispectral estimates (Brillinger and Rosenblatt, 1966) and on the distribution of the estimates (Haubrich, 1964; Yao, 1974).

In analogy to second-order spectral estimates, those of third order show the amount of energy (variance) shared between the various involved processes. Obviously if energy is to be transferred non-linearly from one process to another, one would expect that a significant level of the bispectrum would be observed between those processes. Similarly for energy transfer internal to a process, significant auto-bispectral estimates would be expected at the particular frequencies. However, the auto- and cross-spectrum do not necessarily represent an energy transfer but only a sharing of energy. That energy is transferred between the frequency triplets of a significant auto-bispectral estimate can not be proven or disproven at this time; however, the existence of such triplets will be interpreted as indicating the possibility of an internal energy transfer.

In analyzing the transfer of energy between two different processes, we are more fortunate. The energy transfer functions,
derived by Yao (1974), are employed here in the analysis of several pairs of atmospheric and oceanic data. Besides the significance tests mentioned above for the cross coherence and bicoherence, an additional test is used prior to the calculation of the energy transfer function. The test is based on the idea that the spectral estimate at the Nyquist frequency represents the level of noise present throughout the spectrum. Any values of the spectrum that fall within or below the confidence limits of the Nyquist frequency component are below the noise level of the process and are excluded from the calculation of the energy transfer function.

C. Results

1. Hydrographic Conditions

Figures 1 to 3 depict the hydrographic conditions during the observations. Compare the first two casts. Figure 1 was taken about a week before mooring 298 was implanted and Figure 2 was taken four days after the implantation. Clearly, the entire upper water structure has undergone a large change in a period of about ten days. Such a rapid change might be expected if the entire existing thermal structure experienced an east-west shift (U.S. Navy Oceanographic Office, 1969).

Bathythermograph (BT) measurements taken during this time
also show large changes. The BTs show a warm front entering the area about 24 April and moving northward. Unfortunately these data extend only to the 30th of April. From these observations and from the progressive vector diagrams, Figures 4 and 5, it is postulated that the three data series, 298, 309A, and 309B, describe the formation of a ring. That these conditions describe a ring is not altogether clear (Schmitz, 1974). (However, a meander was detected on mooring 309 (Schmitz, et al., 1970).) During the early portion of the 298 record, the originating meander is seen. The meander moves to the southwest (thus accounting for the changes in the direction of the PVD of 298) and pinches off. When it does separate, the observed speeds fall substantially. The newly formed ring then travels in a northeasterly direction and transits past the 309 mooring near the end of that record.

The existence of such a dynamic phenomena in the region of measurement would not upset the energy transfer calculations. However, the various vertical thermal gradients associated with the different water masses might have a profound effect on the calculation (Halpern, 1974). In Figures 1 and 2 no vertical stratification is seen above the depth of the current meter (14 m) which might inhibit the vertical energy transport. In Figure 3, however, such a stratification is seen. (Notice though that the separation of the Nansen bottles in this case was 49 m.) If the stratification
Figure 1. Hydrographic data for site D taken on 20 April 1969. Depths are given in meters and temperature in degrees centigrade.

Figure 2. Hydrographic data for site D taken on 30 April 1969.

Figure 3. Hydrographic data for site D taken on 14 June 1969.
actually existed as drawn (linear fit between 1 m and 50 m) or if any other thermal gradient existed between the surface and the current meter, then the energy transfer calculation would yield too low a value. On the other hand, if there is a transfer of energy one would expect that such a stratification did not exist. This point will be addressed again later.

Due to the dearth of thermal structure data, little can be conclusively stated about the actual stratification during the current and wind observations. The data on hand do show that potentially disruptive structures are a possibility and that a thermal front passed over the 298 mooring.

2. Progressive Vector Diagrams (PVDs)

Figure 4 shows the PVDs for both wind and current at mooring 309. The large squares on the PVD represent 0000 GMT for each day. (The starting time for the meters was approximately 2000z.)

Oscillations near the beginning of the current PVD appear to be of tidal origin while those later in the plot are of inertial origin. The mean flow for the entire record is almost due North. This is in contrast to Webster's (1969) findings of westerly flow for site D. The unusual shape of the PVD can be accounted for if one supposes that a ring or an anticyclonic eddy from the Gulf
Figure 4. Progressive vector diagrams (PVDs) for the currents at 13 m (4a) and for the winds (4b) on mooring 309. The numbers correspond to the actual date of observation starting with July 13, 1969.
Stream was being advected past the mooring. If a ring were moving on a northeasterly course and intersected the mooring on its left side, one would expect to see first an easterly component, then a northerly component and finally a westerly component, all of approximately the same magnitude. As long as the center of the ring were to the east of the mooring, no southern flow would be experienced and thus the net movement would be northerly. This hypothesis compares favorably with the PVD. 5/

The wind PVD at 309 has a highly erratic behavior initially. In order to more clearly see the wind pattern, the first part of the PVD has been subdivided into three diagrams (Figure 6). As can be seen, the wind field is highly variable for the first month or so of observations. During this period, there is very little directional similarity between the wind and the current fields. The last three weeks of observation of the wind have a much less erratic tendency. Especially in the last two weeks the wind is fairly persistent. In this stable regime, the wind vectors are oriented to the northeast, while the current vectors are pointing to the northwest. Clearly, this is not a normal case of Ekman flow.

The PVD for record 298 is shown in Figure 5. For the first four weeks, the wind is blowing to the east. The current shows a predominantly northerly motion with large velocities. During the next two weeks, the current PVD is southerly and the wind PVD is

5/ For the same time period, the current field at 298, separated by nine miles from 309, exhibits almost exactly the same features.
Figure 5. Progressive vector diagrams (PVDs) for the currents at 14 m (5a) and for winds (5b) on mooring 298.
Figure 6. Progressive vector diagrams for the first 28 days of the 209 wind record. Figure 6a is for the first ten days; 6b is for the next nine days (with an overlap of one day); and 6c is the last ten days (also with an overlap of one day). The small squares on the diagrams represent 0000z hours for each day. The units are given in kilometers.
slightly erratic: it performs a large cyclonic pattern. Again here, as with the 309 data, when the wind is persistent it blows to the right of the current. Could the initial behavior of the current record be due to the meander that generated the ring seen in 309? If the meander crossed the mooring and then receded to the south toward the Gulf Stream's more usual position, the resulting eddy would be in position to pass over the site D moorings as postulated above. Such a behavior would go a long way to describing the curious behavior of the flow field at this site. The hydrographic data tend to support this hypothesis in that they show the passage of an oceanic front. Except for the description of the 309 current as a possible meander (Schmitz et al., 1970), no other data or analysis is known which could prove or disprove this hypothesis.

3. Rotary Spectrum

A) Currents

Figure 7 displays the rotary spectrum of currents from record 309A. The most prominent frequencies are the inertial and the semi-diurnal. The tidal peak is slightly larger than the inertial and is a broad peak. Both of these peaks are seen in the positive as well as negative spectral planes, but at greatly reduced levels. One other noticeable (but not significant) peak is at -.1328 cycles
Figure 7. Rotary current spectrum for 309A. The units of energy are $(\text{mm/sec})^2$ and the frequency is given in cycles per hour.
per hour. The second segment, 309B, displays a different set of conditions (Figure 8). In comparing the two spectra, one sees that the tidal peaks possess almost equal levels of energy. The 309B tidal peak is a narrower peak and is only slightly smaller. However in 309B, the inertial peak is dominant in the spectrum while in 309A it was smaller than both the non-oscillating component and the semi-diurnal component. In the positive plane of 309B (non-significant) spectral features appear at the semi-diurnal frequency and at 0.04687 cycles per hour; the latter frequency is one unit of the resolved frequency band lower than the inertial frequency displayed in the negative plane. The non-oscillatory component of 309A is about twice as large as that of 309B and the variances calculated by the rotary spectral program also show more energy in the first segment. Thus from a study of these spectra, one would conclude that during the first segment the current field was more energetic and was dominated by the semi-diurnal tide; in the second segment the inertial motions dominated and the current field was less energetic. These facts are borne out in the PVD in the auto-correlations, and in the diagram of current speed squared

6/ The auto-correlations are not shown. For the first segment (309A), the auto-correlations have a correlation peak at a lag of about 12 hours (with the east-west component having larger correlations than the north-south component). In the second segment, the north component correlation peaks were lagged by a period of 18 hours. The east component however has a pattern of
Figure 8. Rotary spectrum for the 309B current. The units of energy are $(\text{mm/ sec})^2$ and the frequency units are in cycles per hour.
(Figure 9). Also in the inertial frequency subrange, the total spectrum is dominated by the negative frequencies.

Figure 10 shows the current spectrum for mooring 298. The non-oscillatory frequency component here is much larger than those of either of the segments of 309; the variance is also larger for 298. Frequency components other than the non-oscillatory one have relatively little energy. The inertial peak is very broad and is centered at -.04687 cycles per hour, a slightly lower frequency than for the 309 segments (-.05469 cycles per hour). This observed shift in the inertial frequency is in the proper sense since the 298 mooring was located to the south of the 309 mooring. The only other peak of any importance is found at a frequency of+.03125 cycles per hour. This corresponds to a period of approximately 33 hours. Peaks at this frequency are found in the other spectra in both the positive and negative planes. Webster (1969) attributes such a peak in his spectrum to the beat frequency of a semi-diurnal and inertial frequency interference. Such an explanation fits this data well.

correlations that showed the interference between the 12 hour semi-diurnal tides and the 18 hour inertial current. This pattern is caused by the predominantly east-west direction of the tidal current at side D (Magaard and McKee, 1973).
Figure 9. Current speed squared for mooring 309. The ordinate is the percent of the maximum value (383, 911.75 mm²/sec²).
Figure 10. Rotary current spectrum for 298. The energy units are \((\text{mm/sec})^2 / \text{CPH}\) and the frequency units are in cycles per hour.
B) Wind and Wind Stress

The wind spectra for 309A, 309B, and 298 are shown in Figures 11, 12, and 13. In all three, there appears to be a fairly uniform decay of spectral energy with increasing frequency. The record from mooring 298 has the most energy as judged from both the total spectral variance and from the non-oscillatory component of the rotary spectrum. The wind spectrum for 309B follows next in magnitude and smallest is 309A. At the local inertial frequency, the largest energy density is found in record 298. Notice that of the three data series, the largest inertial frequency component of the wind is associated with the smallest inertial current and the smallest inertial frequency wind level is associated with the largest inertial current.

Wind stress rotary spectra are similar to the spectra of the wind field. There are only slight differences between the wind stress spectra for 309A and 309B. The latter has more energy in the non-oscillatory component, at the inertial frequency, and in the variance, but these differences are not large.

4. Autobispectrum

The autobispectra are displayed here in the same format as in Yao (1974). The origin in all four tri-frequency planes is in the upper right hand corner. The $\sigma_1$ axis is oriented from the origin
Figure 11. Rotary wind spectrum for 309A. Energy units are given in (mm/sec)$^2$ and the frequency units are in cycles per hour.
Figure 12. Rotary wind spectrum for 309B. Energy units are given in \((\text{mm/sec})^2\) and the frequency units are in cycles per hour.
Figure 13. Rotary wind spectrum for 298. Energy units are given in \((\text{mm/ sec})^2/\text{CPH}\) and frequency is in units of cycles per hour.
towards the reader. The $\theta_2$ axis goes from right to left; the $\theta_3$ axis intersects the origin at a 45 degree angle with the other orthogonal axis (in the second and fourth quadrants). The magnitude of the auto-bispectrum has been normalized by the magnitude of the largest component (that value is printed on each of the planes). The small marks or hats on the tri-frequency planes indicate statistically significant peaks. Although there may be a significant interaction between a triplet of frequencies, the interaction is not important if it does not possess sufficient energy. Similarly, peaks with large amounts of energy might not be significant.

A) Current

The autobispectra of the 309A current record is shown in Figure 14. In the positive sum plane (upper right plane) there are no significant peaks in the ridge. In the positive difference plane can be seen a dominant range. This is the manifestation of the $\theta_3 = 0$ axis and will be seen in all the autobispectral plots. Little information is gained from the negative sum (lower left) plane but the negative difference is very interesting. Here the continuation of the $\theta_3 = 0$ axis from the positive difference plane is clearly evident, but also notice the secondary ridge behind it. The separation between the two ridges is equivalent to the semi-diurnal frequency. This coincides with the previously given results from the rotary spectrum and the auto-correlation of this record. Thus the semi-diurnal tide is a dominant process in non-linear inter-
$\mathbf{v}_2 - \mathbf{v}_0 = - \mathbf{v}_f$

$\mathbf{v}_2 + \mathbf{v}_0 = \mathbf{v}_f$

Figure 14. Rotary autobispectrum of 309A currents at 13 m depth.
\[ C_1 - C_2 = C_3 \]

\[ C_1 + C_2 = C_3 \]

\[ -C_1 - C_2 = -C_3 \]

\[ -C_1 + C_2 = C_3 \]

Figure 15. Rotary autobispectrum of the 309B currents at 13 m depth.
actions as is the non-oscillatory component.

For 309B, refer to Figure 15. Notice in comparison to 309A, that the maximum value (used for normalization) is slightly smaller in 309B. The positive sum and difference planes show no interesting features except for the $0_3 = 0$ axis in the latter. Three large and significant peaks are visible in the negative sum plane. The interacting frequency triplets are listed below in order of their magnitude:

\[-0.05469 -0.05469 = -0.1094\]
\[-0.0078 -0.04688 = -0.05469\]
\[-0.07813 -0.07813 = -0.15625\]

Thus the first triplet involves the interaction of the inertial component with itself and with a third frequency. The second involves the inertial frequency in the position of $0_3$. The third is the self interaction of the semi-diurnal tide with the component at twice its frequency.

More interesting is the nature of the internal interactions in the negative difference plane of 309B. In this plot, the double ridge system is seen again. However, the second ridge in 309B is less defined than that in 309A and is slightly closer to the $0_3 = 0$ ridge. The ridge separation is approximately equal to the inertial frequency which again agrees with the previous analysis which showed the stronger influence of the inertial currents in 309B.
However, the reason for the ill-definition of the second ridge is that a small semi-diurnal ridge is also present. If more interacting pairs had been involved with tides, a three ridge system would have clearly arisen.

In this study, the rotary autobispectra are used merely to confirm previous interpretations and analysis. However, the above description shows that, with practice, the autobispectrum could be used for preliminary description of a random process.

As mentioned above, the bispectra, whether auto- or cross-, do not represent energy transfer processes. However, the existence of an important and significant peak in the bispectrum can be interpreted as signifying the possibility of an energy transfer.

Using this reasoning, the following autobispectra triplets which contribute to the inertial current are listed:

<table>
<thead>
<tr>
<th>frequency triplets (cph)</th>
<th>triplet periods (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>309A 0.3125 -0.08594 = -0.05469</td>
<td>32.0, 11.6; 18.3</td>
</tr>
<tr>
<td>309B 0.05469 -1.0938 = -0.05469</td>
<td>18.3, 9.14; 18.3</td>
</tr>
<tr>
<td>0.00781 -0.04688 = -0.05469</td>
<td>12.8, 21.3; 18.3</td>
</tr>
</tbody>
</table>

Thus if internal generation of inertial currents was a factor, the above listed triplets would be expected to be the major contributors.

The triplet from 309A and the first triplet from 309B each involve two interacting frequencies with different signs. This would represent the non-linear interaction between two counterrotating current
components to yield an inertial current. In the second of these two, one of the frequencies involved is the positive inertial frequency and thus, that interaction would be converting energy from the positive to the negative rotating inertial currents. The third triplet is the sum of negative values and could represent a different type of process (e.g., low to high frequency cascade of energy). These findings are highly speculative and are presented only to indicate the possibility of internal generation of inertial currents and of the possible significance of exchanges of energy internal to the current field. This sort of process might become important if it is found that energy is transferred from the atmosphere to the ocean at some predominant frequency (or frequency band) and then redistributed in the current field. Statistical methods for answering these questions do not yet exist.

B) Wind and Wind Stress

Figures 16 and 17 contain the rotary autobispectra of the wind stress for moorings 309A and 309B, respectively. Because of the great similarity between the wind stress autobispectra and

\[7/\] It is possible to trace such a hypothetical energy transfer "path" in the autobispectra. These "paths" are not, in general, direct, but lead through several interacting triplets.
\[ \gamma_1 - \gamma_2 = -\gamma_y \]
\[ \gamma_1 + \gamma_2 = \gamma_y \]

Figure 16. Rotary autobispectrum of the 309A wind stress.
\( \gamma_1 - \gamma_2 = -\gamma_3 \)

\( \gamma_1 + \gamma_2 = \gamma_3 \)

Figure 17. Rotary autobispectrum of the 309B wind stress.
that of the wind, only the former are shown. In making comparisons between the first and second segments, notice that the record is normalized with a much larger value than is the first. In both segments, the $\sigma_3 = 0$ ridge is clearly evident in the positive and negative difference planes. All features in the topography are either clustered around the origin or lie along $\sigma_3 = 0$.

The autobispectra generated by this research compare favorably with those presented by Yao (1974). Although differences do exist, clearly the general topographic features are common to both sets of data.

5. **Rotary Cross Coherence**

A number of combinations of data forms were used in the study of cross coherences. In general very few frequencies had coherences that were significant at the five percent level. A brief discussion of some of these is given below.

In comparing the rotary coherence of winds to currents for the three data sets (309A, 309B, and 298), two common features appear. Significant peaks at a frequency of 0.03125 (both positive and negative angular frequencies) are in common in both 309B and

---

8/ Following is a list of combinations tried: wind to current; wind stress to current; wind stress to current acceleration; current acceleration to wind accelerations; and wind accelerations to current accelerations.
298 (see Figures 18 and 19). Both 309A and 298 display significant peaks at frequencies of .1875 and of .4922 cycles per hour (rotary coherence for 309A is seen in Figure 20). Various other significant peaks are present, but are not held in common. Also of interest is the lack of any coherence at the inertial frequency. If generation of inertial currents by winds were taking place during the periods of these records, the resonance theory would require a coherence at the inertial frequency. \(^9\) This point will be brought up again. \(^10\)

6. **Complex Demodulation**

Complex demodulates of the current were run at the semi-diurnal and at the inertial frequencies. As discussed above, no correlation (leakage) was seen to exist between the demodulates at these frequencies. Figure 21 is the complex demodulate at the east component (309A and 309B), currents at the inertial frequency.

\(^9\) Coherences that were calculated on rectangular coordinates also showed no significant coherence at the inertial frequency.

\(^10\) The only rotary coherence which was significant at the inertial frequency was on the wind stress to current acceleration coherence for the first segment. Thus coherence occurred at the positive angular inertial frequency (whereas the inertial current is a negatively rotating component).
Figure 18. Rotary coherence between the wind and current for 309B.
Figure 19. Rotary coherence between the wind and the current for 298.
Figure 20. Rotary coherence between the wind and current for 309A.

5% significance level
Figure 21. Complex demodulate of the east component of 309 current. The gap separates the 309A segment and the 309B segment. The ordinate is the common logarithm of the complex demodulate.
The common logarithm of the demodulate is plotted on the ordinate. The daily averaged wind speeds (east component) are shown in Figure 22. There appears to be a valid connection between these two plots. Almost all of the features in the inertial demodulate can be traced to features in the wind field. When large changes in the wind direction occur, there are almost always prominent changes in the current: minimum points, maximum points, or areas of large changes in the demodulate amplitude. Only the period of day 32 to day 34 appears to be unresponsive to the wind. This could be due to either the advection of a shallow stably stratified water mass onto the mooring (Figure 3 indicated that this might be the case) or due to the variation in the wind components in such a way that no effective changes are seen by the current (phase lock for instance, Pollard and Millard, 1970). No clear solution can be given to this problem; however, complex demodulates of the wind at frequencies .5260 and .1660 show that the wind scalar components seem to be amplitude compensating: increased energy in one component is accompanied by a corresponding decrease in the other component. Thus this is the preferred explanation.

The wind records were demodulated at various frequencies. By examining one scalar component at a time, a good correlation between the amplitudes of the complex demodulates of wind and
Figure 22. Wind speeds averaged to daily values for the east-west component (above) and for the north-south component (below) of the 309 wind. Speed is given in mm/sec.
current can not be visually established. However, by combining the two scalar components, reasonable fits can be established at the frequencies tested (.05260 and .16660 cycles per hour).

7. **Rotary Cross Bispectrum**

The rotary cross bispectra are presented in the same format as the rotary autobispectra were. See paragraph (4) for details.

A) Wind to Current

Figure 23 is the rotary cross bispectra for wind to current velocities in the first data segment, 309A. Few features are visible in the positive and negative sum planes and in the positive difference plane (except for the ubiquitous $\varnothing_3 = 0$ line). A double ridge system can be faintly seen in the topography of the negative difference plane. The separation of the two ridges is reminiscent of the separation of the first segment autobispectrum. In the rotary cross bispectrum for the second segment, Figure 24, the same general topography is seen. The normalization factor is larger in the first segment cross bispectrum than in the second. The $\varnothing_3 = 0$ line is as prominent as it was for the 309A rotary cross bispectrum and there is some grouping around the origin, especially noticeable in the negative sum plane, as there were for the previous plots. The multiple ridge system in the negative difference plane is more clearly visible than it was above, and as
\[ \gamma_x - \gamma_z = - \gamma_y \]

\[ \gamma_x + \gamma_z = \gamma_y \]

\[ - \gamma_x - \gamma_z = - \gamma_y \]

\[ - \gamma_x + \gamma_z = \gamma_y \]

Figure 23. Rotary cross bispectrum of the winds to the currents for 309A.
Figure 24. Rotary cross bispectrum of the winds to the currents for 309B.
was seen in the rotary autobispectrum, the ridge system is most probably a triple ridge rather than a double ridge. The three lines of constant $\mathcal{U}_3$ are: the zero frequency, the inertial frequency, and the semi-diurnal tidal frequency. Of the eight significant peaks in this negative difference plane, all are associated with either the inertial current or the tidal influence.

B) Wind Stress to Current

The rotary cross bispectrum of the second segment (309B) wind stress to current is shown in Figure 25. In comparing this set of plots to the previous set, a more definite ridge system at $\mathcal{U}_3 = 0$ can be seen in both difference planes. In both cases, the ridge extends further across the tri-frequency plane than it did in the wind to current rotary cross bispectra. The ridging in the negative difference plane more clearly shows the narrow separation which has been previously identified as being inertial oscillation dominated. Again, the trace of a third ridge can be observed.

C) Wind Stress to Current Acceleration

Figure 26 gives the rotary cross bispectral plot of wind stress to current acceleration for the second segment. There are several differences between these displays and those already given. The cross bispectra are noisy and details are more difficult to extract. At low values of $\mathcal{U}_1$, banding can be seen. The effect of the
Figure 25. Rotary cross bispectrum of the wind stress to currents for 309B.
Figure 26. Rotary cross bispectrum of the wind stress to current accelerations for 309B.
derivative operation on the velocity field is seen in the removal of the $\sigma_3 = 0$ ridge. The ridge can be found, but only in the immediate region of the origin. The prominent ridge system seen is a composite of the $\sigma_3 = $ inertial and the $\sigma_3 = $ semi-diurnal.

D) Currents to Winds

The rotary cross bispectrum of currents to winds was calculated but is not shown. No topography was observed except for two peaks in the negative sum plane. These two involved the self interactions of the semi-diurnal tide and the inertial current. Because these two processes so dominate the current regime, the interactions with themselves would be expected to show up in this analysis. Note that no such "favored" frequencies occur in the wind regime and there is no reason to expect such a finding in the atmospheric to oceanic rotary cross bispectra.

8. Energy Transfer Functions

Both linear and non-linear energy transfer functions are calculated. The former will be discussed first. Following that, diagrams of the non-linear transfer will be presented and then the non-linear transfers themselves will be examined.

A) Linear Transfer of Energy

Linear transfer functions were found at several frequencies
in the combinations of data. At $\sigma_3 = .03125$ cph, both 309A and 298 showed strong linear contributions in the wind to current energy transfer (see Figures 27 and 28). Of curious interest is the close fit of the linear transfer at the semi-diurnal frequency for the first segment (Figure 29). Other linear transfer functions were not common to more than one set of data except for the sharing at $\sigma_3 = .1875$ cph in both 298 and 309A. In no case was there found any energy transferred linearly to the inertial current.

B) Topography of Non-linear Transfer

Two dimensional plots of the non-linear energy transfers can be seen in Figure 30. There are no obvious groupings of the frequency components that comprise the non-linear energy transfer except in data set 309B. Here a preponderance of frequency pairs are found between $\sigma_3 = -.03125$ and $\sigma_3 = +.04688$.

For comparison, the topography that was bound by Yao (1974) is illustrated in Figure 31. In this diagram there are more interacting frequency pairs but no pattern can be described. However, the topography compares favorably with the topography for 309A

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11/ These plots show the rotary spectra of the current field and the contributions to that field calculated from either the wind or the wind stress field by the energy transfer functions.
Figure 27. Energy transferred from the wind to the currents for 309B. The vertical bars indicate the level of energy transfer that was calculated. The dotted bars represent the linear transfer and the solid bars represent non-linear transfer: narrow bars for the sum transfer function, and wide bars for the difference transfer function. Also displayed is the 309B current spectrum.
Figure 28. Energy transferred from the wind to the current for 298. The dotted bars represent the linear energy transferred; the narrow solid bars denote the sum non-linear transfer and the wide bars denote the difference non-linear transfer.
Figure 29. Energy transferred from the wind to the current for 309A. The dotted bars represent the linear energy transfer; the solid bars, narrow and wide, represent the sum and difference non-linear energy transfer, respectively.
Figure 30. Topographic plots of the energy transfer functions for 309A and 309B.

Figure 31. Topographic plot of the energy transfer functions found by Yao (1974).
(although the density of transfer triplets is smaller in 309A). A three dimensional plot of Yao's (1974) energy transfer functions was produced but it did not contribute greatly to an understanding of the processes.

Figure 32 presents another comparison between the energy transfer found in this research and that found by Yao (1974). Plotted in this figure are the percentages of the observed currents that are accounted for by the transfer calculation. No pattern can be recognized in both of the data sets.

C) Non-linear Energy Transfer Functions

The total energy density of the current fields that can be accounted for by the energy transfer functions is given in Table 1. Only the frequency elements up to $\sigma_3 = \pm 0.1797$ are used in this calculation in order to permit comparisons with the results of Yao (1974). Thus whereas Yao was able to account for 63 percent of the energy in his current field (at 14 m depth), the percentages for all three sets of Atlantic Ocean data are considerably smaller. Clearly there are differences in the physical environment between the two data sites which must be responsible for this wide discrepancy.

1) Wind to Current

Figures 29, 27, and 28 are the energy transfers for the
Table 1. Contributions (percent).

<table>
<thead>
<tr>
<th>Segment</th>
<th>Positive Frequencies</th>
<th>Negative Frequencies</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Wind to Current</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>309A</td>
<td>4.3</td>
<td>22.0</td>
<td>19.9</td>
</tr>
<tr>
<td>309B</td>
<td>14.1</td>
<td>23.9</td>
<td>23.7</td>
</tr>
<tr>
<td>298</td>
<td>12.6</td>
<td>4.7</td>
<td>10.0</td>
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<tr>
<td>B. Wind Stress to Currents</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>309A</td>
<td>15.7</td>
<td>27.2</td>
<td>26.6</td>
</tr>
<tr>
<td>309B</td>
<td>27.1</td>
<td>16.6</td>
<td>19.5</td>
</tr>
</tbody>
</table>

Table 1A. Percents of the observed current spectrum that can be accounted for by the energy transferred from the wind. These values were obtained by summing up the energy that is transferred on a subrange of the frequency axis (0 - .1797 cph) to facilitate a comparison to the results present in Yao (1974).

Table 1B. Percent of the observed current spectrum that can be accounted for by the transfer of energy from the wind stress.
Figure 32. Comparison between the energy transferred for 309B and that found by Yao (1974). The upper plot is the percentage of the observed current spectrum that is accounted for by the energy transfer calculations. The lower plots are the current spectra for the two data sets. The 309B data is represented by the dotted bars.
wind-to-current for data sets 309A, 309B and 298, respectively. For the first of these, 309A, notice that there is no energy transfer to the inertial current \( (\Omega_3 = -0.05469) \). There is a large quantity of energy transferred linearly at the semi-diurnal frequency and at the adjacent frequency. Compared to the energy transfer function for 309B, 309A has few low-frequency transfers. 309B is also different in that it has a non-linear transfer to the inertial current (which accounts for 22 percent of the observed energy there) and has a non-linear vice linear transfer at the semi-diurnal frequency. The third wind to current energy transfer function depicted is similar to that of 309B in that both possess substantial linear energy transfer at both \( \pm 0.03125 \) cph. However, in 298 there is no energy transferred to either frequencies \( -0.04688 \) or to \( -0.05469 \), the inertial frequencies. At 0.0625 cph, a non-linear transfer does account for about 13 percent of the observed spectrum. Since broadening of the spectral peak is expected in non-linear interactions, could this transfer at higher frequency be part of the inertial motion? Although a possibility, it seems that a similar broadening of the peak should have been observed in the other data spectra.
2) Wind Stress to Currents

In Figures 33 and 34 are the wind stress to current energy transfers for 309A and 309B. The features described for the wind-to-current transfer are also observed in these plots. However the density of transfer along the $0_3$ axis is higher for the wind stress case and some of the transfers account for a larger percentage of the observed energy. For the wind-stress to current transfer in 309B, about 15 percent of the inertial peak can be accounted for.

3) Wind Stress to Current Accelerations

The wind stress to current acceleration energy transfer was calculated but is not shown. The same general features exist in these that did in the previous examples. The fit between the observed current spectrum and the calculated energy transfer was better at higher frequencies than it was for the other energy transfers. Because the differencing operation (used to get acceleration) acts like a high pass filter, only the semi-diurnal and inertial peaks are significant when applying the Nyquist frequency significance test. Thus these plots will not be used.

4) Current to Wind

The energy transfer function was also calculated for energy going from the current system into the wind field. As might be
Figure 33. Energy transferred from the wind stress to the current for 309A. The dotted bars represent the linear energy transfer and the wide and narrow solid bars represent the difference and sum non-linear transfer of energy, respectively.
Figure 34. Energy transferred from the wind stress to the current for 309B. The dotted bars represent the linear energy transfer and the narrow and wide solid bars represent the sum and the difference transfers, respectively.
expected, few frequencies show any transfer of energy. Linear energy transfer however appears much as it did for the wind to current energy transfer. Apparently the transfer function operation is unable to distinguish the direction of the flow of energy for linear transfers. It is interesting to note that both 309A and 309B show non-linear energy transfer in the inertial frequency in the wind. It must be remembered in view these plots that the basic hypothesis of the assignment of cause and effect has (most probably) been broken in this particular analysis.

5) 309A Wind to 309B Current, 309B Wind to 309A Current

Similar reasoning holds for the functions in Figures 35 and 36. These are the energy transfers using the wind of 309A on the current of 309B and vice versa. For the first case, there is an outstanding fit at the inertial frequency (within the confidence limits of the spectrum at that frequency) and there are acceptable fits elsewhere. For the second function, the inertial fit is not so good, but there is a non-linear transfer there. The interpretation given these functions is that the energy contained in either of the wind fields is capable of eliciting a response in the inertial current. This fact by itself sheds new light on the problem. Again the cause and effect hypothesis has also been violated in the energy transfers of the 309A wind to the 309B current and the 309B wind to the 309A current.
Figure 35. Energy transferred from the wind of 309A to the current of 309B. The dotted bars represent the linear energy transfer and the narrow and wide solid bars represent the sum and the difference non-linear transfers, respectively.
Figure 36. Energy transferred from the wind of 309B to the current of 309A. The dotted bars represent the linear energy transfer and the narrow and wide solid bars represent the non-linear sum and difference energy transfers, respectively.
9. **Wind, Wind Stress and Current Accelerations**

By taking the first differences of the current, wind, and wind stress data series, corresponding acceleration series were generated. In general, the acceleration series added little or no new information to that gained from the velocity series. Also, the first differencing scheme, which is essentially a high pass filter, introduces noise into the spectra and bispectra. In the latter, this effect and the natural loss of energy with increasing frequency, cause a banding or broad ridge to form along mid-frequencies of $\Omega_1$. The common $\Omega_3 = 0$ ridge does not appear in the bispectra of first differenced data. The rotary coherence between various combinations (involving one or two acceleration series) of data series showed the same general characteristics that were noted above for coherences, but in some cases, the particular frequencies with significant energy were not the same.

After analyzing the many combinations of data series, it appears that the wind to current and the wind stress to current combinations are the most valuable. Differences appear between these two combinations, but one is not clearly better than the other.
III DISCUSSION

A. Causes of Low Energy Transfer

As noted above, the quantitative results of this research do not compare favorably with the only previous such calculations (Yao, 1974). Because of the wide disparity in results, a comparison of the physical environments in the two situations is in order.

There are several possible explanations for the small energy transfer observed. Eight suggested possibilities are:

1. Mooring induced noise
2. Strength of the wind field
3. Advection of inertial currents past the mooring
4. Stratification
5. Internal generation of inertial currents
6. Resonance generation
7. Non-universal energy transfer functions
8. Persistence of the wind field

1. A valid case might be made that the noise level permitted by the type of surface mooring used in this study degraded the quality of the measurements (through aliasing) to a point where statistical significance could not be obtained. The mooring used by Yao (1974) was the TOTEM buoy which experiences essentially no gravity wave induced motion. Thus there is a contrast between the stability of the two moorings. However, the rotary spectrum and the rotary bispectrum from the two mooring sites are very similar. Both show dominant inertial and semi-diurnal peaks.
High frequencies fall off in both sets and there is no real indication of aliasing. Obviously, the burst sampling method (Webster and Fofonoff, 1971) is effective in eliminating much of the noise. Since many researchers (Halpern, 1974; Webster, 1969; Gonella, 1972) have used this type of mooring with no reported contaminated statistics, it is felt that the mooring type has little to do with the observed behavior.

2. In comparing the wind fields of the two segments, 309A and 309B of the same mooring, one notices in the latter that both the wind strength and the inertial current are stronger and that there is a calculated energy transfer to the inertial current. Thus if some minimum wind strength needs to exist before generation of inertial motion occurs, then these observations would support that hypothesis. However, the wind strength is even stronger in the 298 sequence than it was in 309B, and in 298 there is no energy transferred to the inertial current. Examination of the plot of wind speed squared showed no correlation whatsoever with the current complex demodulate. So it appears as if the intensity of the wind can not account for the observed discrepancies in the energy transfer function.

3. That advection of an inertial current past a current meter can account for observed generation and decay of the current has been argued by Pollard (1974). Undoubtedly such advection occurs,
but it is probably not important here. The PVDs of the two moorings show a homogeneous wind and current regime over a separation of about nine nautical miles. Thus although the current meter might measure inertial currents that were generated at locations other than at its mooring, the other location probably experienced the same physical conditions that existed at the mooring. More conclusive, is the evidence that a qualitative correlation exists between the daily averaged winds and the inertial current complex demodulate (Figures 21 and 22). Thus whether advection exists or does not exist is not an important question in this analysis due to the scale of horizontal homogeneity.

4. That stratification might be important in determining the level of energy transfer has been most recently stated by Halpern (1974). The hydrographic data required to ascertain the stratification at site D is not available. The data that is available shows that a near surface stratification regime was present near the 298 mooring. There is no way to determine what effect, if any, this stratification had on the generation of inertial currents near the mooring, nor is it possible to determine if other such stratifications existed at other times during the observations. However, the daily averaged winds (and especially the daily wind shifts) seem to directly influence the current system for the 309 mooring. Halpern (1974) shows that an influence is not felt by current meters
that are below the well-mixed layer. Thus it appears that throughout the 309 record, the wind field is not prohibited by the hydrographic conditions from influencing the inertial motion. In the 298 record, the correlation between the daily averaged winds and the inertial oscillation, although present, is not as distinct. For this record it might be argued that stratification did play a role in inhibiting the transfer of energy.

5. Since the variation in the daily averaged wind vectors correlates well with the complex demodulate of the inertial current, it seems that internal generation of inertial currents must play, at most a secondary role. Though there must be some modification of the inertial currents by the existing current field, the wind field itself correlates well with the observed behavior in the 309 data set.

6. The hypothesis of resonant generation of inertial currents could possibly account for differences in the strength of those currents. If the period of weak inertial currents were associated with small inertial frequency components in the wind field and periods of large inertial currents with strong inertial winds, an argument in terms of resonance theory would be possible. However in the 309 data record, just the opposite is the case: 309A had more energy at the inertial frequency in the winds and less in the currents than did 309B. Also, no linear coherences were
found linking the inertial current to the wind. Thus it is assumed that there is no resonance effect whatsoever in the generation of the observed inertial currents.

7. It is a possibility that the energy transfer function is not universal. That is, the particular frequency triplets that interact in one locale at one time may be very different from those which interact in a different environment. However, both those functions derived in this study and those found by Yao (1974) show continuum type interactions instead of discrete interactions. No preferred frequencies were discovered in either study (Figures 30 and 31). The conclusion drawn from this is that the energy transfer function is highly dependent upon the structures of the wind and current fields under measurement. However the transfer of energy occurs throughout the tri-frequency plane and although the physical processes involved in the transfer are, in all probability, universal, the transfer frequencies are dependent upon the particular physical situation.

8. The last possibility to be discussed is that of persistence of the wind field. Persistence in this case is meant to define the time scale that a given wind regime remains at a location. Pollard's idea (Pollard, 1970; Pollard, 1974; and Pollard and Millard, 1970) that changes in the wind's direction can build or destroy inertial currents has received support from Halpern (1974)
and Kraus (1972). This study also agrees with that result. The correlation of the daily averaged wind speed with the inertial current is very significant. Shifts in the direction of the wind are quickly manifested in the inertial current. Most of the peaks and troughs in the inertial current demodulate (309) can be accounted for in this manner (Figure 21).

In order to understand why the energy transfer function does not show that much, if any, energy being put into the inertial oscillations, a comparison of this environment is made to that used by Yao (1974). During the period of his measurements, the wind and current fields were essentially constant in speed and direction (the east-west component has significantly less energy than the north-south component.) Throughout that observation, the same physical conditions prevailed. By contrast, during the period of observations in the Atlantic Ocean, a continuous barrage of frontal activity was taking place. The wind field would align itself in one direction for a few days and then switch. This behavior has the effect of building and then destroying an inertial current (Pollard and Millard, 1970). In a statistical analysis of this phenomena, the averaging processes would reduce the calculated energy transfer to an insignificant quantity. That this can occur is due to the fact the frequency analysis is unable to consider the relative phases of the physical processes. In the homogeneous situation used by Yao
(1974), the phase between the wind and current did not change throughout the measurements. The phase is maintained by the continuous energy communication between the wind and current. When an established communication (phase) is interrupted by a different wind field, the inertial currents will probably be destroyed. Of course in some cases, the phase of new wind field might coincide with that of the previous one and the inertial current would be expected to strengthen. However, in general, this must be assumed to be a small possibility as it is difficult to imagine the new wind field having the same relative phases (to the current field) as did the old wind field. The new wind field will re-establish inertial currents at its own phase.

Under this hypothesis, the testing of stationarity of a time series to be analyzed in this manner takes an added importance. Weak stationarity implies strong stationarity only in the case of a Gaussian process. Clearly, it has been established that the processes being dealt with in this study are not Gaussian. Thus, some other assurance must be made of the stationarity of the process to which the energy transfer function is to be applied.
B. **Generation of Inertial Oscillations**

The theoretical models discussed above which describe a resonance type interaction (Belyaev and Kolesnikov, 1967) for the generation of inertial currents are not supported by any empirical results. In particular this study and Yao's study (1974) clearly show that there is not a linear energy transfer from the atmosphere to the oceans at the inertial frequency. Thus, from the data analyzed so far, it can be concluded that resonant generation of inertial currents is not important.

The generation ideas proposed by Pollard (1970, and 1974) however seem to have some empirical support (Pollard and Millard, 1970). Yao (1974) found that there were important energy contributions to the inertial current at the frequencies postulated by Pollard: approximately one third or one fourth of an inertial period. However Pollard and Millard's results were based on frontal activity establishing inertial currents and they postulate that a homogeneous wind field will have destructive as well as creative effects. Clearly this latter idea is not supported by Yao. Although contributions to inertial current come from wind frequencies described above, the wind regime in his analysis is homogeneous in time. If the idea were true, Yao should have observed a very low energy transfer to the inertial current. However he
could account for 100 percent of the observed inertial motion.

The site D wind data used in this research have the characteristics described by Pollard (1974) for being effective inertial current generators, i.e., they describe a series of atmospheric fronts. To evaluate Pollard's hypothesis on these data the phases and rotation of the winds would have to be examined. As has been described, significant changes in the wind's (averaged to daily values) direction coincide with changes in the inertial current complex demodulate. Thus it appears that wind shifts are important in both the creation and destruction of inertial currents, the particular effect being dependent upon some phase relation. However Yao's findings show that a continuous wind adds energy to the inertial current and that it does not remove as much energy as it adds. This is in contrast to the position of Pollard and Millard (1970).

It appears that Pollard's (1970 and 1974) and Pollard and Millard's (1970) idea that wind shifts have significant effects on inertial currents is correct. However in extension of their views, it appears that continuous winds add energy to the existing inertial current structure. Thus a more complicated phase lock theory is required. Once a wind field establishes an inertial current, the phase is maintained between the two processes and energy from the field enhances the energy of that inertial current. New
wind regimes destroy the phase and the existing inertial current and re-establish a new phase and then contribute to the new inertial motion.
IV CONCLUSIONS

From the analysis of data used in this study the following conclusions are drawn.

1. That the rotary autobispectral function is a useful tool in describing the non-linear (quadratic) characteristics of a vector random process. When a process is of sufficient energy to be important, its topography can be located and identified on the tri-frequency planes.

2. That internal generation of inertial currents is possible. This possibility was seen from an examination of the frequency triplets included with the inertial frequency. Also since large linear transfer functions are found at low frequencies of $\omega_3$, it is possible that energy is transferred predominantly by a linear process and redistributed in the current field non-linearly. These hypothesis can not now be evaluated but would not contradict any of these conclusions.

3. That a resonance transfer of energy does not take place although energy may be transferred non-linearly from the inertial frequency in the wind to the inertial current.

4. That energy is transferred from the regime of atmospheric motions to the oceanic current field over a wide range of frequencies. No distribution pattern of transfer functions was noted.
5. That complex demodulation is a useful tool for establishing when the current field and wind field are exchanging energy.

6. That changes of wind fields have profound importance on inertial currents. Continuity of a wind field allows continuity of the inertial oscillations. Thus some sort of phase relation is established between a wind field and the ocean and is maintained until that wind field is replaced by another wind system.

7. That the energy transfer functions derived by Yao (1974) provide an accurate description of the energy transfer only when the meteorological conditions are stationary in time. Because of their inability to take in account the phase relations, non-stationary processes will in general show a much lower energy transfer than actually exists.
BIBLIOGRAPHY


