LAMBERT COORDINATES IN OREGON

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FOREWORD

This paper was originally intended for readers with a knowledge of surveying and mapping. Since it has attracted the attention of a few persons without this knowledge the author gives a few preliminary explanations which will "incline the plane" somewhat for the abrupt bluff ahead.

The surveyor or land engineer usually wants a graphic representation of his work, either as a final product or as an aid in visualizing and calculating changes to be made. If the work covers a small area such as a square mile, distance, angle, and sometimes elevation relationships between points are determined in the field by plane surveying, i. e., without regard to the earth's curvature. (Elevation is considered in this paper only as it affects horizonal distances measured on high mountains with respect to what those distances would have been at sea-level.)

Perhaps the best way of putting these points on the graphic representation or map is through the use of plane rectangular coordinates. The coordinates of different points are found in the following manner: Axes are set at right angles (one north-south and one east-west) at some convenient place, usually to the west and south of the area to be mapped. In practice coordinates are usually assigned to some point; if the bearing (horizonal angle from north or south) of any survey line through this point is known, the origin or crossing point of the axes is fixed. By consideration of the angles and distances between points the perpendicular distance from each axis to any given point is found. The easterly distance from the north-south axis is the "X" coordinate; the northerly distance from the east-west axis is the "Y" coordinate.

When the coordinates are found the points can be plotted to the desired scale (reduction) on the map sheet which has been ruled with a grid whose lines are marked with their "X" and "Y" distances from the axes. If the points are plotted in this manner they are positioned on the map in the same horizonal relationship to each other as they were on the ground.

An illustration using a chessboard may simplify the idea. Place several oversize chessmen at random on a glass-top table. Raise an oversize chess-board under the table and hold stationary under the chessmen. Now, count and record the squares to the right and upward from the lower left hand corner to each chessman. Without looking at the large chessmen but using the numbers of squares, place ordinary chessmen on an ordinary chessboard in the same place they were on the larger one. Compare the two boards and you see the smaller as an approximate reproduction of the larger. You have just used the plane rectangular coordinate principle of map-making.

The above mentioned condition of ideal relationships will not remain if the one square mile area were "extended" to include the county or the state. Due to picturing the curved surface of the earth on a plane surface there are disfigurations and improper relationships of directions and/or distances which are intolerable to the engineer requiring an accurate map. "Map projection" techniques, more complex than the direct calculation of plane coordinates as explained, become necessary. One such projection is the "Lambert conformal conic projection" which first mathematically changes a portion of the earth's surface to a cone then flattens this cone into a plane. This is exemplified by peeling a portion of an apple and laying the peeling flat. Thus in use the Lambert coordinates become ordinary plane coordinates.

In this projection as in many others, geographic coordinates of one or more points are required. Geographic coordinates are simply latitude and longitude. These geographic coordinates may have been so precisely determined as to indicate the position of a point on the earth's surface accurate to about one inch. This position is found by astronomic observation and triangulation which are beyond the scope of this paper. However, Supplement II gives data on "control stations", points whose geographical coordinates have been determined.

Application of Lambert coordinates is simpler than their explanation. Good Luck!

INTRODUCTION

This paper is an attempt to consolidate and simplify general features of the Lambert Coordinate System as they apply to the land surveyor and to the planner of an extensive survey. A sample of computations is included with directions for traverse computation on the Lambert grid.

Since the recent adoption of this system it has slowly found favor from many quarters and is predicted to be of increasing use. An understanding is desireable for one concerned with any survey activity covering a large area now or perhaps only a few sections soon.

SUMMARY

To project a traverse on the Lambert grid one should start at one and preferably end at another control station of first or second order accuracy. This control will usually be triangulation stations of the United States Coast and Geodetic Survey or the United States Army Engineers or stations of the State Highway System. Further control information is in Supplement II.

Using grid azimuths from the control station, azimuths are carried through the traverse and checked at the ending station. Ground lengths are easily converted to grid lengths if necessary. Then with published or easily computed Lambert coordinates of the beginning station, plane coordinates are calculated for traverse stations and checked at the ending station.

HISTORY

Answering a demand from city, county, state highway engineers, and others, the United States Coast and Geodetic Survey in 1932 made a study of various map projections to determine a means of extending simply calculated plane coordinates accurately over large areas. Their senior mathematician, O. S. Adams, found the Lambert Conformal Conic Projection (commonly called Lambert grid, state system, etc.) excellent for areas of greatest extent in an east-west direction. The Survey adopted this projection for states of such shape and divided others approximately square into two or more zones not exceeding 158 miles in width which gives a scale error rarely over 1 in 10,000. Oregon was thus divided into two zones. By 1935 the U. S. C. and G. S. had published tables for calculation of coordinates on this grid and has since computed Lambert coordinates for many stations of its extensive triangulation nets.

Lambert coordinates were incorporated in the Oregon Laws in 1945.

VALUE

Advantages of the system are savings in field survey, easier plotting of maps, and certainty in legal descriptions. Special control surveys of extensive projects, as aerial surveys, are quicker, cheaper, and easier checked through the use of U. S. C. and G. S. stations which are best tied to the survey by coordinates.

Lambert coordinates are far more simple than geographic and polyconic coordinates but are sufficiently accurate and may be extended indefinitely in an east-west direction. Scale distortion is small and more easily figured than in other projections. An advantage to the surveyor is shown by the entitling "conformal" which means that small areas on the map are similar in shape to corresponding areas on the ground. Also grid and geodetic angles between lines up to ten miles long may be considered equal. After an extensive survey of an area the data may be compiled and boundaries correlated without field determination.

Corners with properly calculated coordinates are referenced by other corners and by the national triangulation net. Of course, coordinates cannot outweigh physical evidence of a corner monument. An examination of coordinate descriptions will determine if parcels lotted to a certain tract are actually in that tract. In the east many tax maps have been based on Lambert Coordinates.

A disadvantage of the state system is its inflexibility. Tables are computed for standard parallels and central meridians which cannot be changed to give lower scale and azimuth corrections at the local survey.

Use of the state system for description in public land records and deed records is virtually outlawed in Oregon. The law requires that a point so described shall be within one-half mile of a first or second-order survey station whose geodetic position has been rigidly adjusted on the North America datum of 1927.

The only use many logging operators have for any coordinates are as latitude and departure of points on a single section, so this accurate projection is unnecessary. An Oregon aerial photogrammetrist limits Lambert projection to a twenty-mile north-south survey, but the reasoning is vague if an accuracy of 1 in 10,000 or less is needed.

GENERAL FEATURES

See next page for graphic representation.

Standard parallels were chosen so that two-thirds of the zone area is between them. This means only one-sixth of the zone is outside each standard parallel.

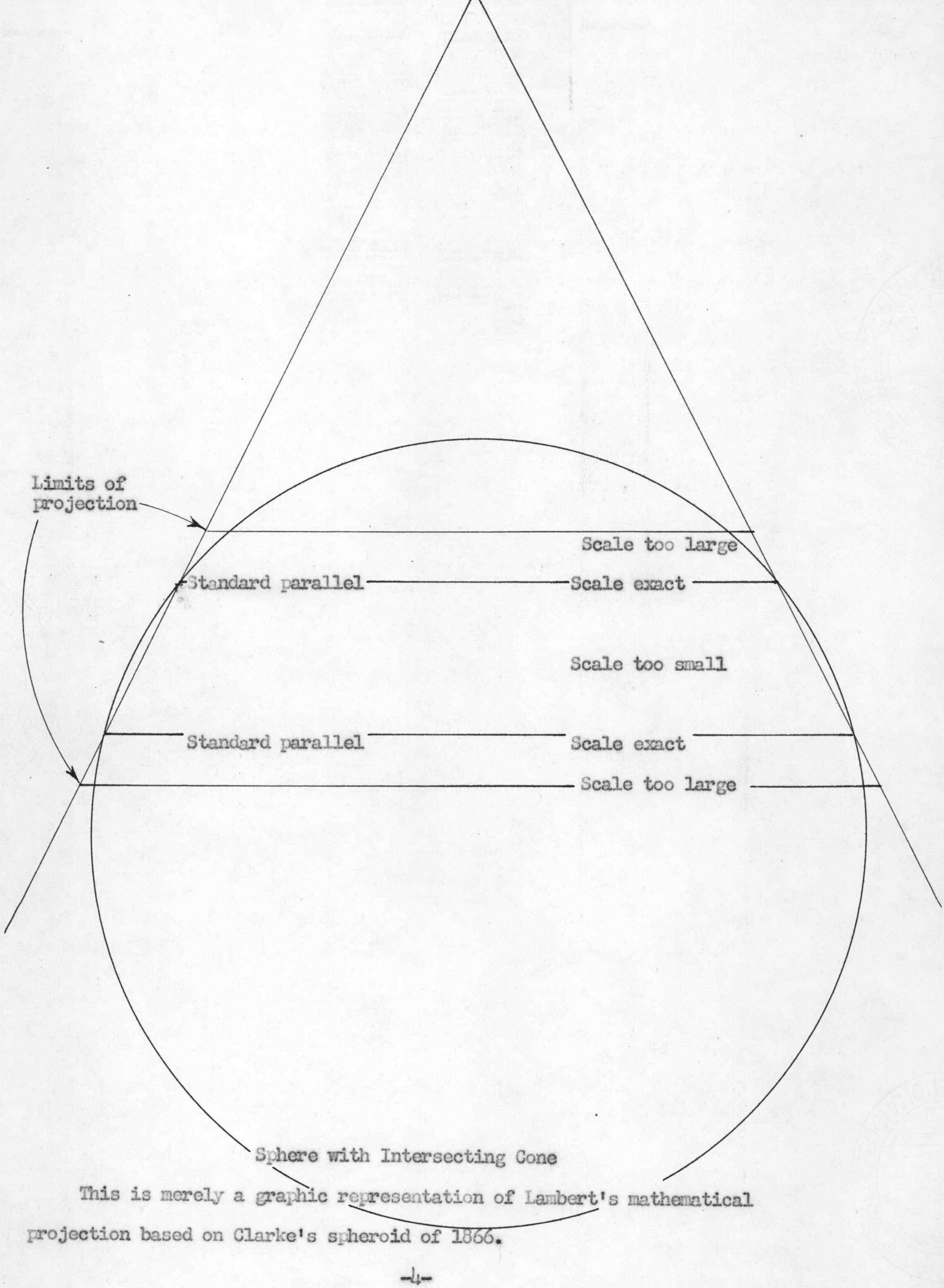
Bases of the Oregon System:

Zone	Standard Parallels	Central Parallel	Scale Ratio	Origin (Longitude)
North	44° 20' & 46° 00'	45° 10'	1:9,500	120° 30'
South	42° 20' & 44° 00'	43° 10'	1:9,500	120° 30'

Considerable overlap is allowed so an area near a zone boundary may be calculated wholly in one zone. Thus both North and South zones include the area from 43° 40' to 45° 00', a north-south distance of about 90 miles.

SCALE

The grid scale ratio is equal for equal latitudes and is given in the tables. An average can usually be taken for the survey, although the average could



Sphere with Intersecting Cone

This is merely a graphic representation of Lambert's mathematical projection based on Clarke's spheroid of 1866.

introduce too large an error. If a survey limits are 43°10' and 43°50' (about 45 miles N-S) this check is made:

Latitude	Scale Ratio
43°10'	.9998946
43°50'	.9999619
Average	.99992825
Maximum error due to use of average ratio	.00003355 or about 1:30,000

If base lengths of the U. S. C. and G. S. are used they will be geodetic lengths, e.g., reduced to sea-level. The Survey makes its measurements and computations in meters, but usually publishes results in feet.

Survey lengths may need to be reduced to sea-level, as a difference of elevation of 1,000 feet reduces the elevation scale factor about one part in 21,000. The elevation and grid factors are sometimes combined to give a negligible reduction, i.e., elevation factor .9998565 (3,000 feet) times geodetic-grid ratio 1.0001005 (North Zone, 44°00') gives a combined factor of .9999560. This shows length corrections may be neglected for grid computation of 1:20,000 accuracy although individually the factors indicate errors of 1:7,000 and 1:10,000.

The scale factor or ratio is multiplied by ground (field measured) length to give corrected length, whether sea-level or grid.

AZIMUTH

The difference between geodetic azimuth and grid azimuth is approximately theta (θ), listed in Table II and called the mapping angle. The sign of θ indicates change from geodetic to grid azimuth.

Oregon Zone	Longitude Limits	θ (maximum in the state)
North	116° 00'	+3°11' 28".8135
	125° 00'	-3°11' 28".8135
South	116° 00'	+3°04' 43".1876
	125° 00'	-3°04' 43".1876

The grid azimuth of a line is usually needed only when starting and ending a traverse. It is always obtained by calculation from coordinates except that θ may be used when the geodetic azimuth mark of a triangulation station is within one mile of the station and if the coordinates of the azimuth mark are unknown.

The U. S. C. and G. S. prefers azimuths reckoned from South instead of bearings. Difference between azimuths of two lines is the angle between them; a known azimuth plus (algebraic clockwise) an angle to a new line is the azimuth of the new line. With care no difficulty arrises in coordinate computation. If a survey is of such accuracy as may be used by the Survey, azimuths should be used.

TRAVERSE COMPUTATION

Coordinate Computation Explanation

The form shown is an aid in converting geographical to grid coordinates. In Oregon many triangulation stations of published geographical position do not have the Lambert coordinates published by the surveying agency.

This form and explanation following are for ten-column machine computation. Logarithms may be used but are somewhat slower and require trigonometric and logarithmic tables to eight decimal places, including S to over 3°. (S is the log of ratio for reducing arc expressed in seconds to Sine.) A machine will probably be more available and/or more economical. Explanation of logarithmic solution is in reference No. 6.

Project ① ______ State ② Oregon ③ North Station ④ College

Latitude ϕ ⑤ 44° 33' 48."449 Longitude λ ⑥ 123° 16' 42."835

Tabular difference of R for 1" of ϕ ⑦ 101.26550

R (for min ϕ) ⑧	21,061,814.72	$\frac{\theta}{2}$ . . . . ⑰	0° 59' 6." 9354
Cor. for sec. of ϕ ⑨	- 4,906.21	sin $\frac{\theta}{2}$ . . . . ⑱	.0171951806
R . . . . ⑩	21,056,908.51	Rsin $\frac{\theta}{2}$ . . . . . . ⑲	362,077.34
θ (for min. of λ) ⑪	-1° 57' 43."4928	Rsin $^2\frac{\theta}{2}$ . . . . . ⑳	6,225.985
Cor. for sec. of λ) ⑫	- 30."37798	2Rsin$^2\frac{\theta}{2}$ . . . . . ㉑	12, 451.97
θ . . . . ⑬	-1° 58' 13."87078	y' (for min. of ϕ) ㉒	322,037.76
Sin θ . . . . ⑭	.03438527657	Cor. for sec. of ϕ ⑨	+ 4,906.21
	2,000,000.00		
Rsin θ . . . . . ⑮	-724,047.62	y' . . . . ㉓	326,943.97
Departure X . . . ⑯	1,275,952.38	2Rsin $^2\frac{\theta}{2}$ . . . . ㉑	+12,451.97
		Latitude Y . . . ㉔	339,395.94

The problem is to find plane coordinates, X and Y.

$$X = 2{,}000{,}000.00 + R\sin\theta$$

$$Y = y' + 2R\sin\frac{2\theta}{2}$$

R, y', and θ are given in Plane Coordinate Projection Tables, Oregon.

The Army Engineers use additional tables and compute Y with logs more easily by the equivalent equation: $Y = y' + R\sin\theta\tan\frac{\theta}{2}$.

Description and geographical coordinates of the triangulation station are on the first and second lines. Items Nos. 7, 8, 11, and 22 are given in the Plane Coordinate Projection Tables, Oregon. Seconds of ø multiplied by No. 7 gives No. 9 which is always subtracted from R (for min. ø) to give R. The constant of sec. θ for 1" of λ given at the top of pages in Table II multiplied by seconds of λ gives No. 12 which is always negative and is added algebraically to No. 11 to give θ. Sine θ to ten decimals is multiplied by R to give No. 15. Item No. 15 is added, algebraically as the sign of θ, to the constant 2,000,000.00 to give the desired departure X.

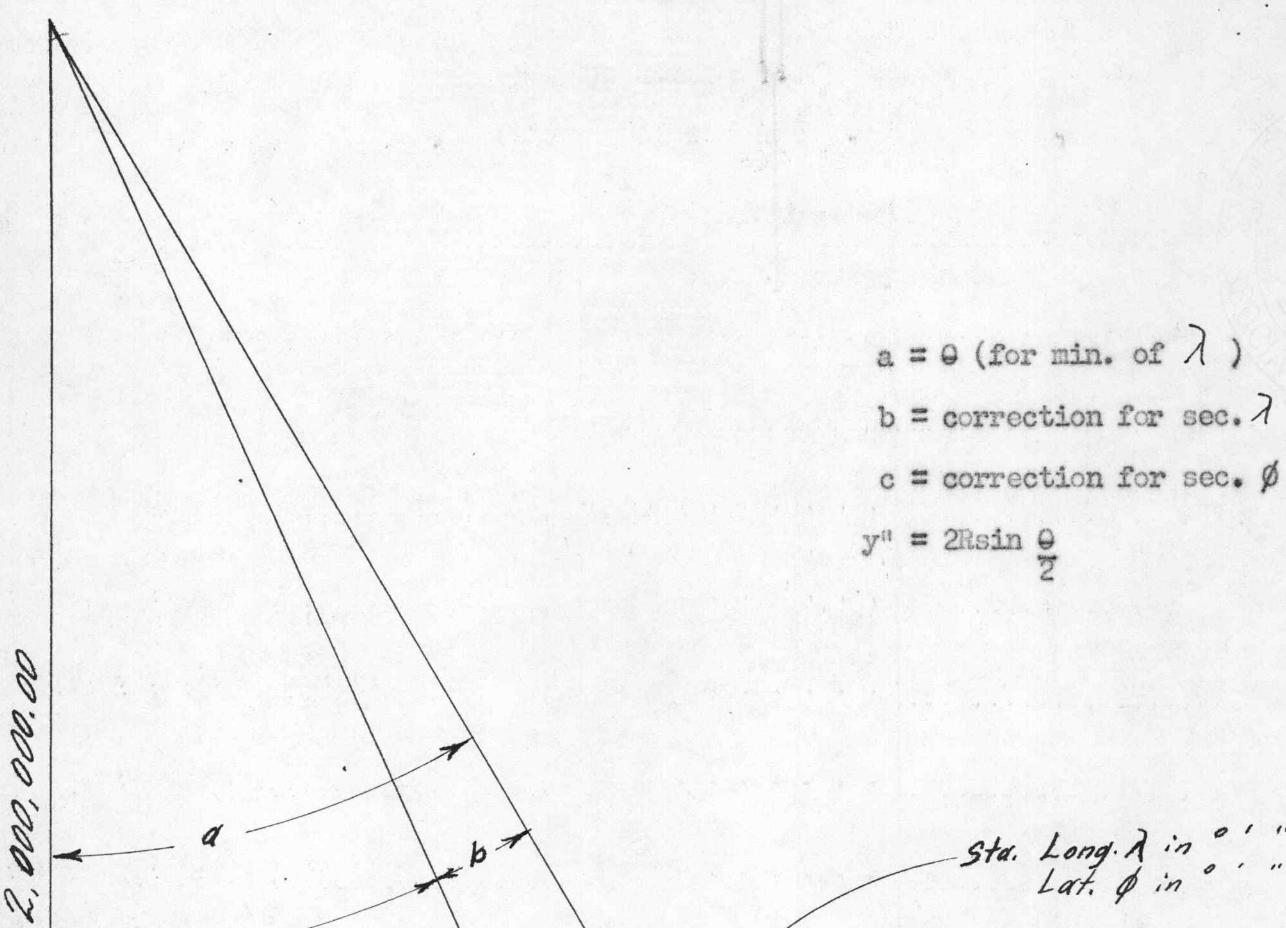
On the right-hand side of the sheet the first five steps are apparent; the order allows easy placement of the decimal. Item No. 22 plus No. 9 gives y' which is added to No. 21 to give the desired latitude Y.

STEPS IN TRAVERSE COMPUTATION

Grid coordinates and grid azimuths at beginning and ending stations are calculated if not available from U. S. C. and G. S. (Inclusion of two triangulation stations in the traverse are recommended.) The field measured grid azimuths are adjusted with closing error distributed equally. Ground lengths are reduced to grid lengths. Ordinary plane coordinates of traverse stations and ending station are figured from adjusted azimuths and grid lengths starting with coordinates of beginning station. Coordinates of traverse stations are adjusted in the familiar manner; in proportion to distance from origin.

Triangulation is widely used, especially by the aerial surveyor.

Adjustments of triangulation in conjunction with the coordinate system are explained in U. S. C. and G. S. Special Publications Nos. 193 and 194.



a = θ (for min. of λ)

b = correction for sec. λ

c = correction for sec. ø

$y'' = 2R\sin\frac{\theta}{2}$

Graphic representation of the coordinate computations.

REFERENCES

1. Breed, C. B. and Hosmer, G. L., The Principles and Practices of Surveying, Vol. II.
2. Clarke, E. C., A Treatise on the Law of Surveying and Boundaries.
3. Mockmore, C. A., et al, Digest of Oregon Land Surveying Laws, Engineering Experiment Station, Oregon State College.
4. U. S. C. and G. S., Special Publication No. 235, The State Coordinate Systems (A manual for Surveyors). Price, .35.*
5. U. S. C. and G. S., Special Publication No. 193, Manual of Plane Coordinate Computation. Price, .45.*
6. U. S. C. and G. S., Special Publication No. 194, Manual of traverse Computation on the Lambert Grid. Price, .45.
7. U. S. C. and G. S., Special Publication No. 227, Horizontal Control Data.
8. U. S. C. and G. S., Special Publication No. 246, Sines, Cosines, and Tangents, Ten Decimal Places, $0^{\circ} - 6^{\circ}$. Price, .20.
9. U. S. C. and G. S., Plane Coordinate Projection Tables, Oregon. Free.
10. Corps of Engineers, U. S. Army, Formulas and Tables For the Transformation of Geographic Positions to Plane Coordinates on the Lambert Projection by use of Logarithms. Portland, Oregon.

*Of particular value to the user of Lambert Coordinates.

SUPPLEMENT I

Lambert Projection for Oregon (North)

Table 1

Lat.	R (feet)	Y' y value on central meridian feet	Tabular difference for 1 sec. of lat. feet	Scale in units of 7th place of logs	Scale expressed as a ratio
46° 01'	20,527,079.02	856,773.46	101.29733	+18.6	1.0000043
02	20,521,001.18	862,851.30	101.29817	+37.5	1.0000086
03	20,514,923.29	868,929.19	101.29900	+56.9	1.0000131
04	20,508,845.35	875,007.13	101.29967	+76.6	1.0000176
05	20,502,767.37	881,085.11	101.30050	+96.7	1.0000223
46° 26'	20,375,118.25	1,008,734.23	101.31867	+604.5	1.0001392
27	20,369,039.13	1,014,813.35	101.31967	+632.8	1.0001457
28	20,362,959.95	1,020,892.53	101.32067	+661.4	1.0001523
29	20,356,880.71	1,026,971.77	101.32150	+690.5	1.0001590
30	20,350,801.42	1,033,051.06		+719.9	1.0001658

$l = 0.70918602$

$\log l = 9.8507601672 - 10$

$\log K = 7.5900831907$

$y_0 = + 547,601.54$

$\log \frac{1}{2\rho_0{}^2 \sin 1''} = 0.3720173 - 10$

$$\text{Geod. Azimuth} - \text{Grid Azimuth} = - \frac{x_2 - x_1}{2\rho_0{}^2 \sin 1''} \left(y_1 - y_0 + \frac{y_2 - y_1}{3}\right) + \theta$$

Lambert Projection for Oregon (North)

1" of Long. = 0".70918602 of θ

Table II

Long.	θ	Long.	θ	Long.	θ
116° 00'	+3° 11' 28".8135				
01	+3 10 46.2624	116° 36'	+2° 45' 56".9717	117° 11'	+2° 21' 07".6811
02	+3 10 03.7112	37	+2 45 14.4206	12	+2 20 25.1299
03	+3 09 21.1600	38	+2 44 31.8694	13	+2 19 42.5788
04	+3 08 38.6089	39	+2 43 49.3182	14	+2 19 00.0276
05	+3 07 56.0577	40	+2 43 06.7671	15	+2 18 17.4764

SUPPLEMENT II

INFORMATION ON CONTROL STATIONS

Finding all or adequate previously established triangulation and traverse stations and their geographic position may greatly reduce control survey in a large area. All likely sources should be investigated. Unfortunately, there is no central clearinghouse for control information.

The beginning of any search will usually be examination of the United States C. and G. S. index map of triangulation for Oregon, free by mail from the Midwestern District Office, Coast and Geodetic Survey, Panama Building, Portland, Oregon. The map shows the location of over 700 triangulation stations whose geographic and state coordinates may be obtained if requested by number and name of station. Some of these stations were not occupied; their description may indicate the station as "the tallest trees", or other features which will be difficult to use. Possible later information may be secured by a personal visit to the Portland office.

The District Engineer, Corps of Engineers, U. S. Army, 628 Pittock Block, Portland, Oregon, can add some information in most areas. Some quadrangle maps and booklets of station descriptions, dated 1943, can be borrowed by mail. These booklets should be used with caution due to typographical errors. Other quadrangle maps may be used and likely more recent information obtained in the Portland office.

The Topographic Division of the U. S. Geological Survey (Department of Interior), 302 Federal Building, Sacremento, California, has done considerable surveying in portions of the state.

The State Highway has, of course, run extensive traverses throughout the state. Until recently, however, they made few geographical ties and some engineers use their data with caution. Apparently their present work is more reliable and Lambert coordinates are used wherever possible. Some data can be secured from the State Highway Engineer, Oregon State Highway Commission, Salem, Oregon.

If the area of the proposed survey is near a National Forest their headquarters or the Regional Forester, North Pacific Region, U. S. Forest Service, Box 4137, Post Office Building, Portland, Oregon, should be contacted.

Counties and c ities usually have surveys which will be helpful, although these sources are variable.

Many private organizations, as power companies, railroads, forest owners, and consulting aerial photogrammetrist or engineers, may have data which can be secured. A round of golf with the vice-president may be a more certain way of getting private survey data than contacting the engineer.

This does not exhaust the sources of control information; but rather, this gives the more common ones and points up the required resourcefulness of the investigator.